THÈSE

En vue de l'obtention du

DOCTORAT DE L'UNIVERSITÉ DE TOULOUSE

Délivré par l'Université Toulouse 1 Capitole

Présentée et soutenue par

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Le 11 juillet 2023

Essais en Organisation Industrielle

Ecole doctorale : **TSE - Toulouse Sciences Economiques**

Spécialité : **Sciences Economiques - Toulouse**

Unité de recherche :
TSE-R - Toulouse School of Economics - Recherche

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Essays in Industrial Organization

PhD Thesis

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Toulouse School of Economics

July, 2023

A mi Chibu

Acknowledgements

First, I would like to massively thank my advisor, Patrick Rey, to whom I am deeply grateful for all his support during my PhD years, for his contributions to my research and his invaluable time through lengthy discussions.

I would also like to thank Bruno Jullien, Juan Pablo Montero and Markus Reisinger for kindly agreeing to be part of the jury for this thesis' defense.

I am grateful as well to many TSE professors for their contributions in different instances, particularly: Daniel Garrett, Patrick Feve, Ulrich Hege, Doh-Shin Jeon, Jean Tirole, Christian Hellwig, Catherine Casamatta, Alexandre de Cornière and Andrew Rhodes.

I want to thank all the administrative staff that helped me in many occasions, with a special mention to Laurence Delorme, who has always been willing to help me with joy. I appreciate as well the support of Nour Meddahi in his role of Director of the Doctoral School.

I am thankful to my parents, for allowing me to choose my own path, and constantly support and encourage me, to my brother Ignacio, for setting a remarkable example and also encouraging me in many ways, and to him and his family for receiving us many times over the years in their home.

I want to thank many friends at TSE, for all the good times shared together, in particular but not exclusively: Celia, Paloma, Matheus and Mariana, Rossi, Miguel, Elia, Filip, Charles and Gosia. Also, I thank my closest friends Germán, Fran, Berni, Cata, Pera, Tama, Tommy, Cocó, Mambri, Jenn and Chamaco.

I acknowledge the financial support of the Agencia Nacional de Investigación y Desarrollo ANID BECAS/DOCTORADO BECAS CHILE 72180519.

Finally, but definitely most importantly, I thank my wife Ali for having the courage to join me in this adventure, for being ever understanding and considerate, and for being a constant support and source of joy. None of this would have been possible without you.

Summary

The present thesis consists of three independent chapters, in the field of Industrial Organization. The first two chapters are related to entry deterrence in different contexts, while the third chapter is concerned with data collection and market tipping in digital economies.

The first chapter develops a dynamic model for the use of all-units-discount (AUD) contracts—also referred to as loyalty rebates—by an incumbent supplier, to deter or delay the entry of a more efficient supplier in the upstream market. This extends the work of Ide, Montero and Figueroa (2016), in which they analyzed the use of such contracts in a static context of *non-contestability*, wherein the incumbent supplier enjoys a non-contestable share of the market, which cannot be served by the entrant. Their main contribution is to show that, contrary to exclusive dealing contracts, AUD contracts cannot foreclose a more efficient entrant in any of the post-Chicago models, and therefore their prevalence is due to reasons other than exclusion. However, the non-contestability constraint reflects the presence of brand loyalty, capacity constraints, or any other mechanism that initially prevents full-market competition, but can be eventually overcome in the future upon entry. Therefore, from a competition perspective, the analysis of the dynamic implications is necessary and relevant. It turns out that when downstream retailers are fierce competitors, AUD contracts can be a profitable tool for the incumbent to deter—or at least delay—entry. The key to this contrasting result is intense downstream competition, as it prevents retailers from capturing any profits, even upon entry, thus the incumbent does not need to compensate them for any forgone future profits, and can transfer its own future profits to oppose entry at the outset.

The second chapter looks into entry deterrence—in the form of a limit price-path—in the context of positive selection. Revisited by Tirole (2016), positive selection refers to the feature in non-durable goods markets where the most motivated customers—those with higher valuations for the good—remain in the market, while the least motivated leave, contrary to the Coasian dynamics of durable goods—negative selection. In such a context, an incumbent facing the threat of entry by a differentiated competitor can implement a sequence of increasing prices, possibly up to its monopoly level, to profitably deter entry. The increasing price-path is explained by the exit of customers least attracted to the incumbent—and most attracted to the entrant—whenever entry does not occur. This leads the way to skimming dynamics, by which the incumbent can modulate the entrant's residual demand and therefore reduce its profits from entry. The increasing limit price-path characterized in this chapter has clear negative consequences for consumer welfare, as prices grow over time, possibly up to the monopoly price, and the lack of entry means less variety for consumers.

The third and last chapter sheds some light on the role of data collection in the competitive process of firms in a digital economy. In the wake of digitization, the role of data in competition has been a very relevant topic of discussion, both from scholars and practitioners. While not dismissing the multiple benefits that more and better data can bring to society, the fact that data can be regarded as a competitive advantage can be a matter of concern. The main interest of this analysis rests in the necessary conditions for market tipping in the long-run, when data allows firms to offer more value to consumers but any data advantage is only transient, as data can become obsolete. In this context, the main insight is that data on its own is not enough for markets to tip in the long-run, precisely because of its obsolescence. Market-tipping requires a firm to have a structural advantage, for example, in the form of more intrinsic value offered to consumers.

Résumé

La présente thèse se compose de trois chapitres indépendants, dans le domaine de l'organisation industrielle. Les deux premiers chapitres sont liés à la dissuasion à l'entrée dans différents contextes, tandis que le troisième chapitre concerne la collecte de données et le basculement du marché ou *market-tipping* dans les économies numériques.

Le premier chapitre développe un modèle dynamique d'utilisation de contrats du type *all-units-discount* (AUD) —ou remises de fidélité— par un fournisseur historique, pour dissuader ou retarder l'entrée d'un fournisseur plus efficace. Cela prolonge le travail d'Ide, Montero et Figueroa (2016), qui analyse l'utilisation de tels contrats dans un contexte statique de *non-contestabilité*, où le fournisseur historique bénéficie d'une part de marché non contestable par l'entrant. Leur contribution est de montrer que, contrairement aux contrats d'exclusivité, les contrats AUD ne peuvent pas exclure un entrant plus efficace dans aucun des modèles post-Chicago, donc leur prévalence est due à des raisons autres que l'exclusion. La contrainte de non-contestabilité reflète tout mécanisme qui empêche initialement la pleine concurrence (par exemple, fidélité à la marque ou contraintes de capacité), mais qui peut être surmontée à l'avenir lors de l'entrée. Dès lors, du point de vue de la concurrence, l'analyse des implications dynamiques est nécessaire et pertinente. Si les détaillants sont des concurrents féroces, les contrats AUD peuvent être un outil rentable pour l'opérateur historique pour s'opposer à l'entrée.

Le deuxième chapitre se penche sur la dissuasion à l'entrée —sous la forme d'une trajectoire de prix limite— dans le contexte de la sélection positive. Revisitée par Tirole (2016), la sélection positive fait référence à la caractéristique des marchés des biens non durables où les clients les plus motivés —ceux dont la valorisation du bien est plus élevée— restent sur le marché, tandis que les moins motivés partent, contrairement à la dynamique coasienne des biens durables —sélection négative. Dans ce contexte, un opérateur historique confronté à la menace d'entrée d'un concurrent différencié peut mettre en œuvre une séquence de prix croissants, éventuellement jusqu'à son niveau de monopole, pour dissuader l'entrée de manière rentable. Les prix augmentent en raison de la sortie des clients les moins attirés par l'opérateur historique chaque fois qu'il n'y a pas d'entrée. Cela ouvre la voie à une dynamique d'écrémage, par laquelle l'opérateur historique peut moduler la demande résiduelle de l'entrant et donc réduire ses bénéfices à l'entrée. Cette stratégie a des conséquences négatives pour le bien-être des consommateurs, car les prix augmentent au fil du temps, et le manque d'entrée signifie moins de variété pour les consommateurs.

Le troisième chapitre apporte un éclairage sur le rôle de la collecte de données dans la concurrence des entreprises dans une économie numérique. Dans le sillage de la numérisation, le rôle des données dans la concurrence a été un sujet de discussion très pertinent, tant de la part des universitaires que des praticiens. Sans négliger les multiples avantages que des données plus nombreuses et de meilleure qualité peuvent apporter à la société, les données peuvent aussi signifier un avantage concurrentiel préoccupant. Le principal intérêt de cette analyse réside dans les conditions nécessaires au basculement du marché —ou *market-tipping*— à long terme: les données permettent aux entreprises d'offrir plus de valeur aux consommateurs, mais tout avantage lié aux données n'est que transitoire, car les données peuvent devenir obsolètes. Dans ce contexte, la principale leçon est que les données à elles seules ne suffisent pas pour que les marchés basculent à long terme, en raison de leur obsolescence. Le basculement du marché exige qu'une entreprise ait un avantage structurel, par exemple sous la forme d'une plus grande valeur intrinsèque offerte aux consommateurs.

Contents

Chapter 1

All-units-discount contracts as a delaying and deterring instrument

Abstract

In the context of an incumbent enjoying a *non-contestable* share of the final market, Ide, Montero and Figueroa (2016) argue that, contrary to Exclusive Dealing contracts, *all-units-discount* (AUD) contracts cannot foreclose a more efficient entrant in any of the post-Chicago models, as the former commit retailers to compensate the incumbent upon trade with the entrant, whereas the latter lack such commitment. Nonetheless, we show that AUD contracts do have some anticompetitive scope—in the form of entry delay or deterrence— in a dynamic model where the entrant can overcome the non-contestability constraint in the future, upon entry. Such contracts remain powerless in the setups of cost uncertainty, and scale economies; however, AUD contracts will be profitable for the incumbent when retailers are downstream competitors, as long as the entrant's cost-efficiency is not too large. Intense competition is key: as retailers' profits are zero after entry, the incumbent need not compensate them for any forgone future profits, and can transfer its own future profits to oppose entry at the outset.

1.1 Introduction

There is a longstanding antitrust debate about the extent to which an incumbent supplier can prevent the entry of a more efficient firm. The Chicago Critique of the original monopoly-leverage theory claims that Exclusive Dealing (or ED) contracts have no anticompetitive potential.[1] This has prompted a series of analyses identifying specific contexts in which efficient entry can nevertheless be foreclosed. These post-Chicago models rely on one of two ingredients: unknown entrant's cost,[2] or economies of scale.[3]

A similar debate has emerged regarding the potential foreclosure effect of discount contracts. One form of such contracts is the *all-units-discount* (or AUD) contract,[4] where the discount applies to *all* units purchased (not just the incremental units) if a certain threshold is reached. AUD contracts may be regarded as a cheaper tool to sustain a strong incumbent position in the market (by preserving a monopoly position, or putting a cap to the market share of smaller rivals).

The European Commission has often argued that the concern of foreclosure arises particularly when part of the incumbent's demand is non-contestable. In this case, the incumbent can leverage its strong position to the contestable part of the demand, creating an artificial barrier to entry for the contestable segment.

The *Michelin II* case provides a good example.[5] Michelin was the main supplier of new and retreaded tyres for heavy trucks in the French market, and sales were mainly made through dealers. Between 1990 and 1998, Michelin offered the dealers, among other features, a quantity rebate, expressed as a percentage of total turnover, conditional on reaching a sales threshold. Therefore, the rebate applied to *all* units sold, and not only those above the threshold. This provided a strong incentive for dealers to favor Michelin over other suppliers. In appeal, the Court of First Instance expressed that "*(...) the quantity rebates which it [Michelin] applied are not merely quantity discounts. It operated a loyalty-inducing discount system (...)*".[6] Similar discount contracts were allegedly at the core of the *AMD v. Intel* case, which started in 2000 with a complaint by AMD, Intel's main competitor (at the time) in the x86 microprocessors' market, and is still ongoing.[7]

[1] See Posner (1976) and Bork (1978).

[2] See Aghion and Bolton (1987).

[3] See Rasmusen, Ramseyer and Wiley (1991) and Segal and Whinston (2000) for the case where buyers are final consumers and economies of scale create externalities across them; see Fumagalli and Motta (2006), Simpson and Wickelgren (2007), Abito and Wright (2008) and Asker and Bar-Isaac (2014) for the case of downstream competition among buyers.

[4] Sometimes also referred to as *loyalty rebates* or *retroactive rebates*.

[5] *Michelin v Commission*, T-203/01, EU:T:2003:250 (Michelin II). See also *Michelin v Commission*, 322/81, EU:C:1983:313 (Michelin I), *British Airways v Commission*, C-95/04 P, EU:C:2007:166, *Tomra Systems ASA and Others v Commission*, C-549/10 P, EU:C:2012:221, *Intel v Commission*, C-413/14 P, EU:C:2017:632 (AMD v. Intel).

[6] Paragraph 309 in Michelin II.

[7] In 2009 the European Commission imposed a €1.06 billion fine on Intel, decision that was upheld in a first judgment by the General Court. However, after Intel's appeal, the European Court

However, Ide, Montero and Figueroa (2016) (henceforth IMF) argue that this concern should not apply to AUD contracts, as even though they have a similar flavour to ED contracts, they have no potential to foreclose a more cost-efficient entrant, in any of the post-Chicago models.[8] Specifically, they consider an entrant with lower total cost (production and entry cost) of supplying the contestable share of the demand, and compare AUD contracts to ED contracts in three versions of the post-Chicago models. In their model, the reason the latter contracts do have foreclosure potential, while the former do not, lies in its implementation commitment. Contrary to ED, where retailers commit themselves (ex-ante) to compensate the incumbent if they interact with the entrant, AUD contracts involve no such commitment (only the discount is forfeited ex-post). Hence, the incumbent cannot profitably lock-in the retailers (as with ED) and therefore cannot foreclose the more efficient entrant. IMF suggest thus a *generalized Chicago Critique*: exclusion of a more efficient competitor cannot arise "*if all relevant parties were to participate simultaneously in the bargaining process, and sufficiently complete contracts (e.g., nonlinear prices) are available for the parties to sign*".

We dispute IMF's claims, as their work focuses on a static setting, and thus ignores the market's dynamics. We show that IMF's insight is not robust in a dynamic environment, where the non-contestability constraint can be overcome in the future. Indeed, an incumbent might worry that if the entrant is let to enter, then it may expand over time and erode the incumbent's strong initial position. As we will show, AUD contracts can have anticompetitive foreclosure effects in this context.

The non-contestability feature can be related, for example, to brand loyalty or to initial capacity constraints while setting up production. In either case, upon entry, the non-contestability constraint can be relaxed over time once the entrant builds a reputation (e.g., through word of mouth, internet reviews or prizes in contests), or augments its production capacity through fresh investment, respectively.[9]

of Justice referred it back to the General Court in 2017. In 2022, the General Court annulled the fine as it found that the Commission failed to "*establish to the requisite legal standard that the rebates at issue were capable of having, or were likely to have, anticompetitive effects (...)*" [General Court of the European Union, Press Release No 16/22]. The Commission has since appealed the General Court's judgement.

[8] Entry deterrence would be achieved if the incumbent can impose an unconditional fee to the retailer upon accepting a discount contract. Also, the no entry deterrence result, in the case of local monopolists, relies partly on the assumption that the entrant offers contracts with *purchasing obligations* for the retailers. This means that retailers, upon accepting an offer from the entrant, are committed to purchase all the units for the contestable segment to the entrant.

[9] A related example is the case of craft beer producers, which are not exactly a "more efficient" competitor to mainstream producers, but might provide a better quality. Often, small craft breweries will try to sell their product through large retailers, such as supermarkets, to reach a larger customers

To capture this, we consider a two-period version of IMF's model, where the contestable demand depends on the time of the entry. Hence, the main departure is that the entrant can overcome the non-contestability constraint in the second period if it enters in the first period.

We analyze this model in all three post-Chicago frameworks reviewed by IMF. For the first two frameworks—namely, uncertain efficiency gains, and economies of scale and downstream local monopolies—IMF's insights carry over: there is no anticompetitive scope for AUD contracts. The reason is that, to prevent entry in the first period, the incumbent would have to compensate retailers for the forgone future profits, and there is no wedge between suppliers for the incumbent to overcome the entrants efficiency advantage.

When retailers are downstream competitors, however, their future profits after entry are transferred away to final consumers, therefore the incumbent need not compensate the retailers. This opens up the possibility for the incumbent to accrue additional resources that may be enough to overcome the entrant's efficiency advantage. Hence, some anticompetitive scope arises for AUD contracts.

The remainder of the paper is organized as follows. Section 1.2 reviews the relevant literature. Sections 1.3, 1.4 and 1.5 develop the analyses for all three post-Chicago frameworks. Section 1.6 presents the concluding remarks and policy implications.

1.2 Literature Review

In recent years, various research has been done regarding the anticompetitive potential of discount contracts. One focus has been the *minimum-share* based contracts, which condition the rebate on the buyer acquiring a minimum share of its needs from the contracting supplier, as opposed to *quantity* based contracts, which condition the rebate on a quantity threshold, independently (at least directly) of the amount acquired from other suppliers. Chen and Shaffer (2014) show that minimum-share contracts can reduce the entry probability of a more efficient rival if price can be committed to as part of the contract. They adopt a setup similar to Aghion and Bolton's (1987). Inderst and Shaffer (2010) compare quantity based AUD contracts (they refer to them as *own-supplier* contracts) to minimum-share based AUD contracts (referred to as *market-share* contracts) in a case of intra- and interbrand competition, where they show that while quantity based AUD contracts cannot dampen both intra-

base. However, it is known that mainstream breweries try to stop this expansion by means of acquisition or by blocking relevant sales channels, as supermarkets, through tailored discount contracts.

and interbrand competition simultaneously, minimum-share based AUD contracts can indeed achieve this outcome.[10]

As mentioned above, Ide, Montero and Figueroa (2016) show that, in contrast to exclusive dealing contracts, AUD contracts cannot deter efficient entry in setups with uncertain efficiency gains (as in Aghion and Bolton (1987)), economies of scale with local monopolies (Rasmusen et al. (1991) and Segal and Whinston (2000)) and with downstream competition (Simpson and Wickelgren (2007), Abito and Wright (2008) and Asker and Bar-Isaac (2014)).

Ordover and Shaffer (2013) consider a two-period setting where a downstream buyer wants two units of a good, which can be supplied by an incumbent and a potential entrant, and preferences are such that the buyer would prefer one unit from each supplier. The first unit, however, can only be supplied by the incumbent for the whole time horizon (i.e., it is non-contestable), whereas the second unit can only be supplied in the second period by the first period's supplier of that unit. If the entrant is financially constrained, exclusion can arise in equilibrium, by means of an incremental price below marginal cost, or even negative, which resembles the use of AUD contracts.

The impact of AUD contracts has also been empirically studied by Conlon and Mortimer (2015), who find that AUD contracts do not implement the product assortment that maximizes social surplus, and that it leads to upstream foreclosure, by tying strong brands to weaker brands.

At first glance, AUD contracts seem to tie the contestable share to the non-contestable share of the market. It is therefore useful to emphasize the differences with the insight of Carlton and Waldman (2002). Their analysis, prompted by the Microsoft case in 1998, relies on the assumption that consumers value the complementary good (contestable market) only if they consume the primary good (non-contestable market). This condition enables the practice of tying to capture the whole bundle surplus. Such an assumption is not considered in this paper. Additionally, we consider an intermediary retailer market, with long-lived firm(s), whereas Carlton and Waldman consider myopic consumers only. This implies that the incumbent does not need to compensate any buyer for future losses in welfare, therefore making it easier to deter entry by tying.

[10] The difference between *own-supplier* and *market-share* contracts in their model is that the former cannot depend on the quantities that a retailer chooses to supply from the competitor, while the latter can.

1.3 Uncertain efficiency gains

1.3.1 Setting

We begin the analysis of AUD contracts in a dynamic environment of contestability with the framework of rent-shifting as in Aghion and Bolton (1987). In order to do so, we consider the extension of IMF's model to a two-period model, where an entrant will overcome the non-contestability constraint in the second period, upon entering in the first period.

There are two periods: $t = 0$ and $t = 1$. Consider an incumbent supplier, I, facing a potential entrant, E. The constant marginal cost of I is $c_I < v$, whereas E has a constant marginal cost of $c_E < v$ but must sink a fixed setup cost of $F > 0$.[11] However, c_E is not known by neither the incumbent nor the retailer; only its distribution is known, and it is such that E is more efficient than I. The actual value becomes public knowledge only in stage (2) of the first period, regardless of the entry decision.

Two levels of cost efficiency will be considered: (i) in case of *strong efficiency*—or *SE*—E is more efficient than I even if entry is for just one period, that is, $F < \lambda(c_I - c_E)$; (ii) in case of *weak efficiency*—or *WE*—E is more efficient than I only if entry is in $t = 0$, that is, $\lambda(c_I - c_E) < F < (\lambda + \delta)(c_I - c_E)$. Hence, under strong efficiency, c_E is distributed according to G^S in $[0, c_I - F/\lambda]$, while under weak efficiency, c_E is distributed according to G^W in $[c_I - F/\lambda, c_I - F/(\lambda + \delta)]$.

In each period, there is a unit mass of (short-lived) consumers with unit demands, all with value v. Consumers are served through a single retailer, R, with no additional costs (other than the cost of the input from the supply side).

If E enters in $t = 0$, at first it can only serve a fraction $\lambda \in (0, 1)$ of the demand (the contestable share), but later on can serve the entire demand. Hence, if E enters in $t = 0$, it can serve the whole demand in $t = 1$, whereas if E enters in $t = 1$, it can only serve the λ share of the demand in that period. All firms have the same discount factor $\delta \in (0, 1]$.[12]

An all-units discount contract (AUD or simply discount contract) consists of a list price p, an off-list discount d, applicable to *all* units purchased, and a threshold $\hat{q}$, which is an amount of units required to obtain the discount. The most effective threshold in this fixed demand model is to require full exclusivity (i.e., $\hat{q} = 1$: a quantity equal to the entire market).[13] In what follows, a full exclusivity all-units

[11] It is assumed that $c_E + F/\lambda < v$.

[12] $\delta = 0$ is ruled out, since this would be the equivalent of a one period model.

[13] In a one-period model, given a price p and a discount d, if $\hat{q} < 1$, then E could aim for the remaining market after the threshold, $1 - \hat{q}$, with a price just short of $p - d$ and profit $(1 - \hat{q})(p - d - c_E) - F$. Therefore, to make this option unprofitable for E, I would need to set

discount contract will be formally referred to as the pair (p, d).

The timing is as follows. In each period $t \in \{0, 1\}$ there are 3 stages. In stage (1), I offers R an AUD contract (p, d).[14] In stage (2), E offers R a wholesale price p_E,[15] and if not already in the market, decides whether to enter, thereby sinking the cost F, or to stay out. In stage (3), R decides how much to buy from each supplier present in the market.[16]

Throughout the paper it will be assumed that there is no inter-temporal commitment, and I cannot commit to contracts contingent on E's actions.

1.3.2 Analysis

In a one-period model (with strong efficiency) IMF showed that the incumbent cannot profitably deter entry: the best it can do is to extract the whole surplus of the non-contestable share. Profits are then

$$\tilde{\Pi}_I = S_{1-\lambda} = (1-\lambda)(v - c_I), \quad \tilde{\Pi}_E = \lambda(c_I - c_E) - F, \quad \tilde{\Pi}_R = \lambda(v - c_I), \tag{1.1}$$

where $S_{1-\lambda}$ is the non-contestable share surplus.

The underlying principle behind this result is the lack of commitment inherent to rebates. With an exclusive dealing contract a penalty is committed to early on, therefore it is sunk from the retailer's perspective at the time it has to decide whether to honour the contract or not. By contrast, with an AUD contract the equivalent penalty—namely the forgone discount—is not already sunk, and it directly affects the retailer's provision decision. This distinction forces the incumbent to offer more surplus to the retailer in order to deter entry with AUD contracts, to the extent that it is not profitable.

In a dynamic environment, it may seem that the incumbent has more incentives to oppose the entrant, given the relaxation of the non-contestability constraint upon entry. By opposition we will refer to either full entry deterrence, or entry delay. However, the insights of IMF follow through in this framework. Under both types of efficiency

$p - d < c_E + F/(1-\hat{q})$, which imposes a constraint on I's contract structure. By setting $\hat{q} = 1$, I can dispense from this constraint.

[14] I might offer $d = 0$, meaning no discount.

[15] Similarly, E might offer a price $p_E > v$, if it is not interested in the offer to be implemented.

[16] This timing replicates the *purchasing obligation* found in IMF, in the sense that E has a last-mover advantage: in their paper, if any retailer accepts E's offer, the former is committed to buy the λ contestable units from the latter. The implications of this condition will become clear later on. In Section B we will allow for I to make a counter-offer, thereby dropping the purchasing obligation condition.

the incumbent cannot do any better than to extract the whole non-contestable surplus in the first period and allow entry.

Proposition 1.1 *In a two-period model with unknown entrant's marginal cost, the incumbent cannot do better than allow entry and extract the whole non-contestable surplus in the first period, regardless of the efficiency advantage considered.*

Proof. We will fix c_E and show that it is not profitable for I to oppose entry. The argument is then by contradiction: if it were profitable to implement some opposition—in the sense that for c_E above a certain threshold such outcomes arise—then for some realizations of c_E it must be certainly profitable.

Let $\pi_R \in [0, v - c_I]$ be the retailer's second-period profit in an opposing candidate equilibrium, and consider a (p, d) offer from I in the first period. For R to forgo the AUD contract, E's price must satisfy:

$$v - (1 - \lambda)p - \lambda p_E + \delta(v - c_I) > v - p + d + \delta\pi_R, \tag{1.2}$$

which is equivalent to

$$p_E < p - \frac{d}{\lambda} + \frac{\delta}{\lambda}(v - c_I - \pi_R), \tag{1.3}$$

and is the analogue of IMF's *effective price*, taking into consideration future profits. This allows E to raise its price vis-a-vis IMF's condition of $p_E < p - d/\lambda$. Observe that similar to I's leverage, represented by the $1/\lambda$ factor multiplying the discount, E is also leveraging its second-period full market presence into the contestable market in the first period.

As c_E is public knowledge in the second period regardless of any outcome in the first period, the entry condition is

$$\Pi_{E|entry} = \lambda(p_E - c_E) - F + \delta(c_I - c_E) \geq \Pi_E^0 = \begin{cases} \delta[\lambda(c_I - c_E) - F], & \text{under SE,} \\ 0, & \text{under WE,} \end{cases} \tag{1.4}$$

where Π_E^0 corresponds to E's outside option of either waiting to enter in the second period under strong efficiency—which will occur as that period would be equivalent to IMF's model—or to be foreclosed altogether under weak efficiency—as it would amount to the exclusion of a less efficient rival in the second period, possible by means of an AUD contract as shown by IMF.

It is clear that I will optimally set $p = v$, as otherwise I can raise p to $p + \Delta$ and d to $d + \lambda\Delta$, thus leaving (1.3) unaltered, and obtain $(1 - \lambda)\Delta$ extra profit.[17] Then,

[17] Note that if p is larger than v, then for R, conditional on trading with E, it is better to only

I's profit of opposing entry (i.e., taking into consideration equations (1.3) and (1.4)) is

$$\begin{aligned} \Pi_{I|oppose} &= v - d - c_I + \delta(v - c_I - \pi_R), & (1.5)\\ &\leq (1-\lambda)(v - c_I) + \Pi_E^0 - [(\lambda+\delta)(c_I - c_E) - F], & (1.6)\\ &< S_{1-\lambda}, & (1.7) \end{aligned}$$

since $\Pi_E^0 \leq \lambda(c_I - c_E) - F < (\lambda+\delta)(c_I - c_E) - F$. It follows that it is not profitable for I to oppose entry for a fixed and known c_E, and *a fortiori* it will not be profitable for any unknown c_E within its support. ■

The reason why any entry opposition is not profitable, despite the apparent additional incentives from the non-contestability constraint relaxation, is a compensation effect. In effect, the extra profits that the incumbent could accrue in the second period, from opposing entry, are equal to what the retailer would forgo from not allowing entry in the first period: $\delta(v - c_I - \pi_R)$, where π_R corresponds to the retailer's second-period profit in a candidate opposing equilibrium. This amount would need to be compensated for by the incumbent, and thus it has no additional resources. On top of that, the entrant can offer an even lower price—vis-a-vis IMF's model—due to the additional resources from future profits.

In conclusion, the incumbent cannot do better in a two-period model, and entry will occur in the first period, as in IMF's static model. It follows that the sole possibility of an expanding entrant is not sufficient for the incumbent to profitably oppose entry.

Remark. Observe that in the basic setup (i.e. in one period only), AUD contracts are optimal for preventing access to the entrant. In effect, consider tariffs $T_I(\cdot)$ and $T_E(\cdot)$, then to optimally prevent entry, I solves

$$\begin{aligned} \max_{T_I(1), T_I(1-\lambda)} \quad & T_I(1) - c_I, \\ s.t. \quad & T_I(1) - \min\{T_I(1-\lambda), (1-\lambda)v\} \leq \lambda c_E + F, \end{aligned}$$

trade with E, reducing I's profit.

where the constraint is to prevent E from profitably entering.[18] The solution is

$$\begin{cases} T_I(1-\lambda) \geq (1-\lambda)v, \\ T_I(1) = (1-\lambda)v + \lambda c_E + F. \end{cases}$$

These tariffs can be achieved, for instance, by the AUD contract ($p = v, d = \lambda(v - c_E) - F$).[19]

1.4 Economies of scale and downstream local monopolies

1.4.1 Setting

We consider now the framework of economies of scale with downstream local monopolies, as in Segal and Whinston (2000). The setting is similar to that in the previous section of uncertain efficiency gains; however, there are three main differences: (i) the entrant's marginal cost is known to all parties, (ii) there are two retailers, R_1 and R_2, each holding a monopoly position in different markets (or different groups of consumers), with a mass 1/2 of consumers each, and (iii) there are economies of scale meaning that the entrant needs to trade with both retailers to cover the fixed entry cost.

The contestable share size λ is the same in both markets. Furthermore, the entrant only overcomes the non-contestability constraint in the markets it has entered (meaning that E needs to enter both markets in the first period to be able to access all consumers in the second period).

The economies of scale feature requires a deeper treatment. In the one-period model of IMF, the assumption amounts to $\lambda(v - c_E)/2 < F$, and it reflects the negative externality that a retailer imposes on the other one when deciding not to trade with the entrant (as in Rasmusen et al. (1991) or Segal and Whinston (2000)). It therefore implies that I needs to lock in just one retailer to avoid entry, that is, induce only one retailer to avoid trade with E. In a two-period model, however, a

[18] E can enter in two ways: by means of R buying $1-\lambda$ units from I and λ units from E, or only the latter, depending on what is optimal for R given $T_I(\cdot)$ and $T_E(\cdot)$. This yields the following necessary condition for E: $\max\{v - T_I(1-\lambda), \lambda v\} - T_E(\lambda) > v - T_I(1)$. The right-hand side of the constraint follows from E's entry profit: $T_E(\lambda) - \lambda c_E - F > 0$.

[19] Kolay, Shaffer and Ordover (2004) show that AUD contracts are superior to two-part tariffs and incremental discount contracts when an upstream monopolist manufacturer (here, the incumbent) faces uncertainty over the downstream monopolist retailer's demand.

stronger condition is necessary to maintain the notion of negative externalities across retailers, because of the future profit that E enjoys conditional on entry. Namely, the condition is that the entrant does not find it profitable to enter just one market in the first period, even after accounting for future profits.

Assumption 1.1 (Negative externality across retailers) *The following conditions hold for each efficiency advantage considered:*

$$\frac{\lambda(v-c_E)}{2} - F + \frac{\delta(1+\lambda)(c_I-c_E)}{2} < \begin{cases} \delta[\lambda(c_I-c_E)-F], & \text{under } SE, \\ 0, & \text{under } WE. \end{cases} \tag{1.8}$$

The left-hand side of Assumption 1.1 corresponds to E's profit of extracting the whole surplus in one market in the first period and then competing for that full market in the second period, plus entering the other market in the second period and competing for the contestable share only; while the right-hand side corresponds, for each case of efficiency advantage, to waiting until the second period to enter both markets and compete for the contestable shares only. Note that this assumption implies the more standard assumption made in IMF.

This assumption is necessary for I to have a chance at profitably opposing entry in this setup: if it is not the case, then I would have to either lock both retailers in, which is equivalent to the previous section with only one retailer and a fixed entrant's marginal cost, or focus only on one of the retailers, which is even less profitable, because I would be facing an entrant that can sink part of the entry cost elsewhere (in the market that I does not focus on); in both cases, neither deterrence nor delay are profitable.

1.4.2 Analysis

In a one-period model (with strong efficiency), IMF showed that deterrence is not profitable, and the payoffs are given by

$$\tilde{\Pi}_I = S_{1-\lambda} = (1-\lambda)(v-c_I), \quad \tilde{\Pi}_E = \lambda(c_I-c_E)-F, \quad \tilde{\Pi}_{R_1} = \tilde{\Pi}_{R_2} = \lambda(v-c_I)/2. \tag{1.9}$$

The intuition behind this result is that both I and E can try to exploit one retailer, say R_2, to bribe the other retailer, R_1. However, due to the cost-efficiency advantage, E can do so to a larger extent than I, thus I does not find it profitable to deter entry. These payoffs are then the second-period profits in a two-period model with strong efficiency if E did not enter in the first period.

In this two-period setup, the same logic applies, and IMF's result still holds true: the best I can do is to extract the whole first-period non-contestable surplus and allow entry, regardless of the efficiency advantage considered.

Proposition 1.2 *In a two-period model of economies of scale and local downstream monopolies, the incumbent cannot do better than allow entry and extract the whole non-contestable surplus in the first period, regardless of the efficiency advantage considered.*

Proof. Let π_{R_1} and π_{R_2} denote the retailers' second-period profits in an opposing candidate equilibrium. conditional on no entry in the first period. The best chance for I to oppose entry in the first period is to offer a contract to one retailer only, say R_1, in the form of (v, d) as already discussed, and exploit the other retailer with a $(v, 0)$ contract.

Assumption 1.1 requires E to trade with both retailers in order to gain access to the market. By an analogous argument to (1.3) in Proposition 1.1, the latter entails prices $p_{E1} < v - d/\lambda + \delta(v - c_I - 2\pi_{R_1})/\lambda$ and $p_{E2} < v + \delta(v - c_I - 2\pi_{R_2})/\lambda$. The entry condition is $\Pi_{E|entry} = \lambda(p_{E1} - c_E)/2 + \lambda(p_{E2} - c_E)/2 - F + \delta(c_I - c_E) > \Pi_E^0$, where again Π_E^0 is E's outside option, and equals $\delta[\lambda(c_I - c_E) - F]$ in the strong efficiency scenario, and 0 in the weak efficiency scenario.

It follows that I's profit from opposing entry is

$$\begin{aligned} \Pi_{I|oppose} &= \frac{v - d - c_I}{2} + \frac{v - c_I}{2} + \delta(v - c_I - \pi_{R_1} - \pi_{R_2}), && (1.10)\\ &\leq (1 - \lambda)(v - c_I) + \Pi_E^0 - [(\lambda + \delta)(c_I - c_E) - F], && (1.11)\\ &< S_{1-\lambda}, && (1.12) \end{aligned}$$

and therefore opposition to entry is not profitable. ■

The reason behind Proposition 1.2 is similar to the one in IMF. If I offers an attractive AUD contract to R_1, and fully exploits R_2, then the latter internalizes that its provision decision is relevant for entry, and therefore it is relevant for future profits. Since E is more efficient than I, it can exploit this retailer to a larger extent than I can do.

The key aspect so far has been that the retailers are forward-looking and strategic about their decisions, therefore IMF's arguments are still valid. However, as we will show in the next section, when downstream retailers compete, a wedge opens up and the incumbent is able to profitably oppose entry under some circumstances.

Remark. It is important to notice that the model, following IMF, considers that if a retailer signs a contract with the entrant, then this retailer is committed to buy

λ units from it, if the latter enters. This is not an assumption made by Rasmusen et al. (1991) nor Segal and Whinston (2000), where incumbent and entrant could compete simultaneously for the "free" buyers (those that did not sign an exclusive contract). IMF argue that these long-term contracts with commitment from the retailers perspective (or as they refer to, "*purchasing obligations*"), are in the interest of both retailers and entrant, *ex-ante* and *ex-post*. A direct consequence of this assumption is that the entrant will also fully exploit R_2 (given that a discount contract was only offered to R_1), since this retailer will anticipate that conditional on no entry, it would get fully exploited by I anyway, and therefore will be willing to sign any contract with E: in the limit, a fully exploiting contract as well. If this purchasing obligation assumption is dropped, however, a wedge opens up between both suppliers: even though both could exploit the second retailer, it is no longer the case that the entrant can do so to a larger extent than the incumbent, as the latter will have a last-mover advantage to make a counter-offer and cap the former's exploitation profit. Moreover, the incumbent may deter entry in the second period (if the entrant remains out of the market) if the entrant's one-period efficiency is not too large. These two aspects play in favour of the incumbent, and as a result, entry deterrence will be profitable provided the entrant's overall efficiency is not too large. Refer to Appendix B for the details.

1.5 Downstream competition

1.5.1 Setting

We consider now the last post-Chicago framework of downstream competition as in Asker and Bar-Isaac (2014) or Abito and Wright (2008). The model is the same as in the previous section, with the exception that both retailers are not in independent markets, but rather belong to the same market and compete à la Bertrand for the final consumers of mass 1. In contrast to the previous section, the entrant needs only to trade with one retailer to gain access to the full market. Following IMF, we will assume that: (i) if two retailers offer the same price, consumers will buy from the one with lower cost; and (ii) retailers can discriminate, among final consumers, between contestable and non-contestable shares, setting possibly different prices for each group.

So far, we have considered the contract's threshold $\hat{q}$ as a certain amount of units to be acquired—a *quantity based* AUD contract. However, we could also consider the threshold to be a percentage of the retailer's total needs— a *minimum-share based* AUD contract. In the previous sections, the distinction between both thresholds is irrelevant, as the sole retailer, or each retailer in the case of local monopolies, holds a monopoly position in its own market (coupled with the unit demand and the absence of demand uncertainty).[20] However, this is no longer the case in the current framework of downstream competition. As it will be shown in more detail, a *quantity* threshold can restore the incumbent's leverage that is otherwise lost due to downstream competition.

1.5.2 Analysis

We proceed to analyse both types of thresholds in turn, starting with the minimum-shared based contracts, which correspond to the off-list rebates considered in IMF.

Minimum-share based contracts

In a one-period model (with strong efficiency) IMF showed that foreclosure is not a profitable strategy for the incumbent when minimum-share based contracts are considered. The reason of this result is as follows: given discount contracts offered by I, E does not need a very low price to convince a retailer to accept its offer, since at least one retailer would earn zero profits in any equilibrium in which E does not enter (either both retailers are offered the same contract, in which case Bertrand competition washes profits away, or one retailer has a better discount contract than the other, leaving the latter without any profit). In effect, consider that I offers (p, d) to one or both retailers, then E needs to set a price just below $p - d$, and not $p - d/\lambda$ (as with the leverage at the core of the previous frameworks); hence, I does not have the leverage between the non-contestable and contestable shares that exists absent downstream competition. Therefore, for I to deter entry, it needs to set a discounted price, $p - d$, below E's relevant cost, $c_E + F/\lambda$, which by strong efficiency is lower than I's marginal cost, c_I, rendering deterrence not profitable. We refer to this as the *loss-of-leverage effect*.

It follows that the best I can do in this one-period case is to extract the whole surplus from the non-contestable share and allow entry. The corresponding profits are (see IMF):

$$\tilde{\Pi}_I = S_{1-\lambda} = (1-\lambda)(v - c_I), \quad \tilde{\Pi}_E = \lambda(c_I - c_E) - F, \quad \tilde{\Pi}_{R_1} = \tilde{\Pi}_{R_2} = 0. \tag{1.13}$$

When we considering a two-period model, a second effect comes into play: due to Bertrand competition, retailers' continuation payoffs after entry in the first period are

[20] If there is a downward sloping demand, or if there is uncertainty over demand, a quantity threshold will not be able to replicate a minimum-share threshold.

zero, therefore I does not need to compensate retailers for any future loss of profits due to entry delay or deterrence. This is the *no-compensation effect*, and it allows the incumbent to borrow from its future profits to offer more attractive AUD contracts in the first period.

The loss-of-leverage and no-compensation effects go in opposite directions, where the first one diminishes I's possibilities of entry opposition, while the second one aids I towards this end.

Proposition 1.3 *In a two-period game of downstream competing retailers with minimum-share based contracts, the incumbent can profitably deter entry if and only if the efficiency advantage is not too large:*

$$\delta[c_E + F/\lambda - c_I] - (1/\lambda)[(\lambda+\delta)(c_I - c_E) - F] \geq (1-\lambda)(v - c_I). \tag{1.14}$$

Proof. See Appendix A. ■

We already know that, under strong efficiency, the second period (conditional on no entry in the first period) is equivalent to IMF's model, and entry will occur. In this context, the loss-of-leverage effect dominates the no-competition effect: even though no compensation is required, I's first-period profit is negative in any delaying equilibrium—and second-period profit is not enough to offset this. The reason for negative first-period profit is the same as in the one-period model: given the loss-of-leverage effect, I has to set a discounted price, $p - d$, below E's relevant cost, which in a two-period model is even lower than $c_E + F/\lambda$, because of future profits upon entry, and therefore it is below c_I.

Under weak efficiency, however, the dominance is potentially reversed if the incumbent's second-period profit from deterrence is large enough. The incumbent is more efficient than the entrant in the second period (conditional on no entry in the first period), and its second-period deterrence profits coincide with the entrant's second-period inefficiency: $c_E + F/\lambda - c_I > 0$. Therefore, if the latter is large enough or, what is the same, the overall efficiency advantage is not too large, then the incumbent will profitably deter entry in the second period, if it has managed to prevent entry in the first period.

Under the previous presumption, it follows that if I manages to deter entry in the first period, then it will deter entry in the second period. Then, from the first period onward, full deterrence will be profitable if future profits net of E's overall cost-efficiency advantage (left-hand side of (1.14)) are larger than I's status quo option represented by the non-contestable surplus in the first period (right-hand side of (1.14)). It is important to remark that E's advantage is adjusted by $1/\lambda$, since downstream competition is preventing the incumbent from leveraging its dominance in the non-contestable share to the contestable one.

Remark. One difficulty that I faces under downstream competition is that E needs to trade with only one retailer to gain access to the final market, and at the same time, I cannot transfer utility to both retailers simultaneously (due to competition). Moreover, suppose I is offering an AUD to R_1 in the first period, then the fact that I loses its leverage power with minimum-share contracts, implies that R_1's expected future profit from deterrence—say, if R_1 believes that upon delay in the first period, deterrence is going to occur through it (R_1)—cannot be used as a leverage by I in the first period. This implies that the incumbent has to rely solely on its future period's profit. Hence, the purchasing obligation feature is playing a relevant role in this case, as it limits such profit. Indeed, if the purchasing obligation were not present, the incumbent could deter entry of a less efficient entrant in the second period and obtain the full market's surplus (in that period), increasing its resources to deter entry in the first period. Refer to Appendix B for the analysis.

Quantity based contracts

When the threshold for the AUD contracts is based on a fixed quantity, then the incumbent can fully restore its leverage power, which was partially absent in the minimum-share based contracts case.

Lemma 1.1 *In a one-period model of downstream competition, the incumbent can restore its full leverage power by offering a (p, d) quantity based contract to one retailer only, say R_1 (with a threshold of $\hat{q} = 1$). Under this scheme, the entrant is indifferent between offering trade to R_1 or R_2.*

Proof. Consider that I offers a (p, d) quantity based contract to R_1, with $\hat{q} = 1$, and $(p, d) = (v, 0)$ to R_2. If E offers a price p_E to R_1, the necessary condition for the latter to accept trade is

$$v - (1 - \lambda)p - \lambda p_E > v - p + d, \tag{1.15}$$

which is the standard leverage condition. On the contrary, if E offers a price p_E to R_2, the necessary condition is that R_2 is able to compete with R_1 in the contestable market. Since a competing R_2 would prevent R_1 from achieving the threshold, the price p_E needs to be sufficiently low so that R_1 prefers to quit the contestable market and exploit the non-contestable one only, that is

$$(1 - \lambda)(v - p) > (1 - \lambda)v + \lambda p_E - p + d. \tag{1.16}$$

Each condition boils down to $p_E < p - d/\lambda$, which reflects the standard leverage power of I, and are therefore equivalent. ∎

Given the Bertrand competition, Lemma 1.1 extends trivially to a two-period model, since retailers' second-period profits after entry are zero, as they are transferred to final consumers, regardless of the efficiency considered (strong or weak).

Corollary 1.1 *In a two-period model of downstream competition, the incumbent can restore its full leverage power by offering a (p, d) quantity based contract to one retailer only. The entrant is indifferent between offering trade to R_1 or R_2 in the first period.*

The case of quantity based contracts was not analyzed by IMF. However, IMF's main argument applies all the same and in a one-period model the incumbent cannot profitably deter entry of a more efficient entrant. Nonetheless, as argued by IMF, the incumbent is able to profitably deter entry of an inefficient entrant.

Claim 1.1 *In a one-period game of downstream competition and quantity based contracts, the incumbent cannot profitably deter entry of a more efficient entrant, but can do so from a less efficient entrant.*

For the dynamic model, the key difference between quantity and minimum-share based contracts is precisely the fact that the former can completely restore the leverage power of the incumbent. Therefore, the loss-of-leverage effect is no longer in play, leaving only the no-compensation effect in place. This difference tilts the balance in favor of the incumbent.

Proposition 1.4 *In a two-period game of downstream competition and quantity based contracts, the incumbent can delay entry under strong efficiency, provided that:*

$$\delta(1-\lambda)(v-c_I) \geq (\lambda+\delta)(c_I-c_E) - F - \delta[\lambda(c_I-c_E)-F]; \tag{1.17}$$

and can deter entry altogether under weak efficiency, provided that:

$$\delta(v-c_I) \geq (\lambda+\delta)(c_I-c_E) - F. \tag{1.18}$$

Proof. See Appendix A. ∎

Observe that the left-hand sides of equations (1.17) and (1.18) correspond to the second-period profits—and thus additional, as there is no need to compensate retailers—that the incumbent can acquire from opposing entry, under strong and weak efficiency, respectively. The right-hand sides of the equations correspond to the entrant's overall efficiency, net of its outside option, Π_E^0, under each efficiency level.

As the incumbent's leverage is restored, both suppliers are effectively competing for the contestable market in the first period. Since there is no compensation to retailers required, I can offer up to its additional profit from delaying or deterring entry to induce exclusivity, through an AUD contract to one retailer only. Similarly, the entrant can offer any retailer up to its overall efficiency, net of its outside option, to access the market.

Hence, entry opposition will arise in equilibrium whenever the incumbent's future profits from opposing entry outweigh the entrant's overall efficiency (net of its outside option). And as has become clear, the key elements to this entry opposition are: (i) the leverage effect that gives an edge to the incumbent, and (ii) the downstream competition that lifts the necessity for I to compensate the retailers.

Remark. Contrary to minimum-share contracts, the purchasing obligation does not play a role when quantity based contracts are considered, because the second-period profit left to R_1 after deterrence is internalized in the first period, precisely because I's leverage is restored.

Quantity based contracts - more than just two periods

So far, we have considered the case of only two periods, and have shown that entry can be opposed by means of quantity based AUD contracts. As will be shown, this feature can be extended to multiple periods. Notably, when strong efficiency is considered, entry can be profitably delayed until the last period without further requirements than the two-period condition. Under weak efficiency, however, further conditions are required for entry to be profitably deterred altogether: either the incumbent's monopolistic margin is larger than the entrant's competitive margin ($v - c_I \geq c_I - c_E$); or a stricter version of the two-period model condition is satisfied. Notice that regardless the number of periods, the notion of strong and weak efficiency remains the same as with two periods (i.e., under weak efficiency, E is less efficient than I if there is only one period left, but is more efficient if there are two or more periods left; whereas under strong efficiency E is more efficient than I even if there is only one period left).

The following propositions establish these ideas formally, as well as characterize the AUD contracts involved.

Proposition 1.5 *In a setup of downstream competition with $t > 2$ periods, strong efficiency and quantity based contracts, the incumbent can profitably delay entry until the last period if and only if it is profitable to do so in a two-period model.*

Proof. See Appendix A. ∎

To understand why the two-period profitability condition is sufficient to delay entry all the way until the last period under strong efficiency, notice that, in each period, the incumbent has to deal only with the entrant's one-period efficiency (net of the entrant's outside option of waiting one period), $(1-\delta)\xi_0 = (1-\delta)[\lambda(c_I - c_E) - F]$, and its next period's profit, $\delta(c_I - c_E)$. Crucially, it need not consider *all* future profits, since future profits other than the very next period's are dealt with in the future periods' "bribes" (whenever there are more than just two periods left). Then, as the contract in the second to last period (which corresponds to the two-period condition) does not rely on any future bribes, it involves a larger discount than previous periods' bribes, rendering the two-period profitability condition a sufficient condition for Proposition 1.5.

The following proposition characterizes the contracts offered along time and illustrates the previous argument.

Proposition 1.6 *In a setup with more than two periods, downstream competing retailers, quantity based contracts and strong efficiency, the incumbent offers one retailer, say R_1, the same contract $(p,d)_\tau = (v, d_{\tau\geq 3})$ for all $\tau \geq 3$ (with τ the number of periods left), and in the second to last period the incumbent offers R_1 the contract $(p,d) = (v, d_{\tau=2})$, with*

$$d_{\tau\geq 3} = \lambda(1-\delta)(v - c_I) + (1-\delta)\xi_0 + \pi_E(2) \tag{1.19}$$

$$d_{\tau=2} = \lambda(v - c_I) + (1-\delta)\xi_0 + \pi_E(2). \tag{1.20}$$

No contract is offered in the last period. Furthermore, the discounted price set by the incumbent is at least c_I in the second to last period, and strictly larger in all previous periods.

Proof. See Appendix A. ∎

The discounts offered by the incumbent to the chosen retailer display a flat pattern over time, with a spike at the end. This contrasts the offer that the entrant needs to make, which is increasing with respect to the periods left: I can distribute its offers over time, whereas E has to transfer all profit at once, as downstream competition washes retailers' profits away after entry. Moreover, the discounted prices are larger than the incumbent's marginal cost, so there is no concern about reaching below marginal cost prices or even negative prices.

We turn now to the case of weak efficiency.

Proposition 1.7 *In a setup of downstream competition with $t > 2$ periods, quantity based contracts and weak efficiency, the incumbent can profitably deter entry in all periods if and only if:*

(i) *it is profitable in a two-period game and $v - c_I \geq c_I - c_E$, or*

$$(ii) \quad \delta(v - c_I) \geq \frac{(1-\delta)}{1-\delta^{t-1}}[\lambda(c_I - c_E) - F] + \delta(c_I - c_E). \tag{1.21}$$

Proof. See Appendix A. ■

Condition (1.21) is stricter than the two-period condition, as weak efficiency implies $\lambda(c_I - c_E) - F < 0$ and $\frac{(1-\delta)}{1-\delta^{t-1}} < 1$. Also, it is worth pointing out that the discounts offered by the incumbent are the same as in the strong efficiency case (except for the last discounts in each case). The reason is that the last discount offered takes into account the entrant's outside option (delay or deterrence), and thus previous periods' discounts just aim at blocking entry until the next period.

1.6 Conclusions

This paper analyzes *all-units-discount* (AUD) contracts' anticompetitive potential in a dynamic two-period environment. In contrast to *Exclusive Dealing* contracts' anticompetitive potential in the post-*Chicago Critique* frameworks,[21] Ide, Montero and Figueroa (2016) (referred to as IMF) have argued that AUD contracts do not have any foreclosing power in the same environments. However, their analysis is static, where an incumbent enjoys non-contestability in a share of the market, while a potential entrant can only compete in the remaining contestable share. In this paper, we study whether an anticompetitive outcome is possible when market dynamics are considered over time: in particular, what happens when the entrant can overcome the non-contestability constraint over time, upon entry.

The key to IMF's argument is that Exclusive Dealing contracts *commit* retailers to trade with the incumbent exclusively, whereas AUD contracts do not have such commitment. This lack of commitment, coupled with the fact that foreclosure of a more efficient entrant is inefficient, as IMF point out, render AUD contracts useless in the attempt of exclusion. However, if the entrant can overcome the non-contestability

[21] Aghion and Bolton (1987) for unknown entrant's marginal cost; Rasmusen, Ramseyer and Wiley (1991) and Segal and Whinston (2000) for economies of scale and downstream local monopolists; Fumagalli and Motta (2006), Simpson and Wickelgren (2007), Abito and Wright (2008) and Asker and Bar-Isaac (2014) for downstream competing retailers.

constraint over time, then a *wedge* may appear between incumbent and entrant, so that the former may bypass the inefficiency nature of exclusion, and profitably deter or delay entry.

We have shown that when retailers are downstream competitors, then the incumbent can profitably delay or deter entry, depending on the level of the entrant's efficiency advantage. The underlying principle is that, after entry has occurred, intense competition transfers retailers' profits to final consumers. Therefore, the incumbent is not required to compensate retailers to prevent entry in the first period, and can thus transfer its own future profits to engage one of the retailers into exclusivity.

The main implication of this paper is that IMF's assertion that AUD contracts are innocuous from an antitrust perspective, is not robust to market dynamics. The possibility of an entrant growing into the market over time, shifts the incumbent's incentives towards early foreclosure. As this practice would hinder new firms to contest the market of incumbents, with the consequences on innovation, efficiency, variety, and ultimately welfare, the analysis of AUD contracts from an antitrust perspective is still valuable and necessary.

Chapter 2

Fending off entry through positive selection

Abstract

I establish the existence of a deterrence—or limit—price-path in the context of positive selection (Tirole (2016)) and horizontal differentiation. Contrary to most limit pricing, the characterized price-path is increasing over time, and can eventually reach the monopoly price. This feature has competitive implications, as entry can be deterred while maintaining high prices, thus reducing consumer welfare.

2.1 Introduction

In the context of entry deterrence, a central idea that has been extensively discussed is that of *limit pricing*. In a nutshell, a limit price is the highest price level—typically below the monopoly price level—by which an incumbent can prevent entry into the market.[1] In the early models, entry probability was assumed to be positively correlated with current price levels, as the latter would serve as an indicator of future profits, upon entry. Hence the notion of a maximal price level that precludes entry.

Friedman (1979) pointed out that if post-entry profits are independent of pre-entry pricing, as is the case in usual complete information models, then limit pricing is futile as it would only reduce pre-entry profits without having an impact on any entry decision. To address this, Milgrom and Roberts (1982) turned to an incomplete

[1] See Bain (1949) for an early discussion of limit pricing, and Gaskins (1971) and Kamien and Schwartz (1971) for early formal analyses.

information model, whereby the incumbent can use its pricing policy as a signal of how competition would ensue upon entry, by means of conveying some information about a private characteristic (e.g., marginal cost). They show that limit pricing need not be detrimental to entry, as the entrant will internalize this opportunistic behaviour. As such, entry probability may be lower, equal or higher than in the absence of limit pricing. The authors thus argue that the apparent trade-off from limit pricing, between pre-entry low prices and delayed or deterred entry, may never actually arise; and conclude, regarding its policy implications, that limit pricing (in the context of their model) "*should not be discouraged, since it means lower prices and cannot, overall, limit entry*".[2]

In this paper we want to revisit the two main aspects of the previous narrative concerning limit pricing: that it has no bite in complete information models, and its implications from a competition standpoint. As we will show, there can be effective limit pricing in the context of complete information, whereby an incumbent manages to affect post-entry competition by means of its pre-entry pricing policy, and therefore deter entry. This has competitive implications concerning the aforementioned trade-off and consumer welfare in general.

We will consider a complete information framework of dynamic screening involving positive selection, as revisited by Tirole (2016). This latter concept, in the context of non-durable goods or services and no commitment, refers to a scenario in which customers with higher valuations remain in the market, contrary to the Coasian dynamics of durable goods—or as Tirole put it, the seller moves up the demand curve, rather than down.

Under the assumption that customers who abstain from buying in any given point of time leave the market forever—absorbing exit—, Tirole argues that in many interesting environments the lack of commitment does not preclude a monopolist from implementing the monopoly pricing rule (i.e., time-consistency holds). The key difference with negative selection (as in Coase (1972)), where time-consistency fails to obtain, lies in how the elasticity of demand changes in each period. Under negative selection, when a price p_0 is set, then customers with higher valuations decide to buy and leave the market, which leads to a more elastic demand in the next period, as the set of infra-marginal customers (for $p \leq p_0$) is reduced. This, in turn, pressures the monopolist to lower its price, which is anticipated by customers, leading to their opportunistic behaviour, and ultimately preventing the monopolist to achieve its commitment solution in the absence of it. On the contrary, under positive selection,

[2]The analyses developed since have been mostly restricted to incomplete information settings, following Milgrom and Roberts' static model, and later on the research has focused on adding dynamics to the environment. See a discussion in Toxvaerd (2017).

next period's demand elasticity remains unchanged, as after a price p_0, the set of infra-marginal customers (for $p \geq p_0$) is the same in both periods. Therefore, in simple terms (deterministic framework), the optimal price does not change over, and the optimal pricing with and without commitment coincide.

When the threat of entry is poised in the context of positive selection with the absorbing exit condition, the incumbent's pre-entry behaviour can indeed affect the post-entry outcome. The idea is that the incumbent can find an increasing sequence of limit prices—or a limit price-path—that modulates, throughout time, the residual demand for the entrant (were it to enter). This limit price-path may even reach the monopoly price level in some circumstances. Therefore, the trade-of between low pre-entry prices and deterred entry is not static, but rather shifts towards lower welfare (through higher prices and limited variety).

The key underlying mechanism is that the value of entry will be increasing in the incumbent's entry-period price, and decreasing in the mass of customers that have exited the market (in some fashion that will be formally defined later on): if the incumbent manages to deter entry while forcing some customers out of the market at the same time, then in the next period the incumbent will be able to deter entry with a strictly higher price, and so the increasing price-path unravels. This mechanism has some flavour of the "divide-and-conquer" equilibrium in Innes and Sexton (1994), as the entrant is never allowed enough critical mass to justify entry.

As a motivating example, consider the brand-name drug markets. To be specific, we will consider a brand-name drug market to be composed of two elements: the drug, that has been initially awarded a patent—that may already have expired—(e.g., Atorvastatin, whose patent expired in 2011)[3], and the big-name pharmaceuticals that produce it (in the example, Pfizer acquired Warner-Lambert, developers of the molecule, in the year 2000—before its patent expired—and sells it under the name of Lipitor; no other big-name pharmaceutical produces the drug). Typically, when the patent of a brand-name drug expires (also referred to as Loss-of-Exclusivity or LoE), a generic market is formed: the drug is copycated by one or more laboratories lacking any renown (i.e., not any big-name pharmaceutical) and is offered at lower prices, vis-à-vis the brand-name drug. The parallel with this paper follows from the notion that brand-name drugs, and its generic counterpart, are not intrinsically very different in terms of ex-post value for a customer, and the difference stems rather from an ex-ante perception (similar to an experience good with highly biased expectations). Under this notion, the generic market acts as the absorbing state, since once a customer experiences the generic drug, and realizes it is not very different

[3]Atorvastatin is a drug developed to reduce the levels of cholesterol in the blood, to help reduce the risk of heart-related diseases.

from the brand-name drug, then there is no point in returning to the latter, given the typically large price differences.

In the current example, the lack of any big-pharmaceutical competition may be explained, at least to some extent, by a limit-pricing argument, where customers gradually exit the brand-name drug's market, and satisfy their needs within the generics market. Moreover, there is evidence of brand-name drugs' prices increasing after the LoE, which has been coined as the *generic competition paradox*.[4] Ching (2010) has documented, in the U.S., that many brand-name drugs' prices tend to increase over time after the patents' expiration. Interestingly, the price increases are not a one-time price adjustment, as would be the case in the logic of pure price discrimination, but it is rather a steady rise. Similar evidence has been presented by Regan (2008) (in the U.S. markets), Vandoros and Kanavos (2013) (in the EU markets, although with some opposite evidence of brand-name prices falling after generics entry), as well as to some extent in Castanheira, Ornaghi and Siotis (2019).

The paper is organized as follows. Section 2.2 lays out the baseline model. Section 2.3 introduces some preliminary concepts and results. Section **??** lays out the skimming mechanism of the limit price-path. Section 2.5 develops an extension of overlapping generations, aiming to relax the absorbing exit condition of the baseline model. Finally, section 2.6 discusses the existence of limit pricing in the context of complete information, as well as its implications for competition policy; and section 2.7 concludes.

2.2 Model

Consider a market for a non-durable good (or service), that spans infinitely over discrete time ($t = 0, 1, 2...$). There is an incumbent monopolist, I, and a potential entrant, E, with entry cost κ. Marginal cost of production is c_i, for $i = I, E$, and there is a common discount factor $\delta \in [1/2, 1)$.[5] Neither party has inter-temporal commitment.

The market structure is in a Hotelling fashion. There is a continuum of customers, indexed by $\theta \in [0, 1]$, according to a smooth distribution $F(\theta)$ with full support, and log-concave density $f(\theta)$. Firm E is located at $\theta = 0$ and firm I is located at $\theta = 1$. Each customer's valuation for the good is v (per period), regardless of the supplier, and there is a cost of transportation τ. The customer's per-period net utility when

[4] See Scherer (1993). Also referred to as *generic entry paradox*.

[5] The lower bound of 1/2 is a sufficient condition; it is possible, however, to admit lower values of δ depending on the parameters of the model.

buying from E at price p_E is $u_E(\theta|p_E) = v - \tau\theta - p_E$, while its per-period net utility when buying from I at price p_I is $u_I(\theta|p_I) = v - \tau(1-\theta) - p_I$. These imply the standard indifferent customer, characterized as $\hat{\theta}(p_I, p_E) = \frac{1}{2} + \frac{p_I - p_E}{2\tau}$, as well as the marginal customer that derives a non-negative (per-period) pay-off from $i = I, E$, at price p_i, denoted as $\theta_i^+(p_i)$.[6]

Similar to Tirole (2016), we will assume that if a customer does not buy in any given period, then it exits the market forever. This is modelled as an outside option that yields a per period utility of v, but requires an upfront investment (e.g., a switching cost) large enough, so that the net present value of the option is small, normalized to 0.[7]

The timing is as follows: for each period,

- if E has not already entered the market, the period is divided into two stages: first, both firms set prices simultaneously, and then, E has to decide whether to enter (sinking the entry cost κ) or stay out;

- if E has already entered, both firms compete simultaneously setting prices in $[0, v]$.

We will focus on Markov strategies, where the state corresponds to $\Theta_t \times \lambda$, the cross product of the set of customers that are active in the market at the beginning of period t, Θ_t, and a binary variable, $\lambda \in \{0, 1\}$, that indicates whether entry has occurred ($\lambda = 1$) or not ($\lambda = 0$). Whenever it is clear from context, the entry state, λ, will be omitted.

Remark. The model is outcome equivalent to an alternative sequential timing for any period in which E has not already entered (if E has already entered the timing remains unchanged—i.e., simultaneous competition): first I sets its price, and then E decides whether to enter or not, and its price.

2.3 Preliminaries

2.3.1 Deterrence equilibrium

Throughout the paper, we will focus on a deterrence equilibrium, that is, pricing and buying decisions such that entry never occurs.

[6] $\theta_E^+(p_E) = \frac{v - p_E}{\tau}$ and $\theta_I^+(p_I) = 1 - \frac{v - p_I}{\tau}$

[7] It is assumed that the good is required by the customers in every period.

Along the deterrence equilibrium path, the following skimming property holds: if in period t an active customer θ buys from I instead of exiting the market, then an active customer $\theta' > \theta$ will do so as well. This follows from the fact that θ' obtains a higher per-period payoff than θ when buying from I, and from period $t+1$ onward, θ' can mimic the behaviour of θ, thus the continuation value of θ' cannot be lower than that of θ.

The previous skimming property implies that in any period with $\lambda = 0$ (which includes the entry period, in case it occurs) the customers' state can be characterized as $\Theta_t = [\underline{\theta}_t(p_I^{t-1}), 1]$, for $\underline{\theta}_t \in [0, 1]$, where p_I^{t-1} is the price set by the incumbent in the previous period (with $\underline{\theta}_0 \equiv 0$). Whenever this is the case, the time- and price-dependency will be dropped, and the customer's state will simply be referred to as $\underline{\theta}$ (unless explicitly stated otherwise).

Formally, the deterrence equilibrium corresponds to state-dependent pricing strategies, $p_i^*(\Theta, \lambda)$, for $i = I, E$; entry decision $e^*(\boldsymbol{p}, \underline{\theta}) \in \{0, 1\}$; and customers' buying decision $b_\theta^*(\boldsymbol{p}, \Theta, \lambda) \in \{I, E, \emptyset\}$,[8] where $\boldsymbol{p^*} = (p_I^*, p_E^*)$. Observe that prices may depend on the general customer's state, Θ, whereas for the entry decision, which is only relevant whenever entry has not occurred, the state is restricted to $\underline{\theta}$.

The proposed equilibrium outcome is a price sequence for the incumbent, denoted $p_I^{(t)}$, for $t \geq 0$, such that entry never occurs.

2.3.2 Monopoly case

Consider first the case when I is a monopolist that does not face the threat of entry, and the state is $\underline{\theta}$. The trivial solution is to set the monopoly price in each period, due to the positive selection (see Tirole (2016)). The per-period monopolist's problem is

$$\max_p \, (p - c_I)[1 - F(\theta_I^+(p))]. \tag{2.1}$$

The monopolist's problem is well-defined since $\theta_I^+(p) = 1 - \frac{v-p}{\tau}$ is strictly increasing in p and $F(\cdot)$ is log-concave (because $f(\cdot)$ is log-concave). The maximum is attained at price p_I^m such that

$$\frac{p_I^m - c}{\tau} = \frac{1 - F(\theta_I^+(p_I^m))}{f(\theta_I^+(p_I^m))}, \tag{2.2}$$

and the per-period monopoly profit is denoted by π_I^m. Note that the previous analysis assumes that $\theta_I^m \equiv \theta_I^+(p_I^m) \geq \underline{\theta}$, otherwise, the monopoly price is simply the price such that $\theta_I^m = \underline{\theta}$.

[8] Buying decision $\emptyset$ represents exit from the market.

In what follows, we will assume that under the incumbent's monopoly price, some customers exit the market. This is necessary for the existence of a skimming dynamic later on.

Assumption 2.1 *At I's monopoly price, p_I^m, some customers obtain a negative per-period net utility when buying from I (i.e., $\theta_I^m > 0$). This is satisfied whenever*

$$\frac{1}{f(0)} > \frac{v - c_I - \tau}{\tau}.$$

2.3.3 Stage-game equilibrium

We now consider the stage-game for any state $\underline{\theta}$. The details can be found in Appendix C. Only the main characterization is presented in this section.

Let $\boldsymbol{p}^s(\underline{\theta})$ denote an equilibrium strategy profile for the stage-game, and define a *shared* equilibrium as follows.

Definition 2.1 (Shared equilibrium) A *shared region* corresponds to prices for which best-responses lead both firms to share the market (*full-market coverage*), and the indifferent customer, $\hat{\theta}$, obtains a strictly positive utility. A *shared equilibrium* is an equilibrium in the shared region.

This definition excludes the uninteresting case of local monopolists, as well as *kink* equilibria.[9] Therefore, to have an interesting problem, we will assume that when $\underline{\theta} = 0$ the stage-game has a shared equilibrium.[10]

Assumption 2.2 *If $\underline{\theta} = 0$, and E is in the market, then there exists a shared stage-game equilibrium, $\boldsymbol{p}^s(0)$, such that $\hat{\theta}(\boldsymbol{p}^s) \in (0, 1)$ and $u_I(\hat{\theta}|p_I^s) = u_E(\hat{\theta}|p_E^s) > 0$.*

An immediate implication of Assumption 2.2, together with the log-concavity of $f(\cdot)$, is that the stage-game for $\underline{\theta} = 0$ has a unique equilibrium—the shared equilibrium.

Proposition 2.1 *For $\underline{\theta} = 0$ the stage-game equilibrium is unique, as well as shared.*

Proof. See Appendix D. ■

[9] *Kink* equilibria correspond to price profiles $\boldsymbol{p}^k$ such that $\theta_I^+(p_I^k) = \theta_E^+(p_E^k)$, that is, firms' demands cover the entire market but do not overlap.

[10] The log-concavity of $f(\theta)$ guarantees that the firms' maximization problems are well-defined.

Uniqueness stems from the standard slopes condition.[11] As is shown, the stage-game equilibrium for $\underline{\theta} > 0$ is still unique, and involves more aggressive strategies (i.e., lower equilibrium prices). Hence, Assumption 2.2 ensures that the stage-game equilibrium is still shared and unique for $\underline{\theta} > 0$.

Proposition 2.2 *For a set of remaining customers characterized by $\underline{\theta} \geq 0$, the stage-game equilibrium, $\boldsymbol{p}^s(\underline{\theta})$, is unique, shared, and exhibits full-market coverage (for the remaining customers). Moreover, equilibrium prices and profits are strictly decreasing in $\underline{\theta}$, for both firms (whenever $p_i^s(\underline{\theta}) > c_i$, for $i = I, E$).*

Proof. See Appendix D. ■

Intuitively, as $\underline{\theta}$ grows larger, the entrant faces a loss of infra-marginal customers, which forces it to price more aggressively (E's stage-game best-response is shifted downward), and due to strategic complementarity (for the market-shared region), the incumbent lowers its price as well, preserving a shared equilibrium. Uniqueness follows for the same reason as in Proposition 2.1. Stage-game profits, denoted by $\pi_i^s(\underline{\theta})$, are strictly decreasing in $\underline{\theta}$ due to a standard strategic complementarity argument.

Remark. When we focus on states of the form $\underline{\theta}$, the stage-game equilibrium preserves the state, meaning that no customer would exit the market under this equilibrium.

Finally, for $\underline{\theta} = 0$ we assume that I's equilibrium price is such that some customers would prefer to exit the market rather than buy from I. Similar to Assumption 2.1, this is necessary for the skimming dynamic later on.

Assumption 2.3 *For $\underline{\theta} = 0$, some customers derive a negative per-period net utility at I's stage-game equilibrium price: $\theta_I^+(p_I^s(0)) > 0$.*

2.4 Entry deterrence

We now proceed to solve the baseline model. The main result is that for entry costs, κ, sufficiently large, there exists an equilibrium in which entry is always deterred, and the incumbent's price is increasing over time (possibly up to its monopoly level). This equilibrium is therefore characterized as an increasing price-path for I—or a limit price-path. It is important to notice that this is not a claim about uniqueness, as there are possibly other equilibria.

[11] Each firm's best-response has a slope strictly lower than 1 (log-concavity ensures this in the market-shared region, and outside the best-response is non-increasing).

Theorem 2.1 *There exists a $\underline{\kappa}$ such that, for all $\kappa \geq \underline{\kappa}$, a deterrence equilibrium exists, that features a price-path $\{p_I^t\}_{t=0}^{\infty}$ whereby entry is deterred and incumbent's prices are strictly increasing whenever they are below p_I^m.*

Theorem 2.1 is obtained by solving the model backwards. The underlying mechanism is that if the incumbent manages to deter entry in the first period, while forcing some customers from the entrant's turf out of the market, then the incumbent can sequentially set larger deterrence prices and force additional customers to exit the market. However, the monopoly level is not always achieved, as the increasing price-path can be bounded below such level (albeit strictly increasing, in this case each increase becomes infinitesimally small).

The following subsections develop the blocks to prove Theorem 2.1.

2.4.1 Post-entry competition equilibrium

Consider that entry has occurred in the previous period (i.e., $\lambda = 1$), and therefore the state is characterized by $\underline{\theta}$. An equilibrium of this post-entry competition game is for firms to price according to the stage-game pricing rule, and for customers to buy from the firm that provides the best per-period utility, whenever its net present value is non-negative. In equilibrium no customer exits the market. Let us denote the post-entry equilibrium strategies as $\boldsymbol{p}^*(\Theta) \equiv \boldsymbol{p}(\Theta, 1)$, and $b_\theta^*(\boldsymbol{p}, \Theta) \equiv b_\theta(\boldsymbol{p}, \Theta, 1)$. Observe, however, that given the customers' strategies, there is only one possible state, $\underline{\theta}$, and therefore we refer to strategies as $\boldsymbol{p}^*(\underline{\theta}) \equiv \boldsymbol{p}^*(\underline{\theta}, 1)$, and $b_\theta^*(\boldsymbol{p}, \underline{\theta}) \equiv b_\theta^*(\boldsymbol{p}, \underline{\theta}, 1)$.

Proposition 2.3 *If entry has occurred in period $t-1$ and the state is $\underline{\theta}$, then a post-entry competition equilibrium is characterized as follows:*

- *Firm i sets $p_i^*(\Theta_{t'}) = p_i^s(\Theta_{t'})$, in every period $t' \geq t$ and any state $\Theta_{t'} \subseteq [\underline{\theta}_t, 1]$, for $i = I, E$;*
- *In each period $t' \geq t$, active customer θ buys from firm i such that $u_i(\theta|p_i) \geq u_j(\theta|p_j)$, for $i = I, E$ and $j \neq i$ (i.e., $b_\theta^*(\boldsymbol{p}, \Theta_{t'}) = i$ and there is no exit), whenever its net present value from staying in the market (anticipating the continuation payoffs under the equilibrium strategies) is non-negative; otherwise it exits the market;*
- *The state remains $\underline{\theta}$ for all $t' \geq t$.*

We first prove the following Lemma, that will be useful throughout the paper.

Lemma 2.1 *Consider the strategies of Proposition 2.3 and define $\tilde{p}_i(p_j)$, for $j \neq i$ and $i = E, I$, as the highest price such that all customers derive a non-negative utility, given p_j (i.e., $\theta_i^+(\tilde{p}_i(p_j)) = \theta_j^+(p_j)$). For any state $\underline{\theta}$ and price profile $\boldsymbol{p}$, if either $p_E^s(\underline{\theta}) \leq \tilde{p}_E(p_I)$ or $p_I^s(\underline{\theta}) \leq \tilde{p}_I(p_E)$, then no customer exits the market.*

Proof. See Appendix D. ∎

When faced with a price profile $\boldsymbol{p}$, Lemma 2.1 provides a condition under which no customer exits the market, given their strategies of the post-entry competition equilibrium, and the firms' continuation strategies. This is trivially satisfied if prices are such that all customers can obtain a non-negative utility. Otherwise, the Lemma requires at least one of the firms' prices not to be too large. The interpretation is that even if a customer would derive a negative utility in a given period, if at least one of the prices is not too large, then the current period's loss can be outweighed by the continuation payoff, as customers are relatively patient ($\delta \geq 1/2$). We prove now Proposition 2.3.

Proof of Proposition 2.3. Consider the candidate equilibrium given by $\boldsymbol{p}^*(\Theta) = \boldsymbol{p}^s(\Theta)$, which always exists, since the strategy space can be restricted to $[0, v]$, a non-empty and compact set, and $\pi_i(p_i, p_j)$ is continuous in p_i and p_j, for $i, j = I, E$, $i \neq j$ (Glicksberg (1952)). Observe that starting from $\Theta_t = [\underline{\theta}_t, 1]$, under the candidate equilibrium the state remains the same, and active customers obtain a strictly positive payoff in every period. This implies that there is only one relevant state to analyze: $\underline{\theta}$.

We analyze a single stage deviation by a firm, say I (deviations by E are analogous). Since E is setting $p_E^s(\underline{\theta})$, and the stage-game equilibrium is shared, it follows that $p_I^s(\underline{\theta}) < \tilde{p}_I(p_E^s(\underline{\theta}))$. Then, by Lemma 2.1, all customers remain in the market and the state remains $\underline{\theta}$. It follows that no deviation can be profitable, as continuation profits remain intact and the candidate equilibrium is the stage-game equilibrium.

Customers' strategies are clearly best-responses, and expectations are fulfilled. ∎

If customers expect firms to revert to the stage-game equilibrium after any firm's deviation, and under the proposed equilibrium each customer expects all other customers to remain in the market, then each customer is better-off remaining in the market, even if it means enduring a "bad" period of negative utility. It follows that under this equilibrium, the post-entry competition game has a value, $V_i(\underline{\theta})$, equal to the repetition of the stage game:

$$V_i(\underline{\theta}) = \frac{\pi_i^s(\underline{\theta})}{1-\delta}, \quad \text{for firm } i = I, E, \tag{2.3}$$

and it is strictly decreasing in $\underline{\theta}$ because the stage-game profit $\pi_i^s(\underline{\theta})$ is so, as stated in Proposition 2.3.

Remark. For Theorem 2.1, all that really matters is that the entrant's post-entry value, $V_E(\underline{\theta})$, is decreasing in $\underline{\theta}$.

2.4.2 Entry threat

Consider now any period in which E is still out of the market (i.e., $\lambda = 0$). Let us denote the equilibrium strategies for this pre-entry regime as $\boldsymbol{p}^e(\underline{\theta}) \equiv \boldsymbol{p}^*(\underline{\theta}, 0)$, $e^*(\boldsymbol{p}, \underline{\theta})$ and $b_\theta^e(\boldsymbol{p}) \equiv b_\theta^*(\boldsymbol{p}, 0)$.

Intuitively, as we are focusing on a deterrence equilibrium, we can restrict attention to $p_I \leq p_I^m$. Then, as a corollary to Lemma 2.1, for a given state $\underline{\theta}$ and any entry price p_E, the state remains $\underline{\theta}$ upon entry.

Corollary 2.2 *If the state is $\underline{\theta}$, and I sets a price $p_I \leq p_I^m$, then for any entry price p_E the state, upon entry, remains $\underline{\theta}$.*

Proof. See Appendix D. ■

We can define now the gross entry value for E, as a function of any price p_E, when the incumbent is setting $p_I \leq p_I^m$ and the state is $\underline{\theta}$:

$$\Pi_E(p_E; p_I, \underline{\theta}) = \pi_E(p_E, p_I; \underline{\theta}) + \delta V_E(\underline{\theta}), \tag{2.4}$$

where $\pi_E(p_E; p_I, \underline{\theta})$ is the per-period profit (as in the stage-game), and the continuation value is precisely the post-entry competition payoff derived earlier, with state $\underline{\theta}$, because of Corollary 2.2. The entry decision is then $e^*(\boldsymbol{p}, \underline{\theta}) = 1$ if and only if $\Pi_E(\boldsymbol{p}, \underline{\theta}) > \kappa$. In equilibrium, entry is therefore governed by its gross entry value, W_E, characterized as

$$W_E(p_I, \underline{\theta}) = \max_p \ \Pi_E(p; p_I, \underline{\theta}). \tag{2.5}$$

Observe that the solution to (2.5) is given by $R_E^s(p_I, \underline{\theta})$, the entrant's stage-game best-response.

In order to focus only on cases in which deterrence is feasible, but also not straightforward, we will make the following assumption:

Assumption 2.4 *The utility $u_I(\theta)$, the density $f(\cdot)$ and the entry cost κ are such that:*

(i) Future profits (i.e., after entry) are never sufficient for E to cover the entry cost:

$$\delta V_E(0) < \kappa. \tag{2.6}$$

(ii) E shall enter if $\underline{\theta} = 0$, and $p_I = p_I^m$ or $p_I = p_I^s(0)$, where p_I^m and $p_I^s(0)$ are I's monopoly and competitive equilibrium price, respectively.[12]

$$\min\{W_E(p_I^m, 0), W_E(p_I^s(0), 0)\} > \kappa. \tag{2.7}$$

Given that future profits are not enough to cover the entry cost, it is clear that, in any equilibrium in which entry occurs, the entry-price is larger than the marginal cost, or

$$R_E^s(p_I, \underline{\theta}) > c_E. \tag{2.8}$$

There are two possible cases to analyze, stemming from Assumption 2.4(ii). If $W_E(p_I^m, 0) \leq W_E(p_I^s(0), 0)$, then, since $R_E^s(p_I^m, 0) > c_E$, it follows that $W_E(p_I^m, 0) > \Pi_E(c_E; p_I^m, 0) = \delta V_E(0)$.[13] On the contrary, if $W_E(p_I^m, 0) > W_E(p_I^s(0), 0)$, then it is straightforward that $W_E(p_I^s(0), 0) = \pi_E(\boldsymbol{p}^s(0); 0) + \delta V_E(0) > \delta V_E(0)$. Therefore, $\min\{W_E(p_I^m, 0), W_E(p_I^s(0), 0)\} > \delta V_E(0)$, and the set of κ values that satisfy both conditions is non-empty.

As for the value function W_E, condition (2.8) guarantees that it is strictly increasing in p_I and strictly decreasing in $\underline{\theta}$. In effect, by the Envelope Theorem:

$$\frac{\partial W_E}{\partial p_I} = \frac{R_E^s - c_E}{2\tau} f(\hat{\theta}) > 0, \tag{2.9}$$

$$\frac{\partial W_E}{\partial \underline{\theta}} = -(R_E^s - c_E) f(\underline{\theta}) + \delta \frac{dV_E}{d\underline{\theta}} < 0. \tag{2.10}$$

These conditions suggest that the incumbent can increase p_I while keeping W_E at a fixed value, as long as $\underline{\theta}$ is also increased.

Lemma 2.2 (Skimming dynamics) *For state $\underline{\theta}$, consider a price p_I^d such that entry is just deterred in that state (i.e., $W_E(p_I^d, \underline{\theta}) = \kappa$) and some customers exit the market (i.e., $\theta_I^+(p_I^d) > \underline{\theta}$). Then, in the ensuing state entry is deterred with slack when price p_I^d is set (i.e., $W_E(p_I^d, \theta_I^+(p_I^b)) < \kappa$).*

Proof. Notice first that since W_E is non-increasing in $\underline{\theta}$, $W_E(p_I^d, \theta_I^+(p_I^d))$ cannot be larger than κ. Let $\theta_I^+ \equiv \theta_I^+(p_I^d)$ and $R_E^s(\cdot) \equiv R_E^s(p_I^d, \cdot)$, as p_I^d is kept fixed. It follows

[12] Given the nature of the Hotelling competition, it is possible to have $p_I^s(0) > p_I^m$. The condition laid out guarantees that in the absence of the absorbing exit condition, entry would occur.

[13] The strict inequality follows from the quasi-concavity of $\pi_E(\cdot\,; p_I, \underline{\theta})$.

that

$$\begin{aligned} W_E(p_I^d, \theta_I^+) &= \Pi_E(R_E^s(\theta_I^+); p_I^d, \theta_I^+), \\ &< \Pi_E(R_E^s(\theta_I^+); p_I^d, \underline{\theta}), \\ &\leq \Pi_E(R_E^s(\underline{\theta}); p_I^d, \underline{\theta}), \\ &= W_E(p_I^d, \underline{\theta}), \\ &= \kappa, \end{aligned}$$

where the first relationship is just by definition; the second one stems from the fact that $\Pi_E(p_E; p_I^d, \underline{\theta})$ is strictly decreasing in $\underline{\theta}$; the third one is due to optimality of $R_E^s(\underline{\theta})$; the fourth one again is just by definition; and the last one is by assumption. ■

In essence, the skimming dynamics presented in Lemma 2.2 refer to the observation that if I manages to deter entry in a given period and at the same time force some customers out of the market, then E's entry prospect is strictly worse in the next period. This property is at the core of the deterrence price-path.

2.4.3 Deterrence price-path

With the previous building blocks, we are now able to prove Theorem 2.1, and describe the skimming dynamics.

Proof of Theorem 2.1. The proposed deterrence equilibrium is supported by the following strategies:

Entry threat

(i) $p_I^*(\underline{\theta}, 0) = p_I^e(\underline{\theta})$ such that $W_E(p_I^e(\underline{\theta}), \underline{\theta}) = \kappa$,

(ii) $p_E^*(\underline{\theta}, 0) = p_E^e(\underline{\theta}) = R_E^s(p_I^e(\underline{\theta}), \underline{\theta})$,

(iii) $e^*(\boldsymbol{p}, \underline{\theta}) = 1$ if and only if $\Pi_E(\boldsymbol{p}, \underline{\theta}) > \kappa$,

(iv) $b_\theta^*(\boldsymbol{p}, \underline{\theta}, 0) = b_\theta^e(\boldsymbol{p}, \underline{\theta}) = \begin{cases} I, & \text{if } u_I(\theta|p_I) \geq 0, \\ \emptyset, & \text{otherwise;} \end{cases}$

Post-entry competition

(v) $\boldsymbol{p}^*(\Theta, 1) = \boldsymbol{p}^s(\Theta)$,

$$\text{(vi)}\ \ b^*_\theta(\boldsymbol{p},\Theta,1) = \begin{cases} i, & \text{such that } u_i(\theta|p_i) \geq u_j(\theta|p_j) \text{ and } \theta\text{'s net present value} \\ & \text{(anticipating the continuation payoffs under the equilib-} \\ & \text{rium strategies) is non-negative,} \\ \emptyset, & \text{otherwise.} \end{cases}$$

The ensuing deterrence price-path is constructed as follows:

1. Consider $\kappa \to \min\{W_E(p_I^m, 0), W_E(p_I^s(0), 0)\}$ from the left, then there exists $p_I^{(0)} \to \min\{p_I^m, p_I^s(0)\}$ such that $W_E(p_I^{(0)}, 0) = \kappa$. Since by Assumption 2.1 $\theta_I^+(p_I^m) > 0$, and by Assumption 2.2 $\theta_I^+(p_I^s(0)) > 0$, by continuity it follows that $\underline{\theta}^{(1)} \equiv \theta_I^+(p_I^{(0)}) > 0$.

2. Under $p_I^{(0)}$ entry is deterred and some customers exit the market, therefore by Lemma 2.2, $W_E(p_I^{(0)}, \underline{\theta}^{(1)}) < \kappa$. Since W_E is strictly increasing in p_I, there exists a price $p_I^{(1)} > p_I^{(0)}$ such that $W_E(p_I^{(1)}, \underline{\theta}^{(1)}) = \kappa$, and since θ_I^+ is strictly increasing, $\underline{\theta}^{(2)} = \theta_I^+(p_I^{(1)}) > \theta_I^+(p_I^{(0)}) = \underline{\theta}^{(1)}$.

3. Lemma 2.2 applies again, and by induction there is a sequence of increasing prices $p_I^{(0)} < p_I^{(1)} < p_I^{(2)} < \ldots$ such that entry is deterred. This sequence either reaches a $p_I^{(t')} > p_I^m$, for some $t' > 0$, point at which the price-path is kept fixed to p_I^m (i.e., $p_I^{(t)} = p_I^m$ for all $t \geq t'$), or it converges to some $\overline{p}_I < p_I^m$.

We proceed to check for deviations for the entry threat regime strategies (the state $\underline{\theta}$ is omitted unless we refer to a specific value of it). The entry decision is trivially optimal, and its entry price, p_E^e, is supported by a *trembling hand* argument. The customers' buying decision is also clearly optimal, as no entry will ever occur in equilibrium and I's price-path is increasing, thus a negative per-period utility when buying from I warrants exit. As for I's strategy, first observe that regardless I's price, if E enters the state remains $\underline{\theta}$. This follows by showing that $p_I^s < \tilde{p}_I(R_E^s(p_I^e))$ and applying Lemma 2.1. In effect, there are two cases to consider. If $p_I^s \leq p_I^e$, then it is straightforward that $p_I^s < \tilde{p}_I(R_E^s(p_I^e))$, as $p_I^e < \tilde{p}_I(R_E^s(p_I^e))$ due to $p_I^e \leq p_I^m$ and the shared equilibrium assumption. If $p_I^s > p_I^e$, we show by contradiction that $p_I^s \leq \tilde{p}_I(R_E^s(p_I^e))$. Suppose $p_I^s > \tilde{p}_I(R_E^s(p_I^e))$, then $\tilde{p}_E(p_I^s) < \tilde{p}_E(\tilde{p}_I(R_E^s(p_I^e))) = R_E^s(p_I^e)$, which together with the shared equilibrium imply that $p_E^s < \tilde{p}_E(p_I^s) < R_E^s(p_I^e)$. This is a contradiction, since $p_I^s > p_I^e$ implies $p_E^s = R_E^s(p_I^s) > R_E^s(p_I^e)$. Now, consider a deviation to $p_I' < p_I^e$. This cannot be profitable as it would still deter entry, but it would reduce its current period profit (since it is a price below the monopoly level and profits are quasi-concave) and shift the price-path weakly downwards. On the other hand, any deviation to $p_I' > p_I^e$ leads to entry, and as such,

I better deviate in the first period, otherwise its post-entry competition payoff is diminished by the exit of customers. The best deviation is therefore under state $\underline{\theta} = 0$ towards $p_I' = R_I^s(p_E^e(0)) > p_I^e(0)$, it's stage-game best-response. However, for κ large enough, such deviation is not profitable. As $p_I^{(0)} = p_I^e(0) < \min\{p_I^m, p_I^s(0)\}$, it follows that $p_E^e(0) = R_E^s(p_I^{(0)}, 0) < R_E^s(p_I^s(0), 0) = p_E^s(0)$ (because it belongs to the shared region), and therefore $\pi_I(R_I^s(p_E^e(0)), p_E^e(0)) < \pi_I(R_I^s(p_E^s(0)), p_E^s(0)) = \pi_I^s(0)$, since in the shared region of the stage-game optimal profits are increasing in the competitor's price.[14] Hence, the deviation payoff is strictly below $\pi_I^s(0)/(1-\delta)$. Whereas considering $\kappa \to \min\{W_E(p_I^m, 0), W_E(p_I^s(0), 0)\}$, the deterrence price-path payoff is at least $\pi_I^s(0)/(1-\delta) - \varepsilon$, for ε arbitrarily small (as measured by the difference between κ and its upper bound).

As κ decreases, both the existence of $p_I^{(0)}$ and the profitability of the deterrence price-path are tightened. First, for lower κ, $p_I^{(0)}$ needs to be lower as well, until eventually $\theta_I^+(p_I^{(0)}) = 0$, whereby a κ_1 is defined. And second, a lower κ shifts the deterrence price-path downward, rendering the strategy less profitable, until eventually I is better off deviating to $R_I^s(p_E^e(0))$ and accommodating entry. This point defines a κ_2. The lower bound is thus defined as $\underline{\kappa} = \max\{\kappa_1, \kappa_2\}$. ■

Theorem 2.1 establishes the existence of a strictly increasing price-path (whenever it is below the incumbent's monopoly level) such that entry is deterred, provided that the entry cost κ is sufficiently large. The price-path can either reach the monopoly level in finite time, or converge to some lower level $\overline{p}_I < p_I^m$. The key mechanism is the skimming dynamic of Lemma 2.2: the incumbent would set a low price in order to deter entry, while forcing some customers from the entrant's turf out of the market; this, in turn, would allow the incumbent to set a higher price in the next period and still deter entry, forcing some more customers out of the market, and so on.

The condition of sufficiently large κ stems from two requirements: (i) initialization of the skimming dynamic, and (ii) profitability of the deterrence price-path. As for the initialization of the skimming dynamic, the deterrence price-path requires the existence of an initial price, $p_I^{(0)}$, such that entry is deterred *and some customers exit the market*. If κ is too low, then I would have to set a rather low $p_I^{(0)}$ to deter entry, and eventually, no customer would exit the market. In such conditions, the skimming dynamic of Lemma 2.2 cannot be put in motion, as the state remains $\underline{\theta} = 0$ after entry deterrence. On the other hand, the deterrence strategy has to be profitable for I, which is not necessarily satisfied for all entry costs. With the deterrence strategy

[14] Recall that I's stage-game optimization is $\pi_I^s(p_E, \underline{\theta}) = \max_{p_I} (p_I - c_I)[1 - F(\hat{\theta}(p_I, p_E))]$, with $\frac{\partial \pi_I^s}{\partial p_E} = \frac{R_I^s - c_I}{2\tau} f(\hat{\theta}) > 0$.

the incumbent sacrifices early profits in exchange for larger future profits (vis-à-vis accomodating entry). Hence, if entry costs are too low, and thus the initial price is low as well, it might be the case that the profits sacrifice prove too large and render the strategy unprofitable.

Remark. If the entrant faces fixed costs whenever production is positive, instead of an entry cost, the model can be interpreted as a margin squeeze price-path. Moreover, the problem is simplified as the deterrence condition would be over single-period profits, rather than inter-temporal profits (in particular, Assumption 2.4(i) would no longer be necessary). The skimming dynamic would be very similar: the incumbent would set a low price to prevent production by its now competitor, while forcing some of its customers to exit the market; this would allow the incumbent to set a higher price in the next period, still prevent production and force additional customers out of the market, and so on.

2.5 Overlapping generations

In this section we consider the presence of different generations of customers, as a way to soften the condition of absorbing exit. Consider the following modification to the main model: in each period, before suppliers' pricing decisions take place, a proportion $\beta \in (0, 1)$ of customers is replaced by new customers, uniformly across the population. This replacement is like for like, in the sense that a fraction β of the mass of customers located at θ, regardless of whether they are still in the market or not, is replaced by new customers (that are in the market).[15]

What the replacement does, is to effectively reduce the mass of customers forced out of the market in any period: only a fraction $1 - \beta$ of the customers located at $\theta < \theta_I^+(p_I^{(t)})$ effectively leave the market. This affects both the post-entry competition profits for E, and its entry-period profits (except for the very first period when all customers are active), enlarging them as β increases. The implications for I are twofold: (i) the price-path will increase at a slower rate; and (ii) reaching the monopoly price becomes less likely.

Whenever entry is deterred, the state of active customers at the beginning of the next period is still characterized by $\underline{\theta}$. However, if entry occurs and the market is fully covered in the post-entry competition, then the distribution of active customers when

[15]This can be interpreted as an inflow of new customers, with the same distribution as the original customers. The convenience of replacing the customers instead of adding new ones, is so that the level of the entry cost, κ, need not to be adjusted proportionally.

the rate of replacement is β and n periods have gone by since entry is characterized as follows:

$$f_{\beta,n}(\theta) = \begin{cases} [1-(1-\beta)^{(n+1)}]f(\theta) & \text{if } \theta < \underline{\theta}, \\ f(\theta) & \text{if } \theta \geq \underline{\theta}. \end{cases}$$

$$F_{\beta,n}(\theta) = \begin{cases} [1-(1-\beta)^{(n+1)}]F(\theta) & \text{if } \theta < \underline{\theta}, \\ F(\theta)-(1-\beta)^{(n+1)}F(\underline{\theta}) & \text{if } \theta \geq \underline{\theta}. \end{cases}$$

Observe that $F_{\beta,n}$ is not log-concave, as $f_{\beta,n}/F_{\beta,n}$ has an upward jump at $\underline{\theta}$. However, $1-F_{\beta,n}$ is log-concave, as the respective discontinuity is a downward jump. Hence, I's stage-game problem is well-defined, and only E's problem requires a more detailed analysis.

The entrant's stage-game problem is

$$\max_{p_E} \, (p_E - c_E) \left[F_{\beta,n}(\min\{\hat{\theta}(p_I, p_E), \theta_E^+(p_E)\}) \right].$$

For $\underline{\theta} = 0$, E's stage-game problem coincides with the one in the baseline model and the equilibrium is shared (thus, $\hat{\theta}^s < \theta_E^+$). As $\underline{\theta}$ is increased, E suffers a loss of customers. Consider as the candidate equilibrium the baseline model equilibrium with an equivalent loss of infra-marginal customers (i.e., replace $\underline{\theta}$ in the baseline model for $\underline{\theta}^{eq}$ such that $F(\underline{\theta}^{eq}) = (1-\beta)^{(n+1)}F(\underline{\theta})$). We already know that in such equilibrium, both firms' prices are decreasing in $\underline{\theta}^{eq}$, and therefore in $\underline{\theta}$, and the condition $\hat{\theta}^s < \theta_E^+$ remains valid, as $\hat{\theta}^s$ increases at a lower rate than θ_E^+, in $\underline{\theta}$.[16] However, for a fixed p_I, E has now an alternative, which is to deviate towards a higher price, $p_E' > R_E^s(p_I)$, such that $\hat{\theta}(p_I, p_E') \leq \underline{\theta}$, and therefore the loss of customers becomes marginal rather than infra-marginal. The entrant's best-response, for replacement β and n periods after entry, is characterized as

$$R_E^{\beta,n}(p_I, \underline{\theta}) = \begin{cases} R_E^s(p_I, 0) & \text{if } p_I \leq p_I^\dagger(\underline{\theta}), \\ R_E^s(p_I, \underline{\theta}^{eq}) & \text{if } p_I \geq p_I^\dagger(\underline{\theta}), \end{cases} \tag{2.11}$$

where $p_I^\dagger(\underline{\theta})$ is the price at which E's alternative is profitable. The best-response coincides with $R_E^s(p_I, 0)$ in the deviation from the candidate equilibrium as $F_{\beta,n}(\hat{\theta})$ is proportional to $F(\hat{\theta})$ if $\hat{\theta} < \underline{\theta}$.

This introduces the possibility that the candidate equilibrium no longer remains an equilibrium. A simple mixed-strategies equilibrium exists; however, such equilibrium

[16] It is verified that $d\hat{\theta}^s/d\underline{\theta} = 0.5(1 - \partial R_I^s/\partial p_E)d\theta_E^+/d\underline{\theta} < d\theta_E^+/d\underline{\theta}$.

is increasing in $\underline{\theta}$.[17] This, in turn, posses a problem in terms of Theorem 2.1, as the condition $V_E(\underline{\theta})$ decreasing may not be valid. There are multiple factors in play, such as whether the profitable deviation exists and whether $V_E(\underline{\theta})$ is indeed non-decreasing for some interval.[18]

In order to obtain more insight from this framework, it is necessary to have more structure. In what follows, we will consider the case of a uniform distribution: $F(\theta) = \theta$. For a state $\underline{\theta}$, and an equivalent loss of infra-marginal customers $\underline{\theta}^{eq}$, the candidate equilibrium is the profile

$$p_I^*(\underline{\theta}^{eq}) = \tau + \frac{2c_I + c_E}{3} - \frac{2\tau\underline{\theta}^{eq}}{3}, \tag{2.12}$$

$$p_E^*(\underline{\theta}^{eq}) = \tau + \frac{c_I + 2c_E}{3} - \frac{4\tau\underline{\theta}^{eq}}{3}. \tag{2.13}$$

A profitable deviation for E occurs when (dropping $\underline{\theta}^{eq}$ from p_I^*)

$$(R_E^s(p_I^*, \underline{\theta}^{eq}) - c_E)\left[\hat{\theta}(p_I^*, R_E^s(p_I^*, \underline{\theta}^{eq}) - \underline{\theta}^{eq}\right] = (R_E^s(p_I^*, 0) - c_E)\left[1 - (1-\beta)^{(n+1)}\right]\hat{\theta}(p_I^*, R_E^s(p_I^*, 0)), \tag{2.14}$$

which implicitly defines a $\underline{\theta}_{min}^{eq}$, and therefore a $\underline{\theta}_{min} = \underline{\theta}_{min}^{eq}/(1-\beta)^{(n+1)}$, necessary for the existence of a profitable deviation. As $\underline{\theta}^{eq}$ is decreasing in n for a fixed $\underline{\theta}$, it is sufficient to consider $n = 1$ (the first period after entry has occurred). Upon inspection, $\underline{\theta}_{min}$ is decreasing in β, and its limit as $\beta \to 1$ is

$$\inf\{\underline{\theta}_{min}\} = \frac{1}{2} + \frac{c_I - c_E}{6\tau}. \tag{2.15}$$

This threshold is larger than θ_I^m, and therefore the candidate equilibrium is indeed

[17] If E's alternative is a profitable deviation from the candidate equilibrium, it would be so for $p_I < p_I^s(0)$, as we know that for $\underline{\theta} = 0$ the candidate equilibrium is indeed the equilibrium; hence, $p_I^\dagger < p_I^s(0)$. The mixed-strategies equilibrium is as follows: I sets $p_I^\dagger$ with probability 1 and E randomizes between $p_E^{(1)} \equiv R_E(p_I^\dagger, \underline{\theta}^{eq})$ and $p_E^{(2)} \equiv R_E(p_I^\dagger, 0)$. The probabilities for E's random strategy need to be such that I is indeed playing a best-response: such probabilities exist since $R_I(p_E^{(1)}, \underline{\theta}) < p_I^\dagger$ and $R_I(p_E^{(2)}, \underline{\theta}) > p_I^\dagger$, for otherwise a pure-strategies equilibrium would exist as $R_I(p_E, \underline{\theta})$ is continuous and increasing in $[p_E^{(1)}, p_E^{(2)}]$. Moreover, the mixed equilibrium features full-market coverage, as $p_I^\dagger < p_I^s(0)$ and $p_E \le R_E(p_I^\dagger, 0) < p_E^s(0)$. E's profit, equal to $(R_E^s(p_I^\dagger, 0) - c_E)[1 - (1-\beta)^n]F(\hat{\theta})$, is increasing in $\underline{\theta}$ as $p_I^\dagger$ is increasing in $\underline{\theta}$.

[18] As $V_E(\underline{\theta})$ is the sum of stage-game equilibrium profits, and the mass of customers, upon entry, is increasing, even if for some periods the stage-game equilibrium is increasing in $\underline{\theta}$, at some point the equilibrium reverts to the pure-strategies equilibrium, and therefore decreasing in $\underline{\theta}$. Therefore, the overall value could be monotonic or not.

an equilibrium in any post-entry competition game along the deterrence price-path.

Claim 2.1 *Under Assumption 2.2 (shared equilibrium),* $\inf\{\underline{\theta}_{min}\} > \theta_I^m$.

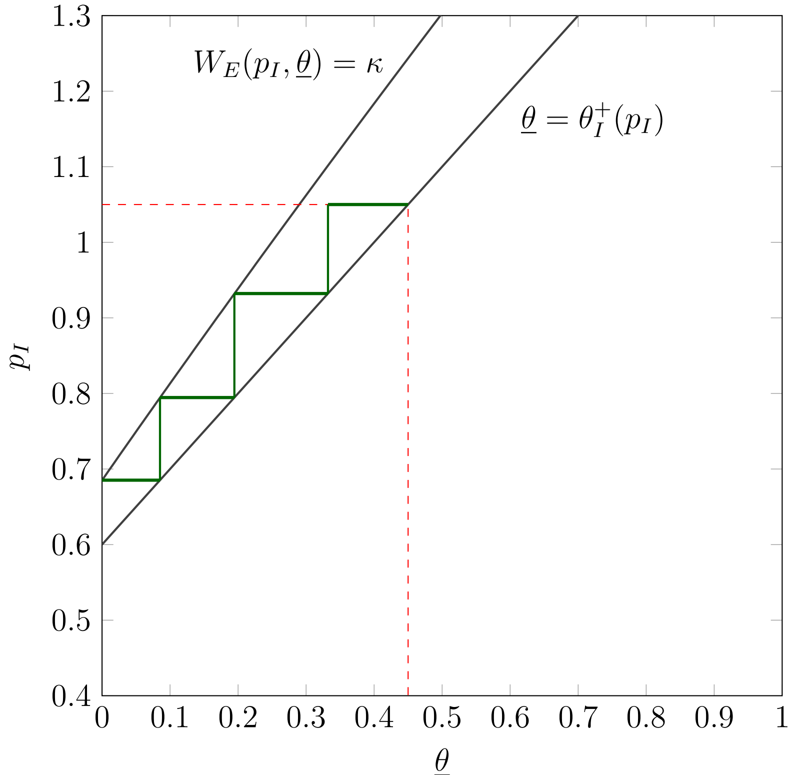
Proof. Direct calculation reveals that $\inf\{\underline{\theta}_{min}\} - \theta_I^m = v - p_I^*(0) > 0$, as Assumption 2.2 ensures a shared stage-game equilibrium and therefore $p_I^*(0) = p_I^s(0) < v$. ■

Figures 2.1 and 2.2 show two examples, with low and high β. In each figure we depict to relevant curves: the iso-curve $W_E(p_I, \underline{\theta}) = \kappa$, determining the largest price that deters entry given a state $\underline{\theta}$; and the evolution of the state $\underline{\theta}$ as a function of p_I, characterized by $\underline{\theta} = \theta_I^+(p_I)$. In the first case, the replacement effect is not too severe, and it only affects the incumbent by slowing down the deterrence price-path, as I is still able to reach its monopoly level. In the second case, the replacement effect is more severe, as it prevents the incumbent from reaching its monopoly level. The best the incumbent can achieve is to deter entry and converge to some intermediate point, which may enable it to exploit some of its monopoly power. The replacement factor β has profitability implications for the incumbent: either a slower price-path to the monopoly level, or even a truncated price-path. As β increases, the iso-curve $W_E(p_I, \underline{\theta}) = \kappa$ rotates clockwise with a pivot point at $W_E(p_I, 0) = \kappa$. As the fraction of customers that is replaced increases, the fraction of customers that effectively are forced out of the market is reduced. Hence, for a given threshold $\underline{\theta}$, I requires a lower price to deter entry. The pivot point stems from the case $\underline{\theta} = 0$, where E's prospect of entry is not affected by β, as all customers are already in the market.

2.6 Discussion

The early analyses of limit pricing were developed under the assumption that current prices could serve as a signal of future profits, by some unspecified mechanism (the idea was that prices could provide information about either the market or the incumbent's stance towards competition). After the argument of independence between pre-entry prices and post-entry competition put forward by Friedman (1979), the analysis of limit pricing has been restricted to the context of incomplete information, where the potential entrants are unaware of the exact realization of some relevant characteristics (such as the incumbent's costs or the actual demand). Nonetheless, in this paper we revisited limit pricing within the scope of complete information, and established the existence of a limit price-path such that entry is deterred.

The key to understand the existence of limit pricing in the absence of information asymmetries lies precisely in Friedman's point: in the context of positive selection, pre-entry prices have a bite into the market's configuration, and therefore can affect the

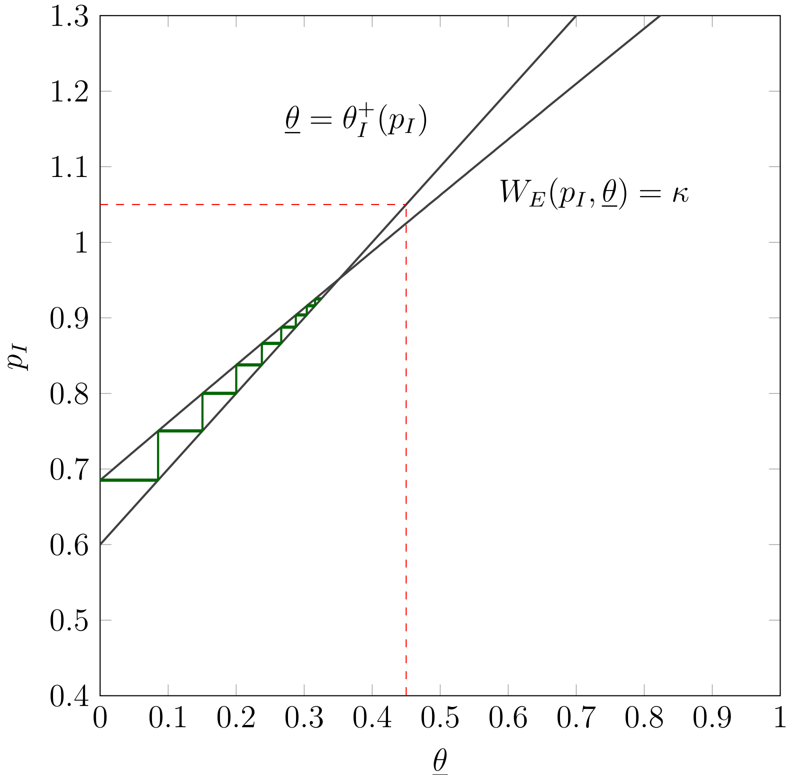


Note: Example corresponds to the uniform distribution. The red-dashed lines indicate I's monopoly level. The dark green line represents the price-path: each increment corresponds to a period. In the 4th period, I reaches its monopoly price and remains at that level.

Figure 2.1: Deterrence price-path for low β.

profits of post-entry competition. More precisely, within the framework of horizontal differentiation, as in the Hotelling line, the incumbent can exploit the nature of positive selection to implement a skimming dynamic and force customers out of the market, particularly those nearest (in preferences) to the entrant. If customers do not return to the market—absorbing exit—or at least not all of them do, as in the overlapping generations extension, then the entrant's prospect of entry is affected and limit pricing can thus arise.

The takeaway is that incomplete information is not a necessary requirement for limit pricing to be a viable strategy for incumbents. Whereas the frameworks of incomplete information have served a great deal to give the narrative of signalling future profits a solid foundation, the frameworks of evolving market structures also



Note: Example corresponds to the uniform distribution. The red-dashed lines indicate I's monopoly level. The dark green line represents the price-path: each increment corresponds to a period. The price-path converges to a level below I's monopoly price.

Figure 2.2: Deterrence price-path for high β.

lead to limit pricing.

A second topic of discussion regarding limit pricing is its implications with respect to competition policy. The early analyses of limit pricing pointed to the presence of a trade-off between low pre-entry prices and delayed or deterred entry. Thus, from a competition policy perspective, it is not entirely clear whether limit pricing is harmful—on the aggregate—or not.

Contrary to the seminal work of Milgrom and Roberts (1982) about limit pricing with incomplete information, where it was argued that the aforementioned trade-off may not actually arise (limit pricing could actually imply lower prices and higher entry probability, and should therefore not be discouraged), in our framework the trade-off can vanish but in the opposite direction: entry can be deterred with prices that do not remain low, but rather increase overtime, possibly up to the monopoly level. The key underlying mechanism is the skimming dynamics of Lemma 2.2: provided that the incumbent can deter entry, while at the same time force some customers out of the market, then it will be able to deter entry in the subsequent period with a higher price.

The increasing limit price-path derived in this paper has clear negative consequences for consumer welfare, as prices grow over time, possibly up to the monopoly price, and the lack of entry means less variety for consumers. From a competition policy perspective, limit pricing should be considered as a viable theory of harm, albeit on a case-by-case basis, paying close attention to market dynamics.

2.7 Conclusion

In this paper, we have explored the existence of a deterrence—or limit—price-path, in the context of positive selection with a non-durable good or service and an absorbing exit condition. An incumbent faces the threat of entry by a differentiated competitor, in a market that spans infinitely over time. The main result is that whenever entry costs are large enough, and customers are relatively patient (as characterized by a discount factor $\delta \in [1/2, 1)$), then entry can be deterred in every period, and the deterring price is increasing over time, possibly up to the incumbent's monopoly level. The increasing price-path is explained by the exit of customers least attracted to the incumbent—and most attracted to the entrant—whenever entry does not occur. This leads the way to *skimming dynamics*, by which the incumbent can modulate the entrant's residual demand and therefore reduce its profits from entry. In order to relax the absorbing exit condition, we consider an extension to overlapping generations, where the mass of customers that effectively exit the market is reduced, as some of the customers that do exit the market are replace by a new generation. The main result is still obtained, albeit the price-path's rate of increase is diminished, as well as the possibility to reach the monopoly level.

The existence of an increasing limit price-path has implications for competition policy. Since the incumbent can deter entry while exploiting some or all of its monopoly position in the long run, consumer welfare can be severely affected through high prices and low variety. Therefore, the dynamics described in this article command a thorough analysis of limit pricing on a case-by-case basis, with emphasis on how the market structure is affected by pricing decisions.

Chapter 3

Data collection is not enough for market tipping

Abstract

We analyze the role of data in the dynamics of competition, with particular interest in the necessary conditions for market-tipping. When data allows firms to offer more value, and data itself becomes partially obsolete over time, market-tipping in the long-run requires more than just a transient data advantage: a structural advantage (e.g., in the form of intrinsic value offered) is necessary.

3.1 Introduction

In the wake of digitization, the role of data in competition has been constantly scrutinized in recent times. With the ever increasing acquisition of user information, data can be regarded as a competitive advantage to offer more valuable products or services, either in terms of quality or fit.[1] However, this can become a matter of concern. The competitive advantage of data can lead to entry barriers when some form of network externality is present. Alternatively, data can be regarded as a means to extract more surplus from consumers. In this case, controlling the stock of data can become an artificial barrier to entry, as in Condorelli and Padilla (2020).[2]

[1]See for instance Hagiu and Wright (forthcoming, 2023) for the role of data-enabled learning in developing a competitive advantage, and Biglaiser et al. (2019) for an analysis of the different channels by which data can become a competitive advantage.

[2]The authors develop a theory of entry deterrence, whereby a monopolist in a data-intensive market preemptively enters a related data-rich market, to control data and deter entry in its primary market.

To address some of these issues, de Cornière and Taylor (2021) have developed a framework to analyze the competitive effects of data, determining conditions for data to be pro- or anti-competitive.

From a normative perspective, there have been many discussions regarding the digital markets' regulation, which touch on the issue of data (see for instance the 2019 Report of the Digital Competition Expert Panel (UK) or the 2019 EU Directorate-General for Competition Report); as well as legislative work, mainly in the U.S. and Europe (v.gr., the EU Digital Markets Act).

An overarching concern with respect to data is the possibility for market-tipping, a phenomenon in which a market is solely—or predominantly—served by only one firm. Despite the nuances in measuring market-tipping,[3] it has been argued that digital markets are prone to market-tipping, as evidenced by the likes of Google, Amazon and Facebook (see Bedre-Defolie and Nitsche (2020) for a discussion of this phenomenon in digital multi-sided platforms). Among several factors pointed out as facilitators of this phenomenon, perhaps the most prominent is positive network externalities.

Prüfer and Schottmüller (2021) have analyzed market-tipping in the context of *data-driven* quality differentiated markets, which feature a form of network externality, and showed that such markets tend to tip very easily. In particular, they consider *data-driven indirect network effects*, where the more user information a firm possesses, the lower the marginal cost of quality production. The key driver for their result is that firms invest in quality, thus there is a permanent aspect to the firms' differentiation through data, which only changes if firms have different incentives to invest.

In this article we want to analyze the role of data in the dynamics of competition, when a data advantage by any firm is not permanent, as data can become obsolete. In particular, we consider a model of data collection and obsolescence, where firms gather data proportionally to the demands they last served, and previous data stock is only partially transferred from period to period. Data is a direct competitive advantage to offer more value, which is a different form of network effects. The aim is to understand when markets tip in this context, and what is the role of data.

The main insight is that data on its own is not enough for markets to tip in the long-run. Data collection exhibits diminishing returns to scale, due to the obsolescence being proportional to the stock of data, therefore it cannot grow unbounded. Market-tipping requires one of the firms to have a structural advantage, and the role of data is to augment that advantage.

The remainder of the paper is organized as follows. Section 3.2 describes the model. Section 3.3 solves the model for interior equilibria in each period. Section 3.4

[3]See Petit and Moreno Belloso (2021) for a discussion on this topic.

characterizes the pure-strategy equilibria, considering border solutions, and determines conditions for market-tipping. Section 3.5 sheds some light on the dynamics for an infinite horizon competition. Lastly, Section 3.6 presents some final remarks.

3.2 Model

Consider two vertically differentiated firms, 1 and 2, with intrinsic values v_1 and v_2, respectively. Let $v \equiv v_1 - v_2 \geq 0$ represent firm 1's value advantage. Firms also differ in the amount of relevant data they posses, and data allows firms to offer more value to consumers. We denote by d_1 and d_2 each firm's data, and let $d \equiv d_1 - d_2$ represent firm 1's data advantage (or disadvantage if $d < 0$). Furthermore, firms are also horizontally differentiated, located at the extremes of a Hotelling line, with differentiation parameter t (firm 1 is at $x = 0$ and firm 2 is at $x = 1$). There are two periods, discounted by $\delta \in [0, 1]$.In each period, firms set prices simultaneously.

Given prices p_1 and p_2, the indifferent consumer is

$$\hat{x}(p_1, p_2) = \frac{1}{2} + \frac{v + d + p_2 - p_1}{2t}, \tag{3.1}$$

which is restricted to the $[0, 1]$ interval.

Firms gather data in proportion to the consumers they have previously served, and its evolution is given by $d_i' = (1-\kappa)d_i + \omega q_i$, where d_i' represents the next period's data level of firm $i \in \{1, 2\}$, d_i and q_i are its current period's data level and consumers served, respectively, and the parameters $\kappa \in [0, 1]$ and $\omega \in (0, \frac{3\sqrt{3}}{2}t)$ represent data obsolescence and collection rates, respectively.[4] Therefore, for first-period prices p_1 and p_2, the data advantage evolves according to

$$d'(p_1, p_2) = (1 - \kappa)d + \omega(2\hat{x}(p_1, p_2) - 1). \tag{3.2}$$

Throughout the analysis, superscripts (1) and (2) will represent first and second period, respectively, and price dependencies are dropped for readability. For any variable y, y' represents its next period's instance.

Remark. For the firms, data advantage is an imperfect substitute of value advantage: even though data and value are perfect substitutes from the consumers' perspective, they are not inter-temporally perfect substitutes for firms, as value is permanent whereas data suffers from obsolescence (represented by the κ parameter).

[4]The upper-bound $\omega < \frac{3\sqrt{3}t}{2}$ is necessary for the interior equilibrium, whenever it exists, to be stable. It is also a sufficient for the firms' problems to be concave.

Remark. It is important to point out that data gathering augments the role of value advantage, in the sense that an increase of v implies an additional increase in next period's data advantage, through $\hat{x}$.

3.3 Interior equilibria

We proceed to solve the model by backward induction. We will focus first on interior—or shared market—solutions for the first-period problem, and then we will characterize border solutions.

3.3.1 Second period

Given a data advantage $d^{(2)}$, the equilibrium is straightforward:

$$\left(p_1^{(2)}, p_2^{(2)}\right)\left(d^{(2)}\right) = \begin{cases} (0, -v - d^{(2)} - t) & \text{if } v + d^{(2)} < -3t, \\ \left(t + \dfrac{v + d^{(2)}}{3}, t - \dfrac{v + d^{(2)}}{3}\right) & \text{if } v + d^{(2)} \in [-3t, 3t], \\ (v + d^{(2)} - t, 0) & \text{if } v + d^{(2)} > 3t, \end{cases} \tag{3.3}$$

the indifferent consumer is

$$\hat{x}^{(2)}(d^{(2)}) = \frac{1}{2} + \frac{v + d^{(2)}}{6t}, \tag{3.4}$$

and profits are

$$\left(\pi_1^{(2)}, \pi_2^{(2)}\right)\left(d^{(2)}\right) = \begin{cases} (0, -v - d^{(2)} - t) & \text{if } v + d^{(2)} < -3t, \\ \left(\dfrac{1}{2t}\left[t + \dfrac{v + d^{(2)}}{3}\right]^2, \dfrac{1}{2t}\left[t - \dfrac{v + d^{(2)}}{3}\right]^2\right) & \text{if } v + d^{(2)} \in [-3t, 3t], \\ (v + d^{(2)} - t, 0) & \text{if } v + d^{(2)} > 3t. \end{cases} \tag{3.5}$$

3.3.2 First period

For simplicity, we consider only non-negative data advantages for the first period: $d^{(1)}$ $ge0$. Consider an interior equilibrium, denoted by $p_1^{(1)}$ and $p_2^{(2)}$. By the data evolution (3.2), the next period's data advantage will be

$$
\begin{aligned}
d^{(2)} &= (1-\kappa)d^{(1)} + \omega(2\hat{x}^{(1)} - 1), \\
&= \left(1 - \kappa + \frac{\omega}{t}\right) d^{(1)} + \frac{\omega}{t}\left(v + p_2^{(1)} - p_1^{(1)}\right).
\end{aligned} \tag{3.6}
$$

Firm 1 solves

$$
\Pi_1(p_2, d^{(1)}) = \max_{p_1} \left[\frac{1}{2} + \frac{v + d^{(1)} + p_2 - p_1}{2t}\right] p_1 + \delta \pi_1^{(2)}(d^{(2)}), \tag{3.7}
$$

whereby its optimality condition (interior solution) is

$$
\frac{1}{2} + \frac{v + d^{(1)} + p_2 - 2p_1}{2t} - \frac{\delta\omega}{3t}\left[1 + \frac{v + \left(1 - \kappa + \frac{\omega}{t}\right) d^{(1)} + \frac{\omega}{t}(v + p_2 - p_1)}{3t}\right] = 0, \tag{3.8}
$$

and similarly for firm 2.[5] Both conditions boil down to

$$
\begin{aligned}
\left(18t - 2\frac{\delta\omega^2}{t}\right) p_1 =& 9t^2 - 6\delta\omega t + \left(9t - 2\frac{\delta\omega^2}{t}\right) p_2 + \left(9t - 2\delta\omega - 2\frac{\delta\omega^2}{t}\right) v \\
&+ \left(9t - 2\delta\omega(1-\kappa) - 2\frac{\delta\omega^2}{t}\right) d^{(1)}, \qquad (3.9) \\
\left(18t - 2\frac{\delta\omega^2}{t}\right) p_2 =& 9t^2 - 6\delta\omega t + \left(9t - 2\frac{\delta\omega^2}{t}\right) p_1 - \left(9t - 2\delta\omega - 2\frac{\delta\omega^2}{t}\right) v \\
&- \left(9t - 2\delta\omega(1-\kappa) - 2\frac{\delta\omega^2}{t}\right) d^{(1)}. \qquad (3.10)
\end{aligned}
$$

As the best-responses are monotonic with slope lower than 1, the interior equilibrium is unique, and the equilibrium strategies are represented by the system

$$
\boldsymbol{p}^{(1)}(d^{(1)}) = \begin{pmatrix} p_1^{(1)} \\ p_2^{(1)} \end{pmatrix}(d^{(1)}) = \begin{pmatrix} t - \frac{2}{3}\delta\omega + \frac{A}{B}v + \frac{A + 2\kappa\delta\omega t}{B} d^{(1)} \\ t - \frac{2}{3}\delta\omega - \frac{A}{B}v - \frac{A + 2\kappa\delta\omega t}{B} d^{(1)} \end{pmatrix}, \tag{3.11}
$$

with $A = 9t^2 - 2\delta\omega t - 2\delta\omega^2$ and $B = 27t^2 - 4\delta\omega^2$.

[5] The (interior) second order condition reads $\frac{1}{t}\left[\frac{\delta\omega^2}{9t^2} - 1\right] < 0$, which is ensured by $\omega < \frac{3\sqrt{3}}{2}t < 3t$ for all values of δ.

Remark. The assumption $\omega < \frac{3\sqrt{3}}{2}t$ ensures the stability of the first-period's interior equilibrium, for all δ.[6] It also implies $B > 0$.

Observe that as firm 1 internalizes the role of data gathering in its future profits, the effect of lowering its first-period price in each of its own per-period demands is stronger for the first-period demand. In effect, fix p_2 in the first period, then for any p_1 in that period, $\hat{x}$ is given by (3.1), which yields a next period data difference

$$d' = (1-\kappa)d^{(1)} + \frac{\omega}{t}(v + d^{(1)} + p_2 - p_1), \tag{3.12}$$

which in turn yields a second-period indifferent consumer in equilibrium equal to

$$\hat{x}' = \frac{1}{2} + \frac{v + (1-\kappa)d^{(1)} + \frac{\omega}{t}(v + d^{(1)} + p_2 - p_1)}{6t}. \tag{3.13}$$

It follows that

$$-\frac{\omega}{6t^2} = \frac{\partial \hat{x}'}{\partial p_1} > \frac{\partial \hat{x}}{\partial p_1} = -\frac{1}{2t}, \tag{3.14}$$

since $\omega < 3t$.

The interior equilibrium demands in the first-period market are characterized by

$$\hat{x}^{(1)} = \frac{1}{2} + \frac{9t + 4\delta\omega}{2B}v + \frac{9t + 4(1-\kappa)\delta\omega}{2B}d^{(1)}, \tag{3.15}$$

which together with (3.6), lead to the next period's data advantage

$$d^{(2)} = \frac{\omega[9t + 4\delta\omega]}{B}v + \frac{3t[9(1-\kappa)t + 3\omega]}{B}d^{(1)}. \tag{3.16}$$

Furthermore, interior equilibrium demands in the second period are then given by

$$\hat{x}^{(2)} = \frac{1}{2} + \frac{9t + 3\omega}{2B}v + \frac{9(1-\kappa)t + 3\omega}{2B}d^{(1)}. \tag{3.17}$$

Observe that $\hat{x}^{(1)}$, $d^{(2)}$ and $\hat{x}^{(2)}$ are all increasing in v and $d^{(1)}$, as well as in δ as B is decreasing in δ, and v and $d^{(1)}$ are non-negative.

Let $\overline{v}^{(\tau)}(\delta)$ denote the largest value advantage in period τ such that the equilibrium in each period is interior (i.e., $\hat{x}^{(\tau)} \in [0,1]$). The thresholds $\overline{v}^{(\tau)}(\delta)$ are obtained by

[6] It stems from imposing that the slope of the best-responses be larger than -1.

setting $\hat{x}^{(\tau)} = 1$, and correspond to

$$\overline{v}^{(1)}(\delta) = \frac{B - [9t + 4(1-\kappa)\delta\omega]d^{(1)}}{9t + 4\delta\omega}, \tag{3.18}$$

$$\overline{v}^{(2)}(\delta) = \frac{B - [9(1-\kappa)t + 3\omega]d^{(1)}}{9t + 3\omega}, \tag{3.19}$$

where the dependence on δ is through the term B. These thresholds, in particular their lower envelope, will help us characterize when the equilibrium ceases to remain interior overall, and the following claim asserts their ordering as of δ.

Claim 3.1 $\overline{v}^{(1)}(\delta) \leq \overline{v}^{(2)}(\delta)$ *if and only if* $\delta \geq \overline{\delta} \equiv \frac{3}{4}\left(1 - \frac{\kappa}{\omega}d^{(1)}\right)$.

Proof. After straightforward algebraic manipulation, the thresholds difference is

$$\overline{v}^{(2)}(\delta) - \overline{v}^{(1)}(\delta) = \frac{B - [9(1-\kappa)t + 3\omega]d^{(1)}}{9t + 3\omega} - \frac{B - [9t + 4(1-\kappa)\delta\omega]d^{(1)}}{9t + 4\delta\omega}, \tag{3.20}$$

$$\propto B\{[4\delta - 3]\omega + 3\kappa d^{(1)}\}, \tag{3.21}$$

which is non-negative if and only if $\delta \geq \overline{\delta} = \frac{3}{4}\left(1 - \frac{\kappa}{\omega}d^{(1)}\right)$, since $B > 0$ and the proportionality constant is the product of the denominators, a positive term. ■

In the following section we will characterize the pure-strategy equilibria for the two main scenarios stemming from Claim 3.1: $\delta \geq \overline{\delta}$ and $\delta < \overline{\delta}$.

3.4 Equilibria characterization

We proceed to characterize the pure-strategy equilibria when $\delta \geq \overline{\delta}$ first. As $\overline{v}^{(1)}(\delta) \leq \overline{v}^{(2)}(\delta)$, the interior equilibrium will break down when the first-period market is cornered, that is, when the value advantage is larger than $\overline{v}^{(1)}(\delta)$. In this regime, it is necessary to determine when the second-period market will be cornered as well, which occurs at the constant threshold

$$\hat{v}^{(2)} = 3t - \omega - (1-\kappa)d^{(1)}. \tag{3.22}$$

This threshold stems from observing that when the first-period market is cornered, then the second-period data advantage is capped at $(1-\kappa)d^{(1)} + \omega$, thus equilibrium demands in the second period are determined by $\hat{x}^{(2)} = \frac{1}{2} + \frac{v + (1-\kappa)d^{(1)} + \omega}{6t}$. Observe that

$$\hat{v}^{(2)} - \overline{v}^{(2)}(\delta) = \frac{[4\delta - 3]\omega + 3\kappa d^{(1)}}{9t + 3\omega},$$

which has the same sign as $\overline{v}^{(2)}(\delta) - \overline{v}^{(1)}(\delta)$.

The following proposition provides a characterization of the equilibria for different levels of value advantage.

Proposition 3.1 *Consider $\delta \geq \overline{\delta}$. For $v \leq \overline{v}^{(1)}(\delta)$, the unique equilibrium is interior in both periods, given by $\boldsymbol{p}^{(1)}(d^{(1)})$ in (3.11). For $v \in (\overline{v}^{(1)}(\delta), \hat{v}^{(2)}]$, in the unique equilibrium the market is served only by firm 1 in the first period, and is shared in the second period. First-period equilibrium prices are*

$$\overline{p}_1^{(1)}(d^{(1)}) = v + d^{(1)} + \overline{p}_2^{(1)} - t, \tag{3.23}$$

$$\overline{p}_2^{(1)}(d^{(1)}) = t - \frac{2}{3}\delta\omega - \frac{A}{B}\overline{v}^{(1)}(\delta) - \frac{A + 2\kappa\delta\omega t}{B}d^{(1)}. \tag{3.24}$$

For $v > \hat{v}^{(2)}$, in the unique equilibrium the market is served only by firm 1 in both periods, and first-period prices are as in (3.23) with $\overline{p}_2^{(1)} = 0$.

Proof. See Appendix F. ■

Remark. There is a continuum of equilibria for $v \in (\overline{v}^{(1)}(\delta), \hat{v}^{(2)}]$, which involve lower prices for firm 2. However, these resort to weakly dominated strategies.

The implications of the previous Proposition can be better understood considering δ and $d^{(1)}$ fixed. Proposition 3.1 establishes that for low levels of value advantage (namely, below $\overline{v}^{(1)}(\delta)$) the equilibrium is interior in each period and both firms share the markets, therefore there is no market tipping. Firm 1, given its value advantage, could be tempted to slash its price, gather more data, and thus tip the market in the second period. However, it has not enough room to do so before cornering the first-period market and reaching its maximal data accumulation: when $p_1^{(1)}$ is reduced, firm 1's first-period market share grows faster than its second-period market share, and as δ is relatively high, the former is already relatively high, therefore the first-period market is cornered before enough data can be accumulated to tip the second-period market. Eventually, for large enough value advantage (namely, above $\overline{v}^{(1)}(\delta)$), firm 2 can no longer compete with firm 1 for the first-period market, which ends up cornered by the latter; however, by the same reasoning as before, the second-period market is not necessarily tipped. For this to be the case, firm 1 requires an even larger value advantage (namely, at least $\hat{v}^{(2)}$).

Clearly, the thresholds discussed are decreasing in $d^{(1)}$, as data advantage is a substitute of value advantage, albeit imperfect. It is readily verified that $\hat{v}^{(2)}$ is increasing in κ and decreasing in ω, as when the data obsolescence rate is higher, or the data collection rate is lower, the cap on the data advantage evolution is tighter, and in order to tip the market firm 1 would need a higher value advantage to substitute for the lower second-period data advantage.

Turning to the case $\delta < \overline{\delta}$, we have now that $\overline{v}^{(1)}(\delta) > \overline{v}^{(2)}(\delta)$, and therefore it is possible to corner the second-period market without cornering the first-period market. Thus, the former is critical market for the interior equilibrium to break down. However, contrary to the previous case, the interior equilibrium will not break down at the relevant threshold, $\overline{v}^{(2)}$, but rather at a lower threshold, referred to as $\tilde{v}^{(2)}(\delta)$. The reason is that for value advantages such that firm 1 is close to cornering the second-period market under the interior equilibrium strategies (i.e., for v close to $\overline{v}^{(2)}$), that firm has incentives to lower its price and effectively corner the second-period market, which has a steeper profit (with respect to that period's data advantage) than the interior equilibrium.

Proposition 3.2 *Consider $\delta < \overline{\delta}$. There exists a threshold $\tilde{v}^{(2)}(\delta) \in [\hat{v}^{(2)}, \overline{v}^{(2)}(\delta)]$, such that for $v \leq \tilde{v}^{(2)}(\delta)$ the unique equilibrium is interior in both periods, given by $\boldsymbol{p}^{(1)}(d^{(1)})$ in (3.11).*

Proof. See Appendix F. ■

As firms are relatively impatient, they do not internalize the impact of its pricing decisions (in its future profits) as much as in the previous case. Therefore, the augmenting effect of data gathering dominates and firm 1's second-period market share is larger than its first-period market share. Thus, firm 1 has more leeway to eventually, for a large value advantage, slash its price and tip the second-period market. This is profitable as the second-period profit in the tipping regime is (discontinuously) steeper (with respect to $d^{(2)}$) than in the interior regime.[7]

Remark. As the value advantage increases beyond $\tilde{v}^{(2)}(\delta)$, no pure-strategy equilibrium exists (only mixed-strategy), until eventually firms revert back to a pure-strategy equilibrium where the second-period market is tipped and firm 2 behaves myopically.

[7]When the second-period market is tipped by firm 1, its profit has a slope of 1, with respect to $d^{(2)}$; whereas the largest slope of the interior profit is 2/3.

Figure 3.1 displays a diagram of the previous characterizations, where there are three distinct regions: (I) interior equilibrium in both periods, (II) cornered equilibrium in the first-period market and interior equilibrium in the second-period market, and (III) tipped equilibrium in the second-period market (regardless of whether the first-period market is interior or not, and possibly with mixed strategies for the case $\delta < \overline{\delta}$, meaning that in some outcomes the second-period market is tipped).

The main takeaway is that for firm 1 to tip the market in the long-run (represented by the second-period market), data gathering alone is not sufficient: a structural advantage is necessary, in this case the value advantage. Moreover, it needs to be large enough, as for $v < \hat{v}^{(2)}$, the long-run equilibrium is interior, regardless of the discount factor.

3.5 Infinite horizon

With the insights of the two-period model, we will examine some qualitative insights when the firms interact indefinitely. In particular, we are interested in an absorbing market-tipping regime.

3.5.1 Absorbing market-tipping regime

Let us look at the possibility of an absorbing regime, in which once the market is tipped by firm 1, it is then tipped by the latter in every period thereafter.

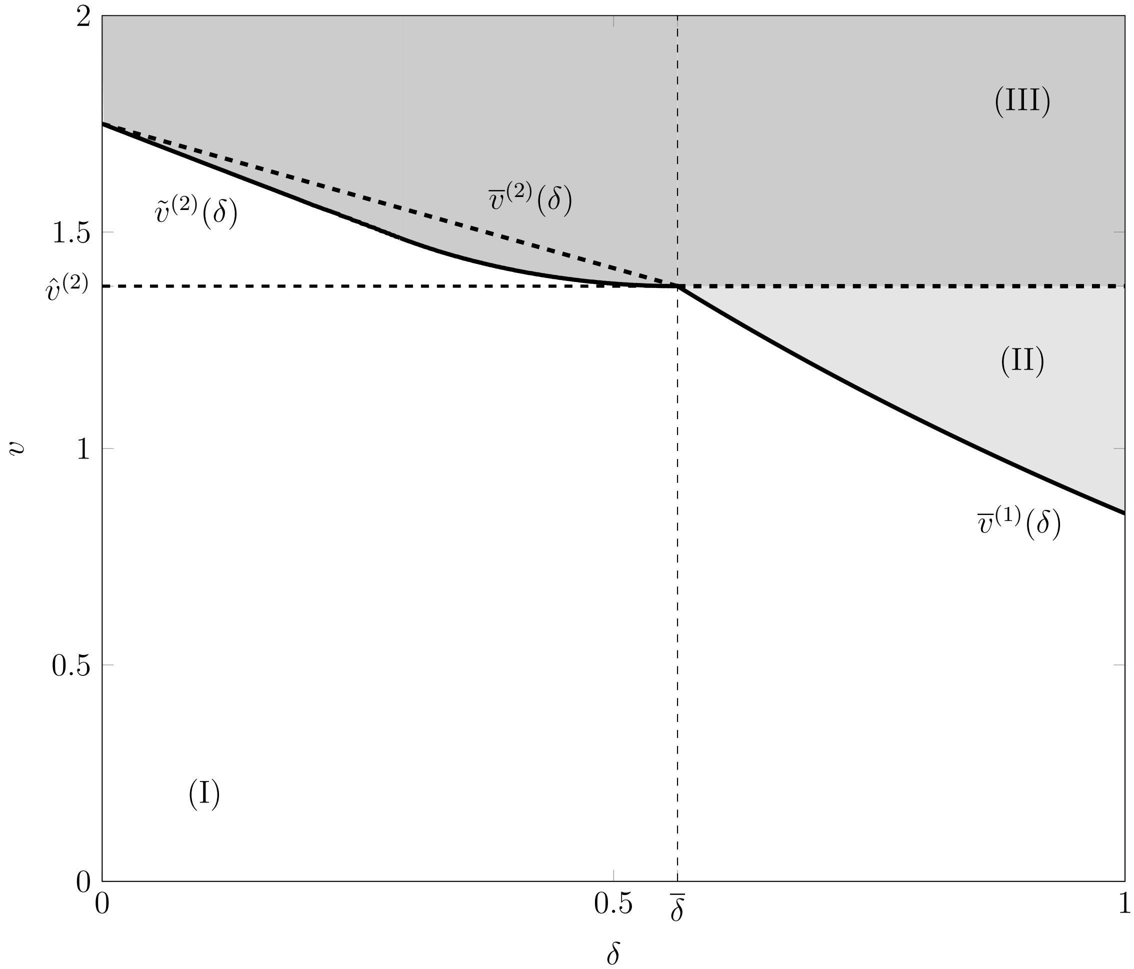
Under this regime, the data evolution is maximal in the sense that in every period an amount ω is accumulated (gross of data obsolescence). Given a current data advantage d, said advantage after τ periods is

$$d^{(\tau)} = \frac{\omega}{\kappa} - (1-\kappa)^{\tau} \left[\frac{\omega}{\kappa} - d\right]. \tag{3.25}$$

The value for firm 1 in this regime, $V_1^*(d)$, is then given by

$$\begin{aligned} V_1^*(d) &= \sum_{\tau=0}^{\infty} \delta^{\tau}(v + d^{(\tau)} - t), & (3.26)\\ &= \frac{v-t}{1-\delta} + \frac{d}{1-\delta(1-\kappa)} + \frac{\delta\omega}{(1-\delta)(1-\delta(1-\kappa))}. & (3.27)\end{aligned}$$

In order for the regime to exist, firm 1 has to have the incentive to corner the



Note: Example corresponds to the parameters $t = 1$, $\omega = 1.5$, $\kappa = 0.75$ and $d^{(1)} = 0.5$. Region (I) displays interior equilibrium in both periods, region (II) displays interior equilibrium in the second period and cornered (by firm 1) in the first period, and region (III) displays market-tipping in the second period (possibly with mixed strategies for $\delta < \overline{\delta}$ and regardless of the first-period market structure).

Figure 3.1: Diagram of equilibria.

market given $p_2 = 0$ (since we are looking at an absorbing regime, firm 2 would not make any profit in it, and therefore it would not set a negative price). Firm 1 solves

$$V_1^*(d) = \max_{p_1} \left[\frac{1}{2} + \frac{v+d-p_1}{2t}\right] p_1 + \delta V_1^* \left((1-\kappa)d + \frac{\omega}{t}(v+d-p_1)\right), \tag{3.28}$$

and the incentive condition boils down to a negative slope for the maximization problem when it sets $p_1 = v + d - t$, the largest price that corners the market when $p_2 = 0$:

$$\frac{3t-v-d}{2t} - \frac{\delta\omega}{t}\frac{1}{1-\delta(1-\kappa)} \leq 0, \tag{3.29}$$

or equivalently:

$$d \geq \overline{d} \equiv 3t - v - \frac{2\delta\omega}{1-\delta(1-\kappa)}. \tag{3.30}$$

For the regime to be indeed absorbing, it is necessary that once $\overline{d}$ has been reached, the data advantage remains above $\overline{d}$. Observe that under the proposed absorbing regime, the data advantage is increasing over time whenever it is below its steady state value of $\frac{\omega}{\kappa}$. Therefore, a necessary and sufficient condition for the regime to be absorbing is that $\overline{d} \leq \frac{\omega}{\kappa}$, or

$$v \geq \hat{v} \equiv 3t - \frac{\omega}{\kappa} - \frac{2\delta\omega}{1-\delta(1-\kappa)}. \tag{3.31}$$

Proposition 3.3 *For* $v \geq \hat{v} \equiv 3t - \dfrac{\omega}{\kappa} - \dfrac{2\delta\omega}{1-\delta(1-\kappa)}$*, firm 1 permanently tips the market whenever the data advantage reaches the level* $\overline{d} \equiv 3t - v - \dfrac{2\delta\omega}{1-\delta(1-\kappa)}$.

Recall that v is a structural characteristic of the model, whereas d is only a state. Therefore, as the threshold $\hat{v}$ is a condition for the tipping regime to be absorbing, it does not depend on the actual data advantage. The threshold is decreasing in δ and ω, and increasing in κ, as expected. As firm 1 becomes more patient, the total advantage—value plus data—necessary to corner the market is lower (see equation (3.30), rearranging v to the left-hand side), and as data advantage is increasing over time (if it is below its steady state level), the necessary value advantage for the absorbing tipping regime is lower as well. The rates of data collection and obsolescence determine the rate at which data advantage evolves, therefore their impact is analogous to a change in the discount factor.

Next, we look at shared-market regimes, to shed some light on the eventual path to an absorbing market-tipping regime.

3.5.2 Shared-market regime

We have already established that for $v < \hat{v}$ there is no absorbing tipping regime. We now turn our focus on a regime where both firms share the market (at least in the long run), starting with no initial data advantage.

We look at optimal strategies under the assumption that the market is shared among the two firms and no initial data advantage. As the profit functions are quadratic, we can guess linear strategies of the form $p_1(d) = z_1 + zd$ and $p_2(d) = z_2 - zd$, and quadratic value functions $V(d) = V_0 + V_1 d + V_2 d^2/2$ and $W(d) = W_0 + W_1 d + W_2 d^2/2$, for firms 1 and 2, respectively. We are interested in the slopes of the strategies, z, as this determines the slope of the data advantage evolution. See Appendix G for the characterization of a system of equations that pins z numerically.

For d such that the market is shared, the data evolution is:

$$d' = (1-\kappa)d + \frac{\omega}{t}(v + d + p_2(d) - p_1(d)), \tag{3.32}$$

$$= d + \frac{\omega}{t}(v + z_2 - z_1) - \left(\kappa - \frac{\omega}{t}(1-2z)\right) d, \tag{3.33}$$

which is contracting for $\kappa - \frac{\omega}{t}(1-2z) > 0$, with a steady state

$$d_S \equiv \frac{\frac{\omega}{t}(v + z_2 - z_1)}{\kappa - \frac{\omega}{t}(1-2z)}. \tag{3.34}$$

A sufficient condition for the data advantage evolution to be contracting is $z \geq 1/2$, which can be numerically verified to be satisfied for a constellation of parameters.

Consider $v \geq \hat{v}$. If $d_S > \overline{d}$, then the interior regime will tend towards the absorbing market-tipping regime, and the transition between both regimes will involve mixed strategies, as for d close to $\overline{d}$, firm 1 will have incentives to undercut its price and reach the absorbing regime sooner. However, if $d_S \leq \overline{d}$, then there two cases to distinguish, stemming from the comparison of $(1-\kappa)d_S + \omega$ and $\overline{d}$. For $(1-\kappa)d_S + \omega > \overline{d}$, firm 1 can deviate and reach the absorbing regime when d is close to the steady state d_S, and may do so if it is profitable. Thus, the pure-strategy interior equilibrium will possibly break down for d close to d_S, and mixing will take place. On the other hand, for $(1-\kappa)d_S + \omega \leq \overline{d}$, firm 1 cannot feasibly reach the absorbing regime under any one-shot deviation, and the equilibrium remains in the interior regime, at least in the long-run.

3.6 Final remarks

In this article we have analyzed the role of data in market-tipping, when data enables firms to offer a more value to consumers. We have shown, both in the baseline two-period model and in the infinite horizon extension, that there is a threshold for the intrinsic value advantage of a firm, such that below it no market-tipping unfolds in the long-run.

This means that when data exhibits obsolescence, it is not capable of tipping the market on its own. Market-tipping will occur when the value advantage of one firm is large enough, meaning that the phenomenon requires one firm to have a structural advantage over the other one.

This is a key contrast to the insight of Prüfer and Schottmüller (2021), where in the context of a permanent data effect, market-tipping arises rather easily. It suggests that data obsolescence is an important aspect in the bigger picture of the role of data in competition.

Appendix A

Proofs of Chapter 1

Proof of Proposition 1.3. Consider first the case of strong efficiency. Given IMF's result, if E does not enter in the first period, it will do so in the second one, and second-period profits are given by (1.13). In the first period, the necessary condition for E to gain access to the market depends on whether I offered only one retailer a discount contract, say R_1, or both retailers the same contract.[1] The necessary condition on the price p_E is

$$p_E < \begin{cases} p - d/\lambda & \text{if } I \text{ offers } (p,d) \text{ to } R_1 \text{ only and } E \text{ offers } p_E \text{ to } R_1, \\ p - d & \text{if } I \text{ offers } (p,d) \text{ to } R_1 \text{ only and } E \text{ offers } p_E \text{ to } R_2, \\ p - d & \text{if } I \text{ offers } (p,d) \text{ to both retailers and } E \text{ offers } p_E \text{ to either } R_1 \text{ or } R_2. \end{cases}$$

The first case is the condition for R_1 to forgo the discount, and the other cases correspond to the prices such that the chosen retailer (by E) can effectively compete against the discounted price in the contestable market. It follows that the relevant necessary condition (the weakest from E's perspective) is $p_E < p - d$, since E can always avoid the leverage imposed by I's contracts. The entry condition is $\Pi_{E|entry} = \lambda(p_E - c_E) - F + \delta(c_I - c_E) > \delta[\lambda(c_I - c_E) - F]$. Setting $p = v$, these conditions imply that I's delaying profit is

$$\begin{aligned} \Pi_{I|delay} &= v - d - c_I + \delta(1-\lambda)(v - c_I), && \text{(A.1)} \\ &\leq -(1-\delta)[c_I - c_E - F/\lambda] - (\delta/\lambda)(c_I - c_E) + \delta(1-\lambda)(v - c_I), && \text{(A.2)} \end{aligned}$$

which is strictly lower than $S_{1-\lambda}$ because of strong efficiency and $\delta \leq 1$.

[1] In practice, I can offer both retailers different AUD contracts, but one is going to be superior, say the one offered to R_1, and then this is equivalent to not offering a contract to R_2.

As for the weak efficiency case, I can profitably deter entry of a less efficient entrant in the second period if and only if

$$c_E + F/\lambda - c_I \geq (1-\lambda)(v - c_I). \tag{A.3}$$

In effect, a less efficient entrant is characterized by $c_E + F/\lambda > c_I$. Given an AUD contract (p, d) to one or both retailers, the necessary condition for E to gain access to the market is $p_E < p - d$, while the necessary condition for it to be profitable is $\Pi_{E|entry} = \lambda(p_E - c_E) - F > 0$. The implied necessary and sufficient condition for I to deter entry boils down to $p - d \leq c_E + F/\lambda$. Then, deterrence will be profitable if and only if the profit under deterrence is larger than the non-contestable share surplus, that is, $\Pi_{I|deter} = c_E + F/\lambda - c_I \geq (1-\lambda)(v - c_I)$.

Now, solving from the first period onward, condition (A.3) is necessary, as otherwise opposing entry in the first period is not profitable: this is a direct consequence of the strong efficiency case already analyzed, as the continuation profits for I and E after opposing entry in this weak efficiency scenario would be the same as in the strong efficiency scenario.

Assume for now that it is profitable to deter entry in the second period conditional on no entry in the first period. Given an AUD contract (p, d) to one or both retailers, the condition for E to gain access to the market is $p_E < p - d$,[2] and the condition for it to be profitable is $\Pi_{E|entry} = \lambda(p_E - c_E) - F + \delta(c_I - c_E) > 0$. Together with $p = v$, these conditions imply that I's deterrence profit is

$$\Pi_{I|deter} = v - d - c_I + \delta[c_E + F/\lambda - c_I] \tag{A.4}$$

$$= (1+\delta)[c_E + F/\lambda - c_I] - (\delta/\lambda)(c_I - c_E), \tag{A.5}$$

which yields condition (1.14) from the Proposition, by requiring $\Pi_{I|deter} \geq S_{1-\lambda}$. Finally, observe that this condition implies (A.3). In effect, condition (1.14) can be rewritten as

$$c_E + F/\lambda - c_I \geq (1-\lambda)(v - c_I) + (\delta/\lambda)[(1+\lambda)(c_I - c_E) - F], \tag{A.6}$$

[2]We are ruling out the case that when only one retailer gets an AUD offer in the first period, say R_1, then the other retailer, R_2, believes that it is going to be offered a deterring contract in the second period after no entry in the first period. This would modify the access condition for E, making it stricter. This kind of belief does not seem very robust, particularly considering that in this circumstance, R_2 would not represent a priori any advantage for I to deter entry through it; and moreover, upon delaying entry in the first period through R_1, there might be some non-modeled benefits for I to keep doing business with R_1 (for instance, there might be trust gains or some specific knowledge that is valuable).

which implies that $c_E + F/\lambda - c_I > (1-\lambda)(v - c_I)$, as $(1+\lambda)(c_I - c_E) > F$ because of weak efficiency. ■

Proof of Proposition 1.4. We analyze the strong efficiency case first. Consider that I offers a (p, d) contract to R_1 only. Strong efficiency implies that deterrence is not profitable in the second period, so entry will occur, I's profit are given by (1.13) and retailers' profits are zero due to competition. The latter, together with Corollary 1.1, imply that the necessary condition for E to gain access is $p_E < p - d/\lambda$. The entry condition is $\Pi_{E|entry} = \lambda(p_E - c_E) - F + \delta(c_I - c_E) > \delta[\lambda(c_I - c_E) - F]$. Letting $p = v$, I's delaying profit under the previous conditions is

$$\begin{aligned}\Pi_{I|delay} &= v - d - c_I + \delta(1-\lambda)(v - c_I) && \text{(A.7)}\\ &= (1-\lambda)(v - c_I) + \delta(1-\lambda)(v - c_I) - (1-\delta)[\lambda(c_I - c_E) - F] - \delta(c_I - c_E), && \text{(A.8)}\end{aligned}$$

from where condition (1.17) is obtained by requiring that $\Pi_{I|delay} \geq S_{1-\lambda}$.

Under weak efficiency, if entry is delayed in the first period, then entry can be deterred in the continuation game, because of Claim 1.1 and weak efficiency. To do so, I will offer R_1 a (v, d) contract such that $d \in [\lambda(v - c_E) - F, \lambda(v - c_I))$, with second-period profits π_I and π_{R_1} for I and R_1, respectively, such that $\pi_I + \pi_{R_1} = v - c_I$. These observations, together with Corollary 1.1, imply that the necessary condition for E to gain access to the market is $p_E < p - d/\lambda - \delta\pi_{R_1}$. Now, the entry condition is $\Pi_{E|entry} = \lambda(p_E - c_E) - F + \delta(c_I - c_E) > 0$. At $p = v$, I's deterrence profit is

$$\begin{aligned}\Pi_{I|deter} &= v - d - c_I + \delta[v - c_I - \pi_{R_1}] && \text{(A.9)}\\ &= (1-\lambda)(v - c_I) + \delta(v - c_I) - [\lambda(c_I - c_E) - F] - \delta(c_I - c_E), && \text{(A.10)}\end{aligned}$$

and condition (1.18) is obtained from I's profitability condition, that is, $\Pi_{I|deter} \geq S_{1-\lambda}$. ■

Proof of Proposition 1.5. The proof follows through by induction. It has already been proven that the proposition is true when only two periods are left, so it remains to be shown that if it is profitable when $\theta - 1$ periods are left, then it is also profitable when θ periods are left.
Consider there are θ periods left, and refer to the value of any relevant variable x, when θ periods are left, as $x(\theta)$. The natural extension of Corollary 1.1 to more than two periods imply that the necessary condition for E to gain access to the market is

$$v - (1-\lambda)p - \lambda p_E(\theta) > \Pi_{I+R_1|delay}(\theta) - \Pi_{I|delay}(\theta), \qquad \text{(A.11)}$$

where $\Pi_{I+R_1|delay}(\theta) = [1 + \delta + \cdots + \delta^{\theta-2} + \delta^{\theta-1}(1-\lambda)](v - c_I)$ is the joint profit of I and R_1 from delaying entry along the considered equilibrium path, $\Pi_{I|delay}(\theta)$ corresponds to I's equilibrium profit from delaying entry, and it is to be noted that R_1's future profits upon entry are zero due to Bertrand competition.
Given the induction hypothesis, E's outside option is to wait until the last period to enter. This implies that the entry condition for E is

$$\Pi_{E|entry}(\theta) = \lambda(p_E(\theta) - c_E) - F + \sum_{i=1}^{\theta-1} \delta^i (c_I - c_E) > \delta^{\theta-1}[\lambda(c_I - c_E) - F]. \quad \text{(A.12)}$$

As before, I optimally sets $p = v$, and conditions (A.11) and (A.12) imply that I's delaying profit is

$$\Pi_{I|delay}(\theta) = \left[1 - \lambda + \sum_{i=1}^{\theta-2} \delta^i + \delta^{\theta-1}(1-\lambda)\right](v - c_I) - (1 - \delta^{\theta-1})\xi_0 - \pi_E(\theta), \quad \text{(A.13)}$$

where $\xi_0 = \lambda(c_I - c_E) - F$ and $\pi_E(\theta) = \sum_{i=1}^{\theta-1} \delta^i (c_I - c_E)$ correspond to E's one-period efficiency and future profits, respectively.
To show that delaying entry is indeed profitable, it is sufficient to show that $\Pi_{I|delay}(\theta)$ is larger than $S_{1-\lambda}$, or equivalently, that

$$\left[\sum_{i=1}^{\theta-2} \delta^i + \delta^{\theta-1}(1-\lambda)\right](v - c_I) \geq (1 - \delta^{\theta-1})\xi_0 + \pi_E(\theta). \quad \text{(A.14)}$$

In effect, let $\phi = \frac{1-\delta^{\theta-1}}{1-\delta} > 1$,[3] then

$$\left[\sum_{i=1}^{\theta-2} \delta^i + \delta^{\theta-1}(1-\lambda)\right](v - c_I) = \left(\phi\delta - \lambda\delta^{\theta-1}\right)(v - c_I), \quad \text{(A.15)}$$

$$> \phi(1-\lambda)\delta(v - c_I), \quad \text{(A.16)}$$

$$\geq \phi[(1-\delta)\xi_0 + \pi_E(2)], \quad \text{(A.17)}$$

$$= (1 - \delta^{\theta-1})\xi_0 + \pi_E(\theta), \quad \text{(A.18)}$$

where inequality (A.16) follows from $\phi > 1$ and $\delta^{\theta-1} < \delta$, and inequality (A.17) stems from the two-period condition (1.17).
The necessity of the two-period condition is straightforward: if it does not hold, then

[3]This assumes $\delta < 1$; the case with $\delta = 1$ follows through in a similar manner and is omitted.

when only two periods are left (provided that E is still out of the market), I will not find it profitable to delay entry. Then, the previous period (when three periods are left) becomes the analogous of a two-period time horizon from I's perspective, however E has even more future profits, and therefore entry delay is again not profitable. The argument unravels for all periods. ■

Proof of Proposition 1.6. When $\tau \geq 3$ periods are left, the discount is defined by $\Pi_{I|delay}(\tau) = v - d_{\theta\geq 3} - c_I + \delta\Pi_{I|delay}(\tau - 1)$. This implies

$$\begin{aligned} d_{\tau\geq 3} &= v - c_I + \delta\Pi_{I|delay}(\tau - 1) - \Pi_{I|delay}(\tau) && \text{(A.19)} \\ &= \lambda(1-\delta)(v - c_I) + (1-\delta)\xi_0 + \pi_E(2). && \text{(A.20)} \end{aligned}$$

For $\tau = 2$, the discount is given by $\Pi_{I|delay}(\tau = 2) = v - d_{\theta=2} - c_I + \delta(1-\lambda)(v - c_I)$, which yields a discount

$$\begin{aligned} d_{\tau=2} &= v - c_I + \delta(1-\lambda)(v - c_I) - \Pi_{I|delay}(2) && \text{(A.21)} \\ &= \lambda(v - c_I) + (1-\delta)\xi_0 + \pi_E(2). && \text{(A.22)} \end{aligned}$$

Notice that

$$d_{\tau=2} - d_{\tau\geq 3} = \delta\lambda(v - c_I) > 0, \tag{A.23}$$

and I's margin for $\theta = 2$ is

$$v - d_{\tau=2} - c_I = (1-\lambda)(v - c_I) - (1-\delta)\xi_0 - \pi_E(2) > 0, \tag{A.24}$$

as $(1-\delta)\xi_0 - \pi_E(2) \leq \delta(1-\lambda)(v - c_I)$ by the two-period condition. ■

Proof of Proposition 1.7. The proof is similar to the strong efficiency case, however the joint profit from opposing entry is now $\Pi_{I+R_1|deter}(\theta) = [1 + \delta + \cdots + \delta^{\theta-1}](v - c_I)$, and E's outside option is nil, which implies the following entry condition

$$\Pi_{E|entry}(\theta) = \lambda(p_E - c_E) - F + \sum_{i=1}^{\theta-1} \delta^i (c_I - c_E) > 0, \tag{A.25}$$

and I's deterrence profit is

$$\Pi_{I|deter}(\theta) = \left[1 - \lambda + \sum_{i=1}^{\theta-1} \delta^i\right](v - c_I) - \xi_0 - \pi_E(\theta). \tag{A.26}$$

Condition (1.21) stems from requiring $\Pi_{I|deter} \geq S_{1-\lambda}$. Alternatively, if it is profitable in a two-period game and $v - c_I \geq c_I - c_E$, then the proposition is straightforwardly verified. Necessity is also straightforward, since if no deterrence is possible in the second to last period, then I has not enough profit to deter entry from a previous period. ■

Appendix B

No Purchasing Obligation

In the main sections, the timing of the model has reflected the *purchasing obligations* from the retailers towards the entrant, by which E has a last-mover advantage. This feature is particularly relevant in the framework of downstream local monopolies, and is not the standard in the works of Rasmusen et al. (1991) or Segal and Whinston (2000), where downstream buyers that did not sign an exclusive contract, are free to trade with either incumbent or entrant.

To consider an environment in which there are no purchasing obligations, we consider the following modified timing of the model. In each period $t = 0, 1$, there are 4 stages. In stage (1), I offers R an AUD contract (p, d). In stage (2), E offers R a price p_E, and if not already in the market, decides whether to enter, thereby sinking the cost F, or to stay out. In stage (3), I can make new offers, but cannot withdraw the contracts offered in stage (1). In stage (4), R decides how much to buy from each supplier in the market.

In this new timing, I can make a counter-offer after E's entry and pricing decision, therefore is I who has a last-mover advantage now, which reflects the absence of purchasing obligations.

This modification, however, is innocuous in the case of one retailer only: to delay or deter entry, I needs to set an effective price lower than E's effective cost, regardless of it being offered at stage (1) or (3).

We will analyze the frameworks of downstream local monopolies and downstream competition.

B.1 Downstream local monopolies

If no purchasing obligation is in place, then this case is altered in the following manner: given a contract (p, d) to R_1 only, and full exploitation of R_2 by the incumbent, the entrant will no longer be able to fully exploit R_2, since R_2 would reject such offer and wait for a new offer from I in stage (3). This modification implies that the resources that the entrant can accrue are lowered, and now it might be profitable for I to deter entry in a one-period game, as opposed to IMF's result of no deterrence.

Proposition B.1 *In a one-period game of local downstream monopolies (with strong efficiency) and no purchasing obligation, the incumbent can profitably deter entry if and only if*

$$\frac{\lambda(v - c_I)}{2} \geq \lambda(c_I - c_E) - F. \tag{B.1}$$

Proof. Given a (p, d) contract to R_1, the necessary condition for E over R_1 is the standard leverage condition, $p_E < p - d/\lambda$, while it will offer R_2 a price of c_I, as R_2 would reject any higher offer and wait for a better counter-offer in stage (3). Entry condition is then $\Pi_{E|entry} = \lambda(p_E - c_E)/2 + \lambda(c_I - c_E)/2 - F > 0$, and I's deterrence profit (setting $p = v$) is $\Pi_{I|deter} = (1-\lambda)(v-c_I) + \frac{\lambda(v-c_I)}{2} - [\lambda(c_I - c_E) - F]$. Therefore, I can profitably deter entry if and only if $\lambda(v - c_I)/2 \geq [\lambda(c_I - c_E) - F]$. ■

In the two-period model, the lack of purchasing obligations reverses the no-opposition results: under both strong and weak efficiency entry can be profitably deterred altogether. As the lack of purchase obligation modifies the continuation game, and the extent to which the entrant can exploit the unfavoured retailer, Assumption 1.1 is modified as follows:

Assumption B.1 (Negative externality across retailers - without PO) *The following condition holds*

$$\frac{\lambda(c_I - c_E)}{2} - F + \frac{\delta(1+\lambda)(c_I - c_E)}{2} < 0. \tag{B.2}$$

Proposition B.2 *In a two-period game of local downstream monopolies, with no purchasing obligations, the incumbent can profitably deter entry altogether if and only if*

$$\frac{(\lambda + \delta)(v - c_I)}{2} \geq (\lambda + \delta)(c_I - c_E) - F. \tag{B.3}$$

Proof. We assume that condition (B.1) holds true, and verify it ex-post. We compute I's deterrence profit in the same way as in Proposition 1.2, however p_{E2} is now at most c_I, as any larger price will be undercut by I in stage (3); and $\Pi_E^0 = \pi_{R_2} = 0$ regardless the efficiency case considered, as entry will be deterred in the second period if E remains out of the market, fully exploiting R_2:

$$\Pi_{I|deter} = v - c_I + \frac{(\delta - \lambda)}{2}(v - c_I) - [(\lambda + \delta)(c_I - c_E) - F]. \tag{B.4}$$

Condition (B.3) follows from imposing $\Pi_{I|deter} \geq S_{1-\lambda}$.
We now check that condition (B.3) implies condition (B.1). In effect, from Condition (B.3)

$$\begin{aligned} \frac{\lambda(v - c_I)}{2} &\geq \frac{\lambda}{\lambda + \delta}[\lambda(c_I - c_E) - F + \delta(c_I - c_E)], && \text{(B.5)} \\ &= \lambda(c_I - c_E) - \frac{\lambda}{\lambda + \delta}F, && \text{(B.6)} \\ &> \lambda(c_I - c_E) - F. && \text{(B.7)} \end{aligned}$$

■

Remark. As $F < \lambda(c_I - c_E)$ under strong efficiency, a necessary condition for Assumption B.1 to hold is that $\delta < \lambda/(1 + \lambda) < 1/2$, a rather low discount factor. As for the weak efficiency case, no such necessary restriction is in place, in the sense that all $\delta \in (0, 1]$ are permissible for F large enough (within its constraints).

B.2 Downstream competition

In the case of downstream competition, the purchasing obligation assumption is only playing a role under weak efficiency when minimum-share based contracts are considered.[1] As anticipated, dropping this assumption makes deterrence in the second period easier (conditional on no entry in the first period).

Proposition B.3 *In a one-period game of downstream competition with weak efficiency, minimum-share based contracts and without purchasing obligation, the incumbent can profitably deter the entry of a less efficient entrant, and obtain the whole market surplus, if and only if*

$$c_E + F/\lambda - c_I \geq (1 - \lambda)(v - c_I). \tag{B.8}$$

[1]Under strong efficiency, deterrence is not profitable in a one-period model even without purchasing obligation, as any offer the incumbent might do in stage (3) would still require $p - d \geq p_E$, as in stage (1).

Proof. If $c_E+F/\lambda-c_I \geq (1-\lambda)(v-c_I)$, then for I to deter entry it is sufficient to offer contracts $(p, d) = (v, 0)$ to both retailers. In effect, after any price offer $p_E \geq c_E+F/\lambda$, I can make a counter-offer (p, d) in stage (3) to any one retailer offered p_E such that $p - d \leq p_E$, as in the proof of Proposition 1.3, which will leave E without any purchases. Anticipating this, E will not decide to enter in stage (2), and I will obtain the whole market surplus. On the contrary, if $c_E + F/\lambda - c_I < (1-\lambda)(v-c_I)$, then I will not find it profitable to set a discounted price in stage (3) of $c_E + F/\lambda$ or lower, so as to discourage entry, nor will I do so in stage (1), therefore entry will not be deterred. ∎

This means that the incumbent has now more second-period profit to achieve deterrence in the first period.

Proposition B.4 *In a two-period game of downstream competition with weak efficiency, market-share based contracts and no purchasing obligation, the incumbent can profitably deter entry if and only if the following two conditions hold simultaneously:*

$$\delta(v - c_I) - (1-\lambda)(v - c_I) \geq (1/\lambda)[(\lambda + \delta)(c_I - c_E) - F], \tag{B.9}$$

$$c_E + F/\lambda - c_I \geq (1-\lambda)(v - c_I). \tag{B.10}$$

Proof. It is the same as Proposition 1.3 but noting that I's second-period profit is $v - c_I$, the full market surplus, instead of $c_E + F/\lambda - c_I$, its cost-efficiency advantage. However, the first condition does not necessarily imply the second condition. Both conditions are satisfied, for instance, for large F or large v ($\lambda + \delta \geq 1$ is in any case a necessary condition). ∎

Comparing condition (B.9) to its counterpart, condition (1.14), it is easy to see that the former is less strict, since $v > c_E + F/\lambda$.

Appendix C

Stage-game equilibrium

For firm i, and prices p_i and p_j, $j \neq i$, the following notation will be used, considering distances from i's position.

- $\theta_i^+(p_i) = \frac{v-p_i}{\tau}$: denotes the farthest customer that would derive a non-negative stage-game net-utility when buying from i, at price p_i.
- $\hat{\theta}_i(p_i, p_j) = \frac{1}{2} + \frac{p_j - p_i}{2\tau}$: denotes the indifferent customer (from i's perspective), at prices p_i and p_j.
- $\underline{\theta}_i$: denotes the closest customer to i still in the market.

Moreover, consider

$$F_i(\theta_i) = \begin{cases} F(\theta_i) & \text{if } i = E, \\ 1 - F(1-\theta_i) & \text{if } i = I, \end{cases}$$

and let $\underline{\boldsymbol{\theta}} = (\underline{\theta}_I, \underline{\theta}_E)$ denote the set of remaining customers, when this is a connected set characterized by $\underline{\theta}_I$ and $\underline{\theta}_E$ (i.e. $\Theta = [\underline{\theta}_E, 1 - \underline{\theta}_I]$). For the stage-game analysis, we will only consider connected sets of remaining customers.

Note that $f(\theta)$ log-concave implies that $F(\theta)$ and $1 - F(\theta)$ are log-concave (Bagnoli and Bergstrom (2005)); hence, $F_i(\theta_i)$ is log-concave for $i = I, E$.

Firm i's maximization problem is

$$\max_{p_i} \ \pi_i(p_i, p_j; \underline{\theta}_i) = \max_{p_i} \ (p_i - c_i) \left[F_i(\min\{\hat{\theta}_i(p_i, p_j), \theta_i^+(p_i)\}) - F_i(\underline{\theta}_i) \right],$$

which is well-defined, as the log-concavity of $F_i(\theta_i)$ (a non-negative and increasing function) ensures that the objective function is log-concave (see Appendix **??**), and therefore quasi-concave.

For a fixed $\underline{\theta}_i$, i's stage-game best-response, $R_i^s(p_j)$,[1] exhibits a *shared* region, for low values of p_j; possibly a *kink* region (where $\hat{\theta}_i = \theta_i^+$), for intermediate values of p_j; and a *monopoly* region, for large values of p_j. Formally, the three ranges are defined by the F.O.C. as follows: let $\tilde{p}_i(p_j)$ be the price such that the customers who derive zero utility from i and j coincide, that is, $\theta_i^+(\tilde{p}_i(p_j)) = \theta_j^+(p_j)$;[2] then

- Shared best-response: $R_i^s(p_j) < \tilde{p}_i(p_j)$, given by

$$F_i(\hat{\theta}_i(R_i^s(p_j), p_j)) - F_i(\underline{\theta}_i) - \frac{(R_i^s(p_j) - c_i)}{2\tau} f_i(\hat{\theta}_i(R_i^s(p_j), p_j)) = 0.$$

- Kink best-response: $R_i^s(p_j) = \tilde{p}_i(p_j)$ and

$$F_i(\hat{\theta}_i(\tilde{p}_i(p_j), p_j)) - F_i(\underline{\theta}_i) - \frac{(\tilde{p}_i(p_j) - c_i)}{2\tau} f_i(\hat{\theta}_i(\tilde{p}_i(p_j), p_j)) \geq 0,$$

$$F_i(\theta_i^+(\tilde{p}_i(p_j))) - F_i(\underline{\theta}_i) - \frac{(\tilde{p}_i(p_j) - c_i)}{\tau} f_i(\theta_i^+(\tilde{p}_i(p_j))) \leq 0.$$

- Monopoly best-response: $R_i^s(p_j) = p_i^m(\underline{\theta}_i)$, given by

$$F_i(\theta_i^+(p_i^m(\underline{\theta}_i))) - F_i(\underline{\theta}_i) - \frac{(p_i^m(\underline{\theta}_i) - c_i)}{\tau} f_i(\theta_i^+(p_i^m(\underline{\theta}_i))) = 0.$$

For the shared region, log-concavity ensures that $R_i^s(p_j, \underline{\theta}_i)$ is increasing in p_j, with slope lower than 1; and decreasing in $\underline{\theta}_i$. In the kink region, the stage-game best-response is increasing in p_j with slope equal to 1; and invariant with respect to $\underline{\theta}_i$. Whereas in the monopoly region, $p_i^m(\underline{\theta}_i)$ does not depend on p_j, but is decreasing in $\underline{\theta}_i$, due to the log-concavity. Notice that $R_i^s(p_j, \underline{\theta}_i)$ is continuous in p_j.

For the following proofs, we let $\underline{\theta}_E \equiv \underline{\theta}$ and $\underline{\theta}_I = 0$.

[1] Whenever θ_i is fixed, R_i^s's dependency on it will be dropped.

[2] This will satisfy the following condition as well: $\hat{\theta}_i(\tilde{p}_i(p_j), p_j) = \theta_i^+(\tilde{p}_i(p_j))$.

Appendix D

Proofs of Chapter 2

Proof of Proposition 2.1. Moving away from the interior equilibrium of Assumption 2.2, the slope of the stage-game best-responses are strictly lower than 1 (lower than 1 in the shared region—due to log-concavity of $f(\cdot)$—, negative 1 in the kink region and 0 in the monopoly region). As the stage-game best responses are continuous, this is sufficient for uniqueness. ■

Proof of Proposition 2.2. Starting with $\underline{\theta} = 0$, Assumption 2.2 guarantees a unique and shared equilibrium. As $\underline{\theta}$ grows larger, E's stage-game best-response is shifted downward, and due to strategic complementarity (for the interior range), I's best-response is lower, preserving an shared equilibrium, and therefore unique, by the same argument as in the proof of Proposition 2.1, with full-market coverage.

The equilibrium price for I can be expressed as

$$\begin{aligned} p_I^s(\underline{\theta}) &= R_I^s(p_E^s(\underline{\theta})), \\ &= R_I^s(R_E^s(p_I^s(\underline{\theta}), \underline{\theta})), \end{aligned}$$

whereas the equilibrium price for E is given by

$$\begin{aligned} p_E^s(\underline{\theta}) &= R_E^s(p_I^s(\underline{\theta}), \underline{\theta}), \\ &= R_E^s(R_I^s(p_E^s(\underline{\theta})), \underline{\theta}). \end{aligned}$$

Taking derivatives against $\underline{\theta}$, we obtain

$$\frac{\partial p_I^s}{\partial \underline{\theta}} = \frac{\frac{\partial R_I^s}{\partial p_E} \cdot \frac{\partial R_E^s}{\partial \underline{\theta}}}{1 - \frac{\partial R_E^s}{\partial p_I} \cdot \frac{\partial R_I^s}{\partial p_E}} < 0, \tag{D.1}$$

$$\frac{\partial p_E^s}{\partial \underline{\theta}} = \frac{\frac{\partial R_E^s}{\partial \underline{\theta}}}{1 - \frac{\partial R_E^s}{\partial p_I} \cdot \frac{\partial R_I^s}{\partial p_E}} < 0, \tag{D.2}$$

and therefore equilibrium prices are strictly decreasing in $\underline{\theta}$.

Finally, firm i's equilibrium profit can be expressed as

$$\pi_i^s(\underline{\theta}) = \max_{p_i} \, (p_i - c_i)[F_i(\hat{\theta}_i(p_i, p_j^s(\underline{\theta}))) - F_i(\underline{\theta}_i)],$$

and by the Envelope Theorem

$$\frac{d\pi_i^s}{d\underline{\theta}}(\underline{\theta}) = (p_i^s(\underline{\theta}) - c_i)\Big[f_i(\hat{\theta}_i)\frac{1}{2\tau}\frac{dp_j^s}{d\underline{\theta}}(\underline{\theta}) - f_i(\underline{\theta})1_{i=E}\Big] \leq 0 \quad (< 0 \text{ if } p_i^s(\underline{\theta}) > c_i).$$

■

Proof of Lemma 2.1. We drop state and price dependencies whenever it is unambiguous. Observe first that the expected continuation payoff is strictly positive for all customers, since in equilibrium all customers derive a strictly positive stage-game utility. If $\boldsymbol{p}$ leads to a full-market coverage in the sense of $\theta_E^+ \geq \theta_I^+$, then no customer exits the market as its current period utility is non-negative and continuation payoff is strictly positive. However, if $\boldsymbol{p}$ does not lead to full-market coverage (i.e., $\theta_E^+ < \theta_I^+$), then there are two cases to analyze: $\hat{\theta}^s \notin (\theta_E^+, \theta_I^+)$ and $\hat{\theta}^s \in (\theta_E^+, \theta_I^+)$, where $\hat{\theta}^s(\underline{\theta})$ is the indifferent customer under stage-game equilibrium prices. Regardless of the case, observe that all customers $\theta \notin (\theta_E^+, \theta_I^+)$ obtain a strictly positive net present value (NPV). We will show that in each case customers $\theta \in (\theta_E^+, \theta_I^+)$ derive a strictly positive NPV as well, and therefore no customer exits the market.

Consider the first case ($\hat{\theta}^s \notin (\theta_E^+, \theta_I^+)$), and suppose without loss of generality that $\hat{\theta}^s \geq \theta_I^+$, so all $\theta \in (\theta_E^+, \theta_I^+)$ would buy from E in equilibrium (if $\hat{\theta}^s \leq \theta_E^+$ the argument is symmetric). As argued, θ_E^+ has a positive NPV, and the difference in NPV between θ and θ_E^+ is only affected by their distance, observing that both customers would buy from E in the continuation equilibrium. This difference is given by

$$-\tau(\theta - \theta_E^+) + \frac{\delta}{1-\delta}\tau(\theta - \theta_E^+) = \frac{2\delta - 1}{1-\delta}\tau(\theta - \theta_E^+) > 0, \tag{D.3}$$

as $\delta \geq 1/2$. Hence, all $\theta \in (\theta_E^+, \theta_I^+)$ have a positive NPV.

In the second case ($\theta \notin (\theta_E^+, \theta_I^+)$), assume the condition over $p_E^s(\underline{\theta})$ holds (the other case is analogous). The customer with the least continuation payoff is $\hat{\theta}^s(\underline{\theta})$. Its per-period continuation payoff is

$$u_E^s(\hat{\theta}^s|p_E^s) = u_I^s(\hat{\theta}^s|p_I^s) \equiv u^s(\hat{\theta}^s) = v - p_E^s - \tau\hat{\theta}^s > v - \tilde{p}_E - \tau\hat{\theta}^s = \tau(\theta_I^+ - \hat{\theta}^s), \quad \text{(D.4)}$$

where the inequality stems from the Lemma's condition and the equality is because

$$u_I(\theta_I^+|p_I) = 0 = u_E(\theta_I^+|\tilde{p}_E) = v - \tilde{p}_E - \tau\theta_I^+.$$

The net present value of $\hat{\theta}^s$ when buying from I in the current period is strictly positive. In effect,

$$\begin{aligned}
NPV_I(\hat{\theta}^s) &= v - p_I - \tau(1-\hat{\theta}^s) + \frac{\delta}{1-\delta}u^s(\hat{\theta}^s), && \text{(D.5)}\\
&= -\tau(\theta_I^+ - \hat{\theta}^s) + \frac{\delta}{1-\delta}u^s(\hat{\theta}^s), && \text{(D.6)}\\
&> \frac{2\delta - 1}{1-\delta}\tau(\theta_I^+ - \hat{\theta}^s), && \text{(D.7)}\\
&> 0. && \text{(D.8)}
\end{aligned}$$

For $\theta \in (\hat{\theta}^s, \theta_I^+)$, its continuation payoff is larger than that of $\hat{\theta}^s$, as well as its current period utility (buying from I). Hence, its NPV is positive as well. For $\theta \in (\theta_I^+, \hat{\theta}^s)$, its NPV is also positive by the same argument as in the first case (in equation (D.3) replace θ by $\hat{\theta}^s$ and θ_E^+ by θ). ■

Proof of Corollary 2.2. There are two possible cases. If $\hat{\theta}^s \geq \theta_I^+$, then $p_E^s < \tilde{p}(p_I^m) < \tilde{p}(p_I)$, and Lemma 2.1 applies. Otherwise, if $\hat{\theta}^s < \theta_I^+$, then $p_I^s < p_I^m$, which implies (together with strategic complementarity and Proposition 2.2) that $p_E^s = R_E^s(p_I^s) < R_E^s(p_I^m) < \tilde{p}_E(p_I^m) \leq \tilde{p}_E(p_I)$, and again Lemma 2.1 applies. ■

Appendix E

Log-concavity

Proposition E.1 *The demands* $D_1(p) = F_i(\hat{\theta}_i(p, p_j)) - F_i(\underline{\theta}_i)$ *and* $D_2(p) = F_i(\theta_i^+(p)) - F_i(\underline{\theta}_i)$ *are both log-concave in* p.

Proof. The log-concavity of $f(\theta)$ implies that $F(\theta)$ and $1 - F(\theta)$ are log-concave (Theorems 1 and 3, Bagnoli and Bergstrom), therefore $F_i(\theta)$ is log-concave for $i = I, E$. As $\hat{\theta}_i(\cdot, p_j)$ and $\theta_i^+(\cdot)$ are linear, it follows that $F_i(\hat{\theta}_i(p, p_j))$ and $F_i(\theta_i^+(\cdot))$ are log-concave in p (Corollary 5, Bagnoli and Bergstrom—log-concavity is preserved under linear transformations).

By the same preservation argument, $D_1(p)$ and $D_2(p)$ is log-concave as the term $-F_i(\underline{\theta}_i)$ corresponds just to a linear transformation. ■

Appendix F

Proofs of Chapter 3

Proof of Proposition 3.1. For $v < \overline{v}^{(1)}(\delta) \leq \overline{v}^{(2)}(\delta)$, as $\delta \geq \overline{\delta}$, the equilibrium path is such that the market is never cornered (in neither period). The only deviations necessary to be considered are towards lower prices such that the second-period market is cornered, since any other price is not a best-response as it would not satisfy the interior optimality condition. However, such deviations are not feasible. For firm 1 to force such continuation in the second period, it would have to set a first-period price, p_1^d, such that $v + d^{(2)}(p_1^d, p_2^{(1)}) > 3t$. However, the lowest p_1^d such that $d^{(2)d} \equiv d^{(2)}(p_1^d, p_2^{(1)})$ is affected is $\underline{p}_1^d$, and satisfies $\hat{x}^{(1)d} \equiv \hat{x}^{(1)}(\underline{p}_1^d, p_2^{(1)}) = 1$, which yields $d^{(2)d} = (1-\kappa)d^{(1)} + \omega$ (any lower p_1^d is clearly dominated by $\underline{p}_1^d$). Then, as $v \leq \overline{v}^{(1)}(\delta)$, we have

$$v + d^{(2)d} \leq \frac{B - [9t + 4(1-\kappa)\delta\omega]d^{(1)}}{9t + 4\delta\omega} + (1-\kappa)d^{(1)} + \omega = 3t\Big[\underbrace{\frac{9t + 3\omega - 3\kappa d^{(1)}}{9t + 4\delta\omega}}_{\leq 1 \text{ as } \delta \geq \overline{\delta}}\Big] \leq 3t.$$

Therefore, it is not feasible for firm 1 to deviate and corner the second-period market. Similarly, for firm 2 to corner the second-period market, it requires $v + d^{(2)d} < -3t$. The lowest possible $d^{(2)d}$ for firm 2 is with a price such that $\hat{x}^{(1)}(p_1^{(1)}, p_2^d) = 0$, which yields $d^{(2)d} = (1-\kappa)d^{(1)} - \omega$. Then

$$v + d^{(2)d} \geq d^{(2)d} \geq -\omega > -3t,$$

since $v \geq 0$, $d^{(1)} \geq 0$, and $\omega < \frac{3\sqrt{3}}{2}t < 3t$, respectively. It follows that it is not possible for firm 2 to corner the second-period market.

For $v \in (\overline{v}^{(1)}(\delta), \hat{v}^{(2)}]$, we consider first deviations for firm 1. Since at the proposed equilibrium the market is cornered in the first period (i.e., $\hat{x}^{(1)} = 1$), a lower price only reduces current period's profit, while not affecting the continuation profit: not a profitable deviation. A higher price would bring the first period to a shared market. As $\overline{p}_1^{(1)} = v + d^{(1)} + \overline{p}_2^{(1)} - t$, the profit's right-derivative (for higher prices) is (analogous to FOC (3.8))

$$\frac{1}{2} + \frac{t - \overline{p}_1^{(1)}}{2t} - \frac{\delta\omega}{3t}\left[1 + \frac{v + (1-\kappa)\, d^{(1)} + \omega}{3t}\right],$$

which we know is equal to 0 for $v = \overline{v}^{(1)}(\delta)$. As v increases, the previous expression becomes negative, therefore a higher price is not a profitable deviation. As for firm 2, a larger price does not change profits: $\hat{x}^{(1)} = 1$ would still hold. A lower price would mean a shared first-period market. Notice that $\overline{p}_1^{(1)} - v - d^{(1)} = \overline{p}_2^{(1)} - t \equiv C$, a constant term that does not depend on v. Therefore, firm 2's left-derivative (for lower prices) is (in a similar way as for firm 1 previously)

$$\frac{1}{2} + \frac{C - 2\overline{p}_2^{(1)}}{2t} - \frac{\delta\omega}{3t}\left[1 - \frac{v + (1-\kappa)\, d^{(1)} + \omega}{3t}\right],$$

which equals 0 for $v = \overline{v}^{(1)}(\delta)$. As v increases, it becomes positive, thus a lower price is not profitable.

Finally, for $v > \hat{v}^{(2)}$, the argument is the same as for the previous range, noting that firm 1's second-period profit is now steeper (with respect to changes in $d^{(2)}$); whereas the opposite is true for firm 2. In effect, for $v > \hat{v}^{(2)}$, firm 1's second-period profit has a slope of 1, whereas its largest second-period profit slope for $v \in (\overline{v}^{(1)}(\delta), \hat{v}^{(2)}]$ is 2/3. Similarly, for $v > \hat{v}^{(2)}$, firm 2's second-period profit slope is equal to 0, same as its lowest second-period profit slope for $v \in (\overline{v}^{(1)}(\delta), \hat{v}^{(2)}]$. Therefore, if there were profitable deviations for $v > \hat{v}^{(2)}$, then there would exist a profitable deviation for $v \in (\overline{v}^{(1)}(\delta), \hat{v}^{(2)}]$, as in the latter firm 1's (firm 2's) second-period profit decreases less (increases more) than in the former. A contradiction. ■

Proof of Proposition 3.2. First, for $v \leq \hat{v}^{(2)} < \overline{v}^{(2)} < \overline{v}^{(1)}$, the equilibrium is interior in both periods. Firm 1 cannot deviate and corner the second-period market. In effect, the largest continuation data advantage is when the first-period market is cornered, and equals $d^{(2)d} = (1-\kappa)d^{(1)} + \omega$, which leads to $v + d^{(2)d} \leq \hat{v}^{(2)} + d^{(2)d} = 3t$, thus no cornering in second-period market. Therefore, firm 1 has no profitable deviation, as within the interior strategies, $p_1^{(1)}$ is optimal. As for firm 2, the argument is the same as in Proposition 3.1, when $v < \overline{v}^{(1)}$.

Second, for $v = \overline{v}^{(2)}$, the interior equilibrium strategies are not an equilibrium as firm 1 has incentives to deviate to a lower price. In effect, if firm 1 lowers its price, the second-period market will be cornered. Therefore, after algebraic manipulation, the left-hand derivative (with respect to p_1) of its overall profit, evaluated at the interior equilibrium strategies, is

$$\frac{\partial_{-}\Pi_1}{\partial p_1}(\boldsymbol{p}^{(1)}) = \frac{t + v + d^{(1)} + p_2^{(1)} - 2p_1^{(1)}}{2t} - \frac{\delta\omega}{t} = -\frac{\delta\omega}{3t} < 0,$$

and thus firm 1 prefers to deviate to a lower price (observe that since $\overline{v}^{(2)(\delta)} < \overline{v}^{(1)}(\delta)$ for $\delta < \overline{\delta}$, there is always room for firm 1 to lower its price while not cornering the first-period market, so the above derivative is indeed to relevant one).

It follows that there must exist a threshold $\tilde{v}^{(2)}(\delta) \in [\hat{v}^{(2)}, \overline{v}^{(2)}(\delta)]$ up to which the interior equilibrium strategies given by $\boldsymbol{p}^{(1)}$ are indeed an equilibrium, and above which it ceases to exist. ■

Appendix G

Infinite horizon – Shared-market regime

In the following formulations, for a variable y, $\dot{y}$ represents its derivative.

Firm 1 solves

$$V(d) = \max_{p_1} \left[\frac{1}{2} + \frac{v + d + p_2(d) - p_1}{2t}\right] p_1 + \delta V \left((1-\kappa)d + \frac{\omega}{t}(v + d + p_2(d) - p_1)\right), \tag{G.1}$$

whose FOC is

$$\frac{t + v + d + p_2(d) - 2p_1(d)}{2t} - \frac{\delta\omega}{t}\dot{V}\left((1-\kappa)d + \frac{\omega}{t}(v + d + p_2(d) - p_1(d))\right) = 0, \tag{G.2}$$

and its envelope condition is

$$\dot{V}(d) = \frac{1 + \dot{p}_2(d)}{2t} p_1(d) + \delta \left[1 - \kappa + \frac{\omega}{t}(1 + \dot{p}_2(d))\right] \dot{V}\left((1-\kappa)d + \frac{\omega}{t}(v + d + p_2(d) - p_1(d))\right). \tag{G.3}$$

For firm 2 the conditions are analogous, minding the anti-symmetry. In particular, it is easy to verify that $V_2 = W_2$. Once we consider the specified functional forms for strategies and value functions, these conditions become polynomial equations and we can equate the coefficients, whereby we obtain the following system of equations that allows us to numerically retrieve z:

$$
\begin{cases}
V_2 &= \dfrac{(1-z)z}{2t\left[1-\delta\left(1-\kappa+\dfrac{\omega}{t}(1-z)\right)\left(1-\kappa+\dfrac{\omega}{t}(1-2z)\right)\right]}, \\
z &= \dfrac{\dfrac{1-z}{2}-\delta\omega\left(1-\kappa+\dfrac{\omega}{t}\right)V_2}{\left(1-\dfrac{2\delta\omega^2 V_2}{t}\right)}.
\end{cases}
\tag{G.4}
$$

References

Abito, J. M. and J. Wright (2008, January). Exclusive dealing with imperfect downstream competition. *International Journal of Industrial Organization 26*(1), 227–246.

Aghion, P. and P. Bolton (1987). Contracts as a Barrier to Entry. *The American Economic Review 77*(3), 388–401.

Asker, J. and H. Bar-Isaac (2014, February). Raising Retailers' Profits: On Vertical Practices and the Exclusion of Rivals. *The American Economic Review 104*(2), 672–686.

Bagnoli, M. and T. Bergstrom (2005). Log-concave probability and its applications. *Economic theory 26*(2), 445–469.

Bain, J. S. (1949). A note on pricing in monopoly and oligopoly. *The American Economic Review*, 448–464.

Bedre-Defolie, Ö. and R. Nitsche (2020). When do markets tip? an overview and some insights for policy. *Journal of European Competition Law & Practice 11*(10), 610–622.

Biglaiser, G., E. Calvano, and J. Crémer (2019). Incumbency advantage and its value. *Journal of Economics & Management Strategy 28*(1), 41–48.

Bork, R. (1978). The antitrust paradox. *Basic Books*.

Carlton, D. W. and M. Waldman (1998, December). The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries. Working Paper 6831, National Bureau of Economic Research. DOI: 10.3386/w6831.

Castanheira, M., C. Ornaghi, and G. Siotis (2019). The unexpected consequences of generic entry. *Journal of Health Economics 68*, 102243.

Chen, Z. and G. Shaffer (2014, March). Naked exclusion with minimum-share requirements. *The RAND Journal of Economics 45*(1), 64–91.

Ching, A. T. (2010). Consumer learning and heterogeneity: Dynamics of demand for prescription drugs after patent expiration. *International Journal of Industrial Organization 28*(6), 619–638.

Coase, R. H. (1972). Durability and monopoly. *The Journal of Law and Economics 15*(1), 143–149.

Commission, E., D.-G. for Competition, Y. Montjoye, H. Schweitzer, and J. Crémer (2019). *Competition policy for the digital era*. Publications Office.

Condorelli, D. and J. Padilla (2020). Data-driven envelopment with privacy-policy tying. *August 31*, 2020.

Conlon, C. T. and J. H. Mortimer (2013). Efficiency and Foreclosure Effects of Vertical Rebates: Empirical Evidence. NBER Working Paper 19709, National Bureau of Economic Research, Inc.

De Corniere, A. and G. Taylor (2021). Data and competition: A simple framework, with applications to mergers and market structure. Technical report, mimeo.

Friedman, J. W. (1979). On entry preventing behavior and limit price models of entry. In *Applied Game Theory: Proceedings of a Conference at the Institute for Advanced Studies, Vienna, June 13–16, 1978*, pp. 236–253. Springer.

Fumagalli, C. and M. Motta (2006). Exclusive Dealing and Entry, when Buyers Compete. *The American Economic Review 96*(3), 785–795.

Gaskins, D. W. (1971, September). Dynamic limit pricing: Optimal pricing under threat of entry. *Journal of Economic Theory 3*(3), 306–322.

Glicksberg, I. L. (1952). A Further Generalization of the Kakutani Fixed Point Theorem, with Application to Nash Equilibrium Points. *Proceedings of the American Mathematical Society 3*(1), 170–174.

Hagiu, A. and J. Wright (2023 (forthcoming)). Data-enabled learning, network effects and competitive advantage. *The RAND Journal of Economics*.

Inderst, R. and G. Shaffer (2010, December). Market-share contracts as facilitating practices. *The RAND Journal of Economics 41*(4), 709–729.

Innes, R. and R. J. Sexton (1994). Strategic buyers and exclusionary contracts. *The American Economic Review*, 566–584.

Kamien, M. I. and N. L. Schwartz (1971). Limit pricing and uncertain entry. *Econometrica: Journal of the Econometric Society*, 441–454.

Kolay, S., G. Shaffer, and J. A. Ordover (2004). All-units discounts in retail contracts. *Journal of Economics & Management Strategy 13*(3), 429–459.

Milgrom, P. and J. Roberts (1982, March). Limit Pricing and Entry under Incomplete Information: An Equilibrium Analysis. *Econometrica 50*(2), 443.

Ordover, J. A. and G. Shaffer (2013, September). Exclusionary discounts. *International Journal of Industrial Organization 31*(5), 569–586.

Petit, N. and N. Moreno Belloso (2021). A simple way to measure tipping in digital markets. *ProMarket, April 6*.

Posner, R. (1976). Antitrust law: An economic perspective. *University of Chicago Press*.

Prüfer, J. and C. Schottmüller (2021). Competing with big data. *The Journal of Industrial Economics 69*(4), 967–1008.

Rasmusen, E. B., J. M. Ramseyer, and J. S. Wiley (1991). Naked Exclusion. *The American Economic Review 81*(5), 1137–1145.

Regan, T. L. (2008). Generic entry, price competition, and market segmentation in the prescription drug market. *International Journal of Industrial Organization 26*(4), 930–948.

Scherer, F. M. (1993). Pricing, profits, and technological progress in the pharmaceutical industry. *Journal of Economic Perspectives 7*(3), 97–115.

Segal, I. R. and M. D. Whinston (2000). Naked Exclusion: Comment. *The American Economic Review 90*(1), 296–309.

Simpson, J. and A. L. Wickelgren (2007). Naked Exclusion, Efficient Breach, and Downstream Competition. *The American Economic Review 97*(4), 1305–1320.

Tirole, J. (2016). From Bottom of the Barrel to Cream of the Crop: Sequential Screening With Positive Selection. *Econometrica 84*(4), 1291–1343.

Toxvaerd, F. (2017). Dynamic limit pricing. *The RAND Journal of Economics 48*(1), 281–306.

Treasury, H. et al. (2019). Unlocking digital competition, report of the digital competition expert panel (2019).

Vandoros, S. and P. Kanavos (2013). The generics paradox revisited: empirical evidence from regulated markets. *Applied Economics 45*(22), 3230–3239.