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Merger Control and Investment: the Role of Efficiency Assessment*

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Abstract

We study how the precision with which merger-specific efficiencies are assessed affects the incentives to invest in such efficiencies before the merger review. With binary investment and an exogenous cutoff, greater precision strengthens investment incentives. We then show how the analysis extends to continuous investment choice and Bayesian approval. A more rigorous merger assessment can therefore encourage efficiency creation.

Keywords: merger control; efficiencies; investment; innovation; information; competition policy.
JEL codes: L13; L41; O31; D82.

1 Introduction

The current debate on EU merger policy places unusual emphasis on investment, innovation and competitiveness. The Draghi Report stresses Europe's need for greater investment and innovation, whereas the European Commission's ongoing review of the Merger Guidelines expressly considers how merger analysis should account for innovation and investment effects (Draghi, 2024; European Commission, 2026). The current European debate often presents

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rigorous merger control as an obstacle to investment. This note points to a channel through which it can instead be a catalyst for investment: a more precise assessment of claimed efficiencies can foster firms' incentives to generate those efficiencies.

The mechanism is simple. Suppose that, before the agency decides on a proposed merger, the parties can undertake preparatory investment that creates merger-specific efficiencies, which will be realized if the merger is approved. The agency only observes a noisy signal of these efficiencies, and authorizes the transaction if the evidence is sufficiently favorable. The precision of that evidence therefore determines how strongly merger approval rewards investment in the project. We show that better information precision can strengthen the incentives of the parties to generate merger efficiencies. The logic is closely related to Holmström's (1979) Informativeness Principle.[1]

Related literature. The paper contributes to the literature on endogenous efficiencies and information in merger control. Banal-Estañol, Macho-Stadler and Seldeslachts (2008) examine how investment and internal organization determine merger efficiencies, whereas Cosnita-Langlais and Tropeano (2012) study how efficiency defenses and remedies affect merger design. Lagerlöf and Heidhues (2005), Dertwinkel-Kalt and Wey (2021), and Bisceglia, Piccolo and Tarantino (2023) analyze incentives to produce or acquire information about efficiencies. Nocke and Whinston (2010) establish conditions under which myopic consumer-surplus-based merger control is dynamically optimal. We provide instead a dynamic incentive rationale for more precise efficiency assessment.

This mechanism complements the literature on mergers and innovation. Federico, Langus and Valletti (2017) examine reduced innovation incentives from internalizing rivalry; Denicolò and Polo (2018) show how reallocating duplicative research can reverse that conclusion. Valletti (2025) clarifies the innovation theory of harm and the separate role of efficiency claims. These contributions underlie the innovation theory of harm, abstracting by construction from merger-specific efficiencies. Instead, we study these efficiencies and how their assessment affects incentives to generate them.

Roadmap. We first present the key intuition in a baseline setting with a discrete investment decision and exogenous efficiency threshold (Section 2). We then show how the analysis extends to continuous investment choice (Section 3) and Bayesian approval (Section 4).

2 Baseline setting

Two firms contemplate a horizontal merger, subject to approval by an antitrust agency. Before the agency decides, they can undertake preparatory investment designed to exploit the integration or combination of the merging parties' assets and businesses after approval; let $\delta \in \{0,1\}$ denote the investment decision. Investing ($\delta = 1$) costs C, drawn from a continuously differentiable distribution $F(C)$ with positive density over $\mathbb{R}_+^*$, and delivers a given efficiency gain $G > 0$; absent investment ($\delta = 0$), no cost is incurred and no efficiency

[1] Holmström (1979) studies how information can improve incentives through the design of optimal contracts. Here, authorization follows a specified merger-review rule rather than an optimally chosen incentive contract.

is generated. By construction, the investment cost is sunk before the merger review, and the resulting benefit is merger-specific: the resulting efficiency gain can be realized only if the merger is implemented.

The timing is substantive. The mechanism requires the parties to invest *before* the merger review, and to forego the return if the merger is blocked. Examples of such investments include the research for synergy avenues, the search or advisory cost incurred to find the most suitable partner for the identified avenues,[2] as well as the specialized internal and external labor resources dedicated to prepare the combination of assets (e.g., to resolve platform-compatibility issues, prepare data and process migration, or otherwise shorten the period of costly duplication following closing).[3] In the same vein, a manufacturer could develop a prototype tailored to a prospective partner's production technology before review, with commercial deployment combining their assets after clearance.

The efficiencies generated by the parties' investment benefit consumers (e.g., cost reductions leading to lower prices, new and improved products) and are supposed to be large enough to counterbalance the loss of competition. The agency, however, does not observe the investment decision δ or the resulting efficiency gain $g \in \{0, G\}$ directly; it observes instead

$$s \equiv g + \frac{\varepsilon}{\tau},$$

where ε is a normally distributed noise – i.e., $\varepsilon \sim N(0,1)$, with c.d.f. $\Phi(\varepsilon)$ and density $\phi(\varepsilon)$ – and $\tau > 0$ measures the precision of the signal observed by the agency.

In this baseline setting, we further assume that the agency uses a fixed signal cutoff as a reduced-form proxy for a consumer-surplus test, $\bar{g}$;[4] that is, it approves the merger if and only if $s \geq \bar{g}$. We focus on the relevant case in which

$$0 < \bar{g} < G.$$

Hence, given the investment decision $\delta \in \{0, 1\}$, the merger is accepted with probability

$$P_\delta(\tau) \equiv \Pr(s \geq \bar{g} \mid \delta) = \Pr(\delta G + \frac{\varepsilon}{\tau} \geq \bar{g}) = 1 - \Phi\left(\tau\left(\bar{g} - \delta G\right)\right).$$

Letting $\Pi_0 > 0$ denote the profit generated by the merger absent investment, and $\Pi_1 > \Pi_0$ the profit in case of investment (gross of investment costs), the firms have an incentive to invest when they face a cost $C < V(\tau)$, where

$$V(\tau) \equiv P_1(\tau)\Pi_1 - P_0(\tau)\Pi_0 \tag{1}$$

denotes the (expected) investment return. The investment probability is therefore

$$I(\tau) \equiv F(V(\tau)).$$

[2]Consistent with this interpretation, Golubov, Petmezas and Travlos (2012) find that top-tier investment banks are better able to identify and structure mergers with higher synergy gains.

[3]Marks and Mirvis (2015) provide a concrete example of such preparatory investment, which they refer as precombination phase of M&As, in which an acquirer committed 10 million to external integration planning before regulatory approval, so as to accelerate implementation if the merger proceeded.

[4]We relax this assumption in Section 4, where the test is endogenously based on the agency's beliefs, at the time of the merger review.

We have:

Proposition 1 (baseline)**.** *With a fixed approval threshold, greater information precision strictly increases the investment return and the investment probability:*

$$V'(\tau) > 0 \text{ and } I'(\tau) > 0.$$

Proof. See Appendix A.1. ■

The result captures the incentive effect of evidentiary precision. When τ is small, approval probabilities are similar under the two investment choices, as the agency has little ability to distinguish mergers that satisfy the consumer-surplus standard from those that do not – indeed, as $\tau \to 0$, both approval probabilities converge to 1/2, and so investing has no impact on the probability of approval. As precision rises, $P_1(\tau)$ increases and $P_0(\tau)$ falls. Approval therefore becomes more sensitive to investment, which in turn fosters investment incentives.

3 Continuous investment

The same mechanism extends locally to continuous investment choice. Let $x \geq 0$ denote the investment, $C(x)$ the associated sunk cost, $G(x)$ the prospective merger-specific efficiency gain, and $\Pi(x)$ the profit generated by the merger if implemented, gross of investment cost. Assume that C, G and Π are twice continuously differentiable, with $C(0) = G(0) = \Pi(0) = 0$, $\lim_{x\to+\infty} G(x) = +\infty$, and

$$C'(\cdot) > 0, \qquad G'(\cdot) > 0, \qquad \Pi'(\cdot) > 0.$$

Let $H(G)$ denote the impact of the merger on consumer surplus, where H is strictly increasing and such that $H(\bar{g}) = 0$. Thus, $G(x) \geq \bar{g}$ would be the full-information consumer-surplus test; the agency applies its noisy counterpart $s \geq \bar{g}$.

The next proposition extends the insights of the previous section to investments that meet the consumer-surplus standard by up to one standard deviation of the signal noise; that is, $G(x) \in [\bar{g}, \bar{g} + 1/\tau]$, or:

$$x \in [\underline{x}, \bar{x}(\tau)] \equiv [G^{-1}(\bar{g}), G^{-1}(\bar{g} + \frac{1}{\tau})],$$

where $\bar{x}(\tau)$ decreasing from $+\infty$ to $\underline{x}$ as τ goes from 0 to infinity.

The approval probability is

$$P(x;\tau) \equiv 1 - \Phi(\tau[\bar{g} - G(x)]),$$

and the net expected value from investing x is $V(x;\tau) - C(x)$, where

$$V(x;\tau) \equiv P(x;\tau)\,\Pi(x).$$

The marginal return from investment is therefore:

$$\frac{\partial V(x;\tau)}{\partial x} = P(x;\tau)\,\Pi'(x) + \frac{\partial P(x;\tau)}{\partial x}\Pi(x). \tag{2}$$

Building on this leads to:

Proposition 2 (continuous investment)**.** *Greater signal precision strictly increases the marginal return to investments meeting the standard by up to one standard deviation:*

$$\frac{\partial^2 V(x;\tau)}{\partial x \partial \tau} > 0 \text{ for any } x \in [\underline{x}, \bar{x}(\tau)].$$

Proof. See Appendix A.2. ■

Proposition 2 shows that, with continuous investments, the insight from the baseline analysis extends locally: precision strengthens investment incentives when the project is close enough to the approval margin. The restriction is a sufficient condition for this monotonicity:[5] the condition $x \geq \underline{x}$, or $G(x) \geq \bar{g}$, ensures that the approval probability increases with τ (i.e., $\partial P/\partial \tau \geq 0$, implying that the first term in (2) increases with precision); the condition $x \leq \bar{x}(\tau)$, or $G(x) \leq \bar{g} + 1/\tau$, ensures instead that the impact of a marginal investment on the approval probability also increases in τ (i.e., $\partial^2 P/\partial x \partial \tau > 0$, implying that the second term in (2) also increases with precision).

It is worth noting that the r+ange $x \in [\underline{x}, \bar{x}(\tau)]$ in which Proposition 2 holds corresponds precisely to those situations where (i) the investment (weakly) benefits consumers (as $x \geq \underline{x}$) but (ii) the investment being close to the threshold (as $x \leq \bar{x}(\tau)$), an imprecise signal is likely to lead to the wrong decision. When instead $x > \bar{x}(\tau)$, the merger is already very likely to be approved (as it should, because efficiencies are very large); greater precision can then reduce the investment margin needed to secure it. For example, with $G(x) = \Pi(x) = x$, $C(x) = x^2/2$ and $\bar{g} = 1$, for $\tau = 8$ the equilibrium investment is approximately 1.286, which exceeds $\bar{x}(\tau) = 1.125$; and if τ is lowered to 4, then the equilibrium investment raises to about 1.442.

4 Bayesian approval

An exogenous threshold provides a transparent benchmark, but a Bayesian agency would endogenously adapt this threshold when precision changes, using its revised belief about investment. We show below that the incentive result survives, as long as the equilibrium is stable.

To see this, let us return to the discrete investment choice of the baseline setting, but assume that the agency approves the merger if, according to its information, consumer surplus is not expected to decrease. Let $-L < 0$ and $S > 0$ denote, respectively, the

[5] A weaker condition for $\partial^2 V/\partial x \partial \tau > 0$ is $\Pi'(x)/[G'(x)\Pi(x)] > \tau^2 [G(x) - \bar{g}] - 1/[G(x) - \bar{g}]$.

impact on consumer surplus absent and with the efficiency gain. If the agency believes that investment has taken place with probability ρ, it approves if and only if $\rho S - (1-\rho)L \geq 0$, which amounts to

$$\rho \geq \bar{\rho} \equiv \frac{L}{L+S}.$$

Equivalently, expected efficiency must satisfy

$$\rho G \geq \bar{g} \equiv \frac{LG}{L+S} \in (0, G).$$

The posterior probability ρ is endogenous: it depends on the prior probability of investment, which we will denote by r, as well as on the information technology. Specifically, upon observing s, using Bayes' rule the agency will update its belief from $r \in [0,1]$ to

$$\rho(s; r, \tau) \equiv \frac{r\phi(\tau(s-G))}{r\phi(\tau(s-G)) + (1-r)\phi(\tau s)}.$$

There always exists a coordination-breakdown equilibrium in which the firms never invest (hence, $r = 0$) and the agency blocks the merger (as $\rho(\cdot; 0, \tau) = 0$). We now look for an interior equilibrium, in which the firms invest with positive probability. Using $\phi(\varepsilon) = [\exp(-\varepsilon^2/2)]/\sqrt{2\pi}$, the approval test $\rho(s; r, \tau) \geq \bar{\rho}$ is equivalent to

$$s \geq \bar{s}(r, \tau) \equiv \frac{G}{2} + \frac{1}{\tau^2 G} \log \frac{(1-r)L}{rS}.$$

Investing raises the approval probability from $\Pr[\varepsilon/\tau \geq \bar{s}(r;\tau)]$ to $\Pr[G + \varepsilon/\tau \geq \bar{s}(r;\tau)]$; hence, the return on investment is

$$V(r;\tau) \equiv \Pi_1[1 - \Phi(\tau[\bar{s}(r;\tau) - G])] - \Pi_0[1 - \Phi(\tau \bar{s}(r;\tau))]. \tag{3}$$

For this extension, we adopt the following assumption:

Assumption A**:**

$$F(\Pi_1 - \Pi_0) < \bar{\rho} = \frac{L}{L+S}.$$

If approval were guaranteed, firms would invest only to obtain the additional profit $\Pi_1 - \Pi_0$. Assumption A asserts that this would generate too little investment to justify unconditional approval based on the corresponding prior. Merger control has therefore a substantive role: expectations of private investment alone do not adequately benefit consumers.

Given the threshold $\bar{s}(r, \tau)$ induced by the agency's prior belief r, firms invest if they face a cost $C \leq V(r;\tau)$, leading to an investment probability $F(V(r;\tau))$; hence, in equilibrium $r = \hat{r}(\tau)$ satisfies

$$\hat{r}(\tau) = F(V(\hat{r}(\tau), \tau)). \tag{4}$$

Building on this leads to:[6]

[6]Stability refers to the usual tâtonnement process; see Appendix A.3 for a precise definition.

Proposition 3. *Under Assumption A, greater precision strictly raises the investment return and thus fosters investment at every stable interior equilibrium.*

Proof. See Appendix A.3. ■

Proposition 3 shows that the insight from the previous sections does not rely on the evidentiary cutoff being exogenous. Keeping the agency's belief fixed, greater precision fosters investment incentives, as in the baseline setting. With Bayesian decision-making, this induces the agency to become more optimistic, which may have an ambiguous impact on investment incentives. However, as long as the equilibrium is stable, the direct effect prevails and greater precision therefore induces higher investment.

5 Policy implication

The key policy implication is that more precise evidence assessment can reward preparatory efforts to generate merger-specific efficiencies. Thus, more precise assessment is not only important to screen out mergers that cause a significant loss of competition, it is also important to strengthen parties' incentives to invest before the deal in generating merger-specific efficiencies.

This implication is relevant to the current European debate. While competitiveness may require more investment, investment incentives can be strengthened by a more rigorous assessment of efficiencies. In this sense, merger control, often presented as an obstacle to investment, can instead act as a catalyst for investment.

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A Appendix

A.1 Proof of Proposition 1

Using $P_1'(\tau) = (G - \bar{g})\phi(\tau(\bar{g} - G))$ and $P_0'(\tau) = -\bar{g}\phi(\tau\bar{g})$, differentiating (1) yields:

$$V'(\tau) = P_1'(\tau)\Pi_1 - P_0'(\tau)\Pi_0 = (G - \bar{g})\phi(\tau(\bar{g} - G))\Pi_1 + \bar{g}\phi(\tau\bar{g})\Pi_0 > 0,$$

where the inequality stems from $G > \bar{g} > 0$, $\phi(\cdot) > 0$, and $\Pi_\delta > 0$ for $\delta = 0, 1$. Furthermore, as $V(\tau)$ strictly increases with τ, so does the resulting probability of investment, $I(\tau) = F(V(\tau))$.

A.2 Proof of Proposition 2

Differentiating (2) with respect to τ yields:

$$\frac{\partial^2 V(x;\tau)}{\partial x \partial \tau} = \frac{\partial P(x;\tau)}{\partial \tau}\Pi'(x) + \frac{\partial^2 P(x;\tau)}{\partial x \partial \tau}\Pi(x), \tag{A.1}$$

where

$$\begin{aligned}
\frac{\partial P(x;\tau)}{\partial \tau} &= -\phi(\tau[\bar{g} - G(x)])[\bar{g} - G(x)], \\
\frac{\partial^2 P(x;\tau)}{\partial x \partial \tau} &= \phi(\tau[\bar{g} - G(x)])G'(x) + \phi'(\tau[\bar{g} - G(x)])\tau[\bar{g} - G(x)]G'(x) \\
&= \left\{1 - \tau^2[\bar{g} - G(x)]^2\right\}\phi(\tau[\bar{g} - G(x)])G'(x),
\end{aligned}$$

where the last equality uses $\phi'(z) = -z\phi(z)$. For $x \in [\underline{x}, \bar{x}(\tau)]$, we have $\bar{g} \leq G(x) \leq \bar{g} + 1/\tau$; hence, $\partial P/\partial \tau \geq 0$ and $\partial^2 P/\partial x \partial \tau \geq 0$, implying $\partial^2 V/\partial x \partial \tau \geq 0$ (as $\Pi(x) > 0$ for $x > (G^{-1}(\bar{g}) \geq) 0$ and $\Pi'(\cdot) > 0$). Furthermore, the first term in (A.1) is positive for $G(x) > \bar{g}$ and the second term is positive for $G(x) < \bar{g} + 1/\tau$. It follows that $\partial^2 V/\partial x \partial \tau \geq 0$ in the entire range $\mathcal{X}(\tau)$.

If $x^*(\tau)$ is a strict investment response lying in $\mathcal{X}(\tau)$, then, from the implicit function theorem:

$$\frac{dx^*(\tau)}{d\tau} = -\frac{\partial^2 V(x^*(\tau);\tau)}{\partial x \partial \tau} / [\frac{\partial^2 V(x^*(\tau);\tau)}{\partial x^2} - C''(x^*(\tau))] > 0,$$

where the inequality follows from $\partial^2 V/\partial x \partial \tau > 0$ for $x \in \mathcal{X}(\tau)$ and the local second-order condition $\partial^2 V(x^*(\tau);\tau)/\partial x^2 - C''(x^*(\tau)) < 0$.

A.3 Proof of Proposition 3

Consider an interior equilibrium $r = \hat{r}(\tau) > 0$, which thus satisfies (4). Differentiating with respect to τ yields:

$$\hat{r}'(\tau) = \left.\frac{f(V(r;\tau))\frac{\partial V(r;\tau)}{\partial \tau}}{1 - f(V(r;\tau))\frac{\partial V(r;\tau)}{\partial r}}\right|_{r=\hat{r}(r)}.$$

Furthermore, the denominator is positive whenever the equilibrium is stable in the usual sense. To see this, consider a standard tâtonnement process in which, in response to a given agency's prior belief r_n, the firms invest if $C \leq V(r_n;\tau)$, leading the agency to revise its belief to $r_{n+1} = F(V(r_n;\tau))$. This tâtonnement process converges if $d\hat{r}_{n+1}/d\hat{r}_n < 1$, which amounts to $f(V(r;\tau))\,\partial V(r;\tau)/\partial r|_{r=\hat{r}(r)} < 1$. As $f(\cdot) > 0$, it follows that the sign of $\hat{r}'(\tau)$ is the same as that of $\partial V(r;\tau)/\partial\tau|_{r=\hat{r}(r)}$.

Differentiating (3) with respect to τ yields:

$$\frac{\partial V(r;\tau)}{\partial\tau} = -\Pi_1\phi\left(\tau\left[\bar{s}(r;\tau) - G\right]\right)\left[\bar{s}(r;\tau) - G + \tau\frac{\partial\bar{s}(r;\tau)}{\partial\tau}\right] + \Pi_0\phi\left(\tau\bar{s}(r;\tau)\right)\left[\bar{s}(r;\tau) + \tau\frac{\partial\bar{s}(r;\tau)}{\partial\tau}\right],$$

where:

$$\frac{\partial\bar{s}(r;\tau)}{\partial\tau} = -\frac{2}{\tau^3 G}\log\frac{(1-r)L}{rS} = \frac{G - 2\bar{s}(r,\tau)}{\tau}.$$

Combining these expressions yields:

$$\frac{\partial V(r;\tau)}{\partial\tau} = \Pi_1\phi\left(\tau\left[\bar{s}(r;\tau) - G\right]\right)\bar{s}(r,\tau) + \Pi_0\phi\left(\tau\bar{s}(r;\tau)\right)\left[G - \bar{s}(r,\tau)\right]. \tag{A.2}$$

If $\bar{s}(r;\tau) \in [0, G]$, then the right-hand side is positive (both terms are non-negative, and at least one of them is positive). If instead $\bar{s}(r;\tau) > G$, then:

$$\frac{\partial V(r;\tau)}{\partial\tau} = \Pi_1\phi\left(\tau\left[\bar{s}(r;\tau) - G\right]\right)G + \left\{\Pi_1\phi\left(\tau\left[\bar{s}(r;\tau) - G\right]\right) - \Pi_0\phi\left(\tau\bar{s}(r;\tau)\right)\right\}\left[\bar{s}(r,\tau) - G\right],$$

where both terms of the right-hand side are positive: this is obvious for the first one, and for the second one, it stems from the working assumption $\bar{s}(r,\tau) > G$ and:

$$\frac{\Pi_1\phi\left(\tau\left[\bar{s}(r;\tau) - G\right]\right)}{\Pi_0\phi\left(\tau\bar{s}(r;\tau)\right)} > \frac{\phi\left(\tau\left[\bar{s}(r;\tau) - G\right]\right)}{\phi\left(\tau\bar{s}(r;\tau)\right)} = \frac{e^{-\frac{\{\tau[\bar{s}(r;\tau)-G]\}^2}{2}}}{e^{-\frac{[\tau\bar{s}(r;\tau)]^2}{2}}} = e^{\frac{\tau^2 G}{2}[2\bar{s}(r;\tau)-G]} > 1,$$

where the inequality follows again from stems from the working assumption $\bar{s}(r,\tau) > G$.

From the above, $\partial V(r;\tau)/\partial\tau > 0$ as long as $\bar{s}\left(\hat{r}(\tau);\tau\right) \geq 0$. Consider the case $\bar{s}\left(\hat{r}(\tau);\tau\right) < 0$. We first note that this implies $\hat{r}(\tau) > \bar{\rho}$, as the agency would then approve the merger upon observing $s = 0 (> \bar{s}\left(\hat{r}(\tau);\tau\right))$, implying:

$$\bar{\rho} < \rho(0;\hat{r}(\tau),\tau) = \frac{\hat{r}(\tau)\,\phi(-\tau G)}{\hat{r}(\tau)\,\phi(-\tau G) + (1-\hat{r}(\tau))\phi(0)} = \frac{\hat{r}(\tau)}{\hat{r}(\tau) + (1-\hat{r}(\tau))\frac{\phi(0)}{\phi(-\tau G)}} < \hat{r}(\tau),$$

where the inequality stems from $\phi(-\tau G) = \phi(\tau G) < \phi(0)$. Furthermore, using $\Phi(-\varepsilon) = 1 - \Phi(\varepsilon)$, $V(r;\tau)$ can be expressed as:

$$V(r;\tau) = \Pi_1\Phi(\tau\left[G - \bar{s}(r;\tau)\right]) - \Pi_0\Phi(-\tau\bar{s}(r;\tau)).$$

It then follows from Assumption A and $F\left(V\left(\hat{r}(\tau);\tau\right)\right) = \hat{r}(\tau) > \bar{\rho}$ that

$$F\left(V\left(\hat{r}(\tau);\tau\right)\right) > F\left(\Pi_1 - \Pi_0\right)$$

implying:

$$\begin{aligned} & \Pi_1\Phi(\tau\left[G-\bar{s}\left(r;\tau\right)\right])-\Pi_0\Phi(-\tau\bar{s}\left(r;\tau\right))>\Pi_1-\Pi_0 \\ \iff & \Pi_0\left[1-\Phi\left(-\tau\bar{s}\left(r;\tau\right)\right)\right]>\Pi_1\left[1-\Phi\left(\tau\left[G-\bar{s}\left(r;\tau\right)\right]\right)\right] \\ \iff & \frac{\Pi_0}{\Pi_1}>\frac{1-\Phi\left(\tau\left[G-\bar{s}\left(r;\tau\right)\right]\right)}{1-\Phi\left(-\tau\bar{s}\left(r;\tau\right)\right)}. \end{aligned}$$

Noting that $\varepsilon\left[1-\Phi\left(\varepsilon\right)\right]/\phi\left(\varepsilon\right)$ is strictly increasing,[7] the last inequality implies[8]

$$\frac{\Pi_0}{\Pi_1}>\frac{-\tau\bar{s}\left(r;\tau\right)\phi\left(\tau\left[G-\bar{s}\left(r;\tau\right)\right]\right)}{\tau\left[G-\bar{s}\left(r;\tau\right)\right]\phi\left(-\tau\bar{s}\left(r;\tau\right)\right)}=\frac{-\bar{s}\left(r;\tau\right)\phi\left(\tau\left[\bar{s}\left(r;\tau\right)-G\right]\right)}{\left[G-\bar{s}\left(r;\tau\right)\right]\phi\left(\tau\bar{s}\left(r;\tau\right)\right)},$$

where the last equality uses $\phi\left(-\varepsilon\right)=\phi\left(\varepsilon\right)$. It follows that the right-hand side of (A.2) is again positive.

Summing-up, Assumption A ensures that $\partial V\left(r;\tau\right)/\partial\tau>0$. Together with equilibrium stability, this implies $\hat{r}'\left(\tau\right)>0$.

[7] $1-\Phi\left(\varepsilon\right)$ can be expressed as:

$$1-\Phi\left(\varepsilon\right)=\int_{\varepsilon}^{\infty}e^{-\frac{x^2}{2}}dx=\int_{0}^{\infty}e^{-\frac{1}{2}\left(\varepsilon+\frac{y}{\varepsilon}\right)^2}\frac{dy}{\varepsilon}=\frac{\phi\left(\varepsilon\right)}{\varepsilon}\int_{0}^{\infty}e^{-y-\frac{y^2}{2\varepsilon^2}}dy,$$

implying

$$\frac{\varepsilon\left[1-\Phi\left(\varepsilon\right)\right]}{\phi\left(\varepsilon\right)}=\int_{0}^{\infty}e^{-y-\frac{y^2}{2\varepsilon^2}}dy,$$

where the right-hand side strictly increases with ε.

[8] Using $G>0$ and the monotonicity of $\varepsilon\left[1-\Phi\left(\varepsilon\right)\right]/\phi\left(\varepsilon\right)$, we have:

$$\frac{\tau\left[G-\bar{s}\left(r;\tau\right)\right]\left[1-\Phi\left(\tau\left[G-\bar{s}\left(r;\tau\right)\right]\right)\right]}{\phi\left(\tau\left[G-\bar{s}\left(r;\tau\right)\right]\right)}>\frac{-\tau\bar{s}\left(r;\tau\right)\left[1-\Phi\left(-\tau\bar{s}\left(r;\tau\right)\right)\right]}{\phi\left(-\tau\bar{s}\left(r;\tau\right)\right)}.$$

The conclusion follows.