Optimal CCS and air capture from heterogeneous energy consuming sectors

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Abstract

We characterize the optimal exploitation paths of two primary energy resources, a non-renewable polluting resource and a carbon-free renewable one. Both resources can supply the energy needs of two sectors. Sector 1 is able to reduce its carbon footprint at a reasonable cost owing to a CCS device. Sector 2 has only access to the air capture technology, but at a significantly higher cost. We assume that the atmospheric carbon stock cannot exceed some given ceiling. We show that there may exist paths along which it is optimal to begin by fully capturing the sector 1's emissions before the ceiling has been reached. Also there may exist optimal paths along which both capture devices have to be activated, in which case the sector 1's emissions are first fully abated and next sector 2 partially abates.

Keywords: Air capture; Carbon stabilization cap; CCS; Fossil resource; Heterogeneity.

JEL classifications: Q32, Q42, Q54, Q58.

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1 Introduction

Among all the alternative abatement technologies aiming at reducing the anthropogenic carbon dioxide emissions, a particular interest should be given to the carbon capture and sequestration (CCS) according to the IPCC (2005, 2007). Even if the efficiency of this technology is still under assessment[1], current engineering estimates suggest that CCS could be a credible cost-effective approach for eliminating most of the emissions from coal and natural gas power plants (MIT, 2007). Along this line of arguments, Islegen and Reichelstein (2009) point out that CCS has considerable potential to reduce CO_2 emissions at a "reasonable" social cost, given the social costs of carbon emissions predicted for a business-as-usual scenario. CCS is also intended to have a major role in limiting the effective carbon tax, or the market price for CO_2 emission permits under a cap-and-trade system. The crucial point is then to estimate how far would the CO_2 price have to rise before the operator of power plants would find it advantageous to install CCS technology rather then buy emission permits or pay the carbon tax. The International Energy Agency (2006) estimates such a break-even price in the range of \$30-90/tCO_2 (depending on technology) but, assuming reasonable technology advances, projected CCS cost by 2030 is around \$25/tCO_2.

However, geologic CCS presents the disadvantage to apply to the sole large point sources of pollution such as power plants or huge manufacturings. This technology is prohibitively costly to filter for instance the CO_2 emissions from transportation as far as the energy input is gasoline or kerosene, small residence heating or scattered agricultural activities. Hence, the ultimate device to abate carbon dioxide fluxes from any concentrated as well as diffuse sources would consist in capturing them directly from the atmosphere.

According to Keith et al. (2006), atmospheric carbon capture – or air capture – differs from conventional mitigation in three key aspects. First, it removes emissions from any part of the economy with equal ease of difficulty, so its cost provides an absolute cap on the mitigation cost. Second, it permits reduction in concentrations faster than the natural carbon cycle. Third, because it is weakly coupled with existing energy infrastructure, air capture may offer stronger economies of scale and smaller adjustment costs than the more

[1] CCS technology consists in filtering CO_2 fluxes at the source of emission, that is, in fossil energy-fueled power plants, by use of scrubbers installed near the top of chimney stacks. The carbon would be sequestered in reservoirs, such as depleted oil and gas fields or deep saline aquifers.

conventional mitigation technologies. As underlined by Keith (2009), though this abatement technology costs more than CCS, it allows one to treat small and mobile emission sources, advantage that may compensate for the intrinsic difficulty of capturing carbon from the air. Finally, deliberately expressing a double meaning, McKay (2009) claims about this alternative that "*capturing carbon dioxide from thin air is the last thing we should talk about*" (p.240). On the one hand, the energy requirements for atmospheric carbon capture are so enormous that, according to McKay, it seems actually almost absurd to talk about it. But on the other hand, "*we should talk about it because capturing carbon from thin air may turn out to be our last line of defense if humanity fails to take the cheaper and more sensible options that may still be available today*" (p.240).

Technically speaking, sucking carbon from thin air can be achieved in different ways.[2] The probably most credible one is to use a chemical process. This involves a technology that brings air into contact with a chemical "sorbent" (an alkaline liquid). The sorbent absorbs CO_2 in the air, and the chemical process then separates out the CO_2 and recycles the sorbent. The captured CO_2 is stored in geologic deposits, just like the CCS from power plant. However, chemical air capture is expensive. Estimates of marginal cost range from \$100-200/tCO_2, which is larger than the cost of alternatives for reducing emissions such as CCS. They are also larger than current estimates of the social cost of carbon, which range from about \$7-85/tCO_2. But, as concluded by Barrett (2009), bearing the cost of chemical air capture can become profitable in the future under constraining cap-and-trade scenarios. Furthermore, we may hope that the cost will decrease, thanks to R&D and learning by doing.

In the present study, we address the question of the heterogeneity of energy users regarding the way their carbon footprint can be reduced. We then consider two abatement technologies and two sectors. The first technology is a conventional emission abatement device (CCS) which is available at a marginal cost assumed to be socially acceptable.

[2]The most obvious approach consists in exploiting the process of photosynthesis by increasing the forestlands or changing the agricultural processes, but this is not the type of device we consider in the present paper. A close idea can be transposed to the oceans. To make them able to capture carbon faster then normal, phytoplankton blooms can be stimulated by fertilizing some oceanic iron-limited regions. A third way is to enhance weathering of rocks, that is to pulverize rocks that are capable of absorbing CO_2, and leave them in the open air. This idea can be pitched as the acceleration of a natural process. Unfortunately, as claimed by Barrett (2009), the effects of all these devices are difficult to verify, their potential is limited in any event, and there are concerns about some unknown ecological consequences.

However, this abatement technology cannot apply to carbon emissions from any type of activity, but only from large point sources of emissions. The second technology directly captures carbon in the atmosphere. Its marginal cost is much higher than the emission capture technology, but it allows to reduce carbon from any sources since the capture process and the generation of emissions are now disconnected. The first sector, in which pollution sources are spatially concentrated, can abate its carbon emissions, but not the second one since energy users are too small and too scattered. The ultimate way for abating pollution is to directly capture carbon in the atmosphere. But since the atmosphere is a public good, this kind of pollution reduction will also benefit to sector 1. Whatever the capture process, we assume that carbon is stockpiled into reservoirs whose size is very large. Then, as in Chakravorty et al. (2006), this suggests a generic abatement scheme of unlimited capacity. Finally, energy in each sector can be supplied either by a carbon-based fossil fuel, contributing to climate change (oil, coal, gas), or by a carbon-free renewable and non biological resource such, as solar energy.

Using a standard Hotelling model for the non-renewable resource and assuming that the atmospheric carbon stock should not exceed some critical threshold – as in Chakravorty et al. (2006) – we characterize the optimal time path of sectoral energy prices, sectoral energy consumptions, emission and atmospheric abatements. The key results of the paper are: i) Irrespective of the availability of the air capture technology, it may happen that it is optimal for the first sector to abate its carbon emissions before the atmospheric carbon concentration cap is attained.[3] ii) Since this type of carbon capture is unable to filter the emissions from the second sector, it is also optimal for the first sector to abate the totality of its own emissions, at least at the beginning. These two first results are at variance with Chakravorty et al. (2006), Lafforgue et al. (2008-a) and (2008-b) who consider a single sector using energy and a single abatement technology. iii) The atmospheric carbon capture is only used when the atmospheric carbon stock reaches the ceiling, maintaining the stock at its critical level. Hence the flow of carbon captured in the atmosphere is lower than the emission flow of the second sector and the whole carbon emissions coming from

[3] This result is in accordance with Coulomb and Henriet (2010) who show that, in a model with a single abatement technology, when technical constraints make it impossible to capture emissions from all energy consumers, CCS should be used before the ceiling is reached if non capturable emissions are large enough.

the two sectors are only partially abated.

The paper is organized as follows. Section 2 presents the model. In section 3, we lay down the social planner program and we derive the optimality conditions. In section 4, we examine the restricted problem in which only the emission carbon capture device is available. In section 5, we examine how the model reacts when the atmospheric carbon capture technology is introduced. We also investigate the time profile of the optimal carbon tax as well as, for each sector, the total burden induced by the mitigation of their emissions. Finally, we briefly conclude in section 6.

2 Model and notations

Let us consider a stationary economy with two sectors, indexed by $i = 1, 2$, in which the instantaneous gross surplus derived from energy consumption are the same.[4] For an identical energy consumption in the two sectors, $q_1 = q_2 = q$, the sectoral gross surplus $u_1(q)$ and $u_2(q)$ are such that: $u_1(q) = u_2(q) = u(q)$. We assume that this common function u satisfies the following standard assumptions. $u : \mathbb{R}_+ \to \mathbb{R}_+$ is a function of class $\mathcal{C}^2$, strictly increasing, strictly concave and verifying the Inada conditions: $\lim_{q \downarrow 0} u'(q) = +\infty$ and $\lim_{q \uparrow +\infty} u'(q) = 0$. We denote by $p(q)$ the sectoral marginal gross surplus function and by $q^d(p) = p^{-1}(q)$, the sectoral direct demand function.

In each sector, energy can be supplied by two primary natural resources: a dirty non-renewable resource (let say oil for instance) and a carbon-free renewable resource (let say solar energy). Let us denote by X^0 the initial oil endowment of the economy, by $X(t)$ the remaining part of this initial endowment at time t, and by $x_i(t)$, $i = 1, 2$, the instantaneous consumption flow of oil in sector i at time t, so that:

$$\dot{X}(t) = -[x_1(t) + x_2(t)], \quad \text{with } X(0) \equiv X^0 \text{ and } X(t) \geq 0 \tag{1}$$

$$x_i(t) \geq 0, \; i = 1, 2. \tag{2}$$

[4] Since the focus of the paper is on the effect of the heterogeneity of the energy consumers regarding the type of abatement technologies they can use, we consider the simple case of two sectors with the same gross surplus function and the same cost structure, excepted the abatement cost. Introducing different demand functions and/or different delivery costs for these sectors would imply a more complex analysis without altering the key message of the paper.

The delivery cost of oil is the same for both sectors. We denote by c_x the corresponding average cost, assumed to be constant and hence equal to the marginal cost. The delivery cost includes the extraction cost of the resource, the cost of industrial processing (refining of the crude oil) and the transportation cost, so that the resource is ready for use by the consumer in the concerned sector. To keep matter as simple as possible, we assume that no oil is lost during the delivery process. Equivalently, the oil stock $X(t)$ may be understood as measured in ready for use units.

Let $Z(t)$ be the stock of carbon within the atmosphere at time t, and Z^0 be the initial stock, $Z^0 \equiv Z(0)$. We assume that a carbon cap policy is prescribed to prevent catastrophic damages which would be infinitely costly. This policy consists in forcing the atmospheric stock to stay under some critical level $\bar{Z}$, with $\bar{Z} > Z^0$.

The atmospheric carbon stock is fed by carbon emission flows resulting from the use of oil. Let ζ be the quantity of carbon which would be potentially released per unit of oil consumption whatever the sector in which the oil is used. Thus, the gross pollutant flow amounts to $\zeta[x_1(t) + x_2(t)]$. However, this gross emission flow can be abated before being released into the atmosphere. We assume that emissions from sector 1 can be abated, but not emissions from sector 2 (or at a prohibitive cost). Emission abatement by carbon capture and sequestration (CCS) can be achieved when burning oil is spatially concentrated, as it is the case for instance in the electricity or cement industries, which are good examples of sector 1's activities. At the other extreme of the spectrum, i.e. in sector 2, there exists some activities with prohibitively costing emission captures since users are too small or too scattered. Transportation by cars, trucks and diesel train are good examples of sector 2's industry.[5]

Let $s_e(t)$ be this part of carbon emissions from sector 1 which is captured and sequestered at some average cost c_e, assumed to be constant. Then the net pollution flow issued from sector 1 amounts to:

$$\zeta x_1(t) - s_e(t) \geq 0, \quad s_e(t) \geq 0. \tag{3}$$

[5] Note that electric traction trains could be good examples of sector 1 users, as well as electric cars (cf. Chakravorty et al., 2010).

In sector 2, the net pollution flow amounts to $\zeta x_2(t)$.

Carbon emission capture is not the unique way to reduce the atmospheric carbon concentration. The other process consists in capturing the carbon present in the atmosphere itself. We denote by $s_a(t)$ the instantaneous carbon flow which is abated owing to this second device, and by c_a the corresponding average cost, also assumed to be constant. Although atmospheric carbon capture seems technically feasible, it is proved to be more costly than emission capture: $c_a > c_e$. The only constraint on this capture flow is:

$$s_a(t) \geq 0. \tag{4}$$

Whatever the capture process, from emissions or from the atmosphere, we assume that carbon is stockpiled into reservoirs whose capacities are unlimited.[6]

Last, there is also some natural self regeneration effect of the atmospheric carbon stock. We assume that the natural proportional rate of decay, denoted by $\alpha > 0$, is constant. Taking into account all the components of the dynamics of $Z(t)$ results into:

$$\dot{Z}(t) = \zeta[x_1(t) + x_2(t)] - [s_a(t) + s_e(t)] - \alpha Z(t), \quad Z(0) \equiv Z^0 < \bar{Z} \tag{5}$$

$$\bar{Z} - Z(t) \geq 0. \tag{6}$$

When the atmospheric carbon stock reaches its critical level, i.e. when $Z(t) = \bar{Z}$, and absent any active capture policy, i.e. $s_a(t) = s_e(t) = 0$, then the total oil consumption $x(t) \equiv x_1(t) + x_2(t)$ is constrained to be at most equal to $\bar{x}$, where $\bar{x}$ is solution of $\zeta x - \alpha\bar{Z} = 0$, that is $\bar{x} = \alpha\bar{Z}/\zeta$. Then, since the two sectors have the same weight, each one consumes the quantity $\bar{x}/2$ of oil at the ceiling when neither CCS nor air capture are activated.

We assume that it may be optimal to abate the pollution for delaying the date of arrival at the critical threshold and for relaxing the constraint on the oil consumption flow, that is: $c_x + c_e < c_x + c_a < u'(\bar{x})$.

The alternative energy source is supplied by the carbon-free renewable resource, the

[6]In order to focus on the abatement options for each sector and their respective costs, we dispense from considering reservoirs of limited capacity. The question of the size of carbon sinks and of the time profile of their filling up is addressed by Lafforgue et al. (2008-a) and (2008-b).

solar energy. We denote by $y_i(t)$ the solar energy consumption in sector i, $i = 1, 2$, and by c_y the average delivery cost of this alternative energy. Because c_x and c_y both include all the costs necessary to deliver a ready for use energy unit to the potential users, then both resources may be seen as perfect substitutes for the consumers, so that we may define the aggregate energy consumption of sector i as $q_i = x_i + y_i$, $i = 1, 2$, as far as the costs c_x and c_y are incurred.

The average cost c_y is assumed to be constant, the same for both sectors, and higher than $u'(\bar{x}/2)$. This last condition implies that the optimal energy consumption paths can be split into two periods: a first one during which only oil is consumed and a second one during which only solar energy is used.[7] We also have to assume that the natural flow of available solar energy, denoted by y^n, is large enough to supply the energy needs in both sectors during the second period described above.[8] Let $\tilde{y}$ be the sectoral energy consumption that it would be optimal to consume at the marginal cost c_y, that is $\tilde{y} = q^d(c_y)$ for which $u'(\tilde{y}) = c_y$. Then we assume that $y^n > 2\tilde{y}$. Under this assumption, no rent has ever to be imputed for using the solar energy. Thus the only constraint on $y_i(t)$ having to be taken into account along any optimal path is a non-negativity constraint:

$$y_i(t) \geq 0, \quad i = 1, 2. \tag{7}$$

Finally, the instantaneous social rate of discount, denoted by ρ, $\rho > 0$, is assumed to be constant over time.

[7] Since both c_x and c_y are set constant, oil and solar cannot be used simultaneously. Using a stock-dependent marginal extraction cost, but a constant marginal cost of the backstop, together with a damage function increasing with the atmospheric carbon stock, Hoel and Kverndokk (1996) and Tahvonen (1997) have shown that there may be a period of simultaneous use of the nonrenewable and the renewable resource. Furthermore, as underlined by Tahvonen (1997), the conjunction of these assumptions gives rise to a multiplicity of possible scenarios.

[8] The case of a rare renewable substitute is analyzed in Lafforgue et al. (2008-b).

3 Social planner problem and optimality conditions

The problem of the social planner consists in maximizing the sum of the discounted net current surplus. Let (P) be this program:

$$(P) \qquad \max_{s_a, s_e, x_i, y_i, i=1,2} \int_0^{\infty} \{u\left[x_1(t) + y_1(t)\right] + u\left[x_2(t) + y_2(t)\right] - c_x\left[x_1(t) + x_2(t)\right] \\ -c_y\left[y_1(t) + y_2(t)\right] - c_a s_a(t) - c_e s_e(t)\} \, e^{-\rho t} dt$$

subject to (1)-(7).

Let us denote by λ_X the costate variable of the state variable X, by λ_Z minus the costate variable of the state variable Z, by γ's the Lagrange multipliers associated with the non-negativity constraints on the command variables, and by ν the Lagrange multiplier associated with the ceiling constraint on Z. As usually done in this kind of problem, we do not take explicitly into account the non-negativity constraint on X. Thus, droping out the time index for notational convenience, we may write the current value Lagrangian $\mathcal{L}$ of problem (P) as follows:

$$\begin{aligned} \mathcal{L} &= u(x_1 + y_1) + u(x_2 + y_2) - c_x(x_1 + x_2) - c_y(y_1 + y_2) - c_a s_a - c_e s_e \\ &\quad -\lambda_X(x_1 + x_2) - \lambda_Z[\zeta(x_1 + x_2) - (s_a + s_e) - \alpha Z] + \nu(\bar{Z} - Z) \\ &\quad + \sum_i \gamma_{x_i} x_i + \sum_i \gamma_{y_i} y_i + \gamma_{s_a} s_a + \gamma_{s_e} s_e + \bar{\gamma}_{s_e}(\zeta x_1 - s_e) \end{aligned}$$

The static and dynamic first-order conditions are:

$$u'[x_1(t) + y_1(t)] = c_x + \lambda_X(t) + \zeta[\lambda_Z(t) - \bar{\gamma}_{s_e}(t)] - \gamma_{x_1}(t) \tag{8}$$

$$u'[x_2(t) + y_2(t)] = c_x + \lambda_X(t) + \zeta\lambda_Z(t) - \gamma_{x_2}(t) \tag{9}$$

$$u'[x_i(t) + y_i(t)] = c_y - \gamma_{y_i}(t), \quad i = 1, 2 \tag{10}$$

$$c_a = \lambda_Z(t) + \gamma_{s_a}(t) \tag{11}$$

$$c_e = \lambda_Z(t) - \bar{\gamma}_{s_e}(t) + \gamma_{s_e}(t) \tag{12}$$

$$\dot{\lambda}_X(t) = \rho\lambda_X(t) \tag{13}$$

$$\dot{\lambda}_Z(t) = (\rho + \alpha)\lambda_Z(t) - \nu(t) \tag{14}$$

together with the associated complementary slackness conditions. Last, the transversality conditions take the following forms:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) \quad = \quad 0 \qquad (15)$$

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) \quad = \quad 0 \qquad (16)$$

Remarks:

1. As expected with a constant marginal delivery cost, the shadow marginal value of the stock of oil, or mining rent, $\lambda_X(t)$, must grow at the social rate of discount ρ. From (13), we get: $\lambda_X(t) = \lambda_{X_0}e^{\rho t}$, with $\lambda_{X_0} \equiv \lambda_X(0)$. Thus the transversality condition (15) reduces to $\lambda_{X_0}\lim_{t\uparrow\infty} X(t) = 0$. If oil is to have some value, $\lambda_{X_0} > 0$, then it must be exhausted along the optimal path.

2. Concerning the shadow marginal cost of the atmospheric carbon stock, $\lambda_Z(t)$, note that before the date $\underline{t}_Z$ at which the ceiling constraint is beginning to be active, we must have $\nu(t) = 0$ since $\bar{Z} - Z(t) > 0$. Then (14) reduces to $\dot{\lambda}_Z = (\rho + \alpha)\lambda_Z$ so that: $t < \underline{t}_Z \;\Rightarrow\; \lambda_Z(t) = \lambda_{Z_0}e^{(\rho+\alpha)t}$, with $\lambda_{Z_0} \equiv \lambda_Z(0)$. Once the ceiling constraint is no more active and forever, $\lambda_Z(t) = 0$. Thus, denoting by $\bar{t}_Z$ the latest date at which $Z(t) = \bar{Z}$, we get: $t > \bar{t}_Z \;\Rightarrow\; \lambda_Z(t) = 0$.[9]

3. In order to simplify the notations in the next sections, it is useful to define the following prices or full marginal costs and the corresponding sectoral consumption levels for which the F.O.C's (8) and (9) relative to $x_1(t)$ and to $x_2(t)$, respectively, are satisfied:[10]

   - Price or full marginal cost of oil and sectoral oil consumption before the ceiling and absent any abatement, whatever the sector under consideration: $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) \equiv c_x + \lambda_{X_0}e^{\rho t} + \zeta\lambda_{Z_0}e^{(\rho+\alpha)t}$ and $\tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) \equiv q^d\left(p^1(t, \lambda_{X_0}, \lambda_{Z_0})\right)$.

   - Price or full marginal cost of oil for consumption in sector 1 given that emissions

[9] This characteristics is standard under the assumption of a linear natural regeneration function of the atmospheric carbon stock. For non linear decay functions, see Toman and Withagen (2000) for instance.

[10] The upper indexes $n = 1, 2, 3$ correspond to the order in which the price p^n and the quantity $\tilde{q}^n$ are appearing along the optimal path. If both $p^n(t, ...)$ and $p^{n+m}(t', ...)$ are appearing along the same path, then it implies that $t < t'$.

from this sector are fully or partially abated, i.e. $s_e(t) > 0$, and corresponding oil consumption of sector 1: $p_e^2(t, \lambda_{X_0}) \equiv c_x + \lambda_{X_0} e^{\rho t} + \zeta c_e$ and $\tilde{q}_e^2(t, \lambda_{X_0}) \equiv q^d \left(p_e^2(t, \lambda_{X_0})\right)$.

- Price or full marginal cost of oil for consumption in sector 2 given that some part of the atmospheric carbon stock is captured, $s_a(t) > 0$, and corresponding consumption in this sector: $p_a^2(t, \lambda_{X_0}) \equiv c_x + \lambda_{X_0} e^{\rho t} + \zeta c_a$ and $\tilde{q}_a^2(t, \lambda_{X_0}) \equiv q^d \left(p_a^2(t, \lambda_{X_0})\right)$.

- Price or full marginal cost of oil once the ceiling constraint $\bar{Z} - Z(t) \geq 0$ is no more active and forever, and corresponding sectoral consumptions, whatever the sector: $p^3(t, \lambda_{X_0}) \equiv c_x + \lambda_{X_0} e^{\rho t}$ and $\tilde{q}^3(t, \lambda_{X_0}) \equiv q^d \left(p^3(t, \lambda_{X_0})\right)$. This last case corresponds to a pure Hotelling regime.

Solving strategy of the social planner:

In order to solve her problem (P), the social planner can proceed as follows. First, she checks whether the most costly device to capture the carbon has ever to be used. The test consists in solving her problem assuming that the atmospheric carbon capture device is not available. This is inducing some path of atmospheric carbon shadow cost $\lambda_Z(t)$. Next, according to the outcome of the first step:

- either this shadow cost is permanently lower than the marginal cost of atmospheric carbon capture, that is $\lambda_Z(t) < c_a$ for any $t \geq 0$, and then the atmospheric carbon capture device has never to be used because too costly;

- or there exists some time interval during which $\lambda_Z(t)$ is higher than c_a so that, in this case, the atmospheric carbon capture device must be activated since the loss in the marginal net surplus induced by not using it is higher than its marginal cost of use.

This test is performed in Section 4. Section 5 deals with the case in which it is optimal to activate the air capture device.

4 Optimal policy without atmospheric carbon capture device

This kind of policies have been investigated and characterized in Chakravorty et al. (2006), and in Lafforgue et al. (2008-a) and (2008-b), but for economies in which any potential emissions can be captured and sequestered irrespective of the oil consumption sector. Thus, in their models, there is a single consumption sector, similar to the sector 1 of the present model. Two important conclusions of these studies are that: i) it is never optimal to abate the potential flow of emissions before attaining the critical level $\bar{Z}$ of atmospheric carbon concentration; ii) along the phase at the ceiling during which it is optimal to abate, only some part of the potential emission flow must be abated. Because abating is never optimal excepted during this phase, then it is never optimal to fully abate the potential flow of emissions along the optimal path.

As we shall show, it may happen in the present context that: i) abating the potential emissions of the sector 1 has to begin before the ceiling level $\bar{Z}$ is attained; ii) when it is optimal to begin to capture the sector 1 potential emissions, before the ceiling is attained, then it is optimal to capture its whole potential emission flow.

4.1 Restricted social planner problem

Assuming that the atmospheric carbon capture technology is not available, the social planner problem reduces to the following restricted problem $(R.P)$:

$$(R.P) \qquad \max_{s_e, x_i, y_i, i=1,2} \int_0^\infty \{u[x_1(t)+y_1(t)] + u[x_2(t)+y_2(t)] - c_x[x_1(t)+x_2(t)] \\ - c_y[y_1(t)+y_2(t)] - c_e s_e(t)\} e^{-\rho t} dt$$

subject to (1), (2), (3), (6), (7) and:

$$\dot{Z}(t) = \zeta[x_1(t)+x_2(t)] - s_e(t) - \alpha Z(t), \quad Z(0) = Z^0 < \bar{Z} \tag{17}$$

The new F.O.C's relative to the command variables, except s_a, and to the state variables are the same then the ones of the unrestricted problem (P), namely (8)-(14). Also the associated complementary slackness condition and the transversality conditions (15) and (16) must hold. We can conclude that remarks 1 and 2 of the previous section 3 also hold in the present restricted context.

The opportunity for sector 1 to fully or partially abate its emissions strongly depends upon the level of c_e. Hence, we have to distinguish the cases of a full abatement phase or a partial abatement phase, before or after being at the ceiling. The next subsections describe these different possibilities.

4.2 Optimal paths along which it is optimal to capture and sequester before being at the ceiling

Let us assume that the initial oil endowment is large enough to justify some period at the ceiling during which $Z(t) = \bar{Z}$, and that there exists some period during which the emissions of sector 1 are abated, $s_e(t) > 0$. Figure 1 below illustrates the optimal price path which is obtained in this case.

[Figure 1]

The optimal price path is a seven phases path. Denoting by $p_i(t)$, for $i = 1, 2$, the price – or full marginal cost – of oil for sector i, these phases are the following:

- Phase 1, before the ceiling and without abatement: $[0, \underline{t}_e)$

During this phase, the oil price is the same for each sector and it is given by $p_1(t) = p_2(t) = p^1(t, \lambda_{X_0}, \lambda_{Z_0})$. The existence of such a phase requires that $\lambda_{Z_0} < c_e$, so $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) < p_e^2(t, \lambda_{X_0})$, that is capturing sector 1's emissions would be too costly. $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) - p_e^2(t, \lambda_{X_0}) = \zeta \left[\lambda_{Z_0} e^{(\rho+\alpha)t} - c_e\right] < 0$, is increasing so that supporting the marginal shadow cost of the atmospheric carbon stock, $\lambda_Z(t) = \lambda_{Z_0} e^{(\rho+\alpha)t}$, is less costly than abating, that is supporting the marginal cost of abating the sector 1's emissions, c_e.

The oil consumption of each sector is given by $x_1(t) = x_2(t) = \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})$.

The common oil price $p^1(t, \lambda_{X_0}, \lambda_{Z_0})$ is increasing at an instantaneous rate which is higher than the rate of growth of $p_e^2(t, \lambda_{X_0})$. At the end of the phase, denoted by $\underline{t}_e$, both prices are equated $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_e^2(t, \lambda_{X_0})$.

Note that, since $p_1(t) = p_2(t) < u'(\bar{x})$ and $Z^0 < \bar{Z}$, then during this phase both $x_1(t)$ and $x_2(t)$ are higher than $\bar{x}$ so that $Z(t)$ is increasing. However, the existence of this phase requires that, at its end, $Z(t)$ is lower than the critical level $\bar{Z}$: $Z(\underline{t}_e) < \bar{Z}$.

- Phase 2, before the ceiling with full abatement of sector 1's emissions: $[\underline{t}_e, \underline{t}_Z)$

From $\underline{t}_e$ onwards, we have $p_e^2(t, \lambda_{X_0}) < p^1(t, \lambda_{X_0}, \lambda_{Z_0})$. Thus it is now strictly less costly for sector 1 to abate than not to abate, hence $p_1(t) = p_e^2(t, \lambda_{X_0})$, implying that $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$.[11] Moreover, since the inequality is strict then the potential sector 1's emissions are fully abated: $s_e(t) = \zeta x_1(t)$.

Sector 2 is not able to abate its emissions and it must support the carbon shadow cost $\zeta \lambda_{Z_0} e^{(\rho+\alpha)t}$ per unit of burned oil, so that $p_2(t) = p^1(t, \lambda_{X_0}, \lambda_{Z_0})$ and $x_2(t) = \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})$.

Note that, during this phase, since $Z(\underline{t}_e) < \bar{Z}$ and $p_2(t) < u'(\bar{x})$, then $x_2(t) > \bar{x}$ and the atmospheric carbon stock increases. Finally, since $p_2(t) > p_1(t)$, the first of these two prices reaching $u'(\bar{x})$ is $p_2(t)$. However, in order that sector 2's consumption begins to be blockaded at $t = \underline{t}_Z$, we must have simultaneously $p_2(t) = u'(\bar{x})$ and $Z(t) = \bar{Z}$ at the end of the phase.

- Phase 3, at the ceiling with sector 2's oil consumption blockaded and sector 1's emissions fully abated: $[\underline{t}_Z, \tilde{t})$

During this phase, the oil price in sector 2 is given by $p_2(t) = u'(\bar{x})$ and the oil consumption of this sector is set to the maximum consumption level allowed by the ceiling constraint, i.e. $x_2(t) = \bar{x}$. Note that this implies that $\lambda_Z(t) = [u'(\bar{x}) - p^3(t, \lambda_{X_0})]/\zeta$ is decreasing over time during the phase.[12]

Since $p_e^2(\underline{t}_Z, \lambda_{X_0}) < u'(\bar{x})$, then $c_e < \lambda_Z(t)$ at the beginning of the phase. Then, once again, abating emissions is proved to be less costly for sector 1 than supporting the shadow cost of the atmospheric carbon stock. Consequently, the sector 1's emissions are fully captured: $s_e(t) = \zeta x_1(t)$. Since $p_1(t) = p_e^2(\underline{t}_Z, \lambda_{X_0})$, we still have $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$.

Given that sector 2's emissions are $\zeta x_2(t) = \zeta \bar{x}$, full abatement in sector 1 implies that, during this phase at the ceiling, the atmospheric carbon stock stays at its critical level: $\dot{Z}(t) = 0$ and $Z(t) = \bar{Z}$. Finally, $p_1(t) = p_e^2(t, \lambda_{X_0})$ is increasing during the phase. At the

[11] Note that during such a phase, because $s_e(t) > 0$ then $\gamma_{s_e}(t) = 0$, so that from (12) we obtain: $\lambda_Z(t) = c_e + \bar{\gamma}_{s_e}(t)$. Substituting for $\lambda_Z(t)$ in (8) and taking into account that $x_1(t) > 0$, hence $\gamma_{x_1}(t) = 0$, and $y_1(t) = 0$, we get: $u'(x_1(t)) = c_x + \lambda_{X_0} e^{\rho t} + \zeta c_e$, from which we conclude that $p_1(t) = p_e^2(t, \lambda_{X_0})$ and $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$.

[12] Since the ceiling constraint is active, then $\nu(t)$ is strictly positive and sufficiently high so that $\dot{\lambda}_Z(t) = (\rho + \alpha)\lambda_Z(t) - \nu(t) < 0$.

end of the phase, $p_e^2(t, \lambda_{X_0}) = u'(\bar{x})$ or, equivalently, $\lambda_Z(t) = c_e$.

- Phase 4, at the ceiling with partial abatement of sector 1's emissions: $[\tilde{t}, \bar{t}_e)$

From time $\tilde{t}$ onwards, $p_e^2(t, \lambda_{X_0})$ becomes higher than $u'(\bar{x})$. Thus, the only way to satisfy simultaneously the F.O.C's (8) and (9) on the x_i's is to set $p_1(t) = p_2(t) = p_e^2(t, \lambda_{X_0})$, which implies $x_1(t) = x_2(t) = \tilde{q}_e^2(t, \lambda_{X_0})$ together with a partial abatement of sector 1's emissions. As far as $p_e^2(t, \lambda_{X_0})$ is staying under $u'(\bar{x}/2)$, then the potential emissions amount to $2\zeta\tilde{q}_e^2(t, \lambda_{X_0}) > \zeta\bar{x} = \alpha\bar{Z}$. As far as $p_e^2(t, \lambda_{X_0})$ is now higher than $u'(\bar{x})$, then the potential emissions $2\zeta\tilde{q}_e^2(t, \lambda_{X_0})$ stays at a lower level than $2\zeta\bar{x}$, so that:

$$\bar{x} < 2\tilde{q}_e^2(t, \lambda_{X_0}) < 2\bar{x}. \tag{18}$$

In order to satisfy the atmospheric carbon constraint $Z(t) = \bar{Z}$, it is sufficient to abate this part $s_e(t)$ of the sector 1's emissions for which $\dot{Z}(t) = 0$. Thus we may have:

$$2\zeta\tilde{q}_e^2(t, \lambda_{X_0}) - s_e(t) = \zeta\bar{x}. \tag{19}$$

Conditions (18) and (19) imply that:

$$s_e(t) = \zeta\left[2\tilde{q}_e^2(t, \lambda_{X_0}) - \bar{x}\right] < \zeta\tilde{q}_e^2(t, \lambda_{X_0}) = \zeta x_1(t). \tag{20}$$

Hence, during this phase, emissions from sector 1 are only partially abated and, since $\tilde{q}_e^2(t, \lambda_{X_0})$ is decreasing through time then the instantaneous rate of capture $s_e(t)$ is also decreasing. This solution may be optimal if and only if abating and supporting the shadow marginal cost of the atmospheric carbon stock are resulting into the same full marginal cost, that is if and only if $\lambda_Z(t)$ is constant and equal to c_e. Since sector 2 cannot abate its emissions, it is supporting the marginal shadow cost of atmospheric carbon and the condition $p_1(t) = p_2(t) = p_e^2(t, \lambda_{X_0}) = c_x + \lambda_{X_0}e^{\rho t} + \zeta\lambda_Z(t)$ guarantees that $\lambda_Z(t) = c_e$ is satisfied.[13]

Since $p_e^2(t, \lambda_{X_0})$ is increasing over time, there exists some date $\bar{t}_e$ at which $p_e^2(t, \lambda_{X_0}) =$

[13] Again, because the ceiling constraint is effective then $\nu(t) > 0$ and, in order that $\dot{\lambda}_Z(t) = 0$, we have: $\nu(t) = (\rho + \alpha)\lambda_Z(t) = (\rho + \alpha)c_e$.

$u'(\bar{x}/2)$. At this date, $x_1(t) = x_2(t) = \bar{x}/2$ and sector 1 ceases to capture its emissions, $s_e(t) = 0$. From $\bar{t}_e$ onwards, we have $p_e^2(t, \lambda_{X_0}) > u'(\bar{x}/2)$ so that the cost of capture of sector 1's emissions becomes prohibitive.

- Phase 5, at the ceiling and without abatement of sector 1's emissions: $[\bar{t}_e, \bar{t}_Z)$

Since abating the sector 1's emissions is now too costly, there is no more abatement and, in order to not overshoot the critical atmospheric carbon level, we must have $p_1(t) = p_2(t) = u'(\bar{x}/2)$ and $x_1(t) = x_2(t) = \bar{x}/2$, so that $\dot{Z}(t) = 0$.

During such a phase, $\lambda_Z(t) = [u'(\bar{x}) - p^3(t, \lambda_{X_0})]/\zeta$ is decreasing. The phase is ending at time $t = \bar{t}_Z$ when $\lambda_Z(t) = 0$, which implies that $p^3(t, \lambda_{X_0}) > u'(\bar{x}/2)$ for $t > \bar{t}_Z$.

- Phase 6, pure Hotelling phase: $[\bar{t}_Z, t_y)$

This phase is the last one during which energy needs are supplied by oil. This is a pure Hotelling phase. The energy price is the same for the two sectors: $p_1(t) = p_2(t) = p^3(t, \lambda_{X_0}) > u'(\bar{x}/2)$, also generating an identical oil consumption in the two sectors: $x_1(t) = x_2(t) < \bar{x}/2 \Rightarrow x(t) < \bar{x}$.

Since $x(t) < \bar{x}$ and $Z(t) = \bar{Z}$ at the beginning of the phase, then $Z(t) < \bar{Z}$ for $t > \bar{t}_Z$ justifying the fact that now $\lambda_Z(t) = 0$ from $\bar{t}_Z$ onwards.[14] Then $\lambda_Z(t)Z(t) = 0$ and the transversality condition (16) is satisfied.

During the phase, the price is ever increasing and there must exist some time $t = t_y$ at which $p^3(t, \lambda_{X_0}) = c_y$. At this time, this level of oil price makes the renewable resource competitive. To be optimal, the switch from the pure Hotelling regime to a pure renewable regime requires that, at time $t = t_y$, $X(t) = 0$ so that from t_y onwards $\lambda_X(t)X(t) = 0$ warranting that the transversality condition (15) relative to X is satisfied.

- Phase 7, carbon-free renewable energy permanent regime: $[t_y, +\infty)$

From t_y onwards, the economy follows a pure renewable energy regime which is free of carbon emissions: $p_1(t) = p_2(t) = c_y$, $x_1(t) = x_2(t) = 0$ and $y_1(t) = y_2(t) = \tilde{y}$. Since

[14] However, note that $Z(t)$ is not necessarily monotonically decreasing during this phase. What is sure is that there exists some critical time interval $(\bar{t}_Z, \bar{t}_Z + \epsilon)$, with ϵ positive and small enough, during which $\dot{Z}(t) < 0$. For $t > \bar{t}_Z + \epsilon$, it may happen that $\dot{Z}(t) > 0$. But, because $x(t) < \bar{x}$, even if $\dot{Z}(t)$ were temporally increasing, it would not be able to go back to $\bar{Z}$.

$x_i(t) = 0$, $i = 1, 2$, then $\dot{Z}(t) = -\alpha Z(t)$ so that $Z(t)$ is permanently decreasing down to 0 at infinity: $Z(t) = Z(t_y)e^{-\alpha(t-t_y)}$.

Determination of the characteristics of the optimal path:

The optimal path described above is parametrized by eight variables whose values have to be determined: λ_{X_0}, λ_{Z_0}, $\underline{t}_e$, $\underline{t}_Z$, $\tilde{t}$, $\bar{t}_e$, $\bar{t}_Z$ and t_y. They are given as the solutions of the following eight equations system.

- Balance equation of non-renewable resource consumption and supply:

$$\begin{aligned} 2\int_0^{\underline{t}_e} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})dt + \int_{\underline{t}_e}^{\underline{t}_Z} \left[\tilde{q}_1(t, \lambda_{X_0}, \lambda_{Z_0}) + \tilde{q}_e^2(t, \lambda_{X_0})\right] dt \\ + \int_{\underline{t}_Z}^{\tilde{t}} \left[\tilde{q}_e^2(t, \lambda_{X_0}) + \bar{x}\right] dt + 2\int_{\tilde{t}}^{\bar{t}_e} \tilde{q}_e^2(t, \lambda_{X_0})dt \\ + \left[\bar{t}_Z - \bar{t}_e\right]\bar{x} + 2\int_{\bar{t}_Z}^{t_y} \tilde{q}^3(t, \lambda_{X_0})dt \quad = \quad X^0. \end{aligned} \tag{21}$$

- Continuity of the carbon stock at time $\underline{t}_Z$:

$$\begin{aligned} Z^0 e^{-\alpha \underline{t}_Z} + 2\zeta \int_0^{\underline{t}_e} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) e^{-\alpha(\underline{t}_Z - t)} dt \\ + \zeta \int_{\underline{t}_e}^{\underline{t}_Z} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0}) e^{-\alpha(\underline{t}_Z - t)} dt \quad = \quad \bar{Z}. \end{aligned} \tag{22}$$

- Price continuity equations:

$$p^1\left(\underline{t}_e, \lambda_{X_0}, \lambda_{Z_0}\right) = p_e^2\left(\underline{t}_e, \lambda_{X_0}\right) \tag{23}$$

$$p^1\left(\underline{t}_Z, \lambda_{X_0}, \lambda_{Z_0}\right) = u'(\bar{x}) \tag{24}$$

$$p_e^2\left(\tilde{t}, \lambda_{X_0}\right) = u'(\bar{x}) \tag{25}$$

$$p_e^2\left(\bar{t}_e, \lambda_{X_0}\right) = u'(\bar{x}/2) \tag{26}$$

$$p^3\left(\bar{t}_Z, \lambda_{X_0}\right) = u'(\bar{x}/2) \tag{27}$$

$$p^3(t_y, \lambda_{X_0}) = c_y. \tag{28}$$

Assuming a positive solution of system (21)-(28), then it is easy to check that all the optimality conditions of the restricted problem $(R.P)$ are satisfied. Reciprocally, it is clear that there exists values of the parameters of the system c_x, c_y, c_e, ζ, α and ρ together with values of initial endowments of oil X^0 and of atmospheric carbon stock Z^0 such that the path described above is the solution of the restricted problem $(R.P)$. However, other solutions may exist, such as the one in which sector 1's emissions have to be captured from the beginning of the planning horizon.

4.3 Paths along which the oil price is the same for the two sectors

4.3.1 Paths along which it is optimal to abate sector 1's emissions

Example of such a path, solution of the restricted problem $(R.P)$, is illustrated in Figure 2 below.

[Figure 2]

This kind of paths is characterized by the fact that, at time $t = \underline{t}_e$ at which $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_e^2(t, \lambda_{X_0})$, then the common value of these two prices is larger than $u'(\bar{x})$ while $Z(\underline{t}_e) = \bar{Z}$ simultaneously.

Because $Z^0 < \bar{Z}$ there must exist a first phase $[0, \underline{t}_e)$ during which the ceiling $\bar{Z}$ is not yet attained and $p_1(t) = p_2(t) = p^1(t, \lambda_{X_0}, \lambda_{Z_0}) < p_e^2(t, \lambda_{X_0})$, hence it is not optimal to abate sector 1's emissions. At the end of this first phase, both $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_e^2(t, \lambda_{X_0})$ and $Z(\underline{t}_e) = \bar{Z}$ so that $\underline{t}_e$ coincides with $\underline{t}_Z$.

The next phase $[\underline{t}_e, \bar{t}_e)$ is a phase at the ceiling during which $p_1(t) = p_2(t) = p_e^2(t, \lambda_{X_0})$. As in the phase 4 of the previous case – $[\tilde{t}, \bar{t}_e)$ of the path illustrated in Figure 1 – because sector 2 cannot abate its emissions, we must have $\lambda_Z(t) = c_e$ during the second phase of the present path. Also because $u'(\bar{x}) < p_e^2(t, \lambda_{X_0}) < u'(\bar{x}/2)$, then only some part of the sector 1's emissions have to be captured (cf. the above equation (20)), $s_e(t) < \zeta \tilde{q}_e^2(t, \lambda_{X_0}) = \zeta x_1(t)$, and the capture intensity $s_e(t)$ diminishes. At the end of this phase, $p_e^2(t, \lambda_{X_0}) = u'(\bar{x}/2)$, $x_1(t) = x_2(t) = \bar{x}/2$ and $s_e(t) = 0$.

The third phase $[\bar{t}_e, \bar{t}_Z)$ is still a phase at the ceiling but without capture of sector 1's emissions: $p_1(t) = p_2(t) = u'(\bar{x}/2)$ and $x_1(t) = x_2(t) = \bar{x}/2$. The phase is ending when $p^3(t, \lambda_{X_0}) = u'(\bar{x}/2)$, that is when $\lambda_Z(t) = 0$. The fourth and fifth phases are respectively the standard pure Hotelling phase $[\bar{t}_Z, t_y)$ and the pure renewable energy phase $[t_y, \infty)$.

4.3.2 Paths along which it is never optimal to capture sector 1's emissions

When the abatement cost c_e is very high, capturing is proved to never be an optimal strategy. In this case, we get a four phases optimal price path as illustrated in Figure 3.

[Figure 3]

In Figure 3, $p_e^2(t, \lambda_{X_0})$ is higher than $p^1(t, \lambda_{X_0}, \lambda_{Z_0})$ along the whole time interval $[0, \underline{t}_Z)$ before the ceiling. Hence, it is never optimal to capture sector 1's emissions. Such optimal paths have been characterized in Chakravorty et al. (2006).

5 Optimal policies requiring to activate both capture devices

In this section, we first determine the conditions under which it is optimal to activate the atmospheric carbon capture device. Next we characterize the optimal paths along which both carbon capture technologies must be used. Last, we discuss about the time profile of the optimal carbon marginal shadow cost, that is the optimal unitary carbon tax, as well as the total burden induced by climate change mitigation policies in each sector, including the tax burden and the abatement cost.

5.1 Checking whether the atmospheric carbon capture device must be used along the optimal path

Let us consider the three kinds of optimal price paths which may solve the planner restricted problem $(R.P)$ and which have been discussed in the previous section. Clearly, since $p_a^2(t, \lambda_{X_0}) > p_e^2(t, \lambda_{X_0})$, then for the two last kinds of optimal paths illustrated in Figures 2 and 3 in subsection 4.3, the price trajectory $p_a^2(t, \lambda_{X_0})$ (not depicted in these figures) is always located above the optimal price path. Hence, it is never optimal to use the atmospheric carbon capture device.

For the optimal path illustrated in Figure 1 in subsection 4.2, it may happen that using the atmospheric carbon capture technology reveals optimal. To check whether this technology is optimal or not, the test runs as follows. Consider the price path $p_a^2(t, \lambda_{X_0})$ (not depicted in Figure 1). Then at time $t = \underline{t}_Z$, either $p_a^2(t, \lambda_{X_0}) < u'(\bar{x})$ or $p_a^2(t, \lambda_{X_0}) \geq u'(\bar{x})$. In the first case, there must exist a time interval around $t = \underline{t}_Z$ such that $p_2(t) >$

$p_a^2(t, \lambda_{X_0})$ and it would be less costly for sector 2 to bear the cost of the atmospheric capture c_a than the burden of the shadow cost of the atmospheric carbon stock $\lambda_Z(t)$. In the second case, using the atmospheric carbon capture technology could not allow to improve the welfare.

5.2 Optimal paths

Let us assume now that the atmospheric carbon capture technology has to be used. Then we may obtain two kinds of optimal paths depending on whether the least costly emission capture technology has to be activated from the beginning or not. The typical optimal path along which it is not optimal to capture the sector 1's emission flows from the start is illustrated in Figure 4 below.

[Figure 4]

The path is an eight phases path and the difference with the trajectory depicted in Figure 1 is that a new phase $[\underline{t}_a, \bar{t}_a)$ – the third one in the present case – appears now during which some of the atmospheric carbon is captured. The seven other phases are similar to the ones which have been described in section 4.2. This new phase begins at $t = \underline{t}_a$ when $p^1(t, \lambda_{X_0}, \lambda_{Z_0}) = p_a^2(t, \lambda_{X_0})$, that is when $\lambda_Z(t) = c_a$. Then for $t > \underline{t}_a$, it becomes less costly for sector 2 to undertake atmospheric carbon capture rather than to pay the social cost of the carbon accumulation within the atmosphere. At the time sector 2's abatement begins, the ceiling is reached, so that $\underline{t}_a$ coincides with $\underline{t}_Z$.

During this phase $[\underline{t}_a, \bar{t}_a)$, each sector uses simultaneously its own abatement technology. We have $p_1(t) = p_e^2(t, \lambda_{X_0})$ and $p_2(t) = p_a^2(t, \lambda_{X_0})$, which implies $x_1(t) = \tilde{q}_e^2(t, \lambda_{X_0})$ and $x_2(t) = \tilde{q}_a^2(t, \lambda_{X_0})$. Since $c_e < c_a$, we also have $p_1(t) < p_2(t)$ and then $x_1(t) > x_2(t)$. Remember that, during this phase, as in the phase 3 of subsection 4.2, sector 1's emissions are fully captured: $s_e(t) = \zeta x_1(t)$. Because this is a phase at the ceiling, sector 2 has just to capture in the atmosphere the necessary amount of carbon in order to maintain the atmospheric carbon stock at its critical level. It is thus optimal for sector 2 to abate at a level which is smaller than its own carbon emissions: $s_a(t) = \zeta x_2(t) - \alpha \bar{Z} < \zeta x_2(t)$. Moreover, since $s_a(t) > 0$, we have $\zeta x_2(t) > \alpha \bar{Z}$, or equivalently, $x_2(t) > \bar{x}$, implying in turns $p_2(t) < u'(\bar{x})$. The price path $p_2(t) = p_a^2(t, \lambda_{X_0})$ being increasing through time, first the amount of abated carbon by the atmospheric device $s_a(t)$ is decreasing, second there must exist a date at which $p_2(t) = u'(\bar{x})$, that is at which $x_2(t) = \bar{x}$ and $s_a(t) = 0$. At that time, denoted by $\bar{t}_a$, since sector 1 still fully abates all its emissions, it is no more optimal for sector 2 to pursue the atmospheric carbon capture. All the efforts to maintain the carbon stabilization cap are now supported by the sole sector 1 and the economy behaves as in section 4.2 from phase 3, that if from the date $\underline{t}_Z$ as depicted in Figure 1.

To the eight variables parameterizing the optimal path in the case without atmospheric capture technology (cf. subsection 4.2), we must here determine the values of two additional variables: $\underline{t}_a$ and $\bar{t}_a$. But because $\underline{t}_a = \underline{t}_Z$, then only one more variable has to be determined. Hence we are left with nine variables that must solve the following nine equations system:

- Balance equation of non-renewable resource consumption and supply:

$$\begin{aligned} 2\int_0^{\underline{t}_e} \tilde{q}^1(t, \lambda_{X_0}, \lambda_{Z_0})dt + \int_{\underline{t}_e}^{\underline{t}_a = \underline{t}_Z} \left[\tilde{q}_1(t, \lambda_{X_0}, \lambda_{Z_0}) + \tilde{q}_e^2(t, \lambda_{X_0})\right] dt \\ + \int_{\underline{t}_a = \underline{t}_Z}^{\bar{t}_a} \left[\tilde{q}_e^2(t, \lambda_{X_0}) + \tilde{q}_a^2(t, \lambda_{X_0})\right] dt + \int_{\bar{t}_a}^{\tilde{t}} \left[\tilde{q}_e^2(t, \lambda_{X_0}) + \bar{x}\right] dt \\ +2\int_{\tilde{t}}^{\bar{t}_e} \tilde{q}_e^2(t, \lambda_{X_0})dt + \left[\bar{t}_Z - \bar{t}_e\right]\bar{x} + 2\int_{\bar{t}_Z}^{t_y} \tilde{q}^3(t, \lambda_{X_0})dt &= X^0. \qquad (29) \end{aligned}$$

- Continuity of the carbon stock at time $\underline{t}_Z$: identical to (22).

- Price continuity equations: identical to (23)-(28) except that (24) is now replaced by the two following equations:

$$p^1(\underline{t}_a, \lambda_{X_0}, \lambda_{Z_0}) = p_a^2(\underline{t}_a, \lambda_{X_0}) \qquad (30)$$

$$p_a^2(\bar{t}_a, \lambda_{X_0}) = u'(\bar{x}) \qquad (31)$$

5.3 Time profile of the optimal carbon tax

The trajectory of the carbon marginal shadow cost corresponding to the optimal path illustrated in Figure 4 is characterized by:

$$\lambda_Z(t) = \begin{cases} \lambda_{Z_0} e^{(\rho+\alpha)t} & , t \in [0, \underline{t}_Z) \\ c_a & , t \in [\underline{t}_Z, \bar{t}_a) \\ \left[u'(\bar{x}) - p^3(t, \lambda_{X_0})\right]/\zeta & , t \in [\bar{t}_a, \tilde{t}) \\ c_e & , t \in [\tilde{t}, \bar{t}_e) \\ \left[u'(\bar{x}/2) - p^3(t, \lambda_{X_0})\right]/\zeta & , t \in [\bar{t}_e, \bar{t}_Z) \\ 0 & , t \in [\bar{t}_Z, \infty) \end{cases} \tag{32}$$

This shadow cost can be interpreted as the optimal unitary tax to be levied on the net carbon emissions. Its time profile is illustrated in Figure 5 below.

[Figure 5]

The unitary tax rate is first increasing but is bounded from above by the highest marginal abatement cost c_a which is attained when it becomes optimal to use this abatement device and, simultaneously, when the atmospheric carbon stock constraint begins to be active, that is at time $t = \underline{t}_a = \underline{t}_Z$. Given that it is always possible to choose to abate rather than release the carbon in the atmosphere, the maximal tax rate of carbon emissions is necessarily determined by the highest marginal cost permitting to avoid polluting carbon releases.

During the ceiling phases, from $\underline{t}_Z$ up to $\bar{t}_Z$, the carbon tax is either constant or decreasing. First, as long as sector 2 abates, that is between $\underline{t}_a$ and $\bar{t}_a$, it is sufficient to set the tax rate equal to c_a to induce an optimal atmospheric capture by sector 2, given that sector 1 fully abates its own emissions. The same applies between $\tilde{t}$ and $\bar{t}_e$ for sector 1 by setting the tax rate equal to c_e, given that sector 2 no more abates. Between these two phases, that is between $\bar{t}_a$ and $\tilde{t}$, and during the last phase at the ceiling, that is between $\bar{t}_e$ and $\bar{t}_Z$, the tax rate strictly decreases. This is due to the oil price increase and to the fact that the emission level is constrained by $\bar{x}$ for sector 2 during $[\bar{t_a}, \tilde{t})$, and by $\bar{x}/2$ for each sector during $[\bar{t}_e, \bar{t}_Z)$.

5.4 Time profile of the tax burdens and the sequestration costs

Assume now that the above tax optimal rate is implemented. Such a tax is inducing a fiscal income $\Gamma_1(t) \equiv [\zeta x_1(t) - s_e(t)]\lambda_Z(t)$ for sector 1 and $\Gamma_2(t) \equiv [\zeta x_2(t) - s_a(t)]\lambda_Z(t)$ for sector 2. The sequestration cost in each sector simply writes as the sequestered carbon flow times the respective marginal cost of sequestration: $S_1(t) \equiv s_e(t)c_e$ and $S_2(t) \equiv s_a(t)c_a$. Then, the total burden of carbon for each sector is the sum of the fiscal burden and the sequestration cost. Denoting by $B_i(t)$ $i = 1, 2$ this total burden, the two following tables detail its components for each sector.

$\Gamma_1(t)$	$S_1(t)$	$B_1(t)$	Phases
$\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$	0	$\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$	$[0, \underline{t}_e)$
0	$\zeta\tilde{q}^2_e(t)c_e$	$\zeta\tilde{q}^2_e(t)c_e$	$[\underline{t}_e, \tilde{t})$
$\zeta\left[\bar{x} - \tilde{q}^2_e(t)\right]c_e$	$\zeta\left[2\tilde{q}^2_e(t) - \bar{x}\right]c_e$	$\zeta\tilde{q}^2_e(t)c_e$	$[\tilde{t}, \bar{t}_e)$
$(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$	0	$(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$	$[\bar{t}_e, \bar{t}_Z)$
0	0	0	$[\bar{t}_Z, \infty)$

Table 1. Decomposition of the total carbon burden for sector 1

$\Gamma_2(t)$	$S_2(t)$	$B_2(t)$	Phases
$\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$	0	$\zeta\tilde{q}^1(t)\lambda_{Z_0}e^{(\rho+\alpha)t}$	$[0, \underline{t}_a)$
$\zeta\bar{x}c_a$	$\zeta[\tilde{q}^2_a(t) - \bar{x}]c_a$	$\zeta\tilde{q}^2_a(t)c_a$	$[\underline{t}_a, \bar{t}_a)$
$\bar{x}\left[u'(\bar{x}) - p^3(t)\right]$	0	$\bar{x}\left[u'(\bar{x}) - p^3(t)\right]$	$[\bar{t}_a, \tilde{t})$
$\zeta\tilde{q}^2_e(t)c_e$	0	$\zeta\tilde{q}^2_e(t)c_e$	$[\tilde{t}, \bar{t}_e)$
$(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$	0	$(\bar{x}/2)\left[u'(\bar{x}/2) - p^3(t)\right]$	$[\bar{t}_e, \bar{t}_Z)$
0	0	0	$[\bar{t}_Z, \infty)$

Table 2. Decomposition of the total carbon burden for sector 2

Their time profile are depicted upon Figure 6 below.

[Figure 6]

Before the ceiling phases, the shapes of the total burden trajectories may be either increasing or decreasing depending upon oil demand elasticity. Once the ceiling is reached, the total burden gradually declines down to zero at the end of the ceiling phase.

For sector 1, the total burden identifies to the sole tax burden as long as abatement is not activated, that is before $\underline{t}_e$. Between $\underline{t}_e$ and $\tilde{t}$, sector 1, fully abating its emissions, does not bear the carbon tax burden ($\Gamma_1(t) = 0$), but bears the sequestration cost $S_1(t)$. During this phase, since sector 1's emissions decrease, so does its sequestration cost and then its total burden. During the next phase, between $\tilde{t}$ and $\bar{t}_e$, it is no more optimal for sector 1 to fully abate its emissions and then, this sector bears a mix of tax burden and abatement cost. Its gross carbon emissions decrease, but its sequestration flow decreases at an even higher rate resulting in an increase in the net emission flow. The cost of sequestration thus decreases. Since the tax rate is constant and equal to the sequestration marginal cost c_e, the fiscal burden rises. The combined effect of these two evolutions results in a declining total carbon burden for sector 1. Over the last ceiling phase, between $\bar{t}_e$ and $\bar{t}_Z$, sector 1 no more abates and bears only the fiscal burden. Then its total burden is declining down to zero when the ceiling constraint becomes no more active, that is at time $\bar{t}_Z$.

During the atmospheric capture phase, that is between $\underline{t}_a$ and $\bar{t}_a$, sector 2 is indifferent between paying the tax and abating from the atmosphere. Since it does not fully abate, it bears both the tax on this part of its emissions which are not captured, and the sequestration cost burden. During this phase, its carbon burden is constant because i) the tax rate is constant and equal to c_a and ii) sector 1 fully abates its emissions and sector 2's net emissions are constrained by $\bar{x}$. Its sequestration effort decreases since gross emissions decline. After $\bar{t}_a$ and during all next phases at the ceiling, the total burden of sector 2 reduces to the sole fiscal burden and it is thus decreasing over time as discussed above.

We conclude by two remarks. First, the total fiscal income, that is $\Gamma_1(t) + \Gamma_2(t)$, jumps down twice at each time when either sector 1 or sector 2 begins to abate. Hence, any environmental policy should take into account the ability of polluters to undertake abatement activities and thus to escape from the tax. Second, since sector 2 is constrained by the higher cost of its abatement technology, its fiscal contribution as well as its total burden are larger or equal than the total burden of sector 1 when pollutive potential intensities and demand functions are the same for both sectors.

6 Conclusion

In a Hotelling model, we have determined the optimal CCS and air capture policies for an economy composed of two kinds of energy users differing by the degree of concentration of their carbon emissions. The concentrated emissions sector has access to geological carbon capture in addition to air capture while the diffuse emissions sector can only abate its emissions through air capture. Both sectors face a global maximal atmospheric carbon concentration constraint.

In this framework, we have shown that carbon sequestration by the first sector must begin strictly before the atmospheric carbon stock reaches its critical threshold and that sector 1's emissions have to be fully abated during a first time phase with constant marginal costs of abatement and a stationary demand schedule. This result stands in contrast with the findings of Chakravorty et al. (2006) that abatement should begin only whence the atmospheric ceiling has been attained in a model with one energy using sector and a single abatement technology.

This difference appears as a consequence of the emission concentration heterogeneity of energy users, CCS being only available for concentrated emissions sectors like thermic electricity plants, steel mills or cement factories and not for the diffuse emissions by transport of house heating. This heterogeneity constrains the potential of CCS to be at most equal to the sole emissions of sector 1 and thus to be always smaller than the total carbon emissions of fossil energy consumers. In a constant CCS cost setting there is no limitation over the amount of abated emissions below the gross emission level and in a case where diffuse emissions alone would drive atmospheric concentration up to its maximum threshold, full abatement by sector 1 of its emissions appears as the only optimal choice for the economy. Furthermore, with or without air capture possibilities, delaying CCS after the atmospheric carbon stock reaches is maximum level is dominated by an earlier development of CCS by sector 1 because of the inability of sector 2 to use carbon sequestration. However, even with air capture availability, the total carbon emission flow from the two sectors remains only partially abated resulting in a time phase during which the atmospheric carbon constraint binds over the fossil fuel consumption possibilities of the two sectors.

Note also that atmospheric capture is undertaken only after the beginning of the atmospheric carbon ceiling phase and that sector 2's abatement effort is always smaller than its gross contribution to carbon emissions, a result which stands now in accordance with Chakravorty et al. (2006). It is interesting to observe that the economy may experience a rather complex dynamic pattern of energy price while being constrained by the atmospheric carbon ceiling. With constant abatement unit costs, the energy price at the consumer stage is composed of a sequence of constant price phases separated by increasing price phases. This complex shape translates to the time profile of the carbon tax implemented to meet the atmospheric concentration objective.

The carbon tax must increase over time before the ceiling but note that sector 1 escapes the tax when fully abating its emissions and bears a comparatively lower sequestration cost, the fiscal burden being transferred over sector 2. Such a discrepancy between sectors is justified by the fact that sector 2 benefits from the carbon sequestration efforts of sector 1, a sort of positive "external" effect of sector 1 upon sector 2. Of course this is not a true external effect since it comes through the carbon price. But this opens interesting policy questions regarding the use of carbon regulation to develop non polluting transportation devices, like the electric car, electricity being provided by plants making use of CCS technologies. During the ceiling phase, the carbon tax has an overall decreasing shape down to zero at the end of the phase. But this general shape is actually composed of a complex sequence of decreasing rates phases separated by constant rates phases, these last phases corresponding respectively to the air capture phase and to the partial carbon sequestration phase by sector 1 which should follow the full carbon abatement phase by this sector. Thus inducing through the carbon tax the optimal sequence of abatement efforts by the two sectors appears as a rather complicated exercise in fiscal policy, the policy maker having to adjust over time the carbon tax rate according to the optimal sequence of abatement phases.

A second source of heterogeneity between sectors comes from the differing availability of the two carbon abatement technologies. As stated before, CCS is only available for sector 1 while air capture may apply to emissions coming from any source. Alternatively we could have assumed that sector 2 abates its emissions at a unit cost c_a through some dedicated technology while sector 1 abates through CCS at a unit cost c_e, $c_e < c_a$, without altering the results of our analysis. To reinforce the heterogeneity argument, it can be shown (Amigues et al., 2011) that, when energy users have a access to a single carbon abatement technology, then even learning or R&D over this technology do not justify to abate before being at the atmospheric ceiling. However, because the time at which the ceiling is attained is endogenous, learning by doing will affect the time profile of the ceiling phase. An interesting extension of the work would be to analyze the effects of learning by doing or dedicated R&D over CCS and air capture in an heterogeneous use framework.

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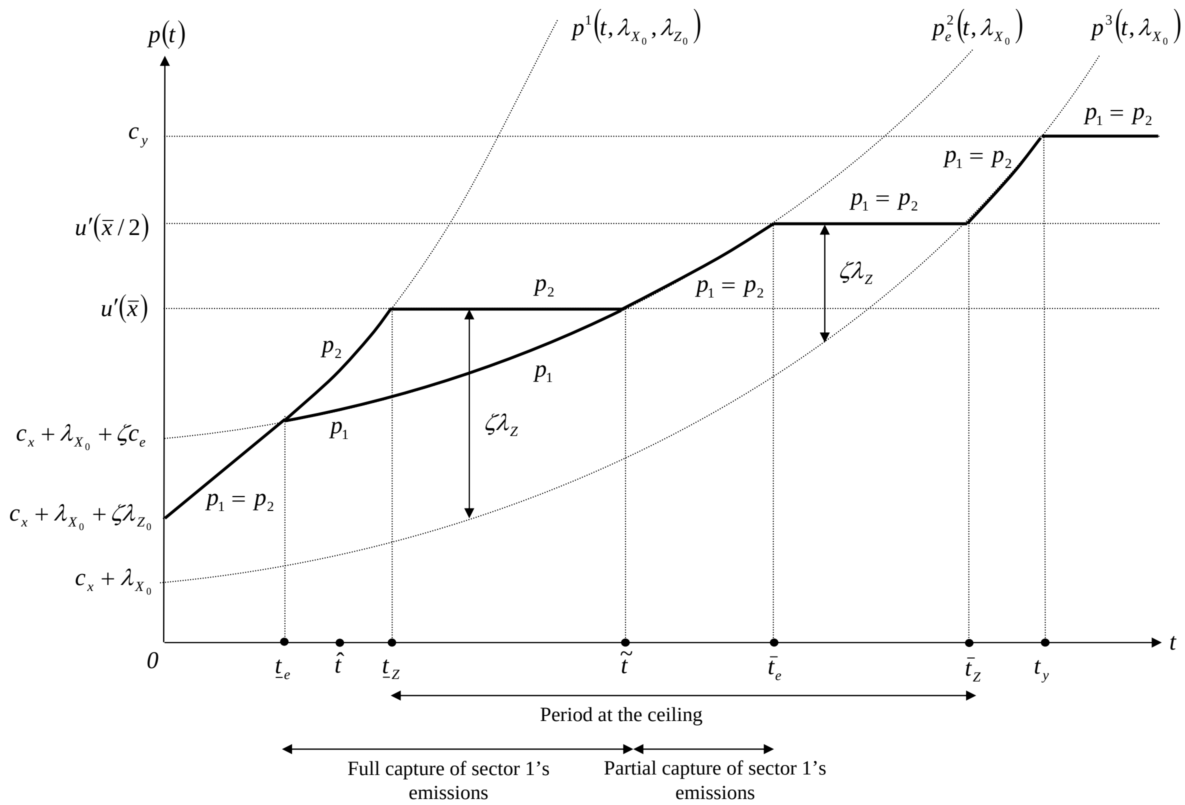

Figure 1: Optimal path along which it is optimal to abate before the ceiling

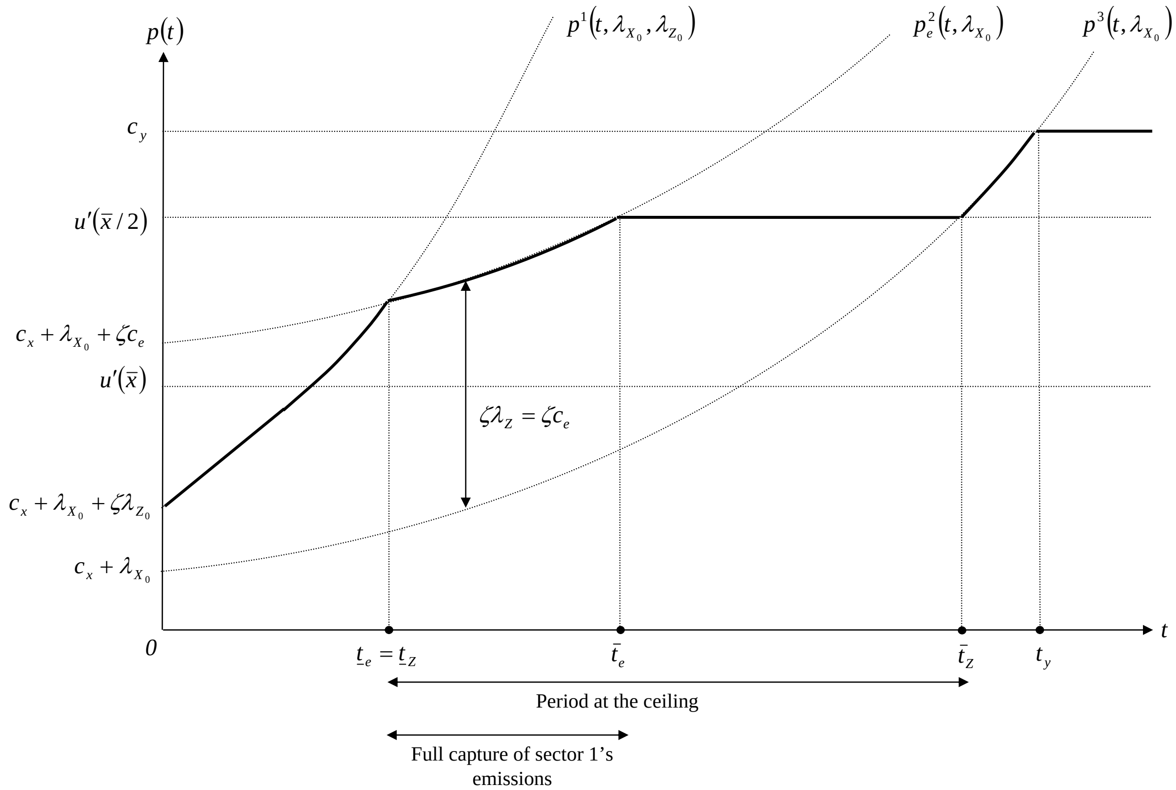

Figure 2: Optimal path along which the energy price is the same for each sector and it is optimal to abate sector 1's emissions

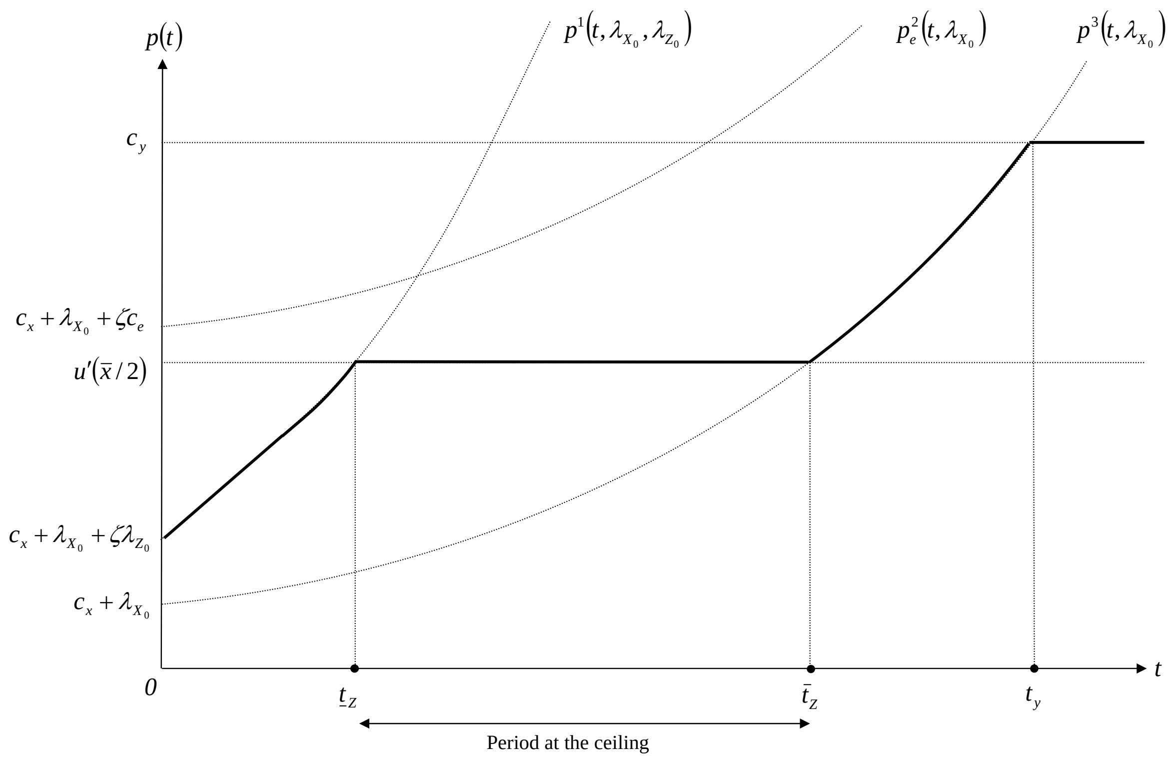

Figure 3: Optimal path along which the energy price is the same for each sector and it is not optimal to abate sector 1's emissions

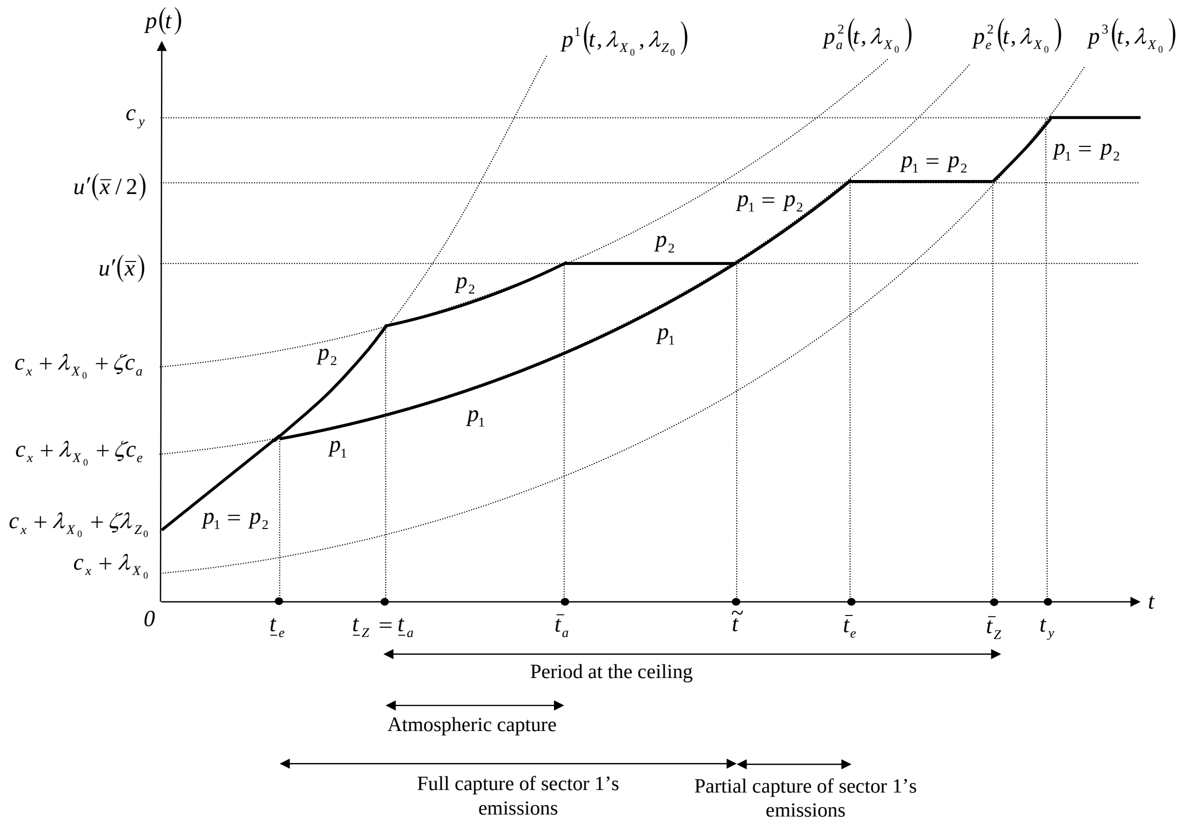

Figure 4: Optimal path requiring to activate the both carbon capture devices

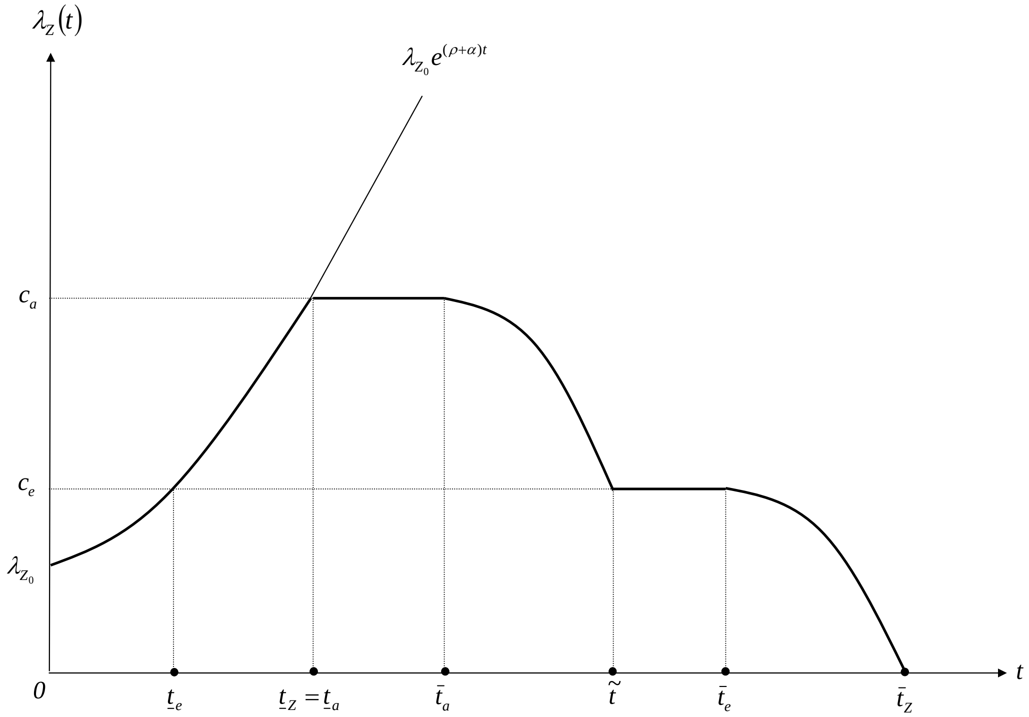

Figure 5: Time profile of the optimal unitary carbon tax

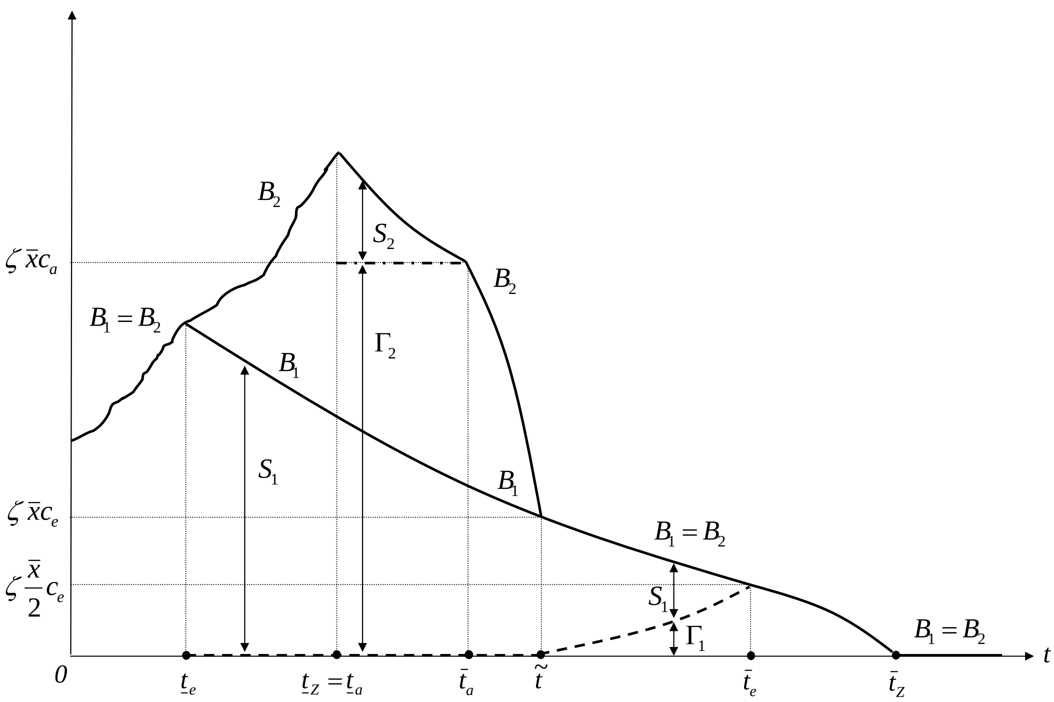

Figure 6: Total burden of carbon for each sector