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“Coordination in the Fight Against Collusion”

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Coordination in the Fight Against Collusion*

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Abstract

While antitrust authorities strive to detect, prosecute, and thereby deter collusive conduct, entities harmed by that conduct are also advised to pursue their own strategies to deter collusion. The implications of such delegation of deterrence have largely been ignored, however. In a procurement context, we find that buyers may prefer to accommodate rather than deter collusion among their suppliers. We also show that a multi-market buyer, such as a centralized procurement authority, may optimally deter collusion when multiple independent buyers would not, consistent with the view that "large" buyers are less susceptible to collusion.

Keywords: collusion, cartel, auction, procurement, reserves, sustainability and initiation of collusion, coordinated effects
JEL Classification: D44, D82, H57, L41

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Collusive conduct has long been recognized as harmful, both in the academic literature and in practice.[1] This has led most major jurisdictions to adopt antitrust laws and set up enforcement agencies dedicated to deterring collusion. In addition to these efforts, competition authorities provide advice to procurement officials on strategies to deter collusion.[2] However, the implications of such delegation of deterrence have largely been ignored. In particular, it is not clear whether procurement officials have the right incentives and tools to deter collusion. Prompted by this, we study the private costs and benefits of deterrence. We show that relying on market participants may result in socially insufficient deterrence, particularly because of free-riding problems among them. In addition, in a setting with multiple independent buyers, even when buyers succeed in deterring collusion, they may fail to coordinate on the optimal deterrence strategy. This suggests that antitrust enforcement by governments and legal authorities may be even more important than hitherto thought.

Specifically, we study a procurement context in which buyers face the possibility of collusion by suppliers, which is a setting of practical relevance for both public and private procurers.[3] We further assume that, if it occurs, collusion takes the form of a market allocation, whereby suppliers coordinate on who refrains from bidding below the reserve in a given market. Such schemes are among the most common collusive practices,[4] and have spurred the development of dedicated policy task forces (e.g., the U.S. Procurement Collusion Strike Force created in 2019) and detailed procurement guidelines.[5] The large and repeated public purchases prompted by the COVID pandemic, as well as recent evidence on the human cost of bid rotation schemes affecting public procurement,[6] further

[1]See, e.g., Smith (1776) and Stigler (1964) and, e.g., the U.S. Federal Trade Commission's discussion of the Sherman Act of 1890 (https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws/antitrust-laws).

[2]The U.S. DOJ (2015*a*) encourages procurement officials to, among other things, expand the list of bidders, require non-collusion affidavits, maintain records that might show a pattern of bid allocation or rotation, and press vendors to explain and justify their prices. The Australian competition authority's guide for government procurement officials provides advice for "competitive tender design" (ACCC, 2019). The OECD has recommendations on fighting bid rigging in public procurement, including a "checklist for designing the procurement process to reduce risks of bid rigging" (OECD, 2016).

[3]Kawai and Nakabayashi (2022) document large-scale collusion by construction firms participating in Japanese government procurement auctions. Executives at optical disk drive manufacturer Hitachi-LG Data Storage pleaded guilty to felony charges that they "conspired with co-conspirators to suppress and eliminate competition by rigging bids for optical disk drives sold to Dell Inc. and Hewlett-Packard Company (HP) and/or fixing prices for optical disk drives sold to Microsoft Corporation" (U.S. Department of Justice Press Release, December 13, 2011, https://www.justice.gov/opa/pr/three-hitachi-lg-data-storage-executives-agree-plead-guilty-participating-bid-rigging-and).

[4]According to the U.S. DOJ's antitrust primer on price fixing, bid rigging, and market allocation schemes, "Most criminal antitrust prosecutions involve price fixing, bid rigging, or market division or allocation schemes" (U.S. Department of Justice, 2015*b*, pp. 1-2). See also U.S. DOJ (2015*a*).

[5]See, e.g., U.S. DOJ (2015*b*), OECD (2016), ACCC (2019), and U.K. CMA (2020).

[6]Barkley (forth.) quantifies the cost in terms of human life of a bid rotation scheme used by a four-

emphasize the relevance of these issues.

Deterrence tools available to buyers include auction format, procurement timing, and reserves, among other things.[7] We assume that buyers use their most advantageous auction format and procurement timing, and focus on the role of reserves in deterring collusion. When taking collusion or competition as given, buyers find it optimal to set a more aggressive reserve in the case of collusion because they end up paying the reserve more often.[8] However, buyers can also *deter* collusion by setting sufficiently aggressive reserves. Because asymmetry impedes collusion, this is best achieved with a differentiated treatment of markets. Still, deterrence comes at the cost of inefficiently low volumes of trade due to the lower reserves. Consequently, deterrence is optimal only when collusion is somewhat fragile, namely, if the discount factor is not too large; otherwise buyers are better off by accommodating collusion and setting the reserve accordingly.

The comparative statics of the buyers' optimal reserves with respect to the discount factor exhibit three regions. For small values of the discount factor, collusion is blockaded by setting the optimal reserve for competition, which is independent of the discount factor. However, as the discount factor increases, collusion is no longer blockaded by the competitive reserve, and deterrence becomes optimal. Because collusion becomes easier as the discount factor increases, the required deterrence reserves decrease with the discount factor. Eventually, for a sufficiently large value of the discount factor, accommodation becomes the best strategy for the buyers, at which point the optimal reserves increase discontinuously and no longer depend on the discount factor.

We also show that independent buyers may not have the correct incentives to deter collusion. Indeed, it is possible that, in any equilibrium, independent buyers only achieve deterrence with suboptimal reserves compared to those that an integrated buyer would choose, or that they fail to deter collusion altogether when an integrated buyer would deter it. Because optimal deterrence entails asymmetric reserves, the interests of independent buyers are not aligned over the set of deterrence reserves. Each buyer favors the reserve that is closest to the competitive one, and the buyer who is supposed to set the less favorable reserve is more prone to switch to accommodation. As a result, the buyers may achieve deterrence only with suboptimal reserves or fail altogether to deter collusion.

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firm bidding ring in the Mexican insulin market, where disjoint procurement divisions used first-price sealed-bid auctions with an identical reserve price. The ring's designated winner submitted a bid at (or just below) the reserve price, while the others submitted slightly higher bids. The government's successful deterrence strategy involved removing entry restrictions and consolidating procurement under a centralized authority.

[7]See, e.g., Aubert, Kovacic and Rey (2006) on the value of enhanced deterrence tools, such as incentives for whistleblowers, that require governmental support.

[8]Specifically, trade takes place under the same circumstances under competition and collusion (namely, when a supplier has a cost below the reserve) but, conditional on trade taking place, the buyer always pays the reserve under collusion, whereas it can benefit from a lower price under competition.

By contrast, there is never a coordination failure in accommodating collusion; hence, an integrated buyer deters collusion in more circumstances than independent buyers would. This is consistent with a view that larger buyers are less vulnerable to collusive conduct by their suppliers.

In addition to analyzing the sustainability of collusion, our focus on market allocations enables us to also study the initiation of collusion. In our setup, a supplier can signal its intent to collude by bidding at the reserve—a credible signal that the supplier had a low enough cost to compete, but chose not to do so, thus indicating a natural allocation of the market. We find that it is profitable to initiate collusion in this way whenever collusion is sustainable.

There is a considerable literature on bid rigging through the use of allocation schemes (see, e.g., Harrington, 2006; Marshall and Marx, 2012).[9] Indeed, Stigler (1964, p. 46) recognizes the effectiveness of customer allocations as a collusive scheme, noting that, relative to fixing market shares, "Almost as efficient a method of eliminating secret price-cutting is to assign each buyer to a single seller." The possibility of deterring bidding rings through defensive measures has also been extensively studied. Our findings related to auction formats and information disclosure are broadly consistent with the literature (e.g., Klemperer, 2002; Kovacic et al., 2006; Marshall and Marx, 2009; Kumar et al., 2015; Marshall, Meurer and Marx, 2014; Marx, 2017), and those on the ability of aggressive reserves to deter bid rigging echo existing results (e.g., Graham and Marshall, 1987; Thomas, 2005; McAfee and McMillan, 1992; Kirkegaard, 2005; Larionov, 2021).[10] However, much less work has been done on the costs of implementing defensive measures and the optimal deployment of defensive measures taking into account both the benefits and costs of doing so,[11] which is the focus of our paper.

An exception is Zhang (2022), who studies optimal collusion in a repeated first-price auction setup, and finds that lower reserves can be used to deter collusion, and that the auctioneer may nevertheless optimally choose to accommodate rather than deter collusion.

[9] Rey and Stiglitz (1995) show that exclusive territories, which are a type of market allocation, can reduce competition. Recent empirical evidence of bid rigging based on geographic market allocations is found in Barrus and Scott (2020). See Kawai et al. (forth.) on using bid rotation and incumbency to detect collusion.

[10] Abdulkadiroğlu and Chung (2003) consider auction design in the face of colluding bidders that optimize their collusive mechanism based on the auction design. In contrast, we take as given a collusive mechanism based on a market allocation.

[11] In a repeated first-price procurement setup in which cartel members observe each other's costs and use an optimal collusive scheme, Chassang and Ortner (2019) show that the buyer can mitigate collusive effects by setting a minimum bid because that restricts the ability of cartel members to use price wars to punish deviations. They focus on the implications of this observation for detecting collusion and do not address the cost to the buyer of imposing minimum bids. Calzolari and Spagnolo (2022) consider a repeated procurement setting with a single buyer in which deterring supplier collusion is costly in terms of reduced noncontractible quality.

Our paper differs in that we focus on a setup with multiple markets and collusion based on a market allocation, allowing us to study, among other things, the value of asymmetric reserves across markets and the possibility of coordination (or miscoordination) of reserves across markets. Furthermore, we analyze the effects of deterrence strategies on buyer surplus and discuss policy implications.

Finally, our analysis also relates to the literature on multi-market contact. In a setting in which firms can collude symmetrically in every market, Bernheim and Whinston (1990) show that multi-market contact helps collusion only if markets are asymmetric.[12] In contrast, we consider a procurement setting in which the winner takes all; as a result, multi-market contact supports collusion even when the markets are symmetric.

The remainder of the paper is organized as follows. Section 1 contains the setup. We analyze tradeoffs between accommodating and fighting collusion in Section 2. In Section 3, we discuss implications for competition policy. Section 4 concludes the paper.

1 Setup

We consider a discrete-time, infinite-horizon setting with two suppliers and two markets.[13] The markets could correspond to distinct customer segments, products, services, or geographic areas; they can be operated by an integrated buyer or two independent buyers with one buyer for each market.

At the beginning of each period, the suppliers draw their costs from a distribution, which is denoted by G and has finite positive continuous density g over the support $[\underline{c}, \overline{c}]$ and increasing reversed hazard rate $G(c)/g(c)$.[14] We assume that each supplier has the capacity to serve both markets and that its cost is the same for both markets.[15] Cost draws are independent across suppliers and time and are the suppliers' private information.

In every period, each buyer wishes to make a purchase, for which it has value $v > \underline{c}$. Buyers rely on first-price auctions, with a reserve equal to $r \in (\underline{c}, \min\{\overline{c}, v\}]$—reserves outside this range are dominated for the buyers.[16] An integrated buyer commits to a pair of reserves (r_1, r_2), one for each market. For the case of two independent buyers, each

[12] Byford and Gans (2014) compare collusion within versus across markets in a similar setup.

[13] In Online Appendix OA-B, we extend the model to allow more than two markets. As we show there, as the number of markets increases, deterrence is optimal for a smaller range of discount factors, as long as one holds fixed whether there is an odd or even number of markets.

[14] This is equivalent to G being log concave. The family of distributions with this property is large and includes most of the "standard" distributions such as the uniform, normal, exponential, power, and extreme value distribution. See Bagnoli and Bergstrom (2005, Table 1) for a more comprehensive list.

[15] The analysis extends to the case of market-specific costs.

[16] While we present our analysis in the context of procurement auctions, it applies equally to the case of sales auctions when suppliers, each one constituting a "market," repeatedly sell scarce resources (e.g., annual wine auctions).

buyer commits to a reserve for its market.

We assume that, in each period, suppliers bid simultaneously, both within and across procurements, and that bids are publicly observed before the next procurement. All agents are risk neutral with quasi-linear utility and discount the future according to the common discount factor $\delta \in [0,1)$. All of the above is common knowledge. The case of $\delta = 0$ corresponds to a one-shot setting, in which case the Bayes Nash equilibrium is unique (Lebrun, 1999).

1.1 Market allocation

We restrict attention to collusive schemes that do not involve the communication of private information and that do not involve transfers between the suppliers. In our setup, as shown below, an optimal collusive scheme within this class is a market allocation, whereby the suppliers alternate in taking the role as the "designated" supplier for each market, and so we focus on such schemes. If, in a given period, supplier 1 is designated for market 1 and supplier 2 is designated for market 2, then they switch roles for the next period, with supplier 1 being designated for market 2 and supplier 2 being designated for market 1. The designated supplier bids slightly below the reserve when its cost lies below it, and at cost otherwise, and the non-designated supplier bids the reserve whenever its cost lies below it, and bids at cost otherwise.[17] Any deviation results in competitive conduct thereafter.

While the joint profit of the suppliers and the payoff of the buyer are the same under a rotation and a fixed market allocation that always designates supplier 1 for market 1 and supplier 2 for market 2 (or vice versa),[18] a rotation is easier to sustain when reserves are asymmetric. A rotation does not eliminate the asymmetry between the two markets, but allows the supplier with the higher gain from a deviation (namely the supplier currently assigned to the less profitable market) to face the higher loss from foregone future collusion because it will be assigned to the more profitable market in the next period. By contrast, in case of a fixed market allocation, that supplier would also face the lower loss. Rotation therefore helps to attenuate the asymmetry among the suppliers' incentives to deviate.

1.2 Payoffs

Suppliers. For a supplier with cost c, let $\pi^m(c,r) \equiv \max\{0, r-c\}$ denote the payoff in a market with reserve r under monopoly and $\pi^c(c,r) \equiv \mathbb{E}_{\tilde{c}}[\max\{0, \min\{r,\tilde{c}\} - c\}]$

[17] Specifying that the non-designated supplier should not bid at all (or, equivalently, bid above the reserve) would reduce expected collusive profits, which would make collusion more difficult to sustain. For further analysis of profitable tacit collusive schemes without explicit communication, see Skrzypacz and Hopenhayn (2004).

[18] The suppliers can share ex ante the expected profit from collusion by randomizing over the initial market designation.

denote the interim expected payoff under competition, where $\tilde{c}$ denotes the rival's cost.[19] Likewise, let

$$\overline{\pi}^m(r) \equiv \mathbb{E}_c\left[\pi^m(c,r)\right] = \int_{\underline{c}}^{r} G(c)dc \quad \text{and} \quad \overline{\pi}^c(r) \equiv \mathbb{E}_c\left[\pi^c(c,r)\right] = \int_{\underline{c}}^{r} G(c)\left[1-G\left(c\right)\right]dc,$$

respectively, denote the supplier's expected per-market payoff under monopoly and competition. The supplier's benefit from collusion is then

$$B(r) \equiv \overline{\pi}^m(r) - \overline{\pi}^c(r) = \int_{\underline{c}}^{r} G^2(c)dc, \tag{1}$$

which is positive and increasing in r.

The designated supplier obtains the monopoly profit and the other supplier, when its cost is c, obtains an interim expected payoff equal to $\pi^n\left(c,r\right) \equiv \left[1-G(r)\right]\pi^m\left(c,r\right)$, which accounts for the probability $1-G(r)$ that the designated supplier's cost exceeds r. The non-designated supplier's expected payoff is therefore $\overline{\pi}^n(r) \equiv \mathbb{E}_c\left[\pi^n\left(c,r\right)\right] = \left[1-G(r)\right]\overline{\pi}^m(r)$, which involves some sacrifice: the cost of collusion for the non-designated supplier is given by

$$C(r) \equiv \overline{\pi}^c(r) - \overline{\pi}^n(r) = \int_{\underline{c}}^{r} [G(r)-G(c)]G(c)dc, \tag{2}$$

which is positive and increasing in r.

Notwithstanding this countervailing effect, the monotonicity of the hazard rate ensures that the benefit of collusion exceeds the cost:

Lemma 1. *A market allocation is jointly profitable for the suppliers: $B(r) > C(r)$.*

Proof. See Appendix A.1.

The proof of Lemma 1 uses the monotonicity assumption on the reversed hazard rate. This corresponds to the condition identified by McAfee and McMillan (1992) for the profitability of optimal collusive mechanisms without communication.[20] Their analysis abstracts from enforcement issues, but accounts for suppliers' private information.

Remark: Optimality of rotation schemes. As shown by Skrzypacz and Hopenhayn (2004) in the context of an infinitely repeated game like ours, when restricting attention to

[19] $\mathbb{E}_{\tilde{c}}\left[\max\{0, \min\{r, \tilde{c}\} - c\}\right]$ is the payoff of a bidder with cost c in the dominant strategy equilibrium of a second-price auction with reserve r; by the payoff equivalence theorem, this is the same as the payoff of the same bidder in a first-price auction.

[20] Allowing for optimal collusive schemes when firms are allowed to communicate, Athey, Bagwell and Sanchirico (2004, Proposition 5) show that sufficiently patient firms bid the reserve price if the distribution is log-concave.

collusive schemes that are independent of history, a market rotation scheme is optimal absent transfers.[21] Thus, without transfers and history-dependent collusive strategies, our focus on market rotation schemes does not restrict the scope for collusion. Furthermore, because of our assumption that G/g is increasing, it follows from Athey, Bagwell and Sanchirico (2004, Section 6) that reversion to static Nash equilibrium induces the worst symmetric perfect public equilibrium.

Buyers. Compared with competitive bidding, collusion does not affect trade, which takes place whenever at least one supplier's cost lies below the reserve; however, it harms the buyer, by forcing it to trade at the reserve.

Specifically, when the suppliers bid competitively, the buyer pays the reserve only when exactly one cost lies below it, which occurs with probability $2[1-G(r)]G(r)$; when instead both costs lie below the reserve, the buyer pays the higher cost, which is distributed with probability density function $2G(c)g(c)$ (derived from a cumulative distribution function equal to $G^2(c)$). The buyer's payoff under competitive bidding is therefore equal to:

$$U^{Comp}(r) \equiv 2(v-r)[1-G(r)]G(r) + 2\int_{\underline{c}}^{r}(v-c)G(c)g(c)dc. \tag{3}$$

By contrast, in case of collusive bidding, the buyer always pays the reserve and its expected payoff is therefore equal to:

$$U^{Coll}(r) \equiv (v-r)\hat{G}(r), \tag{4}$$

where $\hat{G}(c) \equiv 1-[1-G(c)]^2$ is the cumulative distribution function of the lower cost. The monotonicity of the hazard rate ensures that both types of payoff are strictly quasi-concave in the reserve.[22]

Welfare. Collusion also reduces welfare, because the more efficient supplier is no longer always selected. Interestingly, the welfare loss is entirely borne by the non-designated supplier,[23] as the harm to the buyer coincides with the benefit for the designated supplier;

[21] For example, to see why a rotation is easier to sustain than a stationary market allocation, it suffices to note that the supplier currently assigned to the market with the weakly lower reserve—and, thus, weakly more prone to deviate—is rewarded with the weakly more profitable market in the next tender.

[22] We have $dU^{Comp}(r)/dr = \hat{g}(r)[v-r-G(r)/g(r)]$ and $dU^{Coll}(r)/dr = \hat{g}(r)[v-r-\hat{G}(r)/\hat{g}(r)]$, where $\hat{g}$ is the density associated with $\hat{G}$; the monotonicity of the hazard rate $G(r)/g(r)$, which implies that of $\hat{G}(r)/\hat{g}(r)$, thus ensures that $U^{Comp}(r)$ and $U^{Coll}(r)$ are both strictly quasi-concave in r.

[23] Formally, the cost of collusion for the non-designated supplier is $C(r)$. The change in social welfare is equal to the change in the cost of production, which is $\int_{\underline{c}}^{r} cdG(c) + [1-G(r)]\int_{\underline{c}}^{r} cdG(c)$ under collusion and $\int_{\underline{c}}^{r} cd\hat{G}(c)$ under competition. Integrating by parts and simplifying confirms that the difference in costs is indeed $C(r)$.

indeed, integrating (3) by parts, this harm can be expressed as:

$$U^{Comp}(r) - U^{Coll}(r) = \int_{\underline{c}}^{r} G^2(c)dc = B(r). \tag{5}$$

The reason is that the designated supplier benefits from collusion if and only if both costs are below the reserve, which are precisely the instances in which the buyer benefits from competitive bidding. In other words, from the perspective of the designated supplier and the buyer, the market allocation is merely a transfer.

1.3 Scope for collusion

We now characterize the conditions under which collusion can arise, for given reserves. We first study the sustainability of collusion, before addressing initiation.

Sustainability. A market allocation is sustainable if and only if:

$$L(r_1, r_2, \delta) \geq S(r_2) \quad \text{and} \quad L(r_2, r_1, \delta) \geq S(r_1), \tag{6}$$

where the *long-term stake* $L(r_i, r_j, \delta)$ represents the benefit of future collusion for the supplier currently designated for market i, whereas the *short-term stake* $S(r_j)$ reflects the gain from deviating in the other market. Because the supplier will switch to market j in the next period and then keep rotating, the long-term stake is given by:

$$L(r_i, r_j, \delta) \equiv \frac{\delta}{1-\delta^2}\big(B(r_j) - C(r_i) + \delta\left[B(r_i) - C(r_j)\right]\big).$$

The gain from deviating in a market is maximal when the supplier has the lowest possible cost, $\underline{c}$; therefore, the *short-term stake* is

$$S(r_j) \equiv \pi^m(\underline{c}, r_j) - \pi^n(\underline{c}, r_j) = G(r_j)(r_j - \underline{c}). \tag{7}$$

The supplier currently designated for the market with the lower reserve has more to gain from deviating in the other, more profitable market, but also more to lose from forgoing the benefit of collusion in that market in the next period. Intuitively, one would expect the former, immediate effect to dominate the second, delayed one, all the more so when suppliers place less weight on the future. The next lemma confirms these intuitions and shows that, as a result, the temptation to deviate is greater for the supplier currently assigned to the less profitable market and decreasing in the discount factor:

Lemma 2. *Collusion is sustainable if and only if $\delta \geq \hat{\delta}(\mathbf{r})$, where the discount factor threshold $\hat{\delta}(\mathbf{r})$ is the unique solution to $L(\underline{r}, \overline{r}, \delta) = S(\overline{r})$, with $\underline{r}$ and $\overline{r}$, respectively, de-*

noting the lower and the higher of the two reserves.

Proof. See Appendix A.2.

Initiation. To study the incentives to *initiate* a market allocation, suppose that a supplier signals its willingness to engage in a market allocation by bidding the reserve.[24] The long-term stake amounts to switching from competition to collusion in future tenders, and is therefore the same as for sustainability. The short-term sacrifice amounts to forgoing the competitive profit and instead obtaining the profit expected from bidding the reserve when the other supplier bids competitively, $\pi^c(c,r) - \pi^n(c,r)$, which decreases with the initiator's cost. It follows that initiation is profitable whenever it is so for the lowest possible cost, which amounts to the same condition as (6), but with $S(r)$ replaced by $\hat{S}(r) \equiv \pi^c(\underline{c},r) - \pi^n(\underline{c},r)$.[25] Because $\pi^c(\underline{c},r) < \pi^m(\underline{c},r)$, $\hat{S}(r) < S(r)$, we have the following result:

Proposition 1. *Initiation is profitable whenever collusion is sustainable.*

In light of Proposition 1, in what follows we focus on sustainability.

1.4 Defensive measures

Market allocations can be difficult to detect, especially given suppliers' incentives to disguise their conduct. However, suspicions may be aroused by certain bidding patterns, such as bids that are consistently close to the reserve, or a supplier withdrawing from a market. Various tools can be used to "fight back" when suspecting collusion, although buyers may differ in their ability to use these tools based, for example, on their sophistication, commitment ability, and purchase volumes. Three tools particularly emphasized by the literature are auction format, timing of purchasing, and reserves.[26]

With regard to auction format, first-price sealed-bid auctions are generally considered to be more robust to collusion than second-price or ascending-bid auctions because in the latter cases the designated supplier can bid at cost (and yet obtain the monopoly price), which reduces the gain from deviations. To see that it is indeed the case in our setup, suppose that buyers rely instead on second-price sealed-bid auctions. This auction format allows the designated supplier to bid at cost, which limits the other supplier's gain from a

[24] Empirical evidence on initiation of collusion through price signals is also provided by, for example, Alé-Chilet (2017) and Byrne and de Roos (2019).

[25] Sustainability conditions must hold for *every* cost realization, including the lowest one. By contrast, collusion could be initiated for some cost realizations even if it could not be initiated for the lowest cost realization; insisting that initiation must be profitable for every cost realization is thus conservative and may overstate its difficulty.

[26] For a wider ranging discussion of tools for fighting bid rigging, see, e.g., Cassady (1967); Graham, Marshall and Richard (1996); Thomas (2005); Kovacic et al. (2006); Albano et al. (2006).

deviation and reduces the short-term stake, from $S(r)$ to $\hat{S}(r)$; as a result, collusion is sustainable whenever it is profitable to initiate it. With regard to the timing of purchasing, simultaneous procurements are considered to be less prone to collusion than sequential procurements, as deviations can be punished sooner in the latter case.[27] Iossa et al. (2022) confirms this intuition in our setting by showing that, when allowing for arbitrary (constant) lags between the auctions held in the two markets, synchronous procurements are indeed the least prone to a market allocation. Thus, consistent with buyers' adopting a defensive procurement format, we focus on first-price sealed-bid auctions and synchronous procurements.

The literature has recognized that aggressive reserves can be used to respond to and potentially deter collusion, but little attention has been paid so far to the costs of deterring collusion in this way and to whether these costs outweigh the benefits, which is what we analyze in the next section.

Before doing so, we briefly discuss the strand of related literature that derives optimal mechanisms in the face of collusion when the cartel members have access to an internal commitment device and transfers, including Che and Kim (2006, 2009), and Che, Condorelli and Kim (2018). For example, Che and Kim (2006) derive an optimal selling mechanism that is robust to collusion and show that under certain conditions the designer's expected profit is the same as in an optimal auction without collusion. The designer accomplishes this by leveraging the commitment power of the cartel members against themselves. In contrast, in our approach based on market allocation schemes, the cartel members' power derives from the repeated game, while the buyers, taking the auction format as given, can vary the reserve prices to change the incentive compatibility constraint necessary for collusion.

2 To accommodate or fight collusion?

We now study the optimal reserve policy, taking into account its impact on the scope for collusion. To this end, we suppose that, in each market, the buyer initially sets the reserve, which remains in place forever.[28] The suppliers then repeatedly interact, colluding if a

[27]This goes against the view, sometimes expressed in practice, that sequential procurements are less prone to collusion. For example, in the context of the AT&T–Time Warner merger, the judge found that "the *staggered*, lengthy industry contracts would make [coordination] extremely risky" (emphasis added) because one party would have to "jump first" on the hope that the other would do the same later on, concluding that "putting such blind faith in one's chief competitor strikes this Court as exceedingly implausible!" (Leon, 2018, p. 163).

[28]The assumption that the reserve is fixed forever is arguably restrictive. Applying the insights of Frezal (2006) to our setting, collusion can be deterred almost without cost if the buyer can commit to the nonstationary policy of setting the reserve r^C for a predetermined, large number of periods, and preventing trade subsequently for a sufficiently large number of periods. By choosing both numbers large

market allocation is sustainable, and competing otherwise. For the sake of exposition, we adopt the tie-breaking rule that competition prevails in the boundary case where the sustainability condition is binding.[29]

2.1 Competitive and accommodation reserves

Let r^C and r^A, respectively, denote the optimal reserves under competition and under collusion, which we refer to as the *competitive* and *accommodation* reserves. The reserve r^C, which maximizes $U^{Comp}(r)$, corresponds to the monopsony price that a buyer would charge, given its valuation v, if it were to face a single supplier with cost distribution G.[30] Perhaps more surprisingly, r^A, which maximizes $U^{Coll}(r)$, is the reserve that would be optimal if the two suppliers had merged (Loertscher and Marx, 2019), a situation equivalent to perfect collusion. A market allocation does not achieve perfect collusion because production does not necessarily occur at the lowest cost; however, this is immaterial for the buyer: what matters is whether production occurs, in which case the buyer pays the reserve, and it occurs in exactly the same instances as under perfect collusion, namely, when at least one supplier has a cost below the reserve. It follows that r^A is the monopsony price that a buyer would charge when facing a single supplier with the cost distribution $\hat{G}$, which corresponds to the lower of two draws. This distribution has a lower reversed hazard rate than the original distribution G, which makes the supply less elastic and leads to a more aggressive reserve:[31]

Proposition 2. *A buyer's optimal reserve is strictly more aggressive when facing colluding rather than competing suppliers: $r^A < r^C$.*
Proof. See Appendix A.3.

Proposition 2 is consistent with the advice given to practitioners that "[w]hen collusion among suppliers is suspected, the reserve price should be set at a lower value than in the absence of collusion. This simple policy forces the bidding ring to submit a lower bid" (Albano et al., 2006, p. 282). This suggests that collusion may not only result in an inefficient allocation among ring members, but also induce buyers to adopt less efficient procurement practices.

enough, the discounted cost of no trade will be negligible. This nonstationary policy would, however, require a particularly strong form of commitment.

[29] This ensures that optimal deterrence is achieved at the boundary, rather than for reserves "arbitrarily close" to the boundary.

[30] As is well known, the optimal reserve does not depend on the number of suppliers; hence, it maximizes the monopsony payoff $G(r)(v - r)$.

[31] It is interesting to note the similarity and subtle difference relative to Blume and Heidhues (2004). Their analysis implies that in a one-shot, second-price auction, any reserve below $\bar{c}$ eliminates supra-competitive Bayes Nash equilibria, whereas with $r \geq \bar{c}$, there are a continuum of noncooperative Bayes Nash equilibria. In our setting, the optimal reserve in the face of collusion is more aggressive than with competitive bidding—and always strictly lower than the reserve.

Remark: on the nature of collusion. The collusive scheme that we consider enables the non-designated supplier to step in when trade would otherwise not occur. An alternative collusive scheme that requires the non-designated supplier to withdraw regardless of its realized costs amounts instead to a reduction in the number of suppliers. Because the optimal reserve is independent of the number of bidders, buyers' optimal reserves are then the same as under competition. Hence, the optimal reserve depends not only on whether suppliers collude, but also on the nature of the collusive scheme; it can, moreover, be lower under a more efficient collusive scheme.

2.2 Deterrence reserves

More aggressive reserves can also serve a buyer faced with the threat of collusion by making collusion less profitable for the suppliers and thereby unsustainable. For example, for symmetric reserves equal to r, the sustainability condition (6) boils down to

$$\frac{\delta}{1-\delta} \geq \frac{S(r)}{B(r) - S(r)},$$

where the left-hand side is strictly increasing in δ. Furthermore, while decreasing the reserve reduces suppliers' profits under both collusion and competition, for low enough reserves, we show in the proof of Lemma 3 that the *expected* profitability of collusion, $B(r)-C(r)$, decreases at a faster rate than the deviation gain for an efficient supplier, $S(r)$; this is because the reserve is likely to prevent future trade, whereas an efficient supplier always trades. It follows that for sufficiently low reserves, the critical discount factor for symmetric reserves, $\hat{\delta}_S(r) \equiv \hat{\delta}(r,r)$, where $\hat{\delta}(\cdot)$ is defined in Lemma 2, is decreasing in the reserve. A similar argument applies when there is a unique market, which amounts to setting the other reserve to $\underline{c}$, yielding a critical discount factor for a unique market of $\hat{\delta}_U(r) \equiv \hat{\delta}(\underline{c}, r)$.

Lemma 3. *There exists $\overline{r} \in (\underline{c}, \overline{c}]$ such that for $r \in (\underline{c}, \overline{r}]$, both $\hat{\delta}_S(r)$ and $\hat{\delta}_U(r)$ are strictly decreasing in r and $\hat{\delta}_U(r) > \hat{\delta}_S(r)$; furthermore, they both tend to 1 as r tends to $\underline{c}$.*

Proof. See Appendix A.4.[32]

By Lemma 3, for low enough reserves, $\hat{\delta}_U(r) > \hat{\delta}_S(r)$ holds. This means that collusion is easier when the suppliers interact in two markets rather than in only one.[33] However, for both one and two markets, sufficiently aggressive reserves can prevent a market allocation

[32] While the proof uses our assumption that $g(\underline{c}) > 0$, the result extends to the case in which the density is positive only in the interior of the support.

[33] Thus, in contrast with Bernheim and Whinston (1990) for the case where individual markets can be shared, in our procurement setting where the winner takes all, multi-market contact facilitates collusion even when these markets are *symmetric*.

from being sustainable. This resonates with the advice to practitioners in a sales auction context that "In some auctions, the most effective way of overcoming a buyers' ring is to set a reserve price, prohibiting sale of the item below its estimated value and thus impairing the profitability of a collusive operation" (Cassady, 1967, p. 191).

From Lemma 2, collusion is deterred when the non-designated supplier has an incentive to deviate in the more profitable market. It follows that the threshold $\hat{\delta}(\mathbf{r})$ is the solution in δ to $L(\underline{r}, \overline{r}, \delta) = S(\overline{r})$, where $\underline{r} \equiv \min\{r_1, r_2\}$ and $\overline{r} \equiv \max\{r_1, r_2\}$. Furthermore, for δ sufficiently close to 1, the long-term stake $L(r_i, r_j, \delta)$ is increasing in r_i,[34] implying that the critical threshold $\hat{\delta}(\mathbf{r})$ is strictly decreasing in the lower of the two reserves. For simplicity, from now on we will assume that the above three monotonicity properties of the threshold $\hat{\delta}(\mathbf{r})$ hold in the entire range of reserves:

Assumptions (monotonicity):

- Assumption S: $\hat{\delta}_S(r)$ is strictly decreasing in r;
- Assumption U: $\hat{\delta}_U(r)$ is strictly decreasing in r;
- Assumption L: $\hat{\delta}(\mathbf{r})$ is strictly decreasing in $\underline{r}$, the lower of the two reserves.

The first two assumptions extend the monotonicity properties established by Lemma 3 (for low enough reserves or, equivalently, for large enough values of the discount factor threshold $\hat{\delta}(\cdot)$) in the cases of *symmetric reserves* (S) and of a *unique market* (U). Assumption L amounts instead to extending the monotonicity property of the *long-term stake* $L(r_j, r_i, \delta)$ in the range of discount factors where collusion is an issue, i.e., for $\delta > \underline{\delta} \equiv \inf_{\mathbf{r} \in [\underline{c}, \min\{v, \overline{c}\}]^2} \hat{\delta}(\mathbf{r})$. These conditions are satisfied, for example, when costs are distributed over $[0, 1]$ according to $G(c) = c^{1/s}$, where $s > 0$ reflects the strength of the suppliers' cost distribution, and $v \geq 1$.[35]

2.3 Integrated buyer

We now study the optimal reserve policy by first considering the case of an integrated buyer operating both markets. By construction, as long as $\delta \leq \delta^C \equiv \hat{\delta}_S(r^C)$, it is optimal for the buyer to set the competitive reserve in both markets; using terminology from the literature on entry deterrence, collusion can then be said to be *blockaded*, because the optimal reserve absent collusion, r^C, is sufficiently low to deter collusion.

[34] This holds for $\delta > C'(r_i)/B'(r_i)$, where the right side is strictly lower than 1 (see Appendix A.1).

[35] See the Online Appendix. For instance, if costs are uniformly distributed (i.e., $s = 1$), then, as r increases, $\hat{\delta}_U(r)$ decreases from $\hat{\delta}_U(0) = 1$ to $\hat{\delta}_U(1) = (\sqrt{31} - 1)/5 \simeq 0.91$, and $\hat{\delta}_S(r) = 6/(6+r)$ decreases from $\hat{\delta}_S(0) = 1$ to $\hat{\delta}_S(1) = \underline{\delta} = 6/7 \simeq 0.86 > C'(\cdot)/B'(\cdot) = 1/2$.

When instead $\delta > \delta^C$, deterring collusion is costly.[36] As the next proposition shows, minimizing this cost requires *asymmetric reserves.* To see why, starting from symmetric deterrence reserves $r_1 = r_2 = r_S^D(\delta)$, where $r_S^D(\delta) \equiv \hat{\delta}_S^{-1}(\delta)$, consider increasing r_1 by a small amount and reducing r_2 by the same amount. Such a change has no impact on the buyer's expected payoff, as the effect of the two reserves offset each other, but improves deterrence by creating asymmetry in the suppliers' incentives to collude. It follows that there is a nearby (asymmetric) modification of the reserves that improves the buyer's expected payoff while preserving deterrence.

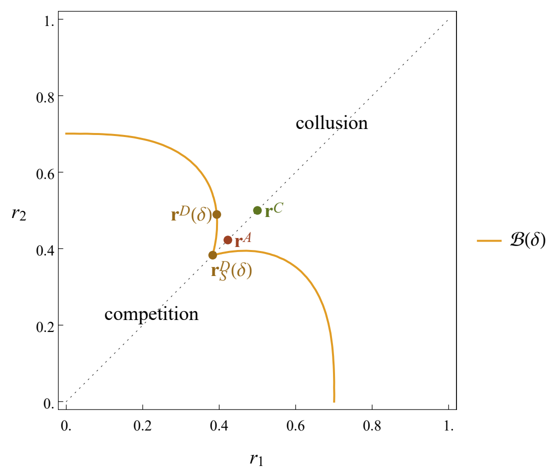

Figure 1: Deterrence boundary and relevant reserves. Assumes that costs are uniformly distributed over $[0, 1]$, $v = 1$, and $\delta = 0.94$.

Furthermore, because the buyer's payoff is strictly quasi-concave in the reserves, the optimal reserves lie on the *boundary* of the set of deterrence reserves.[37] Thus, in the region $r_2 \geq r_1$, it is optimal for the buyer to set the reserves:

$$\mathbf{r}^D(\delta) = (r_1^D(\delta), r_2^D(\delta)) \equiv \underset{\mathbf{r}\in\{\mathbf{r}'\in\mathcal{B}(\delta)|r_1'\leq r_2'\}}{\text{argmax}} \bar{U}^{Comp}(\mathbf{r}),$$

where $\mathcal{B}(\delta)$ denotes the deterrence boundary (formally defined in Appendix A.5), and

$$\bar{U}^{Comp}(\mathbf{r}) \equiv \frac{1}{2}\left[U^{Comp}(r_1) + U^{Comp}(r_2)\right]$$

is the buyer's average per-market payoff under competition. Let

$$U^D(\delta) \equiv \bar{U}^{Comp}\left(\mathbf{r}^D(\delta)\right)$$

[36] As always with deterrence, maintaining a threat begs the question of credibility; although we do not model this explicitly, repeated interaction may here help the buyer to maintain a credible threat.

[37] Starting from any $\mathbf{r}$ in the interior of the deterrence set, moving towards $\mathbf{r}^C$ would enhance the buyer's payoff.

denote the corresponding *deterrence payoff*. We illustrate the deterrence boundary and the optimal deterrence reserves in Figure 1.

Obviously, as long as $\delta \leq \delta_S^A \equiv \hat{\delta}_S(r^A)$, deterring collusion is optimal: symmetric reserves equal to r^A then suffice to deter collusion, implying that $U^D(\delta) > U^{Comp}(r^A) > U^{Coll}(r^A)$. By continuity, this remains the case when the discount factor δ is close to δ_S^A. However, as δ further increases, deterring collusion may become too costly—indeed, as δ tends to 1, the deterrence payoff tends to vanish, as deterrence reserves are close to $\underline{c}$. It is then optimal for the buyer to accommodate collusion and set both reserves equal to r^A. Formally, we have:

Proposition 3. *There exists $\delta^A > \delta_S^A$ such that it is optimal for the buyer to set the competitive reserves if $\delta \leq \delta^C$, to engage in costly deterrence if $\delta^C < \delta \leq \delta^A$, and to accommodate collusion if $\delta^A \leq \delta$. Furthermore, in case of costly deterrence, the optimal deterrence reserves $\mathbf{r}^D(\delta)$ are asymmetric and the deterrence payoff $U^D(\delta)$ is continuous and strictly decreasing in δ.*

Proof. See Appendix A.5.

Figure 2 illustrates the optimal reserves, which are discontinuous in the discount factor as it increases from the region of deterrence to the region of accommodation.

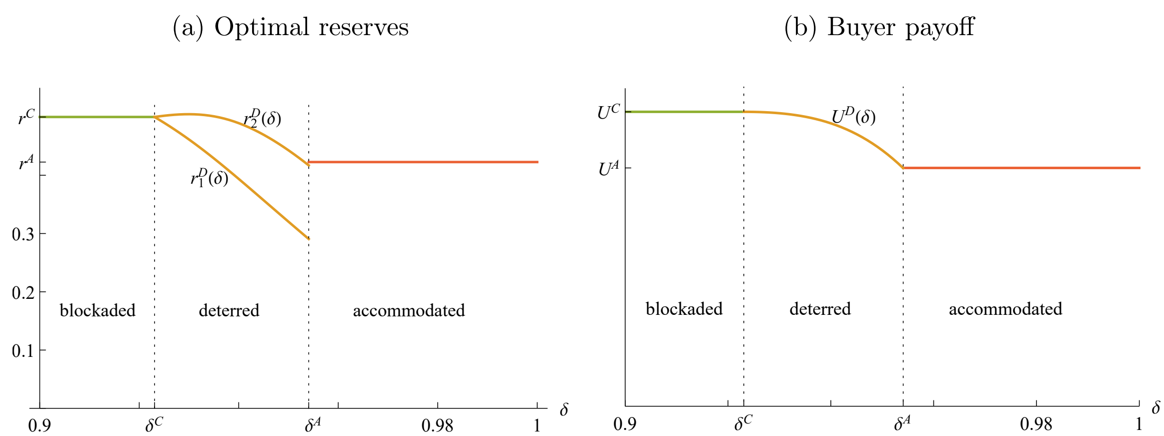

Figure 2: Optimal reserve and expected per-period buyer payoff for an integrated buyer as functions of the discount factor. Dashed lines are for symmetric reserves. Assumes that costs are uniformly distributed over $[0, 1]$ and that $v = 1$, implying that $\delta^C = 0.9231$, $\delta_S^A = 0.9466$, and $\delta^A = 0.9541$.

2.4 Independent buyers

The option of using asymmetric reserves raises the prospect of coordination issues when there are two different buyers, one in each market. To analyze this case, in what follows we assume that two buyers simultaneously and independently set their reserves. To fix ideas, buyer 1 sets the reserve r_1 in market 1, and buyer 2 sets the reserve r_2 in market 2.

Then the suppliers observe the reserves and engage in a market allocation whenever that is sustainable, and otherwise competition occurs. Thus, we analyze the Nash equilibrium reserves in the *reserve-setting game with independent buyers* in which the buyers choose reserves and the payoff of buyer i given strategy profile $\mathbf{r} = (r_1, r_2)$ is

$$U^{Comp}(r_i) \cdot \mathbf{1}_{\mathbf{r} \in \mathcal{D}(\delta)} + U^{Coll}(r_i) \cdot \mathbf{1}_{\mathbf{r} \notin \mathcal{D}(\delta)},$$

where $\mathcal{D}(\delta)$ denotes the set of reserves deterring collusion (formally defined in Appendix A.5). We first note that there always exists a Nash equilibrium:

Lemma 4. *The reserve-setting game with independent buyers has at least one Nash equilibrium in pure strategies.*

Proof. See Appendix A.6.

Having established existence, we now characterize the equilibrium outcomes. Given any r_2, the best response for buyer 1 is either r^C (if that deters collusion), r^A (if accommodation is optimal), or a reserve r_1 such that $(r_1, r_2) \in \mathcal{B}(\delta)$ (if deterrence is optimal, but r^C would allow collusion).[38] Building on this, the next proposition shows that the independent buyers achieve perfect coordination when either collusion is blockaded or accommodation is optimal:

Proposition 4. *If collusion is blockaded (i.e., $\delta \leq \delta^C$) or accommodation is uniquely optimal for an integrated buyer (i.e., $\delta > \delta^A$), then the reserve-setting game with independent buyers has a unique equilibrium, and its outcome matches that for an integrated buyer.*

Proof. See Appendix A.7.

If collusion is blockaded, then setting both reserves equal to r^C clearly constitutes a Nash equilibrium, as each buyer then gets its maximal payoff. Likewise, if accommodation is uniquely optimal for an integrated buyer, then setting both reserves equal to r^A constitutes a Nash equilibrium, as each buyer cannot gain from unilaterally deviating from these optimal reserves. Proposition 4 however goes further by establishing uniqueness. In the case of blockaded collusion, the intuition underlying the proof is that, given any reserve for buyer 1, say, buyer 2 would be best off setting a reserve (namely, r^A if it favors collusion, or a reserve r_2 such that (r_1, r_2) deters collusion) that is so low that, in response, buyer 1 can achieve its maximal payoff by deterring collusion with $r_1 = r^C$. In the case of accommodation, it suffices to note that (i) the average buyer payoff cannot exceed U^A

[38] In the latter case, any reserve lying in the interior of the deterrence region is dominated by another reserve that remains in the interior but is closer to r^C; buyer 1's best response then consists in setting r_1 such that $\mathbf{r}$ lies slightly inside the boundary of $\mathcal{D}(\delta)$.

in that case, and (ii) setting its reserve equal to r^A enables each buyer to obtain U^A (if this triggers collusion) or even more (if it induces competition); it follows that, starting from any $\mathbf{r} \neq (r^A, r^A)$, at least one buyer can profitably deviate.

We now show that the independent buyers may however fail to coordinate their reserve decisions when an integrated buyer would engage in costly deterrence. Specifically, in what follows we say that coordination failure can arise (resp., arises for sure) when there exists a Nash equilibrium outcome that differs (resp., when all Nash equilibrium outcomes differ) from what an integrated buyer would achieve.

To be sure, independent buyers may still adopt the optimal deterrence reserves $\mathbf{r}^D(\delta)$ when δ is close enough to δ^C. By construction, conditional on deterrence, each of these reserves constitutes a best-response to the other one; furthermore, the deterrence payoff remains close to U^C when δ is close to δ^C, implying that both deterrence reserves remain close to r^C, making a deviation to accommodation unprofitable. Yet, as the following proposition shows, coordination problems may induce the buyers to deter collusion with suboptimal reserves and/or to forgo deterrence altogether:[39]

Proposition 5. *In the reserve-setting game with independent buyers, there exists $\delta_N^D \in (\delta^C, \delta^A)$ such that optimal deterrence constitutes an equilibrium for $\delta \leq \delta_N^D$. However, if costly deterrence is uniquely optimal for an integrated buyer (i.e., $\delta^C < \delta < \delta^A$), then generically:*

(*i*) *whenever optimal deterrence constitutes an equilibrium, there also exists a continuum of equilibria with suboptimal deterrence;*

(*ii*) *furthermore, there exist $\tilde{\delta}_N^D$, $\hat{\delta}_N^D$, and δ_N^A satisfying $\delta_N^D \leq \tilde{\delta}_N^D < \hat{\delta}_N^D \leq \delta_N^A < \delta^A$ such that for $\tilde{\delta}_N^D < \delta \leq \hat{\delta}_N^D$, the only deterrence equilibria entail suboptimal reserves and for $\delta > \delta_N^A$, the equilibrium is unique and entails accommodation.*

Proof. See Appendix A.8.

For δ greater than but close to δ^C, collusion is nearly blockaded and thus easy to deter; as a result, the optimal deterrence reserves give each buyer strictly more than the accommodation payoff: $U^{Comp}(r_i^D(\delta)) > U^A$ for $i \in \{1, 2\}$. It follows that the optimal deterrence reserves constitute an equilibrium because, by construction, a unilateral deviation maintaining competition could not be profitable, and a deviation triggering collusion would give a payoff of at most U^A. Conversely, for optimal deterrence to constitute an equilibrium, both buyers must obtain at least the deterrence payoff. Furthermore, because optimal deterrence entails asymmetric reserves, generically one buyer has a strictly lower payoff than the other; hence, one buyer gets at least U^A, and the other buyer gets strictly

[39] Genericness in Proposition 5 refers to G and δ and is formally defined in Lemma A.6 in Appendix A.8.

more than U^A. It follows that, among the nearby reserves on the deterrence boundary, any reserves that are more favorable to the disfavored buyer keep giving both buyers more than U^A, and therefore constitute an equilibrium. Finally, for δ close to δ^A, collusion is difficult to deter and an integrated buyer is close to being indifferent between accommodation and deterrence; the average deterrence payoff is thus close to U^A and, because, optimal deterrence requires asymmetric reserves, generically one buyer obtains less than U^A and would thus have an incentive to deviate.

Figure 3 shows two cases in which deterrence is optimal for an integrated buyer, i.e., $\delta \in (\delta^C, \delta^A)$, but in which coordination failure either arises for sure (panel (a)) or can arise because of multiplicity of Nash equilibria (panel (b)). In Figure 3(a), an integrated buyer would deter collusion, but the only Nash equilibrium involves accommodation. In Figure 3(b), the optimal deterrence reserves constitute a Nash equilibrium of the reserve-setting game, but there is also a continuum of suboptimal deterrence equilibria; hence, while independent buyers deter collusion, they need not use the optimal deterrence reserves.[40]

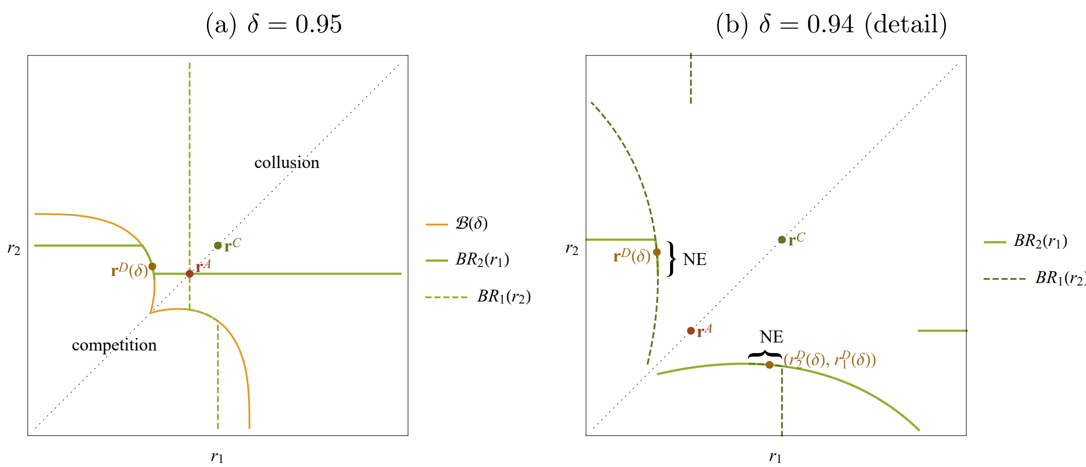

Figure 3: Examples of coordination failure. In both panels, an integrated buyer deters collusion with optimal deterrence reserves $\mathbf{r}^D(\delta)$, while with independent buyers: in panel (a), accommodation is the unique Nash equilibrium, so coordination failure arises for sure; in panel (b), the optimal reserves are not the only Nash equilibrium, so coordination failure can arise. Panel (a) depicts the deterrence boundaries and buyers' best-responses, but for clearer illustration, panel (b) omits the boundaries and "zooms in." Both panels assume that costs are uniformly distributed over $[0, 1]$ and $v = 1$. The discount factor δ is as indicated above the panels. In this setup, $\delta^C = 0.9231$, $\delta_N^D = 0.9475$, $\delta_N^A = 0.9483$, and $\delta^A = 0.9540$, so for panel (a), $\delta \in (\delta_N^A, \delta^A)$, and for panel (b), $\delta \in (\delta^C, \delta_N^D)$.

From Proposition 4, independent buyers always accommodate collusion when it is jointly optimal for them to do so. By contrast, when deterrence is jointly optimal but costly, Proposition 5 shows that independent buyers may fail to deter it, or deter with suboptimal reserves; in particular, for $\delta_N^A < \delta < \delta^A$, an integrated buyer would deter

[40] For additional illustrations see the Online Appendix.

collusion but, with independent buyers, accommodation is the unique equilibrium. In this sense, an integrated buyer deters collusion "more often" as well as more effectively than independent buyers.

Corollary 1. *Compared with an integrated buyer, the equilibrium of the game with independent buyers can never result in over-deterrence, but can result in under-deterrence; furthermore, when the equilibrium still entails deterrence, it may do so in a suboptimal manner.*

For sufficiently well-behaved cost distributions, numerical calculations show that six regions of discount factors can be distinguished (see Figure 4 for an illustration using $G(c) = c^{1/s}$ for $c \in [0, 1]$ and $s \in [0.5, 1.6]$), where the optimal and equilibrium reserves are as follows:

Table 1: Integrated versus independent buyers

	δ^C	$<$	δ_N^D $<$	$\hat{\delta}_N^D$ $<$	δ_N^A $<$	δ^A
integrated:	$\mathbf{r}^C$	$\mathbf{r}^D$	$\mathbf{r}^D$	$\mathbf{r}^D$	$\mathbf{r}^D$	$\mathbf{r}^A$
Nash eqm:	$\mathbf{r}^C$	$\mathbf{r}^D$ & subopt. deterrence	subopt. deterrence	$\mathbf{r}^A$ & subopt. deterrence	$\mathbf{r}^A$	$\mathbf{r}^A$
			coordination failure arises for sure			

The threshold δ_N^D, above which optimal deterrence no longer arises in equilibrium, is such that the buyer with the lower deterrence payoff is indifferent between deterrence and accommodation: $\min_{i \in \{1,2\}} U^{Comp}(r_i^D(\delta_N^D)) = U^A$. The threshold $\hat{\delta}_N^D$, above which accommodation arises in equilibrium, is such that a buyer is indifferent between accommodation and deterrence when the other buyer chooses r^A, that is $U^A = U^{Comp}(\hat{r}(r^A, \hat{\delta}_N^D))$, where $\hat{r}(r, \delta)$ denotes the reserve $r' \leq r$ such that $(r, r') \in \mathcal{B}(\delta)$.[41] For example, for the class of power distributions, the thresholds are as illustrated in Figure 4.

Further, for the class of power distributions used in Figure 4, the strength s of the cost distribution and the buyers' (common) value v affect the optimal reserves and the scope for coordination failure. As shown in Appendix B, weaker suppliers are more likely to be blockaded, whereas stronger suppliers are more likely to be accommodated. Buyers with higher values, who have more to lose from a failure to trade, are more likely to accommodate collusion; conversely, collusion is more likely to be blockaded if the buyer's value is low. With independent buyers, coordination failure is a greater concern when buyers have larger values and when suppliers draw their costs from better distributions in the sense that the range of discount factors such that independent buyers deter collusion only with suboptimal reserves, and the range of discount factors that fail to deter collusion, both increase with v and with s.

[41] For uniformly distributed costs, $\hat{\delta}_N^D = 0.9482$.

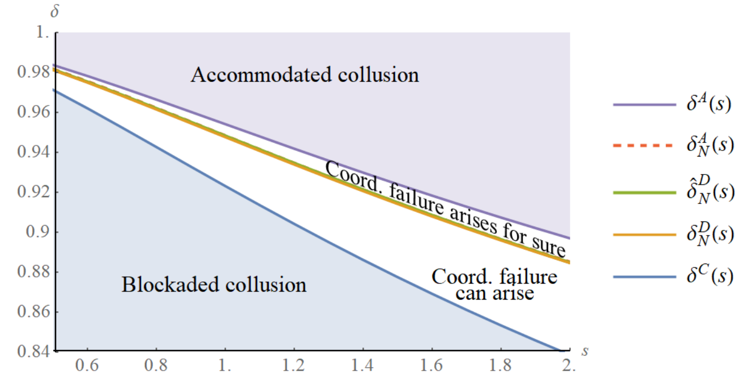

Figure 4: Threshold discount factors. Assumes $G(c) = c^{1/s}$ and $v = 1$. While the regions between δ_N^D and $\hat{\delta}_N^D$ and between $\hat{\delta}_N^D$ and δ_N^A are difficult to discern in the figure (there, coordination failure arises for sure), the relative rankings of the threshold discount factors are maintained throughout.

3 Discussion

The possibility that independent buyers accommodate collusion when an integrated buyer would have deterred it provides a rationale for the popular view that large buyers are less prone to be victims of collusion.[42] To the best of our knowledge, this is the first formalization of this notion. It apparently contrasts with Loertscher and Marx (2019), who show that endowing a buyer with buyer power makes covert collusion more attractive relative to a merger because the merger is a public event and causes the powerful buyer to react in a way that is detrimental to the merging suppliers. The way to reconcile these statements is that *powerful* and *large* buyers are distinct things. Our independent buyers are as powerful as an integrated buyer insofar as they can as well commit to a binding reserve forever. Yet, lacking size, they do not internalize the positive externality of deterring collusion in the other market. Conversely, a buyer cannot use reserves to fight collusion if it cannot credibly maintain them below $\min\{v, \bar{c}\}$. Hence, a buyer must be both powerful and large to fight collusion in an effective manner.[43]

Demand aggregation, or centralized procurement, is already widely used for making purchases and awarding contracts, both in the public and in the private sector.[44] The existing policy and economic literature has however so far highlighted its potential benefit in reducing the total cost of purchases via economies of scale, buyer power, profession-

[42]See, for example, Carlton and Israel (2011) for an expression of this view.

[43]The superior ability of an integrated buyer to deter collusion derives from the asymmetry of the optimal deterrence reserves. As we show in Online Appendix OA-B, this asymmetry does not hinge on the number of markets being two, but rather on it being even.

[44]For the public sector, examples of centralized units include the Government for Service Procurement in the United States, the Crown Commercial Service in the UK, and Consip in Italy. For the private sector, centralization is typically undertaken by creating a procurement department that purchases on behalf of multiple divisions.

alization, infrastructures (e-procurement tools), governance and transaction costs; the potential costs are those typical of delegation.[45] Our findings highlight an additional benefit of centralization, namely, that it enhances the fight against bid-rigging by reducing the scope for potential under-deterrence as well as the cost of actual deterrence.[46]

Yet, creating centralized procurement units is unlikely to fully address bid-rigging problems. Not all procurement can be centralized: for example, independent public and private buyers need to manage directly urgent or non-standardized purchases. Moreover, procurers, and especially centralized procurers, are not final users and their objective may underestimate the benefit of competition for total consumer surplus, thus failing to deter collusion even though doing so would be socially valuable.[47]

These considerations suggest that the fight of bid-rigging cannot be fully delegated to buyers: an effective antitrust enforcement remains desirable. Our results reinforce the call for a close cooperation between public buyers and antitrust authorities, as advocated by the most recent international principles on fighting bid-rigging in public procurement (EC, 2021), and as recently implemented for example by the U.S. DOJ, with the setup of the Procurement Collusion Strike Force.

In the collusive mechanism that we consider, the designated supplier for a market submits a bid just below the reserve, whenever its cost is below that level, and the nondesignated supplier submits a bid at the reserve, whenever its cost is below that level. Thus, the bids will be close whenever both bids are less than or equal to the reserve. This is consistent with research identifying close bids in first-price auctions as potentially reflective of collusion (see, e.g., Marshall and Marx, 2007; Kawai and Nakabayashi, 2022; Kawai et al., forth.) and suggests that the detection mechanism described by Kawai et al. (forth.), which uses the observation that in the absence of collusion, the identity of the winner should be "as-if-random" conditional on close bids, could be useful in our context. This highlights an additional possible advantage of first-price auctions in that effective collusive strategies may require close bids, which then may facilitate detection.

We conclude with a remark on secret reserves. Li and Perrigne (2003, p. 189) note that "The theoretic auction literature is still unclear on the rationale for using a random reserve price."[48] However, in our setup, opting for a *secret*, random reserve creates challenges

[45]See e.g., Dimitri, Dini and Piga (2006); Bandiera, Prat and Valletti (2009); Castellani, Decarolis and Rovigatti (2021).

[46]Related to this, our findings also add to the antitrust analysis of joint purchasing agreements, stressing the efficiency gain that they may generate in terms of fighting collusion among suppliers.

[47]If antitrust authorities were relying on a social welfare criterion, rather than consumer surplus, delegating to buyers the fight against collusion may potentially result in over-deterrence, which is an issue that has so far not been part of the economic debate.

[48]Li and Perrigne (2003) estimate that the French forest service had lower profits as a result of using secret, random reserve prices at timber auctions; however, they assume noncooperative bidding. Given evidence of collusion at U.S. timber auctions (see, e.g., Baldwin, Marshall and Richard, 1997; Athey and

for initiating and sustaining a market allocation. First, secret, random reserves prevent suppliers from signaling their wish to initiate collusion through a bid equal to the reserve, because they do not know what that reserve is. That said, if the reserve is drawn from a distribution with upper bound of the support $\overline{r} < \overline{c}$, then there remains the possibility of signaling initiation with a bid of $\overline{r}$. Second, secret, random reserves inhibit the ability of suppliers to maintain a market allocation while still having positive expected payoffs in their non-designated markets. To see this, note that with a secret, random reserve drawn from a distribution with upper bound of support $\overline{r}$, the only way for the non-designated supplier to ensure that it does not provide meaningful competition for the designated supplier is to bid $\overline{r}$ or above. But in this case, unless it bids exactly $\overline{r}$ and there is an atom in the distribution at that point, it wins with probability zero, and so it has an expected payoff of zero in its non-designated market. As these points suggest, secret, random reserves create challenges for colluding suppliers, and more so if those reserves are drawn from a distribution whose upper support is $\overline{r} = \overline{c}$. By increasing the profitability of deterrence, random reserves could expand the range of values for which a buyer prefers to deter rather than accommodate collusion.

4 Conclusion

Actions that deter collusion impose costs of their own, implying that it can be optimal for buyers to accommodate collusion among their suppliers rather than deter it. We analyze the tradeoff between accommodation and deterrence in a procurement context in which bidders attempt to engage in market allocation. Although aggressive reserves can deter collusion, the use of such reserves comes at the cost of reduced trade. Indeed, for sufficiently high discount factors, collusion is so robust that excessively aggressive reserves would be required to deter it; buyers are then better off setting the optimal reserve in the face of colluding bidders. By contrast, for lower discount factors, buyers optimally adjust reserves to achieve deterrence. Interestingly, a buyer operating two markets optimally sets different reserves across the markets, so as to create an asymmetry that helps to impede collusion. Independent buyers do not internalize the benefit of this asymmetry, and, as a result, may fail to use the optimal deterrence reserves or even fail to deter collusion at all when an integrated buyer would. The greater and more efficient deterrence of collusion by a multi-market buyer, as compared to independent buyers, suggests a benefit of centralized procurement agencies in terms of deterring collusion.[49]

Levin, 2001), the secret, random reserve may have provided benefits in terms of deterring collusion not captured in an analysis based on noncooperative bidding.

[49] In public procurement, this could be done by setting up national or regional procurement authorities that operate on behalf of local offices. In private procurement, it could be achieved by managing some purchases at a central rather than at a division level.

A Proofs

A.1 Proof of Lemma 1

For any reserve $r \in [\underline{c}, \bar{c}]$, the impact of collusion on total profit is equal to:$\Delta\Pi(r) \equiv \overline{\pi}^m(r) + \overline{\pi}^n(r) - 2\overline{\pi}^c(r) = \int_{\underline{c}}^{r} [2G(c) - G(r)]\, G(c)\, dc$. The market is at risk (i.e., collusion is strictly profitable) if and only if $\Delta\Pi(r) > 0$. We have: $\Delta\Pi'(r) = G^2(r) - \int_{\underline{c}}^{r} g(r)\, G(c)\, dc = G(r) \int_{\underline{c}}^{r} g(c)\, dc - \int_{\underline{c}}^{r} g(r)\, G(c)\, dc = \int_{\underline{c}}^{r} G(r)\, G(c) \left[\frac{g(c)}{G(c)} - \frac{g(r)}{G(r)}\right] dc > 0$, where the inequality stems from the monotonicity of the hazard rate. Because $\Delta\Pi(\underline{c}) = 0$, it follows that $\Delta\Pi(r) > 0$ for any $r \in (\underline{c}, \bar{c}]$. ■

A.2 Proof of Lemma 2

The following lemma shows that reserves have more impact on short-term stakes than long-term stakes:

Lemma A.1. *For any $r > \underline{c}$, $S(r) > B(r) + C(r) > 0$, and $S'(r) > B'(r) + C'(r) > 0$.*

Proof. See Online Appendix OA-A.1.

Building on this first lemma yields:

Lemma A.2. *Fix $\mathbf{r} = (r_1, r_2)$ and let $\underline{r} = \min\{r_1, r_2\}$ and $\overline{r} = \max\{r_1, r_2\}$ respectively denote the lower and the higher of the two reserves. We have:*

(i) the more stringent condition in (6) is $L(\underline{r}, \overline{r}, \delta) \geq S(\overline{r})$;

(ii) $L(\underline{r}, \overline{r}, \delta)$ is strictly increasing in δ.

Proof. See Online Appendix OA-A.2.

From Lemma A.2, the relevant sustainability condition is $L(\underline{r}, \overline{r}, \delta) \geq S(\overline{r})$, where the long-term stake is strictly increasing in δ. The conclusion follows. ■

A.3 Proof of Proposition 2

Differentiating the buyer's payoffs yields:

$$\frac{\partial}{\partial r} U^{Comp}(r) = \hat{g}(r)\, [v - \gamma(r)] \quad \text{and} \quad \frac{\partial}{\partial r} U^{Coll}(r) = \hat{g}(r) [v - \hat{\gamma}(r)],$$

where (with $\hat{g}$ denoting the density associated with $\hat{G}$):

$$\gamma(c) \equiv c + \frac{G(c)}{g(c)} \quad \text{and} \quad \hat{\gamma}(c) \equiv c + \frac{\hat{G}(c)}{\hat{g}(c)} = c + \frac{G(c)}{g(c)} \frac{2 - G(c)}{2[1 - G(c)]}$$

are the *virtual costs* associated with the distributions G and $\hat{G}$. The monotonicity of the reversed hazard rate ensures that these virtual costs are strictly increasing in c.[50] It follows that the payoffs are quasi-concave, and the optimal reserves are respectively$r^C \equiv \min\{\gamma^{-1}(v), \bar{c}\}$ and $r^A \equiv \hat{\gamma}^{-1}(v) < \bar{c}$. The conclusion then follows from the fact that, by construction, $\hat{\gamma}(\underline{c}) = \gamma(\underline{c}) = \underline{c}(< v)$ and, for $c > \underline{c}$, we have $\hat{\gamma}(c) > \gamma(c)$. ■

A.4 Proof of Lemma 3

For symmetric reserves equal to r, collusion is deterred if and only if $\phi_S(r,\delta) \geq 0$, where

$$\phi_S(r,\delta) \equiv (1-\delta) S(r) - \delta [B(r) - C(r)],$$

with $B(r)$, $C(r)$, and $S(r)$, respectively, given by (1), (2), and (7). Because $\phi_S(r,\delta)$ is strictly increasing in δ, collusion is deterred for $\delta \leq \hat{\delta}(r,r) \equiv \hat{\delta}_S(r)$, where $\hat{\delta}_S(r)$ is the unique solution to $\phi_S(r,\delta) = 0$, namely:

$$\hat{\delta}_S(r) \equiv \frac{S(r)}{S(r) + B(r) - C(r)}. \tag{A.1}$$

When instead there is a unique market, collusion is deterred if and only if $\phi_U(r,\delta) \geq 0$, where

$$\phi_U(r,\delta) \equiv \left(1-\delta^2\right) S(r) - \delta [B(r) - \delta C(r)].$$

The function $\phi_U(r,\delta)$ is quadratic and convex in δ, equal to $S(r) > 0$ for $\delta = 0$, and to $-[B(r) - C(r)] < 0$ for $\delta = 1$; hence, in the relevant range $\delta \in (0,1)$, collusion is deterred for $\delta \leq \hat{\delta}(\underline{c}, r) \equiv \hat{\delta}_U(r)$, where $\hat{\delta}_U(r)$ is the positive solution to $\phi_U(r,\delta) = 0$, namely:

$$\hat{\delta}_U(r) \equiv \frac{\sqrt{B^2(r) + 4S(r)[S(r) - C(r)]} - B(r)}{2[S(r) - C(r)]}. \tag{A.2}$$

Noting that $B(\underline{c}) = C(\underline{c}) = S(\underline{c}) = 0$, $B'(\underline{c}) = C'(\underline{c}) = S'(\underline{c}) = 0$, $B''(\underline{c}) = C''(\underline{c}) = 0$, $S''(\underline{c}) = 2g(\underline{c}) > 0$, and $B'''(\underline{c}) = 2g^2(\underline{c}) > C'''(\underline{c}) = g^2(\underline{c}) > 0$, for r close to $\underline{c}$ we have (focusing on the dominant terms in both the numerator and denominator):

$$\frac{B(r)}{S(r)} \simeq \frac{B'''(\underline{c}) \frac{(r-\underline{c})^3}{6}}{S''(\underline{c}) \frac{(r-\underline{c})^2}{2}} = g(\underline{c}) \frac{r-\underline{c}}{3}, \tag{A.3}$$

[50] This is obvious for $\gamma(c)$; for $\hat{\gamma}(c)$, it suffices to note that it can be expressed as $\hat{\gamma}(c) \equiv c + \frac{G(c)}{g(c)}\left(1 + \frac{G(c)}{2[1-G(c)]}\right)$, where the expression in large parentheses is positive and increasing.

and

$$\frac{C(r)}{S(r)} \simeq \frac{C'''(\underline{c})\,\frac{(r-\underline{c})^3}{6}}{S''(\underline{c})\,\frac{(r-\underline{c})^2}{2}} = g(\underline{c})\,\frac{r-\underline{c}}{6}. \tag{A.4}$$

Using these approximations, we have for r close to $\underline{c}$:

$$\hat{\delta}_S(r) = \frac{1}{1+\frac{B(r)}{S(r)}-\frac{C(r)}{S(r)}} \simeq \frac{1}{1+\frac{C(r)}{S(r)}} \simeq 1-\frac{C(r)}{S(r)} \simeq 1-g(\underline{c})\,\frac{r-\underline{c}}{6},$$

where the equality uses (A.1), the first approximation uses the implication of (A.3) and (A.4) that $\frac{B(r)}{S(r)} \simeq 2\frac{C(r)}{S(r)}$, the second approximation uses $\frac{1}{1+\varepsilon} \simeq 1-\varepsilon$ for ε close to zero, and the third approximation uses (A.4). This implies that

$$\lim_{r\to\underline{c}} \hat{\delta}_S(r) = 1 \quad \text{and} \quad \lim_{r\to\underline{c}} \hat{\delta}'_S(r) = -\frac{g(\underline{c})}{6} < 0.$$

In addition, for r close to $\underline{c}$:

$$\hat{\delta}_U(r) = \frac{\sqrt{1-\frac{C(r)}{S(r)}+\frac{B^2(r)}{4S^2(r)}}-\frac{B(r)}{2S(r)}}{1-\frac{C(r)}{S(r)}} \simeq \frac{\sqrt{1-\frac{C(r)}{S(r)}+\frac{C^2(r)}{S^2(r)}}-\frac{C(r)}{S(r)}}{1-\frac{C(r)}{S(r)}} \simeq 1-\frac{C(r)}{2S(r)} \simeq 1-g(\underline{c})\,\frac{r-\underline{c}}{12} < 0,$$

where the equality uses (A.2), the first approximation uses the implication of (A.3) and (A.4) that $\frac{B(r)}{2S(r)} \simeq \frac{C(r)}{S(r)}$, the second approximation uses $\frac{\sqrt{1-\varepsilon+\varepsilon^2}-\varepsilon}{1-\varepsilon} \simeq 1-\frac{\varepsilon}{2}$ for ε close to zero, and the final approximation uses (A.4). This implies that

$$\lim_{r\to\underline{c}} \hat{\delta}_U(r) = 1 \quad \text{and} \quad \lim_{r\to\underline{c}} \hat{\delta}'_U(r) = -\frac{g(\underline{c})}{12}.$$

Thus, we have completed the proof that the thresholds $\hat{\delta}_S(r)$ and $\hat{\delta}_U(r)$ both tend to 1 as r tends to $\underline{c}$ and that, for r low enough, they are both strictly decreasing in the reserve. Further, using $\lim_{r\to\underline{c}} \hat{\delta}'_U(r) > \lim_{r\to\underline{c}} \hat{\delta}'_S(r)$, we have shown that for r low enough, $\hat{\delta}_U(r) > \hat{\delta}_S(r)$. ■

A.5 Proof of Proposition 3

We first characterize the set of deterrence reserves. By analogy with $r_S^D(\delta) = \hat{\delta}_S^{-1}(\delta)$, for any $\delta > \hat{\delta}_U(\min\{\bar{c}, v\})$ let

$$r_U^D(\delta) \equiv \hat{\delta}_U^{-1}(\delta)$$

denote the *unique-market* deterrence reserve, and for any $r \in (r_S^D(\delta), r_U^D(\delta)]$, let

$$\hat{r}(r,\delta) \equiv \{r' \leq r \mid \hat{\delta}(r,r') = \delta\}$$

denote the deterrence reserve for the less profitable market, conditional on setting the reserve r on the more profitable one. As shown in the proof of next Lemma, the monotonicity assumption ensures that these reserves are uniquely defined and strictly decreasing in δ. Letting $\underline{\mathbf{c}} \equiv (\underline{c}, \underline{c})$ and $\mathbf{r}_S^D(\delta) \equiv (r_S^D(\delta), r_S^D(\delta))$, we have:

Lemma A.3. *The set of deterrence reserves is $\mathcal{D}(\delta) \equiv \mathcal{D}_S(\delta) \cup \mathcal{D}_1(\delta) \cup \mathcal{D}_2(\delta)$, where $\mathcal{D}_S(\delta) \equiv \{\mathbf{r} \mid \underline{\mathbf{c}} \leq \mathbf{r} \leq \mathbf{r}_S^D(\delta)\}$ and, for $i \neq j \in \{1, 2\}$:*

$$\mathcal{D}_i(\delta) \equiv \{\mathbf{r} \mid r_S^D(\delta) \leq r_i \leq r_U^D(\delta) \text{ and } \underline{c} \leq r_j \leq \hat{r}(r, \delta)\}.$$

Furthermore, as δ increases, $\mathcal{D}(\delta)$ shrinks strictly and continuously.

Proof. See Online Appendix OA-A.3.

It follows from Lemma A.3 that the boundary of the deterrence set can be expressed as $\mathcal{B}(\delta) \equiv \mathcal{B}_1(\delta) \cup \mathcal{B}_2(\delta)$, where

$$\mathcal{B}_i(\delta) \equiv \left\{\mathbf{r} \mid r_i \in [r_S^D(\delta), r_U^D(\delta)] \text{ and } r_j = \hat{r}(r_i, \delta)\right\}.$$

Building on Lemma A.3, we now show that the deterrence payoff is continuously decreasing in δ:

Lemma A.4. *In the range $\delta \geq \delta^C$, $U^D(\delta)$ is continuous and strictly decreasing in δ; furthermore, $U^D\left(\delta^C\right) = U^C \equiv U^{Comp}\left(r^C\right)$ and $\lim_{\delta \longrightarrow 1} U^D(\delta) = 0$.*

Proof. See Online Appendix OA-A.4.

It follows from Lemma A.4 that the deterrence payoff decreases from $U^C > U^A$ to $0 < U^A$ as δ increases from δ^C to 1. Hence, there there exists a unique $\delta^A \equiv \left(U^D\right)^{-1}\left(U^A\right)$ for which the buyer is indifferent between (costly) deterrence and accommodation; by construction, deterrence therefore dominates for $\delta < \delta^A$, and accommodation dominates for $\delta > \delta^A$.

We now show that, whenever collusion is not blockaded (i.e., $\delta > \delta^C$), asymmetric reserves are more effective at deterring it. For $r_i \geq r_j$ the relevant deterrence condition is $\phi(r_j, r_i, \delta) \geq 0$, where:

$$\phi(r_j, r_i, \delta) \equiv \left(1 - \delta^2\right)[S(r_i) - L(r_j, r_i, \delta)] = \bar{\phi}(r_i, \delta) - \underline{\phi}(r_j, \delta),$$

where

$$\bar{\phi}(r, \delta) \equiv \left(1 - \delta^2\right) S(r) - \delta[B(r) - \delta C(r)], \tag{A.5}$$

$$\underline{\phi}(r, \delta) \equiv \delta[\delta B(r) - C(r)]. \tag{A.6}$$

An increase in r_i increases both the short-term stake $S(r_i)$ and the long-term stake $L(r_j, r_i, \delta)$; by contrast, an increase in r_j only affects the long-term stake. It follows from Lemma A.1 that, for symmetric reserves, the impact on short-term stakes prevails. Specifically:

Lemma A.5. *For any $r > \underline{c}$, we have* $\frac{\partial\bar{\phi}(r,\delta)}{\partial r} + \frac{\partial\underline{\phi}(r,\delta)}{\partial r} > 0$.

Proof. See Online Appendix OA-A.5.

With symmetric reserves, when collusion is not blockaded the buyer's optimal reserve is $r_S^D(\delta)$—as the buyer's payoff is strictly quasi-concave in the reserve and $r_S^D(\delta)$, being the maximal symmetric deterrence reserve, is therefore also the closest to r^C. Suppose now that, starting from $\mathbf{r} = \mathbf{r}_S^D(\delta)$, the buyer increases r_1 by $\varepsilon > 0$ and reduces r_2 by the same amount; the change has no first-order effect on the buyer's expected payoff (starting from symmetric reserves, the impact of the two reserves offset each other), but strictly improves deterrence because

$$d\phi = \left(\left.\frac{\partial\bar{\phi}(r,\delta)}{\partial r}\right|_{r=r_S^D(\delta)} + \left.\frac{\partial\underline{\phi}(r,\delta)}{\partial r}\right|_{r=r_S^D(\delta)} \right) \varepsilon > 0,$$

where the inequality uses Lemma A.5. It follows that there exists a nearby (asymmetric) modification of the reserves that improves the buyer's expected payoff while preserving deterrence. ■

A.6 Proof of Lemma 4

We show in the proof of Proposition 4 (see Appendix A.7) that there exists a unique equilibrium in case of blockaded collusion (i.e., $\delta \leq \delta^C$) as well as if accommodation is optimal for an integrated buyer ($\delta \geq \delta^A$). We thus focus here on the intermediate range in which costly deterrence is uniquely optimal for an integrated buyer and consider a given $\delta \in (\delta^C, \delta^A)$.

Obviously, if there is no reserve r such that $(r^A, r) \in \mathcal{D}(\delta)$, then (r^A, r^A) constitutes an equilibrium, as (i) setting r^A maximizes the buyer's payoff under accommodation, and (ii) no buyer can unilaterally deviate to deterrence. Suppose now that there exists r such that $(r^A, r) \in \mathcal{D}(\delta)$, and let $\tilde{U}(\delta)$ denote buyer 1's maximal payoff from a deviation to deterrence, starting from $(r_1, r_2) = (r^A, r^A)$; that is:

$$\tilde{U}(\delta) \equiv \max_{\{r | (r, r^A) \in \mathcal{D}(\delta)\}} U^{Comp}(r).$$

Three cases can be distinguished, depending on the value of $\tilde{U}(\delta)$.

• Case 1: $\tilde{U}(\delta) \leq U^A$. The reserves $(r_1, r_2) = (r^A, r^A)$ then still induce collusion, otherwise we would have $\tilde{U}(\delta) \geq U^{Comp}(r^A) > U^{Coll}(r^A) = U^A$, a contradiction. It follows that these reserves constitute an equilibrium, because: (i) by definition, setting r^A maximizes the buyer's payoff under accommodation, and (ii) because $\tilde{U}(\delta) \leq U^A$, deviations to deterrence are also unprofitable.

• Case 2: $\tilde{U}(\delta) = U^C$. This occurs when $(r^C, r^A) \in \mathcal{D}(\delta)$, implying that the deviation to deterrence yields the maximal buyer payoff. This, in turns, implies that $r^A \leq \hat{r}(r^C, \delta) < r^C$: the inequalities stem from the definition of $\hat{r}(r^C, \delta)$ and the fact that, given $r_2 = r^C$, setting $r_1 = r_A$ would deter collusion whereas, because $\delta > \delta^C$, setting $r_1 = r_C$ would not deter it. It follows that $(r^C, \hat{r}(r^C, \delta))$ constitutes an equilibrium, because: (i) these reserves deter collusion, (ii) buyer 1 therefore obtains its maximal payoff, (iii) conditional on deterrence and on buyer 1 setting r^C, buyer 2 wants to set the reserve that is closest to r^C, which is precisely $\hat{r}(r^C, \delta)$, and (iv) because $r^A \leq \hat{r}(r^C, \delta) < r^C$, the deterrence payoff exceeds that of accommodation (by quasi-concavity of the buyer's payoff, we have: $U^{Comp}(\hat{r}(r^C, \delta)) \geq U^{Comp}(r^A) = U^A$).

• Case 3: $U^A < \tilde{U}(\delta) < U^C$. Let $\tilde{r}(\delta)$ denote a solution to the above problem (that is, $U^{Comp}(\tilde{r}(\delta)) = \tilde{U}(\delta)$). As $\tilde{U}(\delta) < U^C$, $\tilde{r}(\delta)$ must lie on the deterrence boundary—otherwise, starting from $(r_1, r_2) = (\tilde{r}(\delta), r^A)$, slightly moving r_1 towards r^C would improve the buyer's payoff, a contradiction. Two subcases can be distinguished.

Case 3a. Suppose first that $\tilde{r}(\delta) \geq r^A$, implying that $r^A = \hat{r}(\tilde{r}(\delta), \delta)$. It follows that $(r^A, \tilde{r}(\delta))$ constitutes a Nash equilibrium:

- By construction, given $r_1 = r^A$, setting $r_2 = \tilde{r}(\delta)$ is a best-response for buyer 2 because: (i) by definition, $\tilde{r}(\delta)$ maximizes buyer 2's deterrence payoff, and (ii) the resulting payoff exceeds that of accommodation (by construction, $U^{Comp}(\tilde{r}(\delta)) = \tilde{U}(\delta) > U^A$);
- Conversely, given $r_2 = \tilde{r}(\delta)$, setting $r_1 = r^A$ is a best-response for buyer 1, because: (i) by definition, $\hat{r}(\tilde{r}(\delta), \delta) = r^A (< r^C)$ is the closest to r^C in the deterrence region, and thus maximizes the deterrence payoff, and (ii) the resulting payoff dominates that of accommodation (by construction, $U^{Comp}(r^A) > U^{Coll}(r^A) = U^A$).

Case 3b. Suppose now that $\tilde{r}(\delta) < r^A$, implying that $\tilde{r}(\delta) = \hat{r}(r^A, \delta)$. Let $\check{U}(\delta)$ denote buyer 2's maximal payoff from a deviation preserving deterrence, starting from $(r_1, r_2) = (r^A, \tilde{r}(\delta))$; that is:

$$\check{U}(\delta) \equiv \max_{\{r | (\tilde{r}(\delta), r) \in \mathcal{D}(\delta)\}} U^{Comp}(r).$$

By construction, $(\tilde{r}(\delta), r^A) \in \mathcal{D}(\delta)$; it follows from the quasi-concavity of the buyer's payoff that $\check{U}(\delta) \geq U^{Comp}(r^A) > U^{Coll}(r^A) = U^A$. Furthermore, if $\check{U}(\delta) = U^C$ (meaning that

$(\tilde{r}(\delta), r^C) \in \mathcal{D}(\delta))$, then the same reasoning as above shows that $(\hat{r}(r^C,\delta), r^C)$ constitutes an equilibrium. Indeed, buyer 2 then obtains its maximal payoff, and given $r_2 = r^C$, setting $r_1 = \hat{r}(r^C,\delta)$ constitutes a best-response for buyer 1—to see the latter, note first that $(\tilde{r}(\delta), r^C) \in \mathcal{D}(\delta)$ and $\delta > \delta^C$ together imply $\tilde{r}(\delta) \le \hat{r}(r^C,\delta) < r^C$; it follows that buyer 1 obtains the maximal payoff it can achieve from deterrence, and that this payoff exceeds that of accommodation (from the quasi-concavity of the buyer's payoff, we then have $U^{Comp}(\hat{r}(r^C,\delta)) \ge U^{Comp}(\tilde{r}(\delta)) = \tilde{U}(\delta) > U^A$).

Finally, suppose that $U^A < \check{U}(\delta) < U^C$, and let $\check{r}(\delta)$ denote a best-response to $\tilde{r}(\delta)$. By construction, $\check{r}(\delta) \ge r^A$, because: (i) $(\tilde{r}(\delta), r^A) \in \mathcal{D}(\delta)$, implying that $\check{U}(\delta) \ge U^{Comp}(r^A)$, and (ii) from the quasi-concavity of the buyer's payoff, setting $r < r^A (< r^C)$ would generate a payoff lower than $U^{Comp}(r^A)$. It follows that $\tilde{r}(\delta) = \hat{r}(\check{r}(\delta),\delta)$.

To conclude the proof, it suffices to note that $(r_1, r_2) = (\tilde{r}(\delta), \check{r}(\delta))$ then constitutes a Nash equilibrium. Indeed, given $r_1 = \tilde{r}(\delta)$, setting $r_2 = \check{r}(\delta)$ is a best-response for buyer 2 because by definition doing so maximizes its payoff in the deterrence region, and the resulting payoff exceeds that of accommodation (as $\check{U}(\delta) > U^A$). Conversely, given $r_2 = \check{r}(\delta)$, setting $r_1 = \hat{r}(\check{r}(\delta),\delta) = \tilde{r}(\delta)$ is a best-response for buyer 1, because: (i) by construction, it is the largest reserve that buyer 1 can set while preserving deterrence, (ii) it is lower than r^C ($\tilde{r}(\delta) < r^A (< r^C)$), and (iii) the resulting payoff exceeds that of accommodation ($U^{Comp}(\tilde{r}(\delta)) = \tilde{U}(\delta) > U^A$). ■

A.7 Proof of Proposition 4

We study in turn the cases of blockaded collusion and accommodation. In each case, we start with existence before establishing uniqueness.

Blockaded collusion. Consider first the case where $\delta \le \delta^C$, implying that $\mathbf{r}^C = (r^C, r^C) \in \mathcal{D}(\delta)$.

• *Existence.* $\mathbf{r}^C$ obviously constitutes a Nash equilibrium, as it deters collusion and gives each buyer its highest possible payoff.

• *Uniqueness.* Let $\mathbf{r}^N = (r_1^N, r_2^N)$ be an equilibrium, and $\mathbf{U}^N = (U_1^N, U_2^N)$ denote the buyers' associated payoffs. We distinguish three cases, depending on the type of candidate equilibrium.

Case a: $\mathbf{r}^N \notin \mathcal{D}(\delta)$, implying $U_i^N = U^{Coll}\left(r_i^N\right)$. If $r_i^N \neq r^A$ for some buyer $i \in \{1,2\}$, then this buyer would benefit from deviating to $r_i = r^A$, so as to obtain at least

$$\min\left\{U^{Coll}\left(r^A\right), U^{Comp}\left(r^A\right)\right\} = U^{Coll}\left(r^A\right) > U^{Coll}\left(r_i^N\right) = U_i^N,$$

where the inequality stems from $U^{Coll}(\cdot)$ being maximal for $r = r^A$. It follows that $\mathbf{r}^N = \left(r^A, r^A\right)$. But then, any buyer would profitably deviate to r^C: indeed, doing so woud deter collusion (as, under Assumption L, $\mathbf{r}^C \in \mathcal{D}(\delta)$ and $r^A < r^C$ imply $\left(r^A, r^C\right) \in \mathcal{D}(\delta)$), and thus deliver the highest possible payoff.

Case b: $\mathbf{r}^N \in \mathcal{D}(\delta) \backslash \mathcal{B}(\delta)$. As any small change in either reserve keeps deterring collusion, we must have $\mathbf{r}^N = \mathbf{r}^C$ otherwise, as the buyers' payoffs are strictly quasi-concave, any buyer i with $r_i \neq r^C$ would profitably deviate towards r^C.

Case c: $\mathbf{r}^N \in \mathcal{B}(\delta)$; without loss of generality suppose that $\mathbf{r}^N \in \mathcal{B}_1(\delta)$ (i.e., $r_2^N \leq r_1^N$). It follows from Assumption L that reducing r_2 below r_2^N would keep deterring collusion. Hence, $r_2^N \leq r^C$, otherwise buyer 2 would benefit from reducing its reserve to $r_2 = r^C$. This, in turn, implies that $\mathbf{r} = \left(r^C, r_2^N\right) \in \mathcal{D}_1(\delta)$, as $\mathbf{r}^C \in \mathcal{D}(\delta)$ and, from Assumption L, reducing r_2 from r^C to r_2^N thus keeps deterring collusion. But then, we must have $r_1^N = r^C$, otherwise buyer 1 would profitably deviate to $r_1 = r^C$.

Accommodation. Suppose now that accommmodation is uniquely optimal for an integrated buyer (i.e., $\delta > \delta^A$), implying:

$$2U^A > U^{Comp}(r_1) + U^{Comp}(r_2) \text{ for any } \mathbf{r} = (r_1, r_2) \in \mathcal{D}(\delta). \tag{A.7}$$

• *Existence.* Fix $r_i = r^A$ and consider buyer j's best-response, for $i \neq j \in \{1, 2\}$. For any r_j satisfying $\left(r^A, r_j\right) \in \mathcal{D}(\delta)$, we have:

$$U^{Comp}(r^A) + U^A \geq 2U^A > U^{Comp}\left(r^A\right) + U^{Comp}(r_j),$$

where the first inequality stems from $U^{Comp}(\cdot) \geq U^{Coll}(\cdot)$ and the second one from (A.7). Hence, $U^A > U^{Comp}(r_j)$ for any r_j deterring collusion, implying that buyer j's best-response is to accommodate collusion and set $r_j = r^A$.

• *Uniqueness.* Let $\mathbf{r}^N$ be an equilibrium. By deviating to r^A, each buyer could obtain U^A if the deviation induces collusion, and $U^{Comp}\left(r^A\right) \geq U^A$ otherwise. Both buyers must therefore obtain at least $U^A = \max_r U^{Coll}(r)$. It then follows from (A.7) that accommodation must arise in equilibrium, and from the strict quasi-concavity of the buyers' payoffs that $\mathbf{r}^N = \left(r^A, r^A\right)$. ■

A.8 Proof of Proposition 5

We focus on the intermediate range in which costly deterrence is uniquely optimal for an integrated buyer: $\delta \in \left(\delta^C, \delta^A\right)$. Let $\mathbf{r}^C = \left(r^C, r^C\right)$, $\mathbf{r}^A = \left(r^A, r^A\right)$, and $\mathbf{r}^D(\delta) = \left(r_1^D(\delta), r_2^D(\delta)\right)$ denote the pairs of competitive, collusive, and optimal deterrence reserves, and $U^C \equiv U^{Comp}\left(r^C\right)$, $U^A \equiv U^{Coll}\left(r^A\right)$, and $U^D(\delta) \equiv \bar{U}^{Comp}\left(\mathbf{r}^D(\delta)\right)$ denote the associated payoffs.

By construction, $U^D\left(\delta^C\right) = U^C > U^A$; furthermore, from Proposition 3, $U^D(\delta)$ is continuous in δ. It follows that, for δ greater than but close enough to δ^C, $U^D(\delta)$ remains close to U^C. Because U^C is the maximal payoff that a buyer can achieve, this implies that both buyers' payoffs remain close to U^C, and therefore exceed U^A. This, in turn, ensures that the optimal deterrence reserves constitute an equilibrium: by definition, no buyer could benefit from a unilateral deviation maintaining deterrence, and no buyer can benefit from accommodating collusion, which would yield at most U^A.

Before moving to the other two claims of the proposition, we first establish some properties of the optimal deterrence reserves. Recall that, in the range $r_1 \leq r_2$, collusion is deterred if and only if

$$\bar{\phi}(r_2, \delta) \geq \underline{\phi}(r_1, \delta),$$

where $\bar{\phi}(r, \delta)$ and $\underline{\phi}(r, \delta)$ are defined by (A.5) and (A.6) in Appendix A.5, and $\underline{\phi}(r, \delta)$ is strictly increasing in r by Assumption L. It follows that $\mathbf{r}^D(\delta) = \left(r_1^D(\delta), r_2^D(\delta)\right)$ is the solution to:

$$\max_{\mathbf{r}} U^{Comp}(r_1) + U^{Comp}(r_2) \quad \text{s.t.} \quad \bar{\phi}(r_2, \delta) \geq \underline{\phi}(r_1, \delta) \text{ and } r_2 \geq r_1.$$

Furthermore, from Proposition 3, the second constraint is not binding. By contrast, because $\delta > \delta^C$, the first constraint is binding; hence, $\mathbf{r}^D(\delta)$ lies on the boundary, which can be characterized as $\bar{\phi}(r_2, \delta) = \underline{\phi}(r_1, \delta)$ or $r_1 = \hat{r}(r_2, \delta)$, where

$$\frac{\partial \hat{r}}{\partial r}(r, \delta) = \frac{\frac{\partial \bar{\phi}}{\partial r}(r, \delta)}{\frac{\partial \underline{\phi}}{\partial r}(\hat{r}(r, \delta), \delta)}. \tag{A.8}$$

Denoting by $\lambda > 0$ the Lagrangian multiplier associated with the first constraint, the first-order conditions are:

$$\frac{dU^{Comp}}{dr}\left(r_1^D(\delta)\right) = \lambda \frac{\partial \underline{\phi}}{\partial r}\left(r_1^D(\delta), \delta\right), \tag{A.9}$$

$$\frac{dU^{Comp}}{dr}\left(r_2^D(\delta)\right) = -\lambda \frac{\partial \bar{\phi}}{\partial r}\left(r_2^D(\delta), \delta\right), \tag{A.10}$$

where $\lambda > 0$ (as the constraint is binding), $\partial \underline{\phi} / \partial r > 0$ (from Assumption L) and, from the strict quasi-concavity of the buyer's payoff:

$$\frac{dU^{Comp}}{dr}(r) \gtreqless 0 \iff r \lesseqgtr r^C. \tag{A.11}$$

It follows that $\frac{dU^{Comp}}{dr}\left(r_1^D(\delta)\right) > 0$ and $r_1^D(\delta) < r^C$. Furthermore, combining (A.8) with the first-order conditions (A.9) and (A.10) yields:

$$\frac{\partial \hat{r}}{\partial r}\left(r_2^D\left(\delta\right),\delta\right) = -\frac{\frac{dU^{Comp}}{dr}\left(r_2^D\left(\delta\right)\right)}{\frac{dU^{Comp}}{dr}\left(r_1^D\left(\delta\right)\right)}.$$

Combined with $\frac{dU^{Comp}}{dr}\left(r_1^D\left(\delta\right)\right) > 0$, (A.8) and $\partial\underline{\phi}/\partial r > 0$, this leads to:

$$\frac{\partial \hat{r}}{\partial r}\left(r_2^D\left(\delta\right),\delta\right) \gtreqless 0 \Longleftrightarrow \frac{\partial \bar{\phi}}{\partial r}\left(r_2^D\left(\delta\right),\delta\right) \gtreqless 0 \Longleftrightarrow r_2^D\left(\delta\right) \gtreqless r^C. \tag{A.12}$$

The following lemma will be useful:

Lemma A.6. *We have:*

(i) Generically over $G\left(\cdot\right)$, $U^{Comp}\left(r_2^D\left(\delta\right)\right) \neq U^{Comp}\left(r_1^D\left(\delta\right)\right)$ for any $\delta > \delta^C$.

(ii) Generically over δ, $r_2^D\left(\delta\right) \neq r^C$.

Proof. See Online Appendix OA-A.6.

We now proceed to prove the two assertions of the proposition, focusing on the generic cases in which (i) $U^{Comp}\left(r_2^D\left(\delta\right)\right) \neq U^{Comp}\left(r_1^D\left(\delta\right)\right)$ and (ii) $r_2^D\left(\delta\right) \neq r^C$.

• Part (i). Suppose that the optimal deterrence reserves $\mathbf{r}^D(\delta) = \left(r_1^D\left(\delta\right), r_2^D\left(\delta\right)\right)$ constitute an equilibrium outcome. We must then have

$$\min\left\{U^{Comp}\left(r_1^D\left(\delta\right)\right), U^{Comp}\left(r_2^D\left(\delta\right)\right)\right\} \geq U^A, \tag{A.13}$$

otherwise at least one buyer would profitably deviate to r^A, so as to obtain at least $\min\left\{U^{Coll}\left(r^A\right), U^{Comp}\left(r^A\right)\right\} = U^{Coll}\left(r^A\right) = U^A$.

To conclude the argument, we consider in turn the two generic cases, $r_2^D\left(\delta\right) < r^C$ and $r_2^D\left(\delta\right) > r^C$.

If $r_2^D\left(\delta\right) < r^C$, which implies that $r_1^D\left(\delta\right) < r_2^D\left(\delta\right) < r^C$, then, from the strict quasi-concavity of the buyers' payoffs and $(A.13)$, we have

$$U^A \leq U^{Comp}\left(r_1^D\left(\delta\right)\right) < U^{Comp}\left(r_2^D\left(\delta\right)\right).$$

Furthermore, from (A.12), we have $\frac{\partial \hat{r}}{\partial r}\left(r_2^D\left(\delta\right),\delta\right) < 0$ and $\frac{\partial \bar{\phi}}{\partial r}\left(r_2^D\left(\delta\right),\delta\right) < 0$. It follows that $(r_1, r_2) = \left(\hat{r}\left(r_2^D\left(\delta\right) - \varepsilon, \delta\right), r_2^D\left(\delta\right) - \varepsilon\right)$ is a Nash equilibrium for ε positive but small enough to maintain $\frac{\partial \bar{\phi}}{\partial r}\left(r_2, \delta\right) < 0$ and $U^{Comp}\left(r_2^D\left(\delta\right) - \varepsilon\right) > U^A$:

- By construction, $r_2 = r_2^D(\delta) - \varepsilon < r_2^D(\delta) < r^C$ and $r_1 = \hat{r}(r_2) \leq r_2 < r^C$; hence, profitable deviations to deterrence would require an increase in the reserve. But, as $r_1 = \hat{r}(r_2)$, which amounts to $\bar{\phi}(r_2, \delta) = \underline{\phi}(r_1, \delta)$, a unilateral increase in r_1 would violate the deterrence constraint and thus trigger collusion (as $\underline{\phi}(r_1, \delta)$ is strictly increasing from Assumption L, and here $\frac{\partial \bar{\phi}}{\partial r}(r_2, \delta) < 0$).
- By construction, $r_1 = \hat{r}(r_2, \delta)$, where $r_2 = r_2^D(\delta) - \varepsilon < r_2^D(\delta) \left(< r^C\right)$ and $\hat{r}(r, \delta)$ satisfies $\hat{r}(r, \delta) \leq r$ is here strictly decreasing in r; hence, $r_1^D(\delta) < r_1 \leq r_2 < r_2^D(\delta) < r^C$ and $U^{Comp}(r_2) > U^{Comp}(r_1) > U^{Comp}\left(r_1^D(\delta)\right) \geq U^A$, which rules out deviations to accommodation.

If instead $r_2^D(\delta) > r^C$, then from (A.12), $\frac{\partial \hat{r}}{\partial r}\left(r_2^D(\delta), \delta\right) > 0$ and $\frac{\partial \bar{\phi}}{\partial r}\left(r_2^D(\delta), \delta\right) > 0$. Two cases can then be distinguished, depending on which buyer has the lower payoff.

- If $U^{Comp}\left(r_1^D(\delta)\right) > U^{Comp}\left(r_2^D(\delta)\right) \geq U^A$, then $(r_1, r_2) = \left(\hat{r}(r_2^D(\delta) - \varepsilon, \delta), r_2^D(\delta) - \varepsilon\right)$ (implying $r_i < r_i^D(\delta)$ for $i \in \{1, 2\}$) is a Nash equilibrium for ε positive but small enough to maintain $r_2 > r^C$, $\frac{\partial \bar{\phi}}{\partial r}(r_2, \delta) > 0$ and $U^{Comp}(r_1) > U^A$:
  - because $r_1 < r_1^D(\delta) < r^C$ and $r_2 > r^C$, profitable deviations to deterrence would require an increase in r_1, which would trigger collusion as $\bar{\phi}(r_2, \delta) = \underline{\phi}(r_1, \delta)$ and $\underline{\phi}(r, \delta)$ is strictly increasing in r, or a decrease in r_2, which would also trigger collusion because $\bar{\phi}(r, \delta)$ is here strictly decreasing in r;
  - the conditions $U^{Comp}(r_1) > U^A$ and $U^{Comp}(r_2) > U^{Comp}\left(r_2^D(\delta)\right) \left(\geq U^A\right)$ (as $r_2^D(\delta) > r_2 > r^C$) rule out deviations to accommodation.
- If instead $U^{Comp}\left(r_2^D(\delta)\right) > U^{Comp}\left(r_1^D(\delta)\right) \geq U^A$, then $(r_1, r_2) = (\hat{r}(r + \varepsilon, \delta), r + \varepsilon)$ (implying $r_i > r_i^D(\delta)$ for $i = 1, 2$) is a Nash equilibrium for ε positive but small enough to maintain $r_1 < r^C$, $\frac{\partial \bar{\phi}}{\partial r}(r_2, \delta) > 0$ and $U^{Comp}(r_2) > U^A$:
  - because $r_1 < r^C$ and $r_2 > r_2^D(\delta) > r^C$, profitable deviations to deterrence would require again either an increase in r_1 or a decrease in r_2, both of which would trigger collusion;
  - the conditions $U^{Comp}(r_1) > U^{Comp}\left(r_1^D(\delta)\right) \left(\geq U^A\right)$ (as $r_1^D(\delta) < r_1 < r^C$) and $U^{Comp}(r_2) > U^A$ rule out deviations to accommodation.

- Part (ii). We start with (suboptimal) deterrence, before turning to accommodation.

Define

$$\tilde{\delta}_N^D \equiv \max\left\{\delta \in (0, 1) \mid \min\left\{U^{Comp}\left(r_1^D(\delta)\right), U^{Comp}\left(r_2^D(\delta)\right)\right\} \geq U^A\right\}.$$

By construction, for $\delta > \tilde{\delta}_N^D$, optimal deterrence cannot constitute an equilibrium because the buyer with the lower payoff would strictly benefit from deviating to accommodation. Furthermore, by continuity of the deterrence region, we have:

$$\min\{U^{Comp}(r_1^D(\tilde{\delta}_N^D)), U^{Comp}(r_2^D(\tilde{\delta}_N^D))\} = U^A.$$

In addition, it follows from the above analysis that for $\delta = \tilde{\delta}_N^D$, there exists r_2 such that, starting from $(r_1, r_2) = (\hat{r}(r_2, \tilde{\delta}_N^D), r_2)$, each buyer *strictly* prefers not to deviate; by continuity, there exists a deterrence equilibrium for δ slightly above $\tilde{\delta}_N^D$. Specifically:

- if $U^{Comp}(r_1^D(\tilde{\delta}_N^D)) > U^{Comp}(r_2^D(\tilde{\delta}_N^D)) = U^A$, then $(r_1, r_2) = (\hat{r}(r_2^D(\tilde{\delta}_N^D) - \varepsilon, \tilde{\delta}_N^D), r_2^D(\tilde{\delta}_N^D) - \varepsilon)$ is a strict Nash equilibrium for ε positive but small enough;
- if instead $U^{Comp}(r_2^D(\tilde{\delta}_N^D)) > U^{Comp}(r_1^D(\tilde{\delta}_N^D)) = U^A$, then $(r_1, r_2) = (\hat{r}(r_2^D(\tilde{\delta}_N^D) + \varepsilon, \tilde{\delta}_N^D), r_2^D(\tilde{\delta}_N^D) + \varepsilon)$ is a strict Nash equilibrium for ε positive but small enough.

In both cases, by continuity there exists $\hat{\delta}_N^D \in (\tilde{\delta}_N^D, \delta^A)$ such that for $\delta \in (\tilde{\delta}_N^D, \hat{\delta}_N^D]$, $(r_1, r_2) = (\hat{r}(r_2, \delta), r_2)$ is a suboptimal deterrence Nash equilibrium.

We now turn to accommodation. For $\delta = \delta^A$, the optimal deterrence reserves $\mathbf{r}^D(\delta)$ gives the buyers an average deterrence payoff of U^A and, from Lemma A.6, the two buyers obtain different payoffs; hence, the buyer with the lower payoff obtains strictly less than U^A. Furthermore, for any other pair of deterrence reserves, the average buyer payoff is strictly less than U^A, implying that at least one of the buyers obtains strictly less than U^A. It follows that, starting from any pair of deterrence reserves, at least one buyer has a strict incentive to deviate to accommodation. By continuity (see Lemma A.4), there exists $\hat{\delta}_N^A < \delta^A$ such that deterrence cannot be an equilibrium outcome for $\delta \in (\hat{\delta}_N^A, \delta^A)$. To establish existence, we first note that, for $\delta = \delta^A$, starting from (r^A, r^A), no buyer can obtain U^A or more by deviating to deterrence; that is, for any r,

$$(r; r^A) \in \mathcal{D}(\delta^A) \implies U^{Comp}(r) < U^A. \tag{A.14}$$

Indeed, if there existed $\tilde{r}$ satisfying $(\tilde{r}; r^A) \in \mathcal{D}(\delta)$ and $U^{Comp}(\tilde{r}) \geq U^A$, then setting $(\tilde{r}; r^A)$ would enable an integrated buyer to obtain

$$U^{Comp}(\tilde{r}) + U^{Comp}(r^A) \geq U^A + U^{Comp}(r^A) > 2U^A,$$

contradicting the fact that, for $\delta = \delta^A$, the integrated buyer is indifferent between deterrence and accommodation. By continuity, condition (A.14) holds for δ slightly below δ^A. Hence, there exists $\tilde{\delta}_N^A < \delta^A$ such that (r^A, r^A) constitutes a Nash equilibrium in the range $\delta \in (\tilde{\delta}_N^A, \delta^A)$. It follows that, in the range $\delta \in \left(\delta_N^A, \delta^A\right)$, where $\delta_N^A \equiv \max\{\hat{\delta}_N^A, \tilde{\delta}_N^A\}$, there is a unique equilibrium, which entails accommodation. ■

B Comparative statics

Below we provide comparative statics results, assuming that the suppliers' costs are distributed over $[0,1]$ according to $G(c) = c^{1/s}$. A larger s thus corresponds to a better cost distribution in the first-order stochastic dominance sense. We first consider the case of integrated buyers, before turning to the case of independent buyers.

B.1 Integrated buyers

Figure B.1(a) depicts the evolution of the reserves as a function of the buyer's value v (for a particular strength of the suppliers, namely, $s = 1$). It shows that buyers with higher values, who have more to lose from a failure to trade, are more likely to accommodate collusion; conversely, collusion is more likely to be blockaded if the buyer's value is low. Furthermore, in the range of buyer values where deterrence is optimal, the gap between the two asymmetric deterrence reserves increases with v.

Figure B.1(b) focuses instead on the evolution of the reserves as a function of the sellers' strength s (for a particular buyer's value, namely, $v = 1$). It shows that, as suppliers become stronger, implying that collusion is more valuable to them, the reserves required to deter collusion decrease, until eventually the buyer prefers to accommodate. Weaker suppliers are thus more likely to be blockaded, whereas stronger suppliers are more likely to be accommodated.

(a) Reserves as a function of v with $s = 1$ (b) Reserves as a function of s with $v = 1$

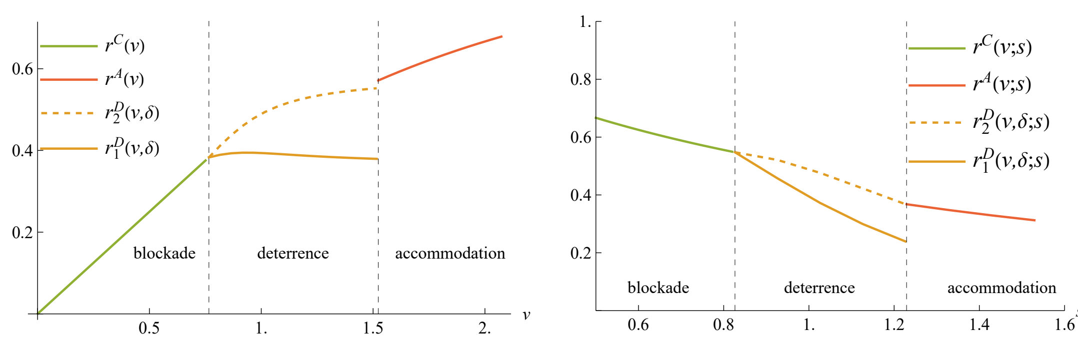

Figure B.1: Optimal reserves for an integrated buyer assuming $G(c) = c^{1/s}$ for $s > 0$. (As s increases the cost distribution becomes "better".) Both panels assume that $\delta = 0.94$.

B.2 Independent buyers

As shown in Figure B.2, the range of discount factors in which independent buyers deter collusion only with suboptimal reserves, and the range of discount factors that fail to deter collusion, are both expanding in v and in s. Thus, here, coordination failure is a greater concern when buyers have larger values and when suppliers draw their costs from better distributions.

(a) Threshold discount factors varying v for $s = 1$ (b) Threshold discount factors varying s for $v = 1$

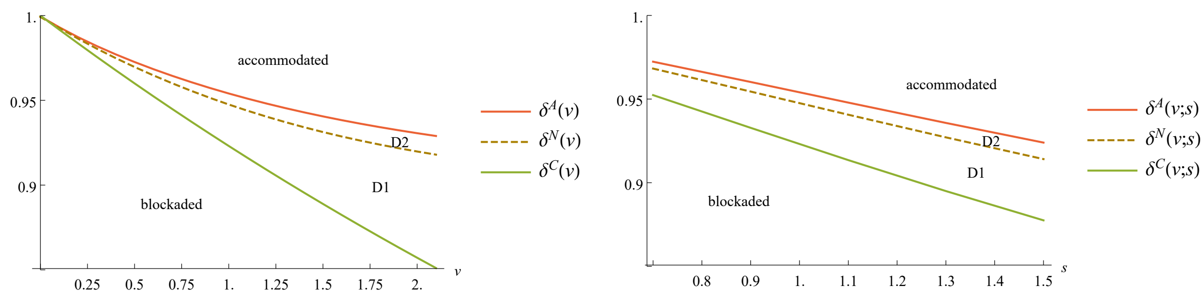

Figure B.2: Effects of changes in the buyer's value and the suppliers' distributional strength on threshold discount factors. Assumes that costs are distributed over $[0, 1]$ with distribution $G(c) = c^{1/s}$. Panel (a) assumes $s = 1$ and varies v; panel (b) assumes $v = 1$ and varies s. In the region labeled "accommodated," both integrated and independent buyers accommodate collusion with the optimal collusive reserve r^A. In the regions labeled D1 and D2, an integrated buyer would deter collusion. In D1, the optimal deterrence reserves are among the multiple Nash equilibria of the reserve-setting game with independent buyers, but in D2 they are not. In the region labeled "blockaded," both integrated and independent buyers deter collusion with the optimal competitive reserve r^C.

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Online Appendix

to accompany

"Coordination in the Fight Against Collusion"

by

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In this Online Appendix, we provide in Section OA-A the proofs of the auxiliary lemmas used in the main Appendix. We then address three points raised in the paper. First, in Section OA-B, we provide an extension of the model to allow more than two markets, as mentioned in footnote 13 in the paper. Second, in Section OA-C, we show that our monotonicity assumptions are satisfied when costs are distributed according to the power distribution, as mentioned in footnote 35 in the paper. Third, in Section OA-D, we provide additional illustrations of coordination and coordination failure with independent buyers, as mentioned in footnote 40 in the paper.

OA-A Proofs of auxiliary lemmas

OA-A.1 Proof of Lemma A.1

We have: $B(r) + C(r) = \overline{\pi}^m(r) - \overline{\pi}^n(r) = G(r)\overline{\pi}^m(r)$, where $G(r)$ and $\overline{\pi}^m(r)$ are both positive for $r > \underline{c}$, and strictly increasing; it follows that $B(r) + C(r)$ is also positive and strictly increasing. Likewise, using $S(r) = \pi^m(\underline{c}) - \pi^n(\underline{c}) = G(r)\pi^m(\underline{c}; r)$, we have: $S(r) - B(r) - C(r) = G(r)\left[\pi^m(\underline{c}; r) - \overline{\pi}^m(r)\right]$, where $G(r)$ and

$$\pi^m(\underline{c}; r) - \overline{\pi}^m(r) = (r - \underline{c}) - \int_{\underline{c}}^{r} G(c)dc = \int_{\underline{c}}^{r} [1 - G(c)]\, dc$$

are both positive for $r > \underline{c}$, and strictly increasing. The conclusion follows. ■

OA-A.2 Proof of Lemma A.2

Part (i). Let $\Psi \equiv \left(1-\delta^2\right)\{[L(\overline{r},\underline{r},\delta)-S(\underline{r})]-[L(\underline{r},\overline{r},\delta)-S(\overline{r})]\}$. Straightforward manipulations yield:

$$\begin{aligned}\Psi &= \left\{\delta\left[B\left(\underline{r}\right)-C\left(\overline{r}\right)\right]+\delta^2\left[B\left(\overline{r}\right)-C\left(\underline{r}\right)\right]-\left(1-\delta^2\right)S\left(\underline{r}\right)\right\}\\ &\quad -\left\{\delta\left[B\left(\overline{r}\right)-C\left(\underline{r}\right)\right]+\delta^2\left[B\left(\underline{r}\right)-C\left(\overline{r}\right)\right]-\left(1-\delta^2\right)S\left(\overline{r}\right)\right\}\\ &= (1-\delta)\left\{(1+\delta)S\left(\overline{r}\right)-\delta\left[B\left(\overline{r}\right)+C\left(\overline{r}\right)\right]\right\}\\ &\quad -(1-\delta)\left\{(1+\delta)S\left(\underline{r}\right)-\delta\left[B\left(\underline{r}\right)+C\left(\underline{r}\right)\right]\right\}\\ &= (1-\delta)\left[\psi\left(\overline{r}\right)-\psi\left(\underline{r}\right)\right],\end{aligned}$$

where $\psi(r) \equiv (1+\delta)S(r)-\delta[B(r)+C(r)]$ is strictly increasing in r, that is $\psi'(r) = (1+\delta)S'(r)-\delta[B'(r)+C'(r)]>0$, where the inequality follows from $\delta \geq 0$ and Lemma A.1. Therefore, $\Psi \geq 0$, implying that the more stringent condition in (6) is $L(\underline{r},\overline{r},\delta) \geq S(\overline{r})$.

Part (ii). We have:

$$\begin{aligned}\frac{\partial L\left(\underline{r},\overline{r},\delta\right)}{\partial\delta} &= \frac{2\delta^2}{\left(1-\delta^2\right)^2}\left\{B(\underline{r})-C(\overline{r})+\delta\left[B(\overline{r})-C(\underline{r})\right]\right\}\\ &\quad +\frac{1}{1-\delta^2}\left\{B(\underline{r})-C(\overline{r})+2\delta\left[B(\overline{r})-C(\underline{r})\right]\right\}\\ &= \frac{(1-\delta)^2\left[B(\underline{r})-C(\overline{r})\right]+2\delta\left[B(\underline{r})-C(\underline{r})+B(\overline{r})-C(\overline{r})\right]}{1-\delta^2}\\ &> 0,\end{aligned}$$

where the second equality rearranges, and the inequality follows from Lemma A.1 and $\underline{r} \leq \overline{r}$, which together imply $B(\underline{r}) > C(\underline{r})$ and $B(\underline{r}) \geq B(\overline{r}) > C(\overline{r})$. ■

OA-A.3 Proof of Lemma A.3

We first establish the existence and properties of the deterrence thresholds $r_S^D(\delta)$ and $r_U^D(\delta) \equiv \hat{\delta}_U^{-1}(\delta)$. From Assumption S and Lemma 3, $\hat{\delta}_S(r) = \hat{\delta}(r,r)$ is strictly decreasing in r and tends to 1 as r tends to $\underline{c}$. Hence, with symmetric reserves, collusion is an issue if $\delta > \hat{\delta}_S(\min\{\bar{c},v\})$, in which case setting the reserves to r deters it if and only if $r \leq r_S^D(\delta) \equiv \hat{\delta}_S^{-1}(\delta)$. Assumption S moreover ensures that $r_S^D(\delta)$ is strictly decreasing in δ.

Similarly, Assumption U and Lemma 3 together ensures that $\hat{\delta}_S(r)$ is also strictly decreasing in r, and tends to 1 as r tends to $\underline{c}$. Hence, in the case of a unique market, collusion is an issue if $\delta > \hat{\delta}_U(\min\{\bar{c}, v\})$, in which case setting the reserve equal to r deters it if and only if $r \leq r_U^D(\delta) \equiv \hat{\delta}_U^{-1}(\delta)$, where $r_U^D(\delta)$ is strictly decreasing in δ.

By construction, $\hat{\delta}\left(\underline{c}, r_U^D(\delta)\right) = \delta$ and $r_U^D(\delta) > \underline{c}$ (as $\hat{\delta}(\underline{c}, \underline{c}) = 1$). Hence, from Assumption L, $\hat{\delta}_S\left(r_U^D(\delta)\right) = \hat{\delta}\left(r_U^D(\delta), r_U^D(\delta)\right) < \delta$; that is, symmetric reserves equal to $r_U^D(\delta)$ would not deter collusion. It then follows from Assumption S that

$$r_U^D(\delta) > r_S^D(\delta).$$

The rest of the proof proceeds in three steps. We start by checking that any $\mathbf{r} \leq \mathbf{r}_S^D(\delta)$ deters collusion (step 1), before characterizing the other deterrence reserves (step 2), and establishing the monotonicity of $\mathcal{D}(\delta)$ in δ (step 3).

- *Step* 1. Fix $\mathbf{r} \leq \mathbf{r}_S^D(\delta)$, and let $\bar{r} = \max\{r_1, r_2\}$ denote the higher of the two reserves. Because $\mathbf{r}_S^D(\delta) \in (\delta)$ and $\bar{r} \leq r_S^D(\delta)$, it follows from Assumption S that $(\bar{r}, \bar{r}) \in \mathcal{D}(\delta)$. And because $\mathbf{r} \leq (\bar{r}, \bar{r})$, it follows from Assumption L that $\mathbf{r} \in \mathcal{D}(\delta)$.

- *Step* 2. Fix $r > r_U^D(\delta)$, which amounts to $\delta > \hat{\delta}_U(r)$; collusion would thus be sustainable if there were a unique market with reserve r. We thus have $S(r) < L(\underline{c}, r, \delta)$ and, from Assumption L, $S(r) < L(r', r, \delta)$ for any $r' \geq \underline{c}$; that is, collusion is sustainable, regardless of the reserve set in the other market. Hence, all deterrence reserves lie below $r_U^D(\delta)$.

Fix now $r \in (r_S^D(\delta), r_U^D(\delta)]$, implying that $(\underline{c}, r)$ deters collusion (i.e., $\hat{\delta}(\underline{c}, r) \geq \delta$) whereas (r, r) does not (i.e., $\hat{\delta}(r, r) < \delta$). From Assumption L, in the range $r' \leq r$, $\delta(r', r)$ is strictly decreasing in r'. Hence, for any δ, conditional on setting the reserve r in one market and a lower reserve $r' \leq r$ in the other market, collusion is deterred if and only if $r' \leq \hat{r}(r_j, \delta)$, where $\hat{r}(r, \delta)$ is the unique solution in r' to $\hat{\delta}(r', r) = \delta$ in the range $r' \leq r$. Assumption L moreover ensures that $\hat{r}(r, \delta)$ is strictly decreasing in δ.

- *Step* 3. As δ increases, the deterrence set $\mathcal{D}(\delta)$ shrinks: for any $\delta \in (0,1)$, $\mathbf{r} \in \mathcal{D}(\delta)$ amounts to $\hat{\delta}(\mathbf{r}) \geq \delta$, which in turn implies $\hat{\delta}(\mathbf{r}) > \delta'$ for any $\delta' < \delta$; hence, $\mathcal{D}(\delta) \subseteq \mathcal{D}(\delta')$ for any $\delta' < \delta$. Furthermore, $\mathcal{D}(\delta)$ is strictly shrinking as δ increases: as $\hat{r}(r, \delta)$ is strictly decreasing in δ, in the range $r_j \leq r_i$, any $\mathbf{r}$ such that $r_i \in \left[r_S^D(\delta), r_U^D(\delta)\right]$ and $r_j = \hat{r}(r_i, \delta)$ deters collusion for δ, but no longer does so for any $\delta' > \delta$. Finally, $\mathcal{D}(\delta)$ shrinks continuously as δ increases. In particular, for any δ and any $\mathbf{r} = (r_1, r_2) \in \mathcal{D}(\delta)$, there exists a nearby $\mathbf{r}'$ that belongs to $\mathcal{D}(\delta')$ for δ' higher but sufficiently close to δ: for instance, in the range $r_2 \leq r_1$, if $r_2 > \underline{c}$, then $\mathbf{r}' = (r_1, r_2')$ would do, for r_2' slightly below r_j; and if instead $r_2 = \underline{c}$, then $\mathbf{r}' = (r_1', r_2)$ would do, for r_1' slightly below r_1. ■

OA-A.4 Proof of Lemma A.4

That $U^D(\delta)$ is continuous follows directly from the Maximum Theorem, as the buyer's competitive payoff $\bar{U}^{Comp}(\mathbf{r})$is continuous in $(\mathbf{r})$ and the deterrence set is compact and continuous in δ.[1] That $U^D(\delta)$ is strictly decreasing follows from the fact that $\mathbf{r}^D(\delta)$ lies in the boundary of the deterrence set, which is strictly shrinking in δ.[2] By construction, $U^D\left(\delta^C\right) = U^{Comp}\left(r^C\right) = \max_r U^{Comp}(r) > U^{Comp}\left(r^A\right) > U^{Coll}\left(r^A\right)$. Finally, $\lim_{\delta \longrightarrow 1} U^D(\delta) = 0$, as the deterrence set converges to $\{(\underline{c}, \underline{c})\}$ as δ tends to 1. ■

OA-A.5 Proof of Lemma A.5

We have: $\frac{\partial \bar{\phi}(r,\delta)}{\partial r} + \frac{\partial \underline{\phi}(r,\delta)}{\partial r} = (1-\delta)\{(1+\delta)S'(r) - \delta[B'(r) + C'(r)]\} > 0$, where the inequality uses $0 < \delta < 1$, and Lemma A.1 (in Appendix A.2). ■

OA-A.6 Proof of Lemma A.6

The proof proceeds in two parts:

• *Part* (*i*). The derivatives involved in the first-order conditions (A.9) and (A.10) are given by:

$$\begin{aligned}
\frac{dU^{Comp}}{dr}(r) &= 2[1-G(r)]g(r)(v-r) - G(r), \\
\frac{\partial \bar{\phi}}{\partial r}(r,\delta) &= \left(1-\delta^2\right)[G(r) + g(r)(r-\underline{c})] - \delta\left[G^2(r) - \delta g(r)\Gamma(r)\right], \\
\frac{\partial \underline{\phi}}{\partial r}(r,\delta) &= \delta\left[\delta G^2(r) - g(r)\Gamma(r)\right],
\end{aligned}$$

where $\Gamma(r) \equiv \int_{\underline{c}}^{r} G(c)dc$. It follows that the first-order conditions (A.9) and (A.10) depend on the cost distribution only through $\{g(r_i^D(\delta)), G(r_i^D(\delta)), \Gamma(r_i^D(\delta))\}_{i=1,2}$.

Let

$$r^D(\delta) \equiv \frac{r_1^D(\delta) + r_2^D(\delta)}{2} \quad \text{and} \quad \Delta^D(\delta) \equiv r_2^D(\delta) - r_1^D(\delta),$$

respectively, denote the mean of and the difference in the optimal deterrence reserves. From Proposition 3, $\Delta^D(\delta) > 0$. Suppose now that $U^{Comp}(r_2^D(\delta)) = U^{Comp}(r_1^D(\delta))$ for some $\delta \in (\delta^C, 1)$, and consider an arbitrary small change in the distribution G that affects $g(\cdot)$, $G(\cdot)$, and $\Gamma(\cdot)$ only in the interval $(r^D(\delta) - \varepsilon, r^D(\delta) + \varepsilon)$, for some $\varepsilon \in (0, \Delta^D(\delta)/2)$, in such a way that:[3]

$$\Delta_\varepsilon^U(\delta) \equiv \int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon} (v-c)\left[g_\varepsilon(c)G_\varepsilon(c) - g(c)G(c)\right]dc \neq 0,$$

where G_ε and g_ε are the distribution and density associated with the change, respectively.

[1] Recall that $\mathcal{D}(\delta) = \mathcal{D}_S(\delta) \cup \mathcal{D}_1(\delta) \cup \mathcal{D}_2(\delta)$, where $\mathcal{D}_S(\delta)$ and each $\mathcal{D}_i(\delta)$ are compact subsets of $[\underline{c}, \min\{\bar{c}, v\}]^2$. For continuity, see step 3 in Appendix A.3.

[2] Specifically, $r_2^D(\delta) = \hat{r}(r_1^D(\delta), \delta)$, where $\hat{r}(\cdot, \delta)$ is strictly decreasing in δ; it follows that $\mathbf{r}^D(\delta) \notin \mathcal{D}(\delta')$ for any $\delta' > \delta$.

[3] We construct an example of such a change in Online Appendix OA-A.7.

By construction, such a change does not affect $\{g(r_i^D(\delta)), G(r_i^D(\delta)), \Gamma(r_i^D(\delta))\}_{i=1,2}$ (and, thus, does not affect the first-order conditions (A.9) and (A.9)); hence, the optimal deterrence reserves $r_1^D(\delta)$ and $r_2^D(\delta)$ remain unchanged. The payoff $U^{Comp}\left(r_1^D(\delta)\right)$ is also unaffected, as it depends on G only in the range $r \leq r_1^D(\delta) < r - \varepsilon$. By contrast, by altering G in the range $\left(r^D(\delta) - \varepsilon, r^D(\delta) - \varepsilon\right) \subset \left(r_1^D(\delta), r_2^D(\delta)\right)$, the change affects $U^{Comp}\left(r_2^D(\delta)\right)$ by an amount that, using (3), is equal to $\Delta_\varepsilon^U(\delta) \neq 0$. Hence, following the change, the optimal deterrence reserves yield different payoffs in the two markets.

- *Part (ii)*. Fix δ such that $r_2^D(\delta) = r^C$. From (A.10) and (A.11), this amounts to:

$$0 = \frac{\partial \bar{\phi}}{\partial r}(r^C, \delta) = \left(1 - \delta^2\right) S'(r^C) - \delta\left[B'(r^C) - \delta C'(r^C)\right]. \tag{OA-A.1}$$

Because $\frac{\partial^2 \bar{\phi}}{\partial r \partial \delta}(r, \delta) = B'(r) + 2\delta\left[S'(r) - C'(r)\right] > 0$, it follows that, for any $\delta' \neq \delta$, the equality (OA-A.1) is violated, implying that the first-order condition (A.10) cannot be satisfied for $r_2 = r^C$ and $\lambda > 0$. Hence, generically over δ, $r_2^D(\delta) \neq r^C$. ■

OA-A.7 Example of a generic alteration of the cost distribution

We construct here an example of a change in the cost distribution, from G to G_ε, satisfying the conditions required in the proof of Lemma A.6 (see footnote 3 in Section OA-A.7), namely:

(i) the change is continuous and affects g, G, and Γ (the primitive of G) in the range $\left(r^D(\delta) - \varepsilon, r^D(\delta) + \varepsilon\right)$ (and only in that range), where

$$r^D(\delta) \equiv \frac{r_1^D(\delta) + r_2^D(\delta)}{2}$$

and ε is an arbitrary number satisfying

$$0 < \varepsilon < \frac{\Delta^D(\delta)}{2},$$

where

$$\Delta^D(\delta)\, r_2^D(\delta) - r_1^D(\delta)$$

denotes the difference in the two deterrence reserves, which is positive from Proposition 3.

(ii) the change affecs buyer 2's payoff, which boils down to

$$\Delta_\varepsilon^U(\delta) \equiv \int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon} (v - c)\left[g_\varepsilon(c) G_\varepsilon(c) - g(c) G(c)\right] dc \neq 0.$$

Consider the following baseline function $f(\cdot)$, defined over $\left[r^D(\delta)-\varepsilon, r^D(\delta)+\varepsilon\right]$:

$$f(c)=\begin{cases} 4+4\frac{c-r^D(\delta)}{\varepsilon} & \text{for } c\in\left[r^D(\delta)-\varepsilon, r^D(\delta)-\frac{3\varepsilon}{4}\right], \\ -2-4\frac{c-r^D(\delta)}{\varepsilon} & \text{for } c\in\left[r^D(\delta)-\frac{3\varepsilon}{4}, r^D(\delta)-\frac{\varepsilon}{4},\right], \\ 4\frac{c-r^D(\delta)}{\varepsilon} & \text{for } c\in\left[r^D(\delta)-\frac{\varepsilon}{4}, r^D(\delta)\right], \\ -4\frac{c-r^D(\delta)}{\varepsilon} & \text{for } c\in\left[r^D(\delta), r^D(\delta)+\frac{\varepsilon}{4}\right], \\ 4\frac{c-r^D(\delta)}{\varepsilon}-2 & \text{for } c\in\left[r^D(\delta)+\frac{\varepsilon}{4}, r^D(\delta)+\frac{3\varepsilon}{4}\right], \\ 4-4\frac{c-r^D(\delta)}{\varepsilon} & \text{for } c\in\left[r^D(\delta)+\frac{3\varepsilon}{4}, r^D(\delta)+\varepsilon\right]. \end{cases}$$

In words, the function $f(\cdot)$ is continuous and, in all segments, its slope is constant and has the same absolute value; furthermore, dividing the interval $\left[r^D(\delta)-\varepsilon, r^D(\delta)+\varepsilon\right]$ into four sub-intervals of equal length, the function $f(\cdot)$ oscillates between -1 and 1 as follows: in the first and last intervals, $\left[r^D(\delta)-\varepsilon, r^D(\delta)-\varepsilon/2\right]$ and $\left[r^D(\delta)+\varepsilon/2, r^D(\delta)+\varepsilon\right]$, it first jumps *up* from 0 to 1, before going back to 0; by contrast, in the two middle intervals $\left[r^D(\delta)-\varepsilon/2, r^D(\delta)\right]$ and $\left[r^D(\delta), r^D(\delta)+\varepsilon/2\right]$, it first jumps *down* from 0 to -1, before going back to 0. It is straightforward to check that this function satisfies (with F denoting the primitive of f, and Φ denoting the primitive of F):

$$\begin{aligned} f\left(r^D(\delta)-\varepsilon\right) &= F\left(r^D(\delta)-\varepsilon\right)=\Phi\left(r^D(\delta)-\varepsilon\right)=0, \\ f\left(r^D(\delta)+\varepsilon\right) &= F\left(r^D(\delta)+\varepsilon\right)=\Phi\left(r^D(\delta)+\varepsilon\right)=0. \end{aligned}$$

It thus satisfies condition (i) above. It follows that any scaled-down function $\rho f(\cdot)$, for any arbitrary small (positive or negative) ρ, also satisfies condition (i)—and for ρ small enough, the modified cost distribution still has a strictly monotone hazard rate.

We now turn to condition (ii). Integrating by parts yields:

$$\begin{aligned} \int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon}(v-c)g(c)G(c)dc &= \left[(v-c)\frac{G^2(c)}{2}\right]_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon}+\int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon}\frac{G^2(c)}{2}dc, \\ \int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon}(v-c)g_\varepsilon(c)G_e(c)dc &= \left[(v-c)\frac{G_\varepsilon^2(c)}{2}\right]_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon}+\int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon}\frac{G_\varepsilon^2(c)}{2}dc. \end{aligned}$$

It follows that $\Delta_{\varepsilon}^{U}(\delta)$ can be expressed as, for $G_{\varepsilon}(\cdot) = G(\cdot) + \rho F(\cdot)$:

$$\begin{aligned}
\Delta_{\varepsilon}^{U}(\delta) &= \frac{1}{2}\int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon} \left[G_{\varepsilon}^{2}(c) - G^{2}(c)\right] dc \\
&= \frac{1}{2}\int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon} \left\{[G(c) + \rho F(c)]^{2} - G^{2}(c)\right\} dc \\
&= \rho \int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon} G(c)F(c)\, dc + \frac{1}{2}\int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon} F^{2}(c)\, dc,
\end{aligned}$$

where the first equality stems from $G_e\left(r^D(\delta) - \varepsilon\right) = G\left(r^D(\delta) - \varepsilon\right)$ and $G_e\left(r^D(\delta) + \varepsilon\right) = G\left(r^D(\delta) + \varepsilon\right)$, implying that the bracketed terms coincide in the previous expressions. In the last expression, the second term is positive. Hence, if

$$\int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon} G(c)F(c)\, dc \geq 0,$$

we have $\Delta_{\varepsilon}^{U}(\delta) > 0$. If instead

$$\int_{r^D(\delta)-\varepsilon}^{r^D(\delta)+\varepsilon} G(c)F(c)\, dc < 0,$$

then $\Delta_{\varepsilon}^{U}(\delta) > 0$ for any $\rho < 0$. Hence, in both cases there exists a change satisfying condition (ii).

OA-B Extension to more than two markets

In this section, we consider an extension that allows for $n \geq 2$ markets. We continue to assume that there are two suppliers and focus on the case with one integrated buyer.

OA-B.1 Setting

Let $\mathcal{N} \equiv \{1, \ldots, n\}$ denote the set of markets and $\mathbf{r} = (r_1, \ldots, r_n)$ the vector of reserves in these markets, with the convention that markets are labeled by decreasing order of the reserves: that is, $r_1(n) \geq \cdots \geq r_n(n)$. As before, each market is characterized by the same value v for the buyer and the same distribution G over $[\underline{c}, \overline{c}]$ for the sellers' constant marginal costs, where cost draws are independent across suppliers and time.

Because symmetry facilitates collusion, for the sake of exposition we focus on market allocation that are as balanced as possible. Hence, if n is even, then each supplier is the designated winner in $n/2$ markets, alternating each period. For example, with $n = 4$, the markets might be divided up as $\{1, 2\}$ and $\{3, 4\}$, with supplier 1 designated for $\{1, 2\}$ in one period and for $\{3,4\}$ in the next period. If n is odd, then suppliers alternate being the designated winner in $(n+1)/2$ and in $(n-1)/2$ markets. For example with $n=3$, the markets might be divided up as $\{1,2\}$ and $\{3\}$, with supplier 1 designated for $\{1,2\}$ in one period and for $\{3\}$ in the next period.[4]

Consider a supplier facing the lowest cost and designated for the markets *other than* $\mathcal{M} \subset \mathcal{N}$. By deviating, the supplier can get the monopoly payoff rather than the non-designated supplier payoff in all markets in $\mathcal{M}$; the associated short-term stake is thus:

$$S_{\mathcal{M}}(\mathbf{r}) \equiv \sum_{j \in \mathcal{M}} S(r_j).$$

Although $S_{\mathcal{M}}(\mathbf{r})$ only depends on $(r_j)_{j \in \mathcal{M}}$, it is notationally convenient to write it as a function of the entire vector $\mathbf{r}$.

The long-term stake for a supplier that would forever be designated for the markets in $\mathcal{M}$ is instead given by

$$L_{\mathcal{M}}(\mathbf{r},\delta) \equiv \frac{\delta}{1-\delta}\left[\sum_{i \in \mathcal{M}} B(r_i) - \sum_{j \in \mathcal{N} \setminus \mathcal{M}} C(r_j)\right],$$

where $B(\cdot)$ and $C(\cdot)$ denote the benefit and cost of collusion, given by (1) and (2). Then the long-term stake for a supplier that is designated for the markets in $\mathcal{M}$ next period, accounting for the rotation over the set of designated markets, is

$$L^R_{\mathcal{M}}(\mathbf{r},\delta) \equiv \frac{1}{1+\delta} L_{\mathcal{M}}(\mathbf{r},\delta) + \frac{\delta}{1+\delta} L_{\mathcal{N}\setminus\mathcal{M}}(\mathbf{r},\delta).$$

Define k to be half the number of markets if there is an even number of markets and that number rounded up if there is an odd number of markets:

$$k \equiv \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

Collusion is not incentive compatible if a supplier has an incentive to deviate in k markets when it is designated for $n-k$ markets. Thus, given reserves $\mathbf{r}$, the suppliers are deterred from collusion if and only if for all $\mathcal{M} \in \mathcal{P}(\mathcal{N},k)$, where $\mathcal{P}(\mathcal{N},k)$ is the set of permutations of subsets of $\mathcal{N}$ containing k elements, either $L^R_{\mathcal{M}}(\mathbf{r},\delta) \leq S_{\mathcal{M}}(\mathbf{r})$ or $L^R_{\mathcal{N}\setminus\mathcal{M}}(\mathbf{r},\delta) \leq S_{\mathcal{N}\setminus\mathcal{M}}(\mathbf{r})$. For example, if $n=4$, then a market allocation in which each supplier alternates between

[4] It can be shown that these market allocations do indeed maximize the scope for collusion for the optimal accommodation and deterrence reserves.

markets $\{1,2\}$ and $\{3,4\}$ is deterred if either

$$L^R_{\{1,2\}}(\mathbf{r},\delta) \leq S_{\{1,2\}}(\mathbf{r}) \text{ or } L^R_{\{3,4\}}(\mathbf{r},\delta) \leq S_{\{3,4\}}(\mathbf{r}).$$

OA-B.2 Optimal reserves

The buyer's optimal reserves conditional on deterrence satisfy:

$$\max_{\mathbf{r}} \sum_{i \in \mathcal{N}} U^{Comp}(r_i)$$

subject to, for all $\mathcal{M} \in \mathcal{P}(\mathcal{N}, k)$, either $L^R_{\mathcal{M}}(\mathbf{r},\delta) \leq S_{\mathcal{M}}(\mathbf{r})$ or $L^R_{\mathcal{N}\setminus\mathcal{M}}(\mathbf{r},\delta) \leq S_{\mathcal{N}\setminus\mathcal{M}}(\mathbf{r})$. The buyer then compares this payoff to $nU^{Coll}(r^{Coll})$ to determine whether to accommodate or deter collusion.

We first note that, as long as the number of markets remains even, asymmetric reserves still help to reduce the cost of deterrence:

Proposition OA-B.1. *If the number of markets is even and collusion is not blockaded, then an integrated buyer's optimal deterrence reserves are asymmetric.*

Proof. Suppose that there are $n = 2k$ markets. We first show that, for symmetric reserves, the scope for collusion is maximized when each supplier is designated for half of the markets. Let r denote the symmetric reserve and suppose without loss of generality that a supplier is currently designated for $n-h$ markets, for some $h \in \mathcal{N}$. The supplier's short-term stake from a deviation in the remaining h markets is then given by

$$S(r,h) \equiv hS(r),$$

whereas its long-term stake is:

$$L(r,h,\delta) \equiv \frac{\delta\left[hB(r) - (n-h)C(r)\right] + \delta^2\left[(n-h)B(r) - hC(r)\right]}{1-\delta^2}.$$

Hence, the supplier has no incentive to deviate if $\phi(r,h,\delta) \geq 0$, where

$$\begin{aligned}\phi(r,h,\delta) &\equiv \left(1-\delta^2\right)\left[L(r,h,\delta) - S(r,h)\right] \\ &= \delta\left[hB(r) - (n-h)C(r)\right] + \delta^2\left[(n-h)B(r) - hC(r)\right] - \left(1-\delta^2\right)hS(r),\end{aligned}$$

which is decreasing in h, that is,

$$\frac{\partial\phi(r,h,\delta)}{\partial h} = (1-\delta)\left[\delta B(r) + \delta C(r) - (1+\delta)S(r)\right] < 0,$$

where the inequality stems from Lemma A.1. Collusion is sustainable if no supplier has an incentive to deviate, that is, if

$$\min\{\phi(r,h,\delta),\phi(r,n-h,\delta)\}\geq 0.$$

It follows that collusion is easiest to sustain when $h = n - h = k (= n/2)$. In particular, collusion is blockaded if $\phi\left(r^C,k,\delta\right)\geq 0$.

Suppose now that collusion is not blockaded. Because the buyer's payoff $U^{Comp}(r)$ is concave in r, the optimal symmetric deterrence reserve, $r_S^D(\delta)$, is then such that $\phi\left(r_S^D(\delta),k,\delta\right) = 0$. Starting from $\mathbf{r}_S^D(\delta) = \left(r_S^D(\delta),...,r_S^D(\delta)\right)$, consider now a small change in reserves in which r_1 is slightly increased by $dr_1 = (n-1)\,dr > 0$, whereas all other reserves are reduced by dr. By construction, this small change in the reserves has no first-order effect on the buyer's overall payoff, as the net impact is given by

$$\sum_{i\in\mathcal{N}}\frac{\partial U^{Comp}}{\partial r_i}(r_i)\,dr_i\bigg|_{r_i=r_S^D(\delta)} = \frac{\partial U^{Comp}}{\partial r}(r)\bigg|_{r=r_S^D(\delta)}[dr_1-(n-1)\,dr]=0.$$

However, for the supplier currently *not* designated for market 1, the short-term stake becomes (where $r\equiv r_S^D(\delta)-dr$)

$$\hat{S}(\mathbf{r})\equiv S(r_1)+(k-1)\,S(r)\,,$$

whereas its long-term stake is

$$\hat{L}^R(\mathbf{r},\delta)\equiv\frac{\delta\left[B(r_1)+(k-1)\,B(r)-kC(r)\right]+\delta^2\left[kB(r)-C(r_1)-(k-1)\,C(r)\right]}{1-\delta^2}.$$

The supplier thus has an incentive to deviate if $\hat{\phi}(\mathbf{r},\delta)\equiv\left(1-\delta^2\right)\left[\hat{L}^R(\mathbf{r},\delta)-\hat{S}(\mathbf{r})\right]<0.$

A first-order approximation yields:

$$
\begin{aligned}
\hat{\phi}(\mathbf{r},\delta) &\simeq \hat{\phi}\left(\mathbf{r}_S^D(\delta),\delta\right)+\\
&\quad \delta\left\{B'\left(r_S^D(\delta)\right)[dr_1-(k-1)\,dr]-kC'\left(r_S^D(\delta)\right)(-dr)\right\}\\
&\quad +\delta^2\left\{kB'\left(r_S^D(\delta)\right)(-dr)-C'\left(r_S^D(\delta)\right)[dr_1-(k-1)\,dr]\right\}\\
&\quad -\left(1-\delta^2\right)S'\left(r_S^D(\delta)\right)[dr_1-(k-1)\,dr]\\
&= \delta\left[B'\left(r_S^D(\delta)\right)+C'\left(r_S^D(\delta)\right)\right]kdr\\
&\quad -\delta^2\left[B'\left(r_S^D(\delta)\right)+C'\left(r_S^D(\delta)\right)\right]kdr\\
&\quad -\left(1-\delta^2\right)S'\left(r_S^D(\delta)\right)kdr\\
&= (1-\delta)\left\{\delta\left[B'\left(r_S^D(\delta)\right)+C'\left(r_S^D(\delta)\right)-(1+\delta)\,S'(r)\right]\right\}kdr\\
&< 0,
\end{aligned}
$$

where the first equality follows from $\hat{\phi}\left(\mathbf{r}_S^D(\delta),\delta\right)=\phi\left(r_S^D(\delta),k,\delta\right)=0$ and $dr_1=(2k-1)dr$, whereas the inequality stems from $\delta\in(0,1)$, $dr>0$ and Lemma A.1. It follows that the change in reserves strictly deters collusion while maintaining the buyer's total payoff. By continuity, there exists a neighboring change in reserves that keeps deterring collusion and enhances the buyer's payoff. ■

To go further, we now focus on the case in which $v=1$ and costs are uniformly distributed over $[0,1]$. Table OA-B.2 reports the buyer's optimal reserve policy for different numbers of markets (from $n=1$ to $n=6$) and a given value of the discount factor ($\delta=0.94$).

Table OA-B.2: Optimal deterrence reserves $\mathbf{r}^D$ for $\delta=0.94$

n	n even	n odd
1		$r_1=0.5$ (blockaded)
2	$r_1=0.4894$, $r_2=0.3937$	
3		$r_1=\cdots=r_3=0.4953$
4	$r_1=0.4849$, $r_2=\cdots=r_4=0.3894$	
5		$r_1=\cdots=r_5=0.4512$
6	$r_1=0.4834$, $r_2=\cdots=r_6=0.3875$	

Note: Assumes $v=1$ and uniformly distributed costs.

Several features can be noted. First, for even numbers of markets, the asymmetry established by Proposition OA-B.1 takes a specific form, where a single reserve is set above the others. Intuitively, treating $n-1$ markets equally enhances the buyer's expected payoff because $U^{Comp}(r)$ is concave in r, and also limits the suppliers' ability to restore symmetry by optimizing over the composition of designated packages.[5] Second, for odd numbers of markets, the optimal reserve policy is instead symmetric.[6] This is because the market allocation itself is necessarily imbalanced (with one supplier designated for $(n+1)/2$ and the other for $(n-1)/2$ markets), to an extent such that there is no need to introduce further asymmetry.[7] Third, for each type of situation, collusion becomes easier as the number of markets increases, which in turn calls for more aggressive reserves. That is, letting $\mathbf{r}^D(n) = (r_1^D(n), \ldots, r_n^D(n))$ denote the optimal deterrence reserves, we have $\mathbf{r}^D(n+2) < \mathbf{r}^D(n)$. To see why, consider first the case of even numbers of markets. If the buyer were restricted to symmetric reserves, then increasing the number of markets would raise proportionally the short-term and long-term stakes, and thus have no impact on the scope for collusion.[8] However, the buyer finds it optimal to introduce an asymmetry by setting one reserve above the others and, given the optimal level of asymmetry, to adjust the overall level of the reserves so as to ensure that the supplier not designated for that market has an incentive to deviate. As the number of markets increases, however, the long-term stake increases proportionally, which in turn calls for more aggressive reserves.

For odd numbers of markets, the optimal deterrence reserves are symmetric (at least for up to six markets), as the market allocation is itself sufficiently asymmetric. However, as the number of markets increases, the relative asymmetry of the market allocation is reduced, which calls again for more aggressive reserves.

[5]For instance, if $n = 4$ and $r_1 > r_2 > r_3 > r_4$, the suppliers can maintain some symmetry by designating one supplier for markets 1 and 4 and the other for markets 2 and 3. By contrast, with $r_1 > r_2 = r_3 = r_4$, one supplier necessarily ends up with a better designated packaged and the buyer moreover perfectly controls the level of asymmetry.

[6]It could therefore be implemented as well by independent buyers, the symmetry of the optimal reserves ensuring that the preference for accommodation versus deterrence is the same for all buyers, integrated or not—and conditional on deterrence, the optimal reserves constitute a Nash equilibrium of the reserve-setting game.

[7]As the number of markets increases, this imbalance however tends to become relatively small; asymmetric reserves may thus become again optimal.

[8]The optimal symmetric reserve is $\hat{\delta}^{-1}(\delta)$, where the condition determining the threshold $\hat{\delta}(\cdot)$ is given by $\frac{\hat{\delta}(r)}{1-\hat{\delta}(r)} = \frac{S(r)}{B(r)-C(r)}$, as increasing the number of markets from 2 to $n = 2k$ leads to multiply both the numerator and denominator of the right-hand side by k.

Table OA-B.3: Threshold discount factor between deterrence and accommodation (δ^A)

n	n even	n odd
1		0.9714
2	0.9541	
3		0.9586
4	0.9501	
5		0.9545
6	0.9489	

Note: Assumes $v = 1$ and uniformly distributed costs.

It follows from the last observation that, for each type of situation (i.e., even or odd number of markets), deterrence becomes more costly as the number of markets increases, and is thus less likely to be optimal. This intuition is confirmed by Table OA-B.3, which reports, for the same numbers of markets as before, the discount factor threshold $\delta^A(n)$ above which accommodation dominates deterrence. We have:

$$\delta^A(n+2) < \delta^A(n).$$

Thus, as the number of markets increases, deterrence is optimal for a smaller range of discount factors.

In contrast, increasing the number of markets from an *even* to an *odd* number introduces an intrinsic asymmetry in the market allocation and can make collusion more fragile, and thus easier to deter. Indeed, deterrence is optimal for a wider range of discounts factors with $n = 3$ or even with $n = 5$ than with $n = 2$.

OA-C Illustration of monotonicity assumptions

In this section, we show that our monotonicity assumptions are satisfied when costs are distributed over $[0, 1]$ according to the power distribution $G(c) = c^{1/s}$ with $s > 0$ and $v \geq 1$. Specifically, we show that the unique-market discount factor threshold, $\hat{\delta}_U(r)$ is decreasing in r.

For this setup, we have:

$$B(r) = \int_0^r G^2(c)\, dc = \frac{sr^{1+\frac{2}{s}}}{2+s}, \quad \text{(OA-C.2)}$$

$$C(r) = \int_0^r [G(r) - G(c)]\, G(c)\, dc = \frac{sr^{1+\frac{2}{s}}}{(1+s)(2+s)}, \quad \text{(OA-C.3)}$$

$$S(r) = G(r)(r - \underline{c}) = r^{1+\frac{1}{s}}.$$

We first show that the critical discount factors thresholds $\hat{\delta}_S(r) = \hat{\delta}(r,r)$ and $\hat{\delta}_U(r) = \hat{\delta}(\underline{c}, r)$ are decreasing in r, before turning to the monotonicity of the long-term stake. For symmetric reserves equal to r, the threshold $\hat{\delta}_S(r)$, given by $(A.1)$, is equal to

$$\hat{\delta}_S(r) = \frac{1}{1+\frac{B(r)-C(r)}{S(r)}} = \frac{1}{1+\frac{s^2 r^{\frac{1}{s}}}{(1+s)(2+s)}},$$

which is strictly decreasing in r over the relevant range $r \in [0,1]$. For a unique market, the threshold $\hat{\delta}_U(r)$, given by $(A.2)$, is equal to

$$\begin{aligned}\hat{\delta}_U(r) &= \sqrt{\frac{S(r)}{S(r)-C(r)} + \frac{B^2(r)}{4[S(r)-C(r)]^2}} - \frac{B(r)}{2[S(r)-C(r)]} \\ &= \sqrt{\frac{(1+s)(2+s)}{(1+s)(2+s)-sr^{\frac{1}{s}}} + \left[\frac{1+s}{2}\frac{sr^{\frac{1}{s}}}{(1+s)(2+s)-sr^{\frac{1}{s}}}\right]^2} - \frac{1+s}{2}\frac{sr^{\frac{1}{s}}}{(1+s)(2+s)-sr^{\frac{1}{s}}}.\end{aligned}$$

Using

$$x(r) \equiv \frac{1+s}{2}\frac{sr^{\frac{1}{s}}}{(1+s)(2+s)-sr^{\frac{1}{s}}}, \tag{OA-C.4}$$

this threshold can be expressed as $\hat{\delta}_U(r) = \delta_U(x(r))$, where:

$$\delta_U(x) \equiv \sqrt{1+\frac{2x}{1+s}+x^2} - x \tag{OA-C.5}$$

is strictly decreasing in x:

$$\delta_U'(x) = \frac{\frac{1}{1+s}+x}{\sqrt{1+\frac{2z}{1+s}+x^2}} - 1 = \frac{\sqrt{\frac{1}{(1+s)^2}+\frac{2x}{1+s}+x^2}}{\sqrt{1+\frac{2x}{1+s}+x^2}} - 1 < 0.$$

Because $x(r)$ is strictly increasing in r, it follows that $\delta_U(x)$ is strictly decreasing in x.

We now show that the long-term stake $L(r_j, r_i, \delta)$ is strictly increasing in r_j the relevant range $\delta > \underline{\delta} \equiv \inf_{\mathbf{r}\in[\underline{c},\min\{v,\overline{c}\}]^2}\hat{\delta}(\mathbf{r})$. We have:

$$\frac{\partial L(r_j, r_i, \delta)}{\partial r_j} = \frac{\delta}{1-\delta^2}\left[\delta B'(r_j) - C'(r_j)\right].$$

It follows that $L(r_j, r_i, \delta)$ is strictly increasing in r_j if and only if $\delta B'(r_j) > C'(r_j)$, which amounts to

$$\delta > \frac{C'(r_j)}{B'(r_j)}.$$

From $(OA-C.2)$ and $(OA-C.3)$, the right-hand side is constant and equal to:

$$\frac{B(r)}{C(r)} = \frac{\frac{sr^{1+\frac{2}{s}}}{(1+s)(2+s)}}{\frac{sr^{1+\frac{2}{s}}}{2+s}} = \frac{1}{1+s}.$$

To conclude the argument, we now show that $\underline{\delta} > 1/(1+s)$. The argument relies on four steps.

• *Step* 1. *For any* $r \in [\underline{c}, \min\{v, \overline{c}\}]$, $\hat{\delta}(r, r) > 1/(1+s)$. Fix $r \in [\underline{c}, \min\{v, \overline{c}\}]$. From the above observations, the threshold $\hat{\delta}(r, r) = \hat{\delta}_S(r)$ is strictly decreasing in r; furthermore, for $r = 1$ it is equal to

$$\hat{\delta}_S(1) = \frac{1}{1 + \frac{s^2}{(1+s)(2+s)}} > \frac{1}{1+s}.$$

The conclusion follows.

• *Step* 2. *For any* $r \in [\underline{c}, \min\{v, \overline{c}\}]$, $\hat{\delta}(\underline{c}, r) > 1/(1+s)$. Fix $r \in [\underline{c}, \min\{v, \overline{c}\}]$. From the above observations, the threshold $\hat{\delta}(\underline{c}, r) = \hat{\delta}_U(r)$ can be expressed as $\delta_U(x)$, for $x = x(r)\,(\geq 0)$ given by $(OA-C.4)$. Furthermore,

$$\begin{aligned}
\delta_U(x) > \frac{1}{1+s} \quad &\Longleftrightarrow \quad \sqrt{1 + \frac{2x}{1+s} + x^2} > x + \frac{1}{1+s} \\
&\Longleftrightarrow \quad 1 + \frac{2x}{1+s} + x^2 > \left(x + \frac{1}{1+s}\right)^2 \\
&\Longleftrightarrow \quad 1 > \frac{1}{(1+s)^2},
\end{aligned}$$

where the first equivalence stems from $(OA-C.5)$ and the second one from $x > 0$ (ensuring that $1 + \frac{2x}{1+s} + x^2$ and $x + \frac{1}{1+s}$ are both positive), and the last inequality holds trivially as $s > 0$. The conclusion follows.

• *Step* 3. *For any* $r \in [\underline{c}, \min\{v, \overline{c}\}]$ *and any* $\tilde{r} \in [\underline{c}, r]$, $\hat{\delta}(r, \tilde{r})$ *is weakly decreasing in* $\tilde{r}$. Fix $r \in [\underline{c}, \min\{v, \overline{c}\}]$ and $\tilde{r} \in [\underline{c}, r]$. From Lemma 2, $L(\tilde{r}, r, \delta)$ is strictly increasing in δ and $\hat{\delta}(r, \tilde{r})$ is the unique solution in δ to

$$L(\tilde{r}, r, \delta) = S(r).$$

As $L(\tilde{r}, r, \delta)$ is twice continuously differentiable in δ and $\tilde{r}$, $\hat{\delta}(r, \tilde{r})$ is continuously differentiable in $\tilde{r}$ and:

$$\left.\frac{\partial \hat{\delta}(r_j, r)}{\partial r_j}\right|_{r_j = \tilde{r}} = -\frac{\left.\frac{\partial L(r_j, r, \delta)}{\partial r_j}\right|_{r_j = \tilde{r}, \delta = \hat{\delta}(r, \tilde{r})}}{\left.\frac{\partial L(r_j, r, \delta)}{\partial \delta}\right|_{r_j = \tilde{r}, \delta = \hat{\delta}(r, \tilde{r})}},$$

where $\partial L(r_j, r, \delta) / \partial \delta|_{r_j=\tilde{r}, \delta=\hat{\delta}(r,\tilde{r})} > 0$ and:

$$\left.\frac{\partial L(r_j, r, \delta)}{\partial r_j}\right|_{r_j=\tilde{r}, \delta=\hat{\delta}(r,\tilde{r})} = \frac{\hat{\delta}(r, \tilde{r})}{1 - \hat{\delta}^2(r, \tilde{r})}\left[\hat{\delta}(r, \tilde{r}) B'(\tilde{r}) - C'(\tilde{r})\right] = \frac{\hat{\delta}(r, \tilde{r}) B'(\tilde{r})}{1 - \hat{\delta}^2(r, \tilde{r})}\left[\hat{\delta}(r, \tilde{r}) - \frac{1}{1+s}\right].$$

It follows that:

$$\left.\frac{\partial \hat{\delta}(r_j, r)}{\partial r_j}\right|_{r_j=\tilde{r}} \lesseqgtr 0 \iff \left.\frac{\partial L(r_j, r, \delta)}{\partial r_j}\right|_{r_j=\tilde{r}, \delta=\hat{\delta}(r,\tilde{r})} \gtreqless 0 \iff \hat{\delta}(r, \tilde{r}) \gtreqless \frac{1}{1+s}. \tag{OA-C.6}$$

Suppose now by way of contradiction that $\left.\partial \hat{\delta}(r_j, r) / \partial r_j\right|_{r_j=\tilde{r}} > 0$ for some $\tilde{r} \in (\underline{c}, r]$, and let $\underline{\tilde{r}} \equiv \inf\left\{\check{r} \in [\underline{c}, \tilde{r}] \mid \left.\partial \hat{\delta}(r_j, r) / \partial r_j\right|_{r_j=\check{r}} > 0\right\}$. From $(OA - C.6)$, $\hat{\delta}(r, \check{r}) < 1/(1+s)$ for any $\check{r} \in (\underline{\tilde{r}}, \tilde{r}]$. Furthermore, from step 2, $\hat{\delta}(r, \underline{c}) > 1/(1+s)$. Hence, $\underline{\tilde{r}} > \underline{c}$ and, by continuity, $\hat{\delta}(r, \underline{\tilde{r}}) = 1/(1+s) > \hat{\delta}(r, \tilde{r})$. It follows that $\check{r} \in (\underline{\tilde{r}}, \tilde{r}]$ such that $\hat{\delta}(r, \check{r}) < 1/(1+s)$ (by continuity) and $\left.\partial \hat{\delta}(r_j, r) / \partial r_j\right|_{r_j=\check{r}} < 0$ (by definition of $\underline{\tilde{r}}$), contradicting $(OA - C.6)$. It follows that $\left.\partial \hat{\delta}(r_j, r) / \partial r_j\right|_{r_j=\tilde{r}} \leq 0$ $\hat{\delta}(r, \tilde{r}) \geq 1/(1+s)$ for any $\tilde{r} \in [\underline{c}, r]$.

- *Step* 4. $\underline{\delta} > 1/(1+s)$. Fix $\mathbf{r} = (r_1, r_2)$ and let $\bar{r} \equiv \max\{r_1, r_2\}$. We have:

$$\hat{\delta}(\mathbf{r}) \geq \hat{\delta}(\bar{r}, \bar{r}) = \hat{\delta}_S(\bar{r}) \geq \hat{\delta}_S(1),$$

where the first inequality stems from step 3 and the symmetry of $\hat{\delta}(\cdot)$ (namely, $\hat{\delta}(r_1, r_2) = \hat{\delta}(r_2, r_1)$), and the second one stems from the monotonicity of $\hat{\delta}_S(r)$. Hence:

$$\underline{\delta} = \hat{\delta}_S(1) = \frac{2 + 3s + s^2}{2 + 3s + 2s^2} = \frac{1}{1 + \frac{s^2}{2+3s+s^2}} > \frac{1}{1+s}$$

where the inequality stems from $s < 2 + 3s + s^2$.

It follows from the above that, for any $\delta \geq \underline{\delta}$, the long-term stake $L(r_j, r_i, \delta)$ is strictly increasing in r_j. This, in turn, implies that the threshold $\hat{\delta}(r_i, r_j)$ is strictly increasing in r_j in the range $r_j \leq r_i$.

Example: uniform distribution. For $s = 1$, we have:

$$\frac{C'(\cdot)}{B'(\cdot)} = \frac{1}{2} < \underline{\delta} = \frac{6}{7},$$

and:

$$\hat{\delta}_U(r) = \frac{\sqrt{36 - 6r + r^2} - r}{6 - r} \text{ and } \hat{\delta}_S(r) = \frac{6}{6 + r},$$

which, as r increases, strictly decrease from $\hat{\delta}_S(0) = \hat{\delta}_U(0) = 1$ to, respectively, $\hat{\delta}_U(1) = \left(\sqrt{31} - 1\right)/5 \simeq 0.91$ and $\hat{\delta}_S(1) = \underline{\delta} = 6/7 \simeq 0.86$.

OA-D Illustrations of coordination and coordination failure with independent buyers

To illustrate two ways in which one obtains no coordination failure, Figure D.1 considers the example in which costs are uniformly distributed over $[0, 1]$ and $v = 1$. The panels depict the deterrence boundary and the buyers' best-responses for different values of the discount factor. Interestingly, the scope for coordination failure is not monotonic in the discount factor. From Proposition 4, when the discount factor is sufficiently high that an integrated buyer accommodates collusion, $\delta > \delta^A$, then the unique Nash equilibrium of the reserve-setting game also involves accommodation, as illustrated in Figure D.1(a). At the other extreme, as illustrated in Figure D.1(b), when the discount factor is sufficiently low that collusion is blockaded, $\delta < \delta^C$, the unique Nash equilibrium of the reserve-setting game has independent buyers both setting a reserve of r^C, just as an integrated buyer would do.

(a) $\delta = 0.975$ (b) $\delta = 0.92$

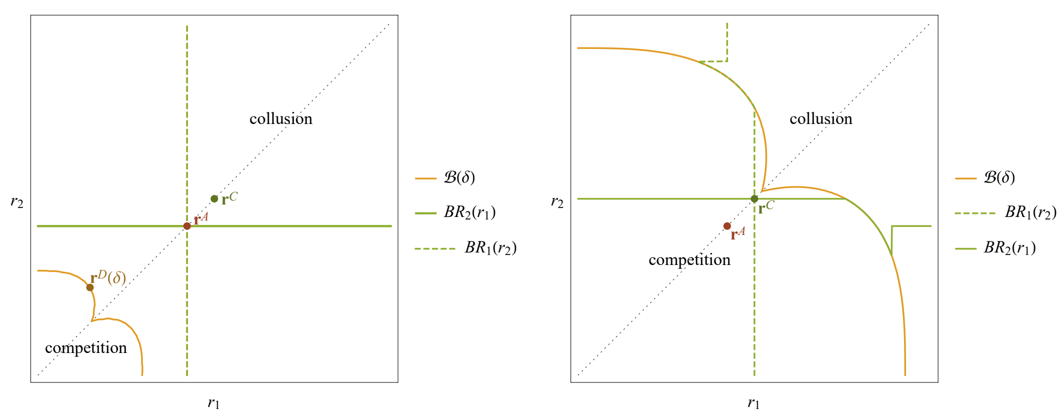

Figure D.1: No coordination failure: an integrated buyer's optimal reserves, $\mathbf{r}^A$ in the case of panel (a) and $\mathbf{r}^C$ in the case of panel (b), are the unique Nash equilibrium of the reserve-setting game. The panels depict the deterrence boundaries and the buyers best-responses over the full relevant range $r_i \in [0, 1]$. Assumes that costs are uniformly distributed over $[0, 1]$, $v = 1$, and δ is as indicated. In this setup, $\delta^C = 0.9231$ and $\delta^A = 0.9540$, so an integrated buyer accommodates collusion in panel (a), and collusion is blockaded in panel (b).

There exists a threshold discount factor $\delta_N^D \in (\delta^C, \delta^A)$ such that coordination failure arises for sure, that is, for $\delta \in (\delta_N^D, \delta^A)$, an integrated buyer would deter collusion using the optimal deterrence reserves, but those optimal deterrence reserves do not constitute a Nash equilibrium of the reserve-setting game with independent buyers.[9] For instance, in Figure 3(a) in the body of the paper, which has $\delta \in (\delta_N^D, \delta^A)$ but close to δ^A, the only Nash equilibrium involves accommodation. Considering a lower δ, but still in the range (δ_N^D, δ^A), Figure D.2(a) shows a case in which there exists an accommodation equilibrium and also a continuum of deterrence equilibria, all of which are suboptimal. For still lower δ, Figure D.2(b) shows a case in which there only exist deterrence equilibria, all of which are suboptimal.

(a) $\delta = 0.94827$ (b) $\delta = 0.94800$ (detail)

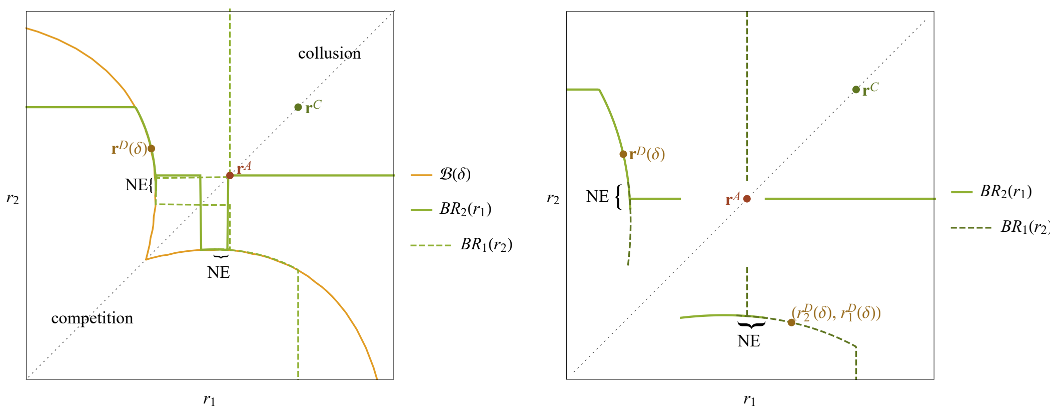

Figure D.2: Coordination failure: an integrated buyer deters collusion with optimal deterrence reserves $\mathbf{r}^D(\delta)$, but in the reserve-setting game with independent buyers, those optimal reserves are not a Nash equilibrium. Panel (a) depicts the deterrence boundaries, buyers' best-responses, and the diagonal; but to reduce clutter, panel (b) shows only the best responses and diagonal. Both panels assume that costs are uniformly distributed over $[0, 1]$ and $v = 1$. The discount factor δ is as indicated above the panels. In this setup, $\delta^C = 0.9231$ and $\delta^A = 0.9540$, so we have $\delta \in (\delta^C, \delta^A)$.

[9] In the examples of Figure 3 in the body of the paper and Figure D.2 here, we have $\delta^C = 0.9231$, $\delta_N^D = 0.9475$, and $\delta^A = 0.9540$.