WORKING PAPERS

N° 1464

August 2023

"Third-Degree Price Discrimination in Two-Sided Markets"

Alexandre de Cornière, Andrea Mantovani and Shiva Shekhar

Toulouse School of Economics

Third-Degree Price Discrimination in Two-Sided Markets*

Alexandre de Cornière[†] Andrea Mantovani[‡] Shiva Shekhar[§]

August 19, 2023

We investigate the welfare effects of third-degree price discrimination by a two-sided platform that enables interaction between buyers and sellers. Sellers are heterogenous with respect to their per-interaction benefit, and, under price discrimination, the platform can condition its fee on sellers' type. In a model with linear demand on each side, we show that price discrimination: (i) increases participation on both sides; (ii) enhances total welfare; (iii) may result in a strict Pareto improvement, with both seller types being better-off than under uniform pricing. These results, which are in stark contrast to the traditional analysis of price discrimination, are driven by the existence of cross-group network effects. By improving the firm's ability to monetize seller participation, price discrimination induces the platform to attract more buyers, which then increases seller participation. The Pareto improvement result means that even those sellers who pay a higher price under discrimination can be better-off, due to the increased buyer participation.

Keywords: Two-sided Markets, Price Discrimination, Network Effects.

JEL Classification: D42, D62, L11, L12.

*We particularly thank Patrick Rey for very detailed comments on a previous version of this paper. We are also grateful to Olivier Bonroy, Joan Calzada, Luis Cabral, Simon Cowan, Bruno Jullien, Heiko Karle, Sarit Markovich, Hakan Ozalp, Geoffrey Parker, Martin Peitz, Flavio Pino, Markus Reisinger, Kevin Remmy, Andrew Rhodes, Robert Somogyi, Tat-How Teh, Mark J. Tremblay, Marshall Van Alstyne, Julian Wright, and seminar participants at Toulouse School of Economics, Laboratoire dÉconomie Appliquée de Grenoble (GAEL), the XXXVI Jornadas de Economia Industrial, the 2022 ASSET Annual Meeting, the 15th Digital Economics Conference (Toulouse School of Economics), the 14th Paris Conference on Digital Economics, the 21st IIOC (Washington DC), the Fifth Economics of Platforms Workshop (Villa Orlandi, Capri), the 11th Oligo Workshop (University of Padova), the Second UB Microeconomics Workshop (Universitat de Barcelona), the 17th Annual CRESSE (Rhodes), and the 2023 Platform Strategy Symposium (Boston University) for helpful comments and suggestions. De Cornière acknowledges funding from ANR under grant ANR-17-EURE-0010 (Investissements d'Avenir program). The usual disclaimer applies.

[†]Toulouse School of Economics, Université Toulouse 1, Toulouse, France. Email: alexandre.de-corniere@tse-fr.eu

[‡]TBS Business School, Toulouse, France. Email: a.mantovani@tbs-education.fr

[§]Tilburg School of Economics and Management, Tilburg, Netherlands. Email: shiva.shekhar.g@gmail.com, s.shekhar_1@tilburguniversity.edu

1. Introduction

Online marketplaces often resort to third-degree price discrimination when dealing with a heterogenous population of sellers. For instance, Amazon and eBay charge different commission rates depending on the product category (electronics, clothes, etc.).[1] Payment card systems such as Mastercard and Visa also apply different fees based on the sector in which a merchant operates, and/or based on its size.[2] In their application stores, Apple and Google discriminate between large and small developers, by charging a higher commission rate (30% instead of 15%) for developers with more than $1m annual revenue. The Microsoft Store applies different fees to games (30%) and non-games applications (usually 15%).[3]

What are the distributional and welfare consequences of such practices? While the effects of third-degree price discrimination have been widely studied (see our literature review below), an interesting feature of marketplaces is that they are two-sided markets, in which the presence of buyers and sellers generates cross-side (sometimes called indirect) network effects. To what extent do the lessons from the standard analysis of third-degree price discrimination apply to two-sided markets? How should a platform design its pricing policy in the presence of network effects? What are the managerial and policy lessons that can be learnt?

To answer these questions, we study a simple model of monopoly price discrimination by a two-sided platform. There are two groups of agents, buyers and sellers. All buyers obtain the same per-seller benefit, but sellers are heterogeneous with respect to their revenue: high-type sellers obtain a larger revenue for each buyer present on the platform than low-type sellers. The platform charges participation fees to buyers and sellers. Agents also differ with respect to their exogenous participation cost (or outside option), which is distributed in such a way as to have linear demand on both sides of the market. We compare the situation where the platform charges the same participation fee to all sellers (uniform pricing) to one in which it can set different fees to high- and low-type sellers (third-degree price discrimination).

Our first result is that seller-side price discrimination leads to an increase in the participation of both buyers and sellers. Intuitively, allowing the monopolist to charge different seller fees allows it to better monetize buyer participation, thereby giving it an incentive to attract more buyers. This in turn attracts more sellers, resulting in overall larger participation on both sides. Second, we show that total welfare increases with price discrimination. Third, we show that price discrimination can constitute a *strict* Pareto improvement: because of increased buyer participation, high-type sellers may be better-off even if they end up paying a higher fee than under uniform pricing.

[1] See `https://sell.amazon.com/pricing` and `https://www.ebay.co.uk/help/selling/fees-credits-invoices/fees-business-sellers?id=4809`, last accessed 12 May 2023.

[2] See, for instance, `https://www.mastercard.us/content/dam/public/mastercardcom/na/us/en/documents/merchant-rates-2022-2023-apr22-2022.pdf`

[3] More examples can be found in Borck et al. (2020).

These results stand in sharp contrast to the "traditional" analysis of third-degree price discrimination. Indeed, with linear demands and no network effects, total output remains constant and welfare goes down, unless the weak market is not served under uniform pricing, in which case discrimination leads to a *weak* Pareto improvement.

Our analysis also delivers insights related to the platform's optimal pricing strategy. We identify several regimes, depending on parameter values. In the first, "typical" case, price discrimination leads to an increase in the fee paid by high-type sellers and to a decrease in the fee paid by low-type ones. Buyers' fee diminishes compared to uniform pricing when buyers' network benefits are relatively small, as the platform needs to increase their participation. When buyers obtain large benefits from sellers' participation, the platform can raise their fee without inducing a drop in participation. Other, more surprising patterns may also emerge in equilibrium. In the second regime, participation fees increase for both seller types under price discrimination, in which case buyers are subsidized (and more so under price discrimination). Alternatively, in the third regime, participation fees decrease for both seller types while buyers pay a higher fee. Interestingly, whether a group (buyers or sellers) pay higher or lower fees does not change the result mentioned above that participation increases for both groups.

For analytical tractability, the baseline model relies on some simplifying assumptions, in particular that buyer benefits are independent of sellers' types, and that the platform charges participation fees. As we show in Section 5, our main insights do not hinge on these assumptions. First, we consider the case in which buyer surplus depends on the seller type. There again, price discrimination increases participation on both sides and may constitute a Pareto improvement. Even though welfare no longer always increases, numerical results indicate that, when it decreases, the loss is very small, while welfare gains can be more substantial. Second, we further investigate the role of network effects in order to compare our results to the extant literature. We show that, though price discrimination still enhances participation on both sides, a sufficiently high degree of network effects is necessary for our main welfare results to hold.

In the third extension, the platform sets ad valorem instead of participation fees. Analytical results are more difficult to obtain, but our main findings concerning the welfare-enhancing effect as well as the possibility of strict Pareto improvement under price discrimination continue to hold. In the fourth extension, we investigate the situation in which the platform cannot charge buyers. Welfare is no longer always higher under price discrimination, due to the fact that the platform has fewer instruments to attract buyers and sellers. We confirm, however, the existence of a parameter region in which price discrimination leads to a strict Pareto improvement.

2. Relevant literature

The analysis of third-degree price discrimination by a monopolist has a long tradition in economics (Pigou 1920, Robinson 1933, Schmalensee 1981, Varian 1985, Aguirre et al. 2010, Bergemann et al. 2015). Because it tends to lead to higher prices in some markets and to lower prices in others, its welfare effects are a priori ambiguous. As shown by Schmalensee (1981) and Varian (1985), a necessary condition for welfare to increase is that total output increases.[4] Failing this, having different consumers face different prices leads to an inefficient "maldistribution of resources" (Robinson 1933). A case of particular interest for its tractability is that of linear demands. There, Pigou (1920) shows that, provided the firm made positive sales to each market under uniform pricing, output would remain the same under price discrimination, and welfare would decrease.

In traditional markets (i.e. without network effects), price discrimination may result in a Pareto improvement for several reasons: when it allows to serve a new market,[5] when profit functions are not single-peaked (Nahata et al. 1990), or in the presence of economies of scale (Hausman & MacKie-Mason 1988). Given the connection between network effects and economies of scale the latter paper is particularly relevant, but the logic is different: in Hausman & MacKie-Mason (1988), price discrimination makes it "cheaper" to attract consumers in the weak market, which reduces the cost to serve the strong market and can induce the firm to lower its price. In our paper, the ability to price discriminate sellers increases the firm's ability to monetize buyer participation, which in turn leads to higher seller participation. A notable difference is that Pareto improvement may happen even when one of the prices increases.

A few recent papers study price discrimination in two-sided markets, though of either the first or second-degree kind. Liu & Serfes (2013) show that first-degree price discrimination can soften competition in a setup where the opposite would happen absent cross-group network effects. In the context of second-degree price discrimination, Böhme (2016) shows that some properties of the optimal contract in traditional markets (e.g. no distortion at the top) no longer hold in two-sided markets. Jeon et al. (2022) provide conditions for pooling to be optimal, and for second-degree price discrimination to increase or decrease welfare. In a related setup, Lin (2020) shows that price discrimination is complementary across sides. In a context where sellers use second-degree price discrimination, D'Annunzio & Russo (2023) study fee discrimination by a platform (or government), based on the quantity purchased by consumers, and show that it can alleviate the distortion induced by sellers' market power. Gomes & Pavan (2016) study price discrimination in matching markets and characterize the optimal many-to-many matching mechanism in the presence of two-sided asymmetric information. Chang et al. (2022) empirically find that price discrimination by Uber increases welfare.[6]

[4] Cowan (2016) identifies families of demand functions for price discrimination to raise total output and welfare.

[5] In that case the Pareto improvement is weak, since consumers in the market that is served under uniform pricing are indifferent.

[6] Bouvard et al. (2022) and Gambacorta et al. (2023) study platform lending as a way to price-discriminate sellers.

Motivated by the app store controversies, Bhargava et al. (2022) study differential revenue sharing schemes, which bear some resemblance but are not equivalent to price discrimination. Indeed, they consider a platform returning to sellers a higher share for revenue contributions up to a predetermined threshold, and a smaller share above that. They find that the platform offering better terms to small developers may benefit large developers (a Pareto improvement), but do not consider the possibility for the platform to raise its commission to one group of developers. Also because of this constraint, the platform does not always gain from adopting a differential sharing scheme, and this represents another difference in comparison to our analysis.

Tremblay (2021) also considers a model of price discrimination by a monopolistic platform (in the absence of network externalities) that charges unit fees to merchants, and finds that perfect fee discrimination is likely to reduce welfare. This result, opposite from what we obtain, stems from a different set of modelling assumptions: we consider a model featuring network externalities, elastic participation on all sides, and a platform that is allowed to charge (or subsidize) buyers, while Tremblay (2021) views the platform as an upstream supplier that only charges merchants, and emphasizes the double marginalization problem. Ding & Wright (2017) study price-discrimination by a payment card issuer, and find ambiguous welfare effects. The key driver of inefficiency in that model is the possibility of excessive intermediation (too many transactions being carried through payment cards), which is absent from ours.

A few papers study third-degree price discrimination in one-sided platforms: Adachi (2005) considers a model where agents from each group enjoy the presence of agents *from the same group*, and shows that welfare can increase with price discrimination even though total output remains the same. Belleflamme & Peitz (2020) analyze the monopoly provision of a network good where *users care about the overall level of participation*; they show that, under particular circumstances, third-degree price discrimination is equivalent to versioning (second-degree price discrimination). Peitz & Reisinger (2022) demonstrate that operating multiple platforms allows to distinguish between single-homing and multi-homing sellers, which enables the platform owner to price discriminate between high-valuation and low-valuation sellers. Closer to us, Hashizume et al. (2021) consider third-degree price discrimination in a one-sided market in which the platform sells a network good in two separate markets. They provide conditions for price discrimination to constitute a Pareto improvement, but do not fully characterize its total welfare effects. Our model allows to investigate interaction between sellers and buyers that connect via the platform, and to consider the effect of increased participation on both sides.

Finally, moved by recent regulatory interventions and proposals, a stream of research theoretically investigates how to regulate platform fees, broadly suggesting to impose fee caps (Gomes & Mantovani 2020, Wang & Wright 2022, Bisceglia & Tirole 2022). The results of our paper suggest that cap regulation, especially if too rigid, may accidentally reduce the benefits brought by price discrimination in the presence of network effects.

3. Model

Consider a monopolist two-sided platform that orchestrates interactions between two groups, which we call buyers and sellers. The structure of the model is similar to Armstrong (2006): all pairs of agents interact, and the platform charges participating fees. While most of the examples of transaction platforms involve ad valorem fees, we focus on participation fees for pedagogical and tractability reasons.[7] We show in Subsection 5.3 that our insights extend to the case of ad valorem fees.

Sellers There are two categories of products, denoted L and H. Each category has a mass 1 of independent products, and each product is offered by a single seller.[8] We do not explicitly model sellers' pricing decisions. Instead, we assume that the seller of a product in category $j \in \{L, H\}$ achieves a variable profit of θ_j for each buyer with which it interacts. We assume that $\theta_H > \theta_L > 0$, and refer to θ_j as the type of sellers in category j.

If we denote the number of buyers on the platform by N_B, the profit of a seller of type θ_j is $\theta_j N_B - f_j$, where f_j is the participation fee paid to the platform. We assume that sellers have an outside option whose value is uniformly distributed over $[0, 1]$, independently of their type. Assuming that all demands are interior (we provide conditions later on), the demand from type j sellers is

$$D_j(N_B, f_j) = \theta_j N_B - f_j.$$

Total seller participation is

$$D_S(N_B, f_L, f_H) = (\theta_L + \theta_H)N_B - f_L - f_H.$$

We will compare two regimes: under uniform pricing, the fees must satisfy $f_L = f_H$, while no such constraint apply under price discrimination.

Buyers' payoffs. There is a mass 1 of buyers. Each buyer obtains a stand-alone value v from using the platform. In addition, they enjoy a benefit b for each seller who is present on the platform.[9] For tractability reasons, in the baseline model we assume that the benefit b is independent of the type of the seller the buyers interacts with. While this assumption can be microfounded,[10] we relax it in Subsection 5.1.

[7]This approach is standard in the literature. See also Jullien et al. (2021), Belleflamme & Peitz (2020), Reisinger (2014), Shekhar (2021), Carroni et al. (2023) among others.

[8]In other words, there is no competition among sellers.

[9]Hence, we assume that the platform only attracts sellers that generate a positive utility for the buyers.

[10]For instance, suppose all marginal costs are zero. A buyer's willingness to pay for the product of a seller of type θ is equal to θ with probability x, and to $\theta + b/(1-x)$ with probability $1-x$, with $x > b/\theta_L$. The optimal price charged by a seller θ is equal to θ. In this case, per interaction profit is indeed equal to θ while buyers' expected per-interaction surplus is b.

If N_S sellers join the platform, a buyer obtains a utility $v + bN_S - p$ from joining the platform, where p is the participation fee set by the platform. Assuming that buyers also have an outside option whose value is uniformly distributed over $[0, 1]$, the participation level of buyers is

$$D_B(N_S, p) = v + bN_S - p. \tag{1}$$

Equilibrium demands and platform's profit. In an equilibrium with rational expectations, participation levels must satisfy

$$N_B = D_B(N_S, p) \quad \text{and} \quad N_S = D_S(N_B, f_L, f_H). \tag{2}$$

Instead of solving the above system to obtain participation levels as a function of fees, we use inverse demands and assume that the platform chooses participation levels and that fees adjust accordingly. In a monopoly setup the quantity approach amounts to assuming away coordination problems and it allows us to convey the logic of our arguments more clearly.

Under uniform pricing, inverting the system (2) with the additional constraint that $f_L = f_H$ leads to the inverse demand system

$$P^U(N_B, N_S) = v + bN_S - N_B \quad \text{and} \quad F^U(N_B, N_S) = \frac{(\theta_L + \theta_H)N_B - N_S}{2}. \tag{3}$$

The platform then chooses N_B and N_S to maximize

$$\Pi^U(N_B, N_S) = N_B P^U(N_B, N_S) + N_S F^U(N_B, N_S). \tag{4}$$

Under price discrimination, on the other hand, inverting the system (2) leads to the following inverse demands, where N_L and N_H denote participation by sellers of type θ_L and θ_H respectively:

$$P^D(N_B, N_L, N_H) = v + b(N_L + N_H) - N_B, \quad F_H^D(N_B, N_H) = \theta_H N_B - N_H, \quad F_L^D(N_B, N_L) = \theta_L N_B - N_L. \tag{5}$$

Price discrimination enables the platform to choose the participation level of each type of seller independently, whereas under uniform pricing the platform can only choose the overall level of participation of sellers, without being able to change the composition of the set of sellers. The platform then chooses N_B, N_L and N_H to maximize

$$\Pi^D(N_B, N_L, N_H) = N_B P^D(N_B, N_L, N_H) + N_L F_L^D(N_B, N_L) + N_H F_H^D(N_B, N_H). \tag{6}$$

Interior solutions. In the first part of the paper we focus on equilibria where all participation levels are strictly between 0 and 1. To ensure this, we make the following assumption on parameter values:

Assumption 1. *$0 < \theta_L < \sqrt{2}$, $\theta_H^2 + \theta_L^2 < 4 - 2v$ and $\max\{0, \frac{\theta_H - 3\theta_L}{2}\} < b < \bar{b}(\theta_H, \theta_L, v) \equiv \frac{1}{2}(\sqrt{8 - 4v - (\theta_H - \theta_L)^2} - \theta_H - \theta_L)$.*

Assumption 1 ensures in particular that some sellers of type θ_L use the platform under uniform pricing. In Subsection 4.4 we discuss the case where these sellers would be excluded.[11]

4. Analysis

4.1. Participation

Uniform pricing. Under uniform pricing, the platform chooses N_B and N_S in order to maximize $\Pi^U(N_B, N_S) = N_B P^U(N_B, N_S) + N_S F^U(N_B, N_S)$. Dropping the arguments to lighten notations, the first-order conditions are:

$$\frac{\partial \Pi^U}{\partial N_B} = 0 \Leftrightarrow P^U + N_B \frac{\partial P^U}{\partial N_B} + N_S \frac{\partial F^U}{\partial N_B} = 0, \tag{7}$$

$$\frac{\partial \Pi^U}{\partial N_S} = 0 \Leftrightarrow F^U + N_S \frac{\partial F^U}{\partial N_S} + N_B \frac{\partial P^U}{\partial N_S} = 0. \tag{8}$$

Beyond the standard marginal revenues, captured by the first two terms on the left-hand side of (7) and (8), the third terms in each equation capture the idea that attracting an extra agent on one side allows the platform to increase its revenue on the other side.

Price discrimination. Under price discrimination, the platform chooses N_B, N_L and N_H to maximize $\Pi^D(N_B, N_L, N_H) = N_B P^D(N_B, N_L, N_H) + N_L F_L^D(N_B, N_L) + N_H F_H^D(N_B, N_H)$. The first-order conditions are

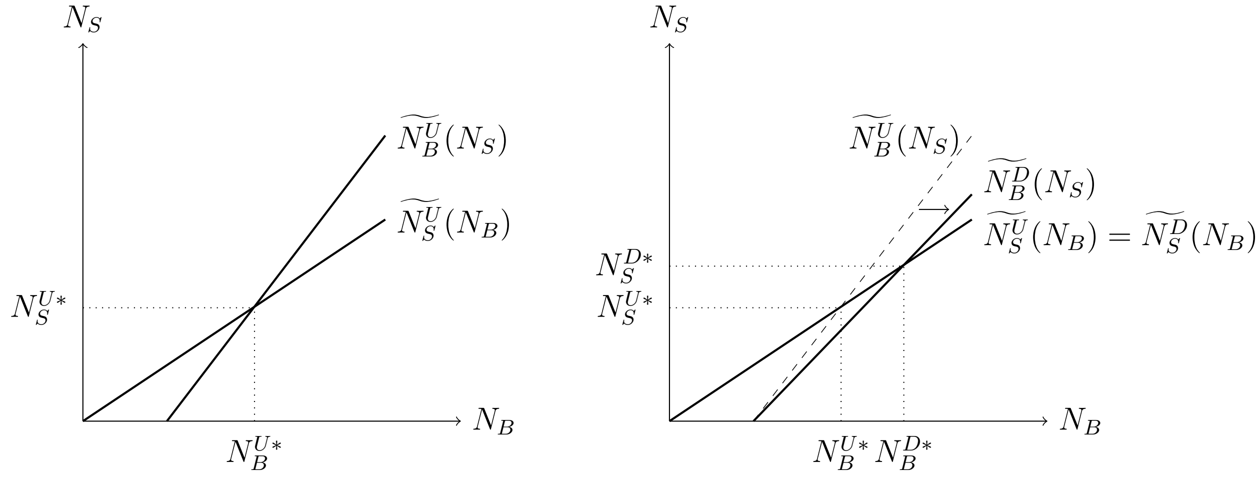
$$\frac{\partial \Pi^D}{\partial N_B} = 0 \Leftrightarrow P^D + N_B \frac{\partial P^D}{\partial N_B} + N_L \frac{\partial F_L^D}{\partial N_B} + N_H \frac{\partial F_H^D}{\partial N_B} = 0, \tag{9}$$

$$\frac{\partial \Pi^D}{\partial N_L} = 0 \Leftrightarrow F_L^D + N_L \frac{\partial F_L^D}{\partial N_L} + N_B \frac{\partial P^D}{\partial N_L} = 0, \tag{10}$$

and

$$\frac{\partial \Pi^D}{\partial N_H} = 0 \Leftrightarrow F_H^D + N_H \frac{\partial F_H^D}{\partial N_H} + N_B \frac{\partial P^D}{\partial N_H} = 0. \tag{11}$$

[11] Full participation on either side, especially on the buyer side, would shut down the feedback loop created by network effects and bring us back to the traditional analysis.



(a) uniform pricing (b) price discrimination

Figure 1: Equilibrium participation

A first result. We are now ready to state our first main result:

Proposition 1. *Under price discrimination, the equilibrium number of both buyers and sellers increases compared to uniform pricing.*

The proof of Proposition 1 can be found in Appendix A.1. Here we provide the intuition for it, illustrated in Figure 1. In the figure, $\widetilde{N_B^U}(N_S)$ is the profit-maximizing participation level for buyers, under uniform pricing, when N_S sellers participate (i.e. the solution to $\max_{N_B} \Pi^U(N_B, N_S)$). The other curves are defined similarly. The proof proceeds in three steps.

First, the participation levels of buyers and sellers are strategic complements from the platform's point of view: increasing the participation level of sellers makes it more profitable for the platform to attract new buyers, and reciprocally. Indeed, as the number of sellers increases, not only can each buyer be charged a higher price (e.g. term $N_B \left(\partial P^U / \partial N_S\right)$ in (8), but attracting a new buyer also allows the platform to increase its price to a larger base of sellers (e.g. term $N_S \left(\partial F^U / \partial N_B\right)$ in (7).[12] On the left panel of Figure 1 the equilibrium under uniform pricing is given by the intersection between the two increasing functions $\widetilde{N_B^U}(N_S)$ and $\widetilde{N_S^U}(N_B)$.

Second, *for a given level of buyer participation*, the profit-maximizing total number of sellers is the same under uniform pricing and discrimination. This result corresponds to the traditional analysis of price-discrimination without network effects and with linear demands: so long as both markets (here, both groups of sellers) are served under uniform pricing, discrimination does not affect total output (Pigou 1920). Formally, this follows from the fact that adding (10) and (11) gives (8). On the right panel of Figure 1, this observation means that $\widetilde{N_S^U}(N_B) = \widetilde{N_S^D}(N_B)$.

[12]Formally, we have $\frac{\partial^2 \Pi^U}{\partial N_B \partial N_S} = \frac{\partial P}{\partial N_S} + \frac{\partial F^U}{\partial N_B} > 0$.

Third, *for a given level of seller participation* $N_S = N_H + N_L$, switching to the discrimination regime induces the platform to attract more buyers. Intuitively, being able to discriminate among sellers allows the firm to fully extract the value generated by each additional buyer on the seller side, which makes it more profitable to attract new buyers. On the right panel of Figure 1, this corresponds to the shift from $\widetilde{N_B^U}(N_S)$ to $\widetilde{N_B^D}(N_S)$.

Put together, these observations imply that equilibrium participation of both sides is higher under price discrimination, driven by the extra incentive to attract buyers.

Discussion and generalization Linear demands offer an ideal benchmark to compare our model to one without cross-side network effects: provided both markets are served under uniform pricing, output would remain the same and welfare would therefore go down. Proposition 1 already shows that the output result is no longer true in two-sided markets. But actually the proposition holds under weaker assumptions. Indeed, its main result about increased participation on both sides continues to be valid if (i) N_B and N_S are strategic complements, (ii) $\widetilde{N_B^D}(N_S) > \widetilde{N_B^U}(N_S)$, and (iii) $\widetilde{N_S^D}(N_B)$ is not that much smaller than $\widetilde{N_S^U}(N_B)$. Conditions (i) and (ii) are fairly natural: having more buyers tends to make attracting an extra seller more profitable (and reciprocally), and being able to extract more profit from sellers through price discrimination makes attracting extra buyers more profitable. Condition (iii) relates to a standard concern in the traditional analysis of third-degree price discrimination, namely the effect of discrimination on total output (see for instance Proposition 4 in Aguirre et al. 2010). Interestingly, the logic of Proposition 1 would hold even if output were to fall slightly in a traditional market, as illustrated in Figure 2.

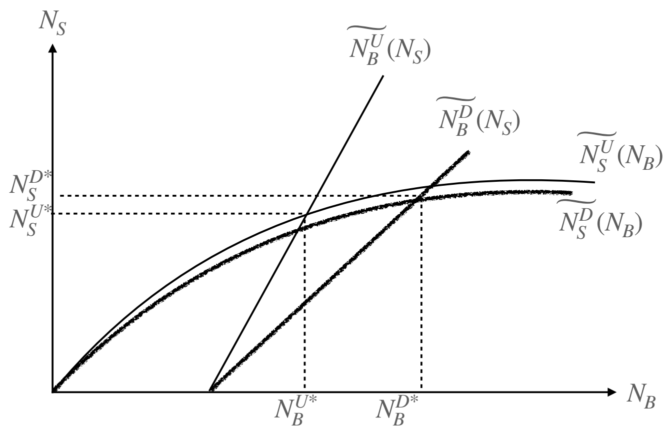


Figure 2: Example with $\widetilde{N_S^D}(N_B) < \widetilde{N_S^U}(N_B)$

4.2. Equilibrium

In order to provide welfare results, we need to explicitly compute the equilibria under uniform pricing and price discrimination.

Uniform pricing. Solving the system of first order conditions (7) and (8), we obtain:[13]

$$N_S^U = \frac{2v(2b+\theta_H+\theta_L)}{8-(2b+\theta_H+\theta_L)^2}, \quad N_B^U = \frac{4v}{8-(2b+\theta_H+\theta_L)^2},$$

which corresponds to equilibrium prices

$$p^U = \frac{v(4-(\theta_H+\theta_L)(2b+\theta_H+\theta_L))}{8-(2b+\theta_H+\theta_L)^2}, \quad f^U = \frac{v(\theta_H+\theta_L-2b)}{8-(2b+\theta_H+\theta_L)^2},$$

and a profit for the platform equal to

$$\Pi^U = \frac{2v^2}{8-(2b+\theta_H+\theta_L)^2}. \tag{12}$$

Price discrimination. Solving the system of first-order conditions (9), (10) and (11), we obtain[14]

$$N_S^D = \frac{v(2b+\theta_L+\theta_H)}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}, \quad N_B^D = \frac{2v}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)},$$

which implies prices

$$p^D = \frac{v(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}, \quad f_j^D = \frac{v(\theta_j-b)}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}, \quad j\in\{H,L\}.$$

The platform's profit is then

$$\Pi^D = \frac{v^2}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}. \tag{13}$$

4.3. Comparison

Welfare analysis. Our main results concern the welfare effects of price discrimination. They are summarized in the proposition below, whose proof is in Appendix A.2:

Proposition 2. *(i) The platform, buyers and low-type sellers are better-off under price discrimination.*

[13] Participation of sellers of type $j\in\{L,H\}$ under uniform pricing is $N_j^U = \frac{v(2b+3\theta_j-\theta_{-j})}{8-(2b+\theta_H+\theta_L)^2}$.

[14] Participation of sellers of type $j\in\{L,H\}$ under price discrimination $N_j^D = \frac{v(b+\theta_j)}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}$.

(ii) Total welfare is higher under price discrimination.

(iii) High-type sellers are better-off under price discrimination if and only if $b > \hat{b}(\theta_H, \theta_L) \equiv \frac{\sqrt{32-7(\theta_H-\theta_L)^2}-3\theta_H-\theta_L}{4}$. In this case, price discrimination constitutes a strict Pareto improvement over uniform pricing.

Part (i) of Proposition 2 follows naturally from Proposition 1. That the platform is better-off follows from a revealed preference argument. Buyers are better-off, as revealed by their increased participation. Interestingly, this may happen even if they pay more (see next proposition for more details), as the augmented seller participation compensates possible fee increases. The result that low-type sellers are better-off follows from inspection of their surplus, as there are instances in which they may end up paying a higher fee (see again next proposition).

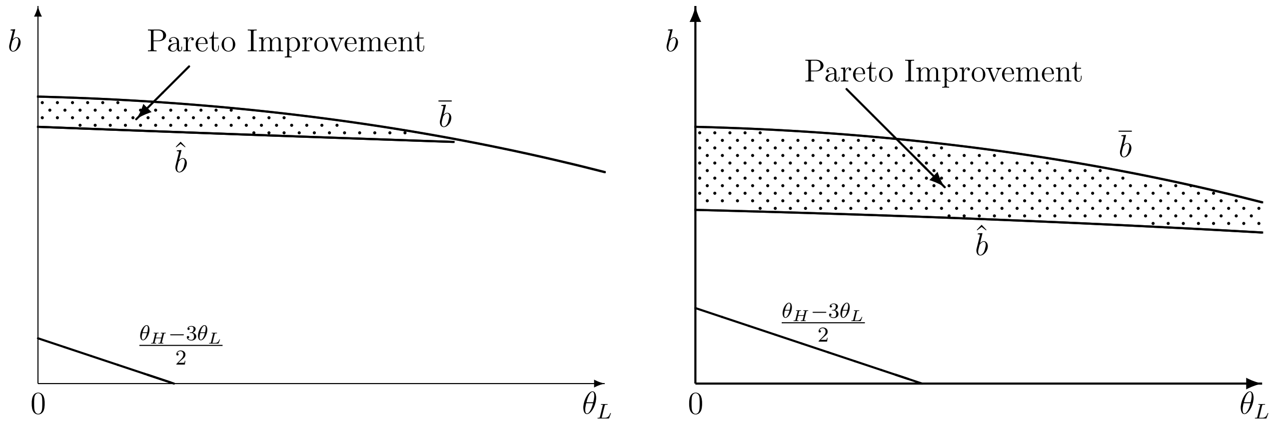
Part (ii) stands in stark contrast with the traditional analysis of price discrimination. Recall that, when demands are linear and both markets are served under uniform pricing, third-degree price discrimination always lowers total welfare. The result is overturned in a two-sided context, thanks to the platform's incentive to increase participation on both sides, as we already explained. As we show in the proof, the increase in participation trumps the misallocation of ressources due to price discrimination, so that the welfare effects are positive.

Part (iii) goes even further: when $b > \hat{b}(\theta_H, \theta_L)$, the high-type sellers benefit from price discrimination, despite having to pay a higher fee than under uniform pricing. In the parametric region in which Pareto improvement holds, two cases can occur, explaining why high-type sellers can be better off. On the one hand, when their per-buyer profit θ_H is relatively low, they may receive a subsidy to join the platform, which becomes even bigger under price discrimination (see next proposition). On the other hand, when θ_H is higher, they end up paying more, but the additional benefit from increased buyer participation induced by price discrimination outweighs the higher participation fee. In this case, the platform can subsidize buyers to join the platform, and the amount of the subsidy increases under price discrimination.

Figure 3 plots the region where price discrimination leads to a Pareto improvement (dotted area), as indicated in part (iii) of Proposition 2.[15] Following Assumption 1, we focus on the region where $\max\{0, \frac{\theta_H-3\theta_L}{2}\} < b < \bar{b}(\theta_H, \theta_L, v)$. We fix $v = 0.1$ and consider two possible values for θ_H to show that the area with Pareto improvement increases with θ_H (this is formally demonstrated in Appendix A.2).[16] This finding implies that we need a sufficiently high combination of network effects for the positive feedback loop induced by price discrimination (more buyer participation and therefore more value for sellers) to generate a Pareto improvement.

[15] In this figure and in the following ones we omit the arguments of the threshold values of b for simplicity.

[16] Also notice that the feasible region decreases in θ_H, as it can be derived from Assumption 1: apart from the evident condition $\theta_H^2 + \theta_L^2 < 4 - 2v$, it can be easily established that $\frac{\partial \bar{b}(\theta_H, \theta_L, v)}{\partial \theta_H} < 0$.



Panel (a): $v = 0.1$ and $\theta_H = 0.5$. Panel (b): $v = 0.1$ and $\theta_H = 0.8$.

Figure 3: Regions with Pareto improvement

Prices. Having stated our main result, it is instructive to take a closer look at the platform's optimal pricing strategy. In Appendix A.3 we formally prove the following results.

Proposition 3. *(i) There exists $\tilde{b}(\theta_H, \theta_L) > 0$ such that $p^D > p^U (> 0)$ if and only if $b > \tilde{b}(\theta_H, \theta_L)$.*

(ii) $f_L^D < f_H^D$ for all parameter values. Depending on the parameter values, we can have: $f^U \leq f_L^D$, $f^U \in (f_L^D, f_H^D)$, or $f^U \geq f_H^D$. When $f^U \leq f_L^D$, we necessarily have that $p^D < p^U < 0$; moreover, $f^U \geq f_H^D$ occurs only when $f^U < 0$.

Part (i) of of Proposition 3 reveals that, if buyers place a high value on seller participation ($b > \tilde{b}(\theta_H, \theta_L)$), the platform increases its price to buyers under price discrimination. In spite of this, buyers are still better-off because of the increased number of sellers under price discrimination. Such a strategy may require subsidizing seller participation, especially if their value for buyer participation is relatively low. Conversely, if b is smaller ($b < \tilde{b}(\theta_H, \theta_L)$), the platform needs to lower its price to buyers in order to trigger the positive feedback loop leading to more participation on each side.

Part (ii) considers the price paid by sellers. Even though the typical case is such that $f_L^D < f^U < f_H^D$, there are regions in the parameter space such that both fees increase or decrease under price discrimination. On the one hand, when sellers scarcely value buyer interaction, the platform may decide to subsidize more (i.e. lowering their negative fees) under price discrimination in order to attract them, thus explaining the region where $f_L^D < f_H^D < f^U < 0$. On the other hand, when sellers highly value buyer interaction, the platform may increase the fees for both of them when it can price discriminate, leading to a parametric region in which $f^U < f_L^D < f_H^D$. We provide more precise conditions in Appendix A.3.

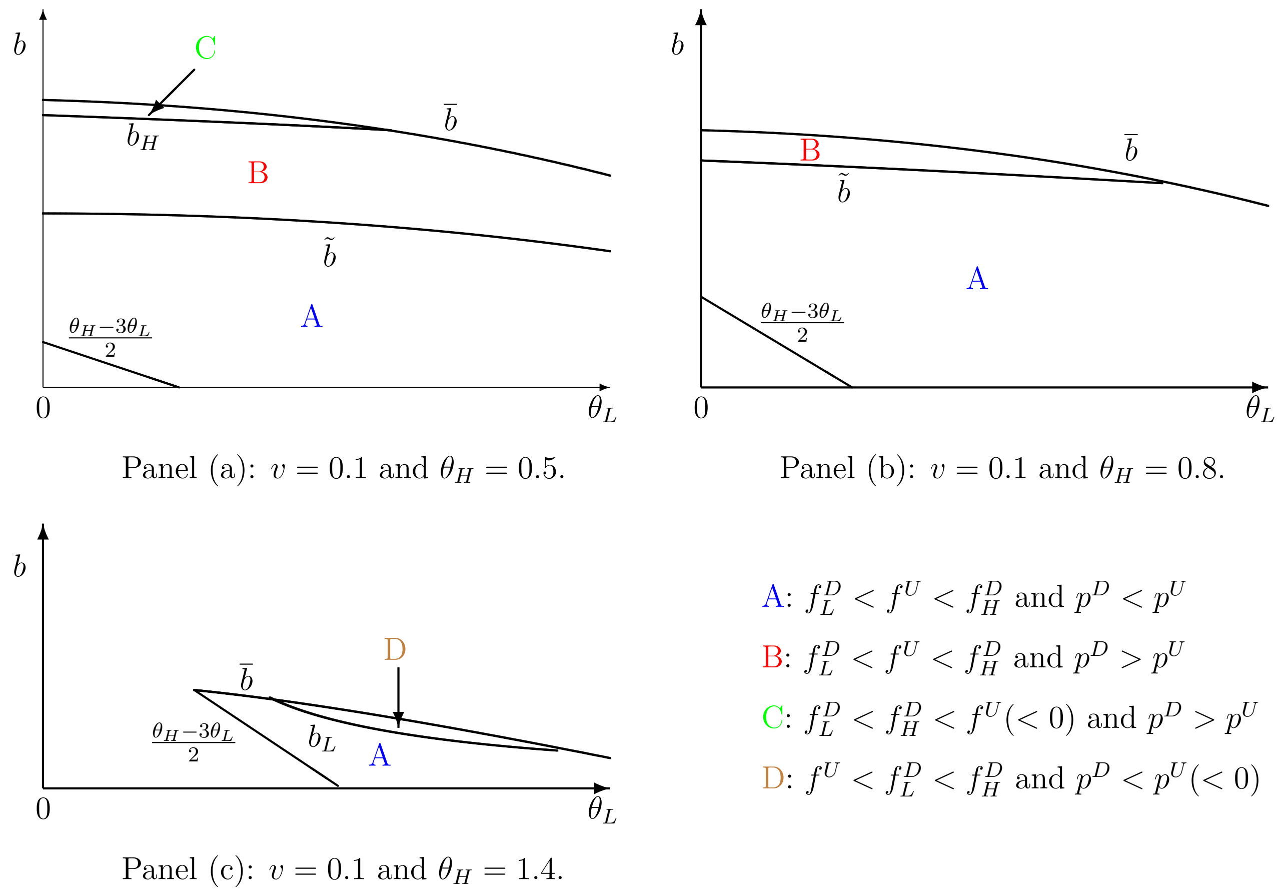


Figure 4: Pricing regimes

In Figure 4 we illustrate the different cases. The standard results are obtained in Region A. In Region B, the platform's optimal strategy is to increase the price paid by buyers, while still moving f_L^D and f_H^D in opposite directions. In region C, sellers get a relatively low per-buyer benefit compared to buyers' per-seller benefit, and are subsidized under both regimes. Price discrimination induces the platform to increase the subsidy to both seller types and to charge a higher price to buyers. In region D, θ_H and θ_L are relatively high compared to b, and the platform increases both seller fees, while at the same time increasing the subsidy to buyers.

4.4. Exclusion of low-type sellers under uniform pricing.

So far we have only discussed the case where both groups of buyers are served (though not fully covered) under uniform pricing. In the analysis of price discrimination without network effects, whether the weak market is served under uniform pricing is important in determining the effects of price discrimination. Indeed, if the weak market is excluded, then price discrimination leads to a *weak* Pareto improvement by enabling to serve the weak market without affecting the strong one. In our model, exclusion of the low-type sellers under uniform pricing occurs when $b < \frac{\theta_H - 3\theta_L}{2}$. We then have:

Proposition 4. *Suppose that Assumption 1 holds except that $b < \frac{\theta_H - 3\theta_L}{2}$. Then price discrimination leads to a strict Pareto improvement over uniform pricing.*

Strict Pareto improvement implies that participation by all groups and total welfare increase, just as in the analysis above. Compared to the standard analysis without network effects, the novelty here is the strictness of the Pareto improvement. High-type sellers benefit from the platform's ability to price-discriminate, because attracting a new type of seller leads to an increase in the participation of buyers.

5. Extensions

5.1. Seller-specific buyers' benefits

In order to obtain analytical results, we have assumed that buyers are indifferent with respect to the type of the sellers they interact with. While this assumption can be microfounded, a more plausible assumption is that buyer surplus depends on the type of the seller, $b(\theta)$, and that buyers prefer to interact with high-type sellers: $b(\theta_H) > b(\theta_L)$. While in the baseline model high-type sellers always pay a higher fee than their low-type peers, this is no longer necessarily the case when $b(\theta_H) > b(\theta_L)$, as the following result shows (formal proof in Appendix B.1.1):

Lemma 1. *Under price discrimination, when $\theta_L - b(\theta_L) > \theta_H - b(\theta_H)$, the platform charges a higher participation fee to the low-type sellers.*

When the above condition holds, even though high-type sellers are willing to pay more, they also generate more benefits to buyers, so that the platform seeks to attract them by offering them a relatively lower price.[17] This is consistent with some strategies used in practice. For example, Steam's commission rate is 30% for games earning less than \$10m, then 25% for earnings between \$10m and \$50m, and 20% for earnings above \$50m.

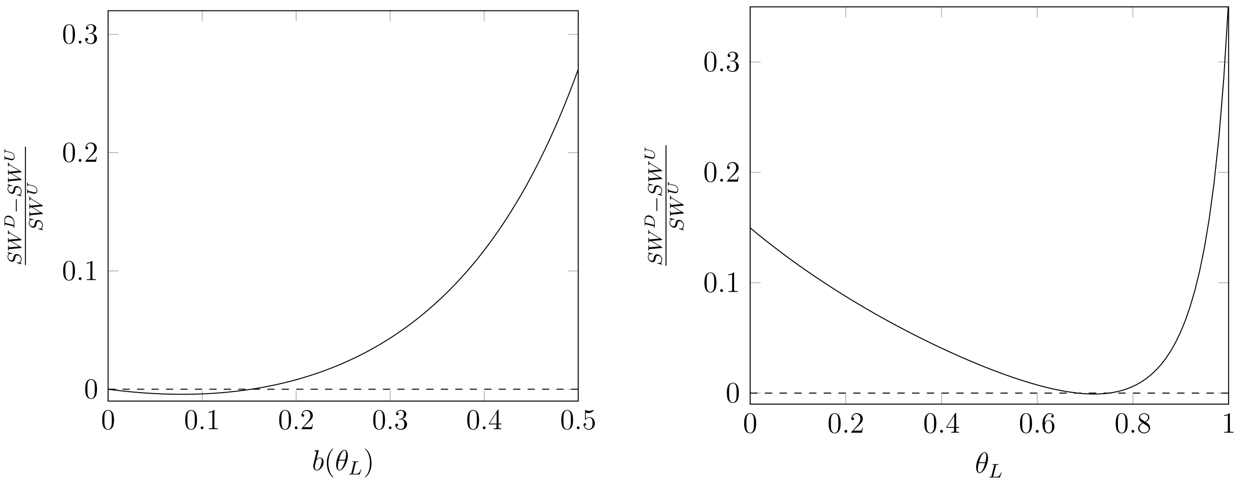
We then have the following result, which is formally demonstrated in Appendix B.1.2:

Proposition 5. *Suppose that parameters are such that the equilibrium is interior. When $b(\theta_H) > b(\theta_L)$, participation on both sides increases under price discrimination.*

[17]This principle is familiar (Armstrong 2006, Belleflamme & Peitz 2021). See also Benzell & Collis (2022) for a recent contribution with an application to social media.

Proposition 5 is a generalization of Proposition 1. Recall that, in Proposition 1, part of the reasoning relied on seller's participation being constant across pricing regimes (for a given N_B). When buyers care about seller type, we need to take into account that, even though N_S is the same for a given N_B, the composition of the set of sellers is different so that buyers may be worse-off everything else being equal. The crux of the proof consists in showing that this composition effect is not enough to offset the platform's incentive to attract more buyers following the improvement of its ability to extract surplus from sellers.

Obtaining clean analytical results in this more general setup is difficult, but numerical simulations indicate that our main insights continue to hold. Even though total welfare may go down with price discrimination, we find that the magnitude of welfare losses is generally small (see Figure 5). There are also parameter regions such that price discrimination leads to a Pareto improvement.



(a) Change in welfare as a function of $b(\theta_L)$. Parameter values: $v = 0.1$, $\theta_L = 0.5$, $\theta_H = 1$ and $b(\theta_H) = 0.5$.

(b) Change in welfare as a function of θ_L. Parameter values: $v = 0.1$, $b(\theta_L) = 0.25$, $\theta_H = 1$ and $b(\theta_H) = 0.5$.

Figure 5: Proportional change in welfare under price discrimination vis-á-vis uniform pricing.

5.2. Importance of network effects

Our baseline model highlights how the existence of network effects can overturn some standard results on the effects of price discrimination. A natural question is whether there is a discontinuity, in that arbitrarily small network effects would be enough to make price-discrimination socially desirable. One issue with our baseline model is that taking θ_H and θ_L to zero eliminates any heterogeneity across sellers, and makes the study of price discrimination meaningless.

In this subsection we instead look at a generalization of the baseline model, where the demand by sellers of type $j \in \{L, H\}$ is $D_j(N_B, f_j) = \alpha_j + \theta_j N_B - f_j$, with $\alpha_H \geq \alpha_L$. This would be the case if sellers' participation costs followed a type-specific distribution.[18] With this specification, it is possible to take θ_H and θ_L to zero while maintaining some heterogeneity across the two groups.

Proposition 1 still applies in this model, because the result that, for a given N_B, total seller participation $N_H + N_L$ is the same under uniform pricing and discrimination continues to hold.

To study the role of network effects on the welfare consequences of price discrimination, we use a scaling parameter λ so that network effects are λb, $\lambda\theta_L$ and $\lambda\theta_H$. Figure 6 illustrates how price discrimination affects welfare depending on the strength of network effects (λ) and a measure of homogeneity across groups (α_L/α_H).

When $\alpha_L = \alpha_H$, welfare always increases with price-discrimination, as in our baseline model. When $\alpha_L < \alpha_H$, there is a tension between the standard distortion due to price discrimination and the effects identified in this paper. Discrimination increases welfare when network effects (measured by λ) are large, and decreases it otherwise.

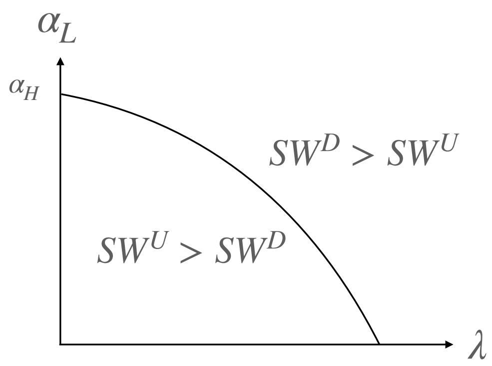


Figure 6: Effects of discrimination on welfare. (Parameter values: $\theta_L = 0.7, \theta_H = 1, b = 0.2, v = 0.1, \alpha_H = 0.3$)

5.3. Ad-valorem pricing

Even though many platforms use ad valorem fees,[19] in our baseline model we focus on participation fees. The main reason is that the use of ad valorem fees in itself constitutes a form of price discrimination, since different sellers end up paying different amounts per transaction. In fact, Wang & Wright (2017) show that ad valorem fees achieve efficient price discrimination when demand is proportional to marginal costs. In addition, in our model with independent firms and no asymmetric information, ad valorem fees would induce a distortion (when marginal costs are positive) and would not be optimal.

[18]For instance, if the participation cost for type j sellers is uniformly distributed over $(-\alpha_j, -\alpha_j + 1)$.
[19]For a detailed overview of different ad-valorem fee charged by platforms, see Borck et al. (2020).

These arguments notwithstanding, in this subsection we show that our main welfare results continue to hold when the platform charges ad valorem fees. Suppose that the platform charges a fee proportional to sellers' revenue, $r_i\theta_i$, for $i \in \{L, H\}$, and that sellers face zero marginal costs, so that their optimal price (and thus gross revenue θ) is not affected by the fee. We compare the uniform pricing regime where the platform charges the same ad-valorem fee to all sellers ($r_H = r_L = r$) to the one where it sets discriminatory prices $r_H \neq r_L$.

Platform profit when respectively employing uniform pricing and discriminatory pricing regimes are

$$\max_{r,p} \Pi_U = (p + r(\theta_H N_H + \theta_L N_L))N_B, \quad \max_{r_H, r_L, p} \Pi_D = (p + r_H \theta_H N_H + r_L \theta_L N_L)N_B.$$

A detailed analysis is presented in Appendix B.1.2, where we also provide the relevant threshold value of b that appears below. In the following proposition, we present the ad-valorem fee counterpart to the results in Proposition 2.

Proposition 6. *In comparison to uniform pricing, under price discrimination we obtain that:*

(i) *Total buyer and seller participation increases.*

(ii) *The platform, buyers and low-type sellers are better-off.*

(iii) *Total welfare is higher.*

(iv) *High-type sellers are better-off if and only if $b > \hat{b}^{ad}(\theta_H, \theta_L)$. In this case, price discrimination constitutes a Pareto improvement over uniform pricing.*

This confirms that our welfare results hold when the platform employs an alternative pricing structure to sellers. The intuitions for these results are similar to those after Proposition 2.

5.4. One-sided pricing

We now consider the case in which the platform does not charge buyers, whereas it still charges the participation fee f to sellers. Most digital platforms, such as the major app-stores, search engines and social networks, usually grant free access to users. This is also common in the lodging sector, in which Online Travel Agencies (OTAs) such as Booking.com and Expedia only charge hotels and lodging establishments. We are therefore interested in the case in which the platform faces an additional constraint in terms of possible cross-subsidization between buyers and sellers.

We compare uniform pricing with price discrimination. The formal analysis is in Appendix B.2, together with the relevant threshold values of b. We present the main results in this proposition.

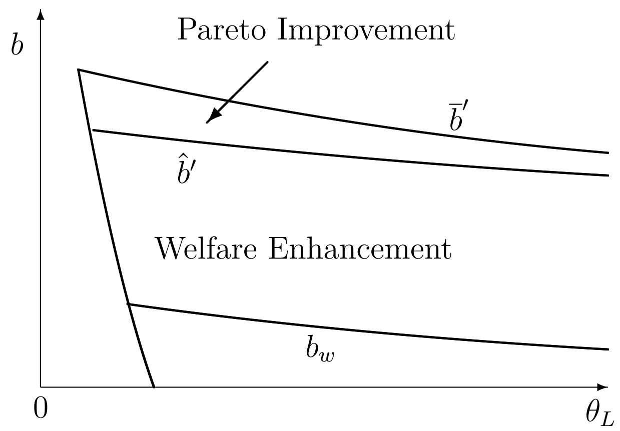


Figure 7: Pareto Improvement and Welfare Enhancement

Proposition 7. *In comparison to uniform pricing, under price discrimination we obtain that:*

(i) *Total buyer and seller participation increases.*

(ii) *The platform, buyers and low-type sellers are better off.*

(iii) *Total welfare is higher if and only if $b > b_w(\theta_H, \theta_L)$.*

(iv) *High-type sellers are better-off if and only $b > \hat{b}'(\theta_H, \theta_L)$, with $\hat{b}'(\theta_H, \theta_L) > b_w(\theta_H, \theta_L)$. In this case, price discrimination constitutes a Pareto improvement over uniform pricing.*

Figure 6 provides the interval regions of interest characterized by $\max\{0, \frac{2\theta_H}{\theta_L(\theta_H-\theta_L)}\} < b < \overline{b}'(\theta_H, \theta_L, v)$. It is plotted for $v = 0.1$ and $\theta_H = 0.5$. As one can easily notice, most of the main results of the benchmark case continue to hold, the most relevant exception being that price discrimination does not always enhance welfare. In fact, when buyer valuation for sellers' participation is not strong enough ($b < b_w(\theta_H, \theta_L)$), total welfare is lower under price discrimination.

Buyers gain with price discrimination, given that sellers participation increases, following the same logic as Proposition 1. The platform obviously gains, and low-type sellers as well, given that $f_L^D < f^U < f_H^D$ always holds. This represents another difference with respect to the benchmark case, in which the standard result that price discrimination raises the price for the high type, while lowering that of the low type, did not always hold.

As per high-type sellers, they lose out under price discrimination when b is low, as they end up paying a higher fee without being compensated by a sufficiently strong increase in buyer participation. When $b < b_w(\theta_H, \theta_L)$, the loss for high-type sellers overcomes the sum of the gains of the other market participants, and price discrimination reduces total welfare.

In comparison to the benchmark case, the platform has fewer instruments to attract buyers and sellers. Only for sufficiently high buyer valuation price discrimination improves social welfare, namely when $b > b_w(\theta_H, \theta_L)$, as buyers value more the presence of sellers. Moreover, when $b > \hat{b}'(\theta_H, \theta_L)$, price discrimination is Pareto improving, as in the benchmark case. Buyers highly value the presence of sellers, and are therefore willing to join the platform; high-type sellers, who in turn value the presence of buyers, are more than compensated for the higher price they end up paying under price discrimination.

6. Managerial and Policy Implications

In this section, we summarize our main results and discuss their managerial and policy implications.

Managerial Insight 1. *Price discrimination among sellers should be accompanied by a pricing policy geared towards increasing buyer participation.*

This is a novel managerial implication that is specific to the platform markets often featuring network effects. The presence of network effects creates externalities between the two types of seller markets which would be absent otherwise. As in the paper, we have shown that seller-side price discrimination increases a platform's ability to monetize buyer participation — i.e., under price discrimination, the platform can more efficiently capture the seller value created by an extra buyer on the platform. Thus, it is intuitive that vis-á-vis no price discrimination, the platform should increase consumer participation under price discrimination and extract the increased seller surplus. A direct implication of this result is that when platforms employ price discrimination on the sellers' side, it should be accompanied by a pricing policy that increases buyer participation.

Managerial Insight 2. *Optimal price discrimination allows a platform to increase its profit as well as total participation. It can make* ***all*** *groups of users better-off, provided that network effects are large enough.*

While the result on profit is standard and applies to traditional markets, the result related to participation is new and specific to platforms. This result may alleviate a perceived tension between short-term and long-term success for a platform. Indeed, it shows that price discrimination is not a purely extractive practice that would come at the expense of its users, but is actually likely to foster overall participation, thereby participating to platform growth.

Interestingly, even the sellers that end up paying a higher fee can benefit from price discrimination, because of the increased participation of buyers. This may also alleviate concerns that price discrimination might drive away too many high-margin sellers, thereby reducing the attractiveness of the platform. With strong network effects, price discrimination no longer entails a trade-off between participation by various groups of sellers.

Policy Insight 1. *Third-degree price discrimination on the seller side enhances buyer surplus.*

In traditional markets, the focus is often only on the impact of price discrimination on the side where such a pricing scheme is employed. Here, we have shown that price discrimination on the seller side always benefits buyers, because the platform seeks to attract more of them. A regulator focused on maximizing consumer surplus should therefore view such a practice favorably.

Policy Insight 2. *Third-degree price discrimination is more likely to increase total welfare in two-sided markets than in traditional markets.*

By now it is well established that third-degree price discrimination has ambiguous welfare effects in standard markets. In this paper, we have shown that, in a setup where price discrimination reduces welfare in a standard market (i.e. with linear demands), the result is overturned once we introduce two-sidedness. Linear demands is of course a special case, but the force that drives this result - namely that increased monetization on one side leads the platform to increase participation on the other and triggers a positive feedback loop - is general and should lead policy makers to adopt a more sanguine view of price discrimination.

Policy Insight 3. *When cross-group network effects are strong, regulatory interventions aimed at capping platform fees may fail to enhance total welfare.*

Regulatory bodies and scholars have recently proposed to cap commission rates, as we saw at the end of Section 2. Our results suggest that, in markets characterized by strong cross-side networks effects, regulating platform fees may hinder the welfare-enhancing effect of price discrimination strategies possibly adopted by platforms.

7. Conclusion

In this paper, we argue that third-degree price discrimination in markets featuring network effects is not only welfare enhancing but can also be Pareto improving. This result arises due to the presence of network externalities as in their absence our analysis would reproduce the well-known findings from traditional markets. In particular, cross-sided network externalities render the multiple sides of a platform interdependent and changes in welfare on one side can have relevant repercussions on the other side.

In the presence of two types of sellers, high-type and low-type, price discrimination enables a platform to profitably and more efficiently extract higher surplus from sellers. To do so, it chooses to enhance their value on the platform by boosting buyer participation. Since demands on the two sides are elastic, the platform only extracts a portion of this increased seller value which results in increased total participation of sellers. Ultimately, we find that price discrimination enhances platform profit, increases buyer surplus and the surplus of low-type sellers, and it can even generate a higher surplus for high-type sellers, thus resulting in a Pareto improvement,

Our analysis is carried out in a simplified setting in which players on both sides pay participation fees to join, and buyers equally value the presence of sellers on the platform. However, we proved that our main results hold when more complex settings are taken into account, such as heterogenity in buyer valuation of sellers, ad valorem fees on the seller side, and buyers freely joining the platform. Finally, we used linear demands for tractability, but the mechanism underpinning our results, namely the increases in the participation of both buyers and sellers generated by price discrimination, holds more generally, as we explained at the end of Subsection 4.1.

Notwithstanding the limitations, the results that we obtain bear important managerial implications for executives of large platforms catering to a wide variety of demand segments. They also offer policy makers precious indications about the possible advantages that platforms can create for society at large when cross-sided network externalities are present. Neglecting these forces may have unintended effects for platform managers contemplating price discrimination strategies as well as for policy makers when designing platform regulation.

References

Adachi, T. (2005), 'Third-degree price discrimination, consumption externalities and social welfare', *Economica* **72**(285), 171–178.

Aguirre, I., Cowan, S. & Vickers, J. (2010), 'Monopoly price discrimination and demand curvature', *American Economic Review* **100**(4), 1601–15.

Armstrong, M. (2006), 'Competition in two-sided markets', *The RAND Journal of Economics* **37**(3), 668–691.

Belleflamme, P. & Peitz, M. (2020), Network goods, price discrimination, and two-sided platforms, Technical report, CORE.

Belleflamme, P. & Peitz, M. (2021), *The Economics of Platforms*, Cambridge University Press.

Benzell, S. G. & Collis, A. (2022), 'Regulating digital platform monopolies: The case of facebook'.

Bergemann, D., Brooks, B. & Morris, S. (2015), 'The limits of price discrimination', *American Economic Review* **105**(3), 921–957.

Bhargava, H. K., Wang, K. & Zhang, X. (2022), 'Fending off critics of platform power with differential revenue sharing: Doing well by doing good?', *Management Science* **68**(11), 8249–8260.

Bisceglia, M. & Tirole, J. (2022), 'Fair gatekeeping in digital ecosystems'.

Böhme, E. (2016), 'Second-degree price discrimination on two-sided markets', *Review of Network Economics* **15**(2), 91–115.

Borck, J., Caminade, J. & von Wartburg, M. (2020), Apple's app store and other digital marketplaces, Technical report, Analysis Group.

Bouvard, M., Casamatta, C. & Xiong, R. (2022), 'Lending and monitoring: Big tech vs banks', *TSE Working paper* .

Carroni, E., Madio, L. & Shekhar, S. (2023), 'Superstars in two-sided markets: exclusives or not?'.

Chang, Y., Winston, C. & Yan, J. (2022), 'Does uber benefit travelers by price discrimination?', *The Journal of Law and Economics* **65**(S2), S433–S459.

Cowan, S. (2016), 'Welfare-increasing third-degree price discrimination', *The RAND Journal of Economics* **47**(2), 326–340.

D'Annunzio, A. & Russo, A. (2023), 'Transaction taxes and second-degree price discrimination', *Mimeo* .

Ding, R. & Wright, J. (2017), ‘Payment card interchange fees and price discrimination’, *The Journal of Industrial Economics* **65**(1), 39–72.

Gambacorta, L., Madio, L. & Parigi, B. (2023), ‘Platform lending and innovation’, *Working Paper* .

Gomes, R. & Mantovani, A. (2020), ‘Regulating platform fees under price parity’, *CEPR Discussion Paper 15048* .

Gomes, R. & Pavan, A. (2016), ‘Many-to-many matching and price discrimination’, *Theoretical Economics* **11**(3), 1005–1052.

Hashizume, R., Ikeda, T. & Nariu, T. (2021), ‘Pareto improving third-degree price discrimination with network effects’, *Institute of Economic Research Faculty of Economics Discussion Papers Series* .

Hausman, J. A. & MacKie-Mason, J. K. (1988), ‘Price discrimination and patent policy’, *The RAND Journal of Economics* pp. 253–265.

Jeon, D.-S., Kim, B.-C. & Menicucci, D. (2022), ‘Second-degree price discrimination by a two-sided monopoly platform’, *American Economic Journal: Microeconomics* **14**(2), 322–69.

Jullien, B., Pavan, A. & Rysman, M. (2021), Two-sided markets, pricing, and network effects, *in* ‘Handbook of Industrial Organization’, Vol. 4, Elsevier, pp. 485–592.

Lin, S. (2020), ‘Two-sided price discrimination by media platforms’, *Marketing Science* **39**(2), 317–338.

Liu, Q. & Serfes, K. (2013), ‘Price discrimination in two-sided markets’, *Journal of Economics & Management Strategy* **22**(4), 768–786.

Nahata, B., Ostaszewski, K. & Sahoo, P. K. (1990), ‘Direction of price changes in third-degree price discrimination’, *The American Economic Review* **80**(5), 1254–1258.

Peitz, M. & Reisinger, M. (2022), ‘Platforms under joint ownership’.

Pigou, A. C. (1920), *The Economics of Welfare*, Macmillan.

Reisinger, M. (2014), ‘Two-part tariff competition between two-sided platforms’, *European Economic Review* **68**, 168–180.

Robinson, J. (1933), *The economics of imperfect competition*, Macmillan and Company.

Schmalensee, R. (1981), ‘Output and welfare implications of monopolistic third-degree price discrimination’, *The American Economic Review* **71**(1), 242–247.

Shekhar, S. (2021), ‘Platform pricing choice: Exclusive deals or uniform prices’, *Review of Network Economics* **20**(3), 159–186.

Tremblay, M. J. (2021), The limits of marketplace fee discrimination, Technical report, CESifo.

Varian, H. R. (1985), 'Price discrimination and social welfare', *The American Economic Review* **75**(4), 870–875.

Wang, C. & Wright, J. (2022), 'Regulating platform fees'.

Wang, Z. & Wright, J. (2017), 'Ad valorem platform fees, indirect taxes, and efficient price discrimination', *The RAND Journal of Economics* **48**(2), 467–484.

A. Omitted proofs: baseline model

A.1. Proof of Proposition 1

In order to provide some intuition along with the proof, it is helpful to study the platform's dual problem of choosing the participation level on each side to maximize profit, while prices adjust accordingly. The idea of the proof is the following: writing $\widetilde{N_j}(N_{-j})$ to denote the profit-maximizing participation level of side $j \in \{B, S\}$ as a function of the total participation on the other side, we will first show that $\widetilde{N_j}$ is increasing in both pricing regimes. Then, we will show that, as in standard models of third-degree price discrimination in one-sided markets, $\widetilde{N_S^U}(N_B) = \widetilde{N_S^D}(N_B)$: taking buyers' participation as given, price discrimination leaves "output" on the seller side unchanged. Finally, we will show that, taking N_S as given, $\widetilde{N_B^U}(N_S) < \widetilde{N_B^D}(N_S)$: because the platform can extract more value from sellers under price discrimination, it has an incentive to attract more buyers. This in turn leads to more sellers joining the platform, and so on until the end of the feedback loop.

Uniform pricing Under uniform pricing, the platform chooses the quantity of buyers N_B and the total quantity of sellers N_S, and the participation fees adjust accordingly. Inverse demands and platform's profit are respectively given by (3) and (4). From first-order conditions (7) and (8) we derive the expression for $\widetilde{N_B^U}(N_S)$ and $\widetilde{N_S^U}(N_B)$:

$$\frac{\partial \Pi^U}{\partial N_B} = 0 \Leftrightarrow P^U + N_B \frac{\partial P^U}{\partial N_B} + N_S \frac{\partial F^U}{\partial N_B} = 0 \Leftrightarrow \widetilde{N_B^U}(N_S) = \frac{v + bN_S}{2} + \frac{(\theta_H + \theta_L)N_S}{4}, \tag{14}$$

$$\frac{\partial \Pi^U}{\partial N_S} = 0 \Leftrightarrow F^U + N_S \frac{\partial F^U}{\partial N_S} + N_B \frac{\partial P^U}{\partial N_S} = 0 \Leftrightarrow \widetilde{N_S^U}(N_B) = \frac{(\theta_L + \theta_H + 2b)N_B}{2}. \tag{15}$$

Price discrimination Under price discrimination, the platform has an extra instrument, and can thus choose N_H and N_L independently. Inverse demands are given by (5), and the platform's profit is given by (6). From the first-order conditions with respect to the number of sellers, namely (10) and (11), we obtain $\widetilde{N_L^D}(N_B)$ and $\widetilde{N_H^D}(N_B)$:

$$\frac{\partial \Pi^D}{\partial N_L} = 0 \Leftrightarrow F_L^D + N_L \frac{\partial F_L^D}{\partial N_L} + N_B \frac{\partial P^D}{\partial N_L} = 0 \Leftrightarrow \widetilde{N_L^D}(N_B) = \frac{(\theta_L + b)N_B}{2}, \tag{16}$$

$$\frac{\partial \Pi^D}{\partial N_H} = 0 \Leftrightarrow F_H^D + N_H \frac{\partial F_H^D}{\partial N_H} + N_B \frac{\partial P^D}{\partial N_H} = 0 \Leftrightarrow \widetilde{N_H^D}(N_B) = \frac{(\theta_H + b)N_B}{2}. \tag{17}$$

Note that adding (17) and (16) gives (15), so that $\widetilde{N_S^D}(N_B) = \widetilde{N_S^U}(N_B)$.

Solving the first-order condition with respect to the number of buyers (9) yields:

$$\frac{\partial \Pi^D}{\partial N_B} = 0 \Leftrightarrow P^D + N_B \frac{\partial P^D}{\partial N_B} + N_L \frac{\partial F_L^D}{\partial N_B} + N_H \frac{\partial F_H^D}{\partial N_B} = 0 \Leftrightarrow N_B = \frac{v + bN_S}{2} + \frac{\theta_H N_H + \theta_L N_L}{2}. \quad (18)$$

In (18), N_B is obtained as a function of N_S, N_H and N_L. But from (17) and (16), we know that $\widetilde{N_H^D}(N_B) = \frac{\theta_H + b}{\theta_H + \theta_L + 2b} \widetilde{N_S^D}(N_B)$ and $\widetilde{N_L^D}(N_B) = \frac{\theta_L + b}{\theta_H + \theta_L + 2b} \widetilde{N_S^D}(N_B)$. Therefore we can rewrite (18) as

$$\widetilde{N_B^D}(N_S) = \frac{v + bN_S}{2} + \frac{\theta_H(\theta_H + b) + \theta_L(\theta_L + b)}{2(\theta_H + \theta_L + 2b)} N_S.$$

Because $\theta_H > \theta_L$, simple algebra then reveals that $\widetilde{N_B^D}(N_S) > \widetilde{N_B^U}(N_S)$: for a given level of seller participation, the platform wants to serve more buyers in the discrimination regime.

Together, these observations imply that discrimination leads first to an increase in N_B, which leads to an increase in N_S, which further increases N_B, etc., until we converge to a point where both buyer and seller participation are higher than under uniform pricing.

A.2. Proof of Proposition 2

We first have to compute buyer surplus, sellers' surplus and total welfare in both scenarios. Platform profits are given by (12) and (13), respectively.

Uniform pricing. When the platform sets a unique fee, buyer surplus and type $j \in \{L, H\}$ sellers' surplus are respectively given by

$$CS^U = \int_0^{N_B^U(p^U, f^U)} (v + b(N_H^U(p^U, f^U) + N_L^U(p^U, f^U)) - p^U - k^B) dk^B = \frac{8v^2}{(8 - (2b + \theta_H + \theta_L)^2)^2},$$

$$DS_j^U = \int_0^{N_j^U(p^U, f^U)} (\theta_j N_B^U(p^U, f^U) - f^U - k^S) dk^S = \frac{v^2(2b + 3\theta_j - \theta_{-j})^2}{2(8 - (2b + \theta_H + \theta_L)^2)^2},$$

for a total welfare of

$$SW^U = CS^U + \Pi^U + \sum_{i=1,2} DS_j^U = \frac{v^2(24 - (2b + 3\theta_H - \theta_L)(2b - \theta_H + 3\theta_L))}{(8 - (2b + \theta_H + \theta_l)^2)^2}.$$

Price discrimination. When the platform charges two different fees, buyer surplus and type $j \in \{L, H\}$ sellers' surplus are respectively given by

$$\begin{aligned} CS^D = & \int_0^{N_B^D(p^D, f_H^D, f_L^D)} (v + b(N_H^D(p^D, f_H^D, f_L^D) + N_L^D(p^D, f_H^D, f_L^D)) - p^D - k^S) dk^S \\ = & \frac{2v^2}{(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2}. \end{aligned}$$

$$DS_j^D = \int_0^{N_j^D(p^D, f_H^D, f_L^D)} (\theta_j N_B^D(p^D, f_H^D, f_L^D) - f_j^D - k^B) dk^B = \frac{v^2(b+\theta_j)^2}{2(4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L))^2},$$

for a total welfare of

$$SW^D = CS^D + \Pi^D + \sum_{i=1,2} DS_j^D = \frac{v^2(12-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}{2(4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L))^2}.$$

We will now prove the three points of Proposition 2, taking into account the admissible parametric region defined by Assumption 1,

(i) By a revealed preference argument, the platform is necessarily better-off under price discrimination. Formally:

$$\Pi^D - \Pi^U = \frac{v^2(\theta_H-\theta_L)^2}{(4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H-\theta_L)(8-(2b+\theta_H+\theta_L)^2)} > 0.$$

That buyers are also better-off is a corollary of Proposition 1. Regarding low-type sellers, one can check that, in the admissible parametric region:

$$DS_L^D - DS_L^U = \frac{1}{2}v^2\left(\frac{(2b+3\theta_L-\theta_H)^2}{(8-(2b+\theta_H+\theta_L)^2)^2} - \frac{(b+\theta_L)^2}{(4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L))^2}\right) > 0.$$

(ii) Turning to total welfare, we obtain that:

$$SW^D - SW^U = \frac{v^2(\theta_H-\theta_L)^2\lambda}{2(4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L))^2(8-(2b+\theta_H+\theta_l)^2)^2}$$

where $\lambda = 32 - 24b^4 - 7\theta_H^4 + 2\theta_H^3\theta_L + 28\theta_L^2 - 7\theta_L^4 - 48b^3(\theta_H+\theta_L) - 2\theta_L\theta_H(12-\theta_L^2) + 14\theta_H^2(2-\theta_L^2) + 2b(\theta_H+\theta_L)(16-13\theta_H^2+2\theta_H\theta_L-13\theta_L^2) + 2b^2(16-25\theta_H^2-22\theta_L\theta_H-25\theta_L^2)$.

Notice that sign of λ determines the sign of the difference in total welfare. Equating λ to 0 and solving for b, we get four solutions but only one is positive and given by:

$$b^{sol}(\theta_H,\theta_L) = \frac{\sqrt{6}\sqrt{16-7(\theta_H-\theta_L)^2+\sqrt{1024+256(\theta_H-\theta_L)^2+(\theta_H-\theta_L)^4}} - 6(\theta_H+\theta_L)}{12}.$$

Further, this $b^{sol}(\theta_H,\theta_L)$ is greater than the upper bound of our feasible region $\overline{b}(\theta_H,\theta_L,v)$.

Next differentiating the expression for λ with respect to b and computing it at $b = b^{sol}$, we get

$$\frac{\partial\lambda}{\partial b}|_{b=b^{sol}} = -\sqrt{\frac{2}{3}}\sqrt{g(16-7(\theta_H-\theta_L)^2+g)} < 0,$$

with $g = \sqrt{1024+256(\theta_H-\theta_L)^2+(\theta_H-\theta_L)^4}$. Regardless of whether λ is convex or concave in b, for

$b < \bar{b}(\theta_H, \theta_L, v) < b^{sol}(\theta_H, \theta_L)$, we must have $\lambda > 0$. We then confirm that the social welfare is higher under price discrimination than under uniform pricing.

(iii) This point follows from the comparison of DS_H^U and DS_H^D. We obtain that:

$$DS_H^D - DS_H^U = \frac{1}{2}v^2 \left(\frac{(2b + 3\theta_H - \theta_L)^2}{(8 - (2b + \theta_H + \theta_L)^2)^2} - \frac{(b + \theta_H)^2}{(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2} \right) > 0$$

if and only if $b > \hat{b}(\theta_H, \theta_L) = \frac{\sqrt{32 - 7(\theta_H - \theta_L)^2} - 3\theta_H - \theta_L}{4}$. Further notice that $\frac{\partial \hat{b}(\theta_H, \theta_L)}{\partial \theta_H} < 0$, thus explaining why the parametric region with Pareto improvement enlarges when θ_H increases, as we can see in Figure 3 when comparing Panel (a) with Panel (b).

A.3. Proof of Proposition 3

Considering the conditions specified on Assumption 1, that define our feasible parametric region, we compare sellers' fees and buyers' participation prices across the two regimes.

Starting from buyers, we find that

$$p^D > p^U \iff b > \tilde{b}(\theta_H, \theta_L) = \frac{\sqrt{16 + (\theta_H + \theta_L)^2} - \theta_H - \theta_L}{4},$$

with $\tilde{b}(\theta_H, \theta_L)$ admissible in the feasible region when θ_H is not very large. Hence, provided the high-type seller's valuation for buyer is not excessive, there exists a threshold value of b above which buyers pay a higher price under price discrimination. This represents another novel result of our analysis, as we prove that buyers may end up paying more under price discrimination. Remember that, by Proposition 1, participation of both sides increases under price discrimination. When b is low, that is when buyers do not highly value seller participation, attracting more of them requires lowering the price, and this could even be achieved through subsidization. On the contrary, when b is high, the increased seller participation is enough to attract more buyers, and the platform can also increase the price buyers have to pay. In other words, the platform and buyers share the increased gross surplus on the buyer side.

Turning to sellers, we first obtain that:

$$f_L^D < f_H^D < f^U \iff b > b(\theta_H)(\theta_H, \theta_L) = \frac{\sqrt{32 + (\theta_H - \theta_L)(9\theta_H + 7\theta_L)} - 3\theta_H - \theta_L}{4},$$

with $b(\theta_H)(\theta_H, \theta_L)$ admissible in the feasible region when both θ_H and θ_L are sufficiently low. Then,

$$f^U < f_L^D < f_H^D \iff b > b(\theta_L)(\theta_H, \theta_L) = \frac{\sqrt{32 - 7\theta_H^2 - 2\theta_H\theta_L} - \theta_H - 3\theta_L}{4},$$

with $b(\theta_L)(\theta_H, \theta_L)$ admissible in the feasible region when both θ_H and θ_L are sufficiently high.

Finally, $f_L^D < f^U < f_H^D$ for all remaining admissible parameter values, which reproduces a well-known result in the traditional one-sided market literature (Robinson 1933): price discrimination raises the price for the high-type, whereas it lowers that of the low type. This applies to the platform context that we consider, provided the sellers' values for buyer participation are neither too small nor too big.

Conversely, if sellers show more extreme attitudes towards the presence of buyers, the conventional result can be overturned. On the one hand, there is a region in which both sellers pay less under price discrimination. More precisely, when $b(\theta_H)(\theta_H, \theta_L)$ is admissible, $f_L^D < f_H^D < f^U < 0$ if and only if $b > b(\theta_H)(\theta_H, \theta_L)$: both types of sellers are subsidized to join the platform, and such subsidy increases under price discrimination. On the other hand, when $b(\theta_L)(\theta_H, \theta_L)$ is admissible, then both sellers pay a higher price under price discrimination if $b > b(\theta_L)(\theta_H, \theta_L) : 0 < f^U < f_L^D < f_H^D$.

By considering together buyers and sellers, we summarize our main results on comparing prices across the two regimes as follows (see also Figure 4):

(i) When θ_H and θ_L are relatively low and $b > b(\theta_H)(\theta_H, \theta_L)$: $f_L^D < f_H^D < f^U < 0$ and $p^D > p^U > 0$. When $b < b(\theta_H)(\theta_H, \theta_L)$, $f_L^D < f^U < f_H^D$ (with subsidies for sellers when b is high enough, and $p^D > p^U$ when $b > \tilde{b}(\theta_H, \theta_L)$).

(ii) For intermediate values of θ_H, we always have $f_L^D < f^U < f_H^D$, and $p^D > p^U$ when $b > \tilde{b}(\theta_H, \theta_L)$.

(iii) When θ_H and θ_L are relatively high and $b > b(\theta_L)(\theta_H, \theta_L)$: $0 < f^U < f_L^D < f_H^D$ and $p^D < p^U < 0$. When $b < b(\theta_L)(\theta_H, \theta_L)$, $f_L^D < f^U < f_H^D$, with $p^D < p^U$ (with subsidies for buyers only when b is high enough).

Starting from point (i), price discrimination enables the platform to charge a high price to buyers, who highly value seller participation, in order to increase the subsidy for both sellers. The fact that the fees are negative implies that the platform can subsidize sellers more than under unique pricing in order to attract them, as their value for buyer participation is particularly low. When b is lower, we obtain the standard result that $f_L^D < f^U < f_H^D$.

Turning to point (ii), when the high-type sellers value for buyer participation is intermediate, we always obtain the standard result that price discrimination increases the price for the high type, while it lowers that of the low type. As per buyers, we still find a region in which they end up paying more with price discrimination (when $b > \tilde{b}(\theta_H, \theta_L)$), but this region shrinks in comparison to point (i).

Finally, when sellers' value for buyer participation is high, we find the interesting case in which price discrimination enables to subsidize buyers more than under uniform pricing, and this is possible as a higher fee is imposed on both sellers. This occurs when $b > b(\theta_L)(\theta_H, \theta_L)$. The fact that this scenario requires a sufficiently high value for b can be explained by the fact that buyers need to have a sufficiently high value for seller participation in order for the platform to decide to increase their subsidy at the expenses of sellers. When $b < b(\theta_L)(\theta_H, \theta_L)$, we obtain the standard result $f_L^D < f^U < f_H^D$, with $p^D < p^U$; buyers are subsidized only when b is high enough. In any case, they pay a lower participation fee (or obtain a higher subsidy) under price discrimination.

A.4. Proof of Proposition 4

When $b < \frac{\theta_H - 3\theta_L}{2}$, the profit-maximizing levels of participation under uniform pricing are $N_L^U = 0, N_H^U = \frac{v(\theta_H+b)}{(4-(\theta_H+b)^2}, N_B^U = \frac{2v}{(4-(\theta_H+b)^2}$. (One can check that the platform cannot do better than that by serving some L-type sellers.) Under price-discrimination, we have $N_L^D = \frac{v(\theta_L+b)}{(4-(\theta_H+b)^2-(\theta_L+b)^2}, N_H^U = \frac{v(\theta_H+b)}{(4-(\theta_H+b)^2-(\theta_L+b)^2}, N_B^U = \frac{2v}{(4-(\theta_H+b)^2-(\theta_L+b)^2}$, all three quantities being larger than in the uniform pricing case.

B. Omitted proofs: extensions to the baseline model

B.1. Seller-specific buyers' benefits

B.1.1. Proof of Lemma 1

Straightforward computation of the equilibrium under price discrimination gives

$$f_i^D = \frac{v(\theta_i - b(\theta_i))}{4 - (\theta_H + b(\theta_H))^2 - (\theta_L + b(\theta_L))^2}.$$

Comparing the fees offered to the two types of sellers, we have

$$f_L^D - f_H^D = \frac{v(\theta_L - b(\theta_L) - (\theta_H - b(\theta_H)))}{4 - (\theta_H + b(\theta_H))^2 - (\theta_L + b(\theta_L))^2}.$$

The above fee difference is positive when $\theta_L - b(\theta_L) > \theta_H - b(\theta_H)$.

B.1.2. Proof of Proposition 5

Similarly to Appendix A.1, we study the platform's dual problem of choosing the participation level on each side to maximize. The aim is again to show that the platform can attract more buyers under price discrimination, and this in turn entices more sellers to join the platform.

Uniform pricing Under uniform pricing, one can view the platform's maximization program as choosing N_B and N_S to maximize profit, without being able to adjust N_L and N_H. For a given fee f, we have

$$N_H = \theta_H N_B - f, \quad \text{and} \quad N_L = \theta_L N_B - f. \tag{19}$$

Adding these two equations, one gets the market clearing uniform price

$$F^U(N_B, N_S) = \frac{\theta_H + \theta_L}{2} N_B - \frac{N_S}{2}.$$

On the buyer side, demand is given by

$$N_B = \theta_H N_H + \theta_L N_L - p. \tag{20}$$

The market clearing price thus depends on the allocation N_H and N_L, not only on the aggregate number of sellers N_S. However, using (19), we know that under uniform pricing N_H and N_L will necessarily satisfy $N_H = N_L + (\theta_H - \theta_L)N_B$. This implies that

$$N_L = \frac{N_S}{2} - (\theta_H - \theta_L)N_B, \quad N_H = \frac{N_S}{2} + (\theta_H - \theta_L)N_B.$$

Plugging this into (20), we obtain the market-clearing buyer price:

$$P^U(N_B, N_S) = \frac{b(\theta_H) + b(\theta_L)}{2} N_S - (1 + (b(\theta_H) - b(\theta_L))(\theta_H - \theta_L))N_B.$$

The platform's profit is

$$\Pi^U(N_B, N_S) = N_S F^U(N_B, N_S) + N_B P^U(N_B, N_S).$$

It is straightforward to check that $\frac{\partial^2 \Pi^U(N_B,N_S)}{\partial N_B \partial N_S} > 0$, so that $\widetilde{N_S^U}(N_B)$ and $\widetilde{N_B^U}(N_S)$ are increasing.

The first-order conditions are

$$\frac{\partial \Pi^U(N_B, N_S)}{\partial N_S} = 0 \Leftrightarrow \widetilde{N_S^U}(N_B) = \frac{(b(\theta_H) + b(\theta_L) + \theta_H + \theta_L)N_B}{2}, \tag{21}$$

$$\frac{\partial \Pi^U(N_B, N_S)}{\partial N_B} = 0 \Leftrightarrow 2\left(1 + (b(\theta_H) - b(\theta_L))(\theta_H - \theta_L)\right) \widetilde{N_B^U}(N_S) = \frac{(b(\theta_H) + b(\theta_L) + \theta_H + \theta_L)N_S}{2}. \tag{22}$$

Price discrimination Under price discrimination the platform can choose N_B, N_L and N_H. Market-clearing prices are given by

$$F_L^D(N_B, N_L) = \theta_L N_B - N_L, \quad F_H^D(N_B, N_H) = \theta_H N_B - N_H,$$

$$P^D(N_B, N_L, N_H) = b(\theta_H)N_H + b(\theta_L)N_L - N_B.$$

The platform's profit is

$$\Pi^D(N_B, N_L, N_H) = N_L F_L^D(N_B, N_L) + F_H^D(N_B, N_H) + N_B P^D(N_B, N_L, N_H).$$

The first-order conditions are

$$\frac{\partial \Pi^D(N_B, N_L, N_H)}{\partial N_B} = 0 \Leftrightarrow 2\widetilde{N_B^D}(N_L, N_H) = (b(\theta_H) + \theta_H)N_H + (b(\theta_L) + \theta_L)N_L, \tag{23}$$

$$\frac{\partial \Pi^D(N_B,N_L,N_H)}{\partial N_H}=0 \Leftrightarrow \widetilde{N_H^D}(N_B)=\frac{b(\theta_H)+\theta_H}{2}N_B, \tag{24}$$

$$\frac{\partial \Pi^D(N_B,N_L,N_H)}{\partial N_L}=0 \Leftrightarrow \widetilde{N_L^D}(N_B)=\frac{b(\theta_L)+\theta_L}{2}N_B. \tag{25}$$

Note that adding (24) and (25) gives $\widetilde{N_S^D}(N_B)=\frac{b(\theta_H)+\theta_H+b(\theta_L)+\theta_L}{2}N_B=\widetilde{N_S^U}(N_B)$ (by (21)): for a given buyer participation level N_B, the optimal seller participation level is the same under the two pricing regimes.

Next, using (24) and (25), we obtain that:

$$\widetilde{N_H^D}(N_B)=\frac{b(\theta_H)+\theta_H}{b(\theta_H)+\theta_H+b(\theta_L)+\theta_L}\underbrace{(\widetilde{N_H^D}(N_B)+\widetilde{N_L^D}(N_B))}_{=\widetilde{N_S^D}(N_B)} \quad \text{and } \widetilde{N_L^D}(N_B)=\frac{b(\theta_L)+\theta_L}{b(\theta_H)+\theta_H+b(\theta_L)+\theta_L}\widetilde{N_S^D}(N_B).$$

Because the optimal ratios N_H/N_S and N_L/N_S are constant, we can rewrite (23) as a function of N_S:

$$2\widetilde{N_B^D}(N_S)=\frac{(b(\theta_H)+\theta_H)^2+(b(\theta_L)+\theta_L)^2}{b(\theta_H)+\theta_H+b(\theta_L)+\theta_L}N_S.$$

Because $b(\theta_H)+\theta_H > b(\theta_L)+\theta_L$, the right-hand side of the previous equation is larger than $\frac{b(\theta_H)+\theta_H+b(\theta_L)+\theta_L}{2}N_S$, which, by (22), is equal to $2\left(1+(b(\theta_H)-b(\theta_L))(\theta_H-\theta_L)\right)\widetilde{N_B^U}(N_S)$. This implies that $\widetilde{N_B^D}(N_S)>\widetilde{N_B^U}(N_S)$.

Putting things together, the facts that (i) all the $\widetilde{N_S}$ functions are increasing, (ii) $\widetilde{N_S^U}(N_B)=\widetilde{N_S^D}(N_B)$, and (iii) $\widetilde{N_B^D}(N_S)>\widetilde{N_B^U}(N_S)$, imply that, in equilibrium, $N_S^D>N_S^U$ and $N_B^D>N_B^U$.

B.2. Proof of Proposition 6

In this extension, we consider the case where the monopolist platform charges sellers ad-valorem fees. As in the benchmark, we compare the uniform pricing regime where the platform charges the same ad-valorem fee to all sellers ($r_H=r_L=r$) to the one where it sets $r_H\neq r_L$.

Sellers' payoffs. Suppose the platform charges ad-valorem fees r_j to sellers of type j. The payoff of a seller from group $j\in\{H,L\}$ with participation cost k^S from affiliating with the platform is

$$\tilde{\pi}_j(k^S)=(1-r_j)\theta_j N_B^e-k^S,$$

where N_B^e is the sellers' expectations on the total mass of buyers affiliating with the platform.

Sellers affiliate with the platform if and only if they obtain positive utility from participating $\tilde{\pi}_j(k^S)\geq 0 \implies k^S\leq(1-r_j)\theta_j N_B^e$ for $j\in\{H,L\}$. Thus, the mass of sellers of type j participating in the platform ecosystem are

$$\tilde{N}_j(N_B^e,f_j)=(1-r_j)\theta_j N_B^e.$$

The total mass of sellers active on the platform under price discrimination is then

$$\tilde{N}_S(N_B^e, r_H, r_L) = ((1 - r_H)\theta_H + (1 - r_L)\theta_L)N_B^e. \tag{26}$$

Under a uniform pricing regime, the total mass of sellers active on the platform is instead

$$\tilde{N}_S(N_B^e, r, r) = (1 - r)(\theta_H + \theta_L)N_B^e. \tag{27}$$

Platform payoffs. Platform profit when employing uniform pricing and discriminatory pricing regimes are respectively given as

$$\max_{r,p} \Pi_U = (p + r(\theta_H N_H + \theta_L N_L))N_B, \quad \max_{r_H, r_L, p} \Pi_D = (p + r_H\theta_H N_H + r_L\theta_L N_L)N_B.$$

Timing and equilibrium concept are the same as in the baseline model, and to ensure an interior solution, we make the following assumption.

Assumption 2. *We assume that buyers' and sellers' valuation for participation on the other side as well as buyer intrinsic valuation are not too large, namely:* $0 < v < \frac{4-2b^2-2b\theta_H-\theta_H^2-2b\theta_L-\theta_L^2}{2}$, $b < \frac{\sqrt{8-(\theta_H-\theta_L)^2}-(\theta_H+\theta_L)}{2} = \bar{b}^{ad}(\theta_H, \theta_L)$, *and* $\theta_L^2 + \theta_H^2 < 4$ *and* $\theta_L < \sqrt{2}$.

Uniform pricing. In this pricing regime, recall the buyer and seller participation from equations (1) and (27), respectively. In a rational expectations equilibrium agents correctly anticipate participation by the other group, so that participation levels $\tilde{N}_B^U$ and $\tilde{N}_S^U$ satisfy

$$\tilde{N}_B^U = v + b\tilde{N}_S^U - p \quad \text{and} \quad \tilde{N}_S^U = (1 - r)(\theta_L + \theta_H)\tilde{N}_B^U.$$

Solving the above system of equations for N_B^U and N_S^U yields buyer participation and seller total participation as functions of prices. We present these demands below.

$$\tilde{N}_B^U(p, r) = \frac{v - p}{1 - b(1 - r)(\theta_H + \theta_L)}, \; \tilde{N}_S^U(p, r) = \frac{(\theta_H + \theta_L)(v - p)(1 - r)}{1 - b(1 - r)(\theta_H + \theta_L)}.$$

Seller demand can be further decomposed into

$$\tilde{N}_H^U(p, r) = \frac{\theta_H(v - p)(1 - r)}{1 - b(1 - r)(\theta_H + \theta_L)}, \; \tilde{N}_L^U(p, f) = \frac{\theta_L(v - p)(1 - r)}{1 - b(1 - r)(\theta_H + \theta_L)}.$$

The platform sets prices to

$$\max_{p,r}(p + r(\theta_H\tilde{N}_H(\cdot) + \theta_L\tilde{N}_L(\cdot)))\tilde{N}_B(\cdot).$$

Differentiating platform profits with respect to p and r and solving the system of first order conditions yields the following prices.

$$\tilde{p}^U = \frac{v(\theta_H^2+\theta_L^2)(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}{2b\theta_H^3+\theta_H^4+2b\theta_H\theta_L(b+\theta_L)-\theta_L^2(4-(b+\theta_L)^2)-\theta_H^2(4-b^2-2b\theta_L-2\theta_L^2)},$$
$$\tilde{r}^U = \frac{\theta_H^2+\theta_L^2-b(\theta_H+\theta_L)}{2(\theta_H^2+\theta_L^2)}.$$

The associated equilibrium seller demands for type $j \in \{L,H\}$, buyer demand, and platform profit are respectively given by:

$$\tilde{N}_j^U(\tilde{p}^U,\tilde{r}^U) = \frac{\tilde{p}^U\theta_j(\theta_H^2+\theta_L^2+b(\theta_H+\theta_L))}{(\theta_H^2+\theta_L^2)(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}, \tilde{N}_B^U(\tilde{p}^U,\tilde{r}^U) = \frac{2\tilde{p}^U}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))},$$
$$\tilde{\Pi}^U = \frac{v\tilde{p}^U}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}.$$

Buyer surplus and type $j \in \{L,H\}$ sellers' surplus are respectively given by

$$\widetilde{CS}^U = \int_0^{\tilde{N}_B^U(\tilde{p}^U,\tilde{r}^U)} (v+b(\tilde{N}_H^U(\tilde{p}^U,\tilde{r}^U)+\tilde{N}_L^U(\tilde{p}^U,\tilde{r}^U))-\tilde{p}^U-k^B)dk^B$$
$$= \frac{2(\tilde{p}^U)^2}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))^2},$$

$$\widetilde{DS}_j^U = \int_0^{\tilde{N}_j^U(\tilde{p}^U,\tilde{r}^U)} ((1-\tilde{r}^U)\theta_j\tilde{N}_B^U(\tilde{p}^U,\tilde{r}^U)-k^S)dk^S = \frac{(\tilde{N}_j^U(\tilde{p}^U,\tilde{r}^U))^2}{2},$$

for a total welfare of

$$\widetilde{SW}^U = \widetilde{CS}^U + \tilde{\Pi}^U + \sum_{i=1,2}\widetilde{DS}_j^U = (\tilde{p}^U)^2\mathcal{X},$$

where

$$\mathcal{X} = \frac{\theta_H^2(2(6-\theta_L^2)-b^2-2b\theta_L)+\theta_L^2(12-(b+\theta_L)^2)-\theta_H^4-2b\theta_H^3-2b\theta_H\theta_L(b+\theta_L)}{2(\theta_H^2+\theta_L^2)(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))^2}.$$

Price discrimination. Under price discrimination, buyer participation is still given as in equation (1), while seller participation is given as in equation (26). Under rational expectations, equilibrium participation thus satisfies the following system:

$$\tilde{N}_B^D = v + b(N_L^D+N_H^D)-p, \quad \tilde{N}_H^D = (1-r_H)\theta_H N_B^D \quad \text{and} \quad \tilde{N}_L^D = (1-r_L)\theta_L N_B^D.$$

Solving the above system of equations for $\tilde{N}_B^D$, $\tilde{N}_H^D$ and $\tilde{N}_L^D$ yields buyer participation and seller participation as functions prices. We present these demands below. The solution is

$$\begin{aligned}
\tilde{N}_B^D(p, r_H, r_L) &= \frac{v-p}{1-b((1-r_H)\theta_H+(1-r_L)\theta_L)}, \\
\tilde{N}_H^D(p, r_H, r_L) &= \frac{(v-p)(1-r_H)\theta_H}{1-b((1-r_H)\theta_H+(1-r_L)\theta_L)}, \\
\tilde{N}_L^D(p, r_H, r_L) &= \frac{(v-p)(1-r_L)\theta_L}{1-b((1-r_H)\theta_H+(1-r_L)\theta_L)}.
\end{aligned}$$

The platform sets prices to maximize profits

$$\max_{p, r_H, r_L} (p + r_H\theta_H\tilde{N}_H^D(\cdot) + r_L\theta_L\tilde{N}_L^D(\cdot))\tilde{N}_B^D(\cdot).$$

Differentiating platform profits with respect to p and r_j, for $j \in \{L, H\}$ and solving the system of first order conditions yields the optimal prices as follows.

$$\tilde{p}^D = \frac{v(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}, \ \tilde{r}_j^D = \frac{\theta_j - b}{2\theta_j}, \ \text{for } j \in \{H, L\},$$

where superscript D indicates the case with price discrimination.[20] The associated equilibrium seller demands for $j \in \{L, H\}$, buyer demand, and platform profit are respectively given as

$$\begin{aligned}
\tilde{N}_j^D = \frac{v(b+\theta_j)}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}, \tilde{N}_B^D &= \frac{2v}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}, \\
\tilde{\Pi}^D &= \frac{v^2}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}.
\end{aligned}$$

Before proceeding further, we make a few observations.

Observation 1. *The following equality holds true.*

- *Under price discrimination, the price charged to buyers remains unchanged regardless of the pricing structure incident on sellers — i.e.,* $p^D = \tilde{p}^D$.
- *Under price discrimination, the total price charged to remains unchanged regardless of the pricing structure incident on sellers — i.e.,* $\tilde{r}_j^D\theta_H\tilde{N}_B^D(\tilde{p}^D, \tilde{r}^D) = f_j^D$.

The above implies that the mass of buyers, sellers and platform profits are identical under price discrimination regime regardless of whether platforms charge sellers a fixed participation price or an ad-valorem fee. As a consequence, consumer surplus and welfare expressions are identical as well.

[20] The denominator is positive by Assumption 1.

Price discrimination vs. uniform pricing. In the following, we show the robustness of the main result obtained in the baseline model.

Comparison of total participation of buyers and sellers. Total participation of both buyers and sellers is higher under price discrimination than under uniform pricing:

$$\tilde{N}_B^D - \tilde{N}_B^U = \frac{2b^2 v(\theta_H - \theta_L)^2}{(4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)) \\ (\theta_H^2(4-b^2-2b\theta_L-2\theta_L^2)-\theta_L^2(4-(b+\theta_L)^2)-2b\theta_H\theta_L(b+\theta_L)-\theta_H^4-2b\theta_H^3)} > 0;$$

$$\tilde{N}_S^D - \tilde{N}_S^U = \frac{v(\theta_H+\theta_L)(\theta_H^2+\theta_L^2+b(\theta_H+\theta_L))}{(\theta_H^2(4-b^2-2b\theta_L-2\theta_L^2)-\theta_L^2(4-(b+\theta_L)^2)-2b\theta_H\theta_L(b+\theta_L)-\theta_H^4-2b\theta_H^3)} > 0.$$

In both inequalities, the numerators are positive, as it can easily seen, and the denominators are positive under Assumption 2.

Comparison of platform profit. Considering platform profits, we obtain that:

$$\Pi^D - \widetilde{\Pi}^U = \frac{b^2 v^2(\theta_H-\theta_L)^2}{(4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)) \\ (\theta_H^2(4-b^2-2b\theta_L-2\theta_L^2)-\theta_L^2(4-(b+\theta_L)^2)-2b\theta_H\theta_L(b+\theta_L)-\theta_H^4-2b\theta_H^3)}.$$

We observe that the sign of the difference in platform profit is determined by the sign of the expressions in the denominator. The two terms in the denominator of the difference in profits are positive as they are just the terms in the denominator of the platform profits in the two pricing regimes. Since Assumption 2 guarantees platform profits are positive, they must be positive as well because the numerator of the profits is always positive.

Comparison of consumer surplus. Comparing consumer surplus under price discrimination with the consumer surplus under uniform pricing yields

$$CS^D - \widetilde{CS}^U = \frac{2}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))^2}\left((p_1^D)^2-(\tilde{p}^U)^2\right).$$

Hence, the difference in buyer prices determines the sign of the difference in consumer surplus.

$$(p_1^D)^2-(\tilde{p}^U)^2 = \mathcal{A}((\theta_L^2+\theta_H^2)(8-3b^2-4b\theta_L-4\theta_L^2)-2b\theta_H\theta_L(b+2\theta_L)-2\theta_H^3(\theta_H+2b)),$$

where $\mathcal{A}$ is a composite term of squared expressions

$$\mathcal{A} = \frac{2b^2v^2(\theta_H-\theta_L)^2}{(4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L))^2 \\ (\theta_H^2(4-b^2-2b\theta_L-2\theta_L^2)-\theta_L^2(4-(b+\theta_L)^2)-2b\theta_H\theta_L(b+\theta_L)-\theta_H^4-2b\theta_H^3)^2} > 0.$$

Therefore, the sign of $(p_1^D)^2 - (\tilde{p}^U)^2$ is determined by the sign of

$$\mathcal{B} = ((\theta_L^2 + \theta_H^2)(8 - 3b^2 - 4b\theta_L - 4\theta_L^2) - 2b\theta_H\theta_L(b + 2\theta_L) - 2\theta_H^3(\theta_H + 2b)).$$

Differentiating $\mathcal{B}$ with respect to b yields

$$\frac{\partial \mathcal{B}}{\partial b} = -2(2(\theta_H + \theta_L)(\theta_H^2 + \theta_L^2) + b(3\theta_H^2 + 3\theta_L^2 + 2\theta_H\theta_L)) < 0.$$

Thus, it is sufficient to show that $\mathcal{B}$ at $b = \bar{b}^{ad}$ is positive.

$$\mathcal{B}\big|_{b=\bar{b}^{ad}} = \frac{(\theta_H - \theta_L)^2(4 + 2\theta_H\theta_L - (\theta_H + \theta_L)\sqrt{(8 - (\theta_H - \theta_L)^2)})}{2}.$$

The second term in the numerator given by $(4+2\theta_H\theta_L - (\theta_H+\theta_L)\sqrt{(8 - (\theta_H - \theta_L)^2)}$ is always positive for $\theta_H > \theta_L > 0$. Thus, we show that consumer surplus is always higher under the price discrimination regime than under a uniform pricing regime.

Comparison of low-type seller surplus. Turning to the low-type sellers, a sufficient statistic for seller surplus is seller participation:

$$N_L^D - \widetilde{N}_L^U = \frac{bv(\theta_H - \theta_L)\mathcal{Z}_L}{\begin{array}{l}(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))\\ \quad(\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)\end{array}}$$

where

$$\mathcal{Z}_L = (\theta_H(4 - (b + \theta_H)^2) - (b + \theta_H)\theta_L(b + \theta_L)).$$

The sign of $N_L^D - \widetilde{N}_L^U$ is determined by the sign of the term $\mathcal{Z}_L$ as all other terms are guaranteed to be positive under Assumption 2.

Differentiating $\mathcal{Z}_L$ with respect to b yields

$$\frac{\partial \mathcal{Z}_L}{\partial b} = -(2\theta_H(b + \theta_H) + \theta_L(2b + \theta_H) + \theta_L^2) < 0.$$

Thus, it is sufficient to show that $\mathcal{Z}_L$ at $b = \bar{b}^{ad}(\theta_H, \theta_L)$ is positive.

$$\mathcal{Z}_L\big|_{b=\bar{b}^{ad}} = \frac{(\theta_H - \theta_L)(4 - \theta_L^2 - \theta_H(\sqrt{(8 - (\theta_H - \theta_L)^2)} - \theta_L)}{2}.$$

The second term in the numerator given by $(4 - \theta_L^2 - \theta_H(\sqrt{(8 - (\theta_H - \theta_L)^2)} - \theta_L)$ is always positive as Assumption 2 ensures $\theta_H > \theta_L > 0$ and $\theta_H^2 + \theta_L^2 < 4$. Thus, we show that the surplus of low-type sellers is always higher under price discrimination than under uniform pricing.

Comparison of total welfare. Comparing total welfare under price discrimination with the total welfare under uniform pricing yields

$$SW^D - \widetilde{SW}^U = \frac{\mathcal{A}}{4}\mathcal{Y}$$

where $\mathcal{Y} \triangleq 4b\theta_H^5 + \theta_H^6 + 2b\theta_H\theta_L(2\theta_L^3 + 5b\theta_L^2 - \theta_L(24-5b^2) - 2b(6-b^2)) + \theta_L^2(80 - 2b^2(18-b^2) - 6b\theta_L(8-b^2) - \theta_L^2(24-7b^2) + 4b\theta_L^3 + \theta_L^4) + \theta_H^4(7b^2 + 4b\theta_L - 3(8-\theta_L^2)) + 2b\theta_H^3(3b^2 + 5b\theta_L - 4(6-\theta_L^2)) + \theta_H^2(80 + 2b^4 + 10b^3\theta_L - 48\theta_L^2 + 3\theta_L^4 - 8b\theta_L(6-\theta_L^2) - 2b^2(18-7\theta_L^2))$. Differentiating $\mathcal{Y}$ twice with respect to b yields

$$\frac{\partial^3\mathcal{Y}}{\partial b^3} = 12(\theta_H + \theta_L)(3\theta_H^2 + 3\theta_L^2 + 2\theta_H\theta_L + 4b(\theta_H + \theta_L)) > 0.$$

Computing the second derivative of $\mathcal{Y}$ with respect to b at $b = \bar{b}^{ad}$ yields

$$\frac{\partial^2\mathcal{Y}}{\partial b^2}\Big|_{b=\bar{b}^{ad}} = -2(\theta_H - \theta_L)^2\left(12 + 2(\theta_H + \theta_L)^2 + 2\theta_H\theta_L - 3(\theta_H + \theta_L)\sqrt{8 - (\theta_H - \theta_L)^2}\right) < 0.$$

Thus, we confirm that $\frac{\partial^2\mathcal{Y}}{\partial b^2}$ is always negative in the feasible region.

Evaluating the first derivative of $\mathcal{Y}$ with respect to b at $b = 0$ yields

$$\frac{\partial\mathcal{Y}}{\partial b}\Big|_{b=0} = -4(\theta_H + \theta_L)(\theta_H^2 + \theta_L^2)(12 - \theta_H^2 - \theta_L^2) < 0.$$

The above is negative as Assumption 2 ensures that $\theta_H^2 + \theta_L^2 < 4$.

Finally, computing $\mathcal{Y}$ at $b = \bar{b}^{ad}$ yields

$$\mathcal{Y}\Big|_{b=\bar{b}^{ad}} = 4(\theta_H - \theta_L)^2\left(4 + 2\theta_L - (\theta_L + \theta_H)\sqrt{8 - (\theta_H - \theta_L)^2}\right) > 0.$$

The above is always positive as Assumption 2 ensures that $\theta_H^2 + \theta_L^2 < 4$. Hence, we show that total welfare is always higher under price discrimination than under uniform pricing.

Comparison of high-type seller surplus. A sufficient statistic for seller surplus is seller participation, which yields:

$$N_H^D - \widetilde{N}_H^U = \frac{bv(\theta_H - \theta_L)\mathcal{Z}_H}{\begin{array}{l}(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)) \\ \quad (\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)\end{array}}$$

where

$$\mathcal{Z}_H = ((b + \theta_L)(\theta_H^2 + \theta_L^2 + b(\theta_H + \theta_L)) - 4\theta_L).$$

The sign of $N_L^D - \widetilde{N}_L^U$ is determined by the sign of the term $\mathcal{Z}_H$ as all other terms are positive under Assumption 2.

Differentiating $\mathcal{Z}_H$ with respect to b yields

$$\frac{\partial \mathcal{Z}_H}{\partial b} = \theta_H^2 + 2\theta_L^2 + \theta_H\theta_L + 2b(\theta_H + \theta_L) > 0.$$

Computing $\mathcal{Z}_H$ at $b = 0$, yields

$$\mathcal{Z}_H|_{b=0} = \theta_L(4 - \theta_L^2 - \theta_H^2) > 0.$$

The above is positive as Assumption 2 ensures that $\theta_H^2 + \theta_L^2 < 4$.

Similarly, computing $\mathcal{Z}_H$ at $b = \bar{b}^{ad}$ yields

$$\mathcal{Z}_H\big|_{b=\bar{b}^{ad}} = \frac{(\theta_H - \theta_L)(4 - \theta_H^2 + \theta_H\theta_L - \theta_L\sqrt{(8 - (\theta_H - \theta_L)^2)}}{2}.$$

The second term in the numerator given by $(4 - \theta_L^2 - \theta_H(\sqrt{(8 - (\theta_H - \theta_L)^2)} - \theta_L)$ is always positive as Assumption 2 ensures $\theta_H > \theta_L > 0$ and $\theta_H^2 + \theta_L^2 < 4$.

Thus, by intermediate value theorem, there must exist a critical level of b denoted by

$$\hat{b}^{ad}(\theta_H, \theta_L) = \frac{1}{2}\left(\frac{\sqrt{\theta_H^4 + 2\theta_H\theta_L(8 - \theta_H^2) + \theta_L^2(16 + \theta_H^2)} - 2\theta_L^2}{\theta_H + \theta_L} - \theta_H\right)$$

where $N_H^D - \widetilde{N}_H^U = 0$. For $b > \hat{b}^{ad}(\theta_H, \theta_L)$, we must have $N_H^D - \widetilde{N}_H^U > 0$ and for $b < \hat{b}^{ad}$, we must have $N_H^D - \widetilde{N}_H^U < 0$.

Thus, we show that the surplus of high type sellers can also increase giving us the result that price discrimination can result in Pareto improvement over uniform pricing.

B.3. Proof of Proposition 7

As in the benchmark case, in order to ensure that the maximization problem is concave, we impose the following conditions:

Assumption 3. *Provided buyer intrinsic valuation as well as sellers' valuations are sufficiently low, we consider the region* $\max\{0, \frac{2\theta_H}{\theta_L(\theta_H - \theta_L)}\} < b < \bar{b}'(\theta_H, \theta_L) = \frac{\sqrt{(v(v-8)+8)(\theta_H^2+\theta_L^2)+2v^2\theta_H\theta_L} + (v-2)(\theta_H+\theta_L)}{(\theta_H-\theta_L)^2}$.[21]

Reproducing the analysis carried out in Section 4, we can easily see that Subsection 4.1 does not change, the only caveat being that we have to consider $p = 0$. As per the modification to Subsections 4.2 and 4.3, we obtain the following results.

[21] More precise conditions on v, θ_H and θ_L can be provided upon request.

Uniform pricing. The platform sets the uniform fee to maximize profits $fN_S^U(f)$, which yields the equilibrium fee:

$$f^U = \frac{v(\theta_H + \theta_L)}{4}.$$

The associated equilibrium seller demands for type $j \in \{L, H\}$, buyer demand, and platform profit are respectively given by:

$$N_j^U = \frac{v(\theta_j(3 - b\theta_j) - \theta_{-j}(1 - b\theta_{-j})}{4 - 4b(\theta_H + \theta_L)}, N_B^U = \frac{v(2 - b(\theta_H + \theta_L))}{2 - 2b(\theta_H + \theta_L)}, \Pi^U = \frac{v^2(\theta_H + \theta_L)^2}{8 - 8b(\theta_H + \theta_L)}.$$

Total participation of the sellers is then $N_S^U = N_L^U + N_H^U = \frac{v(\theta_H+\theta_L)}{2-2b(\theta_H+\theta_L)}$.

Buyer surplus and type $j \in \{L, H\}$ sellers' surplus is respectively given by

$$CS^U = \frac{v^2(2 - b(\theta_H + \theta_L))^2}{8(1 - b(\theta_H + \theta_L))^2}, DS_j^U = \frac{v^2(\theta_j(3 - b\theta_j) - \theta_{-j}(1 - b\theta_{-j}))^2}{32(1 - b(\theta_H + \theta_L))^2}.$$

Total welfare amounts to:

$$SW^U = \frac{v^2(b^2(2 + (\theta_H - \theta_L)^2)(\theta_H + \theta_L)^2 - 2b(\theta_H + \theta_L)\Sigma)}{16(1 - b(\theta_H + \theta_L))^2}.$$

where $\Sigma = (4 + 3\theta_H^2 - 2\theta_H\theta_L + 3\theta_L^2) + 8 + 7(\theta_H^2 + \theta_L^2) - 2\theta_H\theta_L)$.

Price Discrimination. The platform sets two different fees in order to maximize $f_H N_H^D(f_H, f_L) + f_L N_L^D(f_H, f_L)$, which yields at equilibrium

$$f_j^D = \frac{v(2\theta_j(1 - b\theta_j) - b\theta_{-j}(\theta_j - \theta_{-j}))}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}, \text{ for } j \in \{H, L\}.$$

The associated equilibrium seller demands for $j \in \{L, H\}$, buyer demand, and platform profit are:

$$N_j^D = \frac{v(\theta_j(2 - b\theta_j) + b\theta_{-j}^2)}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}, N_B^D = \frac{2v(2 - b(\theta_H + \theta_L))}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2},$$

$$\Pi^D = \frac{v^2(\theta_H + \theta_L)^2}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}.$$

Total seller participation is then given as

$$N_S^D = N_L^D + N_H^D = \frac{v(2(2 + \theta_L) - 2b\theta_L - b\theta_H(2 - \theta_H - \theta_L))}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}.$$

Buyer surplus and type $j \in \{L, H\}$ sellers' surplus are respectively given by

$$CS^D = \frac{2v^2(2 - b(\theta_H + \theta_L))^2}{(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))^2}, DS_j^D = \frac{v^2(\theta_j(2 - b\theta_{-j}) + b\theta_{-j}^2)}{2(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))^2}.$$

Total welfare amounts to:

$$SW^D = \frac{v^2(16 + 12(\theta_H^2 + \theta_L^2) - 8b(\theta_H + \theta_L)(2 + \theta_H^2 + \theta_L^2) - b^2\Delta)}{2(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))^2},$$

where $\Delta = (\theta_H^4 - 2\theta_H^3\theta_L - 4\theta_L^2 + \theta_L^4 - 2\theta_H^2(2 - \theta_L^2) - 2\theta_H\theta_L(4 + \theta_L^2))$.

Price discrimination vs. uniform pricing. Firstly, it is straightforward to show that the platform earns higher profit under price discrimination than under uniform prices.

Before we proceed further, it is informative to keep in mind how seller prices change under price discrimination. Comparing prices, we observe that

$$f^U - f_L^D = \frac{v(2 + b(\theta_H - \theta_L))(\theta_H - \theta_L)(2 - b(\theta_H + \theta_L))}{4(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))} > 0$$

and

$$f^U - f_H^D = -\frac{v(\theta_H - \theta_L)((2 - b\theta_H)^2 - b^2\theta_L^2)}{4(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))} < 0.$$

A corollary from the above price relations is that the low-type sellers are always better off.

Secondly, we find that total seller participation rises:

$$N_S^D - N_S^U = \frac{bv(\theta_H - \theta_L)^2(2 - b(\theta_H + \theta_L)}{2(1 - b(\theta_H + \theta_L))(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))} > 0.$$

The above is always positive because both $(2 - b(\theta_H + \theta_L)$ at the numerator and the expressions at the denominator are positive under Assumption 3. A direct consequence of the above is that buyer surplus rises. This is because the buyer price is set at zero and seller participation increases under price discrimination, thus benefiting buyers. Regarding buyers, their total participation increases, as it can be obtained by investigating the sign of:

$$N_B^D - N_B^U = \frac{2v(4b^3(\theta_H + \theta_L) - b^2(8 - 6\theta_H^2 - 4\theta_H\theta_L - 6\theta_L^2) - b(\theta_H + \theta_L)(8 - (\theta_H + \theta_L)^2) - 2(\theta_H + \theta_L)^2 + 8)}{(8 - (2b + \theta_H + \theta_L)^2)(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))},$$

which is always positive under Assumption 3.

Finally, in order to show that Pareto improvement is a possibility, it is sufficient to find conditions under which the high-type sellers can be better off under price discrimination. A sufficient statistic for this result to hold is to show that the participation of high-type sellers is higher under price discrimination than under uniform pricing despite the fact that participation fee to the high-margin type rises. This can be formally demonstrated as follows. Taking the difference of participation of the high type under price discrimination with its participation under uniform prices yields

$$N_H^D - N_H^U = \frac{v(\theta_H - \theta_L)(2 - b(\theta_H + \theta_L))\Omega}{4(1 - b(\theta_H + \theta_L))(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))},$$

where $\Omega = (2 - b^2(\theta_H + \theta_L) + b(3\theta_H + \theta_L))$. Note that the sign of $N_H^D - N_H^U$ follows that of Ω as all other terms are positive under the assumption that the problem is concave.

Differentiating Ω with respect to b, we observe that

$$\frac{\partial \Omega}{\partial b} = 3\theta_H + \theta_L + 2b(\theta_H - \theta_L)^2 > 0.$$

Further, computing Ω at the two bounds, we find that

$$\Omega|_{b=0} = -2, \ \Omega|_{b=\bar{b}'} = \frac{4(\theta_H^2 + \theta_H\theta_L + 2\theta_L^2) - 2(\theta_H + 3\theta_L)\sqrt{2(\theta_H^2 + \theta_L^2)}}{(\theta_H - \theta_L)^2} > 0.$$

Thus, by the intermediate value theorem, we can state there exists a cut-off denoted by $\hat{b}'$ above which $\Omega > 0$ and negative otherwise.

Equating Ω to zero and solving for b yields the following threshold

$$\hat{b}'(\theta_H, \theta_L) = \frac{\sqrt{17\theta_H^2 - 10\theta_H\theta_L + 9\theta_L^2} - 3\theta_H - \theta_L}{(\theta_H - \theta_L)^2},$$

which is within the admissible parameter bounds, as it can be easily demonstrated.

Finally, comparing social welfare in the two cases, we find that $SW^D > SW^U$ if and only if $b > b_w(\theta_H, \theta_L)$ with $b_w(\theta_H, \theta_L) < \hat{b}'(\theta_H, \theta_L)$; the analytical expression of $b_w(\theta_H, \theta_L)$ is very complex but can be provided upon request.