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"Bidding and Investment in Wholesale Electricity Markets: Discriminatory versus Uniform-Price Auctions"

Bert Willems and Yu Yueting

Toulouse School of Economics

Bidding and Investment in Wholesale Electricity Markets: Discriminatory versus Uniform-Price Auctions
Working Paper

Bert Willems
Université Catholique de Louvain,
Toulouse School of Economics,
Tilburg University

Yueting Yu
Tilburg University

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Abstract
We compare uniform and discriminatory-price auctions in wholesale electricity markets, studying both long-run investment incentives and short-run bidding behaviors. We develop a monopolistic competition model with a continuum of generation technologies ranging from base load to peak load, free entry and uncertain elastic demand. Our findings reveal that discriminatory-price auctions are inefficient because consumers' willingness to pay exceeds the marginal costs and investment incentives are distorted. Despite having an equal total installed capacity, the generation mix under discriminatory-price auctions skews towards a shortage of base-load technologies. Consequently, this results in a lower long-run consumer surplus. (JEL: D44, D47, L94)

1. Introduction

In her 2022 State of the Union Address at the height of the energy crisis, the President of the European Commission emphasized the urgent need for a "deep and comprehensive reform of the electricity market". The primary objective of this reform would be to recoup the benefits from low-cost renewables and to detach the electricity market price from the influence of dominant gas generators (European Union, 2022). Among the various options under consideration by policymakers is a potential reform of the price formation

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E-mail: mail@bertwillems.com (Willems); y.yu@tilburguniversity.edu (Yu)

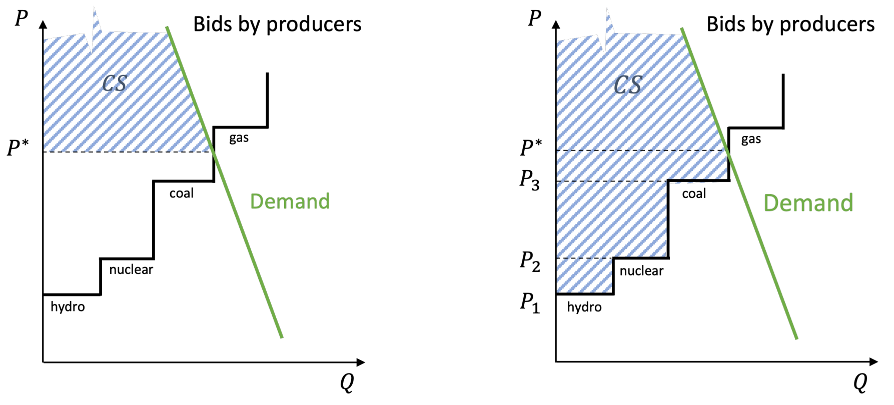

FIGURE 1. Comparison of uniform (left) and discriminatory-price auctions (right).

process in wholesale electricity markets. This would involve replacing the current uniform-price auction with a discriminatory-price auction model.[1]

We compare these two market designs, investigating producers' bidding behaviors and price-cost markups in the short run, as well as investment incentives and the equilibrium generation portfolio in the long run. The model reveals that transitioning to a discriminatory-price auction system could lead to a reduction in the average market price in the short run, ultimately benefiting consumers. However, this comes at the expense of short-run inefficiencies, such as the under-utilization of existing capacities. As a consequence, long-run distortions occur in the generation mix, ultimately harming consumers in the end.

Figure 1 illustrates how these two auctions work. In both auction formats, each bidder submits a bid, consisting of a production quantity paired with a corresponding price. Subsequently, the auctioneer dispatches the production based on increasing price order, also known as the merit order, until the total demand is met. In Figure 1, producers' aggregate bids are visualized as an increasing step function in orange, while the green downward-sloping curve represents the aggregate demand. The market clears at the intersection (p^* in Figure 1), where the aggregate demand precisely matches the supply.

1. The focus here lies on the pan-European day-ahead power exchanges, providing participants with the opportunities to buy and sell electricity in hourly blocks for the entire 24-hour day. Various reform options are being considered, including enhancing the role of long-term contracts and exploring more granular regional markets. For more detailed insights, refer to the market assessment report by The Agency for the Cooperation of Energy Regulators (ACER, 2021), the European Commission's REPowerEU initiative (European Commission, 2022), the CERRE report on market design (Pollitt et al., 2022) and the concurrent review of British electricity trading arrangements (Department for Business, Energy & Industrial Strategy, 2022).

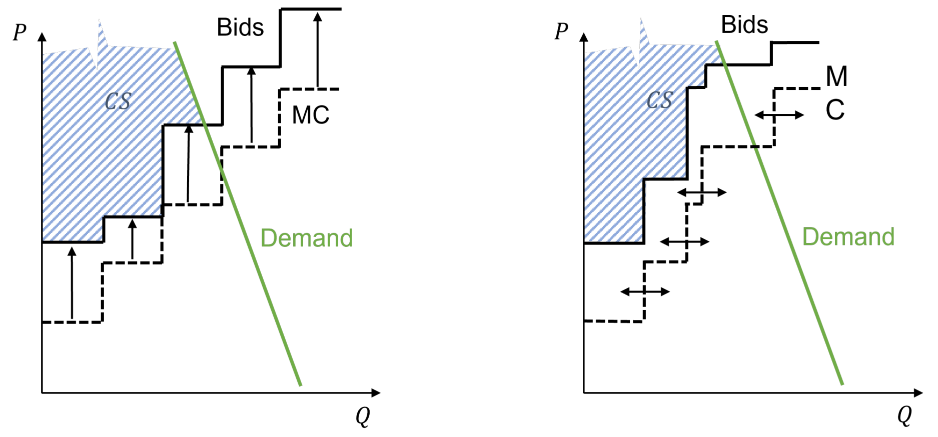

FIGURE 2. Strategic adjustments in discriminatory-price auctions. Left: Bids go above marginal costs. Right: Investment is adjusted.

In a uniform-price auction, also known as a clearing-price auction, all accepted producers receive the market-clearing price. On the other hand, in a discriminatory-price auction also known as a pay-as-bid auction, winning producers are remunerated at their bidding prices (i.e., p_1, p_2 and p_3 in the right figure of Figure 1). The green-shaded region in the figure represents the consumer surplus, which is the difference between consumers' willingness to pay and the price(s) received by the winning producers.

At first glance, Figure 1 suggests that consumers may be better off in a discriminatory-price auction. However, this is not immediately apparent. The auction format can influence how producers bid. For instance, in uniform-price auctions, a bidder may truthfully bid its marginal cost, expecting its inframarginal bid to earn a markup along with the clearing price. However, in discriminatory-price auctions, bidding marginal cost results in zero profit. As a consequence, producers are likely to set higher prices in discriminatory-price auctions, shading their bids as seen in the left figure of Figure 2, where the dotted line represents marginal costs and the solid line represents the corresponding bids. This change in bidding behavior can impact the market-clearing price and technologies selected in the auction.

Moreover, the alternative auction format may also affect producers' long-run investment decisions, corresponding to a horizontal shift of the dotted line in the right figure of Figure 2. Consequently, the overall impact on consumer welfare from the different auction formats becomes ambiguous.

We study the impact of market design on both investments (depicted by the blue line) and bidding behavior (illustrated by the orange line). To achieve this, we develop a variant of the monopolistic competition model to characterize the market equilibrium, capturing essential aspects of wholesale electricity markets. These markets encompass uncertain and elastic demand, with multiple generation technologies ranging from base load to peak load.

In the first stage of our model, producers determine their investment capacities. In the second stage, they simultaneously submit their bids, which are then dispatched in increasing order to meet the demand. The selected and remunerated generators are determined based on either uniform or discriminatory-price auction rules.

Following Tirole (1988), our model maintains three characteristics of monopolistic competition:[2] First, each firm faces a downward-sloping demand. Second, each firm achieves zero profit under the free entry condition. And lastly, a change in one firm's price does not affect the demand of other firms. However, unlike the standard monopolistic competition model, we introduce heterogeneity in production technologies instead of product variety in the electricity market setting.

Our research contributes to the existing literature on equilibrium models in multi-unit auctions utilized in electricity markets (see Table 1). The first three papers compare the short-run performances of both auction formats in scenarios with uncertain demand. Fedrico and Rahman (2003) construct a competitive model incorporating multiple technologies, similar to our paper, while adopting the setup of elastic demand. They find that discriminatory-price auctions lead to less remuneration for infra-marginal plants, but marginal plants receive more compared to uniform-price auctions, resulting in higher consumer surplus. Both Holmberg (2009) and Fabra, von der Fehr, and Harbord (2006) develop models considering market power and single technology. Holmberg (2009) applies a supply function equilibrium model, while Fabra, von der Fehr, and Harbord (2006) consider a duopoly model, where both suppliers submit a single price offer for their entire capacity. These two papers also find that average prices in discriminatory-price auctions are lower than in uniform-price auctions. However, contrary to our findings, they conclude that consumer surplus is higher with discriminatory-price auctions. Our model shares similarities with Fedrico and Rahman (2003), but we extend it by incorporating an investment stage and a more comprehensive formulation of the bidding strategies. We relax the assumptions of linear demand and investments and uniform shock distributions.

In the fourth paper, Fabra, von der Fehr, and de Frutos (2011) expand on their 2006 model by introducing an investment stage and demonstrate that discriminatory-price auctions generally result in lower prices compared to uniform-price auctions while maintaining the same aggregate capacity under reasonable assumptions. However, despite identical aggregate capacity, our introduction of multiple generation technologies reveals that the generation mix

2. Thanks to Patrick Rey for suggesting this link.

Table 1. Comparison of discriminatory-price auctions with uniform-price auctions in multi-unit auctions with uncertain demand.

	Demand	**CS**	**W**	**Invest**	**Model**
Federico & Rahman '03	elastic	+	–	no	perf. comp & monopoly
Holmberg '09	inelastic	+	=	no	supply function equil.
Fabra et al. '06	inelastic	+	=	no	duopoly
Fabra et al. '11	inelastic	+	=	yes, 1 tech	duopoly
Our paper	elastic	–	–	yes, ∞ tech	monopolistic competition

CS: consumer surplus, W: welfare.

becomes distorted with discriminatory-price auctions, leading to a reduction in consumer surplus.

Unlike Fabra, von der Fehr, and Harbord (2006), Fabra, von der Fehr, and de Frutos (2011) and Holmberg (2009), our analysis considers demand responsiveness. In the presence of elastic demand, price distortions cause deadweight losses, resulting in decreased overall welfare. Conversely, there are no welfare effects with perfectly inelastic demand. Given our focus on long-term investment effects, the assumption of elastic demand is reasonable. Moreover, in future low-carbon energy systems aiming to balance intermittent generation, demand elasticity is expected to play a more critical role (Cramton, 2017).

We explore how our findings are influenced by the assumptions of (1) stochastic demand and (2) demand responsiveness and demonstrate that both assumptions combined lead to lower welfare in the discriminatory-price auction.

When demand is inelastic, price distortions in the discriminatory-price auction have no impact on demand levels. Given free entry, the generation portfolio is optimally chosen to meet the stochastic distribution of demand levels. Consequently, the portfolios are the same under both market designs, and both lead to efficient market outcomes.

When demand is perfectly predictable, bidders will mark up their bids in the discriminatory-price auction until they match the clearing price. This leads to equivalent short-run market outcomes in both scenarios, resulting in the same investment levels. However, setting one's bid equal to the clearing price requires that each bidder has perfect information about demand and aggregate production capacity, which may not be the case in practice.

In our model, we do not explicitly account for information asymmetries regarding demand realization or production capacity. Bower and Bunn (2001) employ an agent-based model to analyze the efficiency of uniform and discriminatory-price auctions calibrated for the England and Wales market, finding that the discriminatory-price auction leads to higher prices. This is attributed to agents with significant market share possessing informational advantages in a discriminatory-price auction, facing less competitive pressure. As a result, a dominant producer with informational advantages may be selected, even with high production costs, leading to scheduling power plants out of merit. Furthermore, prediction-related costs impose additional inefficiencies (Kahn et al., 2001). Fabra and Llobet (2021) investigate scenarios where capacities are private information for producers, which may reflect future electricity markets with 100% intermittent renewable production. They find in a short-run model that producers make lower profits under discriminatory-price auctions.

This paper also contributes to the literature on the optimal portfolio choice in wholesale electricity markets. Since electricity is non-storable, a diverse mix of generation technologies is required to efficiently cater to the uncertain and unpredictable demand. Prices for each period reflect the scarcity of production capacity, known as peak-load pricing. The theoretical groundwork for this concept was established in the mid-20th century in a regulated monopoly setting (Boiteux, 1949; Steiner, 1957), forming the basis of our liberalized energy sector and market models (Joskow and Schmalensee, 1988; Joskow and Tirole, 2007). While engineers often use a discrete set of technologies to reflect specific options, for mathematical tractability, Zöttl (2010) employs a continuum of technologies representing the technology frontier for supplying energy under varying capacity factors. We adopt this approach in our model as well.

Zöttl (2010) studies oligopolistic competition in investment in generation portfolios, assuming uniform prices in a reduced-form spot market model. It is shown that producers prefer to invest in a portfolio that follows a hockey stick-shaped aggregate supply function. Strategically, producers tend to overinvest in base load and underinvest in peak capacity. Capacity allocation serves as a strategic commitment device for the subsequent spot market. Our model also assumes a continuum of technologies but places a greater focus on the spot market rules and the bidding process. Additionally, we assume that producers lack strategic investment incentives given the monopolistic competition framework, but each producer correctly predicts the future stochastic price distribution.

There has been an ongoing debate about the relative impact of various factors, such as market structure (number of producers, vertical relations and contracts), market design (auction rules), and behavior (insider trading, collusion) on market outcomes in power markets.[3] Empirical evidence can shed light on bidding behaviors, but measuring the effect of investments can be a complex challenge, and analytical models may be more appropriate for this purpose.

Evans and Green (2003) empirically study the impact of market design on electricity prices in Britain, accounting for market structure and underlying costs. They find that the market design dummy is insignificant. Bushnell,

3. Wilson (2002) coins the term "market architecture" as the combination of market structure and market design which policymakers rely on to create efficient markets.

Mansur, and Saravia (2008) analyze three U.S. wholesale markets (California, PJM and New England) that feature diverse market designs and underlying cost characteristics (e.g., fuel costs and hydro production), and attribute the variation in market competitiveness to both horizontal and vertical market structures. They suggest that spot market design has only a limited impact on market outcomes. In contrast, Fabra and Toro (2003) provide empirical evidence suggesting a correlation between the reduction of the price-cost margins in the British electricity markets and a decline in market concentration, as well as the introduction of new regulations. This indicates that both market design and market structure play a significant role in influencing market outcomes.

The remainder of the paper is organized as follows: Section 2 establishes the model, while Section 3 characterizes the bidding and investment equilibrium for uniform and discriminatory-price auctions, and compares the equilibrium outcomes. In Section 4, specific cases concerning demand assumptions are examined. Finally, Section 5 presents the conclusions.

2. Model

This section presents a competitive power market in both a short-run and long-run setting. In the short-run model, producers participate in a multi-unit spot market auction by submitting a supply bid for their entire installed capacity. In the long-run model, producers first make investment decisions and then participate in the spot market auction. We consider both uniform and discriminatory-price auctions. Consumers are price-takers and are represented by a stochastic demand function. Bids are long-lived, that is, they are assumed to be submitted before demand realization. In the remainder of this section, we first describe the technical assumptions regarding production technologies and the demand-side setup. We explain the market-clearing condition and then characterize the equilibrium bids and investments.

2.1. Supply

We assume an atomistic market structure with a continuum of electricity producers that are price-takers in the spot market auction and are free to enter the market by investing in the generation technology of their choice. There are no entry barriers. Each producer invests in an infinitesimal capacity unit dG, which can produce output dq.

A generation technology is characterized by its marginal cost c and its annualized investment cost k. We assume that a continuum of technologies is available, corresponding to different marginal costs c on the half-open interval $(0, \hat{c}]$, where $\hat{c}$ is the technology with the highest marginal cost. The investment cost of a technology with marginal cost c is the function $k(c)$, which represents the technology frontier of all existing production technologies. We make the following assumption on the shape of the investment cost function, as shown in Figure 3.

Assumption 1. *The technology frontier $k(c)$ is twice continuously differentiable, downward sloping, convex and log-concave on the interval $(0, \hat{c}]$:*

$$\frac{dk(c)}{dc} < 0, \quad \frac{d^2 k(c)}{dc^2} > 0 \quad and \quad \frac{d^2 \ln(k(c))}{dc^2} < 0.$$

At the lower boundary, INADA-like assumptions hold:

$$\lim_{c \to 0} k(c) = \infty, \ \lim_{c \to 0} k'(c) = -\infty.$$

The capital cost at the upper boundary $\hat{c}$ is zero and has a slope of zero:

$$k(\hat{c}) = 0, \ k'(\hat{c}) = 0.$$

The investment cost function has properties that resemble the electricity market. First, investment cost decreasing with marginal cost c, $k'(c) < 0$, implies that power plants with lower operating costs have higher fixed costs and vice versa. For instance, to construct a nuclear power plant, the investment cost $k(c)$ is relatively high, while the marginal cost is relatively low. On the other hand, a natural gas power plant has lower investment costs, but due to higher fuel and carbon prices, operating costs are high. Second, $k(c)$ is convex since it describes the technology frontier. If the function would not be convex, then there exists a technology c which is dominated by a linear combination of two technologies c_1 and c_2. Hence, the technology c should not be part of the frontier. Log-concavity implies that the elasticity of the technology frontier

$$\varepsilon_k(c) = d\ln(k(c))/d\ln(c)$$

is downward sloping, i.e., $\varepsilon_k'(c) < 0$. This will guarantee that bids increase with marginal costs in the discriminatory-price auction, and hence producers with the lowest marginal costs are activated first.

The levelized cost of a technology c with a capacity factor $h \in [0, 1]$, which is the fraction of time a plant is producing, is equal to

$$LC = c + \frac{k(c)}{h}.$$

For a given capacity factor h the technology c that minimizes levelized costs satisfies the condition:

$$k'(c) = -h.$$

The negative relation between fixed investment cost and operating cost implies that none of these technologies is strictly dominated by others, but are optimal

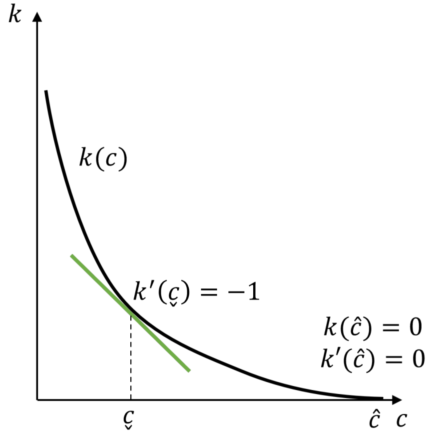

Figure 3. Technology frontier $k(c)$ is downward-sloping and convex. Technology $\underaccent{\check}{c}$ is the "always-on" base-load technology, with $k'(\underaccent{\check}{c}) = -1$. $\hat{c}$ is the Value of Lost Load Technology corresponding to consumer rationing.

for different capacity factors h. The INADA-like assumptions guarantee interior solutions. The zero marginal cost technology $c = 0$ has prohibitively expensive capital costs and will never be used. Define the always-on technology $\underaccent{\check}{c}$ as the technology that is optimal for a capacity factor $h = 1$, hence $k'(\underaccent{\check}{c}) = -1$. The technology $\hat{c}$ is the technology that minimizes levelized costs when the capacity factor $h = 0$. It is common in electricity simulation models to represent consumer rationing (by creating blackouts) as a technology with zero capital costs. The marginal cost $\hat{c}$ thus represents the Value of Lost Load (VOLL), the amount that consumers are willing to pay to avoid blackouts.

The aggregate investment by producers with marginal costs equal to or less than c is represented by the installed capacity function $G(c)$. By design, a producer with technology c can only invest positive amounts, i.e. $dG(c) \geq 0$. We use c_0 to indicate the lowest marginal cost in which anyone invests: $c_0 = \min\{c \mid G(c) > 0\}$. Thus function $G(c)$ is defined on the interval $[c_0, \hat{c}]$. The total installed capacity is $\hat{G} = G(\hat{c})$. In the short-run version of the model, G is exogenous and assumed to have a strictly positive slope $G' > 0$ on the interval $[c_0, \hat{c}]$.

In the spot market auction, a producer with marginal cost c submits a single bid $b(c)$ for its entire infinitesimal capacity $dG(c)$. Hence the producers' investment and bidding strategies can be summarized by two functions $\{G(c), b(c)\}$. See Figure 4. At $c = \hat{c}$, the bidding curve and investment curve are assumed to extend vertically, representing the capacity constraint. We will restrict ourselves to set-ups where the bidding function $b(c)$ is monotonic in equilibrium.

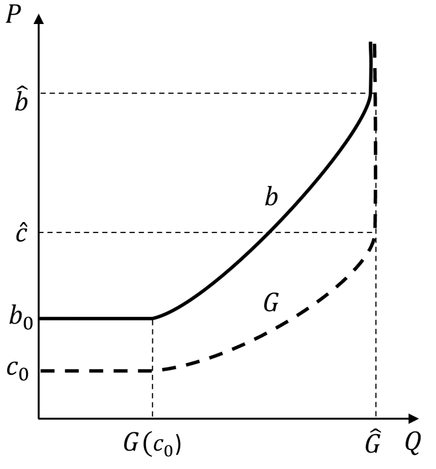

FIGURE 4. Producers' investment function $q = G(c)$ and the supply curve $q = G(b^{-1}(p))$.

2.2. Demand

As for the demand side, we assume that consumers are price-takers represented by a stochastic inverse demand function with additive price shocks.

$$p = P(q) + \varepsilon. \tag{1}$$

This corresponds to the state contingent gross utility function $V(q, \varepsilon)$:

$$V(q, \varepsilon) = \int_0^q P(t)dt + \varepsilon \cdot q.$$

Without loss of generality, we normalize demand by setting $P(0) = 0$, such that the demand shock ε is the intercept of the demand function.

The demand shock ε follows a known cumulative distribution function $F(\varepsilon)$ over the interval $[\underline{\varepsilon}, \bar{\varepsilon}]$. Denote the quantile function of the demand shock by $\mathcal{Q}(\cdot) = F^{-1}(\cdot)$. The deterministic inverse demand $P(q)$ and the demand shock distribution $F(\varepsilon)$ are known before producers submit bids, but the demand shock realization ε is not. We impose the following assumption on the distribution function.

ASSUMPTION 2. *The distribution function of demand shocks $\mathcal{Q}$ and the investment cost k satisfy*

$$\mathcal{Q}'(1 + k'(c)) > \frac{1}{k''(c)} \quad \forall c \in (0, \hat{c}].$$

This condition is in the primitives of the model – the shock distribution ($\mathcal{Q} = F^{-1}$) and the technology frontier $k(c)$.

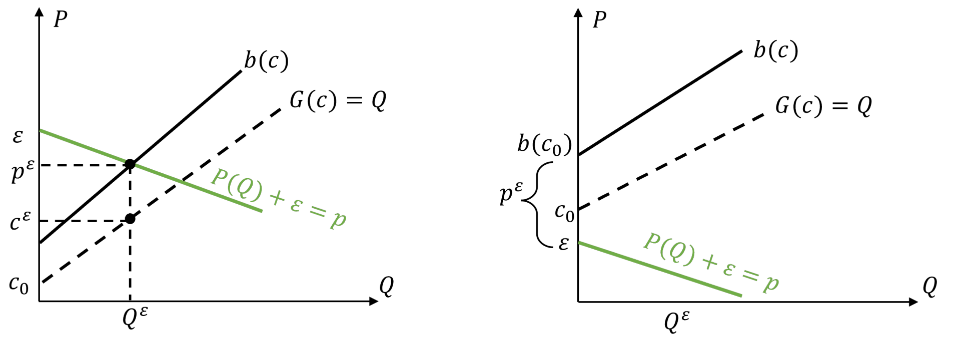

FIGURE 5. Market clearing with realized demand shock ε (left). No market clearing exists with small demand shock ε (right).

2.3. Market clearing

Based on the producers' investment and bidding strategies $\{G(c), b(c)\}$ and after observing the demand shock ε, the auctioneer determines the equilibrium price and aggregate production quantity by clearing the market. This is shown on the left in Figure 5.

The auctioneer clears the market and determines the market-clearing price p^ε, the clearing quantity q^ε, and the marginal technology c^ε by solving:

$$P(q^\varepsilon) + \varepsilon = b(c^\varepsilon) = p^\varepsilon \qquad \text{and} \qquad q^\varepsilon = G(c^\varepsilon).$$

However, for small demand shocks ε (see the right figure in Figure 5), the market does not clear and the production level is zero, $q^\varepsilon = 0$. This happens when the intercept of the demand curve ε is below the intercept of the supply curve $b(c_0)$. That is, the willingness to pay for the first unit of production is lower than the first supply bid. The equilibrium price p^ε is no longer uniquely defined but lies between the intercepts: $p^\varepsilon \in (\varepsilon, b(c_0)]$.

For convenience, from now on, instead of indexing different states of the world by the demand shock ε, we index by the marginal power plant c. Mathematically, the market clearing condition in the state where producers with technology c are marginal then becomes

$$p(c) = b(c) = P(G(c)) + \varepsilon(c), \tag{2}$$

where the function $\varepsilon(c)$ describes the size of the demand shock when technology c is marginal. We define the technology $\underline{c}$ as the technology which is marginal at the lowest possible demand shock, $\varepsilon(\underline{c}) = \underline{\varepsilon}$ and $\bar{c}$ as the technology which is marginal at the highest demand shock $\bar{\varepsilon}$, i.e. $\varepsilon(\bar{c}) = \bar{\varepsilon}$.

A producer with technology c will be selected by the auctioneer to produce when the realized demand shock ε is larger than the shock where its technology is marginal: $\varepsilon > \varepsilon(c)$. The capacity factor $h(c)$ of a producer with technology c

is the likelihood of being selected to produce and is given by

$$h(c) = \Pr[\varepsilon > \varepsilon(c)] = 1 - F(\varepsilon(c)). \tag{3}$$

We can combine the two market clearing conditions in equations (2) and the capacity factor (3) in order to link the bid $b(c)$ to the production level $G(c)$ and the capacity factor $h(c)$:

$$b(c) = P(G(c)) + \mathcal{Q}(1 - h(c)). \tag{4}$$

We use $\underset{\sim}{c}$ and $\tilde{c}$ to refer to the range of technologies that are relevant for market clearing, given demand shock distribution, bids and investment levels. It is the intersection of technologies with positive investments $[c_0, \hat{c}]$ and the interval of technologies that are marginal with positive probability, given the market-clearing process:

$$[\underset{\sim}{c}, \tilde{c}] = [c_0, \hat{c}] \cap [\underline{c}, \bar{c}].$$

In a uniform-price auction, all producers receive the market-clearing price when producing and a producer with technology c collects an expected revenue R^U of

$$R^U(c) = \int_c^\infty b(t)dh(t), \tag{5}$$

while in a discriminatory-price auction, each producer simply sells at its own bid and the producer with technology c receives an expected revenue R^D of

$$R^D(c) = b(c) \cdot h(c). \tag{6}$$

The total expected profit per unit of installed capacity of a producer with technology c is then:

$$\pi(c) = R(c) - c \cdot h(c) - k(c). \tag{7}$$

It is equal to expected revenue minus expected operating costs and investment costs.

The equilibrium price p is stochastic. Let $Z(p)$ be the cumulative distribution function of the price. This price distribution depends on aggregate investment $G(c)$, bids by producers $b(c)$ and the market clearing condition (Equation (2)). It is determined implicitly by the following identity:

$$Z(p(c)) = F(\varepsilon(c)). \tag{8}$$

The consumer surplus in the state ε where technology c is marginal in both auction formats is given by

$$CS^U(c) = V(G(c), \varepsilon(c)) - b(c)G(c),$$
$$CS^D(c) = V(G(c), \varepsilon(c)) - \int_{\underset{\sim}{c}}^{c} b(c)dG(c),$$

and expected consumer surplus is equal to

$$E(CS(c)) = \int_0^\infty CS(c)dh(c).$$

Given the producers' investment and bidding strategies $\{G(c), b(c)\}$, Equations (2) to (8) describe market outcomes, expected profits, and the price distribution.

2.4. Monopolistic Competition

We now formally define the monopolistic competition equilibrium. We assume all producers are risk-neutral, maximize their expected profits in the equilibrium and can freely enter the market.

DEFINITION 1 (Monopolistic Competition). *The set of functions $G(c)$, $b(c)$, $\varepsilon(c)$, $p(c)$ and $Z(p)$ constitutes the* **long-run investment equilibrium**, *if the following three conditions are satisfied.*

(i) market clearing: *prices $p(c)$, demand shocks $\varepsilon(c)$ and the price distribution $Z(p)$ are consistent with the market clearing (Equations (2) and (8) are satisfied).*
(ii) short-run optimum: *taking the price distribution $Z(p)$ as given, a producer with marginal cost c finds it optimal to bid $b = b(c)$;*
(iii) long-run optimum: *a producer with marginal cost c makes zero expected profit $\pi(c) = 0$;*

The set of functions $b(c)$, $\varepsilon(c)$, $p(c)$ and $Z(p)$ constitutes a **short-run bidding equilibrium** *for an exogenous level of investments $G(c)$ if the first two conditions are satisfied.*

The short-run optimality condition (ii) is different for discriminatory and uniform-price auctions. In discriminatory-price auctions, when a producer bids b, it earns a markup $b - c$ and is selected with a probability $Z(b)$. The competitive bid should satisfy:

$$b^D(c) = \arg\max_b (b - c)(1 - Z(b)), \tag{9}$$

In uniform-price auctions, when a producer bids b, it imposes a markup $p - c$ as long as its bid is below the price $p \geq b$. The competitive bid should satisfy:

$$b^U(c) = \arg\max_b \int_b^\infty (p - c)dZ(p). \tag{10}$$

3. Analysis

Now we set out to examine market equilibrium outcomes under the two alternative auction designs. We first start with the short-run competitive bidding equilibrium.

3.1. Bidding equilibrium

Lemma 1 (DPA short-run optimal bid; Fedrico and Rahman 2003). *In discriminatory-price auctions, a producer with technology c charges a markup that is equal to the inverse hazard rate of the equilibrium price distribution:*

$$b(c) = c + \frac{1 - Z(b(c))}{Z'(b(c))}. \tag{11}$$

Proof. Taking the first-order condition of Equation (9) w.r.t. $b(c)$, we obtain the optimal bidding strategy in Lemma 1. □

Relating to single-unit auctions, the discriminatory-price auction is similar to the first-price auction. By choosing $b(c)$, producers are faced with a trade-off between the markup on cost and the probability of getting accepted. Bidding higher increases earnings but lowers the likelihood of the offer being accepted. The optimal bidding strategy implies that all producers with marginal cost $c \in [\underset{\sim}{c}, \tilde{c}]$ set positive markups in discriminatory-price auctions, except for the marginal producer that corresponds to the highest demand realization, who will set the price exactly at the marginal cost $(1 - Z(b(\bar{c})) = 0)$, unless capacity is scarce in which case $Z' = 0$ and we might still have a positive mark-up.

However, the price distribution $Z(p)$ is endogenous and depends on the bid function $b(c)$. Combining the inverse hazard rate formula and the market clearing conditions, we find Proposition 1.

Proposition 1 (DPA short-run optimal bidding strategy). *In a discriminatory-price auction, the equilibrium bid $b(c)$ and the capacity factor $h(c)$ need to satisfy the differential equation*

$$h(c) = -\frac{d}{dc}\left[(b(c) - c)h(c)\right], \tag{12}$$

together with market clearing (4). The boundary condition that determines the solution to this differential equation depends on the installed capacity $\hat{G}$. If capacity is plentiful, then the markup for the highest demand shock is zero, and the highest active cost technology $\tilde{c}$ then clears the market

$$b(\tilde{c}) = \tilde{c} = P(G(\tilde{c})) + \bar{\varepsilon}.$$

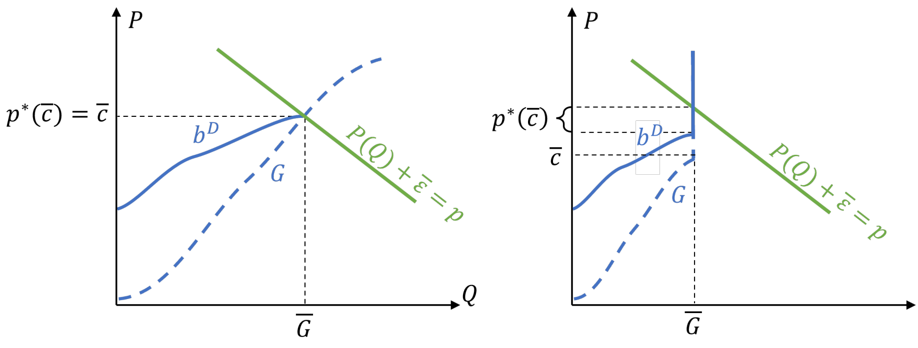

FIGURE 6. The optimal short-run bid under discriminatory-price auctions with spare capacity (left) and scarce capacity (right).

If capacity is scarce, then the markup at the highest demand shock is positive, and the highest bid is

$$b(\hat{c}) = \arg\max_b [1 - F(b - P(\hat{G}))](b - \hat{c}).$$

The functions $\{b(c), h(c)\}$ that satisfy the first order condition 12 and the boundary condition are an equilibrium if $h'(c) \leq 0$.

Proof. See the proof in the Appendix. The first-order condition (12) follows directly from applying the envelope theorem in a direct revelation formulation of the bidding process. A revealed preference argument shows that $h' \leq 0$ is a necessary condition for an equilibrium to exist. The fact that a solution of the first-order condition is globally optimal if $h' \leq 0$, can be proven by showing that profits are weakly increasing for smaller bids and weakly decreasing for larger bids. The boundary condition requires that there is no markup for the highest demand shock with spare capacity. Competition among marginal power plants drives down the markup. When there is scarce capacity, there is no cut-throat competition and the bids reflect a trade-off between mark-ups and capacity factor. See Figure 6. □

The next proposition directly follows from the first-order condition of the profit function for uniform-price auctions (Equation (10)).

PROPOSITION 2 (UPA optimal bidding strategy). *In a uniform-price auction, the optimal bid $b(c)$ is equal to marginal cost, and the capacity factor $h(c)$ is determined by equation (4).*

Proof. The proof can be derived similarly to discriminatory-price auctions. The first-order condition implies that bidding marginal cost is optimal. This can be shown to be always globally optimal. For further details, refer to the proof in the Appendix. □

If we relate to single-unit auctions, uniform-price auctions are similar to second-price auctions. Producers are paid the price of the first rejected bid, rather than the amount that they bid themselves. Bidding higher does not directly reduce the earnings but lowers the likelihood of winning the auction. So the optimal bidding strategy requires that all producers bid exactly their marginal costs. In this way, producers with a lower marginal cost will be first assigned to operate, till the last unit that has a marginal cost equal to the consumer's marginal benefit. Uniform-price auctions are efficient: the most efficient capacity is used first (production efficiency) and the willingness to pay for consumers is equal to the marginal cost of the most expensive generator (allocative efficiency).

3.2. Investment equilibrium

We need the following lemma to construct the investment equilibrium.

Lemma 2 (Local incentive constraint). *Independent of the auction format, the capacity factor satisfies the free entry condition. This implies*

$$h(c) = -\frac{dk(c)}{dc}. \tag{13}$$

Proof. The proof follows from combining the envelop theorem on long-run expected profits and the free entry condition of Definition 1, condition (iii). See the proof in the Appendix. □

This lemma shows that the technology mix is conditionally efficient in the long run, given the capacity factor $h(c)$ of each technology. The derivations in the proof are similar to a screening model where the capacity factor h is used to screen technologies c. Free entry then guarantees that all information rents are zero.

In the following propositions, we characterize the investment equilibrium for the two alternative auction designs respectively.

Proposition 3 (DPA investment equilibrium). *In the investment equilibrium of a discriminatory-price auction, producers with marginal cost $c \in [\underset{\sim}{c}, \tilde{c}]$ invest. The cumulative installed capacity $G(c)$ satisfies*

$$P(G(c)) = c + \frac{k(c)}{h(c)} - \mathcal{Q}(1 + k'(c)), \tag{14}$$

and the bid follows

$$b(c) = c + \frac{k(c)}{h(c)}. \tag{15}$$

Proof. We combine Equation (12) with the free entry condition (13) $h(c) = -k'(c)$ and find:

$$\frac{d}{dc}\left[(b(c)-c)k'(c)\right] = -k'(c).$$

With sufficient capacity, integrating this differential equation over the interval $[c, \tilde{c}]$, and using the boundary condition in Proposition 1, we find that the bidding markup is equal to $-k(c)/k'(c)$:

$$b(c) = c - \frac{k(c)}{k'(c)} = c + \frac{k(c)}{h(c)}.$$

Plugging the optimal bid into the market-clearing condition (2), we obtain the equation that pins down the cumulative installed capacity $G(c)$ in Equation (14). □

We investigate several properties of the equilibrium. First, producers make markups that are decreasing in their marginal costs. This can be seen by combining the derivative of Equation (15) together with the properties of the technology frontier in Assumption 2. The slope of the bidding function

$$b' = \frac{k''k}{k'^2}$$

is between zero and one. Bids increase with the marginal cost, which guarantees that the producers are activated in accordance with their merit order. Second, all technologies will be used in equilibrium, following Assumption 2, that is, G is strictly increasing: $G'(c) > 0 \; \forall c \in [\underline{c}, \tilde{c}]$. To see this, recall that the log-concavity property of the technology frontier in Assumption 1 implies

$$\frac{1}{k''} > \frac{k}{k'^2}.$$

Taking the derivative of (12), and using the fact that $P' < 0$, we see that $G' > 0$ requires

$$\mathcal{Q}'(1+k') > \frac{k}{k'^2},$$

which is satisfied thanks to Assumption 2.

The following lemma follows directly from Proposition 3.

Lemma 3. *The Lerner index in the long-run equilibrium under discriminatory-price auctions is the reciprocal of the elasticity $\varepsilon_k(c)$ of investment costs:*

$$L = \frac{b(c)-c}{c} = \frac{1}{\varepsilon_k(c)}.$$

The Lerner index indicates that producers can charge more than their marginal cost in the competitive equilibrium. However, the markup is not due to demand elasticity, but demand uncertainty. In the long run, it allows for a recoupment of the investment costs.

Proposition 4 (UPA investment equilibrium). *In the investment equilibrium of a uniform-price auction, producers with marginal costs $c \in [\underset{\sim}{c}, \tilde{c}]$ invest. The cumulative installed capacity $G(c)$ satisfies*

$$P(G(c)) = c - \mathcal{Q}(1 + k'(c)). \tag{16}$$

A producer with technology c bids

$$b(c) = c.$$

Proof. The optimal bid follows directly from the first-order condition of Equation (10). Plugging the optimal bid into the market clearing condition (2), we obtain the equation that pins down the cumulative installed capacity $G(c)$ in Equation (16). To show that all technologies that are used, check that $G(c)$ is strictly increasing:

$$G' > 0 \Leftrightarrow \mathcal{Q}'(1 + k') > \frac{1}{k''}.$$

This is guaranteed by Assumption 2. □

3.3. Comparison

After characterizing the short-run and long-run equilibrium outcomes, we now compare both auction formats. In the *short run*, the auction format only affects bidding strategies $b(c)$ as installed capacities $G(c)$ remain fixed. For a given demand realization ε and auction format $i = D, U$, let $p^{\varepsilon,i}$ and $q^{\varepsilon,i}$ represent the equilibrium price and quantity, respectively:

$$p^{\varepsilon,i} = b^i(c^{\varepsilon,i}), \qquad q^{\varepsilon,i} = G(c^{\varepsilon,i}),$$

where $c^{\varepsilon,i}$ is the marginal technology, defined by a one-to-one relationship between demand shocks and marginal technologies $\varepsilon = \varepsilon^i(c^{\varepsilon,i})$. This is illustrated in Figure 7. We can now show the following lemma.

Lemma 4 (Comparison of short-run equilibrium outcomes). *In the short run, for a given demand realization ε, the marginal technology in the discriminatory-price auction is lower than in the uniform-price auction, the equilibrium price is higher, and the equilibrium production quantity is lower:*

$$c^{\varepsilon,D} \leq c^{\varepsilon,U}, \qquad p^{\varepsilon,D} \geq p^{\varepsilon,D} \quad \textit{and} \quad q^{\varepsilon,D} \leq q^{\varepsilon,D}.$$

The inequalities are strict, except for the largest demand shock $\varepsilon = \bar{\varepsilon}$ when spare capacity exists.

Proof. See Appendix. □

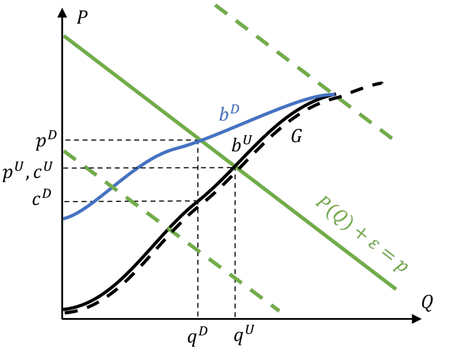

FIGURE 7. Short-run comparison of bidding between discriminatory and uniform-price auctions with spare capacity.

A direct result of this lemma is that the equilibrium capacity factor $h(c) = 1 - F(\varepsilon((c))$ in a discriminatory-price auction is lower than that in a uniform-price auction $h^D(c) < h^U(c)$ for any technology c, except for the technology that is marginal at the largest demand shock when there is spare capacity.

Figure 8 represents consumer surplus (blue), producer surplus (yellow), and deadweight loss (grey) for a given demand shock ε. There is a deadweight loss in the discriminatory-price auction as there is a gap between the marginal willingness to pay and the marginal cost $p^D_\varepsilon > c^D_\varepsilon$. As total surplus decreases, at least one side of the market must be worse off in the discriminatory-price auction.

LEMMA 5 (Comparison of short-run expected profits). *In the short run, the expected profits of generators of technologies c that are active in the market are smaller in the discriminatory-price auction.*

Proof. See Appendix. □

We need to specify particular functional forms for installed capacity G, demand shock distribution F, and demand function P to make more precise statements about expected consumer surplus. There is a trade-off: transitioning to a discriminatory-price auction reduces the expected producer surplus and creates a deadweight loss. Consumers could either benefit or lose out. Fedrico and Rahman (2003) demonstrate that under the assumptions of uniform demand shock distribution $F(\varepsilon)$, linear investment function $G(c)$ and linear demand $P(q)$, expected consumers surplus is higher and the average paid price by consumers, defined as expected expenditure divided by expected consumption, is lower.

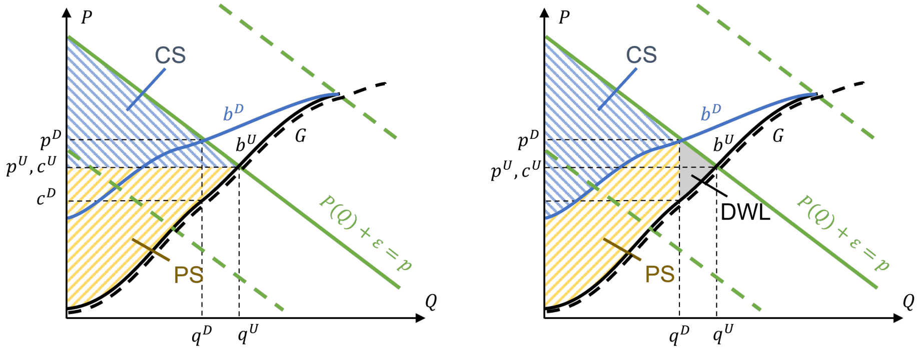

FIGURE 8. Comparison of short-run equilibrium revenue under uniform (left) and discriminatory-price auctions (right) for a specific demand shock ε. The Figure shows a case with spare capacity.

In the *long run*, auction formats affect both bidding decisions $b(c)$ and investment $G(c)$. The following lemma compares both market outcomes.

LEMMA 6 (Comparison of long-run investment). *In the long-run equilibrium, the capacity factor h is the same in both auction formats: $h^U(c) = h^D(c)$. The aggregate investments in uniform-price auctions first-order stochastically dominate investments in discriminatory-price auctions, except for the technology at the top, that is,*

$$G^D(c) \leq G^U(c) \text{ with equality for } c = \bar{c}.$$

Moreover, if the demand function is concave $P'' < 0$, then the marginal investment in each technology $G'(c)$ is larger in the discriminatory-price auction

$$G^{D\prime}(c) \geq G^{U\prime}(c).$$

Proof. See Appendix. □

Figure 9 illustrates this lemma. For a given demand shock ε, the same technology $c^D(\varepsilon) = c^U(\varepsilon) = c$ is marginal. The bids are $b^U(c) = c$ and $b^D(c) = c + k(c)/h(c)$ for uniform and discriminatory-price auctions, respectively.

LEMMA 7. *In the long run, the discriminatory-price auction is less efficient than the uniform-price auction and the expected consumer surplus is lower.*

$$E(CS^D(c)) < E(CS^U(c)).$$

Consumers are better off with the discriminatory-price auction with the highest demand shock $\bar{c}$, but lose out for the lowest demand shock $\underline{c}$.

$$CS^D(\bar{c}) > CS^U(\bar{c}), \quad CS^D(\underline{c}), < CS^D(\underline{c})$$

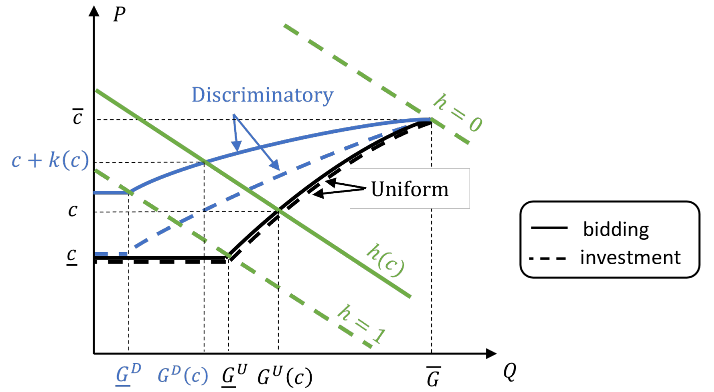

FIGURE 9. Long-run comparison of bidding (solid) and portfolios (dashed) between discriminatory auctions and uniform-price auctions. Note: for a given demand shock ε, the same technology is marginal.

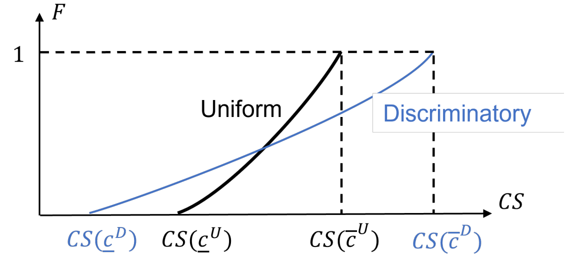

FIGURE 10. Long-run consumer surplus comparison: uniform and discriminatory-price auctions.

Proof. From the standard welfare theorem, we know that a uniform-price auction in a competitive market leads to an efficient market outcome. It generates the standard peak-load pricing as in Boiteux (1960). In the discriminatory-price auction prices and investment levels are distorted, hence total expected surplus must be lower. By the free entry assumption, producers make zero expected profit in both auction formats $\pi^D(c) = \pi^U(c) = 0$, so the expected consumer surplus in the discriminatory-price auction must be lower. For the highest demand shocks, the price in both auction formats is the same $b^U(\hat{c}) = b^D(\hat{c}) = \hat{c}$, and consumers consume the same amount $G(\hat{c})$, but the payment is lower in the discriminatory-price auction. So consumer surplus must be larger. For the lowest demand shock, consumer surplus is lower with the discriminatory pricing as consumers pay a price $b^D(\underset{\sim}{c}) > \underset{\sim}{c}$ for all units consumed, while paying less $b^U(\underset{\sim}{c}) = \underset{\sim}{c}$ in the uniform price auction. $\square$

Figure 10 sketches the cumulative density function of consumer surplus for both auction formats. For small demand shocks, discriminatory-price auctions result in lower consumer surplus due to reduced base load investment and higher prices compared to uniform-price auctions. Conversely, discriminatory-price auctions lead to higher consumer surplus for high demand realizations. In this case, consumers pay average bids that are lower than the market clearing price in uniform-price auctions, with identical volume in both formats. It is worth noting that the variance of consumer surplus is higher under discriminatory-price auctions compared to uniform-price auctions.

Additionally, it is important to note that the revenue equivalence theorem does not hold in our model. Although the rents of producers are the same in both auctions (zero due to free entry), the market surplus differs, and hence, the auctioneer who acts on behalf of consumers, is not indifferent. In our setting, demand is elastic, and the total size of the "cake" (total surplus) therefore changes with the auction format. Consequently, the two auction formats do not yield the same expected expenditure. In Section 4.1 we will explore different setups with inelastic demand or without demand uncertainty where the revenue equivalence theorem holds.

3.4. Example

In this subsection, we consider particular functional forms and derive long-term market outcomes. The demand function is linear $P(q) = -q/\rho$ with $\rho > 0$, and the stochastic demand intercept ε follows a truncated exponential distribution

$$F(\varepsilon) = \frac{\alpha - e^{-\varepsilon/\lambda}}{\beta}$$

on the interval $[\underset{\sim}{\varepsilon}, \bar{\varepsilon}]$ with lower and upper boundaries $\underset{\sim}{\varepsilon} = -\lambda \ln(\alpha)$ and $\bar{\varepsilon} = -\lambda \ln(\alpha - \beta)$ and the quantile function: $\mathcal{Q}(y) = -\lambda \ln(\alpha - \beta y)$. The demand side depends on four parameters: ρ which measures demand slope, α and β which determine the range of demand shocks and λ which measures the thickness of the right tail of the shock distribution.

The convex investment cost function is defined by

$$k(c) = \frac{\hat{c} - \underset{\sim}{c}}{\gamma + 1} \left(\frac{\hat{c} - c}{\hat{c} - \underset{\sim}{c}} \right)^{\gamma+1} \quad \text{with } \gamma > 0,$$

satisfying Assumption 1. It has three parameters: $\hat{c}$ the value of lost load, $\underset{\sim}{c}$ the marginal cost of the always-on technology, and γ the convexity parameter. The optimal portfolio choice, $h(c) = -k'(c)$, determines that the long-term capacity factor of technology c as :

$$h(c) = \left(\frac{\hat{c} - c}{\hat{c} - \underset{\sim}{c}} \right)^{\gamma}.$$

The demand intercept $\varepsilon(c) = \mathcal{Q}(1-h(c))$ that corresponds to the marginal technology c is

$$\varepsilon(c) = -\lambda \ln\left(\alpha - \beta\left[1 - \left(\frac{\hat{c}-c}{\hat{c}-\underset{\smile}{c}}\right)^{\gamma}\right]\right).$$

The long-run aggregate investment, $G^i(c)$, can be calculated by

$$G^i(c) = \rho(\varepsilon(c) - b^i(c)),$$

where the bids $b^i(c)$ are given by:

$$b^U(c) = c \quad \text{and} \quad b^D(c) = c + \frac{\hat{c}-c}{\gamma+1}.$$

If Assumption 2 is satisfied all technologies will be used, $G' > 0$, this is, for instance, the case if $\lambda > (\hat{c} - \underset{\smile}{c})^2/\gamma^2$ and the shock follows a standard exponential distribution, i.e. $\alpha = \beta = 1$ on the interval $[0, \infty]$.

4. Extensions

In Section 3, we compared the equilibrium outcomes of both auction formats assuming demand to be price-responsive and uncertain. However, in this section, we show that disparities in market outcomes vanish when we drop one of these assumptions.

4.1. Inelastic demand

With perfectly inelastic stochastic demand, represented by an inverse demand function with a stochastic price shock $p = P(q) + \varepsilon$ as in Equation (1), is no longer applicable. Instead, we express the demand q by a stochastic quantity variable θ, that is, $q = \theta$. The quantity shock is characterized by a cumulative distribution $F_q(\theta)$, a quantile function $\mathcal{Q}_q(\cdot)$, and has full support on the interval $[\underline{\theta}, \bar{\theta}]$. Market clearing is now determined by the equation

$$\theta(c) = G(c).$$

The capacity factor of technology c is given by

$$h(c) = 1 - F_q(\theta(c)).$$

The function $\theta(c)$ corresponds to the level of demand at which technology c becomes marginal.

The short-term bidding and long-term investment behaviors are described in the next proposition and can be derived as previously discussed.

Proposition 5 (Equilibrium with inelastic demand). *In the short term, i.e. for a given investment pattern $G(c)$, the optimal bids under uniform and discriminatory-price price auctions are as follows $\forall c \in [\underset{\sim}{c}, \tilde{c}]$:*

$$b^U(c) = c,$$
$$b^D(c) = c + \frac{\int_c^{\tilde{c}} 1 - F_q(G(t))dt}{1 - F_q(G(c))}. \tag{17}$$

In the long term, the installed capacity G is the same in both auction formats, and corresponds to the generation portfolio that minimizes total production costs:

$$G^{UP}(c) = G^D(c) = \mathcal{Q}_q(1 + k'(c)).$$

Producers invest in a mix of technologies ranging from the always-on technology $\underset{\sim}{c}$ to the VOLL (Value of Lost Load) technology $\hat{c}$, which are marginal for the lowest demand shock $\underline{\theta}$ and the highest demand shock $\bar{\theta}$, respectively. Therefore, $\bar{c} = \hat{c}$ and $\underline{c} = \underset{\sim}{c} = c_0$. In the long term, the equilibrium bid in discriminatory-price auctions corresponds to levelized cost:

$$b^D(c) = c + \frac{k(c)}{h(c)}.$$

Proof. As previously stated, in a uniform-price auction, the short-run optimal bid is to bid marginal cost, as it remains a dominant strategy. Under discriminatory-price auctions, the short-run optimal bid can be determined by solving the differential equation from Lemma 1:

$$b(c) = c + \frac{1 - Z(b(c))}{Z'(b(c))} = c + \frac{1 - F_q(G(c))}{F_q'(G(c))G'(c)} b'(c),$$

which gives Equation (17). □

Figure 11 illustrates the investment and bidding equilibria with inelastic but stochastic demand, ranging from $\underline{\theta}$ to $\bar{\theta}$. Bids in discriminatory-price auctions are higher than in uniform-price auctions, but the investment levels $G^U(c) = G^D(c)$ are identical.

Although markups are different in the two auction formats, with inelastic demand they do not distort demand levels. Consequently, the optimal generation mix must be the same in both auction formats. This optimal generation mix corresponds to the least-cost investment to meet the (unchanged) stochastic demand levels.

With inelastic demand, there are no deadweight losses, and the total surplus is the same in both auction formats. Given free entry, the expected producer surplus is zero, and the expected consumer surplus must be the same as well. Hence, the revenue equivalence theorem holds in the case of inelastic demand.

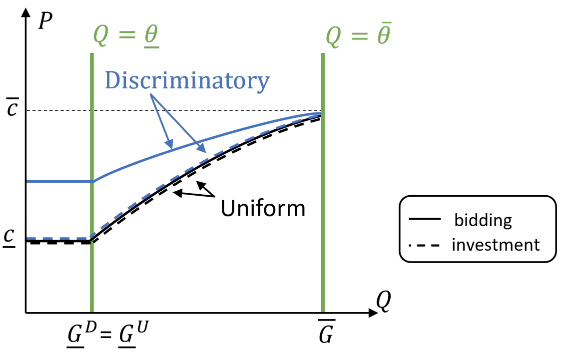

FIGURE 11. Long-run bidding strategy under discriminatory auctions and uniform-price auctions with inelastic demand.

LEMMA 8 (Revenue Equivalence). *With perfectly inelastic demand, consumers receive the same expected pay-off under both auction types.*

Proof. See Appendix. □

4.2. Certain demand

We now consider the scenario where the demand shock is known. The inverse demand function is given as

$$p(q) = P(Q) + \bar{\underline{\varepsilon}},$$

where $\bar{\underline{\varepsilon}}$ is a constant.

In the short run, the installed capacity $G(c)$ is fixed, and producers compete by submitting bids. The short-term equilibrium strategies are not unique, as several strategy profiles can lead to the same market outcome. The following proposition describes the short-term market equilibrium outcome:

PROPOSITION 6 (Short-run equilibrium outcome with certain demand). *Define the technology $\mathring{c}$, at which a perfectly competitive market would clear as:*

$$\mathring{c} = P(G(\mathring{c})) + \bar{\underline{\varepsilon}}. \tag{18}$$

Under both auction formats, any equilibrium outcome is characterized by the fact that producers with lower marginal cost $c \leq \mathring{c}$ produce and are paid $\mathring{c}$, while producers with higher marginal cost $c > \mathring{c}$ do not produce and thus not receive a payment.

In the long-run equilibrium, outcomes under the two auction formats are equivalent. Producers only invest in the base-load technology $\underline{c}$, which is the

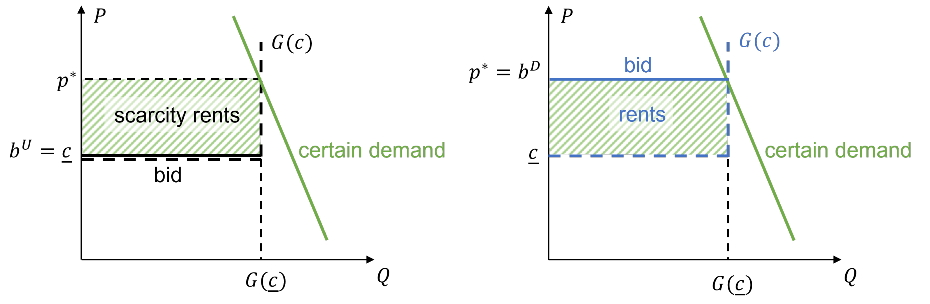

FIGURE 12. Long-run optimal bidding and investment strategies with certain and elastic demand under uniform auctions (left) and discriminatory-price auctions (right).

most efficient technology to meet a deterministic demand level. Since the demand is certain, there is no need to use a portfolio of generation technologies. Producers bid their marginal cost $b = \underline{c}$ under uniform-price auctions, and levelized cost $b = \underline{c} + k(\underline{c})$ under discriminatory-price auctions and earn a short-term rent of $k(\underline{c}) \cdot G(\underline{c})$, which is used to recoup the investment cost. This is illustrated in Figure 12, and formalized in the following Lemma.

LEMMA 9. *With a known demand shock, the long-term equilibrium outcomes for uniform and discriminatory-price auctions are equivalent. The equilibrium price reflects the levelized cost of the base-load technology.*

Proof. Proposition 6 demonstrates that short-term outcomes are equivalent in both auction formats for any investment level $G(c)$. This implies that, with the free-entry condition, the long-term equilibrium outcomes for the two auction formats should also be equivalent.

Any investment in a technology $c \neq \underline{c}$ will be unable to recoup its investment cost when the equilibrium price is $p = \underline{c} + k(\underline{c})$. If the price is above this level, there will be new entries in the base-load investment, while if the price is below the level, even base-load technologies would make a loss and leave the market. □

Hence, this shows that when demand is certain, the revenue equivalence holds. Total invested capacity is the same in both auction formats, and as only the always-on technology is used in both formats, there is no distortion in the generation mix.

5. Conclusions

Our paper draws motivation from recent proposals by authorities advocating for the replacement of the current uniform-price auction format within the wholesale electricity market design. The purpose of this work is to address the question of whether adopting discriminatory-price auctions would enhance market performance, and how auction formats would affect electricity producers' long-run and short-run decision-making and distributional effects. We aim to gain an all-around understanding of the impact of auction formats on market outcomes and social welfare.

To the best of our knowledge, there exists limited literature dedicated to modeling multi-unit investment incentives with a variety of technologies in an auction format. To address this gap, we construct a monopolistic competition model featuring a continuum of generation technologies and consider demand responsiveness and stochasticity.

We find that discriminatory-price auctions are inefficient. In the short run, consumers' willingness to pay is higher than producers' marginal costs, and capacity remains underused. In the long run, producers' investment incentives are affected, resulting in a distortion of the generation technology mix. While the total generation capacity remains the same, it becomes harder for "always-on" base-load producers to generate revenue. Consequently, investments in these technologies decrease in the equilibrium to ensure that they can recoup their investment costs.

Our research demonstrates that inefficiency does not necessarily originate from market power, but can also stem from market design. The interplay of demand uncertainty, diverse generation technologies, and demand elasticity disrupts the revenue equivalence theorem.

Policymakers might be tempted to drastically change the spot market design, driven by the short-term goal of reducing producers' revenues and increasing consumer surplus. However, our model reveals that such actions may compromise efficiency in the short run and erode consumer surplus in the long run. Moreover, other authors have shown that discriminatory auctions can unfairly advantage larger bidders with inside information, potentially leading to activations that deviate from the merit order.

In our model, we assume demand to be elastic, which is appropriate for our focus on the long-run market equilibrium. In this context, consumers have the flexibility to easily adjust their consumption patterns. It is worth noting that with perfectly inelastic demand, both uniform and discriminatory price auctions yield identical outcomes, which would not affect our policy recommendation.[4]

Our monopolistic competition model assumes free entry and zero long-run expected profits. This assumption holds reasonable ground, particularly in the European energy market context, provided there is sufficient investment in cross-border capacity to improve competition, and permitting procedures

4. Introducing different short and long-run demand and supply elasticities, is more realistic, but would require a more complex set-up which explicitly models flexibility and inter-temporal arbitrage and additional reserve markets as in He and Willems (2023).

that do not pose entry barriers.[5] While extending our model to an oligopoly setup would be intriguing, it is likely to be analytically intractable. Such an extension would require a two-stage model wherein producers' strategies are represented by functions defining generation portfolios and technology-specific bidding, akin to the approach in Holmberg and Willems (2015). These models are characterized by nested differential first-order conditions and multiple equilibria. In a companion paper, we study a monopoly version of our model where a single firm invests in a portfolio of technologies and participates in the spot market auction. Our findings reveal price-cost mark-ups in both auction formats and suggest that welfare may be higher in a discriminatory-price auction when demand is relatively inelastic and demand uncertainty is relatively high.

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Appendix

Proof of Proposition 1: Short-run optimal bidding in pay-as-bid auction.

Proof. In a pay-as-bid auction, the expected profit of a firm with technology c bidding as if it was of type c_R, i.e., bidding $b(c_R)$, and obtaining load-factor $h(c_R)$, is equal to:

$$\Pi(c_R, c) = (b(c_R) - c) \cdot h(c_R).$$

Applying the envelope theorem on the profit function at $c = c_R$ we find:

$$\frac{d\Pi(c,c)}{dc} = \frac{\partial \Pi(c,c)}{\partial c}$$
$$\Rightarrow \frac{d}{dc}\left[(b(c) - c)h(c)\right] = -h(c). \tag{A.1}$$

The envelope theorem can be applied as the bid function $b(c)$ and corresponding $h(c)$ are chosen optimally by the firm. In particular, once $b(c)$ is chosen the load factor $h(c)$ is given by the expression

$$h(c) = 1 - Z(b(c)), \tag{A.2}$$

which can be found by combining equations (2), (3) and (8).

The explicit form of functions $\{b(c), h(c)\}$ can be pinned down by Equations (A.1) and (A.2) together with a boundary condition for the highest type $\bar{c}$. At the highest demand shock $\bar{\varepsilon}$, the markup is zero and the market always clears:

$$b(\tilde{c}) = \tilde{c} = P(G(\tilde{c})) + \bar{\varepsilon}.$$

The explicit form of functions $\{b(c), h(c)\}$ can be pinned down by Equations (A.1) and (A.2) together with a boundary condition for the highest type $\tilde{c}$. When the capacity is plentiful, the highest type $\tilde{c} = \bar{c}$ is marginal when the demand shock is of the highest type $\bar{\varepsilon}$. At the highest demand shock $\bar{\varepsilon}$, the markup is zero and the market always clears:

$$b(\tilde{c}) = \tilde{c} = P(G(\tilde{c})) + \bar{\varepsilon},$$

which determines the upper bound. When the capacity is scarce, the markup is still positive with the highest demand shock. The highest type $\tilde{c} = \hat{c}$ submits a bid b to maximize $\Pr(b)(b - \hat{c})$. The first-order condition pins down the highest bid that gives the boundary condition for solving the ordinary differential equation:

$$-f(b - P(\hat{G}))(b - \hat{c}) + 1 - F(b - P(\hat{G})) = 0$$
$$\Rightarrow b^* = \hat{c} + \frac{1 - F(b^* - P(\hat{G}))}{f(b^* - P(\hat{G}))}.$$

Since the highest bid b is in the interval $[\hat{c}, P(\hat{G}) + \bar{\varepsilon}]$, to prove the uniqueness of the highest bid, we examine the monotonicity of the first-order condition. Denote the first-order condition by $g(b)$:

$$\begin{aligned} g(b) &= -f(b - P(\hat{G}))(b - \hat{c}) + 1 - F(b - P(\hat{G})) \\ &= -\frac{1 - F(b - P(\hat{G}))}{f(b - P(\hat{G}))}(b - \hat{c}) + 1, \end{aligned}$$

and then

$$g'(b) = -\Big(\frac{d}{db}\frac{1 - F}{f}\Big)(b - \hat{c}) - \frac{1 - F}{f}.$$

Decreasing hazard rate $\frac{1-F}{f}$ guarantees that the first-order condition is decreasing, implying that b^* is the unique optimum bidding for the highest type of technology $\hat{c}$.

After deriving the first-order condition for the bid functions, we now study the global optimality. The marginal effect on the expected profit for a firm with technology c when bidding as if its technology was c_R is:

$$\frac{\partial \Pi(c_R, c)}{\partial c_R} = b'(c_R)h(c_R) + (b(c_R) - c)h'(c_R). \tag{A.3}$$

The bid and load duration $\{b(c_R), h(c_R)\}$ satisfy the first order condition ((12)) of the firm with technology c_R:

$$\left.\frac{\partial \Pi(c_R, c)}{\partial c_R}\right|_{c=c_R} = b'(c_R)h(c_R) + (b(c_R) - c_R)h'(c_R) = 0. \tag{A.4}$$

Subtracting (A.4) from (A.3) gives:

$$\frac{\partial \Pi(c_R, c)}{\partial c_R} = (c_R - c)h'(c_R).$$

To ensure global optimality, we need

$$\frac{\partial \pi(c_R, c)}{\partial c_R} \geq 0 \quad \forall c_R < c, \text{ and } \frac{\partial \pi(c_R, c)}{\partial c_R} \leq 0 \quad \forall c_R > c,$$

which requires $h'(c_R) < 0$ for all c_R. The solution of the differential equation (12) satisfies this condition if the bid function is increasing and the markup is positive:

$$\frac{-h'(c)}{h(c)} = \frac{b'(c)}{b(c) - c} > 0, \tag{A.5}$$

which follows from rewriting the differential equation.

A necessary condition for any bidding equilibrium is that $h'(c) \leq 0 \; \forall c$. This follows from the incentive compatibility constraints:

$$(b(c_1) - c_0)h(c_1) \leq (b(c_0) - c_0)h(c_0) \tag{A.6}$$

$$(b(c_0) - c_1)h(c_0) \leq (b(c_1) - c_1)h(c_1), \tag{A.7}$$

which can be combined to find that

$$(c_0 - c_1)(h(c_0) - h(c_1)) \leq 0.$$

Hence, the monotonicity of $h(c)$ is a necessary condition for equilibrium and a sufficient condition for a solution of the first-order condition to be an equilibrium.

□

Proof of Proposition 2: Optimal bidding strategy in a uniform-price auction.

Proof. Suppose the report by a firm with technology c is c_R. The expected profit of the firm is

$$\Pi(c_R, c) = \int_{b(c_R)}^{\infty} (p - c) dZ(p).$$

The marginal effect on the expected profit of a firm with technology c to bid, as if it had technology c_R is given by:

$$\frac{\partial \Pi(c_R, c)}{\partial c_R} = -(b(c_R) - c)Z'(b(c_R)).$$

The first-order condition requires that this expression is zero at $c_R = c$, implying that the producer would truthfully report its marginal cost $b(c) = c$. To ensure global optimality, we need

$$\frac{\partial \Pi(c_R, c)}{\partial c_R} > 0 \quad \forall c_R < c, \text{ and } \frac{\partial \Pi(c_R, c)}{\partial c_R} < 0 \quad \forall c_R > c,$$

which requires $Z'(b(c_R)) > 0$ for all c_R. Given that $b'(c_R) = 1$, and using the definition of Z, this is the case if and only if: $h'(c_R) < 0$. This is satisfied when firms bid their marginal cost. To prove this, we take derivatives w.r.t. c on both sides of the market-clearing condition ((4)) $b(c) = c = P(G(c)) + \mathcal{Q}(1 - h(c))$ and find that

$$h'(c) = \frac{P'(G(c))G'(c) - 1}{\mathcal{Q}'(1 - h(c))} < 0.$$

This condition is always true and guarantees that the bidding strategy is globally optimal. □

Proof of Lemma 2: Optimal capacity factor and free entry condition

Proof. Recall that the profit of a producer with technology c is given by

$$\pi(c) = R(c) - c \cdot h(c) - k(c).$$

Taking the total differential, we have

$$\frac{d\pi}{dc} = \frac{\partial \pi}{\partial R} \cdot R'(c) + \frac{\partial \pi}{\partial h} \cdot h'(c) + \frac{\partial \pi}{\partial k} \cdot k'(c) + \frac{\partial \pi}{\partial c}. \tag{A.8}$$

By the envelope theorem the sum of the first two terms on the right-hand side of Equation (A.8) equals zero, as the bidding strategy is chosen optimally. In the long run, all generators earn zero profit due to the free entry:

$$\pi(c) = 0 \Rightarrow \frac{d\pi}{dc} = 0$$

Hence,

$$\frac{\partial \pi}{\partial k} \cdot k'(c) + \frac{\partial \pi}{\partial c} = 0 \;\Rightarrow\; h(c) = -\frac{dk(c)}{dc}.$$

□

Proof of Lemma 4

Proof. For a given technology c, bids in the discriminatory auction are larger than in the uniform-price auctions: $b^D(c) \geq b^U(c)$. It follows from the market clearing condition $b^i(c) = P(G(c)) + \varepsilon^i(c)$ that $\varepsilon^D(c) \geq \varepsilon^U(c)$. Since $\varepsilon^i(\cdot)$ are increasing functions, the marginal technology in the discriminatory auction is smaller than in the uniform-price auction, i.e., $c^{\varepsilon,D} \leq c^{\varepsilon,U}$. Consequently, production capacity is lower in the discriminatory auction, i.e., $q^{\varepsilon,D} = G(c^{\varepsilon,D}) \leq G(c^{\varepsilon,U}) = q^{\varepsilon,U}$. This implies higher prices in the discriminatory auction, ($p^{\varepsilon,D} > p^{\varepsilon,D}$) to compensate for reduced production. □

Proof of Lemma 5

Proof. For the highest demand shock $\bar{\varepsilon}$ and with spare capacity, the marginal technology is the same in both auctions as shown in Figure 8: $\bar{c}^D = \bar{c}^U = \bar{c}$. Firms make the same expected profit in both formats, as they have a zero load factor $h^D(\bar{c}) = h^U(\bar{c}) = 0$.

$$\pi^D(\bar{c}) = \pi^D(\bar{c}) = -k(\bar{c}).$$

Using the envelope theorem, the total derivative of expected profit in auction i is given by $d\pi^i/dc = -h^i(c) - k'(c)$. So we can write the difference in profits for any technology $c < \bar{c}$ as

$$\pi^U(c) - \pi^D(c) = \int_c^{\bar{c}} (h^U(t) - h^D(t))dt > 0.$$

The last inequality follows from the fact that the load factor is higher with uniform price auctions $h^U(c) > h^D(c)$. Firms that are not active in the market $(c > \bar{c})$, do not have any short-term profit as they do not produce and hence $\pi^D(c) = \pi^U(c) = -k(c)$.

If capacity is scarce the expected profit of the highest marginal cost technology $\hat{c}$ is not zero, but given by the following conditions:

$$\pi^D(\hat{c}) = -k(\hat{c}) + \max_{\varepsilon}[(\varepsilon - P(\hat{G}) - \hat{c})(1 - F(\varepsilon))],$$
$$\pi^U(\hat{c}) = -k(\hat{c}) + \int_{\hat{c}-P(\hat{G})}^{\bar{\varepsilon}} (\varepsilon - P(\hat{G}) - \hat{c})dF(\varepsilon).$$

The expected profit in the discriminatory auction is lower than in the uniform one: $\pi^D(\hat{c}) < \pi^U(\hat{c})$. In the uniform price auction the firm captures the consumers' marginal willingness-to-pay $(\varepsilon - P(\hat{G}))$ for each demand shock ε. In the discriminatory auction, the firm produces less often and does not receive the marginal willigness to pay, but only its bid. We can use the envelop theorem again and integrate up to the upper boundary $\hat{c}$ to find that expected profit in the discriminatory auction is lower:

$$\pi^U(c) - \pi^D(c) = \pi^U(\hat{c}) - \pi^D(\hat{c}) + \int_c^{\hat{c}} (h^U(t) - h^D(t))dt > 0.$$

□

Proof of Lemma 6

Proof. In the context of monopolistic competition, the free entry condition (13) ensures efficient investments, which implies $h^U(c) = h^D(c) = h(c)$.

In both auctions, the market clears with Equation (2):

$$b^i(c) = p^i(c) = P(G^i(c)) + \mathcal{Q}(1 - h(c)).$$

Since firms bid a mark-up in discriminatory auctions, the equilibrium price in the discriminatory auction is higher $p^D(c) > p^U(c),\ \forall c \in [\underset{\sim}{c}, \tilde{c}]$. With the same capacity factor $h(c)$, the optimal aggregate capacity is always lower with discriminatory auctions $G^D(c) < G^U(c)$ except for the highest demand realization. Moreover, taking the first order derivatives of market clearing under both auction format and subtracting gives:

$$P'(G^D)G^{D\prime} - P'(G^U)G^{U\prime} = b^{D\prime} - b^{U\prime}.$$

We have shown above that $b^{D\prime} - b^{U\prime} < 0$. If the inverse demand function is concave, $P" < 0$, then $P'(G^U) < P'(G^D)$ and it follows that $G^{U\prime} < G^{D\prime}$. □

Proof of Lemma 8

Proof. We need to prove that consumers' expected expenditures are the same in both auction formats as they consume the same quantities. In a uniform-price auction, consumers pay the bid of the marginal technology $b^U = c$ for the total demand $\theta(c)$. The expected consumer expenditure E^U is

$$E^U = \int_{\underset{\sim}{c}}^{\hat{c}} b^{UP}(c)\,\theta(c)\, dF_q(\theta(c)).$$

In a discriminatory auction, the capacity $d\theta$ receives the price $b^D(c) = c - k(c)/k'(c)$ whenever it produces, which happens with probability $1 - F_q(\theta)$. The expected consumer expenditure E^D is:

$$E^D = b^D(\underset{\sim}{c})\theta(\underset{\sim}{c}) + \int_{\underset{\sim}{c}}^{\hat{c}} b^D(c)(1 - F_q(\theta(c))\, d\theta(c).$$

In the long-run equilibrium, we have $F_q(\theta(c)) = 1 + k'(c)$, which allows us to rewrite the expected equilibrium expenditures as follows:

$$E^U = \int_{\underset{\sim}{c}}^{\hat{c}} c\,\theta(c)k''(c)\, dc,$$

$$E^D = (\underset{\sim}{c} + k(\underset{\sim}{c}))\theta(\underset{\sim}{c}) + \int_{\underset{\sim}{c}}^{\hat{c}} \big(k(c) - ck'(c)\big)\, d\theta(c).$$

Partially integrating the second expression and using the fact that $k(\hat{c}) = k'(\hat{c}) = 0$ and $k'(\underset{\sim}{c}) = -1$, we see that expected expenditures are equal, $E^U = E^D$. □

Proof of Proposition 6

Proof. First, note that $\mathring{c}$ is uniquely determined by Equation (18) since the L.H.S. is an increasing function in $\mathring{c}$, and the R.H.S. is a decreasing function in $\mathring{c}$.

Under uniform-price auctions, one possible equilibrium strategy profile is where all firms bid their marginal cost $b = c$, and inframarginal firms $c \leq \mathring{c}$ produce and receive $\mathring{c}$. However, other equilibrium strategies with equivalent market outcomes exist. For instance, some inframarginal firms $c \leq \mathring{c}$ could bid any price below the competitive price $b < \mathring{c}$, while at least some inframarginal firms bid the competitive price $b = \mathring{c}$. This is an equilibrium since the inframarginal firms have no incentive to change their bidding strategy. They are indifferent to reducing their bids, while they are worse off if they bid more than $\mathring{c}$ and do not produce. Supramarginal firms would not like to bid below cost to get selected, as this results in a loss.

Moreover, there is at least one firm that bids $\mathring{c}$. If firm j is the only firm that bids $\mathring{c}$, and if it deviates downward by ε and bids $b = \mathring{c} - \varepsilon$, it would receive a lower price, which could either be $\mathring{c} - \varepsilon$ or the bid by the inframarginal firm with the highest bid, resulting in a worse-off outcome.

Under discriminatory auctions, inframarginal firms with $c \leq \mathring{c}$ bid $b = \mathring{c}$, while supramarginal firms with $c > \mathring{c}$ bid above: $b > \mathring{c}$. This is an equilibrium. Inframarginal firms receive a lower payment if they bid below $\mathring{c}$, and bidding above $\mathring{c}$ drives them out of merit without any payment. The supramarginal firms can only produce if they bid below costs, which is not optimal. □