“Welfare improving tax evasion”

Chiara Canta, Helmuth Cremer and Firouz Gahvari
Welfare improving tax evasion*

Chiara Cantal, Helmut Cremer, and Firouz Gahvari

Abstract

We study optimal income taxation in a two-group framework where the private cost of misreporting income is positively correlated with productivity. We show that, if high-wage types always reveal their income truthfully, letting low-wage types cheat would lead to Pareto-superior outcomes regardless of the audit costs (as compared to deterring them). When there is no cheating, redistribution takes place on first- or second-best frontiers with the low-wage types always ending up worse off than the high-wage types. Letting low-wage types conceal their income reduces the need to recourse to second-best mechanisms for redistribution. Additionally, it increases the reach of first-best redistribution to outcomes at which low-wage types are better off than high-wage types.

JEL classification: H20, H21, H26

Keywords: Optimal taxation, tax evasion, audits, welfare-improving.

---

*We thank the editor and the reviewers for their helpful comments and suggestions. Helmut Cremer gratefully acknowledges the financial support from Chaire “Marché des risques et création de valeur” of the FdR/SCOR, as well as the funding received by TSE from ANR under grant ANR-17-EURE-0010 (Investissements d’Avenir program).

1Department of Economics and Finance, TBS Business School, 31068 Toulouse, France.
2Toulouse School of Economics, University of Toulouse Capitole, 31015 Toulouse, France.
3Department of Economics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA.
1 Introduction

It is nearly five decades since Mirrlees (1971) and Allingham and Sandmo (1972) launched the literatures on the optimal general income tax and on tax evasion. The two, which have now grown into two of the most fertile subdisciplines in the area of economics of taxation, are both concerned with efficient and fair ways to raise tax revenues for the government. Yet, rather curiously, they have gone their own separate ways. The focus of the optimal tax literature has been on the formulation of income tax schedules, and that of the tax evasion literature on the design of enforcement policies taking the tax schedule as given. One fundamental reason for this parting of ways is no doubt their diametrically different foundational assumptions. Whereas the Mirrleesian optimal taxation literature assumes incomes are publicly observable, the tax evasion literature has unobservability of incomes as its raison d’être.

Over this period of time, the attempts to bring these two literatures together have been few and far between. One, published some twenty five ago, is Cremer and Gahvari (1995) who, using the Stiglitz (1982) two-group reformulation of Mirrlees (1971), allow for incomes to be misreported and observed only through costly audits. They investigate the properties of the resulting optimal policy with a general income tax schedule, an audit policy conditioned on reported incomes, and punishment for misreporters as its instruments. Chander and Wilde (1998) consider nonlinear taxation with a continuum of individuals and establish some properties of the optimal tax schedule for the case where individuals are risk neutral. Schroyen (1997) also allows for non-linear taxation but restricts the penalty to be proportional to the tax evaded.1

The aim of this paper is to cast doubt on a widely-accepted view about tax evasion in the literature and the public. This is the view that tax evasion is a “bad thing”; that is, it lowers social welfare and must be deterred when not too costly. We ask if this is always the case: are there indeed no circumstances under which tax evasion can be a “good thing” which should be glossed over even if it can be deterred at a low cost (indeed costlessly)? The answer lies in a hitherto ignored role that tax evasion might play in the design of an optimal general income tax schedule. This role is due to the existence of private evasion costs and comes into play when high-wage individuals find it more costly to engage in evasion activities than the low-wage individuals. Allowing for it, as we show below, opens up an avenue for tax evasion to become a determining factor for designing optimal tax/deterrence policies. Tolerating some tax evasion can be at times socially optimal.

The key insight comes from the realization that optimal tax systems are inherently distortive

---

1Casamatta (2021) considers costly but legal avoidance so that the issue of auditing does not arise. In his setting, concealment costs are the same for everyone. Gahvari and Micheletto (2020) rule out audits by following a riskless approach to evasion and assume identical concealment technologies.
as they have to ensure certain incentive compatibility constraints are satisfied (unless, of course, the constraints do not bite). Tax evasion can ease these constraints and alleviate the distortions. In this way, tax evasion can improve social welfare. Needless to say, tax evasion is also, by nature, a costly activity that reduces social welfare. In designing an optimal tax system, when incomes are publicly unobservable, both of these positive and negative effects need to be taken into account. An important point to consider though is that it is not just the magnitude of misreporting costs per se that matters in the trade-off between the gains and losses. How the misreporting costs affect individual of different types also plays a crucial role in this calculus. It is this aspect that helps determine which incentive compatibility constraints will loosen and thus the possibility and the extent of redistribution through the tax system.

We consider this problem within the two-group reformulation of Mirrlees (1971) optimal income tax problem by Stiglitz (1982)—a setting we shall refer to as MS (for Mirrlees/Stiglitz). We adopt its informational structure about the public unobservability of ability types and labor supplies, but drop its observability of incomes assumption thus allowing for the possibility of tax evasion. The key feature of our setup is that the two ability groups face different evasion costs. At the most general level, the two sources of heterogeneity (ability and cost of income misreporting) can be uncorrelated or correlated (positively or negatively). The popular perception is that the well-to-do have more opportunities to engage in tax evasion. Others reject this. In a recent study, for example, Gsottbauer et al. (2022) find no support for a negative correlation between social status and ethical behavior. Some believe the relationship goes the other way. Andreoni et al. (2017) have argued that the rich are more likely to behave socially than the poor. They attribute this behavior to the diminishing marginal utility of income. Another argument that supports a correlation of this nature is the stylized fact that low-skill workers are more likely to work in the underground economy, while high-skill workers may find it difficult or even impossible. A number of studies point to a negative relationship between education and informal labor provision; see for instance Pedersen (2003), Oviedo et al. (2009), Heigner et al. (2013), and Kolm and Larsen (2016). To put it differently, the educated rich find it more costly to evade as compared to the uneducated poor.

The aim of this paper is not to take side on this finding—only to explore its implication for devising optimal general income tax policies in the presence of tax evasion. Specifically, we focus on the surprising implications of income misreporting costs for the design of optimal income tax/deterrence policies in a setting where these costs induce low-productivity types to evade but not high-productivity types. A stylized setting in which (i) the misreporting costs for the high-productivity types are large enough to prevent them from income misreporting under any circumstance and (ii) low-skill individuals can misreport their income at no cost, is the simplest
vehicle for this purpose. In what follows, we shall refer to high-productivity individuals who face a prohibitively large evasion cost and thus reveal their income truthfully as “honest”, and to our setting as EL (denoting evasion by the low-wage type).

To see how tax evasion may be welfare-enhancing in our model, consider a redistributive tax system wherein individuals who report a low income face lower taxes (or higher subsidies). Low-skill individuals are in a position to report a low income without incurring any evasion cost which entitles them to pay a low tax or receive a high transfer. They can do this while earning more than they report (by working, for instance, in the informal market). High-skill individuals, by contrast, cannot do this. Evasion imposes a prohibitively high cost on them. To report a low income level, they must also earn a low income level. They may thus prefer to earn/report a high income and to pay a high tax, rather than earn/report a low income in order to pay a low tax. This type of evasion enables the tax system to redistribute a higher amount of resources to low-skill individuals than would otherwise be possible. One should then tolerate it rather than eradicate it through audits (even if the audits entail no costs).

That the simultaneous existence of “honest” in addition to “dishonest” taxpayers matter for equilibrium outcomes and policy design have long been recognized in the literature, but not within the Mirrleesian optimal tax paradigm. Gordon (1987) introduces a “psychic cost of evasion” into Allingham and Sandmo (1972) and study how that changes the latter paper’s results. Erard and Feinstein (1994) posit a game theoretic framework to study and compare the equilibrium solutions for a model with dishonest taxpayers and a model with a mix of honest and dishonest taxpayers. Honesty is defined in terms of truthful reporting of incomes which are exogenously determined. The tax system is given with constant proportional tax and penalty rates. The policy design is limited to audits based on reported incomes.

Alger and Renault (2006) study the importance of honesty in a wider context. They consider a principal and agent framework wherein the agent has certain private information. Agents may or may not feel compelled to reveal their private information truthfully. If they do, they are referred to as honest; otherwise as dishonest. The authors introduce another layer of complication to this setup by assuming that honest agents may or may not feel compelled to reveal that they are honest. They show that the distinction matters significantly and that this latter “conditional” honesty drastically affects the set of implementable allocations.

In our setup, honesty refers only to truthful reporting of incomes; the taxpayer’s type always remains hidden. Incomes are endogenous and not publicly observable. The policy design includes

---

2The intuition we develop in the paper for our results makes it clear that these extreme assumption are not necessary for tax evasion to be welfare-enhancing. Indeed, we present an example in Appendix D where both types face identical misreporting costs and show that there continue to be circumstances under which tax evasion improves welfare.
the tax system, whose sole restriction is incentive compatibility (with respect to the ability type). The tax administration is able to uncover true incomes through an audit policy conditioned on reported incomes. All individuals, regardless of their type, choose their labor supply and the amount of income they want to report. Low-wage individuals have no cost associated with misreporting their income. This is not the case for high-wage individuals, whose cost of misreporting is prohibitive. Consequently, their reported income is the same as their true income. If they want to choose the same consumption/reported-income bundle as low-wage individuals (mimic them as the terminology goes), high-wage individuals will have to work less hours than low-wage individuals in order to actually earn what the latter types report.

We show that allocations that can be implemented in the EL setting include the set of implementable allocations under MS. The inclusion is strict; there are first-best allocations that cannot be implemented in the MS setting with full observability of incomes, but are implementable under EL when low-wage individuals’ incomes are not observable. While these results rely on our specific assumptions about misreporting costs, we show in Appendix D that the mechanism for achieving them remains optimal for a wider range of misreporting costs. Interestingly too, auditing is never optimal in this setting. This is surprising a priori because one would expect less information to yield a worse outcome. The intuition for the result is that, if the income of low-wage individuals is not observable, the tax schedule does not affect their labor supply choice, which can then be set at its first-best level. Hence, their reported income can be distorted down, to relax the incentive constraints of high-wage individuals, at no welfare loss. Put differently, one can decrease the utility of the mimiccker without hurting the mimicked individual. We derive conditions under which the no-audit solution implements the first best.

These results have an interesting methodological implication whose scope goes beyond our specific setting. The existing auditing models, which consider instruments that are restricted only by the information structure (and a cap on penalties), ensure that the revelation principle applies and show that there is truthful reporting of incomes. The fact that the revelation principle and truthful reporting go hand in hand is of course trivial when there is only adverse selection (with individuals’ types being their true incomes). Nevertheless, models that allow for moral hazard as well, as in Mookherjee and Png (1989) or Cremer and Gahvari (1995), have similar properties in that the revelation principle applies and there is truthful reporting of incomes in equilibrium. In our model too, policies are restricted only by the information structure and a

---

3In developing countries high income individuals can and do send their income away from their country. This is not due to their low-cost concealment technology, but to the fact that authorities turn a blind eye on their activities. A necessary requirement for fighting evasion is to have incorruptible tax administrators. Almost all models of tax evasion, including ours, assume a well-meaning and welfare-maximizing tax authority (government). Slemrod (1990) points out, quite correctly, that a well-functioning tax system requires one to specify the enforcement mechanism. We need a completely different theoretical guide for addressing corruption at the top.

maximum penalty, and the revelation principle continues to apply. Yet we do not have truthful reporting of incomes.\textsuperscript{5} The crucial point here is that, because of misreporting costs, incomes are no longer just a message which can be changed at no cost. This result is not quite the same as the one of Green and Laffont (1986), who show that, when the message space is type specific, the revelation principle may not apply. In our setting, it is only the cost of income misreporting which is type specific. Perfectly mimicking another type, by earning their assigned income, bears no cost in itself; its cost arises only when this income is misreported.

2 Related literature

The optimal tax evasion/deterrence literature is vast and even a concise survey of it is beyond the scope of this paper; but a few papers must be mentioned to put our findings in proper perspective.\textsuperscript{6} Our focus in this regard is on the existence of tax evasion in the equilibrium of their models. Most of the early literature on tax evasion, starting with Allingham and Sandmo (1972), where income is exogenously given, and subsequently Sandmo (1981) and Cremer and Gahvari (1994), where income is determined endogenously, restrict the income tax schedule to be linear. They also concentrate on purely random audits where each tax return is audited with a certain probability regardless of the amount that is reported.\textsuperscript{7} These papers do not address the optimality of audits and, by their nature, have tax evasion in equilibrium.

Reinganum and Wilde (1986) took a crucial step forward in the study of optimal audits. They cast the problem within a principal agent setting with exogenous incomes and restrict audits to follow a simple cutoff rule wherein all, and only, the reports below a certain level are audited. They show that this policy dominates purely random audits and that there is tax evasion in equilibrium.\textsuperscript{8} Later, Chander and Wilde (1998) and many subsequent papers, took a more general approach and restricted policies only by the available information.\textsuperscript{9} While the possibility of misreporting in these models restricts the set of feasible policies, the revelation principle applies and there is effectively no tax evasion in equilibrium.\textsuperscript{10}

Among the more recent models is Lehmann et al. (2014), who consider a two-country random-utility multi-principal model wherein there is no tax evasion per se, but individuals can avoid

\textsuperscript{5}Recall that to prove the revelation principle, one shows that every mechanism that involves misreporting can be replicated by a mechanism that induces truth telling. This is achieved by giving the individuals the same payoff in the incentive compatible mechanism as they would have obtained with their (possibly false) report in the initial mechanism.

\textsuperscript{6}We thank a reviewer who suggested this and provided helpful guidance.

\textsuperscript{7}An interesting and critical overview of this literature is provided by Cowell (1985).

\textsuperscript{8}It is of course never optimal to report incomes above the cutoff level.

\textsuperscript{9}A maximum penalty is, however, always imposed and this constraint is binding (the so called “principle of maximum deterrence”).

\textsuperscript{10}Marhuenda and Ortuno-Ortín (1997) show that this depends on the way the penalty function is restricted. For some penalty functions, there will be tax evasion in equilibrium.
taxes through migration. The migration cost in their setting is the counterpart to the concealment cost in ours. They show that the results crucially depend on the income elasticity of this evasion cost. The case where this elasticity decreases with the skill level is similar to our setting where evasion costs increase with wages.\footnote{However, since they consider a continuum of types and, most significantly, introduce strategic interaction between the two principals, their results cannot be directly compared with ours.} A mention must also be made of Dhami and Al-Nowaihi (2007, 2010) who point out that the expected-utility approach the literature has followed fails to account for the fact that most individuals do not cheat even with simple audit strategies that imply a low probability of detection (so that cheating would increase expected utility). They show that this apparent paradox can be resolved by considering prospect theory to model individuals’ choices under uncertainty; see also Piolatto and Trotin (2016).\footnote{They use the same mechanism design problem as in Chandler and Wilde (1998) but with agents who behave under prospect theory.}

These studies share a common message; namely, that the possibility of tax evasion can only lower social welfare. This is the message we challenge in the current paper. Davidson et al. (2007) have previously challenged this message but in a different context. They show that “black markets” may have a positive impact on social welfare and that they should not necessarily be done away with even if audit costs are zero. Their approach differs from ours in that they concentrate on commodity taxes so that the driving factor in their model is the quality of the products that can be sold in the black market.

3 The benchmark model (MS)

Consider an economy with two types of individuals, denoted by $i = h, \ell$, who differ in their productivity $w_h$ and $w_\ell$ with $w_h > w_\ell$. There are $n_h$ individuals of type $h$ and $n_\ell$ individuals of type $\ell$. Preferences over consumption $x$ and labor supply $L$ are represented by the utility function

$$u(x, L),$$

satisfying the standard properties. Denote pre-tax incomes by $I_i = w_i L_i$, tax payments by $T_i$ and assume purely redistributive taxes. The full information Pareto-frontier is obtained by maximizing a weighted sum of utilities with weights $\alpha_h$ and $\alpha_\ell$ such that $\alpha_h + \alpha_\ell = 1$ subject to the resource constraint. It is determined by solving problem $\mathcal{P}_F$ defined as

$$\begin{align*}
\max_{T_h, I_h, T_\ell, I_\ell} & \quad W = \alpha_h u \left( I_h - T_h, \frac{I_h}{w_h} \right) + \alpha_\ell u \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right), \\
\text{s.t.} & \quad n_h T_h + n_\ell T_\ell = 0.
\end{align*}$$
The first-best (FB) allocations, denoted by $[(T_h^*, I_h^*), (T_\ell^*, I_\ell^*)]$, determine the Pareto frontier $PF$ represented in Figure 1, where $u_h$ and $u_\ell$ denote the utility of $h$- and $\ell$-type individuals.\(^1\)

Figure 1: Pareto frontier and implementable allocations under MS and EL.

By the first theorem of welfare economics, the competitive equilibrium is on $PF$; it is shown by point $a$ which is above the 45 degree line because $w_h > w_\ell$. In what follows, we concentrate on the part of the frontier “to the right” of the competitive equilibrium. This implicitly assumes that the weights are such that the solution involves redistribution from the high-wage to the low-wage individuals so that $T_h^* > T_\ell^*$. We know from Stiglitz (1982) that this includes the utilitarian FB obtained when $\alpha_i = n_i$. We also assume $I_\ell^* > 0$.

The Mirrlees-Stiglitz problem assumes that incomes $I_i = w_i L_i$ are publicly observable at no cost, but $w_i$ and $L_i$ are not (for $i = h, \ell$). To determine the MS allocations, one has to add an incentive compatibility constraint (IC) to problem $P_F$, which yields problem $P_{MS}$ defined as

$$
\max_{T_h, I_h, T_\ell, I_\ell} \quad W = \alpha_h u_h \left( I_h - T_h, \frac{I_h}{w_h} \right) + \alpha_\ell u_\ell \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right)
$$

s.t. $u_h \left( I_h - T_h, \frac{I_h}{w_h} \right) - u_\ell \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) \geq 0$, \hspace{1cm} (1)

$$
n_h T_h + n_\ell T_\ell = 0.
$$

It is common practice to refer to the utility of the $h$-type evaluated at the allocation intended for the $\ell$-type in the incentive constraint as the utility of a fictitious “mimicker”—a terminology that we also follow. Stiglitz (1982) has shown that problem $P_{MS}$ can have two types of solution. In one, the incentive compatibility constraint (1) is non-binding and the solution is on the Pareto

---

\(1\) We shall refer to $[(T_h^*, I_h^*), (T_\ell^*, I_\ell^*)]$ as an “allocation” even though, strictly speaking, it corresponds to the allocation $[(x_h^*, I_h^*), (x_\ell^*, I_\ell^*)] = [(I_h - T_h, I_h^*), (I_\ell - T_\ell, I_\ell^*)]$. 

---
frontier (PF). This is depicted by that part of PF in Figure 1 which ends at point $b$. In the other, the solution is given by the FOC of problem $\mathcal{P}_{MS}$ along with the binding incentive compatibility constraint (1) and the resource constraint. This is depicted by the MS curve in Figure 1 that starts from point $b$ and lies everywhere above the 45 degree line. An allocation with $u_l > u_h$ would obviously violate the incentive constraint so that it cannot be achieved under MS; see footnote 18 for a formal proof.

4 The model with low-wage type evaders (EL)

Consider a setting with public unobservability of ability types and labor supplies, as in MS, but with the possibility of tax evasion. Assume that the two groups differ in their cost of concealing their true incomes. High-wage individuals, who cannot be identified by the policy designer, always reveal their true income $I_h = w_h L_h$, because their cost of misreporting is prohibitive. On the other hand, low-wage individuals, who are not identifiable either, can misreport their income $I_l = w_l L_l$ at no cost. Denoting reported income by $\tilde{I}$, we thus have $\tilde{I}_h = I_h$ but $\tilde{I}_l$ may differ from $I_l$. Assume $0 \leq \tilde{I}_l \leq I_l$ to rule out negative- and over-reporting of income. Concentrate again on the case where the binding incentive constraint (if any), is from the $h$-type to the $l$-type. Assuming no audits are performed (we show below that this is in fact the optimal policy), the policy problem is

$$\max_{T_h, I_h, T_l, I_l} W = \alpha_h u \left( I_h - T_h, \frac{I_h}{w_h} \right) + \alpha_l u \left( I_l - T_l, \frac{I_l}{w_l} \right)$$

s.t.

$$u \left( I_h - T_h, \frac{I_h}{w_h} \right) - u \left( \tilde{I}_l - T_l, \frac{\tilde{I}_l}{w_l} \right) \geq 0,$$

$$\tilde{I}_l \geq 0,$$

$$n_l T_l + n_h T_h = 0,$$

and referred to as $\mathcal{P}_l$. The incentive compatibility constraint ensures that high-wage individuals do not mimic low-wage individuals. Since high-wage individuals have a prohibitively high cost of misreporting, if they mimic low-wage individuals they have no choice but to earn exactly the reported income of low-wage individuals, $\tilde{I}_l$.

\textsuperscript{14}The no-overreporting constraint simplifies the expressions; one can easily show that it will not be binding.

\textsuperscript{15}Without loss of generality, we concentrate on self-selecting policies. Formally, proceeding as in Laffont and Martimort (2002, pp. 48-50) or Salanie (1998; pp. 17-18), one can show that, for any policy that does not entail truthful reporting, an incentive compatible policy can be constructed that yields the same outcome. Note that, unlike in much of the literature on optimal auditing, the revelation principle in our setting does not imply that incomes are reported truthfully. It merely implies that both types select the contract designed for them.
While the policy designer does not set $I_\ell$ directly, it is effectively set; albeit indirectly. Low-wage individuals are induced to choose and report $\bar{I}_\ell$ and $T_\ell$. Then, given these values, they choose $I_\ell$ to maximize $u(I_\ell - T_\ell, I_\ell/w_\ell)$. Importantly, $u(I_\ell - T_\ell, I_\ell/w_\ell)$ is the only term in problem (2) that depends on $I_\ell$. Consequently, the optimal choice of $I_\ell$ by low-wage individuals is tantamount to maximizing of $W$ with respect to $I_\ell$. This allow us to reformulate problem $\mathcal{P}_\ell$ by including $I_\ell$ in the list of decision variables. This is represented by problem $\mathcal{P}_\ell'$, which is the same as $\mathcal{P}_\ell$ except that there is an extra decision variable.

The Kuhn-Tucker expression for problem $\mathcal{P}_\ell'$ is

$$
\mathcal{L} = \alpha_h u \left( I_h - T_h, \frac{I_h}{w_h} \right) + \alpha_\ell u \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) + \lambda \left[ u \left( I_h - T_h, \frac{I_h}{w_h} \right) - u \left( \bar{I}_\ell - T_\ell, \frac{\bar{I}_\ell}{w_h} \right) \right] + \gamma \bar{I}_\ell + \mu (n_h T_h + n_\ell T_\ell). \tag{3}
$$

Observe that $\mathcal{P}_\ell'$ is similar to $\mathcal{P}_{MS}$ except that $\bar{I}_\ell$ replaces $I_\ell$ in the utility of the mimicker; see (1) and (2). Importantly, though, problem $\mathcal{P}_\ell'$ contains an extra choice variable in comparison with $\mathcal{P}_{MS}$. This variable is $\bar{I}_\ell$ and can always be set equal to $I_\ell$ to obtain the MS allocation. The idea we develop below is the possibility of choosing $\bar{I}_\ell$ such that $u \left( \bar{I}_\ell - T_\ell, \bar{I}_\ell/w_h \right) < u \left( I_\ell - T_\ell, I_\ell/w_\ell \right)$. If this is possible, reducing $u \left( \bar{I}_\ell - T_\ell, \bar{I}_\ell/w_h \right)$ relaxes the otherwise binding incentive constraint and allows for increased redistribution to enhance welfare. Intuitively, reducing the reported income of the low wage type, $\bar{I}_\ell$, hurts the mimicker (who cannot misreport) while it has no impact on the utility of the mimicked.

It is clear that, in the above expressions, $\mu > 0$ because the resource constraint must be binding. However, it is possible that the other multipliers $\lambda$ and $\gamma$ are equal to zero. Consequently, Problem $\mathcal{P}_\ell'$ may yield different solution regimes depending on the pattern of the binding and non-binding constraints. To study this issue in the most efficient way, we organize our analyses around the results that are already known for the MS problem and examine if or how they may change.

## 5 Solution regimes

As with the MS setting, we distinguish between first- and second-best regimes.

---

16Unobservability of $I_\ell$ disconnects it from the incentive compatibility constraint.
5.1 First best

The solutions under EL will be first best if \( \lambda = \gamma = 0 \) in problem \( P' \). The notable point about them is that they include all first-best allocations that are implementable under MS. First, we know from Stiglitz (1982) that, because the competitive equilibrium satisfies the incentive compatibility constraint with strict inequality, the Pareto efficient allocations in the neighborhood of this equilibrium can be implemented under the information structure in MS. Intuitively, the IC constraint is not violated when the amount of redistribution is “small”. The FB allocations satisfying the IC constraint are represented by the segment \( ab \) on the Pareto frontier in Figure 1; they are such that

\[
\begin{align*}
    u \left( I^*_h - T^*_h, \frac{I^*_h}{w_h} \right) &\geq u \left( I^*_\ell - T^*_\ell, \frac{I^*_\ell}{w_h} \right), \\
    \text{where a “star” denotes the first-best value of a variable. To see that these allocation are also implementable under EL, one can simply duplicate them by setting } & \left( T^*_h, I^*_h \right), \left( T^*_\ell, I^*_\ell \right) = \left( T^*_h, I^*_h \right), \left( T^*_\ell, I^*_\ell \right). \\
    \text{Although low-wage individuals can cheat at no cost, they will not do so because } I^*_\ell = I^*_\ell \text{ maximizes their utility regardless of their report (as long as } T^*_\ell = T^*_\ell). \\
    \text{The more interesting question is whether there are first-best allocations that can be implemented under EL but not under MS. Consider a Pareto efficient allocation } \left( T^*_h, I^*_h \right), \left( T^*_\ell, I^*_\ell \right) \text{ for which } (4) \text{ does not hold and thus is not implementable under MS. Then formulate an EL policy that consists of the tax function } \\
    T(\bar{I}) = T^*_\ell \quad \text{if } \bar{I} = 0, \\
    = T^*_h \quad \text{if } \bar{I} > 0, \\
    \text{and no audits. In words, an individual who reports a zero income pays } T^*_\ell \text{ (which may be negative), while any other reported income is associated with a tax equal to } T^*_h. \text{ The } \ell\text{-types’ best option under this scheme is to report } \bar{I} = 0 \text{ and pay } T^*_\ell \text{ which then leads them to earn } I^*_\ell. \text{ To report } \bar{I} > 0 \text{ and pay } T^*_h > T^*_\ell \text{ will only reduce their utility. As to the } h\text{-types, recall that they cannot misreport their income. Consequently, if they were to report } \bar{I} = 0 \text{ in order to receive } -T^*_\ell, \text{ they must also earn } I^*_h = 0. \text{ Their options are thus either (i) pay } T^*_h \text{ and earn } I^*_h \text{ or (ii) receive and consume } -T^*_\ell \text{ and earn } I^*_h = 0. \\
    \text{The first-best solution will be implementable under EL if and only if } \\
    u \left( I^*_h - T^*_h, \frac{I^*_h}{w_h} \right) \geq u \left( -T^*_\ell, 0 \right). \quad (5) \\
    \text{Now, as long as } I^*_\ell > 0, \text{ the right-hand side of the incentive compatibility constraint } (4) \text{ is larger}.
\end{align*}
\]
than that of (5):
\[ u\left( I^*_\ell - T^*_\ell, \frac{I^*_\ell}{w_h} \right) > u\left( I^*_\ell - T^*_\ell, \frac{I^*_\ell}{w_\ell} \right) > u\left( 0 - T^*_\ell, \frac{0}{w_\ell} \right). \]

Condition (5) is thus strictly weaker than (4). Consequently, there must exist FB allocations that can be implemented under EL even though they are not under MS. These are the allocations that satisfy (5) but not (4). They are represented by segment \( bc \) on the Pareto frontier in Figure 1.

While (5) must hold for some Pareto efficient allocations, it will not hold for all. As long as \( u(-T^*_\ell, 0) > 0 \), there will be a non-empty segment below and to the right of point \( b \) which is implementable under EL. However, when the first-best utility of high-wage individuals gets sufficiently close to zero, the direction of inequality (5) will unavoidably be reversed. In words, when the utility level of high-wage individuals is sufficiently small, they would be better off by exiting the labor market altogether and consume the transfer \(-T^*_\ell\) which is intended for the low type. The point where (5) is satisfied as equality is point \( c \) on the Pareto frontier. Any point to its right cannot be implemented through EL. This is illustrated in the numerical example given in Section 6 below.

Finally, we have thus far assumed that no audits are performed. This is in fact the optimal policy because audits can only do harm. With a positive probability of audits, low-wage individuals might find it optimal to report \( \tilde{I}_\ell > 0 \). Which will then mean that they have to pay \( T^*_h \) rather than \( T^*_\ell \), making the implementation of \( [(T^*_h, I^*_h), (T^*_\ell, I^*_\ell)] \) no longer feasible. Moreover, when \( \tilde{I}_\ell > 0 \), the incentive compatibility constraint (5) would have to be amended, thus making the mimicking option more attractive for the \( h \)-type. This in turn will reduce the set of FB allocations that can be implemented.

The results derived thus far are summarized in the following proposition.

**Proposition 1** Consider the EL and MS settings as defined above. (i) The set of first-best allocations that can be implemented under EL includes the set of allocations that is implementable in the MS setting. (ii) As long as \( I^*_\ell > 0 \), the inclusion is strict so that there exist FB allocations that can be implemented under EL but not under MS. (iii) Not all FB allocations are implementable under EL. (iv) Auditing is never desirable.

That the unobservability of \( I_\ell \) leads to the implementability of first-best allocations, unattainable when \( I_\ell \) is observable at no cost, is a rather striking result. Having less information is expected to bring about a worse outcome, not a better one. To garner intuition for this, remember that, in the MS setting, distorting the low-wage individuals’ labor supply downwards is needed to make mimicking less attractive to high-wage individuals. The distortion is no longer needed
for this purpose, nor is it possible to induce it, under EL. The incentive constraint is manipulated through \( \bar{I}_\ell \) in a way that makes mimicking even more costly than under MS. Low-wage individuals report \( \bar{I}_\ell \) and choose their most desired level of labor supply which would be FB (regardless of the marginal tax rate on \( \bar{I}_\ell \)).

5.2 Second best

We now turn to the case where FB allocations violate condition (5) so that they cannot be implemented. These are the allocations that lie to the right of and below point \( c \) on the Pareto frontier in Figure 1. The Kuhn-Tucker conditions of problem \( P_\ell' \) stated in Appendix A yield second-best solutions at which condition (5) is binding and \( \lambda > 0 \). Of course, with the resource constraint always binding, \( \mu \) must also be positive. Lemma 1, stated and proved in Appendix B, proves that \( \gamma > 0 \) as well. This is because the optimal policy continues to imply \( \bar{I}_\ell = 0 \), though this time as a corner solution. Given \( \bar{I}_\ell = 0 \), it follows from the Kuhn-Tucker conditions that\(^{17}\)

\[
\begin{align*}
    u_c \left( I_h - T_h, \frac{I_h}{w_h} \right) + \frac{1}{w_h} u_L \left( I_h - T_h, \frac{I_h}{w_h} \right) &= 0, \\
    u_c \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) + \frac{1}{w_\ell} u_L \left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) &= 0, \\
    u \left( I_h - T_h, \frac{I_h}{w_h} \right) - u (-T_\ell, 0) &= 0.
\end{align*}
\]

Equations (6)-(8), along with the resource constraint \( n_\ell T_\ell + n_h T_h = 0 \), determine a unique set of values for \( I_h, T_h, I_\ell, T_\ell \). In particular, these values are independent of the weights assigned to \( u^h \) and \( u^\ell \). Moreover, these equations are precisely the same equations that determine the first-best allocation \( [(T_h^c, I_h^c), (T_\ell^c, I_\ell^c)] \) at point \( c \) in Figure 1. Recall that \( c \) is the boundary point on the Pareto frontier satisfying the IC constraint as an equality (beyond it the IC constraint will be binding).

Intuitively, equation (6), along with the resource constraint, implies that \( I_h \) is set at its first-best level. Consequently, equation (8) is simply condition (5) holding as an equality which is precisely the definition of point \( c \). At this point, the high-wage individuals are indifferent between paying the high tax that goes with working and receiving a transfer without working. In other words, increasing \( \alpha_\ell \) above its value at point \( c \) does not change the values of \( [(T_\ell^{c'}, I_\ell^{c'}), (T_h^{c'}, I_h^{c'})] \); any larger value of \( \alpha_\ell \) continues to yield the solution given by point \( c \). When redistribution is limited by a positive value of \( I_\ell \), it will no longer be possible to make the \( \ell \)-types any better-off by decreasing \( I_\ell \) notwithstanding the fact that higher values of \( \alpha_\ell \) call for it. The numerical example of Section 6 below illustrates this point. The floor to redistribution is thus dictated by

\(^{17}\)In Appendix A, set \( \bar{I}_\ell = 0 \) in the Kuhn-Tucker conditions (A1a)-(A1g). This simplifies equations (A1b), (A1e)-(A1f) into equations (6)-(8).
\( \bar{I}_\ell = 0 \) with \( \bar{I}_\ell \leq I_\ell \).

Observe that the EL policy can increase the utility of the \( \ell \)-types beyond what is feasible under MS. There, the \( \ell \)-types always remain less well-off than the \( h \)-types.\(^{18}\) In contrast, point \( c \) is necessarily below the 45 degree line. At point \( c \), where the incentive constraint starts to bind, we have

\[
 u_h = u\left( I_h^*, T_h^*, \frac{I_h^*}{w_h} \right) = u\left( -T_\ell^*, 0 \right) < u_\ell.
\]

The inequality follows because \( \bar{I}_\ell = 0 \) makes the consumption bundle \( (-T_\ell^*, 0) \) available to the \( \ell \)-types under EL. The inequality is strict as long as \( I_\ell^* > 0 \). Compared to the competitive equilibrium, the ranking of utilities is reversed. This property also implies that EL can implement the Rawlsian FB solution (where the 45 degree line intersects the Pareto frontier). It follows that the FB solutions not implementable under EL are rather “extreme” and go beyond the usual notion of income redistribution in the sense that they reverse the order of inequality between the two types.

Finally, observe that auditing has only the effect of decreasing the utility of the mimicked individual without affecting the incentive constraint. Consequently, the optimal policy entails no audits. The results derived above are summarized in the following proposition.

**Proposition 2** In the second best:

(i) The solution under EL, \([ (T_h^{\text{sc}}, I_h^{\text{sc}}), (T_\ell^{\text{sc}}, I_\ell^{\text{sc}}) ] \), is unique. It is represented by point \( c \) in Figure 1 regardless of \( h \)- and \( \ell \)-types’ weights in the social welfare function.

(ii) Point \( c \) is below and to the right of the point where the 45 degree line intersects the Pareto frontier, so that \( u_\ell > u_h \); consequently the MS frontier, which lies above the 45 degree line, must lie everywhere to the left of point \( c \) in Figure 1.

\(^{18}\)To see this, observe that with \( w_h > w_\ell \), and the fact that utility changes negatively with labor supply, we have

\[
 u\left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) > u\left( I_h - T_h, \frac{I_h}{w_h} \right).
\]

Moreover, the IC constraint requires

\[
 u\left( I_h - T_h, \frac{I_h}{w_h} \right) \geq u\left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right).
\]

It follows from these two inequalities that

\[
 u\left( I_h - T_h, \frac{I_h}{w_h} \right) \geq u\left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right) > u\left( I_\ell - T_\ell, \frac{I_\ell}{w_\ell} \right).
\]

In this regard, Figure 2.1 in Stiglitz (1987, p. 2.1) which shows the FB and SB frontiers is misleading.
6 A numerical example

This section illustrates our results through a numerical example. Details of the derivations are presented in Appendix C. Assume preferences are quasilinear and represented by the following utility function:

\[ u = 2[c + 10 \ln(1 - L)]^{0.5}, \]  

(9)

where \(0 < \beta < w_\ell\). The two ability-types are of equal size with the population size being normalized at one. Hence \(n_\ell = n_h = 1/2\). Additionally, set the parameter values \(w_\ell = 14\) and \(w_h = 20\). Given these values, the Pareto frontier is represented by \(u_h^2 + u_\ell^2 = 14.82\) as depicted in Figure 2. The lowest attainable utility level is 0 and the highest 3.85.

![Figure 2: Pareto frontier and implementable allocations under MS and EL. Utilities have the functional form \(u = 2(c + \beta \ln(1 - L))^{0.5}\), with \(w_h = 20\), \(w_\ell = 14\), and \(\beta = 10\).](image)

The laissez-faire allocation is found to be \(I_\ell = c_\ell = 4\) and \(I_h = c_h = 10\) with the corresponding utility levels of \(u_\ell = 1.59\) and \(u_h = 3.50\). This is the FB allocation if \(\alpha_\ell = 1/3\) and is shown by point \(a\) on the Pareto frontier (PF) in Figure 2. Redistribution towards the \(\ell\)-types from \(a\) becomes desirable when \(\alpha_\ell\) exceeds \(1/3\). It limits the possible FB allocations to segment \(af\) on the Pareto frontier. Segment \(ea\) on the PF represents the FB allocations that entail redistribution to the \(h\)-types and are desired when \(\alpha_\ell < 1/3\).

Under MS, the IC constraint (4) is satisfied as a strict inequality if and only if \(\alpha_\ell < 0.42\). Under EL, the IC constraint (5) holds as a strict inequality if and only if \(\alpha_\ell < 0.54\). Consequently, for all \(\alpha_\ell \in [1/3, 0.42]\), the first-best is implementable under both MS and EL settings. These allocations correspond to the points on \(ab\) segment of the Pareto frontier. When \(\alpha_\ell\) exceeds 0.42, the first-best allocations can no longer be implemented under MS. However, as long as
$\alpha_\ell \in [0.42, 0.54]$, the first-best is implementable under EL. These allocations are shown in Figure 2 as segment $bc$ on the PF. When $\alpha_\ell$ exceeds 0.54, the EL setting too cannot implement the corresponding FB allocations. These are shown by segment $cf$ on the PF in Figure 2.

Turning to allocations that are second-best, they will be attained under MS for $\alpha_\ell > 0.42$ and shown in Figure 2 by the $bb'$ curve that lies everywhere below the PF. It approaches the 45 degree line as $\alpha_\ell \to 1$. The limiting $\ell$-types' allocation is $I_\ell = 0, c_\ell = -T_\ell = 1.53$ resulting in $u_\ell = 2.47$. The corresponding values for the $h$-types are $I_h = 10, c_h = 8.48$, and $u_h = 2.49$. Under EL, on the other hand, the second-best allocation is unique and given by point $c$. This is the case for all $\alpha_\ell$ above 0.54. The reason for it is that at $\alpha_\ell = 0.54$, $\tilde{I}_\ell = 0$. Interestingly though, whereas $u_\ell = 2.48$ for $\alpha_\ell = 1$ under MS, $u_\ell = 2.95 > 2.48$ for $\alpha_\ell \geq 0.54$ under EL.

Table 1 illustrates the laissez-faire allocations as well as the first best, MS, and EL allocations for different welfare weights. When low-wage individuals have a welfare weight equal to $\alpha_\ell = 0.35 < 0.42$, the first best is implementable under both MS and EL. When $\alpha_\ell = 0.5 > 0.42$, the first-best allocation continues to be implementable under EL but not MS. This is the Rawlsian FB solution with $u_\ell = u_h = 2.72$. Observe that the second-best MS solution for $\alpha_\ell = 0.5$ entails a lower utility level for the $\ell$-types as compared to what they can attain under EL (2.34 versus 2.72), but the $h$-types enjoy a higher utility level (3.05 versus 2.72.) At $\alpha_\ell = 0.54$, the EL solution remains first-best with $u_\ell = 2.95$ and $u_h = 2.48$. This corresponds to point $c$ on PF and caps the utility level the $\ell$-types can attain under EL. Observe also that $u_\ell > u_h$ at this point.

Raising $\alpha_\ell$ further does not change the optimal allocations under EL—not even as a second-best solution. Table 1 illustrates this point by finding the solution for $\alpha_\ell = 0.65$ for which we continue to have $u_\ell = 2.95$ and $u_h = 2.48$ under EL as we had with $\alpha_\ell = 0.54$. However, for $\alpha_\ell = 0.65$, this allocation is no longer FB. At the FB allocation for this value of $\alpha_\ell$, the $\ell$-types attain a higher utility level equal to $u_\ell = 3.39$ (and the $h$-types a lower utility level equal to $u_h = 1.82$.) The second-best MS allocation at $\alpha_\ell = 0.65$ results in $u_\ell = 2.42$ (and $u_h = 2.93$), which is worse for the $\ell$-types as compared to EL.

7 Discussion: positive and non-prohibitive misreporting costs

The results we have derived on the implementability of FB allocations under EL, rest on the fact that in equilibrium no resources are spent on misreporting (the high-wage does not misreport and the low-wage misreports at zero cost). It is plainly obvious that when solution entails costly misreporting, one cannot attain the FB utility levels. Nevertheless, it may still be possible to retrieve the FB allocations of consumptions and labor supplies.\(^{19}\) The crucial comparison,

\(^{19}\)This depends on whether evasion costs are specified in terms of utility or real resources.
The first extension in Appendix D is to replace the $\ell$-type’s zero cost of misreporting with costly misreporting. As long as the misreporting cost is relatively small, our mechanism continues to be implementable. The reason is that, when low-wage individuals face a small cost of misreporting under our mechanism (for earning a positive income while reporting zero), the cost can fall short of the gain they make by attaining the first-best level of labor supply. Interestingly too, the low-cost of misreporting allows the mechanism to dominate MS. The intuition for this is that the labor supply decision of the low-wage individuals is distorted under MS (due to incentive compatibility constraint). Our mechanism enhances social welfare by eliminating this distortion. At the same time, it lowers social welfare because of the cost of misreporting. As long as the latter costs are relatively small, the gain outweighs the cost.

We also consider the case where both types face positive misreporting costs. We show that our mechanism continues to be feasible and dominates MS if the misreporting cost is neither too large (to ensure that low-wage types misreport), nor too small (to ensure that high-wage types do not misreport), and if the weight of the low-wage types in the social welfare function is not too large (to ensure that the misreporting costs incurred by the $\ell$-types do not lower social welfare by an exceedingly large amount).

---

**Table 1:** Incomes, taxes, and utility levels in first-best, EL, MS, and laissez-faire allocations. Utilities have the functional form $u = 2(c + \gamma \ln(1 - L))^{0.5}$, with $w_\ell = 14$, $w_h = 20$, and $\gamma = 10$.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_\ell = 0.35$</th>
<th>$\alpha_\ell = 0.50$</th>
<th>$\alpha_\ell = 0.54$</th>
<th>$\alpha_\ell = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB</td>
<td>$I_\ell = 4$, $I_h = 10$</td>
<td>$I_\ell = 4$, $I_h = 10$</td>
<td>$I_\ell = 4$, $I_h = 10$</td>
<td>$I_\ell = 4$, $I_h = 10$</td>
</tr>
<tr>
<td></td>
<td>$T = -1.22$</td>
<td>$T = -1.53$</td>
<td>$u_\ell = 3.39$, $u_h = 1.82$</td>
<td>$u_\ell = 2.95$, $u_h = 2.48$</td>
</tr>
<tr>
<td>EL</td>
<td>$I_\ell = 4$, $I_h = 10$</td>
<td>$I_\ell = 4$, $I_h = 10$</td>
<td>$I_\ell = 4$, $I_h = 10$</td>
<td>$I_\ell = 4$, $I_h = 10$</td>
</tr>
<tr>
<td></td>
<td>$u_\ell = 2.72$, $u_h = 2.72$</td>
<td>$u_\ell = 2.95$, $u_h = 2.48$</td>
<td>$u_\ell = 2.95$, $u_h = 2.48$</td>
<td>$u_\ell = 2.95$, $u_h = 2.48$</td>
</tr>
<tr>
<td>MS</td>
<td>$I_\ell = 3.52$, $I_h = 10$</td>
<td>$I_\ell = 3.27$, $I_h = 10$</td>
<td>$I_\ell = 2.67$, $I_h = 10$</td>
<td>$I_\ell = 2.67$, $I_h = 10$</td>
</tr>
<tr>
<td></td>
<td>$T = -0.74$</td>
<td>$T = -0.80$</td>
<td>$T = -0.92$</td>
<td>$T = -0.92$</td>
</tr>
<tr>
<td></td>
<td>$u_\ell = 2.34$, $u_h = 3.05$</td>
<td>$u_\ell = 2.37$, $u_h = 3.02$</td>
<td>$u_\ell = 2.42$, $u_h = 2.93$</td>
<td>$u_\ell = 2.42$, $u_h = 2.93$</td>
</tr>
<tr>
<td>LF</td>
<td>$I_\ell = 4$, $I_h = 10$, $u_\ell = 1.59$,</td>
<td>$u_h = 3.50$</td>
<td>$u_h = 3.50$</td>
<td>$u_h = 3.50$</td>
</tr>
</tbody>
</table>

---

\[^{20}\text{We thank a reviewer for suggesting this extension.}\]
8 Concluding remarks

Three decades ago Slemrod (1990, p. 157) wrote “...in its current state, optimal tax theory is incomplete as a guide to action concerning the questions that began this paper and for other issues in tax policy. It is incomplete because it has not yet come to terms with taxation as a system of coercively collecting revenues from individuals who will tend to resist”. The few attempts that have been made to address this problem, since it was identified by Slemrod, remain far and between. In this paper, we have tried to take a step forward in this direction by integrating one particular factor that enters individuals’ tax evasion decision making into the design of optimal general income tax schedules.

The factor in question is the private cost of evasion; particularly the way it affects different ability types differently (whether in terms of real resource cost to conceal the evasion or in terms of an internal utility cost). These differentials can help in making a redistributive tax system less distortionary. What makes a tax system distortionary are the incentive compatibility constraints embedded into its structure. To the extent that private evasion costs affect different ability types differently, these costs can be used to devise policies that ease the incentive compatibility constraints into a more desired direction that allow for more redistribution.

To bring out the factors that come into play, in the simplest possible way, we have considered a setup in which high-wage types face so high an evasion cost as to make them always report their incomes truthfully. The low-wage types, on the other hand, face no evasion costs. Interestingly, and rather surprisingly, we have found that for every allocation that is feasible when both types are truthful (MS solutions), there exists a corresponding weakly Pareto-superior allocation if the low-wage types cheat but the high-wage types do not (EL solutions). Moreover, auditing is never desirable even if it can be done at no cost. Specifically, we have shown that: (i) Every first-best allocation that can be implemented under MS can also be implemented under EL. (ii) Every utility level that low-wage individuals can have under a second-best MS solution is available to them as a first-best EL solution. (iii) The first-best EL solutions include the Rawlsian outcome as well as outcomes wherein low-wage individuals are better off than high-wage individuals. Neither of these type of solutions are available under MS.

Note that we do not “undo” tax evasion in our analysis; quite the opposite. In most of the audit literature, there is effectively no tax evasion in equilibrium. In these models, the revelation principle implies truthful reporting of incomes but the possibility to evade affects the incentive constraints. We follow this tradition in that we do not impose any ad hoc restrictions on the policy and derive the best allocation that can be achieved given the available information. The interesting feature is that, while the revelation principle continues to apply when it comes to the reporting of types, it no longer implies truthful reporting of income. Consequently, the optimal
policy involves tolerating some evasion rather than getting rid of it by auditing individuals that report a low income.

There are many directions in which this work can be extended to integrate tax evasion into the Mirrleesian optimal income tax framework. The first obvious direction is to relax the assumption of perfect correlation between ability type and unwillingness to misreport income; Appendix D is a tiny step in this direction. Either one of the two individual types, or both, may include honest and dishonest income reporters. Another avenue is to explore the implications of introducing some kind of conditional honesty along the lines of Alger and Renault (2006). The message of Slemrod (1990) remains as relevant today as it was three decades ago.
Appendix

A First-order conditions of Problem (3)

The first-order (Kuhn-Tucker) conditions are

\[
\frac{\partial L}{\partial I_h} = -\left(\lambda + \alpha_h\right) u_e \left( I_h - T_h, \frac{I_h}{w_h}\right) + \mu n_h = 0,
\]
\[
(A1a)
\]
\[
\frac{\partial L}{\partial I_h} = (\lambda + \alpha_h) \left[ u_e \left( I_h - T_h, \frac{I_h}{w_h}\right) + \frac{1}{w_h} u_L \left( I_h - T_h, \frac{I_h}{w_h}\right)\right] = 0,
\]
\[
(A1b)
\]
\[
\frac{\partial L}{\partial I_t} = -\alpha_L u_e \left( I_t - T_t, \frac{I_t}{w_t}\right) + \lambda u_e \left( \tilde{I}_t - T_t, \frac{\tilde{I}_t}{w_t}\right) + \mu n_t = 0,
\]
\[
(A1c)
\]
\[
\frac{\partial L}{\partial I_t} = -\lambda \left[ u_e \left( \tilde{I}_t - T_t, \frac{\tilde{I}_t}{w_t}\right) + \frac{1}{w_t} u_L \left( \tilde{I}_t - T_t, \frac{\tilde{I}_t}{w_t}\right)\right] + \gamma = 0,
\]
\[
(A1d)
\]
\[
\frac{\partial L}{\partial \lambda} = \alpha_L \left[ u_e \left( I_t - T_t, \frac{I_t}{w_t}\right) + \frac{1}{w_t} u_L \left( I_t - T_t, \frac{I_t}{w_t}\right)\right] = 0,
\]
\[
(A1e)
\]
\[
\frac{\partial L}{\partial \gamma} = -\lambda \left[ u \left( I_h - T_h, \frac{I_h}{w_h}\right) - u \left( \tilde{I}_t - T_t, \frac{\tilde{I}_t}{w_t}\right)\right] = 0,
\]
\[
(A1f)
\]
\[
\gamma \frac{\partial L}{\partial \gamma} = \gamma \tilde{I}_t = 0.
\]
\[
(A1g)
\]

B Lemma 1: statement and proof

**Lemma 1** If the Lagrange multiplier \(\lambda\) in Problem \(P_1^t\) is positive, the Lagrange multiplier \(\gamma\) is also positive.

**Proof.** Observe first that \(\tilde{I}_t\) has to be set to minimize the utility of the mimic \(u^h(\tilde{I}_t - T_t, \tilde{I}_t/w_h)\) which is the only term in problem \(P_1^t\) that it affects. Specifically, given our assumptions (in particular that of no over-reporting), we must minimize \(u^h\) over \(\tilde{I}_t \in [0, I_t]\).

Let \(MRS\) denote the marginal rate of substitution between consumption and income. Given the standard assumptions on \(u(\cdot, \cdot)\), \(u^h(\cdot, \cdot)\) is concave with an interior maximum where \(MRS^h(I_t - T_t, \tilde{I_t}/w_t) = w_h\). Now, from the first-order condition (A1c), we have \(MRS^{\ell}(I_t - T_t, I_t/w_t) = w_t\). With \(w_h > w_t\), \(MRS^h(I_t - T_t, \tilde{I_t}/w_h) > MRS^{\ell}(I_t - T_t, I_t/w_t)\). Moreover, from the single-crossing property \(MRS^h(I_t - T_t, I_t/w_h) > MRS^{\ell}(I_t - T_t, I_t/w_t)\). These two inequalities imply

\[
MRS^h(I_t - T_t, \tilde{I_t}/w_h) > MRS^{\ell}(I_t - T_t, I_t/w_h)\,.
\]

(A2)

It follows from inequality (A2) and the concavity of \(u^h\) that \(\tilde{I} > I_t\). Consequently, \(u^h\) is increasing over \([0, I_t]\), which in turn implies that it is minimized at \(\tilde{I} = 0\). This also implies that the bracketed expression in equation (A1d) is positive resulting in a positive solution for \(\gamma\).  ■
C Details of derivations for the example

In the laissez-faire, with \( c_i = w_i L_i \), each individual maximizes

\[
2 \left[ w_i L_i + \beta \ln(1 - L_i) \right]^{0.5},
\]

with respect to \( L_i \). The first-order condition reduces to \( w_i - \beta/(1 - L_i) = 0 \). Assuming \( w_i - \beta > 0 \) for \( i = h, \ell \), this yields interior solutions:

\[
\begin{align*}
L_i^{LF} &= (w_i - \beta)/w_i, \\
c_i^{LF} &= w_i - \beta, \\
w_i^{LF} &= 2 \left[ w_i - \beta + \beta \ln(\beta - \ln w_i) \right]^{0.5}.
\end{align*}
\]

**First-best solution** The Lagrangian expression for the maximization of \( \alpha_h c_h + \alpha_\ell c_\ell \) subject to the resource constraint is

\[
\mathcal{L} = 2\alpha_h \left[ c_h + \beta \ln(1 - L_h) \right]^{0.5} + 2\alpha_\ell \left[ c_\ell + \beta \ln(1 - L_\ell) \right]^{0.5} + \mu (w_h L_h + w_\ell L_\ell - c_h - c_\ell)
\]

The first-order conditions for this problem are, for \( i = h, \ell \),

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial c_i} &= \alpha_i [c_i + \beta \ln(1 - L_i)]^{-0.5} - \mu = 0, \\
\frac{\partial \mathcal{L}}{\partial L_i} &= -\alpha_i \frac{\beta}{1 - L_i} [c_i + \beta \ln(1 - L_i)]^{-0.5} + \mu w_i = 0.
\end{align*}
\]

Substitute for \( \mu \) from (A3) into (A4) and solve for \( L_i \) to get

\[
L_i^* = (w_i - \beta)/w_i, \quad i = h, \ell.
\]

We also have, from (A3),

\[
(\alpha_h)^2 \left[ c_\ell + \beta \ln(1 - L_\ell) \right] = (\alpha_\ell)^2 \left[ c_h + \beta \ln(1 - L_h) \right].
\]
Solving equations (A5)-(A6) along with the resource constraint for \( c_h, c_{\ell}, \) and \( T_{\ell} \) yields
\[
T_{\ell}^* = \frac{\alpha_h^2 [w_{\ell} - \beta + \beta \ln (\beta/w_{\ell})] - \alpha_{\ell}^2 [w_h - \beta + \beta \ln (\beta/w_h)]}{\alpha_{\ell}^2 + \alpha_h^2},
\]
\[
c_h^* = w_h - \beta + T_{\ell}^*,
\]
\[
c_{\ell}^* = w_{\ell} - \beta - T_{\ell}^*.
\]

Given the specification for the utility function (9), and using the first-best values of \( c_h, c_{\ell}, L_h, \) and \( L_{\ell} \) from above equations, we have
\[
u_h^2 + \nu_{\ell}^2 = 4 [w_h + w_{\ell} - 2\beta + \beta \ln(\beta/w_h) + \beta \ln(\beta/w_{\ell})].
\]

The Pareto frontier is found from this equation to be
\[
u_{\ell} = 2 \left\{ w_h + w_{\ell} - 2\beta + \beta \ln(\beta/w_h) + \beta \ln(\beta/w_{\ell}) \right\} - \frac{\nu_{h}^2}{4}.
\]

Using the above values for \( c_h^*, L_h^*, c_{\ell}^*, L_{\ell}^* \) in the utility function (9), we have:
\[
u \left( c_h^*, \frac{L_h^*}{w_h} \right) = 2\alpha_h \left\{ \frac{(w_h + w_{\ell} - 2\beta) + \beta \ln(\beta/w_{\ell}) + \ln(\beta/w_h))}{\alpha_{\ell}^2 + \alpha_h^2} \right\}^{\frac{1}{2}},
\]
\[
u \left( c_{\ell}^*, \frac{L_{\ell}^*}{w_h} \right) = 2 \left\{ \beta \ln \frac{w_h - w_{\ell} + \beta}{w_h} + \frac{\alpha_h^2 [w_h + w_{\ell} - 2\beta + \beta \ln (\beta/w_h)] - \beta \alpha_h^2 \ln (\beta/w_{\ell})}{\alpha_{\ell}^2 + \alpha_h^2} \right\}^{\frac{1}{2}},
\]
\[
u (-T_{\ell}^*, 0) = 2 \left\{ \frac{\alpha_h^2 [w_h - \beta + \beta \ln (\beta/w_h)] - \alpha_{\ell}^2 [w_{\ell} - \beta + \beta \ln (\beta/w_{\ell})]}{\alpha_h^2 + \alpha_{\ell}^2} \right\}^{\frac{1}{2}}.
\]

Comparing (A8) with (A9), one finds that \( \nu (-T_{\ell}^*, 0) < \nu \left( c_{\ell}^*, \frac{L_{\ell}^*}{w_h} \right) \) if and only if
\[
w_{\ell} - \beta + \beta \ln \left( 1 - \frac{w_{\ell} - \beta}{w_h} \right) > 0,
\]

which is true for all \( w_h > w_{\ell} > \beta > 0 \) (as required for \( L_{\ell}^* > 0 \)).\(^{21}\)

\(^{21}\)With \( w_h > w_{\ell} \) and \( w_{\ell} - \beta, \)
\[
w_{\ell} - \beta + \beta \ln \left( 1 - \frac{w_{\ell} - \beta}{w_h} \right) > w_{\ell} - \beta + \beta \ln \left( \frac{\beta}{w_{\ell}} \right) > 0.
\]
Second best under MS

Under MS, the first-best allocation is implementable as long as (A10) is satisfied. If this is not the case, the problem of the social planner is

\[
\begin{align*}
\max_{I_h, I_\ell, T} & \quad 2\alpha_h \left[ I_h + T + \beta \ln \left( 1 - \frac{I_h}{w_h} \right) \right]^{0.5} + 2\alpha_\ell \left[ I_\ell - T + \beta \ln \left( 1 - \frac{I_\ell}{w_\ell} \right) \right]^{0.5} \\
\text{s.t.} & \quad I_h + T + \beta \ln \left( 1 - \frac{I_h}{w_h} \right) \geq I_\ell - T + \beta \ln \left( 1 - \frac{I_\ell}{w_\ell} \right).
\end{align*}
\] (A11)

It is straightforward to show that the second-best allocation is characterized by no distortion at the top, i.e., \(I_{h}^{MS} = w_h - \beta\). Using this condition, the incentive compatibility constraint can be rewritten as

\[
\begin{align*}
w_h - \beta + T + \beta \ln \left( \frac{\beta}{w_h} \right) \geq I_\ell - T + \beta \ln \left( 1 - \frac{I_\ell}{w_\ell} \right),
\end{align*}
\]

which, when binding, implies that

\[
T = \frac{I_\ell + \beta - w_h - \beta \ln(\beta) + \beta \ln(w_h - I_\ell)}{2}.
\]

Substituting this expression for \(T\) and \(I_{h}^{MS} = w_h - \beta\) in (A11), the problem of the social planner can be rewritten as

\[
\begin{align*}
\max_{I_\ell} & \quad 2\alpha_h \left[ \frac{w_h - \beta + \beta \ln(\beta)}{2} - \beta \ln(w_h) + \frac{I_\ell + \beta \ln(w_h - I_\ell)}{2} \right]^{0.5} \\
& \quad + 2\alpha_\ell \left[ \frac{w_h - \beta + \beta \ln(\beta)}{2} + \frac{I_\ell - \beta \ln(w_h - I_\ell)}{2} + \beta \ln \left( 1 - \frac{I_\ell}{w_\ell} \right) \right]^{0.5}
\end{align*}
\]

The first-order condition, for an interior solution, is

\[
\left[ \frac{1}{2} + \frac{\beta}{2(w_h - I_\ell^{MS})} \right] \frac{\alpha_h}{u_h} + \left[ \frac{1}{2} + \frac{\beta}{2(w_h - I_\ell^{MS})} + \frac{\beta}{w_\ell - I_\ell^{MS}} \right] \frac{\alpha_\ell}{u_\ell} = 0.
\]

Second best under EL

The second-best outcome occurs when \(\tilde{\ell} = 0\) and we have a unique solutions with \(I_{h}^{EL}\) and \(I_{\ell}^{EL}\) set at their first-best levels with \(T\) being determined from (A1f) as

\[
T^{EL} = -\frac{I_{h}^{*} + \beta \ln(1 - I_{h}^{*}/w_h)}{2} = -\frac{w_h - \beta + \beta \ln(\beta/w_h)}{2}.
\]
D Positive misreporting costs

D.1 Positive misreporting cost for the ℓ-type

Let \( \delta_i > 0 \) denote the misreporting cost of type \( i \) and assume this is a fixed utility cost which is incurred whenever the individual misreports. In this case, the utility of individual \( i \) reporting type \( i \) is

\[
u_i = \begin{cases} 
  u(I_i - T_i, \frac{I_i}{w_i}) - \delta_i & \text{if } I_i \neq \bar{I}_i \\
  u(I_i - T_i, \frac{I_i}{w_i}) & \text{if } I_i = \bar{I}_i.
\end{cases}
\]

Recall that the EL mechanism consists of the tax function

\[
T(\bar{I}) = T^{\ast}_{\ell} \quad \text{if} \quad \bar{I} = 0,
\]

\[
= T^{\ast}_h \quad \text{if} \quad \bar{I} > 0.
\]

With a prohibitively high \( \delta_h \), the high-wage individuals will never misreport. Our mechanism thus achieves to decentralize the first-best consumption and labor supply levels if low-wage individuals find it optimal to misreport when \( \bar{I}_\ell = 0 \). This requires

\[
u(I^{\ast}_{\ell} - T^{\ast}_{\ell}, \frac{I^{\ast}_{\ell}}{w_{\ell}}) - \delta_{\ell} \geq u(-T^{\ast}_{\ell}, 0)
\]

\[\iff \delta_{\ell} \leq \tilde{\delta}_{\ell} \equiv u(I^{\ast}_{\ell} - T^{\ast}_{\ell}, \frac{I^{\ast}_{\ell}}{w_{\ell}}) - u(-T^{\ast}_{\ell}, 0).\]

Now, because \( \delta_{\ell} > 0 \), there must exist positive levels of misreporting costs such that the mechanism we have characterized decentralizes the first-best consumption and labor levels. However, the first-best utility level cannot be achieved. This is because our mechanism implies misreporting by low-wage individuals and that is costly. Social welfare under this mechanism is then \( W^{FB} - \alpha_\ell \delta_{\ell} \).

Given that social welfare now falls short of \( W^{FB} \), the EL solution no longer necessarily dominates the MS solution under which incomes are observable. Denoting the welfare achievable under MS by \( W^{MS} \), EL dominates only if

\[
W^{FB} - \alpha_\ell \delta_{\ell} \geq W^{MS} \iff \delta_{\ell} \leq \tilde{\delta}_{\ell} \equiv \frac{W^{FB} - W^{MS}}{\alpha_\ell}.
\]

This condition can never be satisfied for values of \( \alpha_\ell \) such that the FB allocation on the Pareto-frontier lies to the left of point \( b \). Such allocation are implementable under MS so that \( W^{MS} = \)
\( W^{FB} \), which implies that \( \delta_\ell = 0 \) and there exists no \( \delta_\ell > 0 \) that satisfies the above condition. To the right of point \( b \), the first-best allocations are not implementable under MS so that \( \delta_\ell \equiv \frac{(W^{FB} - W^{MS})}{\alpha_\ell} > 0 \) and the above condition can be satisfied.

Considering all these restrictions, EL dominates MS only if \( \delta_\ell \leq \min[\delta_\ell, b_\delta \ell] \) and \( \alpha_\ell \) large enough, so that the FB allocation lies on the Pareto-frontier to the left of point \( b \).

**D.2 Positive misreporting cost for both types**

Assume that both types face a utility cost equal to \( \delta \) when misreporting. As previously, low-wage individuals who report \( \tilde{I} = 0 \) will choose to evade if

\[
\begin{align*}
u(I^*_\ell - T^*_\ell, \frac{I^*_h}{w_h}) - \delta &\geq u(-T^*_\ell, 0). \quad (A12)
\end{align*}
\]

Turning to high-wage individuals, they have two options if they were to report \( \tilde{I} = 0 \) and pay \( T^*_\ell \). They can choose not to earn any income or earn an extra income and hide it. The first option gives them a utility level equal to \( u(-T^*_\ell, 0) \), and the second option \( u(I^*_{h\ell} - T^*_\ell, I^*_{h\ell}/w_h) - \delta \) where \( I^*_{h\ell} \) denotes the income they earn in this case. Consequently, high-wage individuals will choose to report \( \tilde{I} > 0 \) and pay \( T^*_h \) if they prefer the \( (I^*_h, T^*_h) \) allocation to either of the two they can have if they report \( \tilde{I} = 0 \). That is,

\[
u(I^*_h - T^*_h, \frac{I^*_{h\ell}}{w_h}) \geq \max \left[ u(I^*_{h\ell} - T^*_\ell, \frac{I^*_{h\ell}}{w_h}) - \delta, u(-T^*_\ell, 0) \right]. \quad (A13)
\]

Suppose first that

\[
u(I^*_{h\ell} - T^*_\ell, \frac{I^*_{h\ell}}{w_h}) - \delta \leq u(-T^*_\ell, 0). \quad (A14)
\]

But since the mimicker has a higher utility than the mimicked, \( u(I^*_{h\ell} - T^*_\ell, I^*_{h\ell}/w_h) > u(I^*_h - T^*_\ell, I^*_h/w_h) \), there cannot exist any positive \( \delta \) for which (A12) and (A14) hold simultaneously. Consequently, if our mechanism is implementable, it must be the case that if high-wage individuals report \( \tilde{I} = 0 \), they will earn a positive income and evade. That is,

\[
u(I^*_{h\ell} - T^*_\ell, \frac{I^*_{h\ell}}{w_h}) - \delta \geq u(-T^*_\ell, 0).
\]

The conditions under which our mechanism is feasible are thus:

\[
u(I^*_h - T^*_h, \frac{I^*_h}{w_h}) \geq u(I^*_{h\ell} - T^*_\ell, \frac{I^*_{h\ell}}{w_h}) - \delta, \quad (A15)
\]

\[
u(I^*_h - T^*_h, \frac{I^*_h}{w_h}) - \delta \geq u(-T^*_\ell, 0). \quad (A16)
\]
Let \( U_{h\ell} \equiv u(I_{h\ell}^* - T_{h\ell}^*, I_{h\ell}/w_h) \), \( U_h \equiv u(I_h^* - T_h^*, I_h^*/w_h) \), and \( U_{\ell} \equiv u(I_{\ell}^* - T_{\ell}^*, I_{\ell}^*/w_{\ell}) \). Using these notations, conditions (A15)–(A16) can be rewritten as

\[
\delta \geq U_{h\ell} - U_h,
\]
\[
\delta \leq U_{\ell} - u(-T_{\ell}^*, 0).
\]

Start first by considering point \( \alpha \) in Figure 1 which corresponds to the competitive equilibrium. At this point, \( T_{\ell}^* = T_{h}^* = 0 \) and \( U_{h\ell} - U_h = 0 \), so that the first condition is always satisfied. It is thus sufficient that the second condition holds at point \( \alpha \) for our mechanism to decentralize the first-best consumption and labor supply levels. This requires \( \delta \) to be “sufficiently” small.

Next, let \( \alpha_{\ell} \) increase from its value at point \( \alpha \), so that the first-best allocations move to the right of point \( \alpha \) along the Pareto frontier. It is easy to show that as \( \alpha_{\ell} \) increases, \( U_{h\ell} \) increases while \( U_h \) decreases. Hence

\[
U_{h\ell} - U_h \leq \delta \leq U_{\ell} - u(-T_{\ell}^*, 0).
\]

Finally, for the EL mechanism to dominate MS when audits are costless, one has to compare the welfare it achieves, \( W^{FB} - \delta \), with the one achievable under MS. Denote the welfare attainable under MS by \( W^{MS} \), misreporting is optimal only if

\[
W^{FB} - \delta \geq W^{MS} \iff \delta \leq \delta \equiv W^{FB} - W^{MS}.
\]
Observe also that at point $a$ and all points to the left of it on the FB frontier, the FB is attainable under MS so that $W^{FB} - W^{MS} = 0$. The EL mechanism cannot then be optimal for low values of $\alpha_\ell$ corresponding to these allocations.
References


