WORKING PAPERS

N° 1460

April 2023

Pricing energy consumption and residential energy-efficiency investment: An optimal tax approach

Claude Crampes, Norbert Ladoux and Jean-Marie Lozachmeur

Toulouse School of Economics

Pricing energy consumption and residential energy-efficiency investment: An optimal tax approach.[1]

Claude Crampes[2], Norbert Ladoux[3] and Jean-Marie Lozachmeur[4]

April 18, 2023

[1]The authors acknowledge funding from ANR under grant ANR-17-EURE-0010 (Investissements d'Avenir programme).

[2]Toulouse School of Economics, University of Toulouse Capitole.

[3]Toulouse School of Economics, University of Toulouse Capitole.

[4]Toulouse School of Economics, CNRS, University of Toulouse Capitole. Corresponding author: Jean-Marie Lozachmeur, 1 Esplanade de l'Université, F-31000 Toulouse. Email: jean-marie.lozachmeur@tse-fr.eu.

Abstract

We analyze a Pareto optimal income tax problem à la Mirrlees (1971) in which households consume three types of goods: energy goods, energy efficient investments and non-energy goods. The two main ingredients of our normative analysis are: *i*) an indirect relationship between energy and the satisfaction of energy needs, as energy-efficient investments transform energy into services such as light, heating, and air conditioning; and, *ii*) imperfect information of the policy designer as regards the level of energy-efficiency of households' housing and their labor market productivity. Each household differs with respect to these two latter characteristics, and the government designs a non-linear income tax combined with energy and energy efficient investment non linear pricing that maximizes a weighted sum of households' utilities. We show that a benevolent social planner should distort energy prices in a way that depends on the difference between the saturation of energy needs and the complementarity between energy and the level of energy efficiency in the provision of energy services. A sufficient condition for energy consumption to be subsidized is that the rebound effect is small. Second, when individuals can invest in energy efficiency on top of energy consumption, these investments should always be subsidized and the marginal subsidy should always be higher than the one on energy consumption.

JEL codes: H21, I38, Q48

Keywords: optimal income taxation, indirect taxation, energy services, energy efficiency, energy consumption.

1 Introduction

Energy poverty due to high energy prices, poor living conditions and limited financial resources is still widespread in developed countries. It is estimated that more than 50 million households are affected by energy poverty in Europe (EU Energy Poverty Observatory, European Commission). In the United States, one household in three has difficulty paying energy bills in 2015 (see the Energy Information Administration, Residential Energy consumption Survey, 2015). At the same time, faced with the challenge of climate change, developed countries have adopted energy efficiency improvement policies. Such measures are usually advocated on the basis of the existence of an energy gap in the sense that households socially under-invest in technologies that reduce the energy bill (e.g. see Gillingham and Palmer, 2014).

Faced with these two problems, governments usually subsidize both energy consumption and the consumption of durable goods that improve households' energy efficiency. The first type of subsidy usually takes the form of specific assistance and is subject to an income test.[1] Some governments also consider that pricing energy by universal increasing block-rates eases redistribution.[2] In the simplest case, it consists of fixing two unit prices for energy purchase: a low unit price (below marginal cost) in order to satisfy basic needs; and a high unit price (above marginal cost) beyond this threshold.[3] However, due to the universal property of this system and the relatively low correlation between income and energy consumption, redistribution has been estimated to be low as compared to a means tested program (e.g. see Borenstein, 2012). Concerning subsidies on energy efficiency improvements, these usually include the form of reduced-rate loans, tax credits and general non linear subsidies usually subject to means testing.[4]

In this paper, we try to find under which circumstances, if any, distributional concerns lead to the subsidization of energy consumption and residential energy efficient investment. Our approach also allows us to determine whether an optimal policy should encourage the consumption of energy relative to residential energy efficient investments. To do so, we deliberately abstract from issues related to environmental externality generated by consumption of energy. Instead we analyze a Pareto optimal income tax

[1] Examples are the CARE program in California, the Warm Home Discount in England or Affordable tariffs (TPN and TSS) in France.

[2] This system has been implemented in most US states but also in Australia or China and is on the political agenda of many European countries.

[3] In California, the system of electrcity prices which had up to five different prices is progressively abandoned.

[4] Examples of such schemes are the "Energy Savings Assistance Program" in California, the "Green Deal & ECO" in the UK or "ANAH" grants and "Programme Habiter Mieux" in France.

problem à la Mirrlees (1971) in which households consume three types of goods: energy goods, energy efficient investments and non-energy goods. The two main ingredients of our normative analysis are: *i*) an indirect relationship between energy and the satisfaction of energy needs, as energy efficient investments transform energy into services such as light, heating, and air conditioning; and, *ii*) imperfect information of the policy designer as regards the level of energy efficiency of households' dwellings and their labor market productivity. Each household differs with respect to these two latter characteristics, and the government designs a non-linear income tax combined with energy and energy efficient investment non linear pricing that maximizes a weighted sum of households' utilities. We first show that for a fixed (but heterogenous) level of energy efficiency, a benevolent social planner should distort energy prices in a way that depends on the difference between the saturation of energy needs and the complementarity between energy and the level of energy efficiency in the provision of energy services. We relate this condition to the so called "rebound effect" discussed in the energy literature. More specifically, a necessary and sufficient condition for energy consumption to be subsidized is that the rebound effect is small enough so that the Jevons paradox is not effective. Second, when individuals can invest in energy efficiency on top of energy consumption, these investments should always be subsidized and the marginal subsidy should always be higher than the one (if any) on energy consumption.

Related literature

Our paper contributes to the literature on redistributive energy pricing in the presence of a non linear labor income tax.[5] We adopt a normative point of view using the approach of optimal income taxation. In this respect, it is related to the literature on direct versus indirect taxation initiated by Atkinson and Stiglitz (1976). The role of commodity taxes is probably one of the most prominent or, at least, one of the oldest issues of taxation policy; see Atkinson (1977). The traditional Ramsey type models which typically advocated non uniform commodity taxes have received a rather fatal blow by the classic contribution of Atkinson and Stiglitz (1976). In their seminal work, they show that under some conditions—weak separability of preferences in labor supply and goods—an optimal nonlinear income tax is sufficient to implement any incentive compatible Pareto-efficient allocation. In other words, commodity taxes/subsidies are redundant (or should be uniform). It is by now well understood though that the Atkinson and Stiglitz result has its limitations. It does not hold under uncertainty (see Cremer

[5] See Feldstein (1972) and Munk (1977) for redistributive electricity pricing without distortionary labor income considerations and the recent paper by Feger and Radulescu (2008) in the presence of linear income and energy consumption distortionary taxes.

and Gahvari, 1995 and the subsequent literature on "new dynamic public finance" resumed by Kocherlakota (2010)), under multi-dimensional heterogeneity, for instance, when individuals differ in preferences (e.g. see Cremer, Gahvari and Ladoux, 2003; or Saez, 2002) or in available market prices (e.g. see Gahvari and Micheletto, 2016). In our model, the celebrated Atkinson and Stiglitz theorem does not hold because the government cannot observe the level of energy efficiency so that the price of energy and energy efficient investment may be distorted for redistributive purposes. It can be interpreted as a model in which individuals differ in preferences for energy consumption due to heterogenous levels of energy efficiency. The main novelty is that heterogenous preferences are endogenous since individuals can invest in energy efficiency. This allows us to compare whether the optimal tax/subsidies scheme should encourage energy consumption relative to energy efficiency investment.

A special feature of our study is that it pays particular attention to tax systems that include nonlinear income and nonlinear commodity taxes. This is important. The feasibility of a particular tax instrument is ultimately determined by the type of information that is available to the tax administration. Restricting income taxes to be linear, as is often done (e.g. see Feger and Radulescu, 2018 for electricity pricing in the context of redistribution), has no basis in theoretic or policy grounds. Similarly, while a linear tax on goods is only feasible for the vast majority of goods (i.e. when transactions are anonymous), the goods we are considering are good examples which can typically be observed by tax administrations and regulators.[6]

One closely related paper is Cremer and Gahavari (2017). They consider a problem of redistribution with a non linear income tax schedules but prices are constrained to be linear (their information structure assumes anonymous transactions). Some of the goods are linearly priced in order to fulfill a break even constraint on the production side. In our analysis, we do assume that energy or durable purchases that improve households' energy efficiency consumption are observable. As a result, the government can (non linearly) price the energy and energy efficiency investments as if it were the price maker. This is actually the case when the market for these goods are perfectly competitive (so that prices equal marginal cost) or when the firms are publicly owned.[7] Moreover, as a policy matter, governments in all countries typically employ graduated income tax schedules as well as implicit graduated subsidy schedules for energy and

[6] Anonymous transactions may concern goods that are environmentally friendly products which could have an impact on energy efficiency (like efficient household appliances, consumer electronics, boilers...). One could then apply differentiated linear tax rates for such efficient products but we are not aware of any such policy in any country (e.g. see Naess-Schmidt et al., 2008, for a discussion on these issues).

[7] We discuss the implications of imperfect competition in the conclusion.

energy efficiency investments.

The next section presents the model. In Section 3, we derive the optimal policy in terms of energy pricing, assuming that the level of energy efficiency is exogenously given. We then extend the model in Section 4 to the case where households may invest in energy efficiency and study its pricing properties. Section 5 discusses some limits of our approach, relates our analysis to empirical studies and concludes.

2 The model

2.1 Assumptions and notations

Each household derives utility from the consumption of a numeraire good x, the use of energy services s (e.g. light, heating or cooking) and disutility from labor supply l. We denote this utility function by $U(x, s, l) = u(x, s) - v(l)$, where u is increasing and strictly concave in its arguments, separable between x and s and v is increasing and strictly convex. Energy services are produced through a function $s = f(e, \phi)$ in which e is the consumption of energy while ϕ is an index denoting the level of energy efficiency hereafter denoted LEE (e.g. quality of housing insulation). A higher ϕ denotes a more energy efficient dwelling. We first assume that ϕ is exogenously fixed. In Section 4, this assumption is relaxed. Energy services s are increasing and concave in e and ϕ. Thus, for any given level of energy services, one has $d\phi/de|_s = -f_e/f_\phi < 0$, where subscripts refer to partial derivatives. In other words, a higher LEE allows the household to have the same level of energy service with less energy consumption. We finally and naturally assume that e and ϕ are complements in the production process, so that $f_{e\phi} \geq 0$.

For the rest of the analytical model, every gross price of goods (the numeraire, energy and LEE) are normalized to one. This has no consequence. One could indeed normalize any prices of e and ϕ to p_e and p_ϕ without changing any of our results. As discussed in the introduction, the underlying assumption is that the market for these goods is perfectly competitive and gross prices are set to their marginal cost. Equivalently, the firm is publicly owned so that the government has full control of energy price. In the latter case, the government can implement any nonlinear tariffs as long as its fiscal revenue is high enough in order to finance fixed cost.

Society is composed of a unit mass of households denoted by an upperscript $i = 1, ..., N$, and each household i is represented by a vector $\left(w^i, \phi^i\right)$ where w^i denotes the household i's labor productivity. The proportion of households of type i is denoted π^i so that $\sum_{i=1}^{N} \pi^i = 1$. Denoting $y = wl$ the household's labor income, each household i's utility is written as:

$$U^i(x,e,y) \equiv u\left(x, f\left(e,\phi^i\right)\right) - v\left(y/w^i\right), \tag{1}$$

where x, e, y are endogenous (we treat the case where ϕ is endogenous in section 4).

The government seeks to maximize a weighted utilitarian social welfare function but does not observe individuals types $\left(w^i, \phi^i\right)$ nor the labor supply l^i and energy service s^i.[8] One may object that the level of energy efficiency may be observable by the government. Some countries like England, France or some states in the US impose the owners of a dwelling to realize an energy efficiency diagnostic that is observable by the government. However, these diagnoses only reveal partial information and are subject to controversies because they are realized ex ante based on engineer's models (see Jacobsen and Kotchen (2013). It does, however, observe gross labor income y^i and energy consumption e^i. Thus, it can design an income tax combined with an energy tariff function $T\left(e^i, y^i\right)$ in order to implement the second best allocation. Deleting upperscripts, the problem solved by each household is:

$$\max_{x,e,y} U(x,e,y)$$
$$\text{s.t.} \quad y - x - e - T(e,y) \geq 0.$$

Denoting T_e and T_y the marginal taxes on energy consumption and income, and substituting x from the budget constraint, the first order conditions with respect to e and y yield:

$$-(1+T_e)\,u_x + f_e u_s = 0 \tag{2}$$
$$(1-T_y)u_x - (1/w)v'(y/w) = 0 \tag{3}$$

where $u_x = \partial u/\partial x$, $u_s = \partial u/\partial s$ and $f_e = \partial f/\partial e$. These two equations can be rewritten as:

$$MRS_{xe} = 1 + T_e, \tag{4}$$
$$MRS_{xy} = 1 - T_y, \tag{5}$$

[8] Households are not required to know precisely their level of LEE. However, as long as they clearly observe their level of energy service and energy consumption, they can infer their level of LEE better than the government. Moreover, as shown recently by Allcott and Greenstone (2017), there is little evidence that individuals misperceive their level of LEE.

where

$$MRS_{xe} = -\left.\frac{dx}{de}\right|_{\bar{U}} = \frac{f_e u_s(x,s)}{u_x(x,s)}, \quad (6)$$

$$MRS_{xy} = \left.\frac{dx}{dy}\right|_{\bar{U}} = \frac{1}{w}\frac{v'(y/w)}{u_x(x,s)}. \quad (7)$$

Equations (4) and (5) describe the usual trade-off stating that the marginal rate of substitution between the numeraire and each other good is equal to the net-of-tax price of that good. In its consumption plan, the household takes into account whether $T_e < 0$ or $T_e > 0$, that is the fact that energy is subsidized or taxed at the margin. In this paper, we are not aiming to characterize the optimal income tax scheme. The marginal income tax rate follows the traditional Mirrleesian properties. Without income heterogeneity, the main results of our paper concerning the properties of the marginal price properties on goods would not change radically. In such a setting a nonlinear tax $T(e)$ would suffice for implementability purposes. Income heterogeneity however allows the pricing of goods to depend on individuals' income reflecting means tested programs discussed in the introduction.

2.2 Indifference curve properties

Before discussing the normative analysis, it will prove useful to study indifference curves properties in plane (x, e). The marginal rate of substitution between x and e measures the amount of numeraire good that the individual is ready to forgo in order to consume more energy for a given level of utility. In other words, it measures the willingness to pay for energy in terms of numeraire good.

The following lemma compares the different marginal rates of substitution for different LEEs.

Lemma 1 *MRS_{xe} is decreasing (increasing) in ϕ iff $f_{e\phi}/f_e f_\phi \leq (>) - U_{ss}/U_s$ while it does not depend on w.*

Proof. Differentiating (6) with respect to ϕ yields:

$$\frac{\partial MRS_{xe}}{\partial \phi} = f_{e\phi}\frac{U_s}{U_x} + f_e f_\phi \frac{U_{ss}}{U_x},$$

then $\partial MRS_{xe}/\partial\phi \lesseqgtr 0$ if and only if:

$$\frac{f_{e\phi}}{f_e f_\phi} \lesseqgtr -\frac{U_{ss}}{U_s}. \quad (8)$$

■

A higher LEE has two opposite effects on the marginal willingness to pay for energy. The first effect as measured by $f_{e\phi}U_s/U_x \geq 0$ is to increase the marginal productivity of energy for the service, which increases the willingness to pay for energy. This effect is higher, the higher the degree of complementarity between energy and LEE ($f_{e\phi} >> 0$). The second effect as measured by $f_e f_\phi U_{ss}/U_x < 0$ is a decrease in the marginal utility from energy service, which decreases the willingness to pay for extra energy consumption.

The coefficient $-U_{ss}/U_s > 0$ can be interpreted as the degree of saturation in the need for energy service, while $f_{e\phi}/f_e f_\phi$ is the coefficient of technical complementarity between energy and LEE in the production of energy service. Lemma 1 shows that if the coefficient of saturation is higher (resp. lower) than the coefficient of complementarity between e and ϕ, the willingness to pay for energy is decreasing (resp. increasing) in the LEE. To illustrate the Lemma, consider a household in a well-insulated dwelling. It does need a higher temperature so that U_s is close to zero. Therefore, the saturation effect is higher than the degree of complementarity between energy and LEE. A more efficient heating system (i.e. an increase in LEE) would make some energy redundant, hence a decrease in the willingness to pay for energy. The opposite stands for people leaving in “thermal sieves”. They would like higher indoor temperature (U_s is high) so that the saturation index is low. A higher LEE is an opportunity to increase s by using a smaller quantity of energy, which translate in a higher willingness to pay.

Another interpretation of this lemma is related to the so called "rebound effect" in the energy literature (e.g. see Gillingham et. al., 2013). A higher LEE changes both household relative prices and preferences for energy services which may change the demand for energy in either way. To see this, fix the level of y and differentiate equation (2) so that:

$$\left.\frac{de}{d\phi}\right|_y = -\frac{f_{e\phi}u_s + f_e f_\phi u_{ss}}{SOC}$$

where we assume that $SOC < 0$ so that the second order condition holds. One can thus easily infer that a *necessary and sufficient* condition for energy consumption to be a decreasing (resp. increasing) function of LEE is that $f_{e\phi}/f_e f_\phi <$ (resp. $>$) $- U_{ss}/U_s$ In other words, $f_{e\phi}/f_e f_\phi < -U_{ss}/U_s$ ensures that the rebound effect is small enough so that the "Jevons paradox" does not apply. We summarize this in the following lemma:

Lemma 2 *A necessary and sufficient condition for the "Jevons paradox" not to hold is that* $f_{e\phi}/f_e f_\phi < -U_{ss}/U_s$

A useful illustration of this condition can be derived if (i) $f(e,\phi)$ is a CES function with an elasticity of substitution equal to ρ and (ii) $u(x,s) = u_1(x) + u_2(s)$ where $u_2 = (1/(1-\sigma))\, s^{1-\sigma}$. In this case, one can easily show that condition (8) is specified as $1/\rho \lesseqqgtr \sigma$.

Finally, the fact that the marginal willingness to pay for energy does not depend on w trivially comes from the separability between consumptions and labor supply.

3 The optimal tax function

3.1 The general case

We characterize the (constrained) Pareto efficient allocations that are obtained by maximizing a weighted sum of utilities subject to the resource constraint and the incentive compatibility constraints. The social weight of a type i household is denoted $\alpha^i\pi^i$ with $\alpha^i \geq 0$ and $\sum_i \alpha^i = 1$. Because types are private information the following incentive compatibility constraints apply for any $i, j = 1, \ldots, N$,

$$U^i = U^i\left(x^i, e^i, y^i\right) \geq U^{ij} = U^i\left(x^j, e^j, y^j\right). \tag{9}$$

In other words, household i equipped with ϕ^i and having productivity w^i must not be able to achieve a (strictly) larger utility level by mimicking household j, i.e. by consuming the consumption bundle designed for household j.

Formally, a constrained Pareto efficient allocation is the solution to the following problem:

$$\max_{\{x^i,e^i,y^i\}} \sum_{i=1}^{N} \alpha^i\pi^i U^i$$

$$\text{s.t.} \sum_{i=1}^{N} \pi^i\left(y^i - x^i - e^i\right) \geq G, \tag{10}$$

$$U^i \geq U^{ij} \quad i,j = 1,...,N, \quad i \neq j, \tag{11}$$

where G is an exogenous revenue requirement while U^i and U^{ij} are defined by (9). Denoting the multipliers of constraints (10) and (11) by μ and λ^{ij} respectively, one can write the Lagrange expression as follows:

$$\Lambda = \sum_{i=1}^{N} \alpha^i\pi^i U^i + \mu\left[\sum_{i=1}^{N} \pi^i\left(y^i - x^i - e^i\right) - G\right] + \sum_{i=1}^{N}\sum_{j=1}^{N} \lambda^{ij}\left[U^i - U^{ij}\right].$$

Using $\sum_{j\neq i}$ as a shorthand for $\sum_{j=1,j\neq i}^{N}$, the first order conditions with respect to x^i, e^i and y^i, $i = 1, ..., N$, are given by:

$$\frac{\partial \Lambda}{\partial x^i} = \left[\alpha^i \pi^i + \sum_{j\neq i} \lambda^{ij}\right] \frac{\partial U^i}{\partial x^i} - \pi^i \mu - \sum_{j\neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i} = 0, \tag{12}$$

$$\frac{\partial \Lambda}{\partial e^i} = \left[\alpha^i \pi^i + \sum_{j\neq i} \lambda^{ij}\right] \frac{\partial U^i}{\partial e^i} - \pi^i \mu - \sum_{j\neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial e^i} = 0, \tag{13}$$

$$\frac{\partial \Lambda}{\partial y^i} = \left[\alpha^i \pi^i + \sum_{j\neq i} \lambda^{ij}\right] \frac{\partial U^i}{\partial y^i} + \pi^i \mu - \sum_{j\neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial y^i} = 0. \tag{14}$$

Denoting $\kappa^i = \alpha^i \pi^i + \sum_{j\neq i} \lambda^{ij}$ and combining equations (12) and (13) yields after some manipulation:

$$\kappa^i \frac{\partial U^i}{\partial x^i} MRS^i_{xe} - \kappa^i \frac{\partial U^i}{\partial x^i} - \sum_{j\neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i} MRS^{ji}_{xe} + \sum_{j\neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i} = 0,$$

where $MRS^{ji}_{xe} = \left(\partial U^{ji}/\partial e^i\right) / \left(\partial U^{ji}/\partial x^i\right)$. After some rearrangements this yields:

$$MRS^i_{xe} = \frac{\kappa^i \frac{\partial U^i}{\partial x^i} - \sum_{j\neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i}}{\kappa^i \frac{\partial U^i}{\partial x^i} - \sum_{j\neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i} \frac{MRS^{ji}_{xe}}{MRS^i_{xe}}}. \tag{15}$$

Comparing the second-best rule (15) to the individual rational choice (4), we can now establish the following proposition.[9]

Proposition 1 *(i) To implement the second-best allocation, the energy unit price must be distorted as follows:* $T^i_e \lesseqqgtr 0$ *if and only if:*

$$\sum_{j\neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i} \left[\frac{MRS^{ji}_{xe}}{MRS^i_{xe}} - 1\right] \lesseqqgtr 0. \tag{16}$$

(ii) A Pareto efficient allocation can be implemented by a tax function such that $T^i_e = 0$, *if and only if condition* $f^i_{e\phi}/f^i_e f^i_\phi = -U^i_{ss}/U^i_s$ *holds for all households* $i = 1, \ldots N$.

[9]Note that since $\pi^i \mu \geq 0$ by equation (12) both the numerator and the denominator in the RHS of (15) are positive at any interior solution.

Proof. (i) By equation (4), one has $T_e^i \lesseqqgtr 0$ iff $MRS_{xe}^i \lesseqqgtr 1$. By equation (15), this is true if:

$$\frac{\kappa^i \frac{\partial U^i}{\partial x^i} - \sum_{j \neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i}}{\kappa^i \frac{\partial U^i}{\partial x^i} - \sum_{j \neq i} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i} \frac{MRS_{xe}^{ji}}{MRS_{xe}^i}} \lesseqqgtr 1.$$

After rearranging, this is true if condition (16) is satisfied.

(ii) A sufficient condition for condition (16) to hold with equality is that $MRS_{xe}^{ji} = MRS_{xe}^i$, which by Lemma 1 is satisfied if and only if $f_{e\phi}^i / f_e^i f_\phi^i = -U_{ss}^i / U_s^i$. ■

Proposition 1 provides a general condition under which a household's demand for energy should not be distorted; specifically this is true when the LHS of (16) is equal to zero. Otherwise, it ought to be distorted, and the direction of the distortion is provided by (16). When the LHS of this condition is negative, households of type i face a marginal subsidy on energy. If it is positive, energy consumption by type-i agents is taxed at the margin. This condition is general in the sense that it is valid whatever the pattern of binding incentive constraints and for any welfare weights. The price to pay for this level of generality is that the condition involves endogenous variables. We show below that (16) reduces to a condition on the primitives of the model in special cases. In those cases, the interpretation will also be facilitated. In the meantime, we look at the interpretation of the general condition.

To do so, let us first compare the choices of (x, e) by households i and j that would prevail in a first best world, that is, when the planner can observe all the households' parameters and decisions so that constraints (11) are not binding. The allocation would, in this case, be described by equations (12) to (14) with $\lambda^{ji} = 0$ for all $i, j = 1...N$. In particular, if $\alpha^i = \alpha^j$ for every $i, j = 1...N$ (which corresponds to the utilitarian case), one would have $x^i = x^j$ since u is separable between x and s. Each household would choose a pair (x, e) that lies at the point where indifference curves have a slope equal to -1, i.e. at the point where $MRS_{xe}^j = MRS_{xe}^i = 1$. Figure 1 illustrates this choice in the (e, x) plane where households j and i choose a different consumption of e with $e^i > e^j$. In the case that is depicted, the marginal rate of substitution between x and e evaluated at point i is lower (in absolute value) for household j than for household i. Now we return to the second best problem and assume that the incentive compatibility constraint preventing household j to mimic household i is binding. Recall that at the point (x^i, e^i), household i has a steeper indifference curve than household j. To make household i's consumption bundle (x^i, e^i) less attractive for household j, it is then desirable to distort the choice of household i towards more e (moving to the right along the indifference curve U^i) i.e. to have a marginal subsidy on e^i. This would have no first order effect on welfare while it relaxes an otherwise binding incentive compatibility constraint. Alternatively, when the marginal rate of substitution is higher (in absolute value) for household j, it is desirable to distort the choice of household i towards less e i.e. to have a marginal tax on e^i.

So far, we have concentrated on the analysis of one pair of households. However, the solution may well imply that several incentive constraints towards type i are binding. Condition (16) considers *all* households j for which the incentive compatibility constraints towards the type i household are binding, i.e. each j such that $\lambda^{ji} > 0$. For each binding incentive compatibility constraint, there is a desirable distortion on the (x^i, e^i) choice. As argued above, the sign of this distortion depends upon the difference between the two households in the marginal rates of substitution. Proposition 1 states that the total distortion on couple i's $\left(x^i, e^i\right)$ trade-off depends upon a weighted sum of distortions imposed by each binding self-selection constraint in which i is the mimicked type.

Insert Figure 1 here

Observe that a household i such that $\lambda^{ji} = 0$ for all j (if it exists) – never faces a distortion (the LHS of (16)) is always equal to zero). This is the counterpart to the traditional *no distortion at the top* result in this multi-dimensional setting. For all other types i (with at least one $\lambda^{ji} > 0$), the LHS of (16) may or may not be zero, depending on the marginal rates of substitution of the mimicker and the mimicked households. A sufficient condition for this to be the case is that $MRS^{ji}_{xe} = MRS^{i}_{xe}$ at the point $\left(x^i, e^i\right)$ for all pairs of households with $\lambda^{ji} > 0$. In other words, this (sufficient) condition requires that all pairs of households linked by a binding incentive constraint have the same marginal rate of substitution (at the consumption bundle of the mimicked household) which is the case if and only if $f^i_{e\phi}/f^i_e f^i_\phi = -U^i_{ss}/U^i_s$ for every i as shown by Lemma 1.

3.2 Example with correlated types

The results obtained so far do not depend on the pattern of binding incentive constraints. We can gain further insights by making some additional assumptions on the distribution of productivities and LEE, and thus ultimately on the pattern of binding incentive constraints.

Assume that households $i = 1, ..., N$ are ordered such that $w^N > ... > w^{i+1} > w^i >$

... $> w^1$ and $\phi^N > ... > \phi^{i+1} > \phi^i > ... > \phi^1$ so that household $i+1$ is richer and has a higher LEE than household i.[10] With this assumption, a single level of w is associated with a single level of ϕ. Put differently, we can express w as an *increasing* function of ϕ. This effectively reduces our problem to a single dimension of heterogeneity, and it is reasonable to assume that incentive compatibility constraints are binding from high ability to low ability households. To simplify notation, further suppose that only *adjacent* downward incentive compatibility constraints are binding, i.e. that $\lambda^{i+1,i} > 0$ and $\lambda^{i+1,j} = 0$ for all $i = 1, ..., N-1$ with $j \neq i$.[11]

In this case, there is no distortion for the household of type N since it is the richest and the best equipped. Furthermore, for $i = 1, \ldots, N-1$, Proposition 1 *(i)* is equivalent to:

$$T_e^i \lesseqqgtr 0 \Leftrightarrow MRS_{x,e}^{i+1,i} \lesseqqgtr MRS_{x,e}^i.$$

To see this, it is sufficient to replace j by $i+1$ in condition (16) while keeping in mind that $\partial U^{ji}/\partial x^i > 0$. Using Lemma 1, we then obtain the following proposition:

Proposition 2 *Assume that w and ϕ are positively correlated and that only adjacent downward incentive compatibility constraints are binding, i.e. that $\lambda^{i+1,i} > 0$ and $\lambda^{i+1,j} = 0$ for all $i = 1...N-1$ with $j \neq i$, one has $T_e^i \lesseqqgtr 0$ if and only if $f_{e\phi}^i / f_e^i f_\phi^i \lesseqqgtr -U_{ss}^i / U_s^i$ $(i = 1, \ldots, N-1)$.*

The intuition for this result is the same as the one developed in the preceding section. A subsidy (resp. tax) on energy consumption is a way to relax an otherwise binding incentive constraint if the willingness to pay for energy is lower (higher) for a rich (and well-equipped) household. As shown by Lemma 1, this is respectively the case if the coefficient of technical complementarity between energy and LEE is lower (higher) than the coefficient of saturation in the need for energy services. As argued at the end of Section 2.2, a very simple case arises if f is a CES function, and the utility for energy service is isoelastic. Moreover, a direct implication of lemma 2 is that energy consumption is always subsidized if the Jevons paradox does not hold for all individuals. We summarize this in the following corollary:

[10] The assumption that richer households have a higher LEE is strongly corroborated by empirical studies e.g. see Drehobl and Ross (2016).

[11] This assumption is stronger than necessary, but it dramatically simplifies the notation. All our qualitative results go through if we assume simply that only downward incentive constraints are binding. To see this, observe that the pairwise comparisons of MRS we perform are valid also when $j \neq i+1$. The translation into marginal tax rates is slightly more complicated because we may have several binding IC constraints towards a given type. However, recall from (16) that the total effect is obtained by adding the pairwise effects. Consequently, when all these effects go in the same direction, the study of the pairwise effects is sufficient.

Corollary 1 *Assume that* w *and* ϕ *are positively correlated and that only adjacent downward incentive compatibility constraints are binding, i.e. that* $\lambda^{i+1,i} > 0$ *and* $\lambda^{i+1,j} = 0$ *for all* $i = 1...N-1$ *with* $j \neq i$*:*

(i) If the Jevons paradox does not hold for all i*, then* $T_e^i \leq 0$ *for all* i*.*

(ii) Assume that $f(e, \phi)$ *is a CES function with an elasticity of substitution equal to* ρ *and* $u(x, s) = u_1(x) + u_2(s)$ *where* $u_2 = (1/(1-\sigma)) s^{1-\sigma}$*. Then* $T_e^i \lesseqqgtr 0$ *if and only if* $1/\rho \lesseqqgtr \sigma$*.*

In this simple example, it is clear that a marginal subsidy (tax) on energy consumption is optimal for all individuals if the coefficient of relative risk aversion – representing the concavity of the utility function for energy service – is higher (lower) than the inverse of the elasticity of substitution between energy and LEE in the production of energy service. Again, this occurs when the Jevons paradox does not hold i.e. that energy consumption is a decreasing function of LEE.

4 Endogenous investment in energy efficiency

Until now, we have treated the LEE as fully exogenous. This section extends the previous analysis by allowing the households to complement their initial LEE.

4.1 The household's problem

Suppose now that the different types of households $i = 1, ..., N$ can increase the efficiency of their initial LEE ϕ^i by investing $\tilde{\phi}$ at a market price normalized to one. The household's i utility is now written as:

$$U^i\left(x, e, \tilde{\phi}, y\right) = u\left(x, f\left(e, \phi^i + \tilde{\phi}\right)\right) - v\left(y/w_i\right). \tag{17}$$

Assuming that the government can observe households expenditures $\tilde{\phi}$ but still does not observe the initial LEE ϕ^i,[12] it can now rely on a tax function $T\left(e^i, \tilde{\phi}^i, y^i\right)$ to implement the second best allocation. Faced with this tax function, the problem of the

[12] Equivalently, one can assume that the government can observe the level of LEE without distinguishing between the initial (unobervable) level ϕ^i and the observable level of investment $\tilde{\phi}^i$ chosen by the individual.

household becomes:[13]

$$\max_{x,e,\tilde{\phi},y} U^i(x,e,y) = u\left(x, f\left(e, \phi^i + \tilde{\phi}\right)\right) - v(y/w_i)$$
$$\text{s.t. } y - x - e - \tilde{\phi} - T\left(e, \tilde{\phi}, y\right) \geq 0.$$

The first order conditions yields:

$$MRS_{x\tilde{\phi}} = 1 + T_{\tilde{\phi}}, \tag{18}$$

together with equations (4) and (5) where:

$$MRS_{x\tilde{\phi}} = -\left.\frac{dx}{d_{\tilde{\phi}}}\right|_{\bar{U}} = \frac{f_\phi u_s(x,s)}{u_x(x,s)}, \tag{19}$$

and the implicit relation between e and $\tilde{\phi}$ is given by:

$$MRS_{e\tilde{\phi}} = \frac{1 + T_{\tilde{\phi}}}{1 + T_e}, \tag{20}$$

where:

$$MRS_{e\tilde{\phi}} = -\left.\frac{de}{d_{\tilde{\phi}}}\right|_{\bar{U}} = \frac{f_\phi}{f_e}. \tag{21}$$

Equation (18) describes the trade-off between the numeraire and additional LEE: if $T_{\tilde{\phi}} < 0$ (resp. > 0), the price of LEE is subsidized (resp. taxed) at the margin. Equation (20) describes the trade-off between energy consumption and LEE expenditures. The marginal rate of substitution between energy consumption and additional LEE states by how much energy a household is ready to forgo in order to invest in LEE (for a given utility level). If $T_{\tilde{\phi}} < T_e$ (resp. $T_{\tilde{\phi}} > T_e$), then LEE expenditures are encouraged relative to energy consumption. When $T_{\tilde{\phi}} = T_e$, there is no distortion in the $\left(e, \tilde{\phi}\right)$ choice. This is true in particular when $T\left(e, \tilde{\phi}, y\right) = T\left(e + \tilde{\phi}, y\right)$ so that the tax/transfer scheme depends on the sum of energy and LEE expenditures.

An alternative view of this distortion is that the choice between e and $\tilde{\phi}$ is distorted towards more LEE if households pay less taxes by increasing their LEE *for a given*

[13] As in section 3, we are not aiming to study the properties of the optimal marginal income tax rate.

level of total expenditures on energy $e + \tilde{\phi}$, that is, when:[14]

$$\left.\frac{\partial T\left(e, \tilde{\phi}, y\right)}{\partial \tilde{\phi}}\right|_{e+\tilde{\phi}} = T_{\tilde{\phi}} - T_e < 0. \tag{22}$$

Using equation (20), it appears that inequality (22) amounts to $MRS_{e\tilde{\phi}} < 1$. Consequently, the two alternative ways to define the distortions are effectively equivalent.

The following lemma compares the marginal willingness to pay for LEE, both in terms of x and e for different values of initial LEE ϕ.

Lemma 3 *$MRS_{x\tilde{\phi}}$ and $MRS_{e\tilde{\phi}}$ are both decreasing in ϕ.*

Proof. Differentiating (19) and (21) with respect to ϕ respectively yields:

$$\frac{\partial MRS_{x\tilde{\phi}}}{\partial \phi} = f_{\phi\phi}\frac{U_s}{U_x} + (f_\phi)^2 \frac{U_{ss}}{U_x} < 0,$$

and

$$\frac{\partial MRS_{e\tilde{\phi}}}{\partial \phi} = \frac{f_{\phi\phi} f_e - f_\phi f_{e\phi}}{(f_e)^2} < 0.$$

■

The willingness to pay for additional LEE in terms of x is decreasing in the initial LEE. This is due to the fact that a higher initial LEE both decreases the marginal productivity of additional LEE and the marginal utility of energy services. In the same way, a higher initial LEE decreases the marginal productivity of additional LEE and increases the marginal productivity of energy, so that the marginal willingness to pay for $\tilde{\phi}$ in terms of energy consumption e is decreasing in the initial LEE.

[14] Symmetrically, the choice between e and $\tilde{\phi}$ is distorted towards more energy consumption when:

$$\left.\frac{\partial T\left(e, \tilde{\phi}, y\right)}{\partial \tilde{\phi}}\right|_{e+\tilde{\phi}} = T_{\tilde{\phi}} - T_e > 0.$$

4.2 The second best optimum

The second best problem of the government is now:

$$\max_{x^i, e^i, \tilde{\phi}^i, y^i} \sum_{i=1}^{N} \alpha^i \pi^i U^i$$

$$\text{s.t.} \sum_{i=1}^{N} \pi^i \left(y^i - x^i - \tilde{\phi}^i - e^i \right) \geq G,$$

$$U^i \geq U^{ij} \quad i, j = 1.......N,$$

where $U^i = u\left(x^i, f\left(e^i, \phi^i + \tilde{\phi}^i\right)\right) - v\left(y^i/w^i\right)$ and $U^{ij} = u\left(x^j, f\left(e^j, \phi^i + \tilde{\phi}^j\right)\right) - v\left(y^j/w^i\right)$. The Lagrangian is:

$$\Lambda' = \sum_{i=1}^{N} \alpha^i \pi^i U^i + \mu \left[\sum_{i=1}^{N} \pi^i \left(y^i - x^i - e^i - \tilde{\phi}^i \right) - G \right] + \sum_{i \neq j} \lambda^{ij} \left[U^i - U^{ij} \right],$$

and the first order conditions are given by:

$$\frac{\partial \Lambda'}{\partial \tilde{\phi}^i} = \left[\alpha^i \pi^i + \sum_{j \neq i}^{N} \lambda^{ij} \right] \frac{\partial U^i}{\partial \tilde{\phi}^i} - \pi^i \mu - \sum_{i \neq j} \lambda^{ji} \frac{\partial U^{ji}}{\partial \tilde{\phi}^i} = 0, \tag{23}$$

together with equations (12) to (14).

Combining (23) with (12) and (23) with (13) gives:

$$MRS^i_{x\tilde{\phi}} = \frac{\kappa^i \frac{\partial U^i}{\partial x^i} - \sum_{i \neq j} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i}}{\kappa^i \frac{\partial U^i}{\partial x^i} - \sum_{i \neq j} \lambda^{ji} \frac{\partial U^{ji}}{\partial x^i} \frac{MRS^{ji}_{x\tilde{\phi}}}{MRS^i_{x\tilde{\phi}}}}, \tag{24}$$

$$MRS^i_{e\tilde{\phi}} = \frac{\kappa^i \frac{\partial U^i}{\partial e^i} - \sum_{i \neq j} \lambda^{ji} \frac{\partial U^{ji}}{\partial e^i}}{\kappa^i \frac{\partial U^i}{\partial e^i} - \sum_{i \neq j} \lambda^{ji} \frac{\partial U^{ji}}{\partial e^i} \frac{MRS^{ji}_{e\tilde{\phi}}}{MRS^i_{e\tilde{\phi}}}}. \tag{25}$$

Comparing (18) and (20) to (19) and (21) respectively, we can now establish the following proposition:

Proposition 3 *Assume that w and ϕ are positively correlated and that only adjacent downward incentive compatibility constraints are binding, i.e. that $\lambda^{i+1,i} > 0$ and $\lambda^{i+1,j} = 0$ for all $i = 1, ..., N-1$, with $j \neq i$. Then Proposition (2) still holds, and ii) $T^i_{\tilde{\phi}} < 0$ and $T^i_{\tilde{\phi}} < T^i_e$ $(i = 1, \ldots, N-1)$.*

Proof. i) The first order conditions (12) and (13) still apply so that the results stated in proposition 2 remain valid. ii) By Lemma 3, $\partial MRS_{x\tilde{\phi}}/\partial\phi < 0$ so that $MRS^{ji}_{x\tilde{\phi}}/MRS^{i}_{x\tilde{\phi}} < 1$. Equating (24) to (18) thus yields $T^i_{\tilde{\phi}} < 0$. Analogously, Lemma 3 yields $\partial MRS_{e\tilde{\phi}}/\partial\phi < 0$ so that equating (25) to (20) yields $T^i_{\tilde{\phi}} < T^i_e$. ■

Since the willingness to pay for additional LEE in terms of x is decreasing in the initial LEE, a marginal subsidy on $\tilde{\phi}$ allows to relax an otherwise binding self-selection constraint. Analogously, a higher subsidy on $\tilde{\phi}$ than on e allows to relax incentive constraints, since the marginal willingness to pay for additional $\tilde{\phi}$ in terms of e is decreasing in ϕ. The second-best redistributive policy thus encourages the consumption of LEE relative to the consumption of energy.

5 Concluding remarks

This paper studies whether energy and energy efficient investments should be subsidized in a redistributive framework. We develop a Mirrleesian model in which households purchase three types of goods: energy goods, energy efficient investments and non-energy goods. An important feature of the model is that the satisfaction generated by the energy good is modeled as a function combining energy consumption and the level of energy efficiency of their housing. Households differ in their labor market productivity and their LEE which are both not observable by the government. The government maximizes a weighted sum of households' utility using a non linear tax that depends on households' observable labor income, energy consumption and energy efficient investments. We first show that the government should distort the price of energy depending upon an index of saturation of energy needs, and an index representing the degree of substitution between energy and LEE in the delivery of energy needs. We show that this condition is related to the size of the rebound effect. Interestingly enough, the condition for which energy should be subsidized at the margin is a sufficient condition guaranteeing that the rebound effect is small. We also show that if households can supplement their initial LEE, the purchase of LEE should always be subsidized and more subsidized than energy consumption.

For tractability, our analysis focuses only on redistributive aspects of energy policy. Its main contribution is to consider both energy retail and energy savings investment in a unified policy framework. It neglects five important points that we now comment.

First, we do not consider environmental externalities generated by energy consumption. This question is important as the social damages caused by pollution from energy consumption has to be borne by the entire population. We thus neglect Pigouvian considerations in our framework. However, as shown by Cremer et al. (2003) in a non linear energy pricing model similar to ours, the Pigouvian element does not alter the progressivity of the energy prices. They indeed show that the non linear price on energy faced by households should also be increasing with respect to income in the presence of externality (a marginal subsidy to poor households may also be optimal if the government is sufficiently adverse to inequality). Our conjecture is that incorporating externalities in our analysis should not change the progressivity of energy prices. Moreover, there would be no reason for energy efficient investment not to be subsidized since redistribution and Pigouvian considerations go hand in hand.

Second, our paper applies to households that are the owners of their dwellings. More specific policies should undoubtedly be implemented in the case of tenants. It is well known that there may be split incentives between landlords and tenants (e.g. see Gillingham et al., 2012) and poor households are more likely to be tenants. An extension of our model should then incorporate the agency problem between landlords and tenants to study this problem.

Third, our paper considers fully rational and informed households. As such it does not take into account that some individuals may make some mistakes in their investment decisions due to inattention or misinformation. Recent papers however find little evidence of this problem (e.g. Allcott et al., 2017) when investing in energy efficient goods. Moreover, more and more countries mandate homeowners (via an audit procedure) to be informed about the energy efficiency level of their dwelling.

Fourth, we have assumed that energy and LEE prices are either set in a competitive market or by a public firm. If the energy good is supplied through a monopolistic market the government may want to "correct" inefficient allocation on top of the redistribution objective and still implement the second best allocation. The resulting implementation through non linear prices may be different in that case. Otherwise, if the firms are constrained to use linear prices and compete on prices (or quantities), the government may want to subsidize this linear price so as to correct for imperfect competition. Now when prices are linear and regulated so that the firm has to break even, the results of Cremer and Gahvari (2017) apply.

Finally, models studying the optimal tax/subsidy of energy consumption are usually framed in a static framework. We regard our analysis as reflecting a long term perspective: investments in energy efficient technonlogy are driven by a lower consumption of energy through substituability in the process of the energy good. One fundamental question that we do not adress here is the longer term decision of housing choice. Taking this choice into account would lead households to choose the type of housing (either as a tenant or an owner). This would be a natural extension of our model since the government would now be able to choose whether to tax or subsidize housing based on the reported level of LEE. An interesting question would then to study whether how energy policies interact with the housing tax treamtent.

References

[1] Akerlof, G. A. (1978), The economics of "tagging" as applied to the optimal income tax, welfare programs, and manpower planning, *The American Economic Review* 68(1), 8-19.

[2] Allcott, H. and M. Greenstone (2017), Measuring the welfare effects of residential energy efficiency programs, NBER Working Paper n° 23386.

[3] Atkinson, A. and J. Stiglitz (1976), The design of the tax structure: direct versus indirect taxation, *Journal of Public Economics* 6, 55-75.

[4] Atkinson, A. (1977), Optimal Taxation and the Direct versus Indirect Tax Controversy, *Canadian Journal of Economics* 10(4), 590-606.

[5] Borenstein, S. (2012), The redistributional impact of non-linear electricity pricing, *American Economic Journal: Economic Policy* 4(3), 56-90.

[6] Cremer, H., Gahvari, F. and N. Ladoux (2003), Environmental taxes with heterogeneous consumers: an application to energy consumption in France, *Journal of Public Economics*, 87, 2791-2815.

[7] Cremer, H. and F. Gahvari (2017), Restoring Ramsey tax lessons to Mirrleesian tax settings: Atkinson–Stiglitz and Ramsey reconciled, *Social Choice and Welfare* 49(1), 11-35.

[8] Drehobl, A. and L. Ross (2016), Lifting the High Energy Burden in America's Largest Cities: How Energy Efficiency Can Improve Low Income and Underserved Communities, ACEEE.

[9] EU Energy Poverty Observatory, European Commission, https://www.energypoverty.eu/about/what-energy-poverty.

[10] Feger, F. and D. Radulescu (2018), Redistribution through Income Taxation and Public Utility Pricing in the Presence of Energy Efficiency Considerations, CESifo Working Paper Series 7195, CESifo Group Munich.

[11] Feldstein, M.S. (1972), Equity and Efficiency in Public Sector Pricing: The Optimal Two-Part Tariff, *The Quarterly Journal of Economics* 86(2), 175-187.

[12] Gahvari, F. and L. Micheletto (2016), Capital income taxation and the Atkinson–Stiglitz theorem, *Economics Letters* 147, 86-89.

[13] Jacobsen, G. D. and M. J. Kotchen (2013), Are building codes effective at saving energy? Evidence from Residential Billing Data in Florida, *The Review of Economics and Statistics* 95(1), 34-49.

[14] Gillingham, K. Harding, M. and D. Rapson (2012), Split incentives and household energy consumption, *Energy Journal* 33(2), 37-62.

[15] Gillingham, K. and K. Palmer (2014), Bridging the energy efficiency gap: policy insights from economic theory and empirical evidence, *Review of Environmental Economics and Policy* 8(1), 18–38.

[16] Kocherlakota, Narayana, R. (2010), The New Dynamic Public Finance. Princeton University Press, 2010.

[17] Mirrlees, J. A. (1971), An exploration in the theory of optimum income taxation, *Review of Economic Studies* 38(114), 175-208.

[18] Munk, K. J. (1977), Optimal Public Sector Pricing Taking the Distributional Aspect into Consideration, *The Quarterly Journal of Economics* 91(4), 639-650.

[19] Naess-Schmidt S., Jespersen, S. T., Termasen, L. B., Winiarczyk, M. and J. Tops (2008), Reduced VAT for environmentally friendly products, Copenhagen Economics.

[20] Saez, E. (2002), The Desirability of Commodity Taxation Under Non-Linear Income Taxation and Heterogeneous Tastes, *Journal of Public Economics* 83(2):217–230.