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## “Imperfect Financial Markets and Investment Inefficiencies”

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# Imperfect Financial Markets and Investment Inefficiencies\*

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## Abstract

We analyze the consequences of noisy information aggregation for investment. Market imperfections create endogenous rents that cause overinvestment in upside risks and underinvestment in downside risks. In partial equilibrium, these inefficiencies are particularly severe if upside risks are coupled with easy scalability of investment. In general equilibrium, the shareholders' collective attempts to boost value of individual firms leads to a novel externality operating through price that amplifies investment distortions with downside risks but offsets distortions with upside risks.

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# 1 Introduction

We analyze the consequences of financial market imperfections for firm decisions and investment when firms maximize shareholder value. Shareholder value maximization is widely viewed as aligning shareholders' private investment returns with social surplus when financial markets are efficient. We instead argue that noisy information aggregation in equity markets can cause shareholders to distort risk-taking and investment decisions in an attempt to capture market rents. We show that even small market imperfections can have severe consequence for investment, either through a high sensitivity of investments to market returns, or through externalities that operate through equity prices and amplify distortions in general equilibrium. Our results suggest a new rationale for regulating financial risk-taking by publicly traded firms even when equity markets operate near efficiency.

Our analysis proceeds in two parts. We first develop a partial equilibrium model of a single firm whose incumbent shareholders make an investment decision prior to selling a fraction of their shares in a financial market populated by informed and noise traders. The share price then emerges as a noisy signal aggregating dispersed investor information about the firm's value.

In our model, the market-clearing share price must partially absorb shocks to demand and supply of securities, since informed traders are not willing or able to perfectly arbitrage perceived gaps between prices and expected fundamental values. This amplifies price fluctuations relative to the information about dividends that is aggregated through the market. The share price is therefore not just a noisy but also a biased estimate of the firm's dividends.

This bias has two important properties: (i) it inherits any asymmetries in underlying cash flow risks, and (ii) it scales with the firm's initial investment decision. Together these two properties result in an endogenous rent-seeking motive for shareholders that distorts corporate investment.

Property (i) implies that expected share prices are generally not an unbiased estimate of expected dividends: if cash flow risks are concentrated on the upside, the excess price fluctuations are primarily on the upside and lead to an upwards bias in average share prices relative to expected dividends. If instead the cash flow risk is

concentrated on the downside, the downside price fluctuations dominate, resulting in a downwards bias of expected share prices. This wedge between the expected market value and the expected dividend value of a firm's equity is a transfer from final to initial shareholders (or vice versa), in other terms, a rent accruing to incumbent shareholders. Importantly, this wedge arises from the way share prices aggregate information, even when there is no firm-specific risk premium embedded in equity returns.

Property (ii) then implies that incumbent shareholders can influence the magnitude of this rent through their investment decision. As our main partial equilibrium result, we show that rent-seeking incentives and investment distortions depend on two characteristics: risk asymmetries and scalability of investments. Firms with upside risks over-invest, while firms with downside risk under-invest. The scalability of investment then determines how flexibly a firm can adjust its investment to the gap between expected fundamentals and market returns. When investment is easy to scale, the surplus from investing is small but the scope for rent-seeking is particularly large. If easy scalability is coupled with upside risk, even small market frictions can induce incumbent shareholders to take excessively large risks purely to capture rents from selling their shares, while the firm in fact generates negative expected surplus. With downside risks, there can be severe under-investment, but surplus always remains positive. We then describe the taxes that implement the efficient investment level.

It is now useful to discuss the interpretation of upside vs. downside risks, and the scalability parameter. Regarding cash-flow risks, what really matters is that the price (being the expectation of the marginal investor) overweights the tails of the realizations of payoffs. In this sense any asymmetry of payoffs in the tails of the cash-flow distribution will result in either upside or downside risks. A natural way to think about this empirically is the comparison between mature value firms versus growth companies, or whether tail risks for firms are more prevalent on the upside (say, an IPO) or on the downside (e.g., bankruptcy). A particular form of upside risk may also come in the form of limited liability and leverage. Our model thus suggests that market imperfections make shareholders prone to over-invest in growth options, but prone to under-value and under-invest in mature firms, or that they value leverage as a way of amplifying the upside risk in share prices.

The scalability parameter can be interpreted as the ability of scaling up investment

to cater to the markets. Investment may not be easily scalable for firms with technologies requiring large fixed capital expenditures, those facing tight collateral constraints, or firms with stronger corporate governance that limits the ease of catering investment to market prices. Investment distortions are then especially pronounced in firms or industries that combine high scalability, or near constant returns to scale, with investment returns that are characterized by upside risks, or high levels of borrowing with limited liability, with financial intermediation a prime example of an industry that combines both features.<sup>1</sup>

In the second part, we embed the single-firm, partial equilibrium setup in an aggregate model with a continuum of heterogeneous firms, each subject to idiosyncratic investment risk. As our main general equilibrium result, we show that equity market imperfections lead to a new externality which operates through share prices and amplifies investment distortions in the case of downside risk but mitigates them in the case of upside risk. The externality arises because shareholders in any given firm do not internalize that by collectively distorting investment to boost their own share prices, they end up with lower aggregate dividends, and lower aggregate market values of equity shares. This introduces an intertemporal wedge that changes investment incentives. We formally derive a connection between the partial and general equilibrium level of investment and show how this wedge changes rent seeking incentives and equilibrium investment depending on the nature of the risks. With downside risk, this wedge reinforces the shareholders' desire to inflate share prices, which amplifies under-investment. With upside risk, it instead reduces the shareholders' desire to inflate market prices, which limits overinvestment and partially restores efficiency. We further show that for highly scalable investments even small incentives to distort investment at the level of each firm may have large consequences in general equilibrium. We then show that the tax that implements the efficient allocation in partial equilibrium has to be modified by a Pigouvian correction to account for the externality in general equilibrium.

Our model of the financial market builds on models of noisy information aggregation (Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981)), or more specifically the formulation in Albagli, Hellwig and Tsyvinski (2022) which characterizes prices for arbitrary securities in a non-linear noisy rational expectations

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<sup>1</sup>See, for example, Philippon (2015).

equilibrium model and argues that such a model can account for cross-sectional asset pricing puzzles. We depart from Albagli, Hellwig and Tsyvinski (2022) in two important aspects. First, we endogenize security cash flows as the outcome of firm’s investment decisions. Second, we embed the firms in a general equilibrium environment. Endogenizing investment and cash flows is challenging even in partial equilibrium because of the interaction between how information aggregation affects investment incentives, and how investment in turn feeds into asset prices, payoffs and information aggregation. These challenges are compounded by the general equilibrium feedback from aggregate share prices to firm level incentives.

Our treatment of general equilibrium effects relates to the growing literature on externalities in financial markets. In our model, the externality results from market imperfections, when individual and aggregate share prices directly enter incumbent shareholder preferences. This is different from the pecuniary externalities commonly identified in the literature on financial constraints, where share prices indirectly affect investment incentives by relaxing or tightening collateral constraints (e.g., Lorenzoni 2008) or incentive constraints (e.g., Farhi, Golosov and Tsyvinski, 2009).<sup>2</sup> With downside risk, our externality has the potential to generate significantly larger aggregate distortions because (i) it affects all firms, rather than a subset of financially constrained firms, and (ii) rather than being the primary source of inefficiency, it amplifies distortions caused by market imperfections. The interaction of trading frictions with externalities operating through price also appears in Asriyan (2021) but in a context where frictions in debt markets amplify balance sheet effects.

We conclude by discussing the empirical relevance of our model. After briefly reviewing evidence of stock market pricing anomalies consistent with our main mechanism, we show that a straightforward extension of our partial equilibrium setting allows us to nest the empirical predictions of two types of models that study the sensitivity of investment to share prices. Specifically, information feedback from stock prices to investment decisions leads to excess price-investment sensitivity and to a negative co-movement of investment with future returns, as information feedback causes shareholders to cater to market expectations of returns. Depending on the realization

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<sup>2</sup>See also Greenwald and Stiglitz (1986), Geanakoplos and Polemarchakis (1986), and Dávila and Korinek (2018), among many other papers.

of fundamentals and liquidity shocks, markets can be overly optimistic or pessimistic about the firm’s return prospects, resulting in excessive investment when share prices are high, and foregone opportunities when share prices are low. Our setting thus delivers empirical predictions of the information feedback models (e.g., Chen, Goldstein and Jiang, 2007) and the catering theory of investment (e.g., Polk and Sapienza, 2009). Finally, David, Hopenhayn and Venkateswaran (2016) argue that noisy information in stock markets can result in large aggregate efficiency losses from misallocation of capital in a general equilibrium economy that combines firm dynamics as in Hopenhayn (1992) with the informational environment of our paper. In contrast we argue that misallocation of capital may be amplified if financial market imperfections cause share prices to provide biased valuations which in turn distort investment incentives.

## 2 Partial equilibrium

In this section, we describe our partial equilibrium model with information frictions in equity markets.

### 2.1 Baseline model

Our model has three stages. In the first stage, incumbent shareholders in a firm decide on an observable investment decision  $k \geq 0$ . In the second stage, they sell a fraction  $\alpha \in (0, 1]$  of the shares to outside investors. At the final stage, the firm’s cash flow  $\Pi(\theta, k) \equiv R(\theta)k - C(k)$  is a function of the investment  $k$  and a stochastic fundamental  $\theta \in \mathbb{R}$ , and paid to the final shareholders. The fundamental  $\theta$  is distributed according to  $\theta \sim \mathcal{N}(0, \lambda^{-1})$ . The return  $R(\cdot)$  on the investment is a positive, increasing function of the firm’s fundamental,  $C(k) = k^{1+\chi}/(1+\chi)$  denotes the cost of investment, and  $\chi \geq 0$  is the scaling parameter that we refer as the firm’s returns to scale or scalability. The expected dividends are given by  $\mathbb{E}(\Pi(\theta, k))$ . The ex-ante efficient investment  $K^*$  maximizes  $\mathbb{E}(\Pi(\theta, k))$ .

**Stage 2: Description of the Market Environment.** There are two types of outside investors: a unit measure of risk-neutral informed traders, and noise traders.

Informed traders (indexed by  $i$ ) observe a private signal  $x_i \sim \mathcal{N}(\theta, \beta^{-1})$ , which is i.i.d. across traders (conditional on  $\theta$ ). After observing  $x_i$ , an informed trader submits a price-contingent demand schedule  $d_i(\cdot) : \mathbb{R} \rightarrow [0, \alpha]$ , to maximize expected wealth  $w_i = d_i \cdot (\Pi(\theta, k) - P)$ . That is, informed traders cannot short-sell, and can buy at most  $\alpha$  units of the shares.<sup>3</sup> An informed trader's strategy is then a function  $d(x_i, P) \in [0, \alpha]$  of the private signal and the price. We assume that demand  $d(x_i, P)$  is non-increasing in price  $P$  which is naturally satisfied if trading takes place through limit orders.

Noise traders place an order to purchase a random quantity  $\alpha\Phi(u)$  of shares, where  $u \sim \mathcal{N}(0, \delta^{-1})$  is independent of  $\theta$ .

The aggregate demand for shares is  $D(\theta, P) = \int d(x, P)d\Phi(\sqrt{\beta}(x - \theta)) + \alpha\Phi(u)$ , where  $\Phi(\sqrt{\beta}(x - \theta))$  represents the cross-sectional distribution of private signals  $x_i$  conditional on  $\theta$ , and  $\Phi(\cdot)$  denotes the cdf of a standard normal distribution. The orders submitted by informed and noise traders are executed at a market-clearing price  $P$  such that  $D(\theta, P) = \alpha$ .

Let  $H(\cdot|x, P)$  denote the traders' posterior cdf of  $\theta$ , conditional on observing a private signal  $x$ , and a market-clearing price  $P$ . A *noisy Rational Expectations Equilibrium* at stage 2 consists of a demand function  $d(x, P)$ , a price function  $P(\theta, u; k)$ , and posterior beliefs  $H(\cdot|x, P)$ , such that  $d(x, P)$  is optimal given the shareholder's beliefs  $H(\cdot|x, P)$ ;  $P(\theta, u; k)$  clears the market for all  $(\theta, u)$  and  $k$ ; and  $H(\cdot|x, P)$  satisfies Bayes' Rule whenever applicable.

**Stage 2: Equilibrium Characterization.** For a given level of investment  $k$ , it is straightforward to characterize the equilibrium share price in the unique noisy Rational Expectations Equilibrium.

**Lemma. *Equilibrium Characterization and Uniqueness.*** Define  $z \equiv \theta + 1/\sqrt{\beta}$ .

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<sup>3</sup>We treat  $\alpha$  as a parameter in the partial equilibrium setting and endogenize it in the general equilibrium setting. It is important to note that all our results carry through if we instead assume symmetric trading bounds, as long as the corresponding modifications to noise trader shocks are made to preserve tractability. In this sense, what matters is that investors face *some* limits to trading, and not whether such limits are more prominent in one direction of trading than the other.



*u. In the unique equilibrium, the market-clearing price function is*

$$P(z, k) = \mathbb{E}(\Pi(\theta, k) | x = z, z). \quad (1)$$

Each informed trader buys a share if the private signal is above a threshold  $\hat{x}(P)$ . The total demand of the informed traders is then

$$\alpha \left( 1 - \Phi(\sqrt{\beta}(\hat{x}(P) - \theta)) \right).$$

Equating the sum of demand of the informed traders and of the uninformed traders ( $\alpha\Phi(u)$ ) with the supply of shares ( $\alpha$ ), a price  $P$  clears the market in state  $(\theta, u)$  if and only if

$$\alpha \left( 1 - \Phi(\sqrt{\beta}(\hat{x}(P) - \theta)) \right) + \alpha\Phi(u) = \alpha,$$

which immediately gives the threshold characterization

$$\hat{x}(P) = \theta + 1/\sqrt{\beta} \cdot u \equiv z.$$

That is, observing  $P$  is informationally equivalent to observing  $z \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$ . Conditional on  $\theta$ ,  $z$  is distributed according to  $z \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$ , while its unconditional distribution is  $z \sim \mathcal{N}(0, \lambda_z^{-1})$ , where  $\lambda_z^{-1} = \lambda^{-1} + (\beta\delta)^{-1}$ .

An intuitive way to understand this result is as follows. The sufficient statistic  $z$  represents the private signal of the trader who must be just indifferent between buying or not buying the stock if the market clears, which summarizes the demand for equity shares through noise traders ( $u$ ) and informed traders ( $\theta$ ). The *identity* of this trader shifts in a systematic way with demand conditions: if informed traders become on average more optimistic (higher  $\theta$ ) or noise trader demand increases (higher  $u$ ), the private signal defining the marginal trader must also increase to keep the market in equilibrium. To keep this marginal trader indifferent, the market price must increase with  $z$  and reveal  $z$  publicly to all market participants or outside observers. Thus,  $z$  acts as a sufficient statistic about the information contained in the price as a public signal, with a precision of  $\beta\delta$ .

The equilibrium share price differs systematically from the expected dividend value  $V(z, k) \equiv \mathbb{E}(\Pi(\theta, k) | z)$ , even though we assumed risk neutrality. Both are character-

ized as expected dividends conditional on the information contained in  $z$ . However, the share price  $P(z, k) = \mathbb{E}(\Pi(\theta, k) | x = z, z)$  also incorporates the market clearing requirement of the equilibrium and thus additionally conditions on  $z$ . That is, it is the expectation of the payoff of an agent who infers  $z$  as the public signal contained in the price and also observes the private signal with the value  $x = z$ . Because the price is equal to the dividend expectations of this marginal trader, it places an additional weight on the signal  $z$  as if it had precision  $\beta + \beta\delta$  (equal to the sum of the private and the price signal precision) compared to the weight of  $\beta\delta$  that would be warranted from its precision as a public signal only when evaluating the expected dividends. Therefore, when  $z$  conveys sufficiently positive news about fundamentals, the price is upwards-biased, while if  $z$  conveys sufficiently negative news the price is biased downwards.

We summarize the discussion of this section as follows. The price represents the expectation of the marginal trader who is indifferent between buying the asset or not. The identity of the marginal trader and hence the aggregate demand is determined by the signal she receives. Market clearing condition requires that the price is therefore a function of the signal of the marginal trader. Thus, the price is equal to the expectation of the dividends conditional on the private signal of the marginal trader and the information content of the public signal (the price) which is also given by the value of the marginal trader's signal. Because the identity of the marginal trader shifts in a systematic way with demand conditions due to market clearing forces (under limited arbitrage), the price reacts more to shocks than the expectation of dividends which uses only the informational content of prices. Notice that this is a general property of noisy REE models, but of little consequence under symmetric payoff commonly studied in the REE literature since the price overreaction for high and low realizations of  $z$  cancels out. With non-linear payoffs however, expected prices and dividends will typically differ, giving way to systematic price premia or discounts and the corresponding distortions to investment decisions, which we explore next.

**Stage 1: Investment Decision** We now describe how the investment decision and the information friction interact.

At the first stage, incumbent shareholders choose  $k$  to maximize the expected value

of their equity:

$$\begin{aligned} & \max_{k \geq 0} \mathbb{E} \{ \alpha P(z; k) + (1 - \alpha) \Pi(\theta, k) \} = \\ & = \max_{k \geq 0} \{ \mathbb{E}(\Pi(\theta, k)) + \alpha \mathbb{E}(P(z; k) - \Pi(\theta, k)) \}, \end{aligned} \quad (2)$$

where  $P(z; k)$  is characterized by (1). The incumbent shareholder's objective differs from expected dividends by the term  $\alpha \mathbb{E}(P(z; k) - \Pi(\theta, k))$ , which is a rent that accrues to incumbent shareholders.

Noisy information aggregation thus introduces a rent-seeking motive into incumbent shareholder preferences. When  $\mathbb{E}(P(z; k) - \Pi(\theta, k)) \neq 0$ , noisy information adds not just noise to stock prices, which would average out from an ex ante perspective, but also a bias. Importantly, the size of the rent is endogenous and its magnitude is influenced by the choice of investment  $k$ .

This rent-seeking motive arises because incumbent shareholders sell a fraction of their equity share at a price that differs in expectation from the shares' expected dividends. In the limit, where the incumbent shareholders keep all their shares (i.e.  $\alpha \rightarrow 0$ ), or in an efficient market (i.e. if  $P(z; k) = V(z; k)$ ), the rent-seeking motive disappears, and incumbent and final shareholder incentives are aligned on maximizing  $\mathbb{E}(\Pi(\theta, k))$ .<sup>4</sup>

The risk-neutral representation of share prices allows us to put additional structure on this rent-seeking motive. Standard arguments of compounding normal distributions imply that

$$\mathbb{E}(P(z; k)) = \int_{-\infty}^{\infty} \Pi(\theta, k) d\Phi(\sqrt{\hat{\lambda}}\theta) \equiv \hat{\mathbb{E}}(\Pi(\theta, k)),$$

for some  $\hat{\lambda}^{-1} > \lambda^{-1}$ . That is, from an ex ante perspective the market attributes too much weight to tail realizations of  $\theta$ , which derives from the fact that the price places larger weight on the signal  $z$  than warranted from its precision as a public signal, as explained above. The parameter  $\hat{\lambda}^{-1}$  depends on  $\beta$ ,  $\delta$ , and  $\lambda$ , and summarizes the severity of market frictions. We will from now on refer to  $\hat{\mathbb{E}}(\cdot)$  as the expectation under the market-implied prior  $\mathcal{N}(0, \hat{\lambda}^{-1})$ .

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<sup>4</sup> $P(z, k) = V(z, k)$  could result for example with free entry of uninformed arbitrageurs as in Kyle (1985), or when there is a public signal  $z$ , but no private information, and no heterogeneity among informed traders, so that they must all be indifferent about buying at equilibrium. This also corresponds to the limiting case of our model with  $\beta \rightarrow 0$ .

The characterization of equilibrium asset prices with noisy information aggregation is by no means specific to the present model with risk-neutral investors and position limit. In Albagli, Hellwig and Tsyvinski (2022), we show that for a general class of noisy REE models –general asset payoffs, investor preferences and position limits that need not be binding– this representation of the share price is equivalent to the conditional expectation of dividends under a risk-neutral probability measure, i.e.  $P(z, k) = \hat{\mathbb{E}}(\Pi(\theta, k) | z) \equiv \mathbb{E}(\Pi(\theta, k) m^I(\theta, z) | z)$ , where  $m^I(\theta, z)$  represents an information-based asset pricing kernel that generalizes the observation that the equilibrium price overweighs the information contained in  $z$ . Hence, the functional form assumptions of risk-neutrality, normal distributions and binding position limits are convenient for comparative statics but not otherwise crucial for our analysis.

We can then define an unconditional pricing kernel  $m^I(\theta) \equiv \mathbb{E}(m^I(\theta, z) | \theta)$ , such that  $\mathbb{E}(P(z; k)) \equiv \hat{\mathbb{E}}(\Pi(\theta, k)) = \mathbb{E}(\Pi(\theta, k) m^I(\theta))$ . In Albagli, Hellwig and Tsyvinski (2022), we argue that under natural regularity conditions  $m^I(\cdot)$  is log-convex, or U-shaped, and thus systematically places higher weight on tail realizations of fundamentals, both in the upper and in the lower tail. As we discuss below, the excess weight placed on tail risks that is captured by  $\hat{\lambda}^{-1} > \lambda^{-1}$  is the key channel through which expected market returns distort investment incentives.

## 2.2 Investment distortions from market frictions

In this section, we characterize investment distortions due to noisy information aggregation in partial equilibrium. Investment and information frictions non-trivially interact. Information frictions engender endogenous rents and thus make payoffs endogenous. Investment decisions depend on and, in turn, also determine the size of the rent. We characterize how this mutual interrelation leads to inefficiencies in investment. We then determine a tax that implements the level of investment that maximizes ex-ante expected dividends.

### 2.2.1 Equilibrium investment distortions

We denote the efficient investment by  $K^*$  such that

$$C'(K^*) = \mathbb{E}(R(\theta)).$$

The initial shareholders instead choose  $\hat{K}$  to equate the marginal cost of investment to a weighted average of expected market return  $\hat{\mathbb{E}}(R(\theta))$  and expected dividend return  $\mathbb{E}(R(\theta))$ :

$$C'(\hat{K}) = \alpha \hat{\mathbb{E}}(R(\theta)) + (1 - \alpha) \mathbb{E}(R(\theta)),$$

or, alternatively,

$$C'(\hat{K}) = \mathbb{E}(R(\theta)) + \alpha \left( \hat{\mathbb{E}}(R(\theta)) - \mathbb{E}(R(\theta)) \right). \quad (3)$$

It then follows that  $\hat{K} \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} K^*$  if and only if

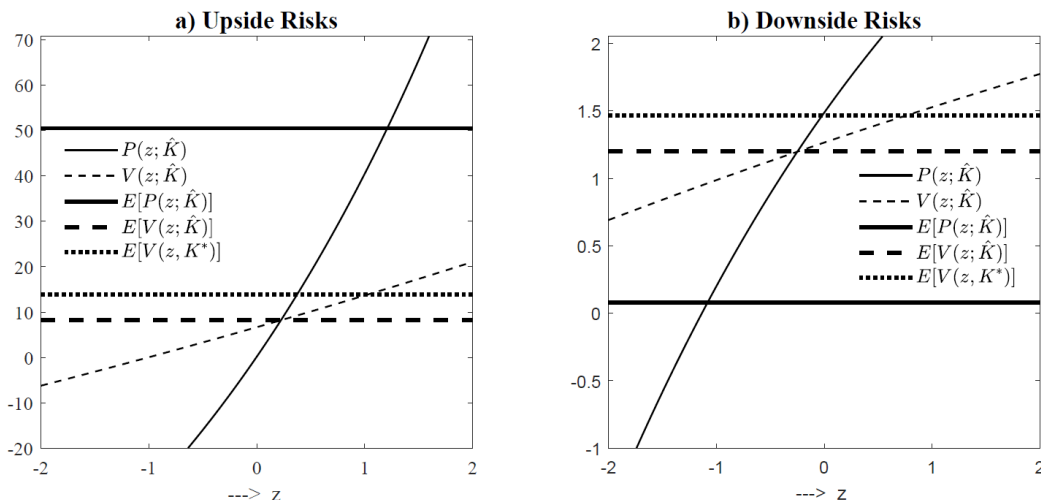
$$\hat{\mathbb{E}}(R(\theta)) \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \mathbb{E}(R(\theta)).$$

Whenever the wedge between the expected price and expected dividends is positive, the initial shareholders find it optimal to overinvest to enhance the over-valuation of their shares. When instead the wedge is negative, the initial shareholders want to under-invest in order to limit the under-valuation of their shares.

We first relate the return ratio and hence the sign of the investment distortion to asymmetry between upside and downside risks. A return  $R(\cdot)$  is *symmetric* if  $R(\theta) - R(0) = R(0) - R(-\theta)$  for all  $\theta > 0$ .  $R(\cdot)$  is *dominated by upside risk* if  $R(\theta) - R(0) \geq R(0) - R(-\theta)$  for all  $\theta > 0$ , and *dominated by downside risk* if  $R(\theta) - R(0) \leq R(0) - R(-\theta)$  for all  $\theta > 0$ . This classification compares gains and losses at fixed distances from the prior median to determine whether risks are concentrated on the upside or on the downside. The differences between upside and downside risks can be determined by comparing firms in different life-cycle stages, such as growth versus value firms, a discussion we return to in Section 4.

If  $R(\cdot)$  is symmetric,  $\hat{\mathbb{E}}(R(\theta)) = \mathbb{E}(R(\theta))$  and investment is undistorted ( $\hat{K} = K^*$ ).

Figure 1: Stage 2 market equilibrium and share mis-pricing



The figure simulates the market equilibrium in stage 2, as a function of  $z$ . For upside risks ( $\Delta > 0$ ) in panel a), the payoff function is  $R(\theta) = a_{up} + \exp(b_{up}\theta)$ , with  $a_{up} = 0.7$  and  $b_{up} = 0.5$ . For downside risks ( $\Delta < 0$ ) in panel b),  $R(\theta) = a_{dn} - \exp(b_{dn}\theta)$ , with  $a_{dn} = 2.5$  and  $b_{dn} = 0.5$ . The rest of parameters are set to:  $\alpha = 0.5, \beta = 1, \lambda = 1$ , and the precision of noise trading  $\delta$  is set to achieve an informational friction parameter of  $\Delta = 0.2$  (upside) and  $-0.2$  (downside).

If  $R(\cdot)$  is dominated by upside risk then  $\hat{\mathbb{E}}(R(\theta)) > \mathbb{E}(R(\theta))$  and the firm over-invests ( $\hat{K} > K^*$ ). If  $R(\cdot)$  is dominated by downside risk then  $\hat{\mathbb{E}}(R(\theta)) < \mathbb{E}(R(\theta))$  and the firm under-invests ( $\hat{K} < K^*$ ).<sup>5</sup> With noisy information aggregation, return asymmetries thus generate a difference between expected share prices and expected dividends, which result in a positive rent (in the case of upside risk) or negative rent (in the case of downside risk): in other words, we interpret the resulting investment distortions as the consequence of a rent-seeking (or rent-avoidance) motive by incumbent shareholders, which arises from the imperfection in equity markets.

Figure 1 shows the price (solid line) and the expected dividend conditional on the market signal  $z$  (dashed line) in stage 2 of our model, as well as their unconditional counterparts (averaging over realizations of  $z$ ). Because the price overweights the market signal  $z$ , it is higher than the expected dividend for high realizations of  $z$ , and lower for low realizations. For the case of upside risks (panel a), this leads to an average

<sup>5</sup>Furthermore, for upside risks, the return ratio  $\hat{\mathbb{E}}(R(\theta)) / \mathbb{E}(R(\theta))$  is strictly increasing in  $\hat{\lambda}^{-1}$ , while for downside risks the return ratio is strictly decreasing in  $\hat{\lambda}^{-1}$ .

price that exceeds the average dividend and an incentive to overinvest by the part of incumbent shareholders at stage 1 in order to exploit the overpricing of shares. This overinvestment amplifies the gap between the expected price and dividend, leading to ex post dividend losses relative to the first-best case without the overinvestment externality. Conversely, downside risks (panel b) lead to an average dividend above the average market price of stage 1, providing incentives to underinvest which also result in dividend losses, on average.

We now represent these investment distortions and the firm's resulting revenue or dividend losses in terms of three easily interpretable parameters: (i) the percentage wedge between market-implied and fundamental returns on investment,  $\Delta \equiv \hat{\mathbb{E}}(R(\theta)) / \mathbb{E}(R(\theta)) - 1$ , (ii) the parameter  $\alpha$ , which captures the share turn-over and thus the weight shareholders give to market prices relative to fundamental values, and (iii) the parameter  $\chi^{-1}$ , which captures the scalability of the firm's investment and thus the extent to which investment responds to the wedge between expected market and fundamental returns. We express the relative over- or under-investment as

$$\frac{\hat{K}}{K^*} = (1 + \alpha\Delta)^{1/\chi},$$

and taking logs,  $\ln(\hat{K}/K^*) \approx \alpha\Delta\chi^{-1}$ , i.e. to a first order the percentage over-investment is given by the product of the percentage return wedge, the share turn-over parameter, and the scalability parameter. Importantly, even small return wedges can generate arbitrarily large over- or under-investment if investments are highly scalable, i.e. the firm operates close to constant returns to scale.

Let  $V(k) = \mathbb{E}(R(\theta)) \cdot k - C(k)$  denote the expected firm value for a given investment level  $k$ . Then we express the expected loss of dividends, relative to the first-best investment  $K^*$ , as

$$\frac{V(\hat{K})}{V(K^*)} = \frac{\hat{K}}{K^*} \left( 1 + \chi^{-1} \left( 1 - \left( \frac{\hat{K}}{K^*} \right)^\chi \right) \right) = (1 + \alpha\Delta)^{1/\chi} (1 - \alpha\Delta\chi^{-1}).$$

It is straight-forward to check that for given  $\chi > 0$  this expression is maximized when  $\alpha\Delta = 0$ . When  $\alpha\Delta < 0$ , dividends remain strictly positive (but they vanish as  $\chi \rightarrow 0$ ),

i.e. the low market-implied returns discourage the incumbent shareholders to take advantage of profitable investment returns. When instead  $\alpha\Delta > 0$ , the firm invests too much and may even generate negative surplus if the over-investment becomes too large, i.e. whenever  $\alpha\Delta > \chi$ .

Again taking logs and using a second-order approximation around  $\ln(\hat{K}/K^*) = 0$ , we obtain  $\ln(V(\hat{K})/V(K^*)) \approx -\frac{1}{2}(1+\chi)\left(\ln(\hat{K}/K^*)\right)^2$ , i.e. the expected loss of dividends due to the distortion is approximately proportional to the square of the investment distortion  $\ln(\hat{K}/K^*) \approx \alpha\Delta\chi^{-1}$ .

We summarize these comparative statics observations in the following proposition

**Proposition 1. *Investment distortions and information frictions in partial equilibrium.***

(i) **Efficient Investment:** *The firm invests efficiently ( $\hat{K} = K^*$ ), if and only if  $\alpha\Delta = 0$ , or in the limit as  $\chi^{-1} \rightarrow 0$  (fixed investment size without scalability).*

(ii) **Downside Distortions:** *If  $\alpha\Delta < 0$ , then  $\hat{K} < K^*$ , and  $V(\hat{K})/V(K^*) \in (0, 1)$ .  $\hat{K}$  is decreasing in  $\chi^{-1}$  and  $|\alpha\Delta|$  with  $\lim_{\chi \rightarrow 0} \hat{K}/K^* = 0$  and  $\lim_{\chi \rightarrow 0} V(\hat{K})/V(K^*) = 0$ .*

(iii) **Upside Distortions:** *If  $\alpha\Delta > 0$ , then  $\hat{K} > K^*$ .  $\hat{K}$  is increasing in  $\chi^{-1}$  and  $\alpha\Delta$  with  $\lim_{\chi \rightarrow 0} \hat{K}/K^* = \infty$  and  $\lim_{\chi \rightarrow 0} V(\hat{K})/V(K^*) = -\infty$ .*

(iv) **Negative Expected Dividends:** *Expected dividends are negative, whenever*

$$\alpha\Delta\chi^{-1} > 1. \tag{4}$$

Proposition 1 shows that the magnitude of investment inefficiencies increases with the expected return wedge  $\Delta$ , the proportion of shares traded  $\alpha$ , and the scalability of investment  $\chi^{-1}$ . The return wedge  $\Delta$  and the proportion of shares traded  $\alpha$  determine the initial shareholders' *incentive* to distort their investment due to the information friction, while the scalability  $\chi^{-1}$  determines their *ability* to do so. Intuitively, scalability may be associated with either technological characteristics or certain corporate/institutional features. For example, firms with higher intangible capital can more easily expand operations, vis-a-vis businesses with large fixed capital expenses and long time-to-build investments which make quick changes in the size of opera-



tions unfeasible. Alternatively, corporate or institutional features such as collateral constraint frictions or stronger corporate governance may limit the ease of catering investment to market prices.<sup>6</sup> With easy scalability (high  $\chi^{-1}$ ), optimal investment is very sensitive to the size of the wedge, and the scope for investment distortions and efficiency losses can become very large. At the other extreme, if marginal costs are very sensitive to  $k$  (low  $\chi^{-1}$ ), investment is not easily scalable, and investment distortions are small.

Furthermore, there is an asymmetry between over- and under-investment. When  $\Delta < 0$  (returns dominated by downside risk) the firm under-invests, i.e. expected market-implied returns are not sufficiently rewarding to induce incumbent shareholders to invest up to the level where marginal investment costs equal marginal fundamental returns on investment. The firm remains profitable (i.e. expected dividends are positive), but it fails to maximize its revenues by under-investing, i.e. it leaves some of its potential surplus on the table, because the market doesn't appropriately value these profitable investment returns.

When instead  $\Delta > 0$  (returns dominated by upside risk), the firm over-invests, i.e. it extends its investment scale beyond the efficient level to “chase” higher expected returns in the stock market. In extreme cases, i.e. when investments are highly scalable, the shareholders' investment incentives become extremely sensitive to small return wedges, and thus the over-investment very large. But in this case, the fundamental surplus of the firm is also very low since marginal costs of investing are nearly constant, and hence the surplus that is lost by over-investing will eventually become so large that the firm turns negative profits. If this is the case, the shareholders knowingly make investments that generate negative expected surplus or “destroy value”, because they expect the market to attach too high rewards to a small chance of high positive returns.<sup>7</sup>

Figure 2 represents investment decisions and their implications for efficiency in Stage 1 of our model, and illustrates the comparative statics for the different cases

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<sup>6</sup>Richardson (2006), for instance, documents how certain corporate governance structures can mitigate inefficient investment of free cash-flows.

<sup>7</sup>The difference between over- and under-investment stems from non-negative investment, i.e. bounded on the downside, but can be arbitrarily large on the upside. The latter implies that profit losses can be arbitrarily large –an arbitrarily large investment scale multiplied by a fixed positive wedge between marginal cost and expected marginal return on investment–, independent of whether the actual return distribution is bounded or unbounded.

described by proposition 1. We plot marginal cost of investment  $C'(k)$ , the expected fundamental returns on investment  $\mathbb{E}(R(\theta))$  and the incumbent shareholders' expected returns on investment  $\mathbb{E}(R(\theta))(1 + \alpha\Delta)$  against the investment level, for high and low values of  $\chi^{-1}$ , and for the cases with  $\Delta > 0$  (over-investment) and  $\Delta < 0$  (under-investment), respectively. The efficient investment  $K^*$  sets  $C'(K^*) = \mathbb{E}(R(\theta))$ , the shareholders' preferred investment  $\hat{K}$  sets  $C'(K^*) = \mathbb{E}(R(\theta))(1 + \alpha\Delta)$ . In all cases, the black triangular area corresponds to the loss in expected dividends that results from the investment distortion. Panel a) (left) considers the case with over-investment (upside risks): the dark grey area corresponds to  $V(K^*)$ , the loss in the expected dividends corresponds to the black area, and the expected dividends  $V(\hat{K})$  to the difference between the dark grey and the black areas. Panel b) (right) considers the case with under-investment (downside risks): the light gray area corresponds to the expected dividends  $V(\hat{K})$ , while the maximal expected dividends  $V(K^*)$  corresponds to the combined light gray and black areas. In both cases, by comparing the panels in the top row with those in the bottom row, we observe that a higher scalability  $\chi^{-1}$  results in a larger gap between  $\hat{K}$  and  $K^*$ , a smaller first-best surplus, and a large loss of surplus due to the investment distortion.

Figure 3 provides further intuition for the comparative statics discussed in Proposition 1, by simulating the ratio between investment in stage 1 and the optimal level ( $\hat{K}/K^*$ ), as well as the implied dividend losses ( $V(\hat{K})/V(K^*)$ ), as a function of the main model parameters. As before, panel a) (left) considers the case with upside risks, and panel b) (right) the case with downside risks. The top part of the figure plots these objects for different values of  $\alpha\Delta$  (recall that both investment and dividends relative to their optimal levels can be expressed as a function of the product  $\alpha\Delta$ ), for a given value of the scalability parameter,  $\chi$ . As  $\alpha\Delta$  increases, the gap between investment and its optimal level (top graphs), as well as the expected dividend losses with respect to the optimum level of investment (bottom graphs), increase. Note that the investment gap and the resulting losses are larger and increase faster as a function of  $\alpha\Delta$  for high scalability (low  $\chi$ ). In the figure, thicker lines are associated with higher scalability of investment (low  $\chi$ ).

To summarize, this section has shown how informational frictions in equity markets distort shareholders' incentives to make risky investments. Investment decisions

Figure 2: Investment distortions and efficiency losses – Conceptual representation

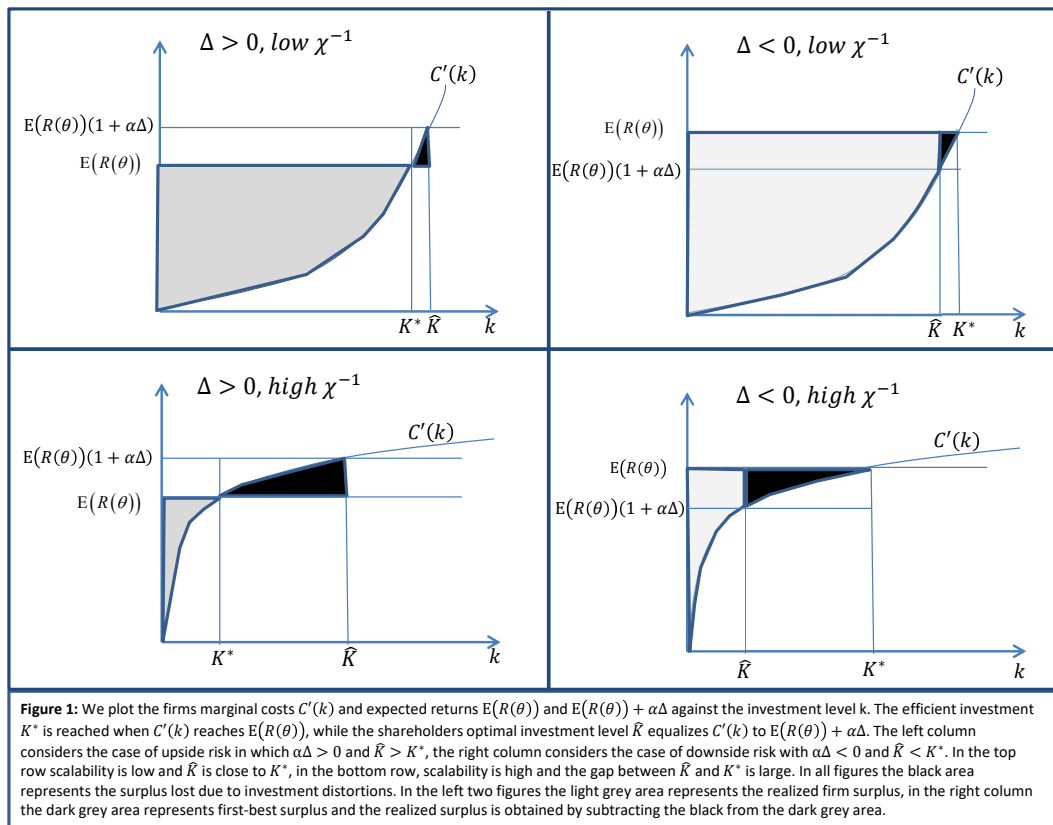
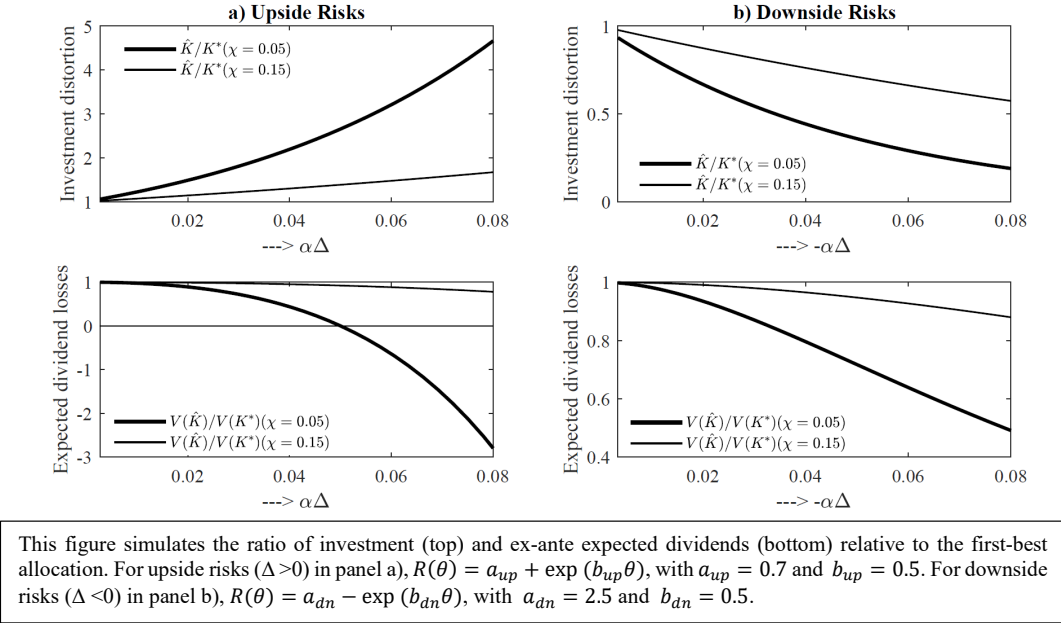


Figure 3: Investment distortions and efficiency losses – Numerical simulation



endogenously determine the size of the rent that arises due to noisy information. The direction and magnitude of the resulting investment distortions and the effects on expected dividends depend on the firms' scalability of investment and on whether returns are characterized by upside or downside risks. If easy scalability is coupled with upside risks, even small frictions in financial markets can have very large consequences – so large, in fact, that the firm may generate negative expected dividends.

### 2.2.2 Implementing efficient investment

In this section, we discuss a simple implementation of the efficient investment with taxes. We first note that trading in the markets can be thought of as a form of a friction in our environment. That is, prohibiting trades in the markets by, for example, setting  $\alpha = 0$  completely eliminates the distortions entailed by the information aggregation frictions. The idea that markets are a form of constraints can be traced to an important paper by Hammond (1987). This is, for example, a relevant restriction in the context of financial intermediation (see Jacklin, 1987, Allen and Gale, 2004, and Farhi, Golosov, and Tsyvinski, 2009). In what follows, we consider policies that indirectly affect the

markets without completely shutting them down.

Consider now a tax,  $\tau$ , that is imposed on the payoff  $R(\theta)k$ . Such a tax modifies the incumbent shareholder's objective function to

$$\begin{aligned} \alpha \hat{\mathbb{E}}((1 - \tau) R(\theta) k - C(k)) + (1 - \alpha) \mathbb{E}((1 - \tau) R(\theta) k - C(k)) &= \\ &= (1 - \tau) (1 + \alpha \Delta) \mathbb{E}(R(\theta)) k - C(k), \end{aligned}$$

and the efficient level of capital  $K^*$  solves the first order condition:

$$(1 - \tau) (1 + \alpha \Delta) \mathbb{E}(R(\theta)) - C'(K^*) = 0.$$

Noting that  $C'(K^*) = \mathbb{E}(R(\theta))$ , we find that a tax  $\tau$  that implements the optimum satisfies

$$\tau = 1 - \frac{1}{1 + \alpha \Delta}. \quad (5)$$

This tax realigns the investment incentives by correcting the effects of the market friction. One can also consider a variety of other policies such as financial transaction taxes that we extensively discuss in the working paper version.

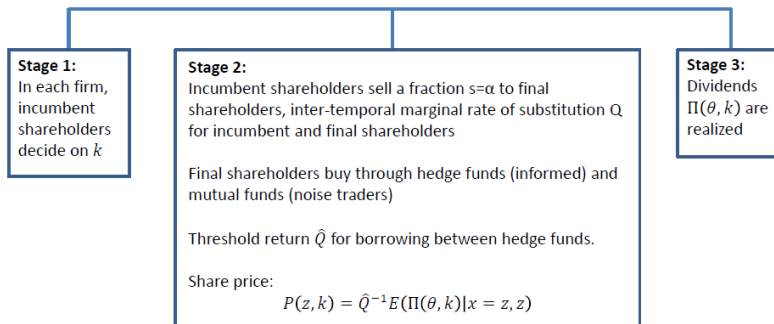
### 3 General equilibrium

In this section, we analyze how individual firms' investment decisions interact in general equilibrium. We identify and characterize a novel externality that operates through share prices, and then we characterize the optimal policy intervention in general equilibrium.

#### 3.1 General equilibrium environment

Our general equilibrium model combines three decision layers: i) the aggregate stock market determines aggregate market value of equity along with the fraction of shares sold; ii) the microstructure of equity markets determines individual firms' share prices; and iii) the incumbent shareholders' stage 1 decision determines investment in each firm. Figure 4 summarizes this timeline of decisions. We now describe each layer.

Figure 4: Timeline of decisions (General Equilibrium model)



There is a unit measure of firms, indexed by  $i$  and characterized by a firm-specific fundamental  $\theta_i$  which is i.i.d. across firms and distributed according to  $\theta_i \sim \mathcal{N}(0, \lambda^{-1})$ , but fully revealed only at the final stage when dividends are realized. These firms are owned and controlled by incumbent shareholders who, in the first stage, choose investment  $k_i$  for each firm. As in Section 2, the firms generate dividends at stage three that are equal to  $\Pi(\theta_i, k_i) \equiv R(\theta_i) k_i - C(k_i)$ .

**Preferences and aggregate market values** At stage 2, incumbent shareholders sell an endogenous fraction  $s$  of shares to final shareholders. This share will, by construction, be the same for all firms, so that at the end, incumbent shareholders hold a fraction  $1 - s$  and final shareholders a fraction  $s$  of the aggregate equity portfolio.

Let the aggregate market value of firms be denoted by

$$T = \int P_i di,$$

the aggregate dividends be denoted by

$$V = \int \Pi_i di.$$

The incumbent shareholders' preferences over stage 2 consumption ( $C_2^I = sT$ ) and stage 3 consumption ( $C_3^I = (1 - s)V$ ) are given by  $v_I(C_2^I) + u_I(C_3^I)$ . The final share-

holders' preferences over stage 2 consumption ( $C_2^F = -sT$ ) and stage 3 consumption ( $C_3^F = sV$ ) is  $C_2^F + u_F(C_3^F)$ , where the functions  $u_I$ ,  $v_I$  and  $u_F$  satisfy standard Inada conditions.

For a given value of  $V$ ,  $T$  and  $s$ , the equilibrium values of  $s$  and  $T$  are uniquely determined from the incumbent and final shareholders' first-order conditions

$$\frac{T}{V} = Q^{-1} = \frac{u'_I((1-s)V)}{v'_I(sT)} = u'_F(sV). \quad (6)$$

Therefore, in the aggregate the financial market aligns the intertemporal marginal rates of substitution of incumbent and final shareholders. The aggregate market value of firms  $T = Vu'_F(sV)$  is equal to aggregate dividends discounted at the incumbent and final shareholders' intertemporal MRS,  $Q^{-1}$ . It will be convenient to assume that incumbent shareholders' preferences are given by  $v_I(C_2^I) + u_I(C_3^I) = \alpha \ln C_2^I + (1-\alpha) \ln C_3^I$  for  $\alpha \in (0, 1)$ , so that their supply of equity shares is inelastic at  $s = \alpha$  and independent of  $T$  and  $V$ .

The main purpose of this part of the microfoundations is to exposit equation (6) that relates the aggregate market value of the firms and the aggregate dividends. This is the only element that is needed for the description of the externality that arises in the general equilibrium setting. One can, otherwise, treat both the intertemporal marginal rate of substitution of the shareholders,  $T/V = 1/Q$ , and fraction of the shares sold,  $s = \alpha$ , as exogenously given.

**Microstructure of the equity markets** We now turn to the micro structure of the equity market. We assume that neither final nor incumbent shareholders have inside information about payoffs, and the incumbent shareholders sell a fixed fraction of each firm, which equals  $\alpha$  given the assumption of log preferences. Final shareholders do not actively manage their investments but invest through two types of funds, mutual funds and hedge funds. As the owner of all hedge funds and mutual funds, the final shareholders indirectly purchase a share  $s$  in the aggregate market portfolio of firms. In equilibrium, the share purchased by the final shareholders  $s$  must equal the share  $\alpha$  sold by the incumbent shareholders. The funds  $sT = s\mathbb{E}(P_i)$  invested by final shareholders are split such that in the aggregate,  $\mathbb{E}(s\Phi(u_i)P_i)$  are invested by mutual funds and

the remainder is allocated to hedge funds.

Mutual funds receive a stochastic inflow of funds and purchase  $s\Phi(u_i)$  fraction of shares in firm  $i$  where  $u_i \sim \mathcal{N}(0, \delta^{-1})$  denotes a random, firm-specific liquidity shock. Our modeling of the mutual funds is similar to that in Allen (1984) who assumes supply noise as that coming from liquidity shocks. One can think of these funds' strategies as purchasing a fixed portfolio of firms' shares, whose overall expenditure varies exogenously with the random inflow/outflow of funds.

Hedge funds on the other hand acquire noisy private information about the different firms' fundamentals and then take positions in specific firms that are deemed sufficiently promising. There is a unit measure of such funds, who each obtain idiosyncratic private signals  $x_i \sim \mathcal{N}(\theta_i, \beta^{-1})$  about each firms' fundamental, after which it decides in which firm to invest. To limit exposure to the risks associated with any individual firm, each hedge fund's positions are limited to no more than  $s$  shares per firm.<sup>8</sup>

Each hedge fund in turn either invests its funds directly in firms by buying up to  $s$  units of equity, or by lending to other hedge funds at a market rate  $\hat{Q}$ . This assumption guarantees that all hedge funds have the same threshold return  $\hat{Q}$  for equity investments. This market rate is a key object of interest in the general equilibrium analysis.

It follows that a hedge fund will purchase  $s$  shares of firm  $i$  if and only if its expectations about that firms' dividend satisfy  $\mathbb{E}(\Pi(\theta_i, k_i) | x, P_i) \geq \hat{Q}P_i$ , resulting in a characterization of an indifference threshold  $\hat{x}(P_i)$  for the private signal that is a monotone function of the price  $P_i$ . As in section 2, a price  $P_i$  clears the market in state  $(\theta_i, u_i)$  if and only if the demand by mutual funds  $s(1 - \Phi(\sqrt{\beta}(\hat{x}(P) - \theta)))$  and hedge funds  $s\Phi(u)$  equals the available supply  $s$ , which immediately gives the threshold characterization  $\hat{x}(P_i) = \theta_i + 1/\sqrt{\beta} \cdot u_i \equiv z_i$ .

The equilibrium price is determined by the indifference of the marginal hedge fund and is represented as a function of  $z_i$ :

$$P_i(z_i, k_i) = \frac{1}{\hat{Q}} \cdot \mathbb{E}(\Pi(\theta_i, k_i) | x = z_i, z_i). \quad (7)$$

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<sup>8</sup>Our assumptions guarantee that the representative final shareholders' equity purchases through hedge funds and mutual funds scale with their aggregate demand for shares.



Suppose for now that all firms make identical investment choices  $k_i = K$ . Aggregating across markets and combining with  $\mathbb{E}(P_i) = T = VQ^{-1}$ , the equilibrium value of  $\hat{Q}$  is

$$\hat{Q} = Q \frac{\hat{\mathbb{E}}(\Pi(\theta_i, K))}{V} = Q \frac{\hat{\mathbb{E}}(\Pi(\theta_i, K))}{\mathbb{E}(\Pi(\theta_i, K))}. \quad (8)$$

Relative to the incumbent and final shareholders' inter-temporal MRS  $Q$ , the threshold return  $\hat{Q}$  is distorted by a wedge  $\hat{\mathbb{E}}(\Pi(\theta_i, K)) / \mathbb{E}(\Pi(\theta_i, K))$  that corresponds to the ratio between the expected market value and the expected dividend of firms.

**Incumbent shareholder's investment decision** Moving back to stage 1, the incumbent shareholders of any given firm  $i$  maximize expected cash flow from equity sales in stage 1 and dividends in stage 2, weighted by their respective marginal utilities and taking the aggregate market values,  $\alpha$ ,  $V$  and  $Q$  as given:

$$\max_{k_i \geq 0} \mathbb{E} \{ \alpha Q P_i(z_i, k_i) + (1 - \alpha) \Pi(\theta_i, k_i) \}. \quad (9)$$

Since firms are ex ante identical, they will all invest the same quantity in equilibrium:  $k = K_{GE}$ .

**Equilibrium definition** General equilibrium allocations are then fully defined by values for  $Q$ ,  $\hat{Q}$ , and  $K_{GE}$ , such that (i)  $k_i = K_{GE}$  solves the incumbent shareholders' first-order condition for the firm's stage 1 investment choice in (9) for given  $Q$ , (ii) the threshold return for hedge funds  $\hat{Q}$  in stage 2 satisfies equation (8), and (iii) the shareholders' intertemporal marginal rate of substitution  $Q$  satisfies the incumbent and final shareholders' first-order condition for aggregate equity sales (6) in stage 2. Given these values, equilibrium share prices in stage 2 satisfy equation (7) for each firm.

### 3.2 Investment distortions in general equilibrium: an externality

We now describe the main result of this section – an externality that arises in the general equilibrium model of information aggregation and endogenous investment.

Substituting (7) in (9), the firm's optimization problem becomes

$$\max_{k_i \geq 0} \left\{ \alpha \frac{Q}{\hat{Q}} \hat{\mathbb{E}}(\Pi(\theta_i, k_i)) + (1 - \alpha) \mathbb{E}(\Pi(\theta_i, k_i)) \right\}. \quad (10)$$

Hence, as in our partial equilibrium model, the incumbent shareholders maximize a weighted average of share price and dividend value. However, the relative weight on these two objectives depends not just on the fraction of shares sold  $\alpha$ , as in the partial equilibrium, but also on the ratio  $Q/\hat{Q}$  which represents the wedge between the shareholders' intertemporal marginal rate of substitution and the interest rate  $\hat{Q}$  faced by hedge funds in equilibrium – a ratio that shareholders take as given when choosing  $k_i$ , but which in turn depends on the aggregate choice  $K_{GE}$  through equation (8). If financial markets were efficient (in the sense that  $P_i(z_i, k_i) = \hat{Q}^{-1} \cdot \mathbb{E}(\Pi(\theta_i, k_i) | z_i)$ ), it would follow that  $\mathbb{E}(P_i(z_i, k_i)) = \hat{Q}^{-1} \cdot \mathbb{E}(\Pi(\theta_i, k_i))$ , and hence  $Q = \hat{Q}$ , i.e., hedge fund's interest rate is aligned with the incumbent shareholders' intertemporal marginal rates of substitution, and incumbent shareholders have an incentive to maximize expected dividends, i.e.  $K = K^*$ . The financial market imperfection from noisy information aggregation is thus key for driving a wedge between  $Q$  and  $\hat{Q}$ .

We now characterize how the informational heterogeneity interacts with the investment decisions in our general equilibrium setting. The incumbent shareholders' first-order condition for investment in stage 1 yields

$$\alpha \frac{Q}{\hat{Q}} \left( \hat{\mathbb{E}}(R(\theta)) - C'(K_{GE}) \right) + (1 - \alpha) \left( \mathbb{E}(R(\theta)) - C'(K_{GE}) \right) = 0.$$

Rearranging the terms and using  $C'(K) = K^\chi$  along with  $\mathbb{E}(R(\theta)) = (K^*)^\chi$  yields

$$\frac{K_{GE}}{K^*} = \left( 1 + \alpha \Delta \frac{Q/\hat{Q}}{1 - \alpha + \alpha Q/\hat{Q}} \right)^{1/\chi}, \quad (11)$$

and taking logs,

$$\ln(K_{GE}/K^*) \approx \underbrace{\alpha \Delta \chi^{-1}}_{\text{PE distortion}} \underbrace{\frac{1}{1 + (\hat{Q}/Q - 1)(1 - \alpha)}}_{\text{GE feedback}}.$$

Comparing to the partial equilibrium setting, the general equilibrium model amplifies investment distortions when  $Q/\hat{Q} > 1$ , and dampens them when  $Q/\hat{Q} < 1$ . The percentage over- or under-investment adjusts the partial equilibrium parameter for share turn-over  $\alpha$  for the general equilibrium feedback generated by  $Q/\hat{Q} \neq 1$ . Equivalently, the shareholders' stage 1 first-order condition for investment can be written as

$$\alpha \underbrace{\left(\frac{Q}{\hat{Q}} - 1\right)}_{\text{GE Wedge}} \left(1 + \Delta - \left(\frac{K_{GE}}{K^*}\right)^x\right) + \underbrace{1 + \alpha\Delta - \left(\frac{K_{GE}}{K^*}\right)^x}_{\text{Partial equilibrium}} = 0. \quad (12)$$

This equation augments the partial equilibrium distortion by an adjustment for the intertemporal wedge that arises in general equilibrium, which satisfies

$$\frac{Q}{\hat{Q}} = \frac{\mathbb{E}(\Pi(\theta_i, K_{GE}))}{\hat{\mathbb{E}}(\Pi(\theta_i, K_{GE}))} = \frac{\chi + 1 - \left(\frac{K_{GE}}{K^*}\right)^x}{(1 + \chi)(1 + \Delta) - \left(\frac{K_{GE}}{K^*}\right)^x}. \quad (13)$$

Equations (11) and (13) describe a system of two equations in two unknowns,  $\frac{K_{GE}}{K^*}$  and  $\frac{Q}{\hat{Q}}$ , for given share turn-over parameter  $\alpha$ , scalability  $\chi^{-1}$  and expected return wedge  $\Delta$ . Here, equation (11) summarizes the optimal investment decisions for a given intertemporal wedge, while equation (13) summarizes the general equilibrium feedback from firm-level investment decisions to the intertemporal wedge. In the appendix we show that there exists a single solution to the pair of equations (11) and (13) that satisfies these additional conditions.

Investment choices in individual firms exert an externality on each other through their effect on the equilibrium interest rate  $\hat{Q}$ . For a given  $\hat{Q}$ , incumbent shareholders in a specific firm gain from distorting investment to increase the market value of their own shares, but they do not internalize that if all firms engage in this behavior, then aggregate dividends will be lower, which lowers the shareholders' intertemporal marginal rate of substitution  $Q$  and the aggregate market value of firms  $T$ . The equilibrium value of  $Q$  feeds back into the firm incentives, amplifying or dampening the rent-seeking incentives through the intertemporal investment wedge  $Q/\hat{Q}$  that influences the relative weight associated with the share price.

Substituting equation (13) into (10), the firm problem is restated as

$$\max_{k_i \geq 0} \left\{ \alpha \frac{\hat{\mathbb{E}}(\Pi(\theta_i, k_i))}{\hat{\mathbb{E}}(\Pi(\theta_i, K_{GE}))} \mathbb{E}(\Pi(\theta_i, K_{GE})) + (1 - \alpha) \mathbb{E}(\Pi(\theta_i, k_i)) \right\}.$$

This expression shows that while individual shareholders in each firm have a strong incentive to raise the equity value of their own firm  $\hat{\mathbb{E}}(\Pi(\theta_i, k_i))$  relative to the aggregate market value of equity  $\hat{\mathbb{E}}(\Pi(\theta_i, K_{GE}))$ , such rent-seeking incentives at the micro-level turn out to be self-defeating in the aggregate. This feedback is similar to the collateral channel in Lorenzoni (2008) or the private trades channel of Farhi, Golosov and Tsyvinski (2009). However, the origin of the externality is different, as it emerges from market imperfections due to information heterogeneity rather than incentive problems. In addition, the share price directly enters the firms' objective thereby affecting incentives to invest, rather than affecting investment indirectly through incentive constraints or financial constraints.

Let  $K_{PE}$  denote the investment level in partial equilibrium with  $Q = \hat{Q}$ . Two possible cases arise. With a positive return wedge ( $\Delta > 0$ , corresponding to upside risks), it is immediate that  $Q < \hat{Q}$ , and therefore the general equilibrium wedge dampens partial equilibrium rent-seeking and investment distortions. As a result, there is less investment compared to the partial equilibrium level, and consequently less over-investment ( $K_{PE} > K_{GE} > K^*$ ).

In contrast, with a negative return wedge ( $\Delta < 0$ , corresponding to downside risks), we have  $Q > \hat{Q}$  and therefore the general equilibrium wedge amplifies rent-seeking and investment distortions. As a result, there is even more under-investment than in partial equilibrium, i.e. we have  $K_{GE} < K_{PE} < K^*$ .

In both cases, the amount of investment is lower than in the partial equilibrium. We summarize these results in the proposition that follows, the proof of which is immediate from the previous arguments.

**Proposition 2. *Investment distortions in general equilibrium.*** *The general equilibrium model has a unique solution. Investment is lower in general equilibrium compared to partial equilibrium:  $K_{GE} < K_{PE}$ . In the case of the upside risks ( $\Delta > 0$ ), overinvestment is offset compared to the efficient investment. In the case of the down-*

side risks ( $\Delta < 0$ ), underinvestment is amplified compared to the efficient investment.

To gain further intuition we now consider the limiting case when investment is highly scalable, or  $\chi$  is close to 0. This case allows us to explore whether the small distortions may lead to large consequences in the general equilibrium setting.

**Proposition 3. Investment distortions for small  $\chi$ .**

(i) **Bounded distortions with upside risk:** If  $\Delta > 0$ , then for small  $\chi$ ,  $\frac{K_{GE}}{K^*} \approx e^\alpha$ ,  $\lim_{\chi \rightarrow 0} V(K_{GE})/V(K^*) = (1 - \alpha)e^\alpha < 1$ , and  $\lim_{\chi \rightarrow 0} Q/\hat{Q} = 0$ .

(ii) **Unbounded distortions with downside risk:** If  $\Delta < 0$ , then for small  $\chi$ ,  $\frac{K_{GE}}{K^*} \approx e^{1-\alpha}(1 + \Delta)^{1/\chi}$ ,  $\lim_{\chi \rightarrow 0} V(K_{GE})/V(K_{PE}) = 0$ , and  $\lim_{\chi \rightarrow 0} Q/\hat{Q} = \infty$ .

Without an intertemporal distortion introduced by general equilibrium, i.e. if  $Q = \hat{Q}$ , firms set marginal costs equal to  $C'(k) = \alpha \hat{\mathbb{E}}(R(\theta)) + (1 - \alpha) \mathbb{E}(R(\theta))$ .

With upside risk ( $\Delta > 0$ ), there is an upwards distortion in price, which implies that the equilibrium interest rate faced by hedge funds is larger than shareholder's marginal intertemporal substitution:  $\hat{Q} > Q$ . The over-investment by firms reduces the incumbent shareholders' intertemporal marginal rate of substitution  $Q$  by more than the threshold return for hedge funds  $\hat{Q}$ , which in turn reduces the weight shareholders attribute to the expected market returns, thus partially offsetting the partial equilibrium rent-seeking motive by increasing the incumbent shareholders' weight on maximizing expected dividends.

Moreover, since expected dividends must remain positive in general equilibrium (otherwise final shareholders will refuse to buy and the equity market no longer fulfills its role of fostering intertemporal trade between incumbent and final shareholders), the extent of over-investment cannot become too large. In other words, marginal costs  $C'(K_{GE})$  cannot stray too far from  $\mathbb{E}(R(\theta))$ , and in the limit as  $\chi \rightarrow 0$ ,  $C'(K_{GE})$  must converge to  $\mathbb{E}(R(\theta))$ . In this limit, the intertemporal wedge becomes large ( $Q/\hat{Q} \rightarrow \infty$ ), dividends remain positive, yet strictly lower than at the first best, and investment remains distorted up by a factor  $e^\alpha$  in the limit. Hence, the general equilibrium effects offset a large part of the partial equilibrium investment distortion but do not restore the expected dividends to the first-best case entirely. Depending on share turn-over  $\alpha$ , the loss relative to first-best can still be very substantial.<sup>9</sup>

<sup>9</sup>Recall that as  $\chi \rightarrow 0$ ,  $V(K_{PE})/V(K^*) \rightarrow -\infty$  and  $\hat{K}/K^* \rightarrow \infty$ . Negative dividends are not

With downside risk ( $\Delta < 0$ ), the distortion in the price is downwards,  $Q < \hat{Q}$ . In this case, underinvestment lowers  $\hat{Q}$  by more than  $Q$ , which in turn pushes incumbent shareholders to shift even more weight towards expected share prices, which reinforces the rent-seeking motive, and the associated externality. In the limit as  $\chi \rightarrow 0$ , marginal costs  $C'(k)$  must converge to  $\hat{\mathbb{E}}(R(\theta))$  in order to keep the aggregate market value of equity positive. The amplification thus becomes so strong that it pushes shareholders to invest as if all their shares were sold in the market. Expected dividends vanish even relative to the partial equilibrium benchmark.<sup>10</sup> For low  $\chi$ , intertemporal trade remains sustainable only with a very large inter-temporal wedge  $Q/\hat{Q}$ , and consequently a large amplification of the under-investment relative to the partial equilibrium setting.

Figure 5 simulates the investment distortions and the efficiency losses that arise in general equilibrium. The upper half of the figure plots comparative statics with respect to the informational friction by varying  $\Delta$ , while the lower panel presents the comparative statics with respect to  $\alpha$ . In each case, the magnitudes are compared with the partial equilibrium counterpart. The simulations confirm the key theoretical take-aways: namely, i) investment inefficiencies and expected dividend losses are increasing in the informational frictions  $\Delta$ , shareholders' short-termism  $\alpha$ , and the ease of investment scalability ( $1/\chi$ ); ii) investment inefficiencies and expected dividend losses are dampened by the general equilibrium externality in the case of upside risks, but magnified in the case of downside risks.

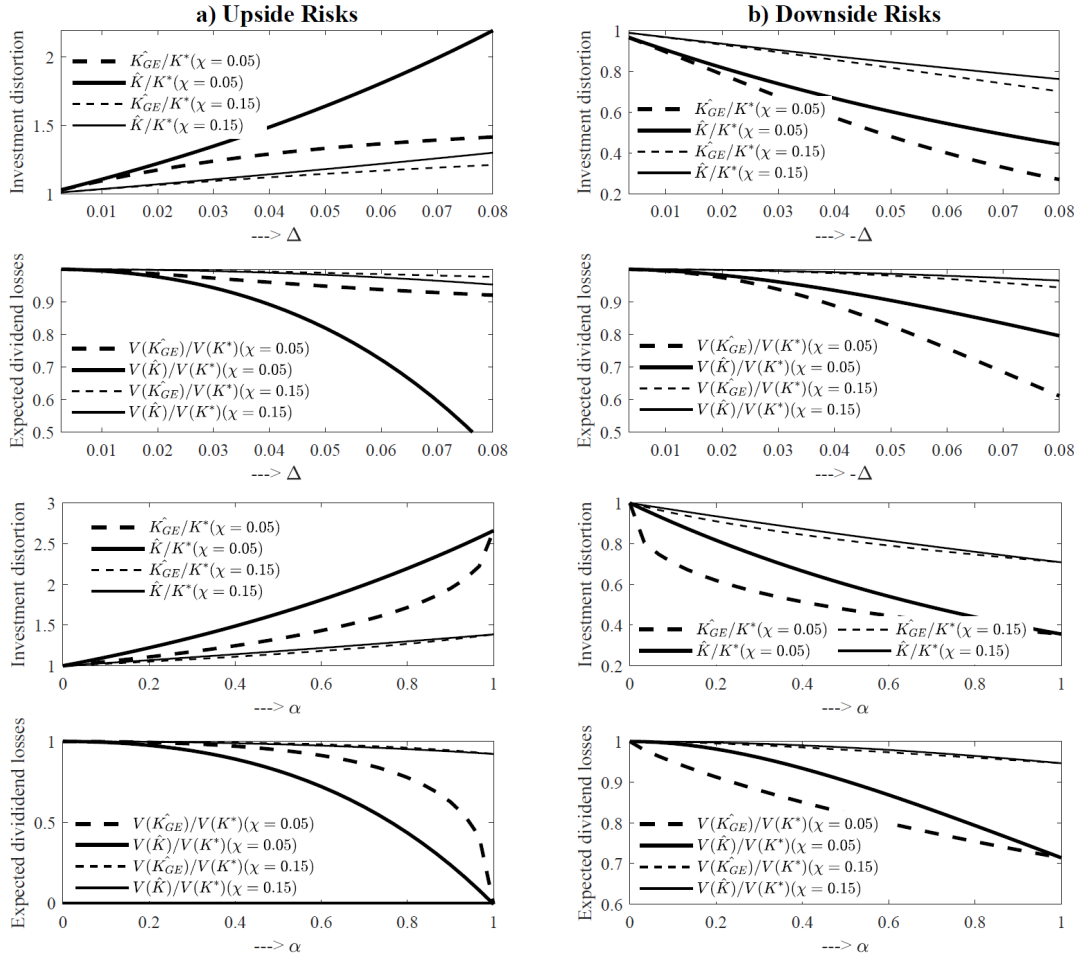
To summarize, in general equilibrium, shareholder rent-seeking generates an externality and an intertemporal investment distortion. The intertemporal distortion partly offsets the externality in the case of upside risk, but reinforces it in the case of downside risk. And once again, scalability of investment determines the severity of investment distortions and the externalities. The limiting results with  $\chi \rightarrow 0$  illustrate that even small imperfections in equity markets can have very dramatic consequences for incentives and investment.

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possible in general equilibrium.

<sup>10</sup>Recall that  $V(K_{PE})/V(K^*) \rightarrow 0$  as  $\chi \rightarrow 0$ . It is straightforward to construct examples in which  $V(K_{PE}) \rightarrow \infty$  as  $\chi \rightarrow 0$ , but  $V(K_{GE}) \rightarrow 0$ , i.e. first-best and partial equilibrium dividends grow infinitely large, yet the realized surplus completely vanishes in general equilibrium.

Figure 5: Investment distortions and efficiency losses – Numerical simulation



The figure simulates the ratio of investment (rows 1 and 3) and ex-ante expected dividends (rows 2 and 4) relative to the first-best allocation. For upside risks ( $\Delta > 0$ ) in panel a,  $R(\theta) = a_{up} + \exp(b_{up}\theta)$ , with  $a_{up} = 0.7$  and  $b_{up} = 0.5$ . For downside risks ( $\Delta < 0$ ) in panel b,  $R(\theta) = a_{dn} - \exp(b_{dn}\theta)$ , with  $a_{dn} = 2.5$  and  $b_{dn} = 0.5$ . In the comparative statics w.r.t  $\Delta$ ,  $\alpha = 0.5$ . In the comparative static w.r.t  $\alpha$ ,  $\Delta = 0.05$  (upside) and  $-0.05$  (downside).

### 3.3 Implementing efficient investment in general equilibrium

We now turn to the analysis of the taxes that implement the efficient allocation. Consider, as in the partial equilibrium, a tax  $\tau$  that is imposed on the payoff  $R(\theta)k$ . Such a tax modifies the incumbent shareholder's objective function to

$$\alpha \frac{Q}{\hat{Q}} \left( (1 - \tau) \hat{\mathbb{E}}(R(\theta))k - C(k) \right) + (1 - \alpha) \left( (1 - \tau) \mathbb{E}(R(\theta))k - C(k) \right).$$

The efficient level of capital  $K^*$  is implemented if the first order conditions for investment are:

$$\alpha \frac{Q}{\hat{Q}} \left( (1 - \tau) (1 + \Delta) \mathbb{E}(R(\theta)) - C'(K^*) \right) + (1 - \alpha) \left( (1 - \tau) \mathbb{E}(R(\theta)) - C'(K^*) \right) = 0.$$

Using  $C'(K^*) = \mathbb{E}(R(\theta))$ , we find that a tax  $\tau$  that implements the optimum satisfies

$$\tau = 1 - \frac{1 - \alpha + \alpha Q/\hat{Q}}{1 - \alpha + \alpha Q/\hat{Q} (1 + \Delta)} = 1 - \frac{1}{1 + \alpha \Delta \left( 1 + (1 - \alpha) \left( \hat{Q}/Q - 1 \right) \right)^{-1}}, \quad (14)$$

where

$$\frac{\hat{Q}}{Q} = 1 + (1 + \chi^{-1}) \Delta$$

represents the intertemporal wedge at the efficient investment level., i.e. when  $K_{GE} = K^*$ . Hence the optimal tax in general equilibrium takes a similar form as in the partial equilibrium model, but adjusts the share turn-over parameter by

$$\frac{1}{1 + \left( \hat{Q}/Q - 1 \right) (1 - \alpha)} = \frac{1}{1 + (1 + \chi^{-1}) (1 - \alpha) \Delta}$$

for the general equilibrium feedback.

The results so far have focused on implementing efficient investment. There still may be an intertemporal wedge in the first order conditions of the initial shareholders, and  $Q$  in general is not equal to  $\hat{Q}$ . Moreover, the discussion here assumes that there are no situations of excess demand or excess supply of equity shares and all sides find



it optimal to trade equity. The working paper version of our work exposits the effects of financial transactions taxes on the shareholders' intertemporal distortions, as well as a variety of other policy interventions.

## 4 Empirical relevance

In this section, we begin with a brief review of the asset pricing literature providing evidence consistent with the relationship between informational frictions and expected returns in stock markets. We then show that a straightforward extension of our partial equilibrium model nests the predictions of two important literatures: the link between real investment and share prices from information feedback theories, and models of catering to investor sentiments. We also discuss recent work which, using our asset market setup, quantifies the general equilibrium consequences of informational frictions for aggregate productivity.

### 4.1 Informational frictions and stock returns

A large empirical asset pricing literature studies the link between informational frictions and stock return anomalies. In an influential paper Diether, Malloy and Scherbina (2002) sort stocks by the dispersion of earnings forecasts across analysts covering each security. They find that stocks in the highest dispersion quintile have monthly returns which are about 0.62% lower than those in the lowest dispersion quintile, amounting to a yearly excess return over 7% for the strategy of going long/short on low/high dispersion stocks. Gebhardt, Lee and Swaminathan (2001) document that an alternative measure of stock risk premia, the cost of capital, is also negatively related to analyst forecast dispersion. Thus, using earnings forecast dispersion as a proxy of informational frictions, these results are consistent with the prediction of our model that larger frictions lead to larger overpricing in securities dominated by upside risks, such as stocks, and thus to lower ex-post returns.

Yu (2011) explores the issue further using bottom-up measures of disagreement for stock portfolios, and then studies the return dynamics associated with time variation in portfolio disagreement. Two results reported in the paper provide support to

our mechanism. First, in line with Diether et al. (2002), an increase in bottom-up portfolio dispersion is associated with a large drop in one-year ahead market returns. Moreover, the paper documents that following an increase in portfolio forecast dispersion, stock prices rise contemporaneously, which is the main driver of ex-post lower portfolio returns, consistent with our mechanism. Second, the author sorts securities into growth and value portfolios, finding that price increases and subsequent low returns are stronger for growth stocks, as predicted by our model if we associate growth firms with cash flows more skewed towards upside risks.

Bassetto and Galli (2019) uses a partial equilibrium setting of our paper to argue for the importance of information in debt crises. There, rather than investment and, hence, rent-seeking endogenously determining the payoffs, it is the repeated interaction and information transmission over time that shapes the payoffs.

## 4.2 Investment feedback and catering theories

A large empirical literature explores the sensitivity of firm decisions, in particular corporate investment, to share prices.<sup>11</sup> One possible explanation for such investment sensitivity to stock prices is information feedback: the share price contains valuable information that helps shareholders and managers make more informed investment decisions.<sup>12</sup> A less positive view on the subject is taken by the catering theory models, stressing how investment managers aim to maximize market valuation by guiding investment towards the opinion of the market, whether such opinion is warranted by fundamentals or not. Indeed, in an influential paper, Polk and Sapienza (2009) show that investment is directly affected by the market deviations from fundamentals. Hoberg and Phillips (2010) also provide evidence for this mechanism, showing that high industry-level stock market valuations coincide with higher investment and new financing, and are subsequently followed by sharply lower operating cash flows and abnormal stock returns in the US, a pattern particularly strong for highly competitive industries.

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<sup>11</sup>See Morck et al. (1990), Baker, Stein and Wurgler (2003), and Gilchrist, Himmelberg, and Huberman (2005).

<sup>12</sup>See Dow and Gorton (1997), Dow and Rahi (2003), Goldstein and Guembel (2008), Foucault and Fresard (2012), and Goldstein, Ozdenoren, and Yuan (2013).

We now allow a direct informational feedback from the share price to investment, and consider how market frictions distort the use of information aggregated through share prices. We modify our benchmark model by assuming that the initial shareholders publicly commit to a price-contingent, or equivalently  $z$ -contingent investment function  $K(z)$ .<sup>13</sup> Market participants perfectly anticipate the investment level that will realize at a given price, and the incumbent shareholders internalize the impact of their decision rule on the share price. We also, for simplicity, set  $\alpha = 1$ , i.e., incumbent shareholders care only about the market value of their equity share.

For a given  $K(z)$ , the equilibrium share price is

$$P(z, K(z)) = \mathbb{E}(R(\theta) | x = z, z) \cdot K(z) - C(K(z)).$$

The expected dividend is

$$V(z, K(z)) = \mathbb{E}(R(\theta) | z) \cdot K(z) - C(K(z)).$$

To illustrate the effect of information feedback, we compare expected dividends and shareholder rents with an increasing investment function  $K(z)$ , with a benchmark in which investment is constant at  $\bar{K} = \mathbb{E}(K(z))$ . The expected dividend takes the form

$$\begin{aligned} & \mathbb{E}(V(z, K(z))) = \\ & = \mathbb{E}(V(z, \bar{K})) + \text{cov}(K(z), \mathbb{E}(R(\theta) | z)) - (\mathbb{E}(C(k(z))) - C(\bar{K})). \end{aligned}$$

The information feedback increases expected dividends by

$$\text{cov}(K(z), \mathbb{E}(R(\theta) | z)) > 0$$

relative to the constant investment case, and it reduces expected dividends by a term

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<sup>13</sup>This requires implicitly that the price function is strictly monotone in  $z$ , a condition that is not automatically satisfied for all  $K(z)$ . Alternatively one may assume that shareholders have the means to infer  $z$  through other means than the price, or that there exists a “non-strategic” component of dividends  $\pi(\theta)$  that is strictly increasing in  $\theta$  and guarantees an upwards-sloping price function. Here we will ignore the invertibility issue, but note that monotonicity is satisfied via an envelope condition for the case of primary interest, where  $\alpha = 1$  and incumbent shareholders maximize expected share price.

due to convexity of costs. The covariance term measures the value of conditioning investment on  $z$ , which strictly exceeds the second term if investment is not too volatile. Expected dividends increase because the information feedback aligns marginal costs and investment more closely with expected returns.

Likewise, we can characterize the effect of information feedback on expected shareholder rents:

$$\begin{aligned} & \mathbb{E}(P(z, K(z))) - \mathbb{E}(V(z, K(z))) = \\ & = \mathbb{E}(P(z, \bar{K})) - \mathbb{E}(V(z, \bar{K})) + \text{cov}(K(z), \mathbb{E}(R(\theta)|x=z, z) - \mathbb{E}(R(\theta)|z)). \end{aligned}$$

If  $R(\cdot)$  is symmetric and  $\mathbb{E}(R(\theta)|x=z, z) \geq \mathbb{E}(R(\theta)|z)$  for  $z \geq 0$ , then this covariance is strictly positive. Information feedback thus generates endogenous upside risk: the firm invests more when  $z$  is high and expected market returns exceed fundamental returns.<sup>14</sup> This reinforces the incumbent shareholders' rent extraction incentive and increases shareholder rents. Moreover, shareholder rents are increasing in the sensitivity of  $K(\cdot)$  to  $z$ . The efficient investment rule sets  $K^*(z)$  such that  $C'(K^*(z)) = \mathbb{E}(R(\theta)|z)$  and incorporates the information contained in the price according to Bayes' Rule. Also, scalability increase the potential value of information feedback, i.e.  $\lim_{\chi \rightarrow 0} \mathbb{E}(V(z, K^*(z))) / V(\bar{K}) = \infty$ , but simultaneously increases shareholder rents, i.e.  $\lim_{\chi \rightarrow 0} \mathbb{E}(P(z, K^*(z)) - V(z, K^*(z))) = \infty$ . Thus even if the original returns are dominated by downside risk, incumbent shareholders in equilibrium capture arbitrarily large positive rents if investment is sufficiently easy to scale up.

Next, we discuss how rent-seeking by incumbent shareholders leads to excess sensitivity of investment to stock prices. Suppose that  $R(\cdot)$  is such that  $\mathbb{E}(R(\theta)|x=z, z) / \mathbb{E}(R(\theta)|z) - 1$  is strictly increasing in  $z$ .

The initial shareholders choose  $\hat{K}(z)$  to satisfy  $C'(\hat{K}(z)) = \mathbb{E}(R(\theta)|x=z, z)$ . Therefore, investment  $K(z)$  is dictated by market expectations of investment returns: investment responds more to  $z$  than would be justified by Bayes' Rule. In effect, information feedback with imperfect equity markets results in a theory of endogenous

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<sup>14</sup>For general return distributions,  $\text{cov}(K(z), \mathbb{E}(R(\theta)|x=z, z) - \mathbb{E}(R(\theta)|z))$  is non-negative and can be arbitrarily large whenever (i)  $K(z)$  is sufficiently responsive to  $z$ , and (ii)  $\mathbb{E}(R(\theta)|x=z, z) > \mathbb{E}(R(\theta)|z)$  for sufficiently large  $z$ . With symmetric or upside risk, information feedback generates or strengthens the upside bias in market prices. With downside risk, information feedback mitigates or overturns downwards bias in prices.

catering effects (see, e.g. Stein 1996). Capital market imperfections distort market valuations, and with information feedback, incumbent shareholders and managers have an incentive to cater investment decisions to these distorted market expectations of returns in an attempt to maximize shareholder rents.

We obtain a positive relation between investment and share prices:

$$\hat{K}(z) = ((1 + 1/\chi) P(z))^{1/(1+\chi)}.$$

Expected returns on equity are

$$\frac{V(z)}{P(z)} - 1 = \frac{1 + \chi}{\chi} \left( \frac{\mathbb{E}(R(\theta) | z)}{\mathbb{E}(R(\theta) | x = z, z)} - 1 \right),$$

and hence decreasing in investment and share price. The following proposition, which follows directly from the derivations above, summarizes the economic effects of information feedback for investment and equity returns.

**Proposition 4. *Information feedback causes excess investment volatility***

- (i) ***Investment is increasing in share prices:***  $cov(\hat{K}(z), P(z)) > 0$ .
- (ii) ***Excess sensitivity of investment to stock prices:***  $\hat{K}(z)/K^*(z)$  is increasing in  $z$ .
- (iii) ***Higher Investment leads to lower equity returns:***

$$cov\left(\hat{K}(z), \frac{V(z) - P(z)}{P(z)}\right) < 0.$$

Our model thus merges the predictions of information feedback theories with models of catering to investor sentiments. Market signals convey valuable information to shareholders. But these signals are not unbiased and result in a catering of investment to market expectations of returns. Information feedback thus results in excess sensitivity of investment, higher expected share prices and shareholder rents, and lower subsequent returns. Proposition 4 summarizes these predictions.

Information feedback gives incumbent shareholders an additional margin along which to optimize their rents. Since shareholder rents are increasing in the sensitivity of investment to  $z$ , they take advantage through an investment rule that caters

to market expectations. This causes excess volatility in investment: on the upside, shareholders over-invest to maximize the rents they extract from inflated share prices. On the downside, they under-invest to limit the losses they incur from the market price being below the fundamental value. Our model thus links investment volatility to stock market volatility by tying investment decisions to market expectations of returns.

Several papers confirm these empirical predictions. First, there is evidence in support of information feedback: Chen, Goldstein and Jiang (2007) find that real investment is more sensitive to share prices in firms whose shares are traded by more informed traders, as measured by PIN (probability of informed trading – Easley et al. (1996)). Roll, Schwartz and Subrahmanayam (2009) provide evidence that deeper options markets for a firm’s share stimulate the entry of informed traders, and that such firms have a higher sensitivity of investment to share prices. These papers suggest that the equity market indeed conveys information about fundamentals that guide corporate investment decisions. Second, Polk and Sapienza (2009) offer direct support for catering effects in corporate investment by estimating the regression coefficients in proposition 4 (i) and (iii). Using discretionary accruals as a proxy for mispricing, they find a positive relation between share overvaluation and investment after controlling for Tobin’s Q.<sup>15</sup> They also find that this relation is stronger for firms with higher share turnover, which can be interpreted as a reasonable proxy for the extent of short-termism in incumbent shareholders, or the fraction  $\alpha$  of shares sold in stage 2 of our model. Moreover, firms with high investment subsequently have low share returns, the more so the larger is their measure of mispricing. This suggests that such investment behavior is indeed inefficient.

Additional support for our theory comes from recent work directly studying the link between proxies of informational frictions and corporate investment efficiency. Chen, Xie and Zhang (2017) document that lower dispersion and/or higher accuracy of analysts’ earnings forecasts increase investment efficiency –increasing (decreasing) investment in firms more likely to under (over) invest. They further show that such effects are stronger in firms with lower institutional stock ownership, another reasonable

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<sup>15</sup>Discretionary accruals measure the extent to which a firm has abnormal non-cash earnings. Firms with high discretionary accruals typically have relatively low share returns in the future, suggesting that discretionary accruals artificially drive up prices temporarily.

proxy for the degree of short-termism in stock trading (parameter  $\alpha$ ).

### 4.3 Information frictions and aggregate efficiency implications

We now address the empirical relevance of informational frictions in general equilibrium. An important paper of David, Hopenhayn and Venkateswaran (2016) augments a general equilibrium model of firm dynamics of Hopenhayn (1992) with the informational environment and the friction of our paper. They carefully calibrate the information friction and argue that it is responsible for 20-50% of the observed dispersion in the marginal (revenue) product of capital, and even a larger fraction if they control for firm-fixed effects.

While their model does not have a rent-seeking motive, such as the one we study, there are important similarities in terms of the general equilibrium mechanism having additional effects compared to those in partial equilibrium. Holding aggregate factors fixed, the informational friction affects aggregate productivity which in turn directly translates to the effects on output. There is, however, an important additional general equilibrium effect – misallocation reduces incentives to accumulate capital and thus amplifies the effects of the informational friction. In our model, an externality operating through the price in general equilibrium has additional effects on capital and the value of the firms compared to the partial equilibrium. Furthermore, we qualify their results showing that depending on the nature of the risk, the general equilibrium effects may either amplify or dampen the partial equilibrium misallocation.

Finally, there is ample evidence that expected stock returns are negatively related to skewness in the cross-section of equity markets (see for instance Conrad, Dittmar and Ghysels, 2013). Salgado, Guvenen and Bloom (2019) document that firm-level employment, sales and productivity growth, along with stock returns, display positive skewness or upside risk in the cross-section during expansions, but negative skewness or downside risk during recessions. Using VAR evidence they further show that an exogenous increase in downside risk tends to be followed by persistent declines in aggregate output and employment. Our model predictions are consistent with these findings, if we interpret an increase in downside risks as a switch from booms to busts throughout the business cycle.

## 5 Conclusion

With unlimited arbitrage, equity markets can be trusted to accurately reflect firm fundamentals. This connection provides the intellectual basis for shareholder value as a measure of social surplus, and for the laissez-faire argument against interference with firm decisions. Its validity as a guiding principle for regulatory policy rests on the unstated assumption that departures from market efficiency cannot be too important, and have at worst minor effects on shareholder incentives.

In this paper we question this assumption by taking a different view of price formation in asset markets that is based on limits to arbitrage and noisy information aggregation. We argue that informational frictions introduce a rent-seeking motive to shareholder value: markets no longer fully align shareholder value with social surplus, and initial shareholders can no longer be trusted to act in the interest of future shareholders or society. Our model links investment incentives to firm-level return asymmetries, share turn-over, and firm-level returns to scale that can be empirically estimated. Importantly, even small departures from market efficiency can have large aggregate consequences either through firm-level scalability of investment, or through the externalities in general equilibrium.

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## 6 Appendix: Proofs

### Proof of Proposition 1:

Results for  $\hat{K}/K^*$  follow directly from the expression  $\hat{K}/K^* = (1 + \alpha\Delta)^{1/\chi}$ . Expected surplus  $\frac{V(\hat{K})}{V(K^*)} = (1 + \alpha\Delta)^{1/\chi} (1 - \alpha\Delta\chi^{-1})$  is maximized when  $\alpha\Delta = 0$ . Moreover,  $\frac{V(\hat{K})}{V(K^*)} \gtrless 0$  whenever  $1 \gtrless \alpha\Delta\chi^{-1}$ . When  $\alpha\Delta > 0$ , it follows that  $\lim_{\chi \rightarrow 0} (1 + \alpha\Delta)^{1/\chi} = \infty$  and  $\lim_{\chi \rightarrow 0} (1 - \alpha\Delta\chi^{-1}) = -\infty$  and therefore  $\lim_{\chi \rightarrow 0} \frac{V(\hat{K})}{V(K^*)} = -\infty$ . When  $\alpha\Delta < 0$ , let  $\psi = -\alpha\Delta\chi^{-1} > 0$  to write  $\frac{V(\hat{K})}{V(K^*)} = (1 - \chi\psi)^{1/\chi} (1 + \psi) \leq e^{-\psi} (1 + \psi)$  and  $\lim_{\chi \rightarrow 0} \frac{V(\hat{K})}{V(K^*)} \leq \lim_{\psi \rightarrow \infty} e^{-\psi} (1 + \psi) = 0$ . ■

### Proof of Propositions 2 and 3:

The first-order condition for  $K_{GE}$  is

$$\alpha \frac{Q}{\hat{Q}} \left( \hat{\mathbb{E}}(R(\theta)) - C'(K_{GE}) \right) + (1 - \alpha) (\mathbb{E}(R(\theta)) - C'(K_{GE})) = 0,$$

which we can rewrite as

$$\left(\frac{K_{GE}}{K^*}\right)^x = \frac{1 - \alpha + \alpha Q/\hat{Q}(1 + \Delta)}{1 - \alpha + \alpha Q/\hat{Q}} = 1 + \Delta \frac{\alpha Q/\hat{Q}}{1 - \alpha + \alpha Q/\hat{Q}}.$$

The inter-temporal wedge  $Q/\hat{Q}$  satisfies

$$\begin{aligned} \frac{Q}{\hat{Q}} &= \frac{(1 + \chi) \mathbb{E}(R(\theta)) - C'(K_{GE})}{(1 + \chi) \hat{\mathbb{E}}(R(\theta)) - C'(K_{GE})} \\ &= \frac{1 + \chi - \left(\frac{K_{GE}}{K^*}\right)^x}{(1 + \chi)(1 + \Delta) - \left(\frac{K_{GE}}{K^*}\right)^x} = 1 - \frac{(1 + \chi) \Delta}{(1 + \chi)(1 + \Delta) - \left(\frac{K_{GE}}{K^*}\right)^x}. \end{aligned}$$

It is then straight-forward to check that when  $\Delta = 0$ , these two conditions hold with  $\left(\frac{K_{GE}}{K^*}\right)^x = \frac{Q}{\hat{Q}} = 1$ .

When  $\Delta > 0$ ,  $\left(\frac{K_{GE}}{K^*}\right)^x \in (1, 1 + \Delta)$  is increasing in  $Q/\hat{Q}$ , with  $\lim_{Q/\hat{Q} \rightarrow 0} \left(\frac{K_{GE}}{K^*}\right)^x = 1$  and  $\lim_{Q/\hat{Q} \rightarrow \infty} \left(\frac{K_{GE}}{K^*}\right)^x = 1 + \Delta$ . Moreover  $Q/\hat{Q} \in (0, \frac{1}{1 + \Delta})$  is increasing in  $\left(\frac{K_{GE}}{K^*}\right)^x$  for  $\left(\frac{K_{GE}}{K^*}\right)^x \in (0, (1 + \chi)(1 + \Delta))$ , with  $\lim_{\left(\frac{K_{GE}}{K^*}\right)^x \rightarrow 0} \frac{Q}{\hat{Q}} = \frac{1}{1 + \Delta}$  and  $\lim_{\left(\frac{K_{GE}}{K^*}\right)^x \rightarrow (1 + \chi)(1 + \Delta)} \frac{Q}{\hat{Q}} = \infty$ . It then follows from continuity that there exists a unique pair  $\left(\left(\frac{K_{GE}}{K^*}\right)^x, \frac{Q}{\hat{Q}}\right)$  that satisfies both conditions, with  $\frac{Q}{\hat{Q}} < \frac{\chi}{\chi + \Delta(1 + \chi)}$  and  $1 < \left(\frac{K_{GE}}{K^*}\right)^x < 1 + \min\left\{\chi, \alpha \Delta \frac{\chi}{\chi + \Delta(1 + \chi)(1 - \alpha)}\right\}$ . In the limit as  $\chi \rightarrow 0$ ,  $\left(\frac{K_{GE}}{K^*}\right)^x \rightarrow 1$  and  $\frac{Q}{\hat{Q}} \rightarrow 0$ .

What's more, combining the two conditions, we have

$$\frac{\alpha}{1 - \alpha} = \frac{\left(\frac{K_{GE}}{K^*}\right)^x - 1}{1 + \Delta - \left(\frac{K_{GE}}{K^*}\right)^x} \frac{(1 + \chi)(1 + \Delta) - \left(\frac{K_{GE}}{K^*}\right)^x}{1 + \chi - \left(\frac{K_{GE}}{K^*}\right)^x}.$$

Since  $\frac{(1 + \chi)(1 + \Delta) - \left(\frac{K_{GE}}{K^*}\right)^x}{1 + \Delta - \left(\frac{K_{GE}}{K^*}\right)^x} \rightarrow 1$  as  $\chi \rightarrow 0$ , it follows that  $\frac{1}{\chi} \left(\left(\frac{K_{GE}}{K^*}\right)^x - 1\right) \rightarrow \alpha$ , and therefore  $\lim_{\chi \rightarrow 0} \left(\frac{K_{GE}}{K^*}\right)^x = e^\alpha$  and  $\lim_{\chi \rightarrow 0} \left(\frac{V(K_{GE})}{V(K^*)}\right) = (1 - \alpha) e^\alpha \in (0, 1)$ .

When  $\Delta < 0$ ,  $\left(\frac{K_{GE}}{K^*}\right)^x \in (1 + \Delta, 1)$  is decreasing in  $Q/\hat{Q}$ , with  $\lim_{Q/\hat{Q} \rightarrow 0} \left(\frac{K_{GE}}{K^*}\right)^x = 1$  and  $\lim_{Q/\hat{Q} \rightarrow \infty} \left(\frac{K_{GE}}{K^*}\right)^x = 1 + \Delta < 1$ . Moreover  $Q/\hat{Q} \in (0, \frac{1}{1 + \Delta})$  is increasing in  $\left(\frac{K_{GE}}{K^*}\right)^x$  for  $\left(\frac{K_{GE}}{K^*}\right)^x \in (0, (1 + \chi)(1 + \Delta))$ , with  $\lim_{\left(\frac{K_{GE}}{K^*}\right)^x \rightarrow 0} \frac{Q}{\hat{Q}} = \frac{1}{1 + \Delta} > 1$  and  $\lim_{\left(\frac{K_{GE}}{K^*}\right)^x \rightarrow (1 + \chi)(1 + \Delta)} \frac{Q}{\hat{Q}} = \infty$ . It then follows from continuity that there exists a unique solution  $\left(\left(\frac{K_{GE}}{K^*}\right)^x, \frac{Q}{\hat{Q}}\right)$  that satisfies both conditions, with  $\frac{Q}{\hat{Q}} > \frac{\chi - \Delta}{\chi(1 + \Delta)}$  and

$1 + \Delta < \left(\frac{K_{GE}}{K^*}\right)^\chi < (1 + \chi)(1 + \Delta)$ . In the limit as  $\chi \rightarrow 0$ ,  $\left(\frac{K_{GE}}{K^*}\right)^\chi \rightarrow 1 + \Delta$  and  $\frac{Q}{\bar{Q}} \rightarrow \infty$ .

Moreover, combining the conditions we obtain

$$\frac{1 - \alpha}{\alpha} = \frac{\left(\frac{K_{GE}}{K^*}\right)^\chi - (1 + \Delta)}{(1 + \chi)(1 + \Delta) - \left(\frac{K_{GE}}{K^*}\right)^\chi} \frac{1 + \chi - \left(\frac{K_{GE}}{K^*}\right)^\chi}{1 - \left(\frac{K_{GE}}{K^*}\right)^\chi}$$

Since  $\frac{1 + \chi - \left(\frac{K_{GE}}{K^*}\right)^\chi}{1 - \left(\frac{K_{GE}}{K^*}\right)^\chi} \rightarrow 1$  as  $\chi \rightarrow 0$ , it follows that  $\frac{1}{\chi} \left(\frac{1}{1 + \Delta} \left(\frac{K_{GE}}{K^*}\right)^\chi - 1\right) \rightarrow 1 - \alpha$ , and therefore  $\lim_{\chi \rightarrow 0} \left(\frac{K_{GE}}{K^*}\right)^\chi = (1 + (1 - \alpha)\chi)(1 + \Delta)$ . To compute the surplus loss in general equilibrium we write  $\frac{V(K_{GE})}{V(K^*)} = \frac{V(K_{GE})V(K_{PE})}{V(K_{PE})V(K^*)}$ , where  $K_{PE} = K^*(1 + \alpha\Delta)^{1/\chi}$  represents the investment level in partial equilibrium, i.e. for  $\frac{Q}{\bar{Q}} = 1$  and fixed  $\alpha$ . Recall from section 2 that  $\frac{V(K_{PE})}{V(K^*)} = (1 + \alpha\Delta)^{1/\chi} (1 - \alpha\Delta\chi^{-1})$  and  $\lim_{\chi \rightarrow 0} \frac{V(K_{PE})}{V(K^*)} = 0$ . To compute  $\frac{V(K_{GE})}{V(K_{PE})}$  note that

$$\begin{aligned} \frac{V(K_{GE})}{V(K_{PE})} &= \left(\frac{K_{GE}}{K_{PE}}\right) \frac{1 + \chi^{-1} \left(1 - \left(\frac{K_{GE}}{K^*}\right)^\chi\right)}{1 + \chi^{-1} \left(1 - \left(\frac{K_{PE}}{K^*}\right)^\chi\right)} \\ &\approx \left(\frac{1 + \Delta}{1 + \alpha\Delta}\right)^{1/\chi} (1 + (1 - \alpha)\chi)^{1/\chi} \frac{1 - \Delta\chi^{-1} - (1 - \alpha)(1 + \Delta)}{1 - \alpha\Delta\chi^{-1}} \end{aligned}$$

Since  $\lim_{\chi \rightarrow 0} (1 + (1 - \alpha)\chi)^{1/\chi} = e^{1 - \alpha}$  and  $\lim_{\chi \rightarrow 0} \frac{1 - \Delta\chi^{-1} - (1 - \alpha)(1 + \Delta)}{1 - \alpha\Delta\chi^{-1}} = \frac{1}{\alpha}$ , it follows from  $\lim_{\chi \rightarrow 0} \left(\frac{1 + \Delta}{1 + \alpha\Delta}\right)^{1/\chi} = 0$  for  $\Delta < 0$  that  $\lim_{\chi \rightarrow 0} \frac{V(K_{GE})}{V(K_{PE})} = 0$ . ■