Compatibility Choices, Switching Costs and Data Portability*

Doh-Shin Jeon† Domenico Menicucci‡ Nikrooz Nasr§

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Abstract

We study mix-and-match compatibility choices of firms selling complementary products in a dynamic setting. Contrary to what happens in a static setting where symmetric firms choose compatibility (Matutes and Régibeau, 1988), when switching costs are high and firms make price discrimination based on past purchases, symmetric firms choose incompatibility to soften future competition if the discount factor is large, which harms consumers. Interoperability increases consumer surplus at least for high switching costs. Data portability, by reducing switching costs, induces the firms to choose compatibility more often but, given a compatibility regime, benefits consumers only if a non-negative pricing constraint binds.

Key words: Compatibility, Switching Cost, Data Portability, Interoperability, Cloud Computing

JEL Codes: D43, L13, L41.

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†Toulouse School of Economics, University of Toulouse Capitole. dohshin.jeon@tse-fr.eu
‡Dipartimento di Scienze per l’Economia e l’impresa, Università degli Studi di Firenze. Via delle Pandette 9, I-50127 Firenze (FI), Italy. E-mail: domenico.menicucci@unifi.it.
§Toulouse School of Economics. E-mail: me@nikrooz.com
1 Introduction

Will the future of the Internet be dominated by incompatible devices and applications? Back in the 90s' when the Internet was at its dawn, openness and compatibility seemed to be the rule. For instance, during the times, Microsoft was the dominant player in the personal computer market but decided to bring two of its most successful software, Internet Explorer and Microsoft Office, to Macs. However, after the turn of the 21st century, we seem to enter a new era in which platforms are becoming "walled gardens" trying to lock-in customers, either by making it hard to move data across platforms or by providing some benefits exclusively to those who use all from the same ecosystem. According to Larry Page, a cofounder of Google:

"The Internet was made in universities and it was designed to interoperate. And as we’ve commercialized it, we’ve added more of an island-like approach to it, which I think is a somewhat a shame for users."1

We provide a theory which shows that competing firms selling complementary products in a dynamic setting tend to embrace incompatibility in markets characterized by high switching costs and behavior-based price discrimination. Consider the market of SaaS (Software as a Service), which is one among three models of cloud computing service.2 In this market, switching from one vendor to another is very costly. For instance, in the cases of productivity software suites and file storage/hosting services, switching from Microsoft Office to Google Docs/Sheets/Slides requires significant efforts to convert all existing files from one format to another and switching from Google Drive to Microsoft OneDrive requires moving all users’ data from one service to another, which can be very costly depending on the size of the client company. As an executive of an Amazon Web Service (AWS) vendor partner put it, "data gravity makes lock-in worse with Amazon",3 implying that as the data stocked in one platform grows, it becomes harder to move from the platform.4 This applies not only to B2B markets such as cloud computing but also to

1http://fortune.com/2012/12/11/fortune-exclusive-larry-page-on-google/
2The other two are PaaS (Platform as a Service) and IaaS (Infrastructure as a Service). In the SaaS model, the user accesses applications managed by the cloud vendor. In the PaaS model, the user builds new applications by accessing services and tools provided by the vendor. In the IaaS model, the user can deploy and run software which includes operating systems and applications while the cloud provider provisions fundamental computing resources. As IaaS offerings have become commoditized, infrastructure providers must offer a range of PaaS and SaaS services to attract users (U.S. House, 2020). In 2019, SaaS represented almost two thirds of total public cloud revenues generated within the EU market, a trend poised to continue until at least 2021 (EC, 2021).
3http://fortune.com/2015/10/08/ aws-lock-in-worry/
4In fact, the recent US House Antitrust Digital Market Report (2020) states that there is high switching cost in the cloud computing market because of high cost of moving data. Similarly, the European Commission (2021) finds that vendor lock-in is one of the most proliferating problems for the EU-based cloud users; the report produced by Deshpande, Stevenson, Virdee and Gunashekar (2021) for the European Commission concerning B2B platforms and emerging cloud services also makes the same point about B2B platforms in general.
B2C markets as each consumer accumulates more and more data in one platform. For instance, 1.2 trillion photos were taken with smartphones in 2017. Google and Apple offer their photo storage services, Google Photos and iCloud Photos. As a consumer accumulates more photos in one of the two platforms, it becomes harder to switch.

An increasingly common feature of the above-mentioned markets is behavior-based price discrimination: firms offer a discount to poach customers from rivals. For instance, in the SaaS market, Microsoft tried to poach Google Drive users "by offering free OneDrive for Business for the remaining term of their existing contract with Google". At the same time, Google offered a similar incentive to Microsoft Office customers by "covering the fees of Google Apps until [their] contract runs out [...] and chip[ing] in on some of the deployment costs". In consumer markets, the rapid advance in information technology makes it easier for sellers to condition their price offers on consumers’ prior purchase behavior. Firms can offer personalized discounts through targeted messages although list prices are publicly quoted (Acquisti and Varian, 2005). The behavior-based price discrimination can also take the form of trade-in, meaning that a vendor offers discounts to a rival’s customers in exchange of their devices. For instance, Google offered up to $600 to iPhone users to switch to Google Pixel, Samsung offered the full price of a Google Pixel in exchange of a Samsung Galaxy, and Microsoft paid $650 to Apple MacBook users to trade-in their MacBook for a Microsoft Surface.

We have two sets of results. First, we attempt to understand firms’ mix-and-match compatibility choices from a dynamic perspective and find that symmetric firms make their products incompatible in order to soften future competition if customer lock-in arises due to high switching costs, the firms practice behavior-based price discrimination and their discount factor is sufficiently large. Our result is opposite to what happens in a static model in which symmetric firms make their products compatible to soften competition (Matutes and Régibeau, 1988). Consistently with our prediction, both the European Commission (2021) and the U.S. House (2020) point out the lack of interoperability in the cloud computing market. One example of incompatibility in a B2C market is Apple’s decision to make iMessage unavailable on Android.

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8https://bgr.com/2019/05/09/google-pixel-3a-deal-iphone-trade-in/
9https://lifehacker.com/samsung-will-pay-you-the-full-price-of-a-google-pixel-3-1836734167
10https://techcrunch.com/2016/10/28/microsoft-apple/
11Unfair commercial practices and a lack of interoperability and data portability between cloud providers create risks of vendor lock-in, undermining users’ trust in cloud computing services and cloud uptake" (EC, 2021, p.93). We note that in the SaaS market, there is some incompatibility between one platform’s file storage service and another’s productivity software suite. For instance, it’s not possible to store Google Docs in Microsoft OneDrive, or to use all functionalities of Microsoft Office, like real-time coauthoring, when stored in Google Drive.
Apple’s senior executives vetoed the initial plan to make it available on Android as this would undermine Apple’s ability to lock users into iOS devices (Sokol and Zhu, 2021). Our analysis also predicts a strong conflict between the compatibility regime chosen by the firms and the one maximizing consumer surplus (or welfare) such that whenever the firms choose incompatibility, it generates a lower consumer surplus and a lower welfare than compatibility.

Second, we study two policy remedies, interoperability and data portability, both of which are hotly discussed across the Atlantic. As the effect of each policy depends on whether a non-negative pricing constraint (NPC) binds or not, we first extend the previous analysis by introducing the constraint. In our two-period model, the firms compete fiercely in period one to build a customer base, which can be exploited in period two due to the switching cost. This competition in period one induces the firms to dissipate the second-period rent from locked-in consumers and may lead to negative prices in period one, which may be impractical due to adverse selection and opportunistic behaviors of consumers (Farrell and Gallini, 1988, Amelio and Jullien, 2012, Choi and Jeon, 2021a). The NPC is more likely to matter in B2C markets than in B2B markets since monitoring a mass of individual customers is much more costly than monitoring business customers. For this reason, we study both the case in which the firms face the NPC and the case in which the NPC does not apply. When the NPC is binding, it limits the dissipation of the rent from locked-in consumers. As the rent is larger under incompatibility than under compatibility, the binding NPC expands the interval of switching costs under which incompatibility is chosen. We find that when the NPC does not apply, interoperability obligations improve both consumer surplus and welfare. We also show that data portability, by lowering the switching cost, typically induces the firms to embrace compatibility more often. Interestingly, we find that given a compatibility regime, whether data portability increases or reduces consumer surplus (and profits) completely depends on whether or not the NPC binds. If the constraint binds, data portability increases consumer surplus but reduces each firm’s profit, whereas the opposite holds when the constraint does not apply.

Our model extends the mix-and-match compatibility model of Matutes and Régibeau (1988) to two periods. They study compatibility choices made by two symmetric firms (A and B) which compete to sell a system of complementary products (x and y). Therefore, under compatibility,
four systems are available \((A, A), (A, B), (B, A), (B, B)\) while under incompatibility, only two pure systems, \((A, A)\) and \((B, B)\), are available. They study a two-stage game in which the first stage of non-cooperative choice between compatibility and incompatibility is followed by the second stage of price competition. As we consider the family of log-concave and symmetric distributions which includes the uniform distribution used by Matutes and Régibeau (1988), the first-period of our model captures the model of Matutes and Régibeau (1988) as a special case. In our model, the firms make their compatibility choices in period one and the compatibility regime determined in period one is maintained in period two. In period two, consumers incur switching costs when they consume products different from those consumed in the first period and each firm competes to poach consumers by offering prices dependent on their past purchase behavior (Chen, 1997b and Fudenberg and Tirole, 2000). Following Villas-Boas (2006) and Doganoglu (2010), we consider experience goods and assume that each consumer discovers the value that she obtains from a product after consuming it.

We assume for simplicity that all consumers incur the same switching cost per product \(s > 0\). Suppose that all products are compatible and consider a submarket in period two which is composed of consumers who bought \(x\) from \(A\). If a consumer wants to switch from \(x\) of \(A\) to \(x\) of \(B\), she should incur \(s\) and therefore firm \(A\) is dominant and firm \(B\) is dominated in this submarket. Similarly, suppose that the products are incompatible and consider the submarket in period two which is composed of the consumers who bought the system \((A, A)\) in period one. If a consumer wants to switch from \((A, A)\) to \((B, B)\), she should incur a switching cost of \(2s\) and therefore firm \(A\) is dominant and firm \(B\) is dominated in this submarket.

When we study how incompatibility affects, relative to compatibility, the firms’ second-period profits from the consumers who bought say \((A, A)\) in period one, we find that the set of values of \(s\) can be partitioned into three regions such that (i) when \(s\) is in the first region (which includes values of \(s\) close to 0), incompatibility reduces the profit of each firm; (ii) when \(s\) is in the third region (which includes values of \(s\) close to an upper bound), incompatibility increases the profit of each firm (iii) when \(s\) is in the second region (which includes some intermediate values of \(s\)), incompatibility increases the profit of the dominant firm but reduces the profit of the dominated firm. For the case in which the second-period valuation is uniformly distributed, the above regions are intervals and there are three thresholds of switching costs \((\bar{s}^1, \bar{s}^2, \bar{s}^3)\) with \(\bar{s}^1 < \bar{s}^2 < \bar{s}^3\) such that the dominant firm’s profit is higher under incompatibility than under compatibility for \(s > \bar{s}^1\), the industry profit is higher under incompatibility for \(s > \bar{s}^2\) and the dominated firm’s profit is higher under incompatibility for \(s > \bar{s}^3\). In other words, when the switching cost is high

\[\text{In our TSE Working paper (Jeon, Menicucci, Nasr, 2020), we study an alternative scenario in which the firms make their compatibility choices each period and find that the main results are similar (see Proposition 13(ii)).}\]

\[\text{In fact, we can allow for heterogenous switching costs. See the paragraph after (1) for more details.}\]
enough, incompatibility softens the second-period competition relative to compatibility. This result can be understood from Hahn and Kim (2012) and Hurkens, Jeon and Menicucci (2019), who extend Matutes and Régibeau (1988) to asymmetric firms (one is dominant and the other is dominated). They find that when the level of dominance is large enough, both the dominant firm and the dominated firm prefer incompatibility since incompatibility softens competition. This mechanism is in place in our model as the level of dominance increases with $s$ (see Section 3.3).

When we study the first-period competition without the NPC, for each given regime of compatibility, each firm charges a price smaller than the static price by the difference equal to the second-period rent from a locked-in consumer. Hence, they completely dissipate this rent. This implies that each firm’s total profit is the sum of the static profit and the profit that a firm realizes in period two if it attracted no consumer in period one. The latter is equal to what we call the profit of the dominated firm. Therefore, for $s > s^3$, if the relative weight of the second-period payoff is large enough, the firms choose incompatibility in period one in order to soften future competition.

As the firms make compatibility choices mainly to soften competition, we find a strong conflict between the compatibility regime chosen by the firms and the one maximizing consumer surplus, regardless of whether the NPC binds. When the NPC does not apply, whenever the firms choose incompatibility, it generates a lower consumer surplus (and a lower welfare) than under compatibility. This generates our first policy recommendation: when the NPC does not apply, interoperability obligations (i.e., mandatory compatibility) strictly improve consumer surplus if the obligations bind. When the NPC binds, the same result applies as long as the switching costs are high; however, for a small range of intermediate switching costs, interoperability obligations can reduce consumer surplus.

Regarding data portability, note first that it is relevant both for B2C markets and B2B markets. In B2C markets, EU’s General Data Protection Regulation (GDPR) provides consumers with the data portability right. According to Article 20 of GDPR,

"the data subject shall have the right to receive the personal data concerning him or her, which he or she has provided to a controller, in a structured, commonly used and machine-readable format and have the right to transmit those data to another controller without hindrance from the controller to which the personal data have been provided."

In B2B markets such as cloud computing, the recent US House Antitrust Digital Market Report (2020) recommends investigating the role of standards in enabling data portability and the European Commission (2020) unveiled a plan to create a cloud rulebook in order to promote data portability among others. We find that data portability, by reducing switching cost, can induce a change in the compatibility regime from incompatibility to compatibility. Interestingly,
we find that given a compatibility regime, data portability increases consumer surplus and reduces each firm’s profit only if the NPC binds; if the NPC does not apply, the reverse holds.

Therefore, our results suggest that the impact of each policy remedy on consumer surplus can differ strikingly depending on whether the NPC binds or not. In B2B markets where the NPC does not apply, we find that interoperability obligations improve consumer surplus whereas data portability tends to reduce it. This finding is very surprising and against the conventional wisdom as policy reports recommend both policies together. In B2C markets where the NPC binds, both policies tend to improve consumer surplus at least for high switching costs, which is in line with the conventional wisdom. However, we are not aware of any policy report which recommends the policies contingent on the prevalence of the NPC.

1.1 Related literature

We merge two different strands of literature, the one on compatibility and the one on poaching. First, as incompatibility is equivalent to pure bundling, our paper is related to the literature on bundling, in particular the one on competitive bundling which studies how bundling affects competition when entry or exit is not an issue (Matutes and Régibeau 1988, 1992, Economides, 1989, Carbajo, De Meza and Seidmann, 1990, Chen 1997a, Denicolo 2000, Nalebuff, 2000, Armstrong and Vickers, 2010, Carlton, Gans, and Waldman, 2010, Thanassoulis 2011). Especially, our paper is related to the line of research on "mix and match" compatibility initiated by Matutes and Régibeau (1988), which has seen some recent development. While Matutes and Régibeau (1988) find that incompatibility intensifies competition in a symmetric duopoly, the extension to asymmetric duopoly by Hahn and Kim (2012) and Hurkens, Jeon and Menicucci (2019) shows that for large asymmetry, incompatibility softens competition. Kim and Choi (2015) and Zhou (2017) consider symmetric oligopoly of more than two firms and find that incompatibility can soften competition when the number of firms is above a threshold which can be small. We contribute to this literature by considering a dynamic setup and showing that even a symmetric duopoly can prefer incompatibility to soften future competition. As we build on Matutes and Régibeau (1988), our model does not include network effects although network effects can be an important factor influencing firms’ compatibility decisions. For instance, Katz and Shapiro (1985), Crémer, Rey and Tirole (2000) and Chen, Doraszelski and Harrington (2009) study com-

patibility choices in the presence of network effects. Note that in these papers, symmetric firms adopt compatibility as they can only benefit from a larger network effect.

Second, our two-period model is similar to those considered in the literature on poaching in the presence of switching costs (Chen, 1997b) or in their absence (Fudenberg and Tirole, 2000). As our paper considers switching costs, it is closer to Chen (1997b) which studies a duopoly model with homogenous products and heterogenous switching costs. Both Chen (1997b) and Fudenberg and Tirole (2000) compare the allocation under poaching with the one without poaching. The main difference between our paper and theirs is that we consider multi-product firms and analyze their compatibility choices under poaching and how data portability affects the choices.

Our paper is also related to the large literature on switching costs. Our model is very similar to that of Doganoglu (2010), which studies competition between two firms producing experience goods over an infinite horizon with overlapping generations of consumers. The utility of a consumer in our model is exactly the same as that of a consumer in Doganoglu (2010). However, Doganoglu (2010) considers neither poaching nor compatibility choices. To some extent, our model is similar to Somaini and Einav (2013), Rhodes (2014), Cabral (2016) and Lam (2017), which assume that consumers’ locations are independently and identically distributed over an Hotelling line across periods. Even if we do not formally make such assumption, a model with such assumption will generate exactly the same predictions as our current model. Our contribution with respect to the literature on switching cost is twofold. First, we embed the mix-and-match compatibility choices into a model of poaching under switching costs and study how switching costs (and data portability) affect the choices. Second, we show that whether a reduction in switching cost increases or reduces consumer surplus crucially depends on whether or not the NPC binds.

To our knowledge, Lam and Liu (2020) is the only economic article studying data portability. They consider an incumbent facing entry and find that data portability can hinder entry instead of facilitating it due to a demand-expansion effect: the possibility of porting data to an entrant induces consumers to provide more data to the incumbent. This can reduce switching as it increases the value of the incumbent’s services based on data analytics and artificial intelligence, which make use of inferred data that is not subject to the portability obligation under the GDPR. Although their result is interesting, the forces generating the result are absent in our

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19 This is because if we consider the Hotelling model for both periods, then the results from Hahn and Kim (2012) and Hurkens, Jeon and Mencucci (2019) apply as the former considers a Hotelling model and the latter considers a family of log-concave distributions which includes the Hotelling model.
model. First, as we consider two symmetric incumbents, asymmetry in data analytic services does not exist. Second, we do not consider consumers’ active choices regarding how much data to provide because in reality most data is generated as a by-product of their consumption activities. Therefore, we assume that data portability reduces switching costs. Our novelty consists in studying the interaction between data portability and compatibility choices, on the one hand, and the one between data portability and the NPC, on the other hand.

The paper is organized as follows. Section 2 describes the baseline model. Section 3 (4) analyzes the second-period (the first-period) price competition given a compatibility regime. Section 5 analyzes compatibility choices. Section 6 provides the analysis of consumer surplus and welfare. Section 7 analyzes how the NPC modifies the previous analysis and studies two policy remedies (interoperability and data portability) by distinguishing the case in which the NPC binds from the case in which it does not. Section 8 provides various extensions to show that our main result (i.e., Proposition 3) holds under more general assumptions. Section 9 provides the conclusion. All the proofs are gathered in Appendix.

2 The baseline model

We here present the baseline model. There are two firms, \( i = A, B \), which produce two perfectly complementary products, \( j = x, y \). Therefore, consumer demand is defined for the system composed of two products. When both firms’ products are compatible, there are four systems available: \((A, A)\), \((A, B)\), \((B, A)\), \((B, B)\). When firm A’s products are not compatible with those of firm B, only two systems are available: \((A, A)\) and \((B, B)\). We consider a two-period model in which consumers have switching costs in the second period.

We assume that each consumer has a unit demand for a system and buys one among the available systems in each period \( t = 1, 2 \) as she obtains a high enough utility from a system. Given the assumption, we set the marginal cost of producing each product to zero for simplicity and interpret prices as margins.

Each firm simultaneously and non-cooperatively chooses between compatibility and incom-

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20 As both firms have significant market shares and hence have inferred data from their consumers, both can provide services based on big data analytics. In contrast, an entrant cannot provide such services as it initially has no stock of inferred data.

21 In fact, we obtain the same results even when the two products can be independently consumed instead of being perfect complements as long as (i) incompatibility is interpreted as pure bundling and (ii) each consumer obtains a high enough utility from each product \( x \) and \( y \) such that the market is fully covered for both products.

22 To make clear the multiproduct feature of cloud computing service, consider a simple photo library app. This app will use the following products from a cloud vendor: (i) storage, to store files (like photos), (ii) database, to store data (like photo’s metadata), (iii) analytics, to provide insight on how the software is being used (iv) compute, to do some computation in cloud (like analyzing photos) (v) networking, to control and guide the traffic to the appropriate resources (vi) identity, to identify a user and manage access to the user’s data.
patibility at the beginning of period one. Compatibility prevails only if both firms choose compatibility; otherwise, incompatibility prevails. The compatibility regime determined in period one is maintained in period two, which makes sense when compatibility choices are embedded into technical design of the products such that undoing the initial design is very costly.

In the first period, consumers have heterogeneous costs of learning to use different products as in Klemperer (1995). Precisely, each consumer is characterized by a pair of locations \((\theta_x, \theta_y)\) \(\in [0, 1]^2\) which determine her learning cost for each product: \(t \theta_j (t (1 - \theta_j))\) is the learning cost for product \(j\) of firm \(A\) (for product \(j\) of firm \(B\)) for \(j = x, y\), for some \(t > 0\). The assumption of negative correlation in the learning costs is made only to make the first-period model similar to that of Matutes and Régibeau (1988). We can allow for independence or positive correlation in the learning costs (see Section 8.1). Hence, a consumer located at \((\theta_x, \theta_y)\) \(\in [0, 1]^2\) incurs a total learning cost of \(t \theta_x + t \theta_y\) to use system \((A, A)\); her learning cost is \(t \theta_x + t (1 - \theta_y)\) for system \((A, B)\). The locations \(\theta_x\) and \(\theta_y\) are i.i.d. over \([0, 1]\), each according to a density \(g_1\) that is logconcave and symmetric around \(\frac{1}{2}\), which implies that \(g_1\) is increasing over \([0, 1/2]\); \(G_1\) is the c.d.f. of \(g_1\).

We consider experience goods as Villas-Boas (2006) and Doganoglu (2010) do. At the beginning of period one, every consumer has the same expected valuation \(2v^e\) for each system. Therefore, depending on the compatibility regime, the first-period utility of a consumer located at \((\theta_x, \theta_y)\) from purchasing \((A, A)\) is given as follows. Under compatibility, it is \(U_1(A, A) = 2v^e - p_{1,x}^A - p_{1,y}^A - t \theta_x - t \theta_y\); under incompatibility, it is \(U_1(A, A) = 2v^e - P_1^A - t \theta_x - t \theta_y\), where \(p_{1,j}^i\) is the price for product \(j\) of firm \(i\) in period one under compatibility and \(P_1^i\) is the price of system \((i, i)\) in period one under incompatibility.\(^{24}\) We assume that \(v^e\) is large enough to make the market fully covered. If there were no second period and \(G_1\) is the uniform distribution, then our model would be identical to that of Matutes and Régibeau (1988).

After a consumer uses product \(j\) (= \(x, y\)) of firm \(i\) in period one, she discovers her own valuation \(v_j^i\) for the product, which she obtains only if the product is consumed together with a

\(^{23}\)This i.i.d. assumption is made to make the first-period of our model similar to the model of Matutes and Régibeau (1988). In reality, positive correlation between \(\theta_x\) and \(\theta_y\) is more likely. But positive correlation weakens the force that makes compatibility soften the first-period competition such that in the case of perfect correlation, the firms are indifferent between compatibility and incompatibility in a static setting. This in turn induces them to choose incompatibility more often in our dynamic setting.

\(^{24}\)For prices, we use lower case letters under compatibility and upper case letters under incompatibility.
compatible product $k$ ($k \neq j$ and $k = x, y$). $v^j_i$ a random draw from a distribution with support $[v, v]$, in which $v > 0$, and a density $g_2$ which is logconcave and symmetric around $v^e = (v + v) / 2$, that is $g_2(v) = g_2(2v^e - v)$ for each $v \in [v, v]$. We assume that the distribution of the valuation is independent across different products and different consumers.

Now we describe what happens in the second period. Consider a consumer who bought product $x$ from $A$ in period one. Then she has learnt her valuation $v^A_x$. Under compatibility, her choice in period two is either to consume the same product and obtain $v^A_x$, or to switch to product $x$ of $B$. In the latter case her gross surplus is $v^e$ minus the switching cost $s > 0$. No correlation between $v^A_x$ and the expected valuation from switching to product $x$ of $B$ is assumed for simplicity but we can allow for positive correlation between the two (see Section 8.1). We assume that each firm can engage in behaviour-based price discrimination to poach consumers: the price a firm charges to a consumer in period two can depend on the product she purchased in period one.

Regarding the switching cost per product $s$, for simplicity we assume that the switching cost is the same for all consumers and products.\(^{25}\) $s$ includes psychological and transactional cost of switching (Farrell and Klemperer, 2007). It also includes the cost of learning to use a different product in the second period, which can be much smaller than the one in the first period as she already learned to use a competing product.\(^{26}\) $s$ also includes the cost of moving data, which can be important for large amount of data. In the context of data-based services, $s$ captures the reduction in the quality of the service offered by the firm to which a consumer switches because the firm has no access to her data generated while she was using the rival’s product in period one. Therefore, data portability can reduce the switching cost (see Section 7.3 for more detailed discussions).

Suppose that the products are compatible and that a consumer who bought $(A, A)$ in the first period switches to $(A, B)$ in the second period. Then, her second-period utility is given by:

$$U_2(A, B) | (A, A) = v^A_x + v^e - p^A_{2,x}(A) - p^B_{2,y}(A) - s,$$

where $p^i_{2,j}(h)$ is the second-period price charged by firm $i$ for product $j$ under compatibility to the consumers who bought product $j$ of firm $h$ in the first period, with $i, h \in \{A, B\}$.\(^{27}\) Suppose

\(^{25}\)In fact, we can allow for a heterogenous switching. More precisely, we can allow for $s$ to be a random variable with a mean $s^e$ provided that the distribution of $v^e + s$ has a density that is logconcave and symmetric around the mean $v^e + s^e$. See the paragraph after (1) for more details.

\(^{26}\)In other words, given $j (= x, y)$, the cost of learning to use both product $j$ of $A$ and product $j$ of $B$ can be much lower than the sum of the cost of learning to use product $j$ of $A$ only and the cost of learning to use product $j$ of $B$ only because of synergy in learning.

\(^{27}\)We may allow $p^i_{2,j}(h)$ to depend not only on the firm $h$ from which the consumer has bought product $j$ in period one, but also on the firm from which the consumer has bought the other product in period one. Since each
now that the products are incompatible and that a consumer who bought \((A, A)\) in the first period switches to \((B, B)\) in the second period. Then her second-period utility is given by:

\[
U_2(B, B)_{|(A, A)} = 2v^e - P_2^B(A, A) - 2s,
\]

where \(P_2^i(h, h)\) is the second-period price charged by firm \(i\) for its system under incompatibility to the consumers who bought \((h, h)\) in the first period with \(i, h \in \{A, B\}\). Our model is similar to Beggs and Klemperer (1992), Klemperer (1995) and Doganoglu (2010) in that the Hotelling differentiation is assumed only in the first period.

All players have a common discount factor \(\delta > 0\); \(\delta\) can be larger than one since it represents the weight assigned to the second-period payoff. All firms have rational expectations. Whether consumers are myopic or forward-looking does not matter in our model. So we consider myopic consumers for our exposition. In fact, the principle from Farrell and Klemperer (2007) that competition between non-myopic firms makes buyer myopia irrelevant applies to our model with symmetric firms.\(^{28}\)

We introduce the following assumption to guarantee that a positive measure of consumers switch in each sub-market in period two under compatibility:

**Assumption 1**: \(g_2(v)(s - \frac{\Delta v}{2}) < 1\), where \(\Delta v \equiv \tau - v > 0\).

If this assumption is not satisfied, then no switching occurs in period two when the products are compatible. Under Assumption 1, a positive measure of consumers switch regardless of the compatibility regime. Notice that if \(g_2(\tau) = 0\), then Assumption 1 holds for any \(s > 0\). If \(g_2(\tau) > 0\), then Assumption 1 is equivalent to imposing on \(s\) an upper bound \(\frac{\Delta v}{2} + \frac{1}{g_2(\tau)}\).\(^{29}\)

The timing in period one is given by:

- **Stage 1**: Each firm simultaneously and non-cooperatively chooses between compatibility and incompatibility.
- **Stage 2**: After observing the compatibility regime determined in Stage 1, each firm simultaneously and non-cooperatively chooses its price(s).
- **Stage 3**: Consumers make purchase decisions.

consumer’s utility function is separable in the utility of the two products, such additional generality is irrelevant.\(^{28}\) The proof of this result is provided in the TSE working paper (Jeon, Menicucci and Nasr, 2020).\(^{29}\) Assumption 1 is made only to avoid dealing with multiple cases. But our main result, Proposition 3, still holds even if \(s\) is such that Assumption 1 is violated. See footnote 32.
In period two, only Stages 2 and 3 occur. Notice that there always exists an equilibrium in which both firms choose incompatibility. We assume that each firm plays its weakly dominant action and therefore compatibility arises if and only if both firms prefer compatibility.

In order to solve this two-period model, we first solve for the firms’ second-period equilibrium behavior for any given first-period compatibility choice and market shares. We find that equilibrium prices and profits are linearly homogenous in $\Delta v$, therefore sometimes it is useful to normalize $\Delta v$ to 1, as the model with $(\Delta v, s)$ is qualitatively equivalent to the one with $(1, s/\Delta v)$.

3 Second-period competition given a compatibility regime

In this section, we first study the second-period competition to poach consumers for a given compatibility regime. Second, we show how incompatibility affects the second-period profits relative to compatibility, which is important to understand the compatibility choices in period one.

3.1 Given compatibility

Suppose that the products are compatible and consider the market for product $j$ composed of the consumers who bought this product from firm $i$ in the first period. We call it market $i_j$. As all four markets $A_x, A_y, B_x, B_y$ are alike, it is enough to analyze just one of them. We normalize the total mass of consumers in market $i_j$ to one.

Consider market $i_j$. Because of the switching cost, firm $i$ has an advantage over firm $h (\neq i)$ and we call firm $i$ the "dominant" firm and firm $h$ the "dominated" firm. Let $p^+_2$ (instead of $p_{2,i}^h(i)$) denote the price charged by the dominant firm and $p^-_2$ (instead of $p_{2,j}^h(i)$) the price charged by the dominated firm. Likewise, let $d^+_2$ denote the demand for product $j$ of the dominant firm and $\pi^+_2 = p^+_2 d^+_2$ its profit; $d^-_2$ and $\pi^-_2 = p^-_2 d^-_2$ are similarly defined.

A consumer with valuation $v^i_j$ for product $j$ of firm $i$ is indifferent between buying again that product and switching to product $j$ of firm $h$ if and only if

$$v^i_j - p^+_2 = v^e - p^-_2 - s.$$  (1)

From (1) we obtain $d^+_2 = 1 - G_2(v^e - s + p^+_2 - p^-_2)$, or equivalently $d^+_2 = G_2(v^e + s - p^+_2 + p^-_2)$, since the symmetry of $g_2$ around $v^e$ implies $G_2(v) + G_2(2v^e - v) = 1$ for each $v \in [v, \bar{v}]$.

Note that we can allow for $s$ to be a random variable with a mean $s^e$ provided that the distribution of $v^i_j + s$ has a density that is logconcave and symmetric around the mean $v^e + s^e$. A sufficient condition for this to occur is that the distribution of $s$ has a density that is logconcave.
and symmetric around the mean $s^e$. Then $s^e$ in the model with heterogeneous $s$ plays the role of $s$ in the baseline model with homogenous $s$.

Now define $F(x)$ as follows:

$$F(x) \equiv G_s(v + \Delta vx). \quad (2)$$

Notice that $F$ is a c.d.f. with support $[0,1]$; basically, $F$ is the c.d.f. of a normalized valuation $(v^j_j - x_i) / \Delta v$. We show that the competition game we are considering can be equivalently seen as an Hotelling duopoly in which (i) the dominant (dominated) firm is located at $x = 0$ (at $x = 1$), (ii) the dominant firm offers a product of which the quality is superior by $s$ to that of the dominated firm, (iii) the transportation cost $t$ is equal to $\frac{1}{2} \Delta v$ and (iv) consumers are distributed over the Hotelling line according to $F$. Given the prices $p^+_2$ and $p^-_2$, in this Hotelling duopoly, the indifferent consumer is located at $\frac{1}{2} + \frac{s - p^+_2 + p^-_2}{\Delta v}$ and the dominant firm’s demand is given by $F\left(\frac{1}{2} + \frac{s - p^+_2 + p^-_2}{\Delta v}\right)$. Because of (2), this coincides with $G_s(v^e + s - p^+_2 + p^-_2)$, which is $d_2^e$ derived from (1).

This equivalence allows us to apply Proposition 1 in Hurkens, Jeon, Menicucci (2019) (HJM from now on), which proves that for each $s \geq 0$ that satisfies Assumption 1, there exists a unique solution $x^*(s)$ to the equation $x = \frac{1}{2} + \frac{s}{\Delta v} + \frac{1 - 2f(x)}{f(x)}$ in the interval $(\frac{1}{2}, 1)$ where $f$ is the density of $F$. The solution $x^*(s)$ is the location of the indifferent consumer in equilibrium and the equilibrium prices and profits can be expressed in terms of $x^*(s)$ as in the next lemma.\(^{30}\)

**Lemma 1.** (corollary of HJM (2019)) Suppose that the products are compatible. Consider the second-period competition in market $i_j$ composed of the consumers who bought product $j$ from firm $i$ in period one. We normalize the total mass of consumers in market $i_j$ to one. There exists a unique equilibrium. Under Assumption 1, the equilibrium prices and profits are given by:

$$p^{+*}_2 = \Delta v \frac{F(x^*(s))}{f(x^*(s))}, \quad p^{-*}_2 = \Delta v \frac{1 - F(x^*(s))}{f(x^*(s))};$$

$$\pi^{+*}_2 = \Delta v \frac{F(x^*(s))^2}{f(x^*(s))}, \quad \pi^{-*}_2 = \Delta v \frac{(1 - F(x^*(s)))^2}{f(x^*(s))},$$

where $F$ is defined in (2) and $x^*(s)$ is the unique solution to $x = \frac{1}{2} + \frac{s}{\Delta v} + \frac{1 - 2f(x)}{f(x)}$ in $(\frac{1}{2}, 1)$.

For the analysis of data portability in Section 7, it is useful to note that an increase in $s$ reduces consumer surplus and increases the joint profit $\pi^{+*}_2 + \pi^{-*}_2$. As $s$ increases, the dominant

\(^{30}\)Precisely, Proposition 1 in HJM (2019) requires that the density $f$ of $F$ is symmetric around $\frac{1}{2}$ and logconcave. Indeed, $f$ is symmetric around $\frac{1}{2}$ since $f(x) = \Delta v g_2(x + \Delta vx)$ and if $x_1$ and $x_2$ are in $[0,1]$ such that $x_1 + x_2 = 1$, then $f(x_1) = \Delta v g_2(x + \Delta vx_1)$ is equal to $f(x_2) = \Delta v g_2(x + \Delta vx_2)$ because $x + \Delta vx = 2v^e - g^e + \Delta vx_1$ and $g_2$ is symmetric around $v^e$. Moreover, $f$ is logconcave as $f'(x) = (\Delta v)^2 g_2'(x + \Delta vx)$, hence $f'(x) = \frac{\Delta v g_2'(x + \Delta vx)}{g_2'(x + \Delta vx)}$ is decreasing with respect to $x$ since $g_2$ is logconcave.
firm has more market power and raises its price, which softens competition such that the sum of 
s and the dominated firm’s price increases. Hence, both a consumer’s payoff upon no switching 
and the one upon switching decrease with s whereas the competition-softening effect raises the 
joint profit.

Corollary 1. In market \( i_j \), as \( s \) increases, each consumer’s payoff strictly decreases and the 
joint profit \( \pi_2^+ + \pi_2^- \) strictly increases.

3.2 Given incompatibility

Suppose that the products are incompatible and consider the market \((i, i)\) composed of the 
consumers who purchased both products from firm \( i \) in the first period. In this market, firm \( i \) 
is the dominant firm and firm \( h \) (\( \neq i \)) is the dominated firm. Let \( P_2^+ \) denote the price charged 
by the dominant firm and \( P_2^- \) the price charged by the dominated firm. Likewise, \( D_2^+, D_2^- \) 
and \( \Pi_2^+, \Pi_2^- \) denote the firms’ demands and profits.

A consumer with valuations \((v_x^i, v_y^i)\) is indifferent between buying system \((i, i)\) and system 
\((h, h)\) if and only if

\[
v_x^i + v_y^i - P_2^+ = 2v_e - P_2^- - 2s
\]

or equivalently

\[
\hat{v}^i - \hat{p}_2^+ = v_e - \hat{p}_2^- - s \tag{3}
\]
in which \( \hat{v}^i = (v_x^i + v_y^i)/2 \) is the average valuation for the bundle of firm \( i \) and \( \hat{p}_2^+ = P_2^+/2 \) and 
\( \hat{p}_2^- = P_2^-/2 \) are the average prices of the bundles. As (3) is analogous to (1), we can determine 
the equilibrium prices and profits by arguing as under compatibility, after replacing \( F \) with \( \hat{F} \) 
(and \( f \) with \( \hat{f} \)) where \( \hat{F} \) is the c.d.f. for a normalized average valuation \((1/2(v_x^i + v_y^i) - v) / \Delta v, \) 
which is the same as the average of two normalized valuations \(1/2 (v_x^i - v) / \Delta v + 1/2 (v_y^i - v) / \Delta v. \) 

Lemma 2. (corollary of HJM (2019)) Suppose that the products are incompatible. Consider the 
second-period competition in market \((i, i)\) composed of the consumers who bought the system \((i, i)\) 
in period one. We normalize the total mass of consumers in this market to one. There exists a 
unique equilibrium. The equilibrium prices and profits are given by:

\[
P_2^+ = 2\Delta v \frac{\hat{F}(\hat{x}^*(s))}{\hat{f}(\hat{x}^*(s))}, \quad P_2^- = 2\Delta v \frac{1 - \hat{F}(\hat{x}^*(s))}{\hat{f}(\hat{x}^*(s))};
\]

\[
\Pi_2^+ = 2\Delta v \frac{(\hat{x}^*(s))^2}{\hat{f}(\hat{x}^*(s))}, \quad \Pi_2^- = 2\Delta v \frac{(1 - \hat{F}(\hat{x}^*(s)))^2}{\hat{f}(\hat{x}^*(s))}.
\]
where \( \hat{x}^*(s) \) is the unique solution to the equation \( x = \frac{x}{2} + \frac{a}{\Delta v} + \frac{1-2F(x)}{f(x)} \) in \((\frac{1}{2}, 1)\).

The effects of a higher switching cost on consumer surplus and the joint profit are qualitatively the same as under compatibility:

**Corollary 2.** In market \((i, i)\), as \( s \) increases, each consumer’s payoff strictly decreases and the joint profit \( \Pi_i^+ + \Pi_i^- \) strictly increases.

### 3.3 The effects of incompatibility: the demand size effect and the demand elasticity effect

In order to analyze how incompatibility affects the second-period profits relative to compatibility, we here apply the finding of HJM (2019) to our model. They study pure bundling (which is equivalent to incompatibility) in a static model with two multi-product firms, assuming that one firm is dominant as it offers products with higher quality than the dominated rival firm. They decompose the effects of incompatibility into a demand size effect and a demand elasticity effect. We here apply their analysis to the second period of our model by considering the market composed of the consumers who bought both products from firm \( A \) (for instance) in period one. We normalize the mass of the consumers of this market to one.

**Demand size effect.** Consider the equilibrium under compatibility, in which firm \( A \) charges \( p_1^+ \) (firm \( B \) charges \( p_2^+ \)) for each product. Let \( \tilde{v}_j^A \) be the valuation of the consumer who is indifferent between switching and no switching; hence the demand for \( A \)'s product is \( 1 - G_2(\tilde{v}_j^A) \). Since \( s > 0 \), we have that \( \tilde{v}_j^A < v^e \), hence \( 1 - G_2(\tilde{v}_j^A) > 1/2 \). Consider now incompatibility and suppose that each firm offers its system at a price equal to the sum of the prices under compatibility, that is \( P_2^+ = 2p_2^+ \) and \( P_2^- = 2p_2^- \). Then the indifferent consumer has the average valuation equal to \( \tilde{v}_j^A \) and the demand for \( A \)'s system is \( 1 - G_2(\tilde{v}_j^A) \), in which \( G_2 \) is the distribution function for the average valuation. An important property of \( G_2 \) is that \( \hat{G}_2 \) is more peaked around \( v^e \) than \( G_2 \): for each \( \varepsilon \in (0, v^e - y) \), we have \( \int_{v^e - \varepsilon}^{v^e + \varepsilon} g_2(s)ds \leq \int_{v^e - \varepsilon}^{v^e + \varepsilon} \hat{g}_2(s)ds \) where \( \hat{g}_2(\cdot) \) is the density of \( \hat{G}_2 \), which means that the distribution of the average valuation is more concentrated around the mean than the distribution of each individual valuation. This implies \( 1 - \hat{G}_2(\tilde{v}_j^A) > 1 - G_2(\tilde{v}_j^A) \) since \( \tilde{v}_j^A < v^e \). Hence, with unchanged prices, incompatibility increases the demand for the dominant firm \( A \) and decreases the demand for the dominated firm \( B \).

\(^{31}\)Under incompatibility, Assumption 1 is not needed to guarantee that there exists a solution to \( x = \frac{x}{2} + \frac{a}{\Delta v} + \frac{1-2F(x)}{f(x)} \) in \((\frac{1}{2}, 1)\). Precisely, the left hand side of the equation is smaller than the right hand side at \( x = \frac{1}{2} \), and the left hand side is greater than the right hand side if \( x \) is close to 1 since it turns out that \( \check{f}(1) = 0 \), which implies \( \lim_{x \uparrow 1} \left( \frac{x}{2} + \frac{a}{\Delta v} + \frac{1-2F(x)}{f(x)} \right) = -\infty \). By contrast, in the case of compatibility, \( f(1) = \Delta v \hat{g}_2(\bar{v}) \) is not necessarily equal to 0; hence Assumption 1 is needed to guarantee the existence of a solution in \((\frac{1}{2}, 1)\). Otherwise the indifferent consumer is located at \( x = 1 \) and the dominant firm wins over the whole market.
Demand elasticity effect. After the regime change from compatibility to incompatibility, the firms will have incentives to choose prices different from \( P_2^+ = 2p_2^+ \) and \( P_2^- = 2p_2^- \). Whether they want to charge higher or lower prices depends on how incompatibility affects the elasticities of the demands, which in turn depends on the valuation of the indifferent consumer and hence on the level of the switching cost. For low levels of switching cost (that is, when \( \tilde{\nu}_j^A \) is not much smaller than \( \nu^e \)), incompatibility makes the demand more elastic: a given decrease in the average price of a system under incompatibility generates a higher boost in demand than the same decrease in the price of each product under compatibility because the distribution of the average valuation is more-peaked around \( \nu^e \) than the distribution of individual valuations. On the other hand, for high levels of switching cost (that is, when \( \tilde{\nu}_j^A \) is close to \( \nu^e \)), incompatibility makes the demand of the dominant firm less elastic – precisely, \( \hat{g}_2(v)/g_2(v) \) converges to zero as \( v \) tends to \( \nu^e \). This induces the dominant firm to increase its price significantly, which increases the demand for the dominated firm and leads also the dominated firm to increase its price as prices are strategic complements. In summary, incompatibility changes the elasticity of demand such that firms compete more aggressively for low levels of switching costs but less aggressively for high levels of switching costs.

Second-period profit comparison. As a result of these two effects, we find that the set of values of \( s \) which satisfy Assumption 1 can be partitioned into three regions such that (i) when \( s \) is in the first region (which includes values of \( s \) close to 0), incompatibility reduces the profit of each firm; (ii) when \( s \) is in the third region (which includes large values of \( s \), close to the upper bound in Assumption 1 if \( g_2(\overline{v}) > 0 \)), incompatibility increases the profit of each firm; (iii) when \( s \) is in the second region (which includes some intermediate values of \( s \)), incompatibility increases the profit of the dominant firm but reduces the profit of the dominated firm. For the case in which \( G_2 \) is the uniform distribution on \( [\nu, \nu + 1] \), the above regions are actually intervals, Assumption 1 reduces to \( s < s_1^0 = 0.701 \); \( 2\pi_2^+ + 2\pi_2^- \geq \Pi_2^+ + \Pi_2^- \) if and only if \( s \leq 1.187 \).

Corollary 3. Suppose that \( G_2 \) is the uniform distribution over \([\nu, \nu + 1]\). Then, there are three threshold values of switching cost, \( \overline{s}^1, \overline{s}^2, \overline{s}^3 \) with \( \overline{s}^1 < \overline{s}^2 < \overline{s}^3 \), such that

\[
2\pi_2^+ \geq \Pi_2^+ \quad \text{if and only if} \quad s \leq \overline{s}^1 (= 0.701);
2\pi_2^- + 2\pi_2^- \geq \Pi_2^+ + \Pi_2^- \quad \text{if and only if} \quad s \leq \overline{s}^2 (= 0.825);
2\pi_2^- \geq \Pi_2^- \quad \text{if and only if} \quad s \leq \overline{s}^3 (= 1.187).
\]

Although each of \( \pi_2^+, \pi_2^-, \Pi_2^+, \Pi_2^- \) depends on \( s \), in what follows we do not highlight this dependence unless it is necessary. Figure 1 shows \( 2\pi_2^+, 2\pi_2^-, \Pi_2^+, \Pi_2^- \) for the case considered in Corollary 3, as a function of \( s \in (0, 3/2) \). There are three thresholds such that
for $s > 0.701$ we have $\Pi_2^+ > 2\pi_2^+$, for $s > 0.825$ we have $\Pi_2^+ + \Pi_2^- > 2\pi_2^+ + 2\pi_2^-$, and for $s > 1.187$ we have $\Pi_2^- > 2\pi_2^-$. 

We can explain this result by using the two effects introduced above. For low switching costs (i.e., $s$ close to zero), the demand size effect is negligible relative to the demand elasticity effect and the latter makes incompatibility intensify competition compared to compatibility. Therefore, incompatibility reduces both firms’ profits. In particular, when $s = 0$, there is only the demand elasticity effect, which explains the result of Matutes and Régibeau (1988) that incompatibility intensifies competition for symmetric firms. For high switching costs (i.e., $s$ close to $3/2$), the demand size effect is again negligible relative to the demand elasticity effect, but now the latter makes incompatibility soften competition compared to compatibility. Therefore, incompatibility increases both firms’ profits. For intermediate level of switching costs, the demand elasticity effect might be neutral but the demand size effect is positive for firm $A$ and negative for firm $B$. Therefore, incompatibility increases $A$’s profit but reduces $B$’s profit.
4 First-period competition given a compatibility regime

We here study the competition in period one given a compatibility regime.

4.1 Given compatibility

Suppose that compatibility was chosen at the beginning of the first period. Given a vector of first-period prices \( (p_{1,x}^A, p_{1,y}^A, p_{1,x}^B, p_{1,y}^B) \), firm \( i \)'s total profit is given as follows:

\[
\pi^i = d_{1,x}^i (p_{1,x}^i + \delta \pi_2^+) + (1-d_{1,x}^i)(\delta \pi_2^-) + d_{1,y}^i (p_{1,y}^i + \delta \pi_2^+) + (1-d_{1,y}^i)(\delta \pi_2^-), \quad \text{for } i = A, B, \tag{4}
\]

where \( d_{1,j}^i \) is the demand for product \( j \) of firm \( i \) in period one (for \( j = x, y \)). Using the indifference condition \( p_{1,j}^A + t\theta_j = p_{1,j}^B + t(1-\theta_j) \), and recalling that \( \theta_j \) is distributed according to a c.d.f. \( G_1 \), we obtain \( d_{1,j}^A \) as follows:

\[
d_{1,j}^A = G_1 \left( \frac{1}{2} + \frac{1}{2t} (p_{1,j}^B - p_{1,j}^A) \right). \tag{5}
\]

We have \( d_{1,j}^B = 1 - d_{1,j}^A \).

Using (5), we derive the equilibrium prices.

**Proposition 1.** Under compatibility, there exists a unique equilibrium, in which the first period equilibrium prices and each firm's total equilibrium profit (per product) are given as follows:

\[
p_{1,j}^* = \frac{t}{g_1(\frac{1}{2})} - \delta (\pi_2^+ - \pi_2^-) \equiv p_1^*, \quad \text{for } i = A, B \text{ and } j = x, y. \tag{6}
\]

\[
\pi^i = \frac{t}{2g_1(\frac{1}{2})} + \delta \pi_2^- \equiv \pi^*, \quad \text{for } i = A, B. \tag{7}
\]

The pricing in (6) is quite intuitive. For \( \delta = 0 \), each firm charges a price per product equal to \( t/g_1(1/2) \) as in a Hotelling model with a symmetric c.d.f. \( G_1(\cdot) \). For \( \delta > 0 \), if firm \( i \) attracts a consumer from the rival in the first period, its expected profit from the customer in the second period is \( \pi_2^+ \). But if the customer stays with the rival, then firm \( i \)'s expected profit from that customer in the second period is \( \pi_2^- \). Therefore, to attract a consumer, each firm is ready to pay the rent from a locked-in consumer \( \delta (\pi_2^+ - \pi_2^-) \), which is dissipated away. Hence, each firm obtains a profit of \( t/[2g_1(1/2)] + \delta \pi_2^- \) as described in (7). What is interesting is that even if there is perfect competition in period one (i.e., \( t = 0 \)), each firm realizes a positive profit.
4.2 Given incompatibility

Suppose now that incompatibility was chosen at the beginning of the first period. Given the period one prices \( (P^A_1, P^B_1) \), firm \( i \)'s total profit is given as follows:

\[
\Pi^i = D^i_1(P^i_1 + \delta \Pi^*_2) + (1 - D^i_1)(\delta \Pi^-_2), \quad \text{for } i = A, B
\]

(8)

where \( D^i_1 \) is firm \( i \)'s market share in period one. The indifference condition is

\[
P^A_1 + t\theta_x + t\theta_y = P^B_1 + t(1 - \theta_x) + t(1 - \theta_y)
\]

which is equivalent to

\[
\hat{p}^A_1 + t\hat{\theta} = \hat{p}^B_1 + t(1 - \hat{\theta}),
\]

where \( \hat{\theta} = (\theta_x + \theta_y)/2 \) and \( \hat{p}^A_1 \) and \( \hat{p}^B_1 \) are prices per good. Then, the demand for firm \( A \) is

\[
D^A_1 = \hat{G}_1(1/2 + \hat{p}^B_1 - \hat{p}^A_1)
\]

(9)

and \( D^B_1 = 1 - D^A_1 \), where \( \hat{G}_1 \) is the c.d.f. of \( (\theta_x + \theta_y)/2 \), with density \( \hat{g}_1 \).

**Proposition 2.** Under incompatibility, there exists a unique equilibrium, in which the first period equilibrium prices and each firm’s total equilibrium profit are given as follows:

\[
P^i_1^* = \frac{2t}{\hat{g}_1(1/2)} - \delta \left( \Pi^+_2 - \Pi^-_2 \right) \equiv P^*_1, \quad \text{for } i = A, B.
\]

(10)

\[
\Pi^i_2^* = \frac{t}{\hat{g}_1(1/2)} + \delta \Pi^-_2 \equiv \Pi^*_2, \quad \text{for } i = A, B.
\]

(11)

Under incompatibility, if \( \delta = 0 \), each firm charges a price for its system equal to \( 2t/\hat{g}_1(1/2) \), which extends the finding of Matutes and Régibeau (1988): they find that \( \Pi^i_2^* \) is equal to \( t/2 \) when \( G_1 \) is the uniform distribution. For \( \delta > 0 \), if firm \( i \) attracts a consumer from the rival in the first period, its expected profit from the customer in the second period is \( \Pi^+_2 \). But if the customer stays with the rival, then firm \( i \)'s expected profit from him in the second period is \( \Pi^-_2 \). Therefore, each firm is ready to pay the rent from a locked-in consumer, \( \delta \left( \Pi^+_2 - \Pi^-_2 \right) \), to attract a consumer. This rent is dissipated away and hence each firm’s equilibrium profit is \( t/\hat{g}_1(1/2) + \delta \Pi^-_2 \).
5 Compatibility choice

We here study the equilibrium compatibility regime, relying on the firms’ profits from the two compatibility regimes in (7) and (11). For the next proposition, we remind that \( \Pi^* - 2\pi^* \) holds for a large \( s \) (see point (ii) in the paragraph preceding Corollary 3 and the corollary itself) and \( \hat{g}_1(\frac{1}{2}) > g_1(\frac{1}{2}) \) always holds as \( \hat{g} \) is more concentrated around \( \frac{1}{2} \) than \( g \). In the next proposition and corollary, we exceptionally highlight the dependence of \( \Pi^* - 2\pi^* \) and \( \pi^* \) on \( s \), which normally we do not do.

**Proposition 3.** If \( \delta (\Pi^*_2(s) - 2\pi^*_2(s)) > \frac{t}{g_1(\frac{1}{2})} - \frac{t}{\hat{g}_1(\frac{1}{2})} \), then in the unique equilibrium both firms choose incompatibility; this inequality holds if \( s \) and \( \delta \) are sufficiently large. Otherwise, there exists an equilibrium in which both firms choose compatibility and this equilibrium weakly Pareto-dominates the incompatibility equilibrium.

**Corollary 4.** Suppose that \( G_2 \) is the uniform distribution over \([v, v+1]\). Then, \( \delta (\Pi^*_2(s) - 2\pi^*_2(s)) > \frac{t}{g_1(\frac{1}{2})} - \frac{t}{\hat{g}_1(\frac{1}{2})} \) holds if \( s > \tilde{\pi}^3(= 1.187) \) and \( \frac{\delta}{t} \) is sufficiently large.

It is well known from Matutes and Régibeau (1988) that incompatibility intensifies competition between symmetric firms in a one-period game. Precisely, when \( \delta = 0 \), incompatibility reduces each firm’s profit from \( \frac{t}{g_1(\frac{1}{2})} \) to \( \frac{t}{\hat{g}_1(\frac{1}{2})} \). Proposition 3 generalizes this result for \( \delta/t \) small enough and/or \( s \) small. First, as long as \( \delta/t \) is small, the compatibility equilibrium emerges because compatibility softens competition in period one and the second-period profits are relatively unimportant. Second, when \( s \) is small, in period two, the dominated firm’s profit is larger under compatibility than under incompatibility. From (7) and (11), we see that the total profit of each firm is the profit in the static model (which is greater under compatibility) plus \( \delta \) times the second-period profit of the dominated firm (which is greater under compatibility); hence both firms choose compatibility.

However, the finding of Matutes and Régibeau (1988) is reversed if both the switching cost and the weight of the second period are large enough. Precisely, recall – from the paragraph immediately before Corollary 3 – that \( \Pi^*_2 - 2\pi^*_2 \) is positive for large \( s \). Then, even though \( \frac{t}{g_1(\frac{1}{2})} - \frac{t}{\hat{g}_1(\frac{1}{2})} > 0 \), the inequality in Proposition 3 is satisfied if \( \frac{\delta}{t} \) is large. Thus, incompatibility emerges for a large \( s \) and high \( \delta/t \). In this case, both firms choose incompatibility as it softens competition in period two. Even if part of the increased second-period profit is dissipated away, each firm retains \( \delta (\Pi^*_2 - 2\pi^*_2) \) in terms of increased profit, which more than compensates the reduction in the first-period profit if \( \delta/t \) is large.\(^{32}\)

\(^{32}\)Notice that this applies even if Assumption 1 is violated, as then \( \pi^*_2 = 0 \) because the dominant firm wins over all the consumers under compatibility. However, we have \( \Pi^*_2 > 0 \), which implies \( \Pi^*_2 - 2\pi^*_2 > 0 \). Therefore incompatibility is the unique equilibrium if both \( s \) and \( \frac{\delta}{t} \) are large.
In Section 8, we provide various extensions to show that Proposition 3 holds under more general assumptions.

6 Consumer surplus and welfare

In this section and the next section, in order to perform analysis of consumer surplus and welfare, we assume that $G_1$ is the uniform distribution on $[0, 1]$ and $G_2$ is the uniform distribution on $[v, v + 1]$; then Assumption 1 is equivalent to $s < \frac{2}{3}$ and it is possible to write the period two equilibrium prices and profits in closed form.

In this section we compare consumer surplus and social welfare under compatibility with those under incompatibility, and the following result is useful to understand the impact of compatibility choices on consumer surplus.

**Corollary 5.** When $G_2$ is the uniform distribution on $[v, v + 1]$, we have

$$\Pi_2^{+*} - \Pi_2^{-*} > 2\pi_2^{+*} - 2\pi_2^{-*} \text{ for any } s \in (0, \frac{3}{2}). \tag{12}$$

Basically, this corollary states that the rent from a locked-in consumer is larger under incompatibility than under compatibility.

6.1 Consumer surplus

In period one, consumer surplus is greater under incompatibility than under compatibility for any $s$ and $\delta/t$ for two reasons. The first reason is known from Matutes and Régibeau (1988): for symmetric firms, incompatibility intensifies the first-period competition because of the demand elasticity effect of incompatibility (see Section 3.3). Even if incompatibility increases the transportation costs incurred by consumers, this effect is dominated by the effect of more intensive competition. The second reason is that from Corollary 5, the rent from a locked-in consumer is larger under incompatibility than under compatibility, implying that the firms dissipate more rent under incompatibility than under compatibility: see (6) and (10).

The comparison for period two depends on $s$. For instance, consider a consumer who bought both products from firm $A$ in period one. Then, for $s$ large, incompatibility softens competition in period two with respect to compatibility such that we have $P_2^{++*} > 2p_2^{+*}$ and $P_2^{--*} > 2p_2^{-*}$. These inequalities and the fact that incompatibility leaves fewer choice opportunities to consumers imply that the consumer is better off under compatibility than under incompatibility for any possible realization of $(v_x^A, v_y^A)$. Conversely, for $s$ small we have $P_2^{++*} < 2p_2^{+*}$ and $P_2^{--*} < 2p_2^{-*}$,
and we find that the second-period consumer surplus is higher under incompatibility if and only if $s < 0.876$.

Let $CS^C$ ($CS^I$) denote the total consumer surplus under compatibility (incompatibility). The previous arguments seem to suggest that $CS^C > CS^I$ when $s > 0.876$ and $\delta/t$ is large. But we need to take into account the fact that $\delta$ affects also the rent from the locked-in consumers that is transferred to consumers through first-period competition, and that the rent is higher under incompatibility because of (12): an increase in $\delta$ reduces $P_1^*$ more than $2p_1^*$. Thus we find $CS^C > CS^I$ if $s > 0.876$ and $\delta/t$ is large. But we need to take into account the fact that $\delta$ affects also the rent from the locked-in consumers that is transferred to consumers through first-period competition, and that the rent is higher under incompatibility because of (12): an increase in $\delta$ reduces $P_1^*$ more than $2p_1^*$. Thus we find $CS^C > CS^I$ if $s > 0.876$ and $\delta/t$ is above a threshold $\delta CS(s)$ specified in the proof of Proposition 4(iii) in the appendix. From Proposition 3, we know that the firms choose incompatibility when $s > \bar{s}_3 (= 1.187)$ and $\frac{\delta}{t} > \frac{1}{2(\Pi_2-\Pi_2-2\pi)}$, and we find that $\delta CS(s) < \frac{1}{2(\Pi_2-\Pi_2-2\pi)}$ holds for $s > \bar{s}_3$. Therefore, whenever the firms choose incompatibility, $CS^C > CS^I$ holds. Likewise, when the firms choose compatibility, it is very often the case that $CS^I > CS^C$. Next proposition summarizes our results.

**Proposition 4. (consumer surplus)**

(i) Consumer surplus under compatibility is

$$CS^C = 2v^e - 2p_1^* - \frac{1}{2}t + \delta(2v^e - 2p_2^{+*} + \pi^{-*})$$ (13)

(ii) Consumer surplus under incompatibility is

$$CS^I = 2v^e - P_1^* - \frac{2}{3}t + \delta(2v^e - P_2^{+*} + \frac{2}{3}\Pi^{-*})$$ (14)

(iii) The inequality $CS^C > CS^I$ holds if and only if $s > 1.168$ and $\delta/t$ is sufficiently large. Whenever incompatibility arises in equilibrium, $CS^C > CS^I$ holds. If compatibility arises in equilibrium, then $CS^I > CS^C$ for $s < 1.168$.

What is remarkable is the conflict between the compatibility regime chosen by the firms and the regime maximizing consumer surplus. This conflict generally arises except for a small range of parameters because the firms choose (in)compatibility in order to soften competition.

### 6.2 Social welfare

We start by describing the first-best allocation. In period one, the first-best requires a consumer with location $\theta_j$ for good $j$ ($j = x, y$) to buy product $j$ of firm $A$ if and only if $\theta_j \leq \frac{1}{2}$. In period two, the first-best requires a consumer who purchased product $j$ of $A$ (for instance) in period one and observes the value $v_j^A$ to keep buying product $j$ from $A$ if $v_j^A \geq v^e - s$, but to switch to $B$ if $v_j^A < v^e - s$. In particular, no switching occurs in the first-best if $s \geq \frac{1}{2}$.
Under compatibility, the first-period allocation coincides with the first-best one, but some inefficiency emerges in the second period since \( p_2^{+*} > p_2^{-*} \) implies that excessive switching occurs. However, this efficiency loss is small if \( s \) is close to zero or if \( s \) is close to \( \frac{3}{2} \). In the former case, \( p_2^{+*} \) is close to \( p_2^{-*} \) hence the switching is only slightly excessive. In the latter case, the proportion of switching consumers (the demand of the dominated firm) tends to 0 as \( s \) tends to \( \frac{3}{2} \).

Incompatibility generates efficiency loss in each period. In period one, consumers cannot mix and match, which increases the learning costs they incur. In period two, likewise, incompatibility forces consumers to make switching decisions only at a system level. As a consequence, we find that total social welfare is higher under compatibility than under incompatibility when \( s \) is close to 0 and when \( s \) is large.

However, we find that for some intermediate values of \( s \) (i.e., for \( s \in (0, 1.535) \)), the second-period social welfare is higher under incompatibility. This occurs because for these values of \( s \), social welfare is maximal if there is no switching, and the market share of the dominant firm is significantly larger under incompatibility than under compatibility because of the demand size effect explained in Section 3.3. As a consequence, incompatibility generates a higher total welfare for \( s \in (0.535, 1.153) \) if \( \delta/t \) is sufficiently large (see the proof of Proposition 5(iii)). From Proposition 3 we know that for \( s \leq \bar{s}^3 \), firms choose compatibility and \( \bar{s}^3 > 1.153 \) holds. Hence it is never the case that incompatibility emerges in equilibrium when it maximizes social welfare. Or equivalently, whenever the firms choose incompatibility, this generates a lower welfare (in addition to a lower consumer surplus) relative to compatibility. When the firms choose compatibility, this choice generates a higher social welfare except for the case in which \( s \in (0.535, 1.153) \) and \( \delta/t \) is large.

**Proposition 5.** (social welfare) (i) Social welfare under compatibility is

\[
SW^C = 2(1 + \delta)v^e - \frac{1}{2}t + \delta(2\pi_2^{+*} + 3\pi_2^{-*} - 2p_2^{+*}).
\]

(ii) Social welfare under incompatibility is

\[
SW^I = 2(1 + \delta)v^e - \frac{2}{3}t + \delta(\Pi_2^{+*} + \frac{5}{3}\Pi_2^{-*} - P_2^{+*}).
\]

(iii) The inequality \( SW^I > SW^C \) holds if and only if \( s \in (0.535, 1.153) \) and \( \frac{\delta}{t} \) is sufficiently large. Hence \( SW^I < SW^C \) holds whenever incompatibility arises in equilibrium.

\[\text{33} \text{We remark that these results do not rely on } G_1 \text{ and } G_2 \text{ being c.d.f. for uniform distributions, but hold generally.}\]
7 Policy remedies

In this section, we consider two different policy remedies: interoperability and data portability. As the effect of each policy remedy depends on whether a non-negative pricing constraint (NPC) binds or not, we start by extending the previous analysis by introducing the constraint.

7.1 Non-negative price constraint (NPC)

As is typical in two-period models with switching costs, the firms compete fiercely in period one to build a customer base, which can be exploited in period two due to the switching cost. This competition may lead to negative prices in period one, which may be impractical due to adverse selection and opportunistic behaviors by consumers (Farrell and Gallini, 1988, Amelio and Jullien, 2012 and Choi and Jeon, 2021a). Therefore in some cases it may be appropriate to consider that firms face an NPC in period one:

\[(NPC) \quad p_{i,j}^1 \geq 0, \quad P_{i,j}^1 \geq 0 \quad \text{for} \quad j = x, y, \quad \text{for} \quad i = A, B. \] (15)

The NPC is likely to be irrelevant in B2B markets, but is likely to matter in B2C markets.

In the case of uniform distribution for both the first-period costs of learning and the second-period valuations, the NPC modifies the results we previously obtained as follows:

Proposition 6. Suppose that the NPC (15) must be satisfied.

(i) Under compatibility, there exists a unique equilibrium, in which the first period equilibrium prices and each firm’s total equilibrium profit (per product) are given as follows:

\[p_{i,j}^* = \max\{t - \delta (\pi_{2,j}^+ - \pi_{2,j}^-), 0\} \equiv p_{1,j}^*, \quad \text{for} \quad i = A, B \quad \text{and} \quad j = x, y.\]

\[\pi^* = \max\{\frac{t}{2} + \delta \pi_{2}^-, \delta (\frac{1}{2} \pi_{2}^+ - \frac{1}{2} \pi_{2}^-)\} \equiv \pi^*, \quad \text{for} \quad i = A, B.\]

(ii) Under incompatibility, there exists a unique equilibrium, in which the first period equilibrium prices and each firm’s total equilibrium profit are given as follows:

\[P_{i,j}^* = \max\{t - \delta (\Pi_{2,j}^+ - \Pi_{2,j}^-), 0\} \equiv P_{1,j}^*, \quad \text{for} \quad i = A, B\]

\[\Pi^* = \max\{\frac{t}{2} + \delta \Pi_{2}^-, \delta (\frac{1}{2} \Pi_{2}^+ - \frac{1}{2} \Pi_{2}^-)\} \equiv \Pi^*, \quad \text{for} \quad i = A, B.\]

\[\text{[34]In particular, Choi and Jeon (2021a) consider a two-period model with switching cost in which tying allows to circumvent the non-negative pricing constraint (see Section 2.B).}\]

\[\text{[35]This is because monitoring a mass of individual consumers is much more costly than monitoring business customers in terms of adverse selection and moral hazard.}\]
(iii) The NPC expands the range of parameter values for which the firms choose incompatibility. Precisely, if \( s > \bar{s}^2 \) and \( \frac{\delta}{t} > \frac{1}{4 \Pi_2^+ (s) + \frac{1}{2} \Pi_2^- (s) - 2 \pi_2^- (s)} \) then in the unique equilibrium both firms choose incompatibility. Otherwise, there exists an equilibrium in which both firms choose compatibility and this equilibrium weakly Pareto-dominates the incompatibility equilibrium.

The basic idea is that in Proposition 1, for a large \( \delta \), the term \( \delta (\pi_2^+ - \pi_2^-) \) in (6) is large and makes \( p_1^\ast \) negative. Then the NPC implies that \( p_1^\ast = 0 \) and each firm’s profit (per product) coincides with its second period profit \( \delta (\pi_2^+ + \pi_2^-)/2 \). Likewise, in Proposition 2, if \( \delta \) is large, the NPC makes \( P_i^\ast \) equal to 0 and each firm’s total profit is equal to the second period profit \( \delta (\Pi_2^+ + \Pi_2^-)/2 \).

In these cases, the binding NPC limits the dissipation of rent from locked-in consumers. As the rent is larger under incompatibility than under compatibility from Corollary 5, the rent dissipation is constrained more under incompatibility than under compatibility. This induces the firms to choose incompatibility more often than when the NPC does not apply. Precisely, since the firms make profits only in period two, they choose incompatibility if and only if \( (\Pi_2^+ + \Pi_2^-)/2 > \pi_2^+ + \pi_2^- \), which is equivalent to \( s > \bar{s}^2 \). By contrast, in the absence of the NPC, a necessary condition for the firms to choose incompatibility is \( \Pi_2^- - 2 \pi_2^- > 0 \) (see Proposition 3), which holds if and only if \( s > \bar{s}^3 \) where \( \bar{s}^3 > \bar{s}^2 \).

Regarding consumer surplus, when the NPC binds under both compatibility regimes, the fact that the binding NPC constrains rent dissipation more under incompatibility than under compatibility makes consumers prefer compatibility more often than when the NPC does not apply. Precisely, \( CS_C > CS_I \) if \( s \geq 0.876 \), or if \( s < 0.876 \) and \( \frac{\delta}{t} \) is sufficiently small whereas, when the NPC does not apply, a necessary condition for \( CS_C > CS_I \) is \( s > 1.168 \). Furthermore, we know that the binding NPC makes the firms choose incompatibility more often. Therefore, the conflict between the compatibility regime chosen by the firms and the one maximizing consumer surplus is preserved.

The NPC has no effect on social welfare given a compatibility regime, as it only affects the distribution of surplus between consumers and firms. However, the NPC affects the comparison between the compatibility regime chosen by the firms and the one maximizing welfare. In fact, we find no clear contrast between the two. For instance, when incompatibility emerges, this is socially optimal if \( s < 1.153 \) and \( \frac{\delta}{t} \) is sufficiently large. When compatibility emerges, this

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36 For simplicity, in our explanation we focused on the case in which the NPC binds for each compatibility regime. But an incompatibility equilibrium can also exist when the NPC binds only under incompatibility. Then, the firms choose incompatibility if and only if \( \delta (\frac{1}{4} \Pi_2^+ + \frac{1}{4} \Pi_2^-) > t + 2 \delta \pi_2^- \), which is equivalent to \( s > \bar{s}^2 \) and \( \frac{\delta}{t} > \frac{1}{4 \Pi_2^+ + \frac{1}{2} \Pi_2^- - 2 \pi_2^-} \), as stated by Proposition 6(iii). For the uniform environment we are considering, the inequality in Proposition 3 boils down to \( \delta (\Pi_2^- - 2 \pi_2^-) > \frac{1}{2} \), which is more restrictive than \( s > \bar{s}^2 \) and \( \frac{\delta}{t} > \frac{1}{4 \Pi_2^+ + \frac{1}{2} \Pi_2^- - 2 \pi_2^-} \).
maximizes social welfare unless $s > 0.535$ and $\frac{\delta}{T}$ is large.

**Proposition 7.** (consumer surplus and welfare under the NPC) Suppose that the NPC (15) must be satisfied and is binding in both compatibility regimes. Then

(i) $CS^C$ is given by (13) with $p_1^* = 0$; $CS^I$ is given by (14) with $p_1^* = 0$.

(ii) The inequality $CS^C > CS^I$ holds if and only if $s \geq 0.876$, or if $s < 0.876$ and $\frac{\delta}{T} < \frac{1}{4H_2^* + 2p_1^* - 6\pi_2^* - 6P_2^*}$. The binding NPC expands the range of parameter values for which the consumers prefer compatibility.

(iii) The compatibility regime chosen by the firms leads to the lower consumer surplus than the other regime for a set of parameters that at least includes all $(s, \frac{\delta}{T})$ such that $s < 0.749$ or $s \geq 0.876$.

(iv) When firms choose incompatibility, this generates the higher social welfare than compatibility if $s \in (0.825, 1.153)$ and $\frac{\delta}{T}$ is large. When firms choose compatibility, this generates the higher social welfare than incompatibility unless $s > 0.535$ and $\frac{\delta}{T}$ is large.

### 7.2 Interoperability

When the NPC does not apply, from Proposition 4 and Proposition 5, it is clear that whenever the firms choose incompatibility, compatibility strictly improves both consumer surplus and welfare. Therefore, banning incompatibility by imposing interoperability obligations improves consumer surplus and welfare. However, the effect of interoperability obligations is less clear-cut when the NPC binds. From Proposition 7(iii) and (iv), such policy does not harm consumers except for a small range of parameters but the welfare effect is very ambiguous.

**Proposition 8.** (i) When the NPC does not apply, banning incompatibility by imposing interoperability obligations strictly improves consumer surplus and welfare when the policy remedy is binding.

(ii) When the NPC binds regardless of the compatibility regimes, banning incompatibility by imposing interoperability obligations strictly improves consumer surplus unless $s \in [0.825, 0.876]$ when the policy remedy is binding.

Our result is consistent with the recommendation of the US House Antitrust Digital Market Report (2020) to foster interoperability among digital platforms. Our result strongly supports interoperability obligations in B2B markets such as the cloud computing market but offers support for the same policy in B2C markets only for high switching costs.
7.3 Data portability

We now study how data portability policy affects consumer surplus, profits and social welfare. Data portability is expected to lower switching costs and thereby to enhance competition among Internet firms.\footnote{Being able to port one’s data directly lowers the cost of moving from one service to another, which in turn causes businesses to compete harder to keep those customers. (the Stigler report, 2019, p.88).} There are two different channels through which data portability lowers switching costs. First, when the sheer volume of data to move is enormous, any protocol which facilitates data portability should lower switching costs. This is relevant to B2B services such as cloud computing. But it can apply to some B2C services since for instance moving a large volume of pictures or emails without any portability protocol can be extremely time-consuming. Second, in the context of B2C services based on big data analytics, data portability can have the effect of lowering switching cost by enabling the firm to which a consumer switches to provide higher quality service. World Economic Forum (2014) distinguishes personal data into three categories: volunteered data, observed data and inferred data.\footnote{Volunteered data refer to data which is intentionally contributed by a user such as name, image, review, post etc. “Observed data” refers to behavioral data obtained automatically from a user’s activity such as location data and web browsing data. “Inferred data” is obtained by transforming in a non-trivial manner volunteered and/or observed data while still related to a specific individual. This includes a shopper’s profiles resulting from clustering algorithms or predictions about a person’s propensity to buy a product. For more details about different categories of data, see the report on data from the Expert Group for the Observatory on the Online Platform Economy (Rodríguez de las Heras Ballell et al., 2021).} Data portability applies to volunteered data and is likely to extend to observed data but not to inferred data (Crémer et al. (2019), p.81). Inferred data matter for the services based on big data analytics and artificial intelligence. As we consider two incumbents with similar market shares, no portability of inferred data is not a concern. In the second-period of our model, each firm already has inferred data from its first-period consumer base and therefore it can use the volunteered and observed data of a switching consumer in order to identify the doppelgängers whose profiles closely match that of the switching consumer and use the inferred data of the identified doppelgängers to provide service based on big data analytics (Stephens-Davidowitz, 2017). Therefore, data portability could significantly lower the reduction in service quality that a switching consumer suffers.

Assuming that data portability reduces switching cost, we first examine the effect of a lower switching cost on the firms’ choice of compatibility regime. When the NPC binds in each regime, compatibility is chosen if and only if \( s < \bar{s}_2 \); hence it is immediate that data portability induces firms to select compatibility more often. When there is no NPC, it is useful to notice that the firms choose incompatibility if and only if \((s, \delta_t)\) lies above the curve \(\delta_t = \frac{1}{2(\Pi_t^{*2} - 2\pi_t^{*2})}\) in Figure 2. Therefore, a reduction in \(s\) makes firms more likely to select compatibility, unless initially \((s, \frac{\delta_t}{2})\) belongs to the small shaded region to the right and below the curve and the reduction in
leads to a point above the curve. The next lemma presents our second result:

**Lemma 3.** (i) Suppose that the NPC does not need to be satisfied. Then, for any given regime of compatibility, each firm’s profit decreases with \( s \) and consumer surplus increases with \( s \).

(ii) Suppose that the NPC binds regardless of the compatibility regime. Then, for any given regime of compatibility, each firm’s profit increases with \( s \) and consumer surplus decreases with \( s \).

The lemma shows that how the switching cost affects profits and consumer surplus completely differs depending on whether or not the NPC binds. Suppose that the constraint binds regardless of the compatibility regime. Then \( s \) affects profits and consumer surplus only in period two. Each firm’s profit is \( \delta \left( \Pi_2^* \right) / 2 \) or \( \delta(\pi_2^* + \pi_2^-) \) depending on the regime, and these profits increase with \( s \) according to Corollary 1 and 2. This is because a higher switching cost relaxes the second-period competition. Conversely, Corollary 1 and 2 establish that consumer surplus decreases with \( s \) under each compatibility regime. Note that these results are not restricted to the case of uniform distribution.

By contrast, when the NPC does not need to be satisfied, because of the full dissipation of the rent from locked-in consumers, each firm’s profit is \( t/2 + \delta \Pi_2^- \) or \( t + \delta \pi_2^- \) depending on the compatibility regime, and each of these profits decreases with \( s \) as the dominated firm’s second-profit profit, \( \Pi_2^- \) or \( \pi_2^- \), decreases with \( s \); this result holds true for general distribution. About consumer surplus, the dissipated rent from locked-in consumers is increasing in \( s \), thus an increase in \( s \) increases period one consumer surplus. Given the uniform distribution in the

\[ 0.44662, 1.5 \]

This can happen in the shaded area since \( \Pi_2^- - 2\pi_2^- \) is decreasing for \( s \) in \( (1.44662, 1.5) \).
second period, this effect dominates the second-period reduction in consumer surplus described above. As a consequence, total consumer surplus increases with $s$.

Therefore, in what follows, we study how data portability affects profits and consumer surplus when it induces a change in compatibility regime. Let us first analyze the effect on profits. Consider first the case in which the NPC does not apply. By Lemma 3(i), data portability increases each firm’s profit for a given compatibility regime. If a regime change occurs, recall that firms choose the regime which generates a higher profit, and the $\max\{t/2 + \delta \Pi_t^{-*}, t + \delta 2\pi_2^{-*}\}$ is a continuous function; thus each firm’s profit increases even if a regime change occurs. Consider now the case in which the NPC always binds. By Lemma 3(ii), firms’ profits decrease if there is no regime change. Then, the previous argument above about the firms’ regime choice and continuity of the maximum function implies that data portability reduces profits even when there is an endogenous regime change.

Finally, we examine the effect of a regime change (induced by data portability) on consumer surplus. Precisely, suppose that $s$ is reduced from $s'(>\bar{s}^2)$ to $s''(<\bar{s}^2)$. We find that the effect on consumer surplus is ambiguous and depends on $(\frac{\delta}{2}, s', s'')$. Here we illustrate the reason for the ambiguity in the case of the binding NPC and relegate a more detailed analysis to the appendix. The key comparison is about $CS^C(\bar{s}^2, \frac{\delta}{2})$ and $CS^I(\bar{s}^2, \frac{\delta}{2})$. From Proposition 7(ii), we find that $CS^C(\bar{s}^2, \frac{\delta}{2}) \geq CS^I(\bar{s}^2, \frac{\delta}{2})$ holds for $\frac{\delta}{2}$ small. Then Lemma 3(ii) implies

$$CS^C(\bar{s}^2, \frac{\delta}{2}) > CS^C(s'', \frac{\delta}{2}) \geq CS^I(s', \frac{\delta}{2}) > CS^I(s'', \frac{\delta}{2}).$$

Hence, data portability increases consumer surplus. By contrast, for $\frac{\delta}{2}$ large, $CS^C(\bar{s}^2, \frac{\delta}{2}) < CS^I(\bar{s}^2, \frac{\delta}{2})$ holds.\(^{40}\) Hence, data portability reduces consumer surplus if $s'$ and $s''$ are both close to $\bar{s}^2$.

Summarizing, we have:

**Proposition 9.** The effects of the data portability policy are as follows.

(i) The policy induces the firms to choose compatibility more often instead of incompatibility, except in the case in which the NPC does not apply and $(s, \frac{\delta}{2})$ lies in the shaded region in Figure 2.

(ii) The policy reduces the firms’ profits if the NPC binds but increases the profits if the constraint does not apply or is slack.

(iii) When the NPC binds, the policy increases consumer surplus if it does not induce any change in compatibility regime; if the regime change from incompatibility to compatibility occurs,\(^{40}\) The level of $\frac{\delta}{2}$ matters because in period one, the possibility to mix and match makes consumer surplus higher under compatibility, but in period two, consumer surplus is higher under incompatibility when $s = \bar{s}^2$ (since $\bar{s}^2 < 0.876$, from Section 6). Hence $CS^C(\bar{s}^2, \frac{\delta}{2}) < CS^I(\bar{s}^2, \frac{\delta}{2})$ holds for large $\frac{\delta}{2}$.\(^{29}\)
the policy increases consumer surplus unless the reduction in switching cost is small and \( \frac{\delta}{T} \) is large. When the NPC does not apply or is slack, the policy decreases consumer surplus except for a set of parameters such that the reduction in \( s \) is small and induces a change in the regime from incompatibility to compatibility.

8 Extensions

In this section we show that our main result, Proposition 3, holds under more general assumptions. For this purpose, we first provide a generalization of our baseline model while keeping the assumption of independence between the first-period valuations and the second-period valuations. Second, we provide an alternative model (which is symmetric to the baseline model), which addresses intertemporal correlation in valuations. Last, we provide an extension of the baseline model to any finite number of periods.

8.1 Generalization of the baseline model

We below generalize the baseline model and show that our main result holds for this generalization. Let \( v^A_{1j}(v^B_{1j}) \) denote a consumer’s first-period valuation for product \( j \) of firm \( A \) (for product \( j \) of firm \( B \)), for \( j = x, y \). In the baseline model, \( v^A_{1j} = v^e - t \theta_j \) and \( v^B_{1j} = v^e - t(1 - \theta_j) \). In period two, consider a consumer who consumed product \( j \) of firm \( A \) in period one and let \( v^A_{2j|A}(v^B_{2j|A}) \) denote her second-period valuation or expected valuation for product \( j \) of firm \( A \) (for product \( j \) of firm \( B \)), for \( j = x, y \). In the baseline model, we have \( v^A_{2j|A} = v^A_j \) and \( v^B_{2j|A} = v^e \).

Consider the case of compatibility. Taking into account the switching cost, the consumer chooses in period two between the competing products by comparing \( v^A_{2j|A} - v^B_{2j|A} + p^A_j(A) - p^B_j(A) - s \). Suppose now that for \( i = A, B \) and \( j = x, y \), the differences \( v^A_{2j|A} - v^B_{2j|A} + p^A_j(A) - p^B_j(A) - s \) are i.i.d. (and independent of the first-period valuations)\(^{41}\), each with support \([-a_2, a_2] \) \((a_2 > 0)\) and density \( h_2 \) that is symmetric around 0 and logconcave; \( H_2 \) is the c.d.f. for \( h_2 \). Then demand for \( A \) is \( 1 - H_2(p^A_{2j}(A) - p^B_{2j}(A) - s) \) and demand for \( B \) is \( H_2(p^A_{2j}(A) - p^B_{2j}(A) - s) \).

For this setting we can prove that the second-period competition is a special case of Hotelling competition in HJM (2019) in which firm \( A \) (firm \( B \)) is located at \( x = 0 \) \((x = 1)\), the quality advantage of firm \( A \) is \( s \), the transportation cost is \( a_2 \), and consumers are distributed over \([0, 1]\) according to \( F(x) = 1 - H_2(a_2 - 2a_2x) \). In this Hotelling duopoly, the indifferent consumer is located at \( \frac{1}{2} + \frac{s - \frac{p^A_{2j}(A) - p^B_{2j}(A)}{2a_2}}{2a_2} \) and the demand for firm \( A \) is \( F(\frac{1}{2} + \frac{s - \frac{p^A_{2j}(A) - p^B_{2j}(A)}{2a_2}}{2a_2}) \), which coincides with \( 1 - H_2(p^A_{2j}(A) - p^B_{2j}(A) - s) \), \( A \)’s demand in the original model. Since \( F(\frac{1}{2} + \frac{s - \frac{p^A_{2j}(A) - p^B_{2j}(A)}{2a_2}}{2a_2}) \)

\(^{41}\)See the next subsection regarding intertemporal correlation of valuations.
is firm A’s demand in HJM, Proposition 1 in HJM applies and establishes that firm A (firm B) earns in equilibrium a profit \( \pi_A^{+*} \) (earns \( \pi_B^{-*} < \pi_A^{+*} \)).

In period one, the consumer chooses between product \( j \) of firm A and product \( j \) of firm B by comparing \( v_{1j}^A - v_{1j}^B \) with \( p_{1j}^A - p_{1j}^B \). Suppose now that for \( j = x, y \), the differences \( v_{1j}^A - v_{1j}^B \) are i.i.d., each with support \([-a_1, a_1]\) \((a_1 > 0)\) and density \( h_1 \) that is symmetric around 0 and logconcave. Then we can determine an equilibrium such that \( p_{1j}^A = p_{1j}^B = \frac{1}{2h_1(0)} - \delta(\pi_A^{+*} - \pi_B^{-*}) \), for \( j = x, y \) and total profit for each firm is \( \frac{1}{2h_1(0)} + 2\delta\pi_A^{-*} \).

Under incompatibility, we can apply the same argument based on \( \hat{H}_2 \), the c.d.f. (with density \( \hat{h}_2 \)) of the average of two independent draws from the c.d.f. \( H_2 \), and \( \hat{H}_1 \), the c.d.f. (with density \( \hat{h}_1 \)) of the average of two independent draws from the c.d.f. \( H_1 \). As a consequence, we determine period-two profits \( \Pi_A^{+*}, \Pi_B^{-*} \) with \( \Pi_A^{+*} > \Pi_B^{-*} \), and the total equilibrium profit for each firm is \( \frac{1}{2h_1(0)} + \delta\Pi_A^{-*} \), with \( \Pi_B^{-*} > 2\pi_A^{-*} \) for a large \( s \).

When we compare the two regimes, we find that \( h_1(0) < \hat{h}_1(0) \), and this implies \( \frac{1}{2h_1(0)} < \frac{1}{2\hat{h}_1(0)} \); thus incompatibility intensifies the first-period competition when \( \delta = 0 \). But \( \Pi_A^{+*} > 2\pi_A^{-*} \) holds for a large \( s \). Therefore if \( \delta \) is large as well, then \( \frac{1}{2h_1(0)} + \delta\Pi_A^{-*} > \frac{1}{2\hat{h}_1(0)} + 2\delta\pi_A^{-*} \) holds and the firms prefer incompatibility.

Notice that in this formulation, it does not matter whether \( v_{1j}^A \) and \( v_{1j}^B \) (respectively, \( v_{2j|i}^{A, A} \) and \( v_{2j|i}^{B, B} \)) are correlated or independently distributed, as long as the conditions specified above are satisfied. For instance, in our model \( v_{1j}^A = v^e - t\theta_j \) and \( v_{1j}^B = v^e - t(1 - \theta_j) \) are perfectly negatively correlated, whereas \( v_{2j|i}^{A, A} = v_{2j|i}^A \) and \( v_{2j|i}^{B, B} = v_{2j|i}^e \) are stochastically independent as \( v_{2j|i}^{B, B} \) is constant. But we would obtain a result analogous to Proposition 3 if \( v_{1j}^A \) and \( v_{1j}^B \) were i.i.d. or positively correlated in such a way to satisfy the above assumption for \( h_1 \) and if \( v_{2j|i}^{A, A} \) and \( v_{2j|i}^{B, B} \) were correlated (in such a way to satisfy the above assumption for \( h_2 \)), for instance with \( v_{2j|i}^{B, B} = \rho v_{2j|i}^{A, A} + (1 - \rho)v^e \) with \( \rho \in (0, 1) \).

### 8.2 Intertemporal correlation

In this subsection, we describe a model with intertemporal correlation between valuations. This model is symmetric to the baseline model in the sense that we make the switching costs heterogenous while assuming homogenous valuations. In the baseline model, we consider heterogenous valuations and homogenous switching costs. We believe that the general case involves both heterogenous valuations and heterogenous switching costs, but for tractability reasons we make one of them homogenous.

Suppose the value from each product is \( v \) and is the same for all consumers, for all products, for all periods. But in the beginning of the second period, each consumer draws a switching cost \( s_j \) for \( j = x, y \). \( s_x \) and \( s_y \) are i.i.d. according to a symmetric logconcave density with c.d.f. \( L \),
over an interval \([s - 1/2, s + 1/2]\) for \(s \geq 1/2\).

Let us start by analyzing period two. Consider compatibility and a consumer who bought good \(j\) from firm \(A\) in period one. This consumer compares \(v - p_{2j}^A(A)\) and \(v - p_{2j}^B(A) - s_j\), and the resulting demand functions are \(1 - L(p_{2j}^A(A) - p_{2j}^B(A))\) for firm \(A\) and \(L(p_{2j}^A(A) - p_{2j}^B(A))\) for firm \(B\). If we define \(H_2\) as \(H_2(k) = L(k + s)\), we see that \(H_2\) is a c.d.f. with support \([-\frac{1}{2}, \frac{1}{2}]\) and the demand for firm \(A\), \(1 - L(p_{2j}^A(A) - p_{2j}^B(A))\), can be written in terms of \(H_2\) as \(1 - H_2(p_{2j}^A(A) - p_{2j}^B(A) - s)\). This shows that the duopoly we are examining can be seen as a special case of the duopoly studied in Subsection 8.1. Thus firm \(A\) earns a profit \(\pi_2^+\), firm \(B\) earns \(\pi_2^-\), and likewise, in each market \(i_j\) the dominant firm earns \(\pi_2^{+*}\), the dominated firm earns \(\pi_2^{-*}\).

A similar analysis applies to the regime of incompatibility, as a consumer who bought goods \(x\) and \(y\) from firm \(i\) in period one compares \(2v - P_i(A, A)\) and \(2v - P_i(A, A) - s_x - s_y\) and the resulting demand function is like the one in the model of Subsection 8.1 with \(H_2\), the c.d.f. of the average of two draws from the c.d.f. \(H_2\). Thus firm \(A\) earns a profit \(\Pi_2^+\) and firm \(B\) earns a profit \(\Pi_2^-\)(< \(\Pi_2^{+*}\)).

In period one, since the products are homogeneous, each consumer buys from the firm with the lower price. Under compatibility, the profit of firm \(i\) from selling product \(j\) to a consumer is \(p_{1j}^i + \delta \pi_2^+\), whereas its profit from not selling product \(j\) to the consumer is \(\delta \pi_2^-\). Therefore, the equilibrium price is \(-\delta (\pi_2^{+*} - \pi_2^{-*})\) (< 0) and the total profit of each firm is \(2\delta \pi_2^+\). Under incompatibility, the profit per consumer of firm \(i\) is \(P_i^i + \delta \Pi_2^{+*}\) if a consumer buys both products from firm \(i\) and is \(\delta \Pi_2^{-}\) if the consumer buys both products from the rival firm. Therefore, the equilibrium price is \(-\delta (\Pi_2^{+*} - \Pi_2^{-*})\) (< 0) and the total profit of each firm is \(\delta \Pi_2^{-}\).

As products are homogenous, when \(\delta = 0\), the first-period profit is zero regardless of the compatibility regime. For this reason, when \(\delta > 0\), \(\delta\) does not affect the profit comparison and the choice of the compatibility regime either. However, for \(s\) sufficiently large, the inequality \(\Pi_2^{+*} > 2\pi_2^{-*}\) holds and both firms choose incompatibility.

Finally, consider extending the model of this subsection by adding a small degree of heterogeneity in valuations: the first period valuations are independently and identically distributed and there is perfect intertemporal correlations. Then, \(\delta\) affects the choice of compatibility regime since if \(\delta = 0\), the firms choose compatibility as this softens the first-period competition. But for \(\delta\) large and \(s\) large, by continuity, the firms prefer incompatibility.

### 8.3 Extension to \(n\) periods

We extend the baseline model of two periods to \(n \geq 3\) periods under an IID assumption: a consumer’s valuation \(v_{t,j}^i\) for product \(j\) of firm \(i\) in period \(t\) is a random draw from a log-concave
symmetric density \( g_2 \) over \([v, \bar{v}]\) with the mean \( v^e = (v + \bar{v})/2 \) and \( v^e_{i,j} \) is independent across \((i, j, t)\). \( G_2 \) is the c.d.f. of \( g_2 \). Although the IID assumption looks restrictive, this is a standard assumption in the literature on switching costs (for instance, see Somani and Einav (2013), Rhodes (2014), Cabral (2016) and Lam (2017)).

At the end of period \( t - 1 \) (\( \geq 1 \)), each consumer discovers her period \( t \) valuations for the products which she consumed in period \( t - 1 \). Moreover, each time a consumer switches from a firm to another, she incurs a switching cost \( s \).\(^{42}\) This has the consequence that the history of a consumer that matters for the competition in period \( t \) is the identity of the firm whose product (or system) the consumer bought in the previous period. The firms and consumers have a common discount factor \( \delta \leq 1 \) which is constant across time. In period one, the setting is the same as the one in the baseline model. Hence, when \( n = 2 \), the current model is identical to the baseline model.

As we have done in Section 3, instead of working with \( G_2 \) (respectively, \( \hat{G}_2 \)), we work with \( F \) introduced in Section 3.1 (respectively, \( \hat{F} \) in Section 3.2). Define \( \pi^- \) as \( \pi^-_2^* \) in Lemma 1 and \( \Pi^- \) as \( \Pi^-_2^* \) in Lemma 2. Then, we have the following result.

**Proposition 10.** Consider the extension of the baseline model to \( n \) periods under the IID assumption.

(i) Under compatibility, there is a unique subgame perfect Nash equilibrium (SPNE) and the equilibrium profit of each firm is equal to

\[
\frac{t}{g_1(\frac{1}{2})} + (\delta + \delta^2 + ... + \delta^{n-1})2\pi^-.
\]

(ii) Under incompatibility, there is a unique SPNE and the equilibrium profit of each firm is equal to

\[
\frac{t}{\hat{g}_1(\frac{1}{2})} + (\delta + \delta^2 + ... + \delta^{n-1})\Pi^-.
\]

(iii) If \((\delta + \delta^2 + ... + \delta^{n-1})(\Pi^- - 2\pi^-) > \frac{t}{g_1(\frac{1}{2})} - \frac{t}{\hat{g}_1(\frac{1}{2})}\) holds, in the unique SPNE both firms choose incompatibility; the inequality holds if \( s \) and \( n \) are large enough and \( \delta \) is close to one.

Proposition 10 extends Proposition 3 to \( n \) periods. As the firms dissipate the rent from locked-in consumers every period from period 1 to period \( n - 1 \), each firm’s per period profit is \( 2\pi^- \) or \( \Pi^- \) from period two on. The firms choose incompatibility when the difference in this stream of profits is larger than the difference in the static profits, which holds if \( s \) is large enough (as that implies \( \Pi^- > 2\pi^- \)) and \( n \) is large enough, \( \delta \) is close to one (that makes \( \delta + \delta^2 + ... + \delta^{n-1} \)

\(^{42}\)Taylor (2003) makes a similar assumption, but supposes that in each period a consumer’s switching cost is the realization of a random variable that is i.i.d. over time.
large). Note that in Proposition 3, we need to allow for $\delta > 1$ in order to make the profit after the first period important enough. But in this extension, we can have $\delta < 1$ but close to one.\footnote{We do not need assume myopic consumers as forward looking consumers buy at each period like myopic consumers because the future expected utility does not depend on which product a consumer buys today.}

9 Conclusion

When moving data across Internet firms is hard, consumer lock-in arises. Then, it is natural for firms to discriminate their offers by making the offer to poach consumers from rivals more attractive than the one to retain locked-in consumers. In those situations, our theory predicts that firms are likely to embrace incompatibility. As the firms choose incompatibility to soften competition, we find a strong conflict between their compatibility choice and the one maximizing consumer surplus such that mandatory interoperability obligations mostly improve consumer surplus. We also find that data portability, by lowering the switching cost, typically induces the firms to embrace compatibility more often. Interestingly, we find that given a compatibility regime, whether data portability increases or reduces consumer surplus (and profits) completely depends on whether or not the non-negative pricing constraint (NPC) binds. If the constraint binds, data portability increases consumer surplus but reduces each firm’s profit, whereas the opposite holds when the constraint does not apply.

Our analysis generates two interesting implications regarding interoperability and data portability policies. First, the impact of each policy on consumer surplus can crucially depend on whether the NPC binds or not. Therefore, policy remedies for B2B markets where the NPC does not apply can be different from the remedies for B2C markets where the NPC binds.\footnote{This message resonates with the message of Choi and Jeon (2021b) obtained in a different context. They study platform design choices in a static two-sided market and find that platform design bias with respect to socially optimal design depends crucially on the existence of a price constraint on the consumer side. They thus conclude that the formulation of optimal antitrust policies towards the platform market can be substantially different for markets where services are provided for free (i.e., the NPC is binding) from those for markets with a positive price (i.e., the NPC is not binding).} Second, interoperability and data portability do not necessarily affect consumer surplus in the same way. In particular, we find that in B2B markets, interoperability improves consumer surplus while data portability tends to reduce it. However, in B2C markets, both policies tend to improve consumer surplus at least when switching costs are high.

Our two-period model is highly stylized and it would be nice to extend our work to more than two periods. We provided an extension to any number of periods under a restrictive IID assumption. It would be interesting to relax this assumption.\footnote{Consider a three-period model in which after using a product (system), a consumer learns its value which remains the same in the next periods. As it is hard to analyze this model, we write only some conjectures. Since the firms dissipate in period one the rent from locked-in consumers, each firm’s profit is equal to the sum of the}
profit when there is only period one and the profit it obtains in period two and three conditional on that all consumers use the rival’s products (system) in period one. So consider the profit that firm A realizes in period two and three from the consumers who bought firm B’s product (system) in period one. In period two, high valuation consumers will keep buying B’s product (system) while low valuation consumers will switch to A’s product (system). Hence, in period three, there are two groups of consumers: those who used the same product (system) in the past (call it the BB group) and those who used both competing products (systems) (call it the BA group). We expect incompatibility to soften competition for the consumers in the BB group when the switching cost is high. Now assume that the switching cost is smaller for the consumers in the BA group than for those in the BB group as the cost of learning is a large part of the switching cost. However, firm A has a double advantage regarding consumers in the BA group: they have not only a (small) switching cost but also low valuations for B’s product (system). If this combined advantage is significant, then incompatibility increases A’s profit. Therefore, incompatibility can increase firm A’s period-three profit from both groups. Hence, if profits from period three are sufficiently important relative to profits from the previous periods, we expect our main result to hold.


European Commission. 2021. Commission Staff Working Document: Strategic Dependencies and Capacities (Accompanying the Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of


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11 Appendix

Proofs of Lemmas 1 and 2

These lemmas are straightforward consequences of Propositions 1 and 2 in HJM (2019).

Proofs of Corollaries 1 and 2

We provide below the proof of Corollary 1. The proof of Corollary 2 is analogous, after replacing $F$ with $\hat{F}$.

We first prove that $\pi_2^{++} + \pi_2^{-s}$ is increasing in $s$. From Lemma 1, it follows that $\pi_2^{++} + \pi_2^{-s} = \Delta v \left( \frac{F^2(x)}{f(x)} + \frac{(1-F(x))^2}{f(x)} \right)$ with $x = x^*(s)$. We notice that $\frac{F^2(x)}{f(x)} + \frac{(1-F(x))^2}{f(x)} = 1 + 2F(x) - 2F(x)^2$ is increasing in the interval $[\frac{1}{2}, 1]$ because the numerator is increasing, the denominator is decreasing (since $f$ is logconcave and symmetric around $\frac{1}{2}$, it is increasing in $[0, \frac{1}{2}]$, decreasing in $[\frac{1}{2}, 1]$). Finally, Proposition 1 in HJM establishes that $x^*(s)$ is increasing in $s$.

We now prove that consumer surplus is decreasing in $s$. We show that as $s$ increases, both $p^{++}$ and $p^{--} + s$ increase, and therefore that each consumer’s utility from each alternative (product) decreases. Precisely, $\frac{F(x)}{f(x)}$ is increasing in $x$ because $f$ logconcave implies that $F$ is logconcave, and $x^*(s)$ is increasing in $s$; thus $p^{++}$ from Lemma 1 is increasing in $s$. About $p^{--} + s$, notice that since $x^*(s)$ is increasing in $s$, fewer consumers choose the dominated product as $s$ increases. This
reveals that such product becomes less convenient with respect to the dominant product, that is
\( p^{−s} + s − p^{+s} \) is increasing. Since \( p^{+s} \) is increasing, it follows that also \( p^{−s} + s \) is increasing.

**Proofs of Propositions 1 and 2**

We provide below the proof of Proposition 1. The proof of Proposition 2 is analogous, after replacing \( G_1 \) with \( \hat{G}_1 \). Under compatibility, the total profit of firm \( A \) from product \( j \) is:

\[
d_{1,j}^A p_{1,j}^A + \delta \left( d_{1,j}^A \pi_2^{+s} + (1 - d_{1,j}^A) \pi_2^{-s} \right)
\]

(16)

where \( d_{1,j}^A \) in (5) is the first-period demand (for \( p_{1,j}^A \in [p_{1,j}^B - t, p_{1,j}^B + t] \)). From (16) we obtain the first order condition

\[
-\frac{1}{2t} g_1(\frac{1}{2}) \left( \frac{p_{1,j}^B - p_{1,j}^A}{2t} \right) (p_{1,j}^A + \delta \pi_2^{+s} - \delta \pi_2^{-s}) + G_1(\frac{1}{2}) \frac{p_{1,j}^B - p_{1,j}^A}{2t} = 0
\]

and imposing \( p_{1,j}^B = p_{1,j}^A \) to look for a symmetric equilibrium yields the first-period equilibrium prices and the profits in Proposition 1(i): see (6), (7).46 This applies as long as (15) does not need to be satisfied, or (15) applies but \( \frac{t}{g_1(\frac{1}{2})} - \delta \left( \pi_2^{+s} - \pi_2^{-s} \right) < 0 \), then we show that the equilibrium price for each firm in period one is zero. In order to see this, consider firm \( A \) and suppose that \( p_{1,j}^A = 0 \). Then \( d_{1,j}^A = 0 \) for each \( p_{1,j}^A \geq t \), and the derivative of firm \( A \)'s profit is \( g_1(\frac{1}{2}) - \frac{p_{1,j}^A}{2t} \left( -\frac{1}{2t} (p_{1,j}^A + \delta \pi_2^{+s} - \delta \pi_2^{-s}) + \frac{G_1(\frac{1}{2}) \frac{p_{1,j}^A}{2t}}{g_1(\frac{1}{2}) \frac{p_{1,j}^A}{2t}} \right) \). This is negative at \( p_{1,j}^A = 0 \) because \( G_1(\frac{1}{2}) = \frac{1}{2} \) and \( \frac{t}{g_1(\frac{1}{2})} - \delta \left( \pi_2^{+s} - \pi_2^{-s} \right) < 0 \), and is negative for \( p_{1,j}^A \in (0, t) \) because the term in the brackets is decreasing with respect to \( p_{1,j}^A \).

**Proof of Proposition 4**

(i) Under compatibility, the second-period consumer surplus in market \( j \) is given by

\[
\int_{v}^{v^e} (v - p_{2}^{−s} - s) dv_j^i + \int_{v}^{v^e} (v - p_{2}^{+s} - s) dv_j^i = v^e - p_{2}^{+s} + \frac{1}{2} \pi_2^{-s}.
\]

The derivative of the profit of firm \( A \) with respect to \( p_{1,j}^A \) can be written as \( g_1(\frac{1}{2}) + \frac{p_{1,j}^B - p_{1,j}^A}{2t} \left( -\frac{1}{2t} (p_{1,j}^A + \delta \pi_2^{+s} - \delta \pi_2^{-s}) + \frac{G_1(\frac{1}{2}) \frac{p_{1,j}^A}{2t}}{g_1(\frac{1}{2}) \frac{p_{1,j}^A}{2t}} \right) \). The logconcavity of \( g_1 \) implies that \( \frac{G_1(\frac{1}{2}) \frac{p_{1,j}^A}{2t}}{g_1(\frac{1}{2}) \frac{p_{1,j}^A}{2t}} \) is decreasing in \( p_{1,j}^A \), hence if the derivative is 0 at \( p_{1,j}^A = p' \), then it is positive for \( p_{1,j}^A < p' \), is negative for \( p_{1,j}^A > p' \). Therefore the first order condition is sufficient to maximize the profit of firm \( A \) with respect to \( p_{1,j}^A \).
The first-period consumer surplus in market \( j \) is \( 2 \int_0^1 (v^e - p_1^t - t\theta_j) d\theta_j = v^e - p_1^t - \frac{1}{3} t \). Hence, the total consumer surplus in market \( j \) is \( v^e - p_1^t - \frac{1}{3} + \delta(v^e - p_2^t + \frac{1}{2} \pi_2^-) \) and the total consumer surplus is given by (13).

(ii) Under incompatibility, the second-period consumer surplus is given by

\[
\begin{align*}
\int_{\Xi} \int_{\Xi} (v^i_x + v^i_y - P_2^{++}) dv^i_x dv^i_y \\
- \int_{\Xi} (2v^e - 2s - P_2^{--} + P_2^{++} - P_2^{+} - v^i_x) (2v^e - P_2^{--} - 2s - (v^i_x + v^i_y - P_2^{++})) dv^i_x dv^i_y \\
= 2v^e - P_2^{++} + \frac{2}{3} \Pi_2^-
\end{align*}
\]

The first-period consumer surplus is \( 2 \int_0^1 \int_0^{1-\theta_x} (2v^e - P_1^t - t\theta_x - t\theta_y) d\theta_y d\theta_x = 2v^e - P_1^t - \frac{2}{3} t \). Hence, the total consumer surplus is given by (14).

(iii) The first period consumer surplus is greater under incompatibility since \( 2v^e - P_1^t - \frac{2}{3} t > 2(v^e - P_1^t - \frac{1}{4} t) \) reduces to \( \frac{5}{2} t + \delta (\Pi_2^{++} - \Pi_2^- - 2\pi_2^- + 2\pi_2^+) > 0 \), which is true because of (12). From Propositions 1 and 2 we insert \( p_1^t = t - \delta(\pi_2^- + \pi_2^+) \) and \( P_1^t = t - \delta(\Pi_2^- + \Pi_2^+) \) into \( CS^C \) and \( CS^I \); we find that \( CS^C > CS^I \) if and only if \( \delta (2\pi_2^{++} - \pi_2^- - 2p_2^{++} + P_2^{++} - \Pi_2^+ + \frac{1}{2} \Pi_2^-) > \frac{5}{2} t \). The left hand side is negative or zero if \( s \leq 1.168 \), thus \( CS^C < CS^I \) in such case. If \( s > 1.168 \), then the left hand side is positive and \( CS^C > CS^I \) if and only if \( \delta > r_{CS}(s) = \frac{12\pi_2^{++} - 8\pi_2^- + 12p_2^{++} + 6\Pi_2^+ - 6\Pi_2^- + 2\Pi_2^-}{2(\Pi_2^- + 2\pi_2^-)} \). Incompatibility arises if and only if \( s > s^3 = 1.187 \) and \( \delta > \frac{1}{2} r_{CS}(s) \), and the latter inequality implies \( \delta > r_{CS}(s) \).

**Proof of Proposition 5**

(i-ii) Social welfare is simply obtained by adding consumer surplus to profits, which we presented in the previous propositions.

(iii) It is immediate to see that \( SW^I - SW^C = -\frac{1}{6} t + \delta(\Pi_2^{++} + \frac{5}{3} \Pi_2^- - P_2^{++} - 2\pi_2^- + 2p_2^{++}) \). It turns out that \( \Pi_2^{++} + \frac{5}{3} \Pi_2^- - P_2^{++} - 2\pi_2^- + 2p_2^{++} \leq 0 \) if \( s \leq 0.535 \) or \( s \geq 1.153 \), hence in this case \( SW^I < SW^C \). If \( s \in (0.535, 1.153) \), then \( \Pi_2^{++} + \frac{3}{5} \Pi_2^- - P_2^{++} - 2\pi_2^- + 3\pi_2^- + 2p_2^{++} > 0 \) and \( SW^I > SW^C \) if \( \delta > \frac{1}{6\Pi_2^{++} + 10\Pi_2^- - 12p_2^{++} - 12\pi_2^- + 12\pi_2^-} \). The firms choose incompatibility only if \( s > s^3 = 1.187 \); hence \( SW^I < SW^C \) when incompatibility emerges.

**Proof of Proposition 6**

(i-ii) The proof of Proposition 6(i) is given in the proof of Proposition 1. The proof of Proposition 6(ii) is analogous, after replacing \( G_1 \) with \( \hat{G}_1 \).
(iii) First notice that under compatibility, the NPC binds if and only if \( \frac{\delta}{t} > \frac{1}{\pi_2^* - \pi_2^*} \); under incompatibility, NPC binds if and only if \( \frac{\delta}{t} > \frac{1}{\Pi_2^* - \Pi_2^*} \). Furthermore, (12) implies \( \frac{1}{\Pi_2^* - \Pi_2^*} < \frac{1}{\pi_2^* - \pi_2^*} \).

If \( s \leq \bar{s}^2 \), then we know from Proposition 3 that each firm prefers compatibility if the NPC does not bind in either regime, that is if \( \frac{\delta}{t} \leq \frac{1}{\Pi_2^* - \Pi_2^*} \). If \( \frac{\delta}{t} \) is between \( \frac{1}{\Pi_2^* - \Pi_2^*} \) and \( \frac{1}{\pi_2^* - \pi_2^*} \), then the NPC binds under incompatibility and each firm’s profit is \( \delta(\frac{1}{2} \Pi_2^* + \frac{1}{2} \Pi_2^*) \), whereas under compatibility each firm’s profit is still \( 2 \delta \pi_2^* \). The inequality \( t + 2 \delta \pi_2^* > \delta(\frac{1}{2} \Pi_2^* + \frac{1}{2} \Pi_2^*) \) is equivalent to \( \frac{1}{2} \Pi_2^* + \frac{1}{2} \Pi_2^* - 2 \pi_2^* \) \( \geq \frac{1}{\pi_2^* - \pi_2^*} \) and is satisfied since \( \frac{\delta}{t} < \frac{1}{\pi_2^* - \pi_2^*} \leq \frac{1}{\Pi_2^* - \Pi_2^*} \) because \( s \leq \bar{s}^2 \). Finally, if \( \frac{\delta}{t} > \frac{1}{\pi_2^* - \pi_2^*} \), then the NPC binds in both regimes and \( \delta(\pi_2^* + \pi_2^*) > \delta(\frac{1}{2} \Pi_2^* + \frac{1}{2} \Pi_2^*) \).

\[ \text{Proof of Proposition 7} \]

(i) The proof is as omitted as it is straightforward.

(ii-iii) After inserting \( P_1^* = 0 \) and \( P_1^* = 0 \) into \( CS^C \) in (13) and \( CS^I \) in (14), we find that \( CS^C > CS^I \) if and only if \( \frac{t}{\delta} > \delta \left( 2 \pi_2^* - \pi_2^* - P_2^* + \frac{2}{3} \Pi_2^* \right) \). The right hand side is negative or zero if \( s \geq 0.876 \), thus \( CS^C > CS^I \) in this case. If \( s < 0.876 \), then the right hand side is positive and \( CS^C > CS^I \) if and only if \( \frac{t}{\delta} > \frac{1}{12 \pi_2^* + 6 \pi_2^* + 4 \Pi_2^*} > \frac{1}{\pi_2^* - \pi_2^*} \). Incompatibility arises if and only if \( s > \bar{s}^2 \) (since we are considering the case in which the NPC binds in both regimes), hence for each \( s > 0.876 \) we have that \( CS^C > CS^I \) but incompatibility emerges. Conversely, if \( s < 0.749 \) then compatibility emerges and \( \frac{\delta}{t} > \frac{1}{\pi_2^* - \pi_2^*} \) (the inequality that implies that the NPC binds in both regimes) implies \( \frac{\delta}{t} > \frac{1}{12 \pi_2^* + 6 \pi_2^* + 4 \Pi_2^*} > \frac{1}{\pi_2^* - \pi_2^*} \), that is \( CS^C < CS^I \).

(iv) From the proof of Proposition 5(iii), we know that \( SW^I < SW^C \) if \( s \leq 0.535 \) or \( s \geq 1.153 \). When \( s \in (0.535, 1.153) \), the inequality \( SW^I > SW^C \) holds if \( \frac{\delta}{t} > \frac{1}{6 \pi_2^* - 2 \pi_2^* + 3 \pi_2^* + 2 \pi_2^*} \). Since the NPC binds, incompatibility emerges if and only if \( s \geq \bar{s}^2 \), and then \( SW^I > SW^C \) if and only if \( s < 1.153 \) and \( \frac{\delta}{t} > \frac{1}{6 \pi_2^* + 4 \Pi_2^* - 2 \pi_2^* + 3 \pi_2^* + 2 \pi_2^*} \). If instead compatibility emerges, this is suboptimal from the point of view of social welfare only if \( s \in (0.535, \bar{s}^2) \) and
\[ \delta > \frac{1}{6(\Pi_2^+ + \frac{1}{4}\Pi_2^- - P_2^+ - 2\pi_2^+ - 3\pi_2^- + 2p_2^-)} \]

**Proof of Lemma 3**

(i) Under compatibility, each firm’s profit is \( t + 2\delta\pi^*_2 \), which is decreasing in \( s \) because \( \pi^*_2 \) is so. Consumer surplus \( CS^C \) is given by Proposition 4(i), and after inserting the equilibrium values from Lemma 1 and Proposition 1 into \( CS^C \), we find an increasing function of \( s \). Under incompatibility, each firm’s profit is \( \frac{1}{2} + \delta\Pi_2^- \), which is decreasing in \( s \) because \( \Pi_2^- \) is so. Consumer surplus \( CS^I \) is given by Proposition 4(ii). After inserting the equilibrium values from Lemma 2 and Proposition 2 into \( CS^I \), we find an increasing function of \( s \).

(ii) Under compatibility, each firm’s profit is \( \delta(\pi^*_2 + \pi^-_2) \), which is increasing in \( s \) by Corollary 1. Consumer surplus is given by Proposition 4(i) with \( p_1^* = 0 \), and is affected by \( s \) only through the second period consumer surplus, which is decreasing by Corollary 1. Under incompatibility, each firm’s profit is \( \delta(\frac{1}{2}\Pi^*_2 + \frac{1}{2}\Pi^-_2) \), which is increasing in \( s \) by Corollary 2. Consumer surplus is given by Proposition 4(ii) with \( P_1^* = 0 \) and is affected by \( s \) only through the second period consumer surplus, which is decreasing by Corollary 2.

**Proof of Proposition 9**

We only need to prove (iii). When the NPC is always binding, the equality \( CS^C(s^2, \delta^2) = CS^I(s^2, \delta^2) \) holds if and only if \( \delta^2 = 4.822 \). Hence, \( CS^C(s^t, \delta^t) > CS^I(s^t, \delta^t) \) if \( \delta^t < 4.822 \). Conversely, \( CS^C(s^2, \delta^t) < CS^I(s^2, \delta^t) \) holds when \( \delta^t > 4.822 \); then \( CS^C(s^t, \delta) < CS^I(s^t, \delta) \) if \( s^t \) is just a bit greater than \( s^2 \) and \( s^t \) is just a bit smaller than \( s^2 \). However, if \( s^t < 0.783 \), the second period consumer surplus under compatibility, given \( s = s'' \), is higher than the one under incompatibility, given \( s = s'' \). Therefore in this case \( CS^C(s'', \delta) > CS^I(s^2, \delta) \) even for \( \delta^t > 4.822 \), given that the first-period consumer surplus is higher under compatibility because of the lower transportation cost it entails. As a consequence, \( CS^C(s'', \delta) \geq CS^I(s^2, \delta) > CS^I(s^t, \delta) \) if \( s'' < 0.783 \).

When the NPC does not apply, if the reduction in \( s \) induces a regime change from compatibility to incompatibility, then the parameters initially belong to the region in which \( CS^I(s, \delta^t) < CS^C(s, \delta^t) \) by Proposition 4(iii). Thus \( CS^I(s'', \delta^t) < CS^I(s', \delta^t) < CS^C(s', \delta^t) \) and the reduction in \( s \) reduces consumer surplus. If instead the reduction in \( s \) induces a regime change from incompatibility to compatibility, then it is possible that \( CS^C(s'', \delta^t) > CS^I(s', \delta^t) \) holds.

\(^{47}\)This is the first of the two reasons mentioned at the beginning of Section 6.1. The second reason does not apply since \( p_1^* = 0 \), \( P_1^* = 0 \).
when \( s'' \) is just slightly smaller than \( s' \) since \( CS^C(s, \frac{\delta}{t}) > CS^I(s, \frac{\delta}{t}) \) when \((s, \frac{\delta}{t})\) is close to the graph of the function \( \frac{1}{2(1-2\pi^2)} \). However, consumer surplus definitely decreases if \( s'' < 1.168 \), since Proposition 4(iii) implies \( CS^C(s'', \frac{\delta}{t}) < CS^I(s'', \frac{\delta}{t}) \) when \( s'' < 1.168 \), and \( CS^I(s'', \frac{\delta}{t}) < CS^I(s', \frac{\delta}{t}) \) holds from Lemma 3(i).

**Proof of Proposition 10**

(i) Under compatibility, we study what happens for a single product, say product \( x \), without loss of generality: since products are symmetric, the results for product \( x \) extend to product \( y \). We call \( A_t \) consumers (\( B_t \) consumers) the consumers who bought product \( x \) from firm \( A \) (from \( B \)) in period \( t \).

Consider period \( n \), which is the last period, and competition in the market of \( A_{n-1} \) consumers; w.l.o.g., we normalize their mass to one. For these consumers, firm \( A \) has an advantage and let \( p^+_n (p^-_n) \) denote the price charged by firm \( A \) (firm \( B \)) to these consumers. Then, a consumer with valuation \( v_{n,x}^A \) is indifferent between buying again the product of firm \( A \) and switching to the product of firm \( B \) if and only if

\[
v_{n,x}^A - p^+_n = v^e - p^-_n - s.
\]

and the demand for \( A \)'s product is \( 1 - G_2(v^e - s + p^+_n - p^-_n) \). As in Section 3.1, we write it as \( F\left(\frac{1}{2} + \frac{s-p^+_n+p^-_n}{\Delta v}\right) \). Then Lemma 1 applies and, under Assumption 1, the equilibrium prices and profits are given by:

\[
p^+_n = \Delta v \frac{F(x^*(s))}{f(x^*(s))}; \quad p^-_n = \Delta v \frac{1 - F(x^*(s))}{f(x^*(s))};
\]

\[
\pi^+_n = \Delta v \frac{F(x^*(s))^2}{f(x^*(s))} \equiv \pi^+; \quad \pi^-_n = \Delta v \frac{(1 - F(x^*(s)))^2}{f(x^*(s))} \equiv \pi^-,
\]

where \( x^*(s) \) is the unique solution to \( x = \frac{1}{2} + \frac{s}{\Delta v} + \frac{1-2F(x)}{f(x)} \) in \((\frac{1}{2}, 1)\). Symmetrically, in the market of \( B_{n-1} \) consumers (with the mass normalized to one), firm \( A \) makes a profit of \( \pi^- \) and firm \( B \) makes a profit of \( \pi^+ \).

Now consider period \( n - 1 \) and the market of \( A_{n-2} \) consumers and normalize their mass to one w.l.o.g. Let \( p^+_{n-1} (p^-_{n-1}) \) denote the price charged by firm \( A \) (firm \( B \)) to these consumers. Then, a consumer with valuation \( v_{n-1,x}^A \) is indifferent between buying again the product of firm \( A \) and switching to the product of firm \( B \) if and only if

\[
v_{n-1,x}^A - p^+_{n-1} = v^e - p^-_{n-1} - s.
\]
The profit of firm A is given by

\[ F\left(\frac{1}{2} + \frac{s - p_{n-1}^+ + p_{n-1}^-}{\Delta v}\right)(p_{n-1}^+ + \delta \pi_{n-1}^+) + (1 - F\left(\frac{1}{2} + \frac{s - p_{n-1}^+ + p_{n-1}^-}{\Delta v}\right)\delta \pi_{n-1}^+), \]

which is equivalent to

\[ F\left(\frac{1}{2} + \frac{s - p_{n-1}^+ + p_{n-1}^-}{\Delta v}\right)(p_{n-1}^+ + \delta \Delta) + \delta \pi_{n-1}^+. \]

in which \( \Delta \equiv \pi^+ - \pi^- \). Likewise, the profit of firm B is given by

\[ (1 - F\left(\frac{1}{2} + \frac{s - p_{n-1}^+ + p_{n-1}^-}{\Delta v}\right))(p_{n-1}^- + \delta \Delta) + \delta \pi_{n-1}^-). \]

The first order conditions are sufficient for profit maximization because of the assumption of log-concavity, and they imply

\[ p_{n-1}^+ = \Delta v\frac{F(x^*(s))}{f(x^*(s))} - \delta \Delta, \quad p_{n-1}^- = \Delta v\frac{1 - F(x^*(s))}{f(x^*(s))} - \delta \Delta. \]

Hence, firm A’s share among \( A_{n-2} \) consumers in period \( n-1 \) is the same as its share among \( A_{n-1} \) consumers in period \( n \).

Therefore, the equilibrium profit of firm A from period \( n-1 \) on is

\[ \pi_{n-1}^+ = \pi^+ + \delta \pi_{n-1}^- = \pi^+ + \delta \pi^- . \]

The equilibrium profit of firm B from period \( n-1 \) on is

\[ \pi_{n-1}^- = \pi^- + \delta \pi_{n-1}^- = \pi^- + \delta \pi^- . \]

Notice that \( \pi_{n-1}^+ - \pi_{n-1}^- \) is equal to \( \Delta \), that is it is equal to \( \pi_{n-1}^+ - \pi_{n-1}^- \).

The previous argument can be extended to any period \( t = 2, ..., n-2 \) and \( A_{t-1} \) consumers. The profit of firm A is given by

\[ F\left(\frac{1}{2} + \frac{s - p_t^+ + p_t^-}{\Delta v}\right)(p_t^+ + \delta \Delta) + \delta \pi_{t+1}^+ , \]

Likewise, the profit of firm B is given by

\[ (1 - F\left(\frac{1}{2} + \frac{s - p_t^+ + p_t^-}{\Delta v}\right))(p_t^- + \delta \Delta) + \delta \pi_{t+1}^- . \]
Therefore, we have \( p_t^+ = p_{n-1}^+ \) and \( p_t^- = p_{n-1}^- \) and firm A’s share among \( A_{t-1} \) consumers in period \( t \) is the same as its share among \( A_{n-1} \) consumers in period \( n \). This implies that the equilibrium profit of firm A (respectively, that of firm B) from period \( t \) on is given as follows:

\[
\begin{align*}
\pi_t^+ &= \pi^+ + \delta \pi_{t+1}^- = \pi^+ + (\delta + \delta^2 + ... + \delta^{n-t}) \pi^- , \\
\pi_t^- &= \pi^- + \delta \pi_{t+1}^+ = \pi^- + (\delta + \delta^2 + ... + \delta^{n-t}) \pi^- .
\end{align*}
\]

Now consider period one. The indifferent consumer’s location \( x \) is determined by

\[
x = \frac{1}{2} + \frac{1}{2t} (p^B_1 - p^A_1).
\]

Firm A’s profit is

\[
G_1(x)(p^A_1 + \delta \Delta) + \delta \pi^-.
\]

Firm B’s profit is

\[
(1 - G_1(x)) (p^B_1 + \delta \Delta) + \delta \pi^-.
\]

From the first-order conditions, it is immediate to obtain a unique solution

\[
p_1^{A*} = p_1^{B*} = -\delta \Delta + \frac{t}{g_1(\frac{1}{2})}.
\]

The equilibrium profit for each firm is equal to

\[
\frac{t}{2g_1(\frac{1}{2})} + \delta \pi^- = \frac{t}{2g_1(\frac{1}{2})} + (\delta + \delta^2 + ... + \delta^{n-1}) \pi^-.
\]

Multiplying the above profit by two gives the total equilibrium profit in Proposition 10(i).

(ii) The analysis of the incompatibility case can be done as we have done above by replacing the c.d.f. and densities \( g_1, G_1, g_2, G_2, f, F \) with \( \hat{g}_1, \hat{G}_1, \hat{g}_2, \hat{G}_2, \hat{f}, \hat{F} \). So we omit it.

(iii) We omit the proof as it is straightforward.