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"Using Consumption Data to Derive Optimal Income and Capital Tax Rates"

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Abstract

We study a Mirrleesian economy with labor income, consumption, and retirement savings or bequests. We derive a novel representation of optimal non-linear income and savings distortions that highlights the role of consumption inequality and consumption responses to tax changes. Our representation establishes a close connection between the formula for top income taxes of Saez (2001) and the uniform commodity taxation theorem of Atkinson and Stiglitz (1976): One cannot be valid without the other, and departures from this joint benchmark lead to a clear trade-off between income and savings taxes. Consumption data in turn discipline the optimal departure from this benchmark. Because consumption is much less concentrated than income, it is optimal to shift a substantial fraction of the top earners' tax burden from income to savings.

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1 Introduction

Optimal Top Income Tax Rate: A Puzzle. Consider a static economy in which workers differ in productivity and choose consumption C and labor income Y given a non-linear income tax schedule. In a milestone contribution to the modern theory of income taxation, Saez (2001) shows that the revenue-maximizing income tax rate on top earners is given by

$$\overline{\tau}_Y^{Saez} = \frac{1}{1 - \bar{\zeta}_Y^I + \bar{\rho}_Y \bar{\zeta}_{Y,\tau_Y}^H}, \tag{1}$$

where $\bar{\rho}_Y > 1$ is the Pareto coefficient of the upper tail of the distribution of taxable income, $\bar{\zeta}_{Y,\tau_Y}^H > 0$ is the Hicksian or compensated elasticity of taxable income with respect to (one minus) the marginal tax rate, and $\bar{\zeta}_Y^I > 0$ is the income effect of a lump-sum tax levy for the highest earners.[1] The appeal of this formula is that it readily lends itself to quantitative evaluations. For example, using empirical estimates of the elasticities $\bar{\zeta}_{Y,\tau_Y}^H = 0.33$ and $\bar{\zeta}_Y^I = 0.25$ and the Pareto coefficient of yearly income $\bar{\rho}_Y = 1.8$, we obtain $\overline{\tau}_Y^{Saez} = 74\%$.

The optimal income tax formula (1) completely abstracts from consumption to focus exclusively on income inequality and labor supply responses to taxes. While this result follows from the objective of maximizing income tax revenue, we find it striking that the marginal costs and benefits of higher income taxes seemingly do not depend on how much redistribution the existing tax system already achieves, that is, on the distribution of consumption rather than pre-tax income. More generally, focusing exclusively on measures of income inequality may paint an incomplete picture of the tradeoff between redistribution and incentives, since in practice agents may insure against labor market risks through other means than the government, such as private insurance, precautionary savings, or intra-family transfers.[2]

The static model studied by Saez (2001) in fact admits an alternative, consumption-based,

[1] Formally, let Y and Y^H denote respectively the uncompensated and compensated labor supplies, expressed in terms of earnings rather than hours. Let also $F_Y(\cdot)$ be the CDF of the income distribution, $T_Y(\cdot)$ the income tax schedule, $\tau_Y \equiv T_Y'(Y)$ the marginal income tax rate, and $\overline{T}_Y \equiv T_Y(Y)/((1-\tau_Y)Y)$ the (normalized) average tax rate. We then define $\bar{\rho}_Y \equiv -\lim \partial \ln(1 - F_Y(Y))/\partial \ln Y$, $\bar{\zeta}_{Y,\tau_Y}^H \equiv \lim \partial \ln Y^H / \partial \ln(1-\tau_Y)$, and $\bar{\zeta}_Y^I \equiv \lim \partial \ln Y / \partial \overline{T}_Y$, where the limits are taken as $Y \to \infty$.

[2] Echoing this observation, equation (1) contrasts with a vast literature that uses consumption data to assess the optimality of risk-sharing arrangements or social insurance design, going back (at least) to Townsend (1994), see, e.g., Ligon (1998) and Kocherlakota and Pistaferri (2009) for applications with asymmetric information or incentive-redistribution tradeoffs of the kind studied in Saez (2001), or the literature on unemployment insurance design following Baily (1978), Gruber (1997), and Chetty (2006a).

representation of revenue-maximizing top income taxes:

$$\overline{\tau}_Y^{Cons} = \frac{1}{\bar{\zeta}_C^I + \bar{\rho}_C \bar{\zeta}_{C,\tau_Y}^H}, \tag{2}$$

where $\bar{\rho}_C > 1$ is the Pareto coefficient of the consumption distribution, $\bar{\zeta}_{C,\tau_Y}^H > 0$ is the highest earners' compensated elasticity of consumption with respect to the retention rate, and $\bar{\zeta}_C^I > 0$ is (minus) the income effect of a lump-sum tax levy on their consumption.[3] We obtain equation (2) from equation (1) and three model-implied identities, $\bar{\rho}_C = \bar{\rho}_Y$, $\bar{\zeta}_{C,\tau_Y}^H = \bar{\zeta}_{Y,\tau_Y}^H$, and $\bar{\zeta}_C^I = 1 - \bar{\zeta}_Y^I$, which all follow from the static budget constraint that equates consumption to after-tax income. In other words, we have two independent data sources—income and consumption—to estimate a single optimal tax relationship. Hence, the role of consumption inequality and consumption responses to taxes is not separately identified from the role of income inequality and labor supply responses. The literature following Saez (2001) has systematically focused on equation (1), but one could equally well use the consumption-based formula (2), or any combination of the two, to determine optimal income taxes, given reliable estimates of the corresponding sufficient statistics.

Using consumption rather than income inequality measures radically changes the inferred optimal top tax rate. Consumption appears to be significantly *more evenly* distributed than income among top earners, with a Pareto coefficient for consumption of $\bar{\rho}_C \approx 3$ (Toda and Walsh 2015; Buda et al. 2022; Gaillard et al. 2023). For the same taxable income elasticities as in the opening paragraph, the optimal top income tax rate falls to 57% if we use consumption inequality rather than income inequality in formula (1), increasing the after-tax income of top earners by 66%.

Unfortunately, the static model provides no guidance about which is the most appropriate measure for estimating income taxes. Moreover, it is inconsistent with the empirical discrepancy between consumption and income inequality. Interpreting the model-implied identities instead as testable over-identifying restrictions, their violation suggests that the static model of Saez (2001) is not well suited to address how consumption or income inequality matter for optimal taxes, or to offer sound, empirically grounded policy prescriptions.

Our Contribution. Motivated by these observations, in this paper we study to what extent consumption inequality and consumption responses to income shocks influence optimal tax design, independently of income inequality and labor supply responses. We consider a Mirrleesian economy in which agents with heterogeneous labor productivities work, consume and save for retirement.[4]

[3] Formally, $\bar{\rho}_C \equiv -\lim \partial \log (1 - F_C(C)) / \partial \log C$, $\bar{\zeta}_{C,\tau_Y}^H \equiv \lim \partial \ln C^H / \partial \ln (1 - \tau_Y)$, and $\bar{\zeta}_C^I \equiv -\lim \partial \ln C / \partial \overline{T}_Y$.

[4] Our baseline model is kept deliberately as simple as possible. In Section 3, we show that our results carry over to much more general settings.

The additional consumption-savings margin allows us to separate consumption from after-tax income, and also introduces capital taxes as a second margin for redistributive taxation. This connects our analysis with another milestone result, the "*zero capital taxation*" (also known as "*uniform commodity taxation*", henceforth UCT) theorem of Atkinson and Stiglitz (1976), which states that it is optimal to leave consumption choices undistorted—and, hence, not to tax savings—if preferences for savings are independent of labor productivity. Savings taxes must thus be rationalized by departures from this benchmark, for example if agents with higher earnings capacity also have a stronger preference for savings relative to consumption (see, e.g., Saez 2002).

We show in Theorem 1 that $\overline{\tau}_Y^{Saez}$ and UCT are two sides of the same coin: the revenue-maximizing top income tax is equal to $\overline{\tau}_Y^{Saez}$ *if and only if* UCT applies and the optimal savings tax is 0, except in the trivial case where savings vanish at the top of the income distribution. Conversely, departures from this joint benchmark identify a trade-off between income and savings taxes: Either it is optimal to tax savings and reduce the income tax below $\overline{\tau}_Y^{Saez}$, or it is optimal to subsidize savings and raise the income tax above $\overline{\tau}_Y^{Saez}$.

Theorem 2 and Corollary 2 then provide novel representations of optimal income and savings taxes in terms of the moments that define the static wedge $\overline{\tau}_Y^{Saez}$ and three *consumption-based* sufficient statistics: (i) the ratio of Pareto coefficients of earnings and consumption $\bar{\rho}_Y/\bar{\rho}_C$, ($ii$) the ratio of compensated elasticities $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ that captures non-homotheticity of preferences and maps into consumption responses to permanent income changes, and (iii) a scaling parameter that is identified by the elasticity of intertemporal substitution (EIS) or consumption responses to wealth tax changes. A simple calibration based on available empirical estimates for these three moments suggests that the optimal policy shifts a significant fraction of the top income earners' tax burden from income to savings taxes, with an optimal savings tax between 12% and 30% and correspondingly a top marginal income tax that drops from $\overline{\tau}_Y^{Saez} = 74\%$ to between 63% and 70%.[5]

What connects the income tax formula (1) to UCT? Saez (2001)'s characterization applies to a static economy with a single labor supply margin. In a dynamic setting, the optimal income tax continues to be equal to $\overline{\tau}_Y^{Saez}$, *if and only if* the design of income taxes can be reduced to a static trade-off between labor supply and after-tax earnings, with no information needed about how the latter is allocated between consumption and savings. But this condition holds if and only if preferences for savings are independent of incentives to work and the conditions for the UCT theorem are satisfied, i.e., iff it is optimal not to tax savings separately from consumption.

[5]These numbers should be interpreted in a life-cycle context: over a 25-year horizon, with annualized returns on savings of 4%, a savings tax of 30% corresponds to a 1.4% annual tax on accumulated wealth, or a 37% capital income tax. If we interpret savings as retirement income or pensions, a wedge of 30% means that top income earners receive a present value of $0.77 in additional pension payments for each additional dollar in social security contributions.

Why is it optimal to shift part of the tax burden from income to savings? If consumption is less concentrated than income, as suggested by estimates of $\bar{\rho}_Y/\bar{\rho}_C$, top earners have a vanishing marginal propensity to consume. This could arise either because savings have a higher income elasticity than consumption for given preferences, i.e., agents have non-homothetic preferences and view savings as a luxury good relative to consumption, or because preferences for savings (relative to current consumption) correlate positively with labor productivity, or a combination of the two. Empirical studies that estimate consumption responses to permanent income changes find pass-through coefficients that are too high to rationalize the gap between consumption and income concentration entirely through non-homothetic preferences. Hence we reject UCT in favor of a shift towards positive savings taxes and lower income taxes.

Consumption data are essential for reaching these conclusions and identifying optimal income and savings taxes. In the scenario where consumption has a thinner tail than income, the highest income earners' labor supply decision is again based on a static tradeoff, but this time between labor supply and savings rather than consumption as in the static model of Saez (2001). Therefore, the static wedge $\overline{\tau}_Y^{Saez}$, and hence income data, continue to matter for optimal policy, but they now determine the optimal *combined* wedge between income and savings. What is more, as the saving share of income converges to one, savings data becomes redundant—and therefore uninformative—relative to income data. As a result, formulas based on the distribution and behavioral responses of *savings*, in the tradition of Saez (2002), are only able to identify the combined wedge and not the two top tax rates separately. Instead, consumption is essential to determine how this combined wedge should be decomposed into an income tax and a savings tax, i.e., how much one should depart from the joint benchmark of Saez (2001) and Atkinson and Stiglitz (1976).

We derive these results from two conditions that optimal income and savings taxes must satisfy. The first is a *no-arbitrage condition* which states that a marginal shift from income to savings taxes (or vice versa) that leaves all agents' utilities unchanged should not raise or lower tax revenues, since it would otherwise lead to a strict Pareto improvement. The no-arbitrage condition shows that the departure from UCT depends on the comparison between the ratio of Pareto coefficients $\bar{\rho}_Y/\bar{\rho}_C$ and the ratio of compensated elasticities $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$. The second optimality condition, called *revenue-spillover condition*, augments the static income tax tradeoff with a fiscal spillover from income taxes to savings tax revenues. This condition implies that the optimal income tax is strictly lower than (resp., equal to or higher than) the static wedge $\overline{\tau}_Y^{Saez}$ if and only if the savings tax is positive (resp., zero or negative). Theorem 1 connects these two optimality conditions and provides a simple test for the departure from Saez (2001) and UCT.

Solving the two optimality conditions, we then derive novel representations of optimal top income and savings taxes in terms of consumption-based sufficient statistics (Theorem 2). These representations generalize the income tax formula (1) to economies with consumption and savings and provide an analogous formula for optimal top savings taxes. Our representation highlights that when agents work, consume, and save, inequality and behavioral elasticities of all three variables matter for optimal tax design, even if they are jointly determined by the optimal allocation and must satisfy over-identifying restrictions analogous to those of the static model.

Finally, we explore several extensions of our baseline model. We generalize the no-arbitrage and revenue spillover conditions, as well as our representation of optimal income and savings taxes and the tradeoff between the two, to multi-good economies with arbitrary planner preferences, a life-cycle economy in which tax perturbations have additional inter-temporal spill-over effects through labor supply to income and savings taxes in other periods, and models with multi-dimensional heterogeneity, in which the optimal tax structure is analogous to that in our baseline setting, except that it is now determined by the co-variation of *average* consumption and savings with income. We finally discuss how our model can accommodate heterogeneous asset returns and dynamic income risks.

Policy Implications. The income tax formula (1) of Saez (2001) and the UCT theorem of Atkinson and Stiglitz (1976) have both been highly influential in shaping tax policies, but they are typically invoked independently from each other: The former features prominently in discussions about income tax design, while debate about savings taxes focuses on the policy relevance of UCT. For example, a highly cited review article by Diamond and Saez (2011) simultaneously makes a case for high top income taxes based on the formula for $\overline{\tau}_Y^{Saez}$ and for positive capital taxes by questioning the assumptions underlying UCT. Theorem 1 shows that policy discussion of income and savings taxes cannot be separated: Both are part of a policy mix that optimally trades off between multiple tax instruments. Hence, the two policy recommendations by Diamond and Saez (2011) are mutually inconsistent. Our characterization of optimal income and savings taxes in Theorem 2 in turn provides empirical guidance on how the policy maker should resolve this tradeoff: Government revenue is maximized by shifting part of the tax burden from income to savings, and consumption data allow us to identify the magnitude of this shift. Hence, we provide empirical support for the second recommendation by Diamond and Saez (2011), while at the same time invalidating their first recommendation.

Related Literature. Our paper contributes to the optimal taxation literature originating with Mirrlees (1971), as well as the sufficient statistics approach towards estimating optimal tax rates that was pioneered by Saez (2001). Our model is based on Atkinson and Stiglitz (1976). Because we allow for arbitrary preferences, their uniform commodity taxation theorem only applies as a special case of our framework.[6] Mirrlees (1976), Saez (2002), and Golosov, Troshkin, Tsyvinski, and Weinzierl (2013) study a similar problem to ours but do not characterize the optimal top tax rates analytically nor express the formulas in terms of empirically observable sufficient statistics.[7]

Scheuer and Slemrod (2021), Gerritsen et al. (2020), Schulz (2021), Ferey, Lockwood, and Taubinsky (2024), and Lefebvre, Lehmann, and Sicsic (2025) characterize or estimate optimal savings taxes using savings-based sufficient statistics.[8] Our contribution here is twofold. First, by combining the revenue-spillover and no-arbitrage conditions for the design of income and savings taxes, we identify the joint benchmark that consists of Saez (2001) for the former and Atkinson and Stiglitz (1976) for the latter, highlight a tradeoff between these instruments away from this benchmark, and most importantly, offer a unified representation of optimal top income and savings tax rates, in a model encompassing these previous studies.[9] Second, and most importantly, we emphasize the role of consumption—rather than savings—data to characterize optimal income and capital taxes. Consumption data are essential for identifying optimal taxes on *top* income earners, who are the main focus of our analysis. In the empirically relevant scenario where consumption is less concentrated than income, the highest income earners' savings co-move one-for-one with their savings, which renders their savings data redundant given the information already contained in their income—thus making savings-based formulae uninformative about the optimal tax rates on top

[6] Christiansen (1984), Jacobs and Boadway (2014), and Gauthier and Henriet (2018) generalize Atkinson and Stiglitz (1976) to non-homothetic preferences, but typically constrain commodity or savings taxes to being linear.

[7] Our work also relates to the literature that uses consumption data to assess optimality of risk-sharing or social insurance design. The first generation of papers (see footnote 2) tended to use consumption to estimate marginal social welfare weights or costs of consumption fluctuations. In this context, Fadlon and Nielsen (2019) and Landais and Spinnewijn (2021) also offer original attempts to infer consumption-smoothing benefits of social insurance from consumption and labor market data. We instead rely on consumption data to minimize the incentive costs of tax distortions, as shown by our no-arbitrage condition. In this regard, we are conceptually closer to Chetty (2008) who uses consumption to isolate incentive costs of UI from liquidity effects, or Hendren (2017), who infers private information about unemployment risks from consumption.

[8] The observation that revenue spillovers affect optimal income tax formulas goes back at least to Golosov, Tsyvinski, and Werquin (2014) and Badel and Huggett (2017), and versions of this condition are used to characterize either income or savings taxes in Ferey, Lockwood, and Taubinsky (2024), Gerritsen et al. (2020), Schulz (2021), and Lefebvre, Lehmann, and Sicsic (2025). The no-arbitrage condition appears as a Pareto-efficiency condition in Scheuer and Slemrod (2021) and Gerritsen et al. (2020), and are discussed also by Ferey, Lockwood, and Taubinsky (2024).

[9] Our model is formally equivalent to Ferey, Lockwood, and Taubinsky (2024), and our characterization complements theirs by focusing specifically on top income earners. Gerritsen et al. (2020) and Schulz (2021) consider models with heterogeneous returns, assuming that preferences satisfy the Atkinson-Stiglitz restrictions. We nest this framework as a special case, but without exploring various micro-foundations of return heterogeneity that are beyond the scope of our analysis. Scheuer and Slemrod (2021) consider a model with heterogeneous initial endowments which is nested by an extension of our baseline model.

earners. Moreover, perturbing one top tax rate at a time no longer suffices to fully determine both optimal top tax rates, since labor supply decisions only respond to the combined wedge between earnings and savings at the top—in other words, both perturbations become colinear at the top and cannot separately identify the two wedges. The no-arbitrage condition, which perturbs both taxes simultaneously holding fixed the combined wedge, is therefore essential in fully characterizing the optimal tax system. We discuss these issues in Section 3.5 when we compare our characterization with Ferey, Lockwood, and Taubinsky (2024) and Scheuer and Slemrod (2021), who are the most closely related to our analysis.

Finally, our calibration builds on the vast empirical literature that estimates consumption responses to income and rate of return changes; we review key contributions in Section 4. In this context, we argue that the important econometric challenge of separately identifying the heterogeneity vs. non-homotheticity of preferences is precisely what underlies the economic rationale to tax savings—namely, the hidden correlation of preferences for savings with permanent income. Many of the existing empirical studies lack the flexibility to identify both mechanisms separately, and hence lead to a biased estimate the pass-through elasticity that is relevant for optimal taxation.

Outline of the Paper. In Section 2, we set up our baseline model and derive our main theoretical results on optimal income and savings taxes (Theorem 2) and the relationship between Saez (2001) and Atkinson and Stiglitz (1976) (Theorem 1). Section 3 discusses several extensions. In Section 4, we present results from a simple calibration. All proofs are collected in an Appendix.

2 Optimal Taxation of Top Earners

In this section, we set up the simplest environment that allows us to decouple consumption from after-tax income and derive our optimal labor and savings tax formulas. In Section 3, we generalize our analysis to much richer environments.

2.1 Environment

There is a continuum of measure 1 of heterogeneous agents indexed by a rank $r \in [0,1]$ uniformly distributed over the unit interval. The preferences of agents of rank r are defined over "labor income" Y, "consumption" C, "savings" S. They are represented by the utility function $U(Y, C, S; r)$.[10] We assume that U is twice continuously differentiable in (Y, C, S) with $U_Y < 0$,

[10]While it is convenient to define preferences in terms of the observables Y, C, and S, it is straightforward to map the preference over labor income Y into a preference over labor effort $N = Y/A(r)$, where $A(r)$ denotes labor

$U_{YY} < 0$, $U_C > 0$, $U_{CC} < 0$, $U_S > 0$, $U_{SS} < 0$ and satisfies the usual Inada conditions as Y, C, or S approach 0 or ∞, for any r. We further assume that $U(Y, C, S; r)$ is non-decreasing in r, for any (Y, C, S). This specification nests the standard case of an intertemporally separable utility function $U_1(Y, C; r) + \beta U_2(S; r)$, where the second-period utility U_2 represents (possibly heterogeneous) preferences over future consumption or bequests; more generally, it also allows for non-separability between current and future consumption, for instance as in models of habit formation.

Assumption 1 (Single-Crossing Conditions). *(i) The marginal rates of substitution (MRS) between consumption and income, $-U_Y/U_C$, and between saving and income, $-U_Y/U_S$, are strictly decreasing in r for all (Y, C, S), i.e.,*

$$\frac{\partial \ln(-U_Y/U_C)}{\partial r} \equiv \frac{U_{Yr}}{U_Y} - \frac{U_{Cr}}{U_C} < 0 \quad \textit{and} \quad \frac{\partial \ln(-U_Y/U_S)}{\partial r} \equiv \frac{U_{Yr}}{U_Y} - \frac{U_{Sr}}{U_S} < 0. \tag{3}$$

Furthermore, the marginal disutility of effort is decreasing in r, $U_{Yr}/U_Y < 0$.

(ii) The MRS between consumption and savings, U_S/U_C, is monotonic in r for all (Y, C, S), i.e.,

$$\frac{\partial \ln(U_S/U_C)}{\partial r} \equiv \frac{U_{Sr}}{U_S} - \frac{U_{Cr}}{U_C} \tag{4}$$

is either non-positive or non-negative everywhere.

The first equality in (3) is the standard single-crossing condition (Mirrlees 1971); the second condition introduces its analogue with regards to savings. These conditions rank agents according to their preferences over leisure, on the one hand, and consumption or savings, on the other hand. On the margin, higher-ranked agents are more willing to work for a given consumption or savings increase. The additional restriction $U_{Yr}/U_Y < 0$ implies that higher ranks r find it less costly to attain a given income level Y; that is, we can associate agents' ranks with their labor productivity.

The second part of Assumption 1 imposes that the inter-temporal MRS is monotonic. If it is increasing and (4) is positive, then higher ranks have a stronger taste for saving (relative to current consumption) than lower ranks. In other words, given the same allocation, those who are the most inclined to work—the higher ranks—are also the most inclined to save. If instead (4) is negative, then those who are the most inclined to work are also those who are the most inclined to spend their incomes on current consumption. Under the restrictions on preferences imposed by Atkinson and Stiglitz (1976), the intertemporal MRS is constant and (4) is uniformly equal to zero.

productivity, and N and $A(r)$ are not directly observable to the planner. The dependence of the utility function on r also allows for heterogeneity of preferences across ranks.

Social planner's problem. Consumption, income, and savings are assumed to be observable, but an individual's productivity rank r is their private information. For now, we assume that the social planner is Rawlsian and maximizes the utility of the lowest-ranked agent; this assumption is without loss of generality for our results on optimal top taxes.[11] The Rawlsian social planner's problem amounts to choosing a tax function $T(Y,S)$ that maximizes government revenue

$$\int_0^1 T\left(Y\left(r\right),S\left(r\right)\right)\mathrm{d}r \tag{5}$$

subject to the constraint that all agents maximize their utility given their budget constraint $C \leq Y - S - T(Y,S)$, that is:

$$\left\{Y\left(r\right),S\left(r\right)\right\} \;\in\; \arg\max_{Y,S}\; U\left(Y, Y-S-T\left(Y,S\right), S; r\right), \tag{6}$$

and a lower bound constraint on the lowest rank's utility

$$U\left(Y\left(0\right),C\left(0\right),S\left(0\right);0\right) \;\geq\; W_0. \tag{7}$$

Following Saez (2001), this formulation allows us to characterize optimal income and savings taxes using simple perturbations of the tax function $T(Y,S)$. With one-dimensional heterogeneity, it is without loss of generality to consider separable tax functions of the form $T(Y,S) \equiv T_Y(Y)+T_S(S)$.

This problem can be transformed into a standard mechanism design problem of choosing an allocation $\{Y(\cdot),C(\cdot),S(\cdot)\}$ that maximizes the government's surplus $\int_0^1\{Y(r)-C(r)-S(r)\}\mathrm{d}r$ subject to incentive compatibility and the lower bound constraint (7). Incentive compatibility implies the envelope condition $V'(r) = U_r(Y(r),C(r),S(r);r)$, where $V(r) \equiv U(Y(r),C(r),S(r);r)$. In the Appendix, we show that this envelope condition is also sufficient for global incentive compatibility, if $Y(\cdot)$, $C(\cdot)$, and $S(\cdot)$ are monotone in r. We then go on to characterize its solution using standard results from optimal control theory.[12]

Marginal income and savings tax rates. Let $\tau_Y(r) \equiv T'_Y(Y(r))$ and $\tau_S(r) \equiv T'_S(S(r))$ denote the marginal income and savings tax rates at rank r. From the first-order conditions implied by equation (6), the marginal income tax rate satisfies $\tau_Y = U_Y/U_C + 1$ and therefore corresponds

[11] We generalize our results to arbitrary planner preferences in Section 3.1. Our top income tax results remain unaffected so long as the marginal utility converges to 0 at infinity, so that top-ranked agents receive vanishing weight in the planner's objective function.

[12] To ease notation, throughout the paper we write $X(r) \equiv X(Y(r),C(r),S(r);r)$ for any function X of both the rank r and the allocation $\{Y(r),C(r),S(r)\}$ assigned to this rank. We further drop the dependence on r whenever this is unambiguous from the context.

to the *labor wedge* at rank r, i.e., the intra-temporal distortion between the marginal product and the marginal rate of substitution between consumption and income. Likewise the marginal savings tax rate satisfies $\tau_S = U_S/U_C - 1$ and therefore corresponds to the *savings wedge* at rank r, i.e., the inter-temporal distortion in the agent's first-order condition for savings. We focus on the top of the income distribution, for which we make the following assumption:

Assumption 2 (Top Tax Rates). *The optimal allocation* $\{Y(\cdot), C(\cdot), S(\cdot)\}$ *is increasing in* r*, for* r *sufficiently close to* 1*. Moreover, optimal labor and savings wedges converge to constants* $\bar{\tau}_Y < 1$ *and* $\bar{\tau}_S > -1$ *as* $r \to 1$*.*

We can interpret our optimal tax system as a combination of income taxes, social security contributions and pension payments ("savings") that are indexed to labor income, without any additional private savings. The savings wedge then represents the marginal shortfall or excess of social security contributions relative to pension payments. Alternatively, we interpret S as "bequests", let C and Y stand for life-time earnings and consumption, and reinterpret the savings wedge as a tax on bequests.

2.2 Sufficient Statistics

Our characterizations of optimal income and savings taxes are based on two sets of sufficient statistics: (i) distributional parameters, such as the Pareto coefficients and the consumption and saving shares of income; and (ii) behavioral elasticities (income and substitution effects) of earnings, consumption, and savings with respect to tax changes.

Distributional parameters. We denote the local Pareto coefficients of the distributions of labor income, consumption, and savings, respectively, by

$$\rho_X(r) \equiv \frac{d\ln(1 - F_X(X(r)))}{d\ln X(r)}$$

for $X \in \{Y, C, S\}$, where F_X denotes the CDF of the distribution of X. We further denote by

$$s_C(r) \equiv \frac{C(r)}{(1-\tau_Y(r))\,Y(r)} \quad \text{and} \quad s_S(r) \equiv \frac{(1+\tau_S(r))\,S(r)}{(1-\tau_Y(r))\,Y(r)}$$

the shares of consumption and savings in retained income at rank r.

Behavioral elasticities. Next, we introduce the substitution and income effects of tax changes on taxable income $(\zeta^H_{Y,\tau_Y}, \zeta^H_{Y,\tau_S}, \zeta^I_Y)$, consumption $(\zeta^H_{C,\tau_Y}, \zeta^H_{C,\tau_S}, \zeta^I_C)$, and savings $(\zeta^H_{S,\tau_Y}, \zeta^H_{S,\tau_S}, \zeta^I_S)$.

Fixing a linear tax function $T(Y,S) = T_0 + \tau_Y Y + \tau_S S$, the substitution effects are defined as the compensated (Hicksian) elasticities with respect to the after-tax wage or asset return, and the income effects are defined as the (weighted) semi-elasticities with respect to lump-sum tax changes:

$$\zeta^H_{X,\tau_Y} \equiv \left.\frac{\partial \ln X}{\partial \ln(1-\tau_Y)}\right|_{\partial U=0}, \; \zeta^H_{X,\tau_S} \equiv \left.\frac{\partial \ln X}{\partial \ln(1+\tau_S)}\right|_{\partial U=0}, \text{ and } \zeta^I_X \equiv (1-\tau_Y)\, Y \left|\frac{\partial \ln X}{\partial T_0}\right|$$

for $X \in \{Y, C, S\}$. These substitution and income effects identify structural properties of the underlying utility function which are fully summarized by the following six preference elasticities:[13]

$$\mathcal{E}_C \equiv -\frac{U_{CC}C}{U_C}, \; \mathcal{E}_S \equiv -\frac{U_{SS}S}{U_S}, \; \mathcal{E}_Y \equiv \frac{U_{YY}Y}{U_Y}, \; \mathcal{E}_{CY} \equiv \frac{U_{CY}Y}{U_C}, \; \mathcal{E}_{SY} \equiv \frac{U_{SY}Y}{U_S}, \; \mathcal{E}_{CS} \equiv \frac{U_{CS}S}{U_C}.$$

We assume that all of these variables converge to constants for top earners:

Assumption 3 (**Convergence**). *The Pareto coefficients* (ρ_Y, ρ_C, ρ_S)*, consumption and savings shares* (s_C, s_S)*, substitution and income effects* $(\zeta^H_{Y,\tau_Y}, \zeta^H_{Y,\tau_S}, \zeta^I_Y)$, $(\zeta^H_{C,\tau_Y}, \zeta^H_{C,\tau_S}, \zeta^I_C)$, $(\zeta^H_{S,\tau_Y}, \zeta^H_{S,\tau_S}, \zeta^I_S)$*, and preference elasticities* $(\mathcal{E}_C, \mathcal{E}_S, \mathcal{E}_Y, \mathcal{E}_{CY}, \mathcal{E}_{SY}, \mathcal{E}_{CS})$ *all converge to finite limits as* $r \to 1$*. For any of these variables, we write* $\bar{X} \equiv \lim_{r\to 1} X(r)$*.*

Model-implied restrictions: a tale of three tails. The Pareto coefficients, spending shares and behavioral elasticities are not independent of each other: They are linked through the agents' budget constraints and first-order conditions. As discussed in the introduction, this additional structure imposes over-identifying restrictions that enter the representation of optimal taxes.

First, differentiating the agents' budget constraint with respect to r yields the following condition linking local Pareto coefficients and spending shares: $1/\rho_Y = s_C/\rho_C + s_S/\rho_S$. This equation relates the concentration in earnings to the concentration in consumption and savings above any rank r. Now, as $r \to 1$, the spending on consumption or savings of top-ranked agents cannot grow faster than their after-tax income, so that $\bar{\rho}_C \geq \bar{\rho}_Y$ and $\bar{\rho}_S \geq \bar{\rho}_Y$. But total spending must grow at the same rate as after-tax income, which in turn implies that $\bar{\rho}_Y = \min\{\bar{\rho}_C, \bar{\rho}_S\}$ and $\bar{s}_C + \bar{s}_S = 1$. In other words, the cross-sectional distribution of labor income must have a weakly thicker upper tail than those of both consumption and savings, but it cannot have a strictly thicker upper tail than more than one of these variables. This relationship is the analogue of the condition $\bar{\rho}_Y = \bar{\rho}_C$ in the static setting. Moreover, if consumption has a strictly thinner tail than income, then the share of consumption in retained earnings must be vanishing, so that $\bar{s}_C = 0$ whenever $\bar{\rho}_C > \bar{\rho}_Y$. Symmetrically, $\bar{s}_S = 0$ if $\bar{\rho}_S > \bar{\rho}_Y$. On the other hand, the shares of consumption and saving can

[13] For future reference, we also define $\mathcal{E}_{SC} \equiv U_{SC}C/U_S = (s_C/s_S)\, \mathcal{E}_{CS}$.

converge to any value in $[0,1]$ when the three upper tails have the same Pareto coefficient.

In addition, income and substitution effects satisfy standard demand-theoretic properties, obtained by differentiating the agents' first order conditions with respect to changes in marginal tax rates and lump-sum income transfers: $\zeta^H_{Y,\tau_X} = s_C \zeta^H_{C,\tau_X} + s_S \zeta^H_{S,\tau_X}$ for $X \in \{Y, S\}$ and $1 - \zeta^I_Y = s_C \zeta^I_C + s_S \zeta^I_S$. These equations generalize the conditions $\zeta^H_{Y,\tau_Y} = \zeta^H_{C,\tau_Y}$ and $1 - \zeta^I_Y = \zeta^I_C$ from the static model and add the analogue condition for substitution effects w.r.t. savings taxes. Moreover, the Slutzky symmetry condition for substitution between earnings and savings imposes $\zeta^H_{Y,\tau_S} = -s_S \zeta^H_{S,\tau_Y}$. Hence, income and substitution effects provide five distinct moments to identify six preference elasticities. The remaining degree of freedom captures the fact that demand functions identify ordinal preferences, not cardinal utilities—but these are sufficient for analyzing the Rawlsian social planner's problem.[14]

Finally, as with the Pareto coefficients, taking the limit as $r \to 1$ imposes additional restrictions on the identification power of these income and substitution effects. If $\bar{s}_C = 1 - \bar{s}_S = 1$, so that savings vanish at the top as a proportion of income, then the substitution and income effects on consumption mirror those on taxable income, i.e. $(\bar{\zeta}^H_{C,\tau_Y}, \bar{\zeta}^H_{C,\tau_S}, \bar{\zeta}^I_C) = (\bar{\zeta}^H_{Y,\tau_Y}, \bar{\zeta}^H_{Y,\tau_S}, \bar{\zeta}^I_Y)$. Therefore, identification requires using the response of top earners' *savings* to tax changes, $(\bar{\zeta}^H_{S,\tau_Y}, \bar{\zeta}^H_{S,\tau_S}, \bar{\zeta}^I_S)$. If $\bar{s}_C = 1 - \bar{s}_S = 0$, so that consumption vanishes at the top, then the income and substitution effects on savings mirror those of income, i.e. $(\bar{\zeta}^H_{S,\tau_Y}, \bar{\zeta}^H_{S,\tau_S}, \bar{\zeta}^I_S) = (\bar{\zeta}^H_{Y,\tau_Y}, \bar{\zeta}^H_{Y,\tau_S}, \bar{\zeta}^I_Y)$, and therefore identification requires using the response of *consumption* to tax changes $(\bar{\zeta}^H_{C,\tau_Y}, \bar{\zeta}^H_{C,\tau_S}, \bar{\zeta}^I_C)$. Finally, if $\bar{s}_C, \bar{s}_S \in (0,1)$, any two sets of substitution and income effects identify the third, along with the preference elasticities.

In sum, our model admits three possible scenarios that we summarize as follows:

1. If $\bar{\rho}_Y = \bar{\rho}_C < \bar{\rho}_S$, then $\bar{s}_C = 1$ and $\bar{s}_S = 0$, so that savings are strictly less concentrated at the top than income and consumption, and top earners consume most of their income in the current period. The Pareto coefficients and behavioral responses of consumption and labor supply then coincide, thus making information from consumption redundant, relative to income data. Preference elasticities and optimal taxes are then identified from Pareto coefficients and behavioral responses of income and *savings*.

2. If $\bar{\rho}_Y = \bar{\rho}_C = \bar{\rho}_S$, then $\bar{s}_C = 1 - \bar{s}_C \in [0,1]$, so that income, consumption, and savings are equally concentrated at the top, and top earners both consume and save non-vanishing fractions of their income. The substitution and income effects of any two of the three variables

[14] Going beyond the Rawlsian tax design problem, the extra degree of freedom summarizes the transformation of ordinal to cardinal preferences, or equivalently, the welfare weights assigned to different agent types.

can then be used to identify the preference elasticities and determine optimal income and savings taxes.

3. If $\bar{\rho}_Y = \bar{\rho}_S < \bar{\rho}_C$, then $\bar{s}_C = 0$ and $\bar{s}_S = 1$, so that consumption is strictly less concentrated at the top than income and savings, and top earners save most of their income. The Pareto coefficients and behavioral elasticities of savings and labor supply coincide, making savings information redundant relative to income data. Preference elasticities and optimal taxes are then identified from Pareto coefficients and behavioral responses of income and *consumption*.

In the sequel, we refer to these cases as *Case 1*, *Case 2*, and *Case 3*, respectively. The evidence cited in the introduction points to *Case 3* with $\bar{\rho}_C > \bar{\rho}_Y$ and $\bar{s}_C = 0$ as the empirically relevant scenario.

2.3 Linking Departures from Seminal Taxation Results

When should income and savings be taxed? It is well-known since Mirrlees (1976) that the optimal labor and savings wedges inherit the signs of $U_{Cr}/U_C - U_{Yr}/U_Y$ and $U_{Sr}/U_S - U_{Cr}/U_C$, respectively. Assumption 1 implies that the former is positive, and hence that it is optimal to tax labor income. Analogously, it is optimal to tax (respectively, subsidize) savings whenever the latter is positive (resp., negative), that is, if higher ranks have a higher (resp., lower) intertemporal MRS U_S/U_C and are thus more inclined to save (resp., consume) their current income than lower ranks. Intuitively, if the more productive ranks have a stronger taste for savings, the planner can screen them—i.e., deter them from mimicking lower ranks—by taxing the savings of lower ranks. This general result nests the uniform commodity taxation setting of Atkinson and Stiglitz (1976) as a special case. When all ranks r have the same intertemporal MRS U_S/U_C, savings taxes are unable to affect the low-productivity workers differently from the more productive ones who mimick them. It is then optimal to set $\tau_S = 0$ for all r, so that redistribution is achieved only through the income tax without further distorting the consumption-savings margin.

This discussion, however, does not shed any light on connections between optimal income and savings taxes. We show that these two tax instruments interact through two distinct channels, and must therefore be jointly determined. First, optimal tax distortions obey a simple *arbitrage principle*: A Pareto efficient tax system must smooth tax distortions across different margins in such a way that a marginal shift from income to savings distortions—or vice versa—that leaves agents' utilities unchanged should also not increase or lower tax revenue. This no-arbitrage condition relates the level of optimal savings taxes or subsidies to the level of optimal income taxes. Second, there are tax *revenue spillovers*: Raising marginal income taxes reduces savings, and thus the revenues or costs from savings taxes or subsidies. Accounting for these revenue spillovers implies that optimal income taxes are a decreasing function of savings taxes, and vice versa. The no-arbitrage and revenue spillover conditions jointly determine the optimal tax system.

By combining these two conditions, we uncover a deep connection between the seminal results of Saez (2001) and Atkinson and Stiglitz (1976), which to our knowledge has not been recognized before: One cannot hold without the other—outside of the "trivial" *Case 1* where savings vanish (as a fraction of earnings) at the top of the income distribution. In other words, the static optimal income tax rate and the uniform commodity taxation theorem are two sides of the same coin. Hence, an expert calling for high income taxes based on $\overline{\tau}_Y^{Saez}$ must also support the recommendation of zero capital taxation, and support for positive savings taxes must be accompanied by less extreme recommendations for the income tax. This connection between optimal income and savings taxation is summarized in the following Theorem:

Theorem 1. *Suppose that $\bar{s}_S > 0$ (Cases 2 and 3). Then,*

$$\overline{\tau}_Y \lesseqqgtr \overline{\tau}_Y^{Saez} \iff \overline{\tau}_S \gtreqqless 0 \iff \bar{\rho}_C/\bar{\rho}_S \gtreqqless \bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{C,\tau_Y}.$$

The revenue-spillover condition implies the first equivalence in Theorem 1, which highlights that the tax authority faces a trade-off between income and savings taxes: Positive taxes (resp., subsidies) on savings must go hand-in-hand with a reduction (resp., increase) of income taxes below (resp., above) the level prescribed by $\overline{\tau}_Y^{Saez}$. These revenue spillovers become negligible only if $\bar{s}_S = 0$ (*Case 1*), in which case the optimal top income tax equals the static optimum. The no-arbitrage principle implies the second equivalence in Theorem 1. It provides a simple empirical test of the joint benchmark of $\overline{\tau}_Y^{Saez}$ and uniform commodity taxation by comparing the ratio of Pareto coefficients of consumption and savings, $\bar{\rho}_C/\bar{\rho}_S$, to the ratio of their inverse substitution effects w.r.t. income tax changes, $\bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{C,\tau_Y}$. Moreover, the difference between these two ratios determines in which direction optimal income and savings taxes must depart from this joint benchmark. The next two subsections derive these two optimality conditions in turn, and Theorem 2 combines them to solve for the optimal tax rates $\overline{\tau}_Y, \overline{\tau}_S$.

2.4 The No-Arbitrage Condition

Our first proposition articulates the no-arbitrage condition, according to which the planner cannot increase revenue by locally shifting tax distortions between income and savings taxes. This result is based on a Pareto efficiency criterion and therefore holds under much weaker conditions than those implied by the optimal planner's solution.

Proposition 1. *Optimal top income and savings taxes satisfy*

$$\frac{\bar{\tau}_S}{1+\bar{\tau}_S} = \frac{\bar{\tau}_Y}{1-\bar{\tau}_Y} \cdot \Sigma, \quad \textit{where} \quad \Sigma \equiv \frac{\bar{\rho}_C \bar{\zeta}^H_{C,\tau_Y} - \bar{\rho}_S \bar{\zeta}^H_{S,\tau_Y}}{\bar{\rho}_C \bar{\zeta}^H_{C,\tau_S} - \bar{\rho}_S \bar{\zeta}^H_{S,\tau_S}}. \tag{8}$$

We now sketch the proof of Proposition 1 and discuss its economic interpretation, first as the solution to the mechanism design problem as in Mirrlees (1971), and second by means of a well-designed tax perturbation in the tradition of Saez (2001).

Proof of Proposition 1 using mechanism design. The first-order conditions to the mechanism design problem imply that at any rank r, the following arbitrage relation must hold between income and savings tax distortions:

$$\frac{\tau_S / (1+\tau_S)}{\tau_Y / (1-\tau_Y)} = \frac{U_{Sr}/U_S - U_{Cr}/U_C}{U_{Cr}/U_C - U_{Yr}/U_Y}. \tag{9}$$

Equation (9) equates the ratio of the resource gains from reducing the savings tax vs. the income tax at rank r, in the left-hand side, to the ratio of changes in marginal information rents $V'(r)$ that such lower tax rates induce, in the right-hand side. The latter capture the respective changes in utility that must be granted to ranks $r' > r$ to deter them from mimicking rank r after the corresponding tax cuts. This arbitrage relation is based on a Pareto efficiency principle that applies regardless of the planner's tastes for redistribution: At the optimum, the planner should not be able to increase tax revenue by perturbing income and savings taxes in such a way that agents' marginal information rents (and hence their utilities) remain unchanged.[15] The following identification lemma relates the changes in marginal information rents to the sufficient statistics in equation (8):

Lemma 1. *For any given system of tax distortions* $\{\tau_Y(r), \tau_S(r)\}$, *we have*

$$(1-r)\left(\frac{U_{Cr}}{U_C} - \frac{U_{Yr}}{U_Y}\right) = \frac{\hat{\mathcal{E}}_C}{\rho_C} + \frac{\hat{\mathcal{E}}_Y}{\rho_Y} + \frac{d\ln(1-\tau_Y)}{d\ln(1-r)} \tag{10}$$

$$(1-r)\left(\frac{U_{Sr}}{U_S} - \frac{U_{Cr}}{U_C}\right) = \frac{\hat{\mathcal{E}}_S}{\rho_S} - \frac{\hat{\mathcal{E}}_C}{\rho_C} - \frac{d\ln(1+\tau_S)}{d\ln(1-r)}, \tag{11}$$

where $\hat{\mathcal{E}}_Y \equiv \mathcal{E}_Y - \mathcal{E}_{CY} - \mathcal{E}_{SY} - \mathcal{E}_{CS}/s_S$, $\hat{\mathcal{E}}_C \equiv \mathcal{E}_C + \mathcal{E}_{SC} - s_C(\mathcal{E}_{CY} - \mathcal{E}_{SY})$, *and* $\hat{\mathcal{E}}_S \equiv \mathcal{E}_S +$

[15] The Uniform Commodity Taxation Theorem of Atkinson and Stiglitz (1976) is a direct corollary of this condition, since equation (9) implies $\tau_S = 0$ whenever $U_{Sr}/U_S - U_{Cr}/U_C = 0$.

$\mathcal{E}_{CS} + s_S(\mathcal{E}_{CY} - \mathcal{E}_{SY})$. *Moreover, the ratios of preference elasticities are identified by the ratios of behavioral elasticities as follows:*

$$\frac{\hat{\mathcal{E}}_Y}{\hat{\mathcal{E}}_C} = -\frac{\zeta^H_{C,\tau_S}}{\zeta^H_{Y,\tau_S}} \quad \text{and} \quad \frac{\hat{\mathcal{E}}_S}{\hat{\mathcal{E}}_C} = \frac{\zeta^H_{C,\tau_Y}}{\zeta^H_{S,\tau_Y}}. \tag{12}$$

Equations (10) and (11) are obtained by totally differentiating the agents' first-order conditions $1-\tau_Y = -U_Y/U_C$ and $1+\tau_S = U_S/U_C$ with respect to r and noting that in each case the differentiation can be decomposed into a component that captures the rank-dependence of preferences for a given allocation (i.e., the heterogeneity of preferences across agents), and a component that captures how the allocations vary with income at a given rank (i.e., the non-homotheticity of preferences for a given agent). The latter is fully identified from preference elasticities and local Pareto tail coefficients, and can thus be used to identify the former. To understand the economics underlying this key identification result, suppose that savings are more concentrated at the top than consumption ($\rho_C > \rho_S$). This could occur either because top-ranked earners face more strongly diminishing marginal utilities of consumption than of savings ($\hat{\mathcal{E}}_C > \hat{\mathcal{E}}_S$), which would constitute a departure from the homotheticity of preferences, or because agents' intrinsic preferences for savings are heterogeneous (i.e., rank-dependent). The ratio ρ_C/ρ_S identifies the concentration of savings relative to consumption observed in the data, while $\hat{\mathcal{E}}_C/\hat{\mathcal{E}}_S$ identifies the concentration of savings relative to consumption that is consistent with rank-independent preferences. Rank-independence is rejected when the former differs from the latter. By comparing ρ_C/ρ_S to $\hat{\mathcal{E}}_C/\hat{\mathcal{E}}_S$, we thus identify how much preference heterogeneity is required to rationalize the gap between consumption and savings inequality, and thus to what extent savings taxes or subsidies are useful to screen and redistribute from higher- to lower-ranked agents, on top of income taxes.[16]

Furthermore, equation (12) identifies the ratios of preference elasticities from the ratios of substitution effects. It shows that $\hat{\mathcal{E}}_S/\hat{\mathcal{E}}_C = \zeta^H_{C,\tau_Y}/\zeta^H_{S,\tau_Y}$, i.e., the substitution effects for consumption and savings with respect to retained earnings are inversely proportional to the preference elasticities. Intuitively, agents pass through earned income changes to consumption and savings with elasticities that are inversely proportional to the respective preference elasticities $\hat{\mathcal{E}}_C$ and $\hat{\mathcal{E}}_S$. Thus, the ratio of

[16] A more formal intuition goes as follows. With constant top savings taxes, agents' consumption and savings must grow with after-tax income at rates that keep the inter-temporal MRS constant, hence $1/U_S$ and $1/U_C$ must have identical upper Pareto tails. With rank-independent intertemporal MRS, this in turn implies that $\hat{\mathcal{E}}_S/\rho_S = \hat{\mathcal{E}}_C/\rho_C$. Conversely, if preferences are rank-dependent ($U_{Sr}/U_S \neq U_{Cr}/U_C$), the upper Pareto coefficients of $1/U_S$ and $1/U_C$ augment the terms $\hat{\mathcal{E}}_S/\rho_S$ and $\hat{\mathcal{E}}_C/\rho_C$, which capture the role of diminishing marginal utilities at a given rank, with additional terms $(1-r)\,U_{Sr}/U_S$ and $(1-r)\,U_{Cr}/U_C$ that capture the rank-dependence of $1/U_S$ and $1/U_C$ at a given allocation. Equating the two Pareto coefficients then implies that the rank-dependence of the intertemporal MRS is equal to the difference between $\hat{\mathcal{E}}_S/\rho_S$ and $\hat{\mathcal{E}}_C/\rho_C$ and is thus fully identified from the latter.

substitution effects from income tax changes identifies the ratio of preference elasticities, and hence the amount of consumption vs. savings concentration at the top that can be rationalized without preference heterogeneity. Combining the two conditions (11) and (12), it follows that the information rent term $U_{Sr}/U_S - U_{Cr}/U_C$, and therefore departures from uniform commodity taxation, depend on the difference between ρ_C/ρ_S and $\zeta^H_{S,\tau_Y}/\zeta^H_{C,\tau_Y}$, or equivalently on $\rho_C\zeta^H_{C,\tau_Y} - \rho_S\zeta^H_{S,\tau_Y}$. The reasoning is similar for the term $U_{Cr}/U_C - U_{Yr}/U_Y$, using the over-identifying restrictions derived in Section 2.1. Proposition 1 then follows from taking limits as $r \to 1$. The ratios of preference elasticities, substitution effects, and Pareto coefficients are therefore natural and transparent sufficient statistics for intrinsic rank-dependence of preferences and hence optimal taxes.

Lemma 1 highlights the importance of non-homothetic preferences for optimal tax design. If preferences are homothetic in consumption and savings, i.e. $\hat{\mathcal{E}}_S = \hat{\mathcal{E}}_C$, consumption and savings of top earners both move one-for-one with after-tax income, implying that $\zeta^H_{C,\tau_Y} = \zeta^H_{S,\tau_Y} = \zeta^H_{Y,\tau_Y}$, and the response of consumption and savings to income tax changes becomes uninformative about optimal savings taxes. Any difference between the concentrations of consumption and savings in the data is then fully attributed to preference heterogeneity, which directly translates into a motive for savings taxes. It is only with non-homothetic preferences that there is a meaningful problem of identifying preference heterogeneity separately from income effects, and only in this latter case that these additional behavioral elasticities are informative about optimal income tax design.

Proof of Proposition 1 using tax perturbation methods. Consider a small perturbation $(\partial\tau_Y, \partial\tau_S)$ that shifts taxes from income to savings for a small interval of types $r' \in [r, r+\partial r)$. Following Saez (2001), these perturbations mechanically change tax payments at rank $r' \geq r + \partial r$ by $\partial\tau_Y\left(Y\left(r+\partial r\right) - Y\left(r\right)\right) + \partial\tau_S\left(S\left(r+\partial r\right) - S\left(r\right)\right) \approx \left(\partial\tau_Y Y/\rho_Y + \partial\tau_S S/\rho_S\right)\partial r/\left(1-r\right)$.[17] They further affect overall tax revenue through the behavioral responses of earnings and savings. These behavioral responses can be decomposed into a substitution effect for types $r' \in [r, r+\partial r)$ and an income effect for types $r' \geq r + \partial r$. We scale the income and savings tax perturbations so that their mechanical effects exactly offset each other: $\partial\tau_Y Y/\rho_Y + \partial\tau_S S/\rho_S = 0$. With this scaling condition, the two tax perturbations also have exactly offsetting income effects for all types $r' > r + \partial r$, leaving their earnings, consumption, and savings (and hence tax payments) unchanged. Therefore, substitution effects for ranks $r' \in [r, r+\partial r)$ are the only possible source of variation in tax revenues. At the optimum, the changes in income and savings tax revenue due to substitution effects must exactly offset each other. The behavioral response of earnings at rank r to the combined change

[17]Throughout the paper, we use the symbol "$\approx$" to denote first-order approximations when $\partial r > 0$ is small. This approximation becomes exact in the limit as $\partial r \to 0$.

of income and savings taxes is $\partial Y = -(\zeta^H_{Y,\tau_Y}\frac{\partial\tau_Y}{1-\tau_Y} - \zeta^H_{Y,\tau_S}\frac{\partial\tau_S}{1+\tau_S})Y = -(\zeta^H_{Y,\tau_Y}\rho_Y - \zeta^H_{S,\tau_Y}\rho_S)\frac{Y}{\rho_Y}\frac{\partial\tau_Y}{1-\tau_Y}$, where we have first substituted the scaling condition $-\partial\tau_Y Y/\rho_Y = \partial\tau_S S/\rho_S$ and then the Slutzky symmetry condition. Likewise, the behavioral response of savings to the combined tax perturbation is $\partial S = (\zeta^H_{S,\tau_S}\rho_S - \zeta^H_{Y,\tau_S}\rho_Y)\frac{S}{\rho_S}\frac{\partial\tau_S}{1+\tau_S}$. The perturbation must leave total tax revenue unchanged, or $\tau_Y\partial Y + \tau_S\partial S = 0$. Since $-\partial\tau_Y Y/\rho_Y = \partial\tau_S S/\rho_S$, it follows that

$$\frac{\tau_S}{1+\tau_S} = \frac{\tau_Y}{1-\tau_Y}\frac{\zeta^H_{Y,\tau_Y}\rho_Y - \zeta^H_{S,\tau_Y}\rho_S}{\zeta^H_{Y,\tau_S}\rho_Y - \zeta^H_{S,\tau_S}\rho_S} = \frac{\tau_Y}{1-\tau_Y}\frac{\zeta^H_{C,\tau_Y}\rho_C - \zeta^H_{S,\tau_Y}\rho_S}{\zeta^H_{C,\tau_S}\rho_C - \zeta^H_{S,\tau_S}\rho_S} \tag{13}$$

where the second equality follows from a few steps of algebra that use the over-identifying restrictions to express the ratio in terms of $(\rho_C, \zeta^H_{C,\tau_Y}, \zeta^H_{C,\tau_S})$ instead of $(\rho_Y, \zeta^H_{Y,\tau_Y}, \zeta^H_{Y,\tau_S})$. This last step matters for *Case 3*, where consumption moments are necessary for identification purposes—notice that the first equality in equation (13) would be equal to "0/0" in this case since savings and earnings are asymptotically indistinguishable. We return to this important point in Section 3.5 below when we discuss the relationship between our paper and Ferey, Lockwood, and Taubinsky (2024).

The tax perturbation proof offers a complementary intuition for the no-arbitrage condition: Shifting tax distortions from earnings to savings increases earnings ($\partial Y > 0$) if and only if $\zeta^H_{Y,\tau_Y}\rho_Y > \zeta^H_{S,\tau_Y}\rho_S$, or the positive substitution effect of lower income taxes outweighs the negative substitution effect of higher savings taxes. If this inequality holds and $\tau_S = 0$, such a compensated shift from income to savings taxes thus raises overall tax revenue, in which case it is optimal to shift part of the tax burden from income to savings. If instead $\zeta^H_{Y,\tau_Y}\rho_Y < \zeta^H_{S,\tau_Y}\rho_S$, subsidizing savings to increase income taxes raises overall tax revenue. Finally, the Uniform Commodity Taxation Theorem applies if and only if the two substitution effects just offset each other and the compensated tax perturbation leaves income and overall tax revenue unchanged. This observation connects the revenue trade-off between income and savings taxes back to the preference conditions that govern the departure from uniform commodity taxation.

2.5 The Revenue Spillover Condition

The second source of interaction between income and savings taxes results from the spillovers that one tax has on the revenue collected from the other. The following proposition derives the optimal top income tax $\bar{\tau}_Y$ for a given savings distortion $\bar{\tau}_S$ on top income earners.

Proposition 2. *For a given top savings distortion $\bar{\tau}_S$, the optimal top income tax $\bar{\tau}_Y$ satisfies*

$$\frac{\bar{\tau}_Y}{1-\bar{\tau}_Y} = \frac{\bar{\tau}_Y^{Saez}}{1-\bar{\tau}_Y^{Saez}}\left(1 - \frac{\bar{\tau}_S}{1+\bar{\tau}_S}\bar{s}_S\Phi_S\right), \quad \textit{where} \quad \Phi_S \equiv \bar{\rho}_Y\bar{\zeta}^H_{S,\tau_Y} + \bar{\zeta}^I_S. \tag{14}$$

where $\bar{\tau}_Y^{Saez}$ is given by (1). In particular, in Case 1 ($\bar{s}_S = 0$), the optimal top income tax rate $\bar{\tau}_Y$ is equal to the static optimum $\bar{\tau}_Y^{Saez}$. In Case 3 ($\bar{s}_S = 1$), the static optimum $\bar{\tau}_Y^{Saez}$ is equal to the combined wedge on income and savings:

$$1 - \bar{\tau}_Y^{Saez} = \frac{1 - \bar{\tau}_Y}{1 + \bar{\tau}_S}. \tag{15}$$

Proposition 2 summarizes how the revenue spillover from income taxes to savings changes the optimal design of income taxes for top income earners. When savings taxes are zero for all types, the optimal income tax is derived from the same trade-off between mechanical increases in tax revenue and labor supply responses as in the static model. Hence, when $\bar{\tau}_S = 0$ the optimal income tax on top earners is equal to $\bar{\tau}_Y^{Saez}$. However, when $\tau_S \neq 0$, an extra term enters the optimal tax trade-off, given by the behavioral response of savings to the income tax change. This tax revenue spillover implies that the optimal income tax is lower than at the static benchmark if savings are taxed, or higher than at the static benchmark if savings are subsidized. The magnitude of this spillover depends on the income and substitution effects of savings in response to income tax changes, as well as the savings share among top earners.

The two polar cases are particularly informative. In *Case 1* ($\bar{s}_S = 0$), top income earners save a vanishing fraction of their income, and the spillover effect to savings taxes or subsidies becomes negligible. Intuitively, savings simply do not matter at the top, and the model converges to the standard static trade-off between income and consumption, so that the optimal income tax is the same as in the static model. On the other hand, in *Case 3* ($\bar{s}_S = 1$), top income earners consume a vanishing fraction of their income, and savings mirror earnings at the top of the distribution, so that $\Phi_S = 1/\bar{\tau}_Y^{Saez}$ and hence $1 - \bar{\tau}_Y^{Saez} = (1 - \bar{\tau}_Y)/(1 + \bar{\tau}_S)$ (equation (15)). That is, the spillover effect from income to savings tax revenue is so strong that higher savings taxes translate one-for-one into lower income taxes, leaving the overall wedge constant at $\bar{\tau}_Y^{Saez}$. Intuitively, if consumption becomes negligible at the top, then the optimal top income tax is again governed by a static trade-off—but it is now between income and *savings*, rather than between income and consumption. The static wedge therefore still matters, but it identifies the combination of the labor earnings and savings wedges—the latter being non-zero whenever the Atkinson-Stiglitz theorem fails to hold.

Proof of Proposition 2 using mechanism design. We decompose the mechanism design problem into two stages. In the first stage, the social planner designs the income tax schedule $T_Y(Y)$ and agents trade off between labor supply and after-tax earnings $M \equiv Y - T_Y(Y)$, as in Saez (2001). In the second stage, the planner designs the savings tax schedule $T_S(S)$

and agents allocate after-tax earnings to consumption and savings. Here, we take the outcome of the second stage as given and focus on the design of income taxes for a given savings tax schedule. Define $\mathcal{U}(Y, M; r) = \max_{C,S} U(Y, C, S; r)$ s.t. $C + S + T_S(S) \leq M$ as the indirect utility function that characterizes the solution to the second-stage problem, and $\mathcal{S}(Y, M; r) = \arg\max_S U(Y, M - S - T_S(S), S; r)$ as the optimal savings decision, given earnings Y and after-tax income $M = Y - T_Y(Y)$. The optimal first-stage allocation $\{Y(\cdot), M(\cdot)\}$ maximizes the aggregate tax revenue $\int_0^1 (T_Y(Y(r)) + T_S(\mathcal{S}(Y(r), M(r); r)))\, dr$ subject to the promise-keeping $V(0) \geq W_0$ and the local incentive compatibility constraint $V'(r) = \mathcal{U}_r(Y(r), M(r); r)$, with $V(r) = \mathcal{U}(Y(r), M(r); r)$. This mechanism design problem departs from the static optimal income tax design problem of Mirrlees (1971) and Saez (2001) only by including a spillover to savings tax revenue in the planner's objective function. We express the resulting optimality condition in terms of observable sufficient statistics to obtain equation (14).

The economic insight of this proof is that the optimal income tax equals $\overline{\tau}_Y^{Saez}$ if and only if the optimal income tax design can be reduced to the first-stage problem, that is, a static problem with preferences directly defined over earnings Y and after-tax income M, with no information needed about how the latter is allocated between consumption and savings. But this condition is met precisely when preferences satisfy the necessary and sufficient condition for the Uniform Commodity Taxation theorem of Atkinson and Stiglitz (1976) and it is optimal to leave savings undistorted; that is, if the MRS between consumption and savings is independent of rank r. By contrast, the reduction to a single decision margin and single tax distortion characterized by Saez (2001) no longer applies when the preference restrictions of Atkinson and Stiglitz (1976) fail to hold. The design of optimal income taxes can no longer ignore how after-tax income is allocated to consumption and savings, since preferences for savings and incentives to work are no longer independent. If higher-ranked agents are more inclined to save, the taxation of savings facilitates redistribution towards lower-ranked agents and allows the social planner to reduce labor supply distortions. Hence, the optimal income tax is strictly lower than the static benchmark $\overline{\tau}_Y^{Saez}$. Conversely, it becomes optimal to subsidize savings and raise the income tax above $\overline{\tau}_Y^{Saez}$ if higher-ranked agents are less inclined to save.

Proof of Proposition 2 using income tax perturbation. Consider a local perturbation of the marginal income tax similar to Saez (2001). Fix a rank $r \in (0, 1)$ and income $Y = Y(r)$, and for small $\partial r > 0$, consider a small increase of the marginal income tax by $\partial \tau_Y > 0$, for ranks $r' \in [r, r + \partial r)$. For ranks $r' \geq r + \partial r$, the marginal income tax is unchanged, but the level of the tax schedule is shifted up by $\partial \tau_Y (Y(r + \partial r) - Y(r))$. Hence, this tax perturbation mechanically raises tax revenues by $\partial\tau_Y\left(Y\left(r+\partial r\right)-Y\left(r\right)\right)\left(1-r\right)\approx\partial\tau_Y\partial r\,Y/\rho_Y$, which at the optimum must be exactly offset by the behavioral responses of earnings and savings in response to the tax change. The behavioral response of earnings can be decomposed into a substitution effect $\partial Y\left(r'\right)=-Y\zeta^H_{Y,\tau_Y}\partial\tau_Y/\left(1-\tau_Y\right)$ for ranks $r'\in\left[r,r+\partial r\right)$, and an income effect $\partial Y\left(r'\right)=\zeta^I_Y\left(r'\right)\left(Y\left(r+\delta r\right)-Y\left(r\right)\right)\partial\tau_Y/\left(1-\tau_Y\left(r'\right)\right)$ for ranks $r'\geq r+\partial r$. Similarly, we decompose the behavioral response of savings into a substitution effect $\partial S\left(r'\right)=-s_SY\zeta^H_{S,\tau_Y}\partial\tau_Y/\left(1+\tau_S\right)$ for ranks $r'\in\left[r,r+\partial r\right)$, and an income effect $\partial S\left(r'\right)=s_S\left(r'\right)\zeta^I_S\left(r'\right)\left(Y\left(r+\partial r\right)-Y\left(r\right)\right)\partial\tau_Y/\left(1+\tau_S\left(r'\right)\right)$ for ranks $r'\geq r+\partial r$. Integrating over $r'>r$ and letting ∂r go to zero yields the following optimality condition for income taxes at rank r:

$$
\begin{aligned}
1 \;=\; & \frac{\tau_Y}{1-\tau_Y}\rho_Y\zeta^H_{Y,\tau_Y}-\mathbb{E}\left[\zeta^I_Y\left(r'\right)\frac{\tau_Y\left(r'\right)}{1-\tau_Y\left(r'\right)}\middle|\,r'>r\right] \\
& +s_S\left(\frac{\tau_S}{1+\tau_S}\rho_Y\zeta^H_{S,\tau_Y}+\mathbb{E}\left[\frac{\tau_S\left(r'\right)}{1+\tau_S\left(r'\right)}\zeta^I_S\left(r'\right)\frac{s_S\left(r'\right)}{s_S\left(r\right)}\middle|\,r'>r\right]\right).
\end{aligned}
\tag{16}
$$

Equation (16) augments the usual tax perturbation trade-off by a spill-over term from the income tax to savings that includes both a substitution effect for ranks $r'\in\left[r,r+\partial r\right)$ and an income effect for ranks $r'\geq r+\partial r$. Proposition 2 then follows from taking limits as $r\to 1$.

2.6 General Representation of Optimal Taxes

We can now solve the system consisting of the revenue spillover and no-arbitrage conditions for $\overline{\tau}_Y$ and $\overline{\tau}_S$. We obtain the following representation of optimal income and savings taxes in terms of sufficient statistics:

Theorem 2. *Optimal top income and savings taxes satisfy*

$$
\overline{\tau}_Y \;=\; \frac{\overline{\tau}^{Saez}_Y}{1+\bar{s}_S\,\Phi_S\,\overline{\tau}^{Saez}_Y\,\Sigma} \quad \text{and} \quad \overline{\tau}_S \;=\; \frac{\Sigma\,\overline{\tau}^{Saez}_Y}{1+\bar{s}_S\,\Phi_S\,\overline{\tau}^{Saez}_Y\,\Sigma-\left(1+\Sigma\right)\overline{\tau}^{Saez}_Y}, \tag{17}
$$

where $\overline{\tau}^{Saez}_Y$ is given by (1), Σ is defined in Proposition 1, and Φ_S is defined in Proposition 2.

Theorem 2 gives our complete characterization of the optimal top income and savings tax rates in terms of Pareto coefficients, income and substitution effects. The first equation in (17) generalizes the top income tax rate formula of Saez (2001), while the second one provides an analogous representation for savings taxes. Our characterization boils down to three (composite) sufficient statistics: the optimal static wedge $\overline{\tau}^{Saez}_Y$, the revenue-spillover coefficient Φ_S, and the tax-arbitrage coefficient Σ whose sign governs both the departure of $\overline{\tau}_Y$ from $\overline{\tau}^{Saez}_Y$ and the sign of the savings wedge $\overline{\tau}_S$. They derive from five behavioral elasticities (two income effects and three substitution effects with respect to changes in income or savings taxes) and the Pareto coefficients of earnings and either consumption or savings.

Without additional structure, all five of these behavioral responses are necessary since they play different roles in the optimal tax formulas: The behavioral responses of earnings ($\bar{\zeta}^I_Y$,$\bar{\zeta}^H_{Y,\tau_Y}$) determine the trade-off between labor supply and after-tax income in the first stage of the tax design problem, and hence the value of $\overline{\tau}^{Saez}_Y$. The behavioral responses of savings ($\bar{\zeta}^I_S$,$\bar{\zeta}^H_{S,\tau_Y}$) determine the spill-over from income taxes to savings, Φ_S. Finally one additional substitution effect with respect to the savings tax is required to fully identify the tax-arbitrage coefficient Σ.

In *Cases 1* and *3*, optimal taxes only depend on the static wedge $\overline{\tau}^{Saez}_Y$ and the tax-arbitrage coefficient Σ. A similar result obtains for *Case 2* if preferences are weakly separable w.r.t. Y:

Corollary 1. *In Case 1 ($\bar{s}_S = 0$), optimal income and savings taxes satisfy*

$$\overline{\tau}_Y \;=\; \overline{\tau}^{Saez}_Y \quad \textit{and} \quad \overline{\tau}_S \;=\; \frac{\Sigma\, \overline{\tau}^{Saez}_Y}{1-(1+\Sigma)\,\overline{\tau}^{Saez}_Y}. \tag{18}$$

In Case 3 ($\bar{s}_C = 0$), optimal income and savings taxes satisfy

$$\overline{\tau}_Y \;=\; \frac{1}{1+\Sigma}\cdot \overline{\tau}^{Saez}_Y \quad \textit{and} \quad \overline{\tau}_S \;=\; \frac{\Sigma}{1+\Sigma}\cdot\frac{\overline{\tau}^{Saez}_Y}{1-\overline{\tau}^{Saez}_Y}. \tag{19}$$

In Case 2, suppose that $\mathcal{E}_{CY} = \mathcal{E}_{SY}$. Then optimal income and savings taxes satisfy

$$\overline{\tau}_Y \;=\; \frac{\overline{\tau}^{Saez}_Y}{1+\bar{s}_S\,(\bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y})\,\Sigma} \quad \textit{and} \quad \overline{\tau}_S \;=\; \frac{\Sigma\,\overline{\tau}^{Saez}_Y}{1+\bar{s}_S\,(\bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y})\,\Sigma-(1+\Sigma)\,\overline{\tau}^{Saez}_Y}. \tag{20}$$

In *Case 1*, the tax revenue spillover vanishes while in *Case 3* it becomes so large as to equal $1/\overline{\tau}^{Saez}_Y$. In both cases, it does not need to be separately identified from $\overline{\tau}^{Saez}_Y$, and therefore the identification problem is summarized by the value of $\overline{\tau}^{Saez}_Y$ and the tax-arbitrage coefficient Σ. In *Case 2*, the revenue-spillover coefficient Φ_S needs to be separately identified, but $\Phi_S\overline{\tau}^{Saez}_Y$ equals $\bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ or $\bar{\zeta}^I_S/(1-\bar{\zeta}^I_Y)$ whenever preferences are weakly separable with respect to earnings. When $\mathcal{E}_{CY} > \mathcal{E}_{SY}$, we obtain $\Phi_S\overline{\tau}^{Saez}_Y \in [\bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y},\, \bar{\zeta}^I_S/(1-\bar{\zeta}^I_Y)]$ and equation (20) provides an upper bound for both $\overline{\tau}_Y$ and $\overline{\tau}_S$. When $\mathcal{E}_{CY} < \mathcal{E}_{SY}$, equation (20) provides a lower bound for $\overline{\tau}_Y$ and $\overline{\tau}_S$.[18]

[18]The opposite bounds for $\overline{\tau}_Y$ and $\overline{\tau}_S$ are obtained by replacing $\bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ with $\bar{\zeta}^I_S/(1-\bar{\zeta}^I_Y)$ in Equation (20).

2.7 Simplifying the Representation

Flynn, Schmidt, and Toda (2023) show that for a wide range of consumption-savings problems, comparative statics w.r.t. changes in rates of return and other primitives depend on the elasticity of intertemporal substitution and an index of non-homotheticity of preferences. In this section, we show that the same insight applies here, and use it to further simplify the optimal tax formulas of Theorem 2 and Corollary 1 by expressing the consumption and savings responses that appear in Σ in terms of these two statistics, for which we have readily available empirical estimates. We discuss *Case 3* here, and *Cases 1 and 2* in the Appendix.

Identifying Preference Elasticities. The two-stage representation of the tax design problem in section 2.5 implies

$$\frac{1}{\bar{\zeta}^H_{Y,\tau_Y}} = \hat{\mathcal{E}}_Y + \widehat{RA}, \quad \text{where} \quad \widehat{RA} \equiv \frac{1}{\bar{s}_S/\hat{\mathcal{E}}_S + \bar{s}_C/\hat{\mathcal{E}}_C}. \tag{21}$$

Therefore, the three preference elasticities $(\hat{\mathcal{E}}_Y, \hat{\mathcal{E}}_C, \hat{\mathcal{E}}_S)$ are fully identified from $(\bar{\zeta}^H_{Y,\tau_Y}, \bar{\zeta}^H_{C,\tau_Y}, \bar{\zeta}^H_{C,\tau_S})$ via equations (12) and (21) and the symmetry condition $\bar{\zeta}^H_{Y,\tau_S} = -\bar{s}_S \bar{\zeta}^H_{S,\tau_Y}$. Following Chetty (2006b), when preferences are completely separable between earnings, consumption, and savings (i.e., $\mathcal{E}_{CY} = \mathcal{E}_{SY} = \mathcal{E}_{CS} = \mathcal{E}_{SC} = 0$), then $\widehat{RA}$ equals $\bar{\zeta}^I_Y/\bar{\zeta}^H_{Y,\tau_Y}$ and represents the agents' *relative risk aversion coefficient* over after-tax income $M = Y - T_Y(Y)$, defined as $RA \equiv -\mathcal{U}_{MM}M/\mathcal{U}_M$, where $\mathcal{U}$ is the indirect utility function for the second-stage problem introduced in Section 2.5. For general preferences, $\widehat{RA}$ differs from RA and satisfies $\bar{\zeta}^I_Y/\bar{\zeta}^H_{Y,\tau_Y} = \widehat{RA} - \mathcal{K} = RA - \mathcal{K}'$, where $\mathcal{K}$ and $\mathcal{K}'$ are composite parameters that summarize the strength of preference complementarities between consumption, savings and earnings. In the empirically relevant *Case 3*, we have $\mathcal{K} = \mathcal{E}_{CS} + \mathcal{E}_{CY}$ and $\mathcal{K}' = \mathcal{E}_{SY}$.[19]

In addition, define the *elasticity of intertemporal substitution* (EIS)

$$EIS \equiv -\frac{\partial \ln(S/C)}{\partial \ln(1+\tau_S)}\bigg|_{Y,\mathcal{U}} = \frac{1}{\bar{s}_C\hat{\mathcal{E}}_S + \bar{s}_S\hat{\mathcal{E}}_C}.$$

The EIS governs how agents' consumption and savings decisions respond to marginal changes in the income tax, holding earnings constant. Since EIS , $\widehat{RA}$, and $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{S,\tau_Y} = \hat{\mathcal{E}}_S/\hat{\mathcal{E}}_C$ only depend on $(\hat{\mathcal{E}}_C, \hat{\mathcal{E}}_S)$, any two of these statistics fully identify $(\hat{\mathcal{E}}_C, \hat{\mathcal{E}}_S)$ along with the third, with $\hat{\mathcal{E}}_Y$ identified by

[19] More generally, we have $\mathcal{K} \geq \mathcal{K}' \geq 0$ if $\mathcal{E}_{CS} \geq 0$ (intertemporal preference complementarity, such as habit formation) and $\mathcal{E}_{CY} \geq \mathcal{E}_{SY} \geq 0$ (preference complementarity between earnings and consumption).

equation (21).[20] This allows us to restate our optimal tax formulae in terms of (i) the compensated labor supply elasticity $\bar{\zeta}^H_{Y,\tau_Y}$; (ii) the non-homotheticity $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$; and (iii) a scaling factor Λ that can be expressed as a function of either EIS, $\widehat{RA}$, $\hat{\mathcal{E}}_Y$, or $\mathcal{K}$.[21] The non-homotheticity and the scaling factor are identified from the consumption elasticities $(\bar{\zeta}^H_{C,\tau_Y}, \bar{\zeta}^H_{C,\tau_S})$.

Optimal Taxes in Case 3. In this case, the top earners face a static trade-off between earnings and savings, with consumption responses to tax changes determined by intertemporal substitution. We obtain the following characterization of optimal income and savings taxes:

Corollary 2. *In Case 3 ($\bar{s}_C = 0$), the optimal income and savings taxes satisfy*

$$\bar{\tau}_Y \;=\; \bar{\tau}_Y^{Saez}\left[1-\Lambda\left(1-\frac{\bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y}}{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y}}\right)\right] \quad \text{and} \quad \bar{\tau}_S \;=\; \frac{\bar{\tau}_Y^{Saez}}{1-\bar{\tau}_Y^{Saez}}\,\Lambda\left(1-\frac{\bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y}}{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y}}\right), \tag{22}$$

where the scaling factor Λ satisfies[22]

$$\Lambda \;=\; \bar{\zeta}^I_Y + \mathcal{K}\,\bar{\zeta}^H_{Y,\tau_Y} \;=\; \frac{\bar{\zeta}^H_{C,\tau_Y}}{EIS} \;=\; \frac{\bar{\zeta}^H_{C,\tau_Y}}{\bar{\zeta}^H_{C,\tau_Y}+\bar{\zeta}^H_{C,\tau_S}}.$$

Moreover, $\Lambda = \bar{\zeta}^I_Y$ if preferences are fully separable, and $\bar{\zeta}^H_{C,\tau_Y} = \bar{\zeta}^H_{Y,\tau_Y}$ if preferences are homothetic.

Corollary 2 clearly identifies the role of consumption moments for optimal taxes. They enter optimal income and savings taxes in three places: (i) the ratio of Pareto coefficients $\bar{\rho}_Y/\bar{\rho}_C$ that describes concentration of consumption vs. earnings and savings; (ii) the ratio of compensated elasticities $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ that identifies the non-homotheticity of preferences over consumption and savings; and (iii) the scaling factor Λ that captures the curvature of preferences w.r.t. consumption and savings and scales the optimal savings tax and the departure of $\bar{\tau}_Y$ from $\bar{\tau}_Y^{Saez}$. While the former is identified from $\bar{\zeta}^H_{C,\tau_Y}$, the latter requires one additional moment which can be obtained by direct estimates of the EIS or the consumption response to savings tax changes, $\bar{\zeta}^H_{C,\tau_S}$.[23]

[20] Since $(\widehat{RA}\cdot EIS)^{-1/2} = \bar{s}_C(\hat{\mathcal{E}}_S/\hat{\mathcal{E}}_C)^{1/2} + \bar{s}_S(\hat{\mathcal{E}}_C/\hat{\mathcal{E}}_S)^{1/2}$, we can infer the degree of non-homotheticity $\hat{\mathcal{E}}_C/\hat{\mathcal{E}}_S$ from $\widehat{RA}\cdot EIS$. Equivalently, we can infer $\widehat{RA}$ using estimates of EIS and the ratio of the compensated elasticities $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{S,\tau_Y}$. In *Case* 3, $\bar{s}_C = 0$ implies $EIS = 1/\hat{\mathcal{E}}_C$ and $RA = \hat{\mathcal{E}}_S$.

[21] Notice that $\widehat{RA}$, $\hat{\mathcal{E}}_Y$ and $\mathcal{K}$ are linked through equation (21) and $\widehat{RA} = \bar{\zeta}^I_Y/\bar{\zeta}^H_{Y,\tau_Y} + \mathcal{K}$.

[22] As described above, we can also express Λ in terms of $\widehat{RA}$ or $\hat{\mathcal{E}}_Y$ as follows: $\Lambda = \widehat{RA}\cdot\bar{\zeta}^H_{Y,\tau_Y} = 1-\hat{\mathcal{E}}_Y\bar{\zeta}^H_{Y,\tau_Y}$.

[23] Moreover, combining an estimate of EIS with estimates of $\widehat{RA}$, $\hat{\mathcal{E}}_Y$ or $\mathcal{K}$ also identifies the non-homotheticity, since $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y} = \widehat{RA}\cdot EIS$. Corollary 2 thus provides comparative statics of optimal taxes w.r.t. the underlying primitives: For given $\bar{\zeta}^H_{Y,\tau_Y}$, $\bar{\zeta}^I_Y$ and $\bar{\zeta}^H_{C,\tau_Y}$, the scaling factor Λ is increasing in $\widehat{RA}$ and $\mathcal{K}$ and decreasing in $\hat{\mathcal{E}}_Y$ and EIS. In turn, $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ is increasing in EIS, $\widehat{RA}$, and $\mathcal{K}$, and decreasing in $\hat{\mathcal{E}}_Y$. Therefore, if $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y} \geq \bar{\rho}_C/\bar{\rho}_Y$, higher values of $\widehat{RA}$ or $\mathcal{K}$, or lower values of $\hat{\mathcal{E}}_Y$ unambiguously increase the gap between $\bar{\tau}_Y$ and $\bar{\tau}_Y^{Saez}$ and the absolute magnitude of $\bar{\tau}_S$. For EIS, the comparative statics are ambiguous and depend on whether EIS is used to identify

The Cases of Complete Separability and Homotheticity. Moreover, with complete separability ($\mathcal{K}=0$), we get $\Lambda=\bar{\zeta}_Y^I$, and we only need one additional moment to identify $\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H$ and therefore determine optimal taxes. Likewise, if preferences are homothetic, $\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H=1$, and only one additional moment is required to determine the scaling factor Λ. If preferences are both homothetic and completely separable—a common assumption in the literature—then equation (22) simplifies even further as $\Lambda=\bar{\zeta}_Y^I$ and $\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H=1$. That is, the ratio of Pareto coefficients $\bar{\rho}_Y/\bar{\rho}_C$ is sufficient to fully identify optimal income and savings taxes, which mechanically revert to $\overline{\tau}_Y^{Saez}$ and 0, respectively, if $\bar{\rho}_Y=\bar{\rho}_C$.

The Role of Preference Complementarities. The role of preference complementarities $\mathcal{K}$ is embedded in the scaling factor $\Lambda=\bar{\zeta}_Y^I+\mathcal{K}\,\bar{\zeta}_{Y,\tau_Y}^H$. In *Case 3*, we have $\mathcal{K}=\mathcal{E}_{CS}+\mathcal{E}_{CY}$, which thus summarizes the impact of savings and earnings on the marginal utility of consumption. Given our identification of Λ from compensated elasticities, $\mathcal{K}$ is then identified from income effects on labor supply: $\bar{\zeta}_Y^I+\mathcal{K}\,\bar{\zeta}_{Y,\tau_Y}^H=\bar{\zeta}_{C,\tau_Y}^H/(\bar{\zeta}_{C,\tau_Y}^H+\bar{\zeta}_{C,\tau_S}^H)$. For a given income effect on labor supply $\bar{\zeta}_Y^I$, a positive value of $\mathcal{K}$ increases Λ, resulting in higher savings taxes and lower income taxes, relative to the baseline case with completely separable preferences ($\mathcal{K}=0$). These comparative statics follow the logic of Corlett and Hague (1953), according to which it is optimal to tax less heavily the good that is more complementary to earnings.

Moreover, if $\mathcal{K}\geq 0$ the scaling factor Λ satisfies $1\geq\Lambda\geq\bar{\zeta}_Y^I$. These two inequalities imply the following inequality restriction on $\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H$ and EIS:

$$\frac{1}{\bar{\zeta}_{Y,\tau_Y}^H}\cdot EIS\;\geq\;\frac{\bar{\zeta}_{C,\tau_Y}^H}{\bar{\zeta}_{Y,\tau_Y}^H}\;\geq\;\frac{\bar{\zeta}_Y^I}{\bar{\zeta}_{Y,\tau_Y}^H}\cdot EIS\quad\text{or}\quad\frac{\bar{\zeta}_{C,\tau_Y}^H}{\bar{\zeta}_Y^I}\;\geq\;EIS\;\geq\;\bar{\zeta}_{C,\tau_Y}^H. \tag{23}$$

In other words, if $\mathcal{K}\geq 0$, each estimate of EIS identifies an interval of admissible values for the non-homotheticity of preferences $\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H$, and each estimate of $\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H$ identifies an admissible range of values for EIS. In combination, these ranges define upper and lower bounds for the departure of optimal taxes from $\overline{\tau}_Y^{Saez}$ and 0. The higher is $\bar{\zeta}_Y^I$, the tighter are the bounds on Λ and therefore on the admissible range of values for EIS and $\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H$. We use these observations in Section 4 to present three different calibrations of our optimal taxes: one with homothetic and separable preferences which does not require any additional consumption moments other than

$\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H$, in which case it amplifies the departure of optimal taxes from $\overline{\tau}_Y^{Saez}$ and 0, or Λ, in which case it dampens it. Notice that risk aversion and intertemporal substitution play conceptually different roles for determining optimal income and savings taxes. The former governs the variation in marginal utilities of after-tax income and thus the strength of the redistribution motive. The latter governs the responses of consumption and savings to savings taxes. Both together identify the non-homotheticity in preferences that drives the magnitude of optimal savings distortions.

Pareto tails; one that matches the pass-through elasticity from permanent income to consumption; and one that matches empirical estimates of EIS obtained from the response of wealth accumulation to wealth tax changes.

3 Extensions

We now generalize our baseline results along several important dimensions, allowing for multiple commodities, multi-period life-cycle economies, heterogeneous initial endowments, multi-dimensional heterogeneity, heterogeneous rates of return, and dynamic stochastic economies. In all of our extensions, we assume suitable generalizations of Assumptions 1, 2, and 3, and we then show how to apply the logic of the no-arbitrage and revenue-spillover conditions, even if their exact form may change. Here, we describe the main results and leave detailed derivations to the Appendix.

3.1 General Preferences over Multiple Commodities

We first extend Theorem 2 to preferences $U(Y, \boldsymbol{C}; r)$ over efficiency units of labor Y, an arbitrary N-dimensional vector $\boldsymbol{C} = (C_1, ..., C_N)$ of consumption goods, and $r \in [0,1]$. To generalize Assumption 1, we define $U_n \equiv \partial U / \partial C_n$ and $U_{nr} \equiv \partial U_r / \partial C_n$ and assume that $U_{Yr}/U_Y < U_{nr}/U_n$ for all n and U_{nr}/U_n is increasing in n. Hence, $-U_Y/U_n$ and U_n/U_m are decreasing in r whenever $m > n$. Each good n is produced at a constant marginal cost of p_n efficiency units of labor.

We consider a general social welfare objective with both rank-dependent Pareto weights $\omega(\cdot)$ such that $\int_0^1 \omega(r)\,\mathrm{d}r = 1$ and an increasing and concave Bergson-Samuelson function $G(\cdot)$ that captures the planner's attitude towards inequality. The planner chooses a tax function $T(Y, \boldsymbol{C})$ that maximizes government revenue $\int_0^1 T(Y(r), \boldsymbol{C}(r))\,\mathrm{d}r$, subject to (i) the promise-keeping constraint $\int_0^1 \omega(r) G(U(Y(r), \boldsymbol{C}(r); r))\,\mathrm{d}r \geq W_0$ for some weighted utility promise W_0 (with associated Lagrange multiplier λ), and (ii) the incentive compatibility constraint $\{Y(r), \boldsymbol{C}(r)\} \in \arg\max_{Y,\boldsymbol{C}} U(Y, \boldsymbol{C}; r)$ s.t. $\sum_{n=1}^N p_n c_n + T(Y, \boldsymbol{C}) \leq Y$. We identify good 1 as the numéraire, and we let $\tau_Y \equiv 1 + U_Y/U_1$ and $t_n \equiv U_n/U_1 - 1$ for $n \geq 2$ denote the marginal income and consumption tax rates.

As in Section 2, we define the Pareto coefficients $\{\rho_Y, \rho_n\}$, spending shares $s_n = -C_n U_n/(U_Y Y)$, and preference elasticities $\{\mathcal{E}_Y, \mathcal{E}_n, \mathcal{E}_{nY}, \mathcal{E}_{nm}\}$. The preference elasticities are identified, up to one degree of freedom, from the income and substitution effects of earnings Y and the N different consumption goods with respect to N different tax changes. The latter can—in principle—be estimated as behavioral responses to income, price, or tax changes.[24] We let $\tilde{\mathcal{E}}_Y = \mathcal{E}_Y - \sum_{n=1}^N s_n \mathcal{E}_{nY} \rho_Y/\rho_n$

[24]Specifically with $N + 1$ different commodities (including efficiency units of labor in the count), there are

and $\tilde{\mathcal{E}}_n = \mathcal{E}_n - \mathcal{E}_{nY}\rho_n/\rho_Y - \sum_{k\neq n}\mathcal{E}_{nk}\rho_n/\rho_k$ for $n = 1, ..., N$.

We derive the generalized no-arbitrage and revenue-spillover conditions that link consumption taxes to income taxes in the Appendix. Solving these conditions yields the following characterization of top income and consumption taxes generalizing Theorem 2 to an arbitrary number of goods and arbitrary planner preferences:

Proposition 3. *The optimal income and consumption taxes are given by:*

$$\bar{\tau}_Y = \frac{1-\bar{\Gamma}}{1/\bar{\tau}_Y^{Saez} - \bar{\Gamma} + \sum_{n=2}^{N} \bar{s}_n \Phi_n \Sigma_n} \quad \textit{and} \quad \frac{\bar{t}_n}{1+\bar{t}_n} = \frac{(1-\bar{\Gamma})\Sigma_n}{1/\bar{\tau}_Y^{Saez} - 1 + \sum_{n=2}^{N} \bar{s}_n \Phi_n \Sigma_n}$$

where $\Sigma_n = \frac{\tilde{\mathcal{E}}_n/\rho_n - \tilde{\mathcal{E}}_1/\rho_1}{\tilde{\mathcal{E}}_1/\rho_1 + \tilde{\mathcal{E}}_Y/\rho_Y}$ *and* $\Phi_n = \bar{\rho}_Y \bar{\zeta}^H_{n,\tau_Y} + \bar{\zeta}^I_n$, *and* $\bar{\Gamma} \equiv \lambda \lim_{r\to 1} \omega(r) G'(U(r)) U_1(r)$.

These expressions have the same structure and interpretation as in the baseline model. Unfortunately, the representation of Σ_n in terms of compensated elasticities is less transparent with $N > 2$ than in the baseline model. Nevertheless, if preferences are separable in earnings ($\mathcal{E}_{nY} = 0$ for all n), we obtain that $\Phi_n \bar{\tau}_Y^{Saez} = \bar{\zeta}^H_{n,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$, generalizing Corollary 1. In addition, if preferences are completely separable ($\mathcal{E}_{nY} = \mathcal{E}_{nm} = 0$ for all n and $m \neq n$), we recover the same identifying relations $\tilde{\mathcal{E}}_n \zeta^H_{n,\tau_Y} = \tilde{\mathcal{E}}_1 \zeta^H_{1,\tau_Y} = 1 - \tilde{\mathcal{E}}_Y \zeta^H_{Y,\tau_Y}$ and $\tilde{\mathcal{E}}_Y \zeta^H_{Y,t_n} = -\tilde{\mathcal{E}}_1 \zeta^H_{1,t_n}$ as in the baseline model, which allows us to express $\bar{\tau}_Y$ and $\bar{t}_n$ only in terms of substitution and income effects and generalize the characterization of optimal income and savings taxes with completely separable preferences to the case with N commodities and arbitrary planner preferences.

3.2 Income and Savings Taxes over the Life Cycle

We now extend our baseline model to multiple periods to explore how income and savings taxes should vary with age. Suppose that households work and consume over a fixed number of periods indexed by $t = 0, \ldots, T$, before enjoying consumption during a final "retirement" period or equivalently leaving a bequest C_{T+1}. Their preferences are given by $\sum_{t=0}^{T} \beta^t U_t(Y_t, C_t; r) + \beta^{T+1} U_{T+1}(C_{T+1}; r)$, where the within-period utility function may vary deterministically with age (for example to capture age-dependence of labor productivity or preferences over consumption), but otherwise satisfies the same restrictions as in our baseline economy. Their initial rank is drawn prior to date $t = 0$ and is their private information. To simplify the analysis, we assume that U_t is separable between consumption and earnings, $U_{CY,t} = 0$ for $t = 0, \ldots, T$.

$(N+1)(N+2)/2$ independent preference elasticities, since the off-diagonal elements must satisfy $s_n\mathcal{E}_{nm} = s_m\mathcal{E}_{mn}$. They are identified from N independent income effects ζ^I_n and $N(N+1)/2$ independent substitution effects $\zeta^H_{n,k}$, accounting for symmetry of the Slutzky matrix and the usual adding-up constraints. The remaining degree of freedom accounts for the fact that income and substitution effects are invariant to monotone transformations of U.

Households and the planner can save at a fixed rate of return $R > 0$, and savings are assumed to be observable and taxable. The households' period-t budget constraint is $RS_{t-1} + Y_t - T_t(Y_t, S_t) \geq C_t + S_t$, where $T_t(Y_t, S_t)$ denotes the total tax payments collected in period t, with $S_{-1}(r)$ given and $C_{T+1} = RS_T$.[25] The planner chooses the sequence of tax functions $\{T_t(\cdot,\cdot)\}_{t=0}^{T}$ to maximize the present discounted value of tax revenue subject to incentive compatibility and a lower bound on the lowest rank's lifetime utility. As in Section 2, we define the age-dependent Pareto coefficients $\{\rho_{Y,t}, \rho_{C,t}\}$, within-period spending shares $\{s_{C,t}, s_{S,t}\}$, and the behavioral elasticities $\zeta^H_{X_s,\tau_{Y,t}}$, $\zeta^H_{X_s,\tau_{S,t}}$, $\zeta^I_{X,t}$ for $X \in \{Y, C, S\}$, where the income effects are in response to transfers within the same period. In addition, let $s_{Y,t} \equiv R^{-t}(1-\tau_{Y,t})Y_t / \sum_{s=0}^{T-1} R^{-s}(1-\tau_{Y,s})Y_s$ denote the share of period-t earnings in net life-time income.

Optimal (revenue-maximizing) income and savings taxes $\overline{\tau}_{Y,t} = \lim_{r\to 1} \partial T_t/\partial Y_t$ and $\overline{\tau}_{S,t} = \lim_{r\to 1} \partial T_t/\partial S_t$ on top income earners are linked through the no-arbitrage conditions

$$\frac{\overline{\tau}_{S,t}}{1+\overline{\tau}_{S,t}} = \Sigma_{S,t} \cdot \frac{\overline{\tau}_{Y,t}}{1-\overline{\tau}_{Y,t}} \quad \text{and} \quad \frac{\overline{\tau}_{Y,t+1}}{1-\overline{\tau}_{Y,t+1}} = \Sigma_{Y,t} \cdot (1+\overline{\tau}_{S,t}) \frac{\overline{\tau}_{Y,t}}{1-\overline{\tau}_{Y,t}},$$

$$\text{where } \Sigma_{S,t} = \frac{\frac{\bar{\rho}_{C,t}}{\bar{\rho}_{C,t+1}} \frac{\zeta^H_{C_t,\tau_{Y,t}}}{\zeta^H_{C_{t+1},\tau_{Y,t}}} - 1}{1 - \frac{\bar{\rho}_{C,t}\zeta^H_{C_t,\tau_{S,t}}}{\bar{\rho}_{Y,t}\zeta^H_{Y_t,\tau_{S,t}}}} \quad \text{and} \quad \Sigma_{Y,t} = \frac{\frac{\bar{\rho}_{C,t}}{\bar{\rho}_{C,t+1}} \frac{\zeta^H_{C_t,\tau_{Y,t}}}{\zeta^H_{C_{t+1},\tau_{Y,t}}} - \frac{\bar{\rho}_{C,t}}{\bar{\rho}_{Y,t+1}} \frac{\zeta^H_{C_t,\tau_{Y,t}}}{\zeta^H_{Y_{t+1},\tau_{Y,t}}}}{1 - \frac{\bar{\rho}_{C,t}}{\bar{\rho}_{Y,t}} \frac{\zeta^H_{C_t,\tau_{S,t}}}{\zeta^H_{Y_t,\tau_{S,t}}}}.$$

These conditions fully characterize optimal top income and savings taxes at all ages for a given value of $\overline{\tau}_{Y,0}$. As a direct consequence, we obtain the following result showing how optimal income and savings taxes should account for age-dependent preferences, income and consumption concentration:

Proposition 4. *Optimal savings taxes are positive (resp., zero, negative), i.e.* $\overline{\tau}_{S,t} \gtreqless 0$*, and income taxes are increasing (resp., constant, decreasing) in age, i.e.* $\overline{\tau}_{Y,t+1} \gtreqless \overline{\tau}_{Y,t}$*, whenever* $\bar{\rho}_{C,t}\zeta^H_{C_t,\tau_{Y,t}} \gtreqless \bar{\rho}_{C,t+1}\zeta^H_{C_{t+1},\tau_{Y,t}}$ *and* $\bar{\rho}_{Y,t}\zeta^H_{Y_t,\tau_{S,t}}/\zeta^H_{C_t,\tau_{S,t}} \gtreqless \bar{\rho}_{Y,t+1}\zeta^H_{Y_{t+1},\tau_{Y,t}}/\zeta^H_{C_t,\tau_{Y,t}}$.

Everything else equal, it is optimal to defer tax distortions to periods with higher levels of income and consumption concentration, or periods with lower elasticity of labor supply and higher risk aversion over consumption, where the gains from redistribution are higher and the costs of incentive distortions lower. Therefore, increasing age-dependent income and consumption concentration provide a rationale for savings taxes and age-dependent income taxes that increase with age. At the same time, if labor supply elasticities are U-shaped, and highest early and late in life, the above result suggests that optimal income taxes may be inverse U-shaped.[26]

[25] Letting S_{-1} depend on rank r allows us to incorporate initial heterogeneity in endowments or wealth.

[26] In contrast to the baseline model, the tax-arbitrage coefficient $\Sigma_{S,t}$ is based on a trade-off between current and

Next, we generalize the revenue-spillover condition using a perturbation of the income tax at any age t holding all other taxes constant:

$$\frac{\overline{\tau}_{Y,t}}{1-\overline{\tau}_{Y,t}} = \frac{\overline{\tau}_{Y,t}^{Saez}}{1-\overline{\tau}_{Y,t}^{Saez}}\left[1-\frac{\overline{\tau}_{S,t}}{1+\overline{\tau}_{S,t}}\bar{s}_{S,t}\Phi_{S_t,\tau_{Y,t}}-\sum_{s\neq t}\frac{\bar{s}_{Y,s}}{\bar{s}_{Y,t}}\left(\frac{\overline{\tau}_{S,s}}{1+\overline{\tau}_{S,s}}\bar{s}_{S,s}\Phi_{S_s,\tau_{Y,t}}+\frac{\overline{\tau}_{Y,s}}{1-\overline{\tau}_{Y,s}}\Phi_{Y_s,\tau_{Y,t}}\right)\right]$$

where $\Phi_{X_s,\tau_{Y,t}} \equiv \bar{\zeta}^H_{X_s,\tau_{Y,t}}\bar{\rho}_{Y,t} + \bar{\zeta}^I_{X,s}$ for $X \in \{Y, S\}$ denote the spillover coefficient from period-t income taxes to period-s savings and earnings, and $\overline{\tau}^{Saez}_{Y,t} = (1 + \bar{\zeta}^H_{Y_t,\tau_{Y,t}}\bar{\rho}_{Y,t} - \bar{\zeta}^I_{Y,t})^{-1}$ the tax that maximizes income tax revenue in period t without internalizing revenue spillovers. The dynamic model admits spillovers within and across periods to both income and savings tax revenue. With separable preferences, we have $\Phi_{S_s,\tau_{Y,t}}/\Phi_{Y_s,\tau_{Y,t}} = -\bar{\zeta}^I_{S,s}/\bar{\zeta}^I_{Y,s}$, and therefore $\Phi_{Y_s,\tau_{Y,t}} \leq 0 \leq \Phi_{S_s,\tau_{Y,t}}$ for all s: When $\overline{\tau}_{Y,t}$ increases, households substitute labor supply and earnings intertemporally from age t towards other ages, which increases income tax revenue in all other periods; this is reinforced by income effects of the tax increase. On the other hand, if preferences are separable, raising $\overline{\tau}_{Y,t}$ reduces savings in all periods. Hence, intertemporal substitution of labor supply generates negative spillovers towards savings but positive spillovers towards income in all other periods.

Because of this additional intertemporal substitution channel, the static tax formula $\overline{\tau}^{Saez}_{Y,t}$ no longer represents the appropriate optimal income tax benchmark in the absence of savings taxation. Define therefore $\{\overline{\tau}^{AS}_{Y,t}\}$ as the sequence of optimal income taxes when $\overline{\tau}_{S,t} = 0$ for all t in the revenue-spillover equation. These are the income taxes that internalize intertemporal revenue spillovers for income taxes; they are the relevant benchmark, rather than the static optimum $\overline{\tau}^{Saez}_{Y,t}$, to characterize the departure of optimal taxes from uniform commodity taxation in a dynamic context. We have $\overline{\tau}^{AS}_{Y,t} \geq \overline{\tau}^{Saez}_{Y,t}$ for all t if $\Phi_{Y_s,\tau_{Y,t}} \leq 0$ for all s, t, i.e., the intertemporal substitution of labor supply unambiguously raises optimal income taxes above the static optimum.

Proposition 5. *If preferences are separable, then $\overline{\tau}_{Y,t} \gtreqless \overline{\tau}^{AS}_{Y,t}$ and $\overline{\tau}_{S,t} \lesseqgtr 0$ for all t if and only if $\bar{\rho}_{C,t+1}\zeta^H_{C_{t+1},\tau_{Y,t}} \gtreqless \bar{\rho}_{C,t}\zeta^H_{C_t,\tau_{Y,t}}$ for all t.*

Proposition 5 extends the equivalence between uniform commodity taxation and optimal income taxation to a dynamic economy. The main difference with the baseline model is that the income tax benchmark for uniform commodity taxation changes to $\{\overline{\tau}^{AS}_{Y,t}\}$ to account for intertemporal

future consumption, not consumption and savings. The change in information rents that is relevant for the one-period savings distortion is $U_{Cr,t+1}/U_{C,t+1} - U_{Cr,t}/U_{C,t}$, which is different from the one that captures the change in marginal information rents over all future periods, unless $C_{t+1}(S_t, r)$ is independent of r, i.e. the cross-sectional variation in savings combined with the causal impact of savings on future consumption fully account for the cross-sectional variation in future consumption. This knife-edge condition is equivalent to requiring that $\bar{\rho}_{C,t+1}\zeta^H_{C_{t+1},\tau_{Y,t}} = \bar{\rho}_{S,t}\zeta^H_{S_t,\tau_{Y,t}}$.

substitution in labor supply. It follows as a direct consequence of Proposition 5 that income taxes exceed $\{\overline{\tau}^{AS}_{Y,t}\}$ and *a fortiori* $\{\overline{\tau}^{Saez}_{Y,t}\}$ whenever savings are subsidized, but revenue spillovers towards savings and earnings have opposite effects on optimal income taxes, and hence the comparison between $\{\overline{\tau}_{Y,t}\}$ and $\{\overline{\tau}^{Saez}_{Y,t}\}$ remains ambiguous whenever it is optimal to tax savings. In addition, departures from uniform commodity taxation must be uniform, i.e., either it is optimal to tax savings in all periods and lower the income tax, or it is optimal to subsidize savings in all periods and increase the income tax above $\overline{\tau}^{AS}_{Y,t}$.[27] In sum, the dynamic economy continues to admit a direct correspondence between the departure from uniform commodity taxation and the level of optimal income taxes, highlighting that positive taxation of savings should go hand in hand with a reduction in income taxes in all periods.

3.3 Multi-Dimensional Heterogeneity

In our baseline model we assumed that agents' intertemporal marginal rates of substitution comove perfectly with their labor productivity. We now extend our two key optimality conditions to a model with multi-dimensional heterogeneity. We adapt the perturbation-based proofs to allow for a non-degenerate joint distribution of income, consumption, and savings. For convenience, we consider tax functions that are asymptotically separable and linear in savings at the top, i.e., $T(Y,S) = T_Y(Y) + \tau_S S$ for income ranks r sufficiently close to 1. For now this is an *ad hoc* restriction on the set of admissible tax policies; below we provide conditions for the optimality of such an asymptotically affine tax structure.

Formally, let $r \in (0,1)$ denote an agent's rank in the income distribution, with $Y(r)$ denoting earnings at rank r. Let $\alpha \in (0,1)$ denote the rank of savings conditional on income, and let $S(r,\alpha)$ denote the savings at rank α, conditional on income rank r. By construction, r and α are mutually independent and uniformly distributed on $(0,1)$. For $X \in \{C,S\}$, let $X(r) \equiv \int X(r,\alpha)\, d\alpha$ denote the average savings and consumption at income rank r. We define the local Pareto coefficient of average savings and average consumption as $\rho_X(r) \equiv -\left(d\log X(r)/d\log(1-r)\right)^{-1}$. Importantly, ρ_S and ρ_C do not represent the Pareto coefficients of savings and consumption, but those of the part of savings and consumption that correlates with income, as $\rho_X(r)/\rho_Y(r) = d\log X(r)/d\log Y(r)$ are the elasticities of average savings and average consumption w.r.t. income. We also define the spending shares $s_S(r,\alpha) \equiv (1+\tau_S)\, S(r,\alpha)/\left((1-\tau_Y(r))\, Y(r)\right)$ and $s_C(r,\alpha) \equiv C(r,\alpha)/\left((1-\tau_Y(r))\, Y(r)\right)$, and the average spending shares $s_X(r) \equiv \int s_X(r,\alpha)\, d\alpha$. Finally, we define substitution and income effects as in Section 2. While $(\zeta^H_{Y,\tau_Y}, \zeta^H_{Y,\tau_S}, \zeta^I_Y)$ are func-

[27] Our characterization still applies when it is optimal to tax savings in some periods and subsidize them in others, but then the sign of revenue spillovers from income to savings taxes is no longer unambiguous.

tions of r only, $(\zeta^H_{C,\tau_Y}, \zeta^H_{C,\tau_S}, \zeta^I_C)$ and $(\zeta^H_{S,\tau_Y}, \zeta^H_{S,\tau_S}, \zeta^I_S)$ may depend on both r and α. We thus let $\zeta^I_X(r) \equiv \int \zeta^I_X(r,\alpha)(s_X(r,\alpha)/s_X(r))\,d\alpha$ denote the average income effects conditional on rank r and weighted by spending shares; the average substitution effects $(\zeta^H_{X,\tau_Y}, \zeta^H_{X,\tau_S})$ are defined analogously. We assume that all these average elasticities converge to finite constants as $r \to 1$.

With these definitions in place, we extend the no-arbitrage and revenue-spillover conditions to the multi-dimensional case as follows:

Proposition 6. *The optimal tax system with multi-dimensional heterogeneity must satisfy the no-arbitrage and revenue-spillover conditions (8) and (14), with tax-arbitrage and revenue-spillover coefficients Σ and Φ_S given by*

$$\Sigma = \frac{\bar{\zeta}^H_{C,\tau_Y}\bar{\rho}_C - \bar{\zeta}^H_{S,\tau_Y}\bar{\rho}_S}{\bar{\zeta}^H_{C,\tau_S}\bar{\rho}_C - \bar{\zeta}^H_{S,\tau_S}\bar{\rho}_S + \frac{\bar{\rho}_C}{\bar{\rho}_Y}\Upsilon} \quad \textit{and} \quad \Phi_S = \lim_{r\to 1}\int \frac{s_S(r,\alpha)}{s_S(r)}\left[\zeta^H_{S,\tau_Y}(r,\alpha)\,\rho_Y(r) + \hat{\zeta}^I_S(r,\alpha)\right]\mathrm{d}\alpha, \tag{24}$$

where $\Upsilon \equiv \lim_{r\to 1} Cov\{\hat{\zeta}^I_S(r,\alpha)\frac{s_S(r,\alpha)}{s_C(r)}, \frac{S_r(r,\alpha)}{S'(r)}|r\}$ and $\hat{\zeta}^I_S(r,\alpha) \equiv \mathbb{E}[\zeta^I_S(r',\alpha)\frac{s_S(r',\alpha)}{s_S(r,\alpha)}|r' > r]$. In Case 1 and Case 3, the (unrestricted) optimal tax policy is asymptotically affine, $\Upsilon = 0$, and Corollary 1 holds exactly given our adjusted definitions of behavioral elasticities and Pareto coefficients.[28]

The main differences with the analysis of Section 2 are in the adjusted definitions of elasticities and Pareto coefficients, which clarify that optimal income and savings taxes depend on the co-movement of consumption and savings with income, not on unconditional measures of consumption or savings inequality. The tax-arbitrage coefficient Σ includes an additional Υ term in the denominator, which appears if income effects on savings vary with α. But the sign of Σ, and hence of $\overline{\tau}_S$, is still determined by that of $\bar{\zeta}^H_{C,\tau_Y}\bar{\rho}_C - \bar{\zeta}^H_{S,\tau_Y}\bar{\rho}_S$, as in our baseline model. Moreover, in the empirically relevant *Case 3*, the formulas of Section 2 continue to hold exactly in the multi-dimensional setting.

Return Heterogeneity. Recent empirical evidence suggests that heterogeneous rates of return, whereby wealthier agents earn higher returns on their savings, are an important component of the observed concentration of wealth; see, e.g., Bach, Calvet, and Sodini (2020), Fagereng et al. (2020), and Gaillard et al. (2023). There are two potential sources of such heterogeneity: type dependence (returns depend on individual types, such as whether they are entrepreneurs or salaried workers), and scale dependence (returns increase with wealth, regardless of an individual's type).

[28]More generally, the optimal tax policy is indeed asymptotically affine only if income and substitution effects of savings for top earners are constant across α, i.e., $\lim_{r\to 1}\frac{\partial}{\partial\alpha}(s_S(r,\alpha)\,\zeta^H_{S,\tau_Y}(r,\alpha)) = \lim_{r\to 1}\frac{\partial}{\partial\alpha}\hat{\zeta}^I_S(r,\alpha) = \lim_{r\to 1}\frac{\partial}{\partial\alpha}\zeta^H_{S,\tau_S}(r,\alpha) = 0$. If this condition fails the tax authority could gain by tailoring the revenue spillover or tax-arbitrage perturbation to the strength of income and substitution effects of tax changes on savings.

Our baseline framework of Section 2 nests these cases assuming that returns depend on savings (scale dependence) or that they correlate perfectly with labor productivity (type dependence), while the extension to multi-dimensional heterogeneity naturally allows for ex-ante heterogeneity in returns and savings that does not perfectly correlate with labor income.

To see this, suppose that agents' preferences $u(Y, C, C_2; \theta)$ are defined over earnings Y, first period consumption C and second-period consumption or final wealth C_2, for some primitive type $\theta \in \Theta \subseteq \mathbb{R}^L$, with $L \geq 1$. Moreover let C_2 be defined by the agents' private returns on savings, $C_2 = R(S; \theta, \varepsilon) S$, where $R(S; \theta, \varepsilon)$ can depend on savings S to capture scale-dependence, on types θ, and a shock ε that captures ex post heterogeneity or idiosyncratic return risk that is uncorrelated with types and savings decisions (S, θ). Substituting for C_2 and taking expectations over ε defines an indirect utility function $U(Y, C, S; \theta) \equiv \mathbb{E}[u(Y, C, R(S; \theta, \varepsilon) S; \theta)]$ as a function of earnings, consumption, and initial savings, given type θ, as in our baseline model. This formulation maps directly into our baseline model if $L = 1$, or the multi-dimensional extension if $L > 1$ and preferences are continuous in $\theta \in \Theta$ with a smooth full-support distribution. It embeds two dimensions of heterogeneity—in labor productivity and in ex-ante returns—through the dependence on θ of the functions u and R. If labor productivity is correlated with expected returns to savings, R will appear to be increasing in the income rank r.

Our optimal tax formulas therefore continue to apply, except that $\bar{\rho}_S$ is defined with regards to initial savings, not final consumption.[29] In particular, in *Case 3*, the budget constraint implies $\bar{\rho}_S = \bar{\rho}_Y$. Since $1/\bar{\rho}_{C_2} = 1/\bar{\rho}_S + 1/\bar{\rho}_R$, where $\bar{\rho}_{C_2}$ and $\bar{\rho}_R$ denote respectively the Pareto coefficients of second-period consumption and rates of return, final wealth can have a thicker tail than labor income—as in the data. Moreover, since our optimal tax formulas are derived from behavioral responses of income, consumption, and savings to tax changes, they apply regardless of the underlying primitives that shape consumption-savings and labor supply decisions. Return heterogeneity (type- or scale-dependence) and co-variation of income and returns thus influence the relevant behavioral elasticities, but conditional on the latter they do not separately affect optimal tax design.[30]

[29] Similar arguments apply if we allow for type-dependent initial endowments $E(\theta)$. Assuming that income and savings are observable while consumption is treated as a residual, the model with endowments is nested by setting $U(Y, C, S; \theta) \equiv \mathbb{E}[u(Y, E(\theta) + C, R(S; \theta, \varepsilon) S; \theta)]$, where $E(\theta) + C$ is total consumption, and C represents the net expenditure on first-period consumption. In this case our formulas apply with $\bar{\rho}_C$ representing the Pareto coefficient on net consumption expenditure in the first period, although $\bar{s}_C$ may be negative if the top income earners consume less than their initial endowments. If endowments and labor productivity are perfectly correlated, this extension reduces to the model of Scheuer and Slemrod (2021), see Section 3.5 below for further discussion.

[30] Optimal tax formulas expressed in terms of ex-post wealth $\bar{\rho}_{C_2}$ rather than savings $\bar{\rho}_S$, such as Schulz (2021), must account for whether heterogeneity of returns is driven by scale- or type-dependence.

3.4 Inverse Euler Equation

We can finally link our results to the "Inverse Euler Equation" that emerges in dynamic Mirrleesian economies with stochastically evolving types (Golosov, Kocherlakota, and Tsyvinski 2003; Farhi and Werning 2013; Golosov, Troshkin, and Tsyvinski 2016). Except for Hellwig (2021), this literature abstracts from heterogeneity in preferences for savings and complementarities between consumption and labor, which rationalize savings taxes in our setting.

Suppose that there are two periods, in which the agent works and consumes, and preferences are additively separable across time. Second-period preferences over consumption C_2 and earnings Y_2 are given by $\beta u_2 (C_2, Y_2; r_2, r)$, where the second period rank $r_2 \in [0,1]$ is uniform and i.i.d. across agents and independent of the first period rank r. By making second-period preferences depend on the first-period rank, we are allowing for labor productivity to be persistent. First-period savings S generate a return $R > 0$. The social planner then sets second-period allocations $\{C_2(\cdot), Y_2(\cdot)\}$ to maximize $U_2(S,r) \equiv \beta \int_0^1 u_2(C_2(r_2,r), Y_2(r_2,r); r_2, r)\, \mathrm{d}r_2$ subject to incentive compatibility constraints $u_2(C_2(r_2,r), Y_2(r_2,r); r_2, r) \geq u_2(C_2(r_2',r), Y_2(r_2',r); r_2, r)$ for all $r_2, r_2' \in [0,1]$ and the break-even constraint $RS \geq \int (C_2(r_2,r) - Y_2(r_2,r))\, \mathrm{d}r_2$. That is, our indirect utility function over savings $U_2(S,r)$ stands for the expected second period utility at given savings S. We can then characterize the optimal labor distortions in both periods $(\tau_{Y,1}, \tau_{Y,2})$ and the wedge between first-period consumption and savings, τ_S, exactly as in our baseline model, using preferences over first-period earnings, consumption and savings defined by $U_1(Y_1, C_1; r) + U_2(S,r)$. In addition, a simple perturbation argument shows that

$$\frac{1}{(1+\tau_S)\, U_{1,C}} = \frac{1}{U_{2,S}} = \mathbb{E}\left[\frac{1}{\beta R\, u_{2,C}(r_2,r)} \cdot m(r_2,r)\right], \tag{25}$$

where we let $m(r_2,r) \equiv \exp\{\int_{1/2}^{r_2} (u_{2,Cr_2}/u_{2,C}) dr_2'\} / \mathbb{E}[\exp\{\int_{1/2}^{r_2} (u_{2,Cr_2}/u_{2,C}) dr_2'\}]$. Thus, the inverse marginal utility of first-period savings $1/U_{2,S}$ is equal to an expected inverse marginal utility of second-period consumption, weighted by an adjustment factor $m(r_2,r)$ that accounts for the non-separability of preferences between consumption and productivity; this adjustment factor is equal to 1 if $u_{2,Cr_2}/u_{2,C} = 0$. In that case, it is well known (see e.g. Golosov, Kocherlakota, and Tsyvinski 2003; Farhi and Werning 2013; Golosov, Troshkin, and Tsyvinski 2016) that the returns to savings must be distributed in proportion to inverse marginal utility so as to raise utilities from savings uniformly, leaving second period incentives to work unaffected by a marginal increase in savings. When instead $u_{2,Cr_2} \neq 0$, returns to savings instead require a further adjustment that takes into account the need to preserve second-period incentive compatibility.[31]

In other words, our characterization of optimal savings wedges naturally extends to a dynamic Mirrleesian economy, which now combines two separate rationales for taxing savings: the heterogeneity in intertemporal marginal rates of substitution or departure from uniform commodity taxation that is captured by τ_S, and the adverse incentive effect of savings that the Inverse Euler equation emphasizes by characterizing the marginal value of savings as a harmonic expectation of second-period marginal utilities. That being said, our identification for separable preferences directly identifies optimal income and savings taxes from Pareto coefficients and behavioral responses of earning, consumption and savings in period 1 without taking a stand on period 2 preferences. Just like the discussion of return heterogeneity, the 2-period model merely offers a structural interpretation of the reduced-form preferences over consumption and savings in period 1, without altering their identification or optimal tax design in the initial period.

We kept the present discussion deliberately simple by assuming that ranks were i.i.d. across time and agents and introducing persistence of types directly in preferences. Hellwig (2021) analyzes a dynamic Mirrleesian economy with arbitrary Markovian shock processes and non-separable preferences, generalizing the characterization of optimal savings wedges in equation (25). The main difference between the fully dynamic characterization and the representation of Theorem 2 is that the sufficient statistics required to compute optimal taxes are now based on the Pareto coefficients of earnings, consumption and savings conditional on the entire type sequence or earnings history.[32]

3.5 Relationship with the Previous Literature

Our results contribute to a recent literature that characterizes optimal savings taxes in terms of observable sufficient statistics. Closest to our work are Ferey, Lockwood, and Taubinsky (2024), henceforth FLT, and Scheuer and Slemrod (2021), henceforth SS. We discuss each paper in turn.

Relationship with Ferey, Lockwood, and Taubinsky (2024). Our paper and FLT's start from a similar two-period model of earnings, consumption and savings. The core intuition for savings distortions in both papers relies on moments that identify preference heterogeneity over savings by separating the cross-sectional variation of savings or consumption with income from the causal impact of income or tax changes on consumption and savings. FLT's analysis follows

[31] See Hellwig (2021) for a general derivation of this incentive adjustment to the Inverse Euler Equation.

[32] Brendon, Hellwig, and Maideu Morera (2024) further show that this characterization offers a rationale for savings subsidies if agents have precautionary savings motives and if productivity shocks are persistent. While inconsistent with our calibration of savings taxes for top income earners, this result may well be more intuitive for low income earners where precautionary savings motives tend to be the strongest.

Saez (2002) to express tax rates in terms of the comparison between the cross-sectional variation of savings with income and the causal effect of earnings on savings. In our case, this separation occurs instead through the comparison of Pareto coefficients and compensated elasticities.

There are, however, important differences, due to the fact that they use different tax perturbations, resulting in a different set of sufficient statistics.[33] While such choices should in principle be equivalent and matter only for interpretation purposes, they turn out to be critical for optimal income and savings taxes on top earners in the empirically relevant *Case 3*, for the following two reasons.

First, as we discussed earlier, if the saving share of income converges to 1, savings data—which include the sufficient statistics used in FLT—become redundant at the top and lose their informational content. The representation of optimal savings taxes in FLT is then no longer well identified for top earners; we show that it converges to a ratio of "0 over 0". Let $S(Y,r) \in \arg\max_S U(Y, Y-S-T(Y,S), S; r)$ denote the optimal savings of a household of rank r given income Y. The optimal savings tax in FLT (equation (19)) at rank r can be expressed as:

$$\frac{\tau_S}{1+\tau_S} = \frac{-\frac{\partial \ln S(Y,r)}{\partial \ln(1-r)}}{-\frac{\partial \ln S(Y,r)}{\partial \ln(1+\tau_S)}\Big|_{Y,T(Y,S)}} \cdot \Psi = \frac{\frac{1}{\rho_S} - \zeta^H_{S,\tau_Y}/\zeta^H_{Y,\tau_Y} \cdot \frac{1}{\rho_Y}}{s_C \cdot EIS} \cdot \Psi,$$

where the denominator represents a compensated elasticity of savings to savings taxes, holding constant the household's income Y and total tax burden $T(Y,S)$, and $\Psi \in (0,1)$ is a constant that depends on the marginal social welfare weights adjusted for income effects. The elasticity $\partial \ln S(Y,r)/\partial \ln(1-r)$ in the first equality captures the effect of preference heterogeneity on savings for a given income and corresponds to $s'_{het} \cdot (1-r)/S$ in FLT. The second equality follows after taking derivatives and expressing $\partial \ln S(Y,r)/\partial \ln(1-r)$ as the difference between the cross-sectional heterogeneity in savings, $1/\rho_S$, and the product of the cross-sectional heterogeneity in earnings, $1/\rho_Y$ and the causal effect of income on savings, $\partial \ln S(Y,r)/\partial \ln Y$, which corresponds to $s'_{inc} \cdot Y/S$ in FLT. The latter in turn is equal to $\zeta^H_{S,\tau_Y}/\zeta^H_{Y,\tau_Y}$.[34] In *Case 3*, the causal effect of earnings on savings, $\partial \ln S(Y,r)/\partial \ln Y = \zeta^H_{S,\tau_Y}/\zeta^H_{Y,\tau_Y}$, mechanically converges to 1, and the Pareto coefficients on savings and earnings also have identical limits. Since s_C also converges to 0, the numerator and the denominator in the above expression both converge to 0, even though their ratio

[33] Other differences include the fact that FLT focus on savings taxes, with income taxes only playing a secondary role in deriving optimal savings tax formulas. We instead emphasize the interaction between both taxes, and show how these interactions shape their joint design.

[34] Perturbing the households' FOC for savings $(1+\tau_S)U_C = U_S$ and the budget constraint as in Lemma 1 yields $\partial \ln S(Y,r)/\partial \ln Y = \hat{\mathcal{E}}_C/(s_S\hat{\mathcal{E}}_C + s_C\hat{\mathcal{E}}_S) = \zeta^H_{S,\tau_Y}/\zeta^H_{Y,\tau_Y}$. The perturbation we used to define EIS leads to $-\partial \ln S(Y,r)/\partial \ln(1+\tau_S)|_{Y,T(Y,S)} = s_C\, EIS$.

converges to a finite constant. Therefore, FLT's formula for savings taxes is not identified at the top. Instead, using the over-identifying restrictions to restate $\partial \ln S(Y,r)/\partial \ln(1-r)$ in terms of consumption data shows that the optimal savings tax is identified even in *Case 3*.[35]

Second, the tax perturbation FLT use to derive optimal savings taxes becomes perfectly colinear with their income tax perturbation at the top. Hence their tax perturbations only identify the *combined* wedge between earnings and savings, but cannot separately identify optimal top income and savings taxes. Consider a perturbation that raises the marginal savings tax for a small interval $[r, r+\partial r]$, holding τ_Y constant.[36] This perturbation results in an optimality condition for $\tau_S(r)$ that trades off the mechanical effect of the tax increase against the income and substitution effects on savings and the revenue spillovers to income taxes. In *Case 3*, savings mirror earnings at the top of the distribution, and therefore the optimality condition for top savings taxes thus reduces to $1-\bar{\tau}_Y^{Saez} = (1-\bar{\tau}_Y)/(1+\bar{\tau}_S)$, which is the same as the revenue spill-over condition (14) for optimal top income taxes.[37] To a first order, top earners only care about the combined tax wedge, not its decomposition into income and savings taxes. Therefore, a higher income tax leads to a one-for-one reduction in the optimal savings tax, and vice versa. By contrast, our no-arbitrage condition considers a shift from income to savings taxes or vice versa, while holding the combined wedge fixed at the top, thus allowing us to separately identify the optimal top income and saving taxes, even when consumption has a strictly thinner upper tail than savings.[38]

In sum, FLT's optimal tax formulas do not identify tax rates on top earners when $\bar{s}_C = 0$ and consumption has a thinner tail than income (*Case 3*); outside this case, their representation coincides with ours, although the papers emphasize different sufficient statistics. Our model-implied over-identifying restrictions allow us to shift between representations based on earnings, consumption and savings data, depending on which data sources are informative of optimal top tax rates.

Relationship with Scheuer and Slemrod (2021). SS consider a two period economy with type-dependent initial endowments $E(r)$ and derive a version of our no-arbitrage condition. We map the model of SS into our baseline model by defining $U(Y,C,S;r) \equiv u(Y, E(r)+C, S; r)$,

[35]The tax-arbitrage coefficient Σ in our formulas also converges to a ratio of "0 over 0" if it is estimated using income and savings data; see the first equality in equation (13). The over-identifying restrictions allow us to identify the tax-arbitrage coefficient using consumption moments, thus leading to the well-identified expression (8).

[36]The savings tax perturbation in FLT is slightly more complicated (see their equation (89)) and amounts to a linear combination of income and savings tax perturbations. This technical complication allows them to handle situations where income and savings are not co-monotonic; see footnote 47 in their Online Appendix. Imposing a co-monotonicity assumption allows us to focus on the simpler "Saezian" perturbation of the savings tax schedule.

[37]Formally, this optimality condition writes $1 = \frac{\bar{\tau}_S}{1+\bar{\tau}_S}(-\bar{\rho}_S\bar{\zeta}^H_{S,\tau_S} + \bar{s}_S\bar{\zeta}^I_S) + \frac{\bar{\tau}_Y}{1-\bar{\tau}_Y}(-\bar{\rho}_S\bar{\zeta}^H_{Y,\tau_S}/\bar{s}_S - \bar{\zeta}^I_Y)$. In *Case 3*, $\bar{\rho}_S = \bar{\rho}_Y$, $-\bar{\zeta}^H_{Y,\tau_S} = -\bar{\zeta}^H_{S,\tau_S} = \bar{\zeta}^H_{Y,\tau_Y}$, and $\bar{s}_S\bar{\zeta}^I_S = 1-\bar{\zeta}^I_Y$, and this condition reduces to $1-\bar{\tau}_Y^{Saez} = (1-\bar{\tau}_Y)/(1+\bar{\tau}_S)$.

[38]The colinearity problem in *Case 3* arises with any linear combination of income and savings tax perturbations unless the combined wedge $(1-\bar{\tau}_Y)/(1+\bar{\tau}_S)$ is kept constant in the limit.

where $E(r)+C$ is total first-period consumption, C represents the net expenditure on first-period consumption, and $u(Y,C_1,C_2;r)$ denote the agents' preferences over earnings and consumption in the two periods, for given rank r. The no-arbitrage condition (9)—expressed in terms of preference elasticities—then becomes

$$\frac{\bar{\tau}_S}{1+\bar{\tau}_S} \;=\; \frac{\bar{\tau}_Y}{1-\bar{\tau}_Y}\cdot\lim_{r\to 1}\frac{U_{Sr}/U_S-U_{Cr}/U_C}{U_{Cr}/U_C-U_{Yr}/U_Y} \;=\; \frac{\bar{\tau}_Y}{1-\bar{\tau}_Y}\cdot\frac{\frac{\hat{\mathcal{E}}_S}{\bar{\rho}_S}-\left(\frac{1}{\bar{\rho}_C}-\frac{E}{C_1}\frac{1}{\bar{\rho}_E}\right)\hat{\mathcal{E}}_C}{\left(\frac{1}{\bar{\rho}_C}-\frac{E}{C_1}\frac{1}{\bar{\rho}_E}\right)\hat{\mathcal{E}}_C+\frac{\hat{\mathcal{E}}_Y}{\bar{\rho}_Y}}$$

where the preference elasticities $\hat{\mathcal{E}}_C$, $\hat{\mathcal{E}}_S$, and $\hat{\mathcal{E}}_Y$ are defined as in the baseline model, and $\bar{\rho}_C$, $\bar{\rho}_S$, $\bar{\rho}_Y$ and $\bar{\rho}_E$ refer to the Pareto coefficients of first-period consumption, savings, earnings, and endowments. In addition, the budget constraint yields the over-identifying restriction $\min\{\bar{\rho}_E,\bar{\rho}_Y\}=\min\{\bar{\rho}_C,\bar{\rho}_S\}$. SS further assume that preferences $u(Y,C_1,C_2;r)$ are separable and homothetic. These restrictions imply that $\hat{\mathcal{E}}_C=\hat{\mathcal{E}}_S=1/EIS=\sigma$, $\bar{\rho}_C=\bar{\rho}_S=\bar{\rho}_E$, and $1/\hat{\mathcal{E}}_Y$ equals the Frisch elasticity of labor supply. We thus obtain Corollary 1 of SS, which states that

$$\frac{\bar{\tau}_S}{1+\bar{\tau}_S} \;=\; \frac{\bar{\tau}_Y}{1-\bar{\tau}_Y}\cdot\frac{\frac{\sigma}{\bar{\rho}_E}\frac{E}{C_1}}{\frac{\hat{\mathcal{E}}_Y}{\bar{\rho}_Y}+\left(1-\frac{E}{C_1}\right)\frac{\sigma}{\bar{\rho}_E}},$$

Using standard values of $\sigma=1$, $1/\hat{\mathcal{E}}_Y=0.3$, $\bar{\rho}_E=1.4$, $\bar{\rho}_Y=1.6$, and approximating E/C_1 by $1+\beta\approx 1.4$, SS provide an estimate of the tax-arbitrage coefficient of 0.56. By assuming homotheticity and separability, the only rationale for taxing savings stems from the information rents associated with type-dependent endowments.

This calibration matches the observation that wealth is more concentrated than income, but it also implies—counterfactually—that first-period consumption is as concentrated as wealth and that the ratio of endowments to earnings E/Y grows arbitrarily large as $r\to 1$ (see footnote 43 for evidence on the latter). However, if first-period consumption is strictly less concentrated than endowments and earnings ($\bar{\rho}_E<\bar{\rho}_Y<\bar{\rho}_C$), then $\lim_{r\to 1}E/C_1=\infty$. But then we obtain that $\lim_{r\to 1}(1-r)(U_{Sr}/U_S-U_{Cr}/U_C)=\infty$ and $\lim_{r\to 1}(1-r)(U_{Cr}/U_C-U_{Yr}/U_Y)=-\infty$ whenever $\hat{\mathcal{E}}_C>0$, so that the single-crossing condition for labor supply is no longer satisfied. Hence it becomes optimal in the limit to have arbitrarily large savings taxes, along with large income subsidies. The reason behind this surprising result is that the richest agents obtain most of their wealth from endowments, hence savings taxes become the main instrument for taxation, and wealth effects on labor supply become so strong that the richest agents also have the lowest inclination to work, even though they are the most productive—implying that labor supply should be subsidized.

Moreover the optimal tax formula is written in terms of preference (rather than behavioral) elasticities. When E/Y and s_C or s_S (or both, if preferences are homothetic) grow arbitrarily large as $r \to 1$, ζ^H_{Y,τ_Y} and ζ^H_{C,τ_Y} or ζ^H_{S,τ_Y} (or both) converge to zero. In other words, the compensated elasticities w.r.t. income tax changes lose their power to identify the preference elasticities for consumption and savings and determine whether preferences are indeed homothetic, as assumed by SS. This is intuitive: If endowments are more concentrated than earnings, the richest agents receive virtually all of their wealth through endowments, and their consumption or savings decisions no longer respond to income tax changes.

4 Calibration

4.1 Parameter Choices

Pareto coefficients ($\bar{\rho}_Y, \bar{\rho}_C$). In Gaillard et al. (2023), we apply a statistical procedure to formally estimate these Pareto coefficients and evaluate the estimated Pareto fit against alternatives. We document that income and consumption both have Pareto tails and estimate their respective coefficients in the U.S. using the 2005 to 2021 waves of the PSID. Evidence of stable Pareto coefficients already appear around the top 10%, and is therefore not restricted to the very top of the distribution. Our model features a unique labor income source and is not designed to fully account for income and wealth concentration at the very top (such as business income, entrepreneurship, capital gains, etc.). As such, it may thus be best interpreted as applying to the "rich", i.e., high-income working professionals in the top 10% of the population, but not the "super-rich" (the top 1% or 0.1%). Our characterization of optimal income and savings taxes remains valid within the top 5-10% if the relevant sufficient statistics are stable within that range.

We find an estimate of the Pareto coefficient for total income equal to $\bar{\rho}_Y = 1.8$ on average across years. The average estimate of the Pareto coefficient on consumption is much larger, $\bar{\rho}_C = 3.1$. Gaillard et al. (2023) show that this result is remarkably robust.[39] We further document that the consumption tail has a similar Pareto coefficient when we restrict the sample to working-age and retired households separately, while the Pareto coefficient for bequests is much lower and close to that of wealth. In light of these findings, there is little doubt that the relevant empirical scenario

[39] In particular, we corrected for under-reporting, performed multiple robustness checks, and considered alternative data sources that provide more direct information on spending patterns of the richest households. Under-sampling of the richest households would result in an upwards bias of the estimated Pareto coefficients, but to the extent that this affects income and consumption equally within a given sample of households, it would not affect the conclusions about relative magnitudes of consumption vs. income concentration, or the ratio $\bar{\rho}_Y/\bar{\rho}_C$. This is consistent with our finding in Gaillard et al. (2023) that $\bar{\rho}_Y/\bar{\rho}_C$ is fairly stable across a range of diverse data sources. Moreover, estimates of Pareto coefficients from income and wealth in the PSID are similar to their analogues from US administrative data, which suggests that under-sampling is not a major concern within the PSID.

is *Case 3*, in which consumption has a substantially thinner tail than labor income, $\bar{\rho}_C > \bar{\rho}_Y$, and the consumption share of income s_C converges to 0 as $r \to 1$.[40,41]

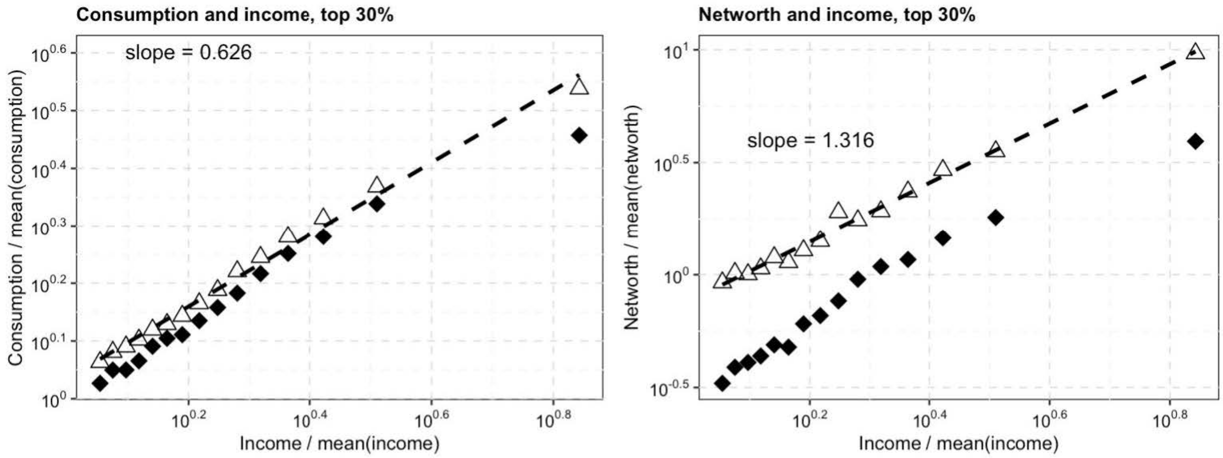
Our baseline model imposes perfect co-monotonicity between earnings and consumption, but the multidimensional generalization of Section 3.3 clarifies that the relevant Pareto coefficients are those of *average* consumption and savings conditional on income. In Figure 1, we therefore compute the mean consumption and net worth of workers within each income quantile. Such averaging removes the variation of consumption and net worth conditional on income rank. The figure reports average consumption and net worth within the 71^{st}; 73^{rd}; 75^{th}; $\ldots$; 99^{th} quantiles on income, and pools together all the years of our data. If the distribution of total income has a Pareto tail, the log-linear relationship between income, on the one hand, and average consumption or average net worth, on the other hand, implies that the latter also admit Pareto tails. Moreover, the slope of the relationship yields an estimate of the ratio of their Pareto coefficients. For consumption, the estimated slope of $\bar{\rho}_Y/\bar{\rho}_C = 0.626$ corresponds to a Pareto coefficient for average consumption equal to $\bar{\rho}_C = 2.88$ when $\bar{\rho}_Y = 1.8$, only slightly below the unconditional Pareto coefficient for consumption of 3.1.

For net worth, the slope estimate of $\bar{\rho}_Y/\bar{\rho}_W = 1.316$ implies, for $\bar{\rho}_Y = 1.8$, a Pareto coefficient of average net worth of $\bar{\rho}_W = 1.37$, which is close to the unconditional Pareto coefficient for wealth of 1.4 reported in Gaillard et al. (2023), and well below the Pareto coefficient for total income (or savings, by virtue of the model's over-identifying restrictions) of 1.8. Expressing average wealth, or second-period consumption, $W(r) = S(r)R(r)$ as the product of average savings and gross returns, the Pareto coefficients of wealth and savings can be reconciled by an elasticity of average returns to income of $\partial \ln R(r)/\partial \ln Y(r) = 0.316$, or a Pareto coefficient on average gross returns of 5.7. This elasticity implies that as household income is doubled, average compounded returns must increase by 13% over a 25-year horizon, and average annual returns increase by roughly 50

[40] These results are consistent also with findings in Toda and Walsh (2015) and Buda et al. (2022).

[41] PSID consumption may not accurately measure the spending patterns of rich and super-rich households, given that we have limited or no information on such items as collectibles or political contributions. The only spending category we could find that is missing from PSID consumption and may be an important share of the richest households' budgets is bequests, which are as concentrated as wealth. Hence it may be reasonable to interpret our non-homothetic preference structure over savings as capturing a warm-glow bequest motive, as in, e.g., De Nardi (2004), Straub (2019), and Daminato and Pistaferri (2024). Alternatively, since "savings" enters as a residual category in our budget constraint, one may also interpret our model as breaking household spending into two bundles, one that is measured as "PSID consumption", and a residual spending category that includes both savings and unmeasured consumption. Our savings tax then represents a wedge between PSID consumption and the residual spending category. This alternative interpretation is especially relevant if PSID consumption data are also used to estimate behavioral responses. If, for instance, residual spending includes political contributions, a value of $\bar{\tau}_S > 0$ suggests for example that the latter should not be exempt from income taxes, but quite the contrary, taxed at a higher rate than regular income. This observation follows directly from the principle that "luxury" goods that cater to the tastes of the richest households should be taxed at a higher rate than regular consumption.

Figure 1: Ratio of Pareto coefficients: consumption (left) and net worth (right) vs. total income



Note: Average (respectively, median) consumption and net worth for a given income level are represented by the white triangles (resp., black diamonds).

basis points. While we are not aware of studies estimating the elasticity of average returns to income, the variation of average returns that is required to close the gap between concentration of income and wealth appears to be well within the range of return variation by wealth that has been documented in the empirical literature (Bach, Calvet, and Sodini 2020; Fagereng et al. 2020; Gaillard et al. 2023).[42] Moreover, defining capital income as $(R(r) - 1) S(r)$, the model-implied Pareto coefficient for capital income is equal to $(R-1)\bar{\rho}_Y/(R-1+R\bar{\rho}_Y/\bar{\rho}_R)$. For a 4% average annual return compounded over 25 years, we obtain a Pareto coefficient for capital income of 1.2 which is almost exactly the same as the value estimated by Gaillard et al. (2023). This back-of-the-envelope calculation suggests that we don't require implausible return heterogeneity and wealth-dependent returns to close the gap between income and wealth inequality when wealth is interpreted as second-period consumption. In other words, a stylized model that abstracts from initial wealth heterogeneity (such as bequests received by age 30, say) still captures the first-order mechanisms leading to the observed levels of wealth inequality in the overall population.[43]

We set $\bar{\rho}_Y = \bar{\rho}_S = 1.8$ and $\bar{\rho}_C = 3$ $(\bar{\rho}_Y/\bar{\rho}_C = 0.6)$ in our baseline calibration, which falls

[42] Daminato and Pistaferri (2024) document a strongly positive rank correlation between earnings and returns, and their estimates also suggest significant variation in returns by income within the top 20%.

[43] This will be the case if the share of financial wealth in total wealth (i.e., the sum of financial and human wealth, where the latter is defined as the net present value of future labor income) is small at age 30. Recent empirical evidence evidence suggests that over a long horizon (i.e., 25 to 30 years) heterogeneous labor income, savings rates and asset returns are indeed the main source of wealth accumulation, not initial financial wealth or inheritance. For example, Black et al. (2024) estimate that inheritances and gifts constitute a small fraction of lifetime resources using administrative data from Norway and remains less important than labor income even for households with parents in the top 0.1% of the wealth distribution. Halvorsen et al. (2023) estimate that inheritances and transfers account for 34% of the excess wealth of the top 0.1% at age 50 and significantly less even within the top $1-10\%$. Feiveson and Sabelhaus (2019) and Bauluz and Meyer (2024) reach similar conclusions using the Survey of Consumer Finances.

between the unconditional estimate of $\bar{\rho}_C = 3.1$, and the estimate of $\bar{\rho}_C = 2.88$ of the Pareto coefficient for average consumption. For robustness, we also consider $\bar{\rho}_Y/\bar{\rho}_C = 0.7$ ($\bar{\rho}_C = 2.57$), a very conservative lower bound to account for potential under-reporting of consumption, and $\bar{\rho}_Y/\bar{\rho}_C = 0.5$ ($\bar{\rho}_C = 3.6$) to provide a range of possible estimates for income and savings taxes.

Taxable income elasticities ($\zeta^H_{Y,\tau_Y}, \zeta^I_Y$). In *Case 3*, the substitution and income effects on labor supply ($\zeta^H_{Y,\tau_Y}, \zeta^I_Y$) identify the combined wedge between income and savings, $\overline{\tau}^{Saez}_Y$. A vast literature estimates the elasticities of taxable income with respect to marginal tax rates and lump-sum transfers. The meta-analysis of Chetty (2012) yields a preferred estimate of the Hicksian elasticity of $\zeta^H_{Y,\tau_Y} = 0.33$, while Gruber and Saez (2002) estimate a value of $\zeta^H_{Y,\tau_Y} = 0.5$ for top income earners. Empirical evidence about the size of the income effects ζ^I_Y is mixed (see, e.g., Keane 2011). Gruber and Saez (2002) find small income effects, while Golosov, Graber, et al. (2021) estimate that \$1 of additional unearned income reduces the pre-tax income by 67 cents in the highest income quartile, which for a top marginal tax rate of 50% translates into an income effect of 0.33. In a large-scale field experiment on universal basic income, Vivalt et al. (2024) estimate an income elasticity of earnings of $\zeta^I_Y = 0.28$ for low income households, which combines both intensive and extensive margin responses.[44] For our baseline calibration, we choose $\zeta^H_{Y,\tau_Y} = 1/3$ for the Hicksian elasticity and $\zeta^I_Y = 1/4$ for the income effect. With an income Pareto coefficient of $\bar{\rho}_Y = 1.8$, these values yield $\overline{\tau}^{Saez}_Y = 74.1\%$. We evaluate the robustness of our quantitative results to the alternative parameter values $\zeta^H_{Y,\tau_Y} = 1/2$ (so that $\overline{\tau}^{Saez}_Y = 60.6\%$) and $\zeta^I_Y = 1/3$ (so that $\overline{\tau}^{Saez}_Y = 78.9\%$).

With completely separable preferences, these baseline values imply a scaling factor of $\Lambda = \zeta^I_Y = 0.25$. If, moreover, preferences are homothetic, Corollary 2 then implies an optimal top income tax rate of $\overline{\tau}_Y = 66.7\%$ and an optimal savings tax rate of $\overline{\tau}_S/(1+\overline{\tau}_S) = 22.2\%$, increasing the top earners' after-tax income by 29 percent but shifting the corresponding tax burden onto savings. These estimates however abstract from the role of non-homothetic or non-separable preferences. We need to turn to the consumption responses to tax changes to determine how these additional channels modify the optimal tax rates prescribed by our model.

Challenges of identifying consumption elasticities. A large literature estimates the consumption responses to permanent income, wage, or rate of return changes net of taxes, in order to assess available risk-sharing or intertemporal substitution channels. Three challenges arise in

[44] Reporting on a similar field experiment run in Germany, Bohmann et al. (2025) report negligible effects of unconditional transfers on participation and hours worked, and small effects on labor earnings.

translating these estimates to our context. First, they typically focus on uncompensated consumption responses, while our optimal tax formulas are based on compensated elasticities. If preferences display a stronger complementarity of earnings with consumption than savings ($\mathcal{E}_{CY} \geq \mathcal{E}_{SY}$), as commonly assumed, then we can show that

$$\frac{\zeta^H_{C,\tau_Y}}{\zeta^H_{Y,\tau_Y}} \;\geq\; \frac{\zeta^H_{C,\tau_Y} + \zeta^I_C}{1 - \zeta^I_Y + \zeta^H_{Y,\tau_Y}} \;\geq\; \frac{\zeta^I_C}{1 - \zeta^I_Y}.$$

Hence, the ratio of income effects $\zeta^I_C/(1-\zeta^I_Y)$ or the uncompensated pass-through elasticity $(\zeta^H_{C,\tau_Y} + \zeta^I_C)/(1-\zeta^I_Y+\zeta^H_{Y,\tau_Y})$ provide lower bounds on $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$. Moreover these bounds hold with equality when preferences are weakly separable in earnings ($\mathcal{E}_{CY} = \mathcal{E}_{SY}$).

Second, $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ and EIS cannot be chosen independently if preferences admit complementarities ($\mathcal{K} \geq 0$). Yet, while many studies estimate each of these statistics, few estimate them jointly, and there is no guarantee that independent estimates of both statistics are consistent with the additional restriction (23) implied by $\mathcal{K} \geq 0$. In the sequel, we therefore propose two different calibrations based on estimates of $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ and EIS, respectively, while picking the other statistic to be consistent with complete separability for our baseline quantification.

Third, and most importantly, we need to identify the role of non-homotheticity independently of preference heterogeneity, as this is the key to identifying the departure of optimal taxes from Uniform Commodity Taxation. For example, structural estimates that impose homotheticity of preferences as part of the model (such as Heathcote, Storesletten, and Violante 2014) impose that the pass-through elasticity from uninsurable after-tax income shocks to consumption equals 1 regardless of the underlying risk preferences, and their estimates of insurable vs. uninsurable income risk are then mechanically driven by the cross-sectional variation in consumption and earnings that is captured by our ratio of Pareto tail coefficients. In other words, their overall pass-through of permanent income shocks to consumption leads to estimates close to the ratio of Pareto coefficients, while their conditional pass-through of uninsurable permanent income shocks is biased towards homotheticity. Similarly, unobserved preference heterogeneity and correlation between permanent income and preferences for savings biases cross-sectional or panel estimates towards the ratio of Pareto tail coefficients $\bar{\rho}_Y/\bar{\rho}_C$.[45]

[45]For the same reason, we also do not introduce specific functional form assumptions on preferences, but prefer to calibrate the relevant elasticities directly from behavioral responses to income and tax changes. For example, the preference structure that underlies the widely used estimates of French (2005) is incompatible with s_C converging to zero and the ratio between consumption and income Pareto coefficients unless labor supply is completely inelastic at the very top, contradicting empirical evidence, or this gap is mechanically attributed to preference heterogeneity. But in that case the calibration would again hard-wire conclusions about optimal savings taxes into the model assumption rather than identifying how much of this gap is attributable to preference heterogeneity rather than income elasticities.

We therefore favor estimates that exploit time-series variation of consumption in response to exogenous income changes at the household level, or estimates from structural models that explicitly allow for non-homothetic preferences *and* household-level heterogeneity in savings behavior.

Consumption responses to earnings shocks ($\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$). Unfortunately, direct evidence on consumption responses of top earners is scarce and complicated by the fact that the top earners' MPCs must converge to 0 if $\bar{\rho}_Y/\bar{\rho}_C < 1$.[46] For low-to-medium income households, the estimates of consumption responses to unconditional transfers in the universal basic income experiment of Bartik et al. (2024) and Vivalt et al. (2024) suggest income effects on consumption ζ^I_C of at least 0.65. Coupled with their estimate of the income effect on earnings of $\zeta^I_Y = 0.28$, we obtain a lower bound for $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ of 0.90. Bohmann et al. (2025) report consumption and savings responses to income transfers for the German UBI experiment that imply a pass-through elasticity of permanent income changes to consumption of 0.75.[47] These pass-through elasticities are consistent with panel estimates that explicitly try to control for preference heterogeneity and correlation between time preferences and permanent income. Straub (2019) estimates a pass-through elasticity of 0.7 instrumenting for permanent income with quasi-differenced future income realizations (if the income process is persistent but stationary) or the first available income observation (with non-stationary processes). Lins (2025) estimates a pass-through elasticity of 0.8 when including education and other proxies to control for heterogeneity in time preferences.[48] Blundell, Pistaferri, and Saporta-Eksten (2016) estimate Marshallian consumption and labor supply elasticities to permanent wage changes in a two-earner unitary household model exploiting the within-household co-movement of spousal labor supply decisions. Since a change in marginal income taxes simultaneously raises the net wage of both household members, we can aggregate their individual elasticity estimates (Table 5) to the household level and find a pass-through elasticity of 0.79.

[46] Field experiments and panel studies that estimate MPCs do not focus specifically on the top of the distribution. Applying these estimates requires a leap of faith that results can be extrapolated from average or below-average income households to the top. Moreover, MPC estimates need to be translated into elasticities (of consumption to permanent income changes). This is not a major concern for average households whose share of consumption spending out of after-tax income is observed and likely to be close to 1, but our model implies that these shares, along with the MPCs, must converge to 0 at the top. This in turn means that the corresponding parameters of interest for our analysis may end up being very different from those implied by the average MPC or pass-through estimates found in the literature.

[47] In comparison to Bartik et al. (2024) and Vivalt et al. (2024), Bohmann et al. (2025) find a much larger increase in savings and correspondingly a smaller consumption response to income transfers.

[48] In both papers, the OLS estimates that do not control for unobserved preference heterogeneity are around 0.6, very close to our estimate of $\bar{\rho}_Y/\bar{\rho}_C$. Straub (2019) interprets the gap between OLS and IV estimates as a "consumption-smoothing bias" that he explicitly attributes to correlation between permanent income and unobserved preferences for savings. Blundell, Pistaferri, and Preston (2008) estimate a pass-through elasticity of 0.64, but they do not control for the consumption-smoothing bias highlighted by Straub.

All these estimates suggest that $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ is less than 1 but higher than the ratio of Pareto tail coefficients of 0.6. We set $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} = 0.75$ as our baseline value. With separable preferences the EIS is then equal to 1.

Elasticity of intertemporal substitution (EIS). The empirical literature providing estimates of the elasticity of intertemporal substitution is too large to adequately review here, and estimates of EIS cover a rather wide range, from values very close to 0 (Hall 1988) to well above 1. The empirical studies that are closest to our model estimate EIS from consumption responses to tax changes. Jakobsen et al. (2020) focus specifically on the behavior of the wealthiest households. Through the lens of a life-cycle model similar to ours, they show that an EIS as large as 2, and even higher for the very wealthy, is necessary to replicate the quasi-experimental evidence on the effects of a large wealth tax reform in Denmark on wealth accumulation. This value is consistent with those of Gruber (2013) and Holm et al. (2024), who estimate $EIS \approx 2$ and 1.6, respectively, by evaluating the spending response to exogenous shifts in the capital income tax rate and a dividend tax news shock, respectively.[49]

High values of EIS in turn also require that $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ becomes large: With our baseline calibration of labor supply elasticities, an EIS estimate of 2 requires $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ to be at least 1.5, if preferences are separable or complementary. In this case, the utility function displays more curvature in savings than in consumption and non-homotheticity reinforces the effects of preference heterogeneity, rather than dampening them—that is, consumption is then the luxury good, i.e. an individual saves a *declining* share of her income as she gets richer. The preference parameter estimates of Jakobsen et al. (2020) are directly consistent with this observation. Their model explicitly allows for non-homothetic end-of-life consumption or bequest motives and separately identifies both from the short- and long-run responses of wealth accumulation to a wealth tax reduction at different ages. For their baseline estimate of EIS between 1.8 and 2.2 for couples within the top two wealth percentiles, the estimated bequest elasticity implies a non-homotheticity estimate of $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} \approx 1.4$, consistent with slightly weaker income effects on labor supply than our baseline value.[50]

[49] Mulligan (2002) also argues for a high value of EIS when estimating intertemporal substitution using tax-adjusted returns to capital in aggregate data.

[50] Jakobsen et al. (2020) find even larger estimates of EIS and non-homotheticity by exploiting another source of variation at the very top of the wealth distribution. Their results are consistent with our observation that estimates of EIS and non-homotheticity are not independent of each other. Moreover, they report an elasticity of wealth to after tax returns of around 0.4. Due to the high exemption thresholds, this elasticity primarily captures the substitution effect $-\zeta^H_{S,\tau_S}$. In *Case 3*, $-\zeta^H_{S,\tau_S}$ converges to ζ^H_{Y,τ_Y} at the top; a value of $-\zeta^H_{S,\tau_S} = 0.4$ is only slightly higher than our baseline calibration of $\zeta^H_{Y,\tau_Y} = 1/3$.

These estimates are near the upper end of the existing range of estimates, possibly because of confounding tax avoidance responses to the policy change. But values of EIS well above 1 are common also in asset pricing, see e.g. Bansal and Yaron (2004) whose preferred calibration of Epstein-Zin preferences uses an EIS of 1.5. These asset pricing estimates focus on preferences of investors, who tend to be among the wealthiest households, just like the estimates based on tax reforms. Moreover, they tend to require that risk aversion is higher than $1/EIS$ to reconcile low and stable risk-free rates with high and variable risk premia. But the latter implies that $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} = \widehat{RA} \cdot EIS > 1$, again providing support for non-homothetic investor preferences with larger curvature over bequests or end-of-life wealth than over consumption.

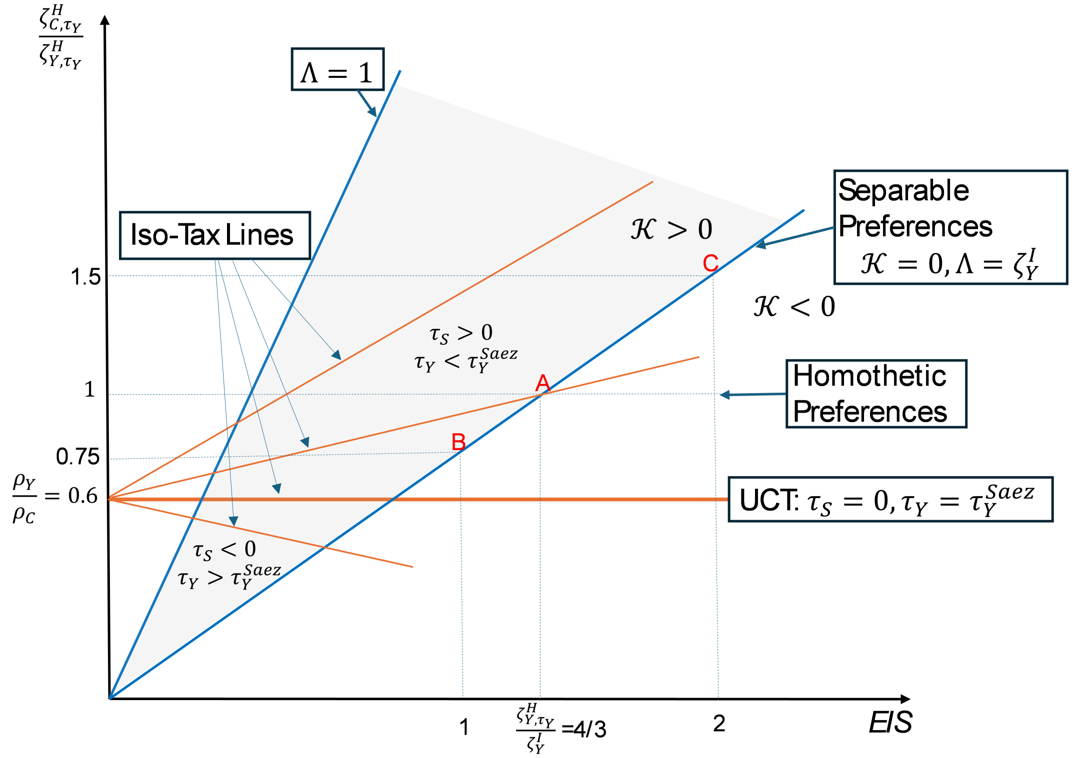
In comparison, macroeconomic estimates used in the business cycle literature tend to be closer to 1 and cross-sectional estimates focused on consumption responses to interest rates are often well below 1 (see Best et al. 2020; Ring 2024 for recent examples). Cross-sectional estimates of the average EIS well below 1 need not be inconsistent with far higher values at the top. Within our model, the EIS of the richest agents is strictly higher than for the average of the population if and only if $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} > 1$. Empirical evidence that the EIS is increasing in wealth (see for example Jakobsen et al. 2020) thus provides further support for a calibration in which non-homotheticity of preferences reinforces rather than dampens the case for positive savings taxes.

Consistent with Jakobsen et al. (2020), we use $EIS = 2$ and $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} = 1.5$ as our baseline calibration of EIS and evaluate the robustness of our results to a wide range of other values.

Preference complementarity ($\mathcal{K}$). Our baseline calibration focuses on completely separable preferences to provide a lower bound on the magnitude of optimal savings taxes and the departure of income taxes from $\overline{\tau}^{Saez}_Y$ when $\mathcal{K} \geq 0$. For positive $\mathcal{K}$, the optimal taxes are higher both because $\mathcal{K}$ increases the scaling factor Λ and because our calibration strategy no longer provides an exact value for $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ but only a lower bound when $\mathcal{E}_{CY} > \mathcal{E}_{SY}$. To explore the role of preference complementarities, we consider the alternative extreme in which $\mathcal{K} = (1 - \zeta^I_Y)/\zeta^H_{Y,\tau_Y}$ and $\Lambda = 1$. This case provides a theoretical upper bound on the departure from uniform commodity taxation.

Summary of the calibration. Figure 2 summarizes our calibration of consumption moments. It represents EIS on the horizontal and $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ on the vertical axis. The grey-shaded area represents the combinations of moments that are consistent with the joint restriction (23); its bound on the right corresponds to the completely separable model with $\mathcal{K} = 0$ and $\Lambda = \zeta^I_Y$. The horizontal line at $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} = 1$ represents the case of homothetic preferences. The orange lines originating from $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} = \bar{\rho}_Y/\bar{\rho}_C$ and $EIS = 0$ represent "iso-tax lines", i.e. preference parameters that

Figure 2: Summary of the Calibration



result in identical values of $(\overline{\tau}_Y, \overline{\tau}_S)$. The horizontal line at $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} = \bar{\rho}_Y/\bar{\rho}_C$ represents the set of preferences under which the optimal taxes are characterized by $\overline{\tau}_Y^{Saez}$ and uniform commodity taxation; the increasing lines for higher values of $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ summarize departures towards positive savings and lower income taxes; and the decreasing lines for lower values of $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y}$ summarize departures towards savings subsidies and higher income taxes.

Focusing on the line with separable preferences, we present optimal taxes for three cases: (i) the case with homothetic preferences indicated by point A in the figure; (ii) the case with a pass-through elasticity of $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} = 0.75$ and $EIS = 1$ that matches evidence on consumption responses to permanent income changes, indicated by point B; and (iii) the case with $EIS = 2$ and $\zeta^H_{C,\tau_Y}/\zeta^H_{Y,\tau_Y} = 1.5$ which matches the evidence on behavioral responses to wealth tax changes, indicated by point C. As we explained above, while point B may be more representative of the overall population, the estimates for point C tend to focus more specifically on the top.

4.2 Results

Table 1 summarizes our quantitative results for the optimal top tax rates on income and savings. While $\overline{\tau}_Y$ represents a marginal labor income tax on gross income, the marginal tax on gross savings

Table 1: Optimal top labor income and savings taxes

		B: $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}=0.75$		**A:** $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}=1$		**C:** $EIS=2$	
	$\overline{\tau}_Y^{Saez}$	$\overline{\tau}_Y$	$\frac{\overline{\tau}_S}{1+\overline{\tau}_S}$	$\overline{\tau}_Y$	$\frac{\overline{\tau}_S}{1+\overline{\tau}_S}$	$\overline{\tau}_Y$	$\frac{\overline{\tau}_S}{1+\overline{\tau}_S}$
Baseline values*	**74.1%**	**70.4%**	**12.5%**	**66.7%**	**22.2%**	**63.0%**	**30.0%**
$\frac{\bar{\rho}_Y}{\bar{\rho}_C}=0.5$	74.1%	67.9%	19.2%	64.8%	26.3%	61.7%	32.2%
$\frac{\bar{\rho}_Y}{\bar{\rho}_C}=0.7$	74.1%	72.8%	4.5%	68.5%	17.6%	64.2%	27.6%
$\bar{\rho}_Y=2$ ($\frac{\bar{\rho}_Y}{\bar{\rho}_C}=\frac{2}{3}$)	70.6%	68.6%	6.3%	64.7%	16.7%	60.8%	25.0%
$\bar{\zeta}^H_{Y,\tau_Y}=\frac{1}{2}$ ($\frac{1}{EIS}\frac{\bar{\zeta}^H_{C,\tau_Y}}{\bar{\zeta}^H_{Y,\tau_Y}}=\frac{1}{2}$)	60.6%	57.6%	7.1%	54.5%	13.3%	54.5%	13.3%
$\bar{\zeta}^I_Y=\frac{1}{3}$ ($\frac{1}{EIS}\frac{\bar{\zeta}^H_{C,\tau_Y}}{\bar{\zeta}^H_{Y,\tau_Y}}=1$)	78.9%	73.7%	20.0%	68.4%	33.3%	60.5%	46.7%
$\Lambda=1$ ($\frac{1}{EIS}\frac{\bar{\zeta}^H_{C,\tau_Y}}{\bar{\zeta}^H_{Y,\tau_Y}}=3$)	74.1%	59.3%	36.4%	44.4%	53.3%	29.6%	63.2%

*Baseline values: $\bar{\rho}_Y=1.8$, $\bar{\rho}_Y/\bar{\rho}_C=0.6$, $\bar{\zeta}^H_{Y,\tau_Y}=1/3$, $\bar{\zeta}^I_Y=1/4$, $\mathcal{K}=0$.

is equal to $\overline{\tau}_S/(1+\overline{\tau}_S)$; this is the variable we report in the table. We set baseline parameter values $\bar{\rho}_Y=1.8$, $\bar{\rho}_Y/\bar{\rho}_C=0.6$ (hence $\bar{\rho}_C=3$), $\bar{\zeta}^I_Y=1/4$, $\bar{\zeta}^H_{Y,\tau_Y}=1/3$, and $\mathcal{K}=0$ (hence $\Lambda=\bar{\zeta}^I_Y$) for the first row, and we vary these parameters one by one in the subsequent rows. The second column reports the static optimum $\overline{\tau}_Y^{Saez}$. The other six columns report top marginal tax rates for the three points labeled B, A, and C in Figure 2, in ascending order of the corresponding EIS and $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ values. For each case we fix $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ or EIS as indicated in the top row, and we set the remaining preference parameter so that $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}=(\Lambda/\bar{\zeta}^H_{Y,\tau_Y})\cdot EIS$. In the first four rows, these values correspond exactly to the ones indicated in Figure 2 where $(\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y})/EIS=0.75$ and $EIS=4/3$ for calibration A, $EIS=1$ for calibration B, and $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}=1.5$ for calibration C. In the fifth row, the line depicting separable preferences becomes flatter ($(\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y})/EIS=0.5$); hence EIS increases to 2 and 1.5 for calibrations A and B, respectively, and $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}=1$ for calibration C (as a result, calibrations B and C coincide). In the sixth row, the line depicting separable preferences becomes steeper ($(\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y})/EIS=1$); hence, the EIS for calibrations A and B decreases to 1 and 0.75, respectively, while $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ increases to 2 for calibration C. In the last row, we set EIS and $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ along the upper limit of the range of admissible preferences at which $\Lambda=1$. This corresponds to a case of very strong preference complementarities.

Our baseline calibration yields a combined wedge $\overline{\tau}_Y^{Saez}$ of 74%. How it is split into income and savings taxes depends on our calibration of inter-temporal substitution and non-homotheticity. With homothetic preferences (Calibration A), the optimal top savings tax is 22%, while the optimal income tax is reduced to 67%. When we calibrate the pass-through elasticity of income to consumption to 0.75 and set EIS to 1 (Calibration B), the savings tax drops to 12.5% while the income tax is at 70.4%. When instead we calibrate the EIS to 2 (Calibration C), the optimal savings tax increases to 30% and the income tax drops to 63%. In this last case, the top earners' after-tax income rises by 43 percent, with a corresponding shift in tax burden towards savings.

To interpret the values of the savings wedge, we translate them into a tax on annualized returns. Considering an annual return of 4% over a 25-year period from working life to retirement, a savings tax of $\overline{\tau}_S/(1+\overline{\tau}_S) = 30\%$ corresponds to a 1.4% annual tax on accumulated wealth, or a 37% capital income tax. Alternatively, if we interpret our model as one of retirement savings, a wedge of 30% means that top income earners will receive a present value of \$0.77 of additional pension payments for each additional dollar in social security contributions.

The next three rows vary the Pareto coefficients. If the Pareto coefficient on consumption changes (second and third rows), $\overline{\tau}_Y^{Saez}$ remains the same, but the split between income and savings taxes varies, corresponding to a vertical shift of the iso-tax lines in Figure 2. If the Pareto coefficient on income changes (fourth row), $\overline{\tau}_Y^{Saez}$ also changes, which affects the scaling of both income and savings taxes. Comparing rows 2-4 with the baseline and Calibration A with Calibration B, we observe the sensitivity of optimal taxes to the ratio of Pareto tail coefficients and pass-through from income to consumption: As the former varies from 0.5 to 0.7 and the latter from 0.75 to 1, the optimal savings tax goes from 4.5% to 26.3% – almost a six-fold increase. This observation illustrates the critical importance of identifying the pass-through elasticity separately from the ratio of Pareto tails, and of controlling for unobserved preference heterogeneity, since it is the latter that determines the strength of the savings tax motive. On the other hand, non-homotheticity and heterogeneity of preferences reinforce each other in Calibration C and therefore the variation in optimal savings taxes across the four rows is much smaller in absolute or relative terms, from 25% to 32.2%.

Rows 5 and 6 vary the earnings elasticities. Higher values of the compensated elasticity of taxable income ζ_{Y,τ_Y}^H reduce the combined wedge on labor income and savings $\overline{\tau}_Y^{Saez}$ (fifth row). In calibrations A and B where $\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H$ and Λ are fixed, this simply scales down both the savings and income tax distortion, while leaving the tradeoff between the two unchanged. In calibration C, we adjust $\bar{\zeta}_{C,\tau_Y}^H/\bar{\zeta}_{Y,\tau_Y}^H$ downwards to maintain EIS constant at 2, which reduces the role of heterogeneity in preferences in accounting for the observed gap between the Pareto coefficients of income and consumption, and thus reduces the scope for savings taxes.

Conversely, higher values of the income effect on labor supply ζ_Y^I raise the combined wedge $\overline{\tau}_Y^{Saez}$, but also shift distortions from income towards savings (sixth row): A higher ζ_Y^I increases both $\overline{\tau}_Y^{Saez}$ and the scaling factor Λ that enters both taxes and determines the magnitude of the shift from income to savings taxes. Both shifts contribute to raising the savings tax, which is unambiguously increasing in ζ_Y^I, but they have opposing effects for income taxes. In calibrations A and B, the increase in $\overline{\tau}_Y^{Saez}$ dominates relative to the shift from income to savings taxes. In Calibration C, $\bar{\zeta}^H_{C,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}$ is increased relative to the baseline to keep EIS constant at 2. In this case the shift towards savings taxes dominates, leading to a reduction of income taxes relative to the baseline.

The last row sets the complementarity $\mathcal{K}$ to its theoretical upper bound so that $\Lambda = 1$. This scenario corresponds to calibrating the model to the upper limit of the gray area in Figure 2. In all three calibrations, the savings taxes are between two and three times as large as in the baseline. Therefore, allowing for intertemporal preference complementarity or complementarity between consumption and earnings further shifts the tax burden towards savings.

We draw three main conclusions from this calibration exercise. First, all our calibrations suggest that it is optimal to shift a part of the overall tax burden from income to savings, for reasonable estimates of the Pareto coefficients and behavioral elasticities of consumption and earnings. Depending on the preferred calibration strategy, it is optimal to tax savings of top earners anywhere between 12.5% and 30%, and to reduce their income tax burden accordingly.

Second, the magnitude of this shift is sensitive to the calibration strategy. Calibrating our formulas to match consumption responses to wealth taxes (Calibration C) leads to much larger optimal savings taxes and lower income taxes than calibrating our formulas to match pass-through of permanent income to consumption (Calibration B), with the homothetic case (Calibration A) falling in between. The pass-through evidence, which is not specifically tailored to top earners, suggests that the average household treats savings as a luxury relative to regular consumption, with a correspondingly higher income elasticity. On the other hand, the estimates based on wealth tax reforms suggest that the richest households treat their consumption as a luxury relative to savings or bequests. In the former case, the non-homotheticity mutes the motive for savings taxes suggested by the gap between consumption and income concentration, while in the latter case, the non-homotheticity reinforces it. We leave it to future research to reconcile these stark differences between the estimated pass-through for average households and the inferred pass-through for top earners, to determine which of these approaches is more policy-relevant.

Third, optimal tax prescriptions are sensitive to precise estimates of the relevant sufficient statistics, and in particular to being able to estimate the cross-sectional concentration of income and consumption separately from the causal impact of income on consumption or savings. The gap between these two measures captures the extent of preference heterogeneity that provides the primitive rationale for savings taxation. As illustrated by the comparison between Calibration

A and B, small errors or biases in the estimates of either the ratio of Pareto coefficients or the pass-through from income to consumption can result in substantial errors in the resulting policy conclusions. Moreover, estimates of the causal impact of income on consumption or savings that do not control for preference heterogeneity are likely to be biased and under-estimate the scope of preference heterogeneity. While unobserved preference heterogeneity has been recognized previously as a source of estimation bias in pass-through elasticities, we believe the connection between this bias and the economic rationale for commodity taxation has not been noted before. Exploiting this connection for econometric tests of (departures from) Uniform Commodity Taxation would be a promising avenue for future research.

Conclusion

This paper argues that income and savings taxes cannot be studied in isolation; they form instead an optimal policy mix that must be characterized jointly. Doing so leads to a stark tradeoff between raising one tax instrument versus the other. If the marginal rate of substitution between consumption and saving is homogeneous across agents, it is optimal to leave savings undistorted, as is well known since Atkinson and Stiglitz (1976), *but also* to set the level of the labor income tax rate at the static optimum given by Saez (2001). Away from this joint benchmark, it is optimal to raise (resp., lower) the savings tax rate above zero *and* simultaneously reduce (resp., increase) the labor income tax rate below the static optimum, if and only if more productive agents have a stronger (resp., weaker) taste for saving relative to current consumption. Our novel optimal tax formulas, expressed in terms of the Pareto coefficients and elasticities of income and consumption, suggest that it is optimal to shift a significant share of the burden of taxes from income to savings. Empirical measures of consumption inequality and consumption responses to taxes play a key role in identifying the direction and magnitude of this shift away from the Saez (2001) and Atkinson and Stiglitz (1976) benchmark. Given such crucial importance of consumption measures for optimal taxes, we believe empirical research should devote as much attention estimating them as has been given to their income counterparts.

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Online Appendix for
"Using Consumption Data to Derive Optimal Income and Capital Tax Rates"

Christian Hellwig and Nicolas Werquin

A Proofs of Section 2 and 3.1

Here, we analyze the optimal tax design problem. To streamline the exposition, we focus on the general model that we analyzed in Section 3.1, and only later specialize to the baseline model with consumption and savings.

Agents' preferences are given by $U(Y, \boldsymbol{C}; r)$ over efficiency units of labor Y and an arbitrary N-dimensional vector $\boldsymbol{C} = (C_1, \ldots, C_N)$ of consumption goods, at rank $r \in [0, 1]$. Let $U_n \equiv \partial U / \partial C_n > 0$ and$U_Y \equiv \partial U / \partial Y < 0$. We further define $U_{nr} \equiv \partial U_r / \partial C_n$ and $U_{Yr} \equiv \partial U_r / \partial Y$ and assume that $U_{Yr}/U_Y < 0$, so that marginal disutility of effort is decreasing in r. For convenience we index leisure $-Y$ as good 0, and write $U_0 = -U_Y$ and $U_{0r}/U_0 = U_{Yr}/U_Y < 0$. Each good n is produced at a constant marginal cost of p_n efficiency units of labor.

Assumption 4 (**Generalized Single-Crossing Condition**). *The marginal rates of substitution (MRS) between any two goods m and n is non-decreasing in r, whenever $m > n$, i.e.*

$$m \gtreqless n \iff \frac{\partial \ln (U_m / U_n)}{\partial r} \equiv \frac{U_{mr}}{U_m} - \frac{U_{nr}}{U_n} \gtreqless 0. \tag{26}$$

The baseline model of Section 2 is a special case of this model with $N = 2$, $\boldsymbol{C} \equiv (C_1, C_2) = (C, S)$ if $U_{Sr}/U_S \geq U_{Cr}/U_C$, and $\boldsymbol{C} \equiv (C_1, C_2) = (S, C)$, if $U_{Sr}/U_S \leq U_{Cr}/U_C$.

The social planner chooses a tax function $T(Y, \boldsymbol{C})$ that maximizes $\int_0^1 T(Y(r), \boldsymbol{C}(r))\, dr$, subject to the incentive compatibility constraint $\{Y(r), \boldsymbol{C}(r)\} \in \arg\max_{Y, \boldsymbol{C}} U(Y, \boldsymbol{C}; r)$ s.t. $\sum_{n=1}^N p_n C_n + T(Y, \boldsymbol{C}) \leq Y$, and the promise-keeping constraint $\int_0^1 \omega(r) G(U(Y(r), \boldsymbol{C}(r); r))\, dr \geq v_0$ for some weighted utility promise v_0, and where the social welfare objective allows for both rank-dependent Pareto weights $\omega(\cdot)$ such that $\mathbb{E}[\omega] = 1$ and an increasing and concave Bergson-Samuelson function $G(\cdot)$ that captures the planner's attitude towards inequality. The Rawlsian social welfare function corresponds to a limiting case with Pareto weights $\omega(r) = 0$ for all $r > 0$, i.e. all the weight is given to the lowest rank, and the promise-keeping constraint reduces to a lower bound constraint on the lowest rank's utility, $G(U(Y(0), \boldsymbol{C}(0); 0)) \geq v_0$.

A.1 Mechanism Design Problem

Statement of the planner's problem. The mechanism design problem consists in choosing an allocation $\{Y(r), \boldsymbol{C}(r)\}$ that maximizes the net present value of tax revenue

$$K(v_0) = \max_{Y(r),\boldsymbol{C}(r)} \int_0^1 \left(Y(r) - \sum_{n=1}^{N} p_n C_n(r) \right) dr \tag{27}$$

subject to the incentive compatibility constraint

$$U(Y(r), \boldsymbol{C}(r); r) \geq U(Y(r'), \boldsymbol{C}(r'); r) \tag{28}$$

for all types r and announcements r', and a lower bound on social welfare

$$\int_0^1 \omega(r) G(W(r)) \, dr \geq v_0 \tag{29}$$

where $W(r)$ is given by the promise-keeping constraint

$$W(r) = U(Y(r), \boldsymbol{C}(r); r). \tag{30}$$

Relaxed problem and sufficient conditions. An allocation$\{Y(r), \boldsymbol{C}(r)\}$ is locally incentive compatible if $W(r)$ is continuous and satisfies the envelope condition

$$W'(r) = U_r(Y(r), \boldsymbol{C}(r); r) \tag{31}$$

almost everywhere. We focus on allocations that are continuously differentiable w.r.t. r. The following lemma shows that local incentive compatibility, coupled with a monotonicity condition on allocations, is sufficient for global incentive compatibility.

Lemma 2 (**Sufficient conditions for incentive compatibility**)**.** *An allocation $\{Y(r), \boldsymbol{C}(r)\}$ satisfies incentive compatibility (6) whenever it satisfies (i) local incentive compatibility (31), and (ii) $\sum_{k=n}^{N} U_k(Y(r''), \boldsymbol{C}(r''); r'') C_k'(r'') \, dr'' \geq 0$ for $n = 1, ..., N$. Conversely, suppose that there exists an open interval $(r_1, r_2) \subset (0,1)$ such that, for all $r'' \in (r_1, r_2)$ and for all $n = 1, ..., N$, we have $\sum_{k=n}^{N} U_k(Y(r''), \boldsymbol{C}(r''); r'') C_k'(r'') \, dr'' \leq 0$ with at least one strict inequality. Then $\{Y(r), \boldsymbol{C}(r)\}$ is not incentive compatible.*

Proof of Lemma 2. For ease of notation, write $\boldsymbol{X}(r) \equiv \{Y(r), \boldsymbol{C}(r)\}$. Consider an allocation

$\boldsymbol{X}(r)$ that is locally incentive compatible and continuously differentiable at every r. We then have

$$\begin{aligned} & U(\boldsymbol{X}(r);r) - U(\boldsymbol{X}(r');r) \\ = & \int_{r'}^{r} \sum_{n=0}^{N} U_n(\boldsymbol{X}(r'');r) C_n'(r'') dr'' \\ = & \int_{r'}^{r} \sum_{n=0}^{N} \frac{U_n(\boldsymbol{X}(r'');r)}{U_n(\boldsymbol{X}(r'');r'')} U_n(\boldsymbol{X}(r'');r'') C_n'(r'') dr''. \end{aligned}$$

This expression can be rewritten as

$$\begin{aligned} & U(\boldsymbol{X}(r);r) - U(\boldsymbol{X}(r');r) \\ = & \int_{r'}^{r} \int_{0}^{1} \frac{U_0(\boldsymbol{X}(r'');r)}{U_0(\boldsymbol{X}(r'');r'')} \sum_{n=0}^{N} U_n(\boldsymbol{X}(r'');r'') C_n'(r'') dr'' \\ & + \sum_{n=1}^{N} \int_{r'}^{r} \left[\frac{U_n(\boldsymbol{X}(r'');r)}{U_n(\boldsymbol{X}(r'');r'')} - \frac{U_{n-1}(\boldsymbol{X}(r'');r)}{U_{n-1}(\boldsymbol{X}(r'');r'')} \right] \sum_{k=n}^{N} U_k(\boldsymbol{X}(r'');r'') C_k'(r'') dr''. \end{aligned}$$

Local incentive compatibility implies $\sum_{n=0}^{N} U_n(\boldsymbol{X}(r'');r'') C_n'(r'') = 0$, and therefore the first line in this last expression equals 0. Moreover, due to single-crossing conditions the term inside the square brackets in the second line is positive (resp., negative), whenever $r > r''$ (resp., $r < r''$). Therefore, $U(\boldsymbol{X}(r);r) \geq U(\boldsymbol{X}(r');r)$ whenever $\sum_{k=n}^{N} U_k(\boldsymbol{X}(r'');r'') C_k'(r'') \geq 0$ for all $n = 1, ..., N$. Conversely, suppose that $\sum_{k=n}^{N} U_k(\boldsymbol{X}(r'');r'') C_k'(r'') dr'' \leq 0$ for all n and all $r'' \in (r_1, r_2)$, with at least one strict inequality. Then $U(\boldsymbol{X}(r_1);r_1) < U(\boldsymbol{X}(r_2);r_1)$ and $U(\boldsymbol{X}(r_2);r_2) < U(\boldsymbol{X}(r_1);r_2)$, hence incentive compatibility is violated. □

The monotonicity is satisfied if for each rank r, there exists a threshold $n(r)$ so that $X_n'(r) \geq 0$ for all goods with index $n \geq n(r)$, and $X_n'(r) \leq 0$ for all goods with index $n < n(r)$, i.e., if the allocation shifts the agents' consumption bundle at higher ranks towards higher indexed goods. However, the allocation doesn't have to be perfectly co-monotonic, i.e., it may be the case that higher-ranked agents start to consume less of the lower indexed goods. However, since $\sum_{k=1}^{N} U_k(Y(r''), \boldsymbol{C}(r'');r'') C_k'(r'') dr'' = -U_Y(Y(r''), \boldsymbol{C}(r'');r'') Y'(r'')$, the sufficient conditions for incentive compatibility hold only if Y is increasing in r.

In our baseline economy, these monotonicity conditions hold if: (i) $U_Y(r'') Y'(r'') \leq 0$, i.e., Y is non-decreasing; and (ii) if $U_{Sr}/U_S > U_{Cr}/U_C$, then $U_Y(r'') Y'(r'') + U_C(r'') C'(r'') \leq 0$, or equivalently $U_S(r'') S'(r'') \geq 0$, i.e., S is non-increasing; or (ii′) if $U_{Sr}/U_S < U_{Cr}/U_C$, then $-U_C(r'') C'(r'') \leq 0$, i.e., C is non-decreasing. Conversely, if $S'(r'') \leq 0$ and $C'(r'') \leq 0$ with one strict inequality, we must also have $Y'(r'') < 0$, which implies a violation of the incentive compatibility constraint. Therefore, local incentive compatibility combined with co-monotonicity of income and either consumption or savings is sufficient for global incentive compatibility (6). Moreover, if $U_{Sr}/U_S = U_{Cr}/U_C$ for all (Y, C, S, r), then the monotonicity of $Y(\cdot)$ and local incentive compatibility are both necessary and sufficient for incentive compatibility—analogous to the well-known necessary and sufficient conditions in mechanism design problems with a single decision margin. If $U_{Sr}/U_S \neq U_{Cr}/U_C$ for some r, monotonicity of Y and either C or S together are sufficient, but not necessary, for incentive compatibility. With two decision margins (labor supply and consumption-savings), incentive compatibility (6) holds if, for any pair of types, the sum of information rents across both margins is non-negative. Monotonicity of Y guarantees that information rents along the labor supply margin are always non-negative, while monotonicity of C or S guarantees that the same holds along the consumption-savings margin. However, (6) may still hold if information rents are positive along only one of the two margins, i.e., if one of the two conditions is violated. The partial converse then says that incentive compatibility must be violated if both conditions are violated over some interval of types, that is, if information rents are negative along both margins. Lemma 2 generalizes this logic to any number of goods, for one-dimensional type spaces.

A.2 Solution to the Relaxed Planner's Problem

Next, we analyze the relaxed planner's problem of maximizing (27) subject to (29), (30), and (31) and express its solution in terms of primitives—i.e., the agent's preferences. Lemma 3 provides a characterization of optimal income and consumption wedges akin to the well-known "ABC" formula (Diamond 1998). We use good 1 as the numeraire and write $\tau_Y \equiv 1 + U_Y/U_1$ and $t_n \equiv U_n/U_1 - 1$.[51]

Lemma 3 (**Optimal tax system**). *The optimal income and consumption wedges satisfy*

$$\frac{\tau_Y(r)}{1-\tau_Y(r)} = (1-r)\left(\frac{U_{1r}(r)}{U_1(r)} - \frac{U_{Yr}(r)}{U_Y(r)}\right)\gamma(r) \tag{32}$$

and

$$\frac{t_n(r)}{1+t_n(r)} = (1-r)\left(\frac{U_{nr}(r)}{U_n(r)} - \frac{U_{1r}(r)}{U_1(r)}\right)\gamma(r) \tag{33}$$

where

$$\gamma(r) = \mathbb{E}\left[\left\{1 - \lambda\omega(r') G'(W(r')) U_1(r')\right\} \frac{U_1(r)}{U_1(r')} \exp\left(\int_r^{r'} \frac{U_{1r}(r'')}{U_1(r'')} dr''\right) \middle| r' \geq r\right] \tag{34}$$

[51]Throughout, we write $X(r) \equiv X(Y(r), \boldsymbol{C}(r); r)$ for any function X of both $\{Y(r), \boldsymbol{C}(r)\}$ and the type r.

with

$$\lambda = \frac{\mathbb{E}\left[\frac{p_1}{U_1(r')}\exp\left(-\int_{r'}^{1}\frac{U_{1r}(r'')}{U_1(r'')}dr''\right)\right]}{\mathbb{E}\left[\omega\left(r'\right)G'\left(W\left(r'\right)\right)\exp\left(-\int_{r'}^{1}\frac{U_{1r}(r'')}{U_1(r'')}dr''\right)\right]}.$$

Thus, Assumption 1 implies that $\tau_Y(r) > 0$, and $t_n(r) > 0$ for all r.[52]

Proof of Lemma 3. Problem (27) subject to (29)-(30)-(31) is an optimal control problem with $W(\cdot)$ as the state variable, and $\boldsymbol{C}(\cdot)$, $Y(\cdot)$, and $S(\cdot)$ as controls. Defining λ, $\psi(r)$, and $\phi(r)$ as the multipliers on these respective constraints, the Hamiltonian is given by:

$$\begin{aligned}\mathcal{H} \;=\; & \sum_{n=1}^{N} p_n C_n(r) - Y(r) + \phi(r) U_r\left(Y(r), \boldsymbol{C}(r); r\right) \\ & + \lambda\omega(r)\left(W_0 - G\left(W(r)\right)\right) + \psi(r)\left\{W(r) - U\left(Y(r), \boldsymbol{C}(r); r\right)\right\}.\end{aligned}$$

The first-order conditions with respect to the allocations $Y(\cdot)$, and $\boldsymbol{C}(r)$ yield:

$$\psi(r) = \frac{1}{-U_Y(r)} + \phi(r)\frac{U_{Yr}(r)}{U_Y(r)} = \frac{p_n}{U_n(r)} + \phi(r)\frac{U_{nr}(r)}{U_n(r)}.$$

The first-order conditions for $Y(\cdot)$, and $\boldsymbol{C}(r)$ define a shadow cost of utility of agents with rank r, $\psi(r)$, which consists of a direct shadow cost $1/U_n(r)$ or $1/(-U_Y(r))$ of increasing rank r utility through higher consumption of good n or lower income, and a second term that measures how such a consumption or income increase affects $U_r(r)$ and thereby tightens or relaxes the local incentive compatibility constraint at r by $U_{nr}(r)/U_n(r)$ or $U_{Yr}(r)/U_Y(r)$. The latter is weighted by the multiplier $\phi(r)$ and added to the former. Combining the first two first-order conditions and rearranging terms then yields the following static optimality conditions:

$$\frac{p_1}{U_1(r)}\frac{\tau_Y(r)}{1-\tau_Y(r)} = \frac{1}{-U_Y(r)} - \frac{p_1}{U_1(r)} = \left(\frac{U_{1r}(r)}{U_1(r)} - \frac{U_{Yr}(r)}{U_Y(r)}\right)\phi(r)$$

and

$$\frac{p_1}{U_1(r)}\frac{t_n(r)}{1+t_n(r)} = \frac{p_1}{U_1(r)} - \frac{p_n}{U_n(r)} = \left(\frac{U_{nr}(r)}{U_n(r)} - \frac{U_{1r}(r)}{U_1(r)}\right)\phi(r).$$

Hence, we obtain equations (32) and (33), with $\gamma(r) = \frac{\phi(r)}{1-r}U_1(r)/p_1$.

The multipliers $\phi(\cdot)$ and λ are derived by solving the linear ODE $\phi'(r) = -\partial\mathcal{H}/\partial W$, after

[52] We use Assumption 4 only to guarantee that local incentive compatibility and monotonicity are sufficient for global incentive compatibility. The results that $\tau_Y(r) > 0$ for each r and $t_n(r) > 0$ inherits the sign of $U_{nr}/U_n - U_{1r}/U_1$ applies even without 4 provided that only local IC constraints are binding.

substituting out $\psi(r)$ using the first first-order condition:

$$\phi'(r) = -\frac{\partial \mathcal{H}}{\partial W} = \lambda\omega(r) G'(W(r)) - \psi(r) = \lambda\omega(r) G'(W(r)) - \frac{p_1}{U_1(r)} - \phi(r)\frac{U_{1r}(r)}{U_1(r)},$$

along with the boundary conditions $\phi(0) = \phi(1) = 0$. Define

$$m_n(r) = \exp\left(-\int_r^1 \frac{U_{nr}(r')}{U_n(r')}dr'\right) \text{ and } m_Y(r) = \exp\left(-\int_r^1 \frac{U_{Yr}(r')}{U_Y(r')}dr'\right)$$

so that $U_{nr}(r)/U_n(r) = m_n'(r)/m_n(r)$ and $U_{Yr}(r)/U_Y(r) = m_Y'(r)/m_Y(r)$. Substituting $m_1(r)$ into the previous ODE and integrating out yields

$$\phi(r) m_1(r) = \int_r^1 (1 - \lambda\omega(r') G'(W(r'))) \frac{p_1}{U_1(r')} m_1(r') dr',$$

or

$$\frac{\phi(r)}{1-r} = \mathbb{E}\left[(1 - \Gamma(r'))\frac{p_1}{U_1(r')}\frac{m_1(r')}{m_1(r)}\middle| r' \geq r\right],$$

where $\Gamma(r) = \lambda\omega(r) G'(W(r)) U_1(r)/p_1$. The boundary condition $\phi(0) = 0$ then gives $\lambda = p_1\mathbb{E}[m_1 U_1^{-1}]/\mathbb{E}[\Gamma m_1 U_1^{-1}]$. Therefore,

$$\gamma(r) \quad = \quad \mathbb{E}\left[\frac{U_1(r)}{U_1(r')}\frac{m_1(r')}{m_1(r)}\middle| r' \geq r\right]\left\{1 - \frac{\mathbb{E}\left[\Gamma(r') \cdot m_1(r')/U_1(r') \middle| r' \geq r\right]}{\mathbb{E}\left[m_1(r')/U_1(r') \middle| r' \geq r\right]}\right\}.$$

Substituting this expression into the static optimality condition then yields equation (34). Applying analogous steps using m_n or m_Y, we obtain the alternative representations $\Gamma(r) = \lambda\omega(r) G'(W(r))$

$$\frac{\phi(r)}{1-r} \quad = \quad \mathbb{E}\left[\frac{1}{-U_Y(r')}\frac{m_Y(r')}{m_Y(r)}\middle| r' \geq r\right] - \lambda\mathbb{E}\left[\omega(r') G'(W(r'))\frac{m_Y(r')}{m_Y(r)}\middle| r' \geq r\right]$$

and

$$\frac{\phi(r)}{1-r} \quad = \quad \mathbb{E}\left[\frac{p_n}{U_n(r')}\frac{m_n(r')}{m_n(r)}\middle| r' \geq r\right] - \lambda\mathbb{E}\left[\omega(r') G'(W(r'))\frac{m_n(r')}{m_n(r)}\middle| r' \geq r\right]$$

with

$$\lambda = \frac{\mathbb{E}\left[\frac{p_n}{U_n(r')} m_n(r')\right]}{\mathbb{E}\left[\omega(r') G'(W(r')) m_n(r')\right]} = \frac{\mathbb{E}\left[\frac{1}{-U_Y(r')} m_Y(r')\right]}{\mathbb{E}\left[\omega(r') G'(W(r')) m_Y(r')\right]}.$$

This concludes the proof. □

Atkinson and Stiglitz (1976) theorem. The theorem of Atkinson and Stiglitz (1976) and its converse follow easily from the proof of Proposition 3.

Lemma 4 (**Uniform commodity taxation**). *The optimal wedges satisfy* $t_m(r) \gtreqless t_n(r)$ *if and only if* $U_{mr}(r)/U_m(r) \gtreqless U_{nr}(r)/U_n(r)$*, which holds if and only if* $m \gtreqless n$*. In particular, in our baseline economy, we obtain* $U_S(r) \gtreqless U_C(r)$*, or* $\tau_S(r) \gtreqless 0$*, if and only if* $U_{Sr}(r)/U_S(r) \gtreqless U_{Cr}(r)/U_C(r)$*.*

Proof of Lemma 4. For any two goods $m \neq n$, the first-order conditions of the planner's problem read

$$\frac{p_m}{U_m(r)} = \frac{p_n}{U_n(r)} + \phi(r)\left(\frac{U_{nr}(r)}{U_n(r)} - \frac{U_{mr}(r)}{U_m(r)}\right),$$

We immediately obtain that $U_n(r)/U_m(r) = p_n/p_m$ or $t_m = t_n$ for all r if and only if $U_{mr}(r)/U_m(r) = U_{nr}(r)/U_n(r)$ for all r, or the MRS $U_m(r)/U_n(r)$ is uniform across types. Conversely, we have $U_m(r)/U_n(r) \gtreqless p_m/p_n$ and $t_m \gtreqless t_n$ iff $U_{mr}(r)/U_m(r) \gtreqless U_{nr}(r)/U_n(r)$. By our ordering of goods according to the generalized single-crossing conditions, this is equivalent to $m \gtreqless n$. □

No Arbitrage conditions. By taking the ratios of equations (32) and (33) we immediately obtain the no-arbitrage conditions

$$\frac{t_n(r)}{1+t_n(r)} = \Sigma_n(r)\frac{\tau_Y(r)}{1-\tau_Y(r)}, \text{ where } \Sigma_n(r) \equiv \frac{U_{nr}(r)/U_n(r) - U_{1r}(r)/U_1(r)}{U_{1r}(r)/U_1(r) - U_{Yr}(r)/U_Y(r)},$$

for any $n = 2, ..., N$. In our baseline model this condition reduces to equation (9). It remains to map Σ_n into preference parameters and behavioral elasticities.

A.3 Identification, Step 1: Optimal taxes in terms of preference elasticities

We finally map the model primitives that appear on the right-hand sides of equations (32) and (33) to our empirically observable sufficient statistics. First, we express the optimal tax formulas in terms of preference elasticities and Pareto coefficients. The following result generalizes Lemma 1 in Saez (2001) to our economy.

Lemma 5 (**Identification: preference elasticities**). *For any given system of tax distortions* $\{\tau_Y(r), t_n(r)\}$*, we have*

$$(1-r)\left(\frac{U_{1r}}{U_1} - \frac{U_{Yr}}{U_Y}\right) = \frac{\tilde{\mathcal{E}}_1}{\rho_1} + \frac{\tilde{\mathcal{E}}_Y}{\rho_Y} + \frac{d\ln(1-\tau_Y)}{d\ln(1-r)} \tag{35}$$

$$(1-r)\left(\frac{U_{nr}}{U_n} - \frac{U_{1r}}{U_1}\right) = \frac{\tilde{\mathcal{E}}_n}{\rho_n} - \frac{\tilde{\mathcal{E}}_1}{\rho_1} - \frac{d\ln(1-t_n)}{d\ln(1-r)}. \tag{36}$$

where $\tilde{\mathcal{E}}_n = \mathcal{E}_n - \mathcal{E}_{nY}\rho_n/\rho_Y - \sum_{k\neq n}\mathcal{E}_{nk}\rho_n/\rho_k$ *for* $n = 1, ..., N,$ *and* $\tilde{\mathcal{E}}_Y = \mathcal{E}_Y - \sum_{n=1}^{N} s_n\mathcal{E}_{nY}\rho_Y/\rho_n$.

Proof of Lemma 5. Totally differentiating $U_n(r)$ and $-U_Y(r)$ yields

$$\frac{\frac{d}{dr}U_n(r)}{U_n(r)} = \frac{U_{nn}(r)\,C_n(r)}{U_n(r)}\frac{C_n'(r)}{C_n(r)} + \sum_{k\neq n}\frac{U_{nk}(r)\,C_k(r)}{U_n(r)}\frac{C_k'(r)}{C_k(r)} + \frac{U_{nY}(r)\,Y(r)}{U_n(r)}\frac{Y'(r)}{Y(r)} + \frac{U_{nr}(r)}{U_n(r)}$$

and

$$\frac{\frac{d}{dr}(-U_Y(r))}{-U_Y(r)} = \frac{U_{YY}(r)\,Y(r)}{U_Y(r)}\frac{Y'(r)}{Y(r)} - \sum_{n=1}^{N}\frac{U_n(r)\,C_n(r)}{-U_Y(r)\,Y(r)}\frac{U_{nY}(r)\,Y(r)}{U_n(r)}\frac{C_n'(r)}{C_n(r)} + \frac{U_{Yr}(r)}{U_Y(r)}.$$

These equations can be rewritten as

$$-\frac{d\ln U_n(r)}{d\ln(1-r)} = (1-r)\frac{U_{nr}(r)}{U_n(r)} - \frac{\mathcal{E}_n(r)}{\rho_n(r)} + \frac{\mathcal{E}_{nY}(r)}{\rho_Y(r)} + \sum_{k\neq n}\frac{\mathcal{E}_{nk}(r)}{\rho_k(r)} = (1-r)\frac{U_{nr}(r)}{U_n(r)} - \frac{\tilde{\mathcal{E}}_n(r)}{\rho_n(r)}$$

and

$$-\frac{d\ln(-U_Y(r))}{d\ln(1-r)} = (1-r)\frac{U_{Yr}(r)}{U_Y(r)} + \frac{\mathcal{E}_Y(r)}{\rho_Y(r)} - \sum_{n=1}^{N}s_n(r)\frac{\mathcal{E}_{nY}(r)}{\rho_n(r)} = (1-r)\frac{U_{Yr}(r)}{U_Y(r)} + \frac{\tilde{\mathcal{E}}_Y(r)}{\rho_Y(r)}.$$

Combining these equations and using the first-order conditions $1-\tau_Y = -U_Y/U_1$ and $1+t_n = U_n/U_1$ yields equations (35) and (36). □

Next, combining equations (35) and (36) with equations (32) and (33), we can express the optimal top tax rates in terms of the preference elasticities:

Lemma 6 (Optimal tax rates on top earners). *Let* $\bar{\Gamma} \equiv \lim_{r\to 1}\lambda\omega(r)\,G'(W(r))\,U_1(r)/p_1$ *denote the marginal social welfare weight of top income earners. Optimal income and consumption taxes on top income earners satisfy*

$$\bar{\tau}_Y = \frac{1-\bar{\Gamma}}{1/\bar{\tau}_Y^* - \bar{\Gamma}} \quad \text{where} \quad \bar{\tau}_Y^* = \frac{\tilde{\mathcal{E}}_1/\rho_1 + \tilde{\mathcal{E}}_Y/\rho_Y}{1+\tilde{\mathcal{E}}_Y/\rho_Y}. \tag{37}$$

and

$$\bar{t}_n = \frac{1-\bar{\Gamma}}{1/\bar{t}_n^* + \bar{\Gamma}} \quad \text{where} \quad \bar{t}_n^* = \frac{\tilde{\mathcal{E}}_n/\rho_n - \tilde{\mathcal{E}}_1/\rho_1}{1-\tilde{\mathcal{E}}_n/\rho_n}. \tag{38}$$

Moreover, the tax arbitrage coefficient satisfies

$$\Sigma_n = \frac{\tilde{\mathcal{E}}_n/\rho_n - \tilde{\mathcal{E}}_1/\rho_1}{\tilde{\mathcal{E}}_1/\rho_1 + \tilde{\mathcal{E}}_Y/\rho_Y}. \tag{39}$$

Proof of Lemma 6. Ignoring the tax progressivity terms which converge to zero at the top, equations (32) and (33) can be rewritten as

$$\frac{\tau_Y(r)}{1-\tau_Y(r)} = \left(\frac{\tilde{\mathcal{E}}_1(r)}{\rho_1(r)} + \frac{\tilde{\mathcal{E}}_Y(r)}{Y(r)}\right) \mathbb{E}\left[(1-\Gamma(r'))\,\frac{U_1(r)}{U_1(r')}\frac{m_1(r')}{m_1(r)}\middle| r' \geq r\right]$$

and

$$\frac{t_n(r)}{1+t_n(r)} = \left(\frac{\tilde{\mathcal{E}}_n(r)}{\rho_n(r)} - \frac{\tilde{\mathcal{E}}_1(r)}{\rho_1(r)}\right) \mathbb{E}\left[(1-\Gamma(r'))\,\frac{U_1(r)}{U_1(r')}\frac{m_1(r')}{m_1(r)}\middle| r' \geq r\right]$$

where

$$\begin{aligned}\frac{U_1(r)}{U_1(r')}\frac{m_1(r')}{m_1(r)} &= \frac{U_1(r)}{U_1(r')}\exp\left(\int_r^{r'}\frac{U_{1r}(r'')}{U_1(r'')}dr''\right)\\ &= \exp\left[\int_r^{r'}\left(\frac{U_{1r}(r'')}{U_1(r'')} - \frac{\frac{d}{dr}U_1(r'')}{U_1(r'')}\right)dr''\right] = \exp\left(\int_r^{r'}\tilde{\mathcal{E}}_1(r'')\,\frac{C_1'(r'')}{C_1(r'')}dr''\right).\end{aligned}$$

Taking the limit as $r \to 1$ and using the fact that C_1 follows a Pareto distribution with tail coefficient ρ_1, we obtain

$$\frac{\bar{\tau}_Y}{1-\bar{\tau}_Y} = (1-\bar{\Gamma})\frac{\tilde{\mathcal{E}}_1/\rho_1 + \tilde{\mathcal{E}}_Y/\rho_Y}{1-\tilde{\mathcal{E}}_1/\rho_1} \quad \text{and} \quad \frac{\bar{t}_n}{1+\bar{t}_n} = (1-\bar{\Gamma})\frac{\tilde{\mathcal{E}}_n/\rho_n - \tilde{\mathcal{E}}_1/\rho_1}{1-\tilde{\mathcal{E}}_1/\rho_1}$$

Equations (37), (38) and (39) follow immediately. □

A.4 Revenue Spillover Condition and Proof of Proposition 2

In this section, we break the mechanism design problem into two stages: a first stage in which the social planner designs the income tax schedule $T_Y(Y)$ and the agent trades off between labor supply and after tax earnings $M \equiv Y - T_Y(Y)$, and a second stage in which the social planner sets the consumption tax schedule $\hat{T}(\boldsymbol{C})$ and the agent allocates after-tax earnings to different consumption goods. Here we take the outcome of the second stage as given and focus on the design of the optimal income tax function for given consumption taxes $\hat{T}(\boldsymbol{C}(r))$.

First-Stage Mechanism Design Problem. Define

$$\mathcal{U}(Y, M; r) \equiv \max_{\boldsymbol{C}} U(Y, \boldsymbol{C}; r) \text{ s.t. } M \geq \hat{T}(\boldsymbol{C}) + \sum_{n=1}^{N} p_n c_n$$

and

$$\boldsymbol{C}(Y, M; r) \quad = \quad \arg\max_{\boldsymbol{C}} U(Y, \boldsymbol{C}; r) \text{ s.t. } M \geq \hat{T}(\boldsymbol{C}) + \sum_{n=1}^{N} p_n c_n$$

as the indirect utility function and the optimal consumption vector that characterize the solution to the second-stage problem, given earnings Y and after-tax income $M = Y - T_Y(Y)$. The optimal first-stage allocation solves

$$\max_{\{Y(\cdot), M(\cdot)\}} \int_0^1 \left(T_Y(Y(r)) + \hat{T}(\boldsymbol{C}(Y(r), M(r); r)) \right) dr$$

subject to promise-keeping $W(r) = \mathcal{U}(Y(r), M(r); r)$, local incentive compatibility $W'(r) = \mathcal{U}_r(Y(r), M(r); r)$, and the ex-ante promise-keeping condition $\int_0^1 \omega(r) G(W(r)) dr \geq W_0$. The Hamiltonian to this program reads

$$\begin{aligned} \mathcal{H} \quad = \quad & M(r) - Y(r) - \hat{T}(\boldsymbol{C}(Y(r), M(r); r)) + \phi(r) \mathcal{U}_r(Y(r), M(r); r) \\ & + \lambda\omega(r)(W_0 - G(W(r))) + \psi(r)\{W(r) - \mathcal{U}(Y(r), M(r); r)\}. \end{aligned}$$

The first-order conditions for M and Y imply

$$\psi(r) = \frac{1}{-\mathcal{U}_Y(r)} \left[1 + \sum_{n=2}^{N} t_n p_n \frac{\partial C_n}{\partial Y} \right] + \phi(r) \frac{\mathcal{U}_{Yr}(r)}{\mathcal{U}_Y(r)} = \frac{1}{\mathcal{U}_M(r)} \left[1 - \sum_{n=2}^{N} t_n p_n \frac{\partial C_n}{\partial M} \right] + \phi(r) \frac{\mathcal{U}_{Mr}(r)}{\mathcal{U}_M(r)},$$

which we solve to find

$$\frac{\tau_Y}{1 - \tau_Y} + \sum_{n=2}^{N} t_n p_n \left(\frac{\partial C_n}{\partial M} + \frac{1}{1 - \tau_Y} \frac{\partial C_n}{\partial Y} \right) = (1 - r) \left(\frac{\mathcal{U}_{Mr}}{\mathcal{U}_M} - \frac{\mathcal{U}_{Yr}}{\mathcal{U}_Y} \right) \frac{\phi(r)}{1 - r} \mathcal{U}_M(r)$$

Defining $s_n \equiv (1 + t_n) p_n C_n / ((1 - \tau_Y) Y)$ and $s_M \equiv M / ((1 - \tau_Y) Y)$, this condition can be rewritten as

$$\frac{\tau_Y}{1 - \tau_Y} + \sum_{n=2}^{N} \frac{t_n}{1 + t_n} s_n \left(\frac{\partial \ln C_n}{\partial \ln M} \frac{1}{s_M} + \frac{\partial \ln C_n}{\partial \ln Y} \right) = (1 - r) \left(\frac{\mathcal{U}_{Mr}}{\mathcal{U}_M} - \frac{\mathcal{U}_{Yr}}{\mathcal{U}_Y} \right) \frac{\phi(r)}{1 - r} \mathcal{U}_M(r).$$

The multiplier $\phi(r)$ solves

$$\phi'(r) = -\frac{\partial \mathcal{H}}{\partial W} = \lambda\omega(r) G'(W(r)) - \psi(r) = \lambda\omega(r) G'(W(r)) - \frac{1}{\mathcal{U}_M} \left[1 - \sum_{n=2}^{N} t_n p_n \frac{\partial C_n}{\partial M} \right] - \phi(r) \frac{\mathcal{U}_{Mr}(r)}{\mathcal{U}_M(r)}$$

with the boundary conditions $\phi(0) = \phi(1) = 0$. Define $m(r) = \exp\left(-\int_r^1 \frac{\mathcal{U}_{Mr}(r')}{\mathcal{U}_M(r')} dr'\right)$, so that $\mathcal{U}_{Mr}(r)/\mathcal{U}_M(r) = m'(r)/m(r)$. Substituting into the previous ODE and integrating out yields

$$\phi(r)\, m(r) = \int_r^1 \left(1 - \lambda\omega(r')\, G'(W(r'))\, \mathcal{U}_M(r') - \sum_{n=2}^{N} t_n p_n \frac{\partial C_n}{\partial M}\right) \frac{1}{\mathcal{U}_M(r')} m(r')\, dr',$$

or

$$\frac{\phi(r)}{1-r}\mathcal{U}_M(r) = \mathbb{E}\left[\left(1 - \Gamma(r') - \sum_{n=2}^{N} \frac{t_n}{1+t_n} s_n \frac{\partial \ln C_n}{\partial \ln M} \frac{1}{s_M}\right) \frac{\mathcal{U}_M(r)}{\mathcal{U}_M(r')} \frac{m(r')}{m(r)} \middle| r' \geq r\right]$$

with $\Gamma(r) = \lambda\omega(r)\, G'(W(r))\, \mathcal{U}_M(r)$ and λ such that $\phi(0) = 0$.[53] The marginal consumption taxes t_n enter in two places: First, the efficiency cost of tax distortions on the left-hand side includes both the direct efficiency cost of the income tax distortion, $\tau_Y/(1-\tau_Y)$, and the additional spill-over effect to consumption tax revenue at rank r, which is given by $\sum_{n=2}^{N} \frac{t_n}{1+t_n} s_n \left(\frac{\partial \ln C_n}{\partial \ln M}\frac{1}{s_M} + \frac{\partial \ln C_n}{\partial \ln Y}\right)$. Second, the shadow price of redistribution $\gamma(r) \equiv \phi(r)\mathcal{U}_M(r)/(1-r)$ is reduced by the fact that some redistribution takes place through consumption taxes.

First-Stage Identification. Define $\check{\mathcal{E}}_M \equiv -\mathcal{U}_{MM}M/\mathcal{U}_M$, $\check{\mathcal{E}}_Y \equiv \mathcal{U}_{YY}Y/\mathcal{U}_Y$, $\check{\mathcal{E}}_{MY} \equiv \mathcal{U}_{MY}Y/\mathcal{U}_M$, $s_M \equiv M/(1-\tau_Y)\,Y$ and $\rho_M(r) \equiv -\left(d\ln M(r)/d\ln(1-r)\right)^{-1}$. We obtain the following first-stage analogue of Lemma 5, already linking the preference elasticities $\check{\mathcal{E}}_M$, $\check{\mathcal{E}}_Y$, and $\check{\mathcal{E}}_{MY}$ to behavioral elasticities ζ_Y^I and ζ_{Y,τ_Y}^H:

Lemma 7 (**Identification (first stage)**). *For any given income tax distortion $\{\tau_Y(r)\}$, we have*

$$-\frac{d\ln\mathcal{U}_M(r)}{d\ln(1-r)} = \frac{\check{\mathcal{E}}_M/s_M - \check{\mathcal{E}}_{MY}}{\rho_Y} + (1-r)\frac{\mathcal{U}_{Mr}}{\mathcal{U}_M} = \frac{\zeta_Y^I}{\rho_Y\zeta_{Y,\tau_Y}^H} + (1-r)\frac{\mathcal{U}_{Mr}}{\mathcal{U}_M} \tag{40}$$

and

$$-\frac{d\ln(-\mathcal{U}_Y(r))}{d\ln(1-r)} = \frac{\check{\mathcal{E}}_Y - \check{\mathcal{E}}_{MY}}{\rho_Y} + (1-r)\frac{\mathcal{U}_{Yr}}{\mathcal{U}_Y} = \frac{1-\zeta_Y^I}{\rho_Y\zeta_{Y,\tau_Y}^H} + (1-r)\frac{\mathcal{U}_{Yr}}{\mathcal{U}_Y} \tag{41}$$

and

$$(1-r)\left(\frac{\mathcal{U}_{Mr}}{\mathcal{U}_M} - \frac{\mathcal{U}_{Yr}}{\mathcal{U}_Y}\right) = \frac{\check{\mathcal{E}}_Y - 2\check{\mathcal{E}}_{MY} + \check{\mathcal{E}}_M/s_M}{\rho_Y} + \frac{d\ln(1-\tau_Y)}{d\ln(1-r)} = \frac{1}{\rho_Y\zeta_{Y,\tau_Y}^H} - \frac{d\ln(1-\tau_Y)}{d\ln(1-r)}. \tag{42}$$

Proof. Since $M = Y - T_Y(Y)$, it follows that $(M'(r)/M(r))\, s_M = Y'(r)/Y(r)$ and $s_M/\rho_M =$

[53] By the envelope condition $\mathcal{U}_M(r) = U_1(r)/p_1$, this definition of $\Gamma(r)$ is the same as in Lemma 6.

$1/\rho_Y$. Differentiating $\mathcal{U}_M$ and $-\mathcal{U}_Y$ w.r.t. r yields

$$\frac{d\ln\mathcal{U}_M}{dr}=\frac{\mathcal{U}_{Mr}}{\mathcal{U}_M}-\check{\mathcal{E}}_M\frac{M'(r)}{M(r)}+\check{\mathcal{E}}_{MY}\frac{Y'(r)}{Y(r)}=\frac{\mathcal{U}_{Mr}}{\mathcal{U}_M}-\left(\check{\mathcal{E}}_M/s_M-\check{\mathcal{E}}_{MY}\right)\frac{Y'(r)}{Y(r)}$$

and

$$\frac{d\ln(-\mathcal{U}_Y)}{dr}=\frac{\mathcal{U}_{Yr}}{\mathcal{U}_Y}+\check{\mathcal{E}}_Y\frac{Y'(r)}{Y(r)}-\check{\mathcal{E}}_{MY}s_M\frac{M'(r)}{M(r)}=\frac{\mathcal{U}_{Yr}}{\mathcal{U}_Y}+\left(\check{\mathcal{E}}_Y-\check{\mathcal{E}}_{MY}\right)\frac{Y'(r)}{Y(r)}$$

and

$$\frac{\mathcal{U}_{Mr}}{\mathcal{U}_M}-\frac{\mathcal{U}_{Yr}}{\mathcal{U}_Y}=\left(\check{\mathcal{E}}_Y-2\check{\mathcal{E}}_{MY}+\check{\mathcal{E}}_M/s_M\right)\frac{Y'(r)}{Y(r)}-\frac{d\ln(1-\tau_Y)}{dr}.$$

Perturbing the first-order condition $1-\tau_Y=-\mathcal{U}_Y/\mathcal{U}_M$ with respect to compensated changes income tax changes $\partial\tau_Y$ yields

$$\frac{\partial\tau_Y}{1-\tau_Y}=\left(\check{\mathcal{E}}_Y-\check{\mathcal{E}}_{MY}\right)\frac{\partial Y}{Y}+\left(\check{\mathcal{E}}_M-s_M\check{\mathcal{E}}_{MY}\right)\frac{\partial M}{M}$$

with $s_M\partial M/M=\partial Y/Y$, and therefore $\zeta^H_{Y,\tau_Y}=(\check{\mathcal{E}}_Y-2\check{\mathcal{E}}_{MY}+\check{\mathcal{E}}_M/s_M)^{-1}$. Substituting this expression into the one for $\mathcal{U}_{Mr}/\mathcal{U}_M-\mathcal{U}_{Yr}/\mathcal{U}_Y$ then yields equation (42).

Similarly, perturbing the budget constraint along with the perturbation of the FOC by a lump sum transfer ∂T yields $s_M\zeta^I_M=1-\zeta^I_Y$ and

$$0=\left(\check{\mathcal{E}}_Y-\check{\mathcal{E}}_{MY}\right)\zeta^I_Y+\left(\check{\mathcal{E}}_M-s_M\check{\mathcal{E}}_{MY}\right)\zeta^I_M$$

from which it follows that

$$\check{\mathcal{E}}_Y-\check{\mathcal{E}}_{MY}=\frac{1-\zeta^I_Y}{\zeta^H_{Y,\tau_Y}}\text{ and }\check{\mathcal{E}}_M/s_M-\check{\mathcal{E}}_{MY}=\frac{\zeta^I_Y}{\zeta^H_{Y,\tau_Y}}.$$

Substituting these expressions into $d\ln\mathcal{U}_M/dr$ and $d\ln(-\mathcal{U}_Y)/dr$ yields equations (40) and (41).

□

The Revenue Spillover Condition on Top Income Earners. Therefore, taking limits as $r\to 1$, we obtain

$$\lim_{r\to 1}\left(\frac{\mathcal{U}_{Mr}}{\mathcal{U}_M}-\frac{\mathcal{U}_{Yr}}{\mathcal{U}_Y}\right)\mathcal{U}_M\phi=\left(1-\bar{\Gamma}-\sum_{n=2}^{N}\frac{t_n}{1+t_n}\bar{s}_n\frac{\partial\ln C_n}{\partial\ln M}\frac{1}{s_M}\right)\frac{1}{\bar{\zeta}^H_{Y,\tau_Y}\bar{\rho}_Y-\bar{\zeta}^I_Y},$$

and the optimality condition for $\bar{\tau}_Y$ can be re-stated as:

$$\frac{\bar{\tau}_Y}{1-\bar{\tau}_Y} = \frac{\bar{\tau}_Y^{Saez}}{1-\bar{\tau}_Y^{Saez}}\left(1-\bar{\Gamma}-\sum_{n=2}^{N}\frac{\bar{t}_n}{1+\bar{t}_n}\bar{s}_n\Phi_n\right)$$

where $\bar{\tau}_Y^{Saez} = (1+\bar{\zeta}^H_{Y,\tau_Y}\bar{\rho}_Y - \bar{\zeta}^I_Y)^{-1}$ and

$$\Phi_n = \frac{\partial \ln C_n}{\partial \ln M}\frac{1}{s_M} + \left(\frac{\partial \ln C_n}{\partial \ln M}\frac{1}{s_M} + \frac{\partial \ln C_n}{\partial \ln Y}\right)\left(\bar{\zeta}^H_{Y,\tau_Y}\bar{\rho}_Y - \bar{\zeta}^I_Y\right).$$

Finally, since $\zeta^H_{n,\tau_Y} = \frac{\partial \ln C_n}{\partial \ln M}\zeta^H_{M,\tau_Y} + \frac{\partial \ln C_n}{\partial \ln Y}\zeta^H_{Y,\tau_Y}$ and $\zeta^H_{Y,\tau_Y} = s_M\zeta^H_{M,\tau_Y}$, we have $\frac{\partial \ln C_n}{\partial \ln M}\frac{1}{s_M} + \frac{\partial \ln C_n}{\partial \ln Y} = \zeta^H_{n,\tau_Y}/\zeta^H_{Y,\tau_Y}$. In addition, $\zeta^I_n = \frac{\partial \ln C_n}{\partial \ln M}\zeta^I_M - \frac{\partial \ln C_n}{\partial \ln Y}\zeta^I_Y$ and $1-\zeta^I_Y = s_M\zeta^I_M$, so that

$$\zeta^I_n = \frac{\partial \ln C_n}{\partial \ln M}\frac{1}{s_M}\left(1-\zeta^I_Y\right) - \frac{\partial \ln C_n}{\partial \ln Y}\zeta^I_Y = \frac{\partial \ln C_n}{\partial \ln M}\frac{1}{s_M} - \frac{\zeta^H_{n,\tau_Y}}{\zeta^H_{Y,\tau_Y}}\bar{\zeta}^I_Y$$

and therefore, we obtain

$$\Phi_n = \frac{\partial \ln C_n}{\partial \ln M}\frac{1}{s_M} - \frac{\zeta^H_{n,\tau_Y}}{\zeta^H_{Y,\tau_Y}}\bar{\zeta}^I_Y + \zeta^H_{n,\tau_Y}\bar{\rho}_Y = \bar{\zeta}^I_n + \zeta^H_{n,\tau_Y}\bar{\rho}_Y.$$

The revenue spill-over condition nests the one for the baseline model with $N=2$ and the spill-over term $\sum_{n=2}^{N}\frac{\bar{t}_n}{1+\bar{t}_n}\bar{s}_n\Phi_n$ on the right-hand side equal to $\frac{\bar{\tau}_S}{1+\bar{\tau}_S}\bar{s}_S(\zeta^H_{S,\tau_Y}\bar{\rho}_Y + \bar{\zeta}^I_S)$. Hence, Proposition 2 follows as a special case.

A.5 Identification, Step 2: from Preference to Behavioral Elasticities

At this stage, it remains to express the preference elasticities $(\tilde{\mathcal{E}}_Y, \tilde{\mathcal{E}}_n)$ and the tax-arbitrage coefficient Σ_n in terms of behavioral elasticities.

Completely separable preferences. If preferences are completely separable, i.e. $\mathcal{E}_{nY} = \mathcal{E}_{nk} = 0$ for all n and $k \neq n$, we obtain $\tilde{\mathcal{E}}_n = \mathcal{E}_n$ and $\tilde{\mathcal{E}}_Y = \mathcal{E}_Y$. Perturbing the FOC $1+t_n = U_n/U_1$ w.r.t. a change in τ_Y or a lump sum transfer ∂T yields $\mathcal{E}_n\zeta^H_{n,\tau_Y} = \mathcal{E}_1\zeta^H_{1,\tau_Y}$ and $\mathcal{E}_n\zeta^I_n = \mathcal{E}_1\zeta^I_1$, and perturbing the FOC $1-\tau_Y = -U_Y/U_1$ w.r.t. a change in t_n or lump sum transfer ∂T yields $-\mathcal{E}_Y\zeta^H_{Y,t_n} = \mathcal{E}_1\zeta^H_{1,t_n}$ and $\mathcal{E}_Y\zeta^I_Y = \mathcal{E}_1\zeta^I_1$. It then follows that

$$\Sigma_n = \frac{\frac{\mathcal{E}_n/\rho_n}{\mathcal{E}_1/\rho_1}-1}{1+\frac{\mathcal{E}_Y/\rho_Y}{\mathcal{E}_1/\rho_1}} = \frac{-\zeta^H_{Y,t_n}\rho_Y}{\zeta^H_{n,\tau_Y}\rho_n}\frac{\zeta^H_{1,\tau_Y}\rho_1 - \zeta^H_{n,\tau_Y}\rho_n}{\zeta^H_{1,t_n}\rho_1 - \zeta^H_{Y,t_n}\rho_Y} = s_n\frac{\rho_Y}{\rho_n}\frac{\zeta^H_{1,\tau_Y}\rho_1 - \zeta^H_{n,\tau_Y}\rho_n}{\zeta^H_{1,t_n}\rho_1 - \zeta^H_{Y,t_n}\rho_Y}.$$

Consumption and Savings ($N = 2$) with unrestricted preferences. When $N = 2$ and preferences are defined over consumption and savings as in our baseline model, we have that

$$\Sigma = \frac{\tilde{\mathcal{E}}_S/\rho_S - \tilde{\mathcal{E}}_C/\rho_C}{\tilde{\mathcal{E}}_C/\rho_C + \tilde{\mathcal{E}}_Y/\rho_Y}$$

with

$$\tilde{\mathcal{E}}_C/\rho_C = \mathcal{E}_C/\rho_C - \mathcal{E}_{CS}/\rho_S - \mathcal{E}_{CY}/\rho_Y \ , \ \tilde{\mathcal{E}}_S/\rho_S = \mathcal{E}_S/\rho_S - \mathcal{E}_{SC}/\rho_C - \mathcal{E}_{SY}/\rho_Y$$

$$\text{and } \tilde{\mathcal{E}}_Y/\rho_Y = \mathcal{E}_Y/\rho_Y - s_C\mathcal{E}_{CY}/\rho_C - s_S\mathcal{E}_{SY}/\rho_S.$$

Moreover the budget constraint implies that $1/\rho_Y = s_C/\rho_C + s_S/\rho_S$, so that we obtain

$$\tilde{\mathcal{E}}_C/\rho_C = \hat{\mathcal{E}}_C/\rho_C - \Delta \ , \ \tilde{\mathcal{E}}_S/\rho_S = \hat{\mathcal{E}}_S/\rho_S - \Delta \text{ and } \tilde{\mathcal{E}}_Y/\rho_Y = \hat{\mathcal{E}}_Y/\rho_Y + \Delta,$$

where $\hat{\mathcal{E}}_Y, \hat{\mathcal{E}}_C, \hat{\mathcal{E}}_S$ are defined in Lemma 1 and $\Delta \equiv s_C/\rho_C\left(\mathcal{E}_{SY} + \mathcal{E}_{SC}/s_C\right) + s_S/\rho_S\left(\mathcal{E}_{CY} + \mathcal{E}_{CS}/s_S\right)$. It follows that $\Sigma = \frac{\hat{\mathcal{E}}_S/\rho_S - \hat{\mathcal{E}}_C/\rho_C}{\hat{\mathcal{E}}_C/\rho_C + \hat{\mathcal{E}}_Y/\rho_Y}$. We finally express $\hat{\mathcal{E}}_Y, \hat{\mathcal{E}}_C, \hat{\mathcal{E}}_S$ in terms of the behavioral elasticities.

Lemma 8 (**Identification: behavioral elasticities**). *The preference elasticities $\hat{\mathcal{E}}_Y, \hat{\mathcal{E}}_C, \hat{\mathcal{E}}_S$ defined in Lemma 1 are jointly identified by the substitution effects $\zeta^H_{Y,\tau_Y}, \zeta^H_{C,\tau_Y}, \zeta^H_{S,\tau_Y}, \zeta^H_{Y,\tau_S}, \zeta^H_{C,\tau_S}, \zeta^H_{S,\tau_S}$ via any three of the four conditions*

$$\hat{\mathcal{E}}_Y\zeta^H_{Y,\tau_Y} + \hat{\mathcal{E}}_C\zeta^H_{C,\tau_Y} = 1, \quad \hat{\mathcal{E}}_C\zeta^H_{C,\tau_Y} - \hat{\mathcal{E}}_S\zeta^H_{S,\tau_Y} = 0,$$
$$\hat{\mathcal{E}}_Y\zeta^H_{Y,\tau_S} + \hat{\mathcal{E}}_C\zeta^H_{C,\tau_S} = 0, \quad \hat{\mathcal{E}}_C\zeta^H_{C,\tau_S} - \hat{\mathcal{E}}_S\zeta^H_{S,\tau_S} = 1.$$

The preference complementarities $\mathcal{E}_{SY} - \mathcal{E}_{CY}, \mathcal{E}_{SY} + \mathcal{E}_{CS}/s_S$ and $\mathcal{E}_{CY} + \mathcal{E}_{SC}/s_C$ are jointly identified by the income effects $\zeta^I_Y, \zeta^I_C, \zeta^I_S$:

$$\hat{\mathcal{E}}_C\zeta^I_C = \hat{\mathcal{E}}_S\zeta^I_S + \mathcal{E}_{SY} - \mathcal{E}_{CY} = \hat{\mathcal{E}}_Y\zeta^I_Y + \mathcal{E}_{SY} + \mathcal{E}_{CS}/s_S,$$

and therefore $\Delta = s_C\zeta^I_C\hat{\mathcal{E}}_C/\rho_C + s_S\zeta^I_S\hat{\mathcal{E}}_S/\rho_S - \zeta^I_Y\hat{\mathcal{E}}_Y/\rho_Y$. Moreover, the preference elasticities $\hat{\mathcal{E}}_Y, \hat{\mathcal{E}}_C, \hat{\mathcal{E}}_S$ and complementarities $\mathcal{E}_{SY} - \mathcal{E}_{CY}, \mathcal{E}_{SY} + \mathcal{E}_{CS}/s_S$ and $\mathcal{E}_{CY} + \mathcal{E}_{SC}/s_C$ are invariant to monotone transformations of U.

Proof of Lemma 8. Consider a labor income tax schedule $T_Y(Y)$ and a savings tax schedule $T_S(S)$. For ease of notation, assume that the tax schedules are locally linear in the top bracket, $T''_Y(Y) = T''_S(S) = 0$. A perturbation of the total tax payment by ∂T and the marginal tax rates by $\partial\tau_Y$ and $\partial\tau_S$ lead to behavioral responses $(\partial Y, \partial C, \partial S)$ that satisfy the perturbed first-order conditions

$$-\frac{U_Y\left[Y+\partial Y, C+\partial C, S+\partial S; r\right]}{U_C\left[Y+\partial Y, C+\partial C, S+\partial S; r\right]} = 1 - T_Y'\left(Y\right) - \partial\tau_Y$$

and

$$\frac{U_S\left[Y+\partial Y, C+\partial C, S+\partial S; r\right]}{U_C\left[Y+\partial Y, C+\partial C, S+\partial S; r\right]} = 1 + T_S'\left(S\right) + \partial\tau_S$$

with

$$\partial C + \left(1 + T_S'\left(S\right)\right)\partial S = \left(1 - T_Y'\left(Y\right)\right)\partial Y - \partial T.$$

Taking first-order Taylor expansions of the two perturbed FOCs around 0 yields:

$$\begin{aligned}
&\left(\mathcal{E}_Y - \mathcal{E}_{CY}\right)\frac{\partial Y}{Y} + \left(\mathcal{E}_C - s_C\mathcal{E}_{CY}\right)\frac{\partial C}{C} - \left(s_S\mathcal{E}_{SY} + \mathcal{E}_{CS}\right)\frac{\partial S}{S} = -\frac{\partial\tau_Y}{1-\tau_Y}\\
&\left(\mathcal{E}_{SY} - \mathcal{E}_{CY}\right)\frac{\partial Y}{Y} + \left(\mathcal{E}_C + \frac{s_C}{s_S}\mathcal{E}_{CS}\right)\frac{\partial C}{C} - \left(\mathcal{E}_S + \mathcal{E}_{CS}\right)\frac{\partial S}{S} = \frac{\partial\tau_S}{1+\tau_S}\\
&s_C\frac{\partial C}{C} + s_S\frac{\partial S}{S} = \frac{\partial Y}{Y} - \frac{\partial T}{\left(1-\tau_Y\right)Y}.
\end{aligned}$$

Using the third equation to substitute respectively $\partial S/S$ and $\partial Y/Y$ into the first two, and using the definitions of $\hat{\mathcal{E}}_Y, \hat{\mathcal{E}}_C, \hat{\mathcal{E}}_S$ leads to:

$$\begin{aligned}
\hat{\mathcal{E}}_Y\frac{\partial Y}{Y} + \hat{\mathcal{E}}_C\frac{\partial C}{C} &= -\frac{\partial\tau_Y}{1-\tau_Y} - \left(\mathcal{E}_{SY} + \mathcal{E}_{CS}/s_S\right)\frac{\partial T}{\left(1-\tau_Y\right)Y}\\
\hat{\mathcal{E}}_C\frac{\partial C}{C} - \hat{\mathcal{E}}_S\frac{\partial S}{S} &= \frac{\partial\tau_S}{1+\tau_S} - \left(\mathcal{E}_{SY} - \mathcal{E}_{CY}\right)\frac{\partial T}{\left(1-\tau_Y\right)Y}
\end{aligned}$$

The solution to this system can be expressed as

$$\begin{aligned}
\frac{\partial Y}{Y} &= -\zeta^H_{Y,\tau_Y}\frac{\partial\tau_Y}{1-\tau_Y} + \zeta^I_Y\frac{\partial T}{\left(1-\tau_Y\right)Y} &= \zeta^H_{Y,\tau_S}\frac{\partial\tau_S}{1+\tau_S} + \zeta^I_Y\frac{\partial T}{\left(1-\tau_Y\right)Y}\\
\frac{\partial C}{C} &= -\zeta^H_{C,\tau_Y}\frac{\partial\tau_Y}{1-\tau_Y} - \zeta^I_C\frac{\partial T}{\left(1-\tau_Y\right)Y} &= \zeta^H_{C,\tau_S}\frac{\partial\tau_S}{1+\tau_S} - \zeta^I_C\frac{\partial T}{\left(1-\tau_Y\right)Y}\\
\frac{\partial S}{S} &= -\zeta^H_{S,\tau_Y}\frac{\partial\tau_Y}{1-\tau_Y} - \zeta^I_S\frac{\partial T}{\left(1-\tau_Y\right)Y} &= \zeta^H_{S,\tau_S}\frac{\partial\tau_S}{1+\tau_S} - \zeta^I_S\frac{\partial T}{\left(1-\tau_Y\right)Y}.
\end{aligned}$$

Substituting these equations into the previous system easily leads to the expressions given in the Lemma 8. □

Derivation of Tax-Arbitrage Coefficient (Proof of Proposition 1). Using Lemmas 5 and 8, we obtain

$$\Sigma = \lim_{r\to 1} \frac{\frac{\hat{\mathcal{E}}_C}{\rho_C}\left(\frac{\rho_C}{\rho_S}\frac{\zeta^H_{C,\tau_Y}}{\zeta^H_{S,\tau_Y}} - 1\right)}{\frac{\hat{\mathcal{E}}_C}{\rho_C}\left(1 - \frac{\rho_C}{\rho_Y}\frac{\zeta^H_{C,\tau_S}}{\zeta^H_{Y,\tau_S}}\right)} = \frac{\frac{1}{\bar{\rho}_S\bar{\zeta}^H_{S,\tau_Y}}\left(\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y} - \bar{\rho}_S\bar{\zeta}^H_{S,\tau_Y}\right)}{\frac{1}{\bar{\zeta}^H_{Y,\tau_S}}\left(\bar{\zeta}^H_{Y,\tau_S} - \frac{\bar{\rho}_C}{\bar{\rho}_Y}\bar{\zeta}^H_{C,\tau_S}\right)}.$$

Using the Slutzky relationship $\bar{\zeta}^H_{Y,\tau_S} = -\bar{s}_S\bar{\zeta}^H_{S,\tau_Y}$ and the over-identifying restrictions $\bar{\zeta}^H_{Y,\tau_S} = \bar{s}_C\bar{\zeta}^H_{C,\tau_S} + \bar{s}_S\bar{\zeta}^H_{S,\tau_S}$ and $1/\bar{\rho}_Y = \bar{s}_C/\bar{\rho}_C + \bar{s}_S/\bar{\rho}_S$, the denominator is equal to

$$\frac{1}{\bar{s}_S\bar{\zeta}^H_{S,\tau_Y}}\left(\left(\frac{\bar{s}_C}{\bar{\rho}_C} + \frac{\bar{s}_S}{\bar{\rho}_S}\right)\bar{\rho}_C\bar{\zeta}^H_{C,\tau_S} - \left(\bar{s}_C\bar{\zeta}^H_{C,\tau_S} + \bar{s}_S\bar{\zeta}^H_{S,\tau_S}\right)\right) = \frac{1}{\bar{\rho}_S\bar{\zeta}^H_{S,\tau_Y}}\left(\bar{\rho}_C\bar{\zeta}^H_{C,\tau_S} - \bar{\rho}_S\bar{\zeta}^H_{S,\tau_S}\right)$$

and therefore

$$\Sigma = \frac{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y} - \bar{\rho}_S\bar{\zeta}^H_{S,\tau_Y}}{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_S} - \bar{\rho}_S\bar{\zeta}^H_{S,\tau_S}}.$$

Similarly, using $\bar{s}_S\bar{\zeta}^H_{S,\tau_Y} = \bar{\zeta}^H_{Y,\tau_Y} - \bar{s}_C\bar{\zeta}^H_{C,\tau_Y}$ and $\bar{s}_S/\bar{\rho}_S = 1/\bar{\rho}_Y - \bar{s}_C/\bar{\rho}_C$, the numerator is equal to

$$\frac{1}{\bar{s}_S\bar{\zeta}^H_{S,\tau_Y}}\left(\left(\frac{1}{\bar{\rho}_Y} - \frac{\bar{s}_C}{\bar{\rho}_C}\right)\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y} - \bar{\zeta}^H_{Y,\tau_Y} + \bar{s}_C\bar{\zeta}^H_{C,\tau_Y}\right) = \frac{1}{\bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_S}}\left(\bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y} - \bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y}\right)$$

and therefore

$$\Sigma = \frac{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y} - \bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y}}{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_S} - \bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_S}}.$$

This concludes the proof.

Proof of Theorem 2. Equations (17) follow easily from equations (8) and (14). □

Proof of Corollary 1. For *Case 1* and *Case 3*, the expressions follow immediately from Theorem 2 and the fact that $\Phi_S = 1/\overline{\tau}^{Saez}_Y$ in *Case 3*. For *Case 2*, we write

$$\Phi_S = \frac{\bar{\zeta}^H_{S,\tau_Y}}{\bar{\zeta}^H_{Y,\tau_Y}}\frac{1}{\overline{\tau}^{Saez}_Y} + \left(1 - \bar{\zeta}^I_Y\right)\left(\frac{\bar{\zeta}^I_S}{1-\bar{\zeta}^I_Y} - \frac{\bar{\zeta}^H_{S,\tau_Y}}{\bar{\zeta}^H_{Y,\tau_Y}}\right) = \frac{\bar{\zeta}^I_S}{1-\bar{\zeta}^I_Y}\frac{1}{\overline{\tau}^{Saez}_Y} - \bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y}\left(\frac{\bar{\zeta}^I_S}{1-\bar{\zeta}^I_Y} - \frac{\bar{\zeta}^H_{S,\tau_Y}}{\bar{\zeta}^H_{Y,\tau_Y}}\right),$$

and therefore $\bar{\zeta}^I_S/\left(1-\bar{\zeta}^I_Y\right) \gtreqless \Phi_S \gtreqless \bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}\cdot 1/\overline{\tau}^{Saez}_Y$ if and only if $\bar{\zeta}^I_S/\bar{\zeta}^H_{S,\tau_Y} \gtreqless \left(1-\bar{\zeta}^I_Y\right)/\bar{\zeta}^H_{Y,\tau_Y}$, and the expressions and bounds for *Case 2* are satisfied if $\bar{\zeta}^I_S/\bar{\zeta}^H_{S,\tau_Y} \gtreqless \left(1-\bar{\zeta}^I_Y\right)/\bar{\zeta}^H_{Y,\tau_Y}$ is equivalent to $\mathcal{E}_{CY} \gtreqless \mathcal{E}_{SY}$. But the identifying restrictions in Lemma 8 imply that

$$\mathcal{E}_{CY} - \mathcal{E}_{SY} = \hat{\mathcal{E}}_S\zeta^I_S - \hat{\mathcal{E}}_C\zeta^I_C = \hat{\mathcal{E}}_C/\bar{s}_C\left(\bar{s}_C\bar{\zeta}^H_{C,\tau_Y}\zeta^I_S/\bar{\zeta}^H_{S,\tau_Y} - \bar{s}_C\zeta^I_C\right)$$

$$= \hat{\mathcal{E}}_C/\bar{s}_C\left(\left(\bar{\zeta}^H_{Y,\tau_Y} - \bar{s}_S\bar{\zeta}^H_{S,\tau_Y}\right)\zeta^I_S/\bar{\zeta}^H_{S,\tau_Y} - \left(1 - \bar{\zeta}^I_Y - \bar{s}_S\bar{\zeta}^I_S\right)\right) = \hat{\mathcal{E}}_C/\bar{s}_C \cdot \bar{\zeta}^H_{Y,\tau_Y}\left(\frac{\bar{\zeta}^I_S}{\bar{\zeta}^H_{S,\tau_Y}} - \frac{1-\bar{\zeta}^I_Y}{\bar{\zeta}^H_{Y,\tau_Y}}\right)$$

from which the result follows immediately. □

A.6 Proofs for results in Section 2.7

Recall the identification from the first-stage problem where $\check{\mathcal{E}}_M \equiv -\mathcal{U}_{MM}M/\mathcal{U}_M, \check{\mathcal{E}}_Y \equiv \mathcal{U}_{YY}Y/\mathcal{U}_Y, \check{\mathcal{E}}_{MY} \equiv \mathcal{U}_{MY}Y/\mathcal{U}_M$ and $s_M \equiv M/\left(\left(1-\tau_Y\right)Y\right)$ satisfy

$$\check{\mathcal{E}}_Y - \check{\mathcal{E}}_{MY} = \frac{1-\zeta^I_Y}{\zeta^H_{Y,\tau_Y}} \text{ and } \check{\mathcal{E}}_M/s_M - \check{\mathcal{E}}_{MY} = \frac{\zeta^I_Y}{\zeta^H_{Y,\tau_Y}}.$$

We first re-state these identifying relationships in terms of the preference elasticities $\hat{\mathcal{E}}_Y$, $\hat{\mathcal{E}}_S$, $\hat{\mathcal{E}}_C$ and one additional preference parameter $\mathcal{K}$ that summarizes preference complementarities.

Lemma 9 (**Identification: labor supply elasticities**)**.** *The income and substitution effects on labor supply ζ^H_{Y,τ_Y} and ζ^I_Y identify $\hat{\mathcal{E}}_Y$ and $\hat{RA} \equiv \hat{\mathcal{E}}_S\hat{\mathcal{E}}_C/(s_S\hat{\mathcal{E}}_C + s_C\hat{\mathcal{E}}_S)$ via the relationships:*

$$\hat{\mathcal{E}}_Y = \frac{1-\bar{\zeta}^I_Y}{\bar{\zeta}^H_{Y,\tau_Y}} - \mathcal{K} \quad \text{and} \quad \hat{RA} = \frac{\bar{\zeta}^I_Y}{\bar{\zeta}^H_{Y,\tau_Y}} + \mathcal{K}$$

where

$$\mathcal{K} = \mathcal{E}_{SC} + \mathcal{E}_{CS} + \frac{s_S\hat{\mathcal{E}}_C\mathcal{E}_{CY} + s_C\hat{\mathcal{E}}_S\mathcal{E}_{SY}}{s_S\hat{\mathcal{E}}_C + s_C\hat{\mathcal{E}}_S}.$$

Proof of Lemma 9. Recall that $\mathcal{U}\left(Y,M;r\right) = U\left(Y,\mathcal{C}\left(Y,M,r\right),\mathcal{S}\left(Y,M,r\right);r\right)$, where $\mathcal{S}\left(Y,M,r\right)$ and $\mathcal{C}\left(Y,M,r\right)$ are the solution to

$$\left(1+\tau_S\right)U_C\left(Y,\mathcal{C},\mathcal{S};r\right) = U_S\left(Y,\mathcal{C},\mathcal{S};r\right) \text{ and } M = \mathcal{C} + \mathcal{S} + T_S\left(\mathcal{S}\right).$$

In addition, by the envelope theorem, we have

$$\mathcal{U}_M\left(Y,M;r\right) = U_C\left(Y,\mathcal{C}\left(Y,M,r\right),\mathcal{S}\left(Y,M,r\right);r\right) \text{ and } \mathcal{U}_Y\left(Y,M;r\right) = U_Y\left(Y,\mathcal{C}\left(Y,M,r\right),\mathcal{S}\left(Y,M,r\right);r\right).$$

Differentiating the envelope condition leads to

$$\begin{aligned}
\check{\mathcal{E}}_Y &= \mathcal{E}_Y - s_C\mathcal{E}_{CY}\frac{\partial \ln \mathcal{C}}{\partial \ln Y} - s_S\mathcal{E}_{SY}\frac{\partial \ln \mathcal{S}}{\partial \ln Y} \\
\check{\mathcal{E}}_{MY} &= \mathcal{E}_{CY} - \mathcal{E}_C\frac{\partial \ln \mathcal{C}}{\partial \ln Y} + \mathcal{E}_{CS}\frac{\partial \ln \mathcal{S}}{\partial \ln Y} \\
\check{\mathcal{E}}_M &= \mathcal{E}_C\frac{\partial \ln \mathcal{C}}{\partial \ln M} - \mathcal{E}_{CS}\frac{\partial \ln \mathcal{S}}{\partial \ln M}.
\end{aligned}$$

Differentiating the second-stage budget constraint and FOC w.r.t. Y yields $s_C \partial \ln \mathcal{C} / \partial \ln Y = -s_S \partial \ln \mathcal{S} / \partial \ln Y$ and $\hat{\mathcal{E}}_C \partial \ln \mathcal{C} / \partial \ln Y - \mathcal{E}_{CY} = \hat{\mathcal{E}}_S \partial \ln \mathcal{S} / \partial \ln Y - \mathcal{E}_{SY}$, which we solve to find

$$\frac{\partial \ln \mathcal{C}}{\partial \ln Y} = \frac{s_S \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} \text{ and } \frac{\partial \ln \mathcal{S}}{\partial \ln Y} = \frac{s_C \left(\mathcal{E}_{SY} - \mathcal{E}_{CY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S}.$$

Differentiating the second-stage budget constraint and FOC w.r.t. M yields $s_C \partial \ln \mathcal{C} / \partial \ln M = s_M - s_S \partial \ln \mathcal{S} / \partial \ln M$ and $\left(\mathcal{E}_C + \mathcal{E}_{SC}\right) \partial \ln \mathcal{C} / \partial \ln M = \left(\mathcal{E}_S + \mathcal{E}_{CS}\right) \partial \ln \mathcal{S} / \partial \ln M$, which we solve to find

$$\frac{\partial \ln \mathcal{C}}{\partial \ln M} \frac{1}{s_M} = \frac{\hat{\mathcal{E}}_S - s_S \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} \text{ and } \frac{\partial \ln \mathcal{S}}{\partial \ln M} \frac{1}{s_M} = \frac{\hat{\mathcal{E}}_C + s_C \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S}.$$

Hence $\check{\mathcal{E}}_Y$, $\check{\mathcal{E}}_{MY}$, and $\check{\mathcal{E}}_M$ satisfy

$$\check{\mathcal{E}}_Y = \mathcal{E}_Y - \frac{s_S s_C \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)^2}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S}$$

and

$$\check{\mathcal{E}}_{MY} = \mathcal{E}_{CY} - \left(\mathcal{E}_C + \mathcal{E}_{SC}\right) \frac{s_S \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S}$$

and

$$\begin{aligned}
\check{\mathcal{E}}_M / s_M &= \mathcal{E}_C \frac{\hat{\mathcal{E}}_S - s_S \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} - \frac{\mathcal{E}_{CS}}{s_S} \frac{s_S \hat{\mathcal{E}}_C + s_S s_C \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} \\
&= \left(\mathcal{E}_C + \mathcal{E}_{SC}\right) \frac{\hat{\mathcal{E}}_S - s_S \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} - \frac{\mathcal{E}_{SC}}{s_C} \frac{s_C \hat{\mathcal{E}}_S - s_C s_S \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} - \frac{\mathcal{E}_{CS}}{s_S} \frac{s_S \hat{\mathcal{E}}_C + s_S s_C \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} \\
&= \left(\mathcal{E}_C + \mathcal{E}_{SC}\right) \frac{\hat{\mathcal{E}}_S - s_S \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right)}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} - \mathcal{E}_{SC} - \mathcal{E}_{CS}
\end{aligned}$$

where we have used the fact that $\mathcal{E}_{CS}/s_S = \mathcal{E}_{SC}/s_C = \mathcal{E}_{SC} + \mathcal{E}_{CS}$. It then follows that

$$\begin{aligned}
\check{\mathcal{E}}_Y - \check{\mathcal{E}}_{MY} &= \mathcal{E}_Y - \mathcal{E}_{CY} + \frac{s_S \hat{\mathcal{E}}_C}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} \left(\mathcal{E}_{CY} - \mathcal{E}_{SY}\right) \\
&= \hat{\mathcal{E}}_Y + \mathcal{E}_{SC} + \mathcal{E}_{CS} + \frac{s_S \hat{\mathcal{E}}_C \mathcal{E}_{CY} + s_C \hat{\mathcal{E}}_S \mathcal{E}_{SY}}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} = \hat{\mathcal{E}}_Y + \mathcal{K}
\end{aligned}$$

and

$$\begin{aligned}
\check{\mathcal{E}}_M / s_M - \check{\mathcal{E}}_{MY} &= \left(\mathcal{E}_C + \mathcal{E}_{SC}\right) \frac{\hat{\mathcal{E}}_S}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} - \mathcal{E}_{SC} - \mathcal{E}_{CS} - \mathcal{E}_{CY} \\
&= \hat{RA} - \mathcal{E}_{SC} - \mathcal{E}_{CS} - \mathcal{E}_{CY} - \frac{s_S \hat{\mathcal{E}}_C \mathcal{E}_{CY} + s_C \hat{\mathcal{E}}_S \mathcal{E}_{SY}}{s_S \hat{\mathcal{E}}_C + s_C \hat{\mathcal{E}}_S} = \hat{RA} - \mathcal{K}.
\end{aligned}$$

This concluses the proof. $\square$

Elasticity of intertemporal substitution. The EIS is defined by

$$EIS \equiv -\frac{\partial \ln (S/C)}{\partial \ln (1+\tau_S)}\bigg|_{Y,U \text{ constant}}.$$

The expenditure minimization problem $\min_{C,S} (C + (1+\tau_S) S)$ s.t. $U(Y,C,S) \geq \bar{U}$ yields the optimality conditions $1+\tau_S = U_S/U_C$ and $U(Y,C,S) = \bar{U}$. Differentiating this optimality condition keeping Y constant leads to

$$\hat{\mathcal{E}}_C \frac{\partial C}{C} - \hat{\mathcal{E}}_S \frac{\partial S}{S} = \frac{\partial \tau_S}{1+\tau_S}.$$

Differentiating $U(Y,C,S) = \bar{U}$ yields $U_C \partial C = -U_S \partial S$, and therefore $\partial C/C = -(s_S/s_C)\,\partial S/S$. Substituting the latter equation into the former and using $\bar{s}_S + \bar{s}_C = 1$ yields

$$EIS = \frac{\frac{\partial C}{C} - \frac{\partial S}{S}}{\frac{\partial (1+\tau_S)}{1+\tau_S}}\Bigg|_{Y,U \text{ constant}} = \frac{1}{\bar{s}_S \hat{\mathcal{E}}_C + \bar{s}_C \hat{\mathcal{E}}_S},$$

which is the expression given in the text.

Optimal Tax Formulas for $N=2$ and Corollary 2. From Corollary 1 , the expression for Σ in Proposition 1, and $\bar{s}_S \bar{\zeta}^H_{S,\tau_Y} = -\bar{\zeta}^H_{Y,\tau_S}$, we write $\bar{\tau}_Y$ as

$$\bar{\tau}_Y = \frac{\bar{\tau}_Y^{Saez}}{1 - \bar{\zeta}^H_{Y,\tau_S}/\bar{\zeta}^H_{Y,\tau_Y}\,\Sigma} = \bar{\tau}_Y^{Saez}\left(1 - \Lambda\left(1 - \frac{\bar{\rho}_Y \bar{\zeta}^H_{Y,\tau_Y}}{\bar{\rho}_C \bar{\zeta}^H_{C,\tau_Y}}\right)\right)$$

where

$$\Lambda = \frac{-\bar{\zeta}^H_{Y,\tau_S}\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y}}{\bar{\zeta}^H_{Y,\tau_Y}\left(\bar{\rho}_C\bar{\zeta}^H_{C,\tau_S} - \bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_S}\right) - \bar{\zeta}^H_{Y,\tau_S}\left(\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y} - \bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y}\right)} = \frac{\bar{\zeta}^H_{C,\tau_Y}}{\bar{\zeta}^H_{C,\tau_Y} - \bar{\zeta}^H_{C,\tau_S}\bar{\zeta}^H_{Y,\tau_Y}/\bar{\zeta}^H_{Y,\tau_S}}.$$

Although this expression focuses on *Case 2*, it also subsumes *Case 1* (in which $\bar{s}_S = -\bar{\zeta}^H_{Y,\tau_S} = 0$) and *Case 3* (in which $\bar{s}_S\bar{\zeta}^H_{S,\tau_Y} = -\bar{\zeta}^H_{Y,\tau_S} = \bar{\zeta}^H_{Y,\tau_Y}$).

Next we restate Λ in terms of the other sufficient statistics. Solving the identifying restrictions $\hat{\mathcal{E}}_Y + \hat{RA} = 1/\bar{\zeta}^H_{Y,\tau_Y}$, $\hat{\mathcal{E}}_C\bar{\zeta}^H_{C,\tau_Y} = \hat{\mathcal{E}}_S\bar{\zeta}^H_{S,\tau_Y}$, and $\hat{\mathcal{E}}_C\bar{\zeta}^H_{C,\tau_S} = -\hat{\mathcal{E}}_Y\bar{\zeta}^H_{Y,\tau_S}$ for $\hat{\mathcal{E}}_C$, $\hat{\mathcal{E}}_S$, $\hat{\mathcal{E}}_Y$ yields $\hat{\mathcal{E}}_C = \Lambda/\bar{\zeta}^H_{C,\tau_Y}$, $\hat{\mathcal{E}}_S = \Lambda/\bar{\zeta}^H_{S,\tau_Y}$, $\hat{RA} = \Lambda/\bar{\zeta}^H_{Y,\tau_Y}$, and $\hat{\mathcal{E}}_Y = (1-\Lambda)/\bar{\zeta}^H_{Y,\tau_Y}$. Combining with $\hat{RA} = \bar{\zeta}^I_Y/\bar{\zeta}^H_{Y,\tau_Y} + \mathcal{K}$, we also obtain $\Lambda = \bar{\zeta}^I_Y + \mathcal{K}\bar{\zeta}^H_{Y,\tau_Y}$. Finally, $1/EIS = \bar{s}_S\hat{\mathcal{E}}_C + \bar{s}_C\hat{\mathcal{E}}_S = \Lambda\bar{\zeta}^H_{Y,\tau_Y}/\left(\bar{\zeta}^H_{C,\tau_Y}\bar{\zeta}^H_{S,\tau_Y}\right)$, and therefore $\Lambda = \bar{\zeta}^H_{C,\tau_Y}\bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}\cdot(EIS)^{-1}$.

Finally, we need to solve for $\bar{\tau}_S$. We use Lemma 6 and the subsequent identification results to write $\bar{\tau}_S$ as

$$\bar{\tau}_S = \frac{\hat{\mathcal{E}}_S/\bar{\rho}_S - \hat{\mathcal{E}}_C/\bar{\rho}_C}{1 - \hat{\mathcal{E}}_S/\bar{\rho}_S + \Delta} = \frac{\hat{\mathcal{E}}_S}{\bar{\rho}_S(1+\Delta) - \hat{\mathcal{E}}_S}\left(1 - \frac{\bar{\rho}_S/\hat{\mathcal{E}}_S}{\bar{\rho}_C/\hat{\mathcal{E}}_C}\right) = \frac{\Lambda}{\bar{\rho}_S\bar{\zeta}^H_{S,\tau_Y}(1+\Delta) - \Lambda}\left(1 - \frac{\bar{\rho}_S\bar{\zeta}^H_{S,\tau_Y}}{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y}}\right).$$

For *Case 3* we have $\bar{\rho}_S = \bar{\rho}_Y < \bar{\rho}_C$ and $\bar{s}_C = 0$, and therefore $\Delta = (\mathcal{E}_{CY} + \mathcal{E}_{CS})/\rho_Y = \mathcal{K}/\rho_Y$. Substituting this expression along with $\bar{\rho}_S\bar{\zeta}^H_{S,\tau_Y} = \bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y}$ the yields

$$\bar{\tau}_S = \frac{1}{\bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y} - \Lambda + \bar{\zeta}^H_{Y,\tau_Y}\mathcal{K}}\Lambda\left(1 - \frac{\bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y}}{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y}}\right) = \frac{\bar{\tau}_Y^{Saez}}{1-\bar{\tau}_Y^{Saez}}\Lambda\left(1 - \frac{\bar{\rho}_Y\bar{\zeta}^H_{Y,\tau_Y}}{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y}}\right).$$

Corollary 2 then follows immediately.

For *Case 1* we have $\bar{\rho}_C = \bar{\rho}_Y < \bar{\rho}_S$ and $\bar{s}_S = 0$, and therefore $\Delta = (\mathcal{E}_{SY} + \mathcal{E}_{SC})/\rho_Y = \mathcal{K}/\rho_Y$. For *Case 2* we have $\bar{\rho}_C = \bar{\rho}_S = \bar{\rho}_Y$, and therefore

$$\bar{\rho}_Y\Delta = \bar{s}_C\zeta^I_C\hat{\mathcal{E}}_C + \bar{s}_S\zeta^I_S\hat{\mathcal{E}}_S - \zeta^I_Y\hat{\mathcal{E}}_Y = \bar{s}_C(\mathcal{E}_{SY} + \mathcal{E}_{SC}/\bar{s}_C) + \bar{s}_S(\mathcal{E}_{CY} + \mathcal{E}_{CS}/\bar{s}_S)$$

$$= \mathcal{E}_{SC} + \mathcal{E}_{CS} + \bar{s}_C\mathcal{E}_{SY} + \bar{s}_S\mathcal{E}_{CY} = \mathcal{K} + \bar{s}_S(\mathcal{E}_{CY} - \mathcal{E}_{SY})\left(1 - \bar{\zeta}^H_{S,\tau_Y}/\bar{\zeta}^H_{Y,\tau_Y}\right).$$

If preferences are completely separable, $\Delta = 0$ and $\Lambda = \bar{\zeta}^I_Y$, and we obtain $\bar{\tau}_S = \frac{\bar{\zeta}^I_Y}{\bar{\rho}_S\bar{\zeta}^H_{S,\tau_Y} - \bar{\zeta}^I_Y}\left(1 - \frac{\bar{\rho}_S\bar{\zeta}^H_{S,\tau_Y}}{\bar{\rho}_C\bar{\zeta}^H_{C,\tau_Y}}\right)$. This concludes the proof.

With $N > 2$ and completely separable preferences, Lemma 6 implies that

$$\frac{\bar{\tau}^*_Y}{1-\bar{\tau}^*_Y} = \frac{\mathcal{E}_Y/\rho_Y + \mathcal{E}_1/\rho_1}{1 - \mathcal{E}_1/\rho_1} \quad \text{and} \quad \frac{\bar{t}^*_n}{1+\bar{t}^*_n} = \frac{\mathcal{E}_n/\rho_n - \mathcal{E}_1/\rho_1}{1 - \mathcal{E}_1/\rho_1}$$

Consider first $\bar{t}^*_n$. Rearranging terms and using Lemma 8 we obtain

$$\frac{\bar{t}^*_n}{1+\bar{t}^*_n} = \frac{\mathcal{E}_1/\rho_1}{1 - \mathcal{E}_1/\rho_1}\left(\frac{\mathcal{E}_n/\rho_n}{\mathcal{E}_1/\rho_1} - 1\right) = \frac{\bar{\zeta}^I_Y}{\bar{\rho}_1\bar{\zeta}^I_Y/\mathcal{E}_1 - \bar{\zeta}^I_Y}\left(\frac{\bar{\rho}_1\bar{\zeta}^H_{1,\tau_Y}}{\bar{\rho}_n\bar{\zeta}^H_{n,\tau_Y}} - 1\right).$$

This expression can be further simplified by noting that $\bar{\zeta}^I_Y/\mathcal{E}_1 = \bar{\zeta}^H_{1,\tau_Y}$, which follows from

$$\frac{\bar{\zeta}^I_Y}{\mathcal{E}_1} = \bar{\zeta}^I_Y\bar{\zeta}^H_{1,\tau_Y} + \frac{\mathcal{E}_Y\bar{\zeta}^I_Y}{\mathcal{E}_1}\bar{\zeta}^H_{Y,\tau_Y} = \bar{\zeta}^H_{1,\tau_Y}\bar{\zeta}^I_Y + \bar{\zeta}^I_1\sum_{k=1}^N \bar{s}_k\bar{\zeta}^H_{k,\tau_Y} = \bar{\zeta}^H_{1,\tau_Y}\left(\bar{\zeta}^I_Y + \sum_{k=1}^N \bar{s}_k\bar{\zeta}^I_k\right) = \bar{\zeta}^H_{1,\tau_Y},$$

where we have used the identities $1 - \bar{\zeta}^I_Y = \sum_{k=1}^N \bar{s}_k\bar{\zeta}^I_k$ and $\bar{\zeta}^H_{Y,\tau_Y} = \sum_{k=1}^N \bar{s}_k\bar{\zeta}^H_{k,\tau_Y}$. We therefore obtain

$$\frac{\bar{t}_n}{1+\bar{t}_n} = \frac{(1-\bar{\Gamma})\bar{\zeta}_Y^I}{\bar{\rho}_1\bar{\zeta}_{1,\tau_Y}^H - \bar{\zeta}_Y^I}\left(\frac{\bar{\rho}_1\bar{\zeta}_{1,\tau_Y}^H}{\bar{\rho}_n\bar{\zeta}_{n,\tau_Y}^H} - 1\right).$$

If $N = 2$, solving for $\bar{\tau}_S = \bar{t}_n^*$ then yields the expression for the separable case in Corollary 2.

Consider next $\bar{\tau}_Y^*$, which we rewrite, using $\mathcal{E}_1 = 1/\bar{\zeta}_{1,\tau_Y}^H - \mathcal{E}_Y\bar{\zeta}_{Y,\tau_Y}^H/\bar{\zeta}_{1,\tau_Y}^H$, as

$$\bar{\tau}_Y^* = \frac{\frac{1}{\bar{\rho}_Y\bar{\zeta}_{Y,\tau_Y}^H} + \left(\frac{\mathcal{E}_Y}{\bar{\rho}_Y} - \frac{1}{\bar{\rho}_Y\bar{\zeta}_{Y,\tau_Y}^H}\right)\left(1 - \frac{\bar{\rho}_Y\bar{\zeta}_{Y,\tau_Y}^H}{\bar{\rho}_1\bar{\zeta}_{1,\tau_Y}^H}\right)}{1+\frac{\mathcal{E}_Y}{\bar{\rho}_Y}} = \frac{1-\left(1-\mathcal{E}_Y\bar{\zeta}_{Y,\tau_Y}^H\right)\left(1-\frac{\bar{\rho}_Y\bar{\zeta}_{Y,\tau_Y}^H}{\bar{\rho}_1\bar{\zeta}_{1,\tau_Y}^H}\right)}{\bar{\rho}_Y\bar{\zeta}_{Y,\tau_Y}^H + \mathcal{E}_Y\bar{\zeta}_{Y,\tau_Y}^H}.$$

Now under complete separability, $\boldsymbol{C}(Y, M; r)$ and $\mathcal{U}_M$ are independent of Y, and therefore

$$\frac{1-\bar{\zeta}_Y^I}{\bar{\zeta}_{Y,\tau_Y}^H} = \check{\mathcal{E}}_Y - \check{\mathcal{E}}_{MY} = \frac{\mathcal{U}_{YY}Y}{\mathcal{U}_Y} - \frac{\mathcal{U}_{MY}Y}{\mathcal{U}_M} = \frac{U_{YY}Y}{U_Y} - 0 = \mathcal{E}_Y.$$

Substituting $\mathcal{E}_Y\bar{\zeta}_{Y,\tau_Y}^H = 1-\bar{\zeta}_Y^I$ then yields

$$\bar{\tau}_Y^* = \bar{\tau}_Y^{Saez}\left(1-\bar{\zeta}_Y^I\left(1-\frac{\bar{\rho}_Y\bar{\zeta}_{Y,\tau_Y}^H}{\bar{\rho}_1\bar{\zeta}_{1,\tau_Y}^H}\right)\right)$$

and solving $\bar{\tau}_Y/(1-\bar{\tau}_Y) = (1-\bar{\Gamma})\bar{\tau}_Y^*/(1-\bar{\tau}_Y^*)$ for $\bar{\tau}_Y$ implies

$$\bar{\tau}_Y = \frac{(1-\bar{\Gamma})\bar{\tau}_Y^{Saez}\left(1-\bar{\zeta}_Y^I\left(1-\frac{\bar{\rho}_Y\bar{\zeta}_{Y,\tau_Y}^H}{\bar{\rho}_1\bar{\zeta}_{1,\tau_Y}^H}\right)\right)}{1-\bar{\Gamma}\bar{\tau}_Y^{Saez}\left(1-\bar{\zeta}_Y^I\left(1-\frac{\bar{\rho}_Y\bar{\zeta}_{Y,\tau_Y}^H}{\bar{\rho}_1\bar{\zeta}_{1,\tau_Y}^H}\right)\right)}.$$

These equations generalize the characterization of optimal income and savings taxes with completely separable preferences to the case with N commodities and arbitrary planner preferences. □

B Proofs of Section 3.2

The mechanism design problem consists in choosing an allocation $\{Y_t(r), C_t(r)\}_{t=0}^T$ that maximizes the net present value of tax revenue

$$K(v_0) = \max_{\{Y_t(r),C_t(r)\}_{t=0}^T; S_T(r)} \int_0^1 \left\{\sum_{t=0}^T R^{-t}\left(Y_t(r) - C_t(r)\right) - R^{-T}S_T(r)\right\} dr \tag{43}$$

subject to the incentive compatibility constraint

$$\sum_{t=0}^{T}\beta^{t}U_{t}\left(Y_{t}\left(r\right),C_{t}\left(r\right);r\right)+\beta^{T+1}U_{T+1}\left(RS_{T}\left(r\right);r\right)\geq\sum_{t=0}^{T}\beta^{t}U_{t}\left(Y_{t}\left(r'\right),C_{t}\left(r'\right);r\right)+\beta^{T+1}U_{T+1}\left(RS_{T}\left(r'\right);r\right) \tag{44}$$

for all types r and announcements r', and a lower bound constraint

$$\sum_{t=0}^{T}\beta^{t}U_{t}\left(Y_{t}\left(0\right),C_{t}\left(0\right);0\right)+\beta^{T+1}U_{T+1}\left(RS_{T}\left(0\right);0\right)\geq W_{0}. \tag{45}$$

Defining $W\left(r\right)$ as

$$W\left(r\right)=\sum_{t=0}^{T}\beta^{t}U_{t}\left(Y_{t}\left(r\right),C_{t}\left(r\right);r\right)+\beta^{T+1}U_{T+1}\left(RS_{T}\left(r\right);r\right) \tag{46}$$

an allocation$\left\{Y_{t}\left(r\right),C_{t}\left(r\right)\right\}_{t=0}^{T};S_{T}\left(r\right)$ is locally incentive compatible if $W\left(r\right)$ is continuous and satisfies the envelope condition

$$W'\left(r\right)=\sum_{t=0}^{T}\beta^{t}U_{r,t}\left(Y_{t}\left(r\right),C_{t}\left(r\right);r\right)+\beta^{T+1}U_{r,T+1}\left(RS_{T}\left(r\right);r\right) \tag{47}$$

The corresponding optimal control problem consists in maximizing (43) s.t. (45)-(46)-(47) is an optimal control problem with $W\left(\cdot\right)$ as the state variable, and $\left\{Y_{t}\left(r\right),C_{t}\left(r\right)\right\}_{t=0}^{T}$ and $S_{T}\left(\cdot\right)$ as controls. Defining $\psi\left(r\right)$ and $\phi\left(r\right)$ as the multipliers on (46)-(47), the Hamiltonian is given by:

$$\begin{aligned}\mathcal{H} = & \sum_{t=0}^{T}R^{-t}\left(C_{t}\left(r\right)-Y_{t}\left(r\right)\right)+R^{-T}S_{T}\left(r\right)\\ & +\psi\left(r\right)\left\{W\left(r\right)-\sum_{t=0}^{T}\beta^{t}U_{t}\left(Y_{t}\left(r\right),C_{t}\left(r\right);r\right)-\beta^{T+1}U_{T+1}\left(RS_{T}\left(r\right);r\right)\right\}\\ & +\phi\left(r\right)\left\{\sum_{t=0}^{T}\beta^{t}U_{r,t}\left(Y_{t}\left(r\right),C_{t}\left(r\right);r\right)+\beta^{T+1}U_{r,T+1}\left(RS_{T}\left(r\right);r\right)\right\}\end{aligned}$$

The first-order conditions with respect to the allocations $Y_{t}\left(\cdot\right)$ and $C_{t}\left(\cdot\right)$ yield:

$$\psi\left(r\right)=\left(\beta R\right)^{-t}\frac{1}{-U_{Y,t}\left(r\right)}+\phi\left(r\right)\frac{U_{Yr,t}\left(r\right)}{U_{Y,t}\left(r\right)}=\left(\beta R\right)^{-t}\frac{1}{U_{C,t}\left(r\right)}+\phi\left(r\right)\frac{U_{Cr,t}\left(r\right)}{U_{C,t}\left(r\right)}.$$

Hence, the optimal labor and savings wedges in period t satisfy

$$\frac{\tau_{Y,t}(r)}{1-\tau_{Y,t}(r)} = (\beta R)^t U_{C,t}(r)\,\phi(r)\left(\frac{U_{Cr,t}(r)}{U_{C,t}(r)} - \frac{U_{Yr,t}(r)}{U_{Y,t}(r)}\right)$$

$$\frac{\tau_{S,t}(r)}{1+\tau_{S,t}(r)} = (\beta R)^t U_{C,t}(r)\,\phi(r)\left(\frac{U_{Cr,t+1}(r)}{U_{C,t+1}(r)} - \frac{U_{Cr,t}(r)}{U_{C,t}(r)}\right)$$

Combining the two conditions yields the static no-arbitrage condition

$$\frac{\tau_{S,t}(r)}{1+\tau_{S,t}(r)} = \Sigma_{S,t}\cdot\frac{\tau_{Y,t}(r)}{1-\tau_{Y,t}(r)}, \text{ where } \Sigma_{S,t} = \frac{\frac{U_{Cr,t+1}(r)}{U_{C,t+1}(r)} - \frac{U_{Cr,t}(r)}{U_{C,t}(r)}}{\frac{U_{Cr,t}(r)}{U_{C,t}(r)} - \frac{U_{Yr,t}(r)}{U_{Y,t}(r)}}.$$

Combining the condition for the optimal labor wedge at $t+1$ and t yields

$$\frac{\tau_{Y,t+1}(r)}{1-\tau_{Y,t+1}(r)} = \Sigma_{Y,t}\cdot(1+\tau_{S,t})\frac{\tau_{Y,t}(r)}{1-\tau_{Y,t}(r)}, \text{ where } \Sigma_{Y,t} = \frac{\frac{U_{Cr,t+1}(r)}{U_{C,t+1}(r)} - \frac{U_{Yr,t+1}(r)}{U_{Y,t+1}(r)}}{\frac{U_{Cr,t}(r)}{U_{C,t}(r)} - \frac{U_{Yr,t}(r)}{U_{Y,t}(r)}}.$$

It now remains to express $\Sigma_{S,t}$ and $\Sigma_{Y,t}$ in terms of preference parameters and compensated elasticities. But these steps are identical to the identification results for the static model with completely separable preferences, implying that

$$\frac{U_{Cr,t}}{U_{C,t}} - \frac{U_{Yr,t}}{U_{Y,t}} = \frac{\mathcal{E}_{C,t}}{\rho_{C,t}} + \frac{\mathcal{E}_{Y,t}}{\rho_{Y,t}} + \frac{d\ln(1-\tau_{Y,t})}{d\ln(1-r)}$$

$$\frac{U_{Cr,t+1}}{U_{C,t+1}} - \frac{U_{Cr,t}}{U_{C,t}} = \frac{\mathcal{E}_{C,t+1}}{\rho_{C,t+1}} - \frac{\mathcal{E}_{C,t}}{\rho_{C,t}} - \frac{d\ln(1+\tau_{S,t})}{d\ln(1-r)}$$

where the identifying relations $\mathcal{E}_{C,t}\zeta^H_{C_t,\tau_{S,t}} = -\mathcal{E}_{Y,t}\zeta^H_{Y_t,\tau_{S,t}}$, $\mathcal{E}_{C,t}\zeta^H_{C_t,\tau_{Y,t}} = \mathcal{E}_{S,t}\zeta^H_{S_t,\tau_{Y,t}} = \mathcal{E}_{C,t+1}\zeta^H_{C_{t+1},\tau_{Y,t}} = -\mathcal{E}_{Y,t+1}\zeta^H_{Y_{t+1},\tau_{Y,t}}$, and $\mathcal{E}_{S,t}\zeta^I_{S,t} = \mathcal{E}_{C,t}\zeta^I_{C,t} = \mathcal{E}_{Y,t}\zeta^I_{Y,t}$ link these preference elasticities to income and substitution effects within and across periods. If Assumption 2 and 3 are satisfied and all these sufficient statistics and optimal taxes converge to finite limits as $r \to 1$, we can then restate the age-dependent tax-arbitrage coefficients as

$$\Sigma_{S,t} \;=\; \frac{\bar{\mathcal{E}}_{C,t+1}/\bar{\rho}_{C,t+1} - \bar{\mathcal{E}}_{C,t}/\bar{\rho}_{C,t}}{\bar{\mathcal{E}}_{Y,t}/\bar{\rho}_{Y,t} + \bar{\mathcal{E}}_{C,t}/\bar{\rho}_{C,t}} \;=\; \frac{\zeta^H_{C_t,\tau_{Y,t}}}{\zeta^H_{C_t,\tau_{S,t}}}\frac{\left(\bar{\rho}_{C,t+1}\zeta^H_{C_{t+1},\tau_{Y,t}}\right)^{-1} - \left(\bar{\rho}_{C,t}\zeta^H_{C_t,\tau_{Y,t}}\right)^{-1}}{\left(\bar{\rho}_{C,t}\zeta^H_{C_t,\tau_{S,t}}\right)^{-1} - \left(\bar{\rho}_{Y,t}\zeta^H_{Y_t,\tau_{S,t}}\right)^{-1}}$$

and

$$\Sigma_{Y,t} = \frac{\bar{\mathcal{E}}_{Y,t+1}/\bar{\rho}_{Y,t+1}+\bar{\mathcal{E}}_{C,t+1}/\bar{\rho}_{C,t+1}}{\bar{\mathcal{E}}_{Y,t}/\bar{\rho}_{Y,t}+\bar{\mathcal{E}}_{C,t}/\bar{\rho}_{C,t}} = \frac{\zeta^H_{C_t,\tau_{Y,t}}}{\zeta^H_{C_t,\tau_{S,t}}}\frac{\left(\bar{\rho}_{C,t+1}\zeta^H_{C_{t+1},\tau_{Y,t}}\right)^{-1}-\left(\bar{\rho}_{Y,t+1}\zeta^H_{Y_{t+1},\tau_{Y,t}}\right)^{-1}}{\left(\bar{\rho}_{C,t}\zeta^H_{C_t,\tau_{S,t}}\right)^{-1}-\left(\bar{\rho}_{Y,t}\zeta^H_{Y_t,\tau_{S,t}}\right)^{-1}}.$$

These conditions fully characterize optimal top income and savings taxes at all ages for a given value of $\overline{\tau}_{Y,0}$. Proposition 4 then follows immediately from the comparative statics of $\Sigma_{S,t}$ and $\Sigma_{Y,t}$ w.r.t. the respective sufficient statistics.

Revenue Spill-Over Conditions. Consider a perturbation $\partial\tau_{Y,t}>0$ of the marginal income tax in period t for ranks $r'\in(r,r+\partial r)$. The mechanical effect of this perturbation on government revenue is this tax perturbation mechanically raises the NPV of tax revenues by

$$R^{-t}\partial\tau_{Y,t}\left(Y_t\left(r+\partial r\right)-Y_t\left(r\right)\right)\left(1-r\right)\approx R^{-t}\partial\tau_{Y,t}\partial r\,Y_t/\rho_{Y,t}.$$

The behavioral response of earnings in period s to the income tax perturbation in period t can be decomposed into a substitution effect $\partial Y_s\left(r'\right)=-\frac{\partial\tau_{Y,t}}{1-\tau_{Y,t}}Y_s\zeta^H_{Y_s,\tau_{Y,t}}$ for ranks $r'\in[r,r+\partial r)$, and an income effect $\partial Y_s\left(r'\right)=\zeta^I_{Y_s}\left(r'\right)\frac{Y_s(r')}{\left(1-\tau_{Y,t}\right)Y_t(r')}\partial\tau_{Y,t}\partial rY_t'\left(r\right)$ for ranks $r'\geq r+\partial r$. The combined effect on the NPV of income tax revenues is

$$\frac{\sum_{s=0}^T R^{-s}\int_r^1\tau_{Y,s}\left(r\right)\partial Y_s\left(r'\right)dr'}{R^{-t}\partial\tau_{Y,t}\partial r\,Y_t/\rho_{Y,t}}\approx-\sum_{s=0}^T\frac{\tau_{Y,s}}{1-\tau_{Y,s}}\frac{s_{Y,s}}{s_{Y,t}}\left\{\zeta^H_{Y_s,\tau_{Y,t}}\rho_{Y,t}-\hat{\zeta}^I_{Y_s}\left(r\right)\right\}$$

where $s_{Y,t}\equiv R^{-t}\left(1-\tau_{Y,t}\right)Y_t/\sum_{s=0}^T R^{-s}\left(1-\tau_{Y,s}\right)Y_s$ represent shares of period t in total life-time earnings, and $\hat{\zeta}^I_{Y_s}\left(r\right)\equiv\mathbb{E}[\zeta^I_{Y_s}\left(r'\right)\big|\,r'>r]$. Taking the limit as $r\to1$, we obtain

$$\lim_{r\to1}\frac{\sum_{s=0}^T R^{-s}\int_r^1\tau_{Y,s}\left(r\right)\partial Y_s\left(r'\right)dr'}{R^{-t}\partial\tau_{Y,t}\partial r\,Y_t/\rho_{Y,t}}=-\sum_{s=0}^T\frac{\overline{\tau}_{Y,s}}{1-\overline{\tau}_{Y,s}}\frac{\bar{s}_{Y,s}}{\bar{s}_{Y,t}}\Phi_{Y_s,\tau_{Y,t}}$$

where $\Phi_{Y_s,\tau_{Y,t}}\equiv\bar{\zeta}^H_{Y_s,\tau_{Y,t}}\bar{\rho}_{Y,t}-\bar{\zeta}^I_{Y,s}$ denotes the spill-over coefficient from period-t income taxes to period s labor supply and earnings.

Similarly, the behavioral response of savings in period s to the income tax perturbation in period t can be decomposed into a substitution effect $\partial S_s\left(r'\right)=-\zeta^H_{S_s,\tau_{Y,t}}S_s\frac{\partial\tau_{Y,t}}{1-\tau_{Y,t}}=-\zeta^H_{S_s,\tau_{Y,t}}s_s\frac{\left(1-\tau_{Y,s}\right)Y_s}{\left(1-\tau_{Y,t}\right)Y_t}\frac{\partial\tau_{Y,t}}{1+\tau_{S,s}}Y_t$ for ranks $r'\in[r,r+\partial r)$, and an income effect $\partial S\left(r'\right)=-\zeta^I_{S_s}\left(r'\right)\frac{S_s(r')}{\left(1-\tau_{Y,t}\right)Y_t(r')}\partial\tau_{Y,t}\partial rY_t'\left(r\right)$ for ranks $r'\geq r+\partial r$. The latter can be re-stated as $\partial S\left(r'\right)=-\zeta^I_{S_s}\left(r'\right)s_{S,s}\left(r'\right)\frac{\left(1-\tau_{Y,s}\right)Y_s(r')}{\left(1-\tau_{Y,t}\right)Y_t(r')}\frac{\partial\tau_{Y,t}}{1+\tau_{S,s}}\partial rY_t'\left(r\right)$.

The combined effect on the NPV of income tax revenues (as $r \to 1$) is

$$\lim_{r\to 1} \frac{\sum_{s=0}^{T} R^{-s} \int_r^1 \tau_{S,s}(r)\, \partial S_s(r')\, dr'}{R^{-t} \partial\tau_{Y,t} \partial r\, Y_t/\rho_{Y,t}} = -\sum_{s=0}^{T} \frac{\overline{\tau}_{S,t}}{1+\overline{\tau}_{S,t}} \frac{\bar{s}_{Y,s}}{\bar{s}_{Y,t}} \bar{s}_{S,t} \Phi_{S_s,\tau_{Y,t}}$$

where $\Phi_{S_s,\tau_{Y,t}} \equiv \bar{\zeta}^H_{S_s,\tau_{Y,t}} \bar{\rho}_{Y,t} + \bar{\zeta}^I_{S,s}$ denotes the spill-over coefficient from period-t income taxes to period-s savings. Combining the terms, the optimal income taxes on top income earners, for a given sequence of savings taxes, must satisfy

$$1 = \sum_{s=0}^{T} \frac{\bar{s}_{Y,s}}{\bar{s}_{Y,t}} \left\{ \frac{\overline{\tau}_{S,t}}{1+\overline{\tau}_{S,t}} \bar{s}_{S,t} \Phi_{S_s,\tau_{Y,t}} + \frac{\overline{\tau}_{Y,s}}{1-\overline{\tau}_{Y,s}} \Phi_{Y_s,\tau_{Y,t}} \right\},$$

or

$$\frac{\overline{\tau}_{Y,t}}{1-\overline{\tau}_{Y,t}} = \frac{\overline{\tau}^{Saez}_{Y,t}}{1-\overline{\tau}^{Saez}_{Y,t}} \left[1 - \frac{\overline{\tau}_{S,t}}{1+\overline{\tau}_{S,t}} \bar{s}_{S,t} \Phi_{S_t,\tau_{Y,t}} - \sum_{s\neq t} \left(\frac{\overline{\tau}_{S,s}}{1+\overline{\tau}_{S,s}} \bar{s}_{S,s} \Phi_{S_s,\tau_{Y,t}} + \frac{\overline{\tau}_{Y,s}}{1-\overline{\tau}_{Y,s}} \Phi_{Y_s,\tau_{Y,t}} \right) \right],$$

where $\overline{\tau}^{Saez}_{Y,t} = (1+\Phi_{Y_t,\tau_{Y,t}})^{-1} = (1+\bar{\zeta}^H_{Y_t,\tau_{Y,t}} \bar{\rho}_{Y,t} - \bar{\zeta}^I_{Y,t})^{-1}$. The dynamic model admits revenue spill-overs within and across periods to both income and savings taxes. In particular, if agents substitute labor supply inter-temporally, i.e. $\bar{\zeta}^H_{Y_s,\tau_{Y,t}} \leq 0$ for $s \neq t$, then raising $\overline{\tau}_{Y,t}$ shifts labor supply from age t towards other ages, thus raising earnings and income tax revenue in all other periods. This effect is reinforced by income effects of the tax increase.

At the same time, if preferences are separable, then $\Phi_{S_s,\tau_{Y,t}}/\bar{\zeta}^I_{S,s} = \bar{\zeta}^H_{S_s,\tau_{Y,t}}/\bar{\zeta}^I_{S,s}\bar{\rho}_{Y,t} + 1 = -\bar{\zeta}^H_{Y_s,\tau_{Y,t}}/\bar{\zeta}^I_{Y,s}\bar{\rho}_{Y,t} + 1 = -\Phi_{Y_s,\tau_{Y,t}}/\bar{\zeta}^I_{Y,s}$, i.e. we generically have that $\Phi_{S_s,\tau_{Y,t}} \geq 0 \geq \Phi_{Y_s,\tau_{Y,t}}$ for all s, so that raising $\overline{\tau}_{Y,t}$ unambiguously lowers savings in all periods. Hence, inter-temporal substitution of labor supply generates negative spillovers towards savings but positive spillovers towards income in all other periods, with an ambiguous net effect on optimal income taxes.

Proof of Proposition 5. We re-state the no-arbitrage and revenue spillover conditions in matrix form as

$$\boldsymbol{T}_S = \hat{\Sigma}_S \cdot \boldsymbol{T}_Y \quad \text{and} \quad \boldsymbol{T}_Y = \hat{T}^{Saez} \left[\mathbf{1} - \hat{\Phi}_S \boldsymbol{T}_S - \hat{\Phi}_Y \boldsymbol{T}_Y \right],$$

where $\boldsymbol{T}_Y$, $\boldsymbol{T}_S$ and $\mathbf{1}$ are T-dimensional vectors with $\overline{\tau}_{Y,t}/(1-\overline{\tau}_{Y,t})$, $\overline{\tau}_{S,t}/(1+\overline{\tau}_{S,t})$, and 1 as their t-th entry, $\hat{T}^{Saez}$ a diagonal $T \times T$ matrix with $\overline{\tau}^{Saez}_{Y,t}/(1-\overline{\tau}^{Saez}_{Y,t})$ as its t-th diagonal element, $\hat{\Sigma}_S$ a diagonal $T \times T$ matrix with $\Sigma_{S,t}$ as its t-th diagonal element, and $\hat{\Phi}_S$ and $\hat{\Phi}_Y$ are $T \times T$ matrices with $\frac{\bar{s}_{Y,s}}{\bar{s}_{Y,t}} \bar{s}_{S,s} \Phi_{S_s,\tau_{Y,t}}$ and $\frac{\bar{s}_{Y,s}}{\bar{s}_{Y,t}} \Phi_{Y_s,\tau_{Y,t}}$ in their t,s-entry, and the diagonal elements of $\hat{\Phi}_Y$ equal to 0. Generalizing Theorem 2, we represent $\boldsymbol{T}_Y$ and $\boldsymbol{T}_S$ in terms of the static optimum $\boldsymbol{T}^{Saez} \equiv \hat{T}^{Saez} \cdot \mathbf{1}$, the revenue spillovers $\hat{\Phi}_S$ and $\hat{\Phi}_Y$ and the tax-arbitrage coefficients $\hat{\Sigma}_S$:

$$\boldsymbol{T}_Y \;=\; \left[\mathbb{I} + \hat{T}^{Saez}\left(\hat{\Phi}_Y + \hat{\Phi}_S\hat{\Sigma}_S\right)\right]^{-1}\boldsymbol{T}^{Saez} \quad \text{and} \quad \boldsymbol{T}_S \;=\; \hat{\Sigma}_S \cdot \left[\mathbb{I} + \hat{T}^{Saez}\left(\hat{\Phi}_Y + \hat{\Phi}_S\hat{\Sigma}_S\right)\right]^{-1}\boldsymbol{T}^{Saez},$$

where $\mathbb{I}$ denotes the T-dimensional identity matrix.

The static optimum $\boldsymbol{T}^{Saez}$ no longer characterizes optimal income taxes even if savings are untaxed. Define $\boldsymbol{T}_Y^{AS} \equiv [\mathbb{I}+\hat{T}^{Saez}\hat{\Phi}_Y]^{-1}\boldsymbol{T}^{Saez}$ as the optimal income tax sequence when $\overline{\tau}_{S,t} = 0$ for all t; these are the income taxes that internalize inter-temporal revenue spillovers for income taxes. Straight-forward matrix algebra implies that $\boldsymbol{T}_Y^{AS} - \boldsymbol{T}^{Saez} = -\hat{T}^{Saez}\hat{\Phi}_Y\boldsymbol{T}_Y^{AS}$, and therefore $\boldsymbol{T}_Y^{AS} \gg \boldsymbol{T}^{Saez}$ if $\Phi_{Y_s,\tau_{Y,t}} \leq 0$ for all s, t, i.e. the inter-temporal substitution of labor supply unambiguously raises optimal income taxes above the static optimum. In addition, $\boldsymbol{T}_Y - \boldsymbol{T}^{Saez} = -\hat{T}^{Saez}(\hat{\Phi}_Y + \hat{\Phi}_S\hat{\Sigma}_S)\boldsymbol{T}_Y$. Hence $\boldsymbol{T}_Y$ may be larger or smaller than $\boldsymbol{T}^{Saez}$, depending on the comparison of spill-over effects to income and savings taxes, given by $\hat{\Phi}_Y + \hat{\Phi}_S\hat{\Sigma}_S$.

The difference between $\boldsymbol{T}_Y$ and $\boldsymbol{T}_Y^{AS}$ directly depends on $\boldsymbol{T}_S$ or equivalently, on $\hat{\Sigma}_S$:

$$\boldsymbol{T}_Y^{AS} - \boldsymbol{T}_Y \;=\; \hat{T}^{Saez}\hat{\Phi}_S\boldsymbol{T}_S \;=\; \hat{T}^{Saez}\hat{\Phi}_S\hat{\Sigma}_S \cdot \boldsymbol{T}_Y.$$

All entries of $\hat{T}^{Saez}$ and $\boldsymbol{T}_Y$ are positive, and all entries of $\hat{\Phi}_S$ are non-positive. it follows that entries of $\hat{\Phi}_S\hat{\Sigma}_S$ (and hence $\boldsymbol{T}_Y^{AS} - \boldsymbol{T}_Y$ and $\boldsymbol{T}_S$) are all non-negative (non-positive), whenever all entries of $\hat{\Sigma}_S$ are non-positive (non-negative). Proposition 5 follows immediately.

C Proofs of Section 3.3

Notation and Preliminary Results. Suppose that $r \in [0,1]$ is uniform and denotes income rank, with $Y(r)$ denoting earnings at rank r, and $\alpha \in [0,1]$ is uniform and denotes the rank of savings, conditional on income. Let $S(r,\alpha)$ and $C(r,\alpha)$ denote the savings and consumption at rank α, conditional on income rank r. The budget constraint implies

$$Y(r) - T(Y(r), S(r,\alpha)) = p_C C(r,\alpha) + S(r,\alpha)$$

where we introduce p_C (set equal to 1) to derive certain relationships between price elasticities. We consider tax functions that are asymptotically separable and linear in savings at the top, i.e. $T(Y,S) = T_Y(Y) + \tau_S S$ for income ranks r sufficiently close to 1. The triplet $(Y(r), C(r,\alpha), S(r,\alpha))$ of earnings, consumption and savings satisfies the first-order conditions $1 - \tau_Y = \frac{-U_Y}{U_C}$, $1 + \tau_S = \frac{U_S}{U_C}$, where the marginal rates of substitution may depend on individual types that map into income and savings ranks r and α, and $\tau_Y = T_Y'(Y)$, along with the budget constraint. Differentiating the budget constraint w.r.t. α implies that $p_C C_\alpha + (1+\tau_S) S_\alpha = 0$.

Now, define $s_C(r,\alpha) \equiv \frac{p_c C(r,\alpha)}{(1-\tau_Y(r))Y(r)}$ and $s_S(r,\alpha) \equiv \frac{(1+\tau_S)S(r,\alpha)}{(1-\tau_Y(r))Y(r)}$, income effects

$$\zeta_Y^I \equiv -\frac{\partial Y}{\partial T}(1-\tau_Y) \ , \zeta_C^I \equiv \frac{\partial C}{\partial T} \cdot \frac{(1-\tau_Y)Y}{C} \text{ and } \zeta_S^I \equiv \frac{\partial S}{\partial T}\frac{(1-\tau_Y)Y}{S}$$

where ∂T represents an increment in unearned income, or a lump-sum reduction in the income tax, as well as uncompensated and compensated elasticities:

$$\zeta_{Z,\tau_Y}^U \equiv \frac{\partial \ln Z}{\partial \ln(1-\tau_Y)}, \zeta_{Z,p_C}^U \equiv \frac{\partial \ln Z}{\partial \ln p_C} \text{ and } \zeta_{T,\tau_S}^U \equiv \frac{\partial \ln Z}{\partial \ln(1+\tau_S)}, \text{ for } Z \in \{Y, C, S\}$$

$$\zeta_{Z,\tau_Y}^H \equiv \frac{\partial \ln Z^C}{\partial \ln(1-\tau_Y)}, \zeta_{Z,p_C}^H \equiv \frac{\partial \ln Z^C}{\partial \ln p_C} \text{ and } \zeta_{T,\tau_S}^H \equiv \frac{\partial \ln Z^C}{\partial \ln(1+\tau_S)}, \text{ for } Z \in \{Y, C, S\}$$

where $Z \in \{Y, C, S\}$ and $Z^C \in \left\{Y^C, C^C, S^C\right\}$ stand for the uncompensated and compensated earnings, income and savings functions, respectively. All these income and substitution effects can in principle be functions of both r and α. For further reference, income and substitution effects satisfy the following well-known relationships:

$$\zeta_{Y,\tau_Y}^U = \zeta_{Y,\tau_Y}^H - \zeta_Y^I, \quad \zeta_{C,\tau_Y}^U = \zeta_{C,\tau_Y}^H + \zeta_C^I, \quad \text{and} \quad \zeta_{S,\tau_Y}^U = \zeta_{S,\tau_Y}^H + \zeta_S^I$$

$$\zeta_{Y,p_C}^U = \zeta_{Y,p_C}^H + s_C\zeta_Y^I, \quad \zeta_{C,p_C}^U = \zeta_{C,p_C}^H - s_C\zeta_C^I, \quad \text{and} \quad \zeta_{S,p_C}^U = \zeta_{S,p_C}^H - s_C\zeta_S^I$$

$$\zeta_{Y,\tau_S}^U = \zeta_{Y,\tau_S}^H + s_S\zeta_Y^I, \quad \zeta_{C,\tau_S}^U = \zeta_{C,\tau_S}^H - s_S\zeta_C^I, \quad \text{and} \quad \zeta_{S,\tau_S}^U = \zeta_{S,\tau_S}^H - s_S\zeta_S^I$$

$$\zeta_{Y,\tau_S}^H = -s_S\zeta_{S,\tau_Y}^H, \quad \zeta_{Y,p_C}^H = -s_C\zeta_{C,\tau_Y}^H, \quad \text{and} \quad s_S\zeta_{S,p_C}^H = s_C\zeta_{C,\tau_S}^H$$

$$1 - \zeta_Y^I = s_C\zeta_C^I + s_S\zeta_S^I$$

$$\zeta_{Y,\tau_Y}^H = s_C\zeta_{C,\tau_Y}^H + s_S\zeta_{S,\tau_Y}^H, \quad \zeta_{Y,p_C}^H = s_C\zeta_{C,p_C}^H + s_S\zeta_{S,p_C}^H, \quad \text{and} \quad \zeta_{Y,\tau_S}^H = s_C\zeta_{C,\tau_S}^H + s_S\zeta_{S,\tau_S}^H.$$

The first three lines are the Slutzky equations for income and savings tax changes, the fourth line holds because of symmetry of substitution effects, the fifth and sixth line follow from marginal perturbations to lump sum taxes, marginal tax rates or p_C. In addition, we show:

$$\frac{\partial \zeta_Y^I}{\partial \alpha} = \frac{\partial \zeta_{Y,\tau_Y}^H}{\partial \alpha} = 0 \quad \text{and} \quad \frac{\partial \zeta_{Y,\tau_S}^H}{\partial \alpha} = -\frac{\partial \zeta_{Y,p_C}^H}{\partial \alpha} = s_S\frac{S_\alpha}{S}\left(1 - \zeta_Y^I\right)$$

$$\frac{\partial \zeta^H_{C,\tau_Y}}{\partial \alpha} = -\frac{C_\alpha}{C}\left(1-\zeta^I_Y+\zeta^H_{C,\tau_Y}\right) \quad \text{and} \quad \frac{\partial \zeta^H_{S,\tau_Y}}{\partial \alpha} = -\frac{S_\alpha}{S}\left(1-\zeta^I_Y+\zeta^H_{S,\tau_Y}\right)$$

$$s_C\frac{\partial}{\partial\alpha}\zeta^I_C + s_S\frac{\partial}{\partial\alpha}\zeta^I_S = s_S\frac{S_\alpha}{S}\left(\zeta^I_C-\zeta^I_S\right)$$

$$s_C\frac{\partial}{\partial\alpha}\zeta^H_{C,\tau_S} + s_S\frac{\partial}{\partial\alpha}\zeta^H_{S,\tau_S} = s_S\frac{S_\alpha}{S}\left(1-\zeta^I_Y-\zeta^H_{S,\tau_S}+\zeta^H_{C,\tau_S}\right)$$

Proof. Consider the approximate budget constraint (ABC) $Y(1-\tau_Y)-T_0 = p_C C + (1+\tau_S) S$, where $\tau_Y = \tau_Y(r)$ and T_0 is chosen so that $T_0+\tau_Y(r)Y(r) = T_Y(Y(r))$, i.e., we use a linear approximation of the tax function around rank r.

Differentiating the ABC w.r.t. T_0 yields $(1-\tau_Y)\frac{\partial Y}{\partial T_0} - 1 = p_C\frac{\partial C}{\partial T_0} + (1+\tau_S)\frac{\partial S}{\partial T_0}$, or $1-\zeta^I_Y = s_C\zeta^I_C + s_S\zeta^I_S = -\frac{\partial}{\partial T_0}\left[p_C C + (1+\tau_S)S\right]$. Differentiating both sides w.r.t. α implies that $\frac{\partial}{\partial\alpha}\zeta^I_Y = -\frac{\partial}{\partial T_0}\left[p_C C_\alpha + (1+\tau_S)S_\alpha\right] = 0$.

Differentiating the ABC w.r.t. $1-\tau_Y$ yields $Y\left(1+\zeta^U_{Y,\tau_Y}\right) = \frac{\partial}{\partial(1-\tau_Y)}\left[p_C C + (1+\tau_S)S\right]$. Differentiating both sides w.r.t. α implies that $\frac{\partial}{\partial\alpha}\zeta^U_{Y,\tau_Y} = 0$, and therefore $\frac{\partial}{\partial\alpha}\zeta^H_{Y,\tau_Y} = \frac{\partial}{\partial\alpha}\left(\zeta^U_{Y,\tau_Y}+\zeta^I_Y\right) = 0$.

Differentiating the ABC w.r.t. $1+\tau_S$ yields $\zeta^U_{Y,\tau_S} = s_S + \frac{1+\tau_S}{(1-\tau_Y)Y}\frac{\partial}{\partial(1+\tau_S)}\left[p_C C + (1+\tau_S)S\right]$, and therefore $\frac{\partial}{\partial\alpha}\zeta^U_{Y,\tau_S} = s_S\frac{S_\alpha}{S}$ and $\frac{\partial}{\partial\alpha}\zeta^H_{Y,\tau_S} = s_S\frac{S_\alpha}{S}\left(1-\zeta^I_Y\right)$.

Differentiating the ABC w.r.t. p_C yields $\zeta^U_{Y,p_C} = s_C + \frac{p_C}{(1-\tau_Y)Y}\frac{\partial}{\partial p_C}\left[p_C C + (1+\tau_S)S\right]$. Differentiating both sides w.r.t. α yields $\frac{\partial}{\partial\alpha}\zeta^U_{Y,p_C} = s_C\frac{C_\alpha}{C}$ and $\frac{\partial}{\partial\alpha}\zeta^H_{Y,p_C} = s_C\frac{C_\alpha}{C}\left(1-\zeta^I_Y\right)$, and from differentiating the budget constraint w.r.t. α, $s_C\frac{C_\alpha}{C} = -s_S\frac{S_\alpha}{S}$.

Differentiating $s_S\zeta^H_{S,\tau_Y} = -\zeta^H_{Y,\tau_S}$ w.r.t. α yields $s_S\left(\frac{S_\alpha}{S}\zeta^H_{S,\tau_Y}+\frac{\partial}{\partial\alpha}\zeta^H_{S,\tau_Y}\right) = -s_S\frac{S_\alpha}{S}\left(1-\zeta^I_Y\right)$, or $\frac{\partial}{\partial\alpha}\zeta^H_{S,\tau_Y} = -\frac{S_\alpha}{S}\left(1-\zeta^I_Y+\zeta^H_{S,\tau_Y}\right)$. Differentiating $s_C\zeta^H_{C,\tau_Y} = -\zeta^H_{Y,p_C}$ w.r.t. α yields $\frac{\partial}{\partial\alpha}\zeta^H_{C,\tau_Y} = -s_C\frac{C_\alpha}{C}\left(1-\zeta^I_Y+\zeta^H_{C,\tau_Y}\right)$.

Differentiating $1-\zeta^I_Y = s_C\zeta^I_C + s_S\zeta^I_S$ w.r.t. α yields $s_C\left(\frac{C_\alpha}{C}\zeta^I_C+\frac{\partial}{\partial\alpha}\zeta^I_C\right) = -s_S\left(\frac{S_\alpha}{S}\zeta^I_S+\frac{\partial}{\partial\alpha}\zeta^I_S\right)$ and differentiating $\zeta^H_{Y,\tau_S} = s_C\zeta^H_{C,\tau_S}+s_S\zeta^H_{S,\tau_S}$ yields $s_C\left(\frac{C_\alpha}{C}\zeta^H_{C,\tau_S}+\frac{\partial}{\partial\alpha}\zeta^H_{C,\tau_S}\right)+s_S\left(\frac{S_\alpha}{S}\zeta^H_{S,\tau_S}+\frac{\partial}{\partial\alpha}\zeta^H_{S,\tau_S}\right) = s_S\frac{S_\alpha}{S}\left(1-\zeta^I_Y\right)$, which we rearrange to find the expressions above. □

As an immediate corollary of these expressions, we obtain that in Case 3 ($\bar{s}_C = 0$),

$$\lim_{r\to1}\frac{\partial}{\partial\alpha}\zeta^I_S = \lim_{r\to1}\frac{\partial}{\partial\alpha}\zeta^H_{S,\tau_Y} = \lim_{r\to1}\frac{\partial}{\partial\alpha}\zeta^H_{S,\tau_S} = 0,$$

which is consistent with the fact that ζ^I_S, ζ^H_{S,τ_Y}, and ζ^H_{S,τ_S} converge to $1-\zeta^I_Y$, ζ^H_{Y,τ_Y}, and ζ^H_{Y,τ_S}, respectively. Likewise, in Case 1 ($\bar{s}_S = 0$), we have $\lim_{r\to1}\frac{\partial}{\partial\alpha}\zeta^I_C = \lim_{r\to1}\frac{\partial}{\partial\alpha}\zeta^H_{C,\tau_Y} = \lim_{r\to1}\frac{\partial}{\partial\alpha}\zeta^H_{C,\tau_S} = 0$.

Finally, differentiating the budget constraint w.r.t. r for given α yields

$$\frac{Y'(r)}{Y(r)} = \frac{C_r(r,\alpha)}{(1-\tau_Y)Y(r)} + \frac{(1+\tau_S)S_r(r,\alpha)}{(1-\tau_Y)Y(r)},$$

or, after integrating over α:

$$\frac{1}{\rho_Y(r)} = s_C(r)\frac{(1-r)C'(r)}{C(r)} + s_S(r)\frac{(1-r)S'(r)}{S(r)} = s_C(r)\frac{1}{\rho_C(r)} + s_S(r)\frac{1}{\rho_S(r)},$$

where $s_C(r) = \int s_C(r,\alpha)\,d\alpha$ and $s_S(r) = \int s_S(r,\alpha)\,d\alpha$ are the average consumption and savings shares at income rank r, $C(r) = \int C(r,\alpha)\,d\alpha$ and $S(r) = \int S(r,\alpha)\,d\alpha$ average consumption and savings at income rank r, $C'(r) = \int C_r(r,\alpha)\,d\alpha$ and $S'(r) = \int S_r(r,\alpha)\,d\alpha$, and $1/\rho_C(r) = (1-r)C'(r)/C(r)$ and $1/\rho_S(r) = (1-r)S'(r)/S(r)$ represent the Pareto coefficient of average consumption and average savings conditional on earnings.

Revenue Spillover Condition. Consider now a tax perturbation that changes the marginal income tax by $\partial\tau_Y(\alpha) > 0$ for all $r' \in [r, r+\partial r)$. The mechanical effect of this tax perturbation is

$$\int \partial\tau_Y(\alpha)\,d\alpha \cdot (1-r)\left(Y(r+\partial r) - Y(r)\right) \approx \frac{\partial r Y(r)}{\rho_Y(r)}\int \partial\tau_Y(\alpha)\,d\alpha.$$

For any (r',α), the behavioral effect of the tax perturbation on earnings is

$$\partial Y(r',\alpha) \approx \begin{cases} -Y(r)\frac{\zeta^H_{Y,\tau_Y}(r)\partial\tau_Y(\alpha)}{1-\tau_Y} & \text{if } r' \in [r, r+\partial r) \\ \partial r Y'(r)\frac{\zeta^I_Y(r')\partial\tau_Y(\alpha)}{1-\tau_Y} & \text{if } r' \geq r+\partial r \end{cases}$$

Integrating over $r' > r$ yields

$$\frac{\tau_Y\int_r^1 \mathbb{E}\left(\partial Y(r',\alpha)\,|r\right)dr'}{\frac{\partial r Y(r)}{\rho_Y(r)}\int \partial\tau_Y(\alpha)\,d\alpha} \approx -\frac{\tau_Y}{1-\tau_Y}\left(\zeta^H_{Y,\tau_Y}(r)\rho_Y(r) - \hat{\zeta}^I_Y(r)\right) = -\frac{\tau_Y}{1-\tau_Y}\frac{1-\tau_Y^{Saez}(r)}{\tau_Y^{Saez}(r)}$$

where $\tau_Y^{Saez}(r) \equiv \left[1 + \zeta^H_{Y,\tau_Y}(r)\rho_Y(r) - \hat{\zeta}^I_Y(r)\right]^{-1}$ and $\hat{\zeta}^I_Y(r) \equiv \mathbb{E}\left(\zeta^I_Y(r')\middle| r' > r\right)$; here we have used the fact that ζ^H_{Y,τ_Y} and ζ^I_Y do not vary across α.

For any (r',α), the behavioral effect of the tax perturbation on savings is

$$\partial S(r',\alpha) \approx \begin{cases} -Y(r)s_S(r,\alpha)\frac{\zeta^H_{S,\tau_Y}(r,\alpha)\partial\tau_Y(\alpha)}{1+\tau_S} & \text{if } r' \in [r, r+\partial r) \\ -\partial r Y'(r)s_S(r',\alpha)\frac{\zeta^I_S(r',\alpha)\partial\tau_Y(\alpha)}{1+\tau_S} & \text{if } r' \geq r+\partial r \end{cases}$$

Integrating over $r' > r$ yields

$$\frac{\tau_S\int_r^1 \mathbb{E}\left(\partial S(r',\alpha)\,|r\right)dr'}{\frac{\partial r Y(r)}{\rho_Y(r)}\int \partial\tau_Y(\alpha)\,d\alpha} \approx -\frac{\tau_S}{1+\tau_S}s_S(r)\Phi_S(r)\left(1 + Cov\left(\frac{s_S(r,\alpha)\Phi_S(r,\alpha)}{s_S(r)\Phi_S(r)}, \frac{\partial\tau_Y(\alpha)}{\int\partial\tau_Y(\alpha)\,d\alpha}\middle| r\right)\right)$$

where $\Phi_S(r,\alpha) \equiv \zeta^H_{S,\tau_Y}(r,\alpha)\rho_Y(r) + \hat{\zeta}^I_S(r,\alpha)$, $\hat{\zeta}^I_S(r,\alpha) \equiv \mathbb{E}\left(\zeta^I_S(r',\alpha)\frac{s_S(r',\alpha)}{s_S(r,\alpha)}\middle| r' > r\right)$, and $\Phi_S(r) \equiv \int \frac{s_S(r,\alpha)}{s_S(r)}\Phi_S(r,\alpha)\,d\alpha$.

Combining terms yields the following necessary condition for optimality of the asymptotically affine tax function:

$$\frac{\tau_Y(r)}{1-\tau_Y(r)} = \frac{\tau_Y^{Saez}(r)}{1-\tau_Y^{Saez}(r)}\left(1 - \frac{\tau_S}{1+\tau_S}s_S(r)\Phi_S(r)\left(1 + Cov\left(\frac{s_S(r,\alpha)\Phi_S(r,\alpha)}{s_S(r)\Phi_S(r)}, \frac{\partial\tau_Y(\alpha)}{\int\partial\tau_Y(\alpha)\,d\alpha}\middle| r\right)\right)\right).$$

This condition generalizes the revenue spill-over condition to the model with multi-dimensional heterogeneity, with the additional co-variance term accounting for income tax perturbations that vary across savings ranks. Within the class of income tax perturbations that $\partial\tau_Y$ that are uniform across α, i.e. within the restricted class of tax policies that are asymptotically separable and linear in savings at the top, the optimal top marginal income tax must satisfy

$$\frac{\tau_Y(r)}{1-\tau_Y(r)} = \frac{\tau_Y^{Saez}(r)}{1-\tau_Y^{Saez}(r)}\left(1 - \frac{\tau_S}{1+\tau_S}s_S(r)\Phi_S(r)\right)$$

which is exactly the same as the revenue spillover condition in our baseline model, with the definitions of the various income and substitution effects suitably adjusted to allow for heterogeneity in savings at each income level.

The term $Cov\left(s_S(r,\alpha)\Phi_S(r,\alpha), \frac{\partial\tau_Y(\alpha)}{\int\partial\tau_Y(\alpha)d\alpha}\middle| r\right)$ adjusts this condition for perturbations that vary across α. A necessary condition for the separable affine tax system to be optimal at the top is that this covariance term is zero for *any* such perturbation, or equivalently that $s_S(r,\alpha)\Phi_S(r,\alpha)$ is constant across α. This must hold for $r \to 1$ in Case 1 (where $\lim_{r\to1} s_S(r) = \lim_{r\to1} s_S(r,\alpha) = 0$ for all α, i.e. the top earners consume almost all their income), as well as in the empirically relevant Case 3 where $\lim_{r\to1} s_S(r) = \lim_{r\to1} s_S(r,\alpha) = 1$, $\lim_{r\to1} s_S(r,\alpha)\zeta^H_{S,\tau_Y}(r,\alpha) = \bar{\zeta}^H_{Y,\tau_Y}$ and $\lim_{r\to1} s_S(r,\alpha)\zeta^I_S(r,\alpha) = 1 - \bar{\zeta}^I_Y$, for all α.[54]

In Case 2, where s_S and s_C both converge to positive, finite limits, $s_S(r,\alpha)\Phi_S(r,\alpha)$ represents the spill-over from income to savings at a given savings rank α, i.e. by how much a marginal increase in the income tax at earnings rank r lowers savings for a given savings rank α. The condition that this term is constant amounts to requiring that this fiscal spill-over of income taxes to savings is uniform across different savings ranks; otherwise the fiscal authority could gain additional revenue with a targeted perturbation that raises the income tax by more for savings ranks with lower fiscal spill-overs, so that the above covariance is negative.

[54] Substituting $s_S\zeta^H_{S,\tau_Y} = \zeta^H_{Y,\tau_Y} - s_C\zeta^H_{C,\tau_Y}$ and $s_S\hat{\zeta}^I_S = 1 - \hat{\zeta}^I_Y - s_C\hat{\zeta}^I_C$ and noting that $\hat{\zeta}^I_Y$ and ζ^H_{Y,τ_Y} are invariant in α, we obtain that $Cov\left(s_S\Phi_S, \frac{\partial\tau_Y(\alpha)}{\int\partial\tau_Y(\alpha)d\alpha}\middle| r\right) = -Cov\left(s_C\left[\zeta^H_{C,\tau_Y}\rho_Y + \hat{\zeta}^{Inc}_C\right], \frac{\partial\tau_Y(\alpha)}{\int\partial\tau_Y(\alpha)d\alpha}\middle| r\right)$.

The restriction to an affine tax system amounts to assuming that the fiscal authority is unable to fine-tune how it internalizes spill-overs from one source of tax revenues to another, and it just internalizes the average spill-overs across top income earners, and the uniform spill-over condition which implies that $s_S(r,\alpha)\,\Phi_S(r,\alpha)$is invariant in α (at least at the top as $r\to 1$) implies that such fine-tuning does not generate additional tax revenue.

No-arbitrage condition. Consider a joint perturbation of income and savings taxes $(\partial\tau_Y(\alpha),\partial\tau_S(\alpha))$ with $\partial\tau_S(\alpha)>0>\partial\tau_Y(\alpha)$ for $r'\in[r,r+\partial r)$, such that $Y'(r)\int\partial\tau_Y(\alpha)\,d\alpha+\int S_r(r,\alpha)\,\partial\tau_S(\alpha)\,d\alpha=0$, or equivalently,

$$\frac{Y(r)}{\rho_Y(r)}\int\partial\tau_Y(\alpha)\,d\alpha=-\frac{S(r)}{\rho_S(r)}\int\partial\tau_S(\alpha)\,d\alpha\left(1+Cov\left(\frac{S_r(r,\alpha)}{S'(r)};\frac{\partial\tau_S(\alpha)}{\int\partial\tau_S(\alpha)\,d\alpha}\middle|\,r\right)\right).$$

Importantly, $\rho_S(r)$ does not represent the Pareto tail on savings, but rather, since $S(r)=\int S(r,\alpha)\,d\alpha$ represents the average savings at income rank r, ρ_S measures the Pareto coefficient of average savings conditional on income. Equivalently, the ratio $\rho_Y(r)/\rho_S(r)$ represents the elasticity of average savings w.r.t. income Y:

$$\frac{\rho_Y(r)}{\rho_S(r)}=\frac{\int S_r(r,\alpha)\,d\alpha/\int S(r,\alpha)\,d\alpha}{Y'(r)/Y(r)}=\frac{\partial\log\mathbb{E}(S|Y(r))}{\partial\log Y(r)}.$$

Since aggregate tax revenue at the top is $T_0+\tau_Y Y(r)+\tau_S\int S(r,\alpha)\,d\alpha$, the mechanical effect is given by

$$-\partial r\int\left[Y'(r)\,\partial\tau_Y(\alpha)+S_r(r,\alpha)\,\partial\tau_S(\alpha)\right]d\alpha=0.$$

For $r'\geq r+\partial r$, the behavioral effect of the tax perturbation on earnings is $\partial Y(r',\alpha)\approx\partial r\zeta_Y^I(r')\left[Y'(r)\frac{\partial\tau_Y(\alpha)}{1-\tau_Y}+S_r(r,\alpha)\frac{\partial\tau_S(\alpha)}{1-\tau_Y}\right]$ and therefore $\int\partial Y(r',\alpha)\,d\alpha=0$ for $r'\geq r+\partial r$. For $r'\in[r,r+\partial r)$ we obtain

$$\partial Y(r',\alpha)\approx-\left[\zeta^H_{Y,\tau_Y}(r)\frac{\partial\tau_Y(\alpha)}{1-\tau_Y}-\zeta^H_{Y,\tau_S}(r,\alpha)\frac{\partial\tau_S(\alpha)}{1+\tau_S}\right]Y(r)$$

$$=-\frac{Y(r)/\rho_Y(r)}{1-\tau_Y}\int\partial\tau_Y(\alpha)\,d\alpha\left\{\zeta^H_{Y,\tau_Y}(r)\,\rho_Y(r)\frac{\partial\tau_Y(\alpha)}{\int\partial\tau_Y(\alpha)\,d\alpha}-\rho_S(r)\frac{\zeta^H_{S,\tau_Y}(r,\alpha)\frac{S(r,\alpha)}{S(r)}\frac{\partial\tau_S(\alpha)}{\int\partial\tau_S(\alpha)d\alpha}}{1+Cov\left(\frac{S_r(r,\alpha)}{S'(r)};\frac{\partial\tau_S(\alpha)}{\int\partial\tau_S(\alpha)d\alpha}\middle|\,r\right)}\right\}$$

where we have substituted $\zeta^H_{Y,\tau_S}=-\zeta^H_{S,\tau_Y}s_S$ and used the above expression for $\frac{Y(r)}{\rho_Y(r)}\int\partial\tau_Y(\alpha)\,d\alpha$.

Integrating w.r.t. α and $r' \geq r$ then yields

$$\frac{\tau_Y \int_r^1 \mathbb{E}\left(\partial Y\left(r',\alpha\right)\mid r\right)dr'}{\frac{\partial r Y(r)}{\rho_Y(r)}\int \partial\tau_Y\left(\alpha\right)d\alpha} \approx -\frac{\tau_Y}{1-\tau_Y}\left\{\zeta_{Y,\tau_Y}^H\left(r\right)\rho_Y\left(r\right)-\zeta_{S,\tau_Y}^H\left(r\right)\rho_S\left(r\right)K_1\left(r\right)\right\},$$

where $\zeta_{S,\tau_Y}^H\left(r\right) \equiv \int \frac{s_S(r,\alpha)}{s_S(r)}\zeta_{S,\tau_Y}^H\left(r,\alpha\right)d\alpha$ and

$$K_1\left(r\right) \equiv \frac{1+Cov\left(\frac{\zeta_{S,\tau_Y}^H(r,\alpha)}{\zeta_{S,\tau_Y}^H(r)}\frac{S(r,\alpha)}{S(r)};\frac{\partial\tau_S(\alpha)}{\int\partial\tau_S(\alpha)d\alpha}\middle| r\right)}{1+Cov\left(\frac{S_r(r,\alpha)}{S'(r)};\frac{\partial\tau_S(\alpha)}{\int\partial\tau_S(\alpha)d\alpha}\middle| r\right)}.$$

For $r' \geq r+\partial r$, the behavioral effect of the tax perturbation on earnings is

$$\partial S\left(r',\alpha\right) \approx -\partial r\zeta_S^I\left(r',\alpha\right)\left[\frac{S\left(r',\alpha\right)}{Y\left(r'\right)}\frac{Y'\left(r\right)\partial\tau_Y\left(\alpha\right)}{1-\tau_Y}+s_S\left(r',\alpha\right)\frac{S_r\left(r,\alpha\right)\partial\tau_S\left(\alpha\right)}{1+\tau_S}\right]$$

$$= \partial r\frac{1}{1+\tau_S}Y'\left(r\right)\int\partial\tau_Y\left(\alpha\right)d\alpha\zeta_S^I\left(r',\alpha\right)s_S\left(r',\alpha\right)\left[\frac{\partial\tau_Y\left(\alpha\right)}{\int\partial\tau_Y\left(\alpha\right)d\alpha}-\frac{S_r\left(r,\alpha\right)\partial\tau_S\left(\alpha\right)}{\int S_r\left(r,\alpha\right)\partial\tau_S\left(\alpha\right)d\alpha}\right].$$

Integrating over $r' \geq r+\partial r$ and α yields

$$\frac{\tau_S\int_{r+\varepsilon}^1\mathbb{E}\left(\partial S\left(r',\alpha\right)\mid r\right)dr'}{\frac{\partial rY(r)}{\rho_Y(r)}\int\partial\tau_Y\left(\alpha\right)d\alpha} \approx -\frac{\tau_S}{1+\tau_S}Cov\left(\hat{\zeta}_S^I\left(r,\alpha\right)s_S\left(r,\alpha\right);\frac{\partial\tau_Y\left(\alpha\right)}{\int\partial\tau_Y\left(\alpha\right)d\alpha}-\frac{S_r\left(r,\alpha\right)\partial\tau_S\left(\alpha\right)}{\int S_r\left(r,\alpha\right)\partial\tau_S\left(\alpha\right)d\alpha}\middle| r\right)$$

where $\hat{\zeta}_S^I\left(r,\alpha\right) \equiv \mathbb{E}\left(\zeta_S^I\left(r',\alpha\right)\frac{s_S(r',\alpha)}{s_S(r,\alpha)}\mid r'>r\right)$. For $r' \in [r, r+\partial r)$ we obtain

$$\partial S\left(r',\alpha\right) \approx \left[\zeta_{S,\tau_S}^H\left(r,\alpha\right)\frac{\partial\tau_S\left(\alpha\right)}{1+\tau_S}-\zeta_{S,\tau_Y}^H\left(r,\alpha\right)\frac{\partial\tau_Y\left(\alpha\right)}{1-\tau_Y}\right]S\left(r,\alpha\right)$$

$$= \frac{Y\left(r\right)/\rho_Y\left(r\right)}{1+\tau_S}\int\partial\tau_Y\left(\alpha\right)d\alpha\cdot\left[\rho_Y\left(r\right)\zeta_{Y,\tau_S}^H\left(r,\alpha\right)\frac{\partial\tau_Y\left(\alpha\right)}{\int\partial\tau_Y\left(\alpha\right)d\alpha}-\rho_S\left(r\right)\zeta_{S,\tau_S}^H\left(r,\alpha\right)\frac{\frac{S(r,\alpha)}{S(r)}\partial\tau_S\left(\alpha\right)}{\int\frac{S_r(r,\alpha)}{S'(r)}\partial\tau_S\left(\alpha\right)d\alpha}\right]$$

where we have substituted the expression for $\int\partial\tau_Y\left(\alpha\right)d\alpha$ and $\zeta_{Y,\tau_S}^H\left(r,\alpha\right) = -\zeta_{S,\tau_Y}^H\left(r,\alpha\right)s_S\left(r,\alpha\right)$. Integrating over α and $r' \in [r, r+\partial r)$then yields

$$\frac{\tau_S\int_r^{r+\varepsilon}\mathbb{E}\left(\partial S\left(r',\alpha\right)\mid r\right)dr'}{\frac{\partial rY(r)}{\rho_Y(r)}\int\partial\tau_Y\left(\alpha\right)d\alpha} \approx \frac{\tau_S}{1+\tau_S}\left[\rho_Y\left(r\right)\zeta_{Y,\tau_S}^H\left(r\right)K_2\left(r\right)-\rho_S\left(r\right)\zeta_{S,\tau_S}^H\left(r\right)K_3\left(r\right)\right]$$

where $\zeta^H_{Y,\tau_S}(r) \equiv \int \zeta^H_{Y,\tau_S}(r,\alpha)\, d\alpha$, $\zeta^H_{S,\tau_S}(r) \equiv \int \zeta^H_{S,\tau_S}(r,\alpha)\, S(r,\alpha)/S(r)\, d\alpha$, and

$$K_2(r) \equiv 1 + Cov\left(\frac{\zeta^H_{Y,\tau_S}(r,\alpha)}{\zeta^H_{Y,\tau_S}(r)} ; \frac{\partial \tau_Y(\alpha)}{\int \partial \tau_Y(\alpha)\, d\alpha} \middle| r \right) \text{ and } K_3(r) \equiv \frac{1 + Cov\left(\frac{\zeta^H_{S,\tau_S}(r,\alpha)}{\zeta^H_{S,\tau_S}(r)} \frac{S(r,\alpha)}{S(r)} ; \frac{\partial \tau_S(\alpha)}{\int \partial \tau_S(\alpha) d\alpha} \middle| r \right)}{1 + Cov\left(\frac{S_r(r,\alpha)}{S'(r)} ; \frac{\partial \tau_S(\alpha)}{\int \partial \tau_S(\alpha) d\alpha} \middle| r \right)}.$$

It follows that

$$\frac{\tau_S \int_r^1 \mathbb{E}\left(\partial S(r',\alpha) \mid r\right) dr'}{\frac{\partial r Y(r)}{\rho_Y(r)} \int \partial \tau_Y(\alpha)\, d\alpha} \approx \frac{\tau_S}{1+\tau_S} \left\{ \rho_Y(r)\, \zeta^H_{Y,\tau_S}(r) - \rho_S(r)\, \zeta^H_{S,\tau_S}(r) + \Upsilon(r) \right\},$$

where

$$\Upsilon(r) \equiv \rho_Y(r)\, Cov\left(\zeta^H_{Y,\tau_S}(r,\alpha) ; \frac{\partial \tau_Y(\alpha)}{\int \partial \tau_Y(\alpha)\, d\alpha} \middle| r \right) - \rho_S(r) \frac{Cov\left(\zeta^H_{S,\tau_S}(r,\alpha) \frac{S(r,\alpha)}{S(r)} - \frac{S_r(r,\alpha)}{S'(r)} \zeta^H_{S,\tau_S}(r) ; \frac{\partial \tau_S(\alpha)}{\int \partial \tau_S(\alpha) d\alpha} \middle| r \right)}{1 + Cov\left(\frac{S_r(r,\alpha)}{S'(r)} ; \frac{\partial \tau_S(\alpha)}{\int \partial \tau_S(\alpha) d\alpha} \middle| r \right)}$$

$$+ Cov\left(\hat{\zeta}^I_S(r,\alpha)\, s_S(r,\alpha) ; \frac{S_r(r,\alpha)\, \partial \tau_S(\alpha)}{\int S_r(r,\alpha)\, \partial \tau_S(\alpha)\, d\alpha} - \frac{\partial \tau_Y(\alpha)}{\int \partial \tau_Y(\alpha)\, d\alpha} \middle| r \right),$$

and the asymptotically affine taxes are optimal only if

$$\frac{\tau_S(r)}{1+\tau_S(r)} = \frac{\tau_Y(r)}{1-\tau_Y(r)} \frac{\zeta^H_{Y,\tau_Y}(r)\, \rho_Y(r) - \zeta^H_{S,\tau_Y}(r)\, \rho_S(r)\, K_1(r)}{\zeta^H_{Y,\tau_S}(r)\, \rho_Y(r) - \zeta^H_{S,\tau_S}(r)\, \rho_S(r) + \Upsilon(r)}$$

for any perturbation $(\partial \tau_Y(\alpha), \partial \tau_S(\alpha))$. If the perturbation is uniform over α, we have $K_1(r) = K_2(r) = K_3(r) = 1$ and $\Upsilon(r) = Cov\left(\hat{\zeta}^I_S(r,\alpha)\, s_S(r,\alpha) ; \frac{S_r(r,\alpha)}{S'(r)} \middle| r \right)$, and therefore the no-arbitrage condition reduces to

$$\frac{\tau_S(r)}{1+\tau_S(r)} = \frac{\tau_Y(r)}{1-\tau_Y(r)} \frac{\zeta^H_{Y,\tau_Y}(r)\, \rho_Y(r) - \zeta^H_{S,\tau_Y}(r)\, \rho_S(r)}{\zeta^H_{Y,\tau_S}(r)\, \rho_Y(r) - \zeta^H_{S,\tau_S}(r)\, \rho_S(r) + \Upsilon(r)}$$

which implies that $\bar{\tau}_S \gtreqless 0$ if and only if $\bar{\zeta}^H_{Y,\tau_Y} \bar{\rho}_Y \gtreqless \bar{\zeta}^H_{S,\tau_Y} \bar{\rho}_S$, as in the model with uni-dimensional heterogeneity. The only difference is that, as discussed above, $\rho_S(r)$ doesn't represent the Pareto coefficient on savings, but $\rho_Y(r)/\rho_S(r) = \frac{\partial \log \mathbb{E}(S(r,\alpha) \mid Y(r))}{\partial \log Y(r)}$ represents the elasticity of average savings to income.

The asymptotically affine tax system can then only be optimal if $K_1(r) = K_2(r) = K_3(r) = 1$ and $\Upsilon(r) = Cov\left(\hat{\zeta}^I_S(r,\alpha)\, s_S(r,\alpha) ; \frac{S_r(r,\alpha)}{S'(r)} \middle| r \right)$ for any non-uniform perturbation $(\partial \tau_Y(\alpha), \partial \tau_S(\alpha))$, otherwise fine-tuning the tax perturbation by savings rank will generate additional revenue.But that in turn holds only if $\zeta^H_{Y,\tau_S}(r,\alpha)$, $\zeta^H_{S,\tau_S}(r,\alpha)$, and $\hat{\zeta}^I_S(r,\alpha)\, s_S(r,\alpha)$ are all invariant in α, in which case we also have $\Upsilon(r) = 0$. Given the above limiting results, this will hold as $r \to 1$ whenever

$s_S = 0$ (Case 1) or $s_C = 0$ (Case 3).

Finally, using $1/\rho_Y(r) = s_C(r)/\rho_C(r) + s_S(r)/\rho_S(r)$, we rewrite the condition for optimal savings taxes in terms of consumption and income or savings:

$$\frac{\tau_S}{1+\tau_S} = \frac{\tau_Y}{1-\tau_Y} \frac{\zeta^H_{C,\tau_Y}(r)\,\rho_C(r) - \zeta^H_{S,\tau_Y}(r)\,\rho_S(r)}{\zeta^H_{C,\tau_S}(r)\,\rho_C(r) - \zeta^H_{S,\tau_S}(r)\,\rho_S(r) + \frac{1}{s_C(r)}\frac{\rho_C(r)}{\rho_Y(r)} Cov\left(\hat{\zeta}^I_S(r,\alpha)\,s_S(r,\alpha), \frac{S_r(r,\alpha)}{S'(r)}\middle| r\right)}$$

Here, ρ_C represents a Pareto coefficient for average consumption at each income rank, hence $\frac{\rho_Y(r)}{\rho_C(r)} = \frac{\partial \log \mathbb{E}(C|Y(r))}{\partial \log Y(r)}$. Using $\hat{\zeta}^I_S(r,\alpha)\,s_S(r,\alpha) = 1 - \hat{\zeta}^I_Y(r) - \hat{\zeta}^I_C(r,\alpha)\,s_C(r,\alpha)$ and $S_r(r,\alpha) = constant - (1+\tau_S)\,C_r(r,\alpha)$, the covariance term in this last expression can also be written as

$$\frac{1}{s_C(r)}\frac{\rho_C(r)}{\rho_Y(r)} Cov\left(\hat{\zeta}^I_S(r,\alpha)\,s_S(r,\alpha), \frac{S_r(r,\alpha)}{S'(r)}\middle| r\right) = \frac{1}{s_S(r)}\frac{\rho_S(r)}{\rho_Y(r)} Cov\left(\hat{\zeta}^I_C(r,\alpha)\,s_C(r,\alpha), \frac{C_r(r,\alpha)}{C'(r)}\middle| r\right)$$

and therefore it follows that this covariance term converges to zero, both when $s_C(r) \to 0$ and when $s_S(r) \to 0$ as $r \to 0$, i.e. in Cases 1 and 3 of our analysis. To conclude in Cases 1 and 3, the optimal top income and savings taxes satisfy exactly the same revenue spillover and no-arbitrage conditions as the uni-dimensional baseline model, with elasticities and Pareto coefficients defined on average savings and consumption conditional on income rank.