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Georges Casamatta and Helmuth Cremer

Toulouse School of Economics

Tax avoidance and commodity tax differentiation*

Georges Casamatta[†] and Helmuth Cremer[‡]

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Abstract

We examine the optimal combination of direct and indirect taxes in the presence of tax avoidance. Our findings indicate that linear commodity taxes should be included in the optimal tax mix, even when they are subject to avoidance and when the conditions of the Atkinson-Stiglitz theorem hold. This is because taxing consumption—despite the possibility of avoidance—enhances the ability to screen *true* income, whereas income taxation alone depends solely on *reported* income. Additionally, we show that when utility is weakly separable, tax rates should be positive and uniform across goods if the subutility function is homothetic, leading to linear Engel curves. However, when Engel curves are nonlinear, commodity taxes need not be uniform. Furthermore, the optimal taxation of luxuries versus necessities depends on the distribution of productivity levels.

Keywords: direct and indirect taxes, avoidance

JEL classification: H21, H26

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[†]UMR CNRS 6240 LISA, University of Corsica Pasquale Paoli, Corte, France, and Toulouse School of Economics; email: casamatta_g@univ-corse.fr

[‡]Toulouse School of Economics, University of Toulouse Capitole.

1 Introduction

The literature on optimal taxation, pioneered by Mirrlees (1971), has traditionally focused on how taxes influence labor supply—what Slemrod (1995) refers to as the *real* response to taxation. However, individuals can also adjust their reported income without altering their labor supply, either legally (tax avoidance) or illegally (tax evasion).

This paper focuses on tax avoidance—legal strategies used to minimize tax liabilities—an issue of growing importance in advanced economies. A 2011 UK Treasury report (*Tackling Tax Avoidance*) estimates that tax avoidance accounts for approximately 17.5

The distinction between avoidance and evasion is largely a matter of interpretation. Following Usher (1986), the cost of tax evasion can be viewed as the expense required to eliminate detection risks when misreporting income.

Recent academic work has increasingly examined tax avoidance, with one strand focusing on the elasticity of taxable income (see Saez et al. (2012)). This measure captures both real and avoidance responses and is central to assessing tax distortions. Other studies analyze the tax avoidance margin. For instance, Slemrod (2001) models income taxation with both labor supply and avoidance responses but does not address optimal taxation. Slemrod and Kopczuk (2002) determines the optimal level of tax avoidance, highlighting that unlike labor supply responses, avoidance can be mitigated by government policy. If avoidance responses are significant, optimal policy may favor broadening the tax base over reducing tax rates, contrary to the standard Mirrleesian approach. Similarly, Roine (2006) examines how avoidance-driven tax distortions influence voting behavior on tax policy.

Despite these contributions, few studies adopt a full-fledged optimal taxation approach. Grochulski (2007) and Casamatta (2021) derive optimal income tax schedules when avoidance is costly. Grochulski (2007) finds that under a sub-additive cost function, individuals should fully report their income at the optimum. Casamatta (2021), assuming a convex cost function, argues that middle-class individuals should be allowed to avoid taxes. However, his results rely on a setting where labor income is the sole tax instrument.[1]

In this paper, we introduce indirect taxation, allowing both labor income and consumption goods to be taxed. We incorporate both labor supply and tax avoidance responses. Our work builds on Boadway et al. (1994), who show that income tax avoidance justifies a role for commodity taxes, even when they would otherwise be redundant. However, their analysis assumes only income taxes can be avoided, making the case for commodity taxation unsurprising. We extend their framework to allow for commodity tax avoidance and generalize their results from a two-type model to a continuous-type setting.

Our analysis contributes to the long-standing debate on direct versus indirect taxation (see Atkinson (1977)). A key result in this debate is the Atkinson-Stiglitz theorem (Atkinson and Stiglitz, 1976), which states that with a nonlinear income tax, taxing consumption is unnecessary if utility is weakly separable between consumption and leisure. This condition is weaker than the requirements in Ramsey taxation, which also demands homothetic subutility for goods. The relevance of the Atkinson-Stiglitz theorem has been questioned in various contexts, including general equilibrium effects (Naito, 1999), multidimensional heterogeneity (Cremer et al., 2001), and income tax evasion (Boadway et al., 1994). We maintain weak separability and abstract from general equilibrium effects and multidimensional heterogeneity, instead focusing on tax avoidance.

A key insight of our study is that tax avoidance disrupts the traditional Atkinson-Stiglitz result.

[1] These findings have been extended to endogenous labor supply settings by Casamatta (2023). Related contributions include Selin and Simula (2020) and Doligalski and Rojas (2023).

Since declared income differs from true income, income taxes alone may be insufficient, creating a role for commodity taxation—even when the assumptions of the Atkinson-Stiglitz theorem hold. Consumption taxes provide better screening of true income than a system relying solely on labor income taxation. Remarkably, this remains true even when commodity taxes themselves can be avoided, providing a novel justification for a mix of direct and indirect taxes.

This reasoning applies to a single consumption good, where in the absence of tax avoidance, the consumption tax can be normalized to zero. However, we also explore the more complex case of multiple consumption goods and the implications for tax differentiation. When Engel curves are linear, we recover the result from Boadway et al. (1994) that tax rates should be uniform across goods if firms do not avoid taxes. However, when firms engage in tax avoidance and have access to the same avoidance technology, uniform taxation requires homothetic preferences over goods. In other words, when both firms and individuals avoid taxes, the conditions for uniform taxation become more stringent, aligning with the Ramsey framework.

With nonlinear Engel curves, optimal commodity taxation is non-uniform but difficult to characterize precisely. In a setting where only individuals avoid taxes, we derive conditions for tax differentiation. Our findings suggest that whether luxuries should be taxed more heavily than necessities depends on the shape of the skill distribution. We illustrate this result using numerical simulations based on the Almost Ideal Demand System (Deaton and Muellbauer, 1980a), estimated on food consumption data.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 solves the social planner's problem. Section 4 examines the case of a single consumption good, while Section 5 extends the analysis to multiple goods.

2 Model

We consider an economy with $1 + n$ goods: leisure and n consumption goods. All individuals have the same utility function $u(\mathbf{c}, l)$, where $\mathbf{c} \equiv (c_1, \ldots, c_n)$ is the vector of goods consumed[2] and l denotes labor supply. They differ in their productivity w, which is distributed according to the distribution function $F(.)$ and the density $f(.)$ on the support $[w_-, w_+]$. Individuals with productivity w generate income $y = wl$. They have the possibility to (legally) reduce their taxable income, which is denoted $\hat{y}$. The cost of hiding Δ euros is $\phi(\Delta)$, such that $\phi(0) = 0$, $\phi'(0) = 0$, $\phi''(.) > 0$.

Firms also have the ability to conceal sales from the authorities. Each unit of output concealed by firms in industry i entails a resource cost of

$$G_i(1 - \alpha),$$

which is an increasing and convex function of the proportion $(1 - \alpha)$ of unreported sales.[3]

Policy instruments consist of a (non-linear) tax $\hat{T}(\hat{y})$ on reported income as well as linear tax rates t_i on each commodity $i = 1, \ldots, n$. [4] Let $R \equiv y - \hat{T}(\hat{y}) - \phi(y - \hat{y})$ denote the net labor income of an individual with true income y and reported income $\hat{y}$.

[2]We assume throughout that these are *normal* goods.

[3]On the firm's side, our specification is inspired by Cremer and Gahvari (1993).

[4]This reflects the by now traditional information structure in mixed taxation models. Reported income is observable at an individual level, while for goods only anonymous transactions are observed.

2.1 Consumption and labor supply decisions

An individual with productivity w and exogenous income I solves the following program:

$$\max_{\mathbf{c},y,\hat{y}} u(\mathbf{c}, y/w)$$
$$\text{st} \sum_k p_k c_k = x, \tag{1}$$

where p_k is the consumer price of good k and $x \equiv R + I$ is disposable income.

Let $U(w)$ denote the level of utility achieved by an individual with productivity w. Applying the envelope theorem, we have

$$\frac{dU}{dw} = -\frac{y}{w^2} u_l, \tag{2}$$

where $u_l \equiv \partial u / \partial l$ is the marginal disutility of labor. This equation represents the local incentive constraint in the government's problem.

2.2 Firms' behavior

The marginal cost is constant and normalized to 1. Firms producing good i maximize

$$\pi_i = (p_i - 1 - \alpha_i t_i - (1-\alpha_i) G_i (1-\alpha_i)) c_i,$$
$$= (p_i - 1 - \alpha_i t_i - g_i (1-\alpha_i)) c_i,$$

where α_i is the reported share of sales and $g_i(1-\alpha_i) = (1-\alpha_i)G_i(1-\alpha_i)$. The first- and second-order conditions with respect to α_i require that

$$g_i'(1-\alpha_i) = t_i$$
$$g_i''(1-\alpha_i) < 0 \tag{3}$$

where (3) is satisfied because G_i is an increasing and convex function. We also assume that $g'(0) = 0$ and $g'(1) = \infty$ to ensure an interior solution.

Defining

$$t_i^e \equiv \alpha_i t_i,$$

the market is in equilibrium when

$$p_i = 1 + g_i + t_i^e.$$

2.3 The social planner's program

According the revelation principle, individuals directly and truthfully report their type w. The government maximizes social welfare, expressed as the sum of a concave transformation $G(.)$ of individual utility levels, subject to resource and incentive constraints. The incentive constraints are given by (2).

The government budget constraint (GBC) is:

$$\int \left(\sum_k t_k^e c_k + T(w) \right) f(w) dw \geq 0, \tag{4}$$

where $T(w)$ is the tax paid by an individual with productivity w.

The planner's program can then be stated as follows:

$$\max_{\mathbf{t},\hat{y}(.),T(.)} \int G(U(w))dF(w) \quad \text{st} \quad (2),(4),$$

where $\mathbf{t} \equiv (t_1, \ldots, t_n)$.

3 Optimal commodity taxes

The Lagrangian associated with the social planner's program is:

$$\mathcal{L} = \int \left[G\left(U(w)\right) + \mu \left(\sum_k t_k^e c_k + T(w) \right) \right] f(w)dw + \int \lambda(w) \left(\frac{dU}{dw} + \frac{y}{w^2} u_l \right) dw,$$

where μ and $\lambda(w)$ are the multipliers associated with CBC constraint and the incentive constraints, respectively. Integrating by parts:

$$\int \lambda(w) \frac{dU}{dw} dw = -\int \lambda'(w)U(w)dw + \lambda(w_+)U(w_+) - \lambda(w_-)U(w_-),$$

the Lagrangian can be rewritten as:

$$\begin{aligned}\mathcal{L} = &\int \left[G(U(w)) + \mu \left(\sum_k t_k^e c_k + T(w) \right) \right] f(w)dw \\ &+ \int \left(\lambda(w) \frac{y}{w^2} u_l - \lambda'(w)U(w) \right) dw + \lambda(w_+)U(w_+) - \lambda(w_-)U(w_-).\end{aligned}$$

The first-order conditions with respect to t_i and T are given by:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial t_i} = &\int \left[G'(U) \frac{dU}{dp_i} \frac{dp_i}{dt_i} + \mu \left(\frac{dt_i^e}{dt_i} c_i + \sum_k t_k^e \frac{dc_k}{dp_i} \frac{dp_i}{dt_i} \right) \right] f(w)dw \\ &+ \int \left[\lambda \frac{d}{dp_i} \left(\frac{y}{w^2} u_l \right) \frac{dp_i}{dt_i} - \lambda' \frac{dU}{dp_i} \frac{dp_i}{dt_i} \right] dw = 0, \; i = 1, \ldots, n \qquad (5)\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial T} = \left[G'(U) \frac{dU}{dT} + \mu \left(1 + \sum_k t_k^e \frac{dc_k}{dT} \right) \right] f(w) + \lambda \frac{d}{dT} \left(\frac{y}{w^2} u_l \right) - \lambda' \frac{dU}{dT} = 0. \qquad (6)$$

Dividing (5) by dp_i/dt_i yields:

$$\int \left[G'(U) \frac{dU}{dp_i} + \mu \left(A_i c_i + \sum_k t_k^e \frac{dc_k}{dp_i} \right) f(w) + \lambda \frac{d}{dp_i} \left(\frac{y}{w^2} u_l \right) - \lambda' \frac{dU}{dp_i} \right] dw = 0, \qquad (7)$$

where

$$A_i \equiv \frac{dt_i^e/dt_i}{dp_i/dt_i}.$$

We now multiply (6) by c_i :

$$c_i \left[G'(U) \frac{dU}{dT} + \mu \left(1 + \sum_k t_k^e \frac{dc_k}{dT} \right) \right] f(w) + \lambda c_i \frac{d}{dT} \left(\frac{y}{w^2} u_l \right) - \lambda' c_i \frac{dU}{dT} = 0,$$

take the integral of this condition and subtract it from (7). Using Roy's identity, $dU/dp_i = c_i(dU/dT)$, we obtain:

$$\int \left[\mu \sum_k t_k^e \left(\frac{dc_k}{dp_i} - c_i \frac{dc_k}{dT} \right) f + \lambda \left(\frac{d}{dp_i} \left(\frac{y}{w^2} u_l \right) - c_i \frac{d}{dT} \left(\frac{y}{w^2} u_l \right) \right) + \mu c_i \left(A_i - 1 \right) f \right] dw = 0.$$

Using the Slutsky equation for consumption goods:

$$\frac{dc_k}{dp_i} = S_{ki} + c_i \frac{dc_k}{dT},$$

this results in:

$$\int \left[\mu \left(\sum_k t_k^e S_{ki} \right) f + \lambda \left(\frac{d}{dp_i} \left(\frac{y}{w^2} u_l \right) - c_i \frac{d}{dT} \left(\frac{y}{w^2} u_l \right) \right) + \mu c_i \left(A_i - 1 \right) f \right] dw = 0. \tag{8}$$

Note that:

$$\frac{d}{d\kappa} \left(\frac{y}{w^2} u_l \right) = \frac{dy}{d\kappa} \left(\frac{1}{w^2} u_l + \frac{y}{w^3} u_{ll} \right) + \frac{y}{w^2} \frac{d^2 u}{d\kappa dl}, \ \kappa = p_i, T.$$

Using the Slutsky equation for labor supply:

$$\frac{dy}{dp_i} = S_{yi} + c_i \frac{dy}{dT},$$

condition (8) becomes:

$$\int \left[\mu \left(\sum_k t_k^e S_{ki} \right) f + \lambda \left(S_{yi} \left(\frac{1}{w^2} u_l + \frac{y}{w^3} u_{ll} \right) + \frac{y}{w^2} \left(\frac{d^2 u}{dp_i dl} - c_i \frac{d^2 u}{dT dl} \right) \right) + \mu c_i \left(A_i - 1 \right) f \right] dw = 0.$$

We differentiate Roy's identity:

$$\frac{d^2 u}{dp_i dl} = \frac{du}{dT} \frac{d\tilde{c}_i}{dl} + c_i \frac{d^2 u}{dT dl},$$

so that the optimality condition becomes:

$$\int \left[\mu \left(\sum_k t_k^e S_{ki} \right) f + \lambda \left(\frac{du}{dT} \frac{dc_i}{dl} + S_{yi} \left(\frac{1}{w^2} u_l + \frac{y}{w^3} u_{ll} \right) \right) + \mu c_i \left(A_i - 1 \right) f \right] dw = 0.$$

The effect of an increase in the tax rate on good i can be decomposed into three terms. First, it creates distortions by inducing individuals to make substitutions between goods (term $\sum_k t_k^e S_{ki}$). Second, it relaxes incentive constraints. This effect has two components (terms $(du/dT)(dc_i/dl)$ and $S_{yi}((1/w^2)u_l + (y/w^3)u_{ll})$). On the one hand, mimicking individuals consume more leisure than mimicked ones. Therefore, goods that are more complementary to leisure should be taxed more. This effect has been studied extensively in the literature (Christiansen, 1984; Jacobs and Boadway, 2014) and is absent with weakly separable preferences (see equation (10) below). On the other hand, under tax avoidance, mimicking individuals earn more income, and thus consume more, than mimicked ones (even though they report the same level of income). It follows that they require more compensation to stay on the same indifference curve when the tax rate is increased. In other words, an increase in the tax rate makes mimicking less desirable. Finally, an increase in the tax rate is detrimental because it leads firms to evade a larger proportion of their tax liability (term $c_i \left(A_i - 1 \right)$).

With a utility function weakly separable between goods and labor:

$$u(\mathbf{c}, l) = \Phi(f(\mathbf{c}), l), \tag{9}$$

the term dc_i/dl is equal to 0 and the optimality condition reduces to:

$$\int \left[\mu \left(\sum_k t_k^e S_{ki} \right) f + \lambda S_{yi} \left(\frac{1}{w^2} u_l + \frac{y}{w^3} u_{ll} \right) + \mu c_i \left(A_i - 1 \right) f \right] dw = 0. \tag{10}$$

For the remainder of the paper we consider weakly separable preferences so that according to the Atkinson and Stiglitz, absent of tax avoidance commodity taxes should be uniform.

4 Uniform tax structure

We ask whether the optimal tax structure is uniform. To answer this question, we assume that all industries offer the same tax avoidance opportunities: $G_i = G$ for all i, and we consider a uniform tax structure: $t_1 = \ldots = t_n \equiv t$. Noticing that in this framework $\alpha_i = \alpha$ and $A_i = A$ for all i, the optimality condition (10) becomes:

$$\int \left[\mu\alpha \left(\sum_k t S_{ki} \right) f + \lambda S_{yi} \left(\frac{1}{w^2} u_l + \frac{y}{w^3} u_{ll} \right) + \mu c_i \left(A - 1 \right) f \right] dw = 0,$$

which implies:

$$t = \frac{\int \lambda \left(S_{yi} \left(\frac{1}{w^2} u_l + \frac{y}{w^3} u_{ll} \right) - \mu c_i \left(A - 1 \right) f \right) dw}{-\mu\alpha \int \left(\sum_k S_{ki} \right) f dw}.$$

We use the following property:[5]

$$\sum_k p_k S_{ki} = (1 - \phi') S_{yi} \tag{13}$$

[5] To arrive at this condition, we differentiate the individual budget constraint (1) with respect to p_i:

$$c_i + \sum_k p_k \frac{dc_k}{dp_i} = \frac{dR}{dp_i}. \tag{11}$$

We then differentiate the individual budget constraint with respect to I:

$$\sum_k p_k \frac{dc_k}{dI^h} = \frac{dR}{dI} + 1.$$

Multiplying this condition by c_i gives:

$$c_i \sum_k p_k \frac{dc_k}{dI} = c_i \frac{dR}{dI} + c_i. \tag{12}$$

Adding up (11) and (12) and using the Slutsky equation, we get:

$$\sum_k p_k S_{ki} = \frac{dR}{dp_i} + c_i \frac{dR}{dI}.$$

Using the FOC on $\hat{y}$, we see that $dR/dp_i = (dR/dy)(dy/dp_i) = (1 - \phi')(dy/dp_i)$ and $dR/dI = (dR/dy)(dy/dI) = (1 - \phi')(dy/dI)$. So, again using Slutsky:

$$\sum_k p_k S_{ki} = (1 - \phi') S_{yi}.$$

This is a well-known property of demand functions with endogenous labor supply (Deaton and Muellbauer, 1980b), where the price of leisure in our setting is $w(1 - \phi')$.

that becomes $\sum_k S_{ki} = (1-\phi')S_{yi}/(1+g+t)$ in the case of a uniform tax structure. Substituting into the previous expression, we obtain:

$$\frac{t}{1+g+t} = \frac{\int \lambda \left(S_{yi}\left(\frac{1}{w^2}u_l + \frac{y}{w^3}u_{ll}\right) - \mu c_i \left(A-1\right) f\right) dw}{-\mu\alpha \int (1-\phi')S_{yi} f dw}.$$

We finally observe that, with a weakly separable utility function (Goldman and Uzawa, 1964; Sandmo, 1974):

$$S_{yi} = \varphi(w)\frac{dc_i}{dx}. \tag{14}$$

With affine Engel curves:[6]

$$c_i = \gamma_i(\mathbf{p}) + \zeta_i(\mathbf{p})x,$$

this becomes

$$S_{yi} = \varphi(w)\zeta_i(\mathbf{p}).$$

Therefore a uniform tax structure is optimal with linear Engel curves if:

$$\frac{t}{1+g+t} = \frac{\int \lambda\varphi(w)\left(\left(\frac{1}{w^2}u_l + \frac{y}{w^3}u_{ll}\right) - \frac{\mu c_i (A-1) f}{\varphi(w)\zeta_i(\mathbf{p})}\right) dw}{-\mu\alpha \int (1-\phi')\varphi(w) f dw}.$$

Two observations are in order. First, when firms do not avoid taxes ($A=1$), a uniform tax structure is optimal, thus restoring the result of Boadway et al. (1994). However, this result no longer holds when firms engage in tax avoidance, since the numerator now depends on i. In such a case, the optimal tax structure is uniform only if consumption is proportional to disposable income, meaning that preferences over produced goods are (strictly) homothetic. Interestingly, the possibility of commodity tax avoidance in addition to income tax avoidance thus strengthens the conditions for obtaining a uniform tax. More precisely, we return to the condition which yields uniform commodity taxes in the Ramsy model, that is absent of income taxation, see Atkinson and Stiglitz (1972).

This leads to the following proposition.

Proposition 1. *With affine Engel curves and firms practicing avoidance, all having access to the same avoidance technology, a uniform commodity tax structure is optimal if and only if preferences over goods are homothetic.*

5 Tax differentiation with general Engel curves

We now specialize the model to the case where only individuals avoid taxes, which implies that $t_i^e = t_i$ and $A_i = 1$, $\forall i = 1, \ldots, n$.

We start from a uniform tax structure with a common tax rate t and asume that the FOC (10) is satisfied for all goods except j. Noting that $p_k = 1+t$ with a uniform tax structure and using (13), this condition can be rewritten as:

$$\int \left[\mu\frac{t}{1+t}(1-\phi')S_{yi}f + \lambda S_{yi}\left(\frac{1}{w^2}u_l + \frac{y}{w^3}u_{ll}\right)\right] dw = 0, \ \forall i \neq j. \tag{15}$$

[6] As pointed out by Boadway et al. (1994) this is the case when the subutility for produced goods is quasi-homothetic, that is homothetic to some arbitrary point which does not need to be the origine

It is then optimal to impose a tax rate higher than t on good j if and only if:

$$\int \left[\mu \frac{t}{1+t}(1-\phi')S_{yj}f + \lambda S_{yj}\left(\frac{1}{w^2}u_l + \frac{y}{w^3}u_{ll}\right)\right] dw > 0$$
$$\Leftrightarrow \frac{\int (1-\phi')S_{yj}f dw}{\int (1-\phi')S_{yi}f dw} < \frac{\int \lambda S_{yj}\left(\frac{1}{w^2}u_l + \frac{y}{w^3}u_{ll}\right) dw}{\int \lambda S_{yi}\left(\frac{1}{w^2}u_l + \frac{y}{w^3}u_{ll}\right) dw}, \quad (16)$$

where the second inequality is obtained by using (15) and substituting μ. This condition says that good j should be taxed more than good i if the relative distortionary effect of good j with respect to good i is less than its relative redistributive effect (relaxation of the incentive constraints). In other words, even if increasing the tax rate on good j creates additional distortions, this negative effect is more than offset by the additional redistributive benefit.

We show in the next proposition that with nonlinear Engel curves and except in some degenerate cases, tax rates should be differentiated at the optimum if the ratio S_{yj}/S_{yi} increases with productivity. Furthermore the desirability of taxing luxuries more heavily than necessities depends on the shape of the distribution of productivities.

Proposition 2. *Consider two goods i and j, where i a necessity and j is a luxury, and assume that: (i) the utility function is weakly separable between goods and labor; (ii) the ratio S_{yj}/S_{yi} increases with productivity. Then, a uniform tax structure is not optimal, except in some degenerate cases. If the density function is sufficiently skewed to the right (resp. left), it is optimal to increase (resp. decrease) the tax rate on good j and to decrease (resp. increase) the tax rate on good i.*

Proof. Under assumption *(ii)*, $\forall w > w_-$:

$$\frac{-S_{yj}(w)}{-S_{yi}(w)} > \frac{-S_{yj}(w_-)}{-S_{yi}(w_-)}$$
$$\Rightarrow -S_{yj}(w) > -S_{yi}(w)\frac{S_{yj}(w_-)}{S_{yi}(w_-)}$$
$$\Rightarrow \int_{w_-}^{w_+} \lambda(-S_{yj})\left(-\frac{1}{w^2}u_l - \frac{y}{w^3}u_{ll}\right) dw > \frac{S_{yj}(w_-)}{S_{yi}(w_-)} \int_{w_-}^{w_+} \lambda(-S_{yi})\left(-\frac{1}{w^2}u_l - \frac{y}{w^3}u_{ll}\right) dw$$
$$\Rightarrow \frac{\int_{w_-}^{w_+} \lambda S_{yj}\left(\frac{1}{w^2}u_l + \frac{y}{w^3}u_{ll}\right) dw}{\int_{w_-}^{w_+} \lambda S_{yi}\left(\frac{1}{w^2}u_l + \frac{y}{w^3}u_{ll}\right) dw} > \frac{S_{yj}(w_-)}{S_{yi}(w_-)}.$$

When $f(w) \to 0, \forall w \neq w_-$ (extremely right skewed distribution), the left side of (16) tends to $S_{yj}(w_-)/S_{yi}(w_-)$. The above inequality then implies that (16) is satisfied and therefore that the tax rate on the luxury good should be increased.

A symmetric argument can be used to show that the tax rate on the necessity good should be increased if the density function is sufficiently skewed to the left. □

Finally, we establish sufficient conditions for assumption *(ii)* to be true. This leads to the following corollary to Proposition 2.

Corollary 1. *Consider two goods i and j, where i is a necessity and j is a luxury, and assume that: (i) the utility function is weakly separable between goods and labor; (ii) disposable income is strictly increasing in productivity; (iii) both goods are normal; (iv) the Engel curve for good j (resp. i) is strictly convex (resp. concave). Then, a uniform tax structure is not optimal, except in some degenerate cases. If the density function is sufficiently skewed to the right (resp. left) it is optimal to increase (resp. decrease) the tax rate on good j and to decrease (resp. increase) the tax rate on good i.*

Proof. With a weakly separable utility function (assumption *(i)*), (14) implies:

$$\frac{S_{yj}}{S_{yi}} = \frac{\frac{dc_j}{dx}}{\frac{dc_i}{dx}}.$$

Assumptions *(ii)*, *(iii)*, and *(iv)* then imply that S_{yj}/S_{yi} increases with productivity. □

6 Application to AIDS preferences

In this section, we continue to assume that only individuals avoid taxes.

6.1 The Almost Ideal Demand System

A convenient representation of preferences for introducing nonlinear Engel curves is the Almost Ideal Demand System (AIDS) developed by Deaton and Muellbauer (1980a). In this model, the demand function for good i is given by:

$$c_i = \frac{x}{p_i}(\alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i(\ln x - \ln P)), \tag{17}$$

where P is a price index defined as:

$$\ln P = \alpha_0 + \sum_k \alpha_k \ln p_k + \frac{1}{2}\sum_k \sum_j \gamma_{kj} \ln p_k \ln p_j,$$

and AIDS parameters meet the following restrictions:

$$\sum_i \alpha_i = 1,\ \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0,\ \sum_i \beta_i = 0,\ \gamma_{ij} = \gamma_{ji}. \tag{18}$$

The indirect utility function v is:

$$v(\mathbf{p}, x) = \frac{\ln x - \ln P}{b}, \tag{19}$$

where

$$b = \beta_0 \prod_i p_i^{\beta_i}.$$

6.2 Analytical derivations

Differentiating (17), we have:

$$\frac{dc_i}{dx} = \frac{c_i}{x} + \frac{\beta_i}{p_i}$$
$$\frac{d^2 c_i}{dx^2} = \frac{1}{x}\frac{\beta_i}{p_i}.$$

Therefore, Engel curves are strictly convex for luxuries ($\beta_i > 0$) and strictly concave for necessities ($\beta_i < 0$). This means that condition *(iii)* of Corollary 1 is satisfied. However, with AIDS preferences, goods may be inferior. Furthermore, it is difficult to check whether assumption *(ii)* is satisfied. Therefore, we rely on numerical simulations in the next section.

6.3 Numerical simulations

We consider an additively separable utility function:

$$u(\mathbf{c}, l) = f(\mathbf{c}) + \varphi(l)$$

where the preferences for goods satisfy AIDS and $\varphi(l) = -l^2/2$. The social welfare transformation is $G(U) = U$. We assume that the cost of avoidance takes the form: $\phi(\Delta) = 0.2\left(\Delta^2/2\right)$.

The Almost Ideal Demand System was estimated by Anderson and Blundell (1983) using data on consumers' expenditures in Canada for the period 1947-1979. Demands for five categories of non-durable goods are considered: food, clothing, energy, transport, and recreation. The estimates of the AIDS parameters are displayed in Table 1. According to these estimates, two goods have negative values of β and are therefore necessities (food and clothing), while the other three goods (energy, transport, and recreation) are luxuries.

Commodity i	α_i	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}
1: Food	0.342	−0.096	0.095	−0.011	0.023	0.007	−0.114
2: Clothing	0.129	−0.102	−0.011	−0.008	0.017	−0.029	0.031
3: Energy	0.055	0.014	0.023	0.017	0.006	−0.030	−0.016
4: Transport	0.147	0.036	0.007	−0.029	−0.030	0.032	0.020
5: Recreation	0.327	0.148	−0.114	0.031	−0.016	0.020	0.079

Table 1: AIDS parameters

Finally, productivites are distributed according to a (truncated) lognormal distribution on the support $[1, 8]$.

β_i	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*
0	0.203	0.203	0.203	0.203	0.203
estimates	0.184	0.194	0.154	0.222	0.226

Table 2: Optimal tax rates

The optimal tax rates are shown in Table 2. In the first row of this table, we set $\beta_i = 0$ for all five goods. So none of them is a necessity or a luxury. We check that the optimal tax rate is the same for all goods. The second row shows the optimal tax rates when β_i are set to the values estimated by Anderson and Blundell (1983). We obtain that one of the luxury goods (energy) should be taxed less than the two necessity goods (food and clothing), while a larger tax rates should be applied to the other two luxury goods (transport and recreation).

7 Conclusion

We analyze the optimal combination of income and commodity taxes when both can be avoided at some cost. Apart from tax avoidance, our framework satisfies the conditions of the Atkinson-Stiglitz theorem, implying that in the absence of avoidance, commodity taxes would be unnecessary. However, we show that when avoidance is possible—even with a single good—both tax instruments become essential.

Commodity taxes should generally be non-uniform unless the subutility function for goods is homothetic, ensuring linear Engel curves that pass through the origin. Notably, the presence of both income and commodity tax avoidance strengthens the conditions for uniform taxation; while linear Engel curves alone suffice when only income tax can be avoided, as shown by Boadway et al. (1994).

When Engel curves are nonlinear, commodity taxes are typically non-uniform, though their optimal structure is complex. Interestingly, it is not always optimal to tax luxuries more than necessities—this depends on the shape of the productivity distribution.

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