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**Abstract**

The producers of electricity using dispatchable plants rely on partially flexible technologies to match the variability of demand and intermittent renewables. We analyse flexibility in a two-stage decision process where production decided at the last moment is more costly than if it is planned in advance. We first determine the first best outputs, prices and gains. We then consider a model where two partially flexible firms compete in quantities to supply a random residual demand. We determine the subgame perfect equilibria corresponding to two market designs: one where all trade occurs in a spot market with known demand, the other where a day-ahead market with random demand is added to the ex-post market, first in a general setting, then using a quadratic specification. We show that when all trade occurs ex post, the least flexible firm is not necessarily disadvantaged. We also show that adding a day-ahead market makes consumers better off and firms worse off by increasing total output. It increases welfare but it also transfers risks from firms to consumers.

JEL codes: C72, D24, D47, L23, L94

Key words: flexibility, electricity, market design, production costs, risk transfer.

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# 1 Introduction

As long as electricity energy cannot be stored at large scale, the equilibrium between production and consumption must be reached in real time. This would be a simple routine if demand was not permanently varying, following predictable cycles (e.g. day-night) and random events (e.g. temperature variations). Moreover, the deployment of intermittent sources of renewable energy (solar, wind) increases the randomness of the residual demand that must be served by dispatchable plants (coal, gas, hydro, nuclear). Buyers' price-responsiveness would ease the balancing of electricity markets but it cannot be a general solution as long as smart meters and appliances are not massively deployed and consumers cannot instantaneously adapt their behavior. The drastic solution of energy blackouts is politically unacceptable in developed countries. Then, under the severe conditions of i) no storage, ii) no demand rationing and iii) no state-dependent pricing, the only way to accomodate variations in residual demand would be to benefit from perfectly flexible technologies able to follow demand in real time. There exist some cases of supply and demand varying in time in a balanced way: it is so in regions where solar energy simultaneously determines the electricity supply from photovoltaïc panels and the demand for air-conditioning. However, cases of perfect positive correlations are the exception. Renewables rather add uncertainty on the exact quantity of residual energy to supply at each moment. The task to match the demand not served by undispatchable renewables is mainly devoted to hydroelectric reservoirs that can instantaneously increase or decrease their output at zero operation cost, and complementarily to less flexible thermal plants that incur fixed starting and stopping costs, plus additional costs for ramping up and down in the very short run (Kök et al., 2020). The flexibility question is often addressed at the electric system level rather than within firms. Investing in gas-fired power plants or flexible CCS plants (Bertsch et al 2016) and energy storage (Bistline, 2017) provides global flexibility as a by-product.

In this paper, we consider the case of production plants that are not fully flexible by assuming that the cost to produce 1kWh is increasing when the time lag to do so is shorter. Our approach differs from what Boyer and Moreaux (1997) call 'technological flexibility' where firms have to make a choice among different equipment, which resuts in different cost configurations. The problem we address is 'flexibility in timing' where costs depend on the decision date. Our analysis belongs to the same strand of literature as Eisenack and Mier (2018). However, contrary to what they do, we do not separate from scratch firms that can only plan their production day ahead from those that can adjust their output in real time. We rather assume like in Crampes and Renault (2019) that each firm can do both, but at different costs. Because our focus is on the supply side characteristics, we assume that all consumers can react to price signals, which excludes any type of rationing as shown in Joskow and Tirole (2007) and Léautier (2019).

Our analysis has common features with the literature on market power in sequential markets (Allaz and Vila ,1993; Ito and Reguant 2016), but with an emphasis on the cost specificities. The question we address is how firms exerting some market power adapt their strategies when their technologies are partially flexible and the demand to serve is both random and price responsive. A correlated question is how the two-market structure that is the standard in most liberalized countries (day-ahead commitment followed by intra-day adjustment) affects the competitors' strategies. Crampes and Renault (2019) show that, when all agents are price-takers and risk-neutral, making competition in the wholesale market efficient given demand uncertainty does not necessitate a day-ahead market. By contrast, when producers have some market power, trading only on ex-post markets or on a combination of ex-ante and ex-post markets is not the same.[1] In this paper, we discuss the elements that determine which market design is the most socially efficient in a Cournot duopolistic structure framework.

The question is sensible in terms of competition policy. Indeed, in the energy field, competition authorities face questions such as "Are units inflexible because they are old and inefficient, because owners have not invested in increased flexibility or because they serve as a mechanism for the exercise of market power?"[2] Our model provides some intuitions that help identifying strategic uses of (in)flexibility in timing. Note also that in the dual market structure, firms that bid in both markets are *de facto* multiproduct producers. Then, one can wonder whether it is possible to use one of the markets as a leverage to exert market power in the other.

The paper is organized as follows. In Section 2 we present the general hypotheses on demand and production, and on the timing of the game. We also specify a quadratic surplus function and a quadratic cost function that will illustrate some of the results. Section 3 presents the basic trade-off between the extra cost of producing a given quantity with little anticipation and the benefits of a better knowledge of the target, first when there is only one production plant, then when production can be allocated to two plants. In Section 4, we switch to the analysis of imperfect competition with a duopoly that can be either symmetric or asymmetric in terms of production cost. In particular, we study how benefits and risks are re-allocated between producers and consumers when the ex-post spot market is complemented by a day-ahead market. We conclude in Section 5.

# 2 Hypotheses

We consider $n$ firms competing to supply residual demand for electricity, that is the demand not served by undispatchable energies like wind, solar and along-the-

[1] Using data from the German market, Goutte and Vassilopoulos (2019) show that the volatility of short term prices provides an additional revenue to the flexible resources able to react quickly as real-time approaches.

[2] Page 104 section 3 in "2018 Quarterly State of the Market Report for PJM", http://www.monitoringanalytics.com/reports/PJM_State_of_the_Market/2018/2018q3-som-pjm-sec3.pdf

river power. Since both supply by renewables and final demand are random, the residual demand is random. Let $S(x,z)$ denote the gross surplus of consumers and green producers, where $z$ is a random variable with expectation $\mathbb{E}(z) = E$ and variance $V = \mathbb{E}(z^2) - E^2$. In the following, we will refer to $z$ as the 'willingness-to-pay' of consumers or the 'market size'. The function $S(x,z)$ is increasing and concave in $x$ and, after reordering, increasing in $z$. Consumers and intermittent producers are price-takers. Because we focus on the partial flexibility of dispatchable producers, we assume that the residual demand is reactive to price changes. Given the exogeneous price $p$ and the state of nature $z$, the residual demand in inverse form is $p(x,z)$.[3] Given the properties of the surplus function, $p(x,z)$ is decreasing in $x$ and increasing in $z$.

Firms can bid in two stages. Day ahead, before knowing the value of $z$, each firm $i$ can bid $Q_i \geq 0$. Later, firm $i$ can bid $q_i \gtreqless 0$ knowing $z$ (as long as $q_i + Q_i \geq 0$) and its competitors' commitment $Q_{-i}$. The cost function is $C_i(Q_i, q_i)$, increasing with its two arguments, and convex. Since delaying decisions until knowing the true state of nature reduces the lag between the decision time and its implementation, the second stage output is more costly than the quantity planned initially. Then $C_i(x,0) < C_i(0,x)$ for any $x > 0$ in each production plant $i$.

In the following, we will write $Q = Q_i + Q_{-i}$, $q = q_i + q_{-i}$, $\vec{Q} = (Q_1, ..., Q_n)$, and $\vec{q} = (q_1, ..., q_n)$.

To illustrate our results, we will use the quadratic specifications

$$\begin{aligned} S(x,z) &= (z - x/2)x \\ C_i(Q_i, q_i) &= (Q_i + q_i)^2 + a_i q_i^2, \ i = 1, ..., n \end{aligned} \tag{1}$$

where $a_i \geq 0$ is the index of (in)flexibility. The quadratic form of the cost function has continuity characteristics that make it easier to handle than the multilinear specification used in Crampes and Renault (2019). Having quadratic specifications for both utility and costs has the advantage to provide explicit results in terms of the average value $E$ and the variance $V$ of the random component of demand without having to specify a distribution of probability. The main drawback is that there is no room for dissymetry and higher statistical characteristics of the random shock in the equilibrium quantities and prices. However, higher statistical moments show up in the variance of equilibrium profits and surplus so that they play a role to explain how market design transfers risks from producers to consumers.

[3]If consumers are perfectly reactive, for each observed pair $(p,z)$, they solve $\max_x S(x,z) - px$. Then, the inverse demand function in state $z$, $p(x,z)$, is derived from the first order condition $\frac{\partial S}{\partial x}(x,z) = p$. Imperfect response can be represented by a parameter $\alpha < 1$ such that the FOC is $\frac{\partial S}{\partial x}(x,z) = \alpha p$. For each $z$, the resulting residual demand curve is located above the optimal one. The lower $\alpha$, the steeper the demand curve. In the extreme case where $\alpha = 0$, the demand curve in state $z$ is a vertical line: consumers are unable to change their consumption when the price of energy varies.

# 3 Wait-and-see gains and costs

The trade-off between the informational benefit of delayed decisions and the extra cost due to shorter delays can be analyzed by maximizing the expected welfare function taking into account the possibility to fix the adjusted production after $z$ is known. Notice that maximising the expected welfare, that is the expected difference between the consumers' surplus and the production cost, implicitly assumes that consumers are risk-neutral to monetary transfers. To facilitate the identification of the elements of the trade-off, in subsection 3.1 we assume that there is one single production plant and in subsection 3.2 we add a second plant.

## 3.1 One production plant

Assume first that all production is coming from one plant. The first-best problem is

$$\max_Q \mathbb{E}_z \max_q W(Q, q, z)$$

where $W(Q, q, z) \overset{def}{=} S(Q + q, z) - C(Q, q)$.

**Ex post.** Upon observing $z$ and knowing the quantity $Q$ already planned, the first order condition of $\max_q S(Q + q, z) - C(Q, q)$ is

$$S'(Q + q, z) = C'_q(Q, q) \tag{2}$$

where $S'(x, z) \overset{def}{=} \partial S(x, z)/\partial x$ is the marginal gross surplus and $C'_q(Q, q)$ the marginal cost of adjustment. Given the hypotheses on surplus and cost, this condition is sufficient to determine the adjustment function $q(Q, z)$. The adjusted quantity is related to the planned output and the random market size by

$$\frac{dq}{dQ} = -\frac{S'' - C''_{qQ}}{S'' - C''_{qq}}, \ \frac{dq}{dz} = -\frac{\partial S'/\partial z}{S'' - C''_{qq}}$$

Since $S'' < 0$ and $C''_{qq} \geq 0$, from the hypothesis $\partial S'/\partial z > 0$ we have that $dq/dz > 0$.

The derivative $dq/dQ$, has the same sign as $S'' - C''_{qQ}$. Then, if $C''_{qQ} \geq 0$, or if $C''_{qQ}$ is negative but small in absolute value, $\frac{dq}{dQ} < 0$. Indeed, with a decreasing marginal surplus ($S'' < 0$), increasing $Q$ decreases the need for a positive adjustment: it is a "saturation effect", or, in a market context, a "competition effect". Additionally, if a larger $Q$ deteriorates the conditions to produce an extra output, i.e. if $C''_{qQ} \geq 0$ ("diseconomies of scope"), the adjusted quantity is a decreasing function of the planned quantity. It is only when there is a large positive technical and/or economical externality between the two production processes, i.e. $C''_{qQ} < S'' < 0$, that a large planned output will induce a larger adjustment. The latter can be the case in thermal plants for small levels of $Q$ since the initial costs of warming-up being already paid to produce $Q > 0$, adjustments will be less expensive. This can be viewed as "economies of scope" encouraging planned production. But this positive effect can be insufficient to offset the "saturation effect". And if $Q$ is large, an increase in $Q$ will most likely increase $C'_q$, resulting in $\frac{dq}{dQ} < 0$. It is the case with the quadratic cost function $C(Q,q) = (Q+q)^2 + aq^2$ since $C''_{qQ} = 2$. Indeed, solving (2) in the quadratic case, we obtain

$$q(Q,z) = \frac{z - 3Q}{3 + 2a} \tag{3}$$

which is decreasing in $Q$. Note that it is also decreasing in $a$ as we could expect.

**Ex ante.** The problem is

$$\max_Q \mathbb{E}_z \left[S(Q + q(Q,z), z) - C(Q, q(Q,z))\right]$$

and, given the adjustment defined by (2), the solution $Q^*$ is such that

$$\begin{aligned} &Q^* \geq 0, \mathbb{E}_z \left[S'(Q^* + q(Q^*,z), z) - C'_Q(Q^*, q(Q^*,z))\right] \leq 0 \\ &Q^* \times \mathbb{E}_z \left[S'(Q^* + q(Q^*,z), z) - C'_Q(Q^*, q(Q^*,z))\right] = 0 \end{aligned} \tag{4}$$

Then, $Q^* = 0$ if $\mathbb{E}_z \left[C'_q(0 + q(0,z), z) - C'_Q(0, q(0,z))\right] \leq 0$. Otherwise, taking account of the adjustment rule (2), $Q^*$ is the solution to the equality between the two expected marginal costs.

In the quadratic case, since $\mathbb{E}_z \left[C'_q(0 + q(0,z), z) - C'_Q(0, q(0,z))\right] = \frac{2a}{3+2a} E > 0$ if $a > 0$, the planned output is positive. From (4), it is the solution to

$$\mathbb{E}_z \left[z - Q - \frac{z - 3Q}{(3 + 2a)} - 2(Q + \frac{z - 3Q}{(3 + 2a)})\right] = 0.$$

We deduce that $Q^* = E/3$ if $a > 0$. From (3) the optimal adjustment is then $q(Q^*, z) = \frac{z-E}{3+2a}$ which is decreasing with both the adjustment cost $a$ (as expected) and the expected demand $E$ since the planned production increases with $E$ and there are diseconomies of scope. The adjustment is positive (resp. negative) for large (resp. small) values of the market size $z$. On average, the adjustment is nil: $\mathbb{E}_z q(Q^*, z) = 0$. Finally, note that if $a = 0$ any appropriate combination of planned and adjusted output is the solution since there is no penalty for delaying production.

**Remark 3.1.** *Notice that with $Q^* + q(Q^*, z) = \frac{2aE+3z}{3(3+2a)}$, the condition $Q^* + q \geq 0$ is satisfied for all non negative $a$ and $z$.*

**Remark 3.2.** *Planning a production $Q^*$ non dependent on the adjustment cost $a \neq 0$ is somewhat counter-intuitive. The same is true as for the zero expected adjustment. This actually is an artifact of our elementary quadratic specification. Indeed, under the slightly more complex function $C(Q,q) = (bQ+q)^2 + aq^2$ where $b$ is a cost index for the base production, the adjustment function is $q(Q,z) =$*

$\frac{z-(1+2b)Q}{3+2a}$, *the optimal planned output is* $Q^* = \frac{1+a-b}{1+a-2b+(1+2a)b^2}E$, *and one can compute that* $\mathbb{E}_z q(Q^*, z) \neq 0$. *The reason is that with* $b \neq 1$ *the forecasted sales* $Q + q$ *are not homogeneous in terms of initial cost in addition to the subsequent adjustment cost. The advantage of setting* $b = 1$, *is that the planned quantities are simple to compute and compare, and we can focus on the adjustment process, then on the benefits and costs of having flexible technologies.*

**Welfare evaluation**. Plotting $Q = \frac{E}{3}$ and $q = \frac{z-E}{3+2a}$ into $W$, we obtain the following welfare value in state $z$ :

$$W^* = \frac{1}{12a + 18}\left(3z^2 + 4azE - 2aE^2\right) \tag{5}$$

and the expected welfare

$$\mathbb{E}W^* = \frac{1}{6}E^2 + \frac{V}{2\,(2a+3)}$$

With $\mathbb{E}W^*$ increasing in $V$, there is a social gain from randomness as was shown by Waugh (1945) for consumers and Oi (1961) for producers, provided they are risk-neutral to monetary transfers. However this benefit decreases when the adjustment cost parameter $a$ increases, and it vanishes when the technology is fully inflexible ($a \longrightarrow \infty$).

**Market mechanism with only ex post transactions.** The first best quantities can be decentralized with competitive price-taking firms and consumers facing exogeneous ex post contingent prices. Since it is a mere application of the second theorem of welfare, let us just check it with our numerical illustration. Ex post, consumers solve $\max_{q+Q} S(Q+q, z) - p(q+Q)$ where $p$ is independent from the quantities. Given (1), demand is $Q + q = z - p$. Producers solve $\max_q pq - C(Q, q)$. Hence the adjustment supply function using (1) is $q = \frac{p}{2(1+a)} - \frac{Q}{(1+a)}$. Equating supply and demand, we deduce the equilibrium price and quantity in state $z$,

$$p\,(Q, z) = 2\frac{z\,(1+a) - aQ}{3+2a}, q(Q, z) = \frac{z - 3Q}{3+2a} \tag{6}$$

Anticipating these quantities and prices, firms launch planned production solving $\max_Q \mathbb{E}\,[p \times (q+Q) - C(Q, q]$. Since they are price-takers, they do not internalize the effect of $Q$ on $p\,(Q, z)$. However, as rational agents, they internalize the effect on $q\,(Q, z)$. Consequently, the first order condition is

$$\mathbb{E}\left[p\,(Q, z)\,(1 + \frac{dq}{dQ}) - \left(C_Q' + C_q'\frac{dq}{dQ}\right)\right] = 0 = \mathbb{E}\left[p\,(Q, z) - C_Q'\right]$$

since $C_q' = p\,(Q, z)$ by the condition on ex post adjustment. Then, simple calculation shows that producers choose $Q = E/3$.

**Trade on two markets.** As shown in Crampes and Renault (2019), if we open a market for ex ante transactions in complement to the ex post market, as it is the case in most organized power markets that combine day-ahead and intra-day trade, under perfect competition the result is the same as when there is one single market like in the former paragraph. Indeed, the ex-post demand function is $q = z - Q - p$ where $Q$ is the ex ante purchase, and supply is the same as in the one single market case. Therefore, the ex post equilibrium is the same as in (6). Let $P$ be the ex ante price. Producers solve $\max_Q PQ + \mathbb{E}\left[p(Q,z)(q(Q,z) - C(Q, q(Q,z))\right]$. Again $\partial p(Q,z)/\partial Q \equiv 0$. Then the FOC reads $P + \mathbb{E}\left[p(Q,z)\frac{\partial q(Q,z)}{\partial Q} - \left(C'_Q + C'_q\frac{\partial q(Q,z)}{\partial Q}\right)\right] = 0$. Given the-ex post adjustment, we obtain the (inverse) supply function for planned production $P = \mathbb{E}\left[\left(C'_Q\right)\right]$ or, with our specification, $P = 2\frac{2aQ+E}{3+2a}$. On the demand side, the consumers solve $\max_Q \mathbb{E}\left[S(Q + q(Q,z)\right] - PQ - \mathbb{E}\left[p(Q,z)\, q(Q,z)\right]$ and their demand function is $\mathbb{E}\left[S'(Q + q(Q,z)\right] = P$, specifically $P = 2\frac{[(1+a)E - aQ]}{(3+2a)}$. At equilibrium between ex ante supply and demand, we obtain $Q = E/3$ as expected, and $P = \frac{2}{3}E$. The latter actually is $\mathbb{E}p(Q,z)$. Indeed, $P = \mathbb{E}p(Q,z)$ prevents any possibility of arbitrage between the two markets.

**Risk neutrality.** It is worthwhile noting that the equivalence of the two market designs results from the quasi-linearity of the consumers' and producers' preferences. Indeed, assuming that the consumers' performance is measured by their net surplus $S_n = S - px$ and the producers' performance by their net profit $\pi = px - C$ implicitly states that they are risk-neutral when facing monetary risks. Consequently, as long as the today price is the same as the expectation of the tomorrow prices, randomness does not affect their decisions.

## 3.2 Two production plants

We now analyse the social gains due to the existence of two production plants. We first consider the general optimization problem, then we determine the explicit solution corresponding to the quadratic specification.

### 3.2.1 General properties

There are two plants producing the same homogenous product with respective cost functions $C_1(Q_1, q_1), C_2(Q_2, q_2)$. Costs are increasing and convex.

The problem to solve is

$$\max_{Q_1,Q_2} \mathbb{E}_z \max_{q_1,q_2} W(\vec{Q}, \overrightarrow{q}, z)$$

- Given $Q_1, Q_2$ and $z$, the adjustments are the solutions to

$$S'(Q + q, z) = C'_{1q}(Q_1, q_1) = C'_{2q}(Q_2, q_2) \tag{7}$$

The adjustment in plant $i$ is then a function of both the market size and the two planned outputs: $q_i(Q_1, Q_2, z)$.

Total differentiation of the two equations in (7) wrt $Q_i$ gives the variation in $q_i$ due to a variation in the planned output of plant $i$:

$$\frac{dq_i}{dQ_i} = D^{-1}\left[S''C''_{-iqq} + \left(S'' - C''_{-iqq}\right)C''_{iqQ}\right]$$

where $D$ denotes the determinant of the full system. $D$ is positive by concavity of the objective function. Then, like in the one-plant case, $C''_{iqQ} \geq 0$ is sufficient for $\frac{dq_i}{dQ_i} \leq 0$. It would take a negative and large in absolute value $C''_{iqQ}$ to obtain the opposite. As for the adjustment in plant $-i$,

$$\frac{dq_{-i}}{dQ_i} = D^{-1}S''\left(C''_{iqq} - C''_{iqQ}\right),$$

which is negative if $C''_{iqQ}$ is negative or positive but small in absolute value, and $\frac{dq_{-i}}{dQ_i} > 0$ when $C''_{iqQ} >> 0$. The reason is that plant 1 and plant 2 compete in the adjustment process. Then, if $C''_{iqQ} < 0$ it is profitable to decrease $q_{-i}$ and leave room to a potential increase in $q_i$. Conversely, if there are strong diseconomies of scope between $Q_i$ and $q_i$ (i.e $C''_{iqQ} >> 0$), efficiency requires to use plant $-i$ for adjustments after an increase in the planned production of plant $i$.

- Ex ante, given the adjustment defined by (7), the solution $Q_1^*, Q_2^*$ is such that for $i = 1, 2$

$$\begin{aligned} Q_i^* \geq 0,\ \mathbb{E}_z\left[S'(Q^* + q(Q_1^*, Q_2^*, z), z) - C'_{iQ}(Q_i^*, q_i(Q_1^*, Q_2^*, z)\right] &\leq 0 \\ Q^* \times \mathbb{E}_z\left[S'(Q^* + q(Q_1^*, Q_2^*, z), z) - C'_{iQ}(Q_i^*, q_i(Q_1^*, Q_2^*, z)\right] &= 0 \end{aligned} \quad (8)$$

Depending on the form of the cost functions, in particular their relative advantage in terms of flexibility, we can obtain solutions where the two plants produce at the two speeds, and others where they must specialize (e.g. $Q_1^* > 0, Q_2^* = 0, q_1(Q_1^*, 0, z) = 0, q_2(Q_1^*, 0, z) > 0$).

### 3.2.2 Quadratic specification

**To share or not to share** From a pure cost-efficiency point of view, the quadratic function $C_i(Q_i, q_i) = (Q_i + q_i)^2 + a_i q_i^2$, $i = 1, 2$ opens the door to a trade-off between specialisation and two-speed production. Indeed, with convex cost functions (∼decreasing returns to scale), we must share any given quantity among several generation units to reduce the total cost. On the other hand, there are "diseconomies of scope", globally $(C_i(Q_i, q_i) - (C_i(Q_i, 0) + C_i(0, q_i)) = 2Q_i q_i > 0)$, and at the margin ($\frac{\partial^2 C_i(Q_i, q_i)}{\partial Q_i \partial q_i} > 0$). Then separating the planned and adjusted productions would lower the marginal cost of both. Actually, since there is no parameter making the planned ouput more or less costly depending on the production unit (see the specification in Remark 3.2 for a counter-example), the trade-off will be solved by asking each plant $i$ to prepare the same ex ante level of production $Q_i$.

**Quantities** To apply the results of subsection 3.2.1, suppose that the social planner can order production from two plants with non-negative inflexibility indices $a_1, a_2$. The utility function is unchanged. The total expected welfare is then

$$W = \mathbb{E}((z - \frac{1}{2}(Q+q))(Q+q) - (Q_1+q_1)^2 - (Q_2+q_2)^2 - a_1 q_1^2 - a_2 q_2^2). \quad (9)$$

From (7) we know that ex post productive efficiency is reached when the two marginal adjustment costs are equal, i.e. given $Q_1, Q_2$ and the total quantity $x = Q + q$ to produce, we have

$$2Q_1 + 2(1+a_1)q_1 = 2Q_2 + 2(1+a_2)q_2 \quad (10)$$

so that

$$(1+a_1)q_1 - (1+a_2)q_2 = Q_2 - Q_1$$

i.e. the larger adjustment will be done in the plant with the smaller planned output and, for a given difference $Q_2 - Q_1$, the larger $a_i$, the smaller $q_i$ and/or the larger $q_{-i}, (i = 1, 2)$. Note that (10) is true for all values of $q_1$ and $q_2$ (in the limit of $q_i \geq -Q_i$) since these quantities can be positive or negative.

Now, equating (10) with marginal utility, we can solve for $q_1$ and $q_2$:

$$\begin{cases} q_1^* &= \gamma^{-1}\left[(1+a_2)z - (4+3a_2)Q_1 - a_2 Q_2\right] \\ q_2^* &= \gamma^{-1}\left[(1+a_1)z - (4+3a_1)Q_2 - a_1 Q_1\right] \end{cases} \quad (11)$$

where $\gamma$ is a constant:

$$\gamma = 4 + 3a_1 + 3a_2 + 2a_1 a_2. \quad (12)$$

As expected, the two quantities decrease with the planned outputs. For a given $z$, it means that the adjusted quantities are positive and smaller and smaller (resp. negative and larger and larger in absolute value) when $Q_1 + Q_2$ is small (resp. large) and increases.

At the planning stage, we know from (8) that the solution $Q_1, Q_2$ must make the two expected marginal costs and the expected marginal utility equal:

$$\mathbb{E}\left[z - Q_1 - Q_2 - q_1^* - q_2^*\right] = \mathbb{E}\left[2(Q_1 + q_1^*)\right] = \mathbb{E}\left[2(Q_2 + q_2^*)\right]. \quad (13)$$

Using (11) and solving, we obtain

$$\begin{array}{c} Q_1^* = \frac{1}{4}E, Q_2^* = \frac{1}{4}E \\ q_1^* = \gamma^{-1}(1+a_2)(z-E), q_2^* = \gamma^{-1}(1+a_1)(z-E) \end{array} \quad (14)$$

**Remark 3.3.** *Contrary to our observation in Remark 3.1, when there are several production plants the internal solution given by (14) does not guarantee that the condition $Q_i^* + q_i^* \geq 0$ holds. Indeed we face the risk of obtaining a solution where one plant adjust downwards to decrease not only its own output but also the output of the other plant, which is technically impossible. Formally, using (12)* *and (14), we must check that* $2a_i\,(1+a_{-i})+(a_i-a_{-i})+(4+4a_{-i})\,z\geq 0, i=1,2$. *We see that the constraint is redundant as long as the difference* $(a_i-a_{-i})$ *is not too large and/or the demand index* $z$ *is never too low. By contrast, suppose that firm 2 is fully inflexible* ($a_2 \longrightarrow +\infty$) *so that* $q_2^*=0$ *and firm 1 can adapt at a finite cost. Then equation (10) is no longer relevant. To determine* $q_1(\overrightarrow{Q},z)$, *we must equate the marginal cost of adjustment in plant 1 with marginal utility:* $2Q_1+2(1+a_1)q_1 = z-(Q_1+Q_2+q_1)$ *and we obtain* $q_1=\frac{z-(3Q_1+Q_2)}{3+2a_1}$. *Solving the stage 1 condition (13) for this adjustment functions, the corner solution is* $Q_1^*=Q_2^*=\frac{1}{4}E, q_1^*=\frac{z-E}{2a_1+3}, q_2^*=0$. *The total output of firm 1 is* $Q_1^*+q_1^*=\frac{1}{4}E+\frac{z-E}{2a_1+3}\geq 0$. *It is non negative if* $z\geq\frac{E}{4}(1-2a_1)$. *We see that the most constraining case is when firm 1 is perfectly flexible* ($a_1=0$). *The condition is then* $z\geq\frac{E}{4}$ *which means that demand forecasts must be accurate enough for the lowest possible level to be at least one fourth of the average, which is not very challenging given the statistical tools available today.*

Apart from the extreme cases evoked in Remark 3.3, the first best (interior) solution is given by (14): both firms are active at the two periods because, under increasing marginal costs, it is efficient to share the load. They produce the same quantity at the planning period because of the peculiarities of our cost function: in each firm, decreasing the planned output $Q_i$ decreases the adjustment marginal cost, and the two firms incur the same cost for the production of $Q_i$. Then the adjusted quantities $q_1, q_2$ only differ from each other because their marginal costs have different slopes $\frac{\partial C_i(Q_1^*,q_1)}{\partial q_i}=\frac{E}{2}+2(1+a_i)q_i$. Ex post, efficiency imposes that $\partial q_i^*/\partial a_i<0, \partial q_i^*/\partial a_{-i}<0$ and $q_1^* \gtreqless q_2^*$ as $a_2 \gtreqless a_1$. Note that the adjustments decrease with the expected demand since planned outputs increase with $E$.

Given these quantities, welfare in state $z$ is

$$W^*=\frac{1}{4\gamma}\left(\left(z^2-E^2\right)a_1+\left(z^2-E^2\right)a_2+\left(a_1+a_2+4\right)z^2-2E^2a_1a_2+2zE\left(a_1+a_2+2a_1a_2\right)\right)$$

resulting in the expected welfare

$$I\!\!EW^*=\frac{E^2}{4}+\frac{2+a_1+a_2}{2\gamma}V$$

To compare these results with those of the one-plant case, assume that $a_1=a_2=a$.Then $q_1^*+q_2^*=\frac{1+a}{2+3a+a^2}\left(z-E\right), Q_1^*+Q_2^*=\frac{1}{2}E$ and $I\!\!EW^*=\frac{E^2}{4}+\frac{1}{2(a+2)}V$. At the two stages of the production process the total quantity is larger because the cost has been alleviated by allocating the output among the two plants. Here again, the expected welfare can be explicitly written as a function of the mean and variance of demand. Both terms are larger than in the one-plant case but, again, there is no benefit from the variance if the parameter $a$ becomes infinite.

**Prices** First best can be decentralized if all agents are price takers. Using the same argument as in the one-plant case, we can compute the adjustment price by fixing it at the value of the first best marginal utility $S' = z - (Q^* + q^*)$,which gives

$$p(z) = z - \gamma^{-1}\left(\gamma\frac{E}{2} + (2 + a_1 + a_2)(z - E)\right).$$

The price of the day-ahead market (if there is one), is obtained by taking the average of the spot price (no-arbitrage argument)

$$P = \mathbb{E}p(z) = \frac{E}{2}$$

to be compared with $P = \frac{2}{3}E$ obtained in the one-plant case. Clearly, sharing the load between two price-taking firms is profitable to consumers since they consume more at a lower price than with one single price-taking producer. As for the firms, the price decrease is offset by a drop in costs, so that the profit of the industry $\frac{1}{8}E^2 + \frac{1+a}{2(2+a)^2}V$ is larger than when there is only one producer, $\frac{1}{9}E^2 + \frac{1+a}{(3+2a)^2}V$.

In order to facilitate the comparison with the results of the next section, we summarize the main outcomes in the following Proposition:

**Proposition 3.4.** *When the gross surplus and cost functions are given by the quadratic specifications (1), at first best with two production plants the ex ante production levels are* $Q_1^* = \frac{1}{4}E, Q_2^* = \frac{1}{4}E$ *and the two expected adjustments are nil. The expected welfare is* $\mathbb{E}W^* = \frac{E^2}{4} + \frac{2+a_1+a_2}{2\gamma}V$*. If the plants are operated by two price-taking private operators, the outcomes are the same as at first best whatever the market design. If one firm is perfectly flexible and the other totally inflexible (* $a_1 = 0$ *et* $a_2 = +\infty$*), when* $V > 0$ *the flexible firm has a higher expected profit than the inflexible one:* $\mathbb{E}\Pi_1^* = \frac{E^2}{16} + \frac{V}{9} > \mathbb{E}\Pi_2^* = \frac{E^2}{16}$.

# 4 Duopoly

We now assume that there are two firms, each managed by an independent private owner who wants to maximize its expected operating profit by fixing the quantities $Q_i$ and $q_i$, taking account of the effect of its decisions on prices, and constrained by its competitor's choices (Cournot competition). In this framework, we know from Crampes and Renault (2019) that the two market organizations presented in the former section are no longer equivalent. In sub-section 4.1 we explain the roots of this no-equivalence and show that the market design has ambiguous effects on the total output and the agents' surplus. In sub-section 4.2, we compare the two market designs under the quadratic specification, and show that the opening of a day-ahead market makes consumers better off and firms worse off. It also transfers risks from firms to consumers.

## 4.1 Strategic behavior

- Ex post, the quantities $Q_1 \geq 0, Q_2 \geq 0$ are fixed and known by the two firms. Either they have been sold on the day-ahead market if it exists, or their production has just been launched if there are only ex post markets.[4] In a subgame-perfect equilibrium, at stage 2 we have a game parameterized by $z$ and $\vec{Q}$. If all quantities are sold at the ex-post price, the payoff of each firm $i$ is given by

$$\Pi_i(q_1, q_2, z) = p(Q+q, z)(q_i + Q_i) - C_i(Q_i, q_i) \tag{15}$$

where $p(Q+q,z) = S'(Q+q,z)$ is the ex-post demand function. When there are two successive markets, $i$'s profit is

$$\widetilde{\Pi}_i(q_1, q_2, z) = PQ_i + p(Q+q, z)q_i - C_i(Q_i, q_i) \tag{16}$$

where $P$ is the ex ante price.

Given its market power, each firm $i$ internalizes that its production will lower the price. In the one-market framework, the FOCs are

$$p(Q+q,z) + (Q_i + q_i)p' = C'_{iq}(Q_i, q_i), \ i = 1, 2 \tag{17}$$

where $p' = \frac{\partial p(q+Q,z)}{\partial q_i} < 0$ since $S'' < 0$. Let $(q_1^C, q_2^C)$ be the unique Cournot equilibrium of this game and $p^C = S'(Q + q^C, z)$ the associated price.

If there are two successive markets, the FOCs are

$$p(Q+\widetilde{q}, z) + \widetilde{q}_i p' = C'_{iq}(Q_i, \widetilde{q}_i), \ i = 1, 2 \tag{18}$$

Let us denote $(\widetilde{q}_1^C, \widetilde{q}_2^C)$ and $\widetilde{p}^C = S'(Q + \widetilde{q}^C, z)$ the equilibrium quantities and price of this game.

In both market frameworks, the equilibrium price and quantities depend on $z$ and $\vec{Q}$. The adjusted quantities are generically decreasing in both $Q_i$ and $Q_{-i}$.

- At stage 1:
  - if all production is sold on the ex-post market the expected profit of $i$ is

$$\Pi_i(Q_1, Q_2) = \mathbb{E}_z\left(p^C(\overrightarrow{Q}, z)(q_i^C(\vec{Q}, z) + Q_i) - C_i(Q_i, q_i^C(\vec{Q}, z)\right) \tag{19}$$

    As detailed in Crampes and Renault (2019), the FOC for the maximisation of $\Pi_i(Q_1, Q_2)$ wrt $Q_i$ (internalizing (17)) is

$$\mathbb{E}_z\left[p^C + \left(Q_i + q_i^C\right)(1 + \frac{\partial q_{-i}^C}{\partial Q_i})p^{C'} - C'_{iQ}\right] = 0. \tag{20}$$

    where $p^{C'} \stackrel{def}{=} S''(Q + q^C, z)$ stands for the slope of the inverse demand function at the equilibrium point. Let $Q_1^C, Q_2^C$ denote the solution.

[4] The assumption that $Q_i$ is known by $-i$ even when there is no ex-ante market-place can be justified in terms of technological and managerial expertise.

– if there are two markets, the expected profit is

$$\widetilde{\Pi}_i(Q_1, Q_2) = P(Q_1+Q_2)Q_i + \mathbb{E}_z\left[\widetilde{p}^C(Q,z)\widetilde{q}_i^C(\vec{Q},z) - C^i\left(Q_i, \widetilde{q}_i^C(\vec{Q},z)\right)\right] \tag{21}$$

where $P(Q_1, Q_2) = \mathbb{E}_z\left(S'(Q + \widetilde{q}_1^C + \widetilde{q}_2^C, z)\right)$ is the ex-ante demand function. Given (18), the FOC is

$$P + P'Q_i + \mathbb{E}_z\left[\widetilde{q}_i^C\left(1 + \frac{\partial \widetilde{q}_{-i}^C}{\partial Q_i}\right)\widetilde{p}^{C'} - C'_{iQ}\right] = 0. \tag{22}$$

Let $\widetilde{Q}_1^C, \widetilde{Q}_2^C$ denote the solution.

- Comparing the two pairs of conditions, we observe two differences:

  – ex post, if the firms have committed in an ex-ante market, increasing their adjustment output has a lower impact on their profit since the drop is $\widetilde{q}_i p'$ in (18) instead of $(Q_i + q_i)p'$ in (17). Then, given the same planned quantities $Q_1 + Q_2$, with two markets the ex post price will be closer to the marginal cost, and quantities larger than without day-ahead trade, which is in line with the role of forward markets in Allaz and Vila (1993) and Ito and Reguant (2016); then this effect pushes towards $\widetilde{q}_i^C > q_i^C$.

  – ex ante, there is no term like $Q_i \frac{\partial q_{-i}^C}{\partial Q_i} p^{C'} > 0$ in (22). The tomorrow reaction of firm $-i$ has no impact on the today's marginal revenue of firm $i$ whereas firm $i$ must consider this response when $Q_i$ and $q_{-i}$ are sold on the same market (see (20)). Consequently, *ceteris paribus* the expected marginal revenue of $i$ in the unique market framework (equation (20)) is higher than in the two-market system. Then, this effect pushes towards $Q_i^C > \widetilde{Q}_i^C$.

  – consequently, without additional information, we cannot predict whether $Q_i^C + q_i^C \lesseqqgtr \widetilde{Q}_i^C + \widetilde{q}_i^C$. The specification below provides an illustration of the outcome from the two antagonistic effects.

## 4.2 Duopoly equilibrium in the quadratic case

Before determining the duopoly equilibrium with and without a day-ahead market, it is useful to gain a better understanding of how the firms can exert their market power with two-speed technologies.

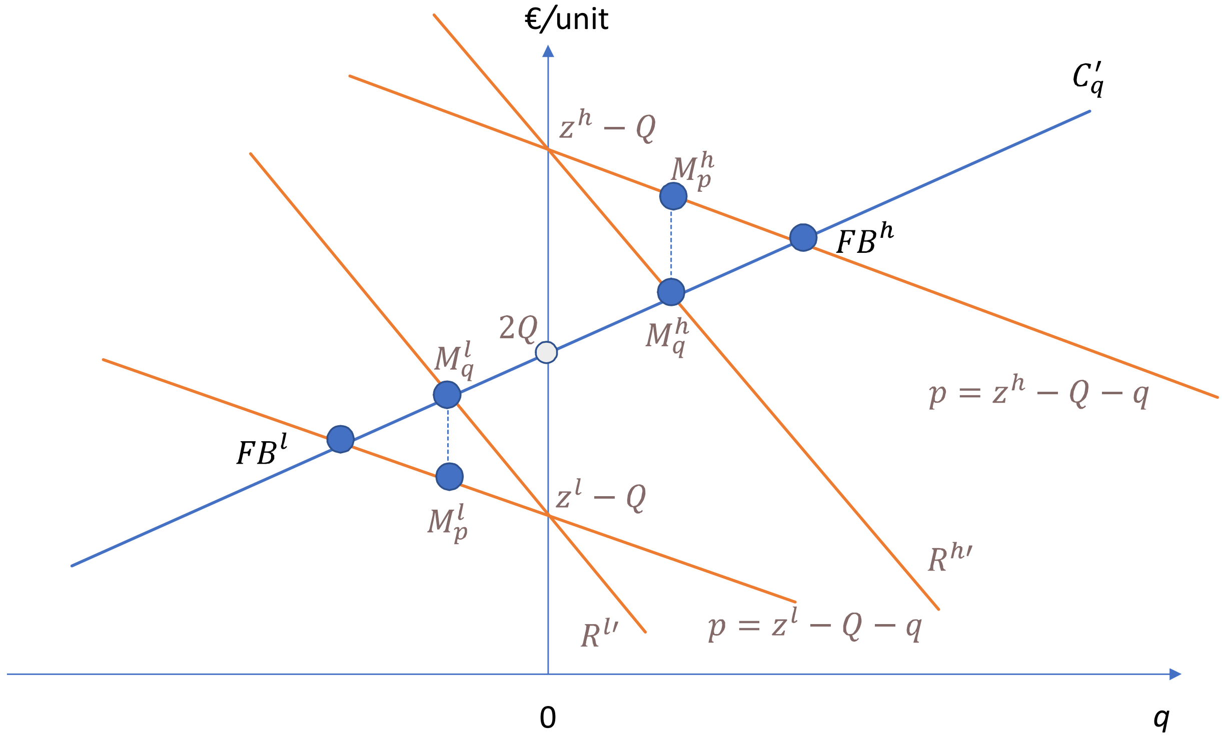


Figure 1: Monopoly adjustment output and price.

#### 4.2.1 Market power

Assume a monopoly with the cost function $C(Q,q) = (Q+q)^2 + aq^2$ and facing the inverse demand function $p = z - (Q+q)$. At stage 2, given $Q$ and after observing $z$, it produces the quantity $q^m(z,Q) = \frac{z-3Q}{2(2+a)}$ that equates the marginal cost $C'_q = 2Q + 2q(1+a)$ and the marginal revenue $R'_q = (z-Q) - 2q$. This is shown in Figure 1 for two values of $z$, a high value $z^h$ and a low value $z^l$. In case $z^h$, the outcome is standard: the monopoly produces less (point $M_q^h$) and sells at a higher price (point $M_p^h$) than what would implement first best (point $FB^h$). It is more interesting to observe that when demand is so low that it necessitates a negative adjustment (case $z^l$), the firm exerts its market power by reducing its production less than what would be optimal, which means that it supplies too much. It fixes a price below the optimal one to trigger a larger consumption.

Since increasing the quantity $Q$ has the double negative consequence of pushing the ex post marginal cost upwards ("diseconomies of scope" effect) and pulling the marginal revenue downwards ("self competition" effect), the planned output produced by the monopolist is the lowest when the extra cost of adjustment $a$ is low.

Now, let us name $i$ the firm we analyze and introduce a second firm $-i$. The output $Q_{-i}$ has no effect on $i$'s marginal cost but it shifts the marginal revenue functions of $i$ downwards, which increases the number of states of nature where $i$ will have to operate negative adjustments. Then firm $i$ is incentivized by its competitor to reduce its planned production, with the positive side effect of decreasing the adjustment cost.

In the following we examine the results of these interactions, first when all the outputs are sold on the spot market, and second when the firms can also trade ex ante.

#### 4.2.2 One market

Two firms $i$ and $-i$ compete in quantities in the dynamic setting depicted above. In the profit function of $i$ (15), the price in state $z$ is $p(Q+q,z) = z-(Q+q)$ and the cost of $i$ is $C_i(Q_i,q_i) = (Q_i+q_i)^2 + a_i q_i^2$.

**Ex post.** From (17), the FOC to determine the adjustment quantity $q_i$ is

$$z - 4Q_i - Q_{-i} - (4+2a_i)q_i - q_{-i} = 0, \; i = 1,2$$

The resulting equilibrium quantities are

$$\begin{cases} q_1^C &= \beta^{-1}\left[(3+2a_2)z - (15+8a_2)Q_1 - 2a_2Q_2\right] \\ q_2^C &= \beta^{-1}\left[(3+2a_1)z - (15+8a_1)Q_2 - 2a_1Q_1\right] \end{cases} \tag{23}$$

and the price is

$$p^C = \beta^{-1}\left[(2a_1+3)(2a_2+3)z - 2\left(3+2a_2\right)a_1Q_1 - 2\left(3+2a_1\right)a_2Q_2\right] \tag{24}$$

where

$$\beta = (4+2a_1)(4+2a_2) - 1.$$

**Ex ante.** To obtain the FOC relative to the ex-ante quantity $Q_i$, let's use (20) where we inject $p^{C'} = -1$, $\frac{\partial q_{-i}^C}{\partial Q_i} = -2a_i\beta^{-1}$ by (23) and $C'_{iQ} = 2(Q_i + q_i^C)$. It results that

$$\mathbb{E}_z\left[p^C - \left(Q_i + q_i^C\right)\left(3 - 2a_i\beta^{-1}\right)\right] = 0, i = 1,2. \tag{25}$$

Then, we inject the adjustment functions given by (23) and solve the two-equation system. The planned quantities we obtain are proportionnal to the expected willingness to pay of demanders (the explicit forms are given by equations (28) in the Appendix 6.1):

$$\begin{aligned} Q_1^C &= k_1 E \\ Q_2^C &= k_2 E \end{aligned} \tag{26}$$

First observe that the two firms will have different ex ante outputs if they differ in terms of cost parameters, contrary to what we found at first best (see (14)). This is because the strategic effect is now at work on top of the cost minimization worry: firms try to gain market shares without decreasing the price too much. Even when they are totally identical ($a_1 = a_2 = a$), the outputs differ qualitatively from first best since $Q_1^C = Q_2^C = \frac{4(2+a)^2}{78a+20a^2+75}E$ is decreasing in $a$

whereas $Q_1^*$ and $Q_2^*$ given in (14) are fixed. Unsurprisingly, the firms exert their market power by restricting quantities: $Q_i^C < Q_i^*$ even when $a = 0$.

When $a_1 \neq a_2$, the analysis of the equilibrium quantities (28) in Appendix 6.1 show that $Q_i^C$ is increasing in $a_i$ and decreasing in $a_{-i}$. Moreover, $Q_1^C \gtrless Q_2^C$ as $a_1 \gtrless a_2$. These results are the consequence of the diseconomies of scope mentioned formerly: the firm with the lower cost of adjustment produces less than its competitor at the first stage because this will limit the negative impact on its second stage cost where it will be more active. The higher the adjustment parameter $a_i$, the higher the planned production $Q_i^C$ because firm $i$ does not intend to intervene intensively at the adjustment stage, hence does not mind about the negative cost externalty.

Consider now the profit of the firms $\Pi_i^C = \left[z - 2\left(Q_i^C + q_i^C\right) - \left(Q_{-i}^C + q_{-i}^C\right)\right](Q_i^C + q_i^C) - a_i q_i^{C^2}$, $i = 1, 2$, and the consumers' net surplus $S_n^C = \frac{(Q^C + q^C)^2}{2}$ in state $z$. Given (23) and (28), these functions can be written under the format $\Pi_i^C = l_i z E + m_i E^2 + n_i z^2, i = 1, 2$ and $S_n^C = l_S z E + m_S E^2 + n_S z^2$ where the positive weights $l_i, m_i, n_i, l_S, m_S, n_S$ only depend on the adjustment coefficients $a_1, a_2$. Consequently, the expected value of $i$'s profit is

$$\mathbb{E}\Pi_i^C = (l_i + m_i + n_i)\, E^2 + n_i V,\ i = 1, 2$$

and, similarly, the expected net surplus is

$$\mathbb{E}S_n^C = (l_S + m_S + n_S)\, E^2 + n_S V$$

They are increasing in $E$ and $V$, at a speed that depends on the values of the adjustment coefficients $a_1, a_2$.

**Numerical illustrations.** To gain insights on how the flexibility question impacts the firms' strategies and the resulting outcomes, in the following we consider four characteristic cases, three for symmetric costs and one for asymmetric costs.[5]

*Cost symmetry*

- When the two firms are identical with a zero additional cost of adjustment ($a_1 = a_2 = 0$), only the total quantity $Q_i + q_i$ can be determined. Among the infinity of sharing rules between $Q_i$ and $q_i$, one is the limit of the subgame-perfect equilibrium when $a_1 = a_2$ goes to 0. In this case, the equilibrium is $Q_i^C = 0.213E$, $q_i^C = 0.2z - 0.213E$, then a total expected output $Q_1^C + Q_2^C + \mathbb{E}(q_1^C + q_2^C) = 2/5$. It is interesting to note that the firms have a negative expected adjustment (contrary to the zero average adjustment at first best in Proposition 3.4), which means that even though each plans to produce less than at first best, the will to gain market shares induces on average pushes the output upwards, which increases the risk of downward adjustment. The expected individual profits are $\mathbb{E}(\Pi_i^C) =$

[5] A larger set of numerical simulations is available in the tables at the end of the paper.

$0.08(E^2+V)$, the expected consumers' net surplus is $\mathbb{E}(S_n^C) = 0.08(E^2+V)$. Consequently, the global performance is $\mathbb{E}(W^C) = 0.24(E^2+V)$. In this extreme case of perfect flexibility, the demand randomness (measured by variance) is as profitable as the average willingness to pay.

- Consider now the opposite: the two fims have fully inflexible technologies: $a_1 = a_2 = +\infty$. Obviously $q_i^C = 0$ for both firms and we have a standard Cournot duopoly. The equilibrium quantities are $Q_i^C = 0.2E$, individual profits amount to $\mathbb{E}(\Pi_i^C) = 0.08E^2$ and the net consumers' surplus is $\mathbb{E}(S_n^C) = 0.08E^2$. Comparing with the perfect flexibility case, we see that everybody is losing the gains from the demand variance.

- Let us switch to an intermediary symmetric case, for example $a_1 = a_2 = 1$. At equilibrium, we obtain

$$\begin{aligned} Q_i^C &\simeq 0.208E,\ q_i^C \simeq \frac{1}{7}z - 0.149E \quad i = 1,2 \\ \mathbb{E}(\Pi_i^C) &\simeq 0.079E^2 + 0.061V \qquad\qquad i = 1,2 \\ \mathbb{E}(S_n^C) &\simeq 0.082E^2 + 0.041V \\ \mathbb{E}(W^C) &\simeq 0.241E^2 + 0.163V \end{aligned} \tag{27}$$

  Unsurprisingly, the result falls in between the two former cases: the planned output takes an intermediary value, the average adjustment is slightly negative and variance matters but less than if the technologies were fully flexible.

- In the three cases above, the firms are symmetric. The case where $a_1 = a_2 = +\infty$ is an interesting bechmark because firms are just competing "à la Cournot" facing an average demand. They both produce $Q_i^C = 0.2E < Q_i^* = 0.25E, q_i^C = q_i^* = 0$ whatever $z$, hence an expected social welfare $\mathbb{E}W^C = 0.24E^2 < \mathbb{E}W^* = \frac{1}{4}E^2$. These are standard results of imperfect competition. We can similarly observe that Cournot profits are larger than perfect competition profits and the opposite for consumers'net surplus. In the two other cases where the $a_i$ are finite, we observe that $Q_i^C$ is larger than when $a_i = \infty$ and that the average adjustments are negative. This is a Stackelberg effect: each firm has the incentive to invade the market to decrease the market share of its competitor. But doing so, it is competing against itself in the adjustment stage which explains the average decrease in $q_i$. This is a strategy à la Allaz and Vila (1993) with the trade-off between the gains of pushing the price up by restricting supply and the drawback of leaving a larger market share to the competitor.

*Cost asymmetry*

- There is an infinity of possibilities to depict cost dissymmetry. Suppose that $a_2 > a_1$ so that firm 2 is less efficient than firm 1 at adjusting its output.

Then anticipating it will produce a small ex post quantity, it can increase its planned production without impairing too much its ex-post cost. Facing this more agressive ex ante strategy, firm 1 reduces its planned production and participates more in the adjustment process. As expected, there occurs a partial specialisation of the firms.

- Let us illustrate it with the extreme case where firm 1 is perfectly flexible and firm 2 fully inflexible: $a_1 = 0, a_2 = +\infty$. When $a_1 = 0$, there exist several equilibria defined by $Q_1^C + q_1^C = \frac{z}{4} - \frac{3E}{56}$. The following one is the equilibrium obtained when $a_1$ tends to 0:[6]

$$\begin{aligned} Q_1^C &\simeq 0.196E,\ Q_2^C \simeq 0.214E, \\ q_1^C &= \frac{1}{4}(z-E),\ q_2^C = 0, \\ \mathbb{E}(\Pi_1^C) &\simeq 0.077E^2 + 0.125V,\ \mathbb{E}(\Pi_2^C) \simeq 0.080E^2 \\ \mathbb{E}(S_n^C) &\simeq 0.084E^2 + 0.03V \\ \mathbb{E}(W^C) &\simeq 0.242E^2 + 0.156V \end{aligned}$$

Since firm 2 cannot adjust its production to the revelation of $z$ (in Figure 1 its marginal cost is just the vertical axis), firm 1 is a monopoly at the second stage. And since firm 1 can produce at the same cost at the two stages, it would like to share its output equally to prevent being penalized by increasing marginal costs. However, it would not be profitable to do so because of the opportunism of firm 2 that would increase its ex ante production. Then we have $Q_1^C < Q_2^C < Q_1^* = Q_2^*$. Consider now the expected profits. Firm 1's profit $\mathbb{E}(\Pi_1^C)$ is increasing in both the average and the variance of demand. By contrast, since firm 2 does not participate in the adjustment market, only the average value of the willingness-to-pay appears in $\mathbb{E}(\Pi_2^C)$, but with a higher coefficient than in $\mathbb{E}(\Pi_1^C)$. It means that, contrary to what we had under perfect competition (see Proposition 3.4), for a small variance, the inflexibility of firm 2 is an advantage over the flexibility of firm 1. Indeed, firm 2 contrary to firm 1 can credibly commit that it will not adjust ex post. Then it has a Stackelberg market power pushing away firm 1 from the ex ante process, a result in line with the worry of competition authorities quoted in footnote 1.

To sum up:

**Proposition 4.1.** *When the gross surplus and cost functions are given by the quadratic specifications (1) and there is no day-ahead market, two firms competing in quantities produce less than at first best. If the firms have the same cost function, the higher the adjustment cost, the lower the planned outputs and the higher the (negative) expected adjustment. If the firms are asymmetrical, the less flexible firm plans a higher level of output and adjusts less ex post than its*

[6] Notice that the condition $q_1^C + Q_1^C \geq 0$ is met if $z \geq \frac{3}{14}E$, which is satisfied given the restriction $z \geq \frac{E}{4}$ set in Remark 3.3.

*competitor. When the demand variance is low, being inflexible is more profitable than being flexible.*

**Remark 4.2.** *In the symmetric case $a_1 = a_2 = a$, it is somewhat couterintuitive to have $Q_i^C$ decreasing and $\mathbb{E}(q_i^C)$ increasing in $a$. On cost grounds, it is obviously inefficient. Then the explanation is to be sought in terms of strategic effect: each firm is seeking to appropriate a large market share (market stealing effect), but doing so it increases the set of states of nature where it will have to adjust downwards (note that the expected adjustment is always negative). When the adjustment cost $a$ increases, the firm decreases its planned production in order to limit the cost of the negative expected adjustment. The higher $a$, the closer to $0$ the negative $\mathbb{E}(q_i^C)$, and the global game looks more and more like a one-shot game with an outcome converging to a standard Cournot equilibrium. As a byproduct, the expected total quantity $Q_i^C + \mathbb{E}(q_i^C)$ is not monotonous in $a$: it is increasing (resp. decreasing) when $a$ is small (resp. large).*

#### 4.2.3 Addition of a day-ahead market

Assume now that the two firms $i = 1, 2$ do not just decide on the production of $Q_i$ at stage 1. They also sell it on a day ahead market at a price equal to the expectation of the ex post prices. On the intra-day market, fim $i$ only sells $q_i$. In the following, we first consider two cases of identical costs: $a_1 = a_2 = 1$ and $a_1 = a_2 = +\infty$ to emphasize the drastic changes due to the opening of a day-ahead market, then the case $a_1 = 0, a_2 = +\infty$ to illustrate the potential of firms' specialization.

**Finite identical costs**: $a_1 = a_2 = 1$

Applying the quadratic specification to the first order condition (18) and assuming that $a_1 = a_2 = 1$ we obtain the following equilibrium outcome (the explicit solution is in Appendix 6.2):

$$\widetilde{Q}_i^C \simeq 0,184E,\ \widetilde{q}_i^C \simeq 0,143z - 0,105E,\ i = 1,2$$

The average profit of $i$ is

$$\mathbb{E}(\widetilde{\Pi}_i^C) \simeq 0,073E^2 + 0,061V,$$

the average consumers' net surplus is

$$\mathbb{E}(\widetilde{S}_n^C) = 0,098E^2 + 0,041V.$$

and the resulting average welfare is

$$\mathbb{E}(\widetilde{S}_n^C + \widetilde{\Pi}^C) \simeq 0,244E^2 + 0,163V.$$

Comparing with (27), we can observe that adding a day-ahead market has the following effects:

each firm produces less at the first stage ($\widetilde{Q}_i^C < Q_i^C$) but its average total production is larger ($\widetilde{Q}_i^C + \mathbb{E}\widetilde{q}_i^C > Q_i^C + \mathbb{E}q_i^C$).

expected prices are lower: $\mathbb{E}(\widetilde{p}^C) \simeq 0.556E < \mathbb{E}(p^C) \simeq 0.595E$,

consumers are better off ($\mathbb{E}\widetilde{S}_n^C > \mathbb{E}S_n^C$) and firms are worse off ($\mathbb{E}\widetilde{\Pi}_i^C < \mathbb{E}\Pi_i^C$)

total welfare is higher ($\mathbb{E}\widetilde{W}^C > \mathbb{E}W^C$).

With $\widetilde{Q}_i^C < Q_i^C$ but $\widetilde{q}_i^C > q_i^C$, we have an illustration of the two opposite effects due to the opening of a day-ahead market that we have identified at the end of subsection 4.1. Here the conflict ends out with $\widetilde{Q}_i^C + \mathbb{E}\widetilde{q}_i^C > Q_i^C + \mathbb{E}q_i^C$, so that opening a day-ahead market is profitable to consumers, detrimental to producers, and socially beneficial. However, notice that all the gains and losses result from an increase in the coefficient of the mean demand, whereas the variance effect remains unchanged. What if we take a closer look at the random performance values of the second stage? We observe that $\widetilde{W}^C > W^C$ if and only if $z < 0.96E$ and $\widetilde{S}_n^C > S_n^C$ if and only if $z > 0,94E$. In words, if $z$ is small, the firms get a bonus and the consumers a malus, because at stage 1 the firms have sold a smaller quantity at a relatively high price. By contrast, if $z$ is high, the firms get a malus and the consumers get a bonus, because the consumers have bought at stage 1 products at a relatively low price. The reason why the consumers are better off and the firms are worse off in expectation is that with convex cost functions, high values of $z$ matter more than low values.

Interestingly, the minimum profit is larger with a day-ahead market ($\simeq 0,069E^2$, reached when $z \simeq 0,738E$) than without ($\simeq -0,033E^2$, reached when $z = 0$). Indeed the day-ahead market yields insurance to the firms in case of small $z$. This is confirmed by the observation that the addition of the day-ahead market induces a transfer of risks (measured by profit and surplus variances) from firms to consumers. Specifically, applying the formulae in Appendix 6.2.3, we have that

$$V(\widetilde{\Pi}_i) - V(\Pi_i) \simeq 0,0227VE^2 + 0,0172E^4 - 0,0172E\mathbb{E}(z^3)$$

$$V(\widetilde{S}_n) - V(S_n) \simeq 0,0712VE^2 - 0,0225E^4 + 0,0225E\mathbb{E}(z^3)$$

To assess the sign of these differences, we need the following Lemma:

**Lemma 4.3.** *Let $z$ be a random variable taking values in $\mathbb{R}_+$. Recalling that $E = \mathbb{E}(z)$, $V = \mathbb{E}(z^2) - E^2$, then:*

$$\mathbb{E}(z^3) \geq E^3 + \frac{3}{2}EV.$$

*Proof:* Consider the function $f : x \mapsto x^{3/2}$, $f$ is convex on $\mathbb{R}_+$ so by Jensen's inequality the expectation of $f(z^2)$ is at least $f(\mathbb{E}(z^2))$, that is: $\mathbb{E}(z^3) \geq (E^2 + V)^{3/2}$. But $(E^2 + V)^{3/2} = (E^2(1 + \frac{V}{E^2}))^{\frac{3}{2}} = E^3(1 + \frac{V}{E^2})^{\frac{3}{2}}$. Since $(1 + x)^{\frac{3}{2}} \geq 1 + \frac{3}{2}x$ for all $x$, we obtain $(1 + \frac{V}{E^2})^{\frac{3}{2}} \geq 1 + \frac{3}{2}\frac{V}{E^2}$ and $\mathbb{E}(z^3) \geq E^3(1 + \frac{V}{E^2})^{\frac{3}{2}} \geq E^3 + \frac{3}{2}EV$.

Applying the Lemma, we obtain

$$V(\widetilde{\Pi}_i) - V(\Pi_i) \leq -0,0031VE^2 < 0$$

$$V(\widetilde{S}_n) - V(S_n) \geq 0,105VE^2 > 0$$

Note that this is true − under the quadratic specification (1) − for any distribution of probabilities of the willingness-to-pay $z$.

**Infinite identical costs**: $a_1 = a_2 = +\infty$

This transfer of risks can be emphasized by the case of total inflexibility for both firms. When $a_1 = a_2 = +\infty$, we obtain $q_i^C = \widetilde{q}_i^C \equiv 0$ and $Q_i^C = \widetilde{Q}_i^C = 0.2E$, $i = 1, 2$. Even though the outcomes seem identical, the following table shows that profits and net surplus are differently affected:

| intraday market only | day-ahead + intraday market |
|---|---|
| $p^C = z - 0.4E$ | $P^C = 0.6E$ |
| $\Pi_i^C = \frac{1}{25}(5z - 3E)E$ | $\widetilde{\Pi}_i^C = 0.08E^2$ |
| $S_n^C = 0.08E^2$ | $\widetilde{S}_n^C = 0.4zE - 0.32E^2$ |

Profits have equal expected values in the two market designs. The same for the consumer's net surplus. However, when all trade can only occur ex post, the quantity $Q^C = 0.4E$ is produced ex ante and sold at the random price $p^C = z - 0.4E$ so that all risks are on the shoulders of firms and consumers are fully insured. Symmetrically, when trade occurs day ahead, consumers buy the quantity $\widetilde{Q}_i^C = 0.4E$ but they pay a random price which provides full insurance to the producers. In our model, this transfer is innocuous since both consumers and producers are risk neutral. But it is worthwhile emphasizing it because in the real world most consumers (at least households) are risk averse and many entrepreneurs are risk lovers, so that the opening of a day-ahead market may have some detrimental effect on welfare.

**Asymmetric competition:** $a_1 = 0, a_2 = +\infty$.

The detailed solution for the case where firm 1 is perfectly flexible and firm 2 is totally inflexible is in Appendix 6.3. The first obvious result is that $\widetilde{q}_2^C = 0$ whatever $z$ and $Q_1 + Q_2$. Then, firm 1 is a monopoly on the ex post market, and since it can adjust its output at the same cost as ex ante, its best choice is $\widetilde{Q}_1^C = 0$, abandonning the initial stage to firm 2. The result is full specialization with two successive monopolies where firm 2 sells $\widetilde{Q}_2^C = \frac{3}{14}E$ day-ahead and firm 1 sells $\widetilde{q}_1^C = \frac{1}{4}z - \frac{3}{56}E$ on the spot market.[7]

We could deduce that the addition of the day ahead market has very dam-ageable consequences for competition, then for consumers, since firm 1 has now complete freedom in the ex-post market. However, the ex post market is shrinked since the quantity $\widetilde{Q}_2^C = \frac{3}{14}E$ has already been sold. It results a spot price $\widetilde{p} \simeq \frac{3}{4}z - 0.161E$ to be compared with $p \simeq z - 0.411E$ in subsection 4.2.2. We see that $\widetilde{p}$ is smaller (resp. larger) than $p$ for large (resp.small) values of $z$ and, on average, $\mathbb{E}\widetilde{p} = \mathbb{E}p = 0.589$. Since the quantity traded by each firm in each state of nature is the same as when everything is sold ex post ( $\widetilde{Q}_i^C + \widetilde{q}_i^C = Q_i^C + q_i^C$,

[7] Notice that this is the unique equilibrium of the game whereas when there is no day-ahead market there is a multiplicity of equilibria since firm 1 can produce at the same cost ex ante and ex post and everything is sold ex post as seen formerly.

$i = 1, 2$), the expected profits of the two firms and the expected net surplus and welfare are unchanged after the adjunction of the day-ahead market.

However, the average values hide some subtle changes. Firm 2 earns the same average profit, but its profit is not random since all its production is sold ex ante at an average price. In the same vein, the expectation of the spot price is the same under the two market designs but the spot price is less varying with the random shock when there is a day-ahead market. Clearly, the ex ante sales have a stabilizing effect on the ex post trade. The most interesting feature is the role of consumers, already mentioned in the cases $a_1 = a_2$. Their net surplus is higher in the day ahead setup if and only if $z > E$. Here again, the variance of the consumers' net surplus increases with the opening of the day-ahead market. Indeed, from subsection 6.3.3 in the Appendix, we have

$$V(\widetilde{S}_n) - V(S) \simeq 0,029VE^2 - 0,01E^4 + 0,01E\mathbb{E}(z^3)$$

which is non-negative by applying the Lemma.

As for firm 1, we have that $\Pi_1^C = \widetilde{\Pi}_1^C = \frac{1}{8}(z - \frac{3}{14}E)^2$ and, for firm 2, $\Pi_2^C = \frac{9}{56}E(z - \frac{1}{2}E) \neq \widetilde{\Pi}_2^C = \frac{9}{112}E^2$. Then the opening of the day-ahead market changes nothing as regards the financial risks held by the flexible firm and provides full insurance to the inflexible firm at the expense of the consumers.

**Proposition 4.4.** *Under the specification (1), when a firm is totally inflexible and the other is perfectly flexible, if a day-ahead market is added to the spot market the expected gains of all agents remain unchanged but the inflexible firm is fully insured by consumers, whatever the distribution of probablities of demand.*

# 5 Conclusion

The analysis of strategic behavior when electricity producers use partially flexible technologies becomes more and more necessary given the deployment of intermittent renewable sources at large scale. The paper provides some general results on the trade-off faced by dispatchable generation plants between planning low-cost production beforehand and benefitting from last minute accurate information on residual demand. It also shows how the opening of a day-ahead market in addition to ex post trade changes the strategic choices of two firms competing in quantities. To obtain precise results it is necessary to use specific forms of cost and surplus functions. In this paper, we have assumed a quadratic surplus function and a quadratic cost function with diseconomies of scope between the planned and the adjusted outputs. With these specifications we have shown that being inflexible can be a strategic advantage since it confers credibility when planning to produce large quantities. We also have shown that the opening of a day-ahead market is always socially beneficial, profitable for consumers and detrimental to producers. However, it also transfers risks from firms to consumers.

Clearly, these results cannot be generalized since the cost and surplus functions we have used are very specific. However, they provide some hints on the paths to explore in the analysis of supply flexibility and the design of electricity markets. We must relax at least four hypotheses to investigate the robustness of the results. On the supply side, considering non quadratic cost functions would allow to introduce statistical moments higher than variance, in particular skewness since adjustment is generally more costly upwards than downwards. The case of economies of scope (that is, a negative cross second derivative of the cost function) should also be considered because starting costs and warming-up costs are essential in thermal plants. On the demand side, we have assumed linearity without analysing how the results vary if the slope of the demand function changes. With more responsiveness to price variations on the consumer side, technical flexibility becomes less essential. Finally, the transfer of risks to consumers when a day-ahead market is opened is sufficiently intriguing to justify the analysis of the case where consumers are risk averse instead of risk neutral. A discount on the day-ahead price would allow efficient risk sharing between producers and consumers.

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# 6 Appendix

## 6.1 Duopoly game without day ahead market

After substituting for the adjustment functions $q_1^C, q_2^C$ given by (23), solving the two-equation system.

$$\begin{array}{l} \mathbb{E}_z\left[p^C - \left(Q_1 + q_1^C\right)\left(3 - 2a_1\beta^{-1}\right)\right] = 0 \\ \mathbb{E}_z\left[p^C - \left(Q_2 + q_2^C\right)\left(3 - 2a_2\beta^{-1}\right)\right] = 0 \end{array}$$

gives the planned quantities

$$\boxed{\begin{array}{l} Q_1^C = \frac{E}{D}4(a_1+2)(a_2+2)\left((15+8a_1)(45+24a_1+46a_2+24a_1a_2)+8a_2^2(2+a_1)(11+6a_1)\right) \\ Q_2^C = \frac{E}{D}4(a_1+2)(a_2+2)\left((15+8a_2)(45+46a_1+24a_2+24a_1a_2)+8a_1^2(2+a_2)(11+6a_2)\right) \\ \text{where } D = \left(8a_1(2+a_2)(7+4a_2)+(15+8a_2)^2\right)\left(8a_2(2+a_1)(7+4a_1)+(15+8a_1)^2\right) - \\ \qquad 64a_1a_2(2+a_1)^2(2+a_2)^2. \end{array}} \tag{28}$$

They are denoted $Q_i^C = k_i E$, $i = 1, 2$ in subsection 4.2.2.

The equilibrium adjustments are given by

$$\left\{ \begin{array}{lll} bq_1^* & = & (3+2a_2)z - (15+8a_2)Q_1^* - 2a_2Q_2^* \\ bq_2^* & = & (3+2a_1)z - 2a_1Q_1^* - (15+8a_1)Q_2^* \end{array} \right.$$

## 6.2 Subgame perfect equilibrium in the quadratic case with $a_1 = a_2 = 1$ when there are two markets

### 6.2.1 Stage 2 equilibrium

At stage 2, given $\vec{Q} = (Q_1, Q_2)$ and $z$, the price will be $p(Q+q, z) = z - (Q+q)$. Each firm $i$ chooses $q_i$ so as to maximize:

$$\Pi_i(q_1, q_2) = (z - (Q+q))q_i - (Q_i + q_i)^2 - q_i^2 + P(Q_1, Q_2)Q_i.$$

Notice that the last term does not depend on $q_i$ and does not matter at this stage. This is concave in $q_i$, and differentiating gives:

$$\begin{cases} 6q_1 + q_2 &= z - 3Q_1 - Q_2 \\ q_1 + 6q_2 &= z - Q_1 - 3Q_2. \end{cases} \tag{29}$$

We obtain the equilibrium of stage 2:

$$\begin{cases} \widetilde{q}_1(z, \overrightarrow{Q}) &= \frac{1}{7}z - \frac{17}{35}Q_1 - \frac{3}{35}Q_2, \\ \widetilde{q}_2(z, \overrightarrow{Q}) &= \frac{1}{7}z - \frac{3}{35}Q_1 - \frac{17}{35}Q_2, \\ Q_1 + \widetilde{q}_1(z, \overrightarrow{Q}) &= \frac{1}{7}z + \frac{18}{35}Q_1 - \frac{3}{35}Q_2, \\ Q_2 + \widetilde{q}_2(z, \overrightarrow{Q}) &= \frac{1}{7}z - \frac{3}{35}Q_1 + \frac{18}{35}Q_2. \end{cases} \tag{30}$$

As a consequence,

$$p(Q + \widetilde{q}, z) = \frac{5}{7}z - \frac{3}{7}Q_1 - \frac{3}{7}Q_2,$$

and the demand function at stage 1 is:

$$P(Q_1, Q_2) = I\!E(p(Q + \widetilde{q}, z)) = \frac{5}{7}E - \frac{3}{7}Q_1 - \frac{3}{7}Q_2.$$

### 6.2.2 Stage 1 equilibrium

At stage 1, each firm $i$ chooses $Q_i$ in order to maximize:

$$\begin{aligned} \Pi_i(Q_i, Q_{-i}) &= P(Q_i, Q_{-i})Q_i + I\!E_z\Big(p(Q + \widetilde{q}_i(\vec{Q}, z), z)\widetilde{q}_i(\vec{Q}, z) - C_i(Q_i, \widetilde{q}_i(\vec{Q}, z)\Big) \\ &= (\frac{5}{7}E - \frac{3}{7}Q_i - \frac{3}{7}Q_{-i})Q_i \\ &\quad +(\frac{5}{7}z - \frac{3}{7}Q_i - \frac{3}{7}Q_{-i})(\frac{1}{7}z - \frac{17}{35}Q_i - \frac{3}{35}Q_{-i}) \\ &\quad -(Q_i + \frac{1}{7}z - \frac{17}{35}Q_i - \frac{3}{35}Q_{-i})^2 - (\frac{1}{7}z - \frac{17}{35}Q_i - \frac{3}{35}Q_{-i})^2. \end{aligned}$$

Differentiating with respect to $Q_i$ and combining the best response functions, we obtain the equilibrium outputs

$$\widetilde{Q}_1^C = \widetilde{Q}_2^C = \frac{73}{397}E \simeq 0,184E.$$

Then, one can compute the prices

$$\widetilde{P}^C \simeq 0,557E, \quad \widetilde{p}^C \simeq 0,714z - 0,158E.$$

The intraday quantities are

$$\widetilde{q}_1^C = \widetilde{q}_2^C \simeq 0,143z - 0,105E$$

and the total quantity produced by $i$ is equal to

$$\widetilde{Q}_i^C + \widetilde{q}_i^C \simeq 0,143z + 0,079E, \qquad i = 1,2$$

The resulting profit is

$$\widetilde{\Pi}_i^C \simeq 0,102E^2 - 0,090zE + 0,061z^2 \tag{31}$$

and its expected value is

$$\mathbb{E}(\widetilde{\Pi}_i^C) \simeq 0,073E^2 + 0,061V.$$

The consumers' net surplus is

$$\widetilde{S}_n^C = ((z-(\widetilde{Q}^C+\widetilde{q}^C)/2)(\widetilde{Q}^C+\widetilde{q}^C)-\widetilde{P}^C\widetilde{Q}^C-\widetilde{p}^C\widetilde{q}^C \simeq -0,251E^2+0,041z^2+0,308zE \tag{32}$$

and its expected value is

$$\mathbb{E}(\widetilde{S}_n^C) = 0,098E^2 + 0,041V.$$

Finally, the expected welfare value is

$$\mathbb{E}(\widetilde{S}_n^C + \widetilde{\Pi}^C) \simeq 0,244E^2 + 0,163V.$$

### 6.2.3 Variances when $a_1 = a_2 = 1$

The variance of profits and surplus is

- without day-ahead trade:

$$V(\Pi_i) \simeq -0,004V^2 + 0,004\mathbb{E}(z^4) - 0,011VE^2 - 0,01E^4 + 0,006E\mathbb{E}(z^3) \tag{33}$$

$$V(S_n) \simeq -0,002V^2 + 0,002\mathbb{E}(z^4) - 0,005VE^2 - 0,004E^4 + 0,003E\mathbb{E}(z^3) \tag{34}$$

- with a day-ahead market:

$$V(\widetilde{\Pi}_i) \simeq -0,004V^2 + 0,004\mathbb{E}(z^4) + 0,012VE^2 + 0,007E^4 - 0,011E\mathbb{E}(z^3) \tag{35}$$

$$V(\widetilde{S}_n) \simeq -0,002V^2 + 0,002\mathbb{E}(z^4) + 0,066VE^2 - 0,027E^4 + 0,025E\mathbb{E}(z^3) \tag{36}$$

### 6.3 Subgame perfect equilibrium in the quadratic case with $a_1 = 0, a_2 = +\infty$ when there are two markets

#### 6.3.1 Stage 2 equilibrium

Given the cost structure, its obvious that $\widetilde{q}_2^C = 0$. At stage 2, given $\vec{Q} = (Q_1, Q_2)$ and $z$, firm 1 chooses $q_1$ so as to maximize:

$$\Pi_1(\vec{Q}, q_1, 0, z) = (z - (Q_1 + Q_2 + q_1))q_1 - (Q_1 + q_1)^2 + P(Q_1, Q_2)Q_1.$$

which is concave in $q_1$. The first order condition is $z - 3Q_1 - Q_2 - 4q_1 = 0$, from which we derive the equilibrium of stage 2:

$$\begin{aligned} \widetilde{q}_1(\vec{Q}, z) &= \frac{z - 3Q_1 - Q_2}{4} \\ \widetilde{q}_2(\vec{Q}, z) &= 0. \end{aligned}$$

Then, the spot price is

$$\widetilde{p}(Q + \widetilde{q}_1, z) = z - (Q + \widetilde{q}_1) = \frac{3}{4}z - \frac{1}{4}Q_1 - \frac{3}{4}Q_2,$$

and the price of stage 1 is

$$\widetilde{P}(Q_1, Q_2) = \mathbb{E}(\widetilde{p}(Q + \widetilde{q}_1, z)) = \frac{3}{4}E - \frac{1}{4}Q_1 - \frac{3}{4}Q_2.$$

#### 6.3.2 Stage 1 equilibrium

At stage 1, firm 1 chooses $Q_1$ in order to maximize:

$$\begin{aligned} \mathbb{E}\widetilde{\Pi}_1(Q_1, Q_2) &= \widetilde{P}(Q_1, Q_2)Q_1 + \mathbb{E}_z\widetilde{p}(Q + \widetilde{q}_1, z)\widetilde{q}_1(\vec{Q}, z) - C_1(Q_1, \widetilde{q}_1(\vec{Q}, z)) \\ &= (\frac{3}{4}E - \frac{1}{4}Q_1 - \frac{3}{4}Q_2)Q_1 + \\ &(\frac{3}{4}z - \frac{1}{4}Q_1 - \frac{3}{4}Q_2)(\frac{1}{4}z - \frac{3}{4}Q_1 - \frac{1}{4}Q_2) - (z/4 + Q_1/4 - Q_2/4)^2. \end{aligned}$$

Differentiating with respect to $Q_1$, we find $16\frac{\partial \mathbb{E}(\Pi_1)}{\partial Q_1} = -4Q_1 < 0$, so that at equilibrium $\widetilde{Q}_1^C = 0$.

Firm 2 chooses $Q_2$ in order to maximize the non random profit

$$\widetilde{\Pi}_2 = (3/4E - Q_1/4 - 3/4Q_2)Q_2 - Q_2^2.$$

At equilibrium we obtain

$$\widetilde{Q}_2^C = \frac{3}{14}E.$$

Plotting these values in the adjustment function of firm 1, we obtain

$$\widetilde{q}_1^C = \frac{1}{4}z - \frac{3}{56}E,$$

The prices are consequently

$$\widetilde{P}^C \simeq 0,557E, \widetilde{p}^C(z) \simeq 0,75z - 0,161E$$

The random profit and the expected profit of firm 1 are

$$\widetilde{\Pi}_1^C = \frac{1}{8}(z - \frac{3}{14}E)^2, I\!E\widetilde{\Pi}_1^C \simeq 0.077E^2 + 0.125V$$

and the gains of firm 2 are

$$\widetilde{\Pi}_2^C = \frac{9}{112}E^2 = I\!E\widetilde{\Pi}_2^C.$$

The consumers' net surplus is

$$\widetilde{S}_n^C = \frac{1}{32}(z^2 + \frac{45}{7}zE - \frac{927}{14^2}E^2),$$

then, on average

$$I\!E\widetilde{S}_n^C \simeq 0.084E^2 + 0.03V.$$

Finally, the expected welfare is

$$W = S_n + \Pi = \frac{5}{32}z^2 + \frac{33}{224}zE + \frac{81}{6272}E^2.$$

$$I\!E(\widetilde{\Pi}^C + \widetilde{S}_n^C) \simeq 0.242E^2 + 0.156V$$

### 6.3.3 Variances when $a_1 = 0, a_2 = \infty$

When there is a day-ahead market, the variance of consumers' net surplus is

$$V(\widetilde{S}_n) \simeq -0,001V^2 + 0,001I\!E(z^4) + 0,026VE^2 - 0,013E^4 + 0,012EI\!E(z^3)$$

to be compared with

$$V(S) \simeq -0,001V^2 + 0,001I\!E(z^4) - 0,003VE^2 - 0,003E^4 + 0,002EI\!E(z^3)$$

when the whole ouput is sold ex post.

## Flexibility: Equilibrium quantities with a single market (Crampes and Renault, 22032021)

| $a_1$ | $a_2$ | $Q_1/E$ | $Q_2/E$ | coeff in z of $q_1$ | coeff in E of $q_1$ | coeff in z of $Q_1+q_1$ | coeff in E of $Q_1+q_1$ | Lower bound for z/E induced by firm1 | coeff in z of $q_2$ | coeff in E of $q_2$ | coeff in z of $Q_2+q_2$ | coeff in E of $Q_2+q_2$ | Lower bound for z/E induced by firm 2 | $(Q_1+q_1)/E$, expected | $(Q_2+q_2)/E$, expected | (Q+q)/E, expected | price/E, expected | coeff in V of **E**($\pi_1$) | coeff in $E^2$ of **E**($\pi_1$) | coeff in V of **E**($\pi_2$) | coeff in $E^2$ of **E**($\pi_2$) | coeff in V of **E**(Sn) | coeff in $E^2$ of **E**(Sn) | coeff in V of **E**(W) | coeff in $E^2$ of **E**(W) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | **0** | 0,2133 | 0,2133 | 0,2000 | -0,2133 | 0,2000 | 0,0000 | 0,00 | 0,2000 | -0,2133 | 0,2000 | 0,0000 | 0,00 | 0,2000 | 0,2000 | 0,4000 | 0,6000 | 0,0800 | 0,0800 | 0,0800 | 0,0800 | 0,0800 | 0,0800 | 0,2400 | 0,2400 |
| **1** | **1** | 0,2081 | 0,2081 | 0,1429 | -0,1486 | 0,1429 | 0,0595 | -0,42 | 0,1429 | -0,1486 | 0,1429 | 0,0595 | -0,42 | 0,2023 | 0,2023 | 0,4046 | 0,5954 | 0,0612 | 0,0795 | 0,0612 | 0,0795 | 0,0408 | 0,0819 | 0,1633 | 0,2408 |
| **2** | **2** | 0,2058 | 0,2058 | 0,1111 | -0,1143 | 0,1111 | 0,0915 | -0,82 | 0,1111 | -0,1143 | 0,1111 | 0,0915 | -0,82 | 0,2026 | 0,2026 | 0,4051 | 0,5949 | 0,0494 | 0,0794 | 0,0494 | 0,0794 | 0,0247 | 0,0821 | 0,1235 | 0,2410 |
| **5** | **5** | 0,2031 | 0,2031 | 0,0667 | -0,0677 | 0,0667 | 0,1354 | -2,03 | 0,0667 | -0,0677 | 0,0667 | 0,1354 | -2,03 | 0,2021 | 0,2021 | 0,4041 | 0,5959 | 0,0311 | 0,0796 | 0,0311 | 0,0796 | 0,0089 | 0,0817 | 0,0711 | 0,2408 |
| **10** | **10** | 0,2018 | 0,2018 | 0,0400 | -0,0404 | 0,0400 | 0,1614 | -4,04 | 0,0400 | -0,0404 | 0,0400 | 0,1614 | -4,04 | 0,2014 | 0,2014 | 0,4028 | 0,5972 | 0,0192 | 0,0797 | 0,0192 | 0,0797 | 0,0032 | 0,0811 | 0,0416 | 0,2406 |
| **100** | **100** | 0,2002 | 0,2002 | 0,0049 | -0,0049 | 0,0049 | 0,1953 | -40,04 | 0,0049 | -0,0049 | 0,0049 | 0,1953 | -40,04 | 0,2002 | 0,2002 | 0,4004 | 0,5996 | 0,0024 | 0,0800 | 0,0024 | 0,0800 | 0,0000 | 0,0802 | 0,0049 | 0,2401 |
| **1000** | **1000** | 0,2000 | 0,2000 | 0,0005 | -0,0005 | 0,0005 | 0,1995 | -400,04 | 0,0005 | -0,0005 | 0,0005 | 0,1995 | -400,04 | 0,2000 | 0,2000 | 0,4000 | 0,6000 | 0,0002 | 0,0800 | 0,0002 | 0,0800 | 0,0000 | 0,0800 | 0,0005 | 0,2400 |
| **0** | **1** | 0,2075 | 0,2136 | 0,2174 | -0,2260 | 0,2174 | -0,0186 | 0,09 | 0,1304 | -0,1393 | 0,1304 | 0,0743 | -0,57 | 0,1988 | 0,2047 | 0,4036 | 0,5964 | 0,0945 | 0,0791 | 0,0510 | 0,0801 | 0,0605 | 0,0814 | 0,2060 | 0,2406 |
| **0** | **2** | 0,2046 | 0,2138 | 0,2258 | -0,2322 | 0,2258 | -0,0276 | 0,12 | 0,0968 | -0,1035 | 0,0968 | 0,1104 | -1,14 | 0,1982 | 0,2071 | 0,4053 | 0,5947 | 0,1020 | 0,0786 | 0,0375 | 0,0802 | 0,0520 | 0,0822 | 0,1915 | 0,2409 |
| **0** | **5** | 0,2010 | 0,2140 | 0,2364 | -0,2400 | 0,2364 | -0,0389 | 0,16 | 0,0545 | -0,0584 | 0,0545 | 0,1556 | -2,85 | 0,1975 | 0,2102 | 0,4076 | 0,5924 | 0,1117 | 0,0780 | 0,0208 | 0,0803 | 0,0423 | 0,0831 | 0,1749 | 0,2413 |
| **0** | **10** | 0,1991 | 0,2141 | 0,2421 | -0,2442 | 0,2421 | -0,0451 | 0,19 | 0,0316 | -0,0338 | 0,0316 | 0,1803 | -5,71 | 0,1970 | 0,2119 | 0,4089 | 0,5911 | 0,1172 | 0,0776 | 0,0120 | 0,0803 | 0,0375 | 0,0836 | 0,1666 | 0,2415 |
| **0** | **100** | 0,1967 | 0,2143 | 0,2491 | -0,2493 | 0,2491 | -0,0526 | 0,21 | 0,0037 | -0,0039 | 0,0037 | 0,2103 | -57,14 | 0,1965 | 0,2140 | 0,4105 | 0,5895 | 0,1241 | 0,0772 | 0,0014 | 0,0804 | 0,0319 | 0,0843 | 0,1574 | 0,2418 |
| **0** | **1000** | 0,1965 | 0,2143 | 0,2499 | -0,2499 | 0,2499 | -0,0535 | 0,21 | 0,0004 | -0,0004 | 0,0004 | 0,2139 | -571,42 | 0,1964 | 0,2143 | 0,4107 | 0,5893 | 0,1249 | 0,0772 | 0,0001 | 0,0804 | 0,0313 | 0,0843 | 0,1564 | 0,2419 |
| **1** | **2** | 0,2054 | 0,2084 | 0,1489 | -0,1532 | 0,1489 | 0,0522 | -0,35 | 0,1064 | -0,1107 | 0,1064 | 0,0977 | -0,92 | 0,2011 | 0,2041 | 0,4052 | 0,5948 | 0,0665 | 0,0792 | 0,0453 | 0,0797 | 0,0326 | 0,0821 | 0,1444 | 0,2409 |
| **1** | **5** | 0,2020 | 0,2088 | 0,1566 | -0,1590 | 0,1566 | 0,0430 | -0,27 | 0,0602 | -0,0627 | 0,0602 | 0,1461 | -2,42 | 0,1996 | 0,2063 | 0,4059 | 0,5941 | 0,0736 | 0,0787 | 0,0254 | 0,0800 | 0,0235 | 0,0824 | 0,1225 | 0,2411 |
| **1** | **10** | 0,2002 | 0,2090 | 0,1608 | -0,1622 | 0,1608 | 0,0380 | -0,24 | 0,0350 | -0,0364 | 0,0350 | 0,1726 | -4,94 | 0,1988 | 0,2076 | 0,4064 | 0,5936 | 0,0776 | 0,0785 | 0,0147 | 0,0801 | 0,0192 | 0,0826 | 0,1114 | 0,2412 |
| **1** | **100** | 0,1980 | 0,2093 | 0,1660 | -0,1661 | 0,1660 | 0,0318 | -0,19 | 0,0041 | -0,0043 | 0,0041 | 0,2050 | -50,15 | 0,1978 | 0,2091 | 0,4069 | 0,5931 | 0,0827 | 0,0782 | 0,0017 | 0,0803 | 0,0145 | 0,0828 | 0,0988 | 0,2413 |
| **1** | **1000** | 0,1977 | 0,2093 | 0,1666 | -0,1666 | 0,1666 | 0,0311 | -0,19 | 0,0004 | -0,0004 | 0,0004 | 0,2089 | -502,24 | 0,1977 | 0,2093 | 0,4070 | 0,5930 | 0,0833 | 0,0782 | 0,0002 | 0,0803 | 0,0139 | 0,0828 | 0,0974 | 0,2413 |
| **5** | **10** | 0,2014 | 0,2034 | 0,0687 | -0,0693 | 0,0687 | 0,1321 | -1,92 | 0,0388 | -0,0394 | 0,0388 | 0,1640 | -4,23 | 0,2008 | 0,2028 | 0,4036 | 0,5964 | 0,0330 | 0,0794 | 0,0181 | 0,0798 | 0,0058 | 0,0815 | 0,0568 | 0,2407 |

Table 1

## Flexibility: Equilibrium quantities with two markets (Crampes and Renault, 22032021)

| $a_1$ | $a_2$ | $Q_1/E$ | $Q_2/E$ | coeff in z of $q_1$ | coeff in E of $q_1$ | Lower bound for z/E induced by firm1 | coeff in z of $q_2$ | coeff in E of $q_2$ | Lower bound for z/E induced by firm 2 | coeff in z of Q_1+q_1 | coeff in E of Q_1+q_1 | coeff in z of Q_2+q_2 | coeff in E of Q_2+q_2 | $(Q_1+q_1)/E$, expected | $(Q_2+q_2)/E$, expected | (Q+q)/E, expected | price/E, expected | coeff in V of **E**($\pi_1$) | coeff in $E^2$ of **E**($\pi_1$) | coeff in V of **E**($\pi_2$) | coeff in $E^2$ of **E**($\pi_2$) | coeff in V of **E**(Sn) | coeff in $E^2$ of **E**(Sn) | coeff in V of **E**(W) | coeff in $E^2$ of **E**(W) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | **0** | 0,0526 | 0,0526 | 0,2000 | -0,0421 | 0,21 | 0,2000 | -0,0421 | 0,21 | 0,2000 | 0,0105 | 0,2000 | 0,0105 | 0,2105 | 0,2105 | 0,4211 | 0,5789 | 0,0800 | 0,0776 | 0,0800 | 0,0776 | 0,0800 | 0,0886 | 0,2400 | 0,2438 |
| **1** | **1** | 0,1839 | 0,1839 | 0,1429 | -0,1051 | 0,74 | 0,1429 | -0,1051 | 0,74 | 0,1429 | 0,0788 | 0,1429 | 0,0788 | 0,2217 | 0,2217 | 0,4433 | 0,5567 | 0,0612 | 0,0728 | 0,0612 | 0,0728 | 0,0408 | 0,0983 | 0,1633 | 0,2439 |
| **2** | **2** | 0,1914 | 0,1914 | 0,1111 | -0,0851 | 0,77 | 0,1111 | -0,0851 | 0,77 | 0,1111 | 0,1063 | 0,1111 | 0,1063 | 0,2174 | 0,2174 | 0,4348 | 0,5652 | 0,0494 | 0,0742 | 0,0494 | 0,0742 | 0,0247 | 0,0945 | 0,1235 | 0,2430 |
| **5** | **5** | 0,1963 | 0,1963 | 0,0667 | -0,0524 | 0,79 | 0,0667 | -0,0524 | 0,79 | 0,0667 | 0,1440 | 0,0667 | 0,1440 | 0,2106 | 0,2106 | 0,4213 | 0,5787 | 0,0311 | 0,0765 | 0,0311 | 0,0765 | 0,0089 | 0,0887 | 0,0711 | 0,2418 |
| **10** | **10** | 0,1981 | 0,1981 | 0,0400 | -0,0317 | 0,79 | 0,0400 | -0,0317 | 0,79 | 0,0400 | 0,1664 | 0,0400 | 0,1664 | 0,2064 | 0,2064 | 0,4128 | 0,5872 | 0,0192 | 0,0779 | 0,0192 | 0,0779 | 0,0032 | 0,0852 | 0,0416 | 0,2410 |
| **100** | **100** | 0,1998 | 0,1998 | 0,0049 | -0,0039 | 0,80 | 0,0049 | -0,0039 | 0,80 | 0,0049 | 0,1959 | 0,0049 | 0,1959 | 0,2008 | 0,2008 | 0,4016 | 0,5984 | 0,0024 | 0,0797 | 0,0024 | 0,0797 | 0,0000 | 0,0806 | 0,0049 | 0,2401 |
| **1000** | **1000** | 0,2000 | 0,2000 | 0,0005 | -0,0004 | 0,80 | 0,0005 | -0,0004 | 0,80 | 0,0005 | 0,1996 | 0,0005 | 0,1996 | 0,2001 | 0,2001 | 0,4002 | 0,5998 | 0,0002 | 0,0800 | 0,0002 | 0,0800 | 0,0000 | 0,0801 | 0,0005 | 0,2400 |
| **0** | **1** | 0,0335 | 0,1949 | 0,2174 | -0,0501 | 0,23 | 0,1304 | -0,0946 | 0,73 | 0,2174 | -0,0167 | 0,1304 | 0,1002 | 0,2007 | 0,2306 | 0,4313 | 0,5687 | 0,0945 | 0,0738 | 0,0510 | 0,0767 | 0,0605 | 0,0930 | 0,2060 | 0,2436 |
| **0** | **2** | 0,0249 | 0,2037 | 0,2258 | -0,0513 | 0,23 | 0,0968 | -0,0731 | 0,76 | 0,2258 | -0,0264 | 0,0968 | 0,1306 | 0,1994 | 0,2274 | 0,4268 | 0,5732 | 0,1020 | 0,0745 | 0,0375 | 0,0775 | 0,0520 | 0,0911 | 0,1915 | 0,2431 |
| **0** | **5** | 0,0141 | 0,2098 | 0,2364 | -0,0525 | 0,22 | 0,0545 | -0,0422 | 0,77 | 0,2364 | -0,0383 | 0,0545 | 0,1675 | 0,1980 | 0,2221 | 0,4201 | 0,5799 | 0,1117 | 0,0756 | 0,0208 | 0,0787 | 0,0423 | 0,0882 | 0,1749 | 0,2426 |
| **0** | **10** | 0,0082 | 0,2119 | 0,2421 | -0,0530 | 0,22 | 0,0316 | -0,0246 | 0,78 | 0,2421 | -0,0448 | 0,0316 | 0,1873 | 0,1973 | 0,2189 | 0,4162 | 0,5838 | 0,1172 | 0,0763 | 0,0120 | 0,0794 | 0,0375 | 0,0866 | 0,1666 | 0,2423 |
| **0** | **100** | 0,0010 | 0,2140 | 0,2491 | -0,0535 | 0,21 | 0,0037 | -0,0029 | 0,79 | 0,2491 | -0,0525 | 0,0037 | 0,2112 | 0,1965 | 0,2148 | 0,4114 | 0,5886 | 0,1241 | 0,0771 | 0,0014 | 0,0802 | 0,0319 | 0,0846 | 0,1574 | 0,2419 |
| **0** | **1000** | 0,0001 | 0,2143 | 0,2499 | -0,0536 | 0,21 | 0,0004 | -0,0003 | 0,79 | 0,2499 | -0,0535 | 0,0004 | 0,2140 | 0,1964 | 0,2143 | 0,4108 | 0,5892 | 0,1249 | 0,0772 | 0,0001 | 0,0803 | 0,0313 | 0,0844 | 0,1564 | 0,2419 |
| **1** | **2** | 0,1819 | 0,1929 | 0,1489 | -0,1096 | 0,74 | 0,1064 | -0,0814 | 0,77 | 0,1489 | 0,0724 | 0,1064 | 0,1115 | 0,2213 | 0,2179 | 0,4392 | 0,5608 | 0,0665 | 0,0736 | 0,0453 | 0,0735 | 0,0326 | 0,0965 | 0,1444 | 0,2435 |
| **1** | **5** | 0,1797 | 0,1989 | 0,1566 | -0,1151 | 0,74 | 0,0602 | -0,0472 | 0,78 | 0,1566 | 0,0646 | 0,0602 | 0,1517 | 0,2212 | 0,2119 | 0,4331 | 0,5669 | 0,0736 | 0,0747 | 0,0254 | 0,0744 | 0,0235 | 0,0938 | 0,1225 | 0,2429 |
| **1** | **10** | 0,1786 | 0,2010 | 0,1608 | -0,1182 | 0,73 | 0,0350 | -0,0276 | 0,79 | 0,1608 | 0,0604 | 0,0350 | 0,1733 | 0,2213 | 0,2083 | 0,4296 | 0,5704 | 0,0776 | 0,0754 | 0,0147 | 0,0749 | 0,0192 | 0,0923 | 0,1114 | 0,2426 |
| **1** | **100** | 0,1773 | 0,2029 | 0,1660 | -0,1219 | 0,73 | 0,0041 | -0,0033 | 0,80 | 0,1660 | 0,0554 | 0,0041 | 0,1997 | 0,2213 | 0,2037 | 0,4251 | 0,5749 | 0,0827 | 0,0763 | 0,0017 | 0,0756 | 0,0145 | 0,0903 | 0,0988 | 0,2422 |
| **1** | **1000** | 0,1771 | 0,2031 | 0,1666 | -0,1223 | 0,73 | 0,0004 | -0,0003 | 0,80 | 0,1666 | 0,0548 | 0,0004 | 0,2028 | 0,2214 | 0,2032 | 0,4245 | 0,5755 | 0,0833 | 0,0764 | 0,0002 | 0,0756 | 0,0139 | 0,0901 | 0,0974 | 0,2422 |
| **5** | **10** | 0,1957 | 0,1986 | 0,0687 | -0,0539 | 0,79 | 0,0388 | -0,0307 | 0,79 | 0,0687 | 0,1418 | 0,0388 | 0,1679 | 0,2104 | 0,2067 | 0,4171 | 0,5829 | 0,0330 | 0,0773 | 0,0181 | 0,0771 | 0,0058 | 0,0870 | 0,0568 | 0,2414 |

Table 2