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"Information disclosure in preemption races: Blessing or (winner's) curse?"

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Information disclosure in preemption races: Blessing or (winner's) curse?*

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Abstract

Firms receiving independent signals on a common-value risky project compete to be the first to invest. When firms are symmetric and competition is winner-take-all, rents are fully dissipated in equilibrium and the extent to which signals are publicly disclosed is irrelevant for welfare. When disclosure of signals is asymmetric, welfare is highest when firms are most asymmetric, and policies that uniformly promote disclosure may backfire, especially when competition is severe. When firms strategically select their disclosure policies, a moderate subsidy for disclosure induces a low correlation between firms' policies, and thus maximizes welfare.

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1 Introduction

■ Priority races (patent or R&D races, scientific or academic research, entry games...) are often characterized by both highly skewed rewards in favor of the first mover, and by opportunities to learn from the experiences of rivals. Firms (or researchers) then face a tension between the need to act quickly and the desire to wait to gather information. As an illustration, Krieger (2021) empirically documents that, upon observing a rival exiting an R&D race, pharmaceutical companies with a project using a similar technology are more likely to discontinue their own projects in spite of a lowered preemption risk. Whereas firms quite accurately observe the progress of their rivals in the drugs industry (health regulators like the FDA or the EMA impose pharmaceutical companies to report their clinical trial results at all phases of the drug development process), other priority races are characterized instead by little information disclosure. For instance, in scientific or academic research, failed attempts (null, negative, or inconclusive results) are rarely disclosed.[1] The concern that such a "file drawer problem" (Rosenthal, 1979) might harm the overall efficiency of the scientific process has triggered recent changes going in the direction of greater transparency.[2] At the policy level, there has been as well a recent push towards more information disclosure. The World Health Organization has notably taken a strong stance in its 2015 Statement on Public Disclosure of Clinical Trial Results:

> "*Researchers have a duty to make publicly available the results of their research* [...] *Negative and inconclusive as well as positive results must be published or otherwise made publicly available.*"[3]

Although an implicit rationale behind such initiatives and regulations is that more information disclosure improves welfare, one may wonder if this is actually the case in environments already plagued by a strong inefficiency due to intense competition. In line with this idea, the objective of this article is to examine the interplay between preemption concerns (payoff externalities) and the desire to learn from others (information externalities), and to assess the welfare impact of policies fostering or mandating disclosure.

We consider a model of investment timing that features both competition and learning. Firms observe conditionally independent processes that may stochastically reveal bad news

[1] For instance, Franco, Malhotra, and Simonovits (2014) show that studies that yield null results are 40% less likely to be published, and 60% less likely to be written up in any form than those with statistically significant results.

[2] For instance, several journals, institutions, and grants request the preregistration of scientific studies in a public registry. Furthermore, a new type of journals (such as the Missing Pieces Collection by *PLOS One*) specializing in the publication of negative, null, or inconclusive results, has emerged.

[3] See `https://www.who.int/news/item/09-04-2015-japan-primary-registries-network`.

about the common value of a project they contemplate, and compete to be the first to invest, if they ever do so. Whereas firms learn from their own signals, the extent to which they also observe their rivals' signals (or exit) varies between the two polar cases of public and private learning. We establish that, as soon as signals are not always publicly disclosed, firms face a *winner's curse*: when seeing that no rival has invested yet, they become wary that such lack of investment might result from some rival having privately learned that the project fails; being the first one to invest is then all else equal bad news. This winner's curse provides an incentive to delay investment in order to gain extra confidence on the project through one's own signal. In any case, under winner-take-all competition, rents are fully dissipated in equilibrium, and the publicity of signals is accordingly irrelevant to welfare: with public news, firms learn faster but also invest sooner, leaving the NPV of the project constant and null at any investment date. Regulating information disclosure with the aim of promoting information spillovers is thus vain: the benefits arising from these spillovers are fully eroded by competition anyway.

In the second part of the article, we show that this irrelevance result no longer holds when firms have asymmetric rates of disclosure. Specifically, we study a two-player asymmetric preemption race where firms in addition receive a payment both when their rival successfully invests (thereby relaxing winner-take-all competition), and when they publicly disclose their own signals. First, we highlight that a firm known to be less likely to disclose its signal obtains a competitive advantage in the preemption race, which provides a micro-foundation for the cost of disclosure. Second, we show that there is typically an interior optimal level of publicity in that policies promoting disclosure boost welfare up to a certain point but then depress it. This optimal level of publicity decreases as competition gets more severe. This suggests a pecking order whereby the optimal regulation should first and foremost address winner-take-all competition, and promote transparency only once competition is less severe. Finally, this extended framework allows us to analyze strategic disclosure decisions and to derive the optimal subsidy that a planner offers to firms to incentivize disclosure. By committing to disclosing its signal, a firm can secure the disclosure reward in case bad news arises, but incurs the risk of being at a competitive disadvantage when its rival does not disclose. We then characterize a symmetric (possibly mixed-strategy) equilibrium in which firms optimally choose their disclosure policies. Although a higher reward increases average disclosure, the correlation between disclosure policies across firms is nonmonotonic in the reward: too large and too small rewards will induce firms to either both disclose or not disclose. Instead, a moderate reward makes them most likely to be ex-post asymmetric, and accordingly maximizes welfare.

□ **Related literature.** This article relates to the literatures on preemption and learning externalities in timing games. Since the seminal works of Reinganum (1981) and Fudenberg and Tirole (1985) on preemption races, several authors have generalized the basic preemption framework in terms of payoff functions (Hoppe and Lehmann-Grube, 2005), number of firms (Argenziano and Schmidt-Dengler, 2014), and uncertainty about the presence of competitors (Bobtcheff and Mariotti, 2012; Bobtcheff, Bolte, and Mariotti, 2017). Besides, following Chamley and Gale (1994), who highlight how the desire to learn from others' investment decisions causes inefficient delays, another stream of articles have studied timing games with no preemption but with information externalities (Décamps and Mariotti, 2004; Murto and Välimäki, 2011; Margaria, 2020; Kirpalani and Madsen, 2023).

Our model combines both payoff and information externalities, as in Chen, Ishida, and Mukherjee (2023). However, they only consider private signals, whereas we stress the comparison between public and private news to highlight the welfare impact of transparency. This focus also relates our article to a series of works that compare public and private learning in timing games. Hopenhayn and Squintani (2011) analyze the impact of players being privately informed about their payoff from exiting. However, because they consider a private-value setup, there is no information externality, and hence no winner's curse. Moscarini and Squintani (2010) study the impact of private signals in a model where private information is on the arrival rate of payoffs, so that staying in the game signals positive information to the competitor. Akcigit and Liu (2015) also focus on the inefficiency created by private information. Whereas observable signals take the form of breakthroughs (good news) in their model, we consider bad-news learning, which generates a winner's curse. As a result, we emphasize the downside of public information, a concern that is absent in their setup where public information is always optimal. Wagner and Klein (2022) consider a setup where private signals can dominate public signals, because the opportunity for signaling brought about by private information allows to encourage investment, thereby mitigating free riding. Finally, Hoppe-Wewetzer, Katsenos, and Ozdenoren (2023) compare private and public learning in a discrete-time race with good-news learning. As a consequence, there is no winner's curse and public news may lead to longer experimentation, in contrast to this article. Notice to conclude that none of these works on private versus public learning considers strategic disclosure.

The complementarity between the publicity of learning and the prize-sharing rule we highlight also relates this article to Halac, Kartik, and Liu (2017) and Ely et al. (2023), who study how a principal optimally designs a contest both in terms of the allocation of the prize and of the feedback given to contestants. A key difference, however, is that we consider a stopping problem where investment stops the game for everyone, and these preemption externalities generate a completely different kind of inefficiency.

The article is organized as follows. In Section 2, we describe the model. In Section 3, we characterize equilibrium behavior and profits, and examine welfare. In Section 4, we consider asymmetric and strategic disclosures, and derive the optimal subsidy for information disclosure. Section 5 concludes. Proofs that are not in the text are in Appendices A–B.

2 A preemption race

■ The building blocks of our model are as follows.

Actions and payoffs. Time is continuous and indexed by $t \geq 0$. Each of finitely many firms, indexed by $i = 1, \ldots, n$, contemplates investing in a project of ex-ante unknown but common quality. The quality of the project is either high, with probability p_0, or low, with probability $1 - p_0$. Each firm decides when to invest in the project, if it ever does. Investment involves an irreversible cost $I \in (p_0, 1)$, so that the ex-ante NPV of the project is negative. Upon investing, a firm obtains a revenue of 1 if and only if it is the first to invest and the project is of high quality. In case several firms simultaneously attempt to invest, a tie-breaking rule randomly selects one of them and the others do not incur the investment cost I, as in Dutta and Rustichini (1993). Specifically, to every firm i is associated a weight $\alpha^i > 0$, and if a subset $\mathcal{I}$ of firms simultaneously attempt to invest, every firm $i \in \mathcal{I}$ is selected with probability $\frac{\alpha^i}{\sum_{j \in \mathcal{I}} \alpha^j}$. In all the other cases, the revenue is 0. Firms are risk-neutral and discount future revenues and costs at rate r.

Personal signals. As long as it has not invested, each firm learns about the quality of the project by observing for free a personal signal. This signal generates a single failure at a date that is exponentially distributed with rate $\lambda > 0$ if the project is of low quality, and 0 otherwise. Thus, a failure conveys conclusive bad news about the quality of the project. Instead, whereas one becomes increasingly optimistic as long as nothing is observed, it is impossible for any firm to ever know for certain that the quality of the project is high.[4] Personal signals are conditionally independent across firms given the quality of the project.

Information structure. That a firm's personal signal has generated a failure may or may not be disclosed to its rivals. Specifically, we assume that, with probability x, a failure is publicly and immediately disclosed, whereas, with probability $1 - x$, it remains forever

[4] Notice that none of our results would be affected if our model also featured fully revealing good news as long as observing a fully revealing signal is more likely in the bad state than in the good state. In particular, it is clear that the optimal strategy for a firm who receives fully revealing good news is to invest right away.

private.[5] In addition, investment by any firm becomes immediately public. The information set of each firm at date t thus possibly consists of three elements: whether or not it has observed a failure from its personal signal, whether or not the failure of some other firm has been publicly disclosed, and whether or not some other firm has invested.

Strategies. As soon as its information set is nonempty—that is, either a rival has invested, or a failure has been personally observed or publicly disclosed—a firm cannot do better than giving up. Therefore, a pure strategy for each firm just specifies at which date t to invest, if any, conditional on observing neither a failure nor any of its rivals investing by then. Because conditional beliefs at all future dates can be perfectly anticipated at date 0, if planning to invest at date t is optimal from a firm's perspective at date 0, it is optimal for the firm to invest at date t as long as it has observed nothing by then. So there is no loss in considering that strategies are chosen at date 0 and simply implemented whenever nothing is observed by the planned investment date. Likewise, if a firm plays according to a mixed strategy, it randomly draws at date 0 an investment date t from a distribution F, and investment is then implemented at date t as long as the firm has observed nothing by then. Our equilibrium concept is perfect Bayesian equilibrium.

□ **Discussion.** Though stylized, our model is meant to capture a variety of situations in which players (firms, researchers) face a tradeoff between the desire to act quickly to preempt their rivals and the value of waiting to learn from their own but also from their rivals' experience. Whereas we define x as the probability of observing a failure from any rival's personal signal when it arises, an equivalent interpretation is that x captures the probability that exit is public (as long as exit is observed with no delay), as bad news is fully revealing and thus leads players to discontinue their projects (exit). For instance, because of mandatory disclosure, drug companies often publicly announce themselves when they terminate one project (high x); instead, scientists hardly know whether other scientists are still working on some unpublished project or have secretly abandoned it (low x).

3 Equilibrium analysis

■ In this section, we develop the equilibrium analysis of our basic model.

□ **Pure-strategy equilibria.** We first study under which conditions our preemption race admits a pure-strategy equilibrium. This allows to get a first sense of the role of payoff and information externalities.

[5] Although we view x at this stage as an exogenous market parameter common to all firms, we will allow firms to endogenously choose their (possibly asymmetric) disclosure policies in Section 4.

Proposition 1 *The following holds*:

(i) If $x < 1$, there exists no pure-strategy equilibrium.

(ii) If $x = 1$, there exist pure-strategy equilibria. In any such equilibrium, $m \geq 2$ firms plan to invest at $\hat{t}$ such that

$$\frac{p_0}{p_0 + (1 - p_0)\mathrm{e}^{-\lambda n \hat{t}}} = I, \tag{1}$$

and the other $n - m$ firms each plan to invest after $\hat{t}$ or to stay inactive. All firms earn zero profit in equilibrium.

The intuition for the case of partial disclosure ($x < 1$) is as follows. For any firm planning to invest at t, the relevant belief is the probability that the project is of high quality conditional on observing no failure by t *and* winning the race. When $x < 1$, the mere fact of winning the race brings bad news. Indeed, one is more likely to win the race in the bad state as some other firm planning to invest at t or before may have privately observed a failure and, hence, dropped out from the race. Accordingly, there is a *winner's curse*. For the sake of exposition, let us illustrate how this winner's curse precludes the existence of a pure-strategy equilibrium in the case of two firms and a fair tie-breaking rule.

Suppose that both firms are supposed to invest at t in case they have observed nothing by then. Conditional on observing no failure and winning the race, each firm's belief at date t that the project is of high quality is

$$p(t) = \frac{\frac{1}{2}p_0}{\frac{1}{2}p_0 + (1 - p_0)\mathrm{e}^{-\lambda t}\left[\frac{1}{2}\mathrm{e}^{-\lambda t} + (1 - x)(1 - \mathrm{e}^{-\lambda t})\right]}.$$

In the good state, no firm ever observes bad news, so both firms attempt to invest at t, each winning with probability $\frac{1}{2}$. In the bad state, instead, a firm attempts to invest at t only if it has observed no failure from its personal signal by t, which happens with probability $\mathrm{e}^{-\lambda t}$; it then wins the race with probability $\frac{1}{2}$ when its rival has not observed a failure either and thus attempts to invest at t, which again happens with probability $\mathrm{e}^{-\lambda t}$; finally, it wins the race with probability 1 when its rival has *privately* observed a failure, which happens with probability $(1 - x)(1 - \mathrm{e}^{-\lambda t})$. In turn, if a firm deviates by planning to invest at $s < t$, there is never a tie and its belief at date s that the project is of high quality is

$$p(s) = \frac{p_0}{p_0 + (1 - p_0)\mathrm{e}^{-\lambda s}[\mathrm{e}^{-\lambda s} + (1 - x)(1 - \mathrm{e}^{-\lambda s})]}.$$

It is clear that, as soon as $x < 1$, $p(t) < \lim_{s \uparrow t} p(s)$. Thus, conditional on winning the race, beliefs that the project is of high quality discontinuously fall at the date at which the other firm plans to invest. As a consequence, if investing at t yields a nonnegative NPV (a necessary condition for equilibrium), then any firm strictly benefits from planning to invest slightly earlier, because this both increases the likelihood of winning the race and allows one to avoid the winner's curse.

In the case of perfect disclosure ($x = 1$), there is no winner's curse: winning the race is a pure matter of luck and carries no informational content. Firms all share the same belief about the quality of the project at any date and, pushed by the Bertrand logic, they must earn zero profit in equilibrium. Indeed, if the NPV upon investment were positive, firms would be willing to undercut their rivals, decreasing only marginally their confidence in the project at the benefit of an upward jump in the probability of winning the race.

The analysis of pure-strategy equilibria thus highlights the role of the two forces of our model: on the one hand, preemption motives due to winner-take-all competition drive equilibrium profits to zero; on the other hand, private signals generate a winner's curse in that lack of investment by other firms becomes all else equal bad news. In what follows, we outline how these payoff and information externalities interact to shape equilibrium behavior in mixed-strategy equilibria.

□ **Mixed-strategy equilibria.** Recall that firm j playing a mixed strategy initially draws an investment date t from a distribution F^j and actually attempts to invest at t as long as it has observed nothing by then. We allow for $\lim_{t\to\infty} F^j(t) < 1$ to capture that firm j may plan to never invest with positive probability, in which case it earns zero profit.

If no F^j, $j \neq i$, has an atom at t, then firm i's expected profit if it plans to invest at t is[6]

$$V^i(t) \equiv \mathrm{e}^{-rt}\Bigg(p_0(1-I)\prod_{j\neq i}[1-F^j(t)] \\ - (1-p_0)I\mathrm{e}^{-\lambda t}\prod_{j\neq i}\left\{(1-x)\int_0^t \lambda\mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda t}[1-F^j(t)]\right\}\Bigg). \tag{2}$$

In the good state, no firm ever observes a failure from its personal signal, so that firm i wins the race if all other firms' planned investment dates turn out to be posterior to t, which happens with probability $\prod_{j\neq i}[1-F^j(t)]$. In the bad state, firm i wins the race when it has observed no failure from its personal signal by t, which happens with probability $\mathrm{e}^{-\lambda t}$, and none of its rivals has yet invested or disclosed a failure, which happens with probability

$$\prod_{j\neq i}\left\{(1-x)\int_0^t \lambda\mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda t}[1-F^j(t)]\right\},$$

where the bracketed term is the probability that firm j either has privately observed a failure from its personal signal prior to the date at which it planned to invest, or has observed no

[6] As argued in the discussion of pure-strategy equilibria, atoms generate downward jumps in the profit functions of other firms. We show in Proposition 2 that equilibrium distributions are atomless when $x < 1$.

failure from its personal signal and has not invested yet by t.

The expression (2) highlights that planning to invest at or after a date by which one rival would have invested for sure had he observed nothing must yield a nonpositive profit; formally, if $F^j(t) = 1$ for some j and t, then $V^i(s) \leq 0$ for all $i \neq j$ and $s \geq t$. This has the following implication.

Lemma 1 *In any equilibrium, all firms earn zero profit.*

To lay out intuition, suppose that, say, firm 1 makes a positive profit in equilibrium, and let $\underline{t}^1$ and $\overline{t}^1$ be the infimum and supremum of the support of its equilibrium distribution F^1, respectively. The argument is then twofold. On the one hand, all other firms must earn zero profit in equilibrium. Indeed, because firm 1 makes a profit by planning to invest at $\overline{t}^1$, it must be that all other firms planned to invest with positive probability after that date or to stay inactive, which gives them at most zero profit—in both cases, because of winner-take-all competition. On the other hand, at $\underline{t}^1$, firm 1 must be weakly more pessimistic about the project than any other firm. Indeed, its behavior up to $\underline{t}^1$ does not impose any information externality onto its rivals—that it had not invested carries no information, so that there is no winner's curse.[7] This implies that if firm 1 can make a profit by planning to invest at $\underline{t}^1$, then so can all other firms. To sum up, because of winner-take-all competition, at most one firm can make a profit in equilibrium, and because of the winner's curse (or common beliefs when $x = 1$), either all or no firm makes a profit in equilibrium. Whereas winner-take-all competition typically leads to rent dissipation (Fudenberg and Tirole, 1985), this discussion highlights how information and payoff externalities combine to generate full dissipation of rents in the presence of learning as well.

Lemma 1 implies that $\min_{i=1,..,n} \underline{t}^i = \underline{t}$, where $\underline{t}$ is the first date at which the NPV of the project is nonnegative given that no firm could have possibly invested beforehand; that is,

$$\frac{p_0}{p_0 + (1 - p_0)\mathrm{e}^{-\lambda \underline{t}}(1 - x + x\mathrm{e}^{-\lambda \underline{t}})^{n-1}} = I. \tag{3}$$

Observe from (3) that $\underline{t}$ is a decreasing function of x. We are now ready to characterize all the mixed-strategy equilibria of the preemption race.

Proposition 2 *If $x < 1$, the following holds:*

(i) For each $m \geq 2$, there exists an essentially unique equilibrium in which $m \geq 2$ firms are active and plan to invest according to a common continuously differentiable increasing distribution over $[\underline{t}, \infty)$, whereas $n - m$ firms remain inactive.

[7] Notice that this argument is also valid when $x = 1$. In this case, all firms have the same beliefs at any date. What is key is that a firm making a profit cannot be strictly more optimistic than its rivals.

(ii) If $x < x' < 1$, then, in the mixed-strategy equilibria under x and x' in which all firms are active, the investment date under x' is smaller than the investment date under x in the hazard-rate order.

Proposition 2 establishes that the only source of equilibrium multiplicity is the number of firms that are active in equilibrium, and that all active firms must play symmetric strategies. As is the case for pure-strategy equilibria, at least two firms must be active, but given that all firms earn zero profit, up to $n-2$ firms may equally want to stay inactive.[8] Notice that, if $0 < x < 1$, these inactive firms are not out of the market altogether, because they still receive personal signals that may be disclosed to their rivals.[9] The most natural equilibrium to consider is the symmetric one in which all n firms are active, and in which the equilibrium strategy $F(t)$ is as follows:[10]

$$F(t) = 1 - \mathrm{e}^{-\frac{1}{n-1}\lambda(t-\underline{t})} \left[\frac{\left(\frac{p_0}{1-p_0}\,\frac{1-I}{I}\right)^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda \underline{t}}}{\left(\frac{p_0}{1-p_0}\,\frac{1-I}{I}\right)^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda t}} \right]^{\frac{1+(n-1)x}{n}}, \quad t \geq \underline{t}. \tag{4}$$

As x goes to 1, $\underline{t}$ goes to $\hat{t}$, so that $\left(\frac{p_0}{1-p_0}\,\frac{1-I}{I}\right)^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda \underline{t}}$ goes to 0 by (1). Hence, by (4), for any sequence $(x_k)_{k\in\mathbb{N}}$ converging to 1, the corresponding sequence of equilibrium distributions $(F_{x_k})_{k\in\mathbb{N}}$ converges weakly to the jump distribution $\mathbb{1}_{[\hat{t},\infty)}$. That is, as failures from personal signals become close to being perfectly observable, the sequence of mixed-strategy equilibria converges to the pure-strategy equilibrium described in Proposition 1(ii).

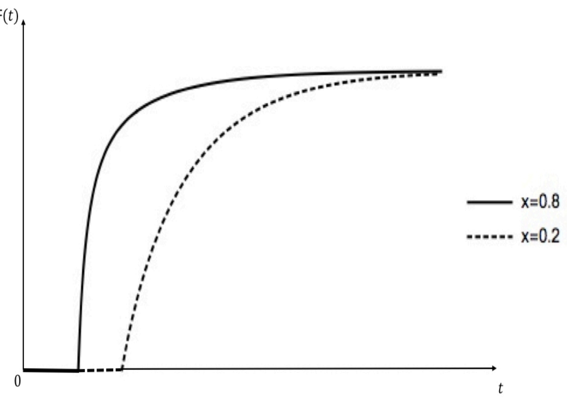

Figure 1: Equilibrium distributions for $x = 0.2$ and $x = 0.8$ [$p_0 = 0.5, I = 0.7, \lambda = 0.8, n = 2$].

Proposition 2 also establishes that equilibrium distributions can be ranked in the hazard-rate order as a function of the publicity of news: when failures are less likely to be disclosed,

[8]Specifically, Proposition 2 implies that equilibria with $m < n$ active firms are entry-proof in the sense that, if all the m active firms play according to their equilibrium strategies, then the remaining $n-m$ firms are better off staying inactive.

[9]Only if $x = 0$ is being inactive equivalent to being out of the market.

[10]To keep notation light, we do not make explicit that $\underline{t}$ and F depend on x and n.

firms respond by investing later in equilibrium, as illustrated in Figure 1. Intuitively, the winner's curse problem worsens, which provides incentives to further delay investment so as to gain extra confidence about the project through one's personal signal.[11]

□ **Welfare.** We now draw the welfare implications of our analysis, focusing for simplicity on the fully symmetric equilibrium with n active firms.

An irrelevance result. Using the fact that firms' beliefs upon observing nothing by date t are given by

$$p(t) = \frac{p_0[1-F(t)]^{n-1}}{p_0[1-F(t)]^{n-1} + (1-p_0)\mathrm{e}^{-\lambda t}\left\{(1-x)\int_0^t \lambda \mathrm{e}^{-\lambda s}[1-F(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda t}[1-F(t)]\right\}^{n-1}}$$

for all $t \geq \underline{t}$, and that $\frac{p_0[1-F(t)])^{n-1}}{p(t)}$ is thus the total probability for a given firm to reach date t without observing anything, we derive that the random variable equal to the first date at which investment occurs is distributed according to $n\,\frac{p_0[1-F(t)]^{n-1}}{p(t)}\,\mathrm{d}F(t)$. As a consequence, total expected welfare is given by

$$W \equiv \int_{\underline{t}}^{\infty} \mathrm{e}^{-rt}[p(t)-I]\,n\,\frac{p_0[1-F(t)]^{n-1}}{p(t)}\,\mathrm{d}F(t). \tag{5}$$

By Lemma 1, each firm earns zero profit in equilibrium, so that the project has zero NPV given its beliefs at any date at which it contemplates investing; that is, $p(t) = I$ for all $t \geq \underline{t}$. Hence the following result.

Proposition 3 *W is null no matter the publicity of news x and the number of firms n.*

Notice that the probability that investment ever takes place, $\int_{\underline{t}}^{\infty} n\,\frac{p_0[1-F(t)]^{n-1}}{p(t)}\,\mathrm{d}F(t)$, is also independent of x and n. Indeed, using $p(t) = I$, it reads $\frac{p_0}{I}\int_{\underline{t}}^{\infty} n[1-F(t)]^{n-1}\,\mathrm{d}F(t) = \frac{p_0}{I}$ for all x and n. Because the probability of investment is always 1 in the good state, this notably implies that the probability of investing in a project of low quality is also independent of the publicity of news x. Indeed, higher publicity generates information spillovers, improving the quality of learning holding firms' strategies fixed; but, on the other hand, firms respond to higher publicity by planning to invest sooner, and the net effect on the total probability of investment in the bad state is null.

The cooperative benchmark. To see how the irrelevance result arises from the preemption motive, it is useful to consider what would be the planner's (or cooperative) solution. A planner that could impose the timing of investment—but not information disclosure—would choose t to maximize the expected value of the project,

$$\mathrm{e}^{-rt}\left\{p_0(1-I) - (1-p_0)I\left[(1-x+\mathrm{e}^{-\lambda t})^n - (1-x)^n(1-\mathrm{e}^{-\lambda t})^n\right]\right\}.$$

[11] Notice that a lower publicity also decreases the speed at which firms learn even before they start investing, which, as shown by Bobtcheff and Levy (2017), reinforces the incentives to delay investment.

It is easy to check that this objective function attains its maximum at some finite date posterior to $\underline{t}$ that optimally solves the tradeoff between discounting and learning. Observing that this objective function is increasing in x and n, we infer from the envelope theorem that its maximum value increases with x and n. Put differently, because competition is muted in the cooperative solution, firms optimally respond to more publicity, enabling the social planner to reach a higher welfare level thanks to information spillovers. Our irrelevance result is thus an equilibrium feature, that is, a consequence of preemption forces.

Conditions for irrelevance. More generally, our irrelevance result holds both because the planner is assumed to have preferences that are aligned with those of firms, and because rents are fully dissipated in equilibrium.

To figure out the role of aligned preferences, notice that, if the planner gets instead a payoff W_S in case of success and $-W_F$ in case of failure, (5) becomes

$$\tilde{W} \equiv \int_{\underline{t}}^{\infty} \mathrm{e}^{-rt}\{p(t)W_S - [1-p(t)]W_F\}n\,\frac{p_0(1-F(t))^{n-1}}{p(t)}\,\mathrm{d}F(t),$$

that is, given that $p(t) = I$ for all $t \geq \underline{t}$,

$$\tilde{W} = \underbrace{[IW_S - (1-I)W_F]}_{\text{Planner's NPV upon investment}} \times \underbrace{\int_{\underline{t}}^{\infty} \mathrm{e}^{-rt} n\,\frac{p_0[1-F(t)]^{n-1}}{I}\,\mathrm{d}F(t)}_{\text{Present value of obtaining 1 at the first investment date}}$$

Because investment arises sooner when x increases, $\int_{\underline{t}}^{\infty} \mathrm{e}^{-rt} n\,\frac{p_0[1-F(t)]^{n-1}}{I}\,\mathrm{d}F(t)$, the present expected value of obtaining 1 at the first investment date, is increasing in x. This implies that, when $\frac{W_S}{W_F} > \frac{1-I}{I}$, that is, when the planner is relatively keener on investing than the firms, he prefers investment to take place as early as possible, hence a preference for public signals ($x = 1$). By contrast, when $\frac{W_S}{W_F} < \frac{1-I}{I}$, that is, when the planner is more conservative than the firms, he prefers investment to be delayed, hence a preference for private signals ($x = 0$).[12] This provides a possible rationale against mandatory disclosure when the planner primarily cares about avoiding investments in the bad state.

As the proof of Proposition 3 shows, our irrelevance result follows from the full dissipation of rents. Much like the Bertrand paradox, this is an extreme result due in particular to the fact that firms are symmetric. In addition, it is easy to see that firms obtain rents when a firm that does not invest (a second mover, or a firm that observed a failure) receives a positive profit.[13] In the next section, we consider a richer model where we relax the two key assumptions of symmetry and winner-take-all competition, and spell out how the publicity of signals then becomes relevant to welfare.

[12] When $\frac{W_S}{W_F} = \frac{1-I}{I}$, the planner's preferences are aligned with the firms' and we retrieve our irrelevance result.

[13] It is for instance immediate to see that such a firm could secure a positive profit by waiting forever.

4 Endogenous disclosure

■ In this section, we generalize the baseline model along three dimensions: (a) we allow firms to have asymmetric probabilities of disclosure; (b) we relax winner-take-all competition by assuming that a preempted firm obtains a revenue $L \geq 0$ whenever the project is of high quality; (c) we introduce a reward $D \geq 0$ for firms that publicly disclose failures from their personal signals. We assume for simplicity that a firm observing a failure can secure D by disclosing it even when such disclosure brings no extra social value (that is, if the bad state was already revealed either by some unsuccessful investment or by another firm disclosing a failure from its personal signal).[14] To simplify the analysis and the exposition, we restrict attention to the case of two firms in this section, and we assume that $r = 0$.[15]

We first characterize the equilibrium in the asymmetric preemption race, holding the disclosure probabilities (x^1, x^2) and the payoffs L and D fixed, and derive the corresponding welfare. Building on this, we derive how firms choose x^1 and x^2 as a function of D, and how the planner then optimally sets D to incentivize disclosure. In so doing, we treat $L \in (0, 1 - I)$ as a parameter measuring the severity of competition: indeed, between the two polar cases of winner-take-all competition ($L = 0$) and of no preemption risk ($L = 1-I$), priority races often feature a significant first-mover advantage but also nontrivial payoffs for the follower.[16] Our analysis accordingly allows us to derive how the optimal policy varies with the magnitude of the first-mover advantage.

In what follows, it will prove convenient to define

$$y \equiv \frac{p_0}{1 - p_0} \frac{1 - I - L}{I}, \tag{6}$$

which provides a measure of the willingness of firms to win the preemption race given the risk of failure it entails and the outside option to wait for the other firm to move first. As y increases, the cutoff belief above which a firm is willing to invest increases. Notice in particular that y is decreasing in L, the revenue of the second mover.

In addition, the case $\frac{p_0}{1-p_0} \frac{1-I}{I} > \frac{1}{2}$ turns out to yield to the richest set of configurations, and we thus henceforth make the following assumption.

Assumption 1 $\frac{p_0}{1-p_0} \frac{1-I}{I} > \frac{1}{2}$.

□ **Equilibrium.** We first look at the preemption race with asymmetric disclosure rates, assuming with no loss of generality that $x^1 \leq x^2 \leq 1$.

[14] Alternative assumptions are possible but would only make the analysis more cumbersome without any essential changes in the results.

[15] Notice that none of the results of Section 3 depends on the value of r. A convenient feature of assuming $r = 0$ is that it does not matter when the payoff L accrues.

[16] For instance, Hill and Stein (forthcoming) argue that competition between scientists in molecular biology is far from winner-take-all, and estimate that "scooped" papers receive only 20% fewer citations.

Proposition 4 *Suppose that $x^1 \leq x^2 \leq 1$. Then the following holds:*

(i) If $x_1 < 1$, then there exists a mixed-strategy equilibrium in which both firms plan to start investing at $\underline{t}(x^1)$, where

$$\frac{p_0(1-L)}{p_0 + (1-p_0)\mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - x^1 + x^1 \mathrm{e}^{-\lambda \underline{t}(x^1)}]} = I. \tag{7}$$

If $x^1 < x^2$, firm 2 stays inactive with probability $\frac{(x^2-x^1)[1-\mathrm{e}^{-\lambda \underline{t}(x^1)}]}{1-x^1+x^1\mathrm{e}^{-\lambda \underline{t}(x^1)}}$.

(ii) If $x^1 = 1$, then there exists a unique, pure-strategy equilibrium in which both firms plan to invest at $\underline{t}(1)$.

(iii) In any equilibrium, the firms' equilibrium profits are

$$V^{1*}(x^1, x^2) = p_0 L + (1-p_0)x^1 D + (x^2 - x^1)(1-p_0) I \mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}], \tag{8}$$
$$V^{2*}(x^1, x^2) = p_0 L + (1-p_0)x^2 D. \tag{9}$$

In the asymmetric race that arises when $x^1 < x^2$, firm 1 has a comparative advantage: it learns at a faster rate than firm 2 because it is more likely to observe firm 2's personal signal than firm 2 is to observe firm 1's. Besides, firms could guarantee themselves the profit of the second mover in the good state by staying inactive, that is, $p_0 L$. Thus, as long as firm 2's beliefs do not allow it to secure this amount, firm 1 faces no preemption risk. This is why firm 1 in equilibrium plans to start investing only at date $\underline{t}(x^1)$, which leaves firm 2 with the profit it would get from waiting indefinitely. This logic is reminiscent of that of Bertrand competition with asymmetric costs.[17] Notice also that $\underline{t}(x^1)$ is decreasing in x^1 and increasing in L. Indeed, when the informational advantage of firm 1 increases, that is, when x^1 decreases, it takes longer for firm 2 to be optimistic enough to even contemplate investing. Likewise, when competition becomes less stringent, that is, when L increases, firm 2 will not invest unless the perceived value of the project is high enough, insulating firm 1 longer from the risk of preemption.

□ **Welfare.** Total welfare depends on whether the payoffs L and D are transfers made by the planner (for instance, in the form of subsidies), or underlying parameters of the environment. As mentioned above, we take the view that L is a parameter governing the severity of competition. Instead, in line with the idea that competitors typically do not disclose bad news absent any incentive, and motivated by the policy debate regarding

[17]Notice, though, that, when $x^1 < x^2$, firm 1 must also randomize in equilibrium. Indeed, if firm 1 were to invest at $\underline{t}(x^1)$ for sure, firm 2 would never invest because lack of investment at this date by firm 1 would reveal that the project is of low quality, so that firm 1 would be better off waiting longer.

transparency regulations, we view D as a reward (or subsidy) offered by the planner to incentivize disclosure. Therefore, when $x_1 \leq x_2$, we take welfare to be equal to

$$\begin{aligned} W(x^1, x^2) &\equiv V^{1*}(x^1, x^2) + V^{2*}(x^1, x^2) - (1-p_0)(x^1 + x^2)D \\ &= 2p_0 L + (x^2 - x^1)(1-p_0) I \mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}], \end{aligned} \tag{10}$$

where the equality follows from (8)–(9). Notice that considering D as a transfer makes it neutral to welfare, so that any welfare impact of D comes from its sole impact on disclosure policies, as we show later.[18]

Proposition 5 *As long as $x^1 \leq x^2$, $W(x^1, x^2)$ is decreasing in x^1 and increasing in x^2.*

Proof. That $W(x^1, x^2)$ is increasing in x^2 as long as $x^1 \leq x^2$ is immediate from (10). To see why $W(x^1, x^2)$ is decreasing in x^1, notice, using (7), that $p_0 L + (x^2 - x^1)(1-p_0) I \mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}] = p_0(1-I) - (1-p_0) I \mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - x^2 + x^2 \mathrm{e}^{-\lambda \underline{t}(x^1)}]$, which is decreasing in x^1 as $\underline{t}(x^1)$ is decreasing in x^1. Hence the result. □

Welfare is thus highest when the asymmetry between firms is maximum. This result echoes the irrelevance result derived in Section 3 that welfare does not depend on the publicity of news as long as firms have symmetric disclosure probabilities. One prediction from Proposition 5 is then that the optimal policy should be asymmetric: encourage one firm to disclose, while deterring the other. In many instances, however, policies must uniformly apply to all, that is, increase or decrease x^1 and x^2 by the same amount. To understand the welfare implications of such transparency policies, let us consider $\Delta \equiv x^2 - x^1 > 0$ and look at how $W(x^1, x^1 + \Delta)$ varies with x^1, holding Δ fixed.

Proposition 6 *There exist (L_0, L_1) with $0 < L_0 < L_1 < 1 - I$ such that*

(i) if $L \leq L_0$, then $W(x^1, x^1 + \Delta)$ is decreasing in x^1, with a maximum reached at $x^{1*} = 0$;

(ii) if $L_0 < L < L_1$, then $W(x^1, x^1 + \Delta)$ is single-peaked in x^1, with a maximum reached at $x^{1*} = 2(1 - 2y)$;

(iii) if $L \geq L_1$, then $W(x^1, x^1 + \Delta)$ is increasing in x^1, with a maximum reached at $x^{1*} = 1 - \Delta$.

Given (6), Proposition 6 implies that the welfare-maximizing level of disclosure x^{1*} is nondecreasing in L, as illustrated in Figure 2.[19] In particular, maximal disclosure may be

[18] Welfare would more generally read $W(x^1, x^2) \equiv V^{1*}(x^1, x^2) + V_2^*(x^1, x^2) - \eta_L p_0 L - \eta_D (1-p_0)(x^1 + x^2) D$, where η_L and η_D capture the costs of policy interventions on L and D. We thus take the stance that $\eta_L = 0$ and $\eta_D = 1$. It is easy to see how our results would be modified under different configurations.

[19] Notice that $L_0 > 0$ is implied by Assumption 1. If Assumption 1 does not hold, the region where $x^{1*} = 0$ simply does not exist, but nothing is otherwise changed.

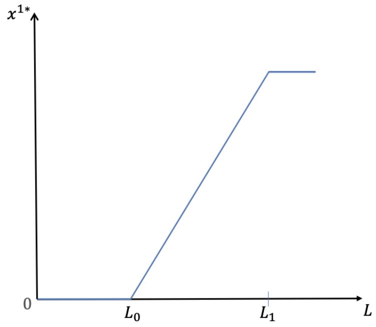

Figure 2: The optimal level of disclosure.

detrimental to welfare under winner-take-all competition, but may become optimal once preemption concerns become milder. Accordingly, promoting disclosure can be efficient as a complement to a policy that relaxes winner-take-all competition; however, in the impossibility to resort to such policies, fostering transparency may actually backfire. One also remarks that there are parameter values such that V^{1*} is increasing in x^1, keeping $\Delta = x^2 - x^1$ constant. In the spirit of raise-your-rival-cost strategies, some firms (here firm 1) would then possibly have an incentive to lobby for more stringent disclosure requirements when this improves their relative competitive situations.

□ **Strategic disclosure.** In this section, we let firms endogenously select their disclosure policies as a function of the reward they receive for disclosing their signals, and we then explore the optimal incentive policy of the planner given firms' equilibrium responses.

The timing of the game now has three stages:

1. the planner sets D;

2. the firms simultaneously and publicly commit to their disclosure policies x^1 and x^2;

3. the preemption race starts.

In this strategic-disclosure game, we assume that every firm i commits to disclose with probability x^i and with no delay any failure generated by its personal signal. In addition, we assume that personal signals are verifiable, so that firms can choose to disclose them or not, but cannot report a failure that they did not observe.[20]

The assumption that firms can ex-ante commit to a disclosure policy is clearly restrictive.[21] A plausible alternative would be that firms strategically disclose bad news upon observing

[20] In a cheap-talk game, firms would always pretend that they know the project to fail to discourage others, hence no information transmission. This contrasts with Wagner and Klein (2022), who consider a model with pure information externalities (no preemption) where signaling aims at encouraging the rival to invest.

[21] Notice that considering time-invariant disclosure policies is restrictive as well, but a more general analysis would be beyond the scope of this work.

it. But unless there is a direct or strategic disclosure cost (not modeled here), firms would be always tempted to disclose bad news in the presence of a reward. Besides, our analysis shows that firms known to disclose less than their rivals have a comparative advantage in the preemption race. Our approach thus allows to micro-found the cost of disclosure on the basis of the ingredients of the model only. Focusing on ex-ante disclosure choices highlights in a simple way the tradeoff between the benefit of disclosing (the reward) and the benefit of concealing (the competitive advantage).

Recall from (9) that $V^{2*}(x^1, x^2)$ is increasing in x^2 for $x^1 \leq x^2$, which implies that the best response to x^1 is either $x^2 < x^1$ or $x^2 = 1$. Because the best response to $x^2 = 0$ is $x^1 = 1$, $x^i = 1$ must be played with positive probability by every firm i in any equilibrium, which allows us to pin down the equilibrium profit of $p_0 L + (1 - p_0)D$.

Proposition 7 *There exists a symmetric (possibly mixed-strategy) equilibrium in which each firm earns a profit $p_0 L + (1 - p_0)D$. Moreover, there exist $D_0(y)$ and $D_1(y)$ such that*

(i) if $D \leq D_0(y)$, then the average probability of disclosure in equilibrium is $\mathbf{E}[\tilde{x}^*] = \frac{D}{y(1-y)I}$;

(ii) if $D_0(y) < D < D_1(y)$, then the average probability of disclosure $\mathbf{E}[\tilde{x}^*] \in (\frac{D_0(y)}{y(1-y)I}, 1)$ is increasing in D;

(iii) if $D \geq D_1(y)$, both firms fully disclose, that is, $\mathbf{E}[\tilde{x}^*] = 1$.

Quite intuitively, the average probability of disclosure increases with the reward D. When $D = 0$, there is no benefit from disclosing, and not disclosing possibly gives a comparative advantage in the preemption race, so no firm discloses ($\mathbf{E}[\tilde{x}^*] = 0$). Conversely, if $D \geq D_1(y)$, the reward is sufficiently large to outweigh the benefit of any comparative advantage in the preemption race, so both firms fully disclose ($\mathbf{E}[\tilde{x}^*] = 1$.) For intermediate values of D, any symmetric equilibrium must be in mixed strategies. We show in the proof of Proposition 8 that the expected welfare equals

$$W = 2p_0 L + 2(1 - p_0)\{1 - \mathbf{E}[\tilde{x}^*]\}D.$$

We can now derive how the planner should optimally set the subsidy D to maximize welfare.

Proposition 8 *The optimal subsidy is $D^* \equiv \frac{y(1-y)I}{2}$ and is such that $\mathbf{E}[\tilde{x}^*] = \frac{1}{2}$.*

The intuition behind Proposition 8 is as follows: if D is small or large, both firms have minimal or maximal incentives to disclose, respectively. This makes it likely that they ex post choose similar disclosure policies. As we have seen, welfare is low in symmetric environments (Proposition 3), and maximized when firms are most asymmetric (Proposition 5). Instead, when D is neither too small nor too large, firms face a nontrivial tradeoff between the comparative advantage they can obtain by disclosing less than their rival and the temptation to disclose to get the reward. Specifically, a key dimension is that disclosure policies are strategic substitutes (at least over some range). When the rival discloses, not disclosing is attractive as it allows to reap a maximal comparative advantage in the ensuing preemption race. In turn, if one's rival does not disclose, there is no possible competitive advantage, and one should disclose to get the reward. This strategic substitutability leads firms to opt for asymmetric policies, resulting in rents. Even focusing on a symmetric equilibrium in which firms ex-ante play identical strategies, this equilibrium involves mixed strategies and firms are more likely to be ex-post asymmetric for intermediate values of the subsidy, hence a higher welfare. Indeed, at the optimal subsidy, $\mathbf{E}[\tilde{x}^*] = \frac{1}{2}$, which corresponds to the lowest possible correlation of disclosure probabilities across firms (each firm chooses $x = 0$ and $x = 1$ with equal probabilities). To conclude, we derive that the optimal subsidy is nonmonotonic in the severity of competition L.

Proposition 9 *The optimal subsidy D^* is increasing in L for $L \leq L_0$ and decreasing for $L \geq L_0$.*

Figure 3 illustrates this nonmonotonicity. The intuition has to do with the way a higher competitive pressure impacts the willingness of firms to acquire a competitive advantage in the preemption race by committing to disclosing less. When competition is fierce (small L), the second-mover profit is low, and learning faster than one's rival is very attractive. In addition, as L increases, the less efficient (slow-learning) firm is less concerned about being preempted, which lowers the competitive pressure faced by the more efficient firm, to its primary benefit. Such an increase in L thus expands the willingness to acquire a comparative advantage even further, hence lower incentives to disclose. In turn, the planner must increase disclosure incentives to decrease the correlation of policies.[22] As L gets large, however, being the second mover becomes attractive and firms no longer seek to obtain a competitive advantage, because there is little competition in the first place. Hence, they will both tend to disclose too often, and the planner should then reduce disclosure incentives by lowering the reward D.

[22]As mentioned above, $L_0 > 0$ only under Assumption 1. Thus, the optimal reward D^* decreases in L in case Assumption 1 does not hold.

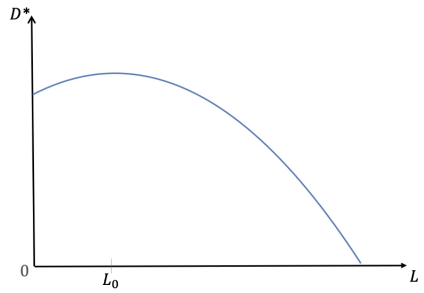

Figure 3: The optimal subsidy D^*.

5 Conclusion

We consider a preemption race between firms that learn from each others' signals and investment decisions. In such a context, preemption concerns provide incentive to accelerate investment, especially under winner-take-all competition, but the opportunities to learn from others generates incentives to wait, because of the winner's curse that arises when other firms' signals are not publicly observed. Whereas welfare is invariant to the publicity of signals when firms are symmetric, we highlight how welfare gains arise when firms have asymmetric disclosure policies. As a result, transparency regulations or excessive incentives that induce universal disclosure may be welfare-reducing.

Appendix A

Proof of Proposition 1. Let us fix a pure-strategy equilibrium $(t^i)_{i=1,\dots,n}$, assuming one exists, and let $\underline{t} \equiv \min_{i=1,\dots,n} t^i$. For each i and t, and holding the strategies of the firms other than i fixed, we denote by $P^i(t)$ the probability that firm i planning to invest at t observes nothing by t and wins the race at t, and by $N^i(t)$ its expectation of the NPV of the project at t conditional on its observing nothing by t and winning the race at t. Hence firm i's expected profit from planning to invest at t is $V^i(t) \equiv \mathrm{e}^{-rt} P^i(t) N^i(t)$.

(i) Fix $x < 1$ and suppose, by way of contradiction, that a pure-strategy equilibrium exists. Let $\mathcal{I} \equiv \{i = 1, \dots, n : t^i = \underline{t}\}$, with $m \equiv |\mathcal{I}|$. Then, for each $i \in \mathcal{I}$,

$$\begin{aligned} &P^i(\underline{t}) \\ &\equiv \alpha_0^i p_0 + (1-p_0)\mathrm{e}^{-\lambda\underline{t}}(1-x+x\mathrm{e}^{-\lambda\underline{t}})^{n-m} \sum_{k=0}^{m-1} \binom{m-1}{k} (1-x)^k (1-\mathrm{e}^{-\lambda\underline{t}})^k \mathrm{e}^{-\lambda(m-1-k)\underline{t}}\, \alpha_k^i, \end{aligned} \tag{A.1}$$

where, for each $k = 0, \dots, m-1$,

$$\alpha_k^i \equiv \frac{1}{\binom{m-1}{k}} \sum_{\mathcal{K} \subset \mathcal{I}\setminus\{i\},\, |\mathcal{K}|=k} \frac{\alpha^i}{\sum_{j \in \mathcal{I}\setminus\mathcal{K}} \alpha^j} \tag{A.2}$$

is the probability that firm i wins the race at $\underline{t}$ conditional on k firms in $\mathcal{I} \setminus \{i\}$ having dropped out from the race after privately observing a failure by $\underline{t}$. In turn, we have

$$N^i(\underline{t}) \equiv \frac{\alpha_0^i p_0}{P^i(\underline{t})} - I. \tag{A.3}$$

Because $\sum_{j\in\mathcal{I}\setminus\mathcal{K}} \alpha^j < \sum_{j\in\mathcal{I}} \alpha^j$ if $\mathcal{K} \neq \emptyset$, it follows from (A.2) that $\alpha_k^i > \frac{\alpha^i}{\sum_{j\in\mathcal{I}}\alpha^j} = \alpha_0^i$ for all $k = 1, \dots, m-1$ if any such k exists, that is, if $m \geq 2$. Then, using (A.1), (A.3), and the binomial formula, we obtain

$$\begin{aligned} &N^i(\underline{t}) \\ &\leq \frac{p_0}{p_0 + (1-p_0)\mathrm{e}^{-\lambda\underline{t}}(1-x+x\mathrm{e}^{-\lambda\underline{t}})^{n-m} \sum_{k=0}^{m-1}\binom{m-1}{k}(1-x)^k(1-\mathrm{e}^{-\lambda\underline{t}})^k \mathrm{e}^{-\lambda(m-1-k)\underline{t}}} - I \\ &= \frac{p_0}{p_0 + (1-p_0)\mathrm{e}^{-\lambda\underline{t}}(1-x+x\mathrm{e}^{-\lambda\underline{t}})^{n-1}} - I, \end{aligned} \tag{A.4}$$

with a strict inequality if $m \geq 2$.

Claim 1. $m = 1$.

Proof. Suppose, by way of contradiction, that $m \geq 2$. Then, by (A.4), we have

$$N^i(\underline{t}) < \lim_{t\uparrow\underline{t}} \frac{p_0}{p_0 + (1-p_0)\mathrm{e}^{-\lambda t}\left(1-x+x\mathrm{e}^{-\lambda t}\right)^{n-1}} - I = \lim_{t\uparrow\underline{t}} N^i(t) \tag{A.5}$$

for all $i \in \mathcal{I}$, where the equality follows from the fact that no firm plans to invest before $\underline{t}$, so that winning the race at $t < \underline{t}$ brings no additional information to firm i. Moreover, as $\alpha^i_k < 1$ for all $k = 0, \ldots, m-2$, using again (A.1), the binomial formula, and the fact that no firm plans to invest before $\underline{t}$ yields

$$\begin{aligned} P^i(\underline{t}) &< p_0 + (1-p_0)\mathrm{e}^{-\lambda \underline{t}}(1 - x + x\mathrm{e}^{-\lambda \underline{t}})^{n-m} \sum_{k=0}^{m-1} \binom{m-1}{k} (1-x)^k (1-\mathrm{e}^{-\lambda \underline{t}})^k \mathrm{e}^{-\lambda(m-1-k)\underline{t}} \\ &= \lim_{t \uparrow \underline{t}} p_0 + (1-p_0)\mathrm{e}^{-\lambda t}(1 - x + x\mathrm{e}^{-\lambda t})^{n-1} \\ &= \lim_{t \uparrow \underline{t}} P^i(t). \end{aligned} \tag{A.6}$$

Because $N^i(\underline{t}) \geq 0$ as firm i must earn a nonnegative profit in equilibrium, it follows from (A.5)–(A.6) and the definition of V^i that $V^i(\underline{t}) < \lim_{t \uparrow \underline{t}} V^i(t)$, a contradiction as this implies that any firm $i \in \mathcal{I}$ has a profitable deviation. The claim follows. □

To complete the proof of (i), it suffices to remark that $m = 1$ is inconsistent with equilibrium. Indeed, suppose that only firm i plans to invest at $\underline{t}$. If $N^i(\underline{t}) = 0$, then firm i would be better off slightly delaying its planned investment date, a contradiction. If $N^i(\underline{t}) > 0$, then, by similar arguments as above, any firm $j \neq i$ would be better off planning to invest slightly before $\underline{t}$, once again a contradiction. We conclude that no pure-strategy equilibrium exists when $x < 1$.

(ii) We first claim that, if $x = 1$, then $\underline{t} = \hat{t}$ in any pure-strategy equilibrium, where $\hat{t}$ is defined by (1). Because $x = 1$, all firms share at any date the same belief about the quality of the project. Hence

$$N^i(\underline{t}) = \frac{p_0}{p_0 + (1-p_0)\mathrm{e}^{-n\lambda \underline{t}}} - I$$

for all $i \in \mathcal{I}$. The same arguments as in the proof of (i) show that at least two firms must plan to invest at $\underline{t}$; moreover, if $N^i(\underline{t}) > 0$, then any firm is strictly better off planning to invest slightly before $\underline{t}$, a contradiction. Thus it must be that $\underline{t} = \hat{t}$, as claimed, and that each firm earns zero profit in equilibrium. Finally, the tie-breaking rule ensures that such an equilibrium can be sustained by letting at least two firms plan to invest at $\hat{t}$, and, off path, if any date $t > \hat{t}$ is reached with no firm having invested by t, by letting all firms immediately attempt to invest at t. Hence the result. □

Proof of Lemma 1. The proof consists in showing that the fact that one firm makes a profit simultaneously implies that all other firms must earn zero profit and that they all must make a profit, which is impossible. Hence, suppose, by way of contradiction, that, say, firm 1 makes a profit in an equilibrium $(F^i)_{i=1,\ldots,n}$, and let $\underline{t}^i$ and $\overline{t}^i$ be the infimum and supremum of the support of every firm i's equilibrium distribution F^i, respectively.

Claim 1. If $x < 1$, then every firm $j \neq 1$ earns zero profit.

Proof. Suppose first that $F^j(\bar{t}^1) < 1$ for all $j \neq 1$, where we set $F^j(\bar{t}^1) \equiv \lim_{t\to\infty} F^j(t)$ in case $\bar{t}^i = \infty$. As mentioned in the main text, $F^1(\bar{t}^1) = 1$ implies that $V^j(t) \leq 0$ for all $t \geq \bar{t}^1$ and $j \neq 1$. Moreover, because $F^j(\bar{t}^1) < 1$ for all $j \neq 1$, every firm $j \neq 1$ must with positive probability plan to invest at dates $t > \bar{t}_1$ or to stay inactive. This implies that all firms $j \neq 1$ earn zero profit.

Suppose next that, say, $F^2(\bar{t}^1) = 1$, so that $\bar{t}^2 \leq \bar{t}^1$. We must in fact have $\bar{t}^2 = \bar{t}^1$, for, otherwise, $\limsup_{t\uparrow\bar{t}^1} V^1(t) \leq 0$, a contradiction as $\bar{t}^1$ is the supremum of the support of F^1 and firm 1 makes a profit in equilibrium. Reordering the players if necessary, a simple inductive argument shows that there exists $m \geq 2$ such that $\bar{t}^i = \bar{t}^1$ for all $i \leq m$, and $\bar{t}^i > \bar{t}^1$ for all $i > m$. If F^j is continuous at $\bar{t}^1$ for at least one $j \leq m$, $j \neq 1$, then, applying (2) to firm 1, we obtain that V^1 is upper semicontinuous at $\bar{t}^1$, so that $\limsup_{t\uparrow\bar{t}^1} V^1(t) \leq V^1(\bar{t}^1) \leq 0$, once again a contradiction. Hence every distribution F^j, $j \leq m$, $j \neq 1$, must have an atom at $\bar{t}^1$. A winner's curse argument similar to the one used in the proof of Proposition 1(i) implies that F^1 cannot also have an atom at $\bar{t}^1$, and that, in fact, $m = 2$. Thus, applying (2) to firm 2, we obtain that V^2 is upper semicontinuous at $\bar{t}^1$, so that $\limsup_{t\uparrow\bar{t}^1} V^2(t) \leq V^2(\bar{t}^1) \leq 0$, which implies, as $\bar{t}^1$ is the supremum of the support of F^2, that firm 2 earns zero profit. Finally, that all firms $j > 2$ earn zero profit is immediate. The claim follows. □

Claim 2. Every firm $j \geq 2$ makes a profit of zero.

Proof. Let $l^i(t)$ be firm i's likelihood ratio that the project is of high rather than of low quality given that firm i has observed nothing by date t, that is

$$l^i(t) = \frac{p_0 \prod_{j\neq i}[1 - F^j(t)]}{(1-p_0)\mathrm{e}^{-\lambda t} \prod_{j\neq i}\left\{(1-x)\int_0^t \lambda \mathrm{e}^{-\lambda s}[1 - F^j(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda t}[1 - F^j(t)]\right\}}. \tag{A.7}$$

Let us compare $l^1(\underline{t}^1-)$ and $l^i(\underline{t}^1-)$ for $i \neq 1$. We have $F^1(\underline{t}^1-) = 0$ and hence

$$\frac{l^i(\underline{t}^1-)}{l^1(\underline{t}^1-)} = \frac{(1-x)\int_0^{\underline{t}^1} \lambda \mathrm{e}^{-\lambda s}[1 - F^i(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda \underline{t}^1}[1 - F^i(\underline{t}^1-)]}{(1-x)\int_0^{\underline{t}^1} \lambda \mathrm{e}^{-\lambda s}[1 - F^i(\underline{t}^1-)]\,\mathrm{d}s + \mathrm{e}^{-\lambda \underline{t}^1}[1 - F^i(\underline{t}^1-)]} \geq 1. \tag{A.8}$$

Because V^1 is right-continuous at $\underline{t}^1$, the infimum of the support of F^1, and because firm 1 makes a profit in equilibrium, it must be that $V^1(\underline{t}^1) > 0$. As every firm $i \neq 1$ is, by (A.8), weakly more optimistic than firm 1 about the quality of the project at $\underline{t}^1-$, it can guarantee itself a strictly positive profit by planning to invest at $\underline{t}^1$ if $F^1(\underline{t}^1) = 0$, or slightly before $\underline{t}^1$ if $F^1(\underline{t}^1) > 0$. The claim follows. □

If $x < 1$, the contradiction between Claims 1 and 2 implies that all firms must earn zero profit in equilibrium. If $x = 1$, a standard Bertrand argument similar to the one used to prove Proposition 1 implies that all firms cannot make a profit in equilibrium, as predicted by Claim 2 as soon as one firm makes a profit. The result follows. $\square$

Proof of Proposition 2. The proof consists of a series of claims.

Claim 1. In equilibrium, there cannot be an interval $[a, b]$, $0 \leq a < b$, such that $V^i(a) = 0$ for some i and a is in the support of F^i, and such that $F^j(b) = F^j(a) < 1$ for all $j \neq i$.

Proof. Suppose, by way of contradiction, that such an interval exists. Then, by (2), V^i is differentiable over (a, b) and, for each $t \in (a, b)$,

$$\begin{aligned}\dot{V}^i(t) = &- rV^i(t)\\ &+ \lambda(1-p_0)I\mathrm{e}^{-(r+\lambda)t}\prod_{j\neq i}\left\{(1-x)\int_0^t \lambda\mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda t}[1-F^j(a)]\right\}\\ &- (1-p_0)I\mathrm{e}^{-(r+\lambda)t}\frac{\mathrm{d}}{\mathrm{d}t}\prod_{j\neq i}\left\{(1-x)\int_0^t \lambda\mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda t}[1-F^j(a)]\right\}\end{aligned}$$

Now, $\frac{\mathrm{d}}{\mathrm{d}t}\left\{(1-x)\int_0^t \lambda\mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda t}[1-F^j(a)]\right\} = -x\lambda\mathrm{e}^{-\lambda t}[1-F^j(a)] \leq 0$ for all $j \neq i$ and $t \in (a, b)$. From $V^i(a) = 0$, we then derive that $\dot{V}^i(a+) > 0$. That it, i would make a positive profit by planning to invest slightly after a, which we know is impossible by Lemma 1. The claim follows. $\square$

Claim 2. In equilibrium, no F^i can have an atom at a finite date. In particular, $V^i(t) = 0$ for all i and t in the support of F^i.

Proof. Let us suppose, by way of contradiction, that some F^k has an atom at some finite date t, and let $\delta \equiv F^k(t) - \lim_{s\uparrow t} F^k(s) > 0$. For the sake of simplicity, and with no loss of generality, we assume that firm k is the only firm whose equilibrium distribution has an atom at t.[23] The undiscounted expected profit of a firm $i \neq k$ from planning to invest at t is

$$\begin{aligned}&p_0(1-I)\left[1-F^k(t)+\frac{\alpha^i}{\alpha^i+\alpha^k}\,\delta\right]\prod_{j\neq i,k}[1-F^j(t)]\\ &\quad-(1-p_0)I\mathrm{e}^{-\lambda t}\left\{(1-x)\int_0^t \lambda\mathrm{e}^{-\lambda s}[1-F^k(s)]\,\mathrm{d}s+\mathrm{e}^{-\lambda t}\left[1-F^k(t)+\frac{\alpha^i}{\alpha^i+\alpha^k}\,\delta\right]\right\}\\ &\quad\quad\prod_{j\neq i,k}\left\{(1-x)\int_0^t \lambda\mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s+\mathrm{e}^{-\lambda t}[1-F^j(t)]\right\}. \qquad \text{(A.9)}\end{aligned}$$

In turn, computing firm i's undiscounted expected profit from planning to invest at a date

[23] It is straightforward to see that nothing in the following argument hinges on this restriction. The case of multiple atoms at t can be ruled out along the lines of the proof of Proposition 1(i).

$t-\varepsilon<t$ that is not an atom of the distributions F^j, $j \neq i$, and taking the limit as $\varepsilon>0$ goes to 0 yields

$$\begin{aligned} & p_0(1-I)[1-F^k(t)+\delta] \prod_{j \neq i,k} [1-F^j(t)] \\ & \quad - (1-p_0) I \mathrm{e}^{-\lambda t} \left\{ (1-x) \int_0^t \lambda \mathrm{e}^{-\lambda s} [1-F^k(s)] \, \mathrm{d}s + \mathrm{e}^{-\lambda t} [1-F^k(t)+\delta] \right\} \\ & \quad \prod_{j \neq i,k} \left\{ (1-x) \int_0^t \lambda \mathrm{e}^{-\lambda s} [1-F^j(s)] \, \mathrm{d}s + \mathrm{e}^{-\lambda t} [1-F^j(t)] \right\}. \end{aligned} \tag{A.10}$$

We now show that (A.9) is less than (A.10), so that there is a downward discontinuity in firm i's expected profit at t. Because $\delta>0$, it is easily seen that the difference between (A.9) and (A.10) has the same sign as

$$(1-p_0) I \mathrm{e}^{-2\lambda t} \prod_{j \neq i,k} \left[(1-x) \int_0^t \lambda \mathrm{e}^{-\lambda s} \frac{1-F^j(s)}{1-F^j(t)} \, \mathrm{d}s + \mathrm{e}^{-\lambda t} \right] - p_0(1-I), \tag{A.11}$$

where we have taken advantage of the fact that $F^j(t)<1$ for all $j \neq k$, for, otherwise, by (2) and $x<1$, firm k would make a loss upon investing at t. Because F^k has an atom at t, it must be that $V^k(t)=0$ by Lemma 1, that is,

$$p_0(1-I) = (1-p_0) I \mathrm{e}^{-\lambda t} \prod_{j \neq k} \left[(1-x) \int_0^t \lambda \mathrm{e}^{-\lambda s} \frac{1-F^j(s)}{1-F^j(t)} \, \mathrm{d}s + \mathrm{e}^{-\lambda t} \right],$$

from which it follows as $x<1$ that (A.11) is negative. Given the interpretation of (A.11), we deduce that for each $j \neq k$, F^j is constant and less than 1 over a right-neighborhood of t; moreover, $V^k(t)=0$ and t clearly belongs to the support of F^k. However, by Claim 1, we know that these three properties cannot be simultaneously satisfied, a contradiction. To conclude the proof, observe that the continuity of all the distributions F^i implies that all the functions V^i are continuous. Because firms earn zero profit in equilibrium by Lemma 1, this implies that, for each i, $V^i=0$ over the support of F^i. The claim follows. □

Claim 3. In equilibrium, $\bar{t}^i=\infty$ for all i.

Proof. Suppose, by way of contradiction, that $\bar{t}^i<\infty$ for some i. By Claim 2, $V^i(\bar{t}^i)=0$ and thus $F^j(\bar{t}^i)<1$ for all $j \neq i$ by (2). By (2) again, $V^j(t)<0$ for all $j \neq i$ and $t>\bar{t}^i$. This implies that no firm $j \neq i$ plans to invest after $\bar{t}^i$. By Claim 1, this is impossible, as firm i would then strictly gain by planning to invest slightly after $\bar{t}^i$. The claim follows. □

Claim 4. In equilibrium, $\min_{i=1,\ldots,n} \underline{t}^i = \underline{t}$, where $\underline{t}$ is defined by (3).

Proof. By construction, $\underline{t}$ is the first date t at which the project has zero NPV, conditional on no firm investing before t. No firm would ever be willing to invest before $\underline{t}$, as it would thereby make a loss. Conversely, if $\min_{i=1,\dots,n} \underline{t}^i > \underline{t}$, at least one firm would make a profit, which we know is impossible by Lemma 1. The claim follows. $\square$

Claim 5. In equilibrium, $\underline{t}^i = \underline{t}$ or $\underline{t}^i = \infty$ for all i. Moreover, there are at least two firms i such that $\underline{t}^i = \underline{t}$.

Proof. Suppose, by way of contradiction, that, say, $\underline{t} = \underline{t}^i < \underline{t}^1 < \infty$. (We know from Claim 4 that there must be at least one such firm i.) From (A.8), and using the fact that $F^i(\underline{t}^1) > F^i(\underline{t})$, we obtain $l^i(\underline{t}^1) > l^1(\underline{t}^1)$. Thus, as the project has zero NPV at date $\underline{t}_1$ from the perspective of firm 1, it must have positive NPV at date $\underline{t}_1$ from the perspective of firm i. This contradicts that firm i must make zero profit in equilibrium. Thus every firm i either plans to start investing at $\underline{t}$ ($\underline{t}^i = \underline{t}$) or stays inactive ($\underline{t}^i = \infty$). That at least two firms plan to start investing at $\underline{t}$ is an immediate consequence of Claim 1. The claim follows $\square$

Claim 6. In any situation where exactly $m \geq 2$ firms, say, firms $1, \dots, m$ are active, and play according to continuous distributions F^i whose supports are included in $[\underline{t}, \infty)$, with $\underline{t}^i = \underline{t}$ and $\bar{t}^i = \infty$, if $V^i(t) \leq 0$ for some $i \leq m$ and $t > \underline{t}$, then $V^k(t) < 0$ for all $k > m$.

Proof. The expected profit of a firm $i \leq m$ from planning to invest at $t > \underline{t}$ is

$$V^i(t) = \mathrm{e}^{-rt} \prod_{j \neq i, j \leq m} [1 - F^j(t)] \Bigg\{ p_0(1 - I) \tag{A.12}$$
$$- (1 - p_0) I \mathrm{e}^{-\lambda t} \prod_{j \neq i, j \leq m} \left[(1 - x) \int_0^t \lambda \mathrm{e}^{-\lambda s} \frac{1 - F^j(s)}{1 - F^j(t)} \, \mathrm{d}s + \mathrm{e}^{-\lambda t} \right] (1 - x + x\mathrm{e}^{-\lambda t})^{n-m} \Bigg\},$$

whereas the expected profit of a firm $k > m$ from planning to invest at $t > \underline{t}$ is

$$V^k(t) = \mathrm{e}^{-rt} \prod_{j \leq m} [1 - F^j(t)] \Bigg\{ p_0(1 - I) \tag{A.13}$$
$$- (1 - p_0) I \mathrm{e}^{-\lambda t} \prod_{j \leq m} \left[(1 - x) \int_0^t \lambda \mathrm{e}^{-\lambda s} \frac{1 - F^j(s)}{1 - F^j(t)} \, \mathrm{d}s + \mathrm{e}^{-\lambda t} \right] (1 - x + x\mathrm{e}^{-\lambda t})^{n-m-1} \Bigg\},$$

where we have used the fact that, by Claim 3, $F^j < 1$ over $\mathbb{R}_+$ for all j. Now, for each $t > \underline{t}$,

$$\begin{aligned} &\frac{\prod_{j \neq i, j \leq m} \left[(1 - x) \int_0^t \lambda \mathrm{e}^{-\lambda s} \frac{1 - F^j(s)}{1 - F^j(t)} \, \mathrm{d}s + \mathrm{e}^{-\lambda t} \right] (1 - x + x\mathrm{e}^{-\lambda t})^{n-m}}{\prod_{j \leq m} \left[(1 - x) \int_0^t \lambda \mathrm{e}^{-\lambda s} \frac{1 - F^j(s)}{1 - F^j(t)} \, \mathrm{d}s + \mathrm{e}^{-\lambda t} \right] (1 - x + x\mathrm{e}^{-\lambda t})^{n-m-1}} \\ &= \frac{1 - x + x\mathrm{e}^{-\lambda t}}{(1 - x) \int_0^t \lambda \mathrm{e}^{-\lambda s} \frac{1 - F^i(s)}{1 - F^i(t)} \, \mathrm{d}s + \mathrm{e}^{-\lambda t}} \\ &< 1, \end{aligned}$$

where the strict inequality follows from $x < 1$ along with the fact that, as $t > \underline{t}$ and F^i

is continuous, with $\underline{t}^i = \underline{t}$, $F^i(s) < F^i(t)$ for all s in a right-neighborhood of $\underline{t}$. From (A.12)–(A.13), this implies that $V^k(t) < V^i(t) \leq 0$ for all $t > \underline{t}$. The claim follows. □

Claim 7. In equilibrium, if exactly $m \geq 2$ firms, say, firms $1, \ldots, m$ are active, then their equilibrium distributions have the same support $[\underline{t}, \infty)$.

Proof. Suppose, by way of contradiction, that, say, firm 1's equilibrium distribution has not full support $[\underline{t}, \infty)$. By Claims 2–3, there exists a maximal interval $[a, b] \subset (\underline{t}, \infty)$ such that a and b belong to the support of F^1 and $F^1(a) = F^1(b)$. Moreover, by Claim 1, there exists a firm $i \leq m$ such that $F^i(a) < F^i(b)$ and a belongs to the support of F^i. By Claim 3, we can rewrite the likelihood ratio (A.7) as

$$l^i(t) = \frac{p_0}{(1-p_0)\mathrm{e}^{-\lambda t}\prod_{j\neq i}\left\{(1-x)\int_0^t \lambda \mathrm{e}^{-\lambda s}\frac{1-F^j(s)}{1-F^j(t)}\,\mathrm{d}s + \mathrm{e}^{-\lambda t}\right\}},$$

and similarly for firm 1. Because a belongs to the supports of F^1 and F^i, we must have $l^i(a) = l^1(a) = \frac{I}{1-I}$, which implies, thanks to the above observation,

$$\int_0^a \lambda \mathrm{e}^{-\lambda s}\frac{1-F^i(s)}{1-F^i(a)}\,\mathrm{d}s = \int_0^a \lambda \mathrm{e}^{-\lambda s}\frac{1-F^1(s)}{1-F^1(a)}\,\mathrm{d}s. \tag{A.14}$$

In turn,

$$\begin{aligned}\frac{l^i(b)}{l^1(b)} &= \frac{(1-x)\int_0^b \lambda \mathrm{e}^{-\lambda s}\frac{1-F^i(s)}{1-F^i(b)}\,\mathrm{d}s + \mathrm{e}^{-\lambda b}}{(1-x)\int_0^b \lambda \mathrm{e}^{-\lambda s}\frac{1-F^1(s)}{1-F^1(b)}\,\mathrm{d}s + \mathrm{e}^{-\lambda b}}\\ &= \frac{(1-x)\left[\int_0^a \lambda \mathrm{e}^{-\lambda s}\frac{1-F^i(s)}{1-F^i(b)}\,\mathrm{d}s + \int_a^b \lambda \mathrm{e}^{-\lambda s}\frac{1-F^i(s)}{1-F^i(b)}\,\mathrm{d}s\right] + \mathrm{e}^{-\lambda b}}{(1-x)\left[\int_0^a \lambda \mathrm{e}^{-\lambda s}\frac{1-F^i(s)}{1-F^i(a)}\,\mathrm{d}s + \int_a^b \lambda \mathrm{e}^{-\lambda s}\,\mathrm{d}s\right] + \mathrm{e}^{-\lambda b}}\\ &> 1,\end{aligned}$$

where the second equality follows from (A.14) and $F^1(a) = F^1(b)$, and the inequality follows from $F^i(a) < F^i(b)$. Hence $l^i(b) > l^1(b)$. Because $l^i(b) \leq \frac{I}{1-I}$ as firm i earns zero profit, this implies that $l^1(b) < \frac{I}{1-I}$, which is impossible by Claim 2 as b belongs to the support of F^1. The claim follows. □

Claim 8. In equilibrium, if exactly $m \geq 2$ firms, say, firms $1, \ldots, m$ are active, then their equilibrium distributions F^i, $i = 1, \ldots, m$, coincide and are continuously differentiable.

Proof. By Claims 2 and 7, for each $i \leq m$, $V^i = 0$ over $[\underline{t}, \infty)$. Hence, by (A.12),

$$\prod_{j\neq i, j\leq m}\left[(1-x)\int_0^t \lambda \mathrm{e}^{-\lambda s}\frac{1-F^j(s)}{1-F^j(t)}\,\mathrm{d}s + \mathrm{e}^{-\lambda t}\right] = \frac{p_0(1-I)}{(1-p_0)I\mathrm{e}^{-\lambda t}(1-x+x\mathrm{e}^{-\lambda t})^{n-m}}$$

for all $i \leq m$ and $t \geq \underline{t}$. Taking this equation for two different players $i_1, i_2 \leq m$ and dividing, we derive, as $x < 1$, that

$$\int_0^t \lambda e^{-\lambda s} \frac{1 - F^{i_1}(s)}{1 - F^{i_1}(t)} \, ds = \int_0^t \lambda e^{-\lambda s} \frac{1 - F^{i_2}(s)}{1 - F^{i_2}(t)} \, ds$$

for all $t \geq \underline{t}$. As a result,

$$e^{-\lambda t}(1 - x + x e^{-\lambda t})^{n-m} \left[(1 - x) \int_0^t \lambda e^{-\lambda s} \frac{1 - F^i(s)}{1 - F^i(t)} \, ds + e^{-\lambda t} \right]^{m-1} = \frac{p_0}{1 - p_0} \frac{1 - I}{I} \quad \text{(A.15)}$$

for all $i \leq m$ and $t \geq \underline{t}$, from which it readily follows that each F^i is continuously differentiable. Differentiating (A.15) leads to

$$(1 - x) \int_0^t \lambda e^{-\lambda s} \frac{1 - F^i(s)}{1 - F^i(t)} \, ds \left[(m - 1) \frac{\dot{F}^i(t)}{1 - F^i(t)} - \lambda - \frac{\lambda(n - m) x e^{-\lambda t}}{1 - x + x e^{-\lambda t}} \right]$$
$$= \lambda e^{-\lambda t} \left[1 + (m - 1)x + \frac{(n - m) x e^{-\lambda t}}{1 - x + x e^{-\lambda t}} \right] \quad \text{(A.16)}$$

Letting $y_0 \equiv \frac{p_0}{1-p_0} \frac{1-I}{I}$ and using (A.15) to replace $(1 - x) \int_0^t \lambda e^{-\lambda s} \frac{1-F^i(s)}{1-F^i(t)} \, ds$ in (A.16) yields

$$(m - 1) \frac{\dot{F}^i(t)}{1 - F^i(t)} = \lambda + [1 + (m - 1)x] \frac{\lambda e^{-\frac{m}{m-1}\lambda t}(1 - x + x e^{-\lambda t})^{\frac{n-m}{m-1}}}{y_0^{\frac{1}{m-1}} - e^{-\frac{m}{m-1}\lambda t}(1 - x + x e^{-\lambda t})^{\frac{n-m}{m-1}}}$$
$$+ \frac{\lambda(n - m) x e^{-\lambda t}}{1 - x + x e^{-\lambda t}} \left[1 + \frac{e^{-\frac{m}{m-1}\lambda t}(1 - x + x e^{-\lambda t})^{\frac{n-m}{m-1}}}{y_0^{\frac{1}{m-1}} - e^{-\frac{m}{m-1}\lambda t}(1 - x + x e^{-\lambda t})^{\frac{n-m}{m-1}}} \right] \quad \text{(A.17)}$$

for all $i \leq m$ and $t \geq \underline{t}$. From the ordinary differential equation (A.17) and the initial condition $F^i(\underline{t}) = 0$, we conclude from a straightforward integration that all F^i, $i = 1, \ldots, m$, coincide with a function F_m.[24] It is clear from (A.17) that $\dot{F}_m > 0$ over $[\underline{t}, \infty)$, and from (A.15) that $\lim_{t \to \infty} F_m(t) = 1$. The claim follows. □

Claim 9. An equilibrium exists.

Proof. It follows readily from Claims 6 and 8 that, for each $m \geq 2$, there exists a unique equilibrium in which m firms are active and play according to the distribution F_m, whereas the remaining $n - m$ firms stay inactive. The claim follows. □

When n firms are active, that is, $m = n$, (A.17) simplifies to

$$(n - 1) \frac{\dot{F}_n(t)}{1 - F_n(t)} = \lambda + [1 + (n - 1)x] \frac{\lambda e^{-\frac{n}{n-1}\lambda t}}{y_0^{\frac{1}{n-1}} - e^{-\frac{n}{n-1}\lambda t}}. \quad \text{(A.18)}$$

Remarking that

$$\int_{\underline{t}}^t \frac{\lambda e^{-\frac{n}{n-1}\lambda s}}{y_0^{\frac{1}{n-1}} - e^{-\frac{n}{n-1}\lambda s}} \, ds = \frac{n - 1}{n} \ln \left(\frac{y_0^{\frac{1}{n-1}} - e^{-\frac{n}{n-1}\lambda t}}{y_0^{\frac{1}{n-1}} - e^{-\frac{n}{n-1}\lambda \underline{t}}} \right) \quad \text{(A.19)}$$

[24] We only emphasize the dependence on m, but it should be clear that F_m also depends on n and x.

and taking advantage of $F_n(\underline{t}) = 0$, we obtain

$$F_n(t) = 1 - \mathrm{e}^{-\frac{1}{n-1}\lambda(t-\underline{t})} \left(\frac{y_0^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda \underline{t}}}{y_0^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda t}} \right)^{\frac{1+(n-1)x}{n}}, \quad t \geq \underline{t}. \tag{A.20}$$

To conclude, we infer from (A.18) along with the fact that $\underline{t}$ is decreasing in x that, if $x < x' < 1$, the investment date under x' is smaller than the investment date under x in the hazard-rate order. Hence the result. □

Proof of Proposition 4. The expected profit of firm i from planning to invest at a date t that is not an atom of F^j is

$$\begin{aligned} V^i(t) = {} & p_0 L + (1-p_0)x^i D + p_0(1-I-L)[1-F^j(t)] \\ & - (1-p_0)I\mathrm{e}^{-\lambda t}\left\{(1-x^j)\int_0^t \lambda \mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda t}[1-F^j(t)]\right\}. \end{aligned} \tag{A.21}$$

If $x^1 = x^2 \equiv x$, we remark that, up to the constant $p_0 L + (1-p_0)xD$, the firms' profits have the same structure as in (2), except that the reward in case of success is $1 - I - L$ instead of $1 - I$. Therefore, as long as $L < 1 - I$, exactly the same analysis as in Propositions 1–2 applies. In particular, because there are only two firms, there exists a unique equilibrium, which is symmetric. If $x = 1$, then this equilibrium is in pure strategies, and given by (**??**). If $x < 1$, then this equilibrium is in mixed strategies. In any case, each firm makes a profit $p_0 L + (1-p_0)xD$ in equilibrium.

With no loss of generality, let us then hereafter focus on the case $0 \leq x^1 < x^2 \leq 1$, and let F^1 and F^2 be the firms' equilibrium distributions. The proof consists of a series of claims.

Claim 1. $\operatorname{supp} F^2 \cap \mathbb{R}_+ \subset \operatorname{supp} F^1 \cap \mathbb{R}_+$, with equality if $x^2 < 1$.

Proof. Suppose, by way of contradiction, that t belongs to $(\operatorname{supp} F^2 \cap \mathbb{R}_+) \setminus (\operatorname{supp} F^1 \cap \mathbb{R}_+)$. Then there exists an open interval centered around t that does not intersect the support of F^1. From $x^1 < 1$ and (A.21) for $i = 2$ and $j = 1$, it is easy to check that V^2 must be increasing over this interval, which contradicts that t is in the support of F^2. By the same argument, the reverse equality holds if $x^2 < 1$. The claim follows. □

Claim 2. Neither F^1 nor F^2 can have an atom at a finite date.

Proof. Let us suppose, by way of contradiction, that some F^j has an atom at some finite date t, and let $\delta \equiv F^j(t) - \lim_{s\uparrow t} F^j(s) > 0$. The expected profit of firm $i \neq j$ from planning to invest at t is

$$p_0(1-I-L)\left[1 - F^j(t) + \frac{\alpha^i}{\alpha^i + \alpha^j}\,\delta\right] + p_0 L + (1-p_0)x^i D \tag{A.22}$$

$$- (1-p_0)I\mathrm{e}^{-\lambda t}\left\{(1-x^j)\int_0^t \lambda \mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s + \mathrm{e}^{-\lambda t}\left[1-F^j(t)+\frac{\alpha^i}{\alpha^i+\alpha^j}\,\delta\right]\right\}.$$

In turn, computing firm i's expected profit from planning to invest at a date $t-\varepsilon<t$ that is not an atom of the distribution F^j and taking the limit as $\varepsilon>0$ goes to 0 yields

$$\begin{aligned} &p_0(1-I-L)[1-F^j(t)+\delta]+p_0L+(1-p_0)x^iD \\ &\quad - (1-p_0)I\mathrm{e}^{-\lambda t}\left\{(1-x^j)\int_0^t \lambda \mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s+\mathrm{e}^{-\lambda t}[1-F^j(t)+\delta]\right\}. \end{aligned} \tag{A.23}$$

Because $\delta>0$, it is easily seen that the difference between (A.22) and (A.23) has the same sign as

$$(1-p_0)I\mathrm{e}^{-2\lambda t}-p_0(1-I-L). \tag{A.24}$$

We now distinguish two cases.

Case 1. Suppose first that $j=2$. Because F^2 has an atom at t, it follows from (A.21) that $V^2(t)\geq p_0L+(1-p_0)x^2D$, that is,

$$p_0(1-I-L)\geq(1-p_0)I\mathrm{e}^{-\lambda t}\left\{(1-x^1)\int_0^t \lambda\mathrm{e}^{-\lambda s}\,\frac{1-F^1(s)}{1-F^1(t)}\,\mathrm{d}s+\mathrm{e}^{-\lambda t}\right\}>(1-p_0)I\mathrm{e}^{-2\lambda t},$$

where we have taken advantage of the fact that $x^1<1$ and $F^1(t)<1$, for, otherwise, by (A.21), firm 2's profit would be less than $p_0L+(1-p_0)x^2D$, a contradiction. Hence (A.24) is negative if $j=2$, so that there is a downward discontinuity in firm 1's profit at t. Thus F^1 is constant and less than 1 over a right-neighborhood of t, from which we conclude from (A.21) that firm 2 could strictly increase her profit by planning to invest slightly after t, a contradiction. Hence F^2 cannot have an atom at a finite date, and is thus continuous over $\mathbb{R}_+$.

Case 2. If $j=1$ and $x^2<1$, the same argument as in Case 1 applies and F^1 cannot have an atom at a finite date. Thus suppose that $x^2=1$. As t is an atom of F^1, firm 1's equilibrium profit is

$$V^1(t)=p_0L+(1-p_0)x^1D+[p_0(1-I-L)-(1-p_0)I\mathrm{e}^{-2\lambda t}][1-F^2(t)]. \tag{A.25}$$

If $F^2(t)<1$, we conclude that $p_0(1-I-L)\geq(1-p_0)I\mathrm{e}^{-2\lambda t}$. If this inequality is strict, that is, if (A.24) is strictly negative, then, switching the roles of firms 1 and 2, the same argument as in Case 1 leads to a contradiction. Thus, if $F^2(t)<1$, it must be that $p_0(1-I-L)=(1-p_0)I\mathrm{e}^{-2\lambda t}$. But then, by (A.22) and $x^1<1$, the expected profit of firm 2 from planning to invest at t must be less than $p_0L+(1-p_0)x^2D$. As firm 2 can always guarantee this profit by (A.21), F^2 is constant and less than 1 over a right-neighborhood of t, from which we conclude from (A.21) that firm 1 could strictly increase her profit by planning to invest slightly after t, a contradiction. Hence the only remaining case to consider is $F^2(t) = 1$, which implies from (A.25) that firm 1's equilibrium profit is $p_0 L + (1 - p_0)x^1 D$. As F^2 is continuous over $\mathbb{R}_+$, it follows from (A.25) that

$$p_0(1 - I - L) = (1 - p_0)I\mathrm{e}^{-2\lambda s} \tag{A.26}$$

for all $s \in \operatorname{supp} F^1 \cap \mathbb{R}_+ \cap (F^2)^{-1}([0, 1))$. This set is thus empty, or a singleton $\{s\}$. In the first case, $F^1_{|[0,t]} = \gamma^1 \mathbb{1}_{\{t\}}$ for some $\gamma^1 \in (0, 1]$, a contradiction as F^2 is continuous over $[0, t]$ and $F^2(t) = 1$, and $\operatorname{supp} F^2 \cap \mathbb{R}_+ \subset \operatorname{supp} F^1 \cap \mathbb{R}_+$ by Claim 1. The second case leads to a contradiction along the same lines as above. The claim follows. □

Claim 3. The following holds:

(i) $\underline{t}^1 \equiv \inf \operatorname{supp} F^1 = \underline{t}^2 \equiv \inf \operatorname{supp} F^2 \equiv \underline{t} < \infty$.

(ii) $\overline{t}^1 \equiv \sup \operatorname{supp} F^1 \cap \mathbb{R}_+ = \overline{t}^2 \equiv \sup \operatorname{supp} F^2 \cap \mathbb{R}_+ = \infty$.

Proof. (i) By Claim 1, $\underline{t}^1 \leq \underline{t}^2$. Suppose, by way of contradiction, that $\underline{t}^1 < \underline{t}^2$. Then, by (A.21) for $i = 1$ and $j = 2$, V^1 is strictly increasing in a right-neighborhood of $\underline{t}^1$, a contradiction as $\underline{t}^1 \equiv \inf \operatorname{supp} F^1 < \infty$. Thus $\underline{t}^1 = \underline{t}^2 \equiv \underline{t}$. To conclude, we must rule out the case $\underline{t} = \infty$, that is, $F^1 = F^2 = \mathbb{1}_\infty$. In that case, every firm i would earn a profit $(1 - p_0)x^i D$, which is less, by (A.21), than the profit she would secure by planning to invest at a large finite date t, a contradiction.

(ii) By Claim 1, $\overline{t}^1 \geq \overline{t}^2$, so that we only need to show that $\overline{t}^2 = \infty$. Suppose, by way of contradiction, that $\overline{t}^2 < \infty$. We distinguish two cases.

Case 1. Suppose first that $x^2 < 1$. Then, by (A.21) for $i = 1$ and $j = 2$, V^1 is strictly increasing over $[\overline{t}^2, \infty)$, a contradiction because $\overline{t}^2 \in \operatorname{supp} F^1 \cap \mathbb{R}_+$ by Claim 1 and V^1 is continuous over $\mathbb{R}_+$ by Claim 2.

Case 2. Suppose next that $x^2 = 1$. As $\overline{t}^2 \in \operatorname{supp} F^1 \cap \mathbb{R}_+$ by Claim 1 and V^1 is continuous over $\mathbb{R}_+$ by Claim 2, firm 1's equilibrium profit is equal to $V^1(\overline{t}^2) = p_0 L + (1 - p_0)x^1 D$. As in Case 2 of the proof of Claim 2, we deduce that (A.26) holds for all $s \in \operatorname{supp} F^1 \cap [0, \overline{t}^2)$. Because this set is not empty as $\operatorname{supp} F^2 \cap \mathbb{R}_+ \subset \operatorname{supp} F^1 \cap \mathbb{R}_+$ by Claim 1 and both F^1 and F^2 are continuous over $[0, \overline{t}^2]$ by Claim 2, it is a singleton $\{s\}$. But then F^1 has an atom at s, a contradiction by Claim 2. The claim follows. □

Claim 4. Each firm i's equilibrium profit is

$$V^{i*}(x^1, x^2) \equiv \lim_{t \to \infty} V^i(t) = p_0 L + (1 - p_0)x^i D + p_0 \beta^j (1 - I - L), \tag{A.27}$$

where $\beta^j \equiv 1 - \lim_{t\to\infty} F^j(t)$ is positive if F^j has an atom at ∞, that is, firm j stays inactive with positive probability.

Proof. Immediate from (A.21) and Claim 3. □

Claim 5. For each j, $\beta^j > 0$ implies $\beta^i = 0$.

Proof. Suppose $\beta^j > 0$. If firm i stays inactive with positive probability, that is, if $\beta^i > 0$, firm i earns a profit of $p_0 L(1-\beta^j) + (1-p_0)x^i D$, which is less than its equilibrium profit of $p_0 L + (1-p_0)x^i D + p_0\beta^j(1-I-L)$ by Claim 4, a contradiction. The claim follows. □

Claim 6. The equilibrium profits are given by (8)–(9).

Proof. For $t \leq \underline{t}$, $F^1(t) = F^2(t) = 0$, and thus, by (A.21),

$$V^i(t) = p_0(1-I) + (1-p_0)x^i D - (1-p_0)I\mathrm{e}^{-\lambda t}(1 - x^j + x^j \mathrm{e}^{-\lambda t}).$$

Because F^1 and F^2 are continuous over $[\underline{t}, \infty)$, where $\inf \operatorname{supp} F^1 = \inf \operatorname{supp} F^2 \equiv \underline{t}$ by Claim 3, it follows that

$$V^{i*}(x^1, x^2) = p_0(1-I) + (1-p_0)x^i D - (1-p_0)I\mathrm{e}^{-\lambda \underline{t}}(1 - x^j + x^j \mathrm{e}^{-\lambda \underline{t}}). \tag{A.28}$$

Equating (A.27) to (A.28) yields, for each j,

$$p_0(1-\beta^j)(1-I-L) = (1-p_0)I\mathrm{e}^{-\lambda \underline{t}}(1 - x^j + x^j \mathrm{e}^{-\lambda \underline{t}}), \tag{A.29}$$

from which it follows that $\beta^2 > \beta^1$ as $x^2 > x^1$, which implies in turn by Claim 5 that $\beta^2 > 0$ and $\beta^1 = 0$. It then follows from (A.27) for $i = 2$ that $V^{2*}(x^1, x^2) = p_0 L + (1-p_0)x^2 D$, which is (9), from (7) and (A.29) for $j = 1$ that $\underline{t} = \underline{t}(x^1)$, and from (A.28) for $i = 1$ and (A.29) for $j = 1$ that $V^{1*}(x^1, x^2) = p_0 L + (1-p_0)x^1 D + (x^2 - x^1)(1-p_0)I\mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}]$, which is (8). The claim follows. □

Claim 7. An equilibrium exists.

Proof. Consider first firm 2. According to Claim 6, it is enough to find a distribution F^1 such that $F^1(\underline{t}(x^1)) = 0$ and $V^2(t) = p_0 L + (1-p_0)x^2 D$ for all $t \geq \underline{t}(x^1)$. Proceeding as in the proof of Proposition 2, and in analogy with (A.20), this yields

$$F^1(t) = 1 - \mathrm{e}^{-\lambda[t - \underline{t}(x^1)]}\left[\frac{y - \mathrm{e}^{-2\lambda \underline{t}(x^1)}}{y - \mathrm{e}^{-2\lambda t}}\right]^{\frac{1+x^1}{2}}, \quad t \geq \underline{t}(x^1). \tag{A.30}$$

Consider next firm 1. According to Claim 6, it is enough to find a distribution F^2 such that $F^2(\underline{t}(x^1)) = 0$ and $V^1(t) = p_0 L + (1-p_0)x^1 D + (1-p_0)IC$ for all $t \geq \underline{t}(x^1)$, where $C \equiv (x^2 - x^1)\mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}]$. This yields

$$\mathrm{e}^{-\lambda t}\left[(1-x^2)\int_0^t \lambda \mathrm{e}^{-\lambda s}\,\frac{1-F^2(s)}{1-F^2(t)}\,\mathrm{d}s + \mathrm{e}^{-\lambda t}\right] = y - \frac{C}{1-F^2(t)}, \tag{A.31}$$

which implies

$$(y - \mathrm{e}^{-2\lambda t})\dot{F}^2(t) = \lambda(y + x^2\mathrm{e}^{-2\lambda t})[1 - F^2(t)] - \lambda C. \tag{A.32}$$

By now standard computations, when $C = 0$, the general form of the solution to (A.32) is given, for some positive constants $a > \frac{\ln(y)}{2\lambda}$ and K_0, by

$$F_0^2(t) = 1 - K_0 \exp\left(-\lambda t - (1+x^2)\int_a^t \frac{\lambda \mathrm{e}^{-2\lambda s}}{y - \mathrm{e}^{-2\lambda s}}\,\mathrm{d}s\right) = 1 - K_0\mathrm{e}^{-\lambda t}\left(\frac{y - \mathrm{e}^{-2\lambda a}}{y - \mathrm{e}^{-2\lambda t}}\right)^{\frac{1+x^2}{2}}.$$

Thus, using the Lagrange method, let us try a solution to (A.32) of the form

$$F^2(t) = 1 - K_C(t)\exp\left(-\lambda[t - \underline{t}(x^1)] - (1+x^2)\int_{\underline{t}(x^1)}^t \frac{\lambda\mathrm{e}^{-2\lambda s}}{y - \mathrm{e}^{-2\lambda s}}\,\mathrm{d}s\right). \tag{A.33}$$

Differentiating (A.33), identifying terms with (A.32), and imposing $F^2(\underline{t}(x^1)) = 0$ yields

$$K_C(t) = \lambda C \int_{\underline{t}(x^1)}^t \frac{1}{y - \mathrm{e}^{-2\lambda s}} \exp\left(\lambda[s - \underline{t}(x^1)] + (1+x^2)\int_{\underline{t}(x^1)}^s \frac{\lambda\mathrm{e}^{-2\lambda u}}{y - \mathrm{e}^{-2\lambda u}}\,\mathrm{d}u\right)\mathrm{d}s + 1.$$

and thus

$$F^2(t) = 1 - \frac{\lambda C \int_{\underline{t}(x^1)}^t \frac{1}{y-\mathrm{e}^{-2\lambda s}} \exp\left(\lambda[s - \underline{t}(x^1)] + (1+x^2)\int_{\underline{t}(x^1)}^s \frac{\lambda\mathrm{e}^{-2\lambda u}}{y-\mathrm{e}^{-2\lambda u}}\,\mathrm{d}u\right)\mathrm{d}s + 1}{\exp\left(\lambda[t - \underline{t}(x^1)] + (1+x^2)\int_{\underline{t}(x^1)}^t \frac{\lambda\mathrm{e}^{-2\lambda s}}{y-\mathrm{e}^{-2\lambda s}}\,\mathrm{d}s\right)}. \tag{A.34}$$

We need to show that $\overline{F}^2 \equiv 1 - F^2$ is decreasing and converges to β^2. From (A.34), $\dot{\overline{F}}^2(t)$ has the same sign as

$$\begin{aligned} H(t) = {} & C\exp\left(\lambda[t - \underline{t}(x^1)] + (1+x^2)\int_{\underline{t}(x^1)}^t \frac{\lambda\mathrm{e}^{-2\lambda s}}{y - \mathrm{e}^{-2\lambda s}}\,\mathrm{d}s\right) \\ & - \left[\lambda C \int_{\underline{t}(x^1)}^t \frac{1}{y - \mathrm{e}^{-2\lambda s}} \exp\left(\lambda[s - \underline{t}(x^1)] + (1+x^2)\int_{\underline{t}(x^1)}^s \frac{\lambda\mathrm{e}^{-2\lambda u}}{y - \mathrm{e}^{-2\lambda u}}\,\mathrm{d}u\right)\mathrm{d}s + 1\right] \\ & (y + x^2\mathrm{e}^{-2\lambda t}). \end{aligned} \tag{A.35}$$

Using that $C = (x^2 - x^1)\mathrm{e}^{-\lambda\underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda\underline{t}(x^1)}]$ and $y = \frac{p_0(1-I-L)}{(1-p_0)I} = \mathrm{e}^{-\lambda\underline{t}(x^1)}[1 - x^1 + x^1\mathrm{e}^{-\lambda\underline{t}(x^1)}]$ by (A.29) for $j=1$, we obtain that $H(\underline{t}(x^1)) = \mathrm{e}^{-\lambda\underline{t}(x^1)}\{x^2[1 - 2\mathrm{e}^{-\lambda\underline{t}(x^1)}] - 1\} < 0$. Moreover, from (A.35), it is easy to verify that $\dot{H}(t) > 0$ for all $t \geq \underline{t}(x^1)$. Thus, to prove that $\dot{\overline{F}}^2(t) < 0$ for all $t \geq \underline{t}(x^1)$, we only need to verify that $\lim_{t\to\infty} H(t) \leq 0$. Observe that, by (A.19),

$$\mathrm{e}^{\lambda t} - \lambda y \int_{\underline{t}(x^1)}^t \frac{1}{y - \mathrm{e}^{-2\lambda s}} \exp\left(\lambda s - (1+x^2)\int_s^t \frac{\lambda\mathrm{e}^{-2\lambda u}}{y - \mathrm{e}^{-2\lambda u}}\,\mathrm{d}u\right)\mathrm{d}s$$

$$= \mathrm{e}^{\lambda t} - y \int_{\underline{t}(x^1)}^{t} \frac{\lambda \mathrm{e}^{\lambda s}}{(y - \mathrm{e}^{-2\lambda s})^{\frac{1-x^2}{2}} (y - \mathrm{e}^{-2\lambda t})^{\frac{1+x^2}{2}}} \,\mathrm{d}s$$

$$\leq \mathrm{e}^{\lambda t} - \frac{y}{y - \mathrm{e}^{-2\lambda t}} \int_{\underline{t}(x^1)}^{t} \lambda \mathrm{e}^{\lambda s} \,\mathrm{d}s$$

$$= \frac{y \mathrm{e}^{\lambda \underline{t}(x^1)} - \mathrm{e}^{-\lambda t}}{y - \mathrm{e}^{-2\lambda t}}. \tag{A.36}$$

In light of (A.35) and (A.36), we obtain that $\lim_{t\to\infty} H(t) \leq 0$ is equivalent to

$$C \exp\left((1 + x^2) \int_{\underline{t}(x^1)}^{\infty} \frac{\lambda \mathrm{e}^{-2\lambda s}}{y - \mathrm{e}^{-2\lambda s}} \,\mathrm{d}s \right) \leq y,$$

that is, by (A.19), $C\left[\frac{y}{y-\mathrm{e}^{-2\lambda \underline{t}(x^1)}}\right]^{\frac{1+x^2}{2}} \leq y$. Because $C = (x^2 - x^1)\mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}]$, the left-hand side of this inequality is maximized for $x^2 = 1$, in which case it is easy to verify using $y = \mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - x^1 + x^1 \mathrm{e}^{-\lambda \underline{t}(x^1)}]$ that it becomes an equality. We conclude that $H < 0$ over $[\underline{t}(x^1), \infty)$, from which it follows that $\overline{F}^2$ is decreasing over this interval. It remains only to check that $\lim_{t\to\infty} \overline{F}^2(t) = \beta^2$, where $\beta^2 = \frac{(x^2-x^1)[1-\mathrm{e}^{-\lambda \underline{t}(x^1)}]}{1-x^1+x^1\mathrm{e}^{-\lambda \underline{t}(x^1)}}$ by (A.29). Applying L'Hôpital's rule to (A.34) yields $\lim_{t\to\infty} \overline{F}^2(t) = \frac{C}{y}$, which is the desired result given the expressions of C and y. The claim follows. □

The proof of Proposition 4 is now complete. Hence the result. □

Appendix B

Proof of Proposition 6. For fixed Δ, welfare as a function of x^1 is given by

$$W(x^1, x^1 + \Delta) = 2p_0 L + \Delta(1 - p_0) I \mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}].$$

Therefore, because $\underline{t}(x^1)$ is decreasing in x^1, $\frac{\mathrm{d}W}{\mathrm{d}x^1}(x^1, x^1 + \Delta)$ has the same sign as $1 - 2\mathrm{e}^{-\lambda \underline{t}(x^1)}$. Let L_0 and L_1 be such that

$$\frac{p_0(1 - I - L_0)}{(1 - p_0)I} = \frac{1}{2} \quad \text{and} \quad \frac{2\,\frac{p_0(1-I-L_1)}{(1-p_0)I}}{\Delta + \sqrt{\Delta^2 + 4(1-\Delta)\,\frac{p_0(1-I-L_1)}{(1-p_0)I}}} = \frac{1}{2}. \tag{B.1}$$

Assumption 1 ensures that $L_0 > 0$, and one can verify from (B.1) that $1 - \Delta > L_1 > L_0$. Using (7), one easily derives that

$$\mathrm{e}^{-\lambda \underline{t}(x^1)} = \frac{2y}{1 - x^1 + \sqrt{(1 - x^1)^2 + 4yx^1}}. \tag{B.2}$$

This gives $\mathrm{e}^{-\lambda \underline{t}(0)} = y$. Hence, at $L = L_0$, we have $\mathrm{e}^{-\lambda \underline{t}(0)} = \frac{1}{2}$. Likewise, $\mathrm{e}^{-\lambda \underline{t}(1-\Delta)} = \frac{2y}{\Delta+\sqrt{\Delta^2+4(1-\Delta)y}}$. Hence, at $L = L_1$, we have $\mathrm{e}^{-\lambda \underline{t}(1-\Delta)} = \frac{1}{2}$. We can thus conclude:

(i) If $L \le L_0$, then $\mathrm{e}^{-\lambda \underline{t}(0)} \ge \frac{1}{2}$, which implies, as $\underline{t}$ is decreasing, that $\mathrm{e}^{-\lambda \underline{t}(x^1)} > \frac{1}{2}$ for all $x^1 \in [0, 1-\Delta]$. Hence $W(x^1, x^1+\Delta)$ is decreasing in x^1;

(ii) If $L_0 < L < L_1$, then $\mathrm{e}^{-\lambda \underline{t}(0)} < \frac{1}{2} < \mathrm{e}^{-\lambda \underline{t}(1-\Delta)}$. Hence $W(x^1, x^1+\Delta)$ is single-peaked in x^1, with a maximum reached at x^{1*} such that $\mathrm{e}^{-\lambda \underline{t}(x^{1*})} = \frac{1}{2}$, that is, using (B.2), $x^{1*} = 2(1-2y)$;

(iii) If $L \ge L_1$, then $\mathrm{e}^{-\lambda \underline{t}(1-\Delta)} \le \frac{1}{2}$, which implies, as $\underline{t}$ is decreasing, that $\mathrm{e}^{-\lambda \underline{t}(x^1)} < \frac{1}{2}$ for all $x^1 \in [0, 1-\Delta]$. Hence $W(x^1, x^1+\Delta)$ is increasing in x^1.

Hence the result. $\square$

Proof of Proposition 7. Before proving the proposition, it is useful to first derive the best response of a firm when the other firm chooses $x = 1$. Let x_1^* denote such a best response, which is generically unique. By (8), $x_1^* \in \arg\max_{x \in [0,1]} \pi(x)$, where $\pi(x) \equiv p_0 L + (1-p_0)Dx + (1-p_0)I(1-x)\mathrm{e}^{-\lambda \underline{t}(x)}[1-\mathrm{e}^{-\lambda \underline{t}(x)}]$.

Lemma B.1 *The following holds*:

(i) If $y \le \frac{1}{4}$, then $x_1^* = 0$ if $D \le 2y^2(1-y)I$, $0 < x_1^* < 1$ if $2y^2(1-y)I < D < \sqrt{y}(1-\sqrt{y})I$, and $x_1^* = 1$ if $D \ge \sqrt{y}(1-\sqrt{y})I$;

(ii) If $\frac{1}{4} < y < \frac{1}{2}$, then $x_1^* = 0$ if $D \le 2y^2(1-y)I$, $0 < x_1^* < 2-4y$ if $2y^2(1-y)I < D < \frac{I}{4}$, and $x_1^* = 1$ if $D \ge \frac{I}{4}$;

(iii) If $y \ge \frac{1}{2}$, then $x_1^* = 0$ if $D < y(1-y)I$, $x_1^* = 1$ if $D > y(1-y)I$, and x_1^* is indifferently 0 or 1 if $D = y(1-y)I$.

Proof. One easily checks, using (7), that $\pi'(x) = (1-p_0)D - 2(1-p_0)I\,\frac{\mathrm{e}^{-2\lambda \underline{t}(x)}[1-\mathrm{e}^{-\lambda \underline{t}(x)}]}{2x\mathrm{e}^{-\lambda \underline{t}(x)}+1-x}$. The marginal benefit of increasing x is thus increasing in D, which immediately implies that x_1^* is nondecreasing in D.

Claim 1. $\pi'(x)$ is strictly quasiconvex in x.

Proof. Simple computations show that $\pi''(x)$ has the same sign as $\phi(x) \equiv 8x\mathrm{e}^{-2\lambda \underline{t}(x)} - 9x\mathrm{e}^{-\lambda \underline{t}(x)} + 5\mathrm{e}^{-\lambda \underline{t}(x)} + 3(x-1)$. We thus only need to show that ϕ is increasing. Simple computations show that $\phi'(x)$ has the same sign as $\psi(x) \equiv -x\mathrm{e}^{-2\lambda \underline{t}(x)} + 6x\mathrm{e}^{-\lambda \underline{t}(x)} + 3\mathrm{e}^{-2\lambda \underline{t}(x)} - 4\mathrm{e}^{-\lambda \underline{t}(x)} + 3(1-x) = Q(\mathrm{e}^{-\lambda \underline{t}(x)})$, where $Q(Z) \equiv (3-x)Z^2 + 2(3x-2)Z + 3(1-x)$ for $Z \in [0,1]$. If $x > \frac{2}{3}$, then Q is increasing in Z over $[0,1]$, so that, as $Q(0) = 3(1-x) \ge 0$, we have $Q(Z) > 0$ for all $Z \in (0,1]$, which is the desired result. If $x \le \frac{2}{3}$, then the discriminant $6x^2 - 5$ of Q is negative, so that Q has no real root, which again implies the desired result.

The claim follows. $\square$

Observe from (7) that $e^{-\lambda \underline{t}(0)} = y$ and $e^{-\lambda \underline{t}(1)} = \sqrt{y}$. We now distinguish three cases.

Case 1. Suppose first that $\pi''(1) \leq 0$. By Claim 1, this is equivalent to $y \leq \frac{1}{4}$, and π is strictly concave over $[0,1]$. Then $x_1^* = 0$ if $\pi'(0) \leq 0$, that is, $2y^2(1-y)I \geq D$, $x_1^* = 1$ if $\pi'(1) \geq 0$, that is, $D \geq \sqrt{y}(1-\sqrt{y})I$, and $0 < x_1^* < 1$ otherwise. This proves (i).

Case 2. Suppose next that $\pi''(0) \geq 0$. By Claim 1, this is equivalent to $y \geq \frac{3}{5}$, and π is strictly convex over $[0,1]$. Then $x_1^* = 0$ if $\pi(0) > \pi(1)$, that is, $y(1-y)I > D$, $x_1^* = 1$ if $\pi(0) < \pi(1)$, that is, $y(1-y)I < D$, and x_1^* is indifferently 0 or 1 if $\pi(0) = \pi(1)$, that is, $D = y(1-y)I$. This proves (iii) for $y \geq \frac{3}{5}$.

Case 3. Suppose finally that $\pi''(1) > 0 > \pi''(0)$. By Claim 1, this is equivalent to $\frac{1}{4} < y < \frac{3}{5}$, and π is first strictly concave and then strictly convex.

If $\pi'(0) \leq 0$, that is, $2y^2(1-y)I \geq D$, then $x = 0$ maximizes $\pi(x)$ over the interval where π is concave, which implies that x_1^* is either 0 or 1. If $\frac{1}{4} < y < \frac{1}{2}$, then we have $2y^2(1-y)I < y(1-y)I$. Hence $y(1-y)I > D$, so that $\pi(0) > \pi(1)$ and $x_1^* = 0$. This proves the first statement in (ii). If, instead, $\frac{1}{2} \leq y < \frac{3}{5}$, then we have $2y^2(1-y)I \geq y(1-y)I$, with equality at $y = \frac{1}{2}$. This implies that $x_1^* = 0$ if $D < y(1-y)I$, $x_1^* = 1$ if $y(1-y)I < D \leq 2y^2(1-y)I$, and x_1^* is indifferently 0 or 1 if $\pi(0) = \pi(1)$, that is, $y(1-y)I = D$. This proves (iii) for $\frac{1}{2} \leq y < \frac{3}{5}$ and $2y^2(1-y)I \geq D$.

If $\pi'(0) > 0$, that is, $D > 2y^2(1-y)I$, then π is increasing at 0. If $\frac{1}{2} \leq y < \frac{3}{5}$, then $2y^2(1-y)I \geq y(1-y)I$ and, as shown above, $x_1^* = 1$ if $y(1-y)I < D \leq 2y^2(1-y)I$. Because x_1^* is nondecreasing in D, then a fortiori $x_1^* = 1$ for $D > 2y^2(1-y)I$. This concludes the proof of (iii). Suppose finally that $\frac{1}{4} < y < \frac{1}{2}$. In such a case, $\overline{x} \equiv 2 - 4y \in (0,1)$, and using (B.2), one remarks that $e^{-\lambda \underline{t}(\overline{x})} = \frac{1}{2}$. In addition, when $D = \frac{I}{4}, \pi'(\overline{x}) = 0$ and $\pi''(\overline{x})$ has the same sign as $2\overline{x} - \frac{9}{2}\overline{x} + 3\overline{x} - \frac{1}{2} = -\frac{1}{2}(1-\overline{x}) < 0$, so $\overline{x}$ is the maximum of π on the part where π is concave. Finally, $\pi(\overline{x}) = p_0 L + (1-p_0)D\overline{x} + (1-p_0)\frac{I}{4}(1-\overline{x}) = p_0 L + (1-p_0)D = \pi(1)$ if $D = \frac{I}{4}$. Hence, in that case, x_1^* is indifferently $\overline{x}$ or 1. Because x_1^* is nondecreasing in D, we infer that $x_1^* = 1$ for $D > \frac{I}{4}$, and that $x_1^* \in (0, 2-4y)$ for $D \in (2y^2(1-y)I, \frac{I}{4})$. This concludes the proof of (ii). The result follows. $\square$

Now, let

$$D_1(y) \equiv \begin{cases} \sqrt{y}(1-\sqrt{y})I & \text{if } y \leq \frac{1}{4} \\ \frac{I}{4} & \text{if } \frac{1}{4} < y < \frac{1}{2} \\ y(1-y)I & \text{if } y \geq \frac{1}{2} \end{cases}.$$

By Lemma B.1, if $D \geq D_1(y)$, then, for each i, $x^i = 1$ is a best response to $x^j = 1$. Thus there exists a symmetric pure-strategy equilibrium in which both firms disclose with probability 1, and $\mathbf{E}[\tilde{x}^*] = 1$. Both firms earn a profit $p_0 L + (1-p_0)D$. This proves (iii).

Suppose next that $D < D_1(y)$. We distinguish two cases.

Case 1. Suppose first that $y \geq \frac{1}{2}$. We know from Lemma B.1 that, in that case, when $D < D_1 = y(1-y)I$, $x_i = 0$ is a best response to $x_j = 1$, that is, $\pi(0) \geq \pi(x)$ for all $x \in [0,1]$. In such a case, there exists a mixed-strategy equilibrium in which each player plays $x = 1$ with probability $\alpha \equiv \frac{\pi(1) - p_0 L}{\pi(0) - p_0 L} = \frac{D}{y(1-y)I}$ and $x = 0$ with probability $1 - \alpha$. The argument is twofold. First, taking advantage of (8)–(9) and of the fact that a firm that plays $x = 1$ obtains $\pi(1)$ no matter what the other firm does, it is easy to check that each firm is then indifferent between playing $x = 0$ and $x = 1$. Second, if a firm deviates to $x \in (0,1)$, then its profit becomes $h(x) \equiv \alpha\pi(x) + (1-\alpha)[p_0 L + (1-p_0)xD]$ by (9). Because $h'(x) = \alpha\pi'(x) + (1-\alpha)(1-p_0)D$, we have $h'(0) = \alpha\pi'(0) + (1-\alpha)(1-p_0)D = (1-p_0)D - 2\alpha(1-p_0)Iy^2(1-y) = (1-p_0)(1-2y)D \leq 0$. Moreover, because $h'' = \alpha\pi''$ and, as shown in the proof of Lemma B.1, π is either convex or concave and then convex over $[0,1]$ if $y \geq \frac{1}{2}$, so is h. Because $h'(0) \leq 0$ and $h(0) = h(1)$, this implies that there is no $x \in (0,1)$ such that $h(x) > h(0)$. Hence we have characterized a mixed-strategy equilibrium in which $\mathbf{E}[\tilde{x}^*] = \alpha = \frac{D}{y(1-y)I}$.

Case 2. Suppose now that $y < \frac{1}{2}$. We construct a symmetric mixed-strategy equilibrium in which each firm randomizes over $x \in [x_a, x_b] \cup \{1\}$, where $x_b \equiv \min\{\overline{x}, 1\}$ and x_a is yet to be determined. Recall that $\overline{x} = 2 - 4y$, which yields $x_b = 1$ for $y \leq \frac{1}{4}$ and $x_b = \overline{x} = 2 - 4y \in (0,1)$ for $\frac{1}{4} < y < \frac{1}{2}$. Notice that for each $x \leq x_b$, we have $\mathrm{e}^{-\lambda \underline{t}(x)} \leq \frac{1}{2}$. Indeed, if $y \leq \frac{1}{4}$, then $\mathrm{e}^{-\lambda \underline{t}(x)} \leq \mathrm{e}^{-\lambda \underline{t}(1)} = \sqrt{y} \leq \frac{1}{2}$ for all $x \in [0,1]$, and if $\frac{1}{4} < y < \frac{1}{2}$, then $x_b = \overline{x}$ so that $\mathrm{e}^{-\lambda \underline{t}(x)} \leq \mathrm{e}^{-\lambda \underline{t}(\overline{x})} = \frac{1}{2}$ for all $x \leq \overline{x}$.

Suppose that firm 2 plays according to a distribution G with support $[x_a, x_b] \cup \{1\}$ that is continuous over $[x_a, x_b]$. Then, in equilibrium, the profit from choosing $x^1 \in [x_a, x_b]$, that is, by (8),

$$p_0 L + (1-p_0)x^1 D + (1-p_0)I\mathbf{P}[\tilde{x}^2 \geq x^1]\mathbf{E}[\tilde{x}^2 - x^1 \,|\, \tilde{x}^2 \geq x^1]\mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}], \quad \text{(B.3)}$$

must be the same as the payment from choosing $x^1 = 1$, that is, $p_0 L + (1-p_0)D$. Hence, for each $x^1 \in [x_a, x_b]$, it must be that

$$\mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}]\left\{\int_{x^1}^{x_b} (x^2 - x^1)\,\mathrm{d}G(x^2) + [1 - G(x_b)](1 - x^1)\right\} = \frac{(1-x^1)D}{I}. \quad \text{(B.4)}$$

Differentiating (B.4) and plugging (B.4) into the resulting equation yields

$$G(x) = 1 - \frac{2D}{\mathrm{e}^{-\lambda \underline{t}(x)}[1 - \mathrm{e}^{-\lambda \underline{t}(x)}]I}\left[1 - \frac{\mathrm{e}^{-\lambda \underline{t}(x)}}{2x\mathrm{e}^{-\lambda \underline{t}(x)} + 1 - x}\right], \quad x \in [x_a, x_b]. \quad \text{(B.5)}$$

To make sure that this yields an equilibrium, we need to check that the following conditions are met: (a) G is increasing over $[x_a, x_b]$, (b) $G(x_b) \leq 1$, (c) $G(x_a) \geq 0$, (d) Deviating to $x \in (x_b, 1)$ cannot be strictly profitable (whenever relevant), (e) Deviating to $x \in [0, x_a)$ cannot be strictly profitable (whenever relevant). We prove each of these claims in turn.

(a) Because $\mathrm{e}^{-\lambda \underline{t}(x)} \leq \frac{1}{2}$ for all $x \leq x_b$ and $\underline{t}(x)$ is decreasing in x, $\mathrm{e}^{-\lambda \underline{t}(x)}[1 - \mathrm{e}^{-\lambda \underline{t}(x)}]$ is increasing in $x \leq x_b$. So if $1 - \frac{\mathrm{e}^{-\lambda \underline{t}(x)}}{2x\mathrm{e}^{-\lambda \underline{t}(x)} + 1 - x}$, which is positive, is decreasing in $x \leq x_b$, then $G(x)$ must be increasing over $[x_a, x_b]$. The derivative of $x \mapsto 1 - \frac{\mathrm{e}^{-\lambda \underline{t}(x)}}{2x\mathrm{e}^{-\lambda \underline{t}(x)} + 1 - x}$ with respect to x has the same sign as $\mathrm{e}^{-\lambda \underline{t}(x)}[2\mathrm{e}^{-\lambda \underline{t}(x)} - 1] - (1 - x)\frac{\mathrm{d}}{\mathrm{d}x}\,\mathrm{e}^{-\lambda \underline{t}(x)}$. Because $\mathrm{e}^{-\lambda \underline{t}(x)} \leq \frac{1}{2}$ for all $x \leq x_b$ and $\underline{t}(x)$ is decreasing in x, this mapping is thus decreasing. Hence G is increasing over $[0, x_b]$, as claimed.

(b) Because $1 - \frac{\mathrm{e}^{-\lambda \underline{t}(x)}}{2x\mathrm{e}^{-\lambda \underline{t}(x)} + 1 - x}$ is positive for all $x \in [0, x_b]$, it follows from (B.5) that $G(x_b) \leq 1$, as claimed.

(c) It follows from (B.5) that $G(0) \geq 0$ if and only if $D \leq \frac{yI}{2}$. In this case, we set $x_a \equiv 0$. Suppose now that $\frac{yI}{2} < D < D_1(y)$. In this case, $G(0) < 0$. It is easy to check that $G(x_b) > 0$ when $D \leq D_1(y)$. Indeed, if $y \leq \frac{1}{4}$, then $x_b = 1$ and $G(1) = 1 - \frac{D}{\sqrt{y}(1-\sqrt{y})I} = 1 - \frac{D}{D_1(y)} > 0$, and if $\frac{1}{4} < y < \frac{1}{2}$, then $x_b = \overline{x}$ and $G(\overline{x}) = 1 - \frac{4D}{I} = 1 - \frac{D}{D_1(y)} > 0$. By continuity of G, there exists a unique $x_a \in (0, x_b)$ such that $G(x_a) = 0$, that is,

$$\frac{2D}{\mathrm{e}^{-\lambda \underline{t}(x_a)}[1 - \mathrm{e}^{-\lambda \underline{t}(x_a)}]I}\left[1 - \frac{\mathrm{e}^{-\lambda \underline{t}(x_a)}}{2x_a\mathrm{e}^{-\lambda \underline{t}(x_a)} + 1 - x_a}\right] = 1 \tag{B.6}$$

(d) This is relevant only if $x_b = \overline{x} < 1$, that is, $\frac{1}{4} < y < \frac{1}{2}$. If a firm deviates to $x \in (\overline{x}, 1)$, then its profit becomes $\tilde{h}(x) \equiv [1 - G(\overline{x})]\pi(x) + G(\overline{x})[p_0 L + (1 - p_0)xD]$. Observe that $\tilde{h}'(\overline{x}) = [1 - G(\overline{x})]\pi'(\overline{x}) + G(\overline{x})(1 - p_0)D$. Using $\mathrm{e}^{-\lambda \underline{t}(\overline{x})} = \frac{1}{2}$, we infer that $G(\overline{x}) = 1 - \frac{4D}{I}$ as in (c). In addition, $\pi'(\overline{x}) = (1 - p_0)(D - \frac{I}{4})$. This yields $\tilde{h}'(\overline{x}) = 0$. Moreover, because $\tilde{h}'' = [1 - G(\overline{x})]\pi''$ and, as shown in the proof of Lemma B.1, π is concave and then convex over $[0, 1]$ if $\frac{1}{4} < y < \frac{1}{2}$, so is h. Because $\tilde{h}'(\overline{x}) \leq 0$ and $\tilde{h}(\overline{x}) = \tilde{h}(1)$, this implies that there is no $x \in (\overline{x}, 1)$ such that $\tilde{h}(x) > \tilde{h}(0)$. If, in addition, we have

$$D \leq D_0(y) \equiv \frac{yI}{2},$$

so that $x_a = 0$, we have characterized a symmetric mixed-strategy equilibrium in which, by equating (B.3) for $x^1 = 0$ to $p_0 L + (1 - p_0)D$, $\mathbf{E}[\tilde{x}^*] = \frac{D}{y(1-y)I}$.

(e) This is relevant only if $x_a > 0$, that is, $D_0(y) < D < D_1(y)$. If a firm, say, firm 1, deviates to $x^1 < x_a \leq \mathbf{E}[\tilde{x}^*]$, then its profit becomes

$$p_0 L + (1 - p_0)x^1 D + (1 - p_0)\{\mathbf{E}[\tilde{x}^*] - x^1\}I\mathrm{e}^{-\lambda \underline{t}(x^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(x^1)}] = V^{1*}(x^1, \mathbf{E}[\tilde{x}^*]).$$

Claim 2. The mapping $x \mapsto V^{1*}(x, \mathbf{E}[\tilde{x}^*])$ is strictly concave over $[0, x_a]$.

Proof. Simple computations show that $\frac{\partial^2 V^{1*}}{\partial x^2}(x, \mathbf{E}[\tilde{x}^*])$ has the same sign as the bilinear form $B_Z(x, E) \equiv E(1-5Z+5Z^2)-(1-2Z)+x[1-3Z+3Z^2-(1-2Z)^3E]$, where E stands for $\mathbf{E}[\tilde{x}^*]$ and Z for $\mathrm{e}^{-\lambda \underline{t}(x)}$. Because $\mathrm{e}^{-\lambda \underline{t}(x)} < \frac{1}{2}$ for all $x \leq x_a < x_b$, we have $0 < Z < \frac{1}{2}$. Moreover, because $x \leq x_a \leq \mathbf{E}[\tilde{x}^*]$, we have $0 \leq x \leq E \leq 1$. We now show that, for any $Z \in (0, \frac{1}{2})$, $\max_{0 \leq x \leq E \leq 1} B_Z(x, E) < 0$, which implies the result. Observe first that $\frac{1-3Z+3Z^2}{(1-2Z)^3} > 1$ for all $Z \in (0, \frac{1}{2})$, so that the term multiplying x in $B_Z(x, E)$ is positive as $E \leq 1$. Thus $B_Z(x, E)$ is maximized for $x = E$, leading to $B_Z(E, E) = -(1-2Z)^3E^2 + (2-8Z+8Z^2)E - (1-2Z) = -(1-2Z)[E(1-2Z)-1]^2 < 0$ as $Z \in (0, \frac{1}{2})$. The claim follows. □

Now, a simple computation yields

$$\begin{aligned}
\frac{\partial V^{1*}}{\partial x^1}(x_a, \mathbf{E}[\tilde{x}^*]) &\propto D - \mathrm{e}^{-\lambda \underline{t}(x_a)}[1-\mathrm{e}^{-\lambda \underline{t}(x_a)}]I \\
&\quad + \frac{\mathrm{e}^{-\lambda \underline{t}(x_a)}[1-\mathrm{e}^{-\lambda \underline{t}(x_a)}]I[1-2\mathrm{e}^{-\lambda \underline{t}(x_a)}]}{2x_a\mathrm{e}^{-\lambda \underline{t}(x_a)}+1-x_a}\{\mathbf{E}[\tilde{x}^*]-x_a\} \\
&= D - \mathrm{e}^{-\lambda \underline{t}(x_a)}[1-\mathrm{e}^{-\lambda \underline{t}(x_a)}]I + \frac{1-2\mathrm{e}^{-\lambda \underline{t}(x_a)}}{2x_a\mathrm{e}^{-\lambda \underline{t}(x_a)}+1-x_a}(1-x_a)D \\
&= D - 2D\left[1-\frac{\mathrm{e}^{-\lambda \underline{t}(x_a)}}{2x_a\mathrm{e}^{-\lambda \underline{t}(x_a)}+1-x_a}\right] + \frac{1-2\mathrm{e}^{-\lambda \underline{t}(x_a)}}{2x_a\mathrm{e}^{-\lambda \underline{t}(x_a)}+1-x_a}(1-x_a)D \\
&= 0, \qquad \text{(B.7)}
\end{aligned}$$

where the first equality follows from (B.4) at $x^1 = x_a$ and the second equality follows from (B.6). Because $V^{1*}(x^1, \mathbf{E}[\tilde{x}^*])$ is strictly concave in $x^1 \in [0, x_a]$, this implies that $V^{1*}(x^1, \mathbf{E}[\tilde{x}^*])$ is increasing in x^1 over $[0, x_a]$. This implies that no deviation to some $x \in [0, x_a)$ can be strictly profitable, and thus, together with (d), that we have characterized a mixed-strategy equilibrium when $D_0(y) < D < D_1(y)$.

Notice that $\mathbf{E}[\tilde{x}^*]$ is continuous in D. If $y \geq \frac{1}{2}$, this follows from the fact that $\mathbf{E}[\tilde{x}^*] = \frac{D}{y(1-y)I}$ if $D < D_1(y) = y(1-y)I$ and $\mathbf{E}[\tilde{x}^*] = 1$ otherwise. If $y \leq \frac{1}{2}$, that $\mathbf{E}[\tilde{x}^*]$ is continuous at $D_0(y) = \frac{yI}{2}$ follows from the fact that $\mathbf{E}[\tilde{x}^*] = \frac{D}{y(1-y)I}$ if $D < D_0(y)$ and that $x_a = 0$ if $D = D_0(y)$, which, using (B.4) at $x^1 = 0$ along with $\mathrm{e}^{-\lambda \underline{t}(0)} = y$, yields $\mathbf{E}[\tilde{x}^*] = \frac{D_0(y)}{y(1-y)I}$. If $y \leq \frac{1}{4}$, that $\mathbf{E}[\tilde{x}^*]$ is continuous at $D_1(y) = \sqrt{y}(1-\sqrt{y})I$ follows from the fact that, if $D < D_1(y)$ goes to $D_1(y)$, then x_a goes to 1 by (B.6), so that $\mathbf{E}[\tilde{x}^*]$ goes to 1. Finally, if $\frac{1}{4} < y < \frac{1}{2}$, that $\mathbf{E}[\tilde{x}^*]$ is continuous at $D_1(y) = \frac{I}{4}$ follows from the fact that, if $D < D_1(y)$ goes to $D_1(y)$, then $x_b = \overline{x}$ goes to 1 and $G(\overline{x})$ goes to zero, so that $\mathbf{E}[\tilde{x}^*]$ goes to 1.

To complete the proof of the proposition, we need to verify that $\mathbf{E}[\tilde{x}^*]$ is strictly increasing in D. The only case where this does not immediately follow from the description of the equilibrium is when $x_a > 0$. In that case, because $x^1 = x_a$ and $x^1 = 1$ are both played at equilibrium, it must be that

$$V^{1*}(x_a, \mathbf{E}[\tilde{x}^*]) = p_0 L + (1 - p_0)D. \tag{B.8}$$

It is easy to see from (B.6) that x_a is differentiable in D, and from (B.8) and the definition of V^{1*} that $\mathbf{E}[\tilde{x}^*]$ is differentiable in D. Differentiating (B.8) with respect to D yields

$$\frac{\partial V^{1*}}{\partial x^1}(x_a, \mathbf{E}[\tilde{x}^*])\,\frac{\mathrm{d}x_a}{\mathrm{d}D} + \frac{\partial V^{1*}}{\partial x^2}(x_a, \mathbf{E}[\tilde{x}^*])\,\frac{\mathrm{d}\mathbf{E}[\tilde{x}^*]}{\mathrm{d}D} = 1 - p_0.$$

Because $\frac{\partial V^{1*}}{\partial x^1}(x_a, \mathbf{E}[\tilde{x}^*]) = 0$ by (B.7), we derive from the definition of V^{1*} that

$$\frac{\mathrm{d}\mathbf{E}[\tilde{x}^*]}{\mathrm{d}D} = \frac{1}{\mathrm{e}^{-\lambda \underline{t}(x_a)}[1 - \mathrm{e}^{-\lambda \underline{t}(x_a)}]I} > 0. \tag{B.9}$$

Hence the result. □

Proof of Proposition 8. Using (10), we derive that welfare equals

$$\begin{aligned} W &= 2p_0 L + 2\mathbf{E}_{\tilde{x}^1}[\mathbf{P}_{\tilde{x}^2}[\tilde{x}^1 \leq \tilde{x}^2]\{\mathbf{E}_{\tilde{x}^2}[\tilde{x}^2 \,|\, \tilde{x}^2 \geq \tilde{x}^1] - x^1\}(1 - p_0)I\mathrm{e}^{-\lambda \underline{t}(\tilde{x}^1)}[1 - \mathrm{e}^{-\lambda \underline{t}(\tilde{x}^1)}]] \\ &= 2p_0 L + 2\mathbf{E}_{\tilde{x}^1}[(1 - p_0)(1 - \tilde{x}^1)D] \\ &= 2p_0 L + 2(1 - p_0)\{1 - \mathbf{E}[\tilde{x}^*]\}D, \end{aligned}$$

where the first equality leverages symmetry, and the second equality follows from (B.4). First notice that, because $\mathbf{E}[\tilde{x}^*]$ is continuous in D, so is W. When $D \leq D_0(y)$, we have $\mathbf{E}[\tilde{x}^*] = \frac{D}{y(1-y)I}$, so W is maximized at $D^* \equiv \frac{y(1-y)I}{2} < \frac{yI}{2} = D_0(y)$ over this range, for which $\mathbf{E}[\tilde{x}^*] = \frac{1}{2}$. Notice also that, for $D \geq D_1(y)$, $\mathbf{E}[\tilde{x}^*] = 1$ and thus $W = 0$, so the optimal subsidy must be lower than $D_1(y)$. Now consider the interval $[D_0(y), D_1(y)]$. By continuity, it is clear that, if the global solution lies within this range, it must belong to its interior. Using (B.4) for $x^1 = x_a$ and (B.9), we derive

$$\frac{\mathrm{d}W}{\mathrm{d}D} = 2(1 - p_0)\left\{1 - \mathbf{E}[\tilde{x}^*] - D\,\frac{\mathrm{d}\mathbf{E}[\tilde{x}^*]}{\mathrm{d}D}\right\} = 2(1 - p_0)\left\{1 - x_a - \frac{(2 - x_a)D}{\mathrm{e}^{\lambda \underline{t}(x_a)}[1 - \mathrm{e}^{\lambda \underline{t}(x_a)}]I}\right\}.$$

Thus, if an interior solution exists, it must be given by the first-order condition $D = \frac{\mathrm{e}^{\lambda \underline{t}(x_a)}[1-\mathrm{e}^{\lambda \underline{t}(x_a)}](1-x_a)I}{2-x_a}$. However, inserting $D = \frac{\mathrm{e}^{\lambda \underline{t}(x_a)}[1-\mathrm{e}^{\lambda \underline{t}(x_a)}](1-x_a)I}{2-x_a}$ into (B.6) yields $\frac{2(1-x_a)}{2-x_a}\left[1 - \frac{\mathrm{e}^{-\lambda \underline{t}(x_a)}}{2x_a \mathrm{e}^{-\lambda \underline{t}(x_a)} + 1 - x_a}\right] = 1$, which is clearly impossible. So an interior solution cannot exist over $[D_0(y), D_1(y)]$, and we conclude that $D^* = \frac{y(1-y)I}{2}$ reaches a maximum of W. Hence the result. □

Proof of Proposition 9. By definition of L_0, $y > \frac{1}{2}$ for $L < L_0$. Thus increasing L decreases y and accordingly increases $D^* = \frac{y(1-y)I}{2}$. Conversely, $y < \frac{1}{2}$ for $L > L_0$. Thus increasing L decreases y and accordingly decreases $D^* = \frac{y(1-y)I}{2}$. Hence the result. □

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