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Abstract

Because of risk aversion, any sensible investment valuation system should value less projects that contribute more to the aggregate risk. In theory, this is done by adjusting discount rates to consumption betas. But in reality, most public institutions use a discount rate that is rather insensitive to the risk profile of their investment projects. The economic consequences of the implied misallocation of capital are severe. I calibrate a Lucas model in which the investment opportunity set contains a constellation of projects with different expected returns and risk profiles. The model matches the traditional financial and macro moments, together with the observed heterogeneity of assets' risk profiles. The welfare loss of using a single discount rate is equivalent to a permanent reduction in consumption that lies somewhere between 15% and 45% depending upon which single discount rate is used.

Keywords: Discounting, investment theory, asset pricing, carbon pricing, Arrow-Lind theorem, WACC fallacy, rare disasters, capital budgeting.

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1 Introduction

It is an enduring common practice in most western countries to value public investments and policies by measuring the present value of their flow of expected social benefits using a single discount rate. As already noticed by Bazelon and Smetters (1999) and Cherbonnier and Gollier (2023) for example, this means that no insurance value is recognized to policies that hedge the macroeconomic risk such as improving earthquake-resistant construction norms, increasing pandemic-treatment capacities, or building a strategic petroleum reserve. Symmetrically, no penalty is imposed to policies involving benefits materializing mostly in good states of nature, such as expanding the capacity of energy and transportation infrastructures. It is never too late to change this inefficient practice. In this paper, I estimate its social cost. It is large.

Any investment decision criterion that recognizes risk aversion should value less projects that raises the aggregate risk borne by the decision-maker, everything else unchanged. Modern asset pricing and investment theories translated this simple idea into practice by recommending that discount rates be adjusted for the projects' betas, which measure their contribution to the aggregate risk. The Consumption-based Capital Asset Pricing Model (CCAPM) pioneered by Rubinstein (1976), Lucas (1978) and Breeden (1979), and its extensions (Bansal and Yaron, 2004; Barro, 2006) to solve the asset pricing puzzles (Mehra and Prescott, 1985; Weil, 1989), provide a normative framework to justify this methodology. The large market risk premium observed over the last century suggests that the risk-adjustment embedded in the efficient discounting system should play a crucial role in the investment evaluation process. In a recent international survey among professional economists, Gollier et al. (2023) document a consensus in which more than 75% of the respondents supported a risk-adjustment mechanism to evaluate project-specific discount rates.

But, as explained in the next section, most governments and international organizations use a single discount rate that is not sensitive to the risk profile of the decision under scrutiny. This dogma of a single discount rate for the public sector has long been believed to be supported by the influential Arrow-Lind theorem (Arrow and Lind, 1970), which claims that "the government invests in a greater number of diverse projects and is able to pool risks to a much greater extent than private investors", thereby washing out risk completely. Many people interpreted this result as meaning that all public investment projects should be discounted at the risk-free interest rate. But, as stated by Sandmo (1972), Lucas (2014), Baumstark and Gollier (2014) and the CCAPM literature in general, this result is valid only for projects with a zero CCAPM beta. Notice that Arrow and Lind mentioned this point in their paper.[1] Because a vast majority of projects have a positive beta, the use of the risk-free rate as the discount rate implies an excess of positive NPV projects compared to the capacity of public funding, thereby often forcing governments to impose a capital rationing scheme on top of the valuation process.

In this paper, I measure the welfare loss associated with using a single discount rate when performing the benefit-cost analysis to determine the optimal allocation of capital, either at the individual level, or in the economy as a whole. Contrary to the standard endowment economy that is used in the CCAPM (Lucas (1978), Martin (2013)), I examine a dynamic

[1] "The results ... depend on returns from a public investment being independent of other components of national income." (p. 373)

model in which the profiles of investments are endogenously selected in an opportunity set with heterogeneous risk profiles and expected benefits. At the beginning of each period, identical infinitely-lived expected-utility agents must determine what share of their wealth should be consumed, and which investments should be implemented. The equilibrium investment rule entails a CCAPM discounting system in which project-specific discount rates are linearly risk-adjusted, using an equilibrium risk-free rate and aggregate risk premium. The model is calibrated in Section 4 to fit the following empirical moments: interest rate, aggregate risk premium, and expected growth and volatility. My calibration is also disciplined to match the observed diversity of CCAPM betas of implemented projects in the economy. This is a key ingredient for the estimation of the welfare loss generated by inefficient discounting systems. In order to solve the standard asset pricing puzzles, I introduce infrequent macroeconomic catastrophes à la Barro (2006).

In Section 5, I first measure the welfare loss incurred by an isolated agent who would use a single discount rate to value assets and to determine this agent's portfolio. If this agent uses the average cost of capital prevailing in the rational equilibrium as the unique discount rate to value all projects, the welfare loss is equivalent to an immediate reduction of this agent's wealth by 27%. In Section 6, I move to an equilibrium analysis in which all agents use the same inefficient discounting system with a single discount rate equaling their average cost of capital. The welfare loss is estimated to around 15% of global wealth in that case. Finally, I examine an equilibrium model in which all agents use the equilibrium interest rate as their single discount rate. Because the set of potential projects passing the test of a positive NPV is very large in that case, no such equilibrium exists without imposing a rationing scheme for the capital allocation. If one combines this single discount rate with a rationing scheme in which only 60% of the projects with a positive NPV are implemented, an equilibrium exists with a welfare loss equivalent to a 45% drop in initial global wealth compared to the rational equilibrium with a first-best discounting system. This is a reminder of the importance of the allocation of capital in our economy to generate collective prosperity.

2 Public discounting in practice

France is currently the only country in the world in which public investment projects must be evaluated using a discount rate that is sensitive to the project's risk profile (Guesnerie (2023)). The French discounting system is based on the CCAPM with a risk-free discount rate of 1.2% and a systematic risk premium of 2%.[2] The evaluators are thus required to estimate the consumption beta of their project, which is defined as the elasticity of the project's net benefit to changes in aggregate consumption. Personal anecdotes suggest that lobbies from high-beta sectors have periodically attempted to go back to a single discount rate, or to reduce the level of the aggregate risk premium by referring to the equity premium puzzle.

Between 1997 and 2012, Norway used a simplified version of the CCAPM to evaluate large public investment projects, with project-specific discount rates ranging from 3.5% to 8% depending upon the project's beta. But a report published in 2012 (Hagen et al. (2012)) claimed that "considerable room for discretionary assessments with regard to estimates as

[2]All discount rates discussed in this paper are real discount rates. I limit this description to short-term discount rates. The French system also imposes a smaller risk-free discount rate and a larger risk premium for longer maturities.

to project-specific risk ... may offer incentives to choose assumptions that may influence the outcome of the analysis in the direction favored by various interest parties... These circumstances suggest that it may be preferable to recommend simple and transparent rules that capture the most important aspects of the matter, without being too complex to understand or to apply" (page 77). Consequently, the report recommends the use of a single discount rate of 4%. It has been determined by combining a risk-free rate of 2.5% and an average risk premium of 1.5%.

More recently, the Netherlands has adopted three public discount rates (Rijksoverheid (2020)): An all-purpose discount rate of 2.25%, with two exceptions. A lower discount rate of 1.6% should be used for "costs that are largely or wholly independent of usage (i.e. fixed costs)". A larger discount rate of 2.9% should be used for "benefits that are highly non-linear relative to usage, where usage, moreover, depends on the state of the economy." This could be interpreted as a simplified version of the CCAPM discounting system, with the partition of the investment opportunity set into three beta segments.

All other countries that have published a discounting guideline have been using since a long time – and are still using – a single discount rate. In the United Kingdom, the official discount rate has been 3.5% since 2003, using the Ramsey rule (Treasury (2018)). In the European Union, it is equal to 5.5% for the "Cohesion countries" (basically the more recent member states) and 3.5% for the others (Florio (2008)). The discounting system prevailing in the United States is a special case. For a long time, two discount rates of 3% and 7% were used in that country (OMB, 2003). This official document justifies these two rates as respectively the "real rate of return on long-term government debt" and the "average before-tax rate of return to private capital in the U.S. economy". Nordhaus (2013) claimed that "the OMB discussion is completely confused... because the difference is not the difference between investment and consumption" but instead "the risk premium on leveraged corporate capital" (quoted by Sunstein (2014)). This confusion and the absence of guideline about which of these two discount rates should be used in practice represents a procedural failure that has been used by the Trump administration to increase the discount rate for carbon pricing to 7 percent, yielding a large drop in the tutelar carbon price. After more than 20 years with this discounting system, OMB (2023a) decided to reduce the discount rate to 2%. This rate is based on the average real risk-free interest rate over the last 30 years. In this new Circular A-4, the OMB recommends to use this rate to discount the flow of certainty equivalent net benefits. This methodology is supported by the theory as explained for example by Bazelon and Smetters (1999), but it fails to fit observed investment decisions because of the standard asset pricing puzzles that prevail in expected utility theory without extreme macroeconomic events. The Circular A-94 (Appendix D) of OMB (2023b) also recommends to use a "real discount rate of 2.0% if the benefits or costs reflect certainty-equivalent valuations and 3.1% if they do not". This default discount rate of 3.1% combines the risk-free discount rate of 2% with a single risk-adjustment based on a deleveraged equity premium of 2.5% and a correlation coefficient of 0.45, "which captures the correlation in equity markets for economic sectors closest to government investment".

The absence of consensus on the Social Cost of Carbon (SCC) in our profession illustrates the mess in which economists and practitioners have to survive under this inefficient public discounting system. In climate economics since the publication of the Stern Review (Stern (2007)), most proponents to the debate used the Ramsey rule (Ramsey, 1928) to evaluate the rate at which future climate damages should be discounted.[3] The problem is that the Ramsey rule and its extension to uncertainty (Hansen and Singleton, 1983) characterize the rate at which *safe* benefits should be discounted. The first reference to the necessity to adjust the climate discount rate to the risk profile of the climate damages emerged when the Obama administration convened a commission aimed at making recommendation on the SCC. The Technical Support Document (Interagency Working Group on Social Cost of Carbon, 2010) used three discount rates: 2.5%, 3% and 5%, this latter rate reflecting "the possibility that climate damages are positively correlated with market returns." Dietz et al. (2018) showed that in the DICE model of Nordhaus (2008), the CCAPM beta of climate damages is close to unity: In the business-as-usual scenario, future climate damages will be larger if the future will be more prosperous. This implies that the entire debate on the SCC has long been misleading by ignoring the crucial risk-adjustment of the climate discount risk.[4]

Whether the private sector uses more efficient investment decision rules remains an open question. On one side, standard textbooks in finance strongly recommend the CCAPM rule to evaluate investment projects (Bodie and Merton, 2000; Brealey et al., 2017), and most CFOs claim to use it (Graham and Harvey, 2001; Jacobs and Shivdasani, 2012). On the other side, there is ample evidence in observed asset prices that the CCAPM pricing rule is only partially able to explain them. The Security Market Line – which links expected returns to betas – is too flat (Fama and French, 1992). This generates a problem similar to the one observed in the public sector, with low-beta projects being undervalued, and large-beta projects being overvalued. Dessaint et al. (2021) confirm this finding by examining a large sample of mergers and acquisitions. Another standard misunderstanding in this field is what Krueger et al. (2015) have termed the "WACC fallacy". It consists in using the Weighted Average Cost of Capital (WACC) of an institution as the single discount rate used by this institution to evaluate its investment opportunities. In a sense, this is the private sector version of the fallacious interpretation of the Arrow-Lind theorem.

3 The model

The model is an adaptation of the CCAPM in which the dynamics of heterogenous capital allocation is endogenous. There is a single consumption good that can be either consumed or invested. An investment project is characterized by a pair $(\theta, \beta) \in \mathbb{R}^2$, and the investment opportunity set in the economy is described by a distribution function F over this pair. This distribution is stationary. For simplicity, capital is short-lived. One unit of capital invested in project (θ, β) at date $t-1$ generates a single benefit $x_t(\theta, \beta)$ that materializes at date t, with

$$x_t(\theta, \beta) = \theta + \beta y_t + \tilde{\varepsilon}_t(\theta, \beta), \tag{1}$$

with $E_{t-1}\tilde{\varepsilon}_t = 0$. We assume that risks $\tilde{\varepsilon}(\theta, \beta)$ are idiosyncratic, in the sense that $\tilde{\varepsilon}(\theta, \beta)$ and $\tilde{\varepsilon}(\theta', \beta')$ are statistically independent for all $(\theta, \theta', \beta, \beta')$. The project-specific benefit $x_t(\theta, \beta)$ is sensitive to the realization of a common factor whose realization y_t at date t is unknown

[3] See for example Arrow (2007), Nordhaus (2007), Dasgupta (2008) and Weitzman (2010).

[4] EPA (2023) addressed this issue by using the Stochastic Discount Factor (SDF) approach to the SCC. The SDF approach is an alternative to the CCAPM in which the value of an asset under uncertainty is defined as the expectation of its contingent present value, using the relevant Ramsey-discount rate in each contingency or scenario. The SDF and CCAPM approaches are equivalent in the Gaussian world.

at date $t-1$, with $E_{t-1}y_t = 0$. We assume that $(y_0, y_1, y_2, ...)$ is a vector of independent and identically distributed random variables that are independent of the idiosyncratic risks $\tilde{\varepsilon}$. To sum up, a project is characterized by its expected gross return on investment (ROI) θ and by its sensitivity β to the common factor y. Without loss of generality, we assume that the average β among all projects belonging to the investment opportunity set is equal to unity:

$$\iint \beta dF(\theta, \beta) = 1. \tag{2}$$

The decision variable $\alpha_t(\theta, \beta)$ represents the capital invested in projects (θ, β) at date t. If the investment strategy α_t is chosen at date t, it generates total wealth z_{t+1} at date $t+1$ which is equal to

$$z_{t+1} = \iint \alpha_t(\theta, \beta) x_{t+1}(\theta, \beta) dF(\theta, \beta) = \overline{\theta}_t + \overline{\beta}_t y_{t+1}, \tag{3}$$

with

$$\overline{\theta}_t = \iint \alpha_t(\theta, \beta)\theta dF(\theta, \beta) \tag{4}$$

and

$$\overline{\beta}_t = \iint \alpha_t(\theta, \beta)\beta dF(\theta, \beta). \tag{5}$$

Observe from equation (3) that idiosyncratic risks $\tilde{\varepsilon}_t(\theta, \beta)$ associated to investing in the family of projects (θ, β) are washed out by diversification. Consumption at date t equals $c_t = z_t - \overline{\alpha}_t$, where $\overline{\alpha}_t$ is total investment expenditure at date t, with

$$\overline{\alpha}_t = \iint \alpha_t(\theta, \beta) dF(\theta, \beta) \tag{6}$$

I assume that the capital which can be invested at date t in any project (θ, β) in the economy is constrained to be non-negative and smaller than capacity z_t/η, with $\eta \in [0, 1]$. Without this constraint, investors would invest their entire wealth in the single project (θ, β) with the best risk-return profile, as idiosyncratic risk ε can be washed out by diversification. This capacity constraint means that individuals are forced to disperse their investments (to full capacity) into a fraction η of projects available in the investment opportunity set.

There is a continuum of infinitely-lived agents in the economy. They are endowed with the same initial wealth and they all face the same opportunity set of investment projects. They maximize the discounted expected utility of their flow of consumption. Their preferences are characterized by their common utility discount factor δ and by their increasing and concave utility function u over consumption. I assume a CRRA utility function with $u(c) = c^{1-\gamma}/(1-\gamma)$, with $\gamma > 0$. In the calibration section of this paper, I will solve the asset pricing puzzles by assuming rare disasters in the distribution of y.

4 The rational equilibrium

4.1 Characterization

A rational equilibrium is an allocation in which all agents follow the investment strategy that maximizes their discounted expected utility. Because all agents have the same preferences and the same initial endowment, autarky is an equilibrium. I first characterize the optimal investment strategy in this economy. It solves the following recursive problem:

$$V(z_t) = \max_{\alpha_t:\mathbb{R}^2\to[0,z_t/\eta]} u(z_t - \overline{\alpha}_t) + \delta EV(\overline{\theta}_t + \overline{\beta}_t y). \tag{7}$$

The first-order condition for the investment decision in project (θ, β) can be written as follows: For all (θ, β) such that $dF(\theta, \beta) > 0$,

$$u'(z_t - \overline{\alpha}_t^*) = \delta E\left[(\theta + \beta y)V'(\overline{\theta}_t^* + \overline{\beta}_t^* y)\right] + \psi_t(\alpha, \beta) \tag{8}$$

with

$$\psi_t(\theta, \beta) \begin{cases} \geq 0 & \text{if} \quad \alpha_t^*(\theta, \beta) = 0, \\ = 0 & \text{if} \quad \alpha_t^*(\theta, \beta) \in \ ]0, z_t/\eta[, \\ \leq 0 & \text{if} \quad \alpha_t^*(\theta, \beta) = z_t/\eta. \end{cases} \tag{9}$$

I consider the following guess solution:

$$V^*(z) = h^* \frac{z^{1-\gamma}}{1-\gamma} \tag{10}$$

Proposition 1 describes the solution of this problem, which is based on this guess solution. It is easy to check that this solution satisfies the equilibrium conditions (7)-(9).

Proposition 1. *If it exists, the rational equilibrium investment strategy* $\alpha_t^*(\theta, \beta) = \alpha^*(\theta, \beta)z_t/\eta$ *is such that*

$$\alpha^*(\theta, \beta) \begin{cases} = 0 & \text{if} \quad \theta \leq R^* + \beta\pi^*, \\ \in \ [0, 1] & \text{if} \quad \theta = R^* + \beta\pi^*, \\ = 1 & \text{if} \quad \theta \geq R^* + \beta\pi^*, \end{cases} \tag{11}$$

The risk-free rate R^* *and the aggregate risk premium* π^* *are defined respectively as*

$$R^* = \frac{\eta^{1-\gamma} - \delta E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{1-\gamma}}{\delta(\eta - \overline{\alpha}^*)E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{-\gamma}}, \tag{12}$$

and

$$\pi^* = -\frac{Ey\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{-\gamma}}{E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{-\gamma}}, \tag{13}$$

where the triplet $(\overline{\alpha}^*, \overline{\theta}^*, \overline{\beta}^*) \in \mathbb{R}^3$ *is such that* $\overline{\alpha}_t^* = \overline{\alpha}^* z_t/\eta$, $\overline{\theta}_t^* = \overline{\theta}^* z_t/\eta$ *and* $\overline{\beta}_t^* = \overline{\beta}^* z_t/\eta$. *The welfare measure* h^* *at equilibrium equals*

$$h^* = \frac{(\eta - \overline{\alpha}^*)^{1-\gamma}}{\eta^{1-\gamma} - \delta E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{1-\gamma}}. \tag{14}$$

This rational equilibrium exists if and only if $\eta^{1-\gamma}$ *is larger than* $\delta E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{1-\gamma}$.

Function $\alpha^*(\theta, \beta)$ characterized by equation (11) describes the optimal investment strategy. Projects (θ, β) are implemented at full capacity if and only if their expected rate of return θ is larger than the project-specific discount rate $R^* + \beta\pi^*$. Variable $\overline{\alpha}^*$ can thus be interpreted as the proportion of projects in the investment opportunity set that are implemented. It implies a constant consumption/wealth ratio equaling $1 - \overline{\alpha}^*/\eta$.

Along this optimal stationary investment strategy, the growth process entails serially independent shocks:

$$\frac{z_t}{z_{t-1}} = \frac{c_t}{c_{t-1}} = \frac{\overline{\theta}^*}{\eta} + \frac{\overline{\beta}^*}{\eta} y_t. \tag{15}$$

In equation (1), we expressed the return of any project (θ, β) as a linear function of the artificial common factor y. Using the above equation, we can rewrite this return as a linear function of the growth rate of aggregate consumption:

$$x_t(\theta, \beta) = \left(\theta - \beta \frac{\overline{\theta}^*}{\overline{\beta}^*}\right) + \beta \frac{\eta}{\overline{\beta}^*} \frac{c_t}{c_{t-1}} + \tilde{\varepsilon}_t(\theta, \beta). \tag{16}$$

Notice that this equation is the classical CCAPM regression in which the return of a project (θ, β) is regressed on the growth rate of consumption. It implies that the CCAPM beta of this project is defined as follows:

$$\beta^{CCAPM}(\beta) = \beta \frac{\eta}{\overline{\beta}^*}. \tag{17}$$

The optimal intertemporal welfare is measured by $V^*(z_0)$. Normalizing z_0 to unity, it can be more intuitively measured by the permanent equivalent consumption level c^{pe} that generates the same intertemporal utility, yielding

$$c^{pe*} = \left((1-\delta)h^*\right)^{\frac{1}{1-\gamma}}. \tag{18}$$

This variable is a convenient measure of optimal intertemporal welfare. A similar policy evaluation instrument has been used by Epstein et al. (2014) in another context.

4.2 Calibration

In order to derive market implications of the model presented in the previous section, I calibrate it using an annual frequency. I discipline the choice of the parameters to match key moments observed on financial markets. These moments are summarized in Table 1. The most important moment to match for this exercise is undoubtedly the aggregate risk premium, that is the excess expected return of a claim on GDP. I use its estimation at 2.2% by Lustig et al. (2008). Assuming a CCAPM beta for equity around 3, as suggested by Bansal and Yaron (2004) for example, this corresponds to an equity premium around 6.6%. I also want to match the risk-free interest rate. Jorda et al. (2019) estimated a post-1950 mean return on bills at 0.88%. The mean and the standard deviation of the growth rate of consumption are also crucial to match in order to have a realistic representation of the macroeconomic stochastic dynamics. I use here the estimation by Bansal and Yaron (2004), with an expected growth rate of 1.8% and a standard deviation of 2.9%. I also want to duplicate the kind of catastrophic macroeconomic events that have been documented by Barro (2006), as explained later on in this section. Finally, an important ingredient to evaluate the welfare loss of an inefficient risk-adjustment of the discounting system relies on the distribution of CCAPM betas in the economy. If all projects would have the same beta, they should all be optimally evaluated with a single discount rate. There would be no inefficiency associated with the WACC and the fallacious interpretation of the Arrow-Lind theorem in that case. To estimate the heterogeneity of these betas, we use the Fama-French dataset of annual value-weighted returns of 49 industries over the period 1930-2018. The list of estimated CCAPM betas for these 49 industries is given in Table 2. The empirical standard deviation of these betas is equal to 0.63.

The parameters of the benchmark calibration that closely replicate these moments are summarized in Table 3. I assume a constant relative risk aversion of $\gamma = 3$ and a utility discount factor of $\delta = 0.99$, which are in line with the literature. I assume that, in the constellation of feasible investments, the mean payoff θ and the sensitivity β to the common factor are independently distributed.[5] I also assume that they are normally distributed, with $\theta \sim N(\mu_\theta, \sigma_\theta^2)$ and $\beta \sim N(\mu_\beta, \sigma_\beta^2)$. The expected return of feasible projects in the investment opportunity set has a mean of 3%, and a standard deviation of 2%. Parameter σ_β measures the heterogeneity of the investment risk profiles in the investment opportunity set. I take $\sigma_\beta = 0.5$. I also assume that $\eta = 0.5$, which means that the entire wealth in the economy would be able to finance 50% of all possible investment projects. Finally, in order to solve the classical asset pricing puzzles that prevail in the standard CCAPM, I use the Barro's approach based on the possibility of macroeconomic catastrophes (Barro (2006), Martin (2013)). I assume that the common factor y is distributed as random variable $\exp(Y) - E\exp(Y)$ with

$$Y \sim \left(N(\mu_{bau}, \sigma_{bau}^2), 1-p; N(\mu_{cat}, \sigma_{cat}^2), p\right). \tag{19}$$

With probability $1-p$, the state is business-as-usual (bau) and the distribution of Y conditional to that state is normal with mean μ_{bau} and volatility σ_{bau}. But with a small probability p, the catastrophic state occurs, and the distribution of Y conditional to that state is normal with mean $\mu_{cat} << 0$ and volatility σ_{cat}. The calibration of these parameters is documented in Table 3. They will be justified later on in comparison to the standard Barro's calibration. I use Barro's estimated probability of macroeconomic catastrophes at $p = 1.7\%$ per year.

I numerically characterize the rational equilibrium described in Proposition 1. Because R^* and π^* depend upon the triplet $(\overline{\alpha}^*, \overline{\theta}^*, \overline{\beta}^*)$ that is determined by the optimal investment strategy $\alpha^*(.,.)$, this proposition describes the optimal solution only implicitly. I solve this problem by observing that this optimal strategy is a function of pair (R^*, π^*), so are $\overline{\theta}^*(R^*, \pi^*)$, $\overline{\beta}^*(R^*, \pi^*)$ and $\overline{\alpha}^*(R^*, \pi^*)$, using respectively equations (4), (5) and (6). Thus, equations (12) and (13) can be interpreted as a system of two equations with two unknowns, R^* and π^* that I solve numerically.

The rational investment strategy and its implication in terms of risk, return and intertemporal welfare are described in Table 4. The aggregate risk premium and the risk-free discount rate and are respectively equal to 2.22% and 0.86%, which match their empirical moments almost perfectly. Under this calibration, the dynamic equation (15) for consumption can be

[5] Because of the optimal selection process, they will be positively correlated within the family of implemented projects.

rewritten as follows:

$$\frac{c_t - c_{t-1}}{c_{t-1}} = 0.0151 + 0.778y. \tag{20}$$

The parameters $(\mu_{bau}, \sigma_{bau})$ governing the growth process in the business-as-usual state imply that, conditional to that state, the expected growth rate of wealth and consumption equals 1.87% and its volatility equals 3.12%, close to Barro's calibration using respectively 2% and 3.5%. The parameters $(\mu_{cat}, \sigma_{cat})$ governing the growth process in the catastrophe state implies an expected drop in consumption of 19.5%, and a standard deviation of 23.5%. This is in the order of magnitude of catastrophic events documented by Barro (2006). Unconditionally, wealth and consumption grow at a trend of 1.51%, with a volatility of 2.76%, in line with the moments to be matched. Finally, the equilibrium investment selection process is determined by the optimality test $\theta \geq R^* + \beta\pi^*$ for invesment. One can translate this into a distribution for the CCAPM betas of these projects by using equation (17). This distribution is described by the smooth curve in Figure 1. These CCAPM betas have a mean of 1.03 and and a standard deviation of 0.59. The relative concordance of the distribution of the CCAPM betas predicted by the model with this empirical distribution provides an additional support to this calibration exercise.

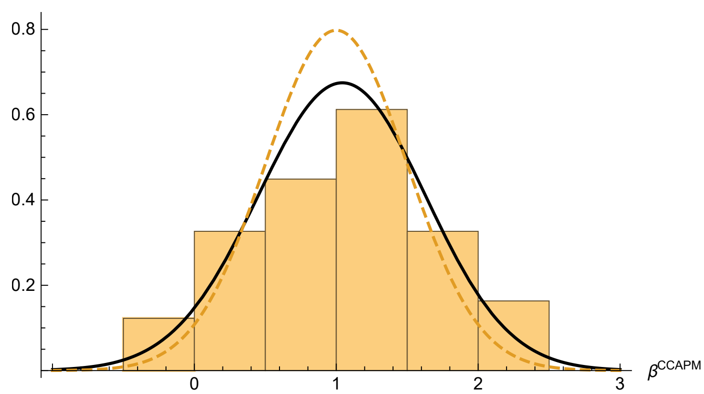

Figure 1: Histogram of the OLS estimators of the CCAPM betas of the 49 Fama-French industries of the US economy, based on industry-specific value-weighted equity returns (Table 2). Its standard deviation equals 0.63. The plain curve describes the density function of the distribution of the CCAPM betas of the implemented project predicted by the model. The standard deviation of these CCAPM betas predicted by the model is 0.59. The dashed curve is the density function $N(\mu_\beta, \sigma_\beta^2)$ of the betas of the projects in the opportunity set.

Other observations are noteworthy. The rational selection of projects allows for both an increase in the mean expected return and a reduction in the mean sensitivity of the selected projects compared to their distribution in the opportunity set. The mean sensitivity is 1 in the opportunity set, and is only 0.80 among implemented projects. The mean expected return is 3% in the opportunity set, and it increases to 4.43% among implemented projects. It yields a price-earning ratio of 22.57. Given the optimal discounting system, 48.60% of the investment projects pass the test of a positive NPV. Because each implemented project requires two units of wealth ($\eta = 1/2$), 97.20% of total wealth is reinvested every period, yielding a consumption-wealth ratio of 2.80%. Notice that we did not attempt to match this variable with real data given the lack of consensus surrounding the estimation of wealth in the economy (Lustig et al., 2008). Finally, observe that the intertemporal welfare obtained from following this optimal investment strategy at equilibrium is equivalent to consuming a constant flow of 4.64% of initial wealth z_0.

4.3 Sensitivity analysis

I also performed a sensitivity analysis to test the fitness of the match in the selected calibration. This exercise is summarized in Table 5. As explained earlier, the heterogeneity of the betas in the investment opportunity set is the central source of inefficiency of valuation rules that ignore the risk-adjustment of discount rates. In the calibration, I assume $\sigma_\beta = 0.5$. If I raise it to 1, the standard deviation of the betas of the implemented projects goes up to 1.43, which is more than twice its observed value. Moreover, the aggregate risk premium goes down to 1.17%, which is vastly smaller than its observed value of 2.2%. This is because investors can take advantage of the much safer projects in the opportunity set to reduce the risk of their optimal portfolio, thereby reducing the equilibrium price of risk. Finally, the reduced macroeconomic risk raises the equilibrium interest rate to an unrealistically large level.

An alternative way to make the investment opportunity set more diverse is to increase the standard deviation of the expected return θ from $\sigma_\theta = 2\%$ to 4%. The optimal selection procedure allows investors to raise the expected return of their portfolio, at a cost of an increase in risk. This alternative calibration fails to match the aggregate risk premium, which is too large, and the interest rate, which is too small. I also examined the impact of an increase in the mean expected gross return from $\mu_\theta = 1.03$ to 1.05. The main impact of this change is to raise the equilibrium interest rate to an unrealistically large 2.98%.

Finally, I analyzed the impact of a change in the capacity constraint related to η. Remember that $\eta = 0$ corresponds to the unconstrained case in which investors are able at the limit to pick the single project with the best risk-return tradeoff. In the selected calibration, I assume $\eta = 0.5$, which means that individuals are forced to disperse their investments to half of the opportunity set (in the absence of consumption). If I reinforce this capacity constraint to $\eta = 0.55$, this makes the choice problem too bad to be realistic. Indeed, the interest rate goes down to 0.48%.

5 An irrational agent in the rational equilibrium

In this section, I consider the case of an irrational agent who uses a single discount rate to determine his dynamic investment strategy. All other agents behave optimally as described in the previous section. Therefore, the existence of this marginal agent has no effect on the equilibrium. The dynamics of the economy, and therefore on equilibrium asset prices, are the ones that have been examined in the previous section. The irrational agent uses the following decision rule based on his single discount rate ρ:

$$\alpha^*(\theta, \beta) \begin{cases} = 0 & \text{if} \quad \theta \leq \rho, \\ \in \ [0,1] & \text{if} \quad \theta = \rho, \\ = 1 & \text{if} \quad \theta \geq \rho. \end{cases} \tag{21}$$

This rule has the advantage of not requiring the irrational agent to estimate the beta of the projects under scrutiny, but it implies an inefficient portfolio allocation. Given the ρ selected by the agent, one can characterize his investment portfolio and his wealth and consumption dynamics. It yields a triplet $(\overline{\alpha}_\rho, \overline{\theta}_\rho, \overline{\beta}_\rho)$ similar to what has been described earlier, expect that this triplet is now a function of ρ. Since it is assumed that θ and β are independently distributed, the investment decision rule (21) implies that the mean beta of the implemented projects will be equal to unity, so that $\overline{\beta}_\rho$ equals $\overline{\alpha}_\rho$ in this model. The intertemporal welfare of this agent with initial wealth z_0 and using the single discount rate ρ is denoted $V_\rho(z_0)$. It is defined recursively as follows:

$$V_\rho(z) = u\left(z\left(1 - \frac{\overline{\alpha}_\rho}{\eta}\right)\right) + \delta E V_\rho\left(\frac{z}{\eta}(\overline{\theta}_\rho + \overline{\beta}_\rho y)\right). \tag{22}$$

Using this decision rule implies that the intertemporal welfare of the isolated agent is equal to $V_\rho(z_0) = h_\rho z_0^{1-\gamma}/(1-\gamma)$ where h_ρ satisfies the following condition:

$$h_\rho = \frac{(\eta - \overline{\alpha}_\rho)^{1-\gamma}}{\eta_\rho^{1-\gamma} - \delta E\left(\overline{\theta}_\rho + \overline{\beta}_\rho y\right)^{1-\gamma}}. \tag{23}$$

5.1 The isolated WACC strategy

This solution is a function of the single discount rate ρ that is used by the irrational agent As a benchmark, let us first examine the "WACC strategy" which consists in using the average cost of capital in the economy as the all-purpose discount rate used by the irrational agent. The agent knows that the average beta of the projects that he will implement is equal to 1. Because all other agents behave rationally, the equilibrium asset returns are as described in the previous section, with $r^* = 0.86\%$ and $\pi^* = 2.22\%$. The WACC of the irrational agent will thus be equal to $r^* + \pi^* = 3.08\%$. He selects this rate as the single discount rate for investment evaluation. In Table 6, I describe the outcome of this investment strategy and I compare it to the optimal strategy already described in the previous section. The two investment strategies are described in Figure 2. The irrational agent invests in approximately the same number of projects (48.4%) than the rational agents (48.6%). However, the compositions of the portfolio are quite different. The irrational agent undertakes too many risky projects (those in the north-east red quadrant in Figure 2 should not be implemented), and too few safe projects (those in the south-west red quadrant should be implemented). This yields more uncertainty about future consumption, with a volatility of wealth and consumption growth going up from 2.76% to 3.44% for rational investors. This is only partially compensated by a larger expected portfolio return (4.65% up from 4.43%). The bottom line is a massive 27% reduction in the measure of intertemporal welfare. Indeed, the permanent equivalent consumption level c^{pe} goes down from 0.0464 to 0.0339.

It is useful to examine how this estimate of the welfare loss is sensitive to the value of the parameters of the model. To do this, I used the same changes in the parameters that I used in Table 5. This robustness analysis is summarized in Table 7. Without surprise, increasing the dispersion of betas in the opportunity set from $\sigma_\beta = 0.5$ to 1 raises the welfare cost of inefficient valuation methods and capital allocation. In this case, the first-best strategy leads to a welfare of $c^{pe} = 0.0517$, whereas the alternative WACC strategy reduces it to 0.0258, which corresponds to a 50.0% reduction in welfare. This mostly doubles it compare to the 27%

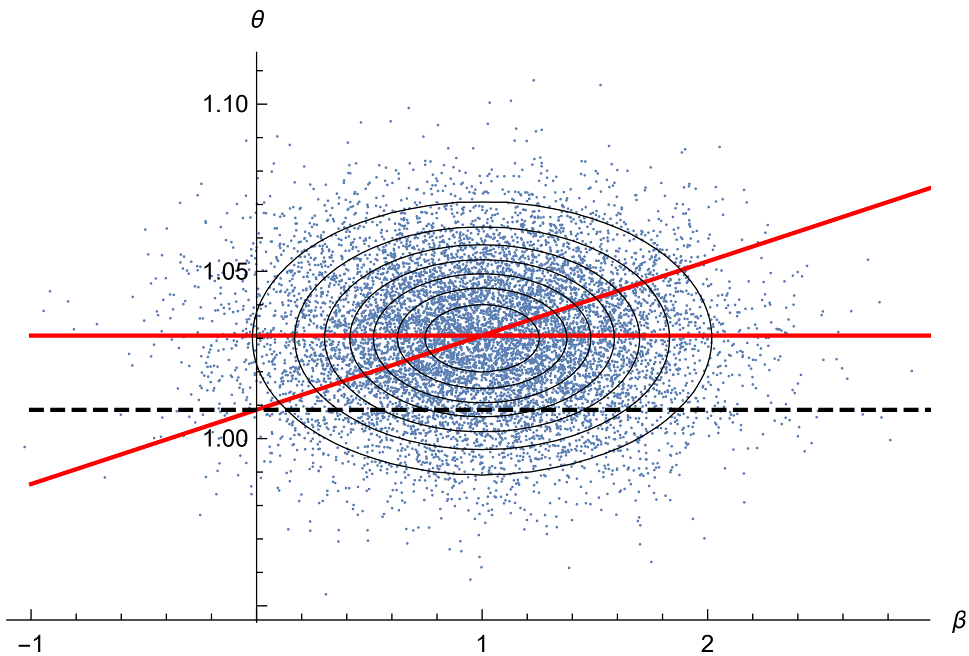

Figure 2: Comparison of the "optimal", the "WACC" and the "Arrow-Lind" investment strategies. We draw a sample of 10.000 projects from the joint normal distribution of (β, θ), using the benchmark calibration described in Table 3. The ellipses are iso-density curves of this joint distribution. The oblique and horizontal plain lines describe respectively the optimal and WACC frontiers, with the set of implemented projects above these lines. The dashed line corresponds to the Arrow-Lind strategy.

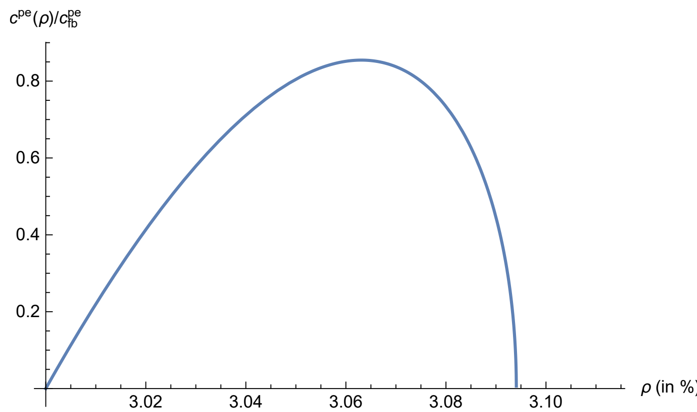

Figure 3: Ratio of the permanent equivalent consumption under a single discount rate ρ to the first-best permanent equivalent consumption c^{pe}_{fb}. Among these second-best strategies, the optimal single discount rate is $\rho_1 = 3.06\%$.

reduction in the benchmark calibration. But remember that this alternative calibration fails to match observed asset prices and dispersion of consumption betas. The other (unrealistic) changes in parameters examined in this table generate a welfare loss generated by using the suboptimal WACC strategy between 9% and 25% of initial wealth.

5.2 The isolated Arrow-Lind strategy

One could alternatively examine the "Arrow-Lind strategy" which would consist in using the risk-free interest rate $r^* = 0.86\%$ as the single discount rate. However, this discount rate is too small to yield a feasible solution. Indeed, implementing this investment evaluation procedure would imply that 86% of the investment projects would yield a positive NPV, implying that the irrational agent should spend every period 172% of his wealth to invest.

In fact, the irrational agent has a very narrow interval of possible single discount rates to choose from in order to generate a positive intertemporal welfare. More specifically, if he chooses a single discount rate smaller than 3%, consumption would be negative, as in the Arrow-Lind strategy. If he chooses a discount rate larger than 3.09%, early consumption is too large and capital accumulation too small to support a positive permanent consumption equivalent. In Figure 3, I show how the intertemporal welfare of the irrational agent is related to the choice of the single discount rate.

6 The WACC equilibrium

In this section, I assume that *all* agents in the economy use the same single-DR strategy (21). Contrary to the previous section, the fact that all agents follow the same inefficient investment strategy means that the dynamics of growth and thus the equilibrium asset prices are affected by the irrationality of the agents. The WACC equilibrium is defined as a dynamic allocation of capital is which all individuals select their portfolio based on decision rule (21), where $\rho = \rho_1$ is the average cost of capital in the economy, i.e., ρ_1 equals $R_1 + \pi_1$. In that economy, the risk-free interest rate and the risk premium must satisfy the following equilibrium conditions:

$$R_1 = \frac{(\eta - \overline{\alpha}_{\rho_1})^{-\gamma}}{\delta h_{\rho_1} E\left(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y\right)^{-\gamma}}, \tag{24}$$

and

$$\pi_1 = \frac{-Ey\left(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y\right)^{-\gamma}}{E\left(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y\right)^{-\gamma}}. \tag{25}$$

By replacing h_{ρ_1} by its expression in (23), we can rewrite the WACC condition $\rho_1 = R_1 + \pi_1$ as follows:

$$\rho_1 = \frac{\eta^{1-\gamma} - \delta E(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y)^{1-\gamma}}{\delta(\eta - \overline{\alpha}_{\rho_1}) E(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y)^{-\gamma}} + \frac{-Ey(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y)^{-\gamma}}{E(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y)^{-\gamma}}. \tag{26}$$

This equation, which is solved numerically, characterizes the WACC equilibrium. It is described in Table 8. The global WACC is equal to $\rho_1 = 3.06\%$. Notice that this single discount rate combines a risk-free interest rate and a risk premium that are very different from the individual WACC strategy described in the previous section in the isolated case. Indeed, under this equilibrium in which all agents behave irrationally, the equilibrium interest rate goes down to -1.48% because of precautionary savings, and the equilibrium risk premium goes up to 4.54% because of the larger macroeconomic uncertainty. Again, under this single discount rate rule, the decision-maker overinvests in risky projects and underinvests in relatively safer ones. This absence of selectivity on the risk dimension implies that the average beta is equal to 1.00, to be compared to only 0.80 under the equilibrium with rational agents. The good news is that the average expected return equals 4.64% under the second-best strategy, to be compared to only 4.43% under the first best. Also, people invest a larger fraction of their wealth in projects, so that the consumption-wealth ratio is reduced from 2.80% to 2.52%. This is due to a precautionary effect, since the volatility of consumption growth is markedly increased from 2.76% to 3.46%. The bottom line is again an important deterioration of intertemporal welfare. The permanent equivalent consumption drops from 0.0464 to 0.0396, a permanent 15% reduction in consumption. Notice that the small difference between the single discount rate used in the individual WACC solution and in the WACC equilibrium implies a sizeable effect on welfare. This is because the marginally smaller discount rate in the isolated case marginally increases the saving rate. But because the consumption-wealth ratio is small, this has a sizeable effect to reduce the consumption rate, yielding an important impact on intertemporal welfare.

It is useful to search for the single discount rate that maximizes the intertemporal welfare of irrational agents that use a single-DR strategy. In other words, what is the ρ that maximizes $V_\rho(z_0) = h_\rho u(z_0)$? The answer to this question is obtained by using equation (23). It is easy to check that the first-order condition to this problem is given by equation (26). In short, the equilibrium WACC $\rho_1 = 3.06\%$ is the single discount rate that corresponds to the maximum in Figure 3. This result is summarized in the following proposition.

Proposition 2. *Suppose that all agents in the economy use the same single-discount-rate rule to determine their investment strategy. The single discount rate that minimizes the welfare cost of this irrational behavior is the equilibrium WACC* $\rho_1 = R_1 + \pi_1$*, in which* R_1 *and* π_1 *are respectively the equilibrium interest rate and the equilibrium risk premium in this economy.*

Of course, the WACC equilibrium is dominated by the rational equilibrium, but if all agents in the economy apply the same single discount rate rule, using the equilibrium WACC as the all-purpose discount rate is the rule that maximizes intertemporal welfare in the set of single-discount-rate allocations.

7 The rationed Arrow-Lind equilibrium

In this section, I examine an economy in which all agents believe in the fallacious interpretation of the Arrow-Lind theorem consisting in using the equilibrium interest rate as the all-purpose discount rate to evaluate investment projects. As we know from the previous two sections, the equilibrium interest rate is typically too small to be use as a single discount rate, so that the capital necessary to finance all investment projects that pass the test of a positive NPV is larger than aggregate wealth. No equilibrium exists under this approach. In practice, experts who have been using the Ramsey rule to estimate the public discount rate often addressed the excess demand for public funds that this solution generated by proposing a capital rationing scheme.[6] In practice, the inability to fund all positive-NPV projects under a too low public discount rate has offered discretion to politicians and high-ranked public servants to prioritize public investments.

I hereafter characterize a family of rationed Arrow-Lind equilibria. Such equilibria are parametrized by a scalar q which denotes the probability for a project with a positive NPV to be implemented. The fact that q is less than 1 means that capital is rationed in the economy. So, an Arrow-Lind equilibrium with rationing q is defined by the fact that all agents use the equilibrium interest rate in the economy as a all-purpose discount rate, but only a proportion q of non-negative-NPV projects are actually implemented. The equilibrium Arrow-Lind discount rate $\rho_{AL}(q)$ must thus satisfy the following equilibrium condition:

$$\rho_{AL}(q) = \frac{\left(\frac{\eta}{q}\right)^{1-\gamma} - \delta E(\overline{\theta} + \overline{\beta} y)^{1-\gamma}}{\delta(\frac{\eta}{q} - \overline{\alpha}) E(\overline{\theta} + \overline{\beta} y)^{-\gamma}} = R. \tag{27}$$

I describe in Table 9 two rationed AL equilibria, respectively with rationing ratio $q = 0.6$ and $q = 0.8$. Of course, it is inefficient to randomize the access to capital for good projects to compensate for a single discount rate that is too small. This implies for example that when we allow only $q = 60\%$ of the non-negative-NPV projects to be implemented, the permanent equivalent consumption level is limited to 0.0253, a catastrophic 45% permanent reduction in consumption compared to the rational strategy. The risk-free interest rate in this economy (and thus the single discount rate) is equal to 1.18%. The demand for capital is 63.6% larger than total wealth in the economy, but only 60% of the demand is satisfied, which leaves 1.84% of wealth for consumption. Financial risk and economic growth are highly volatile in this economy.

[6] This is illustrated for example by the last report in France that recommended a single discount rate, where the Ramsey rule was used, combined with a public capital rationing scheme (Lebègue (2005), pp. 72-76).

8 Concluding remarks

One of the most puzzling feature of the experts' debate on the public discount rate is its reliance on its misleading cornerstone, the Ramsey rule. This rule, adjusted for the uncertainty affecting economic growth, provides the right basis to estimate the rate at which risk-free benefits and costs should be discounted. Using that rule to recommend an all-purpose discount rate in the economy does not only represent a very dangerous interpretation of the theory, as explained in this paper. It also makes it impossible to initiate a constructive debate about how to value the future. As long as one ignores the necessity to adjust discount rates to risk characteristics, all sorts of difficulties materialize, from the WACC fallacy to the rationing of public investments with a positive NPV. Over the last two decades, the remarkable stalemate prevailing in the Stern/Nordhaus debate on the social cost of carbon is another vivid illustration of our collective inability to transform our consensual asset pricing theory into practical evaluation rules. I show in this paper that the social cost of this failure is huge.

The recent decision by the U.S. OMB (OMB, 2023b) to either (i) use the risk-free interest rate of 2% to discount the flow of certainty equivalent net benefits, or (ii) use a single risk-adjusted discount rate of 3.1% to discount the flow of expected net benefits provides only a marginal improvement to this situation. Strategy (ii) is inefficient as explained in this paper, and the welfare loss is in the order of magnitude of 20% of the value of the investment portfolio. The inefficiency of strategy (i) will depends upon how U.S. evaluators will estimate certainty equivalent net benefits. If they use an expected utility model with gaussian risks to perform this task, they will be confronted to the traditional equity premium puzzle, and to the incoherence of their risk premia with respect to the titular risk-free rate of 2% (risk-free rate puzzle). In that case, the welfare loss of strategy (i) will be even worse than for strategy (ii), as the single discount rate of 2% is far off the weighted-average cost of capital in the economy.

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Moment	value	source
Aggregate risk premium	2.2%	Lustig et al. (2008)
Interest rate	0.88%	Jorda et al. (2019)
Expected growth rate of consumption	1.8%	Bansal and Yaron (2004)
Volatility of the growth rate of consumption	2.9%	Bansal and Yaron (2004)
Standard deviation of CCAPM betas	0.63	own estimation

Table 1: Targeted moments used to calibrate the model, and their estimated value to be matched.

Table 2 of the consumption betas using sectoral equity returns

CCAPM beta	Fama-French Industry
0.36	Agriculture
0.44	Food Products
0.43	Candy & Soda
0.74	Beer & Liquor
-0.09	Tobacco Products
1.27	Recreation
1.91	Entertainment
2.35	Printing and Publishing
0.97	Consumer Goods
0.68	Apparel
0.02	Healthcare
1.15	Medical Equipment
0.26	Pharmaceutical Products
0.63	Chemicals
1.19	Rubber and Plastic Products
1.15	Textiles
0.65	Construction Materials
1.70	Construction
1.41	Steel Works Etc
0.42	Fabricated Products
1.61	Machinery
1.68	Electrical Equipment
1.22	Automobiles and Trucks
1.01	Aircraft
1.10	Shipbuilding, Railroad Equipment
0.21	Defense
-0.25	Precious Metals
1.01	Non-Metallic and Industrial Metal Mining
1.04	Coal
1.07	Petroleum and Natural Gas
0.64	Utilities
0.84	Communication
2.23	Personal Services
1.03	Business Services
1.00	Computers
-0.10	Software

2.20	Electronic Equipment
0.97	Measuring and Control Equipment
1.55	Business Supplies
0.36	Shipping Containers
1.44	Transportation
1.74	Wholesale
0.72	Retail
1.57	Restaurants, Hotels, Motels
0.90	Banking
1.21	Insurance
2.15	Real Estate
1.63	Trading
0.58	Almost Nothing

Table 2: Estimation of the CCAPM betas of the 49 Fama-French industries of the US economy. The CCAPM beta of an industry is the OLS estimator of the regression of the industry-specific value-weighted return on the growth rate of real GDP/cap, using annual data from 1930 to 2018. Source: Own computations using the Fama-French database.

parameter	value	description
γ	3	relative risk aversion
δ	0.99	utility discount factor
$1/\eta$	2	investment capacity per project
μ_θ	1.03	mean expected payoff per unit of capital
σ_θ	0.02	standard deviation of expected payoff per unit of capital
μ_β	1	mean payoff sensitivity to the common factor
σ_β	0.5	standard deviation of payoff sensitivity to the common factor
p	1.7%	annual probability of a macroeconomic catastrophe
μ_{bau}	0	technical parameter of the common factor
σ_{bau}	0.04	technical parameter of the common factor
μ_{cat}	-0.40	technical parameter of the common factor
σ_{cat}	0.40	technical parameter of the common factor

Table 3: Benchmark calibration of the model.

Moment	observed value	equilibrium value	variable
Aggregate risk premium	2.2%	2.22%	π^*
Interest rate	0.88%	0.86%	$r^* = R^* - 1$
Expected growth rate of consumption	1.8%	1.51%	$(\overline{\theta}^*/\eta) - 1$
Volatility of the growth rate of consumption	2.9%	2.76%	$\overline{\beta}^* \sigma_y/\eta$
Standard deviation of CCAPM betas	0.63	0.59	numerical estimation
Permanent equivalent consumption		0.0464	c^{pe*}

Table 4: Description of the rational equilibrium under the benchmark calibration, and comparison with the observed moments to be matched.

Moment	observed value	benchmark value	σ_β $0.5 \to 1$	σ_θ $2\% \to 4\%$	μ_θ $1.03 \to 1.05$	η $0.5 \to 0.55$
aggregate risk premium	2.2%	2.22%	1.17%	2.65%	2.14%	2.32%
interest rate	0.88%	0.86%	1.92%	0.55%	2.98%	0.48%
E[growth]	1.8%	1.51%	1.28%	2.13%	2.14%	1.49%
growth volatility	2.9%	2.76%	2.02%	2.97%	2.74%	2.81%
St.dev. β^{CCAPM}	0.63	0.59	1.43	0.58	0.60	0.58
c^{pe*}		0.0464	0.0517	0.0725	0.0811	0.0436

Table 5: Sensitivity analysis of the rational equilibrium.

	Optimal strategy	WACC strategy	Arrow-Lind strategy
discount rate	$0.86\% + \beta \times 2.22\%$	0.86%+ 2.22% $= 3.08\%$	0.86%
aggregate risk premium	2.22%	2.22%	2.22%
interest rate	0.86%	0.86%	0.86%
E[growth]	1.51%	1.30%	
growth volatility	2.76%	3.44%	
fraction of projects implemented	48.60%	48.40%	85.76%
consumption/wealth ratio	2.80%	3.20%	
E[return]	4.43%	4.65%	
E[sensitivity]	0.80	1.00	1.00
c^{pe}	0.0464	0.0339	

Table 6: Comparisons of outcomes in an economy in which all agents use the optimal investment strategy of the rational equilibrium, except one isolated agent who uses a single discount rate. In the "WACC strategy" column, this discount rate ρ is selected to be the WACC $R^* + \pi^*$ of the portfolio of investments undertaken by this agent. In the "Arrow-Lind strategy" column, the discount rate is R^*, yielding an infeasible solution. The "Optimal strategy" column is copy-pasted from Table 4.

	benchmark value	σ_β $0.5 \to 1$	σ_θ $2\% \to 4\%$	μ_θ $1.03 \to 1.05$	η $0.5 \to 0.55$
c^{pe*} under optimal strategy	0.0464	0.0517	0.0725	0.0811	0.0436
c^{pe*} under WACC strategy	0.0339	0.0258	0.0663	0.0679	0.0329
welfare loss (in %)	27	50	9	16	25

Table 7: Robustness analysis of the welfare loss of an isolated agent using the average cost of capital in the rational equilibrium as that agent's unique discount rate to evaluate investment projects and the portfolio allocation (WACC strategy).

	Rational equilibrium	WACC equilibrium
discount rate	$0.86\% + \beta \times 2.22\%$	-1.48%+ 4.54% = 3.06%
aggregate risk premium	2.22%	4.54%
interest rate	0.86%	-1.48%
E[growth]	1.51%	2.00%
growth volatility	2.76%	3.46%
fraction of projects implemented	48.60%	48.70%
consumption/wealth ratio	2.80%	2.52%
E[return]	4.43%	4.64%
E[sensitivity]	0.80	1.00
c^{pe}	0.0464	0.0396

Table 8: Comparisons of outcomes of the rational equilibrium and the WACC equilibrium.

	Rational equilibrium	Rationed Arrow-Lind equilibrium	
		$q = 0.8$	$q = 0.6$
discount rate	$0.86\% + \beta \times 2.22\%$	2.44%	1.18%
aggregate risk premium	2.22%	4.61%	4.73%
interest rate	0.86%	2.44%	1.18%
E[growth]	1.51%	2.74%	6.96%
growth volatility	2.76%	4.34%	5.81%
fraction of projects implemented	48.60%	48.88%	49.08%
consumption/wealth ratio	2.80%	2.24%	1.84%
E[return]	4.43%	4.25%	3.65%
E[sensitivity]	0.80	1.00	1.00
c^{pe}	0.0464	0.0339	0.0253

Table 9: Description of two rationed Arrow-Lind equilibria. Parameter q is the proportion of non-negative-NPV projects that are implemented.