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in highly regulated economies”

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Abstract

The existence of transaction taxes reduces transactions, and in the case of housing, reduces household mobility and affects the costs of downsizing in dire times. We construct and estimate an overlapping generation model in which households are heterogeneous in age and earnings, and prudential regulation and the tax system are modeled in fine detail. These housing and public policies are likely to affect markets globally, and clearing both rental and property markets is important when evaluating them. We use the institutional and data setting of France, where transactions taxes are some of the highest in Europe, and evaluate the counterfactual impact of reducing transaction taxes from 14% to 6%, similar to US levels. The impact on transactions is strong, but the impact on welfare remains limited.

Keywords: Heterogenous agents, dynamic structural models, general equilibrium, housing, transaction taxes.

JEL codes: C68, D15, D58, H31, R21, R31

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1 Introduction¹

Although it has been known for some time (e.g., Diamond and Mirrlees 1971) that taxes on real estate transactions can have a significant distorting effect on these transactions and homeownership choices, these taxes have retained their appeal for decision makers with respect to, say, property taxes. Real estate transaction prices are indeed much easier to observe accurately than are property values when properties have not been on the market for a while (OECD, 2022). As transaction taxes remain prevalent, increasing attention has been paid in the recent economic literature to an analysis of the impacts of transaction taxes on the real estate market. Their main distortion might be their adverse effect on residential mobility and, therefore, indirectly on the labor market, making it more difficult for unemployed individuals to accept a job in other regions (Causa and Pichelmann, 2020). Namely, one can expect such transaction taxes to induce mismatched housing situations; for example, households might be discouraged from downsizing after a decrease in income or household size (a “lock-in” effect; see Slemrod et al. 2017) while young households may give up buying a home at an early stage. In turn, these effects distort the ratio of prices across rental and owner markets by increasing the demand on the former and reducing it on the latter (Han et al., 2022).

Economic evaluations of the effects of transaction taxes on transactions and tenure mostly use natural experiments and provide short- and medium-term estimates of treatment effects (e.g., Dachis et al. 2012; Best and Kleven 2018; Slemrod et al. 2017). When policy changes remain at a small scale, for instance, at the city level in Dachis et al. (2012), effects on the government budget or general equilibrium aspects are negligible. However, most changes in housing tax and benefit policies, such as an alteration to transaction taxes, are likely to involve equilibrium effects because they cause housing prices and rents to adjust to changes in demand and supply in both housing and rental markets. Moreover, their evaluation is likely to involve dynamic effects for agents. Buying a house is an investment, and it is key to consider life-cycle effects in which households move up and down the housing ladder—that is, they buy a larger or smaller house depending

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on the evolution of their income and the size of their family. Sommer and Sullivan (2018) is the seminal paper that develops an overlapping generation dynamic model with heterogeneous agents in which housing policies can be evaluated in a general equilibrium framework calibrated using US data (see also Cho et al. 2021, that we review below).

In this paper, we extend this line of research that models household choices of consumption, housing services, tenure, and housing transactions over their life cycle of overlapping generations of households with heterogeneous earnings. Households can accumulate financial assets through bank deposits and have access to mortgage financing. Housing and rental markets are balanced by housing prices and rents, which are linear in size.

Our first original contribution is to derive the model's solution in a heavily regulated economic environment. Specifically, the heavy regulation refers to the household credit used to finance housing investments. In our model, prudential regulations take three forms: loan-to-value, or downpayment, constraint (fixed at 20%); maximal payment-to-income ratio (say, 30%); and minimum repayment rule per period (say, 5%). These forms account for the prevalence of fixed-duration loans. Dealing with these prudential regulations make this dynamic and heterogeneous economic model quite intricate to solve.

The solution to this dynamic model is achieved by designing algorithms that find the fixed point of the value functions first and balancing rental and owner markets second. Our next contribution is that we address the nonconvexities arising because of discrete choices in this dynamic model using the elegant solution proposed by Karaivanov and Townsend (2014). These authors argue that households can have access to random lotteries by which they increase their welfare and allow nonconvexities to be smoothed out. We also show how to adapt the technique of endogenous grids proposed by Carroll (2006) to solve the model in the continuous state variable in the case of many prudential regulations that constrain consumption choices, housing services and housing investment behavior.

Our third contribution is to use data from a country in which transaction taxes are extremely high. Housing transaction taxes that comprise transfer taxes, fees paid to brokers and recording agencies can be as high as 14% of property prices in Belgium, France or Greece (Andrews et al., 2011), a rate three times higher than what is observed in northern European countries. Consequently, France is a good setting in which to experiment with a general equilibrium evaluation of

the impact of transaction taxes on a rich set of outcomes.² Furthermore, we adapt to this specific environment by accounting for not only the progressive tax schedule and inheritance taxes but also other government housing policies, such as the generous housing allowances for low-income households for both tenants and homeowners with a mortgage.

We estimate preference and cost parameters using the method of simulated moments. Preference parameters include risk aversion, preferences for housing services and ownership, a mobility cost for owners and a maintenance cost for landlords. Parameters are fitted using empirical moments describing ownership and landlordship rates, mobility between renting and ownership and down or upsizing, as well as the share of rents in income for renters. Those empirical counterparts are extracted from the French Housing Survey collected in 2006. The distributions of household earnings and demographics are calibrated, as are other parameters that index taxes and prudential regulations.

We can finally turn to the counterfactual evaluation of the impact of a decrease in transaction taxes using the fitted model. More precisely, transaction costs are reduced from 8% when buying and 6% when selling to 3% in both cases to match the usually retained values in northern European countries (Andrews et al., 2011). We keep the government budget fixed by using several tax schemes and compare the steady states before and after the change to evaluate the long-run effects of such a change in taxes. It is very unlikely that our results could be obtained using natural experiments. In our main scenario, we balance the budget by using a proportional tax on properties, a quite popular proposal in the recent literature (Bonnet et al., 2021). Another scenario adjusts budgets using a proportional tax on earnings, which is mostly nondistortive in our setting. We also study the robustness of the results in changes in housing supply in a third counterfactual in which we assume a low elasticity of supply equal to 0.4. This low elasticity of supply is one of the main explanations of the movement of the rent-to-price ratio (beyond changes in interest rates and irrational bubbles, cf. Glaeser et al. 2008) and an explanation particularly attractive in the case of France, according to Caldera and Johansson (2013).

Quite expectedly, reducing transaction taxes increases access to ownership, from 53.1% to 54.9% in the main scenario, which balances the budget through the increase in property taxes.

²With a slight abuse of language, we refer to this as a “transaction tax” because most of its elements determined by tax policies; however, it must be recognized that part of this figure covers costs (in particular, those linked to a lack of competition in the services sector).

This effect on the homeownership rate is much weaker than in a few similar studies (Ma and Zubairy, 2021; Schmidt, 2022; Kaas et al., 2021). This can be explained by the fact that we balance both the owning and rental markets, making prices and rents endogenous, whereas these papers assume that the rental market is competitive—as a consequence, the relation between rents and prices is linear. Indeed, in our estimations, reduced transaction taxes results in less demand for rentals so that prices and rents undergo a change in opposite directions—an increase of 2.4% in housing prices and a decrease in rents of 2.0% in our benchmark counterfactual. When balancing the budget is based on a nondistorting tax, the increase in housing prices is more than twice that observed in our benchmark counterfactual, while the ownership rate increases to 55.5%. The orders of magnitude obtained regarding price changes are similar to those obtained by Cho et al. (2021), who also considers that house prices and rents freely adjust. We obtain a lesser effect on the homeownership rate than did the latter paper, which can be explained by the fact that we model a greater set of credit constraints arising from prudential regulations.

The decrease in transaction taxes leads to a strong increase in trading volume. In particular, the frequency at which a tenant (resp. an owner) accesses ownership (resp. goes back to renting) increases by 40% (resp. is multiplied by three). This finding indicates that a reduction in transaction taxes allows households to better manage an adverse shock to their income by selling their property and becoming a renter again. In other words, lower transaction taxes make houses more liquid in the sense of Kaplan et al. (2014). By anticipation, this flexibility leads them to opt for homeownership at a younger age. Thus, homeownership rates decline sharply among poor, young households (from 11% to 6%) and increase sharply among wealthy, young households (from 59% to 88%). This better match between housing demands and the needs and financial capacities of young households has a positive effect on their well-being. Their average utility increases in all three counterfactual scenarios, while the utility of older households decreases due to higher housing prices. When considering the expected utility of newborns, the former positive effect prevails in counterfactual A with a positive impact of +0.2%. This welfare impact is twice as large when we assume an elastic supply of housing and negative when the budget is balanced using a proportional tax on earnings.³ The fact that balancing the budget using a supposedly less distortive

³Notably, in our study, we do not account for a potentially important effect of transaction costs, namely, a reduction in mobility on the labor market—see Oswald (1996); Halket and Vasudev (2014); Oswald (2019).

earning tax that makes the welfare impact negative is also observed in other studies (Cho et al., 2021; Schmidt, 2022). This observation is due here to the sharp increase in prices in this scenario, which penalizes homeowners and partly benefits institutional owners (e.g., banks). Quite surprisingly, the impact on inequality of the three counterfactuals—measured by Gini indices relating to consumption and wealth—is negligible. This reflects the fact that the effects observed are more differentiated according to generations than according to income level. More specifically, young households gain while older households lose, regardless of their income level.

The outline of the paper is as follows. We first complete this introduction with a brief review of the literature. Next, Section 2 presents the primitives of the model, which are household preferences, the tax and benefit system and prudential regulations. Section 3 presents the techniques and algorithms used to solve the model. In Section 4, we detail the empirical strategy used to estimate parameters using the simulated method of moments. Section 5 reports the results of the counterfactual experiments analyzed. Section 6 concludes.

1.0.1 Literature review

Different methods have been employed in the literature to assess the effect of transaction costs on real estate markets, from empirical approaches to dynamic general equilibrium life-cycle models. We successively review recent works based on the three main approaches.

Natural experiments

The empirical literature has recently estimated the effects of real estate transaction taxes on household behavior related to housing by exploiting natural experiments generated by policy changes in those taxes. The outcomes of interest are not only transaction levels, house prices and rents but also ownership and mobility rates.

These papers generally estimate short-term effects, among which the most salient are an anticipation of transactions before an increase (say) in transaction taxes, followed by a decline in the volume of transactions in the months that follow. The effects on prices are much more elusive. Regression discontinuity or difference-in-differences designs have been used in Canada (Dachis et al., 2012), France (Bérard and Trannoy, 2018), Germany (Fritzsche and Vandrei, 2019), the United Kingdom (Besley et al., 2014; Best and Kleven, 2018) and the US (Slemrod et al., 2017). The last paper is an exception in finding small or no effects on transaction levels. Dachis et al. (2012)

stands out in that it considers an unanticipated evolution of transaction costs. More precisely, in the paper, a natural experiment arises from the decision of Toronto’s local authorities to introduce a land transfer tax equal to approximately 1.1% of the sale price and equivalent to an increase of approximately 15% in mobility cost. According to this study, the transfer tax induced a 14% decrease in mobility and a decline in housing prices approximately equal to the tax.

Among recent studies, Eerola et al. (2021) exploit a tax reform in Finland that raised the transfer tax from 1.6% to 2% for apartments but not houses, a seemingly attractive framework with treated and controls. They argue, however, that this reform also affected the control group because of spillover effects at the market level between houses and apartments. Their main result is that the transfer tax increases on apartments from 1.5% to 2% induced a 7.2% reduction in mobility. The welfare loss is of the same order of magnitude as the additional tax revenue proxied by the tax change times the average housing price. They also show that the spillover effects on mobility rates are indeed significant and may explain that some quasiexperimental strategies may underestimate the effects of transfer taxes on mobility.

In Dolls et al. (2021), the authors use a very large database from Germany on house transactions between 2005 and 2019 and, as a quasiexperiment, a constitutional reform in 2006 that enables states to set their transaction taxes. This allows the authors to capture the medium- and long-term effects of reforms. They find a strong effect on prices—a one-point increase in taxes that implies a 3% increase in prices or a semielasticity of 3 (an effect called *overshifting*). Price increases are larger for apartments and in growing housing markets. The authors argue that sellers have enough bargaining power to pass through the tax increase and provide a simple bargaining model to rationalize these results. However, they do not examine the changes in ownership, which appear to account for the contrasting effect observed in other empirical studies and our own work—a decrease in transaction costs typically leading to an increase in house prices and ownership rates.

In our work, we also find a significant increase in mobility following a sharp decline in transaction taxes. More precisely, the rate of access to ownership increases by approximately 40%, whereas moving back to renting is increased by a multiple of three. In total, the number of transactions increases by approximately 20%, which is lower than what is observed in the natural experiments previously mentioned. This is likely due to the delayed adjustments in prices upward and rents downward, which natural experiments cannot completely account for.

Search and assignment models

Another strand of the literature models households' decisions to rent, own, move in and out in an assignment or search-and-matching setup whose parameters are calibrated or estimated. Such works point out a significant welfare loss induced by increasing transaction taxes. Määttänen and Terviö (2022) build an assignment model in which households are matched one-to-one with houses, and all households agree on the ranking of house values. Households may wish to change houses after an income shock, although they are refrained from doing so because of transaction taxes. As a result, some houses are misallocated, and welfare is degraded. These studies calibrate their models using data from the Helsinki metropolitan region. They consider a reform whereby the transaction tax whose value is 2% is replaced with a revenue-equivalent property tax. They observe a very significant impact on transaction volume, nearly twice what we obtain, which induces a Laffer effect—with the Laffer curve peaking at a transaction tax of approximately 10%. Welfare gains are moderate—approximately 13% of the tax revenue—although these gains quickly increase with transaction taxes. We do not find such an effect on welfare in our model because our measure of welfare impact considers the comparison between steady states. Thus, we observe a positive impact on newborns' expected utility and a negative effect on elderly and wealthy households' utility, resulting in an overall negative effect on the average utility of the whole population. In contrast, Määttänen and Terviö (2022) find a positive effect on rich and elderly households in particular because they benefit from rising prices during the transition phase.

Han et al. (2022) revisits the impact of natural experiment in Toronto in 2008 explored by Dachis et al. (2012) and reviewed above, in which transaction taxes are increased. However, the authors use a much longer time-span of 12 years. They show that transactions by owner occupiers decrease, that house prices decrease while rents increase and that buy-to-rent becomes more prevalent. To explain this evidence, they propose a search-and-matching model in which households are renters or owner occupiers, and investors rent homes. Owner and rental markets are both balanced by house prices and rents. Transaction taxes make renting more attractive, resulting in higher rents, lower house transactions and a lower ownership rate. We obtain the same results, although we accommodate intensive margins for housing services as consumption and prudential regulations. Specifically, their search-and-matching framework implicitly presumes that consumption cannot be smoothed over time. This absence might explain why our welfare

evaluations are much less sizeable than theirs.

General equilibrium models]

Last but not least, a strand of the literature modeling household life-cycle decisions in general equilibrium is closer to our paper. A vein of papers in partial equilibrium before 2014 has been summarized in Davis and Van Nieuwerburgh (2015). Furthermore, Floetotto et al. (2016) and Sommer and Sullivan (2018) embed household life-cycle models into overlapping general equilibrium frameworks calibrated using US data and analyze the impact of changes in housing policies, such as the tax deductibility of mortgage repayments.

A few recent papers continue in this direction. In particular, Ma and Zubairy (2021) calibrate a dynamic general equilibrium life-cycle model using the US Current Population Survey over a long period (1995–2015). Their model imposes prudential regulations in the form of loan-to-value and payment-to-income ratios. They predict the counterfactual impact of a decrease in transaction costs of 20% when buying and selling. The ownership rate increases and the housing size decreases when buying costs decrease, and the reverse occurs when selling costs decrease. Kaas et al. (2021) investigates the impact of tax changes on ownership in Germany, a country with a low ownership rate and large social housing. They are the first to model social housing, although its rents are pegged to rents in the private market and to prices because of a competitive private rental market. A strong decrease in transaction taxes is one counterfactual experiment that the authors analyze—from 5% to 0.33%. They predict very significant effects—and stronger than ours—in particular on the rate of ownership, which moves up by 6–14 percentage points across all working-age groups. Next, Schmidt (2022) build the same type of model calibrated on Dutch data and compare the effect of removing transfer duties depending on whether tax neutrality is achieved by an increase in income or property taxes. They obtain a result similar to ours on the welfare of newborns—a negative effect in the first case and a positive effect in the latter case. However, they obtain very different price movements from what we observe in our work—a strong increase in both housing prices and rents when the reform is financed by an increase in income taxes and the opposite of weak fluctuations when the reform is financed by an increase in property taxes.

These differences can be explained by the fact that we balance both owning and rental markets and make prices and rents endogenous, as in Sommer and Sullivan (2018). Those three papers above assume that the rental market is competitive and that real estate firms can freely enter. As

a consequence, the relation between rents and prices is linear, in contrast to the approach in this paper. The fact that we find significantly smaller effects on mobility and homeownership shows that general equilibrium effects on both house prices and rents play a considerable role and should not be neglected.

In this regard, the closest work to ours is Cho et al. (2021). They consider a life-cycle model but only with a loan-to-value prudential regulation. In contrast, they are the first to introduce random preference shocks in addition to income shocks that allow them to argue that the model better fits the mobility rates of households. They embed the life-cycle model within an overlapping generation model and balance both rental and owner markets, as we do. They calibrate the model using Australian data. They obtain similar results to ours regarding the effect on well-being as measured from the expected utility of newborns—a positive effect when the budget is balanced through an increase in property taxes and a negative effect when it is balanced through a general income tax. Kaplan et al. (2014) found indeed that Australia and France were close in terms of illiquid asset holding. What distinguishes our paper from theirs is that we account for a more diverse set of prudential constraints, including a minimal payment-to-income ratio and the minimum repayment constraint that prevents households from using mortgages to finance consumption. This may explain why we obtain the same order of magnitude regarding price movements (house prices and rents) but half the size of the effects on homeownership rates.

The second difference with Cho et al. (2021) is how we address the numerous technical issues in these nonconvex dynamic models. We argue that adding preference shocks only partly addresses the issue of nonconvexities. A few methods are proposed in the literature. Fella (2014) addresses this issue by checking whether the first-order condition is necessary and sufficient using the discretized value function and provides a more efficient algorithm than the original version that Carroll proposed. Iskhakov et al. (2017) show that sufficiently large preference shocks can help smooth out nonconvexities, as in Cho et al. (2021), although these conditions are difficult to check. We do not follow these routes and use the solution proposed by Karaivanov and Townsend (2014), in which the authors allow households to randomize across nonconvexities. This increases household welfare and makes value functions, concave.

2 A heterogenous-agent housing model

Our structural model is built on different blocks that we detail in this section. All model variables and parameters are listed and defined in Appendix A.1.

2.1 Demography and labor earnings

The population is composed of overlapping generations of households that age stochastically. Households are grouped into 4 age groups. For working households, the first three age groups range from 20 to 35 years, from 35 to 50, and from 50 to 65, while retired households form a group above 65. The probability of moving from one age category to the next, or aging, is equal to χ_1 , χ_2 and χ_3 , respectively, for the first three age categories, whereas the probability of dying (only for the fourth category) is equal to χ_4 . These parameters are calibrated by setting, for instance, life expectancy to 80 years (i.e., 60 periods in the model). When a household dies, it is replaced by a household in the first age category, and bequests are accidental but taxed, as described below.

Household earnings vary randomly between and within age groups. Within each age category, we consider three earnings groups—L(ower), M(edium) and U(pper)—that are defined with respect to thresholds of means-tested housing policies and whose values capture increasing and concave labor earnings (see Appendix A.2). Moving across earnings groups (for instance, L to M or U) within each age category is stochastic. In contrast, no moves are allowed across earnings groups within the last age category (retirement) since pensions are fixed. Moreover, earnings groups remain the same when a household ages, that is, moves to the next age category. Earnings of the young household that replaces a deceased household are randomly drawn in $\{L, M, U\}$ given the group of the latter. Both households thus form a “dynasty” in the particular sense of having accidental bequests and a human capital level, or earnings, passed on stochastically to the heir.

Stochastic transitions between age and earnings groups ensure that the population is stable and that the model is stationary. More precisely, the population size is set to unity, and each household belongs to an age and an earnings group. There are twelve such groups ($J = 4 \times 3$), and stochastic aging and earnings mobility are translated into a transition matrix of dimension 12. The values of the transition matrix are calibrated to match the stationary probabilities within each state as measured from the 2006 French Housing Survey and the transition matrix is derived from a single cohort of French workers in the private sector (Magnac et al., 2013). The details of this

procedure are provided in Appendix A.4. Household earnings levels are denoted as $\{w_k\}_{k=1,\dots,J}$.

2.2 Housing and preferences

We adopt the setup of Sommer and Sullivan (2018) with some minor variations in the structure.

House size and tenure House size, denoted h , takes K ordered discrete values, $\{h_1, \dots, h_K\}$, where $h_1 > 0$. Housing services, denoted s , are also discrete and belong to the set of values $\{h_1, \dots, h_{K-1}\}$. The exclusion of the last value is adopted to ensure that some owners are landlords since the last value of h , $h_K > s$ for all s .

Housing tenure takes three values: Renters are households who rent and do not own a house. We denote the size of the house that they own as $h = 0$, which is less than s , the housing services that they consume. Owner-occupiers own their home and no more ($h = s \geq h_1$). Landlords are households that invest in additional real estate ($h > s$). We suppose that landlords live in a house that they own such that owner-occupiers and landlords satisfy the restriction:

$$h \geq s \text{ if } h > 0. \quad (1)$$

Landlords receive a rent equal to $\rho(h - s)$, and the yearly rent that renters pay per square meter, ρ , is constant over s . House prices per square meter, denoted as q , are also constant for different areas, h . Buyers and sellers of a house whose size is h pay or receive qh before taxes.

We also include two additional parameters related to housing: parameter c_m enters the budget constraint and describes mobility costs for owners. Second, landlords pay a fixed cost ϕ for maintenance and management.

Preferences In each period, every household decides on its consumption level, c , the quantity of housing services, s , and the level of next period housing ownership, h' . The current period utility function of a household is given by:

$$U(c, s; h) = e \frac{(c/e)^{1-\sigma}}{1-\sigma} (s/e)^\alpha + \xi \min(h, s) \quad (2)$$

in which parameter $e > 0$ is an equivalence scale for each age category. Specification (2) imposes that utility is homogenous of degree one in current period expenditures, $c + \pi s$, in which π is the shadow or actual price of housing services. Parameter $\frac{\alpha}{1+\alpha-\sigma}$ measures the housing service share of expenditures, $(\sigma - \alpha)$ is the risk aversion parameter. Furthermore, parameter ξ captures the

preference for ownership—because when $s \leq h \neq 0$, homeowners do not obtain the same utility as that of renters ($h = 0$) for the same housing services. As owner-occupier housing maintenance costs are hardly identified from the ownership preference, parameter ξ stands for the ownership preference net of maintenance costs for owner-occupiers.

Equivalence scales, e , are hump-shaped over the lifecycle, and their calibrated values are displayed in Appendix A.2. The three other parameters are estimated. We impose the following constraints on these parameters to obtain a properly defined increasing and concave utility function:

$$\xi > 0, (\alpha, \sigma) \in (0, 1)^2 \text{ or } \sigma > 1, \alpha < 0.$$

2.3 Financial assets and mortgages

Households can accumulate financial assets through bank deposits at a fixed interest rate. The only borrowing channel is a mortgage for buying a house. The mortgage rate is r^m , while the deposit rate for financial assets is r . The ranking between these rates, $r^m > r$, rationally prevents a household that has a mortgage from having any financial deposit and conversely. Consequently, decisions can be cast in terms of net financial wealth denoted as b , which can be positive or negative. We impose $b \geq 0$ if $h = 0$ such that no borrowing for consumption motives is allowed. This implies that negative values for b are interpreted as the opposite of mortgage levels.

As one of our original contributions, we impose three types of restrictions on mortgages that make the resolution of the economic model slightly involved. The first two are macroprudential regulations on the loan-to-value (LTV) ratio or the payment-to-income (PTI) ratio (Kuttner and Shim, 2016). The third specifies that mortgages should be repaid within a fixed time frame (typically 15 or 20 years in Europe) and imposes minimal reimbursement in each period. Remortgaging is always nonetheless possible if a new house is bought. We also adopt the prevalent rule that retired households cannot have access to a new mortgage, although existing mortgages continue to be paid off.

We now detail these constraints and their consequences.

Loan-to-value ratio Let h' denote the next-period ownership level. Buying a house (i.e., $h' \neq h$) might be a first-time purchase ($h = 0, h' \neq 0$) or moving between houses. Because of loan-to-value ratio regulations, the household should have enough financial assets to cover a minimum

fraction, θ , of the total value qh' before contracting a mortgage. Consequently, the financial position after buying a house, b' , denotes that the next-period financial wealth should satisfy the constraint:

$$b' \geq -(1 - \theta)qh', \text{ if } h' \neq h. \quad (3)$$

since next-period net wealth, $b' + qh'$, should be greater than the downpayment, $\theta qh'$. This constraint also encompasses the borrowing constraint when $h' = 0$.

Minimum repayment When the owner remains in the same house in the next period and has a mortgage ($b < 0$), we assume that a fixed fraction, δ , of the mortgage, or of the value of the house, qh , should be paid back, whichever is the largest.

$$b' - (1 + r^m)b \geq \delta \max(-b, qh) \text{ if } b' < 0, h' = h. \quad (4)$$

Note that we did not condition on $b < 0$ since doing so is redundant with the condition that $b' < 0$ because $\max(-b, qh) \geq 0$ and as a consequence of equation (4):

$$0 > b' \geq (1 + r^m)b.$$

This minimum repayment constraint (4) prevents paybacks from becoming smaller and smaller when the mortgage level is close to zero. It also has the advantage of avoiding maintaining high mortgages for pure consumption motives. In other words, home equity is not used to finance higher consumption paths. To have some bite in this dimension, we impose a large enough ratio of repayment, δ :

$$\mathbf{Condition\ S:} \quad r^m(1 - \theta) < \delta.$$

which means that at least a fraction of the mortgage capital is paid back every period.

Payment to income (PTI) ratio The last constraint refers to the maximal mortgage repayment that should not exceed a certain threshold:

$$\frac{\delta qh}{w} \leq \tau_e, \quad (5)$$

which effectively forbids large housing investments relative to permanent income.

We can now derive the consequences of these different constraints and Condition S. In particular, we show that the minimal repayment when the mortgage is positive is δqh and that the

downpayment condition (3) is always verified even if $h' = h$, as in Sommer and Sullivan (2018). It is here a consequence of constraints (3), (4) and condition S.

Proposition 1 *Under Conditions (3), (4) and S, then*

$$(i) \quad b' - (1 + r^m)b \geq \delta qh \text{ if } b' < 0, h' = h. \quad (6)$$

$$(ii) \quad b \geq -(1 - \theta)qh \text{ for all } h. \quad (7)$$

Proof. See Appendix A.5 ■

The full set of “prudential” constraints is therefore described by the downpayment equation (7), the minimal repayment equation (6) and the payment-to-income ratio (5).

2.4 The tax and benefit system

Household income includes earnings and landlord revenues and is taxed using a progressive schedule. There are also taxes on housing services and ownership, as well as transaction taxes on purchases and sales of houses. The tax system can be described by general and housing-specific taxes. The first category includes the following:

- The income of a household is the sum of labor earnings (w) and rental income for landlords ($\rho(h - s)$). The tax schedule is discrete, $t(\cdot)$, and includes payroll and progressive income taxes.
- Deceased household assets (financial assets and housing) are taxed at 20% by the government. The remaining 80% are stochastically allocated to the young household starting its life as a replacement for the deceased household.

Second, some taxes are specific to housing and ownership:

- A residence tax, τ^r , is paid on the capitalized value of the occupied dwelling, qs .
- A property tax τ^h is a tax based on the value of the housing stock qh .
- Transaction taxes are paid on the value of the purchase, qh' , or sale qh , and are equal to τ^b for the buyer and τ^s for the seller.

Furthermore, the main government policy regarding housing affordability is to provide housing allowances to low-income households. The values that we retain are described in Appendix A.3. We show there how we adapt the groupings of households according to the receipt of housing allowances, which is means-tested and depends on geographical location.⁴ A slight complication arises from the fact that housing allowances are also given to owner-occupiers ($h = s, h > 0$) with a mortgage ($b < 0$). This makes housing allowances, denoted as $\eta(w, h, h', s, b)$, depend on the existence or nonexistence of a mortgage and, more importantly, a discontinuous function in b at 0. This discontinuity breaks the result that the value function is continuous at $b = 0$ and, as a consequence, the contraction property in the Bellman equation is unlikely to hold. Since housing allowances are likely to be fuzzy around 0, we smooth out this discontinuity in a way that is explained in Appendix A.6.

In summary, the tax function is written as:

$$\begin{aligned}
T(w, h, h', s, b) = & \underbrace{t(w + \rho(h - s))}_{\text{Income and payroll taxes}} + \underbrace{\eta(w, h, h', s, b)}_{\text{Housing allowances}} \\
& + \underbrace{\tau^r qs}_{\text{Residential tax}} + \underbrace{\tau^h qh}_{\text{Property tax}} \\
& + \underbrace{\mathbf{1}\{h' \neq h, h > 0\} \tau^s qh}_{\text{Housing sale tax}} + \underbrace{\mathbf{1}\{h' \neq h, h' > 0\} \tau^b qh'}_{\text{Housing purchase tax}}.
\end{aligned} \tag{8}$$

Overall, we assume that these taxes are collected by the government and consolidated with any other agent, such as real estate agents or the institutional owners of other housing property. Government money is spent on a public good that is not modeled here and whose distribution across generations does not change in the counterfactuals. Any reform is evaluated by imposing a change in other taxes in such a way that the government's budget (i.e., including any agent other than households) remains constant.

3 Dynamic optimization and optimal value functions

Given the economic primitives presented in the previous section, we set up the dynamic programming problem and explain how we solve for stationary value functions by iterating a two-step,

⁴Another financial incentive for first-time house buyers is a zero-interest loan that may cover 20% of the total financing costs up to a specific ceiling. Because of its complicated consequences on the dynamics of the model, we leave the analysis of this loan for further research.

fixed-point procedure that handles in turn policy functions concerning continuous choices, i.e., consumption and discrete choices, i.e., housing services and future housing stocks. Consumption functions are obtained using an endogenous grid method adapted to the intricacy of the decision problem considered with many prudential constraints. Next, as discrete choices make the resulting value function non concave, we introduce lotteries that agents can play and that enhance their utilities and make the resulting value functions concave. We finish this section by providing the fixed-point algorithm that solves the dynamic problem and by setting up the general equilibrium problem that solves for rental and house prices.

3.1 The dynamic programming problem

State variables: The dynamics in the model are derived from three transitional processes: (1) age and earnings; (2) financial assets and the intertemporal budget constraint; (3) housing. Consequently, it is enough for each household to be identified with three state variables that govern their current and future decisions about consumption, savings, housing services, tenure and housing stocks, as well as mortgages. Those state variables are as follows:

- w : age and discrete earnings level (J groups).
- b : net financial wealth (a continuous variable).
- h : house size in $K + 1$ groups (from which tenure is derived, e.g., $h = 0$ for renters).

The Bellman equation: Denote β as the discount factor, and let $V(w, b, h)$ be the value function that satisfies the following Bellman equation:

$$V(w, b, h) = \max_{s, c, h'} \left(U(c, s; h) + \beta \sum_{w'} \Pi(w'|w) V(w', b', h') \right), \quad (9)$$

in which w', b', h' are the next period state variables, and $\Pi(w'|w)$ denotes the transition matrix of earnings and age over time.

The optimization program is implemented under the following constraints. The first ones are restrictions for owner-occupiers and landlords given by equation (1), minimum repayment given by equation (6), downpayment given by equation (7) and payment-to-income given by equation

(5). Second, the intertemporal budget constraint is given by:

$$\begin{aligned}
c + b' &= \zeta(h', s; w, h, b) = (1 + r(b))b + w \\
&+ \underbrace{\rho(h - s)}_{\text{Net rent}} - \underbrace{\phi \mathbf{1}\{h > s\}}_{\text{Landlord fixed cost}} - \underbrace{q(h' - h)}_{\text{Net price of new house}} \\
&- T(w, h, h', s, b) - c_m \mathbf{1}\{h \neq h'\},
\end{aligned} \tag{10}$$

in which we denote:

$$r(b) = r \mathbf{1}\{b \geq 0\} + r^m \mathbf{1}\{b < 0\},$$

and $T(w, h, h', s, b)$ is defined in equation (8). Function ζ in this expression is a piecewise linear function, and its left derivative with respect to b is:

$$\frac{\partial \zeta}{\partial b} = 1 + r(b) - \frac{\partial T}{\partial b} > 0.$$

This dynamic program is stationary in the state variables w , h and b because no parameter is time dependent and new generations replace old ones. We thus iterate over the dynamic program and obtain a time-independent value function $V(w, b, h)$, as the fixed point of Bellman equation (9) can be written in abstract form as:

$$V(w, b, h) = \mathcal{L}(V(w', b', h'))$$

in which \mathcal{L} is an operator on functions of state variables (w, b, h) for any discrete value of w, h and of the continuous variable, b . The range of the continuous variable, b , is slightly restricted below by considering piecewise linear functions.

A two-step procedure: We split the maximization program with respect to the discrete (s, h') and continuous variables, c , as doing so facilitates computation. The Bellman equation is rewritten as a discrete choice across housing services and housing size and tenure:

$$V(w, b, h) = \max_{s, h'} (V^I(h', s; w, b, h)), \tag{11}$$

subject to constraint (1) and in which the interim value function, $V^I(\cdot)$, is given by continuous consumption choices:

$$V^I(h', s; w, b, h) = \max_c \left(U(c, s; h) + \beta \sum_{w'} \Pi(w'|w) V(w', b', h') \right) \tag{12}$$

under constraints (6) and (7), and under the binding budget constraint (10).

In other words, we solve the dynamic program by looking for a fixed point of the composition of two functional operators denoted \mathcal{L}_D and \mathcal{L}_I and are given by equations (11) and (12), respectively, under their associated constraints. We start by describing how we operationalize the second operator \mathcal{L}_I using an endogenous grid method (Carroll, 2006). We then turn to the issue that the first operator \mathcal{L}_D given by equation (11) delivers non convex functions and explain how we convexify it using an economic argument. In turn, this justifies why we can use the endogenous grid method in the first step.

3.2 Consumption choices

We briefly explain how we numerically solve equation (12) under constraints, although the intricacy of constraints (5), (6) and (7) makes us move the complete set of derivations to Appendix B.

We now sketch out how to construct the operator in the simple case in which constraints (5) and (6) are absent. The remaining constraints are the budget constraint (10) and the downpayment equation (7). Moreover, we assume that the term on the right-hand side of equation (12), $\sum_{w'} \Pi(w'|w)V(w', b', h')$, is concave, which is the result of the convexification argument that we use in the next subsection. We also fix the other arguments (h, h', s, w) to their values in this subsection.

First, we choose to restrict the space of consumption functions to piecewise linear functions defined on grids of values, $b \in \mathcal{G}(h)$ that are held constant over the whole fixed point procedure. The grid for b is specific to each h because of the downpayment constraint (7). The evaluation of the value function, $V(w, b, h)$, is exact for points on the grid, $b \in \mathcal{G}(h)$, and intra or extrapolated otherwise. We allow the value functions to be equal to $-\infty$ or, more exactly, a large negative number.

Second, we use the endogenous grid method of Carroll (2006). Instead of solving the problem forward, it is solved backward by using the grid for the end-of-period asset b' , e.g., $\mathcal{G}(h')$. Indeed, solving forward requires solving for b' , the nonlinear Euler equation for every grid point in b , and this is too costly in terms of computing time. The grid is denoted $\mathcal{G}(h') = \{b'_1, \dots, b'_g, \dots, b'_G\}$ in which points are ordered, and the first point of this grid, as follows

$$b'_1 = -(1 - \theta)qh',$$

is chosen such that the downpayment constraint (7) is satisfied by any b'_g in the grid. Grid $\mathcal{G}(h')$

should also contain point $b = 0$ since it could be a kink point for the value function. The remaining points are arbitrary. We varied their construction and chose to have as many negative as positive values except when $h = 0$.

The consumption value, say c_g , that solves equation (12) under the budget constraint is given by the standard Euler equation ((B.2) in the Appendix) equating the marginal utility of consumption to the future discounted expected marginal value of assets. The resulting current value of b , say b_g , is then derived from c_g and b'_g using the budget constraint. However, it could be that b_g does not satisfy downpayment constraint (7). In that case, consumption c_g is constrained at the value given by the downpayment constraint (7).

The rather tedious complete algorithm when we account for all prudential constraints (5), (6) and (7) is developed in Appendix B, in which we show that it is important to distinguish three cases: (1) households selling or buying a house in the next period, i.e., $h \neq h'$, (2) households who remain renters next period, i.e., $h = h' = 0$, and (3) the immobile homeowners, i.e., $h = h', h \neq 0$. The endogenous grid method is shown to apply to these different cases.

3.3 Housing services and housing buying and selling

We start by showing that households can improve their welfare obtained in equation (11) using lotteries, which makes the resulting value function concave in its arguments. We next show how to solve for the optimal value function.

3.3.1 Convexifying the discrete choice decision program

In a well-behaved stochastic dynamic optimization problem, the first-order condition with respect to consumption is necessary and sufficient because the utility function is strictly increasing and concave and because the future value function is increasing and concave.

In our case, households make discrete choices for their housing services, s , and their housing stock, h' , by taking the supremum of functions $V^I(\cdot)$. This causes nonconcavities in the resulting value function and, thus, does not warrant that the first-order condition is sufficient for describing an interior optimum (Iskhakov et al., 2017). Our model is, however, highly nonconcave due to discrete choices and because tax and benefit schedules are not convex such that the solutions in Fella (2014) and Iskhakov et al. (2017) are costly to implement.

We use the approach of Karaivanov and Townsend (2014) by assuming that households randomize their decisions using lotteries over all possible choices. This has two consequences. First, households improve their welfare by using these lotteries. Second, the resulting value function is concave. In other words, the discrete maximization operator in equation (11) is replaced by the following function:

$$V(w, b, h) = \max_{\pi(h', s|w, b, h)} \sum_{h', s} \pi(h', s, s; w, b, h) V^I(h', s; w, b, h), \quad (13)$$

in which $\pi(\cdot)$ is the probability of choosing h', s for a household. Now, if we assume that V^I is an increasing and concave function of b , the resulting $V(w, b, h)$ is concave with respect to b since it is the upper envelope of concave functions. In practice, it is coded by computing the increasing and concave envelope of functions $V^I(h', s; w, b, h)$ over the values of b for any w and h .

3.3.2 The optimal value function

To approximate the optimal value function, the interim value functions $V^I(\cdot)$ need to be written on a common grid for b as a function of (h, w) , and a new grid depending only on h needs to be chosen for backward induction. The grid for each (h', s) , $\mathcal{G}^I(h', s; h, w)$, computed in the previous section, has G points, and the number of pairs (h', s) is less than $K_2 = (K + 1)^2$. The algorithm is decomposed into the following steps.

The encompassing grid It is formed by intermingling all grids, i.e., by sorting:

$$\mathcal{G}_0^*(h, w) = (b_1^* = \min_{(s, h') \in \mathcal{D}_{h, w}} \mathcal{G}^I(h', s; h, w), \dots, b_M^* = \max_{(s, h') \in \mathcal{D}_{h, w}} \mathcal{G}^I(h', s; h, w)).$$

By construction, this grid is adapted to constraints (5), (6) and (7). In particular, the value function might take value $-\infty$ or a very large negative number if at any point in this grid, the constraints are not satisfied.

We construct, by linear optimization, the value function $V(b, h, w)$, for any (h, w) defined in equation (13). For any $b \in [b_1^*, b_{G^*}^*]$, we denote $v_g^* = V^I(h', s; w, b_g^*, h)$, which can be computed by inter or extrapolation, and we define $V(b, h, w)$ as:

$$V(b, h, w) = \max_{\substack{\lambda_g \in [0, 1] \\ \sum_g \lambda_g = 1 \\ b = \sum_g \lambda_g b_g^*}} \sum_g \lambda_g v_g^*. \quad (14)$$

We can compute $V(b, w, h)$ for any point on the original grid b_g that lies between b_1^* and $b_{G^*}^*$. It amounts to computing the convex hull of all values v_g^* . The range of variations in b is bounded from below. For all points above $b_{G^*}^*$, we extrapolate all functions linearly using the last two points $b_{G^*-1}^*$ and $b_{G^*}^*$.

Furthermore, the derivative of $V(\cdot)$ is given by

$$V_b(b_g, h, w) = \sum_g \lambda_g^* dv_g,$$

where dv_g is the collection of derivatives calculated in equation (B.5) in the Appendix, λ_g^* is the solution for program (14) for any point b_g in the interval $[b_1^*, b_{G^*}^*]$. For any point outside this range, the value function is extrapolated by a line, and the derivative $V_b(\cdot)$ at these points is the slope of this line.

The reduction and update of the grid There are two important points to address when updating the grid over iterations. First, because the value function is obtained by a fixed point routine, it is impossible to let the number of points in the grid multiply as the iterations progress. Second, the grid should be written only as a function of h since we used a common grid for all w because w is unknown in the previous period. If the grid is $\mathcal{G}(h') = \{b_1(h'), \dots, b_G(h')\}$, the one-step backward revision delivers a set of extreme points, say $\tilde{b}_g(h, w)$ of the concave function $V(h, w, \tilde{b}_g)$. By interpolation using this set of extreme points, we derive the values $V(h, w, b_g)$ for each element of the original grid (up to a change from h' to h) $\mathcal{G}(h) = \{b_1(h), \dots, b_G(h)\}$.

3.4 The fixed-point algorithm

The value function needs to be calculated as the fixed point of the general operator defined by the Bellman equation (9) in which we allowed, in the previous subsection, the use of random lotteries as a convexification device.

We initialize the fixed point iteration process by using a specific value function for retired households. We show in Appendix F that a reasonable approximation for consumption and housing services for retired households, neglecting taxes, is to assume that they all sell out their housing property at retirement. For all h and $b \geq -(1 - \theta)qh$, this yields:

$$\begin{aligned} c^* &= w + r(b + (1 - \tau^s)qh) - \rho s^*, \\ s^* &= \arg \max_s (u(w + r(b + (1 - \tau^s)qh) - \rho s, s), \end{aligned}$$

and therefore:

$$V^{(1)}(w, b, h) = \frac{u(c^*, s^*)}{1 - \beta}.$$

We use this initial condition for the values of (w, b, h) regarding retired households and then solve by backward induction the dynamic program (9) for other generations and by iterating over the Bellman operator \mathcal{L} . The complete algorithm used to find the functional fixed point is described in Section C in the Appendix.

3.5 Steady state distribution function

Given this optimal value function, we next turn to the computation of the steady state of the economy given these value and policy functions. We initialize the distribution of age and earnings groups at their steady state values and iterate over the set of policy functions obtained above until reaching a minimal tolerable level of convergence to the ergodic distribution function $dP(w, b, h)$ of state variables (w, h, b) on the discrete set of values consisting of $\{w_j\}_{j=1,..,J} \times \{(h_k, \mathcal{G}(h_k))\}_{k=0,..,K}$. The algorithm used is detailed in Section D in the Appendix.

4 Empirical strategy

Our empirical strategy consists of two nested loops. In the inner loop, we compute the values of housing prices, q , and rental price, ρ , that balance supply and demand in the housing and rental markets, given parameter values θ . In the outer loop, we estimate the main parameters by fitting theoretical moments to empirical moments derived from the housing survey using a simulated method of moments. Other parameters are calibrated. We describe both procedures in the following.

4.1 Market clearing

We compute rental price ρ and housing price q by equating excess supply to zero in two markets: private housing and the rental market. We explain in this section how we use the results developed above in Sections 3.4 and 3.5 and how we calibrate the unknowns in housing and rental supply from the housing survey.

4.1.1 Clearing housing markets

First, the supply of private housing is fixed and equal to \bar{h}_{PV} . The expression equating ownership to housing supply is:

$$\mathbb{E}(h_i) = \int h(w, b, h; q, \rho) dP(w, b, h; q, \rho) = \bar{h}_{PV}, \quad (15)$$

in which the demand for housing $h(w, b, h; q, \rho)$ is derived using what is developed in Section 3.4, and the discrete probability measure of households at values of the state variables, i.e., wage w , net financial assets, b and housing level, h , $dP(w, b, h; q, \rho)$ is derived as in Section 3.5.

Second, the equilibrium on the rental market is given by:

$$\int (s(w, b, h; q, \rho) - h(w, b, h; q, \rho)) dP(w, b, h; q, \rho) = \mathbb{E}(s_i - h_i) = \bar{h}_{PUB}, \quad (16)$$

in which the average demand for housing services, s_i , in excess of private supply, h_i , is equal to \bar{h}_{PUB} , the average area supplied by public institutions.

Clearing markets by solving equations (15) and (16) depends on the average housing volumes available in the market, \bar{h}_{PUB} and \bar{h}_{PV} that we calibrate as presented next.

4.1.2 Calibrating housing supply

We distinguish four subpopulations: renters in the public housing sector (Public), renters in the private market (Private), Owner-occupiers and Landlords. We consider that Public also includes the fraction of the private market owned by institutions (banks, insurance companies, etc.), as well as social housing. Also key is that we assume that rents are the same in the public and private markets. The actual difference in the data is assumed to reflect an adjustment in the quality of housing services.

We assess the proportion of each subpopulation from the housing survey and calibrate average housing services, s , expressed in square meters per household in the population, as follows:⁵

This calibration strikes a balance between the distance between volumes and prices predicted by the model, on the one hand, and those observed from the housing survey, on the other hand. The former volumes and prices are underestimates of the latter by a 10 to 15% margin.

⁵The housing survey does not provide robust information on areas rented out by landlords. We assume that they are as large as that for renters.

| Sector | Current Proportion | Average house size (s , square meters) |
|-----------------|--------------------|---|
| Public | 0.17 | 55 |
| Private | 0.26 | 55 |
| Owner-occupiers | 0.49 | 80 |
| Landlords | 0.08 | 80+55=135 |

Table 1: Frequencies and average size by housing tenure.

In what refers to \bar{h}_{PV} and \bar{h}_{PUB} , the volume of privately owned properties, \bar{h}_{PV} , is consequently calibrated as:

$$\bar{h}_{PV} = \underbrace{0.08 * 135}_{\text{Landlords}} + \underbrace{0.49 * 80}_{\text{Owner-occupiers}} \simeq 50.$$

Second, the volume of housing properties that are either publicly owned or owned by institutions, \bar{h}_{PUB} , is derived by first assessing the volume of properties rented out by institutions in the private market:

$$\underbrace{0.26 * 55}_{\text{Private market}} - \underbrace{0.08 * 55}_{\text{Fraction owned by landlords}} \simeq 10,$$

and using:

$$\bar{h}_{PUB} = \underbrace{0.17 * 55}_{\text{Public housing}} + \underbrace{10}_{\text{Fraction rented out by institutions}} \simeq 20.$$

The total housing volume is thus $\bar{h}_{PV} + \bar{h}_{PUB} = 70$ square meters per household in the population.

When we evaluate counterfactuals in Section 5, we assess the robustness of our conclusions to relaxing the condition that the housing supply is fixed. The elasticity of supply in France, ε_s , is in the low range, and it is reasonable to fix it at a value of 0.4 (OECD, 2022). We also assume that this elasticity does not depend on whether properties are private or public. In other words, $\bar{h}_{PV} = 50 * (q/q^*)^{0.4}$ and $\bar{h}_{PUB} = 20 * (q/q^*)^{0.4}$ in which q^* is the initial equilibrium price.

4.1.3 Solving for market clearing prices

Given these empirical counterparts, let the vector of excess demands on the rental and housing markets be:

$$E_d = [\mathbb{E}(s_i) - \mathbb{E}(h_i) - \bar{h}_{PUB}, \mathbb{E}(h_i) - \bar{h}_{PV}]$$

and if Ω is a weighting matrix, the criterion function $E_d \Omega^{-1} E_d'$ is minimized over (ρ, q) . For Ω , we use either $\Omega = I$ or $\Omega = \text{Var}(E_d)$. After some experimentation, we decide to set $\Omega = I$.

A few technical notes are in order. First, because we discretize the dynamic problem and randomize to smooth out non convexities of the value function, the demand for housing in the owner market, h_i , and in the rental market, s_i , is discontinuous, and the equilibrium in this economy might not exist. We decide to smooth out these discontinuities by first computing the demands on finite grids for prices (ρ, q) and then estimate smooth demands using a nonparametric kernel or nearest neighbors methods. The asymptotic framework that we entertain is that discontinuities in demand disappear if the number of points of support of heterogeneity tends to infinity (see, for instance, Han (2019) for such approximations used in a continuous context). We perform these computations by using thinner and thinner grids.

4.2 Outer loop: Calibration and estimation of parameters

Our model has two sets of parameters listed in Section A.1 in the Appendix. The first set comprises parameters that can be calibrated externally using the current environment or legislation. Their calibrated values are reported in Section A.1. The second set is composed of parameters governing the utility function (three parameters), the maintenance cost for landlords, ϕ , and the mobility cost, c_m , estimated using the simulated moments method.

To estimate their values, we chose the moment equations of interest that describe the most important dimensions of our economic model in terms of housing tenure, mobility and share of housing expenditures in household budgets. We consider five such empirical first-order moments, as described in the following Table, estimated using the 2006 French housing survey.

| Statistic | Model | Data | Housing Survey 2006 |
|-----------------------------------|-----------------------------------|--------|---------------------|
| Proportions | | | |
| Owners | $E(\mathbf{1}\{h > 0\})$ | 0.57 | SOC4 |
| Landlords | $E(\mathbf{1}\{h > s\})$ | 0.082 | MR FON or XLN |
| New homeownership | $E(\mathbf{1}\{h' > 0\} h = 0)$ | 0.0416 | SOC4, VSOC and SAA |
| Net sellers each period | $E(\mathbf{1}\{h' < h\})$ | 0.019 | VLR, SAA1 and VSOC |
| Shares of housing expenses | | | |
| Renters | $E(\frac{ps}{w} h > 0)$ | 0.215 | LMLM and MRTOTA |

Table 2: Empirical moments to be matched

The simulated moments method that we use minimizes the weighted distance between the vectors of empirical and theoretical moments in which the weights are the inverse of the squares of these empirical moments. Theoretical moments at a certain value of parameter θ are obtained by computing the value function $V(w, b, h, \rho, q; \theta)$ —as in Section 3.4 and the steady-state distribution function $dP(w, b, h; q, \rho)$ derived in Section 3.5—at the equilibrium prices ρ and q obtained in the previous Section 4.1.

The final value of the criterion is equal to 0.0175, and estimates are reported in Table 3. Estimates for risk aversion and housing preferences fall in the range of macroeconomic estimates. First, as $\sigma > 1$, we necessarily have $\alpha < 0$. From Section 2, they translate into a risk aversion parameter equal to $1.76 + 0.18 \simeq 2$, and the instantaneous budget share of housing expenditures is equal to $\frac{-0.18}{-0.95} \simeq 19\%$. Furthermore, ownership preferences are quite small. For an owner-occupier of a 90-square meter house, the money metric utility value is an additional 300 euros (or 1.5%) in consumption (evaluated at the mean). Landlords' maintenance costs are also quite small, while owners' mobility costs are much higher and equal to approximately three-quarters of the yearly rent paid for an average-size house.

| | Parameters | Estimated Value |
|----------|---------------------------|---------------------|
| σ | Consumption risk aversion | 1.76 |
| α | Housing preferences | -0.18 |
| ξ | Preference for ownership | $3.9 \cdot 10^{-8}$ |
| ϕ | Landlord fixed cost | 19 €/year |
| c_m | Mobility cost for owners | 3850 €/year |
| Prices | | |
| ρ | Rent | 68 €/sq.meters*year |
| q | Housing price | 890€/sq.meters |

Table 3: Estimated parameters and equilibrium prices

The goodness-of-fit of empirical moments with model predictions at the minimized value is quite good. Errors in housing tenure are less than 10%. Even if new home ownership is less well predicted, errors remain in a reasonable range. This is also true for the share of housing expenses for renters.

| Moments | Model | Data | Relative error |
|-----------------------------------|--------|--------|----------------|
| Proportions | | | |
| Owners | 0.53 | 0.57 | -0.07 |
| Landlords | 0.074 | 0.082 | -0.10 |
| New homeownership | 0.0262 | 0.0416 | -0.46 |
| Net sellers each period | 0.019 | 0.019 | -0.01 |
| Shares of housing expenses | | | |
| Renters | 0.235 | 0.215 | 0.09 |

Table 4: Goodness of fit

4.3 Robustness: Nonfitted moments

We can also assess the accuracy of moments that are not fitted by the procedure. The two following tables show the extent to which two of the moments used are correctly predicted by age groups and earning levels. We can see that the model significantly overestimates the homeownership rate, particularly those with high income. Similarly, the rate of high-income, middle-aged and senior households moving into ownership is exaggerated by the model. In practice, moving from a middle-income class to a high-income class usually does not happen overnight. When that happens, becoming a homeowner still takes time. These progressive steps are naturally not accounted for by the model, in which the transitions from one group to another are modeled from evolutions observed over 15-year periods. Thus, the very high rate in both tables for high-income groups simply reflects the fact that a middle-aged household reaching such income usually becomes a homeowner after a few years.

Table 7 shows housing size (variable s) by tenants' age group and income level. Again, we see some differences, particularly lower house size for low- and middle-income households, which is very significant for senior or retired households. This can be partly explained by the fact that, in the model, flats and houses are assumed to be of the same quality with a single price per square meter. We indeed observe in the French housing survey that a gap of approximately 40% exists between the prices per square meter of houses of low- and high-income households and that this gap is greater for senior and retired households and nearly absent for middle-aged households.

| | Junior | Middle-aged | Senior | Retired |
|---------------|-----------------|-----------------|-----------------|-----------------|
| Low income | 8.00% / 11.28% | 19.85% / 17.98% | 15.87% / 19.53% | 3.89% / 5.78% |
| Middle income | 23.10% / 45.50% | 54.77% / 50.34% | 67.90% / 54.51% | 70.98% / 54.52% |
| High income | 55.89% / 58.85% | 79.50% / 96.25% | 82.69% / 98.17% | 87.78% / 98.63% |

Table 5: Homeownership rate (data/model)

| | Junior | Middle-aged | Senior | Retired |
|---------------|-----------------|------------------|----------------|---------------|
| Low income | 0.86% / 0.04% | 1.01% / 0.44% | 0.22% / 0.04% | 0.02% / 0.00% |
| Middle income | 4.03% / 4.95% | 5.79% / 0.02% | 3.78% / 0.04% | 1.10% / 0.00% |
| High income | 12.63% / 16.04% | 12.73% / 100.00% | 7.44% / 38.99% | 2.16% / 0.00% |

Table 6: Moving-into-ownership rate (data/model)

| | Junior | Middle-aged | Senior | Retired |
|---------------|---------|-------------|-----------|-----------|
| Low income | 58 / 49 | 78 / 62 | 70 / 38 | 59 / 35 |
| Middle income | 64 / 51 | 90 / 69 | 97 / 54 | 82 / 50 |
| High income | 85 / 86 | 111 / 106 | 114 / 123 | 107 / 106 |

Table 7: Average housing size (data/model)

5 Counterfactuals

Given these calibrated parameters, we now evaluate the impact of counterfactual policies that aim to increase mobility by reducing transaction taxes. More precisely, over the period considered, when buying (resp. selling) a house, current taxes are 8% (resp. 6%) of the value of the house. We decrease these amounts to 3% and 3% to match the usually retained values in the US (Sommer and Sullivan, 2018). The following analysis compares the corresponding steady states of the economy when we hold constant across steady states, the budget of the government and all other housing institutions (i.e., real estate institutions and public investors in public housing). In the main counterfactual (denoted “counterfactual A”), budget equivalence is achieved by increasing the property tax on homeowners (based on the value of their property), a quite popular policy proposal at the moment (Bonnet et al., 2021).

We also consider two other counterfactuals that depart slightly from the previous one. In counterfactual B, we achieve budget equivalence by using a presumably weakly distortive instrument, that is, a proportional tax on income, which we denote by its French acronym, *CSG* (i.e.,

“Generalized social contribution”).⁶ In counterfactual C, denoted as “elastic supply”, we relax the assumption that the supply of housing is fixed and retains an elasticity of supply $\epsilon_s = 0.4$. For the latter counterfactual, we still achieve budget equivalence by increasing the property tax on homeowners.

5.1 Impact on house prices and homeownership

We start by describing the main equilibrium effects on prices and housing ownership. The first row of Table 8 displays the homeownership rate for each scenario. Quite expectedly, reducing transaction taxes increases access to ownership, from 53.11% to 54.95% in counterfactual A. As explained earlier and in further detail in the next subsection, the reduction in transaction taxes makes it easier for households to sell their property to buy a new one of a different size or to become a tenant again. Therefore, a reduction in transaction taxes makes it less risky to become an owner or to buy a larger house—the risk involved is to incur at any point in the life cycle a negative shock on income requiring the resale of the house.

This results in less demand for rentals, and prices and rents move in the opposite direction, with an increase of 2.36% in housing prices and a decrease in rents of 1.59% in counterfactual A, while the budget is balanced by increasing the property tax from 0.3 to 0.5%.

The effects on housing prices and homeownership rates in counterfactuals B and C are also expected, although with a weaker price increase of +2.0% when the supply of housing is slightly elastic (Counterfactual C) and a strong increase of 5.53% when the budget balance is based on an almost lump-sum tax (Counterfactual B). In the latter case, the increase in housing prices is more than twice that observed in counterfactual A. The decline in rents is significantly greater in counterfactual C with an elastic supply (−2.53%) and is slightly greater in the other (−1.75%). As expected, the choice of the tax that balances the government’s budget has significant implications for price levels.

In the three counterfactuals, the share of landlords in the population declines sharply. This can be explained by the fact that, in the baseline scenario, a significant proportion of landlords are middle-income households that have suffered a negative income shock and have opted to keep their home and sublet part of it. As shown in Table 9, lower transaction costs in the counterfactuals

⁶As labor supply and retirement decisions are not modeled here, any tax on labor earnings is nondistortive and acts as a lump-sum tax. There is, however, some distortion arising because this tax is also levied on landlords’ rental income, which is endogenous in the model, although its impact remains limited.

| | Baseline | Counterfactuals | | |
|-----------|----------|-----------------|--------|--------|
| | | A | B | C |
| Ownership | 53.11% | 54.95% | 55.46% | 54.17% |
| Price | 890.3€ | 911.3€ | 939.6€ | 908.3€ |
| Rent | 67.94€ | 66.86€ | 66.75€ | 66.21€ |
| Landlords | 7.44% | 3.44% | 3.23% | 3.13% |

Table 8: Housing tenure rate, housing and rental prices, share of Landlords.

Note: The baseline designates the model calibrated on the French housing survey, whereas counterfactual A designates the simulation of the new steady-state economy once transaction taxes have been lowered, with a fixed housing supply and a budget balanced by increasing the property tax on homeowners. Counterfactual B is similar but with a budget balanced by increasing the proportional tax on income. Counterfactual C differs from counterfactual A in that housing supply is assumed to be elastic.

allow them to sell their properties more easily and to buy a smaller property. The evolution of the landlord rate among high-income households is more complicated to explain because several opposing factors come into play: given a decline in transaction costs, real estate becomes both more liquid and less profitable (since housing prices increase and rents decrease).

| | Junior | Middle-aged | Senior | Retired |
|---------------|-----------------|----------------|----------------|---------------|
| Low income | 0.58% / 1.18% | 0.16% / 0.20% | 0.26% / 0.36% | 0.00% / 0.00% |
| Middle income | 16.65% / 23.36% | 4.94% / 15.04% | 1.76% / 22.12% | 0.19% / 2.33% |
| High income | 9.38% / 4.86% | 1.38% / 1.78% | 1.75% / 3.64% | 5.27% / 2.86% |

Table 9: Share of landlords (main counterfactual/baseline)

5.2 Impact on housing tenure transitions over the life cycle

We now move on to an analysis of mobility between renting and ownership. As shown in the first row of Table 10, the decrease in transaction taxes leads to a strong increase in trading volume: the proportion of households carrying out a real estate transaction increases from 2.66% to 3.21% in counterfactual A, an increase of approximately 21%. However, this is not enough to induce a Laffer effect since the average transaction tax declines by more than 50%.

Reducing transaction taxes strongly increases mobility along the life cycle in both ways: moving to ownership increases by approximately 40%, whereas moving back to renting is multiplied by three. As mentioned in the introduction, this reflects the fact that smaller transaction taxes allow an owner to better manage an adverse shock to household income by selling property and

| | Baseline | Counterfactuals | | |
|--------------------------|----------|-----------------|-------|-------|
| | | A | B | C |
| Housing transactions | 2.66% | 3.21% | 3.23% | 3.18% |
| Average transaction cost | 4439€ | 2003€ | 2082€ | 1997€ |
| Move to ownership | 2.26% | 3.12% | 3.17% | 3.04% |
| Move to rent | 0.34% | 0.96% | 0.94% | 0.95% |
| Change in property size | 2.68% | 2.33% | 2.34% | 2.35% |

Table 10: Transactions, tenure change and changes in housing sizes
(cf. caption of Table 8 for a detailed definition of the baseline and of the counterfactuals).

becoming a renter again. In anticipation, this flexibility leads households to opt more often for homeownership. However, there is very little change in the size of houses among owner-occupiers, except for among those who are also landlords. The rate of change is close to zero (rows 5 and 6 of Table 10), whether due to moving to a larger or a smaller apartment. This finding seems to indicate that the mobility costs (calibrated parameter $c_m = 3850\text{€}/\text{year}$) combined with low transaction costs are sufficient to deter such relocations. The decrease in the frequency at which owners change the size of their housing properties is explained by the decline in the proportion of landlords (who tend to change their real estate savings more often than do owner-occupiers).

Next, Table 11 provides more details of the various points mentioned in the previous paragraphs. We focus on the comparison between the calibrated baseline and the main variant (Counterfactual A). The first row shows a very sharp decline in the rate of ownership for low-income households. This is primarily because reduced transaction costs make selling their houses easier for households experiencing an unexpected decline in income. The second row shows a very strong increase in the rate of ownership for high-income junior households because reduced transaction costs allow rich households to move more quickly into homeownership. The third row highlights a better match between housing needs and the house size of high-income homeowners when transaction costs are low. Indeed, we observe that house size varies more over the life cycle in the counterfactual for high-income homeowners and is more closely aligned with household size—in particular, given an average house size of seventeen square meters lower for junior households.

5.3 Impact on household welfare

A better match between the demand for housing and the needs and financial capacities of households, observed in the previous section, has potentially a positive impact on welfare. In the

| (main counterfactual/baseline) | Junior | Middle-aged | Senior | Retired |
|------------------------------------|----------------|-----------------|----------------|---------------|
| ownership of low-income group | 6.31% / 11.28% | 12.59% / 17.98% | 6.74% / 19.53% | 2.40% / 5.78% |
| ownership of high-income group | 88.5% / 58.8% | 97.4% / 96.2% | 97.7% / 98.2% | 98.3% / 98.6% |
| house size (high-income homeowner) | 84 / 101 | 109 / 107 | 130 / 124 | 101 / 106 |

Table 11: Ownership and house size for different income groups by age.

following, we measure this effect by computing, as in Cho et al. (2021) and Schmidt (2022), the impact of the counterfactual change in transaction taxes on the expected utility of newborn households when they have no initial wealth. We also compute the monetary equivalent of such a counterfactual, that is, the additional initial wealth that induces the same variation in expected utility. More precisely, we calculate the lump-sum cash transfer necessary at the start of life to equate the household’s expected discounted utility in the baseline economy to that in the counterfactual economy. To achieve this, since the change in expected utility is marginal, we construct the function that associates a given level of initial wealth (for a young household with no property assets) with the expected utility over the entire life cycle. We then divide the impact of the reform on expected utility by the slope of this curve near zero.

The results are presented in Table 12 and show a positive impact on welfare—but only when the decrease in transaction taxes is balanced by an increase in property taxation in the government budget (counterfactuals A and C). At first sight, it may seem surprising that financing using a distortive tax on housing improves well-being relative to financing in counterfactual B using the nondistortive CSG tax. However, it is well known, in particular since Samuelson (1958), that overlapping-generations models with infinite time horizons are not dynamic efficient in the sense that a competitive equilibrium is not always Pareto optimal and may involve an overaccumulation of savings, which a tax on real estate wealth can help to reduce.

Furthermore, the impact on welfare is positive and nearly twice as great in the variant with an elastic supply (counterfactual C), which can be explained by the fact that housing supply responds to the increased demand, inducing lower rental and housing prices than those in counterfactual A.

On the other hand, the penultimate row in Table 12 shows that the impact on the whole population is always negative. This contrasts with the effect on newborns discussed in the previous paragraph and results from the fact that younger households benefit from those reforms (as shown in the last row of Table 12), whereas all of the other age categories are penalized. More precisely, as mentioned in the previous section, reduced transaction costs allow young households to better

adapt their status as owners or tenants according to the evolution of their income but penalize older households due to the resulting increase in house prices. It should be noted that we are comparing steady states here, and we do not take into account the effect of a reduction in transaction taxes on existing households in the initial steady state. In particular, we found that it induces an increase in real estate prices, and therefore generates added value gains for the homeowners. As mentioned in the literature review above, studies that account for these capital gains find on the contrary that the overall effect on the population of reducing transaction taxes is positive.

| | Counterfactuals | | |
|-------------------------|-----------------|----------------|---------------|
| | A | B | C |
| Newborns (poor) | 0.25% / 1457€ | -0.10% / -591€ | 0.45% / 2580€ |
| Newborns (middle class) | 0.25% / 1593€ | -0.10% / -613€ | 0.40% / 2566€ |
| Newborns (wealthy) | 0.17% / 1306€ | -0.12% / -939€ | 0.25% / 1937€ |
| Whole population | -0.20% / -5.6€ | -0.42% / -6.3€ | -0.04% / -4€ |
| Young households | 0.34% / 0.78€ | 0.23% / 0.55€ | 0.48% / 1.3€ |

Table 12: Welfare impact

Note: The first term is the percentage change in expected utility, the second term is the monetary equivalent. Regarding the impact on the whole population, the monetary equivalent is the compensating variation

(cf. caption of table 8 for a detailed definition of the baseline and of the counterfactuals).

It is then natural to wonder about the impact on inequalities, which can be measured using Gini indices. Since wages remain unchanged, the Gini index related to income is unaffected and is equal to 0.29 in the baseline and counterfactuals, which is consistent with what was measured for France in 2006 by the World Bank.⁷ We display in Table 13 the impact of counterfactuals on Gini indices related to consumption and wealth (defined as the sum of the net financial wealth and the market value of real estate properties of a household). The first row of Table 13 provides the Gini index for consumption. The second row also provides the Gini index on current period expenditures—consumption expenditures increased by housing expenditures (the rent paid directly or the imputed rent in the case of a homeowner). The last row provides the Gini index on wealth. In all cases, we find that the impact on the inequalities of the reforms that we consider remains marginal. This is confirmed by Lorenz curves as shown in Figure 1 since baseline and counterfactuals are

⁷Cf. <https://data.worldbank.org/indicator/SI.POV.GINI?locations=FR>, which provides a Gini index for income in France equal to 0.297 in 2006.

undistinguishable.

| | Baseline | Counterfactuals | | |
|--------------|----------|-----------------|-------|-------|
| | | A | B | C |
| Consumption | 0.295 | 0.296 | 0.297 | 0.295 |
| Expenditures | 0.278 | 0.277 | 0.278 | 0.278 |
| Wealth | 0.707 | 0.705 | 0.704 | 0.707 |

Table 13: Gini indexes

(cf. caption of table 8 for detailed definition of the baseline and of the counterfactuals).

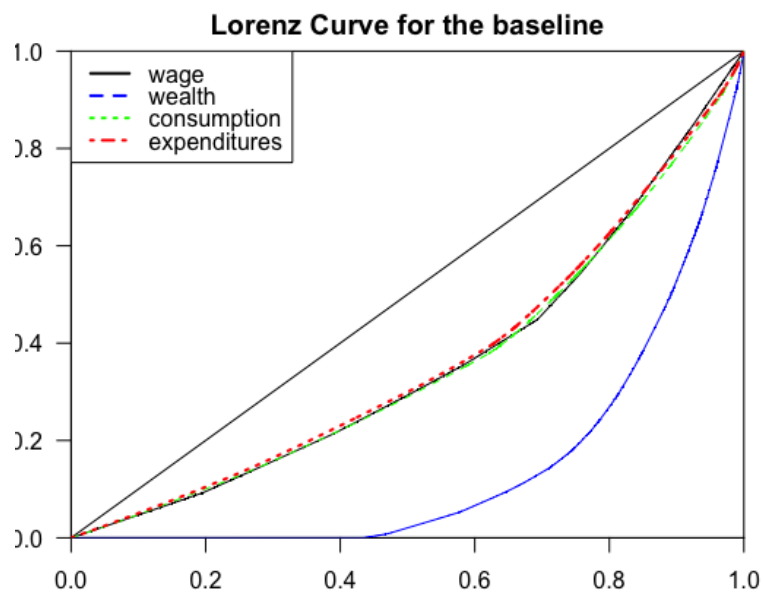


Figure 1: Lorenz curve in the baseline calibrated model for wage, consumption, wealth and expenditures (Lorenz curves for the baseline and the counterfactuals are undistinguishable).

6 Conclusion

In this paper, we constructed a life-cycle model of housing decisions taken by heterogeneous households belonging to overlapping generations. Those households face strong prudential regulations in terms of downpayment, payment-to-income ratios and mortgage repayment rates that correspond to a highly regulated economy such as France. In addition, transaction taxes in France are among the highest in Europe.

We solved and calibrated the resulting general equilibrium rental and owner prices which clear rental and owner markets. This type of construction leads to many nonconvexities that we dealt with by introducing random lotteries that households are given access to smooth their value functions across their many discrete choices.

Using these estimates, we computed the impacts of reducing transaction taxes when different instruments are used to balance the government's budget through property or income taxes, and in the case in which housing supply is fully or somewhat inelastic.

As expected, reducing transaction taxes have two main effects: first, consumption and housing choices are less distorted over the life-cycle; second, housing prices increase, and the larger housing supply elasticity, the less so. The former effect on distortions results from increased liquidity of housing as put forward in the recent macroeconomic literature (Boar et al., 2022). Liquidity make households more willing to downsize (or upsize) in case of negative (respectively positive) income shocks. In equilibrium their willingness to home ownership also increases as a result, and rental prices decrease.

The combination of these effects on homeownership, prices and rents lead to modest gains of welfare when welfare is computed over the life cycle by comparing newborn households across steady states. These gains are comparable to what is found in Australia (Cho et al., 2021). They are slightly larger than in Germany where welfare effects are found to be negative in a somewhat different model (Kaas et al., 2021). It is probably because homeownership is much lower and social housing is more important in Germany than in France. A further cross-country comparison would be interesting to pursue in future work.

These modest welfare gains, and the null impact on inequalities across income groups, do not advance the agenda of the policy relevance of a reduction in transaction taxes since those taxes are much easier to estimate and collect than property taxes. In the absence of transactions of a property, the estimation of the property value would have to rely on hedonic price estimates of properties sold in its neighborhood with a large degree of uncertainty. This trade-off remains unexplored to our knowledge in the literature.

A Data Appendix and Proofs

A.1 Dictionnary

Variables and functions of the structural model

| Variable or function | Name | Values |
|----------------------|---|----------------------------|
| w | Earnings or productivity | J values |
| c | Consumption | \mathbb{R}^+ |
| s | Housing services | $\{0, h_1, \dots, h_K\}^*$ |
| h | Housing stock | $\{0, h_1, \dots, h_K\}^*$ |
| h' | Next period housing stock | $\{0, h_1, \dots, h_K\}^*$ |
| r | Deposit rate | Calibrated |
| r^m | Mortgage rate | Calibrated |
| b | Net financial wealth | \mathbb{R} |
| b' | Next period net financial wealth | \mathbb{R} |
| ρ | Rental rate | \mathbb{R}^+ |
| q | Housing price | \mathbb{R}^+ |
| t | Income tax | \mathbb{R}^+ |
| T | Full taxes | \mathbb{R}^+ |
| ζ | Next period net assets or liabilities: $c + b'$ | \mathbb{R} |
| ε | Preference shocks on value function | \mathbb{R} |
| Ψ | Surplus function for discrete program | \mathbb{R} |
| V | Value function | \mathbb{R} |
| V^I | Interim value function | \mathbb{R} |

*We set $K = 6$ and $h_1 = 35$, $h_2 = 65$, $h_3 = 95$, $h_4 = 125$, $h_5 = 155$, $h_6 = 185$ (in square meters).

Parameters of the structural model

| Parameter | Name | Values |
|------------------|--|---------------------------------|
| J | Age and earnings groups: number | 12 |
| N_A | Age groups: Number | 4 |
| χ_j | Transition probability from age j to age $j + 1$ and if $j = 4$ to death | Estimated (see Appendix A.4) |
| p_j | Transition prob. between productivity groups indexed by the number of scales upward | Estimated (see Appendix A.4) |
| $\Pi(w' w)$ | Transition matrix between productivity groups | |
| K | Number of housing groups | 5 |
| γ | Equivalence scale | See Table 1 |
| σ | Consumption risk aversion | Estimated |
| α | Share of housing expenditures | Estimated |
| ξ | Preference for ownership | \mathbb{R}^+ ; Estimated |
| ϕ | Landlord fixed cost | \mathbb{R}^+ ; Estimated |
| c_m | Mobility cost for owners | \mathbb{R}^+ ; Estimated |
| θ | Downpayment rate | 20% |
| τ_e | Maximal repayment to income ratio | 30% |
| δ | Minimum payoff rate | 5% |
| τ^r | Residence tax including housing benefits | Calibrated: 0.002 |
| τ^h | Property tax | Calibrated: 0.003 |
| τ^b | Housing purchase tax | Calibrated: 0.08 |
| τ^s | Housing sale tax | Calibrated: 0.06 |
| β | Discount rate | 97% |
| H | Housing stock supply | See Section 4.1 |
| \mathcal{G}, G | Grid for b, b_0, \dots, b_G ; Number of grid points | 50 |
| L | Number of pairs $(w, h) : J(K + 1)$ | 72 |

A.2 Earnings levels

The first earnings category within each age group comprises households benefiting from housing subsidies (see below Appendix A.3). The second earnings category represents households eligible for a capped zero-interest loan but not eligible for housing subsidies. The calibration is performed using the 2006 Housing Survey and is summarized in the following table, which can be read as follows: the subcategory of households from 35 to 50 years benefiting from housing subsidies represents 9.5% of the population, have a median income equal to 13384 euros and an average household size, or equivalent scale, e , of 3.3. We assume that the last age category (older than 65) is entitled to housing subsidies for tenants and homeowners with existing mortgages but not to the zero-interest rate loan.

| Age Category | Earning Group | j | Median Earnings | Household Size | Population Share |
|--------------|---------------|----|-----------------|----------------|------------------|
| Junior | L | 1 | 10248€ | | 0.108 |
| | M | 2 | 16183€ | 2.50 | 0.072 |
| | U | 3 | 36771€ | | 0.06 |
| Middle | L | 4 | 13384€ | | 0.095 |
| | M | 5 | 18406€ | 3.30 | 0.088 |
| | U | 6 | 43529€ | | 0.081 |
| Senior | L | 7 | 10341€ | | 0.047 |
| | M | 8 | 15576€ | 2.40 | 0.104 |
| | U | 9 | 41307€ | | 0.104 |
| Retired | L | 10 | 10110€ | | 0.031 |
| | M | 11 | 13587€ | 1.50 | 0.131 |
| | U | 12 | 32056€ | | 0.079 |

Table A-1: Median earnings, equivalent scales and frequencies by age and earning groups

A.3 Housing allowances

Since 2001, housing allowances have been based on the same system and denoted AL in this appendix. The value of those subsidies is based on a complex formula (see Bozio et al. (2015)) that accounts for household characteristics (earnings and size), geographical location and housing expenses. For a given household and a given location, it can be approximated by a piecewise-affine function, i.e., the sum of a constant and a fraction of a capped housing repayment, that is,

$AL = c + \tau \times \min(P, P_{\max})$. Both tenants and homeowners with mortgages can benefit from this policy: in the previous formula, payment P can be either the rent or the loan repayment. The parameters c , τ and P_{\max} were obtained using a comprehensive database managed by the French ministry of housing and calibrated to obtain the best fit. Regarding homeowners with mortgages, the ceiling on loan repayment is quite low such that housing subsidies are nearly flat. This result is consistent with what is observed using the 2006 French housing survey—a very low correlation between subsidies and loan repayments. The data in the table below are monthly values.

$$\begin{aligned}
\text{age} \in [20, 35] : & \quad AL = 71 + 0.5 \times \min(R, 310) & \text{or} & \quad AL = 92 + 0.11 \times \min(L, 430) \\
\text{age} \in [35, 50] : & \quad AL = 100 + 0.42 \times \min(R, 370) & \text{or} & \quad AL = 116 + 0.06 \times \min(L, 780) \\
\text{age} \in [50, 65] : & \quad AL = 105 + 0.33 \times \min(R, 340) & \text{or} & \quad AL = 105 + 0.086 \times \min(L, 1290) \\
\text{age} > 65 : & \quad AL = 39 + 0.49 \times \min(R, 230) & \text{or} & \quad AL = 52 + 0.14 \times \min(L, 710)
\end{aligned}$$

in which R is the rent, and L is the loan repayment.

A.4 Earnings transition matrix

The values are calibrated to match the stationary probabilities within each state measured from the French Housing Survey (INSEE, 2006), as well as transition probabilities as described in Magnac et al. (2013). The authors report estimates of a transition matrix using a very long panel for a single cohort of male French wage earners working in the private sector and observed from 1977 to 2007. They provide transition matrices between quintiles and in two subperiods, the first one being defined between the labor market entry (1977) and 15 years later (1992), and the second one between 1993 and 2007. The approach does not fit perfectly with our age and revenue breakdown, but we can approximate that by considering that our first age category corresponds to the first two quintiles, the third to the fifth quintile, and the two different periods corresponding, respectively, to our first- and second-age categories (hereafter called 'junior' and 'middle-aged'). The following Table shows the transition probabilities between the two first and the last quintile for the first and the second period during 1997–2007.

| | L to L | L to U | U to U | U to P | (U to M) |
|-------|--------|--------|--------|--------|----------|
| Young | 58, 6% | 8, 83% | 50, 9% | 9, 2% | (40%) |
| Older | 74, 4% | 1, 95% | 72, 8% | 3, 9% | (23%) |

(A.1)

The yearly transition matrix is fitted to match both the stationary probabilities within each state and the 15-year transition probability described in the four first columns of the table. The

last is used for the consistency check that we obtain a transition matrix that fits this last column as well.

The estimated transition matrix is as follows:

| $i \backslash j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | 0.890 | 0.043 | 0.000 | 0.067 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.869 | 0.064 | 0.000 | 0.067 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.017 | 0.078 | 0.839 | 0.000 | 0.000 | 0.067 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 0.916 | 0.023 | 0.000 | 0.061 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.927 | 0.013 | 0.000 | 0.061 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.003 | 0.017 | 0.920 | 0.000 | 0.000 | 0.061 | 0.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.923 | 0.024 | 0.000 | 0.053 | 0.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.913 | 0.012 | 0.000 | 0.075 | 0.000 |
| 9 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.003 | 0.017 | 0.938 | 0.000 | 0.000 | 0.042 |
| 10 | 0.036 | 0.000 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.933 | 0.000 | 0.000 |
| 11 | 0.036 | 0.000 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.933 | 0.000 |
| 12 | 0.036 | 0.000 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.933 |

Element (i, j) represents $\Pi(w' = w_j | w = w_i)$ in which w and w' are the earnings of two consecutive periods.

A.5 Proof of Proposition 1

- We first prove (i).

If $h' = h$ and $b' < 0$, then necessarily, $h \neq 0$. Indeed, renters can not have negative financial wealth.

Assume now, that $h = 0$ and $h' > 0$, i.e., the household bought a house $h' \neq 0$ in period t . Because of the downpayment constraint (3) in period t , we have

$$b' \geq -(1 - \theta)qh' \geq -qh'.$$

Suppose that the household remained in this house in period $t+1$, $h'' = h'$. Note that constraint (4) is written for $b'' < 0$:

$$b'' - (1 + r^m)b' \geq \delta \max(-b', qh').$$

Combining the last two inequalities leads to:

$$b'' - (1 + r^m)b' \geq \delta qh'.$$

which is Equation (6).

This also implies that:

$$\begin{aligned}
b'' &\geq \delta q h' + (1 + r^m) b' \\
&\geq \delta q h' - (1 + r^m)(1 - \theta) q h' \quad \text{from (3)} \\
&\geq -((1 + r^m)(1 - \theta) - \delta) q h' \\
&\geq -q h'' = -q h'''.
\end{aligned}$$

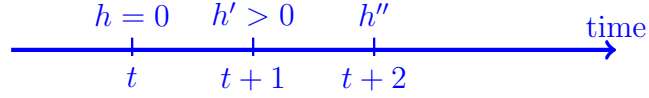
The last inequality is derived from Condition S, which implies $(1 + r^m)(1 - \theta) - \delta < 1 - \theta < 1$.

By repeating the same argument by forward induction if $h''' = h''$, this further implies that equation (6) applies to b''' and that $b''' \geq -q h'''$. This proves that for any sequence of periods in which housing remains constant, condition (6) applies.

- We now prove (ii).

This is trivially true for $h = 0$.

Consider the following sequence of housing choices:



$b' \geq -(1 - \theta) q h'$ because of the borrowing constraint (3).

If $h'' = h' > 0$, we know from above that Condition S implies

$$(1 + r^m)(1 - \theta) - \delta < 1 - \theta.$$

Equation (6) applied to b'' and b' leads to

$$\begin{aligned}
b'' &\geq \delta q h' + (1 + r^m) b' \\
&\geq \delta q h' - (1 + r^m)(1 - \theta) q h' \quad \text{from (3)} \\
&\geq -((1 + r^m)(1 - \theta) - \delta) q h' \\
&\geq -(1 - \theta) q h' = -(1 - \theta) q h''.
\end{aligned}$$

If $h'' \neq h'$. Either $h'' = 0$ and, therefore, $b'' \geq 0$ is trivially satisfied. Or, $h'' > 0$. If $h' > h'' > 0$:

$$\begin{aligned}
b'' &\geq (1 + r^m) b' + \delta q h' - (h'' - h') q \\
&\geq -(1 - \theta) q h' - (h'' - h') q \\
&\geq -(1 - \theta) q h''.
\end{aligned}$$

If $h'' > h' > 0$, the borrowing constraint imposes to increase the mortgage by $(1 - \theta)q(h'' - h')$ at the maximum. Therefore,

$$\begin{aligned} b'' &\geq (1 + r^m)b' + \delta qh' - (1 - \theta)(h'' - h')q \\ &\geq -(1 - \theta)qh' - (1 - \theta)(h'' - h')q \\ &\geq -(1 - \theta)qh''. \end{aligned}$$

Therefore Equation (7) is true at $t + 1$ in any case, and we have shown that it implies it is true in all configurations at $t + 2$.

By forward induction, this proves (ii).

■

A.6 Smoothing out the discontinuity in housing allowances

As explained in the text, housing allowances $\eta(w, h, h', s, b)$ are also given to owner-occupiers ($h = s, h > 0$) with a mortgage ($b < 0$). As $\eta(w, h, h', s, b)$ depends on the existence of a mortgage, it is discontinuous, which may lead to a discontinuous value function at $b = 0$. Since housing allowance rules are fuzzy, we make this dependence continuous by assuming that for owner-occupiers (1), the absence of a mortgage makes $\eta(h', s; w, h, b)$ constant for $b \geq 0$ and equal to $\eta^+(h', s; w, h)$; (2) $\eta(h', s; w, h, b)$ is constant, equal to $\eta^-(h', s; w, h)$, for $b < -\Delta qs\delta$ since mortgages are greater in absolute value than around $\Delta < 2$ annuities of reimbursement; (3) $\eta(h', s; w, h, b)$ is linear and decreasing between $-\Delta qs\delta$ and 0 since $\eta^-(h', s; w, h) > \eta^+(h', s; w, h)$ because of housing benefits. We denote the slope in absolute value

$$\sigma_b = \frac{\eta^-(h', s; w, h) - \eta^+(h', s; w, h)}{\Delta qs\delta} > 0.$$

Parameter Δ is introduced so that the right-hand side of equation (B.3) remains increasing in b_g . This is achieved if $\sigma_b(\Delta) < 1 + r_m$ and can be achieved by manipulating Δ . In the code, we set $\Delta = 1.5$, which means that the decreasing section of function η_0 is close to zero (within a 1.5-year mortgage reimbursement).

A full description of the effect it has on the budget constraint is developed in the online Appendix E. In particular, note that the derivative of the full income function $\zeta(h', h, s, w, b)$ defined in equation (10) is equal to $1 + r(b)$, which is obtained by:

$$r(b) = r1\{b \geq 0\} + r_m1\{b < -\Delta qs\delta\} - \sigma_b1\{b \in [-\Delta qs\delta, 0)\}.$$

B Consumption and endogenous grids

In this subsection, we explain how to map the operator \mathcal{L}_I from the future expected value function to the current interim value function. This is why we fix the values of (h', s) and (h, w) .

B.1 Unconstrained problem

As in the text, we start by looking at the solutions that do not necessarily verify the downpayment constraint (7) $b \geq -(1 - \theta)qh$ or the minimum repayment constraint (6) on payoffs:

$$b' \geq b(1 + r^m) + \delta qh \quad \text{if } h' = h, \quad b' < 0.$$

B.1.1 The unconstrained solution

The unconstrained solution applies in particular to cases $h' \neq h$ but is also used as a reference in other cases. Absent any constraints other than the budget constraint, the first-order condition for consumption in the interim decision program is given by:

$$U_c(c, s; h) = \beta \mathbb{E} V_b(w', b', h')$$

denoting the derivative of a function $f(x, y)$ with respect to x by f_x . For any point b'_g in the grid, consumption c_g solves:

$$c_g = U_c^{-1}(\beta \mathbb{E} V_b(w', b'_g, h'), s; h) = \left(\frac{\beta \mathbb{E} V_b(w', b'_g, h')}{\gamma} s^{-\alpha} \right)^{-\frac{1}{\sigma}}, \quad (\text{B.2})$$

in which U_c^{-1} is the inverse of U_c with respect to its first argument, and $V_b(w', b'_g, h')$ is the derivative of the value function with respect to b' evaluated at each point of the grid $b'_g \in \mathcal{G}(h')$. Note that because the expected future value function is concave, consumption is a nondecreasing function of b'_g .

We obtain a point in the current grid, b_g , by solving the budget constraint (10) in b :

$$c_g + b'_g = \zeta(h', h, s, w, b_g) = \zeta_0(h', s; w, h) + (1 + r(b_g))b_g \quad (\text{B.3})$$

which exists and is unique:

$$(1 + r(b_g))b_g = c_g + b'_g - \zeta_0(h', s; w, h)$$

in which $r(b_g)$ is either r or r^m depending on whether the right-hand side is positive or negative.

B.1.2 Graphs of value and policy functions

Note that current period financial wealth b_g is an increasing function of b'_g either by construction or because of the assumption $\sigma_b(\Delta) < 1 + r_m$.

Points (b_g, c_g) describe the graph of the policy function

$$c = c(b, h', s, h, w).$$

The interim value function at these points:

$$v_g = V^I(h', s; w, b_g, h) = u(c_g, s; h) + \beta \mathbb{E}V(w', b'_g, h') \quad (\text{B.4})$$

defines a point (b_g, v_g) on the graph of

$$V^I(h', s; w, b, h) = \max_c \{u(c, s; h) + \beta \mathbb{E}V(w', b', h')\}$$

subject to:

$$\begin{aligned} c + b' &= \zeta(h', h, s, w, b), \\ b' &\geq -(1 - \theta)qh'. \end{aligned}$$

Moreover, we can also compute its derivative in b_g :

$$dv_g = V_b^I(h', s; w, b_g, h) = \beta(1 + r(b))\mathbb{E}V_b(w', b'_g, h'). \quad (\text{B.5})$$

However, this unconstrained grid (b_1, \dots, b_G) of solutions above neither imposes the downpayment constraint (7) $b_g \geq -(1 - \theta)h$ nor the repayment constraint (6) on (b'_g, b_g) . This is what we now turn to.

B.2 Imposing constraints on financial assets

We can summarize when and how constraints bind in the following Table if $H = \{0, \dots, h_K\}$:

| Case | Housing variables | Constraints on (b, b') |
|-----------------------|--------------------------|---|
| (1) Selling or buying | $h \neq h'; h, h' \in H$ | $b' \geq -(1 - \theta)qh', b \geq -(1 - \theta)qh$ |
| (2) Renters | $h = h' = 0$ | $b \geq 0, b' \geq 0$ |
| (3) Immobile owners | $h = h', h \neq 0$ | $\begin{cases} b' \geq b(1 + r^m) + \delta qh, b \geq -(1 - \theta)qh & \text{if } b' < 0 \\ b \geq -(1 - \theta)qh & \text{if } b' \geq 0 \end{cases}$ |

It is natural to impose that one point on the grid is such that $b'_g = 0$ regardless of the values of (h, h') .

We address these three cases one by one.

B.2.1 Changing housing status or level: $h \neq h'$

Consider first the solution corresponding to $b'_1 = -(1 - \theta)qh'$ derived from equation (B.3) that is:

$$(1 + r(b_1))b_1 = c_1 - (1 - \theta)qh' - \zeta_0(h', s; w, h, b_1).$$

If $h \neq 0$, the condition $b_1 \geq -(1 - \theta)qh$ is equivalent to:

$$c_1 - (1 - \theta)qh' - \zeta_0^-(h', s; w, h) \geq -(1 + r^m)(1 - \theta)qh$$

because $\zeta_0(h', s; w, h, b_1)$ is constant when b_1 is close to the lower bound $-(1 - \theta)qh$. This is also true when $h = 0$ since the RHS is equal to zero and $\zeta_0(h', s; w, h, b_1)$ is constant for $h = 0$.

This constraint is expressed as:

$$c_1 \geq (1 - \theta)qh' + \zeta_0(h', s; w, h, b) - (1 + r^m)(1 - \theta)qh \equiv c_0. \quad (\text{B.6})$$

Therefore, there are two cases:

- if c_1 satisfies constraint (B.6), then $b_1 \geq -(1 - \theta)qh$. We should then complete the grid of bs between $b = -(1 - \theta)qh$ and $b = b_1$. In this range, because of the future borrowing constraint in terms of b' , consumption is constrained and equal to the following linear function of b :

$$c(b) = (1 - \theta)qh' + \zeta_0(h', s; w, h, b) + (1 + r(b))b.$$

However, because consumption $c > 0$, this case is further decomposed into:

- If $c_0 = (1 - \theta)qh' + \zeta_0(h', s; w, h, b) - (1 + r^m)(1 - \theta)qh > 0$, we can thus add a point $(c_0, b_0 = -(1 - \theta)qh)$ to the graph. Furthermore,

$$\begin{aligned} v_0 &= u(c_0, s, h) + \beta EV(-(1 - \theta)qh', h', w'), \\ dv_0 &= (1 + r(b_0))u'_c(c_0, s, h). \end{aligned}$$

- If $c_0 < 0$, we associate to the point $b_0 = -(1 - \theta)qh$ the value function $v_0 = -\infty$ and $dv_0 = +\infty$. We can also complete the grid by choosing an arbitrary, small positive value c_+ and define $b_+ \in (-(1 - \theta)h, b_1)$ as:

$$\begin{aligned} c_+ &= (1 - \theta)qh' + \zeta_0(h', s; w, h, b) + (1 + r(b_+))b_+, \\ v_+ &= u(c_+, s, h) + \beta EV(-(1 - \theta)qh', h', w'), \\ dv_+ &= (1 + r(b_+))u'_c(c_+, s, h). \end{aligned}$$

- The second case is when c_1 does not satisfy constraint (B.6) and $b_1 < -(1 - \theta)qh$. Being outside the constraints, this cannot be a solution. This also applies to any point in the grid such that $b_g < -(1 - \theta)qh$ as derived from equation (B.3).

We create a new intermediate value $b_0 = -(1 - \theta)qh$ and compute by interpolation the consumption value c_0 . We can thus derive from equation (B.3) the solution for the next period's wealth b'_0 :

$$(1 + r(b_0))b_0 = c_0 + b'_0 - \zeta_0(h', s; w, h, b_0),$$

and value v_0 :

$$v_0 = u(c_0, s, h) + \beta EV(b'_0, h', w'),$$

in which $EV(b'_0, h', w')$ is computed by interpolation. We also associate with the values $b_g < -(1 - \theta)qh$ the solutions:

$$v_g = -\infty, dv_g = +\infty.$$

B.2.2 Renters: $h' = h = 0$

This case is very similar to the previous one. Note that the income function $\zeta_0(0, s; w, 0)$ is constant.

Consider again the solution corresponding to $b'_1 = 0$ derived from equation (B.3), that is:

$$(1 + r(b_1))b_1 = c_1 - \zeta_0(0, s; w, 0).$$

There are two cases:

- if $c_1 > \zeta_0(0, s; w, 0)$, then $b_1 > 0$. We should then complete the grid of bs between $b = 0$ and $b = b_1$. In this range, because of the borrowing constraint, consumption is constrained and equal to the linear function of b :

$$c(b) = \zeta_0(0, s; w, 0) + (1 + r)b.$$

The formula (10) for $\zeta_0(0, s; w, 0)$ simplifies into:

$$\zeta_0(0, s; w, 0) = w - \rho s - T(w, 0, 0, s),$$

and can be either positive or negative. This again leads to two subcases according to the sign of $c_0 = \zeta_0(0, s; w, 0)$:

– If $c_0 > 0$, we can add a point $(c_0, b_0 = 0)$ to the graph. Furthermore,

$$\begin{aligned} v_0 &= u(c_0, s, 0) + \beta EV(0, 0, w'), \\ dv_0 &= (1 + r)u'_c(c_0, s, 0). \end{aligned}$$

– If $c_0 < 0$, we associate with the point $b_0 = 0$ the value function $v_0 = -\infty$ and $dv_0 = +\infty$. We can also complete the grid by choosing an arbitrary, small, positive value c_+ and define $b_+ \in (0, b_1)$ as:

$$\begin{aligned} c_+ &= \zeta_0(0, s; w, 0) + (1 + r)b_+, \\ v_+ &= u(c_+, s, 0) + \beta EV(0, 0, w'), \\ dv_+ &= (1 + r)u'_c(c_+, s, 0). \end{aligned}$$

- The second case is when $c_1 < \zeta_0(0, s; w, 0)$ and $b_1 < 0$. Being outside the constraints, this cannot be a solution. This also applies to any point in the grid such that $b_g < 0$, as derived from equation (B.3).

We create a new, intermediate value $b_I^* = 0$ and compute by interpolation the value for next period financial wealth b_I^* . We can thus derive from equation (B.3) the solution for c_I^* :

$$0 = c_I^* + b_I^* - \zeta_0(0, s; w, 0),$$

and value v_I^* and derivatives, dv_I^* :

$$v_I^* = u(c_I^*, s, 0) + \beta EV(b_I^*, 0, w'), \quad dv_I^* = \beta(1 + r)EV_b(b_I^*, 0, w').$$

We also associate with the values $b_g < 0$ the solutions:

$$v_g = -\infty, \quad dv_g = +\infty$$

These boundary issues possibly lead to the rewriting of an augmented grid from b_0 to b_G .

B.2.3 Immobile owners: $h' = h \neq 0$

This is the more elaborate case, and the logic should take into account that the additional constraint (6)

$$b' \geq b(1 + r^m) + \delta qh \quad \text{if } h' = h, \quad b' < 0.$$

is not necessarily satisfied for all (b_g, b'_g) if $b'_g < 0$. Additionally, we need to ensure that the consumption calculated in (B.2) is compatible with the state variable.

Consider $h = h' \neq 0$ and note that if $b' < 0$, then $b < 0$ by equation (6). By continuity, we have that $b' = 0$ implies that $b \leq 0$ such that only negative b s matter. We use the expression of the income function $\zeta_0(h, h, s, w, b)$ that can be derived from Appendix E :

$$\zeta_0(h, h, s, w, b) = \zeta_0^- - (\sigma_b + \Delta\delta qh)1\{b \in [-\Delta\delta qh, 0]\} \text{ if } b < 0.$$

Preliminaries First, denote the index g_0 such that $b'_{g_0} = 0$, and note that $g_0 > 1$ because $h = h' > 0$. We associate with this value unconstrained consumption c_{g_0} and the derived value b_{g_0} , given by the budget constraint:

$$c_{g_0} = \zeta_0(h, s; w, h, b_{g_0}) + (1 + r(b_{g_0}))b_{g_0}.$$

Note that necessarily $b_{g_0} > -(1 - \theta)qh$ since it is impossible when $h \neq 0$ to reimburse in one period the whole mortgage taken in the previous period under reasonable values of parameters and variables. Furthermore, if $g \geq g_0$, note that $b'_g \geq 0$ satisfies constraint (6); therefore, we follow the standard grid from $g = g_0$ onwards. We possibly modify the uncensored grid (c_g, b_g) if $g < g_0$ or, equivalently, only when $b'_g < 0$ only.

Second, in the positive domain for consumption, the budget constraint is written when $b' < 0$

$$\begin{aligned} 0 &< c = \zeta(h, h, s, w, b) - b' \\ &= \zeta_0^- - \sigma_b(b + \Delta\delta qh)1\{b \in [-\Delta\delta qh, 0]\} + (1 + r_m)b - b' \\ &\leq \zeta_0^-(h, h, s, w) - \sigma_b(b + \Delta\delta qh)1\{b \in [-\Delta\delta qh, 0]\} - \delta qh \end{aligned}$$

in which we use equation (B.3) and Appendix E in the second line and constraint (6) in the third line. This implies that:

$$c + \sigma_b(b + \Delta\delta qh)1\{b \in [-\Delta\delta qh, 0]\} \leq \zeta_0^-(h, h, s, w) - \delta qh \text{ if } b' < 0. \quad (\text{B.7})$$

Conversely, if constraint (B.7) is satisfied, then the reimbursement constraint (6) is satisfied.

In the following, we denote (c_g^*, b_g^*) the constrained solution for which the constrained financial wealth is:

$$b_g^* = \frac{b'_g - \delta qh}{1 + r^m},$$

and the constrained consumption is:

$$c_g^* = \zeta_0^-(h, h, s, w) - \delta qh - \sigma_b(b_g^* + \Delta\delta qh)1\{b_g^* \in [-\Delta\delta qh, 0]\},$$

which is nonconstant only in the neighborhood of 0 and for any g , $c_g^* \leq c_1^*$.

The constraint (B.7) is binding whenever:

$$c = c_g^* < c_g \text{ and } b = b_g^* < b_g. \quad (\text{B.8})$$

In the following, we repetitively use the fact that the uncensored solutions c_g and b_g are increasing with b' and, thus, with an increasing index g in the grid. We also use the fact that (c_1, b_1) cannot be a solution because $b_1 < -(1 - \theta)qh$ as $b'_1 = -(1 - \theta)qh$ and by condition S, $\delta > r^m(1 - \theta)$, (see the proof of Proposition 1). This implies that (c_1^*, b_1^*) cannot be a solution because

$$b_1^* \leq b_1 < -(1 - \theta)qh.$$

We set the main classification variable below to:

$$c_1^* = \zeta_0^-(h, h, s, w) - \delta qh$$

since $b_1^* < -\Delta\delta qh$.

Algorithm The analysis can now proceed by decomposing the problem into four regimes:

1. If $c_1^* \leq 0 < c_1$, for all $g < g_0$ (and therefore $b'_g < 0$), the choice $(h' = h, s)$ (given w) is impossible for a household because of the constraint (B.8) that makes consumption negative. Necessary payoffs exceed the resource capacity of the household. We set $v_g = -\infty$ for all $g < g_0$.
2. Otherwise, if $c_1^* \in (0, c_1)$, for all $g < g_0$, the constraint (B.8) is binding since:

$$c_g^* \leq c_1^* < c_1 < c_g.$$

We thus modify the grid as:

- (a) For all g such that $b_g^* < -(1 - \theta)qh$ (which is true for $g = 1$ by the above), the constrained solution is unattainable, and we set $v_g = -\infty$.
- (b) Create $b_0 = -(1 - \theta)qh$ ($\in (b_1^*, b_{g_0})$ by the above) and set the constrained solution $c_0 = c_1^*$ and the corresponding value function:

$$v_0 = u(c_0, s, h) + \beta EV^I(b'_0, h', w'),$$

in which $b'_0 = (1 + r^m)b_0 + \delta qh$ and $EV^I(b'_0, h', w')$ is interpolated.

- (c) For all other larger $g < g_0$, $b_g^* > b_0$, (c_g^*, b_g^*) are the solutions. We change the value function into:

$$v_g^* = u(c_g^*, s, h) + \beta EV^I(b'_g, h', w').$$

3. Else if $c_1^* \in [c_1, c_{g_0})$:

- (a) Suppose there exists $1 < g_1 < g_0$ such that $b_{g_1} \geq -(1 - \theta)qh$ and $c_{g_1} \leq c_{g_1}^*$. If g_1 is not unique; take the minimum.

- i. For all $g < g_1$, $b_g < -(1 - \theta)qh$, the solution is unattainable and $v_g = -\infty$ (for $g = 1$ this is always the case).
- ii. Set $b_0 = -(1 - \theta)qh$, interpolate c_0 between c_{g_1-1} and c_{g_1} and compute b'_0 using the budget constraint such that:

$$v_0 = u(c_0, s, h) + \beta EV^I(b'_0, h', w'),$$

in which $EV^I(b'_0, h', w')$ is interpolated.

- iii. Consider all $g_1 \leq g < g_0$ such that $c_g \leq c_g^*$. The solution is (c_g, b_g) .
- iv. As c_g is an increasing sequence, consider all $g_1 < g < g_0$ such that $c_g > c_g^*$. Because $g > g_1$, $b_g^* > b_{g_1}^* > b_{g_1} > -(1 - \theta)qh$ since b_g^* is an increasing sequence. We continue with grid (c_g^*, b_g^*) until $g = g_0 - 1$ and:

$$v_g^* = u(c_g^*, s, h) + \beta EV^I(b'_g, h', w').$$

- (b) In the alternative case in which g_1 does not exist, we replace c_g with the constrained solution c_g^* (and b_g^*) because of constraint (B.8).

- i. For all $g < g_0$, such that $b_g^* < -(1 - \theta)qh$, the solution is unattainable and $v_g = -\infty$ (for $g = 1$. This is always the case).
- ii. We set $b_0 = -(1 - \theta)qh$, $c_0 = c_1^*$ and the value function is:

$$v_0 = u(c_0, s, h) + \beta EV^I((1 + r^m)b_0 + \delta qh, h', w'),$$

in which $EV^I((1 + r^m)b_0 + \delta qh, h', w')$ is interpolated.

- iii. We continue the grid with $b_g^* > -(1 - \theta)qh$ with grid (c_g^*, b_g^*) until $g = g_0 - 1$ and:

$$v_g^* = u(c_g^*, s, h) + \beta EV^I(b'_g, h', w').$$

4. Else if $c_1^* \geq c_{g_0}$:

- (a) Consider all $g < g_0$, and set $v_g = -\infty$ when $b_g < -(1 - \theta)qh$.
- (b) Define $b_0 = -(1 - \theta)qh$ and interpolate c_0 with the consumption defined by the bounds in bs containing b_0 . The value function is given by:

$$v_0 = u(c_0, s, h) + \beta EV^I(b'_0, h', w').$$

in which $b'_0 = (1 + r^m)b_0 + \zeta_0^-(h, h, s, w, b) - c_0$ and in which $EV^I(b'_0, h', w')$ is interpolated.

- (c) Continue with (c_g, b_g) until $g = g_0 - 1$

C Fixed point resolution of value and policy functions

A pseudocode for the algorithm runs as follows:

1. Choose a set of grids $\mathcal{G}(h)$ according to the constraints on b , i.e., $b \geq -(1 - \theta)qh$ and containing 0 (see text). This grid is fixed over the fixed-point routine. Set the value function to the initial condition $V^{(1)}(b', h', w')$ seen in Section 3.4 and the counter of iterations, $k = 1$.
2. At iteration k : For all productivity and housing stocks (w, h) apply:
 - (a) For all (h', s) , apply the EGM algorithm detailed in Section B
 - i. Compute for every grid point on b'_g the uncensored c_g using equation (B.2).
 - ii. Use the different regimes developed in Section B and compute the final b_g and final consumption value.
 - iii. For each (h', s) construct the interim value function $V^I(h, w, h', s, b_g(h, w, h', s))$ using equation (B.4).
 - (b) For each (h, w) , compute the convex hull of all values $V^I(h, w, h', s, b_g(h, w, h', s))$. Define extreme points of this convex hull as $(\tilde{b}(h, w), \max_{(h', s)} V^I(h, w, h', s, \tilde{b}))$. The fact that \tilde{b} depends on (h, w) and that many such points remain implicit in the following keeps the notation simple. However, note that the first extreme point is necessarily the first point in the grid $b_1(h)$. We also impose that the resulting value function $V(h, w, \tilde{b}) = \max_{(h', s)} V^I(h, w, h', s, \tilde{b})$ is increasing over the set of extreme points. Note also that a single (h', s) is associated with each of those extreme points \tilde{b} with probability one. This is the optimal discrete choice (h', s) at \tilde{b} .

(c) Interpolate on the flat segments between extreme points \tilde{b} (and possibly above the maximum of these values) the value function $V^{(k+1)}(b_g, h, w)$ at any b_g on the fixed grid $\mathcal{G}(h)$.

3. Verify the convergence criterion: $\|V^{(k+1)}(b, h, w) - V^{(k)}(b, h, w)\| < \epsilon$ in which the norm is:

$$\|V^{(k+1)}(b, h, w) - V^{(k)}(b, h, w)\| = \max_{w, h, b_g \in \mathcal{G}(h)} \left[(V^{(k+1)}(b, h, w) - V^{(k)}(b, h, w))^2 \right].$$

4. If the condition is not true, start a new iteration $k = k + 1$ by updating $V^{(k)}(b, h, w)$ on the grid $\mathcal{G}(h)$.

5. Exit if the condition is true and store $V^*(\tilde{b}, h, w)$ on the final set of extreme points $\tilde{b}(h, w)$ derived in step 2b.

Knowing $V^*(\tilde{b}, h, w)$, we can now compute the values of the optimal policy functions obtained at the extreme points \tilde{b} : $\tilde{c}, \tilde{h}', \tilde{s}$ as could be derived at step 2b. Those are necessarily unique. This means that for any (h, w, \tilde{b}) , we are able to compute the policy functions \tilde{c}, \tilde{h}' and \tilde{s} , as well as the probabilistic transition functions to w' and the deterministic function $\tilde{b}' = \tilde{b}'(\tilde{b}, \tilde{c}, \tilde{h}', \tilde{s})$.

The extension of this computation procedure of the policy functions for any point (h, w, b) in the feasible set is straightforward. The only difficulty is that b is generally between two extreme points in the grid specific to (h, w) . Suppose that $b \in (\tilde{b}_e, \tilde{b}_{e+1})$ in which $\tilde{b}_{E+1} = +\infty$, and E is the number of extreme points. Denote:

$$\lambda_e = \frac{b - \tilde{b}_e}{\tilde{b}_{e+1} - \tilde{b}_e} \text{ if } e < E, \lambda_e = \frac{b - \tilde{b}_E}{\tilde{b}_E - \tilde{b}_{E-1}} \text{ if } e = E.$$

The policy functions are given by $e < E$:

$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} c(b, h, w) = (1 - \lambda_e)c(\tilde{b}_e, h, w) + \lambda_e c(\tilde{b}_{e+1}, h, w) \\ \tilde{h}'(b, h, w) = \tilde{h}'_e = \tilde{h}'_{e+1}, \\ \tilde{s}'(b, h, w) = \tilde{s}_e = \tilde{s}_{e+1}, \end{array} \right. \quad \text{if } (\tilde{h}'_e, \tilde{s}_e) = (\tilde{h}'_{e+1}, \tilde{s}_{e+1}), \\ \left\{ \begin{array}{l} \left\{ \begin{array}{l} c(b, h, w) = c(\tilde{b}_e, h, w) \\ \tilde{h}'(b, h, w) = \tilde{h}'_e \\ \tilde{s}'(b, h, w) = \tilde{s}_e \end{array} \right. \quad \text{with probability } (1 - \lambda_e) \\ \left\{ \begin{array}{l} c(b, h, w) = c(\tilde{b}_{e+1}, h, w) \\ \tilde{h}'(b, h, w) = \tilde{h}'_{e+1} \\ \tilde{s}'(b, h, w) = \tilde{s}_{e+1} \end{array} \right. \quad \text{with probability } \lambda_e \end{array} \right. \quad \text{if } (\tilde{h}'_e, \tilde{s}_e) \neq (\tilde{h}'_{e+1}, \tilde{s}_{e+1}). \end{array} \right.$$

in which the expression in the first bracket means that the interim functions are the same but are different in the second brackets. In the case $e = E$, we extrapolate linearly above \tilde{b}_E by using:

$$\begin{aligned} c(b, h, w) &= c(\tilde{b}_E, h, w) + \lambda_E(c(\tilde{b}_E, h, w) - c(\tilde{b}_{E-1}, h, w)) \\ \tilde{h}'(b, h, w) &= \tilde{h}'_E, \\ \tilde{s}'(b, h, w) &= \tilde{s}_E. \end{aligned}$$

D Finding the steady state

Given prices $p = (\rho, q)$ and parameters θ , we start:

- with a value function obtained using the fixed point algorithm and constructed on an endogenous grid in which for any (h, w) , the extreme grid points are

$$\mathcal{G}_e(h, w) = \{b_k^{(h,w)}\}_{k=1,..,K(h,w)}.$$

- a probability distribution function denoted μ_0 . Only extreme points, denoted $l = (b_k^{(h,w)}, h, w)$, are charged with a probability mass $\mu_0^{(l)}$. We denote $b(l) = b_k^{(h,w)}$.

Strategy: We establish the new value of μ_{t+1} from the value μ_t in the following way.

- Start from $l = (b_k^{(h,w)}, h, w)$ and find (b'_k, h'_k, s_k) the policy functions associated with $b_k^{(h,w)}$.
- Denote w' as the future wage and $\pi(w' | w)$ as its probability. Associate probability $\mu^{(l)} * \pi(w' | w)$ with the point (b'_k, h'_k, w') . This is not, however, an extreme point in the grid (h', w') .
- Find l'_- and l'_+ the adjacent points in grid $\mathcal{G}_e(h', w')$ such that $b(l'_-) \leq b'_k \leq b(l'_+)$. If l'_- is the upper right point, then set $l'_+ = l'_-$. These are the closest extreme points.
- Associate a probability mass with each of them, proportional to the distance between them and b'_k . This means that a nonextreme point is a weighted combination of two extreme points:

$$\begin{aligned} - l'_+ &\text{ gets a mass of } \mu^{(l)} * \pi(w' | w) * \frac{b'_k - b(l'_-)}{b(l'_+) - b(l'_-)} \\ - l'_- &\text{ gets a mass of } \mu^{(l)} * \pi(w' | w) * \frac{b(l'_+) - b'_k}{b(l'_+) - b(l'_-)} \end{aligned}$$

– Note that as l'_+ and l'_- are derived from the same interim function, the randomization is not necessary but is equivalent to the solution.

- This is the transition matrix between μ_t and μ_{t+1} .

Algorithm:

Initialization:

- A list of elements (w, h) and two vectors: first, b containing extreme points of $V(w, h, b)$, their number varying with (w, h) . Second, a vector of probabilities $\mu^{(0)}(w, h, b)$ of the same length than b .
- A list of elements (w, h) , one vector b containing extreme points of $V(w, h, b)$, and a list of possible futures (w', h', b'_+) and (w', h', b'_-) , b'_+ and b'_- being of the same length as b and being the b -coordinates of points l'_+ and l'_- defined above. Associated with these vectors are the vectors $\pi(w' | w) * \frac{b'_k - b(l'_-)}{b(l'_+) - b(l'_-)}$ and $\pi(w' | w) * \frac{b(l'_+) - b'_k}{b(l'_+) - b(l'_-)}$ defined above. This list of lists defines the nonempty rows of the transition matrix between (w, h, b) and (w', h', b') and does not depend on the marginal distributions μ . We denote it as $K((w', h', b') | (w, h, b))$.

Recursion:

- Starting from $\mu^{(k)}$, we apply the operator $K((w', h', b') | (w, h, b))$ to compute the marginal probabilities of each future history after (w, h, b) . This results in lists describing (w', h', b') and its associated probabilities
- Sum over (w, h, b) to recover the marginal probability of any (w', h', b') . Denote it as $\mu^{(k+1)}$.

Given the fixed point values (V, μ) , in which μ is the ergodic distribution, we are now able to compute prices in this economy and calibrate their parameters.

Appendix not for publication

E Piecewise linear income function

In Appendix A.6 we smoothed out the discontinuity using function $\eta(\cdot)$ for the housing allowances. From this definition, we extend the same construction for any $h > 0$ and $h = s$ to the income function $\zeta_0(h', s; w, h, b)$ defined in equation (B.3) in Appendix B.2, which is a linear function of $\eta(\cdot)$. It is a piecewise linear income function at negative values of b near 0. It is described as, using notations ζ_0^+ , ζ_0^- and Δ ,

$$\zeta_0(h', s; w, h, b) = \begin{cases} \zeta_0^+ & \text{if } b \geq 0, \\ \zeta_0^+ - \frac{b}{\Delta\delta qh}(\zeta_0^- - \zeta_0^+) & \text{if } b \in (-\Delta\delta qh, 0), \\ \zeta_0^- & \text{if } b \leq -\Delta\delta qh. \end{cases}$$

For $h = 0$ or $h \neq s$, we have $\zeta_0(h', s; w, h, b) = \zeta_0^+ = \zeta_0^-$ is independent of b .

The slope of ζ_0 , in absolute value as a function of b on the inner interval is denoted in the text $\sigma_b = \frac{\zeta_0^- - \zeta_0^+}{\Delta\delta qh}$ and Δ is chosen such that $\sigma < 1 + r_m$. Rewrite ζ_0 as:

$$\zeta_0(h', s; w, h, b) = \begin{cases} \zeta_0^+ & \text{if } b \geq 0, \\ \zeta_0^+ - \sigma_b b & \text{if } b \in (-\Delta\delta qh, 0), \\ \zeta_0^- & \text{if } b \leq -\Delta\delta qh. \end{cases}$$

We now have to solve equation (B.3) in b :

$$(1 + r(b))b = \underbrace{b' + c}_{\text{Cash-in-hand}} - \zeta_0(h', s; w, h, b)$$

in which $r(b) = r_1\{b \geq 0\} + r_m\{b < 0\}$. As $\zeta(b)$ is a decreasing function of slope $s_\zeta < 1 + r_m$, the previous equation has a unique solution in b and is continuous in $b' + c$.

First, note that if $b \geq 0$, then $(1 + r)b = b' + c - \zeta_0^+$. The condition $b \geq 0$ is equivalent to $b' + c \geq \zeta_0^+$ and $b = \frac{b' + c - \zeta_0^+}{1 + r}$.

Second, note that if $b < -\Delta\delta qh$, then $(1 + r_m)b = b' + c - \zeta_0^-$. The condition $b < -\Delta\delta qh$ is equivalent to $b' + c < -\Delta\delta qh(1 + r_m) + \zeta_0^-$ and $b = \frac{b' + c - \zeta_0^-}{1 + r_m}$.

Finally, the last regime is given by $b' + c \in (-\Delta\delta qh(1 + r_m) + \zeta_0^-, \zeta_0^+)$ and $b = \frac{b' + c - \zeta_0^+}{1 + r_m - \sigma_b}$.

Note that by the condition $\sigma_b < 1 + r_m$, the interval in the last regime is well defined.

Reciprocally:

$$b' + c = (1 + r(b))b + \zeta_0(h', s; w, h, b)$$

and therefore:

$$b' + c = \begin{cases} (1+r)b + \zeta_{0p} & \text{if } b \geq 0, \\ (1+r_m - s_\zeta)b + \zeta_{0p} & \text{if } b \in (-\Delta\delta qh, 0), \\ (1+r_m)b + \zeta_0 & \text{if } b \leq -\Delta\delta qh. \end{cases}$$

F Initializing terminal value functions and parameters

F.1 The simple case

Consider the simplified program in which e is an equivalence scale that is constant over time:

$$\begin{aligned} V(b_t) &= \max_{c_t} \left[u\left(\frac{c_t}{e}\right) + \beta V(b_{t+1}) \right] \\ \text{s.t. } b_{t+1} &= (1+r)b_t + w - c_t, b_t \geq 0. \end{aligned}$$

in which we do not consider the probability of survival and w is pension income.

The first-order condition yields:

$$\frac{1}{e} u'\left(\frac{c_t}{e}\right) = \beta V'(b_{t+1})$$

and the derivative of the state equation:

$$V'(b_t) = \beta(1+r)V'(b_{t+1}).$$

This yields the Euler equation:

$$u'\left(\frac{c_t}{e}\right) = \beta(1+r)u'\left(\frac{c_{t+1}}{e}\right).$$

F.1.1 Stationary case

Assume first:

$$\beta(1+r) = 1$$

to avoid nonstationarities. This implies that:

$$c_t = c_{t+1} = c^*.$$

Furthermore:

$$V(b_t) = \max_{\{c_t, \dots\}} \sum_{\tau=0}^{\infty} \beta^\tau u\left(\frac{c_{t+\tau}}{e}\right) = \frac{u\left(\frac{c^*}{e}\right)}{1-\beta}.$$

Financial wealth $b_t = b_0$ is therefore constant and:

$$c^* = w + rb_0.$$

Note also that $\beta = \frac{1}{1+r}$, which yields:

$$\beta V'(b) = \frac{1}{e} \frac{1}{1+r} \frac{ru'(\frac{c^*}{e})}{1 - \frac{1}{1+r}} = \frac{1}{e} u'(\frac{c^*}{e}).$$

F.1.2 Nonstationary case

Assume that:

$$\beta(1+r) = \lambda < 1,$$

and note that it applies in particular to retired households in which $\beta \equiv \beta(1 - \chi_4)$. This implies that:

$$u'(c_t) = \lambda u'(c_{t+1}),$$

and if $u'(c_t) = c_t^{-\sigma}$ that:

$$c_t = \lambda^{-1/\sigma} c_{t+1}.$$

The value function becomes:

$$\begin{aligned} V_t(b) &= \sum_{\tau=0}^{\infty} \beta^\tau \frac{(c_{t+\tau})^{1-\sigma}}{1-\sigma} = \sum_{\tau=0}^{\infty} \beta^\tau \frac{(\lambda^{\tau/\sigma} c_t)^{1-\sigma}}{1-\sigma} \\ &= \frac{(c_t)^{1-\sigma}}{1-\sigma} \sum_{\tau=0}^{\infty} (\beta \lambda^{(1-\sigma)/\sigma})^\tau = \frac{(c_t)^{1-\sigma}}{1-\sigma} \frac{1}{1 - \beta \lambda^{(1-\sigma)/\sigma}}. \end{aligned} \quad (\text{F.9})$$

Denote:

$$\begin{aligned} \mu &= \beta \lambda^{(1-\sigma)/\sigma} = \frac{\beta \lambda^{1/\sigma}}{\lambda} \\ &= \frac{\beta \lambda^{1/\sigma}}{\beta(1+r)} = \frac{\lambda^{1/\sigma}}{(1+r)}, \end{aligned}$$

and note that $\mu < 1$. Moreover, the budget constraint writes:

$$\begin{aligned} \sum_{\tau=0}^{\infty} \frac{1}{(1+r)^\tau} c_{t+\tau} &= (1+r)b + \sum_{\tau=0}^{\infty} \frac{1}{(1+r)^\tau} w, \\ \iff \sum_{\tau=0}^{\infty} \frac{1}{(1+r)^\tau} \lambda^{\tau/\sigma} c_t &= (1+r)b + \frac{(1+r)w}{r}, \\ \iff \frac{1}{1-\mu} c_t &= \frac{1}{1 - \frac{\lambda^{1/\sigma}}{1+r}} c_t = (1+r)b + \frac{(1+r)w}{r}. \end{aligned}$$

Replacing the above yields the value function:

$$V_t(b) = \frac{((1-\mu) \frac{1+r}{r} (rb+w))^{1-\sigma}}{(1-\sigma)(1-\mu)}.$$

This is what we obtained above when $\lambda = 1$.

Note that by equation (F.9), we have:

$$V_t(b) = \frac{(c_t)^{1-\sigma}}{(1-\sigma)(1-\mu)}$$

such that:

$$c_t = (1-\mu) \frac{1+r}{r} (rb + w).$$

F.2 Housing services

In the stationary case, generalize the previous setting to allow renting housing services, s_t in the case $h = 0$

$$\begin{aligned} V(b_t) &= \max_{c_t, s_t} [u(c_t, s_t) + \beta(1-\chi)V(b_{t+1})] \\ \text{s.t. } b_{t+1} &= (1+r)b_t + w - c_t - \rho s_t, b_t \geq 0. \end{aligned}$$

If s_t is continuous, note that the first-order condition yields:

$$\frac{u'_s(c_t, s_t)}{u'_c(c_t, s_t)} = \rho$$

such that if u'_c is constant, u'_s is also. Given the strict concavity of u , which means that c_t and s_t are constant.

If s_t is discrete, the same argument applies, and s_t and c_t are constant and because

$$V(b_t) = \max_{\{c_t, s_t, \dots\}} \sum_{\tau=0}^{\infty} [\beta(1-\chi)]^\tau u(c_{t+\tau}, s_{t+\tau}) = \frac{u(c^*, s^*)}{1-\beta(1-\chi)},$$

b_t is constant and equal to b_0 . Thus

$$\begin{aligned} c^* &= w + rb_0 - \rho s^*, \\ s^* &= \arg \max_s u(w + rb_0 - \rho s, s). \end{aligned}$$

F.3 Housing stocks

In the stationary case, introduce now owners of h units of housing that is supposed to be constant over time. We have:

$$\begin{aligned} V(b_t) &= \max_{c_t, s_t} [u(c_t, s_t) + \xi \min(h, s) + \beta(1-\chi)V(b_{t+1})] \\ \text{s.t. } b_{t+1} &= (1+r)b_t + w - c_t + \rho(h - s_t) - \phi \mathbf{1}\{s_t < h\}, b_t \geq 0. \end{aligned}$$

The only thing that changes is the way constant c^* and s^* are determined:

$$\begin{aligned} c^* &= w + rb_0 + \rho h - \rho s^*, \\ s^* &= \arg \max_s (u(w + rb_0 + \rho h - \rho s - \phi \mathbf{1}\{s < h\}, s) + \xi \min(h, s)). \end{aligned}$$

An alternative strategy is to sell the stock of housing, which yields:

$$\begin{aligned} c^* &= w + r(b_0 + (1 - \tau^s)qh) - \rho s^*, \\ s^* &= \arg \max_s (u(w + r(b_0 + (1 - \tau^s)qh) - \rho s, s)). \end{aligned}$$

The last solution also applies when b_0 is negative.

In the nonstationary case, consumption decreases as housing services do. If those are assumed continuous, this a balanced decreasing path for both consumption and housing services. Considering that $\beta \equiv \beta(1 - \chi_4)$ we have:

$$\begin{aligned} \mu &= \beta(1 - \chi_4) [\beta(1 - \chi_4)(1 + r)]^{(1-\sigma)/\sigma} \\ &= \frac{[\beta(1 - \chi_4)(1 + r)]^{1/\sigma}}{1 + r} \end{aligned}$$

and therefore as written in the text, the set of initial conditions for retirees is:

$$\begin{aligned} c_t &= (1 - \mu) \frac{1 + r}{r} (rb + w) - \rho s_t, \\ s_t &= \arg \max_s (u(w + r(b_t + (1 - \tau^s)qh_t) - \rho s_t, s_t)). \end{aligned}$$

F.4 Initial parameter values

If the elasticity of intertemporal substitution is -0.7 (see Blundell, Browning and Meghir, 94), $\sigma = 1/0.7 \simeq 1.3$. As

$$\frac{U'_s}{U'_c} = \rho = \frac{\alpha c}{(1 - \sigma)s}$$

we have:

$$\frac{\rho s}{c} = \frac{\alpha}{(1 - \sigma)}.$$

In the data, expenditures on housing are approximately 1/4 of total expenditures. so that $\frac{\rho s}{c} = 1/3$. We thus obtain $\alpha = -0.1$.

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