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Abstract

The producers of electricity using dispatchable plants rely on partially flexible technologies to match the variability of both production from renewables and final demand. We analyse upward and downward flexibility in a two-stage decision process where firms compete at low cost in quantities planned before knowing the demand function and adjust the output at high cost when the true state of demand is revealed. We first compute the first best and competitive outcomes. Then we consider the outcome of imperfect competition. We begin with an analysis of the monopoly case, then we determine the duopoly subgame perfect equilibria corresponding to two market designs: one where all trade occurs in an intra-day market with known demand, the other where a day-ahead market with random demand is added to the intra-day market. We show that being inflexible can be more profitable than being flexible. We also show that adding a day-ahead market to the intra-day market increases welfare but transfers risks from firms to consumers. The transfer is all the more important as technologies are not very flexible.

JEL codes: C72, D24, D47, L23, L94

Key words: flexibility, electricity, market design, intra-day market, day-ahead market, risk transfer.

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1 Introduction

As long as electricity energy cannot be stored at large scale, the equilibrium between production and consumption must be reached in real time. This would be a simple routine if demand was not permanently varying, following predictable cycles (e.g. day-night) and random events (e.g. temperature variations). Moreover, the deployment of intermittent sources of renewable energy (solar, wind) increases the randomness of the residual demand that must be served by dispatchable plants (coal, gas, hydro, nuclear). Buyers' price-responsiveness eases the balancing of electricity markets but it cannot be a general solution as long as smart meters and appliances are not massively deployed and consumers cannot instantaneously adapt their behavior. Energy blackouts are a drastic solution politically unacceptable in developed countries. Then, under the severe conditions of i) no storage, ii) no demand rationing and iii) low demand-response, the only way to accomodate variations in residual demand is to benefit from flexible technologies able to follow demand in real time. There exist some cases of supply and demand varying in time in a balanced way: it is so in regions where solar energy simultaneously determines the electricity supply from photovoltaïc panels and the demand for air-conditioning. However, cases of perfect positive correlations are the exception. Renewables rather add uncertainty on the exact quantity of residual energy to supply at each moment.[1] The task to match the demand not served by undispatchable renewables is mainly devoted to hydroelectric reservoirs that can instantaneously increase or decrease their output at zero operation cost, and complementarily to less flexible thermal plants that incur fixed starting and stopping costs, plus additional costs for ramping up and down in the very short run (Kok et al., 2020). The flexibility question is often addressed at the system level rather than within firms.[2] Investing in gas-fired power plants or flexible Carbon Capture and Storage plants (Bertsch et al 2016) and energy storage (Bistline, 2017) provides global flexibility as a by-product.

In this paper, we consider the case of production plants that are not fully flexible by assuming that the cost to produce 1kWh is increasing when the time lag to do so is shorter. Our approach differs from what Boyer and Moreaux (1997) call 'technological flexibility' where firms have to make a choice among different equipment, which results in different cost configurations. The problem we address

[1] See the "Duck Chart" in Denholm et al. (2015). Considering the need for reliability, most system operators place the problem of flexibility at the center of their concerns. For example, each year, the California Independent System Operator conducts a technical study to determine the flexible capacity needs of the system for up to three years into the future (CAISO, 2021). See also ACER (2022, page 3): "price volatility in the electricity system is likely to increase in the years ahead, indicating increasing flexibility needs of the system."

[2] See Carlsson (1989) for a wider view and a historical presentation of the concept of flexibility in the economic literature. In the electricity industry, given the current priority to de-carbonation, the focus is shifting towards "low carbon flexibility technologies", including low carbon flexible generation, storage, interconnection to other countries, and devices and technologies which shift or reduce demand (see chapter 7 in DBEIS, 2022)

is 'flexibility in timing' where costs depend on the decision date. Our analysis belongs to the same strand of literature as Eisenack and Mier (2019) who model flexibility in timing with three independent technologies differentiated by their dispatchability. In our approach, we do not separate from scratch firms that can only plan their production day-ahead from those that can adjust their output in real time to meet demand. We rather assume like in Crampes and Renault (2019) that each firm can do both with the same technology, but at different costs. Because our focus is on the supply side characteristics, we assume that all consumers can react to price signals[3], which excludes any type of rationing as shown in Joskow and Tirole (2007) and Léautier (2019).

Our analysis has strong common features with the literature on market power in sequential markets (Allaz and Vila, 1993; Ito and Reguant, 2016), but with an emphasis on the cost specificities.[4] The question we address is how firms exerting some market power adapt their strategies when their technologies are partially flexible and the demand to serve is both random and price responsive. A correlated question is how the two-market structure that is the standard in most liberalized countries (day-ahead commitment followed by an intra-day market)[5] affects the competitors' strategies.[6] Crampes and Renault (2019) show that, when all agents are price-takers and risk-neutral to monetary transfers, making competition in the wholesale market efficient given demand uncertainty does not necessitate a day-ahead market.[7] By contrast, when producers have some market power, trading only on intra-day markets or on a combination of day-ahead and intra-day markets is not the same.[8] In this paper, we discuss the elements that determine which market design is the most socially efficient in a Cournot duopolistic structure framework.

Our contribution to the literature is threefold. First, we show that, contrary to Allaz and Vila (1993) on forwards, a monopolist will enter the day-ahead market

[3]In its "Final Assessment of the EU Wholesale Electricity Market Design", ACER (2022), classifies the improvement of demand flexibility among 13 measures for the consideration of policymakers.

[4]The Allaz and Vila's model of forward trading desirability has also been enlarged to the case of limited observability of forward positions by Hughes and Kao (1997) and to an environment with multiple firms and increasing marginal costs by Bushnell (2007). Note that it has been challenged by Mahenc and Salanié (2004): when firms compete in prices instead of quantities, forward trading is not socially desirable because it increases prices.

[5]Australia is an exception: there is no day-ahead market (https://www.aer.gov.au/wholesale-markets). Notice that producers can also sign Power Purchase Agreements that are negotiated with large consumers or retailers on a bilateral basis.

[6]We do not consider a framework encompassing a separate flexibility market. For a literature review, see Schittekatte and Meeus (2020).

[7]Two-speed production process is not the only possible explanation for the two-stage organization of electricity markets. It can also be justified in terms of transaction costs (see Küenneke et al., 2010 on modes of organization in infrastructures) or risk management (Redl et al. 2009).

[8]Using data from the German market, Goutte and Vassilopoulos (2019) show that the volatility of short term prices provides an additional revenue to the flexible resources able to react quickly as real-time approaches.

to benefit from technical economies despite the resulting loss of market power on the intra-day. Second, we reinforce Allaz-Vila's (1993) result, for a duopoly by showing that sequential markets reduce the extent of market power and increase social welfare even if the technologies contain technical diseconomies. Our third result is more worrisome as it points out that the opening of a day-ahead market transfers risks from producers to consumers, in particular it fully insures inflexible firms, a transfer that policy makers may consider inappropriate.

The flexibility question is sensible for the electricity system security, but also as regards competition policy. Indeed, in the energy field, competition authorities face questions such as "*Are units inflexible because they are old and inefficient, because owners have not invested in increased flexibility or because they serve as a mechanism for the exercise of market power*?"[9] Our model provides some intuitions that help identifying strategic uses of (in)flexibility in timing. Note also that in the two-market structure, firms that sell in both markets are *de facto* multiproduct producers. Then, one can wonder whether it is possible to use one of the markets as a leverage to exert market power in the other.

The paper is organized as follows. In Section 2 we present the general assumptions on demand and production, and the timing of the game. We also specify a quadratic surplus function and a quadratic cost function that will illustrate the results. Section 3 presents the basic trade-off between the extra cost of producing a given quantity with little anticipation and the benefits of a better knowledge of the target, first when there is only one production plant, then when production can be allocated to two plants. In Section 4, we switch to the analysis of imperfect competition, first in the monopoly case, then with a duopoly that can be either symmetric or asymmetric in terms of production cost. In particular, we study how benefits and risks are re-allocated between producers and consumers when the intra-day market is complemented by a day-ahead market. We conclude in Section 5.

2 Assumptions

We consider firms competing to supply residual demand for electricity, that is the demand not served by undispatchable energies like wind, solar and along-the-river power. As long as supply by renewables and final demand are not perfectly positively correlated, the residual demand is random. We first set the general assumptions on cost and surplus. Then we introduce a quadratic specification.

[9] Page 104 section 3 in "2018 Quarterly State of the Market Report for PJM", http://www.monitoringanalytics.com/reports/PJM_State_of_the_Market/2018/2018q3-som-pjm-sec3.pdf

2.1 General assumptions

Let $S(x,z)$ denote the gross surplus of energy consumers, where x is the total quantity consumed and z is a positive random variable with finite expectation $\mathbb{E}(z)=E$ and variance $V=\mathbb{E}(z^2)-E^2$. To guarantee the existence of at least one maximum, the function $S(x,z)$ is assumed twice continuously differentiable, for all z strictly concave in x ($S''_{xx}(x,z)<0$) with $\lim_{x\to\infty} S(x,z)=-\infty$, and

$$S'_x(0,z)>0 \quad \forall z. \tag{H1}$$

We will refer to z as the "willingness-to-pay" of consumers or the "market size". Given this interpretation, we assume that $S'_z(x,z)\geq 0$, $\forall x$, and to simplify the analysis we also assume that the marginal surplus is increasing in z: $S''_{xz}(x,z)>0$.

Let us now consider the cost function. Firms can produce in two stages. Day ahead, before knowing the value of z, the representative firm can prepare the production of $Q\geq 0$. Later, it can decide on an additional $q \gtreqless 0$ knowing z, as long as $q+Q\geq 0$. It allows the producer to adjust its initial plan Q to demand so that $Q+q=x$. When there is a intra-day market, i.e. a market where trade occurs after learning the exact value of z, q is the additional production (if $q>0$) to increase total delivery up to $Q+q>Q$ or the reduction in production (if $q<0$) to decrease total delivery down to $Q+q<Q$.

The cost function $C(Q,q)$ is assumed twice continuously differentiable, non negative, increasing in Q, and convex in both Q and q. Since q can be negative, we do not assume that $C(Q,q)$ is increasing in q. Delaying decisions until knowing the true state of nature reduces the lag between the decision time and its implementation. Therefore the final output is more costly than the quantity planned initially. Formally, we write it as follows

$$C'_q(0,q)>C'_Q(0,q) \;\; \forall q>0 \tag{H2}$$

Additionnally, we assume that totally cancelling the planned output $Q>0$ is more costly than if a small volume $\varepsilon>0$ is ultimately produced, that is $C(Q,-Q+\varepsilon)\leq C(Q,-Q)$, or

$$C'_q(Q,-Q)\leq 0 \;\; \forall Q\geq 0. \tag{H3}$$

We also assume

$$\lim_{Q\to\infty, q\to-\infty} C(Q,q)=\infty \tag{H4}$$

meaning that the cost of planning a very large output followed by the decision to cancel it is huge.

Finally, the equilibrium of energy flows imposes the equality of demand and supply in each state of nature, that is $x=Q+q$ for all z. We will denote the welfare function by W: it is the difference between the gross surplus of consumers $S(x,z)$, and the cost to serve them $C(Q,q)$.

2.2 Quadratic specification

To illustrate our results and to derive closed-form solutions, we will use the following quadratic specifications:

$$S(x,z) = (z - \frac{x}{2})x \quad (1)$$
$$C(Q,q) = (Q+q)^2 + aq^2 \quad (2)$$

where $a \geq 0$ is the index of (in)flexibility. One can easily check that these functions satisfy all the assumptions of the former subsection.

The quadratic form of the cost function has properties that make it easier to handle than the piecewise linear specification used in Crampes and Renault (2019). Having quadratic specifications for both utility and costs has the advantage to provide explicit results in terms of the average value E and the variance V of the random component of demand. With this specification, we also obtain results that do not depend on the shape of the probability distribution of demand. One drawback is that there is no room for dissymetry and higher statistical characteristics of the random shock in the equilibrium quantities and prices. However, we will see that higher statistical moments show up in the variance of equilibrium profits and surplus so that they play a role to explain how market design transfers risks from producers to consumers.

3 Wait-and-see gains and costs

The trade-off between the informational benefit of delayed decisions and the extra cost due to shorter delays can be analyzed by maximizing the expected welfare function taking into account the possibility to fix the adjusted production q after z is known.[10] To facilitate the identification of the elements of the trade-off, in subsection 3.1 we assume that there is one single production plant and in subsection 3.4 we add a second plant.

3.1 Social optimum with one representative producer

Assume that all production comes out of one plant. The first-best problem is

$$\max_{Q \geq 0} \mathbb{E}_z \max_{q \geq -Q} W(Q,q,z),$$

where $W(Q,q,z) \stackrel{def}{=} S(Q+q,z) - C(Q,q)$.

As usual, we solve the problem backwards.

[10] Notice that maximising the expected welfare, that is the expected difference between the consumers' surplus and the production cost, implicitly assumes that both consumers and producers are risk-neutral to monetary transfers.

3.1.1 Ex post

Upon observing z and knowing the quantity Q already planned, the problem to solve is $\max_{q \geq -Q} S(Q+q,z) - C(Q,q)$. From hypotheses H1 and H3, we have that $S_x^{'}(0,z) > 0 \geq C_q^{'}(Q,-Q)$. Consequently, the unique solution is interior and we denote it by $q^*(Q,z) > -Q$, determined by the first order condition

$$S_x^{\prime}(Q+q^*,z) = C_q^{\prime}(Q,q^*) \tag{3}$$

Hereafter, the superscript * will denote socially optimal values. The adjusted quantity is related to the planned output and the random market size by

$$\frac{\partial q^*}{\partial Q} = -\frac{S_{xx}^{''} - C_{qQ}^{''}}{S_{xx}^{''} - C_{qq}^{''}}, \text{ and } \frac{\partial q^*}{\partial z} = -\frac{S_{xz}^{''}}{S_{xx}^{''} - C_{qq}^{''}}$$

Since $S_{xx}^{''} < 0$ and $C_{qq}^{''} \geq 0$, from the assumption $S_{xz}^{''} > 0$ we have that $\partial q^* / \partial z > 0$.

Consider now the derivative $\partial q^*/\partial Q$. It has the same sign as $S_{xx}^{''} - C_{qQ}^{''}$. Then, $\frac{\partial q^*}{\partial Q} < 0$ if $C_{qQ}^{''} \geq 0$, or if $C_{qQ}^{''}$ is negative but small in absolute value. Indeed, with a decreasing marginal surplus ($S_{xx}^{''} < 0$), increasing Q decreases the need for a positive adjustment: it is a "saturation effect", or, in a market context, a "competition effect". Additionally, if a larger Q deteriorates the conditions to produce an extra output, i.e. if $C_{qQ}^{''} \geq 0$ ('technical diseconomies"), the adjusted quantity is a decreasing function of the planned quantity. It is only when there is a large positive technical and/or economical externality between the two production processes, i.e. $C_{qQ}^{''} < S_{xx}^{''} < 0$, that a large planned output will induce a larger adjustment. The latter can be the case in thermal plants for small levels of Q since the initial costs of warming-up being already paid to produce $Q > 0$, adjustments will be less expensive. These 'technical economies" encourage planned production. But this positive effect can be insufficient to offset the 'saturation effect". And if Q is large, an increase in Q will most likely increase $C_q^{'}$, resulting in $\frac{\partial q^*}{\partial Q} < 0$.

These effects are illustrated in Figure 1 for two values of z, a high z^h and a low z^l. For a given $Q > 0$, the initial solutions defined by (3) are at points A_1^h (positive adjustment) and A_1^l (negative adjustment) respectively. Now, observe the consequences of an increase $\Delta Q > 0$ in the planned output. The two curves of marginal surplus are shifted downwards since $S_{xx}^{''} < 0$. Under technical diseconomies $C_{qQ}^{''} > 0$, the marginal cost curve moves upwards so that the new optimal points A_2^h and A_2^l are located to the left of the initial ones A_1^h and A_1^l respectively: the adjustment $q(Q + \Delta Q, z)$ will be smaller if $z = z^h$ and larger (in absolute value) if $z = z^l$. Under technical economies $C_{qQ}^{''} < 0$, the marginal cost curve move downwards. The new optimal points A_3^h and A_3^l can be located to the left of the initial ones A_1^h and A_1^l if the decrease in marginal cost is not important, or, as shown in the Figure, to the right.

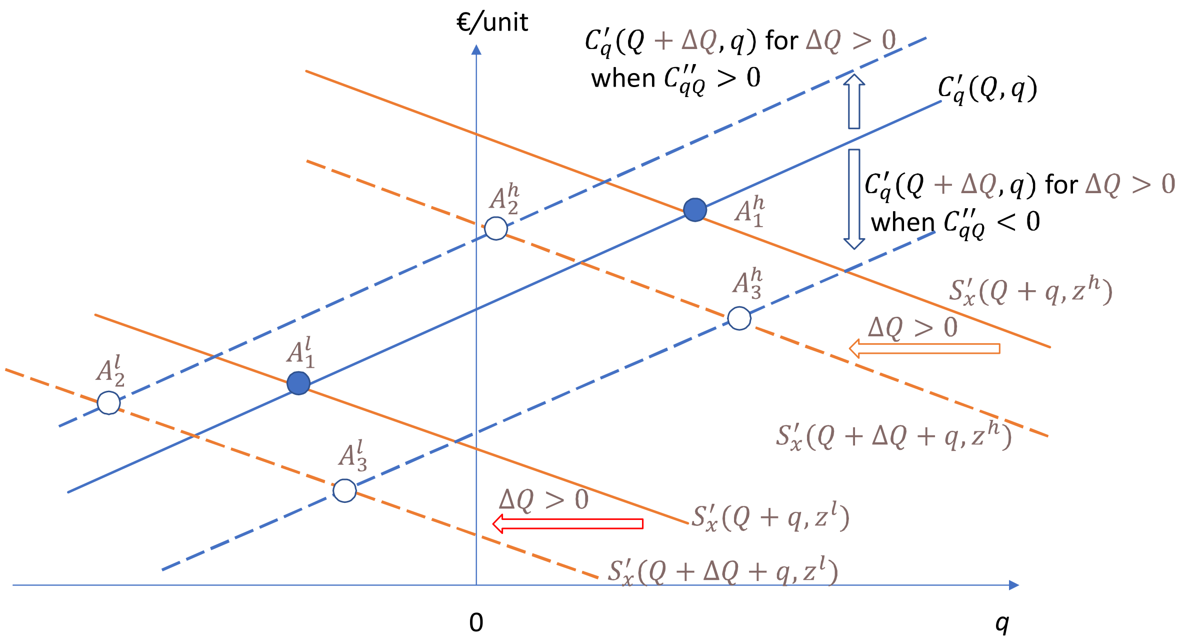

Figure 1: First-best adjustment

3.1.2 Ex ante

The planned output Q is the solution to

$$\max_{Q\geq 0} \mathbb{E}_z \left[S(Q+q^*(Q,z),z) - C(Q,q^*(Q,z)) \right].$$

Combining the ex post adjustment (3) and hypothesis H2 we can write

$$S'_x(q(0,z),z) = C'_q(0,q^*(0,z)) > C'_Q(0,q^*(0,z)) \text{ for all } z. \tag{4}$$

Consequently, since H4 holds the ex ante problem has an interior solution $Q^* > 0$. Taking into account the ex post adjustment (3), Q^* is simply determined by the first order condition

$$\mathbb{E}_z \left[S'_x(Q^*+q^*(Q^*,z),z) - C'_Q(Q^*,q^*(Q^*,z)) \right] = 0. \tag{5}$$

3.2 Application to the quadratic case

Solving (3) in the quadratic case (1)-(2), we obtain the ex post adjustment

$$q^*(Q,z) = \frac{z-3Q}{3+2a} \tag{6}$$

which is larger than $-Q$ and decreasing in Q. Note that $|q(Q,z)|$ is also decreasing in the adjustment parameter a as we could expect.

Applying (5), the optimal planned output Q^* is the solution to

$$I\!E_z\left[z - 3(Q + \frac{z - 3Q}{3 + 2a})\right] = 0.$$

We deduce that $Q^* = E/3$ if $a > 0$. From (6) the optimal adjustment is then $q^*(Q^*, z) = \frac{z-E}{3+2a}$ which is decreasing with both the adjustment cost a (as expected) and the expected demand E since the planned production increases with E and there are technical diseconomies. The adjustment is positive (resp. negative) for large (resp. small) values of the market size z. On average, the adjustment is nil: $I\!E_z q(Q^*, z) = 0$. Finally, note that if $a = 0$ any appropriate combination of planned and adjusted output is the solution since there is no penalty for delaying the production decision.

Remark 3.1. *Notice that with* $Q^* + q^*(Q^*, z) = \frac{2aE+3z}{3(3+2a)}$*, the condition* $Q^* + q^* \geq 0$ *is satisfied for all non negative* a *and* z*.*

Remark 3.2. *Planning a production* Q^* *not depending on the adjustment cost* $a \neq 0$ *is somewhat counter-intuitive. The same is true as for the zero expected adjustment. This actually is an artifact of our elementary quadratic specification. Indeed, under the slightly more complex function* $C(Q, q) = (bQ + q)^2 + aq^2$ *where* b *is a cost index for the base production, the adjustment function is* $q^*(Q, z) = \frac{z-(1+2b)Q}{3+2a}$ *and the optimal planned output is*

$$Q^* = \frac{E}{3}\left(1 + (1 - b)\frac{6(2 + b) + 4a(5 + 4b) + 8a^2(1 + b)}{6(1 - b)^2 + 2a(5 - 4b + 8b^2) + 4a^2(1 + 2b^2)}\right).$$

This shows that $Q^* > \frac{E}{3}$ *if and only if* $b < 1$*, which is natural since a small* b *implies a smaller cost of planed production. One can also compute that with* $b \neq 1$*,* $I\!E_z q^*(Q^*, z) \neq 0$*. The reason is that the forecasted sales* $Q + q$ *are not homogeneous in terms of initial cost in addition to the subsequent adjustment cost. The advantage of setting* $b = 1$*, is that the planned quantities are simple to compute and compare, and we can focus on the adjustment process, then on the benefits and costs of having flexible technologies.*

To evaluate the first best welfare, let us insert $Q^* = \frac{E}{3}$ and $q^* = \frac{z-E}{3+2a}$ into W. We obtain the following welfare value in state z,

$$W^* = \frac{1}{12a + 18}\left(3z^2 + 4azE - 2aE^2\right)$$

and the expected welfare

$$I\!EW^* = \frac{1}{6}E^2 + \frac{V}{2\,(2a + 3)} \tag{7}$$

With $I\!EW^*$ increasing in V, flexibilty reduces the social costs from randomness as was shown by Waugh (1944) for consumers and Oi (1961) for producers, provided they are risk-neutral to monetary transfers and perfectly rational. However this benefit decreases when the adjustment cost parameter a increases, and it vanishes when the technology is fully inflexible ($a \longrightarrow \infty$).

3.3 Market implementation

The first best quantities can be decentralized with competitive price-taking firms and consumers trading ex post at prices contingent to the state of nature. Since it is a mere application of the second theorem of welfare, let us just illustrate it with the quadratic specification.

3.3.1 Market mechanism with only ex post transactions

Ex post, that is knowing z, the representative consumer solves $\max_{q+Q} S(Q+q,z) - p(q+Q)$ where p is independent from the quantities. Therefore, given (1), demand is $Q+q = z-p$. The representative producers solves $\max_q pq - C(Q,q)$. Hence, using (2), the adjustment supply function is $q = \frac{p}{2(1+a)} - \frac{Q}{(1+a)}$. Equating supply and demand, we deduce the equilibrium price and quantity in state z,

$$p\left(Q,z\right) = 2\frac{z\left(1+a\right)-aQ}{3+2a}, \quad q(Q,z) = \frac{z-3Q}{3+2a} \tag{8}$$

Anticipating these contingent quantities and prices, the representative firm plans its production Q solving $\max_Q \mathbb{E}\left[\left(q\left(Q,z\right)+Q\right).p - C(Q,q\left(Q,z\right))\right]$. Since it is a price-taker, it does not internalize the effect of Q on $p\left(Q,z\right)$. However, as a rational agent, it internalizes the effect on $q\left(Q,z\right)$. Consequently, the first order condition is

$$\mathbb{E}\left[p\left(Q,z\right)(1+\frac{dq}{dQ}) - \left(C_Q' + C_q'\frac{dq}{dQ}\right)\right] = 0 = \mathbb{E}\left[p\left(Q,z\right) - C_Q'\right]$$

since $C_q' = p\left(Q,z\right)$ in every state z by the condition on ex post adjustment. Then, simple calculation shows that producers choose $Q = E/3$, that is the first-best output Q^*.

3.3.2 Trade on two markets.

As shown in Crampes and Renault (2019), if we open a market for ex ante transactions in complement to the ex post market, as it is the case in most organized power markets that combine day-ahead and intra-day trade (Borggrefe and Neuhoff, 2011; ECA, 2015), under perfect competition the result is the same as when there is one single market like in the former paragraph. Indeed, the ex-post inverse demand function knowing z is $p = S_x'(Q+q,z)$ (or $p = z-Q-q$ with our specification) where Q is the day-ahead quantity, and supply is the same as in the one single market case. Therefore, the intra-day equilibrium is the same as in (8).

On the day-ahead market, the price is not state contingent: let us denote it by P. The representative producer solves $\max_Q PQ + I\!E\left[p(Q,z)\,q(Q,z) - C(Q, q(Q,z)\right]$ where $\partial P/\partial Q \equiv 0$ and $\partial p(Q,z)/\partial Q \equiv 0$. Then the FOC reads

$$P + I\!E\left[p(Q,z)\frac{\partial q(Q,z)}{\partial Q} - \left(C'_Q + C'_q\frac{\partial q(Q,z)}{\partial Q}\right)\right] = 0.$$

Given the ex-post adjustment, we obtain the (inverse) supply function for planned production $P = I\!E\left[C'_Q\right]$ or, with the quadratic specification, $P = 2\frac{2aQ+E}{3+2a}$.

On the demand side, the consumer solves $\max_Q I\!E\left[S(Q + q(Q,z)\right] - PQ - I\!E\left[p(Q,z)\,q(Q,z)\right]$. Since it is a price-taker, using the ex post adjustment the demand function is $I\!E\left[S'_x(Q + q(Q,z)\right] = P$, that is $P = 2\frac{[(1+a)E - aQ]}{(3+2a)}$ with the quadratic specification. At the day-ahead equilibrium between supply and demand, we obtain $Q = E/3$ as expected, and $P = \frac{2}{3}E$. The latter actually is $I\!Ep(Q,z)$. Indeed, $P = I\!Ep(Q,z)$ prevents any possibility of arbitrage between the two markets.[11]

Remark 3.3. ***Risk neutrality.*** *It is worthwhile noting that the equivalence of the two market designs whatever the shape of the cost function results from the quasi-linearity of the consumers' and producers' preferences. Indeed, assuming that the consumers' performance is measured by their net surplus $S - px$ and the producers' performance by their net profit $\pi = px - C$ implicitly states that they are risk-neutral when facing monetary risks. Consequently, as long as the day-ahead price is the same as the expectation of the intraday price, randomness does not affect their decisions whatever the cost function.*

3.4 Social optimum with two representative producers

In order to prepare the analysis of the duopoly in section 4, we now consider the social gains due to the existence of two production plants. We first consider the general optimization problem, then we determine the explicit solution corresponding to the quadratic specification.

3.4.1 General properties

There are two plants producing the same homogenous product with respective cost functions $C_i(Q_i, q_i)$, $i = 1, 2$. These costs satisfy the assumptions set in section 2, in particular they are increasing in Q_i and convex in both Q_i and q_i. In the following, we will write $Q = Q_1 + Q_2$, $q = q_1 + q_2$, $\vec{Q} = (Q_1, Q_2)$, and $\vec{q} = (q_1, q_2)$.

[11] The assumption of perfect arbitrage between the day-ahead market and the intra–day market is quite common in the literature. However, empirical studies show that arbitrage remain imperfect in electricity markets. Joint with market power it generates a price premium (Ito and Reguant, 2016).

The problem to solve is

$$\max_{Q_1 \geq 0, Q_2 \geq 0} \mathbb{E}_z \max_{q_1 \geq -Q_1, q_2 \geq -Q_2} W(\vec{Q}, \vec{q}, z)$$

where $W(\vec{Q}, \vec{q}, z) = S(Q + q, z) - C_1(Q_1, q_1) - C_2(Q_2, q_2)$.

Ex post, given $\vec{Q}$ and z, the intra-day quantities are the solutions to

$$S_x'(Q + q, z) = C_{1q}'(Q_1, q_1) = C_{2q}'(Q_2, q_2) \tag{9}$$

The output q_i in plant $i = 1, 2$ is then a function of both the market size and the two planned outputs: $q_i(Q_1, Q_2, z)$.

Total differentiation of the two equations in (9) w.r.t. Q_i gives the variation in q_i due to a variation in the planned output of plant i:

$$\frac{\partial q_i}{\partial Q_i} = D^{-1} \left[S_{xx}^{''} C_{-iqq}^{''} + \left(S_{xx}^{''} - C_{-iqq}^{''} \right) C_{iqQ}^{''} \right] \qquad i = 1, 2 \qquad , -i \neq i \tag{10}$$

where D denotes the determinant of the full system. D is positive by concavity of the objective function. Then, like in the one-plant case, $C_{iqQ}^{''} \geq 0$ is sufficient for $\frac{\partial q_i}{\partial Q_i} \leq 0$. It would take a negative and large in absolute value $C_{iqQ}^{''}$ to obtain the opposite. As for the adjustment in plant $-i \neq i$,

$$\frac{\partial q_{-i}}{\partial Q_i} = D^{-1} S_{xx}^{''} \left(C_{iqq}^{''} - C_{iqQ}^{''} \right), \quad i = 1, 2 \tag{11}$$

which is negative if $C_{iqQ}^{''}$ is negative, or positive but small in absolute value, and $\frac{\partial q_{-i}}{\partial Q_i} > 0$ when $C_{iqQ}^{''} >> 0$. The reason is that plant 1 and plant 2 compete at the adjustment stage. Then, if $C_{iqQ}^{''} < 0$ it is profitable to decrease q_{-i} and leave room to a potential increase in q_i. Conversely, if there are strong technical diseconomies between Q_i and q_i (i.e $C_{iqQ}^{''} >> 0$), efficiency requires to use plant $-i$ for adjustments after an increase in the planned production of plant i.

Ex ante, **d**ifferentiating $\mathbb{E}_z W(\vec{Q}, q_1(\vec{Q}, z), q_2(\vec{Q}, z), z)$ wrt $\vec{Q}$ and using (9), the solution (Q_1^*, Q_2^*) is such that for $i = 1, 2$

$$\begin{aligned} &Q_i^* \geq 0, \ \mathbb{E}_z \left[S_x'(Q^* + q(Q_1^*, Q_2^*, z), z) - C_{iQ}'(Q_i^*, q_i(Q_1^*, Q_2^*, z) \right] \leq 0 \\ &\mathbb{E}_z \left[S_x'(Q^* + q(Q_1^*, Q_2^*, z), z) - C_{iQ}'(Q_i^*, q_i(Q_1^*, Q_2^*, z) \right] .Q^* = 0 \end{aligned} \tag{12}$$

Depending on the form of the cost functions, in particular their relative advantage in terms of flexibility, we obtain solutions where the two plants are active at both stages, and others where they must specialize, e.g. $Q_1^* > 0, Q_2^* = 0, q_1(Q_1^*, 0, z) = 0, q_2(Q_1^*, 0, z) > 0$.

3.4.2 Quadratic technologies

From a pure cost-efficiency point of view, the quadratic function $C_i(Q_i, q_i) = (Q_i + q_i)^2 + a_i q_i^2$, $i = 1, 2$ opens the door to a trade-off between specialisation and production at the two stages. Indeed, with convex cost functions (i.e. decreasing returns to scale), it is efficient to share any given quantity among several generation units to reduce the total cost. On the other hand, since our specification entails "technical diseconomies" both globally $(C_i(Q_i, q_i) - (C_i(Q_i, 0) + C_i(0, q_i)) = 2Q_i q_i > 0)$, and at the margin $(C''_{iqQ} = 2)$, separating the planned and adjusted productions would lower the marginal cost of both. In the following, we analyze how the social planner will balance these two effects.

Quantities sharing With the quadratic specification (1)-(2) and two plants, the total expected welfare is

$$W = \mathbb{E}\left[(z - \frac{1}{2}(Q+q))(Q+q) - (Q_1+q_1)^2 - (Q_2+q_2)^2 - a_1 q_1^2 - a_2 q_2^2\right]. \tag{13}$$

Ex post,.from (9) we know that ex post productive efficiency is reached when the two marginal adjustment costs are equal, i.e. given Q_1, Q_2 and the total quantity $x = Q + q$ to produce, we have

$$2Q_1 + 2(1+a_1)q_1 = 2Q_2 + 2(1+a_2)q_2 \tag{14}$$

so that

$$(1+a_1)q_1 - (1+a_2)q_2 = Q_2 - Q_1$$

i.e. the larger adjustment will be done in the plant with the smaller planned output and, for a given difference $Q_2 - Q_1$, the larger a_i, the smaller q_i and/or the larger q_{-i}, $(i = 1, 2)$. Note that (14) is true for all values of q_1 and q_2 (constrained by of $q_i \geq -Q_i$) since these quantities can be positive or negative.

Now, equating (14) with marginal utility, we can solve for q_1 and q_2:

$$\begin{cases} q_1^* &= \gamma^{-1}\left[(1+a_2)z - (4+3a_2)Q_1 - a_2 Q_2\right] \\ q_2^* &= \gamma^{-1}\left[(1+a_1)z - (4+3a_1)Q_2 - a_1 Q_1\right] \end{cases} \tag{15}$$

where γ is a constant:

$$\gamma = 4 + 3a_1 + 3a_2 + 2a_1 a_2. \tag{16}$$

As expected, the two quantities decrease with the planned outputs. For a given z, it means that the adjusted quantities are positive and smaller and smaller (resp. negative and larger and larger in absolute value) when $Q_1 + Q_2$ is small (resp. large) and increases.

At the planning stage, we know from (12) that if $Q_1 > 0$ and $Q_2 > 0$, the two expected marginal costs must be set equal to the expected marginal utility:

$$\mathbb{E}\left[z - Q_1 - Q_2 - q_1^* - q_2^*\right] = \mathbb{E}\left[2(Q_1 + q_1^*)\right] = \mathbb{E}\left[2(Q_2 + q_2^*)\right]. \tag{17}$$

Using (15) and solving, we obtain

$$\begin{gathered} Q_1^* = \tfrac{1}{4}E, Q_2^* = \tfrac{1}{4}E \\ q_1^* = \gamma^{-1}(1+a_2)(z-E), q_2^* = \gamma^{-1}(1+a_1)(z-E) \end{gathered} \tag{18}$$

Remark 3.4. *Contrary to our observation in Remark 3.1, when there are several production plants the interior solution given by (18) does not guarantee that the condition* $Q_i^* + q_i^* \geq 0$ *holds. Indeed we face the risk of obtaining a solution where one plant adjust downwards to decrease not only its own output but also the output of the other plant, which is technically impossible. Formally, using (16) and (18), we must check that* $2a_i(1+a_{-i}) + (a_i - a_{-i}) + (4+4a_{-i})z \geq 0, i = 1, 2$. *We see that the constraint is redundant as long as the difference* $(a_i - a_{-i})$ *is not too large and/or the demand index* z *is never too low.*

In the first best (interior) solution given by (18), the two representative firms are active at the two periods because, under increasing marginal costs, it is efficient to share the load. They produce the same quantity at the planning period because of the peculiarities of our cost function: in each firm, decreasing the planned output Q_i decreases the adjustment marginal cost, and the two firms incur the same cost for the production of Q_i. Then the adjusted quantities q_1, q_2 only differ from each other because their marginal costs $C'_{iq}(Q_i^*, q_i) = \frac{E}{2} + 2(1+a_i)q_i$ have different slopes. Ex post, efficiency imposes that $\partial q_i^*/\partial a_i < 0, \partial q_i^*/\partial a_{-i} < 0$ and $q_1^* \gtreqless q_2^*$ as $a_2 \gtreqless a_1$. Note that the adjustments decrease with the expected demand since planned outputs increase with E.

Given these quantities, welfare in state z is

$$W^* = \frac{1}{4\gamma}\left(\left(z^2 - E^2\right)a_1 + \left(z^2 - E^2\right)a_2 + (a_1 + a_2 + 4)z^2 - 2E^2 a_1 a_2 + 2zE(a_1 + a_2 + 2a_1 a_2)\right)$$

resulting in the expected welfare

$$\mathbb{E}W^* = \frac{E^2}{4} + \frac{2 + a_1 + a_2}{2\gamma}V$$

To compare these results with those of the one-plant case, assume that $a_1 = a_2 = a$. Then $q_1^* + q_2^* = \frac{1+a}{2+3a+a^2}(z - E), Q_1^* + Q_2^* = \frac{E}{2}$ and $\mathbb{E}W^* = \frac{E^2}{4} + \frac{1}{2(a+2)}V$. At the two stages of the production process the total quantity is larger because the cost has been alleviated by allocating the output among the two plants. Here again, the expected welfare is an explicit function of the demand mean and variance. Both terms are larger than in the one-plant case (see (7)) but, again, there is no benefit from the variance if the parameter a becomes infinite.

Specialization. Suppose now that firm 2 is fully inflexible ($a_2 \longrightarrow +\infty$) so that $q_2^* = 0$ and firm 1 can adapt at a finite cost. Then equation (14) is no longer relevant. To determine $q_1(\overrightarrow{Q}, z)$, we must equate the marginal cost of adjustment in plant 1 with marginal utility: $2Q_1 + 2(1+a_1)q_1 = z - (Q_1 + Q_2 + q_1)$ and we obtain $q_1 = \frac{z-(3Q_1+Q_2)}{3+2a_1}$. Solving the stage 1 condition (17) for this adjustment functions, the corner solution is $Q_1^* = Q_2^* = \frac{1}{4}E, q_1^* = \frac{z-E}{2a_1+3}, q_2^* = 0$. The total output of firm 1 is $Q_1^* + q_1^* = \frac{1}{4}E + \frac{z-E}{2a_1+3}$. It is non negative if $z \geq \frac{E}{4}(1-2a_1)$. We see that the most constraining case is when firm 1 is perfectly flexible ($a_1 = 0$). The condition is then $z \geq \frac{E}{4}$ which means that demand forecasts must be accurate enough for the lowest possible level of z to be at least one fourth of its average, which is not very challenging given the statistical tools available today.

It is noteworthy that when $a_1 = 0$ and $a_2 \longrightarrow +\infty$, the expected profits of the two firms are $\mathbb{E}\Pi_1^* = \frac{E^2}{16} + \frac{V}{9} > \mathbb{E}\Pi_2^* = \frac{E^2}{16}$, which shows that flexibility gives a financial advantage as long as $V > 0$.

Decentralization First best can be decentralized if all agents are price takers. Using the same argument as in the one-plant case, we can compute the intra-day price by fixing it at the value of the first best marginal utility $S' = z - (Q^* + q^*)$,which gives

$$p(z) = z - \gamma^{-1} \left(\gamma \frac{E}{2} + (2 + a_1 + a_2)(z - E) \right).$$

The price of the day-ahead market (if there is one), is obtained by taking the average of the intra-day price (no-arbitrage argument)

$$P = \mathbb{E}p(z) = \frac{E}{2}$$

to be compared with $P = \frac{2}{3}E$ obtained in the one-plant case. Clearly, sharing the load between two price-taking firms is profitable to consumers since they consume more at a lower price than with one single price-taking producer. As for the firms, the price decrease is offset by a drop in costs, so that the profit of the industry $(= \frac{1}{8}E^2 + \frac{1+a}{2(2+a)^2}V)$ is larger than when there is only one producer $(= \frac{1}{9}E^2 + \frac{1+a}{(3+2a)^2}V.)$

In order to facilitate the comparison with the results of the next section, we summarize the main results in the following paragraph:

First best results When the gross surplus and cost functions are given by the quadratic specification (1)-(2), at first best with two active production plants the ex ante production levels are $Q_1^* = \frac{1}{4}E, Q_2^* = \frac{1}{4}E$, and the two expected adjustments are nil. The expected welfare is $\mathbb{E}W^* = \frac{E^2}{4} + \frac{2+a_1+a_2}{2\gamma}V$. If the plants are operated by two price-taking private operators, the results are the same as at first best whatever the market design. If one firm is perfectly flexible and the other totally inflexible ($a_1 = 0$ et $a_2 = +\infty$), when $V > 0$ the flexible firm has a higher expected profit than the inflexible one.

4 Imperfect competition

We now switch to imperfect competition with and without a day-ahead market and analyze how the firms can exert their market power when their production facilities allow to produce and sell at the two stages. We first consider the monopoly case because the savings from early production create an incentive to produce on the day-ahead market whereas in Allaz and Vila (1993, section 3.4) a monopoly has no advantage from selling forward. We then focus on the duopoly case where each firm is managed by an independent private owner who maximizes its expected operating profit by fixing the quantities Q_i and q_i, taking into account the effect of its decisions on prices, and constrained by its competitor's choices (Cournot competition). In this framework, we know from Crampes and Renault (2019) that the two market organizations presented in the former section are no longer equivalent. In sub-section 4.2 we explain the roots of this no-equivalence and show that the market design has ambiguous effects on the total output and the agents' surplus. In sub-section 4.3, we compare the two market designs under the quadratic specification, and demonstrate that the opening of a day-ahead market makes consumers better off and firms worse off, which shows that the result of the Allaz-Vila model (1993) remains true when the technology exhibits technical diseconomies between production stages. More importantly, we also show that although the two-market framework has the advantage of increasing global welfare at the expense of firms, it has the important drawback of transferring risks from firms to consumers.

4.1 The monopoly case

It is interesting to contrast the result of Allaz-Vila in the monopoly case with what occurs in our setting. Indeed, when there is only one way to produce and two ways to sell (forward and on the spot), a monopoly has no reason to compete against itself by committing on forwards since these sales alleviate its market power on the spot market. This is clearly stated in Allaz and Vila (1993, section 3.4). By contrast, in our setting with a two-stage production function and two markets, the monopoly is better off by preparing production and selling at stage 1 to benefit from the lower cost.

To prove it, we consider the case of one single production plant. For a given state of nature z, knowing that consumers maximize their net surplus by $S'_x(Q+q,z)=p$, a monopoly maximizes its profit $qp(Q+q,z)-C(Q,q)$ given Q by fixing $q^m(Q,z)>-Q$ such that[12]

$$p(Q+q,z)+qp'_x(Q+q,z)=C'_q(Q,q), \tag{19}$$

or

$$S'_x(Q+q^m(Q,z),z)+q^m(Q,z)S''_{xx}(Q+q^m(Q,z),z)=C'_q(Q,q^m(Q,z)).$$

Comparing with the first best ex post condition (3) that determines $q^*(Q,z)$, since $S''_{xx}(Q+q,z)<0$, for a given pair (Q,z) we have $q^m(Q,z)>q^*(Q,z)$

[12] Indeed, the condition that permits to set $q>-Q$ at first best (that is $S'_x(0,z)>C'_q(Q,-Q)$) is satisfied in the monopoly case since the marginal revenue $p(q+Q,z)+qp'_x(Q+q,z)$ is equal to $S'_x(0,z)-QS''_{xx}(0,z)>S'_x(0,z)$ when $q+Q=0$. Then, if $Q+q>0$ for all z at first best, it is also true for the monopoly.

when adjustments are negative, and $q^m(Q,z) < q^*(Q,z)$ for positive adjustments. When $Q=0$, adjustments can only be positive so that $q^m(0,z) < q^*(0,z)$.

At the time to trade on the day-ahead, the monopoly's expected profit is

$$\widetilde{\Pi}^m(Q) = P(Q)Q + I\!E_z\left[p^m(Q+q^m(Q,z),z)q^m(Q,z) - C\left(Q, q^m(Q,z)\right)\right]$$

where $P(Q) = I\!E_z\left[S'_x(Q+q^m(Q,z),z)\right]$ is the inverse ex ante demand and $p^m(Q+q^m(Q,z),z)$ is the monopoly price on the intra-day market. Here and below, the sign $\sim$ identifies when necessary the two-market framework What is the sign of $\frac{d\widetilde{\Pi}^m(Q)}{dQ}$ at $Q=0$?

Using (19), the expected marginal revenue from the planned production is

$$\frac{d\widetilde{\Pi}^m(Q)}{dQ} = P'(Q)Q + P(Q) + I\!E_z\left[q^m(Q,z)p_x^{m\prime}(Q+q^m(Q,z),z) - C'_Q\left(Q, q^m(Q,z)\right)\right],$$

so that

$$\frac{d\widetilde{\Pi}^m(Q=0)}{dQ} = I\!E_z\left[S'_x(q^m(0,z),z) + q^m(0,z)p_x^{m\prime}(q^m(0,z),z) - C'_Q\left(0, q^m(0,z)\right)\right].$$

Then, given that $S'_x(q^m(0,z),z) = p^m(0,z)$, using (19) one more time, we obtain

$$\frac{d\widetilde{\Pi}^m(Q=0)}{dQ} = I\!E_z\left[C'_q\left(0, q^m(0,z)\right) - C'_Q\left(0, q^m(0,z)\right)\right]$$

which is strictly positive by H2, so that the monopoly is better off by selling day-ahead: $Q^m > 0$. The trade-off between the loss in future revenues (Allaz-Vila's effect, $q^m(0,z)p_x^{m\prime}(q^m(0,z),z) < 0$) and the today lower cost turns out to the advantage of the latter.

Notice that the monopoly could improve its performance by planning a positive output ($Q^m > 0$) without selling it foreward, as if there was no day-ahead market. Doing so, it would benefit from lower costs without alleviating its market power. However this behavior could be attacked by competition authorities as a refusal to deal (art. 102 of the Treaty on the Functioning of the European Union and section 2 of the Sherman Act).

4.2 Strategic behavior in a duopoly

We now consider the two-stage game between two independent private firms endowed with the partially flexible technologies analyzed in the former sections. For simplicity, we assume from scratch the existence and uniqueness of Nash-Cournot equilibria under our general assumptions of section 2. Under the quadratic specification, existence and uniqueness are guaranteed. As usual, the game is solved backwards.

4.2.1 Intra-day market.

The quantities $Q_1 \geq 0, Q_2 \geq 0$ are fixed and known by firms 1 and 2. Either they have been sold on the day-ahead market if it exists, or their production has just been launched if there are only intra-day markets.[13] In a subgame-perfect equilibrium, at stage 2 we have a game parameterized by z and $\vec{Q}$ where the firms set q_1, q_2 independently of each other. If all quantities are sold at the intra-day price, the payoff of each firm i is given by

$$\Pi_i(\overrightarrow{Q}, q_1, q_2, z) = p(Q+q, z)(q_i + Q_i) - C_i(Q_i, q_i) \qquad (20)$$

where $p(Q+q,z) = S'_x(Q+q,z)$ is the ex-post demand function given z. When there are two successive markets, i's profit is

$$\widetilde{\Pi}_i(\overrightarrow{Q}, q_1, q_2, z) = PQ_i + p(Q+q, z)q_i - C_i(Q_i, q_i) \qquad (21)$$

where P is the day-ahead price.

Given its market power, each firm i internalizes that its production q_i will lower the price. In the one-market framework, the FOC for the maximization of (20) with respect to q_i is

$$p(Q+q, z) + (Q_i + q_i)p'_x = C'_{iq}(Q_i, q_i),\ i = 1, 2 \qquad (22)$$

where $p'_x = \frac{\partial p(x,z)}{\partial x} < 0$ since $S''_{xx} < 0$. Let $\overrightarrow{q}^C = (q_1^C(\overrightarrow{Q}, z), q_2^C(\overrightarrow{Q}, z))$ be the unique Cournot equilibrium of this game and $p^C = S'_x(Q + q^C, z)$ the associated price.

If there are two successive markets, the FOC is

$$p(Q+\widetilde{q}, z) + \widetilde{q}_i p'_x = C'_{iq}(Q_i, \widetilde{q}_i),\ i = 1, 2 \qquad (23)$$

Let us denote $(\widetilde{q}_1^C, \widetilde{q}_2^C)$ and $\widetilde{p}^C = S'_x(Q + \widetilde{q}^C, z)$ the equilibrium quantities and price of this game.

In both market frameworks, the equilibrium price and quantities depend on z and $\vec{Q}$. By using (10) and (11) with marginal revenue replacing marginal surplus, we see that the adjusted quantities are generically decreasing in both Q_i and Q_{-i}.

4.2.2 Day-ahead.market

When all production is sold on the intra-day market the expected profit of i is

$$\Pi_i(Q_1, Q_2) = \mathbb{E}_z \left(p^C(\overrightarrow{Q}, z)(q_i^C(\vec{Q}, z) + Q_i) - C_i(Q_i, q_i^C(\vec{Q}, z) \right) \qquad (24)$$

[13]The assumption that Q_i is observed by $-i$ even when there is no ex-ante market-place can be justified in terms of technological and managerial expertise. See also Hughes and Kao (1997).

As detailed in Crampes and Renault (2019), the FOC for the maximisation of $\Pi_i(Q_1, Q_2)$ wrt Q_i (internalizing (22)) is

$$I\!E_z \left[p^C + \left(Q_i + q_i^C\right)(1 + \frac{\partial q_{-i}^C}{\partial Q_i}) p_x^{C'} - C'_{iQ} \right] = 0. \tag{25}$$

where $p_x^{C'} \overset{def}{=} S''_{xx}(Q + q^C, z)$ stands for the slope of the inverse demand function at the equilibrium point. Let Q_1^C, Q_2^C denote the solution.

If there are two markets, the expected profit is

$$\widetilde{\Pi}_i(Q_1, Q_2) = P(Q_1 + Q_2)Q_i + I\!E_z \left[\widetilde{p}^C(Q, z)\widetilde{q}_i^C(\vec{Q}, z) - C^i\left(Q_i, \widetilde{q}_i^C(\vec{Q}, z)\right)\right] \tag{26}$$

where $P(Q_1 + Q_2) = I\!E_z \left(S'_x(Q + \widetilde{q}_1^C + \widetilde{q}_2^C, z)\right)$ is the day-ahead demand function. Given (23), the FOC is

$$P + P'Q_i + I\!E_z \left[\widetilde{q}_i^C \left(1 + \frac{\partial \widetilde{q}_{-i}^C}{\partial Q_i}\right) \widetilde{p}_x^{C'} - C'_{iQ} \right] = 0. \tag{27}$$

Let $\widetilde{Q}_1^C, \widetilde{Q}_2^C$ denote the solution.

4.2.3 Larger or smaller output?

Comparing the two pairs of conditions, we observe two differences:

ex post, if the firms have committed on a day-ahead market, increasing their adjustment output $\widetilde{q}$ has a lower impact on their profit since the drop is $\widetilde{q}_i\widetilde{p}'_x$ in (23) instead of $(Q_i + q_i)p'_x$ in (22). Then, given the same total planned quantity $Q_1 + Q_2$, with two markets the ex post price will be closer to the marginal cost, and quantities larger than without day-ahead trade, which is in line with the role of forward markets in Allaz and Vila (1993) and Ito and Reguant (2016); then this effect pushes towards $\widetilde{q}_i^C > q_i^C$;

ex ante, there is no term like $Q_i \frac{\partial q_{-i}^C}{\partial Q_i} p_x^{C'} \geq 0$ in (27). The tomorrow reaction of firm $-i$ has no impact on the today's marginal revenue of firm i whereas firm i must consider this response when Q_i and q_{-i} are sold on the same market (see (25)). Consequently, *ceteris paribus* the expected marginal revenue of i in the unique market framework (equation (25)) is higher than in the two-market design. Then, this effect pushes towards $\widetilde{Q}_i^C < Q_i^C$.

Consequently, without additional information, we cannot predict whether $\widetilde{Q}_i^C + \widetilde{q}_i^C \lesseqgtr Q_i^C + q_i^C$. Obviously, the anticipation of the competitor's flexibility capacity $\frac{\partial q_{-i}^C}{\partial Q_i}$ plays a pivotal role. The quadratic specification below provides an illustration for one possible outcome from the two antagonistic effects.

4.3 Duopoly equilibrium in the quadratic case

We first compute and discuss the Cournot-Nash equilibrium when the two firms can only sell their output on the intra-day market, then we examine the consequences of the opening of a day-ahead market.

4.3.1 No day-ahead market

Subgame perfect equilibrium Firms 1 and 2 compete in quantities and all their output is sold on the intra-day market. In the profit function (20) for $i = 1, 2$, the inverse demand function in state z is $p(Q + q, z) = z - (Q + q)$ and the cost of i is $C_i(Q_i, q_i) = (Q_i + q_i)^2 + a_i q_i^2$.

Intra-day,.from (22), the FOC to determine the adjustment quantity q_i is

$$z - 4Q_i - Q_{-i} - (4 + 2a_i)q_i - q_{-i} = 0, \ i = 1, 2$$

The resulting equilibrium quantities are

$$\begin{cases} q_1^C &= \beta^{-1}\left[(3 + 2a_2)z - (15 + 8a_2)Q_1 - 2a_2 Q_2\right] \\ q_2^C &= \beta^{-1}\left[(3 + 2a_1)z - (15 + 8a_1)Q_2 - 2a_1 Q_1\right] \end{cases} \tag{28}$$

and the price is

$$p^C = \beta^{-1}\left[(2a_1 + 3)(2a_2 + 3)z - 2\left(3 + 2a_2\right)a_1 Q_1 - 2\left(3 + 2a_1\right)a_2 Q_2\right] \tag{29}$$

where

$$\beta = (4 + 2a_1)(4 + 2a_2) - 1.$$

Day-ahead**,** to obtain the FOC relative to the quantity Q_i, we use (25) where we insert $p_x^{C'} = -1$, $\frac{\partial q_{-i}^C}{\partial Q_i} = -2a_i\beta^{-1}$ by (28) and $C'_{iQ} = 2(Q_i + q_i^C)$. It results that

$$I\!E_z\left[p^C - \left(Q_i + q_i^C\right)\left(3 - 2a_i\beta^{-1}\right)\right] = 0, i = 1, 2. \tag{30}$$

Then, we insert the adjustment functions given by (28) and solve the two-equation system. The planned quantities we obtain are proportionnal to the expected willingness to pay of consumers (the explicit forms are given by equations (35) in the Appendix 6.1):

$$\begin{matrix} Q_1^C = k_1 E \\ Q_2^C = k_2 E \end{matrix} \tag{31}$$

First observe that the two firms will have different ex ante outputs if they differ in terms of cost parameters, contrary to what we found at first best (see (18)). This is because the strategic effect is now at work on top of the cost minimization concern: firms try to gain market shares without decreasing the price too much. Even when they are fully identical ($a_1 = a_2 = a$), the outputs differ qualitatively from first best since $Q_1^C = Q_2^C = \frac{4(2+a)^2}{78a+20a^2+75}E$ is decreasing in a whereas Q_1^* and Q_2^* given in (18) are fixed. Indeed, the higher a the lower the profitability of operating on the intra-day market and the game becomes closer to a one-shot Cournot game with no advantage from being a first-mover. Unsurprisingly, the firms exert their market power by restricting quantities: $Q_i^C < Q_i^*$ even when $a = 0$.

When $a_1 \neq a_2$, the analysis of the equilibrium quantities (35) in Appendix 6.1 show that Q_i^C is increasing in a_i and decreasing in a_{-i}. Moreover, $Q_1^C \gtrless$

Q_2^C as $a_1 \gtrless a_2$. These results are the consequence of the technical diseconomies mentioned formerly: the firm with the lower cost of adjustment produces less than its competitor at the first stage because this will limit the negative impact on its cost at the second stage where it will be more active. The higher the adjustment parameter a_i, the higher the planned production Q_i^C because firm i does not intend to intervene intensively intra-day, hence does not mind about the cost diseconomies.

Consider now the profit of the firms $\Pi_i^C = \left[z - 2\left(Q_i^C + q_i^C\right) - \left(Q_{-i}^C + q_{-i}^C\right)\right](Q_i^C + q_i^C) - a_i {q_i^C}^2$, $i = 1, 2$, and the consumers' net surplus $S_n^C = \frac{(Q^C + q^C)^2}{2}$ in state z. Given (28) and (35) in the Appendix, these functions can be written under the format[14] $\Pi_i^C = l_i zE + m_i E^2 + n_i z^2, i = 1, 2$ and $S_n^C = l_S zE + m_S E^2 + n_S z^2$ where the weights $l_i, m_i, n_i, l_S, m_S, n_S$ only depend on the adjustment coefficients a_1, a_2. Consequently, the expected value of i's profit is

$$I\!E\Pi_i^C = (l_i + m_i + n_i)\, E^2 + n_i V,\ i = 1, 2$$

and, similarly, the expected net surplus of consumers is

$$I\!E S_n^C = (l_S + m_S + n_S)\, E^2 + n_S V$$

As we can see in the right part of Table 1 in the Appendix, $n_i > 0, n_S > 0, l_i + m_i + n_i > 0$, and $l_S + m_S + n_S > 0$. Therefore the expected profits and the expected net surplus are increasing in E and V, at a speed that depends on the values of the adjustment coefficients a_1, a_2.

Numerical illustrations. To gain insights on how the flexibility question impacts the firms' strategies and the market equilibria, in Table 1 of the Appendix we have computed the outcomes of the game corresponding to 19 different values of the pair a_1, a_2. The simulation allows to find out some important results such as that being inflexible can be more profitable than being flexible and that adding a day-ahead market to the intra-day market increases welfare but transfers risks from firms to consumers.

In the following we only discuss 4 characteristic cases out of the 19, three for symmetric costs and one for asymmetric costs.

Cost symmetry When the two firms are identical with a zero additional cost of adjustment ($a_1 = a_2 = 0$), only the total quantity $Q_i + q_i$ can be determined. Among the infinity of sharing rules between Q_i and q_i, one is the limit

[14] Indeed, by inserting (31) into (28) we obtain adjustment quantities as linear functions of z and E with weights that only depend on the adjustment coefficients a_1 and a_2. Then, given the quadratic specification of $\Pi_i(.)$ and $S(.)$, it is straightforward to obtain the profits and the net surplus as linear functions of E^2, z^2 and zE.

of the subgame-perfect equilibrium when $a_1 = a_2$ goes to 0. In this case, the equilibrium is $Q_i^C = 0.213E$, $q_i^C = 0.2z - 0.213E$, then a total expected output $Q_1^C + Q_2^C + \mathbb{E}(q_1^C + q_2^C) = 2/5$. It is interesting to note that the firms have a negative expected adjustment, contrary to the zero average adjustment at first best (3.4.2). It means that even though they both plan to produce less than at first best, the will to gain market shares pushes the output upwards, which increases the risk of downward adjustment. The expected profit of i and the expected consumers' net surplus have the same value $\mathbb{E}(\Pi_i^C) = \mathbb{E}(S_n^C) = 0.08(E^2 + V)$. Consequently, the global performance is $\mathbb{E}(W^C) = 0.24(E^2 + V)$. In this extreme case of perfect flexibility, the demand randomness (measured by variance) is as profitable as the average willingness to pay for both consumers and producers.

Consider now the opposite: the two fims have fully inflexible technologies: $a_1 = a_2 = +\infty$. Obviously $q_i^C = 0$ for both firms and we have a standard static Cournot duopoly, i.e. the equilibrium quantity of i is $Q_i^C = 0.2E$ bexause it does not fear ex post competition. The individual profit and the net consumers' surplus amount to $\mathbb{E}(\Pi_i^C) = 0.08E^2$. Comparing with the perfect flexibility case, we see that everybody is losing the gains from the demand variance, unequivocally implying that flexibility is socially desirable even under imperfect competition.

Let us switch to the intermediary symmetric case $a_1 = a_2 = 1$ (for other values, see the upper part of Table 1 in the Appendix). At equilibrium, we obtain

$$\begin{aligned}
Q_i^C &\simeq 0.208E,\ q_i^C \simeq \frac{z}{7} - 0.149E \quad i = 1, 2 \\
\mathbb{E}(\Pi_i^C) &\simeq 0.079E^2 + 0.061V \qquad\qquad i = 1, 2 \\
\mathbb{E}(S_n^C) &\simeq 0.082E^2 + 0.041V \\
\mathbb{E}(W^C) &\simeq 0.241E^2 + 0.163V
\end{aligned} \tag{32}$$

Unsurprisingly, the result falls in between the two former cases: the planned output takes an intermediary value, the average adjustment is slightly negative and the variance of demand matters but less than when technologies are fully flexible.

In the three cases above, the firms have identical technologies. The case where $a_1 = a_2 = +\infty$ is an interesting benchmark because firms are just competing "à la Cournot" facing an average demand. They both produce $Q_i^C = 0.2E < Q_i^* = 0.25E, q_i^C = q_i^* = 0$ whatever z, hence an expected social welfare $\mathbb{E}W^C = 0.24E^2 < \mathbb{E}W^* = \frac{1}{4}E^2$. These are standard results of imperfect competition. We can similarly observe that Cournot profits are larger than under perfect competition and the opposite holds for the consumers'net surplus. In the two other cases where the a_i are finite, we observe that Q_i^C is larger than when $a_i = \infty$ and that the average adjustments are negative. This is a Stackelberg effect: each firm has the incentive to invade the market to decrease the market share of its competitor. But doing so, it is competing against itself in the adjustment stage, both technically and commercially, which explains the average decrease in q_i. Day-ahead, the trade-off is between the gains of pushing the price up and the adjustment marginal cost down by restricting supply on the one hand, and the drawback of leaving a larger market share to the competitor on the other hand.

Cost asymmetry There is an infinity of possibilities to depict cost dissymmetry. Suppose that $a_2 > a_1$ so that firm 2 is less efficient than firm 1 at adjusting its output. Then firm 2 anticipates it will not change its initial planned production Q_2 very much, so that it can increase Q_2 without impairing too much its ex-post cost. Facing this more agressive ex ante strategy, firm 1 reduces its planned production and participates more in the adjustment process. As expected, there occurs a partial specialisation of the firms.

Let us illustrate it with the extreme case where firm 1 is perfectly flexible and firm 2 fully inflexible: $a_1 = 0, a_2 = +\infty$. Since $a_1 = 0$, there exist several equilibria defined by $Q_1^C + q_1^C = \frac{z}{4} - \frac{3E}{56}$. The following one is the equilibrium obtained when a_1 goes to 0:[15]

$$\begin{aligned}
Q_1^C &\simeq 0.196E,\ Q_2^C \simeq 0.214E, \\
q_1^C &= \frac{1}{4}(z-E),\ q_2^C = 0, \\
\mathbb{E}(\Pi_1^C) &\simeq 0.077E^2 + 0.125V,\ \mathbb{E}(\Pi_2^C) \simeq 0.080E^2 \\
\mathbb{E}(S_n^C) &\simeq 0.084E^2 + 0.03V \\
\mathbb{E}(W^C) &\simeq 0.242E^2 + 0.156V
\end{aligned}$$

Since firm 2 cannot adjust its production to the revelation of z, firm 1 is a monopoly for the adjustment to demand on the intra-day market. And since firm 1 can produce at the same cost at the two stages, it would like to share its output equally to prevent being penalized by increasing marginal costs. However, it would not be profitable to do so because of the opportunism of firm 2 that would increase its ex ante production. Then we have $Q_1^C < Q_2^C < Q_1^* = Q_2^*$. Consider now the expected profits. Firm 1's profit $\mathbb{E}(\Pi_1^C)$ is increasing in both the average and the variance of demand. By contrast, since firm 2 does not participate in the intra-day market, only the average value of the willingness-to-pay appears in $\mathbb{E}(\Pi_2^C)$, but with a higher coefficient than in $\mathbb{E}(\Pi_1^C)$. It means that, contrary to what we had under perfect competition (see Result 3.4.2), for a small variance, the inflexibility of firm 2 is an advantage over the flexibility of firm 1. Indeed, firm 2 contrary to firm 1 can credibly commit that it will not adjust ex post. Then it has a Stackelberg market power pushing away firm 1 from the ex ante process, a result in line with the worry of competition authorities quoted in footnote 1.

To sum up:

Duopoly results without a day-ahead market

When the gross surplus and cost functions are given by the quadratic specifications (1)-(2) and there is no day-ahead market, two firms competing in quantities

[15]Notice that the condition $q_1^C + Q_1^C \geq 0$ is met if $z \geq \frac{3}{14}E$, which is satisfied given the restriction $z \geq \frac{E}{4}$ set in Remark 3.4.

produce less than at first best. If the firms have the same cost function, the higher the adjustment cost, the lower the planned outputs (see Remark 4.1 below) and the higher the (negative) expected adjustment. If the firms are asymmetrical, the less flexible firm plans a higher level of output and adjusts less ex post than its competitor. When the demand variance is low, being inflexible is more profitable than being flexible.

Remark 4.1. *In the symmetric case $a_1 = a_2 = a$, we see in the upper part of Table 1 that Q_i^C is decreasing and $\mathbb{E}(q_i^C)$ is increasing in a.*[16] *This is inefficient on cost grounds. Then the explanation is to be sought in terms of strategic effect: by setting a large Q_i^C, each firm is seeking to appropriate a large market share (market stealing effect). But doing so it increases the set of states of nature where it will have to adjust downwards (note that the expected adjustment is always negative). Consequently, when a increases, the firms want to reduce the adjustments, and since they are negative, in means to increase their expected value $\mathbb{E}(q_i^C)$. And less downward adjustment means a lower planned output Q_i^C. As a byproduct, the expected total quantity $Q_i^C + \mathbb{E}(q_i^C)$ is not monotonous in a: it is increasing (resp. decreasing) when a is small (resp. large).*

4.3.2 Addition of a day-ahead market

Assume now that the two firms $i = 1, 2$ do not just decide on the production of Q_i at stage 1. They also sell it on a day ahead market at a price equal to the expectation of the random ex post prices On the intra-day market, fim i only sells q_i. In the following, we first consider two cases of identical costs: $a_1 = a_2 = 1$ and $a_1 = a_2 = +\infty$ to emphasize the drastic changes due to the opening of a day-ahead market, then the case $a_1 = 0, a_2 = +\infty$ to illustrate the potential for firms' specialization. The outcomes of 16 other pairs of coefficients are listed in Table 2.

Finite identical costs: $a_1 = a_2 = 1$ Applying the quadratic specification to the first order condition (23) and assuming that $a_1 = a_2 = 1$ we obtain the following equilibrium outcome (the explicit solution is in Appendix 6.2):

$$\widetilde{Q}_i^C \simeq 0,184E,\ \widetilde{q}_i^C \simeq 0,143z - 0,105E,\ i = 1,2$$

The average profit of i is

$$\mathbb{E}(\widetilde{\Pi}_i^C) \simeq 0,073E^2 + 0,061V,$$

the average consumers' net surplus is

$$\mathbb{E}(\widetilde{S}_n^C) = 0,098E^2 + 0,041V.$$

[16] It is obvious as for Q_i. Concerning $\mathbb{E}(q_i)$, since i) q_i is linear in z and E, and ii) $E \overset{def}{=} \mathbb{E}(z)$, we see that $\mathbb{E}(q_i)$ is increasing in a by adding the coefficients in column "coeff in z of q_i" and those in column "coeff in E of q_i" in Table 1.

and the resulting average welfare is

$$\mathbb{E}(\widetilde{S}_n^C + \widetilde{\Pi}^C) \simeq 0,244E^2 + 0,163V.$$

Comparing with (32), we can observe that adding a day-ahead market has the following effects:

i) each firm produces less at the first stage ($\widetilde{Q}_i^C < Q_i^C$) but its average total production is larger ($\widetilde{Q}_i^C + \mathbb{E}\widetilde{q}_i^C > Q_i^C + \mathbb{E}q_i^C$).

ii) expected prices are lower: $\mathbb{E}(\widetilde{p}^C) \simeq 0.556E < \mathbb{E}(p^C) \simeq 0.595E$,

iii) consumers are better off ($\mathbb{E}\widetilde{S}_n^C > \mathbb{E}S_n^C$) and firms are worse off ($\mathbb{E}\widetilde{\Pi}_i^C < \mathbb{E}\Pi_i^C$)

iv) total welfare is higher ($\mathbb{E}\widetilde{W}^C > \mathbb{E}W^C$).

With $\widetilde{Q}_i^C < Q_i^C$ but $\widetilde{q}_i^C > q_i^C$, we have an illustration of the two opposite effects due to the opening of a day-ahead market that we have identified at the end of subsection 4.2. Here the conflict ends out with $\widetilde{Q}_i^C + \mathbb{E}\widetilde{q}_i^C > Q_i^C + \mathbb{E}q_i^C$, so that opening a day-ahead market is profitable to consumers, detrimental to producers, and socially beneficial. However, notice that all the gains and losses result from an increase in the coefficient of the mean demand, whereas the variance effect remains unchanged. Let us take a closer look at the random performance values of the second stage. We observe that $\widetilde{W}^C > W^C$ if and only if $z < 0.96E$ and $\widetilde{S}_n^C > S_n^C$ if and only if $z > 0,94E$. In words, if z is small, the firms get a bonus and the consumers a malus, because day-ahead the firms have sold a smaller quantity at a relatively high price. By contrast, if z is high, the firms get a malus and the consumers get a bonus, because day-ahead the consumers have bought products at a relatively low price. The reason why the consumers are better off and the firms are worse off in expectation is that with convex cost functions, high values of z matter more than low values.

Interestingly, the minimum profit is larger with a day-ahead market ($\simeq 0,069E^2$, reached when $z \simeq 0,738E$) than without ($\simeq -0,033E^2$, reached when $z = 0$). Indeed the day-ahead market yields insurance to the firms in case of small z. This is confirmed by the observation that the addition of the day-ahead market induces a transfer of risks (measured by the variances of profit and surplus) from firms to consumers. Specifically, applying the formulae in Appendix 6.2.3, we have that

$$V(\widetilde{\Pi}_i^C) - V(\Pi_i^C) \simeq 0,0227VE^2 + 0,0172E^4 - 0,0172E\mathbb{E}(z^3)$$

$$V(\widetilde{S}_n^C) - V(S_n^C) \simeq 0,0712VE^2 - 0,0225E^4 + 0,0225E\mathbb{E}(z^3)$$

Using the Lemma 6.2.4 in the Appendix, we obtain

$$V(\widetilde{\Pi}_i^C) - V(\Pi_i^C) \leq -0,0031VE^2 < 0 \tag{33}$$

$$V(\widetilde{S}_n^C) - V(S_n^C) \geq 0,105VE^2 > 0 \tag{34}$$

Note that this is true $-$ under the quadratic specification (1)-(2) $-$ for any distribution of probabilities of the willingness-to-pay z.

Infinite identical costs: $a_1 = a_2 = +\infty$ The transfer of risks is emphasized when both firms are totally inflexible. With $a_1 = a_2 = +\infty$, we obtain $q_i^C = \widetilde{q}_i^C \equiv 0$ and $Q_i^C = \widetilde{Q}_i^C = 0.2E$, $i = 1, 2$. Even though the results seem identical to the case of no day-ahead market, the following table shows that profits and net surplus are differently affected:

intraday market only	day-ahead + intraday market
$p^C = z - 0.4E$	$P^C = 0.6E$
$\Pi_i^C = \frac{1}{25}(5z - 3E)E$	$\widetilde{\Pi}_i^C = 0.08E^2$
$S_n^C = 0.08E^2$	$\widetilde{S}_n^C = 0.4zE - 0.32E^2$

Profits have equal expected values in the two designs. The same for the consumer's net surplus. However, when all trade can only occur intraday, the quantity $Q^C = 0.4E$ is planned day-ahead and sold at the random price $p^C = z - 0.4E$ so that all risks are on the shoulders of firms (Π_i^C depends on z) and consumers are fully insured (S_n^C only depends on E). Symmetrically, when some trade occurs day ahead at price P^C, consumers buy the quantity $\widetilde{Q}_i^C = 0.4E$ but they pay a random price which provides full insurance to the producers. In our model, this transfer is innocuous since both consumers and producers are risk neutral when facing monetary lotteries. But it is worthwhile emphasizing it because in the real world most consumers (at least households) are risk averse and many entrepreneurs are risk lovers, so that the opening of a day-ahead market may have some detrimental effect on welfare.[17]

To conclude on the symmetric case, observe that the social advantage of adding a day-ahead market to the intra-day market measured by the difference $I\!E\widetilde{W}^C - I\!EW^C$ is vanishing when $a_1 = a_2 = a$ is increasing (compare the last columns in the uppert part of Tables 1 and 2). This makes sense since when a increases, adapting quantities on the intra-day market becomes more expensive. Consequently the firms rather rely on the response of demand to price. It also means that the social advantage of trading ex ante à la Allaz and Vila (1993) relies on an implicit assumption of some flexibility in the production process.

Asymmetric competition: $a_1 = 0, a_2 = +\infty$. The detailed solution for the case where firm 1 is perfectly flexible and firm 2 is totally inflexible is in Appendix 6.3. The first obvious result is that $\widetilde{q}_2^C = 0$ whatever z and $Q_1 + Q_2$. Then, firm 1 is a monopoly on the intra-day market, and since it can adjust its output at the same cost as ex ante, its best choice is $\widetilde{Q}_1^C = 0$, abandonning the initial stage to firm 2. The result is full specialization with two successive monopolies where firm 2 sells $\widetilde{Q}_2^C = \frac{3}{14}E$ day-ahead and firm 1 sells $\widetilde{q}_1^C = \frac{1}{4}z - \frac{3}{56}E$ on the intra-day

[17] Before the 2022 energy crisis, the European authorities published a text encouraging real-time electricity pricing (art 11 page 148 of Directive (EU) 2019/944), that is risky contracts for consumers. But during the crisis, national and European policy-makers did everything possible to ensure that the electricity prices paid by consumers varied as little as possible over time.

market.[18]

We could deduce that the addition of the day ahead market has very damageable consequences for competition, then for consumers, since firm 1 has now complete freedom in the ex-post market. However, the ex post market is shrinked since the quantity $\widetilde{Q}_2^C = \frac{3}{14}E$ has already been sold. It results a intra-day price $\widetilde{p} \simeq \frac{3}{4}z - 0.161E$ to be compared with $p \simeq z - 0.411E$ we had in subsection 4.3.1. We see that $\widetilde{p}$ is smaller (resp. larger) than p for large (resp. small) values of z even though, on average, $\mathbb{E}\widetilde{p} = \mathbb{E}p = 0.589E$. Since the quantity traded by each firm in each state of nature is the same as when everything is sold ex post ($\widetilde{Q}_i^C + \widetilde{q}_i^C = Q_i^C + q_i^C$, $i = 1, 2$), the expected profits of the two firms and the expected net surplus and welfare are unchanged after the adjunction of the day-ahead market.

However, the average values hide some subtle changes. Firm 2 earns the same average profit, but its profit is not random since all its production is sold ex ante at an average price. In the same vein, the expectation of the intra-day price is the same under the two market designs but the intra-day price is less varying with the random shock z when there is a day-ahead market. Clearly, the ex ante sales have a stabilizing effect on the ex post trade. The most interesting feature is the role of consumers, already mentioned in the cases $a_1 = a_2$. Their net surplus is higher in the day ahead setup if and only if $z > E$. Here again, the variance of the consumers' net surplus increases with the opening of the day-ahead market. Indeed, from subsection 6.3.3 in the Appendix, we have

$$V(\widetilde{S}_n) - V(S) \simeq 0,029VE^2 - 0,01E^4 + 0,01E\mathbb{E}(z^3)$$

which is non-negative by applying the Lemma of subsection 6.2.4.

Profits are $\Pi_1^C = \widetilde{\Pi}_1^C = \frac{1}{8}(z - \frac{3}{14}E)^2$ and $\Pi_2^C = \frac{9}{56}E(z - \frac{1}{2}E) \neq \widetilde{\Pi}_2^C = \frac{9}{112}E^2$. Then the opening of the day-ahead market changes nothing as regards the financial risks held by the flexible firm and provides full insurance to the inflexible firm at the expense of the consumers. To sum up:

Risk transfer in the case of specialization. Under specification (1)-(2), when a firm is totally inflexible and the other is perfectly flexible, if a day-ahead market is added to the intra-day market the expected gains of all agents remain unchanged but the inflexible firm is fully insured by consumers, whatever the distribution of probabilities of the demand size.

5 Conclusion

The energy mix needs to be adpated to the multiple challenges of green transition, security of supply, rollercoaster primary energy prices, energy savings, and so on. In the electricity industry, the main problem is the intermittency of green

[18]Notice that this is the unique equilibrium of the game whereas when there is no day-ahead market there is a multiplicity of equilibria since, when $a_1 = 0$, firm 1 can produce at the same cost ex ante and ex post and everything is sold ex post as seen formerly.

sources (solar, wind, along-the-river hydro) facing weak storage capacities and demand dependency on weather conditions rather than on intra-day prices. Then, encouraging the flexibility of production units able to serve residual demand is mandatory. As stated by ACER (2022, p. 30), *"flexibility needs arise at every possible timeframe, from seconds to weeks to years".* Our paper is a contribution to the economics of daily flexibility, that is when dispatchable generation plants face a trade-off between planning low-cost production day-ahead and benefitting from intra-day accurate information on the demand that cannot be served by planned production and green producers. We also recall that before any market redesigning it is important to assess the consequences of potential strategic behavior. For example, we show that in a duopoly the non flexible firm can earn higher profits than its flexible competitor because non-flexibility confers credibility when planning to produce large quantities. Consequently, when competition is imperfect, promoting flexibility can necessitate additional payments.

We also show that the addition of a day-ahead market to the real-time market is socially beneficial, profitable for consumers and detrimental to producers. This is a extension of the well-known result of Allaz and Vila (1993) to the case of complex technologies. A side effect of this social benefit is the transfer of risks from firms to consumers thanks to day-ahead trade. This form of collective insurance can be viewed negatively by governments.

To obtain precise results it is necessary to use specific forms of cost and surplus functions. This is why we have assumed a quadratic gross surplus function and a quadratic cost function with technical diseconomies between the planned and the adjusted outputs. Consequently, all results cannot be generalized. However, they provide some hints on the paths to explore in the analysis of supply flexibility and the design of electricity markets. To investigate the robustness of the results, we must relax at least four assumptions.

On the supply side, considering non quadratic cost functions would allow to introduce statistical moments higher than variance, in particular skewness since adjustment is generally more costly upwards than downwards. The case of technical economies (that is, a negative crossed second derivative of the cost function) should also be considered because starting costs and warming-up costs are essential in thermal plants.

On the demand side, we have assumed perfectly rational consumers, which means full demand responsiveness. Actually, small consumers are not able to react to variations in intra-day prices. With inertia on the demand side, supply flexibility is mandatory to avoid blackouts. This could change with the use of IoT devices at consumption locations. With more responsiveness to price variations on the consumer side, technical flexibility becomes less essential. Finally, the transfer of risks to consumers when a day-ahead market is opened entails the need to analyse the case where consumers are risk averse instead of risk neutral. Using a continuity argument we can conjecture that the risk reallocation effect will persist if consumers are slightly risk averse, still facing risk-neutral producers. But it remains to prove it and to investigate whether the negative risk reallocation effect can be large enough to outweigh the welfare-enhancing effect of a day-ahead market and how large it is depending on the degree of flexibility of producers.

References

[1] ACER (2022), "Final Assessment of the EU Wholesale Electricity Market Design", April.

[2] Allaz B. and J. L. Vila (1993), "Cournot Competition, Forward Markets and Efficiency", *Journal of Economic Theory*, Volume 59, Issue 1, February, Pages 1-16

[3] Bertsch J., C. Growitsch, S. Lorenczik and S. Nagl (2016), "Flexibility in Europe's power sector — An additional requirement or an automatic complement?", *Energy Economics* 53, 118–131

[4] Bistline, J.E. (2017), "Economic and technical challenges of flexible operations under large-scale variable renewable deployment", *Energy Economics* 64, 363–372.

[5] Borggrefe, F. and K. Neuhoff (2011), "Balancing and Intraday Market Design: Options for Wind integration", *DIW Berlin Discussion Paper*, 1162, http://climatepolicyinitiative.org/wp-content/uploads/2011/12/Intraday-and-Wind-Integration-Paper.pdf.

[6] Boyer, M. and M. Moreaux (1997), "Capacity commitment vs. flexibility", *Journal of Economics & Management Strategy*, Volume 6, Number 2, Summer, 347–376.

[7] Bushnell, J. (2008), "Oligopoly Equilibria in Electricity Contract Markets", *Journal of Regulatory Economics* 32, pp. 225-245.

[8] California Independent System Operator (2021), "Final Flexible Capacity Needs Assessment for 2022", May 14, http://www.caiso.com/InitiativeDocuments/Final2022FlexibleCapacityNeedsAssessment.pdf

[9] Crampes C. and J. Renault (2019), "How many markets for wholesale electricity when supply is partially flexible?", *Energy Economics*, 81, 465-478

[10] Denholm P., M. O'Connell, G. Brinkman, and J. Jorgenson (2015), "Overgeneration from Solar Energy in California: A Field Guide to the Duck Chart", *National Renewable Energy Laboratory*, https://www.nrel.gov/docs/fy16osti/65023.pdf

[11] DBEIS - Department for Business Energy and Industrial Strategy (2022), "Review of Electricity Market Arrangements. Consultation Document", July, https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1098100/review-electricity-market-arrangements.pdf

[12] ECA - Economic Consulting Associates (2015) "European Electricity Forward Markets and Hedging Products – State of Play and Elements for Monitoring", Final Report, September.

[13] Eisenack, K. and M. Mier (2019), "Peak-Load Pricing with Different Types of Dispatchability", *Journal of Regulatory Economics*, volume 56, 105–124.

[14] Goutte S. and P. Vassilopoulos (2019), "The value of flexibility in power markets", *Energy Policy*, Volume 125, February, 347-357.

[15] Hughes, J. S. and J. L. Kao (1997), "Strategic Forward Contracting and Observability, *International Journal of Industrial Organization* 16, pp.121-133.

[16] Ito K. and M. Reguant (2016), "Sequential Markets, Market Power, and Arbitrage", *American Economic Review*, vol. 106, n¬° 7, July, pp. 1921-57.

[17] Joskow, P. and J. Tirole. (2007), "Reliability and competitive electricity markets", *RAND Journal of Economics* 38(1): 60-84.

[18] Küenneke, R., J. Groenewegen, and C. Ménard. (2010). "Aligning Modes of Organization with Technology: Critical Transactions in the Reform of Infrastructures" Journal of Economic Behavior and Organization 75 (3): 494-505.

[19] Kok A.G, K. Shang and S. Yucel (2020), "Investments in Renewable and Conventional Energy: The Role of Operational Flexibility", *Manufacturing & Service Operations Management*, Vol. 22, No. 5.

[20] Léautier T-O (2019), "Imperfect Markets and Imperfect Regulation. An Introduction to the Microeconomics and Political Economy of Power Markets", MIT Press.

[21] Mahenc P. and F. Salanié (2004), "Softening Competition through Forward Trading", *Journal of Economic Theory*, vol. 116, 2, p. 282–293.

[22] Oi W. (1961), "The desirability of price instability under perfect competition", *Econometrica*, January, Vol. 29, 1, p. 58-64.

[23] Redl, C., Haas, R., Huber, C., and Böhm, B. (2009), "Price formation in electricity forward markets and the relevance of systematic forecast errors. Energy Economics", 30, p.356-364.

[24] Schittekatte T. and L. Meeus (2020), "Flexibility markets: Q&A with project pioneers", Utilities Policy 63, p. 1-11

[25] Waugh F. V.(1944), "Does the consumer benefit from price instability?", *Quarterly Journal of Economics*, p. 602-614.

6 Appendix

6.1 Planned quantities in the duopoly game without day ahead market

After substituting for the adjustment functions q_1^C, q_2^C given by (28), solving the two-equation system

$$\begin{array}{l} \mathbb{E}_z\left[p^C - \left(Q_1 + q_1^C\right)\left(3 - 2a_1\beta^{-1}\right)\right] = 0 \\ \mathbb{E}_z\left[p^C - \left(Q_2 + q_2^C\right)\left(3 - 2a_2\beta^{-1}\right)\right] = 0 \end{array}$$

gives the planned quantities

$$\begin{array}{l} Q_1^C = \frac{E}{D}4(a_1+2)(a_2+2)\big((15+8a_1)(45+24a_1+46a_2+24a_1a_2)+8a_2^2(2+a_1)(11+6a_1)\big) \\ Q_2^C = \frac{E}{D}4(a_1+2)(a_2+2)\big((15+8a_2)(45+46a_1+24a_2+24a_1a_2)+8a_1^2(2+a_2)(11+6a_2)\big) \\ \text{where } D = \left(8a_1(2+a_2)(7+4a_2)+(15+8a_2)^2\right)\left(8a_2(2+a_1)(7+4a_1)+(15+8a_1)^2\right) - \\ \qquad 64a_1a_2(2+a_1)^2(2+a_2)^2. \end{array} \tag{35}$$

They are denoted $Q_i^C = k_iE$, $i = 1, 2$ in subsection 4.3.1.

6.2 Subgame perfect equilibrium in the quadratic case with $a_1 = a_2 = 1$ when there are two markets

6.2.1 Stage 2 equilibrium

At stage 2, given $\vec{Q} = (Q_1, Q_2)$ and z, the price will be $p(Q+q, z) = z - (Q+q)$. Each firm i chooses q_i to maximize the concave function

$$\Pi_i(q_1, q_2) = (z - (Q+q))q_i - (Q_i + q_i)^2 - q_i^2 + P(Q_1, Q_2)Q_i.$$

Since the last term does not depend on q_i, it does not matter at this stage. Differentiating wrt q_i gives:

$$\left\{\begin{array}{rcl} 6q_1 + q_2 &=& z - 3Q_1 - Q_2 \\ q_1 + 6q_2 &=& z - Q_1 - 3Q_2. \end{array}\right. \tag{36}$$

We obtain the equilibrium of stage 2:

$$\left\{\begin{array}{rcl} \tilde{q}_1(z, \overrightarrow{Q}) &=& \frac{1}{7}z - \frac{17}{35}Q_1 - \frac{3}{35}Q_2, \\ \\ \tilde{q}_2(z, \overrightarrow{Q}) &=& \frac{1}{7}z - \frac{3}{35}Q_1 - \frac{17}{35}Q_2. \end{array}\right. \tag{37}$$

As a consequence,

$$p(Q+\widetilde{q},z)=\frac{5}{7}z-\frac{3}{7}Q_1-\frac{3}{7}Q_2,$$

and the demand function at stage 1 is:

$$P(Q_1,Q_2)=\mathbb{E}(p(Q+\widetilde{q},z))=\frac{5}{7}E-\frac{3}{7}Q_1-\frac{3}{7}Q_2.$$

6.2.2 Stage 1 equilibrium

At stage 1, each firm i chooses Q_i in order to maximize:

$$\begin{aligned}\Pi_i(Q_i,Q_{-i}) &= P(Q_i,Q_{-i})Q_i+\mathbb{E}_z\Big(p(Q+\widetilde{q}_i(\vec{Q},z),z)\widetilde{q}_i(\vec{Q},z)-C_i(Q_i,\widetilde{q}_i(\vec{Q},z))\Big)\\ &= (\frac{5}{7}E-\frac{3}{7}Q_i-\frac{3}{7}Q_{-i})Q_i\\ &\quad +(\frac{5}{7}z-\frac{3}{7}Q_i-\frac{3}{7}Q_{-i})(\frac{1}{7}z-\frac{17}{35}Q_i-\frac{3}{35}Q_{-i})\\ &\quad -(Q_i+\frac{1}{7}z-\frac{17}{35}Q_i-\frac{3}{35}Q_{-i})^2-(\frac{1}{7}z-\frac{17}{35}Q_i-\frac{3}{35}Q_{-i})^2.\end{aligned}$$

Differentiating with respect to Q_i and combining the best response functions, we obtain the equilibrium outputs

$$\widetilde{Q}_1^C=\widetilde{Q}_2^C=\frac{73}{397}E\simeq 0,184E.$$

Then, one can compute the prices

$$\widetilde{P}^C\simeq 0,557E,\quad \widetilde{p}^C\simeq 0,714z-0,158E.$$

The intraday quantities are

$$\widetilde{q}_1^C=\widetilde{q}_2^C\simeq 0,143z-0,105E.$$

The resulting profit is

$$\widetilde{\Pi}_i^C\simeq 0,102E^2-0,090zE+0,061z^2 \tag{38}$$

and its expected value is

$$\mathbb{E}(\widetilde{\Pi}_i^C)\simeq 0,073E^2+0,061V.$$

The consumers' net surplus is

$$\widetilde{S}_n^C=((z-(\widetilde{Q}^C+\widetilde{q}^C)/2)(\widetilde{Q}^C+\widetilde{q}^C)-\widetilde{P}^C\widetilde{Q}^C-\widetilde{p}^C\widetilde{q}^C\simeq -0,251E^2+0,041z^2+0,308zE \tag{39}$$

and its expected value is

$$\mathbb{E}(\widetilde{S}_n^C)=0,098E^2+0,041V.$$

Finally, the expected welfare value is

$$\mathbb{E}(\widetilde{S}_n^C+\widetilde{\Pi}^C)\simeq 0,244E^2+0,163V.$$

6.2.3 Variances when $a_1 = a_2 = 1$

The variance of profits and surplus is

- without day-ahead trade:

$$V(\Pi_i) \simeq -0,004V^2 + 0,004\mathbb{E}(z^4) - 0,011VE^2 - 0,01E^4 + 0,006E\mathbb{E}(z^3) \quad (40)$$

$$V(S_n) \simeq -0,002V^2 + 0,002\mathbb{E}(z^4) - 0,005VE^2 - 0,004E^4 + 0,003E\mathbb{E}(z^3) \quad (41)$$

- with a day-ahead market:

$$V(\widetilde{\Pi}_i) \simeq -0,004V^2 + 0,004\mathbb{E}(z^4) + 0,012VE^2 + 0,007E^4 - 0,011E\mathbb{E}(z^3) \quad (42)$$

$$V(\widetilde{S}_n) \simeq -0,002V^2 + 0,002\mathbb{E}(z^4) + 0,066VE^2 - 0,027E^4 + 0,025E\mathbb{E}(z^3) \quad (43)$$

6.2.4 Proof of the inequalities (33) and (34)

To assess the sign of the variation in variances (33) and (34), we use the following Lemma:

Lemma 6.1. *Let z be a random variable taking values in $\mathbb{R}_+$. Recalling that $E = \mathbb{E}(z)$, $V = \mathbb{E}(z^2) - E^2$, then:*

$$\mathbb{E}(z^3) \geq E^3 + \frac{3}{2}EV.$$

Proof: Consider the function $f : x \mapsto x^{3/2}$, f is convex on $\mathbb{R}_+$ so by Jensen's inequality the expectation of $f(z^2)$ is at least $f(\mathbb{E}(z^2))$, that is: $\mathbb{E}(z^3) \geq (E^2 + V)^{3/2}$. But $(E^2+V)^{3/2} = (E^2(1+\frac{V}{E^2}))^{\frac{3}{2}} = E^3(1+\frac{V}{E^2})^{\frac{3}{2}}$. Since $(1+x)^{\frac{3}{2}} \geq 1+\frac{3}{2}x$ for all x, we obtain $(1+\frac{V}{E^2})^{\frac{3}{2}} \geq 1+\frac{3}{2}\frac{V}{E^2}$ and $\mathbb{E}(z^3) \geq E^3(1+\frac{V}{E^2})^{\frac{3}{2}} \geq E^3 + \frac{3}{2}EV$.

6.3 Subgame perfect equilibrium in the quadratic case with $a_1 = 0, a_2 = +\infty$ when there are two markets

6.3.1 Stage 2 equilibrium

Given the cost structure, it is obvious that $\widetilde{q}_2^C = 0$. At stage 2, given $\vec{Q} = (Q_1, Q_2)$ and z, firm 1 chooses q_1 so as to maximize:

$$\Pi_1(\vec{Q}, q_1, 0, z) = (z - (Q_1 + Q_2 + q_1))q_1 - (Q_1 + q_1)^2 + P(Q_1, Q_2)Q_1.$$

which is concave in q_1. The first order condition is $z - 3Q_1 - Q_2 - 4q_1 = 0$, from which we derive the equilibrium of stage 2:

$$\begin{aligned}\widetilde{q}_1(\vec{Q},z) &= \frac{z-3Q_1-Q_2}{4}\\ \widetilde{q}_2(\vec{Q},z) &= 0.\end{aligned}$$

Then, the intra-day price is

$$\widetilde{p}(Q+\widetilde{q}_1,z) = z-(Q+\widetilde{q}_1) = \frac{3}{4}z-\frac{1}{4}Q_1-\frac{3}{4}Q_2,$$

and the price of stage 1 is

$$\widetilde{P}(Q_1,Q_2) = \mathbb{E}(\widetilde{p}(Q+\widetilde{q}_1,z)) = \frac{3}{4}E-\frac{1}{4}Q_1-\frac{3}{4}Q_2.$$

6.3.2 Stage 1 equilibrium

At stage 1, firm 1 chooses Q_1 in order to maximize:

$$\begin{aligned}\mathbb{E}\widetilde{\Pi}_1(Q_1,Q_2) &= \widetilde{P}(Q_1,Q_2)Q_1+\mathbb{E}_z\widetilde{p}(Q+\widetilde{q}_1,z)\widetilde{q}_1(\vec{Q},z)-C_1(Q_1,\widetilde{q}_1(\vec{Q},z))\\ &= (\frac{3}{4}E-\frac{1}{4}Q_1-\frac{3}{4}Q_2)Q_1+\\ &(\frac{3}{4}z-\frac{1}{4}Q_1-\frac{3}{4}Q_2)(\frac{1}{4}z-\frac{3}{4}Q_1-\frac{1}{4}Q_2)-(z/4+Q_1/4-Q_2/4)^2.\end{aligned}$$

Differentiating with respect to Q_1, we find $16\frac{\partial \mathbb{E}(\Pi_1)}{\partial Q_1} = -4Q_1 < 0$, so that at equilibrium $\widetilde{Q}_1^C = 0$.

Firm 2 chooses Q_2 in order to maximize the non random profit

$$\widetilde{\Pi}_2 = (3/4E-Q_1/4-3/4Q_2)Q_2-Q_2^2.$$

At equilibrium we obtain

$$\widetilde{Q}_2^C = \frac{3}{14}E.$$

Plotting these values in the adjustment function of firm 1, we obtain

$$\widetilde{q}_1^C = \frac{1}{4}z-\frac{3}{56}E.$$

Consequently the prices are

$$\widetilde{P}^C \simeq 0,557E, \widetilde{p}^C(z) \simeq 0,75z-0,161E.$$

The random profit and the expected profit of firm 1 are

$$\widetilde{\Pi}_1^C = \frac{1}{8}(z-\frac{3}{14}E)^2, \mathbb{E}\widetilde{\Pi}_1^C \simeq 0.077E^2+0.125V$$

and the gains of firm 2 are

$$\widetilde{\Pi}_2^C = \frac{9}{112}E^2 = \mathbb{E}\widetilde{\Pi}_2^C.$$

The consumers' net surplus is

$$\widetilde{S}_n^C = \frac{1}{32}(z^2 + \frac{45}{7}zE - \frac{927}{14^2}E^2),$$

then, on average

$$\mathbb{E}\widetilde{S}_n^C \simeq 0.084E^2 + 0.03V.$$

Finally, the welfare is

$$\widetilde{W}^C = \widetilde{S}_n^C + \widetilde{\Pi}^C = \frac{5}{32}z^2 + \frac{33}{224}zE + \frac{81}{6272}E^2$$

and, on average,

$$\mathbb{E}(\widetilde{W}^C) \simeq 0.242E^2 + 0.156V.$$

6.3.3 Variances when $a_1 = 0, a_2 = \infty$

When there is a day-ahead market, the variance of consumers' net surplus is

$$V(\widetilde{S}_n^C) \simeq -0,001V^2 + 0,001\mathbb{E}(z^4) + 0,026VE^2 - 0,013E^4 + 0,012E\mathbb{E}(z^3)$$

to be compared with

$$V(S_n^C) \simeq -0,001V^2 + 0,001\mathbb{E}(z^4) - 0,003VE^2 - 0,003E^4 + 0,002E\mathbb{E}(z^3)$$

when the whole ouput is sold ex post.

Table 1: Equilibrium quantities with a single market

a_1	a_2	Q_1/E	Q_2/E	coeff in z of q_1	coeff in E of q_1	Lower bound for z/E induced by firm 1	coeff in z of q_2	coeff in E of q_2	Lower bound for z/E induced by firm 2	$(Q_1+q_1)/E$, expected	$(Q_2+q_2)/E$, expected	$(Q+q)/E$, expected	price/E, expected	coeff in V of $\mathbf{E}(\pi_1)$	coeff in E^2 of $\mathbf{E}(\pi_1)$	coeff in V of $\mathbf{E}(\pi_2)$	coeff in E^2 of $\mathbf{E}(\pi_2)$	coeff in V of $\mathbf{E}(Sn)$	coeff in E^2 of $\mathbf{E}(Sn)$	coeff in V of $\mathbf{E}(W)$	coeff in E^2 of $\mathbf{E}(W)$
0	0	0,2133	0,2133	0,2000	-0,2133	0,0000	0,2000	-0,2133	0,0000	0,2000	0,2000	0,4000	0,6000	0,0800	0,0800	0,0800	0,0800	0,0800	0,0800	0,2400	0,2400
1	1	0,2081	0,2081	0,1429	-0,1486	-0,4162	0,1429	-0,1486	-0,4162	0,2023	0,2023	0,4046	0,5954	0,0612	0,0795	0,0612	0,0795	0,0408	0,0819	0,1633	0,2408
2	2	0,2058	0,2058	0,1111	-0,1143	-0,8232	0,1111	-0,1143	-0,8232	0,2026	0,2026	0,4051	0,5949	0,0494	0,0794	0,0494	0,0794	0,0247	0,0821	0,1235	0,2410
5	5	0,2031	0,2031	0,0667	-0,0677	-2,0311	0,0667	-0,0677	-2,0311	0,2021	0,2021	0,4041	0,5959	0,0311	0,0796	0,0311	0,0796	0,0089	0,0817	0,0711	0,2408
10	10	0,2018	0,2018	0,0400	-0,0404	-4,0350	0,0400	-0,0404	-4,0350	0,2014	0,2014	0,4028	0,5972	0,0192	0,0797	0,0192	0,0797	0,0032	0,0811	0,0416	0,2406
100	100	0,2002	0,2002	0,0049	-0,0049	-40,0394	0,0049	-0,0049	-40,0394	0,2002	0,2002	0,4004	0,5996	0,0024	0,0800	0,0024	0,0800	0,0000	0,0802	0,0049	0,2401
1000	1000	0,2000	0,2000	0,0005	-0,0005	-400,0399	0,0005	-0,0005	-400,0399	0,2000	0,2000	0,4000	0,6000	0,0002	0,0800	0,0002	0,0800	0,0000	0,0800	0,0005	0,2400
0	1	0,2075	0,2136	0,2174	-0,2260	0,0855	0,1304	-0,1393	-0,5697	0,1988	0,2047	0,4036	0,5964	0,0945	0,0791	0,0510	0,0801	0,0605	0,0814	0,2060	0,2406
0	2	0,2046	0,2138	0,2258	-0,2322	0,1222	0,0968	-0,1035	-1,1403	0,1982	0,2071	0,4053	0,5947	0,1020	0,0786	0,0375	0,0802	0,0520	0,0822	0,1915	0,2409
0	5	0,2010	0,2140	0,2364	-0,2400	0,1646	0,0545	-0,0584	-2,8535	0,1975	0,2102	0,4076	0,5924	0,1117	0,0780	0,0208	0,0803	0,0423	0,0831	0,1749	0,2413
0	10	0,1991	0,2141	0,2421	-0,2442	0,1862	0,0316	-0,0338	-5,7100	0,1970	0,2119	0,4089	0,5911	0,1172	0,0776	0,0120	0,0803	0,0375	0,0836	0,1666	0,2415
0	100	0,1967	0,2143	0,2491	-0,2493	0,2111	0,0037	-0,0039	-57,1379	0,1965	0,2140	0,4105	0,5895	0,1241	0,0772	0,0014	0,0804	0,0319	0,0843	0,1574	0,2418
0	1000	0,1965	0,2143	0,2499	-0,2499	0,2140	0,0004	-0,0004	-571,4235	0,1964	0,2143	0,4107	0,5893	0,1249	0,0772	0,0001	0,0804	0,0313	0,0843	0,1564	0,2419
1	2	0,2054	0,2084	0,1489	-0,1532	-0,3504	0,1064	-0,1107	-0,9182	0,2011	0,2041	0,4052	0,5948	0,0665	0,0792	0,0453	0,0797	0,0326	0,0821	0,1444	0,2409
1	5	0,2020	0,2088	0,1566	-0,1590	-0,2745	0,0602	-0,0627	-2,4247	0,1996	0,2063	0,4059	0,5941	0,0736	0,0787	0,0254	0,0800	0,0235	0,0824	0,1225	0,2411
1	10	0,2002	0,2090	0,1608	-0,1622	-0,2361	0,0350	-0,0364	-4,9361	0,1988	0,2076	0,4064	0,5936	0,0776	0,0785	0,0147	0,0801	0,0192	0,0826	0,1114	0,2412
1	100	0,1980	0,2093	0,1660	-0,1661	-0,1917	0,0041	-0,0043	-50,1451	0,1978	0,2091	0,4069	0,5931	0,0827	0,0782	0,0017	0,0803	0,0145	0,0828	0,0988	0,2413
1	1000	0,1977	0,2093	0,1666	-0,1666	-0,1866	0,0004	-0,0004	-502,2381	0,1977	0,2093	0,4070	0,5930	0,0833	0,0782	0,0002	0,0803	0,0139	0,0828	0,0974	0,2413
5	10	0,2014	0,2034	0,0687	-0,0693	-1,9246	0,0388	-0,0394	-4,2268	0,2008	0,2028	0,4036	0,5964	0,0330	0,0794	0,0181	0,0798	0,0058	0,0815	0,0568	0,2407

In a very few cases a slight assumption on the law of z is needed: for instance if $a_1=0$ and $a_2=1$,
then we need that almost surely z is at least 0.0855 E and not only that z is positive (see column 7).

Table 2: Equilibrium quantities with two markets

a_1	a_2	Q_1/E	Q_2/E	coeff in z of q_1	coeff in E of q_1	Lower bound for z/E induced by firm 1	coeff in z of q_2	coeff in E of q_2	Lower bound for z/E induced by firm 2	$(Q_1+q_1)/E$, expected	$(Q_2+q_2)/E$, expected	(Q+q)/E, expected	price/E, expected	coeff in V of **E**(π_1)	coeff in E^2 of **E**(π_1)	coeff in V of **E**(π_2)	coeff in E^2 of **E**(π_2)	coeff in V of **E**(Sn)	coeff in E^2 of **E**(Sn)	coeff in V of **E**(W)	coeff in E^2 of **E**(W)
0	**0**	0,0526	0,0526	0,2000	-0,0421	0,2105	0,2000	-0,0421	0,2105	0,2105	0,2105	0,4211	0,5789	0,0800	0,0776	0,0800	0,0776	0,0800	0,0886	0,2400	0,2438
1	**1**	0,1839	0,1839	0,1429	-0,1051	0,7355	0,1429	-0,1051	0,7355	0,2217	0,2217	0,4433	0,5567	0,0612	0,0728	0,0612	0,0728	0,0408	0,0983	0,1633	0,2439
2	**2**	0,1914	0,1914	0,1111	-0,0851	0,7655	0,1111	-0,0851	0,7655	0,2174	0,2174	0,4348	0,5652	0,0494	0,0742	0,0494	0,0742	0,0247	0,0945	0,1235	0,2430
5	**5**	0,1963	0,1963	0,0667	-0,0524	0,7853	0,0667	-0,0524	0,7853	0,2106	0,2106	0,4213	0,5787	0,0311	0,0765	0,0311	0,0765	0,0089	0,0887	0,0711	0,2418
10	**10**	0,1981	0,1981	0,0400	-0,0317	0,7924	0,0400	-0,0317	0,7924	0,2064	0,2064	0,4128	0,5872	0,0192	0,0779	0,0192	0,0779	0,0032	0,0852	0,0416	0,2410
100	**100**	0,1998	0,1998	0,0049	-0,0039	0,7992	0,0049	-0,0039	0,7992	0,2008	0,2008	0,4016	0,5984	0,0024	0,0797	0,0024	0,0797	0,0000	0,0806	0,0049	0,2401
1000	**1000**	0,2000	0,2000	0,0005	-0,0004	0,7999	0,0005	-0,0004	0,7999	0,2001	0,2001	0,4002	0,5998	0,0002	0,0800	0,0002	0,0800	0,0000	0,0801	0,0005	0,2400
0	**1**	0,0335	0,1949	0,2174	-0,0501	0,2306	0,1304	-0,0946	0,7256	0,2007	0,2306	0,4313	0,5687	0,0945	0,0738	0,0510	0,0767	0,0605	0,0930	0,2060	0,2436
0	**2**	0,0249	0,2037	0,2258	-0,0513	0,2274	0,0968	-0,0731	0,7552	0,1994	0,2274	0,4268	0,5732	0,1020	0,0745	0,0375	0,0775	0,0520	0,0911	0,1915	0,2431
0	**5**	0,0141	0,2098	0,2364	-0,0525	0,2221	0,0545	-0,0422	0,7738	0,1980	0,2221	0,4201	0,5799	0,1117	0,0756	0,0208	0,0787	0,0423	0,0882	0,1749	0,2426
0	**10**	0,0082	0,2119	0,2421	-0,0530	0,2189	0,0316	-0,0246	0,7799	0,1973	0,2189	0,4162	0,5838	0,1172	0,0763	0,0120	0,0794	0,0375	0,0866	0,1666	0,2423
0	**100**	0,0010	0,2140	0,2491	-0,0535	0,2148	0,0037	-0,0029	0,7852	0,1965	0,2148	0,4114	0,5886	0,1241	0,0771	0,0014	0,0802	0,0319	0,0846	0,1574	0,2419
0	**1000**	0,0001	0,2143	0,2499	-0,0536	0,2143	0,0004	-0,0003	0,7857	0,1964	0,2143	0,4108	0,5892	0,1249	0,0772	0,0001	0,0803	0,0313	0,0844	0,1564	0,2419
1	**2**	0,1819	0,1929	0,1489	-0,1096	0,7356	0,1064	-0,0814	0,7651	0,2213	0,2179	0,4392	0,5608	0,0665	0,0736	0,0453	0,0735	0,0326	0,0965	0,1444	0,2435
1	**5**	0,1797	0,1989	0,1566	-0,1151	0,7352	0,0602	-0,0472	0,7841	0,2212	0,2119	0,4331	0,5669	0,0736	0,0747	0,0254	0,0744	0,0235	0,0938	0,1225	0,2429
1	**10**	0,1786	0,2010	0,1608	-0,1182	0,7349	0,0350	-0,0276	0,7905	0,2213	0,2083	0,4296	0,5704	0,0776	0,0754	0,0147	0,0749	0,0192	0,0923	0,1114	0,2426
1	**100**	0,1773	0,2029	0,1660	-0,1219	0,7344	0,0041	-0,0033	0,7962	0,2213	0,2037	0,4251	0,5749	0,0827	0,0763	0,0017	0,0756	0,0145	0,0903	0,0988	0,2422
1	**1000**	0,1771	0,2031	0,1666	-0,1223	0,7344	0,0004	-0,0003	0,7968	0,2214	0,2032	0,4245	0,5755	0,0833	0,0764	0,0002	0,0756	0,0139	0,0901	0,0974	0,2422
5	**10**	0,1957	0,1986	0,0687	-0,0539	0,7854	0,0388	-0,0307	0,7921	0,2104	0,2067	0,4171	0,5829	0,0330	0,0773	0,0181	0,0771	0,0058	0,0870	0,0568	0,2414

We need assumptions on the law of z to avoid corner solutions: for instance if $a_1=0$ and $a_2=1$, then we need that almost surely z is at least 0.7256 E (see column 10).