WORKING PAPERS

N° 1626

March 2025

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Endogenous altruism and long term care policies in a Mirrleesian setting[*]

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March 2025

[*]We have written this paper to honor our colleague, coauthor, and friend Pierre Pestiau. We thank him for all he has taught us over our academic careers. Helmuth Cremer gratefully acknowledges financial support from the Chaire "*Marché des risques et creation de valeur*" of the FdR/SCOR as well as the funding received by TSE from ANR under grant ANR-17-EURE-0010 (Investissements d'Avenir program).

Abstract

This study contributes to the long-term care policy literature by exploring how, in an uncertain environment, redistributive tax policies and long-term care program design interact with informal care incentives, shaping long-term caregiving outcomes. The analysis is done within an overlapping-generations model in the steady state under full and asymetric information. Altruistic children provide informal care to their elderly parents if dependent. Not all children are altruistic. Children's level of altruism is shaped by the time and attention they received in childhood. Key findings, under asymetric information, include: (i) Allocations are distorted for redistributive purposes, except for savings, (ii) marginal income tax rates are positive, aligning with standard nonlinear income taxation models, and (iii) a consequence of government's redistributive policies is to encourage time spent with children thus incresing family caregiving. These three findings apply to both "opting out" and "topping up" schemes. (iv) Savings must be subsidized in an opting out system due to fiscal externalities; (v) if public assistance carries a stigma, it may have to be distorted upward; the opting-out policy welfare dominates the topping-up policy. Finally, if long term care provision carries no stigma, opting out is more cost-effective than topping up in both first- and second-best.

JEL classification: H2, H5.

Keywords: Long term care, uncertain altruism, opting out, topping up, public insurance.

1 Introduction

The growing demand for long-term care (LTC) due to an aging population presents a significant societal challenge. In recent decades, most developed countries have experienced rapid population aging, leading to a rising number of elderly individuals with cognitive and physical impairments. This trend is expected to accelerate in the coming years. LTC needs increase sharply after the age of 80, and this demographic is expanding faster than any other segment of the population. The primary drivers of this growth are increased life expectancy and declining birth rates. Additionally, the baby boomer generation is approaching the age at which care needs become most pressing. As a result, the number of elderly individuals requiring care in the European Union (EU-27) is projected to rise from approximately 21 million in 2007 to around 44 million by 2060 (EC, 2009). A similar trend is expected in the United States.[1]

Long-term care dependency poses a substantial financial risk, with only limited coverage provided by social insurance. Private LTC insurance remains scarce, a phenomenon known as the "LTC insurance market puzzle," which has been attributed to factors such as adverse selection and a preference for informal care. Consequently, many individuals rely on personal savings or unpaid care from family members.[2]

Informal care provided by relatives plays a critical role in LTC services, though its precise contribution is difficult to quantify. Studies suggest that informal care constitutes a significant share of total care hours, with considerable variation across countries.[3] Bolin et al. (2008), using data from the Survey of Health, Aging, and Retirement in Europe (SHARE), find that elderly individuals with at least one child are more likely to

[1] See Cremer *et al.*, (2012) or Grabovski *et al.* (2012) for extensive overviews of the LTC need projections.

[2] The literature has presented a number of explanations for this "LTC insurance market puzzle", including adverse selection (Finkelstein and McGarry, 2006) and the parents' preference for informal care (Pauly, 1990).

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receive informal care. More recent and precise estimates by Barczyk and Kredler (2019) indicate that informal care accounts for 22% of LTC in Northern Europe (Belgium, Denmark, the Netherlands, and Sweden), 43% in Central Europe (Austria, France, and Germany), 81% in Southern Europe (Italy and Spain), and 54% in the United States.

Informal care offers several advantages—it is often seen as compassionate, familiar, and cost-effective. Many elderly individuals prefer home-based care over institutional care due to its emotional and psychological benefits. However, informal care also has significant limitations. Its availability is uncertain, as demographic shifts such as declining birth rates, increased geographic mobility, and evolving social norms may leave elderly individuals without family caregivers. Moreover, some children may be unable or unwilling to provide care, particularly if the financial or time burden is too great. With rising female labor force participation and changing societal expectations, the number of elderly individuals lacking access to informal care is expected to increase in the future.

The motivations behind informal caregiving are complex, often driven by a combination of altruism, implicit exchanges, and social norms. Many caregivers feel a sense of duty or even guilt when they are unable to assist their aging parents.[4] Analyses of SHARE data by Alessie et al. (2014) and Tomini et al. (2016) indicate that both altruistic and exchange-based motives influence caregiving decisions, with variations based on individual and family characteristics. Regional studies using SHARE data also reveal significant differences in caregiving motivations across Europe.[5]

Given the limitations of informal care and the absence of a robust private LTC insurance market, there is a strong rationale for well-designed public LTC policies. However, these policies can influence informal care dynamics, and their design must account for this effect. One key concern is the potential for "Crowding out," where

[4] See Cremer et al. (2012), Klimaviciute and Pestieau (2023) or Klimaviciute et al. (2017).

[5] See, for instance, Klimaviciute *et al.*, 2017). Arrondel and Masson (2006) provides a detailed survey of the empirical literature.

publicly funded LTC reduces the provision of family care.[6] Crowding out can make public LTC less effective for some individuals and increase overall costs. The magnitude of this effect depends on whether caregiving is primarily motivated by altruism, implicit exchange, or social norms. When altruism is the driving force, public LTC support—depending on its structure—may significantly reduce informal caregiving.[7] On the other hand, if caregiving is shaped by social expectations, public support is likely to have a more limited impact. Public LTC assistance generally follows one of two models. The "opting-out" (OO) policy provides public LTC benefits but prohibits individuals from supplementing them with personal funds—those seeking higher-quality care must forgo public support entirely. The "topping-up" (TU) policy, in contrast, allows individuals to enhance public LTC benefits with private resources. In our analysis, we will examine both approaches.

Various long-term care (LTC) programs can coexist within a country, though most systems typically follow the topping-up (TU) model.[8] Public LTC programs include cash transfers—often means-tested—such as France's Allocation Personnalisée d'Autonomie (APA), Germany's Pflegegeld, Italy's Assegno d'Accompagnamento, and the Supplemental Security Income program in the United States. In-kind benefits, such as subsidized or free services (e.g., meals on wheels and formal home care), are common across Europe, as is the Supplemental Nutrition Assistance Program in the US. Some systems, like those in Scandinavian countries and under Medicaid in the US, offer a choice between receiving formal care at home or in institutions. While elderly residents in institutional care often pay rent, they may also receive a personal-need allowance to cover additional expenses. However, even in Scandinavian countries—where LTC insurance is predominantly structured around formal care—dependent individuals continue to rely

[6] See, for instance, Cremer *et al.* (2012) and Grabowski *et al.* (2012).

[7] See Cremer et al. (2017) and Canta et al. (2020).

[8] For an overview of different policies and financing models in the EU, see Lipszyc et al. (2012) and European Commission (2013).

heavily on informal care.[9]

The closest real-world example of an OO scheme is the formal care provided by nursing homes, although this is frequently supplemented by informal care from relatives (such as visits or meal assistance). Pure OO policies serve as extreme theoretical constructs for policy design as real-world systems are far from optimal and are continuously evolving. Many public nursing facilities struggle with quality issues and chronic understaffing due to insufficient funding and caregiver shortages. As a result, numerous countries have adopted hybrid models that combine institutional care as a last resort with incentives for home care when dependency is less severe. Some European nations have even implemented an "opting-out-cum-transfer" model which includes a TU component. Examples include LTC leave policies in the Netherlands that help working children balance caregiving with employment; cash transfers in Germany for elderly individuals receiving family care; and comprehensive support measures in Sweden—such as caregiver training, support services, and respite care programs—for families managing dependent relatives.[10]

This paper contributes to the long-term care policy literature by exploring how, in an uncertain environment, redistributive tax policies and long-term care program design interact with informal care incentives to shape caregiving outcomes over the long term. The analysis centers on the well-being of a generation of adults with varying earning capacities across three life stages. Some individuals are altruistic, offering support to their elderly parents—but only if those parents become dependent—while others provide no assistance under any circumstances. The degree of altruism in these parents is largely predetermined by the care and attention they received in childhood, However, for their own retirement, they must decide how much time to devote to their children and how much to save. These decisions are made under two layers of uncertainty: the probability of becoming dependent in old age and the likelihood of having altruistic children.

[9] See Karlsson et al. (2010).

[10] For a survey of these policies in OECD countries, see Gori et al. (2016).

These uncertainties also shape government decisions on public long-term care provision. Moreover, as parents differ in income, redistributive concerns become a key consideration in tax policy. This creates a link between tax policies and the time parents invest in their children, which, in turn, shapes the type of long-term care support available to them in later life.

Previous research has examined the impact of uncertainty on informal care and its policy implications; see Cremer et al. (2017), Canta et al. (2020), and Canta and Cremer (2021).[11] Yet, these studies overlook three key aspects that we address in this paper. First, although LTC dependency poses a major financial risk, tax-cum-LTC policy serves not only as insurance but also plays an important redistributive role. Wealthier individuals can self-insure through private savings without greatly reducing current consumption, whereas poorer individuals face significant challenges in doing so, making the alleviation of this burden a critical objective.[12] Second, while parents cannot control their fertility or their children's future mobility, they may influence their children's willingness to provide care by spending more time with them. Third, public LTC benefits might carry a stigma depending on the program's design, which can affect their uptake.

The study most closely related to ours is Cremer et al. (2014), which investigates endogenous and uncertain altruism among heterogeneous individuals. However, that work only considers linear income tax policies and opting-out schemes. In contrast, our study focuses on nonlinear tax policies—where instruments are constrained solely by the information structure and available resources—and their interactions with LTC

[11] Canta and Cremer (2019, 2023) also assume that parents lack precise knowledge of their children's degree of altruism (or the cost of providing care). However, they focus on exchange-based informal care, where children receive transfers in return for their assistance. Because parents do not know their children's care costs, they cannot simply reimburse these costs but must instead design a nonlinear transfer scheme that effectively screens for them, thereby leaving a rent to the most altruistic children.

[12] For instance, average daily costs of nursing homes in the US in 2017 is $235 (but is typically closer to $400 in the Northeast). The average stay in a nursing home is 835 days, according to the National Care Planning Council, which brings the average total cost to about $200,000 (and twice that amount in some states). See https://www.payingforseniorcare.com/longtermcare/paying-for-nursing-homes.html

policies. Moreover, we study both topping-up and opting-out policies comparing their various implications.

We build on prior research on LTC policy design by focusing on its financing within a Mirrleesian framework that accounts for informational asymmetries. In this setting, we evaluate the effectiveness of both topping-up and opting-out schemes in addressing LTC challenges. Two key uncertainties drive the LTC provision problem we examine. One concerns the health status of parents in old age—specifically, whether they will remain independent or become dependent. The second is the availability of family care. Some will never provide care; others, motivated by altruism, will; though the extent of their care depends on past parental actions. Parents allocate their time between working and child-rearing, with greater time investment fostering stronger altruistic bonds in later years.

Our analysis centers on a generation of young parents with varying earning capacities, observed across three life stages. In childhood, they make no independent decisions, and their welfare is embedded in that of their parents. In adulthood, they determine consumption and plan for the future, with these plans unfolding in old age. Assuming that future generations follow the same life cycle and that a steady state prevails, we investigate tax-and-public-LTC policies aimed at maximizing this generation's welfare. The tax policy is designed to address societal redistributive concerns, while public LTC programs—whether based on OO or TU models—are intended to support dependent parents without available family care. We exclude private insurance markets for two reasons. First, private LTC insurance is both rare and expensive, typically burdened by high loading factors, making it unrepresentative of real-world conditions. Second, incorporating private insurance would add unnecessary complexity to the analysis without significantly altering the main conclusions.

The key findings of the paper include: Under full information (i) policy decisions are efficiency-driven, with lump-sum taxes handling redistribution; (ii) savings should be subsidized in an OO system—not for redistributive purposes but because they generate a positive fiscal externality. Additional savings by dependent parents reduce the government's need to fund public assistance for those who will receive no family assistance. This fiscal externality does not exist in TU systems, so savings remain undistorted.

Under asymmetric information: (i) Allocations are distorted for redistributive purposes, except for savings, (ii) marginal income tax rates are positive, in line with standard nonlinear income taxation models, and (iii) a consequence of government's redistributive policies is to encourage time spent with children thus increasing family caregiving. These three findings apply to both OO and TU schemes. (iv) As with full information, savings must be subsidized in an OO system due to fiscal externalities; (v) if public assistance under OO carries a stigma, and the stigma is larger for higher wage individuals, public LTC is distorted upward.

Finally, comparing OO and TU programs, we find that if OO carries little or no stigma, it is more cost-effective than TU in both first- and second-best environments. Under OO, those receiving informal care will not seek public assistance as long as it offers less support than family caregiving. This self-targeting ensures that only individuals who genuinely need public LTC receive it, thereby limiting government spending without harming overall welfare.

2 The common framework

Consider an overlapping generations model comprising old retired parents, young working parents, and small children in steady-state equilibrium. Each generation consists of N parents with identical tastes but different earning abilities (wages). They are indexed by i and ranked in ascending order of wages, such that a higher i corresponds to a higher wage, w_i. The government's objective is to to design tax-cum-LTC policies that maximize the welfare of young working parents with wages w_i, who are the decision makers in this model at given time t. The welfare of children is embedded in their parents' utility, while the welfare of the elderly is shaped partly by their past choices and partly by the decisions of the current generation of young parents.

Parents are endowed with one unit of time, which they allocate between working and spending time with their children. Neither activity carries inherent utility or disutility. Work generates income, while time spent with children fosters potential family assistance in old age. However, only altruistic children provide care, and not all children are altruistic. Altruism is a trait that becomes observable only when children reach adulthood. In old age, parents may either remain healthy or become dependent, receiving assistance only in the latter case.

Parents have quasilinear preferences over consumption in both their youth and old age.[13] If altruistic, they also derive utility from their own parents' well-being. Denote the utility function associated with old-age consumption by $\varphi(\cdot)$ if healthy and by $\phi(\cdot)$ if dependent, the probability of dependency by π, the probability of having altruistic children by p, consumption level when young by c, consumption level when old and healthy, equal to savings, by s, expected assistance provided by altruistic children in the future by a^*, additional resources on top of children assistance by x, and the total LTC care parents receive if the children do not help by d.[14] Setting the rate of interest on savings at zero, parents' preferences over their own consumption can be expressed by means of the expected utility function

$$U = c + (1-\pi)\varphi(s) + \pi\left[p\phi(x+a^*) + (1-p)\phi(d)\right] \tag{1}$$

This formulation implicitly assumes that public assistance does not exceed family care. Without this assumption, the government cannot run an "opting out" policy as discussed below. We further assume that $\varphi(\cdot)$ and $\phi(\cdot)$ are increasing and concave in their arguments and that $\phi'''(\cdot) \leq 0$.

[13]All our key results, except for value comparisons, remain valid if quasilinearity assumption is dropped as long as preferences remain separable

[14]For ease in notation, we have dropped the i index referring to a particular parent.

Additionally, altruistic parents care about the utility of their own parents according to

$$\psi = f(\widehat{e})\,\phi(\widehat{x}+a) - a,$$

where $\widehat{e}$ denotes the time young parents have spent in their childhood with the older parents, and $\widehat{x}$ denotes old parents' current resources determined when they were young, and $f(\widehat{e})$ denotes the degree of altruism young parents feel towards their old parents. We assume that $f(\cdot)$ is increasing and concave. Those who are not altruistic towards their parents have a $f(e) \equiv 0$ so that they always choose $a = 0$ and never provide any care. The parents' overall utility function is then represented by

$$u = c + (1-\pi)\,\varphi(s) + \pi\left[p\phi(x+a^*) + (1-p)\,\phi(d)\right] + f(\widehat{e})\,\phi(\widehat{x}+a) - a \qquad (2)$$

Observe that, at any given time t, $\widehat{e}$ and $\widehat{x}$ are predetermined (having been determined at $t-1$), and a^* is to be determined in $t+1$. Consequently, the variables of interest that are to be determined at t are c, s, x, a, in light of government's policy including d.

2.1 The young parents' choice of family assistance

Consider first the parents' choice of a. The first-order condition (FOC) of the maximization of u results in

$$\phi'(\widehat{x}+a) = \frac{1}{f(\widehat{e})}.$$

It follows from the above relationship that

$$\widehat{x}+a = (\phi')^{-1}\left(\frac{1}{f(\widehat{e})}\right) \equiv m(\widehat{e})\,. \qquad (3)$$

Hence the young young parents supplement the resources of their own parents', $\widehat{x}$, by an amount a that is just enough to bring $\widehat{x}$ up to a level $\widehat{x}+a$ that depends only on the predetermined value $\widehat{e}$ (the time they have spent together). This aggregate amount is independent of how much the old parent has saved or any public assistance that they may receive.[15]

With each generation behaving the same as the previous generation, and the perfect foresight assumption, the young parents, in case they become dependent in old age, expect to receive family support from their altruistic children governed by the same relationship as (3). That is,

$$x + a^* = (\phi')^{-1} \left(\frac{1}{f(e)} \right) \equiv m(e), \tag{4}$$

where e denotes the time the young parents will spend with their children. *Importantly, property (4) holds for both* OO *and* TU *policies and regardless of what the government's tax-cum-in-kind policy is.* Government's policy affects $x + a^*$ only through the parents' choice of e. Lemma 1, proved in Appendix A shows the properties of $x + a^* \equiv m(e)$.

Lemma 1 *Old Parents on family assistance have resources at their disposal,* $x + a^* \equiv m(e)$*, that are an increasing and concave function of the time spent with their children* e.

Observe that altruistic children choose a^* if and only if it gives them more utility than the option of no assistance. In what follows, we restrict our attention to the equilibrium that entails children actually helping their parents (albeit probabilistically). That is, public LTC is never sufficient enough for dependent parents of altruistic children not to accept help from their children.[16] Armed with Lemma 1, we are in a position to

[15] Needless to say, the level of other resources are crucial in determining the older parents' utility if they remain healthy.

[16] This requires

$$y_i - a_i + f(e_i)\,\phi(x_i + a_i) > y_i + f(e_i)\,\phi(d_i).$$

Or

$$f(e_i)\left[\phi(x_i + a_i) - \phi(d_i)\right] > a_i.$$

This condition can only be satisfied if

$$\phi(x_i + a_i) > \phi(d_i),$$

investigate first-best and second-best tax-cum-LTC policies. We shall start with the OO scheme followed by TU.

Concluding this section, note that, with parents anticipating the children's decision, we can substitute $h(e)$ for $\phi(x+a^*)$, $h(\widehat{e})$ for $\phi(\widehat{x}+a)$, and $m(\widehat{e})-\widehat{x}$ for a in (2) to rewrite it as

$$u = c + (1-\pi)\varphi(s) + \pi\left[ph(e) + (1-p)\phi(d)\right] + f(\widehat{e})h(\widehat{e}) - m(\widehat{e}) + \widehat{x}.$$

Now, with $f(\widehat{e})h(\widehat{e}) - m(\widehat{e}) + \widehat{x}$ being predetermined and thus a constant, the objective function we want to maximize is

$$U = c + (1-\pi)\varphi(s) + \pi\left[ph(e) + (1-p)\phi(d)\right] \tag{5}$$

This is the case for both opting out and topping up policies. We also observe from Lemma 1 that $h(e) \equiv \phi(x+a^*)$, utility of consumption in old-age dependency when receiving family assistance, increasing and concave in e. We have

$$\begin{aligned} h'(e) &= \phi'(x+a^*)m'(e) > 0, \\ h''(e) &= \phi''(x+a^*)m'(e) + m''(e)\phi'(x+a^*) < 0. \end{aligned}$$

3 Opting out

Under an opting-out scheme, recipients are prohibited from supplementing their public assistance allotment. They are restricted to consuming only what the public sector provides. If they desire more, they must forgo public assistance entirely and rely on their savings and family support. On the other hand, if they opt into public assistance, they cannot supplement it with personal resources and must hand in their savings to

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$\Rightarrow$

$$x_i + a_i > d_i.$$

the government. We thus have $x_i = s_i$ and $d_i = z_i$ which allows equation (5) to be rewritten as

$$U_i = c_i + (1 - \pi)\,\varphi(s_i) + \pi\,[ph(e_i) + (1 - p)\,\phi(z_i)]\,.$$

Accepting public assistance under OO policies often carries a stigma, both internally and externally, due to feelings of shame. To account for this, we assume that parents i value z_i dollars worth of public assistance not by $\phi(z_i)$; instead, only by $\gamma_i\phi(z_i)$ where $\gamma_i < 1$ with stigma and $\gamma_i = 1$ without. Allowing for stigma, parents i's utility function is rewritten as

$$U_i = c_i + (1 - \pi)\,\varphi(s_i) + \pi\,[ph(e_i) + (1 - p)\,\gamma_i\phi(z_i)]\,. \tag{6}$$

One also expects that the extent of stigma to differ across individuals and that the wealthier parents experience a higher degree of stigma when going for public assistance. Thus, denoting the parents i's wage by w_i, the higher is w_i the lower will be γ_i.

3.1 First-best policy

3.1.1 Problem and first-order conditions

Given full public observability, we can determine the first-best policy by directly determining the values of c_i, s_i, e_i, and z_i. To ensure redistributive policy concerns in the face of quasilinear preferences, we consider an optimization problem based on an increasing and concave transformation of U_i rather than U_i itself. This is denoted by $V(U_i)$ where $V'(\cdot) > 0$ and $V''(\cdot) < 0$. Index parents in increasing order of wages so that $i > j$ if and only if $w_i > w_j$. Denote the population size of parents of type i by n_i and associate a positive welfare weight, δ_i, to each type with the normalization $\sum_i \delta_i = 1$. Then maximize $\sum n_i\delta_i V(U_i)$ with respect to c_i, s_i, e_i, and z_i subject to the government's budget constraint

$$\sum_i n_i\,[w_i\,(1 - e_i) - c_i - s_i] \geq \pi(1 - p)\sum_i n_i(z_i - s_i). \tag{7}$$

The Lagrangian expression associated with this problem is

$$\mathcal{L} = \sum_i n_i \left\{ \delta_i V\left(U_i\right) + \mu \left[w_i \left(1 - e_i\right) - c_i - s_i - \pi(1-p)(z_i - s_i) \right] \right\}.$$

where U_i is given by equation (6).

The first-order conditions (FOC) are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_i} &= n_i \left[\delta_i V'\left(U_i\right) - \mu \right] = 0, \\
\frac{\partial \mathcal{L}}{\partial s_i} &= n_i \left\{ \delta_i V'\left(U_i\right)(1-\pi)\varphi'(s_i) + \mu \left[-1 + \pi(1-p) \right] \right\} = 0, \\
\frac{\partial \mathcal{L}}{\partial e_i} &= n_i \left[\delta_i V'\left(U_i\right) \pi p h'\left(e_i\right) - \mu w_i \right] = 0, \\
\frac{\partial \mathcal{L}}{\partial z_i} &= n_i \left\{ \delta_i V'\left(U_i\right) \left[\pi(1-p) \right] \gamma_i \phi'\left(z_i\right) - \mu \pi (1-p) \right\} = 0.
\end{aligned}$$

Simplifying yields

$$V'\left(U_i^{FB}\right) = \frac{\mu}{\delta_i}, \tag{8}$$

$$\varphi'(s_i^{FB}) = \frac{1 - \pi\left(1-p\right)}{1-\pi} = 1 + \frac{\pi p}{1-\pi}, \tag{9}$$

$$h'\left(e_i^{FB}\right) = \frac{w_i}{\pi p}, \tag{10}$$

$$\phi'\left(z_i^{FB}\right) = \frac{1}{\gamma_i} \geq 1. \tag{11}$$

3.1.2 Interpretation and comparative statics

Expression (8) follows a standard principle: transfers are designed so that the social marginal utility of income is equalized across individuals. Equation (9) indicates that all parents save the same amount: the RHS does not depend on i and $s_i^{FB} = s^{FB}$. The uniformity in savings is due to the quasi-linearity of preferences and the fact that all parents face the same probabilities of becoming dependent and of having uncaring children. The expression itself demonstrates that, at the optimum, the marginal social cost and marginal social benefit of savings are equalized. The marginal social cost of savings is simply its private cost, which is one. The private marginal benefit of savings is $(1-\pi)\,\varphi'(s_i^{FB})$ because, as shown in equation (6), savings only matter in old age if parents remain healthy which happens with probability $(1-\pi)$. However, the marginal social benefit of savings is higher than its marginal social benefit by $\pi\,(1-p)$ bringing it up $(1-\pi)\,\varphi'(s_i^{FB})+\pi\,(1-p)$. This is then equalized to the marginal social cost of savings which is one.

The reason the marginal social benefit exceeds the marginal private benefit by $\pi\,(1-p)$ is that savings generate a positive fiscal externality of the same amount. A fraction of the parents, $\pi\,(1-p)$ of them, become dependent with no family assistance. Because their savings cover part of the expenditures on public LTC, any increase in their savings reduces the amount of taxes that is required to finance public assistance. Given that the fraction parents who become dependent without caring children is $\pi(1-p)$, the expected fiscal benefit to the government from one additional dollar of savings is $\pi(1-p)$ dollars.

Equation (10) describes how the time spent with children e_i^{FB} is determined. Parents choose their time allocation so that they are indifferent between working more—earning an additional w_i in income—and spending more time with their children, which increases the likelihood of receiving support in old age by $\pi p h'\left(e_i^{FB}\right)$. And, not surprisingly, e_i^{FB} moves negatively with w_i,

$$\frac{\partial e_i^{FB}}{\partial w_i}=\frac{1}{\pi p h''\left(e_i^{FB}\right)}<0. \tag{12}$$

The reason is that while time spent with children has the same marginal benefit for all parents regardless of their wage, its opportunity cost increases with wage. This also means that in the absence of stigma more productive parents spend less time with their children; but this can be reversed if stigma is sufficiently larger for these parents.

Finally, equation (11) indicates that because of stigma public LTC is non-uniform.

More precisely, the higher is the degree of stigma the smaller is public LTC:

$$\frac{\partial z_i^{FB}}{\partial \gamma_i} = \frac{-1}{(\gamma_i)^2 \phi''\left(z_i^{FB}\right)} \geq 0.$$

This makes sense. Parents who have a smaller degree of stigma, value public LTC more and should be assigned more of it. In the absence of stigma, on the other hand, public LTC is uniform. Neither marginal cost nor marginal benefit depends on w_i, and lump-sum taxes take care of redistribution so that under full information there is no reason to differentiate z_i across types. Observe also that z_i does not depend on p nor on π. With social marginal utilities equalized via lump-sum taxes, marginal social cost and benefits of z_i are simply proportional to $\pi(1-p)$ and these two variables play no role for the determination of individual public LTC benefits.

3.1.3 Decentralization

Decentralization of the first-best outcome requires lump sum taxes to equalize social marginal utilities of income across parents as specified by condition (8). More interestingly, whereas the time parents spend with their children requires no tampering with, their savings must be subsidized by $\pi(1-p)$. To see this, consider the optimization of parents i with no tax or subsidy on e_i and $\pi(1-p)$ subsidy on savings. They maximize

$$U_i = c_i + (1-\pi)\varphi(s_i) + \pi\left[ph(e_i) + (1-p)\gamma_i\phi(z_i)\right]$$

subject to the budget constraint

$$c_i + \left[1 - \pi(1-p)\right]s_i = w_i(1-e_i) - T_i,$$

where T_i is the lump-sum tax levied on parents i. Substituting for c_i in U_i and maximizing the resulting equation with respect to s_i and e_i results in the FOC

$$\frac{\partial U_i}{\partial s_i} = -\left[1-\pi(1-p)\right] + (1-\pi)\varphi'(s_i) = 0, \quad (13)$$

$$\frac{\partial U_i}{\partial e_i} = -w_i + \pi p h'(e_i) = 0. \quad (14)$$

Rearranging, we obtain

$$\varphi'(s_i) = 1 + \frac{\pi p}{1-\pi},$$
$$h'(e_i) = \frac{w_i}{\pi p},$$

which correspond to the FB conditions (9) and (10).

The reason for a savings subsidy is to internalize the positive fiscal externality it generates (discussed earlier). To maximize utility, parents set the marginal private of benefits, $(1-\pi)\varphi'(s_i)$, equal to the marginal cost of savings which, in the absence of subsidy, is equal to one. This results in a less than optimal level of savings. To correct this, a Pigouvian subsidy at the rate of $\pi(1-p)$, equal to the fiscal externality of savings, is levied. The subsidy lowers the marginal private cost of savings to $1-\pi(1-p)$ which they will then equalize to the marginal private benefits of an extra dollar of savings $(1-\pi)\varphi'(s_i)$. This results in $\varphi'(s_i) = 1 + \pi p/(1-\pi)$.[17]

3.2 Second-best policy

3.2.1 Optimal allocation

Assume now that a parent's type, w_i, and the time he spends with his children, e_i, are not publicly observable; but his income, $I_i \equiv w_i(1-e_i)$, consumption, c_i, and saving, s_i, are. This allows the government to tax incomes and savings at a personal level. To characterize the optimal tax system, we consider the standard equivalent problem of the government first choosing optimal allocations subject to resource balance and self-selection constraints. Having derived the optimal allocation, we then describe the tax structure that can implement it.

[17] We have

$$1-\pi(1-p) = (1-\pi)\varphi'(s_i)$$

$\Rightarrow$

$$\varphi'(s_i) = 1 + \frac{\pi p}{1-\pi}$$

Formally, the government chooses c_i, s_i, I_i, and z_i to maximize $\sum n_i \delta_i V(U_i)$ subject to its budget constraint

$$\sum_i n_i (I_i - c_i - s_i) \geq \pi(1-p) \sum_i n_i (z_i - s_i),$$

and the self-selection constraints

$$U_i \geq U_{ik} \text{ for all } i \neq k;\ \ i, k = 1, 2, \ldots, N, \tag{15}$$

where

$$U_i \;=\; c_i + (1-\pi)\varphi(s_i) + \pi \left[ph\left(1 - \frac{I_i}{w_i}\right) + (1-p)\gamma_i \phi(z_i) \right], \tag{16}$$

$$U_{ik} \;=\; c_k + (1-\pi)\varphi(s_k) + \pi \left[ph\left(1 - \frac{I_k}{w_i}\right) + (1-p)\gamma_i \phi(z_k) \right]. \tag{17}$$

Equation (17) shows the utility of a person with productivity w_i who chooses the (c_k, s_k, I_k, z_k) bundle meant for one with productivity w_i. In the language of optimal tax theory, this person is called a "mimicker".

The Lagrangian expression associated with this problem is[18]

$$\begin{aligned} \mathcal{L} \;=\; & \sum_i n_i \left\{ \delta_i V(U_i) + \mu \left[I_i - c_i - s_i - \pi(1-p)(z_i - s_i) \right] \right\} + \\ & \sum_i \sum_{k \neq i} \lambda_{ik} (U_i - U_{ik}), \end{aligned}$$

where λ_{ik}'s are (non-negative) Lagrangian multipliers associated with the self-selection constraints (15). We show in Appendix B that the FOC of this problem can be written

[18] To simplify notation, we use $\sum_i$ for $\sum_{i=1}^N$ and $\sum_{k \neq i}$ for $\sum_{\substack{k=1 \\ k \neq j}}^N$ throughout the paper.

as

$$\delta_i n_i V'(U_i) = \mu n_i + \sum_{k \neq i} \lambda_{ki} - \sum_{k \neq i} \lambda_{ik}, \tag{18}$$

$$(1-\pi)\varphi'(s_i) = 1 - \pi(1-p), \tag{19}$$

$$\mu n_i \left[h'(e_i) - \frac{w_i}{\pi p} \right] = \sum_{k \neq i} \lambda_{ki} \left[\frac{w_i}{w_k} h'(e_{ki}) - h'(e_i) \right], \tag{20}$$

$$\mu n_i \gamma_i \phi'(z_i) - \mu n_i = \sum_{k \neq i} \lambda_{ki} (\gamma_k - \gamma_i) \phi'(z_i). \tag{21}$$

These conditions define the optimal allocation: s_i^{SB} (saving), e_i^{SB} (time spent with children), I_i^{SB} (income), and z_i^{SB} (public LTC provision).

3.2.2 Decentralization

Allocation (19)–(21) can be decentralized with a nonlinear income tax $T(I_i)$ and a Pigouvian subsidy equal to $\tau_s = \pi(1-p)$. To see this, consider the optimization problem of parents i who maximize

$$U_i = c_i + (1-\pi)\varphi(s_i) + \pi \left[ph\left(1 - \frac{I_i}{w_i}\right) + (1-p)\gamma_i \phi(z_i) \right]$$

subject to

$$c_i + [1 - \pi(1-p)] s_i = I_i - T(I_i),$$

Substituting for c_i from the above equation into the equation for U_i and maximizing the resulting equation with respect to s_i and I_i yields,

$$\begin{aligned} \frac{\partial U_i}{\partial s_i} &= -\pi p + (1-\pi)\left[\varphi'(s_i) - 1\right] = 0, \\ \frac{\partial U_i}{\partial I_i} &= 1 - T'(I_i) - \frac{1}{w_i} \pi p h' \left(1 - \frac{I_i}{w_i}\right) = 0. \end{aligned}$$

Simplifying results in

$$\varphi'(s_i) = 1 + \frac{\pi p}{1-\pi}, \tag{22}$$

$$T'(I_i) = 1 - \frac{1}{w_i} \pi p h' \left(1 - \frac{I_i}{w_i}\right). \tag{23}$$

With the mechanism designer setting

$$1-\frac{1}{w_i}\pi p h'\left(1-\frac{I_i}{w_i}\right)=\frac{-1}{\mu n_i w_i}\pi p\sum_{k\neq i}\lambda_{ki}\left[\frac{w_i}{w_k}h'\left(1-\frac{I_i}{w_k}\right)-h'\left(1-\frac{I_i}{w_i}\right)\right],$$

parents' private optimization solution satisfies (19)–(20).

3.2.3 Properties and interpretation

The properties of the income tax function $T(I_i)$ which implements the optimal allocation, and the comparison between the second-best and first-best allocations are presented in the following proposition proved in Appendix C.

Proposition 1 *(i) If upward self-selection constraints are non-binding, marginal income tax rates are positive:*

$$\begin{aligned} T'(I_i) &> 0, \quad i=1,2,\ldots,N-1, \\ T'(I_N) &= 0. \end{aligned}$$

(ii) Assume upward self-selection constraints are non-binding. It then follows that for all $i=1,2,\ldots,N-1$,

$$I_i^{SB} < I_i^{FB}, \tag{24}$$

$$e_i^{SB} > e_i^{FB}, \tag{25}$$

$$z_i^{SB} \geq z_i^{FB}. \tag{26}$$

and for $i=N$

$$\begin{aligned} I_N^{SB} &= I_N^{FB}, \\ e_N^{SB} &= e_N^{FB}, \\ z_N^{SB} &= z_N^{FB}. \end{aligned}$$

(iii) The second-best value of savings is the same as its FB value.

$$s_i^{SB}=s_i^{FB}$$

Proposition 1 indicates that the traditional no-distortion-at-the-top result holds for our model. Moreover, because of the quasilinearity of preferences, this applies not only to the optimal rules but also to the levels of variables as shown by equality of first- and second-best values of e_N, I_N, and z_N. However, for the rest of the population, except for savings, the second-best allocation entails distortions.

Equation (24) indicates a downward distortion in I (and thus a positive marginal income tax rate $T'(I) > 0$). This happens because, with mimicker being more productive than the mimicked, reducing I is more costly for the mimicker. The same reasoning explains equation (25) which indicates an upward distortion in e. That a larger e goes hand-in-hand with a smaller I follows from the relationship $I = w(1-e)$. It is interesting to note that a by-product of the government's redistributive policy is to encourage parents to spend more time with their children.

Turning to equation (26), it shows that in the absence of stigma, first- and second-best levels of public LTC are equal. This is because without stigma mimicker and mimicked have the same marginal willingness to pay for z. On the other hand, when public LTC carries a stigma, the mimicker (with a smaller γ_i) has a smaller willingness to pay for z so that the upward distortion weakens the incentive constraint.

Finally, savings remain at their first-best value with the same positive fiscal externality property. The reason is the fact that savings are valued the same by the mimicker and the mimicked so that a distortion does not relax the incentive constraint.

3.2.4 Second-best with unobservable savings

If savings are unobservable, they cannot be taxed or subsidized and the mechanism designer takes its value as given. Under this assumption, the value of savings is determined through the maximization of the parents' expected utility (16) which yields

$$\varphi'(s_i) = \frac{1}{1-\pi}.$$

However, given the additivity of references, this does not affect the SB characterization of the other variables we found with observability of savings. Now because with observable savings, the SB value was given by $\varphi'(s_i) = 1/(1-\pi) - \pi(1-p)/(1-\pi)$, unobservability results in less savings. And while it is optimal to subsidize it, we cannot do so. This also means that while, for other variables, the *rules* are unaffected the *levels* are not .

4 Topping up

Under this scheme, public assistance is provided to everyone while allowing recipients to supplement their allotment if they so wish. We thus have $d_i = s_i + z_i$ which allows equation (5) to be rewritten as

$$U_i = c_i + (1-\pi)\varphi(s_i) + \pi[ph(e_i) + (1-p)\phi(s_i + z_i)]. \tag{27}$$

There is no impediment to receiving public assistance and no benefit to refusing it. Moreover, since it is a universal program receiving assistance carries no stigma. The possibility of topping up also changes the specification of the government's budget constraint. With all dependent parents, with or without family support, receiving aid,and keeping their savings, the government needs to raise enough taxes to cover all expenditures on public LTC which is equal to $\pi \sum_i n_i z_i$. The government's budget constraint is thus given by

$$\sum_i n_i [I_i - c_i - s_i] \geq \pi \sum_i n_i z_i. \tag{28}$$

4.1 First-best policy

4.1.1 Problem and first-order conditions

As with the OO program, start by determining the optimal allocation. To do this, maximize $\sum n_i \delta_i V(U_i)$ with respect to c_i, s_i, e_i, and z_i subject to the government's budget constraint

$$\sum_i n_i \left[w_i \left(1 - e_i \right) - c_i - s_i \right] \geq \pi \sum_i n_i z_i.$$

The Lagrangian expression associated with this problem is

$$\mathcal{L} = \sum_i n_i \left\{ \delta_i V \left(U_i \right) + \mu \left[w_i \left(1 - e_i \right) - c_i - s_i - \pi z_i \right] \right\}, \tag{29}$$

where U_i is given by equation (27).

We show in Appendix D that the FOC can be rewritten as follows

$$\begin{aligned} \delta_i V' \left(U_i \right) &= \mu, & (30) \\ \varphi'(s_i) &= 1 & (31) \\ h' \left(e_i \right) &= \frac{w_i}{\pi p}, & (32) \\ \phi' \left(s_i + z_i \right) &= \frac{1}{1-p}. & (33) \end{aligned}$$

4.1.2 Interpretation and comparative statics

Expression (30) indicates that transfers are designed to equalize the social marginal utility of incomes across individuals. This is the same condition that applies under OO, as stated in. (8). With full information, this condition remains optimal whether topping up is allowed or not.

Expression (31) establishes that all parents save the same amount: the RHS does not depend on i and $s_i^{FB} = s^{FB}$. This is once again due to the quasi-linearity of preferences and the fact that all parents face the same probability of becoming dependent and of having uncaring children. Moreover, comparing equation (31) with its counterpart under OO, equation (9), shows that $\varphi'(s^{TU}) < \varphi'(s^{OO})$. Given the concavity of $\varphi(\cdot)$, It follows that $s^{TU} > s^{OO}$: savings are larger under TU than under OO. This makes sense because, under OO, savings are taxed away when parents receive public LTC, whereas under TU, savings are used to supplement z.

Expression (31) itself, $\varphi'(s_i) = 1$, reflects the optimality condition of equal marginal social cost and marginal social benefit of savings. The marginal social cost is simply its private cost (which is one). The marginal social benefit is also equal to its marginal private benefit, because, unlike under OO, saving under TU does not create a fiscal externality. To see this, recall that the source of fiscal externality is the LTC recipients' handing over their savings to the government and thus freeing up part of the taxes required to finance LTC expenditures. This does not happen under TU as all parents keep their savings. However, not all parents benefit from keeping their savings. Any increase in savings by dependent parents receiving family assistance is fully offset by a reduction in the assistance they receive. Consequently, the marginal private benefits of savings is only $(1-\pi)\varphi'(s_i) + \pi(1-p)\phi'(s_i + z_i)$.

Next, observe that the government provides public LTC at a level at which its marginal cost and benefit are equalized. The marginal cost of LTC is π dollars, as it is provided to all dependent parents. The benefit of z_i, on the other hand, is enjoyed only by dependent elderly with uncaring children, i.e., $\pi(1-p)$ percent of all parents. Public assistance does not change the consumption level of dependent parents who receive family assistance, as any government-provided aid is offset by a reduction in family assistance. As a result, the marginal benefit of z is $\pi(1-p)\phi'(s_i + z_i)$. Setting π equal to $\pi(1-p)\phi'(s_i + z_i)$ leads to $(1-p)\phi'(s_i + z_i) = 1$. This makes the private marginal benefit of saving equal to $(1-\pi)\varphi'(s_i) + \pi$. Setting it equal to one, which is the marginal cost of saving, results in $\varphi'(s_i) = 1$.

Comparing equation (32) with (10) reveals that the time parents spend with their children, e_i^{FB}, remains unaffected by the possibility of topping up. This is because the marginal cost (lost earnings) and the marginal benefit of e_i, $\pi p h'\left(e_i^{FB}\right)$, are identical under both OO and TU systems. Consequently, equation (12) remains valid, and e_i decreases with w_i for the same reasons as under OO.

Finally, because the RHS of (33) does not depend on i, total savings and public assistance, $s_i^{FB} + z_i^{FB}$, are the same for all individuals. Because s_i^{FB} is constant, z_i^{FB} must also be uniform: $z_i^{FB} = z^{FB}$. This uniformity arises because neither the marginal cost nor the marginal benefit of public assistance depends on w_i. Recall that the same reason was behind the uniformity of z_i under OO in the absence of stigma. However, whereas z_i is independent of π and p under OO, here it continues to be independent of π but depends on p. The difference arises because under OO, both the benefit and cost of z are proportional to the fraction of dependent parents who have uncaring children, i.e., $\pi(1-p)$. In contrast, under TU, the cost of z is proportional to π because LTC is provided to all dependent parents, whereas its benefit is proportional to dependent elderly with uncaring children, i.e., $\pi(1-p)$. Dependent parents with caring children also receive z but they do not benefit from it since it is fully offset by an equivalent reduction in care from children. Thus, an increase in p, the proportion of dependent parents with caring children, reduces the marginal benefit of z and with it its optimal level. This is confirmed by differentiating (33) with respect to p which yields

$$\frac{\partial z^{FB}}{\partial p} = \frac{1}{(1-p)^2 \phi''\left(s^{FB} + z^{FB}(p)\right)} < 0.$$

4.2 Decentralization

The fact that saving under TU does not create fiscal externality implies that, once public LTC is provided at the appropriate level, personal lump-sum taxes are sufficient to decentralize the FB. No taxes or subsidies on s and e are required. This is easily seen by considering the optimization problem of parents i. They maximize their expected utility

$$U_i = c_i + (1-\pi)\varphi(s_i) + \pi\left[ph(e_i) + (1-p)\phi(s_i + z_i)\right],$$

subject to the budget constraint

$$c_i + s_i = w_i(1-e_i) - T_i.$$

Substituting for c_i in U_i and maximizing the resulting equation with respect to s_i and e_i yields the FOC

$$\frac{\partial U_i}{\partial s_i} = -1 + (1-\pi)\,\varphi'(s_i) + \pi\,(1-p)\,\phi'(s_i+z_i) = 0,$$
$$\frac{\partial U_i}{\partial e_i} = -w_i + \pi p h'(e_i) = 0.$$

With the government anticipating s_i and setting z_i such that $(1-p)\,\phi'(s_i+z_i) = 1$, the above equations simplify to

$$\begin{aligned}\varphi'(s_i) &= 1,\\ h'(e_i) &= \frac{w_i}{\pi p},\end{aligned}$$

which are the optimal conditions (31)–(32).

4.3 Second-best policy choice

4.3.1 Optimal allocation

We follow the same informational structure as in the OO scheme and assume that incomes, $I_i \equiv w_i(1-e_i)$, consumption levels, c_i, and savings, s_i, are publicly observable while types, w_i, and times spent with children, e_i, are not. We also follow the same procedure to characterize the optimal tax system. The government chooses c_i, s_i, I_i, and z_i to maximize $\sum n_i \delta_i V(U_i)$ subject to its budget constraint

$$\sum_i n_i (I_i - c_i - s_i) \geq \pi \sum_i n_i z_i,$$

and the self selection constraints

$$U_i \geq U_{ik} \text{ for all } i \neq k;\ i,k = 1,2,\ldots,N,$$

where

$$U_i = c_i + (1-\pi)\,\varphi(s_i) + \pi\left[ph\left(1-\frac{I_i}{w_i}\right) + (1-p)\,\phi(s_i+z_i)\right], \tag{34}$$
$$U_{ik} = c_k + (1-\pi)\,\varphi(s_k) + \pi\left[ph\left(1-\frac{I_k}{w_i}\right) + (1-p)\,\phi(s_k+z_k)\right]. \tag{35}$$

Summarize the government's problem by the Lagrangian expression

$$\begin{aligned}\mathcal{L} &= \sum_i n_i \left[\delta_i V\left(U_i\right) + \mu\left(I_i - c_i - s_i - \pi z_i\right)\right] + \\ &\quad \sum_i \sum_{k \neq i} \lambda_{ik}(U_i - U_{ik}),\end{aligned}$$

where μ and λ_{ik}'s are non-negative Lagrangian multipliers associated with the government's budget constraint and the self-selection constraints. Rearrange the terms and rewrite $\mathcal{L}$ as

$$\begin{aligned}\mathcal{L} &= \sum_i \left[n_i \delta_i V\left(U_i\right) + \sum_{k \neq i} \lambda_{ik} U_i\right] + \mu \sum_i n_i \left(I_i - c_i - s_i - \pi z_i\right) \\ &\quad - \sum_i \sum_{k \neq i} \lambda_{ik} U_{ik},\end{aligned} \tag{36}$$

We show in Appendix (E) that the FOC associated with (36) can be written as

$$\delta_i n_i V'\left(U_i\right) = \mu n_i + \sum_{k \neq i} \left(\lambda_{ki} - \lambda_{ik}\right), \tag{37}$$

$$\varphi'\left(s_i\right) = 1, \tag{38}$$

$$\left[\frac{w_i}{\pi p} - h'\left(e_i\right)\right] = \frac{1}{\mu n_i} \sum_{k \neq i} \lambda_{ki} \left[h'\left(1 - \frac{I_i}{w_i}\right) - \frac{w_i}{w_k} h'\left(1 - \frac{I_i}{w_k}\right)\right], \tag{39}$$

$$(1 - p)\, \phi'\left(s_i + z_i\right) = 1. \tag{40}$$

4.3.2 Decentralization

Decentralization of the optimal allocation can be achieved with a nonlinear income tax $T\left(I_i\right)$. Unlike OO, no tax or subsidy on savings is required. Consider the optimization problem of parents i who maximize

$$U_i = c_i + (1 - \pi)\, \varphi\left(s_i\right) + \pi \left[ph\left(1 - \frac{I_i}{w_i}\right) + (1 - p)\, \phi\left(s_i + z_i\right)\right],$$

subject to the budget constraint,

$$c_i + s_i = I_i - T\left(I_i\right).$$

Substitute for c_i from the budget constraint into the expression for U_i and maximizing the resulting equation with respect to s_i and I_i yields

$$\begin{aligned}\frac{\partial U_i}{\partial s_i} &= -1+(1-\pi)\,\varphi'(s_i)+\pi(1-p)\,\phi'(s_i+z_i),\\ \frac{\partial U_i}{\partial I_i} &= 1-T'(I_i)-\frac{1}{w_i}\pi p h'\left(1-\frac{I_i}{w_i}\right)=0.\end{aligned}$$

Simplifying

$$\varphi'(s_i) = \frac{1-\pi(1-p)\,\phi'(s_i+z_i)}{1-\pi}, \tag{41}$$

$$T'(I_i) = 1-\frac{1}{w_i}\pi p h'\left(1-\frac{I_i}{w_i}\right). \tag{42}$$

With the mechanism designer setting

$$\begin{aligned}\pi(1-p)\,\phi'(s_i+z_i) &= 1,\\ \left[1-\frac{1}{w_i}\pi p h'\left(1-\frac{I_i}{w_i}\right)\right] &= \frac{\pi p}{\mu n_i w_i}\sum_{k\neq i}\lambda_{ki}\left[h'\left(1-\frac{I_i}{w_i}\right)-\frac{w_i}{w_k}h'\left(1-\frac{I_i}{w_k}\right)\right],\end{aligned}$$

Parents' private optimization satisfies second-best conditions (38)–(39).

4.3.3 Properties and interpretation

The main properties of the decentralizing policy and the second-best allocation, particularly its comparison with the first-best, are stated in the following proposition proved in Appendix C.

Proposition 2 *(i)The SB value of savings uniform and is identical to its FB value.*

$$s_i^{SB}=s^{SB}=s_i^{FB}=s^{FB}$$

(ii)Assume upward self-selection constraints are non-binding. It then follows that for all $i=1,2,\ldots,N-1$,

$$I_i^{SB} < I_i^{FB}, \tag{43}$$

$$e_i^{SB} > e_i^{FB}. \tag{44}$$

and for $i = N$

$$I_N^{SB} = I_N^{FB},$$
$$e_N^{SB} = e_N^{FB}.$$

(iii) LTC is uniformly provided with $z_i^{SB} = z^{SB}$ for all $i = 1, 2, \ldots, N$. Furthermore $z^{SB} = z^{FB}$ so that the level of social LTC is identical to its FB level.(iv) If upward self-selection constraints are non-binding, marginal income tax rates are positive:

$$T'(I_i) > 0, \quad i = 1, 2, \ldots, N-1,$$
$$T'(I_N) = 0.$$

Not surprisingly Proposition 2 shows that, as with OO, we have the traditional no-distortion-at-the-top result. Given the quasilinearity, this applies not only to the rules for optimality but also to the levels of variables as shown by point *(iii) in above.* For other individuals we have distortions. Equations (43) and (44) are equivalent; a smaller I goes hand-in-hand with a larger e. This follows from the relationship $I = w(1-e)$. Similar to OO, the downward distortion in I, and the positive marginal income tax rate $T'(\cdot)$, arise because, with the mimicker being more productive than the mimicked, reducing I is more costly for the mimicker. This also explains the upward distortion in e. Again, it is interesting to find that a by-product of the government's redistributive policy is to encourage parents to spend more time with their children. Concerning point *(iv),* recall that when public assistance under OO carries no stigma, first- and second-best levels of z are equal. Since the TU policy entails no stigma, we have the same result. The reason is the same: mimicker and mimicked have the same marginal willingness to pay for z and distortion cannot weaken the incentive constraint. Finally, s is also valued in the same way by mimicker and mimicked so that a distortion cannot relax the incentive constraint. Therefore the level of s is the same as under full information.

4.4 Second-best: Unobservable savings

Under this assumption, the value of savings is determined through the maximization of the parents' expected utility (16) which yields

$$(1-\pi)\varphi'(s_i) + \pi\,(1-p)\,\phi'\,(s_i + z) = 1. \tag{45}$$

This is the same equation we had with observable savings. With all other SB characterizations remaining the same as we had under observable savings assumptions, we will have the same outcome. Consequently, the government will set z such that $(1-p)\,\phi'\,(s_i+z) = 1$ so that $\varphi'(s_i) = 1$. This makes sense in that even with observable savings, savings were not taxed. It is in contrast with OO under which, unobservability resulted in less savings and, with it, different values for other variables.

5 Opting out versus topping up

We have already shown that $s^{TU} > s^{OO}$ and that this holds both for the FB and the SB. Similarly since $\phi'\,(s_i+z_i) = 1/\,(1-p)$ under TU and $\phi'\,(z_i) = 1$ under OO, it follows that

$$\phi'\left(s^{TU} + z^{TU}\right) > \phi'\left(z^{OO}\right) \Rightarrow s^{TU} + z^{TU} < z^{OO} \Rightarrow z^{TU} < z^{OO}.$$

Expressed in words, the level of social LTC is smaller under topping up than under opting out. This makes sense because providing the same level of z to all recipients costs more with the TU program under which public assistance is offered to all parents and not just those without family assistance.

In comparing these two policies, the most interesting and policy-relevant question relates to the level of overall welfare under each. The following proposition demonstrates that OO dominates TU. Specifically, it shows that the optimal TU allocation can be implemented under OO in a way that utility in all states of nature and for all types is the same but the budgetary cost is smaller. Interestingly, this holds both for the FB and the SB solution. For the latter case this requires that the considered allocation also satisfies the incentive constraints and we show that this is true.

Proposition 3 *Assume that $\gamma_i = 1$ so that the OO policy does not carry stigma.*

(i) FB - Consider the TU first-best solution $c_i^{TU}, s_i^{TU}, e_i^{TU}, z_i^{TU}$, as defined by expressions (30)–(A12). The OO policy $(c_i, s_i, e_i, z_i) = (c_i^{TU}, s_i^{TU}, e_i^{TU}, s_i^{TU} + z_i^{TU})$ gives the same utility level to all parents at smaller cost.

(ii) SB - Take the second-best TU solution $c_i^{TU}, s_i^{TU}, I_i^{TU}, z_i^{TU}$, now redefined by expressions (37)–(40). Now use and OO scheme $(c_i, s_i, I_i, z_i) = (c_i^{TU}, s_i^{TU}, I_i^{TU}, s_i^{TU} + z_i^{TU})$. This policy yields the same utility for everyone, is less expensive and continues to satisfy the incentive constraints (so that it can be implemented under OO).[19]

Proof. (i) The additional expenditures government makes on parents without altruistic children, by raising z_i^{TU} under TU to $s_i^{TU} + z_i^{TU}$ under OO, are offset by its taxing their private savings, s_i^{TU}, away. Under OO, the government saves the public LTC it provides to parents with altruistic children, z_i^{TU}. Formally, substitute $(c_i^{TU}, s_i^{TU}, e_i^{TU}, s_i^{TU} + z_i^{TU})$ into (6) and compare it to (27) evaluated at $(c_i^{TU}, s_i^{TU}, e_i^{TU}, z_i^{TU})$. This shows that utilities are the same under both policies. Furthermore, since the budget constraint must be binding at the *optimal TU* policy, we have

$$\sum_i n_i \left[w_i \left(1 - e_i^{TU}\right) - c_i^{TU} - s_i^{TU} \right] = \pi \sum_i n_i z_i^{TU} > \pi(1-p) \sum_i n_i (s_i^{TU} + z_i^{TU} - s_i^{TU})$$

so that the budget constraint under TU (7) is satisfied with strict inequality. The government will then have

$$\pi \sum_i n_i z_i^{TU} - \pi(1-p) \sum_i n_i (s_i^{TU} + z_i^{TU} - s_i^{TU}) = p \sum_i n_i z_i^{TU},$$

in extra resources that can be used to make everyone better off.

[19] Not to clutter notation, we do not use different notation for the variables in the FB and SB.

(ii) The argument presented in (i) continues to apply here. To show that the policy continues to be feasible under OO, we have to additionally show that the incentive constraints are also satisfied. To do this, evaluate the utility of (a) the mimicker (equation (16)), and the mimicked (equation (17)), under OO at $(c_i^{TU}, s_i^{TU}, I_i^{TU}, s_i^{TU} + z_i^{TU})$, and (b) their counterparts under TU (equations (34) and (35)) evaluated at $(c_i^{TU}, s_i^{TU}, I_i^{TU}, z_i^{TU})$. Comparing (a) to (b) shows that utilities of mimicker and mimicked are the same in the two cases.[20] Consequently the replicated policy continues to satisfy the IC constraint under OO. ■

The results make sense. As demonstrated in the previous section, public LTC under an OO scheme is directed toward parents with uncaring children. That is, the public system effectively provides insurance against the failure of altruism—something not possible under a TU scheme. Parents with uncaring children continue to receive the same amount of social LTC care and thus maintain their utility level. Parents with caring children receive no social LTC care, but they are not disadvantaged, as their children compensate for the shortfall. The government's savings from not funding LTC for these parents represent a net benefit for the generation in question the cost of which is passed on to the children. Observe that Proposition 3 assumes the OO policy carries no stigma ($\gamma_i = 1$). By continuity property, however, the results of the proposition hold when γ_i's are close to one. That is, when the stigma associated with the OO policy is

[20] We have, under OO at $(c_i^{TU}, s_i^{TU}, I_i^{TU}, s_i^{TU} + z_i^{TU})$,

$$\begin{aligned} U_i &= c_i^{TU} + (1-\pi)\varphi\left(s_i^{TU}\right) + \pi\left[ph\left(1 - \frac{I_i^{TU}}{w_i}\right) + (1-p)\phi\left(s_i^{TU} + z_i^{TU}\right)\right], \\ U_{ik} &= c_k^{TU} + (1-\pi)\varphi\left(s_k^{TU}\right) + \pi\left[ph\left(1 - \frac{I_k^{TU}}{w_i}\right) + (1-p)\phi\left(s_k^{TU} + z_k^{TU}\right)\right], \end{aligned}$$

and under TU at $(c_i^{TU}, s_i^{TU}, I_i^{TU}, z_i^{TU})$

$$\begin{aligned} U_i &= c_i^{TU} + (1-\pi)\varphi\left(s_i^{TU}\right) + \pi\left[ph\left(1 - \frac{I_i^{TU}}{w_i}\right) + (1-p)\phi\left(s_i^{TU} + z_i^{TU}\right)\right], \\ U_{ik} &= c_k^{TU} + (1-\pi)\varphi\left(s_k^{TU}\right) + \pi\left[ph\left(1 - \frac{I_k^{TU}}{w_i}\right) + (1-p)\phi\left(s_k^{TU} + z_k^{TU}\right)\right] \end{aligned}$$

sufficiently small.

6 Concluding remarks and suggestions for future research

Longer life expectancy, rising healthcare costs, expensive and inadequate insurance, and a decline in family caregiving—due to increased female workforce participation and increased mobility that helps children to move away—have created a major societal challenge in many developed countries: the long-term care (LTC) crisis. The LTC problem is complex and requires a mix of policy, economic, and social interventions to ensure sustainable, equitable care for aging populations. Over the past three decades, a vast body of research has examined the economics of LTC from various perspectives. Our paper contributes to this literature by exploring how, in an uncertain environment, redistributive tax policies and LTC program design interact with informal care incentives, shaping long-term caregiving outcomes.

The paper has focused on the well-being of a generation of adults with varying earning abilities across three life stages. These individuals make decisions about savings and time to spend spent with their children in the face of two key uncertainties: the risk of becoming dependent in old age and the likelihood of having altruistic children. The government, in designing its LTC policies—either through an OO or a TU approach—faces the same uncertainties. Additionally, the government formulates its tax policy to mitigate the inequality of incomes. Policy choices are constrained only by the availability of resources and information.

One key finding of our study is that if no stigma is attached to public LTC assistance, OO policies are more cost-effective than TU policies in both first- and second-best environments. OO is self-targeted in that only those genuinely in need seek public assistance. Informal care recipients do not turn to public support as long as it provides less assistance than family does. This limits government spending without reducing overall welfare. Another important result is that, under an OO system, saving should be subsidized due to the positive fiscal externality it generates. Other findings, derived under asymmetric information and for both OO and TU systems, include: Allocations other than savings are distorted for redistributive purposes; marginal income tax rates are positive; policies should encourage time spent with children enhancing family caregiving.

This paper, like most papers on LTC, has concentrated on a single motive for providing informal care—altruism in our case. In reality, informal care may be the result of several intertwined motives as well as implicit or explicit exchanges. For example, parents may offer intergenerational transfers as an incentive to encourage their children to provide more care than they would out of pure altruism. In other words, informal care may in part be exchange-based where children provide care in return for financial compensation such as gifts or bequests[21] Incorporating this dynamic into our model would be a valuable extension of our analysis. It could serve as an alternative mechanism for ensuring care when altruism fails while also mitigating the extent to which public long-term care displaces informal caregiving. Additionally, it would introduce another dimension in policy design as it relates to possible taxation or subsidization of intergenerational transfers.

Canta and Cremer (2019, 2021, 2023) study some of these questions but do not explicitly consider altruism. Neither do they consider *ex ante* heterogeneous. Yet, in exchange-based care, redistribution is a crucial consideration. Wealthier individuals are not only better able to afford formal care but are also in a better position to provide gifts or bequests in exchange for formal care.

We have also considered a simplified model of dependency, categorizing elderly individuals as either healthy or dependent. In reality, dependency is a gradual process. Initially, individuals tend to experience mild dependency—often measured by the Katz index—requiring only informal or formal home care. Over time, however, many progress

[21] See Canta and Cremer (2019) for a discussion and references.

to a more severe stage, necessitating institutional care in a nursing home. Borsenberger et al. (2024) consider a model incorporating varying degrees of dependency severity, but their focus is on insurance contract design for *ex ante* identical individuals, without considering informal care. Extending our framework to incorporate mild and severe dependency would provide deeper insights into the interaction between long-term care (LTC) design and tax policy, particularly regarding the effectiveness of opt-out (OO) and top-up (TU) programs.

Informal care plays a crucial role in the early stages of dependency which diminishes in importance as the need for formal nursing home care intensifies. Because informal care is typically provided at home, top-up policies tend to be more beneficial for individuals in the initial stages of dependency. In contrast, nursing home care—being extremely costly—cannot be universally provided for free, making opt-out policies the more viable approach. This also underscores the necessity of redistribution in the context of nursing home care, as its high costs are unaffordable for most individuals without public assistance or subsidies.

Another simplification in our model is the assumption that individuals accurately assess their risk of dependency and the likelihood of their children's altruism. In reality, risk misperception is widespread.[22] Many individuals underestimate their dependency risk due to myopia or a reluctance to acknowledge unpleasant future scenarios. Likewise, they may hold overly optimistic or pessimistic views regarding their children's willingness to provide care. Incorporating risk misperception into our analysis would be a valuable extension. If the government is paternalistic and seeks to maximize individuals' true preferences, misperception would not alter the first-best allocation but would introduce a Pigouvian correction in policy implementation. However, in a second-best setting, misperception would impact incentive constraints, requiring careful analysis—particularly if it is correlated with income.

[22] See Cremer and Roeder (2013) for discussin and references.

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Appendix

A Proof of Lemma 1

Differentiating (3) with respect to e_i and arranging the terms yields

$$\frac{d(x+a^*)}{de} = m'(e) = \frac{1}{\phi''(x+a^*)} \frac{-f'(e)}{(f(e))^2} > 0. \tag{A1}$$

Next differentiate equation (A1) wrt e to get

$$\begin{aligned} m''(e) &= \frac{-\phi'''(x+a^*)\, m'(e)}{\left[\phi''(x+a^*)\right]^2} \frac{-f'(e)}{(f(e))^2} - \\ &\quad \frac{1}{\phi''(x+a^*)} \frac{f''(e)(f(e))^2 - 2f(e)(f'(e))^2}{(f(e))^4}. \end{aligned}$$

Substitution for $m'(e_i)$ from (A1) in above and simplifying

$$\begin{aligned} m''(e) &= \frac{-\phi'''(x+a^*)}{\left[\phi''(x+a^*)\right]^2} \left[\frac{1}{\phi''(x+a^*)} \frac{-f'(e)}{(f(e))^2}\right] \frac{-f'(e)}{(f(e))^2} - \\ &\quad \frac{1}{\phi''(x+a^*)} \frac{f''(e)(f(e))^2 - 2f(e)(f'(e))^2}{(f(e))^4} \\ &= -\frac{\phi'''(x+a^*)}{\left[\phi''(x+a^*)\right]^3} \left[\frac{f'(e)}{(f(e))^2}\right]^2 - \frac{1}{\phi''(x+a^*)} \frac{f''(e) f(e) - 2(f'(e))^2}{(f(e))^3} \\ &= -\frac{1}{(f(e))^3 \phi''(x+a^*)} \left\{ \frac{\phi'''(x+a^*)}{\left[\phi''(x+a^*)\right]^2} \frac{(f'(e))^2}{f(e)} + f''(e) f(e) - 2\left(f'(e)\right)^2 \right\} < 0. \end{aligned}$$

B Proof of Equations (18)–(21)

Rearrange the terms to rewrite the Lagrangian expression as

$$\begin{aligned} \mathcal{L} &= \sum_i \left[n_i \delta_i V(U_i) + \sum_{k \neq i} \lambda_{ik} U_i \right] \\ &\quad + \mu \sum_i n_i \left[I_i - c_i - s_i - \pi(1-p)(z_i - s_i) \right] - \sum_i \sum_{k \neq i} \lambda_{ik} U_{ik}. \end{aligned} \tag{A2}$$

This yields the following first-order conditions for $i = 1, 2, \ldots, N$,[23]

$$\frac{\partial \mathcal{L}}{\partial c_i} = \left[\delta_i n_i V'(U_i) + \sum_{k \neq i} \lambda_{ik}\right] \frac{\partial U_i}{\partial c_i} - \mu n_i - \sum_{k \neq i} \lambda_{ki} \frac{\partial U_{ki}}{\partial c_i} = 0, \tag{A3}$$

$$\frac{\partial \mathcal{L}}{\partial s_i} = \left[n_i \delta_i V'(U_i) + \sum_{k \neq i} \lambda_{ik}\right] \frac{\partial U_i}{\partial s_i} + \mu n_i \left[-1 + \pi(1-p)\right] - \sum_{k \neq i} \lambda_{ki} \frac{\partial U_{ki}}{\partial s_i} = 0, \tag{A4}$$

$$\frac{\partial \mathcal{L}}{\partial I_i} = \left[n_i \delta_i V'(U_i) + \sum_{k \neq i} \lambda_{ik}\right] \frac{\partial U_i}{\partial I_i} + \mu n_i - \sum_{k \neq i} \lambda_{ki} \frac{\partial U_{ki}}{\partial I_i} = 0, \tag{A5}$$

$$\frac{\partial \mathcal{L}}{\partial z_i} = \left[n_i \delta_i V'(U_i) + \sum_{k \neq i} \lambda_{ik}\right] \frac{\partial U_i}{\partial z_i} - \mu \pi(1-p) n_i - \sum_{k \neq i} \lambda_{ki} \frac{\partial U_{ki}}{\partial z_i} = 0. \tag{A6}$$

Partially differentiating U_i and U_{ki} with respect to c_i, s_i, I_i, and z_i yields

$$\begin{aligned}
\frac{\partial U_i}{\partial c_i} &= 1, \\
\frac{\partial U_i}{\partial s_i} &= (1-\pi)\varphi'(s_i), \\
\frac{\partial U_i}{\partial I_i} &= -\frac{1}{w_i} \pi p h'(e_i), \\
\frac{\partial U_i}{\partial z_i} &= \pi (1-p) \gamma_i \phi'(z_i),
\end{aligned}$$

$$\begin{aligned}
\frac{\partial U_{ki}}{\partial c_i} &= 1, \\
\frac{\partial U_{ki}}{\partial s_i} &= (1-\pi)\varphi'(s_i), \\
\frac{\partial U_{ki}}{\partial I_i} &= -\frac{1}{w_k} \pi p h'(e_{ki}), \\
\frac{\partial U_{ki}}{\partial z_i} &= \pi (1-p) \gamma_k \phi'(z_i).
\end{aligned}$$

Substituting these values in (A3)–(A6) and simplifying, we successively obtain

[23] Observe that the derivative of $\sum_i \sum_{k \neq i} \lambda_{ik} U_{ik}$ with respect to a variable g_i is $\sum_{k \neq i} \lambda_{ki} \frac{\partial U_{ki}}{\partial g_i}$ so that it results in the transposition of their i and k indices. This is due to the fact that for $k \neq i$, $\partial U_{ik}/\partial g_i = 0$ while at the same time $\partial U_{ki}/\partial g_i \neq 0$.

$$\left[\delta_i n_i V'(U_i) + \sum_{k\neq i}\lambda_{ik}\right] - \mu n_i - \sum_{k\neq i}\lambda_{ki} = 0,$$

$$\left[n_i\delta_i V'(U_i) + \sum_{k\neq i}\lambda_{ik}\right](1-\pi)\varphi'(s_i) + \mu n_i\left[-1+\pi(1-p)\right] - \sum_{k\neq i}\lambda_{ki}(1-\pi)\varphi'(s_i) = 0,$$

$$\left[n_i\delta_i V'(U_i) + \sum_{k\neq i}\lambda_{ik}\right]\left(-\frac{1}{w_i}\pi p h'(e_i)\right) + \mu n_i - \sum_{k\neq i}\lambda_{ki}\left(-\frac{1}{w_k}\pi p h'(e_{ki})\right) = 0,$$

$$\left[n_i\delta_i V'(U_i) + \sum_{k\neq i}\lambda_{ik}\right]\pi(1-p)\gamma_i\phi'(z_i) - \mu n_i\pi(1-p) - \sum_{k\neq i}\lambda_{ki}\pi(1-p)\gamma_k\phi'(z_i) = 0.$$

or

$$\left[\delta_i n_i V'(U_i) + \sum_{k\neq i}\lambda_{ik}\right] = \mu n_i + \sum_{k\neq i}\lambda_{ki},$$

$$\left[\mu n_i + \sum_{k\neq i}\lambda_{ki}\right](1-\pi)\varphi'(s_i) = -\mu n_i\left[-1+\pi(1-p)\right] + \sum_{k\neq i}\lambda_{ki}(1-\pi)\varphi'(s_i),$$

$$\left[\mu n_i + \sum_{k\neq i}\lambda_{ki}\right]\left(\frac{1}{w_i}\pi p h'(e_i)\right) = \mu n_i + \sum_{k\neq i}\lambda_{ki}\left(\frac{1}{w_k}\pi p h'(e_{ki})\right),$$

$$\left[\mu n_i + \sum_{k\neq i}\lambda_{ki}\right]\pi(1-p)\gamma_i\phi'(z_i) = \mu n_i\pi(1-p) + \sum_{k\neq i}\lambda_{ki}\pi(1-p)\gamma_k\phi'(z_i).$$

Simplifying and rearranging yields expressions (18)–(21).

C Proof of Proposition 1

(i) With $\lambda_{ki} = 0$ for all $k < i$, equation (20) simplifies to

$$\mu n_i\left[h'(e_i) - \frac{w_i}{\pi p}\right] = \sum_{k>i}\lambda_{ki}\left[\frac{w_i}{w_k}h'\left(1-\frac{I_i}{w_k}\right) - h'\left(1-\frac{I_i}{w_i}\right)\right], \quad i = 1,2,\ldots,N \tag{A7}$$

$$\mu n_N\left[h'(e_N) - \frac{w_N}{\pi p}\right] = 0. \tag{A8}$$

We have, for $k > i$, $w_k > w_i \Rightarrow$

$$1 - \frac{I_i}{w_k} > 1 - \frac{I_i}{w_i} \Rightarrow h'\left(1 - \frac{I_i}{w_k}\right) < h'\left(1 - \frac{I_i}{w_i}\right).$$

This inequality along with the fact that $w_k > w_i$ then implies

$$\frac{w_i}{w_k} h'\left(1 - \frac{I_i}{w_k}\right) < h'\left(1 - \frac{I_i}{w_k}\right) < h'\left(1 - \frac{I_i}{w_i}\right).$$

It follows from this relationship that the RHS of (A7) is negative which in turn implies that its left-hand-side (LHS) is also negative. That is,

$$h'(e_i) < \frac{w_i}{\pi p}, \quad i = 1, 2, \ldots, N-1.$$

Moreover, it follows from (A8) that

$$h'(e_N) - \frac{w_N}{\pi p}.$$

Substituting $h'(e_i) < w_i/\pi p$, for $i = 1, 2, \ldots, N-1$, and $h'(e_N) = w_N/\pi p$ in (23) results in

$$\begin{aligned} T'(I_i) &> 0, \quad i = 1, 2, \ldots, N-1, \\ T'(I_N) &= 0. \end{aligned}$$

(ii) First, with $\lambda_{ki} = 0$ for all $k < i$, we have already shown that

$$\begin{aligned} h'\left(e_i^{SB}\right) &< \frac{w_i}{\pi p}, \quad i = 1, 2, \ldots, N-1, \\ h'\left(e_N^{SB}\right) &= \frac{w_N}{\pi p}. \end{aligned}$$

Previously, we had shown that at the FB allocation

$$h'\left(e_i^{FB}\right) = \frac{w_i}{\pi p} \quad i = 1, 2, \ldots, N.$$

Comparing the SB with FB allocation indicates

$$\begin{aligned} h'\left(e_i^{SB}\right) &< h'\left(e_i^{FB}\right), \quad i = 1, 2, \ldots, N-1, \\ h'\left(e_N^{SB}\right) &= h'\left(e_N^{FB}\right). \end{aligned}$$

It then follows from the concavity of $h(\cdot)$ that

$$\begin{aligned} e_i^{SB} &> e_i^{FB}, \quad i = 1, 2, \ldots, N-1, \\ e_N^{SB} &= e_N^{FB}. \end{aligned}$$

Second, the properties

$$\begin{aligned} I_i^{SB} &< I_i^{FB}, \quad i = 1, 2, \ldots, N-1, \\ I_N^{SB} &= I_N^{FB}, \end{aligned}$$

follow immediately from the definition of I_i

$$I_i = w_i (1 - e_i),$$

and the properties $e_i^{SB} > e_i^{FB}, i = 1, 2, \ldots, N-1$, and $e_N^{SB} = e_N^{FB}$.

Third, with $\lambda_{ki} = 0$ for all $k < i$, equation (21) simplifies to

$$\begin{aligned} &\mu n_i \gamma_i \phi'(z_i) - \mu n_i = \phi'(z_i) \sum_{k>i} \lambda_{ki} (\gamma_k - \gamma_i) \leq 0, \quad i = 1, 2, \ldots, N-1, \\ &\mu n_N \gamma_N \phi'(z_N) - \mu n_N = 0, \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} &\phi'(z_i) \leq \frac{1}{\gamma_i}, \quad i = 1, 2, \ldots, N-1, \\ &\phi'(z_N) = \frac{1}{\gamma_N}, \end{aligned}$$

In the FB, on the other hand, we have

$$\phi'\left(z_i^{SB}\right) = \frac{1}{\gamma_i}, \quad i = 1, 2, \ldots, N.$$

$\Rightarrow$

$$\begin{aligned} \phi'\left(z_i^{SB}\right) &\leq \phi'\left(z_i^{FB}\right), \quad i = 1, 2, \ldots, N-1, \\ \phi'\left(z_N^{SB}\right) &= \phi'\left(z_N^{FB}\right). \end{aligned}$$

The concavity of $\phi(\cdot)$ then implies,

$$
\begin{aligned}
z_i^{SB} &\geq z_i^{FB}, \quad i = 1, 2, \ldots, N-1, \\
z_N^{SB} &= z_N^{FB}.
\end{aligned}
$$

(iii) The proof follows directly from (9) and (19).

D Proof of Equations (30)–(33)

The FOC are

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_i} &= n_i \left[\delta_i V'(U_i) \frac{\partial U_i}{\partial c_i} - \mu \right] = 0, \\
\frac{\partial \mathcal{L}}{\partial s_i} &= n_i \left[\delta_i V'(U_i) \frac{\partial U_i}{\partial s_i} - \mu \right] = 0, \\
\frac{\partial \mathcal{L}}{\partial e_i} &= n_i \left[\delta_i V'(U_i) \frac{\partial U_i}{\partial e_i} - \mu w_i \right] = 0, \\
\frac{\partial \mathcal{L}}{\partial z_i} &= n_i \left[\delta_i V'(U_i) \frac{\partial U_i}{\partial z_i} - \mu \pi \right] = 0.
\end{aligned}
$$

From the specification of U_i in (27), we have

$$
\begin{aligned}
\frac{\partial U_i}{\partial c_i} &= 1, \\
\frac{\partial U_i}{\partial s_i} &= (1-\pi) \varphi'(s_i) + \pi (1-p) \phi'(s_i + z_i), \\
\frac{\partial U_i}{\partial e_i} &= \pi p h'(e_i), \\
\frac{\partial U_i}{\partial z_i} &= \pi (1-p) \phi'(s_i + z_i).
\end{aligned}
$$

Substituting in above:

$$
\frac{\partial \mathcal{L}}{\partial c_i} = n_i \left[\delta_i V'(U_i) - \mu \right] = 0, \tag{A9}
$$

$$
\frac{\partial \mathcal{L}}{\partial s_i} = \delta_i V'(U_i) \left[(1-\pi) \varphi'(s_i) + \pi (1-p) \phi'(s_i + z_i) \right] - \mu = 0, \tag{A10}
$$

$$
\frac{\partial \mathcal{L}}{\partial e_i} = \delta_i V'(U_i) \pi p h'(e_i) - \mu w_i = 0, \tag{A11}
$$

$$
\frac{\partial \mathcal{L}}{\partial z_i} = \delta_i V'(U_i) \pi (1-p) \phi'(s_i + z_i) - \mu \pi = 0. \tag{A12}
$$

$\Rightarrow$

$$
\begin{aligned}
&\delta_i V'(U_i) = \mu, \\
&\left[(1-\pi)\varphi'(s_i) + \pi(1-p)\phi'(s_i+z_i)\right] = 1, \\
&\pi p h'(e_i) = w_i, \\
&(1-p)\phi'(s_i+z_i) = 1.
\end{aligned}
$$

Rearranging these expressions yields (30)–(33).

E Proof of Equations (37)–(40)

This yields the following first-order conditions for $i = 1, 2, \ldots, N$,

$$\frac{\partial \mathcal{L}}{\partial c_i} = \left[\delta_i n_i V'(U_i) + \sum_{k \neq i} \lambda_{ik}\right] \frac{\partial U_i}{\partial c_i} - \mu n_i - \sum_{k \neq i} \lambda_{ki} \frac{\partial U_{ki}}{\partial c_i} = 0, \tag{A13}$$

$$\frac{\partial \mathcal{L}}{\partial s_i} = \left[n_i \delta_i V'(U_i) + \sum_{k \neq i} \lambda_{ik}\right] \frac{\partial U_i}{\partial s_i} - \mu n_i - \sum_{k \neq i} \lambda_{ki} \frac{\partial U_{ki}}{\partial s_i} = 0, \tag{A14}$$

$$\frac{\partial \mathcal{L}}{\partial I_i} = \left[n_i \delta_i V'(U_i) + \sum_{k \neq i} \lambda_{ik}\right] \frac{\partial U_i}{\partial I_i} + \mu n_i - \sum_{k \neq i} \lambda_{ki} \frac{\partial U_{ki}}{\partial I_i} = 0, , \tag{A15}$$

$$\frac{\partial \mathcal{L}}{\partial z_i} = \left[n_i \delta_i V'(U_i) + \sum_{k \neq i} \lambda_{ik}\right] \frac{\partial U_i}{\partial z_i} - \mu n_i \pi - \sum_{k \neq i} \lambda_{ki} \frac{\partial U_{ki}}{\partial z_i} = 0. \tag{A16}$$

We have

$$
\begin{aligned}
\frac{\partial U_i}{\partial c_i} &= 1 \\
\frac{\partial U_i}{\partial I_i} &= -\frac{1}{w_i} \pi p h'(e_i) \\
\frac{\partial U_i}{\partial s_i} &= (1-\pi)\varphi'(s_i) + \pi(1-p)\phi'(s_i+z_i) \\
\frac{\partial U_i}{\partial z_i} &= \pi(1-p)\phi'(s_i+z_i)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial U_{ki}}{\partial c_i} &= 1 \\
\frac{\partial U_{ki}}{\partial I_i} &= -\frac{1}{w_k}\pi p h'\left(1-\frac{I_i}{w_k}\right) \\
\frac{\partial U_{ki}}{\partial s_i} &= (1-\pi)\,\varphi'(s_i)+\pi(1-p)\,\phi'(s_i+z_i) \\
\frac{\partial U_{ki}}{\partial z_i} &= \pi(1-p)\,\phi'(s_i+z_i)
\end{aligned}
$$

Substituting these values in (A13)–(A16) and simplifying⇒

$$
\begin{aligned}
&\left[\delta_i n_i V'(U_i)+\sum_{k\neq i}\lambda_{ik}\right]=\mu n_i+\sum_{k\neq i}\lambda_{ki}, \\
&\left[\mu n_i+\sum_{k\neq i}\lambda_{ki}\right]\big[(1-\pi)\,\varphi'(s_i)+\pi(1-p)\,\phi'(s_i+z_i)\big] \\
&-\mu n_i-\sum_{k\neq i}\lambda_{ki}\big[(1-\pi)\,\varphi'(s_i)+\pi(1-p)\,\phi'(s_i+z_i)\big]=0, \\
&\left[\mu n_i+\sum_{k\neq i}\lambda_{ki}\right]\left[-\frac{1}{w_i}\pi p h'(e_i)\right]+\mu n_i \\
&-\sum_{k\neq i}\lambda_{ki}\left[-\frac{1}{w_k}\pi p h'\left(1-\frac{I_i}{w_k}\right)\right]=0, \\
&\left[\mu n_i+\sum_{k\neq i}\lambda_{ki}\right]\pi(1-p)\,\phi'(s_i+z_i) \\
&-\mu n_i\pi-\sum_{k\neq i}\lambda_{ki}\pi(1-p)\,\phi'(s_i+z_i)=0.
\end{aligned}
$$

$\Rightarrow$

$$\delta_i n_i V'(U_i) = \mu n_i + \sum_{k \neq i} (\lambda_{ki} - \lambda_{ik}), \tag{A17}$$

$$\mu n_i \left[(1-\pi)\varphi'(s_i) + \pi(1-p)\phi'(s_i+z_i)\right] + \left[(1-\pi)\varphi'(s_i) + \pi(1-p)\phi'(s_i+z_i)\right]\sum_{k \neq i}\lambda_{ki} \tag{A18}$$

$$-\mu n_i - \left[(1-\pi)\varphi'(s_i) + \pi(1-p)\phi'(s_i+z_i)\right]\sum_{k \neq i}\lambda_{ki} = 0,$$

$$\left[\mu n_i + \sum_{k \neq i}\lambda_{ki}\right]\left[-\frac{1}{w_i}\pi p h'(e_i)\right] + \mu n_i - \sum_{k \neq i}\lambda_{ki}\left[-\frac{1}{w_k}\pi p h'\left(1 - \frac{I_i}{w_k}\right)\right] = 0, \tag{A19}$$

$$\left[\mu n_i \pi(1-p)\phi'(s_i+z_i) + \pi(1-p)\phi'(s_i+z_i)\sum_{k \neq i}\lambda_{ki}\right] \tag{A20}$$

$$-\mu n_i \pi - \pi(1-p)\phi'(s_i+z_i)\sum_{k \neq i}\lambda_{ki} = 0.$$

$\Rightarrow$

$$\begin{aligned}
&\delta_i n_i V'(U_i) = \mu n_i + \sum_{k \neq i}(\lambda_{ki} - \lambda_{ik}),\\
&\left[(1-\pi)\varphi'(s_i) + \pi(1-p)\phi'(s_i+z_i)\right] = 1,\\
&\mu n_i\left[1 - \frac{1}{w_i}\pi p h'(e_i)\right] = \sum_{k \neq i}\lambda_{ki}\left[\frac{1}{w_i}\pi p h'\left(1 - \frac{I_i}{w_i}\right) - \frac{1}{w_k}\pi p h'\left(1 - \frac{I_i}{w_k}\right)\right],\\
&(1-p)\phi'(s_i+z_i) = 1.
\end{aligned}$$

With $(1-p)\phi'(s_i+z_i) = 1$, the second equation above simplifies to $\varphi'(s_i) = 1$ for all $i = 1, 2, \ldots, N$. Multiplying the third equation by $w_i/\pi p \mu n_i$ we have (37)–(40).

F Proof of Proposition 2

(i) The proof is the same as that of point (i) in Proposition 1.

(ii) The proof follows directly from (31) and (38).

(iii) With $\lambda_{ki} = 0$ for all $k < i$, we have already shown in the proof of *(i)* that

$$\begin{aligned} h'\left(e_i^{SB}\right) &< \frac{w_i}{\pi p}, \quad i = 1, 2, \ldots, N-1, \\ h'\left(e_N^{SB}\right) &= \frac{w_N}{\pi p}. \end{aligned}$$

And previously, we had shown that

$$h'\left(e_i^{FB}\right) = \frac{w_i}{\pi p} \quad i = 1, 2, \ldots, N.$$

$\Rightarrow$

$$\begin{aligned} h'\left(e_i^{SB}\right) &< h'\left(e_i^{FB}\right), \quad i = 1, 2, \ldots, N-1, \\ h'\left(e_N^{SB}\right) &= h'\left(e_N^{FB}\right). \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} e_i^{SB} &> e_i^{FB}, \quad i = 1, 2, \ldots, N-1, \\ e_N^{SB} &= e_N^{FB}. \end{aligned}$$

The properties

$$\begin{aligned} I_i^{SB} &< I_i^{FB}, \quad i = 1, 2, \ldots, N-1, \\ I_N^{SB} &= I_N^{FB}, \end{aligned}$$

then follow immediately from

$$e_i = 1 - \frac{I_i}{w_i}.$$

(iv) The proof follows directly from (ii) together with equations (33) and (40)