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"Negative results in science: Blessing or (winner's) curse"

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Negative results in science: Blessing or (winner's) curse ?*

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Abstract

Two players receiving independent signals on a risky project with common value compete to be the first to innovate. We characterize the equilibrium of this preemption game as the publicity of signals varies. Private signals create a *winner's curse*: investing first implies that the rival has abstained from investing, possibly because he has privately received adverse information about the project. Since players want to gather more evidence in support of the project as a compensation, they invest later when signals are more likely to be private. Because of preemption, the NPV of investment is zero at equilibrium regardless of the publicity of signals. However, for a conservative planner who cares about avoiding unprofitable investments, this implies that investment arises too early at equilibrium, and such a planner then prefers signals to be private. This provides a rationale against the mandatory disclosure of negative results in science, notably when competition is severe. Our results suggest that policy interventions should primarily tackle winner-takes-all competition, and regulate transparency only once competition is sufficiently mild.

1 Introduction

There is abundant evidence that scientific negative results suffer from little publicity. Franco, Malhotra and Simonovits (2014), for instance, show that studies that yield null results are 40% less likely to be published, and 60% less likely to be written up in any form than those with statistically significant results.[1] The sparsity of negative results in academic publishing is arguably harmful to the efficiency of the scientific process. First, it may bias

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[1] In addition, Fanelli (2012) shows that the share of academic articles showing positive support for the hypothesis tested has consistently increased since 1990 in all disciplines and countries, at the expense of negative results. Kanaan et al. (2011) points that, although some negative results are actually published, they are disproportionately more likely to be published in journals with lower impact factors, hence are less visible.

the interpretation of existing knowledge. In particular, meta-analyses supposed to offer a good representation of the state of the art then report positive results in a disproportionate way.[2] Such biases may also have important consequences since public policy (e.g., health policy, economic policy...) is more likely to be responsive when there is an apparent strong consensus among scientists.[3] Second, other researchers unaware of past failed trials may engage in a socially wasteful duplication of efforts. While this problem has been well-known since Rosenthal (1979) coined it as the "file drawer problem", it has received renewed interest in the recent years. In particular, there has been a recent push for the systematic publication of negative results. The World Health Organization has notably taken a strong stand on this issue in its 2015 Statement on Public Disclosure of Clinical Trial Results:

> "*Researchers have a duty to make publicly available the results of their research... Negative and inconclusive as well as positive results must be published or otherwise made publicly available.*"[4]

In line with this objective, scientific research has recently experienced several major changes going in the direction of greater transparency. First, several journals, institutions and grants now request the preregistration of scientific studies in a public registry. Preregistration has become a standard in medical research, and is increasingly important in most disciplines.[5] Second, some journals have started to promote a new editorial policy whereby the acceptance decision can be based on the empirical strategy of an article and not necessarily on its results. For instance, the Journal of Development Economics offers authors the opportunity *"to have their prospective empirical projects reviewed and approved for publication before the results are known.*[6] Third, and relatedly, there is an increasing pressure on authors

[2]Turner et al. (2008), for instance, showed that, while 94% of published studies of antidepressants used by the FDA to make approval decisions had positive results, this fraction fell to 51% once one included unpublished studies.

[3]For instance, in a meta-analysis on the impact of the minimum wage, Card and Krueger (1995) suggest that the literature relying on time series data may have been affected by publication bias, leading to an overrepresentation of statistically significant results in the published literature. In a similar spirit, Ashenfelter and Greenstone (2004) find that publication bias may explain a strong upward bias in the reported estimates of the value of statistical life, a key determinant of health and environment regulation.

[4]See `https://www.who.int/ictrp/results/WHO_Statement_results_reporting_clinical_trials.pdf`.

[5]Top medical journals only publish studies that have been registered, notably on the most widely used registry clinicaltrials.org. In economics, the AEA RCT registry created in 2012 receives an increasing number of registrations.

[6]In a similar spirit, 8 health economics journals editors have issued a common statement that *"studies that utilize appropriate data in a sound and creative manner (....) have potential scientific and publication merit regardless of whether such studies' empirical findings do or do not reject null hypotheses that may be specified"*. See `https://www.cambridge.org/core/journals/health-economics-policy-and-law/article/editorial-statement-on-negative-findings/E7326C8D32F70D3461655D58B7CAE679`.

to publish a pre-analysis plan so as to avoid cherry-picking of results.[7] Finally, several new journals specializing in the publication of negative results have emerged (e.g., "Missing pieces Collection" by PLOS One, Journal of Negative Results, Journal of Pharmaceutical Negative Results...). All these evolutions undoubtedly contribute to greater transparency and a more systematic dissemination of negative results.

However, these initiatives also arise in a context where the incentives to publish for academic researchers have become extremely salient ("publish or perish"), which allegedly biases incentives towards positive results even further. In this paper, we examine the welfare impact of public information in a preemption game, and argue that promoting the publication of negative results may actually have an adverse effect on welfare, especially when competition is fierce. Beyond the example of negative results in science, the question that this paper addresses is also relevant for innovation or patent races. Indeed, when a firm receives bad news about the value of the innovation and thus exits the race, the possibility for its rivals to observe exit generates an information spillover. However, we show that this positive spillover is not per se enough to guarantee that observing exit does actually increase welfare.

We consider a model of investment timing that features both competition and learning. Two players (firms, researchers...) observe (conditionally) independent processes that may stochastically reveal bad news about the common value of the project which they contemplate.[8] They compete by choosing the date at which they invest, if they ever do. A player gets a positive payoff upon investing if and only if he is the first one to invest (winner-takes-all competition), and the project is of high quality (hence the learning rationale). We allow the extent to which each player observes his rival's signal to vary between the two polar cases of public and private signals. The public signals case corresponds to a situation where negative results are systematically disclosed (alternatively, exit is observable); the private signals case to one where they never are. We establish that, as soon as signals are not public, the possibility of preemption creates, on top of the usual payoff externality, an information externality. Indeed, when seeing that one's rival has not invested yet, one becomes wary that lack of investment might result from the rival having privately observed bad news on the project. This creates a *winner's curse*: being the first one to invest then becomes all else equal bad news. Since this effect is stronger when signals are more likely to be private,

[7] Pre-analysis plans notably specify the primary outcome variable, the statistical model specification, and the set of covariates to be included in the study. See Olken (2015) on this issue, and Christensen and Miguel (2018) for a more general survey on publication bias.

[8] One possible interpretation is that such arrival of bad news is a *negative result*.

investment then arises later in equilibrium. Actually, the winner's curse provides an incentive to delay investment in order to gain extra confidence on the project through one's own signal.

In terms of welfare, the impact of the publicity of signals on the quality of decision-making is a priori ambiguous. Conditional on a given investment strategy, players are better informed when signals are public thanks to the information spillover. However, since players then tend to invest earlier, they accumulate less confidence in the project. Overall, because competition fully dissipates rents, the NPV of the project is equal to zero at any equilibrium investment date: by investing later, a player gains exactly as much when the project is bad (because he is then more likely to find out and, hence, save the investment cost) than he loses from delaying investment when it is good, so that the two effects exactly compensate each other. Regulating information disclosure in the aim of promoting information spillovers is thus vain: the benefits arising from these spillovers are always fully eroded by competition. However, a social planner with different preferences over type I and type II errors might strictly prefer one environment to the other. In particular, a planner who cares more than players about the costs of unsuccessful investments and is thus more conservative prefers an environment where signals are private, as investment then arises later. This provides a possible rationale against the mandatory disclosure of negative results.

We then extend our analysis to n players, and show that all our results carry over. An interesting insight emerging from this analysis is the complementarity between the extent of competition and the publicity of signals in the planner's welfare. Specifically, we establish that more competition (a higher number of players) increases welfare as long as the regime of publicity of signals is optimal from the planner's perspective. For instance, if the planner is more conservative, hence prefers signals to be private, more competition increases his welfare when signals are actually private, but it instead decreases welfare when signals are public. Accordingly, competition magnifies differences of welfare between the two regimes of disclosure, which suggests that the question of the optimal level of publicity of negative results of is all the more critical as the environment is competitive.

Finally, we examine the impact of policy interventions aiming at fostering the publication of negative results (e.g., subsidies). A natural intuition is that such subsidies would provide incentives to experiment longer because obtaining a negative result is now rewarded. Since negative results would then be public, such a policy would arguably allow to achieve the best of two worlds: reap the benefit of information spillovers without this leading to excessively early investment. It happens that this intuition is incorrect. Indeed, even when negative results are rewarded, the equilibrium investment strategy remains independent of the subsidy (though the equilibrium payoff increases) because of competition. Investment strategies are indeed only driven by the fear of preemption – actually, they are pinned down by the condition that one must be indifferent between winning the preemption race or not. Accordingly, promoting the disclosure of negative results matters only to the extent that signals become public. To address the question of the optimal regime of publicity of signals in a more general context, we also relax the assumption of winner-takes-all competition, and show that public signals are more likely to be optimal when competition becomes less fierce. Public signals may notably be suboptimal for the planner under winner-takes-all competition, but become optimal when the second mover gets a sufficiently large payoff. Accordingly, policies aiming at regulating information disclosure or transparency may be useless – and even detrimental – when competition is too severe. This suggests a pecking order in regulation whereby policy-makers should primarily tackle winner-takes-all competition (e.g., through policies related to patents and intellectual property), and regulate transparency only once competition is softer.

Our paper relates to the literatures on preemption and learning externalities in timing games. Since the seminal papers of Reinganum (1981) and Fudenberg and Tirole (1985) on preemption races, several papers have generalized the basic preemption framework in terms of payoff functions (Hoppe and Lehmann-Grube, 2005), number of firms (Argenziano and Schmidt-Dengler, 2014) and uncertainty about the presence of competitors (Bobtcheff and Mariotti, 2012; Bobtcheff, Bolte and Mariotti, 2017). Another stream of papers consider timing games where preemption concerns are absent and, on the contrary, players have incentives to wait in the hope of learning from others (Décamps and Mariotti, 2004; Murto and Välimäki, 2011; Kirpalani and Madsen, 2019; Margaria, 2020).

Our model combines both preemption and learning externalities, as in Chen, Ishida and Mukherjee (2018). However, their analysis does not cover the case of winner-takes-all competition which is at the core of our analysis. In addition, they only consider private signals while we stress the comparison between public and private news to highlight the pros and cons of the publication of negative results. This focus also relates our paper to a series of papers that compare public and private learning in timing games. Hopenhayn and Squintani (2011) analyze the impact of players being privately informed about their payoff from exiting. However, since they consider a private value setup, there is no information externality, hence no winner's curse. Moscarini and Squintani (2010) consider the impact of private signals in a model where private information is on the arrival rate of payoffs, so that staying in the game signals positive information to the competitor. Akcigit and

Liu (2015) also consider the inefficiency created by private information. While observable signals take the form of breakthroughs (good news) in their model, we consider bad news learning, which generates a winner's curse. As a result, we are able to exhibit the downside of public information, a concern which is absent in their setup where public information is always optimal. Hoppe-Wewetzer, Katsenos and Ozdenoren (2019) consider pure-strategy equilibria in a discrete-time preemption race. Because they consider good news learning, there is no winner's curse in their model either, while we show that private news induce a winner's curse problem, which in turn precludes the existence of pure-strategy equilibria, giving rise to a different dynamics of beliefs.

Finally, some papers show that private learning can dominate public learning in different contexts. Klein and Wagner (2018) consider a setup with learning externalities in which the possibility of signaling which private information brings allows to encourage investment, thereby mitigating free-riding. Henry (2009) and Herresthal (2017) consider environments where an agent running experiments can voluntarily disclose the outcomes of his tests to a principal, and establish that hidden testing may be superior to transparency.

The paper is organized as follows. In Section 2, we introduce the model and solve for the benchmark case of a single decision-maker. We then characterize the equilibrium of the two-player preemption game in Section 3, and examine its welfare properties in Section 4. In Section 5, we study the n-player game. Finally, in Section 6, we relax the assumption of winner-takes-all competition and examine whether rewarding negative results can improve welfare. Section 7 concludes.

2 The Model

2.1 An investment timing game

Time is continuous and indexed by $t \geq 0$. Two players (firms, researchers...) contemplate investing in a project of ex ante unknown but common quality. The quality of the project is high with probability p_0, and low with probability $1 - p_0$. Investment involves an irreversible cost $I \in (p_0, 1)$. Each player decides at which time to invest in the project, if he ever does. Upon investing, a player obtains a revenue of 1 if and only if the project is of high quality and he is the first player to invest.[9] In all other cases, his revenue is zero. Both players are risk-neutral and discount future revenues and costs at rate r.

[9] If both players attempt to invest at the same time, a fair public device randomly selects one of them, as in Dutta and Rustichini (1993).

As long as he has not invested, each player learns about the quality of the project by observing for free a personal signal. If the project is of low quality, this signal generates a failure at a time that is exponentially distributed with rate $\lambda > 0$. In the example of scientific research, one can interpret such a failure as a negative result, for instance, a counterexample to a theorem. In the case of a pharmaceutical firm, such a failure could be interpreted as the detection of significant side-effects that preclude the marketing of the drug regardless of its effectiveness.[10] Instead, when the project is good, nothing is ever observed. Thus, players become increasingly optimistic about the quality of the project as long as no failure has been observed.[11] The players' signals are conditionally independent given the quality of the project. We assume that a player that observes a failure immediately exits the game, that is, each player's signal generates at most one failure.

That a player observes a failure, and thus exits, may or may not be observed by his rival. Specifically, we assume that exit is public (hence immediately observed) with probability x, and private with probability $1-x$, in which case it will never be discovered by the other player. Notice that since the learning technology can only produce failures, and failures immediately trigger exit, observing the personal signal of one's rival is equivalent to observing his exit (as long as exit is observed with no delay). The two polar cases have a natural interpretation. The case $x = 1$ corresponds to public signals, i.e., both players observe a common signal that is twice more informative than the signal observed in isolation. Alternatively, each player observes his own signal only, but immediately observes when the other exits, which implies that both players have the same beliefs at any point in time. The case $x = 0$ corresponds to private signals (or unobserved exit), in which case each player is unsure whether the rival has exactly the same beliefs or is out of the game.

To allow for a finer comparative statics analysis, we let x vary continuously between these two polar cases. In a first stage, we take x to be an exogenous parameter reflecting the informational environment. For instance, drug development is highly regulated in that pharmaceutical firms are mandated to register their clinical trials and to disclose their results, so that it seems appropriate to think of this market as one with high x. At any point in time, competitors can observe whether or not a given firm is moving to the next phase of trials, and

[10] In these interpretations, the irreversible investment I could consist of submitting one's paper in the example of academic research, and of marketing one's product in the example of an innovation (drug). Notice that, in the example of scientific research, the fact that one obtains a payoff of 1 when the project is good and 0 otherwise implies that the refereeing process is assumed to be perfect. Therefore, we abstract from issues related to the quality of certification or refutability that are studied, e.g., in Bernard (2020).

[11] In such a bad news model, it is therefore impossible to obtain conclusive news that the project is good. If we allowed for conclusive good news, it is easy to see that it would always be optimal to invest as soon as good news arises in the preemption game.

thus interpret the fact of not moving further as exiting even if exit is not publicly announced. In the example of scientific research, incentives to publish negative results are low, so that it is typically difficult to observe whether other researchers have moved to another topic. This case corresponds to low values of x. Studying how welfare varies as a function of x will allow us to draw policy implications regarding the promotion of transparency between competitors. In a second stage, we allow a social planner to subsidize the publication of negative results, thereby endogenizing the publicity of signals.

To conclude the description of the model, we assume that investment is immediately observable to the rival. This implies that a strategy for each player specifies at what time to invest conditional on observing neither a failure – from his or his rival's signal – nor the rival investing. Our equilibrium concept is perfect Bayesian equilibrium.

2.2 The Single-player Benchmark

Let us first consider the benchmark case where there is a single player (monopolist) and, hence, no preemption concerns.[12] Let $V^m(t)$ denote the expected payoff of the monopolist viewed from date 0 when his strategy is to invest at date t conditional on having observed no bad news by then. There is a probability $p_0 + (1-p_0)\mathrm{e}^{-\lambda t}$ that no failure arises before date t, and conditional on reaching t without a failure, the probability of success is $\frac{p_0}{p_0+(1-p_0)\mathrm{e}^{-\lambda t}}$, so this expected payoff reads

$$\begin{aligned} V^m(t) &= e^{-rt}\left(p_0 + (1-p_0)\mathrm{e}^{-\lambda t}\right)\left(\frac{p_0}{p_0+(1-p_0)\mathrm{e}^{-\lambda t}} - I\right) \\ &= \mathrm{e}^{-rt}\left(p_0(1-I) - (1-p_0)\mathrm{e}^{-\lambda t} I\right). \end{aligned}$$

This expression illustrates the tradeoff between discounting and learning: while waiting delays the realization of the payoff, it allows to learn. The value of learning increases with λ, that is, with the probability that a failure reveals that the project is of low quality, in which case the decision maker avoids inefficiently sinking the investment outlay I. The optimal investment time t^m reflects this tradeoff:

$$t^m \equiv -\frac{1}{\lambda}\ln\left(\frac{rp_0(1-I)}{(r+\lambda)(1-p_0)I}\right) > 0. \tag{1}$$

It is clear that t^m is decreasing in p_0 and r, and increasing in I. Besides, as shown in Bobtcheff and Levy (2017), t^m is decreasing in λ because the ex-ante NPV of the project is negative ($p_0 < I$).[13]

[12]The monopoly benchmark is the same as in Décamps and Mariotti (2004) and Bobtcheff and Levy (2017).

[13]This last result is not immediate. Indeed, there are two effects at play: on the one hand, when λ increases

3 Competition with two players

3.1 Preliminaries

Strategies As in the single-player case, a (pure) strategy for player i consists of a date at which player i invests conditional on having observed nothing, i.e., neither a failure – from his or j's signal – nor j investing. Since future beliefs can be perfectly anticipated at date 0, if the strategy t is optimal viewed from date 0, it will also be optimal to invest at t if this date is reached before anything is observed. So there is no loss in considering that strategies are chosen at date 0 and simply executed whenever nothing happens. Likewise, a mixed strategy is a distribution of investment dates $F^i(t)$ chosen at date 0. A date t is drawn at date 0 from the distribution F^i and investment is implemented at date t as long as player i has observed nothing by then. Because a failure or investment by player j effectively terminates the game, the distribution F^i fully captures player i's strategy. Notice that one must behave in the same way when the project is of high quality and when it is of low quality but no failure has been observed since one cannot tell these two events apart.

Payoffs In order to express the objective function of player i, let us first derive the ex ante probability that player i turns out to invest in the project if he chooses a strategy t, which we denote $N^i(t)$. As long as F^j has no atom at t, this probability is just the probability that date t is reached without i observing either a failure – from his own or player j's signal – or j investing.[14] It is the sum of the probabilities of two disjoint events. First, if the project is of high quality, no failure will arise and the probability that nothing happens by date t is simply the probability that player j does not invest by then, that is, $1 - F^j(t)$. Second, if the project is of low quality, player i observes no failure from his own signal with probability $\mathrm{e}^{-\lambda t}$. Regarding player j, we distinguish two disjoint sub-events. Either player j has observed no failure from his signal and has not invested by time t; this occurs with probability $\mathrm{e}^{-\lambda t}[1-F^j(t)]$. Or player j has observed a failure from his signal at a date before the date at which he was supposed to invest, but this failure (or the ensuing exit) has not been observed by player i; this occurs with probability $(1-x)\int_0^t \lambda \mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s$. We

(faster learning), one reaches a given level of optimism on the project sooner, which provides incentives to invest early; on the other hand, improved learning opportunities increase the value of learning further, which tends to delay investment. If the project had positive ex-ante NPV ($p_0 \geq I$), then t^m would be single-peaked in λ (Bobtcheff and Levy, 2017).

[14] At atoms of F^j, the probability that i invests is possibly determined by the public randomization device, which complicates the expression of N^i. This will become apparent when we discuss the existence of pure-strategy equilibria. Except for the particular case where a pure-strategy equilibrium exists, we will show that equilibrium distributions must be atomless, so that these considerations are irrelevant and we ignore them in the exposition for clarity.

derive

$$N^i(t) = p_0[1-F^j(t)]+(1-p_0)e^{-\lambda t}\left(e^{-\lambda t}[1-F^j(t)]+(1-x)\int_0^t \lambda \mathrm{e}^{-\lambda s}[1-F^j(s)]\,\mathrm{d}s\right). \quad (2)$$

Let us denote by p_t^i the posterior probability that player i assigns to the project being good conditional on having observed nothing at date t. From Bayes' rule, this probability is equal to

$$p_t^i = \frac{p_0[1-F^j(t)]}{N^i(t)}. \quad (3)$$

Overall, the expected discounted payoff of player i from investing at time t reads

$$V^i(t) \equiv \mathrm{e}^{-rt}N^i(t)\left(p_t^i - I\right). \quad (4)$$

Winner's curse As usual in preemption games, investment by the rival prevents from investing in one's project, which creates a payoff externality. In addition, the possibility for the rival to invest creates an information externality. To see this, let us remark that, as soon as $F^j(t) < 1$, one can rewrite p_t^i as follows.

$$p_t^i = \frac{p_0}{p_0+(1-p_0)e^{-\lambda t}\left(e^{-\lambda t}+(1-x)\int_0^t \lambda \mathrm{e}^{-\lambda s}\frac{1-F^j(s)}{1-F^j(t)}\,\mathrm{d}s\right)} \quad (5)$$

$$= \frac{p_0}{p_0+(1-p_0)e^{-\lambda t}\left(1-x+xe^{-\lambda t}+(1-x)\int_0^t \lambda \mathrm{e}^{-\lambda s}\frac{F^j(t)-F^j(s)}{1-F^j(t)}\,\mathrm{d}s\right)}. \quad (6)$$

Let us first look at what happens as long as the competitor is not supposed to invest, i.e., for $F^j(t) = 0$. In that case, beliefs at date t are equal to $\frac{p_0}{p_0+(1-p_0)e^{-\lambda t}\left(1-x+xe^{-\lambda t}\right)}$, an increasing function of x, reflecting the fact that, whenever the signals received by the competitor (or his exit) become more observable, each player learns at a faster pace. However, as soon as $F^j(t)$ is positive, player i should infer additional information from the absence of investment of his rival. Indeed, the investment decision of player j depends on some underlying information which i does not observe perfectly (as long as $x < 1$), namely j's signal. Actually, lack of investment of j is bad news for i when j is supposed to invest with positive probability because it may be that player j has observed a failure (hence has exited) before the time at which his strategy prescribed him to invest. This is analogous to the winner's curse problem in common-value auctions.

As is clear from (6), the term $(1-x)\int_0^t \lambda \mathrm{e}^{-\lambda s}\frac{F^j(t)-F^j(s)}{1-F^j(t)}\,\mathrm{d}s$ captures this additional informational content. This term is obviously zero when $x = 1$ (public signals). Indeed, since i then observes whatever j observes, lack of investment by j cannot signal anything relevant that i would not know of, so there is no winner's curse. In that case, $p_t^i = \frac{p_0}{p_0+(1-p_0)e^{-2\lambda t}}$ and each player actually learns twice faster than in the single-player case. However, one remarks from inspecting (6) that if $x=0$, one has

$$p_t^i = \frac{p_0}{p_0 + (1-p_0)e^{-\lambda t} + (1-p_0)e^{-\lambda t}\int_0^t \lambda \mathrm{e}^{-\lambda s}\frac{F^j(t)-F^j(s)}{1-F^j(t)}\,\mathrm{d}s} < \frac{p_0}{p_0+(1-p_0)e^{-\lambda t}},$$

that is, a player is less optimistic upon seeing nothing before t than when alone: this is because not only he cannot observe the other's signal, hence learn from it, but the winner's curse imposes to shade his beliefs downward to take into account that lack of investment by the rival is bad news.

Overall, an increase in $F^j(t)$ has both a real effect, by reducing the probability that player i is the first to invest, and an informational effect, by increasing the winner's curse. This can be best seen by rewriting the objective function V^i as follows:

$$\begin{aligned} V^i(t) &= e^{-rt}\left(1-F^j(t)\right)\left(p_0(1-I)-(1-p_0)Ie^{-\lambda t}(1-x+xe^{-\lambda t})\right) \\ &- (1-x)(1-p_0)Ie^{-(r+\lambda)t}\int_0^t \lambda \mathrm{e}^{-\lambda s}[F^j(t)-F^j(s)]\,ds \end{aligned} \tag{7}$$

One sees that as long as $F^j=0$, the payoff is $e^{-rt}\left(p_0(1-I)-(1-p_0)Ie^{-\lambda t}(1-x+xe^{-\lambda t})\right)$, which corresponds to the single-player payoff once one takes into account that the decision maker observes a signal of quality λ with probability $1-x$ and of quality 2λ with probability x. The presence of the rival imposes a payoff externality in that this payoff is obtained with probability $1-F^j(t)$ only, i.e., as long as the rival does not preempt, as well as an information externality captured by the winner's curse term $e^{-(r+\lambda)t}(1-x)(1-p_0)I\int_0^t \lambda \mathrm{e}^{-\lambda s}[F^j(t)-F^j(s)]\,ds$. While the payoff externality induces players to hurry investment, the winner's curse provides incentives to delay investment. In what follows, we further explore the interplay between these two effects and examine more specifically the role of the publicity of signals x on investment incentives.

3.2 Equilibrium analysis

The first natural avenue to look for is whether there are pure-strategy equilibria. In any such equilibrium, it must be the case that each player invests at the first date at which the expected value of the project conditional on no failure becomes nonnegative, that is, at $\hat{t}(x)$ such that

$$\frac{p_0}{p_0+(1-p_0)e^{-\lambda\hat{t}(x)}\left(1-x+xe^{-\lambda\hat{t}(x)}\right)} = I. \tag{8}$$

To see that both players must invest at $\hat{t}(x)$ in a pure-strategy equilibrium, consider what would happen otherwise. First, investing before $\hat{t}(x)$ yields a negative profit, hence cannot be optimal as one could always secure a payment of 0 by never investing; second, if one had $t_j > t_i > \hat{t}(x)$, then j would strictly increase his profit by investing at any date between $\hat{t}(x)$ and t_i; third, if $t_j > \hat{t}(x) = t_i$, then i would strictly increase his profit by investing at any date between $\hat{t}(x)$ and t_j; finally, if $t_i = t_j > \hat{t}(x)$, any player could increase his payoff by slightly undercutting his rival.[15]

Let us look at whether both players investing at $\hat{t}(x)$ is an equilibrium, and consider first the case $x < 1$. When F^j has an atom at t, the formulae given by (2), (3) and (4) do not readily apply since both players attempt to invest at the same date with a non-zero probability, in which case the "winner" of the preemption race is determined by the tie-breaking rule. In such a situation, the relevant belief p_t^i is the probability that the project is good conditional on observing nothing by date t *and* winning the race.[16] In the pure-strategy equilibrium we consider, each player turns out to win with probability $\frac{1}{2}$ upon observing nothing by $\hat{t}(x)$. This means that, conditioning on winning the race and observing no failure, each player has beliefs

$$\begin{aligned} p^i_{\hat{t}(x)} &= \frac{\frac{1}{2}p_0}{\frac{1}{2}p_0 + (1-p_0)e^{-\lambda\hat{t}(x)}\left(\frac{1}{2}e^{-\lambda\hat{t}(x)} + (1-x)(1-e^{-\lambda\hat{t}(x)})\right)} \\ &< \frac{p_0}{p_0 + (1-p_0)e^{-\lambda\hat{t}(x)}\left(1 - x + xe^{-\lambda\hat{t}(x)}\right)} \\ &= I. \end{aligned}$$

Therefore, each player gets a negative expected payoff so this cannot be an equilibrium. The reason why it is not is the winner's curse problem. Since in the bad state the competitor might have observed a failure and thus exited, one is more likely to win in the bad state than in the good state, which implies that winnng is bad news.[17] Correcting for this winner's curse, the expected value of the project – that is arbitrarily close to 0 right before $\hat{t}(x)$ – jumps and becomes negative at $\hat{t}(x)$. Therefore, such a pure-strategy equilibrium cannot exist. Notice that this holds only if $x < 1$. In the case where $x = 1$, there is no winner's curse, that is, even conditioning on the fact of winning, one still has $p^i_{\hat{t}(x)} = I$. Indeed, winning is then only a matter of luck. In this case, a pure-strategy equilibrium exists: both players invest at $\hat{t}(1)$ and get an expected payoff of 0.

[15]Undercutting indeed increases the likelihood of being the one who invests, and increases the probability of success conditional on investing. That is, both $N^i(t)$ and p_t^i have a downward discontinuity at $t = t_j$. See the proof of Lemma 1 in the Appendix for details.

[16]When a player's strategy is to invest at a date t that is not an atom of F^j, he wins the race with probability 1 conditional on observing nothing by date t, so conditioning on winning does not change beliefs further.

[17]Notice that this argument would hold for any tie-breaking rule in case of simultaneous investment.

It is therefore the presence of the winner's curse that precludes the existence of pure-strategy equilibria if $x < 1$. We now establish that, in that case, there exists a unique mixed-strategy equilibrium where players randomize over the interval $[\hat{t}(x), \infty)$.

Proposition 1 *The game admits a unique equilibrium in which both players get an expected payoff of zero:*

- *If $x = 1$, both players invest at $\hat{t}(1)$ such that $p_0(1-I) - (1-p_0)Ie^{-2\lambda\hat{t}(1)} = 0$.*
- *If $x < 1$, both players follow a mixed strategy described by a cdf F_x given by*

$$F_x(t) = 1 - \mathrm{e}^{-\lambda[t-\hat{t}(x)]} \left(\frac{p_0(1-I) - (1-p_0)I\mathrm{e}^{-2\lambda\hat{t}(x)}}{p_0(1-I) - (1-p_0)I\mathrm{e}^{-2\lambda t}} \right)^{\frac{1+x}{2}} \tag{9}$$

for all $t \geq \hat{t}(x)$.

Proof: In the Appendix.

Notice that because $p_0(1-I)-(1-p_0)I\mathrm{e}^{-2\lambda\hat{t}(1)} = 0$, for any sequence $(x_\tau)_{\tau\in\mathbb{N}}$ converging to 1, the corresponding sequence of distribution functions $(F_{x_\tau})_{\tau\in\mathbb{N}}$ given by (9) converges weakly to the jump distribution $\mathbb{1}_{[\hat{t}(1),\infty)}$. That is, as signals (or exit) become close to perfectly observable, the sequence of mixed-strategy equilibria converges to the pure-strategy equilibrium. We now derive how the equilibrium investment strategy changes as the publicity of signals x varies.

Proposition 2 *If $x' < x < 1$, the investment time under x is smaller than the investment time under x' in the hazard-rate order.*

Proof: In the Appendix.

Proposition 2 notably implies that equilibrium distributions can be ranked in terms of (first order) stochastic dominance: if $x > x', F_{x'}(t) \leq F_x(t)$ for all t. In other words, players invest sooner when signals are more observable. Notice also that, since $\hat{t}(1) < \hat{t}(x)$, investment occurs sooner when $x = 1$ than for any $x < 1$. The result of Proposition 2 is illustrated in Figure 1, where we depict the equilibrium distribution of investment dates for two different values of x.

The intuition that investment arises sooner when signals are more likely to be publicly observed rests on three distinct effects. First, learning is faster when the publicity of signals increases. Holding the investment strategy of his rival fixed, a player is more confident about the project upon seeing no failure as x increases since he learns from overall more

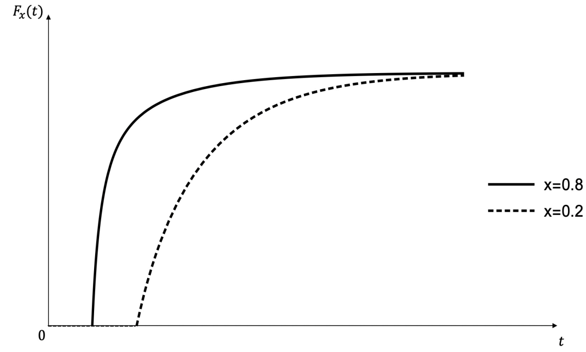

Figure 1: Equilibrium distribution for $x = 0.2$ and $x = 0.8$ $[p_0 = 0.5, I = 0.7, \lambda = 0.8]$

informative signals. This direct effect would be present even in the absence of competition, as an increase in the speed of arrival of news unambiguously leads to earlier investment when there are no preemption concerns.[18] This direct effect is exacerbated by the strategic interaction between players, which generates two additional forces. On the one hand, because each player should expect his rival to invest earlier, he should in turn be more aggressive for fear of being preempted. This is due to the strategic complementarity in preemption games, which arises even in the absence of information externality. On the other hand, as x decreases, the winner's curse problem worsens, which provides incentives to further delay investment so as to gain extra confidence on the project through one's own signal. These three effects compound to generate earlier investment when signals are more likely to be public. To underline that earlier investment is not simply the mechanical consequence of more information being available to each player, let us compare what happens when signals are public ($x = 1$) and the rate of arrival of bad news is $\frac{\lambda}{2}$ for each player on the one hand, and when signals are private ($x = 0$) and the arrival rate of bad news is λ on the other hand, so that the total arrival rate is the same (equal to λ) in the two situations. In the first scenario, both players invest at t such that $p_0(1-I) - (1-p_0)e^{-2\frac{\lambda}{2}t} = 0$, that is, at $\hat{t}(0)$. In the second scenario, players randomize over all dates $t \geq \hat{t}(0)$. This shows that the presence of the winner's curse alone is enough to generate later investment when signals are private. We now turn to the analysis of the welfare impact of such delayed investment.

[18] An increase in x is here qualitatively similar to an increase in λ in the single-player case. As we observed in Section 2.2, t^m decreases in λ when the ex-ante NPV of the project is negative.

4 Welfare

In Proposition 1, we establish that players get an expected payoff of zero regardless of the publicity of signals. As usual in winner-takes-all models of competition, preemption leads to full dissipation of rents in equilibrium (Fudenberg and Tirole, 1985). The intuition in the pure-strategy equilibrium ($x = 1$) is standard: if the equilibrium payoff was positive, it would be possible for one player to increase his profit by slightly undercutting his rival.[19] When $x < 1$, the intuition reflects the winner's curse problem. In such a case, the equilibrium can only be in mixed strategy and the support of the distribution of investment dates F_x must be unbounded. Otherwise, the payoff from investing at the upper bound of the support would be strictly negative, because each player could reflect that this upper bound may be reached without his rival investing only if the latter has observed a failure that has not been publicly observed. Because of discounting, this implies that the equilibrium payoff must exactly be zero.

A property of the equilibrium strategy is thus that players' beliefs remain constant – specifically, $p_t^i = I$ – over the whole support of investment dates. Relatedly, the probability that investment takes place conditional on the project being bad is independent of x. Denoting α this probability and f_x the density of the distribution of investment dates, one indeed can check:[20]

$$\begin{aligned}
\alpha &= \frac{1}{1-p_0}\int_{\hat{t}(x)}^{\infty} 2\left(N(t) - p_0[1-F_x(t)]\right) f_x(t)\, dt \\
&= \frac{1}{1-p_0}\int_{\hat{t}(x)}^{\infty} 2\frac{1-p_t}{p_t} p_0[1-F_x(t)] f_x(t)\, dt \\
&= \frac{p_0}{1-p_0}\frac{1-I}{I}\int_{\hat{t}(x)}^{\infty} 2[1-F_x(t)] f_x(t)\, dt \\
&= \frac{p_0}{1-p_0}\frac{1-I}{I}
\end{aligned}$$

The first equality uses (3), the second follows from $p_t = I$ for all $t \geq \hat{t}(x)$. Therefore, because rents are fully dissipated anyway, imposing different standards of disclosure for negative results has no impact on the quality of investments, and policy interventions that affect x would have no efficiency impact.

However, in the applications we have in mind, such as drug development or scientific research, the social planner's objective function may differ from the players' in several ways.

[19]Of course, such undercutting strategy is possible because no player ever invests at date 0 (the ex ante NPV is negative). If we had $p_0 > I$, there would be a pure-strategy equilibrium where both players invest at date 0 and make positive profits.

[20]Since the equilibrium is symmetric, we simplify the notation: $N^i(t) = N^j(t) = N(t)$, and $p_t^i = p_t^j = p_t$.

For instance, the planner may not fully internalize the profits of pharmaceutical companies, but may instead care about consumer surplus. The social value of investment may also differ from its private value when investment generates positive externalities, as is the case for the development of a vaccine, or when part of the investment is financed by public funds, e.g., for scientific research. Finally, the social cost of unsuccessful investments is often significantly larger than the investment costs borne by innovating firms. For instance, in the case of drug development, the social cost may capture the cost of public funds if an ineffective drug is reimbursed by a public health insurance scheme, or the health costs generated by a drug's negative side effects. In academic research, there is no reason to expect the private value of a publication (better career prospects, prestige...) to match its social value.

To be as flexible as possible, let us assume that the planner's payoffs in case of successful and unsuccessful investment are respectively $1 - I + \Delta_S \geq 0$ and $-(I + \Delta_F) \leq 0$. Thus Δ_S and Δ_F parametrize how social objectives depart from private ones in each state. With such preferences, a planner that would have access to the signal would invest at a date t^* such that

$$t^* \equiv \max\left\{-\frac{1}{\lambda}\ln\left(\frac{rp_0(1-I+\Delta_S)}{(r+\lambda)(1-p_0)(I+\Delta_F)}\right), 0\right\}. \tag{10}$$

As in Section 2.2, the investment strategy reflects the tradeoff between discounting and learning. One remarks that

$$t^m < t^* \Leftrightarrow \frac{1-I+\Delta_S}{I+\Delta_F} < \frac{1-I}{I} \Leftrightarrow \frac{\Delta_S}{\Delta_F} < \frac{1-I}{I}.$$

If $(1 - I + \Delta_S)/(I + \Delta_F) < (1 - I)/I$, the planner cares relatively more about avoiding failures as compared to players, thus has a stronger motive for learning. As a consequence, his investment policy is more conservative and he invests later.

With this premise in mind, let us go back to the preemption game and derive how the welfare of the planner at equilibrium varies with the publicity of signals x.

Proposition 3 *More frequently disclosing negative results (a higher x) decreases the planner's welfare if $\Delta_S/\Delta_F < (1-I)/I$, and increases the planner's welfare if $\Delta_S/\Delta_F > (1-I)/I$.*

Proof: Let $W(x)$ denote the planner's welfare at the equilibrium of the preemption game. If $x < 1$, $W(x)$ is given by

$$(1-I+\Delta_S)\int_{\hat{t}(x)}^{+\infty} 2e^{-rt}p_0[1-F_x(t)]f_x(t)\,dt - (I+\Delta_F)\int_{\hat{t}(x)}^{\infty} 2e^{-rt}\left(N(t) - p_0[1-F_x(t)]\right)f_x(t)\,dt$$

Using (3) as well as $p_t = I$ for all $t \geq \hat{t}(x)$, one derives

$$W(x) = p_0\left(\Delta_S - \Delta_F\,\frac{1-I}{I}\right)\int_{\hat{t}(x)}^{\infty} 2\mathrm{e}^{-rt}[1-F_x(t)]f_x(t)\,dt. \tag{11}$$

From Proposition 2, for all $x' < x < 1$, $F_{x'}$ first-order stochastically dominates F_x, which implies

$$\int_{\hat{t}(x')}^{\infty} 2\mathrm{e}^{-rt}[1-F_{x'}(t)]f_{x'}(t)\,dt < \int_{\hat{t}(x)}^{\infty} 2\mathrm{e}^{-rt}[1-F_x(t)]f_x(t)\,dt.$$

If $x = 1$,

$$\begin{aligned} W(1) &= e^{-r\hat{t}(1)}\left(p_0(1-I+\Delta_S) - (1-p_0)(I+\Delta_F)e^{-\lambda\hat{t}(1)}\right) \\ &= p_0\left(\Delta_S - \Delta_F\,\frac{1-I}{I}\right)e^{-r\hat{t}(1)}. \end{aligned}$$

Since $\hat{t}(1) < \hat{t}(x)$, $\int_{\hat{t}(x)}^{\infty} 2\mathrm{e}^{-rt}[1-F_x(t)]f_x(t)\,dt < e^{-r\hat{t}(1)}$ for any $x < 1$.

We can thus conclude that $W(x)$ is increasing in x if and only if $\Delta_S/\Delta_F > (1-I)/I$. ■

In the presence of competition, the tradeoff between discounting and learning we highlight in the single-player case is muted, since the fear of preemption induces players to stop learning as soon as the NPV of investment is zero. Since this holds regardless of x, the preference of the planner over the publicity of signals simply reflects his time preferences given that players will invest at their zero-NPV cutoff. This can be best seen by decomposing the planner's welfare $W(x)$ as follows:

$$W(x) = \underbrace{(I\Delta_S - (1-I)\Delta_F)}_{\text{Planner's NPV upon investment}} \times \underbrace{\int_{\hat{t}(x)}^{\infty} 2\frac{p_0}{I}e^{-rt}[1-F_x(t)]f_x(t)\,dt}_{\text{Present value of obtaining 1 at the first investment date}} \tag{12}$$

Let us first look at the first term. At any equilibrium investment date, a player has beliefs $p_t = I$. With such a probability of success, the NPV of investment from the perspective of the planner is

$$p_t(1-I+\Delta_S) - (1-p_t)(I+\Delta_F) = I\Delta_S - (1-I)\Delta_F.$$

To figure out the interpretation of the second term (the integral), let us denote by G_x the cdf of the random variable equal to the first date at which investment arises at equilibrium. If one player invests, then the other will never invest so that $G_x(t)$ is simply worth twice the probability that a given player invests by date t, that is, $G_x(t) = 2\int_{\hat{t}(x)}^{t} N(s)f_x(s)\,ds$. Using $p_t = I$ for all $t \geq \hat{t}(x)$ together with (3) yields

$$G_x(t) = \frac{p_0}{I}\int_{\hat{t}(x)}^{t} 2[1-F_x(s)]f_x(s)\,ds. \tag{13}$$

$2\frac{p_0}{I}(1-F_x)f_x$ is thus the density of the first investment date, and $\int_{\hat{t}(x)}^{\infty} 2\frac{p_0}{I}e^{-rt}[1-F_x(t)]f_x(t)\,dt$ is the present value of obtaining 1 whenever investment first occurs.

In this context, the intuition for Proposition 3 is as follows. When $\Delta_S/\Delta_F > (1-I)/I$, the planner is more prone to investing than players, so, from his perspective, the NPV when players invest is positive. Since the NPV is also the same at any equilibrium investment date, he would like investment to take place as early as possible, hence a preference for public signals. However, if $\Delta_S/\Delta_F < (1-I)/I$, the planner is more conservative: his NPV upon investment is negative (and constant), so that he prefers investments to be delayed, hence a preference for private signals. In the special case $r = 0$, the publicity of signals is irrelevant to the planner, since the probability that investment ever takes place is independent of x. Notice in this respect that, since F_x is independent of r, nothing in the above analysis hinges on the assumption that the social planner has the same discount rate as the players.[21]

In a nutshell, while public signals generate a positive information spillover (better learning), the benefits created by these spillovers are fully eroded by competition. The publicity of signals is thus irrelevant to players, and can matter only to a planner that has different preferences over type I and type II errors. In particular, a planner who cares a lot about the costs of failed investments prefers more conservative investment policies, hence private signals. Indeed, the resulting winner's curse provides incentives to delay investment, thereby softening the incentives to accelerate investment caused by preemption concerns. This provides a possible rationale against the mandatory disclosure of negative results.

5 Competition with n players

We have so far assumed that there are only two players. In this section, we examine the impact of an increase in competition on investment strategies, and hence, on welfare. Suppose that there are now n identical players. As in the previous section, it can be easily shown that there cannot be a pure-strategy equilibrium as soon as $x < 1$. In the case of public signals ($x = 1$), there exists a pure-strategy equilibrium in which all players invest at the first date at which the NPV becomes nonnegative, that is, at $\hat{t}(1, n)$ such that

$$p_0(1-I) - (1-p_0)I\mathrm{e}^{-\lambda n \hat{t}(1,n)} = 0 \tag{14}$$

[21] In particular, this implies that if the only conflict of interests between players and the planner was different preferences for the present, the welfare of the planner would be independent of the publicity of news. Formally, if the planner has discount rate $r^P \neq r$ and $\Delta_F = \Delta_S = 0$, then $W(x) = 0$ for all x.

More generally, we establish in the following Proposition that all the properties of the equilibrium characterized in Proposition 1 go through with n players.[22]

Proposition 4 *The game admits a unique symmetric equilibrium in which all players get an expected payoff of zero:*

- *If $x = 1$, all players invest at $\hat{t}(1, n)$.*
- *If $x < 1$, all players follow a mixed strategy described by a cdf $F_{x,n}$ such that*

$$F_{x,n}(t) = 1 - \mathrm{e}^{-\frac{1}{n-1}\lambda[t-\hat{t}(x,n)]} \left(\frac{\left(\frac{p_0}{1-p_0}\frac{1-I}{I}\right)^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda\hat{t}(x,n)}}{\left(\frac{p_0}{1-p_0}\frac{1-I}{I}\right)^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda t}} \right)^{\frac{1+(n-1)x}{n}} \quad (15)$$

for all $t \geq \hat{t}(x, n)$, where $\hat{t}(x, n)$ is defined by

$$p_0(1 - I) - (1 - p_0)I\mathrm{e}^{-\lambda\hat{t}(x,n)}\left[1 - x + x\mathrm{e}^{-\lambda\hat{t}(x,n)}\right]^{n-1} = 0 \quad (16)$$

If $x > x'$, the investment time under x is smaller than the investment time under x' in the hazard-rate order.

Finally, the planner's welfare $W(x, n)$ is nondecreasing in x iff $\frac{\Delta_S}{\Delta_F} \geq \frac{1-I}{I}$.

Proof: In the Appendix.

Based on the results of Proposition 4, let us now study the welfare impact of competition, that is, how $W(x, n)$ varies with n (holding x fixed). For the sake of simplicity, let us compare the two polar cases $x = 0$ and $x = 1$ (private and public news).

Proposition 5 *Suppose $\frac{\Delta_S}{\Delta_F} \geq \frac{1-I}{I}$ (resp. $\frac{\Delta_S}{\Delta_F} \leq \frac{1-I}{I}$). If $x = 1$, more competition (a higher n) increases (resp. decreases) the planner's welfare. If $x = 0$, more competition decreases (resp. increases) the planner's welfare.*

Proof: In the Appendix.

Proposition 5 basically states that, as long as we are in the optimal regime of publicity of signals for the planner, more competition increases the planner's welfare. However, in a regime where the publicity of signals is not optimal, more competition will exacerbate the inefficiency. Suppose for instance that $\frac{\Delta_S}{\Delta_F} \leq \frac{1-I}{I}$, that is, the planner prefers signals to be private, but that signals are instead public ($x = 1$). In that case, an increase in competition

[22] The only difference is that we show equilibrium uniqueness in the two-player case, while we only show that there is a unique symmetric equilibrium in the n-player case.

will prompt investment even further, which hurts the planner. An increase in competition in the market for drugs may accordingly have an adverse effect for a planner who primarily cares about avoiding side-effects. In the same vein, a planner who does not internalize the cost for researchers of having their papers rejected but just cares about positive results being published will suffer from an increase of competition. An immediate corollary of Proposition 5 is that $|W(1, n) - W(0, n)|$ is increasing in n. That is, competition amplifies differences of welfare between the two regimes of disclosure. This suggests that reaching the optimal level of publicity is all the more critical as the environment is competitive. While the ongoing initiatives going towards more transparency in scientific research may accordingly be an optimal response to increased competition, our paper suggests that, because transparency is not necessarily optimal, they might also create even more damage.

6 Should negative results be rewarded?

In line with the main question of the paper, we examine in this section whether the planner could increase welfare by promoting the publicity of signals, for instance, by subsidizing the publication of negative results. Specifically, we assume that a player obtains a prize $c \geq 0$ from publishing a negative result if he is the first to publish one and if no one has invested in the project (otherwise failure reveals that the project is bad and there is no reward for a negative result that proves what is already known). Moreover, we also relax the assumption of winner-takes-all competition and assume that a player obtains a payoff $L \geq 0$ from moving second (if and only if the project is good). The analysis of this more general case allows to examine how the optimal regime of publicity of signals may change as competition increases. In terms of policy implications, it also allows to understand whether a planner can increase welfare by subsidizing second movers and/or negative results, and which of the two policy instruments is more effective.

In order to build intuition on the possible role of c, let us first look at the benchmark case of a single decision maker. Given a prize c, the decision maker maximizes

$$e^{-rt}\left(p_0(1-I) - (1-p_0)Ie^{-\lambda t}\right) + (1-p_0)c\int_0^t \lambda e^{-(r+\lambda)s}\, ds. \tag{17}$$

One easily checks that this function is maximum at

$$t^m(c) = -\frac{1}{\lambda}\ln\frac{rp_0(1-I)}{(1-p_0)((\lambda+r)I+\lambda c)} > 0. \tag{18}$$

Hence the decision maker invests later as c increases: indeed, learning longer now brings the extra benefit that a negative result could be obtained. If the planner could set c, he would choose $c = 0$ if $\Delta_S/\Delta_F > (1-I)/I \Leftrightarrow t^* < t^m$ since the decision maker already invests too late from the planner's perspective even with $c = 0$. However, if $\Delta_S/\Delta_F < (1-I)/I \Leftrightarrow t^* > t^m$, the planner would choose c such that $t^m(c) = t^*$ and thus realign the investment strategy of the decision maker with what we would himself do.[23]

With this in mind, a natural intuition is that rewarding negative results could allow the planner to get the best of two worlds: on the one hand, it promotes information sharing, which generates positive spillovers; one the other hand, c could be fine-tuned to increase players' incentives to delay investment so as to counteract the adverse effect that public signals entail earlier investment. It happens that this intuition is incorrect, and that increasing c has indeed no direct impact on investment strategies, hence on welfare, because of competition. To see this formally, let us assume that when $c > 0$ all players publish their negative results whenever they have one, while they withhold it if $c = 0$.[24] In that context, we derive the following proposition:

Proposition 6

- *Suppose $c > 0$ and $x = 1$. There is an equilibrium in pure strategies whereby each player invests at $t_1(L)$ such that*

$$p_0(1-I) - (1-p_0)Ie^{-2\lambda t_1(L)} = p_0 L$$

- *Suppose $c = 0$ and $x = 0$. There exists a cutoff $L_{max} < 1 - I$ such that, as long as $L \leq L_{max}$, there is a symmetric equilibrium in mixed strategy whereby each player invests following a cdf $\tilde{F}$ with support $[\underline{t}(L), \overline{t}(L)]$.*[25]

Proof: In the Appendix.

Proposition 6 generalizes the equilibrium of Proposition 1 to the case where moving second and negative results are rewarded. As in Proposition 1, the equilibrium is in pure strategies as soon as signals are public, while with private signals, the presence of the winner's curse imposes to have a mixed-strategy equilibrium.[26] Proposition 6 formally establishes

[23] One can check that the optimal c is then $\frac{\lambda+r}{\lambda}\frac{(1-I)\Delta_F - \Delta_S I}{1-I+\Delta_S} > 0$.

[24] This is equivalent to assuming an infinitesimal cost of publishing negative result. More realistically, one could assume that the probability of publishing a negative result x increases continuously with c, which would arise, for instance, when players have a random outside option. We abstract from these considerations for conciseness and to better illustrate our point that c does not impact equilibrium strategies beside its impact on the rate of disclosure x.

[25] Since the expression of $\tilde{F}$ brings no intuition, we do not report it here for conciseness, and refer to the Appendix, notably Eq. (A.23).

[26] Notice that the reason why such a mixed-strategy equilibrium exists only when L is sufficiently small is that the game changes from a preemption game to a war of attrition as L gets too large. In this case, investing second becomes preferable to investing first since it does not entail the risk of failure.

that, as long as signals are public, the equilibrium strategy (and, hence, the planner's welfare) is independent of c. Indeed, investment strategies are driven by the fear of preemption, which forces players to invest as soon as they are indifferent between winning or losing the preemption race, that is, at a date which is independent of the prize from publishing a negative result. Allegedly, if the subsidy is costly to the planner, he will thus choose to set either $c = 0$ or c arbitrarily close to 0 according to the regime of publicity of signals he wants to promote.

In this respect, let us now compare the equilibrium strategies, payoffs and welfare in the two regimes where signals are public and private.

Proposition 7 *In the equilibrium with public signals* ($x = 1$ *and* $c > 0$), *players invest sooner and obtain a larger expected payoff than in the equilibrium with private signals* ($x = 0$ *and* $c = 0$). *The NPV upon investment is larger with public signals. Finally, there exists a cutoff* $\overline{\Delta} < \frac{1-I}{I}\Delta_F - (1 + \frac{\Delta_F}{I})L$ *such that the planner's welfare is larger with public signals if and only if* $\Delta_S \geq \overline{\Delta}$.

Proof: In the Appendix.

Like in the case where $L = 0$, the presence of the winner's curse implies that investment always takes place earlier when signals are public: $\underline{t}(L) > t_1(L)$. When $L > 0$, we derive the additional result that players get a positive payoff at equilibrium, and that this equilibrium payoff is strictly larger with public signals (even if c is arbitrarily close to 0) than with private signals. The intuition is also related to the winner's curse. The winner's curse captures the negative information externality which players impose onto each other by not disclosing their negative results. Accordingly, players are better off when such an externality is absent. When $L = 0$, whether signals are public or not is irrelevant because rents are fully dissipated in both regimes, but as soon as L becomes positive and players obtain rents, they prefer the regime where negative results are disclosed in which the rent erosion is milder. Notice finally that increasing c increases the equilibrium payoff (without affecting equilibrium strategies), hence reinforces the preference of players for public signals.

The most noteworthy finding we draw from this analysis has to do with welfare. As in Section 4, the planner prefers negative results to be public as soon as Δ_S is sufficiently large (alternatively, Δ_F is sufficiently small). In addition, the cutoff $\overline{\Delta}$ is strictly smaller than the counterpart cutoff under winner-takes-all competition, that is, $\Delta_F \frac{1-I}{I}$. This implies that whenever the planner prefers public signals under winner-takes-all competition ($L = 0$), he a fortiori prefers public signals once the second-mover gets a positive prize L. However, there are situations where the planner would strictly prefer private signals under winner-takes-all competition, but strictly prefers news to be public once competition is milder. This is the case when, e.g., $-(1+\frac{\Delta_F}{I})L < \Delta_S - \frac{1-I}{I}\Delta_F < 0$. This condition is more likely to be satisfied as L increases. Essentially, a larger L, by softening competition, increases the relative merit of public signals in two respects. First, because there is no winner's curse, the NPV of investment is larger in the public signals case. Investment is then more efficient, which is valuable to the planner regardless of his preferences for type I and type II errors. Second, softer competition reduces incentives to hurry investment, thereby attenuating the additional incentives to prompt investment which public signals bring.

A few conclusions emerge from this analysis. While rewarding negative results may be optimal for the planner when there is a single decision maker, it becomes less relevant as soon as there is competition. Indeed, with competition, c matters only to the extent that it determines whether signals are public, but otherwise affects neither investment strategies nor welfare. In this regard, while public signals may be detrimental to welfare under winner-takes-all competition, they may become optimal once preemption concerns become milder. Accordingly, mandating the disclosure of negative results can be efficient as a complement to a policy that relaxes winner-takes-all competition, but in the impossibility to resort to such policies, mandating the disclosure of negative results may actually backfire. This suggests a pecking order whereby the optimal regulation should first and foremost address winner-takes-all competition, and promote transparency only once competition is less severe.

7 Conclusion

In this paper, we consider a model of investment timing in which firms (or researchers) face a tradeoff between learning and discounting. Waiting allows to learn about the value of the project, which potentially allows to avoid unprofitable investments, but delays the payoff when investing is profitable. In the case where the planner cares a lot about the social costs of failed investments, hence would invest later than firms (researchers) in the absence of preemption, the presence of competition is essentially bad for the planner, since the fear of preemption induces earlier investment. In this context, we show that private signals can be optimal, because the resulting winner's curse provides incentives to delay investment, thereby softening the negative impact of preemption from the planner's perspective. In such situations, mandating (or incentivizing) the disclosure of negative results backfires, notably when competition is too fierce. Accordingly, our analysis suggests that a public policy aiming at increasing the disclosure of negative results has a positive impact when it is paired with a policy aiming at reducing winner-takes-all competition, but may have an adverse impact on welfare otherwise.

Notice also that, as information in our model comes for free, the social value of disclosing negative results does not stem from any wasteful duplication of costs in case signals are private, an argument often put forward to promote the mandatory disclosure of negative results. Instead, the possible optimality of public signals in our model results from the fact that investment takes place earlier when signals are public, which is valuable when the planner cares about getting investment done. Therefore, our paper on the one hand raises doubts as to the fact that mandatory disclosure of negative results might be universally optimal, but on the other hand provides an alternative and complementary reason for why it might be.

Appendix

Proof of Proposition 1.

Let us recall from (7) that the expected payoff of player i from investing at a date t that is not an atom of F^j reads

$$\begin{aligned} V^i(t) &= e^{-rt}\left(1-F^j(t)\right)\left(p_0(1-I)-(1-p_0)Ie^{-\lambda t}(1-x+xe^{-\lambda t})\right) \\ &- (1-x)(1-p_0)Ie^{-(r+\lambda)t}\int_0^t \lambda e^{-\lambda s}[F^j(t)-F^j(s)]\,ds \end{aligned} \quad (A.1)$$

Before deriving equilibrium strategies, we first establish two lemmas which provide a set of conditions which any equilibrium must satisfy. Let S_i denote the support of player i's investment dates.

Lemma 1 *Suppose $x < 1$. In any equilibrium, $S_i = S_j = S = [\hat{t}(x), +\infty)$ and F^i is continuous and strictly increasing on S. All players get an equilibrium payoff of 0.*

The proof involves several claims, which we prove sequentially. Let $\underline{t}_i$ and $\bar{t}_i$ be the smallest and largest elements of S_i.

<u>Claim 1</u>: $\underline{t}_1 = \underline{t}_2$ and $\bar{t}_1 = \bar{t}_2$.

Proof: Suppose that $\bar{t}_i < \bar{t}_j$. Then, using (A.1), for any $t \geq \bar{t}_i$,

$$V^j(t) = -(1-x)(1-p_0)Ie^{-(r+\lambda)t}\int_0^{\bar{t}_i} \lambda e^{-\lambda s}[1-F^i(s)]\,ds < 0.$$

Thus, dates $t \in (\bar{t}_i, \bar{t}_j]$ cannot belong to the support S_j. This proves $\bar{t}_1 = \bar{t}_2 = \bar{t}$.

Suppose now that $\underline{t}_i < \underline{t}_j$, and let $a(t) = e^{-rt}\left(p_0(1-I)-(1-p_0)Ie^{-\lambda t}(1-x+xe^{-\lambda t})\right)$. It is easy to see that a is single-peaked. Let $t^*(x)$ denote the argmax of $a(t)$.

Using (A.1), we observe that $V^i(t) = a(t)$ for all $t \leq \underline{t}_j$ and $V^i(t) < a(t)$ for all $t > \underline{t}_j$. Therefore, if $\underline{t}_i > t^*(x)$, investing at $\underline{t}_i$ cannot be profit-maximizing for i since investing at $t^*(x)$ does strictly better. If $\underline{t}_i < t^*(x)$, then player i could strictly increase profit by investing at $\underline{t}_i + \epsilon$. If $\underline{t}_i = t^*(x)$, player j whose payoff $V^j(\underline{t}_j)$ is bounded away from $a[t^*(x)]$ could get a payoff arbitrarily close to $a[t^*(x)]$ by investing slightly before $t^*(x)$. This proves $\underline{t}_1 = \underline{t}_2 = \underline{t}$.

<u>Claim 2</u>: If t is an atom of F^j then t does not belong to the support S_i.

Proof: Consider a date t_0 that is an atom of F^j and let $F^j(t_0) - \lim_{\epsilon\to 0} F^j(t_0-\epsilon) = a > 0$.

When i invests at t_0, N^i and p^i_t are modified as follows:

$$N^i(t_0) = p_0[1-F^j(t_0)+\frac{1}{2}a]+(1-p_0)e^{-\lambda t_0}\left((1-x)\int_0^{t_0}\lambda e^{-\lambda s}[1-F^j(s)]\,ds + e^{-\lambda t_0}[1-F^j(t_0)+\frac{1}{2}a]\right)$$

$$p^i_{t_0} = \frac{p_0[1 - F^j(t_0) + \frac{1}{2}a]}{N^i(t_0)}$$

Recalling $V^i(t_0) = e^{-rt}N^i(t_0)(p^i_{t_0} - I)$, we infer

$$V^i(t_0) = e^{-rt_0}\left\{p_0(1-I)[1-F^j(t_0)+\frac{1}{2}a] - (1-p_0)Ie^{-\lambda t_0}\left((1-x)\int_0^{t_0}\lambda e^{-\lambda s}[1-F^j(s)]\,ds + e^{-\lambda t_0}[1-F^j(t_0)+\frac{1}{2}a]\right)\right\}$$

In turn, for $\epsilon > 0, V^i(t_0 - \epsilon)$ reads

$$e^{-r(t_0-\epsilon)}\left\{p_0(1-I)[1-F^j(t_0-\epsilon)] - (1-p_0)Ie^{-\lambda(t_0-\epsilon)}\left((1-x)\int_0^{t_0-\epsilon}\lambda e^{-\lambda s}[1-F^j(s)]\,ds + e^{-\lambda(t_0-\epsilon)}[1-F^j(t_0-\epsilon)]\right)\right\}$$

We derive

$$\begin{aligned} V^i(t_0) - \lim_{\epsilon\to 0} V^i(t_0-\epsilon) &= e^{-rt_0}\left(p_0(1-I) - (1-p_0)Ie^{-2\lambda t_0}\right)\left(1 - F^j(t_0) + \frac{1}{2}a - 1 + \lim_{\epsilon\to 0} F^j(t_0-\epsilon)\right) \\ &= -\frac{1}{2}ae^{-rt_0}\left(p_0(1-I) - (1-p_0)Ie^{-2\lambda t_0}\right) \\ &< 0. \end{aligned}$$

The last inequality makes use of the fact that one must have

$$e^{-rt}\left(p_0(1-I) - (1-p_0)Ie^{-2\lambda t_0}\right) > 0.$$

Indeed, since $V^i(t) < e^{-rt}\left(p_0(1-I) - (1-p_0)Ie^{-2\lambda t}\right)$ for all t, i and $x < 1$, if we had $e^{-rt}\left(p_0(1-I) - (1-p_0)Ie^{-2\lambda t_0}\right) \leq 0$, t_0 could not be in the support S_j.

Therefore, if t_0 is an atom of F^j, the objective function of player i has a downward discontinuity at t_0, and it cannot maximize player i's payoff to invest at t_0.

Claim 3: There cannot be an atom at $\underline{t}$ or $\bar{t}$.

Proof: Claim 3 is an immediate corollary from Claims 1 and 2. Let us remark that Claim 3 proves that there cannot be a pure-strategy equilibrium as soon as $x < 1$.

Claim 4: In any equilibrium, both players get an expected payoff of zero.

Proof: To prove the result, we prove that $\bar{t} = +\infty$. Suppose, by way of contradiction, that $\bar{t}$ is finite. Since F^j has no atom at $\bar{t}$, one can write

$$V^i(\bar{t}) = -(1-x)(1-p_0)I\mathrm{e}^{-(r+\lambda)\bar{t}}\int_0^{\bar{t}}\lambda\mathrm{e}^{-\lambda s}[1-F(s)]\,\mathrm{d}s < 0.$$

The equilibrium payoff cannot be negative since the player could secure 0 by never investing. This implies that the support of F^j must unbounded, i.e., $\bar{t} = \infty$. The equilibrium payoff must thus be equal to $\lim_{t\to\infty} V^i(t) = 0$.

In turn, this implies that $\underline{t} = \hat{t}(x)$. Indeed, $V^i(t) < 0$ for any $t < \hat{t}(x)$, and if one had $\underline{t} > \hat{t}(x)$, it would be possible for one player to obtain a strictly positive profit by investing at date between $\hat{t}(x)$ and $\underline{t}$.

Claim 5: For each player, the support of investment dates is an interval.

Proof: Suppose that there exist t_a and t_b with $\hat{t}(x) < t_a < t_b$ such that $t_a \in S_i$ and $(t_a, t_b) \not\subset S_i$. Then for any $t \in (t_a, t_b)$, $F^i(t) = F^i(t_a)$, hence

$$V^j(t) = e^{-rt}[1 - F^i(t_a)] \left(p_0(1-I) - (1-p_0)Ie^{-\lambda t}\left(b + xe^{-\lambda t}\right)\right),$$

where $b \equiv (1-x)\left(1 - \int_0^{t_a} \lambda e^{-\lambda s} \frac{F^i(t_a) - F^i(s)}{1 - F^i(t_a)}\, ds\right) > 0$.

We derive

$$V^{j\prime}(t) = -rV^j(t) + \lambda(1-p_0)Ie^{-(r+\lambda)t}[1 - F^i(t_a)]\left(b + 2xe^{-\lambda t}\right).$$

Since at equilibrium $V^j(t) \le 0$ for all t and j, we derive that $V^{j\prime}(t) > 0$ on $t \in (t_a, t_b)$, which implies that the interval (t_a, t_b) cannot belong to the support S_j either.

In turn (using the same argument), this implies that $V^i(t)$ is increasing on $t \in (t_a, t_b)$.

In addition, $t_a \in S_i$, so $V^i(t_a) = 0$. From Claim 2, $t_a \in S_i$ also implies that F^j cannot have an atom at t_a. Therefore, V^i is continuous at t_a, and there must be a neighbourhood $[t_a, t_a + \epsilon]$ on which V^i is positive, which contradicts that the equilibrium payoff is zero. This implies that the support of each player's startegy is an interval.

Claim 6: For all i, F^i has no atom.

Proof: Suppose that F^i has an atom at t. From the proof of Claim 2, V^j has a downward discontinuity at t. Since V^j can never have an upward discontinuity, this implies that there is an ϵ such that the interval $[t, t+\epsilon]$ does not belong to S_j. From Claim 5, this is impossible.

From all these claims, we conclude that $S_i = S_j = S = [\hat{t}(x), +\infty)$ and F^i is continuous and strictly increasing on S. ■

Lemma 2 *Suppose $x = 1$. In the only equilibrium, both players invest at $t = \hat{t}(1)$.*

Proof: In the case of public news ($x = 1$), each player maximizes

$$V^i(t) = e^{-rt}\left(p_0(1-I) - (1-p_0)Ie^{-2\lambda t}\right)\left(1 - F^j(t)\right).$$

Let us first show that the equilibrium payoff must be 0. Suppose first that $\bar{t}_1 = \bar{t}_2$. In that case, the equilibrium payoff must be $V^i(\bar{t}_i) = 0$, so the result is immediate. Suppose instead that $\bar{t}_i > \bar{t}_j$. This implies that the equilibrium payoff of player i is $V^i(\bar{t}_i) = 0$. Suppose now that the equilibrium payoff of player j is positive. This implies that $\underline{t}_j > \hat{t}(1)$. In that case, player i could secure a positive payoff from investing at a date $t \in [\hat{t}(1), \underline{t}_j]$. A contradiction. This proves that both players must have an equilibrium payoff of 0.

This in turn implies that, for any t that belongs to the support of investment dates, $\left(p_0(1-I) - (1-p_0)Ie^{-2\lambda t}\right)(1 - F^j(t)) = 0$. Suppose $F^j[\hat{t}(1)] < 1$. This implies that there is a value $t > \hat{t}(1)$ such that $F^j(t) < 1$, so that $V^i(t) > 0$. This contradicts the fact that the equilibrium payoff must be zero. So $F^j[\hat{t}(1)] = 1$. Since it is clear that investment never takes place before $\hat{t}(1)$, we derive that a necessary equilibrium condition is that both players invest at $\hat{t}(1)$. It is immediate to see that this is indeed an equilibrium. ■

Let us now derive the equilibrium strategies in the case $x < 1$. From Lemma 1, the equilibrium payoff is zero. Therefore, for any $t \geq \hat{t}(x), p_t^i = I$, which, using (5), can be written

$$e^{-\lambda t}\left(e^{-\lambda t} + (1-x)\int_0^t \lambda e^{-\lambda s}\frac{1-F^j(s)}{1-F^j(t)}\,ds\right) = \frac{p_0}{1-p_0}\frac{1-I}{I}. \tag{A.2}$$

Differentiating (A.2) over $[\hat{t}(x), \infty)$ yields

$$(1-x)e^{-\lambda t}\int_0^t \lambda e^{-\lambda s}\frac{1-F^j(s)}{1-F^j(t)}\,ds\left[\frac{f^j(t)}{1-F^j(t)} - \lambda\right] = (1+x)\lambda e^{-2\lambda t} \tag{A.3}$$

for all $t \geq \hat{t}(x)$, where f^j is the density of F^j. Substituting (A.3) into (A.2) and simplifying, we obtain that, for any such t,

$$\frac{f^j(t)}{1-F^j(t)} = \lambda + \frac{(1+x)(1-p_0)I\lambda e^{-2\lambda t}}{p_0(1-I) - (1-p_0)Ie^{-2\lambda t}}. \tag{A.4}$$

Integrating between $\hat{t}(x)$ and t, we obtain

$$\ln[1 - F^j[\hat{t}(x)]] - \ln[1 - F^j(t)] = \lambda(t - \hat{t}(x)) + \frac{1+x}{2}\ln\frac{p_0(1-I) - (1-p_0)Ie^{-2\lambda t}}{p_0(1-I) - (1-p_0)Ie^{-2\lambda\hat{t}(x)}} \tag{A.5}$$

Using $F^j[\hat{t}(x)] = 0$, we can now conclude that the solution to the problem is given by

$$F^j(t) = 1 - e^{-\lambda[t-\hat{t}(x)]}\left(\frac{p_0(1-I) - (1-p_0)Ie^{-2\lambda\hat{t}(x)}}{p_0(1-I) - (1-p_0)Ie^{-2\lambda t}}\right)^{\frac{1+x}{2}}$$

It is clear that F^j is nondecreasing in t, is worth 0 at $t = \hat{t}(x)$ and goes to 1 as $t \to \infty$, so F^j is indeed a cdf. Since all that precedes holds for all $j = 1, 2$, the equilibrium is symmetric and we denote the solution $F^1 = F^2 = F_x$.

F_x is the only equilibrium in the class of equilibria such that F_x is continuous and strictly increasing over its support. Using Lemma 1, this equilibrium is therefore unique. ■

Proof of Proposition 2. Suppose $x' < x < 1$. From (A.4), we have $\frac{f_x(t)}{1-F_x(t)} > \frac{f_{x'}(t)}{1-F_{x'}(t)}$. Note that this inequality also holds on the interval $[\hat{t}(x), \hat{t}(x')]$ where $f_x(t) > 0 = f_{x'}(t)$. This immediately implies that the investment time under x is smaller than the investment time under x' in the hazard-rate order.

■

Proof of Proposition 4. Let us first remark that the various claims of Lemma 1 equally hold with n players, so at equilibrium all players get 0 and play according to a continuous and increasing cdf. For simplicity, let us focus here on symmetric equilibria where all players follow the same strategy, which we denote $F_{x,n}$.

Let us denote by $V_{x,n}(t)$ the expected payoff from investing at t when all other players follow the strategy $F_{x,n}$. It equals

$$V_{x,n}(t) = e^{-rt}\left\{p_0(1-I)[1-F_{x,n}(t)]^{n-1} - (1-p_0)Ie^{-\lambda t}\left((1-x)\int_0^t \lambda e^{-\lambda s}[1-F_{x,n}(s)]\,ds + e^{-\lambda t}[1-F_{x,n}(t)]\right)^{n-1}\right\}$$

As long as $F_{x,n}(t) = 0$,

$$V_{x,n}(t) = e^{-rt}\left\{p_0(1-I) - (1-p_0)Ie^{-\lambda t}\left((1-x)(1-e^{-\lambda t}) + e^{-\lambda t}\right)^{n-1}\right\}.$$

Therefore, it must be the case that the lower bound of the support is the first date at which the NPV of the project becomes nonnegative, that is, $\hat{t}(x,n)$ such that

$$p_0(1-I) - (1-p_0)I\mathrm{e}^{-\lambda\hat{t}(x,n)}\left[1-x+x\mathrm{e}^{-\lambda\hat{t}(x,n)}\right]^{n-1} = 0$$

In addition, $V_{x,n}(t) = 0$ over $[\hat{t}(x,n), \infty)$, that is,

$$p_0(1-I)[1-F_{x,n}(t)]^{n-1} - (1-p_0)Ie^{-\lambda t}\left((1-x)\int_0^t \lambda e^{-\lambda s}[1-F_{x,n}(s)]\,ds + e^{-\lambda t}[1-F_{x,n}(t)]\right)^{n-1} = 0$$

$$\Leftrightarrow e^{-\lambda t}\left((1-x)\int_0^t \lambda e^{-\lambda s}\frac{1-F_{x,n}(s)}{1-F_{x,n}(t)}\,ds + e^{-\lambda t}\right)^{n-1} = \frac{p_0}{1-p_0}\frac{1-I}{I}. \quad \text{(A.6)}$$

Differentiating (A.6) over $[\hat{t}(x,n), \infty)$ yields

$$(1-x)\int_0^t \lambda e^{-\lambda s}\frac{1-F_{x,n}(s)}{1-F_{x,n}(t)}\,ds\left[(n-1)\frac{f_{x,n}(t)}{1-F_{x,n}(t)} - \lambda\right] = [1+(n-1)x]\lambda e^{-\lambda t} \quad \text{(A.7)}$$

for all $t \geq \hat{t}(x,n)$, where $f_{x,n}$ is the density of $F_{x,n}$. Substituting (A.7) into (A.6) and using $\alpha = \frac{p_0}{1-p_0}\frac{1-I}{I}$, we obtain that, for any such t,

$$(n-1)\frac{f_{x,n}(t)}{1-F_{x,n}(t)} = \lambda + [1+(n-1)x]\frac{\lambda\mathrm{e}^{-\frac{n}{n-1}\lambda t}}{\alpha^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda t}}$$

Let us remark at this stage that, if $x > x'$, we can derive from the previous equality that $\frac{f_{x,n}(t)}{1-F_{x,n}(t)} > \frac{f_{x',n}(t)}{1-F_{x',n}(t)}$, which implies that the investment date under x is smaller than the investment time under x' in the hazard-rate order.

Finally, remarking that $\int_{\hat{t}(x,n)}^{t} \frac{\lambda \mathrm{e}^{-\frac{n}{n-1}\lambda s}}{\alpha^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda s}} ds = \frac{n-1}{n} \ln \frac{\alpha^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda t}}{\alpha^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda \hat{t}(x,n)}}$, and using $F[\hat{t}(x,n)] = 0$, we conclude

$$F_{x,n}(t) = 1 - \mathrm{e}^{-\frac{1}{n-1}\lambda[t-\hat{t}(x,n)]} \left(\frac{\alpha^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda \hat{t}(x,n)}}{\alpha^{\frac{1}{n-1}} - \mathrm{e}^{-\frac{n}{n-1}\lambda t}} \right)^{\frac{1+(n-1)x}{n}} \tag{A.8}$$

If $x < 1$, the expected welfare $W(x,n)$ equals

$$\begin{aligned} & p_0(1 - I + \Delta_S) \int_{\hat{t}(x,n)}^{+\infty} n e^{-rt} \left[1 - F_{x,n}(t)\right]^{n-1} f_{x,n}(t)\, dt \\ - \quad & (1-p_0)(I + \Delta_F) \int_{\hat{t}(x,n)}^{+\infty} n e^{-(r+\lambda)t} \left((1-x) \int_0^t \lambda e^{-\lambda s}[1 - F_{x,n}(s)]\, ds + e^{-\lambda t}[1 - F_{x,n}(t)] \right)^{n-1} f_{x,n}(t)\, dt \end{aligned}$$

Using the fact that $V_{x,n}(t) = 0$ for all $t \geq \hat{t}(x,n)$, we derive:

$$W(x,n) = p_0 \left(\Delta_S - \Delta_F \frac{1-I}{I} \right) \int_{\hat{t}(x,n)}^{+\infty} n e^{-rt}[1 - F_{x,n}(t)]^{n-1} f_{x,n}(t)\, dt \tag{A.9}$$

$\int_{\hat{t}(x,n)}^{+\infty} n e^{-rt}[1 - F_{x,n}(t)]^{n-1} f_{x,n}(t)\, dt$ is the value at 0 of receiving 1 at the first date at which investment takes place conditional on the project being good. Suppose $x' < x < 1$. Since the investment date under x is smaller than the investment time under x' in the hazard-rate order, we have

$$\int_{\hat{t}(x,n)}^{+\infty} n e^{-rt}[1 - F_{x,n}(t)]^{n-1} f_{x,n}(t)\, dt > \int_{\hat{t}(x',n)}^{+\infty} n e^{-rt}[1 - F_{x',n}(t)]^{n-1} f_{x',n}(t)\, dt.$$

If $x = 1$,

$$\begin{aligned} W(1,n) &= e^{-r\hat{t}(1,n)} \left(p_0(1 - I + \Delta_S) - (1-p_0)(I + \Delta_F) e^{-\lambda \hat{t}(1,n)} \right) \\ &= p_0 \left(\Delta_S - \Delta_F \frac{1-I}{I} \right) e^{-r\hat{t}(1,n)}. \end{aligned}$$

Since $\hat{t}(1,n) < \hat{t}(x,n)$ for all $x < 1$, we conclude from all the above that

$$W(x,n) > W(x',n) \Leftrightarrow \frac{\Delta_S}{\Delta_F} > \frac{1-I}{I}.$$

$\blacksquare$

Proof of Proposition 5. Let us first start with the case of public signals ($x = 1$). As we have seen,

$$W(1,n) = p_0(\Delta_S - \Delta_F \frac{1-I}{I})e^{-r\hat{t}(1,n)}$$

Since $\hat{t}(1,n)$ decreases in n, $W(1,n)$ is increasing in n if $\Delta_S \geq \Delta_F \frac{1-I}{I}$ and decreasing otherwise.

Let us now turn to the case of private signals ($x = 0$). One checks that $\hat{t}(0,n)$ is such that $p_0(1-I) = (1-p_0)Ie^{-\lambda\hat{t}(0,n)}$, that is, $\hat{t}(0,n) = \hat{t}(0)$ for all n.

$$W(0,n) = p_0\left(\Delta_S - \Delta_F \frac{1-I}{I}\right)\int_{\hat{t}(0)}^{+\infty} ne^{-rt}[1-F_{0,n}(t)]^{n-1}f_{0,n}(t)\,dt \tag{A.10}$$

Integrating by parts, one rewrites the integral as

$$e^{-r\hat{t}(0)} - r\int_{\hat{t}(0)}^{+\infty} e^{-rt}[1-F_{0,n}(t)]^n\,dt \tag{A.11}$$

Using (A.8), we derive

$$\begin{aligned} [1-F_{0,n}(t)]^n &= e^{-\lambda\frac{n}{n-1}(t-\hat{t}(0))}\frac{\alpha^{\frac{1}{n-1}} - e^{-\lambda\frac{n}{n-1}\hat{t}(0)}}{\alpha^{\frac{1}{n-1}} - e^{-\lambda\frac{n}{n-1}t}} \\ &= \frac{\alpha^{\frac{1}{n-1}}e^{\lambda\frac{n}{n-1}\hat{t}(0)} - 1}{\alpha^{\frac{1}{n-1}}e^{\lambda\frac{n}{n-1}t} - 1} \\ &= \frac{\frac{1}{\alpha} - 1}{\alpha^{\frac{1}{n-1}}e^{\lambda\frac{n}{n-1}t} - 1} \end{aligned}$$

Differentiating the denominator with respect to n, one gets

$$\frac{1}{(n-1)^2}\alpha^{\frac{1}{n-1}}e^{\lambda\frac{n}{n-1}t}\left(-\lambda t - \ln\alpha\right).$$

Since $t \geq \hat{t}(0)$, we must have $-\lambda t \leq -\lambda\hat{t}(0) = \ln\alpha$. This implies that $[1-F_{0,n}(t)]^n$ is increasing in n. We can now conclude that $W(0,n)$ is decreasing in n if $\Delta_S \geq \Delta_F\frac{1-I}{I}$ and increasing otherwise. ■

Proof of Proposition 6 Let us first consider the situation where c is positive and signals are public. Suppose that j invests at $t_1(L)$ whenever no one observes a failure. The expected payoff from investing at t for i is thus

$$\begin{cases} e^{-rt}\left(p_0(1-I) - (1-p_0)Ie^{-2\lambda t}\right) + (1-p_0)c\int_0^t \lambda e^{-(r+2\lambda)s}\,ds & \text{if } t < t_1(L) \\ e^{-rt_1(L)}\left(\frac{p_0(1-I)-(1-p_0)Ie^{-2\lambda t_1(L)}}{2} + \frac{p_0 L}{2}\right) + (1-p_0)c\int_0^{t_1(L)} \lambda e^{-(r+2\lambda)s}\,ds & \text{if } t = t_1(L) \\ e^{-rt}p_0 L + (1-p_0)c\int_0^{t_1(L)} \lambda e^{-(r+2\lambda)s}\,ds & \text{if } t > t_1(L) \end{cases}$$

Using the definition of $t_1(L)$, this expected payoff is continuous.

One can check that $e^{-rt}\left(p_0(1-I)-(1-p_0)Ie^{-2\lambda t}\right)$ is single-peaked in t and increasing at $t=t_1(L)$, so we can infer that $e^{-rt}\left(p_0(1-I)-(1-p_0)Ie^{-2\lambda t}\right)+(1-p_0)c\int_0^t \lambda e^{-(r+2\lambda)s}\,ds$ is a fortiori increasing on $[0,t_1(L)]$. We can conclude that investing at $t_1(L)$ maximizes the expected payoff of player i, so such a pure-strategy equilibrium exists.

Let us now turn to the case where $c=0$ and signals are private. We look for a *regular symmetric equilibrium* in which the common distribution of investment dates $\tilde{F}$ conditional on observing neither a failure nor one's rival's investing is continuous over $[0,\infty)$ and strictly increasing and differentiable over the interior of its supporting interval. Let $[\underline{t},\overline{t}]$ denote the support. In such an equilibrium, one should have for all $t\in[\underline{t},\overline{t}]$:

$$e^{-rt}\left\{p_0(1-I)[1-\tilde{F}(t)]+p_0L\tilde{F}(t)-(1-p_0)Ie^{-\lambda t}\left(\int_0^t \lambda e^{-\lambda s}[1-\tilde{F}(s)]\,ds+e^{-\lambda t}[1-\tilde{F}(t)]\right)\right\}=\tilde{V}, \tag{A.12}$$

where $\tilde{V}$ denote the equilibrium payoff.[27] Differentiating with respect to t yields

$$-r\left\{p_0(1-I)[1-\tilde{F}(t)]+p_0L\tilde{F}(t)-(1-p_0)Ie^{-\lambda t}\left(\int_0^t \lambda e^{-\lambda s}[1-\tilde{F}(s)]\,ds+e^{-\lambda t}[1-\tilde{F}(t)]\right)\right\}$$

$$-p_0(1-I)\tilde{F}'(t)+p_0L\tilde{F}'(t)+\lambda(1-p_0)Ie^{-\lambda t}\left(\int_0^t \lambda e^{-\lambda s}[1-\tilde{F}(s)]\,ds+e^{-\lambda t}[1-\tilde{F}(t)]\right)+(1-p_0)Ie^{-2\lambda t}\tilde{F}'(t)=0$$

Using (A.12), one can rewrite this as

$$\left(p_0(1-I-L)-(1-p_0)e^{-2\lambda t}I\right)\tilde{F}'(t)=\lambda p_0(1-I-L)[1-\tilde{F}(t)]+\lambda p_0L-(\lambda+r)e^{rt}\tilde{V} \tag{A.13}$$

Before going further, let us derive the following lemma, which provides conditions on $\underline{t}$ and $\overline{t}$.

Lemma 3 *$\underline{t}$ and $\overline{t}$ must satisfy the following conditions:*

1. $\underline{t}>t_1(L)$
2. $\overline{t}=h(\underline{t})\equiv\frac{1}{r}\ln\frac{\lambda p_0L}{(\lambda+r)V^m(\underline{t})}$

[27] Notice that, in this formulation, we implicitly assume that the second-mover obtains the payoff of L at the date at which his strategy prescribes him to invest, and not at the date at which the first mover actually invests. This allows to find a solution to the problem, which would be impossible otherwise. If L were obtained at the date of the first investment, incentives to invest later would be increased, so that the main result we underline, namely that investment takes place later with private signals, would be even reinforced. Note that we made a similar assumption in deriving the pure-strategy equilibrium in the case $x=1$ for consistency, but in that case it is irrelevant.

Proof: Let us first remark that, as long as $t < \underline{t}$, the expected payoff of a player equals the single-player payoff $V^m(t)$, which implies that $\tilde{V} = V^m(\underline{t})$. To show the first part, let us take (A.13) at $t = \underline{t}$, which gives

$$\begin{aligned}\left(p_0(1-I-L) - (1-p_0)e^{-2\lambda \underline{t}} I\right) \tilde{F}'(\underline{t}) &= -rp_0(1-I) + (\lambda + r)(1-p_0)Ie^{-\lambda \underline{t}} \\ &= e^{r\underline{t}} V^{m\prime}(\underline{t})\end{aligned}$$

A necessary equilibrium condition is $\underline{t} \leq t^m$. Otherwise, investing at t^m would yield strictly more than investing at $\underline{t}$. This implies $V^{m\prime}(\underline{t}) > 0$.

Hence, $\tilde{F}'(\underline{t}) \geq 0 \implies p_0(1-I-L) - (1-p_0)e^{-2\lambda \underline{t}} I \geq 0$, that is, $\underline{t} \geq t_1(L)$.

To show the second point, let us take (A.13) at $t = \bar{t}$. This gives

$$\left(p_0(1-I-L) - (1-p_0)e^{-2\lambda \bar{t}} I\right) \tilde{F}'(\bar{t}) = \lambda p_0 L - (\lambda + r)e^{r\bar{t}} \tilde{V} \tag{A.14}$$

Since $\bar{t} \geq \underline{t} \geq t_1(L)$, $\tilde{F}'(\bar{t}) \geq 0$ imposes

$$\lambda p_0 L - (\lambda + r)e^{r\bar{t}} \tilde{V} \geq 0 \tag{A.15}$$

Now let us consider the objective function of a player for $t \geq \bar{t}$. It is equal to

$$e^{-rt}\left(p_0 L - (1-p_0)Ie^{-\lambda t} \int_0^{\bar{t}} \lambda e^{-\lambda s}[1 - \tilde{F}(s)]\, ds\right)$$

Since we are looking for a regular equilibrium where the objective function is continuous, an equilibrium condition is that it is locally decreasing in the right neighbourhood of $\bar{t}$ for otherwise $\bar{t}$ could not be payoff-maximizing. This reads

$$(\lambda + r)(1-p_0)Ie^{-(r+\lambda)\bar{t}} \int_0^{\bar{t}} \lambda e^{-\lambda s}[1 - \tilde{F}(s)]\, ds - rp_0 L e^{-r\bar{t}} \leq 0$$

But since by definition the objective function is worth $\tilde{V}$ at $\bar{t}$, we can write

$$(1-p_0)Ie^{-(r+\lambda)\bar{t}} \int_0^{\bar{t}} \lambda e^{-\lambda s}[1 - \tilde{F}(s)]\, ds = e^{-r\bar{t}} p_0 L - \tilde{V}$$

Accordingly, we derive from the two previous equations

$$\lambda p_0 L - (\lambda + r)e^{r\bar{t}} \tilde{V} \leq 0 \tag{A.16}$$

Taking together (A.15) and (A.16), we derive

$$\lambda p_0 L - (\lambda + r)e^{r\bar{t}} \tilde{V} = 0 \tag{A.17}$$

Notice that (A.17) is equivalent to $\tilde{F}'(\bar{t}) = 0$.

Using $\tilde{V} = V^m(\underline{t})$ we derive the second point.

In order for $\bar{t}$ to be actually defined by $h(\underline{t})$, one needs to make sure that $h(\underline{t}) \geq \underline{t}$. It is easy to see that h is decreasing. Since $h[\hat{t}(0)] = \infty$ and $h(t^m) = \frac{1}{r}\ln\frac{L}{1-I} + t^m < t^m$, there exists a cutoff date t^c such that $h(t^c) = t^c$. Since we must have $\underline{t} > t_1(L)$, we also need to check that $t_1(L) < t^c$. This inequality holds true if $t_1(L) - h[t_1(L)] < 0 \Leftrightarrow (\lambda + r)\left(p_0(1-I) - \sqrt{p_0(1-p_0)I(1-I-L)}\right) - \lambda p_0 L < 0$. This function is either increasing or U-shaped, and negative at $L = 0$ so there exists a value L_{max} such that $t_1(L) \leq h[t_1(L)]$ for all $L \leq L_{max}$. We assume henceforth that $L \leq L_{max}$. ■

Let $b(t) \equiv \frac{\lambda p_0(1-I-L)}{p_0(1-I-L)-(1-p_0)Ie^{-2\lambda t}}$ and let $C(t)$ be the function such that

$$1 - \tilde{F}(t) = C(t)e^{-\int_{\underline{t}}^{t} b(s)\,ds} \tag{A.18}$$

Differentating (A.18) wrt t yields

$$-\tilde{F}'(t) = C'(t)e^{-\int_{\underline{t}}^{t} b(s)\,ds} - [1 - \tilde{F}(t)]b(t)$$

Using (A.13), one derives that

$$C'(t)e^{-\int_{\underline{t}}^{t} b(s)\,ds} = -\frac{\lambda p_0 L - (\lambda + r)e^{rt}\tilde{V}}{p_0(1-I-L) - (1-p_0)e^{-2\lambda t}I}$$

This gives

$$C(t) = C(\underline{t}) - \int_{\underline{t}}^{t} \frac{\lambda p_0 L - (\lambda + r)e^{rz}\tilde{V}}{p_0(1-I-L) - (1-p_0)e^{-2\lambda z}I} e^{\int_{\underline{t}}^{z} b(s)\,ds}\,dz$$

From (A.18), it is clear that $\tilde{F}(\underline{t}) = 0 \implies C(\underline{t}) = 1$.
It follows that

$$1 - \tilde{F}(t) = \left(1 - \int_{\underline{t}}^{t} \frac{\lambda p_0 L - (\lambda + r)e^{rz}\tilde{V}}{p_0(1-I-L) - (1-p_0)e^{-2\lambda z}I} e^{\int_{\underline{t}}^{z} b(s)\,ds}\,dz\right) e^{-\int_{\underline{t}}^{t} b(s)\,ds} \tag{A.19}$$

Recalling $\tilde{V} = V^m(\underline{t})$, and using $F(\bar{t}) = 1$, we conclude that $\underline{t}$ is determined by the solution of the following equation:

$$g(\underline{t}) \equiv \int_{\underline{t}}^{h(\underline{t})} \frac{\lambda p_0 L - (\lambda + r)e^{rz}V^m(\underline{t})}{p_0(1-I-L) - (1-p_0)e^{-2\lambda z}I} e^{\int_{\underline{t}}^{z} b(s)\,ds}\,dz = 1 \tag{A.20}$$

Differentiating g, we derive

$$g'(t) = b(t) - V^{m'}(t)\left(\frac{e^{rt}}{p_0(1-I-L)-(1-p_0)Ie^{-2\lambda t}} + \int_t^{h(t)} \frac{(\lambda+r)e^{rz}e^{\int_t^z b(s)\,ds}\,dz}{p_0(1-I-L)-(1-p_0)Ie^{-2\lambda z}}\right) - b(t)g(t)$$

At the solution $\underline{t}$, we have $g(\underline{t}) = 1$, which implies

$$g'(\underline{t}) = -V^{m'}(\underline{t})\left(\frac{e^{r\underline{t}}}{p_0(1-I-L)-(1-p_0)Ie^{-2\lambda\underline{t}}} + \int_{\underline{t}}^{h(\underline{t})} \frac{(\lambda+r)e^{rz}e^{\int_{\underline{t}}^z b(s)\,ds}\,dz}{p_0(1-I-L)-(1-p_0)Ie^{-2\lambda z}}\right)$$

Since we have seen that $\underline{t} < t^m$, we know that $V^{m'}(\underline{t}) > 0$, which implies that $g'(\underline{t}) < 0$. Since g is continuous in t, this implies that if g has a root, it is unique.

Let us show that there is a root in $[t_1(L), t^c]$.

Remarking that

$$\int_{\underline{t}}^{z} b(s)ds = \frac{1}{2}\ln\frac{(1-p_0)I - p_0(1-I-L)e^{2\lambda z}}{(1-p_0)I - p_0(1-I-L)e^{2\lambda\underline{t}}} \tag{A.21}$$

one can rewrite

$$g(t) = \frac{1}{\sqrt{p_0(1-I-L)-(1-p_0)e^{-2\lambda t}I}}\int_t^{h(t)} \frac{\lambda p_0 L - (\lambda+r)e^{rz}V^m(t)}{\sqrt{p_0(1-I-L)-(1-p_0)e^{-2\lambda z}I}} e^{\lambda(z-t)}\,dz \tag{A.22}$$

This implies that $g[t_1(L)] = \infty$. In turn, $g(t^c) = 0$. Because g is continuous, there exists a root to $g(t) = 1$ in $[t_1(L), t^c]$.

It is clear that from (A.19) that $\tilde{F}$ is nondecreasing in t. By construction, it is worth 0 at $t = \underline{t}$ and to 1 at $\bar{t}$ so $\tilde{F}$ is indeed a cdf.

Notice that, rearranging (A.19), and relabelling $\underline{t} = \underline{t}(L)$, $\tilde{F}$ can be rewritten

$$\begin{aligned}\tilde{F}(t) &= 1 - e^{-\lambda[t-\underline{t}(L)]}\sqrt{\frac{p_0(1-I-L)-(1-p_0)Ie^{-2\lambda\underline{t}(L)}}{p_0(1-I-L)-(1-p_0)Ie^{-2\lambda t}}} \\ &+ \frac{e^{-\lambda t}}{\sqrt{p_0(1-I-L)-(1-p_0)Ie^{-2\lambda t}}}\int_{\underline{t}(L)}^{t} \frac{\lambda p_0 L e^{\lambda z} - (\lambda+r)e^{(r+\lambda)z}V^m[\underline{t}(L)]}{\sqrt{p_0(1-I-L)-(1-p))Ie^{-2\lambda z}}}\,dz\end{aligned} \tag{A.23}$$

■

Proof of Proposition 7 That investment takes place sooner with public news for all c is a direct consequence of $t_1(L) < \underline{t}(L)$, which is proven in Lemma 3 above.

Let us now compare equilibrium payoffs. In the public news case, the equilibrium payoff is given by

$$e^{-rt_1(L)}p_0 L + (1-p_0)c\frac{\lambda}{r+2\lambda}\left(1 - e^{-(r+2\lambda)t_1(L)}\right),$$

In the private news case, using (A.17), the equilibrium payoff $\tilde{V}$ equals

$$\frac{\lambda}{\lambda+r}p_0Le^{-r\bar{t}(L)}.$$

Since $\bar{t}(L) \geq \underline{t}(L) \geq t_1(L)$, we derive

$$\begin{aligned}\frac{\lambda}{\lambda+r}p_0Le^{-r\bar{t}(L)} &\leq \frac{\lambda}{\lambda+r}p_0Le^{-rt_1(L)} \\ &\leq p_0Le^{-rt_1(L)} \\ &\leq p_0Le^{-rt_1(L)} + (1-p_0)c\frac{\lambda}{r+2\lambda}\left(1-e^{-(r+2\lambda)t_1(L)}\right).\end{aligned}$$

Therefore, for any c, the equilibrium payoff is larger with public signals.

Let us now compare the NPV of investment in the two regimes. With public signals, beliefs at equilibrium are

$$\frac{p_0}{p_0+(1-p_0)e^{-2\lambda t_1(L)}} = \frac{I}{1-L}, \tag{A.24}$$

using the definition of $t_1(L)$.

With private signals, using (5), beliefs are given by

$$\frac{p_0}{p_0+(1-p_0)e^{-\lambda t}\left(e^{-\lambda t}+\int_0^t \lambda \mathrm{e}^{-\lambda s}\frac{1-\tilde{F}(s)}{1-\tilde{F}(t)}\,\mathrm{d}s\right)} \tag{A.25}$$

The difference (A.25)-(A.24) has the same sign as

$$p_0(1-I-L)-(1-p_0)Ie^{-\lambda t}\left(e^{-\lambda t}+\int_0^t \lambda \mathrm{e}^{-\lambda s}\frac{1-\tilde{F}(s)}{1-\tilde{F}(t)}\,\mathrm{d}s\right)$$

From (A.12), this is equal to $\frac{e^{rt}\tilde{V}-p_0L}{1-\tilde{F}(t)}$.
In turn, using (A.17), this equals $\frac{p_0L}{1-\tilde{F}(t)}\left(\frac{\lambda}{\lambda+r}e^{r(t-\bar{t}(L))}-1\right) \leq 0$.
This proves that the NPV is larger with public signals.

Let us now turn to the comparison of the planner's welfare across regimes.
With public signals, the welfare of the planner reads

$$\begin{aligned}& e^{-rt_1(L)}\left(p_0(1-I+\Delta_S)-(1-p_0)e^{-2\lambda t_1(L)}(I+\Delta_F)\right) \\ =\;& p_0e^{-rt_1(L)}\left(\Delta_S-\frac{1-I}{I}\Delta_F+(1+\frac{\Delta_F}{I})L\right) \\ \equiv\;& \tilde{W}(1,L).\end{aligned}$$

Note that the planner's welfare does not depend on c since $t_1(L)$ is independent of c.

Let $\tilde{W}(0,L)$ denote the welfare of the planner when signals are private ($x = c = 0$). It equals

$$p_0(1-I+\Delta_S)\int_{\underline{t}(L)}^{\bar{t}(L)} 2e^{-rt}[1-\tilde{F}(t)]\tilde{f}(t)\,dt - (1-p_0)(I+\Delta_F)\int_{\underline{t}(L)}^{\bar{t}(L)} 2e^{-(r+\lambda)t}\left(\int_0^t \lambda e^{-\lambda s}[1-\tilde{F}(s)]\,ds + e^{-\lambda t}[1-\tilde{F}(t)]\right)\tilde{f}(t)\,dt$$

Using (A.12), we can rewrite it

$$\begin{aligned}\tilde{W}(0,L) &= p_0(1-I+\Delta_S)\int_{\underline{t}(L)}^{\bar{t}(L)} 2e^{-rt}[1-\tilde{F}(t)]\tilde{f}(t)\,dt - \frac{I+\Delta_F}{I}\int_{\underline{t}(L)}^{\bar{t}(L)} 2\left(e^{-rt}[p_0(1-I-L)(1-\tilde{F}(t))+p_0L]-\tilde{V}\right)\tilde{f}(t)\,dt\\ &= p_0\left(\Delta_S - \frac{\Delta_F}{I}(1-I)+(1+\frac{\Delta_F}{I})L\right)\int_{\underline{t}(L)}^{\bar{t}(L)} 2e^{-rt}[1-\tilde{F}(t)]\tilde{f}(t)\,dt - 2(1+\frac{\Delta_F}{I})\left(p_0L\int_{\underline{t}(L)}^{\bar{t}(L)} e^{-rt}\tilde{f}(t)\,dt-\tilde{V}\right)\end{aligned}$$

Let us write the difference in the planner's welfare in the two situations:

$$\begin{aligned}\tilde{W}(1,L)-\tilde{W}(0,L) &= p_0\left(\Delta_S - \frac{1-I}{I}\Delta_F + (1+\frac{\Delta_F}{I})L)\right)\left(e^{-rt_1(L)} - \int_{\underline{t}(L)}^{\bar{t}(L)} 2e^{-rt}[1-\tilde{F}(t)]\tilde{f}(t)\,dt\right)\\ &+ 2(1+\frac{\Delta_F}{I})\left(p_0L\int_{\underline{t}(L)}^{\bar{t}(L)} e^{-rt}\tilde{f}(t)\,dt - \tilde{V}\right).\end{aligned}$$

Since investment takes place earlier when signals are public, i.e., $t_1(L) < \underline{t}(L)$, we must have

$$e^{-rt_1(L)} > \int_{\underline{t}(L)}^{\bar{t}(L)} 2e^{-rt}[1-\tilde{F}(t)]\tilde{f}(t)\,dt.$$

We derive that $\tilde{W}(1,L)-\tilde{W}(0,L)$ is increasing in Δ_S. This implies that there exists a cutoff $\overline{\Delta}$ such that

$$\tilde{W}(1,L) \geq \tilde{W}(0,L) \Leftrightarrow \Delta_S \geq \overline{\Delta}.$$

In addition, using (A.17), we note that

$$\tilde{V} = \frac{\lambda}{\lambda+r}p_0Le^{-r\bar{t}(L)} \leq \frac{\lambda}{\lambda+r}p_0L\int_{\underline{t}(L)}^{\bar{t}(L)} e^{-rt}\tilde{f}(t)dt \leq p_0L\int_{\underline{t}(L)}^{\bar{t}(L)} e^{-rt}\tilde{f}(t)dt.$$

This implies that $\overline{\Delta} < \frac{1-I}{I}\Delta_F - (1+\frac{\Delta_F}{I})L$.

■

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