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“Welfare Implications of Supplier Encroachment With Consumer Shopping Costs”

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Welfare Implications of Supplier Encroachment With Consumer Shopping Costs*

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Abstract

In this paper, we study supplier encroachment in competition with multi-product retailers and its effects on retail profits under endogenous consumer shopping behavior. We find that supplier encroachment (weakly) increases both supplier and retailer profits, as the retailer benefits from better consumer segmentation and price discrimination despite (weakly) higher wholesale prices. The effect of encroachment on consumers is more nuanced: when the competitive product's value is high, consumers benefit. Instead, when the value of the competitive product is low, consumers buying exclusively from the multi-product retailer are worse off while consumers who mix and match across stores are better off. Overall, supplier encroachment can improve market outcomes if the value of the supplier's product offering is sufficiently high.

JEL Classification: L13, L22, L42, and L81.

Keywords: Supplier Encroachment, Vertical Contracting, Downstream Competition, and Consumer Shopping Costs.

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1 Introduction

The recent decades have seen the growing dominance of powerful big-box retailers that stock a large assortment of products. Such a store format is particularly interesting for consumers as it offers them the benefits of buying a bundle of products in one place. Specifically, consumers benefit from lower shopping costs through one-stop shopping in large multi-product stores. This reduction in shopping costs creates complementarities even among independent products within the store. Therefore, the mushrooming of such a format is not surprising.

Interestingly and in contrast to this trend, there is another recent trend in the retail industry. Specifically, suppliers are entering retail market directly through their own specialty stores. For instance, L'Oreal sells beauty products through super markets as well as its own Boutique stores. Similarly, there are many FMCG firms that offer direct to consumer sales options as well.[1] This has results in the polarization of store size with large multi-product retailers competing with small specialty stores or hard-discount chains (Griffith & Krampf (1997) or more recently, Igami (2011)). The presence of these direct to consumer boutique stores implies that the multi-product retailers face fiercer competition and consumers may be better off with supplier encroachment. In this paper, we study the welfare implications of supplier encroachment by offering a direct sales channel.

We consider a setting with a multi-product retailer where consumers are heterogeneous according to their shopping costs. One of the good in the assortment of the multi-product retailer is supplied by supplier U that offers a linear fee. In such a market environment, we first consider the *No-Encroachment* regime where the supplier is not active in the retail market and earns only through the multi-product retailer channel. In this case, all consumers are single-homers. We compare this regime with a market regime where the supplier is active in the retail market through its own boutique store. We denote this as the *Supplier Encroachment* regime. To capture

[1] For instance, Nestlé through Nespresso has been a pioneer in direct to consumer sales with its online store, boutiques, and club memberships.

competition in this market regime between these polarized retail formats, we employ the model developed by Chen & Rey (2012). The retailer attracts one-stop shoppers, and the multi-stop shoppers mix and match to get the best deals across stores. Comparing the outcomes in the two market regimes, we find that the supplier encroachment regime (weakly) increases both supplier profit and retailer profit. The result on retailer profit is interesting because it demonstrates that (direct channel) competition between such formats can be beneficial to retailers. This is because the presence of these asymmetric retail channels allows the retailer to better screen consumers into their types and price discriminate. Furthermore, the profit of the retailer (weakly) increases despite higher wholesales price in the encroachment regime. Focusing on consumer surplus, we find that the introduction of the competing direct channels has nuanced effects on consumers. When the value generated by the competitive product is high enough, consumers are better off as the multi-stop shopping consumers benefit from a high value product being introduced in the market. Instead when the value of the competitive product is low, there are two opposing effects on consumer surplus. Specifically, the single-stop shoppers are worse off while the multi-stop shoppers are better. The sum of these opposing effects determines the impact on consumer surplus. This elicits the interesting insight that the introduction of competition in the retail market through the direct sales channel of the supplier can hurt consumers. This occurs when the value of the competitive product is not sufficiently high. Nevertheless, we demonstrate that the introduction of the competitive direct sales channel can be Pareto improving when the value of the competitive good is sufficiently high.

The rest of this paper is organized as follows. In Section (2), the literature review is discussed and then followed by the introduction of model in Section (3). The *No-Encroachment* benchmark is presented in Section (4) followed by the *Supplier Encroachment* regime in Section (5). In Section (6), the welfare implications of supplier encroachment are discussed and the conclusions are present in Section (7). All proofs are available in the appendix.

2 Literature Review

Our work is related to various streams of literature.

We contribute to the burgeoning literature on supplier encroachment. There is no consensus yet in this strand of literature on the effects of supplier encroachment on retailers and consumers. On the one hand, there is a large strand of literature that states supplier encroachment is harmful to the retailers (Pan (2018), Ronayne & Taylor (2022), Yenipazarli (2024)). On the other hand, another strand of this literature that shows encroachment can be beneficial to retailer (Chiang et al. (2003), Tsay & Agrawal (2004), Arya et al. (2007), Arya & Mittendorf (2013), Liu & Zhang (2006), Liu et al. (2021) and Tian et al. (2023)). Our paper differs from these works as it considers a setting where consumers may be interested in buying multiple goods and can mix and match (under supplier encroachment). Specifically, Arya et al. (2007) find that a reduction in the wholesale fee after supplier encroachment can lower the double marginalization inefficiency and create benefits for the market participants. We show that Pareto gains arise as well under encroachment. Interestingly, the result in our work arises even without changes in the wholesale fee after encroachment or even higher wholesale fees. As will be evident later, the mechanism is different in our setting and arises from the supplier offering a higher quality product through its direct sales channels which benefits the multi-stop shopping consumers.

Our work is also related to the literature on vertical contracting with shopping costs (Caprice & von Schlippenbach (2013), Johansen & Nilssen (2016)). Caprice & von Schlippenbach (2013) show that, when one-stop shopping behavior is considered, slotting fees may emerge as a result of a rent-shifting mechanism in a three-party negotiation framework, where a monopolistic retailer negotiates sequentially with two competing or independent suppliers about two-part tariff contracts. The wholesale price negotiated with the first supplier is distorted upwards, and the first supplier may pay a slotting fee, as long as its bargaining power vis-à-vis the retailer is not too large. One-stop shopping behavior involves complementarity between products. This allows the retailer and the first supplier to extract rent from the second supplier.

Johansen & Nilssen (2016) study a merger game between retailing stores to look into the incentives of independent stores to form a big store when some consumers have preferences for one-stop shopping. They show that one-stop shopping behavior may lead to an improvement in the bargaining position of the merged entity vis-à-vis producers, through the creation of an inside option that small stores do not have. In the present paper, we are interested in how the introduction of a competing channel by the supplier affects market participants when consumers face shopping costs. We elicit that competition between the large retailer and direct sales channel of the supplier can lead to higher retail profit and benefit both the supplier and the retailer.

Finally, we also contribute to the small but interesting literature on negative value products. While there is a large literature that confirms retailers often introduce negative value products (Xu et al. (2017), Singh, Shakya, Singh & Biswas (2018), Singh, Singh, Kumar & Biswas (2018), Lin et al. (2016), Krishnamoorthy (2018)), this phenomenon is not yet well studied. Heidhues et al. (2016) show that firms may find it profitable to introduce products that are socially wasteful by deceiving consumers. In contrast to their work, we do not allow the retailer to deceive consumers. Caprice & Shekhar (2019) find that selling negative market value products can be a sustainable and a profitable strategy for multi-product retailers. In spite of some modeling similarities, our research makes this point through a richer model. Specifically, Caprice & Shekhar (2019) exogenously assume that there is a negative value product. Instead in the present work, negative value (from the retailer's perspective) is obtained endogenously in the supplier encroachment regime.

3 The Model

Players and environment

The relationships between a supplier, retailers and consumers are modeled as follows. There are two levels of the market: the upstream and the downstream market. In the upstream market, a branded supplier U incurs a per-unit manufacturing cost $c > 0$ and supplies its product B in the market to a large multi-product retailer R. The branded supplier may also choose to sell directly to consumers via its specialized direct sales channel $\mathfrak{D}$ which we refer to as the *Supplier Encroachment Regime* . In the downstream market, the multi-product retailer R sells two products, A and B. When the supplier encroaches by setting up its direct sales channel, then it competes with R for sales of product B. Therefore, in our setting, R is a monopolist when selling product A and competes (under supplier encroachment) with the direct sales channel, $\mathfrak{D}$, when selling product B. We consider two retail market structures and study pricing to present insights.

(i) No-Encroachment Regime: Retailer R sells products A and B.

(ii) Supplier Encroachment Regime: Retailer R sells products A and B and supplier sells B through its direct sales channel $\mathfrak{D}$.

We assume that the contract between the supplier U and the large retailer R takes the form of linear contracts. Let w be the linear wholesale fee which is paid to the supplier by the retailer R.

Consumer demand

The microfoundation for the consumer demand is modeled à la Chen & Rey (2012) where consumers are distributed according to their shopping cost s which follows the distribution function $F(\cdot)$ and a density function $f(\cdot)$ over the support $s \in [0, \infty)$. Consumers incur a shopping cost s when buying from a retailing channel.

Consumer basic value for the product B is given by $v_B > c > 0$.[2] To differentiate from the product sold at the multi-product retailer R, the branded supplier innovates and offers additional value for the product B sold through its direct channel. Therefore, the composite value of product B sold through the direct sales channel is denoted by $v_{\mathfrak{D}} \triangleq v_B + \Delta$ where $\Delta > 0$ is the additional service value offered by the branded supplier U in its direct channel

[2]This assumption ensures that the product adds value in the market.

under the encroachment regime — $v_{\mathfrak{D}} > v_B > 0$.[3] We refer to the market for product B as the (potentially) competitive market as product B can be sold by both the multi-product retailer R and the direct sales channel $\mathfrak{D}$. In contrast, product A, with utility $v_A > 0$, is sold exclusively by the multi-product retailer and is referred to as the monopoly market.

Consumers' utility from consumption of the set of products sold by retailer R (products A and B) and the product B sold by U through its direct sales channel $\mathfrak{D}$ is given as

$$U_A(p_A) \triangleq v_A - p_A, \qquad U_B(p_B) \triangleq v_B - p_B, \qquad \text{and } U_{\mathfrak{D}}(p_{\mathfrak{D}}) \triangleq v_B + \Delta - p_{\mathfrak{D}},$$

where p_A, p_B and $p_{\mathfrak{D}}$ are the respective price for product A, product B sold by R and (the augmented) product B sold by the branded supplier through its direct channel $\mathfrak{D}$.

In the following, we characterize demand under the two regimes we study.

Demand in the No-Encroachment Regime. In this regime, consumers buy both products from the large retailer and single-stop shop, they incur shopping costs once which yields consumer utility as —

$$U_{AB}(p_A, p_B) - s \triangleq U_A(\cdot) + U_B(\cdot) - s.$$

This allows us to pin down the consumer type that is indifferent between buying at R and not buying as

$$U_{AB}(\cdot) \geq 0 \implies s < v_A + v_B - p_A - p_B.$$

For brevity, we define $v_{AB} \triangleq v_A + v_B$ and $p_{AB} \triangleq p_A + p_B$ This yields the total demand as $F(v_{AB} - p_{AB})$. The following figure presents the demand distribution under the no-encroachment regime.

[3]For instance, consumers have a better experience at the Apple store than at Walmart when buying Apple products.

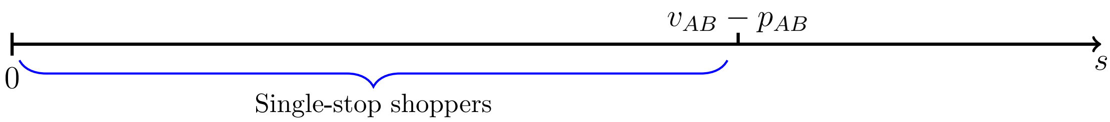

Figure 1: Shopping decisions according to shopping costs

Demand in Supplier Encroachment Regime. In this regime, the supplier enters the market for good B and offers a higher quality product through its direct sales channel. In this case, some consumers may find it profitable to mix and match the products at the two stores at the expense of increased shopping costs incurred by them — i.e., $2s$. We refer to these consumers as *multi-stop shoppers*. The utility obtained by these consumers can be written as

$$U_{A\mathfrak{D}}(p_A, p_{\mathfrak{D}}, \Delta) - 2s \triangleq U_A(\cdot) + U_{\mathfrak{D}}(\cdot) - 2s.$$

Consumers will buy the product mix obtained through multi-stop shopping only when the value derived from doing so is higher than the value derived from single-stop shopping at the retailer R. Formally,

$$U_{A\mathfrak{D}}(\cdot) - 2s \geq U_{AB}(\cdot) - s \implies s \leq \Delta - p_{\mathfrak{D}} + p_B.$$

The above expression presents the arbitrage value derived by consumers when they multi-stop shop across stores for the best deal. All consumers who have a shopping cost below the threshold $\Delta - p_{\mathfrak{D}} + p_B$ will choose to multi-home. Thus, the mass of consumers that mix and match is $F(\Delta - p_{\mathfrak{D}} + p_B)$. Further, to ensure positive demand, the multi-product retailer must set prices to ensure that the set of products on its store are at least as attractive as buying product B from the direct sales channel of the brand ($\mathfrak{D}$). This results in the fact that the total mass of consumers that are active in the market depends on two cases. Specifically, the multi-product retailer must set prices to ensure that following holds — i.e., $U_A(p_A) + U_B(p_B) - s \geq U_{\mathfrak{D}}(\cdot) - s$. We focus on the parameter constellation such that this inequality holds. Then, the total mass of consumers active in the market is pinned down by the following.

$$U_{AB} \geq 0 \implies s < U_A(\cdot) + U_B(\cdot) = v_{AB} - p_{AB}.$$

This gives the total demand as $F(v_{AB} - p_{AB})$. Keeping the above in mind and recalling the expression for multi-stop shopping demand, the mass of single-stop shoppers is given as $F(v_{AB} - p_{AB}) - F(\Delta - p_{\mathfrak{D}} + p_B)$. The following figure presents the demand distribution. Intuitively, consumers with a high s favor

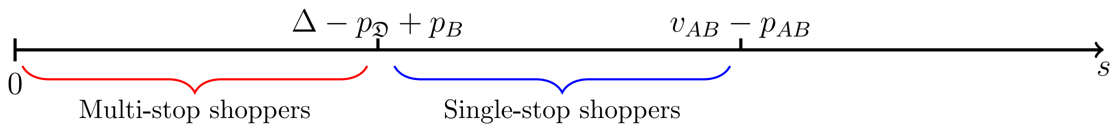

Figure 2: Shopping decisions according to shopping costs

single-homing, whereas those with a lower s can take advantage of multi-homing; the mix of single-homers and multi-homers is, however, endogenous and depends on R's prices, p_A and p_B and branded supplier's price, $p_{\mathfrak{D}}$.

Payoffs and Timing. We present the payoff of the supplier and the multi-product retailer in the two regimes below.

No-Encroachment regime. In this regime the multi-product retailer serves all consumers active in the market. In this case, the retailer's payoff is given as

$$\Pi_R(p_A, p_B, w) \triangleq \underbrace{(p_{AB} - w)F(v_{AB} - p_{AB}).}_{\substack{\text{Revenue from} \\ \text{single-stop shoppers}}}$$

Profit of the supplier U arises from wholesale revenues by supplying to the multi-product retailer R and is given as

$$\Pi_U(p_B, p_A, w) \triangleq \underbrace{(w - c)F(v_{AB} - p_{AB})}_{\text{Wholesale revenue}},$$

where $c > 0$ is the marginal cost of supplying product B.

Supplier Encroachment regime. In this regime, the multi-product retailer serves all consumers who single-stop shop. The retailer's payoff is given as

$$\Pi_R(p_A, p_B, p_{\mathfrak{D}}, w) \triangleq \underbrace{(p_{AB} - w)(F(v_{AB} - p_{AB}) - F(\Delta - p_{\mathfrak{D}} + p_B))}_{\text{Revenue from single-stop shoppers}} + \underbrace{p_A F(\Delta - p_{\mathfrak{D}} + p_B).}_{\substack{\text{Revenue from} \\ \text{multi-stop shoppers}}}$$

The profit of the multi-product retailer consists of sales to single-stop shoppers who buy both the products at R and sales of its exclusive product A to multi-stop shoppers.

Profit of the branded supplier U consists of revenues from direct sales in the retail market through its subsidiary $\mathfrak{D}$ and wholesale revenues by supplying to the multi-product retailer R and is given as

$$\Pi_U(p_{\mathfrak{D}}, p_B, p_A, w) \triangleq \underbrace{(w - c)(F(v_{AB} - p_{AB}) - F(\Delta - p_{\mathfrak{D}} + p_B))}_{\text{Wholesale revenue}} + \underbrace{(p_{\mathfrak{D}} - c)F(\Delta - p_{\mathfrak{D}} + p_B).}_{\substack{\text{Direct sales} \\ \text{revenues}}}$$

where $c > 0$ is the marginal cost of supplying product B. The branded supplier sells product B to multi-homers through its direct sales channel and supplies the multi-product retailer who caters to single-homers.

Timing, contracts and equilibrium concept. The timing of the game is as follows.

(t=1) (**Contracting:**) At stage one, the branded supplier sets the linear wholesale fee (w) to the multi-product retailer R.

(t=2) (**Retail pricing stage:**) Multi-product retailer R either accepts or rejects the contract offers. If R accepts the contract offer, it sets prices p_A and p_B. In case of supplier encroachment, the supplier sets price for its direct channel $p_{\mathfrak{D}}$ simultaneously with the prices set by R. Consumers buy and profits are realized.

Assumption 1 *We impose the following technical restrictions.*

(1). The inverse hazard rate, $g(\cdot) \triangleq F(\cdot)/f(\cdot)$, *is strictly increasing.*

(2.) $v_B \geq v_A > \Delta > 0$.

The first assumption ensures the quasi-concavity of the multi-product retailer R's profit.[4] The second assumption restricts the advantage of the supplier's direct sales channel. This ensures that the multi-product retailer offers some advantage when offering both the products.

We solve the game backwards to obtain the subgame perfect equilibrium of this game. We start by presenting the outcome of the no-encroachment regime and then compare it with the outcome in supplier encroachment regime.

4 The No-Encroachment Setting

In this case, only the multi-product retailer R is active in the retail market. To make our point clear, we present a useful benchmark.

A useful benchmark. Suppose only product A was available in the market. In this case, demand for product A is just $F(v_A - p_A)$ and the profit of R is $\Pi_A = p_A F(v_A - p_A)$. Differentiating the profit with respect to p_A and solving yields that the optimal price is characterized by $p_A^M = \frac{F(v_A - p_A^M)}{f(v_A - p_A^M)} = g\left(v_A - p_A^M\right)$. The associated optimal profit of R is then $\Pi_A^M = p_A^M F(v_A - p_A^M)$.

Retail Pricing Stage: In this stage, the multi-product retailer R sets its price to maximize its profits as presented below.

$$\max_{p_A, p_B} \Pi_R(p_{AB}, w) \triangleq (p_{AB} - w)F(v_{AB} - p_{AB}). \tag{1}$$

Differentiating the above with respect to p_A and p_B yields the following first-order conditions

$$\frac{\partial \Pi_R(\cdot)}{\partial p_k} = F(v_{AB} - p_{AB}) - (p_{AB} - w)f(v_{AB} - p_{AB}) = 0, \text{ for } k \in \{A, B\}. \tag{2}$$

[4]See Chen & Rey (2012) for details on this.

The above first-order condition presents the classical margin and volume trade-off faced by a monopolist when setting prices. Solving the above equation yields the condition that characterizes the price as a function of linear wholesale fees. Specifically, The optimal bundle price as a function of wholesale fee w is characterized as $p^\star_{AB}(w) = w + g(v_{AB} - p^\star_{AB}(w))$.

Lemma 1 *The optimal bundle price is increasing in w — i.e.,* $\frac{\partial p^\star_{AB}(w)}{\partial w} = \frac{1}{1+g'(v_{AB}-p^\star_{AB}(w))} \geq 0$.

Substituting these optimal prices into the demand and the profit expression yields the profit of the retailer as

$$\Pi^\star_R(w) \triangleq (p^\star_{AB}(w) - w)F(v_{AB} - p^\star_{AB}(w)). \tag{3}$$

Contracting Stage: In the contracting stage, the supplier sets the linear fees to maximize profits given by

$$\max_w \Pi^\star_U(w) \triangleq (w - c)F(v_{AB} - p^\star_{AB}(w)) \text{ subject to } \Pi^\star_R(w) \geq \Pi^M_A.$$

For any contract to be accepted by the retailer, the retailer's profit must be higher or equal to its outside option of selling just good A. Note that the constraint on retailer profit is binding for $w = \overline{w} = v_B$. This is because the retailer R will never accept negative margins on good B.
Differentiating the above profit with respect to w yields the following first-order condition

$$\underbrace{F(v_{AB} - p^\star_{AB}(w))}_{\text{Margin Effect}} \underbrace{-(w-c)f(v_{AB} - p^\star_{AB}(w))\frac{\partial p^\star_{AB}(w)}{\partial w}}_{\text{Volume Effect}} = 0. \tag{4}$$

The above first-order condition presents classical volume and margin trade-off faced by a firm setting fees. Dividing the above first-order condition by $f(v_{AB} - p^\star_{AB}(w))$ and recalling from Lemma (1) that $\frac{\partial p^\star_{AB}(w)}{\partial w} = \frac{1}{1+g'(v_{AB}-p^\star_{AB}(w))} > 0$, we can rewrite the above first condition as

$$g(v_{AB} - p^\star_{AB}(w))[1 + g'(v_{AB} - p^\star_{AB}(w))] - (w - c) = 0. \tag{5}$$

The (interior solution) optimal fee solves the above first-order condition and is denoted by $w^\star$. There also exists a corner solution and we present the conditions for both outcomes. For ease of presentation, let us define $\Gamma(\cdot) \triangleq g(\cdot)[1 + g'(\cdot)]$. A corner solution exists if the above first-order condition is positive for $w = v_B$ and otherwise there exists an interior solution.[5]

Lemma 2 *When $v_B - c \geq \Gamma(v_A - p_A^M)$, the optimal fee is an interior solution characterized by*

$$w^\star - c = \Gamma(v_{AB} - p_{AB}^\star(w^\star)).$$

Otherwise, we have a corner solution with $w_c^\star = v_B$.

Substituting these wholesale fees into the bundle pricing strategy of R yields

$$\begin{cases} p_{AB}^\star(w^\star)) & \text{if } v_B - c \geq \Gamma(v_A - p_A^M), \\ p_A^M + v_B & \text{if } v_B - c < \Gamma(v_A - p_A^M). \end{cases}$$

Substituting this bundle price into the demand expression yields the demand outcome for the interior and the corner case as follows.

$$\begin{cases} F(v_{AB} - p_{AB}^\star(w^\star)) & \text{if } v_B - c \geq \Gamma(v_A - p_A^M), \\ F(v_A - p_A^M) & \text{if } v_B - c < \Gamma(v_A - p_A^M). \end{cases}$$

The associated profit expression of the multi-product retailer R is given as follows.

$$\begin{cases} \Pi_R^\star(p_{AB}^\star(w^\star), w^\star) & \text{if } v_B - c \geq \Gamma(v_A - p_A^M), \\ \Pi_A^M & \text{if } v_B - c < \Gamma(v_A - p_A^M). \end{cases}$$

The associated profit expression of the supplier U is given as follows.

$$\begin{cases} \Pi_U^\star(w^\star) & \text{if } v_B - c \geq \Gamma(v_A - p_A^M), \\ \Pi_U^\star(w_c^\star) = (v_B - c)F(v_A - p_A^M) & \text{if } v_B - c < \Gamma(v_A - p_A^M). \end{cases}$$

[5] A direct consequence of $w = v_B$ is that $p_B = v_B$ implying that $p_{AB} - w = p_A$.

5 The Supplier Encroachment Setting

In the following, we discuss the case when the supplier decides to enter and compete with R. Supplier chooses to do this through a direct sales channel denoted by $\mathfrak{D}$. In this direct sales channel, the supplier offers a better value for the product B and this differentiates its offering vis-à-vis the sales with R. Specifically, consumer value associated with purchases of B from the direct channel is $v_{\mathfrak{D}} = v_B + \Delta$. This differentiation results in some consumers multi-stop shopping presented in the Section (3). Consumer with low shopping costs multi-stop shop and mix-and-match their purchase to benefit from the augmented offering of product B through the direct sales channel $\mathfrak{D}$. Instead, consumers who face high shopping costs save on them. The mix of single-homers and multi-homers is, however, endogenous and depends on R's prices, p_A and p_B, and U's price $p_{\mathfrak{D}}$. For reference on the demand split, we refer the reader to Figure (2).

Retail Pricing Stage: In this stage, both the supplier U and the multi-product retailer R set prices to maximize their profits as presented below.

$$\begin{aligned}
\max_{p_{\mathfrak{D}}} \Pi_U(p_{\mathfrak{D}}, p_B, p_A, w) &\triangleq \underbrace{(w-c)\left[F(v_{AB}-p_{AB}) - F(\Delta - p_{\mathfrak{D}} + p_B)\right]}_{\text{Wholesale revenue}} \\
&\quad + \underbrace{(p_{\mathfrak{D}} - c)F(\Delta - p_{\mathfrak{D}} + p_B),}_{\substack{\text{Direct sales}\\ \text{revenues}}} \\
\max_{p_A, p_B} \Pi_R(p_A, p_B, p_{\mathfrak{D}}, w) &\triangleq \underbrace{(p_{AB}-w)\left[F(v_{AB}-p_{AB}) - F(\Delta - p_{\mathfrak{D}} + p_B)\right]}_{\text{Revenue from single-stop shoppers}} \\
&\quad + \underbrace{p_A F(\Delta - p_{\mathfrak{D}} + p_B).}_{\substack{\text{Revenue from}\\ \text{multi-stop shoppers}}}
\end{aligned}$$

To compare with the no-encroachment setting, maximization problem of the multi-product retailer can be rewritten as follows.

$$\max_{p_{AB}, p_B} \Pi_R(p_{AB}, p_B, p_{\mathfrak{D}}, w) \triangleq (p_{AB}-w)F(v_{AB}-p_{AB}) - (p_B - w)F(\Delta - p_{\mathfrak{D}} + p_B).$$

Differentiating the above expression with respect to p_{AB} and p_B yields the following set of first conditions

$$-(p_{AB} - w) + g(v_{AB} - p_{AB}) = 0, \quad (6)$$
$$-(p_B - w) - g(\Delta - p_{\mathfrak{D}} + p_B) = 0. \quad (7)$$

The above first-order condition characterizes the best response of the retailer to a change in the price at U's direct sale channel. Note that the bundle price p_{AB} is independent of $p_{\mathfrak{D}}$ and is not affected by the pricing strategy of U at its direct sales channel. Instead, the pricing strategy for the good B supplied by U is affected by the price of $U's$ direct sales channel. In addition, as in Chen & Rey (2012) the margin of good B at R is negative — i.e., $(p_B - w) = -g(\Delta - p_{\mathfrak{D}} + p_B) < 0$ as $g(\cdot) > 0$. This feature plays an important role in our discussion later.

Differentiating the profit of the supplier (as presented above) with respect to $p_{\mathfrak{D}}$ yields the following first-order condition.

$$-(p_{\mathfrak{D}} - w) + g(\Delta - p_{\mathfrak{D}} + p_B) = 0. \quad (8)$$

The above first-order condition presents the optimal pricing strategy of the supplier U. Comparing the best response of the supplier's pricing strategy and retailer's pricing strategy, we notice that $\frac{\partial p_B^{BR}(p_{\mathfrak{D}})}{\partial p_{\mathfrak{D}}} = \frac{\partial p_{\mathfrak{D}}^{BR}(p_B)}{\partial p_B} = \frac{g'(\Delta - p_{\mathfrak{D}} + p_B)}{1 + g'(\Delta - p_{\mathfrak{D}} + p_B)} \in (0, 1)$. The above comparative static is straightforward and follows directly from the fact that both the retailer R and the supplier U are competing in the market for product B.

Solving the above first-order conditions presented in equations (6), (7) and (8) simultaneously yields the optimal price as a function of wholesale fees denoted by $\widehat{p}^\star_{AB}(w)$, $\widehat{p}^\star_B(w)$ and $\widehat{p}^\star_{\mathfrak{D}}(w)$. Performing some comparative statics with respect to w, we present the results in the following Lemma.

Lemma 3 *The optimal price of good B at retailer R, the price at the direct sales channel of U and bundle price increase in w — i.e.,* $\frac{\partial \widehat{p}^\star_B(w)}{\partial w} = \frac{\partial \widehat{p}^\star_{\mathfrak{D}}(w)}{\partial w} = 1$, $\frac{\partial \widehat{p}^\star_{AB}}{\partial w} = \frac{1}{1 + g'(v_{AB} - \widehat{p}^\star_{AB}(w))} \in [0, 1]$.

The intuition for the above results is as follows. An increase in the wholesale fee charged by the supplier U to the multi-product retailer is an increase in the marginal cost of selling product B which increases the market price $(\widehat{p}^{\star}_{B}(\cdot))$ as well. Further as established before that prices are strategic complements, an increase in wholesale fee also increases the price of product B sold by the supplier U through its direct sales channel $\mathfrak{D}$. Interestingly, an increase in the wholesale fee, w, increases the price of the bundle at R by a smaller amount than the increase in price of good B at R. This implies that an increase in the wholesale fee lowers the price charged for product A by the large retailer. This is because an increase in w increases the price of product B at R implying that the value of the basked for single-homers falls. To avoid loss in volume of demand from single-homers, the retailer R finds it profitable to lower p_A to maximize profits.

Substituting these optimal prices into the profit expression of the supplier and the retailer R yields the profits $\widehat{\Pi}^{\star}_{U}(w) \triangleq \Pi_U(\widehat{p}^{\star}_{\mathfrak{D}}(w), \widehat{p}^{\star}_{AB}(w), \widehat{p}^{\star}_{B}, w)$ and $\widehat{\Pi}^{\star}_{R}(w) \triangleq \Pi_R(\widehat{p}^{\star}_{AB}(w), \widehat{p}^{\star}_{B}(w), \widehat{p}^{\star}_{\mathfrak{D}}(w), w)$. Before, we move forward it is worth discussing a few details. Keeping wholesale fees fixed (and in an interior solution), the profit of retailer R is higher in the encroachment regime.

$$\widehat{\Pi}^{\star}_{R}(w) - \Pi^{\star}_{R}(w) = -(\widehat{p}^{\star}_{B}(w) - w)F(\Delta - \widehat{p}^{\star}_{\mathfrak{D}}(w) + \widehat{p}^{\star}_{B}(w)) > 0. \tag{9}$$

As the retailer chooses a negative margin for sales on good B, the above expression is positive. This result follows from Chen & Rey (2012) and helps clarify ideas later.

Contracting Stage: In the contracting stage, the supplier sets the linear fee w to maximize profits given by

$$\max_{w} \widehat{\Pi}^{\star}_{U}(w) \text{ subject to } \widehat{\Pi}^{\star}_{R}(w) \geq \Pi^{M}_{A}.$$

For any contract to be accepted by the retailer, the retailer must be better off or equal to its outside option of selling just good A. Let us denote the wholesale fee $\widehat{\overline{w}}$ such that the retailer's constraint is binding — i.e., $\widehat{\Pi}^{\star}_{R}(\widehat{\overline{w}}) = \Pi^{M}_{A}$. It must be that this binding fee $\widehat{\overline{w}} > \overline{w} = v_B$. This follows from two observations. First, keeping wholesale fees fixed, retailer profit under encroachment is higher than under no-encroachment (see the discussion after Equation (9)). Second, $\widehat{\Pi}^{\star}_R(w)$ *is decreasing in* w, and in the no-encroachment case, the binding wholesale fee is given by $\overline{w} = v_B$.

For ease of comparison of both, $\widehat{\overline{w}}$ and $\overline{w}$, let us define μ, which is given by $\mu \triangleq \widehat{\overline{w}} - \overline{w}$, with $\overline{w} = v_B$. We present later a discussion on μ.

Differentiating the supplier's profit with respect to w and employing the envelope theorem yields the following first-order condition

$$\underbrace{F(v_{AB} - \widehat{p}^{\star}_{AB}(\cdot)) - F(\Delta + \widehat{p}^{\star}_{B}(\cdot) - \widehat{p}^{\star}_{\mathfrak{D}}(\cdot))}_{\text{Margin Effect } (+)} \underbrace{-(w-c) f(v_{AB} - \widehat{p}^{\star}_{AB}(w)) \frac{\partial \widehat{p}^{\star}_{AB}(w)}{\partial w}}_{\text{Volume Effect } (-)}$$
$$+ \underbrace{(\widehat{p}^{\star}_{\mathfrak{D}}(\cdot) - w) f(\Delta + \widehat{p}^{\star}_{B}(w) - \widehat{p}^{\star}_{\mathfrak{D}}(w)) \frac{\partial \widehat{p}^{\star}_{B}(w)}{\partial w}}_{\text{Strategic Effect } (+)} = 0. \tag{10}$$

The above first-order condition presents how supplier profit changes with a unit change in fees. As in the no-encroachment case (see Equation (4)), the supplier faces the classical margin and volume trade-off. In addition, there is also the strategic effect arising solely due to encroachment by the supplier U. Specifically, a unit increase in fees (w) lowers the value of the bundle as the price of the bundle increases. This encourages some consumers who were single-homing at the multi-product retailer to multi-home and buy good B from the direct sales channel of U. This increases the volume of direct sales and thus encourages the supplier to set higher fees. We simplify the above first-order condition by recalling two points: (i) from Equation (8) that $(\widehat{p}^{\star}_{\mathfrak{D}}(w) - w) = g(\Delta + \widehat{p}^{\star}_{B}(w) - \widehat{p}^{\star}_{\mathfrak{D}}(w))$ (ii) from Lemma (3) that $\frac{\partial \widehat{p}^{\star}_{B}(w)}{\partial w} = \frac{\partial \widehat{p}^{\star}_{\mathfrak{D}}(w)}{\partial w} = 1$ and simplifying yields the following first-order condition.

$$-(w-c) f(v_{AB} - \widehat{p}^{\star}_{AB}(w)) \frac{\partial \widehat{p}^{\star}_{AB}(w)}{\partial w} + F(v_{AB} - \widehat{p}^{\star}_{AB}(w)) = 0.$$

Dividing the above first-order condition by $f(v_{AB} - \widehat{p}^{\star}_{AB}(w))$ and recalling that $\frac{\partial \widehat{p}^{\star}_{AB}(w)}{\partial w} = \frac{1}{1+g'(v_{AB} - \widehat{p}^{\star}_{AB}(w))} > 0$ and $\Gamma(\cdot) = g(\cdot)[1 + g'(\cdot)]$, we can rewrite the above first condition as

$$\Gamma(v_{AB} - \widehat{p}^{\star}_{AB}(w)) - (w-c) = 0. \tag{11}$$

Note that the above expression is analogous with the expression in Equation (5). The (interior solution) optimal fee solves the above first-order condition and is denoted by $\widehat{w}^{\star}$. There also exists a corner solution, i.e., $\widehat{\overline{w}} = v_B + \mu$ and we present the conditions for both outcomes. Before presenting the conditions for both outcomes, we now explain how we obtain μ.

A discussion on μ. To be more specific, μ satisfies $\widehat{\Pi}^{\star}_R (v_B + \mu) = \Pi^M_A$. Moreover, keeping wholesale fees fixed, the profit of retailer R is higher in the encroachment setting than in the no-encroachment setting. Let us define this difference in the retailer's profit as a function of wholesale fees as

$$\Pi_{MSS} \triangleq \widehat{\Pi}^{\star}_R(w) - \Pi^{\star}_R(w) = -(\widehat{p}^{\star}_B(w) - w)F(\Delta - \widehat{p}^{\star}_{\mathfrak{D}}(w) + \widehat{p}^{\star}_B(w)) > 0.$$

Before proceeding further, it is useful to note that $\Pi_{MSS}(w)$ is constant in w. Recall that $\frac{\partial \widehat{p}^{\star}_B(w)}{\partial w} = \frac{\partial \widehat{p}^{\star}_{\mathfrak{D}}(w)}{\partial w} = 1$, the result is immediate. By using (7) and (8), we can write,

$$\Pi_{MSS}(w) = g\left(\Delta - \widehat{p}^{\star}_{\mathfrak{D}}(w) + \widehat{p}^{\star}_B(w)\right) F(\Delta - \widehat{p}^{\star}_{\mathfrak{D}}(w) + \widehat{p}^{\star}_B(w))$$

where $\widehat{p}^{\star}_B(w) - \widehat{p}^{\star}_{\mathfrak{D}}(w) = -2g\left(\Delta - \widehat{p}^{\star}_{\mathfrak{D}}(w) + \widehat{p}^{\star}_B(w)\right)$, that is invariant in w. Using this result and (6), we can rewrite $\widehat{\Pi}^{\star}_R(w)$ as,

$$\widehat{\Pi}^{\star}_R(w) = g\left(v_{AB} - \widehat{p}^{\star}_{AB}(w)\right) F(v_{AB} - \widehat{p}^{\star}_{AB}(w)) + \Pi_{MSS}(w),$$

where $\Pi_{MSS}(w)$ is defined above. In the following, we omit w in the expression of $\Pi_{MSS}(.)$ because it is invariant in w.

Now employing our previous definition of μ, we can state that, μ satisfies

$$\left| g\left(v_{AB} - \widehat{p}^{\star}_{AB}(v_B + \mu)\right) F(v_{AB} - \widehat{p}^{\star}_{AB}(v_B + \mu)) - \underbrace{g\left(v_A - p^M_A\right) F(v_A - p^M_A)}_{\Pi^M_A} \right| = \Pi_{MSS}.$$

The expression on the left hand side of the equality represents the losses arising from one-stop shopping behavior (one-stop shoppers and multi-stop shoppers), whereas the expression on the right hand side term presents the profit value from the multi-stop shoppers. Having explained how we obtain μ,[6] we now present the conditions for both outcomes (i.e., an interior solution or a corner solution) with respect to the optimal fee.

Lemma 4 *When $v_B - c \geq \Gamma(v_{AB} - \widehat{p}^{\star}_{AB}(\widehat{\overline{w}})) - \mu$, the optimal fee is an interior solution characterized by*

$$\widehat{w}^{\star} - c = \Gamma(v_{AB} - \widehat{p}^{\star}_{AB}(\widehat{w}^{\star})).$$

Otherwise, we have a corner solution with $\widehat{w}^{\star}_c = \widehat{\overline{w}} = v_B + \mu$.

This Lemma helps us characterize fees in the interior and the corner solution case. It is straightforward to note that in the interior solution case, the wholesale fee charged is identical to the (interior solution) fee under no-encroachment.[7]

Substituting these wholesale fees into the bundle pricing strategy of R yields

$$\begin{cases} \widehat{p}^{\star}_{AB}(\widehat{w}^{\star}) & \text{if } v_B - c \geq \Gamma(v_{AB} - p^{\star}_{AB}(\widehat{\overline{w}})) - \mu, \\ \widehat{p}^{\star}_{AB}(\widehat{\overline{w}}) & \text{if } v_B - c < \Gamma(v_{AB} - p^{\star}_{AB}(\widehat{\overline{w}})) - \mu. \end{cases}$$

Substituting this bundle price into the demand expression yields the total demand in the market for the interior and the corner case as follows.

$$\begin{cases} F(v_{AB} - \widehat{p}^{\star}_{AB}(\widehat{w}^{\star})) & \text{if } v_B - c \geq \Gamma(v_{AB} - p^{\star}_{AB}(\widehat{\overline{w}})) - \mu, \\ F(v_{AB} - \widehat{p}^{\star}_{AB}(\widehat{\overline{w}})) & \text{if } v_B - c < \Gamma(v_{AB} - p^{\star}_{AB}(\widehat{\overline{w}})) - \mu. \end{cases}$$

The associated profit expression of the multi-product retailer R is given as follows.

$$\begin{cases} \widehat{\Pi}^{\star}_{R}(\widehat{w}^{\star}) & \text{if } v_B - c \geq \Gamma(v_{AB} - p^{\star}_{AB}(\widehat{\overline{w}})) - \mu, \\ \Pi^{M}_{A} & \text{if } v_B - c < \Gamma(v_{AB} - p^{\star}_{AB}(\widehat{\overline{w}})) - \mu. \end{cases}$$

[6] Note that we also used this characterization of μ later to discuss how consumer surplus is affected by encroachment.

[7] To focus on our main point, we impose that $\widehat{p}^{\star}_{B}(v_B + \mu) < v_B$. This is imposed to ensure that one-stop shoppers buy the good B for any value of w, that we consider. As $\widehat{p}^{\star}_{B}(w)$ is increasing in w, we can claim the following: if it is true for $w = v_B + \mu$, it is true for any $w < v_B + \mu$.

The associated profit expression of the supplier U is given as follows.

$$\begin{cases} \widehat{\Pi}^\star_U(\widehat{w}^\star) & \text{if } v_B - c \geq \Gamma(v_{AB} - p^\star_{AB}(\widehat{\overline{w}})) - \mu, \\ \widehat{\Pi}^\star_U(\widehat{\overline{w}}) & \text{if } v_B - c < \Gamma(v_{AB} - p^\star_{AB}(\widehat{\overline{w}})) - \mu. \end{cases}$$

6 The Welfare Implications of Encroachment

In the following, we discuss how supplier encroachment affects the market.

Before proceeding with the comparison of the two regimes, it is informative to define three regions.

Region 1: $\Gamma(v_A - p^M_A) \leq v_B - c$**.** In this region, the wholesale fees are characterized by an interior solution in both the regimes. Observing the first-order conditions on the two regimes, it is straightforward that the wholesales fees are identical in the two regime — i.e., $w^\star = \widehat{w}^\star$.

Region 2: $\Gamma(v_{AB} - \hat{p}^\star_{AB}(\widehat{\overline{w}})) - \mu \leq v_B - c < \Gamma(v_A - p^M_A)$**.** In this region, the wholesale fees are characterized by a corner solution in the no-encroachment regime and an interior solution in the encroachment regime. Interestingly, fees under encroachment are higher than under no-encroachment. Specifically, the interior encroachment fee is larger than even the market value of good B at retailer B — i.e., $\widehat{w}^\star > \overline{w} = v_B$.

Region 3: $0 < v_B - c < \Gamma(v_{AB} - \hat{p}^\star_{AB}(\widehat{\overline{w}})) - \mu$**.** In this region, wholesale fees in both regimes are characterized by their respective corner solutions. In this case as well, fees under encroachment are higher than under no-encroachment. — i.e., $\widehat{\overline{w}} > \overline{w} = v_B$ with $\widehat{\overline{w}} = v_B + \mu$.

The following figure presents the wholesale fee comparison in the two regimes. Having discussed the impact of encroachment on wholesale fees, it is now useful to discuss how the total bundle profit is affected.

Proposition 1 (Price and Demand Comparison.) *Comparing consumer prices in the two regimes, we find the following.*

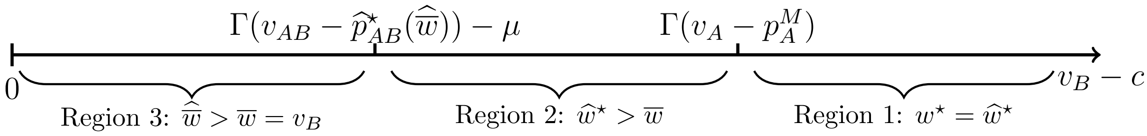

Figure 3: Wholesale fees comparison in the three regions.

- *In region 1, bundle price and total market demand are identical in the two regimes — i.e.,* $p^{\star}_{AB}(w^{\star}) = \widehat{p}^{\star}_{AB}(\widehat{w}^{\star})$.
- *In region 2, bundle price under no-encroachment is* $p_A^M + v_B$ *and under encroachment is given as* $\widehat{p}^{\star}_{AB}(\widehat{w}^{\star})$ *with the total bundle price being higher under encroachment — i.e.,* $\widehat{p}^{\star}_{AB}(\widehat{w}^{\star}) > p_A^M + v_B$*. total demand is lower in the encroachment regime.*
- *In region 3, bundle price under no-encroachment is* $p_A^M + v_B$ *and under encroachment is given as* $\widehat{p}^{\star}_{AB}(\widehat{\widehat{w}})$ *with the total bundle price being higher under encroachment — i.e.,* $\widehat{p}^{\star}_{AB}(\widehat{\widehat{w}}) > p_A^M + v_B$*. Total demand is lower in the encroachment regime.*

The above result follows through from the comparative static of the total price with respect to w. Specifically, we demonstrated earlier that the total bundle price is rising in w. As we know that the wholesale fees are weakly higher in encroachment regime, it follows directly that the total bundle price is also higher. This result has direct implications on total demand in the market.

The result on consumer demand follows directly from the results on total bundle price. As the total bundle price is higher under encroachment in region (2) and region (3), it is straightforward that the total demand must be lower in these two regions.

Now, we are in a position to discuss retail profits. An important observation from the above is that wholesale fee charged to the retailer R is weakly higher in the encroachment regime vis-à-vis the no-encroachment regime. This leads to weakly higher total bundle price which implies total demand is lower under encroachment. Further, in the encroachment regime, the supplier competes with the good B offering of retailer R. As a consequence of this, one would expect that the retailer's profit falls in the encroachment regime. Interestingly, we find instead that the retailer R 's profit (weakly) increases after supplier encroachment. The following proposition discusses these results.

Proposition 2 (Retail and Supplier Profit Comparison.) *Supplier U's profit is always higher under encroachment. Comparing retail profit in the two regimes, we find the following.*

- *In region 1, the retailer's profit is higher under encroachment — i.e.,* $\widehat{\Pi}_R^\star(\widehat{w}^\star) > \Pi_R^\star(w^\star)$.
- *In region 2, the retailer's profit is higher under encroachment — i.e.,* $\widehat{\Pi}_R^\star(\widehat{w}^\star) > \Pi_A^M$.
- *In region 3, retailer's profit is identical in the two regimes; the retailer earns* Π_A^M.

It is not surprising that the supplier's profit is higher under encroachment. The supplier could always organize its fees to earn at least the profit under no-encroachment. Under encroachment, it has more tools to control value creation and thus is always better off.

Consumer Surplus Analysis. Let $U_{AB}(\cdot) = v_{AB} - p_{AB}(\cdot)$ denote the consumer value of single-stop shopping (without accounting for shopping costs). The expression for consumer surplus under no-encroachment is given as

$$CS(\cdot) = \underbrace{\int_0^{U_{AB}(\cdot)} (U_{AB}(\cdot) - s)dF(s)}_{\text{One-stop shopping surplus}}.$$

Plugging in the optimal bundle price (as a function of w) in the no-encroachment case into the above expression yields the consumer surplus as $CS^\star(w) \triangleq CS(p_{AB}^\star(w))$. In addition, we define utilities as a function of w as $U_{AB}^\star(w) = U_{AB}(p_{AB}^\star(w))$.

Under encroachment, it is useful to breakdown the consumer value into the sum of two terms: the value of single-shopping and the arbitrage value in case of multi-stop shopping. Thus, we can write the utility from multi-stop shopping as $U_{A\mathfrak{D}}(\cdot) = U_{AB}(\cdot) + U_{\mathfrak{D}}(\cdot) - U_B(\cdot)$ where $U_{\mathfrak{D}}(\cdot) - U_B(\cdot)$ is the arbitrage value of multi-stop shopping (without accounting for shopping costs).

The total consumer surplus under encroachment can be written as

$$\widehat{CS}(\cdot) = \underbrace{\int_0^{U_{AB}(\cdot)} (U_{AB}(\cdot) - s)dF(s)}_{\text{One-stop shopping surplus } (CS(\cdot))} + \underbrace{\int_0^{U_{\mathfrak{D}}(\cdot)-U_B(\cdot)} \left(U_{\mathfrak{D}}(\cdot) - U_B(\cdot) - s\right) dF(s)}_{\text{Arbitrage Surplus for Multi-stop shoppers}},$$

where the first term represents the value of one-stop shopping (one-stop shoppers and multi-stop shoppers), whereas the second term is the arbitrage value enjoyed by multi-stop shopping (multi-stop shoppers only). Plugging in the optimal prices as a function of w, yields $\widehat{CS}^{\star}(w) \triangleq \widehat{CS}(\widehat{p}^{\star}_{AB}(w), \widehat{p}^{\star}_B(w), \widehat{p}^{\star}_{\mathfrak{D}}(w))$. Further, we define as utilities for given w as $\widehat{U}^{\star}_{AB}(w) = U^{\star}_{AB}(w)$, $\widehat{U}^{\star}_B(w) = U_B(\widehat{p}^{\star}_B(w))$ and $\widehat{U}^{\star}_{\mathfrak{D}}(w) = U_{\mathfrak{D}}(\widehat{p}^{\star}_{\mathfrak{D}}(w))$.

From a cursory glance at the expressions for consumer surplus in the two settings, keeping w fixed (across settings), consumer surplus under encroachment is unambiguously higher than under no-encroachment. Note that the first term into the consumer surplus under encroachment corresponds to the consumer surplus under no-encroachment (keeping w fixed). Further, $CS^{\star}(\cdot)$ is decreasing in w as $U^{\star}_{AB}(\cdot)$ decreases when w increases. The second term into the expression of consumer surplus under encroachment corresponds to the arbitrage surplus for multi-stop shoppers. Note this term, that is positive is constant as w increases ($\frac{\partial \widehat{p}^{\star}_B(w)}{\partial w} = \frac{\partial \widehat{p}^{\star}_{\mathfrak{D}}(w)}{\partial w} = 1$).[8]

Proposition 3 (Consumer Surplus Comparison.) *Comparing consumer surplus in the two regimes, we find the following.*

[8]The expression $\int_0^{\widehat{U}^{\star}_{\mathfrak{D}}(\cdot)-\widehat{U}^{\star}_B(\cdot)} \left(\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_B(\cdot) - s\right) dF(s)$ is invariant in w as $\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_B(\cdot) = \Delta - \widehat{p}^{\star}_{\mathfrak{D}}(w) + \widehat{p}^{\star}_B(\cdot)$ does not depend on w. We have $\frac{\partial \widehat{p}^{\star}_B(w)}{\partial w} = \frac{\partial \widehat{p}^{\star}_{\mathfrak{D}}(w)}{\partial w} = 1$, See Lemma (3).

- *In region 1, consumer surplus is higher under encroachment.*
- *In region 2 and 3, consumer surplus is higher under encroachment if and only if the surplus gains from multi-stop shopping are greater than the surplus loss in case of one-stop shopping.*

We consider successively the two parameter constellations, that we distinguished above in the Proposition, i.e., $v_B - c \geq \Gamma(v_A - p_A^M)$ and $v_B - c < \Gamma(v_A - p_A^M)$.

$\underline{v_B - c \geq \Gamma(v_A - p_A^M)}$: The wholesale sale price is unchanged when the manufacturer encroaches. Encroachment benefits lower shopping costs consumers, who are now better off by mixing and matching, whereas the consumer surplus of one-stop shopping does not change. There is an increase in total consumer surplus, which is given by

$$\Delta_{CS} = \int_0^{\widehat{U}_{\mathfrak{D}}^\star(w^\star)-\widehat{U}_B^\star(w^\star)} \left(\widehat{U}_{\mathfrak{D}}^\star(w^\star) - \widehat{U}_B^\star(w^\star) - s\right) dF\left(s\right) > 0.$$

It means consumer surplus increases in the encroachment setting in region 1.

$\underline{v_B - c < \Gamma(v_A - p_A^M)}$: In this case, two regions have to be studied: region 2 and region 3. In region 2, the wholesale price in case of no-encroachment is given by v_B, whereas in the encroachment case we get $\widehat{w}^\star$ with $\widehat{w}^\star \in (v_B, v_B + \mu)$. In region 3, the encroachment's wholesale price equals $v_B + \mu$, which is larger than the no-encroachment's wholesale price, that is v_B. In both regions, encroachment results in a higher wholesale price compared the no-encroachment case. We merge the analysis of the two regions. Consumer value from one-stop shopping falls (as w is higher), whereas lower shopping costs consumers now benefit from the arbitrage value of multi-stop shopping, when the manufacturer encroaches. By the above construction of the utility of multi-stop shoppers (i.e., $U_{AB} + U_{\mathfrak{D}} - U_B$), it becomes clear that all consumers are worse off in one-stop shopping (in case of encroachment) and only multi-stop shoppers enjoy the augmented value of multi-stop shopping (that is, $U_{\mathfrak{D}} - U_B$).

We first focus on the change of the value of one-stop shopping (analysis of the augmented value of multi-stop shopping follows). Let Δ_{OSS} (< 0) denote the loss in the value of one-stop shopping due to encroachment — i.e.,

$$\Delta_{OSS} = \left(\widehat{U}^{\star}_{AB}(\widehat{w}^{\star}) - U^{\star}_{AB}(v_B)\right) F\left(\widehat{U}^{\star}_{AB}(\widehat{w}^{\star})\right) - \int_{\widehat{U}^{\star}_{AB}(\widehat{w}^{\star})}^{U^{\star}_{AB}(v_B)} \left(U^{\star}_{AB}(v_B) - s\right) dF(s) < 0.$$

We know that $\widehat{U}^{\star}_{AB}(\widehat{w}^{\star}) < U^{\star}_{AB}(v_B)$ (as $\widehat{w}^{\star} > v_B$ increases in the encroachment setting). Thus, consumers with a shopping cost exceeding $\widehat{U}^{\star}_{AB}(\widehat{w}^{\star})$ do not consume, while in the no-encroachment setting they obtain just the consumption utility from good A — i.e., $U^{\star}_{AB}(w^{\star}) - s = U^{M}_{A} = v_A - p^{M}_{A}$ in consumption value. The second term represents this loss. The first term is the difference in the values of one-stop shopping in the two settings (No-Encroachment versus Encroachment setting). All consumers face a loss in one-stop shopping when the manufacturer encroaches.

We now study the arbitrage value of multi-stop shopping. Let us denote this arbitrage value as Δ_{MSS} (> 0), which is given by

$$\Delta_{MSS} = \int_{0}^{\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot)} \left(\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot) - s\right) dF(s).$$

Remember this term is invariant in w (See above, when we described the consumer surplus expression under the encroachment setting).

Overall, when the manufacturer encroaches, the consumer surplus changes by $\Delta_{CS} = \Delta_{OSS} + \Delta_{MSS}$ and consumer surplus decreases if $|\Delta_{OSS}| > \Delta_{MSS}$. The condition is given by

$$\left(U^{\star}_{AB}(\cdot) - \widehat{U}^{\star}_{AB}(\cdot)\right) F\left(\widehat{U}^{\star}_{AB}(\cdot)\right) + \int_{\widehat{U}^{\star}_{AB}(\cdot)}^{U^{\star}_{AB}(\cdot)} \left(U^{\star}_{AB}(\cdot) - s\right) dF(s) > \int_{0}^{\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot)} \left(\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot) - s\right) dF(s),$$

where the expression on the left hand side of the inequality, that is, $|\Delta_{OSS}|$ is increasing in w, and the right term is constant in w .

To study the changes in the consumer surplus in the parameter constellation, that we consider, i.e., $v_B - c < \Gamma(v_A - p^{M}_{A})$, we can also breakdown the changes according to the following three groups of consumers:

$s \leq \widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot)$, $\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot) < s < \widehat{U}^{\star}_{AB}$ and $\widehat{U}^{\star}_{AB} < s < U^{\star}_{AB}$. We can provide a figure with respect to s for the three groups of consumers. Figure, to be done.

We can thus write

$$\begin{aligned}\Delta_{CS} \;=\; & \underbrace{\int_0^{\widehat{U}^{\star}_{\mathfrak{D}}(\cdot)-\widehat{U}^{\star}_{B}(\cdot)} \left(\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot) - s\right) dF\left(s\right) - \left(U^{\star}_{AB} - \widehat{U}^{\star}_{AB}\right) F\left(\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot)\right)}_{s \leq \widehat{U}^{\star}_{\mathfrak{D}}(\cdot)-\widehat{U}^{\star}_{B}(\cdot)} \\ & \underbrace{-\left(U^{\star}_{AB} - \widehat{U}^{\star}_{AB}\right)\left[F\left(\widehat{U}^{\star}_{AB}\right) - F\left(\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot)\right)\right]}_{\widehat{U}^{\star}_{\mathfrak{D}}(\cdot)-\widehat{U}^{\star}_{B}(\cdot)<s<\widehat{U}^{\star}_{AB}} \\ & \underbrace{-\int_{\widehat{U}^{\star}_{AB}}^{U^{\star}_{AB}} \left(U^{\star}_{AB} - s\right) dF\left(s\right).}_{\widehat{U}^{\star}_{AB}<s<U^{\star}_{AB}}\end{aligned}$$

We see clearly that, when $s \leq \widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot)$ consumers who were one-stop shoppers in the No-Encroachment setting become multi-stop shoppers in the Encroachment setting. This is reflected in the expression above for the group $s \leq \widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot)$, where we have the augmented value of multi-stop shopping and the difference in the value of one-stop shopping. For the group $\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_{B}(\cdot) < s < \widehat{U}^{\star}_{AB}$, one-stop shopping prevails, but the value of one-stop shopping decreases. Lastly, for the group $\widehat{U}^{\star}_{AB} < s < U^{\star}_{AB}$, consumers, who buy in the No-Encroachment setting now do not buy in the Encroachment setting.

To go further in the changes of the consumer surplus for this parameter constellation, we can also define the wholesale price in the encroachment setting, that is $w = v_B + \widetilde{\mu}$ (with $\widetilde{\mu} > 0$) for which $\Delta_{CS} = 0$ — i.e., $\widehat{CS}^{\star}\left(v_B + \widetilde{\mu}\right) = CS^{\star}\left(v_B\right)$, where $CS^{\star}\left(v_B\right) = CS^{M}_{A}$, that is, $CS^{M}_{A} = \int_0^{U^M_A} (U^M_A - s) dF\left(s\right)$ with $U^M_A = v_A - p^M_A$. Remember in regions 2 and 3, encroachment results in higher wholesale price compared to the no-encroachment setting: we get $\widehat{w}^{\star} \in (v_B, v_B + \mu)$ under encroachment, while we get $w^{\star} = v_B$ (in the no-encroachment setting). Thus, depending on the ranking of $\widetilde{\mu}$ and μ, we can conclude. Results in terms of consumer surplus are summarized in the Corollary below.

Corollary 1 (Depending on the ranking between μ and $\tilde{\mu}$.) *If $\widetilde{\mu} > \mu$, consumer surplus is higher under encroachment for any parameter constellation.*

If $\widetilde{\mu} \leq \mu$, there exists a threshold in $v_B - c \in [\Gamma(v_{AB} - \hat{p}^{\star}_{AB}(\widehat{\overline{w}})) - \mu, \Gamma(v_A - p^M_A)]$ below which, consumer surplus is lower under encroachment and above which, consumer surplus is higher under encroachment.

Example: Assuming uniform distribution of shopping costs, we get $\mu = \widetilde{\mu} = v_A - \frac{1}{3}\sqrt{(3v_A + 2\Delta)(3v_A - 2\Delta)}$ — i.e., consumer surplus does not decrease under encroachment (Calculations are available upon request).

To sum-up, encroachment may decrease the consumer surplus, as in some conditions, it decreases the value of one-stop shopping. These conditions may arise when $v_B - c < \Gamma(v_A - p^M_A)$. We provide above a condition according to which the loss in one-stop shopping is larger than the gain in the augmented value of multi-stop shopping. Such condition arises for $\widetilde{\mu} \leq \mu$ — i.e., if $\widetilde{\mu} \leq \mu$, there exists a threshold in $v_B - c \in [\Gamma(v_{AB} - \hat{p}^{\star}_{AB}(\widehat{\overline{w}})) - \mu, \Gamma(v_A - p^M_A)]$ below which, consumer surplus is lower under encroachment. It means that for these values of $v_B - c$, we get a wholesale price $\widehat{w}^{\star}$ under encroachment, that is larger than $v_B + \widetilde{\mu}$, which results in $\Delta_{CS} < 0$; saying it differently, we have $|\Delta_{OSS}| > \Delta_{MSS}$ for these values of $v_B - c$. Further, note that the surplus of the high-shopping costs consumers always decreases when $v_B - c < \Gamma(v_A - p^M_A)$ because the wholesale price increases under encroachment. By contrast, when $v_B - c \geq \Gamma(v_A - p^M_A)$, all consumers (weakly) benefit in the Encroachment setting (because the value of one-stop shopping does not change): low-shopping costs consumers strictly benefit under encroachment while the surplus of high-shopping costs consumers does not change in both settings.

7 Conclusion

In this work, we considered the welfare implications of supplier encroachment. We found that supplier encroachment can make the supplier and the retailer (weakly) better off. This is because supplier encroachment allows screening of consumers into single-stop shoppers and multi-stop shoppers which allows the multi-product retailer to more efficiently extract consumer utility. Interestingly, we find that despite such an outcome arises consumers are better off when the value of the competitive good is large enough. This is because the introduction of the high-value direct sales channel of the supplier makes multi-stop shoppers better off while one-stop shoppers are not affected by encroachment. This leads to the interesting result that supplier encroachment can be Pareto improving when the value of the competitive product is large enough. Instead, in the case when the value of the competitive product is sufficiently low, one-stop shoppers are worse-off while multi-stop shopping consumers are better off. In this parameter constellation, consumers can be worse-off despite increased competition in the retail market. Our work offers clear policy implications to policy makers on the regulation of supplier encroachment strategies.

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A Proofs

Proof of Lemma 1. Recalling that the optimal bundle price as a function of the wholesale price w satisfies,

$$-\left(p^{\star}_{AB}\left(w\right)-w\right)+g\left(v_{AB}-p^{\star}_{AB}\left(w\right)\right)=0.$$

Differentiating this price relation with respect to w yields,

$$-\left(\frac{\partial p^{\star}_{AB}\left(w\right)}{\partial w}-1\right)-\frac{\partial p^{\star}_{AB}\left(w\right)}{\partial w}g'\left(v_{AB}-p^{\star}_{AB}\left(w\right)\right)=0.$$

Solving the above equation for $\frac{\partial p^{\star}_{AB}(w)}{\partial w}$, we get,

$$\frac{\partial p^{\star}_{AB}\left(w\right)}{\partial w}=\frac{1}{1+g'\left(v_{AB}-p^{\star}_{AB}\left(w\right)\right)}\in\left(0,1\right)\ \text{ with } g'\left(\cdot\right)>0.$$

The expression of the retailer's profit is given by $\Pi^{\star}_{R}(w)$, i.e.,

$$\Pi^{\star}_{R}(w)\triangleq\left(p^{\star}_{AB}\left(w\right)-w\right)F\left(v_{AB}-p^{\star}_{AB}\left(w\right)\right).$$

Differentiating this expression with respect to w (and using the envelop theorem) yields

$$\frac{d\Pi^{\star}_{R}(w)}{dw}=-F\left(v_{AB}-p^{\star}_{AB}\left(w\right)\right)<0,$$

which results in $\Pi^{\star}_{R}(w)$ decreasing in w.

SOC: We check the SOC with respect to p_{AB},

The FOC is,

$$-\left(p_{AB}-w\right)+g\left(v_{AB}-p_{AB}\right)=0,$$

differentiating this expression with respect to p_{AB} yields $-1-g'\left(v_{AB}-p_{AB}\right)$. The result is: the SOC is satisfied if $g'\left(\cdot\right)>-1$. Recalling that, by assumption, $g'\left(\cdot\right)>0$, we can claim that the SOC satisfied. Q.E.D. ■

Proof of Lemma 2. Let us start by the FOC, which determines the interior solution in w, $w^{\star}$ satisfies

$$-\left(w-c\right)+\left[1+g'\left(v_{AB}-p^{\star}_{AB}\left(w\right)\right)\right]g\left(v_{AB}-p^{\star}_{AB}\left(w\right)\right)=0.$$

For ease of presentation, we have defined $\Gamma\left(\cdot\right)\triangleq\left[1+g'\left(\cdot\right)\right]g\left(\cdot\right)$, $w^{\star}$ thus satisfies

$$-\left(w-c\right)+\Gamma\left(v_{AB}-p^{\star}_{AB}\left(w\right)\right)=0.$$

Before proceeding further, we check the SOC with respect w; the SOC is satisfied if $\Gamma'(\cdot) > -1$ with $\frac{\partial p^{\star}_{AB}(w)}{\partial w} \in (0,1)$ (the inequality is obtained in differentiating the above expression with respect to w). Then, differentiating $\Gamma(\cdot)$, we get the SOC is satisfied if

$$g''(\cdot)\, g(\cdot) + [1 + g'(\cdot)]\, g'(\cdot) > -1,$$

that is,

$$g''(\cdot) > -\frac{1 + [1 + g'(\cdot)]\, g'(\cdot)}{g(\cdot)}.$$

Recalling $g'(\cdot) > 0$, we can claim $g''(\cdot) \geq 0$ implies $\Gamma'(\cdot) > -1$. We can interpret $g''(\cdot) \geq 0$, as a sufficient condition for the SOC to be satisfied.

Then, the existence of a corner solution relies to the participation constraint of the retailer, i.e., $\Pi^{\star}_{R}(w) \geq \Pi^{M}_{A}$ in which $\Pi^{\star}_{R}(w)$ is decreasing in w, and in which Π^{M}_{A} is given by $\Pi^{M}_{A} \triangleq p^{M}_{A} F\left(v_A - p^{M}_{A}\right)$ (recalling p^{M}_{A} satisfies $-p^{M}_{A} + g\left(v_A - p^{M}_{A}\right) = 0$). From the participation constraint of the retailer, we can claim the retailer will never accept w larger than v_B. Replacing w by v_B in the FOC, it is easy to see that,

- if $-(w-c) + \Gamma\left(v_{AB} - p^{\star}_{AB}(w)\right)\big|_{w=v_B} < 0$, the optimal wholesale price satisfies $-(w-c) + \Gamma\left(v_{AB} - p^{\star}_{AB}(w)\right) = 0$,
- whereas we get $w = v_B$ if $-(w-c) + \Gamma\left(v_{AB} - p^{\star}_{AB}(w)\right)\big|_{w=v_B} \geq 0$ (the participation constraint is tight).

A direct consequence of $w = v_B$ is that $p_B = v_B$, which implies $p^{\star}_{AB}(v_B) = p^{M}_{A}$; using this, we can rewrite the conditions above:

- if $-(v_B - c) + \Gamma\left(v_A - p^{M}_{A}\right) < 0$, the optimal wholesale price satisfies $-(w-c) + \Gamma\left(v_{AB} - p^{\star}_{AB}(w)\right) = 0$,
- by contrast, if $-(v_B - c) + \Gamma\left(v_A - p^{M}_{A}\right) \geq 0$, the optimal wholesale price is given by $w = v_B$ (corner solution).

Q.E.D. ■

Proof of Lemma 3. The price relations, that define the equilibrium in retail prices are,

$$\begin{aligned}
-\left(\widehat{p}^{\star}_{AB}(w) - w\right) + g\left(v_{AB} - \widehat{p}^{\star}_{AB}(w)\right) &= 0, \\
-\left(\widehat{p}^{\star}_{B}(w) - w\right) - g\left(\Delta - \widehat{p}^{\star}_{\mathfrak{D}}(w) + \widehat{p}^{\star}_{B}(w)\right) &= 0, \\
-\left(\widehat{p}^{\star}_{\mathfrak{D}}(w) - w\right) + g\left(\Delta - \widehat{p}^{\star}_{\mathfrak{D}}(w) + \widehat{p}^{\star}_{B}(w)\right) &= 0.
\end{aligned}$$

Differentiating these relations with respect to w yields,

$$\begin{aligned} -\left(\frac{\partial\widehat{p}_{AB}^{\star}(w)}{\partial w}-1\right)-\frac{\partial\widehat{p}_{AB}^{\star}(w)}{\partial w}g'\left(v_{AB}-\widehat{p}_{AB}^{\star}(w)\right) &= 0, \\ -\left(\frac{\partial\widehat{p}_{B}^{\star}(w)}{\partial w}-1\right)+\left[\frac{\partial\widehat{p}_{\mathfrak{D}}^{\star}(w)}{\partial w}-\frac{\partial\widehat{p}_{B}^{\star}(w))}{\partial w}\right]g'\left(\Delta-\widehat{p}_{\mathfrak{D}}^{\star}(w)+\widehat{p}_{B}^{\star}(w)\right) &= 0, \\ -\left(\frac{\partial\widehat{p}_{\mathfrak{D}}^{\star}(w)}{\partial w}-1\right)+\left[\frac{\partial\widehat{p}_{B}^{\star}(w))}{\partial w}-\frac{\partial\widehat{p}_{\mathfrak{D}}^{\star}(w)}{\partial w}\right]g'\left(\Delta-\widehat{p}_{\mathfrak{D}}^{\star}(w)+\widehat{p}_{B}^{\star}(w)\right) &= 0. \end{aligned}$$

Solving the first equation in $\frac{\partial\widehat{p}_{AB}^{\star}(w)}{\partial w}$, we get

$$\frac{\partial\widehat{p}_{AB}^{\star}(w)}{\partial w}=\frac{1}{1+g'\left(v_{AB}-\widehat{p}_{AB}^{\star}(w)\right)};$$

recalling that $g'(\cdot)>0$ yields $\frac{\partial\widehat{p}_{AB}^{\star}(w)}{\partial w}\in(0,1)$.

Then, solving the two latter equations simultaneously in $\frac{\partial\widehat{p}_{B}^{\star}(w)}{\partial w}$ and $\frac{\partial\widehat{p}_{\mathfrak{D}}^{\star}(w)}{\partial w}$ yields $\frac{\partial\widehat{p}_{B}^{\star}(w)}{\partial w}=\frac{\partial\widehat{p}_{\mathfrak{D}}^{\star}(w)}{\partial w}=1$.

The profit of the retailer is

$$\widehat{\Pi}_{R}^{\star}(w)\triangleq\left(\widehat{p}_{AB}^{\star}(w)-w\right)F\left(v_{AB}-\widehat{p}_{AB}^{\star}(w)\right)-\left(\widehat{p}_{B}^{\star}(w)-w\right)F\left(\Delta-\widehat{p}_{\mathfrak{D}}^{\star}(w)+\widehat{p}_{B}^{\star}(w)\right).$$

Differentiating this expression with respect to w yields (and using the envelop theorem)

$$\frac{d\widehat{\Pi}_{R}^{\star}(w)}{dw}=\underbrace{\frac{\partial\widehat{\Pi}_{R}^{\star}(w)}{\partial\widehat{p}_{\mathfrak{D}}^{\star}(w)}}_{=\left(\widehat{p}_{B}^{\star}(w)-w\right)f\left(\Delta-\widehat{p}_{\mathfrak{D}}^{\star}(w)+\widehat{p}_{B}^{\star}(w)\right)}\frac{\partial\widehat{p}_{\mathfrak{D}}^{\star}(w)}{\partial w}+\underbrace{\frac{\partial\widehat{\Pi}_{R}^{\star}(w)}{\partial w}}_{=-\left[F\left(v_{AB}-\widehat{p}_{AB}^{\star}(w)\right)-F\left(\Delta-\widehat{p}_{\mathfrak{D}}^{\star}(w)+\widehat{p}_{B}^{\star}(w)\right)\right]}$$

i.e.,

$$\begin{aligned} \frac{d\widehat{\Pi}_{R}^{\star}(w)}{dw} &= \left(\widehat{p}_{B}^{\star}(w)-w\right)f\left(\Delta-\widehat{p}_{\mathfrak{D}}^{\star}(w)+\widehat{p}_{B}^{\star}(w)\right)\frac{\partial\widehat{p}_{\mathfrak{D}}^{\star}(w)}{\partial w} \\ &\quad -\left[F\left(v_{AB}-\widehat{p}_{AB}^{\star}(w)\right)-F\left(\Delta-\widehat{p}_{\mathfrak{D}}^{\star}(w)+\widehat{p}_{B}^{\star}(w)\right)\right]. \end{aligned}$$

Recalling $\frac{\partial\widehat{p}_{\mathfrak{D}}^{\star}(w)}{\partial w}=1$ and $-\left(\widehat{p}_{B}^{\star}(w)-w\right)-g\left(\Delta-\widehat{p}_{\mathfrak{D}}^{\star}(w)+\widehat{p}_{B}^{\star}(w)\right)=0$, the expression above resumes to

$$\frac{d\widehat{\Pi}_{R}^{\star}(w)}{dw}=-F\left(v_{AB}-\widehat{p}_{AB}^{\star}(w)\right).$$

It is thus straightforward to claim that $\frac{d\widehat{\Pi}^{\star}_{R}(w)}{dw} < 0$. Q.E.D. ■

Proof of Lemma 4. Let us start by recalling that $\widehat{w}^{\star}$ satisfies

$$-\left(w-c\right)+\Gamma\left(v_{AB}-\widehat{p}^{\star}_{AB}\left(w\right)\right)=0,$$

with $\Gamma\left(\cdot\right)\triangleq\left[1+g'\left(\cdot\right)\right]g\left(\cdot\right)$.

The participation constraint of the retailer is, $\widehat{\Pi}^{\star}_{R}(w)\geq\Pi^{M}_{A}$ in which, $\widehat{\Pi}^{\star}_{R}(w)$ and Π^{M}_{A} are given by

$$\begin{aligned}\widehat{\Pi}^{\star}_{R}(w) &\triangleq \left(\widehat{p}^{\star}_{AB}\left(w\right)-w\right)F\left(v_{AB}-\widehat{p}^{\star}_{AB}\left(w\right)\right)-\left(\widehat{p}^{\star}_{B}\left(w\right)-w\right)F\left(\Delta-\widehat{p}^{\star}_{\mathfrak{D}}(w)+\widehat{p}^{\star}_{B}(w)\right),\\ \Pi^{M}_{A} &\triangleq p^{M}_{A}F\left(v_{A}-p^{M}_{A}\right).\end{aligned}$$

$\widehat{\Pi}^{\star}_{R}(w)$ is decreasing in w and $\widehat{\Pi}^{\star}_{R}(v_B)>\Pi^{M}_{A}$, we can define $\mu>0$ such that,

$$\widehat{\Pi}^{\star}_{R}(v_B+\mu)=\Pi^{M}_{A},$$

i.e.,

$$\begin{aligned}&\underbrace{\left[\left(\widehat{p}^{\star}_{AB}\left(v_B+\mu\right)-v_B+\mu\right)F\left(v_{AB}-\widehat{p}^{\star}_{AB}\left(v_B+\mu\right)\right)-\Pi^{M}_{A}\right]}_{<0}\\ &\qquad\underbrace{-\left(\widehat{p}^{\star}_{B}\left(v_B+\mu\right)-v_B+\mu\right)F\left(\Delta-\widehat{p}^{\star}_{\mathfrak{D}}(v_B+\mu)+\widehat{p}^{\star}_{B}(v_B+\mu)\right)}_{>0}=0\end{aligned}$$

in which the first term is negative with $\mu>0$ and the second term is positive with $\left(\widehat{p}^{\star}_{B}\left(w\right)-w\right)<0$.

Then, considering the FOC, the existence of a corner solution depends on whether the participation constraint is tight or not,

- if $\left.-\left(w-c\right)+\Gamma\left(v_{AB}-\widehat{p}^{\star}_{AB}\left(w\right)\right)\right|_{w=v_B+\mu}<0$, the participation constraint is slacked and the solution in w is given by the FOC (interior solution),

- if $\left.-\left(w-c\right)+\Gamma\left(v_{AB}-\widehat{p}^{\star}_{AB}\left(w\right)\right)\right|_{w=v_B+\mu}\geq 0$, the participation constraint is tight and w is given by $w=v_B+\mu$.

We can rewrite the above conditions as follows,

- if $v_B-c>\Gamma\left(v_{AB}-\widehat{p}^{\star}_{AB}\left(v_B+\mu\right)\right)-\mu$, interior solution,

- if $v_B-c\leq\Gamma\left(v_{AB}-\widehat{p}^{\star}_{AB}\left(v_B+\mu\right)\right)-\mu$, corner solution, i.e., $w=v_B+\mu$.

■

Proof of Proposition 1. See the main text. ■

Proof of Proposition 2 . See the main text. ■

Proof of Proposition 3. Comparing consumer surplus in the two parameter constellations, we find the following.

In region 1, consumer surplus is higher under encroachment.

In regions 2 and 3, consumer surplus does not decrease under encroachment if $\mu \leq \widetilde{\mu}$.

When $\mu > \widetilde{\mu}$, there exists a threshold in $v_B - c, \in \left(\Gamma\left(v_{AB} - \widehat{p}^{\star}_{AB}\left(v_B + \mu\right)\right) - \mu, \Gamma\left(v_A - p^M_A\right)\right)$ below which consumer surplus is lower. (In regions 2 and 3, consumer surplus can decrease under encroachment.)

It means, we need to define properly $\widetilde{\mu}$.

As we have defined $\mu > 0$ such that,

$$\widehat{\Pi}^{\star}_R(v_B + \mu) = \Pi^M_A,$$

we can define $\widetilde{\mu}$ such that $\widehat{CS}(v_B + \widetilde{\mu}) = CS^M_A$.

$\widehat{CS}(w)$ is decreasing in w and $\widehat{CS}(v_B) > CS^M_A$, we can define $\widetilde{\mu} > 0$ such that,

$$\widehat{CS}(v_B + \widetilde{\mu}) = CS^M_A,$$

i.e.,

$$\underbrace{\left(\widehat{U}^{\star}_{AB}(\cdot) - U^M_A\right) F\left(\widehat{U}^{\star}_{AB}(\cdot)\right) - \int_{\widehat{U}^{\star}_{AB}(\cdot)}^{U^M_A} \left(U^M_A - s\right) dF\left(s\right)\bigg|_{w=v_B+\widetilde{\mu}}}_{<0}$$
$$+ \underbrace{\int_0^{\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_B(\cdot)} \left(\widehat{U}^{\star}_{\mathfrak{D}}(\cdot) - \widehat{U}^{\star}_B(\cdot) - s\right) dF\left(s\right)\bigg|_{w=v_B+\widetilde{\mu}}}_{>0} = 0$$

in which the first term is negative with $\widetilde{\mu} > 0$ and the second term is positive.

Then, Proposition 3 is obtained in comparing μ and $\widetilde{\mu}$.

If $\mu \leq \widetilde{\mu}$, it means that the optimal wholesale price is lower than $v_B + \widetilde{\mu}$ in any region, it is straightforward to claim that consumer surplus does not decrease under encroachment.

By contrast, if $\mu > \widetilde{\mu}$, it means that consumer surplus may decrease under encroachment. Recalling in region 3, the optimal wholesale price is $w = v_B + \mu$, consumer surplus is lower (with $\mu > \widetilde{\mu}$), whereas in region 1 consumer surplus is higher (in this region, the optimal wholesale is lower than v_B, that is lower than $v_B + \widetilde{\mu}$, which results in higher consumer surplus). Opposing the results in region 3 and in region 1, we can claim, there exists a threshold in $v_B - c$ below which consumer decreases (the optimal wholesale is higher than $v_B + \widetilde{\mu}$), and above which consumer increases under encroachment (the optimal wholesale is lower than $v_B + \widetilde{\mu}$). $\blacksquare$