WORKING PAPERS

N° 1188

February 2021

# Public Safety under Imperfect Taxation

Nicolas Treich and Yuting Yang

Toulouse School of Economics

# Public Safety under Imperfect Taxation*

Nicolas Treich[†1,2] and Yuting Yang[‡1,3]

[1]Toulouse School of Economics, University Toulouse Capitole

[2]INRAE

[3]University of New Mexico

**Abstract**

Standard benefit-cost analysis often ignores distortions caused by taxation and the heterogeneity of taxpayers. In this paper, we theoretically and numerically explore the effect of imperfect taxation on the public provision of mortality risk reductions (or public safety). We show that this effect critically depends on the source of imperfection as well as on the individual utility and survival probability functions. Our simulations based on the calibration of distributional weights and applied to the COVID-19 example suggest that the value per statistical life, and in turn the optimal level of public safety, should be adjusted downwards because of imperfect taxation. However, we also identify circumstances under which this result is reversed, so that imperfect taxation cannot generically justify less public safety.

**Keywords:** Public safety, environmental health, imperfect taxation, value per statistical life, distortionary taxation, wealth inequality, risk aversion.

**JEL Classification:** D61, H21, H41, I18, Q51

*The authors would like to thank Matt Adler, Werner Antweiler, Helmuth Cremer, Philippe De Donder, James Hammitt, Liqun Liu, François Salanié, Fred Schroyen, Nicolas Werquin, the editor and three anonymous reviewers as well as the participants at FAERE and EGRIE conferences in 2017 and SBCA and WCERE conferences in 2018 for their useful comments. Nicolas Treich acknowledges research support from ANR under grant ANR-17-EURE-0010 (Investissements d'Avenir program), and the FDIR chair at the Toulouse School of Economics (TSE-P).

†Email address: nicolas.treich@inrae.fr

‡Email address: yutingyang@unm.edu

# 1 Introduction

Mortality reduction represents a significant part of the benefit of many environmental projects. For instance, it has been estimated to account for more than 90% of the monetized benefit of the Clean Air Act (U.S. Environmental Protection Agency 2011). A standard practice to evaluate this benefit is to use benefit cost analysis (BCA) based on the willingness to pay approach. In the context of the evaluation of a mortality risk reduction project, this amounts to using the value of statistical life (VSL) approach.[1] Importantly, BCA and thus the VSL approach traditionally assumes that the financing of a project is "perfect" in the sense that taxation optimally accounts for the heterogeneity of taxpayers, and does not create distortions such as labor supply distortions. In this paper, we relax this assumption and examine how the imperfections of the taxation system affect the optimal level of public safety, and in turn whether adjustments in the standard VSL approach are warranted.

Accounting for imperfect taxation in the evaluation of mortality reduction benefits in public safety projects is important for several reasons. First, it is well documented that the taxation system is imperfect in both developed and developing countries and that the degree of imperfection varies widely across the world (Tanzi and Zee 2001).[2] Second, from a policy perspective, various guidelines encourage policy evaluations to also include in BCA "distributive impacts", "equity", or "environmental justice" (European Commission 2009; U.S. Environmental Protection Agency 2016a). But it is also well known that concrete methodologies for evaluating such additional impacts remain undeveloped (Adler 2008). Moreover, safety issues usually raise strong equity concerns that call for a careful and systematic analysis of distributive impacts, as illustrated by the economic policy discussions about the COVID-19 pandemic (Adler 2020). Third, the literature in public economics has long debated in general settings the issue of the optimal provision of public goods under distortionary taxes and individual heterogeneities (Atkinson and Stiglitz 1980). It thus seems useful to examine a specific but important domain of application such as public safety provision. A starting point to do so is to develop a comparative

[1]Environmental Protection Agency (EPA) guidelines recommend using a VSL of $9.7 million in 2013 U.S dollars (U.S. Environmental Protection Agency 2010). In 2016, the U.S. Department of Transportation (DOT) uses a VSL of $9.6 million for their analyses (U.S. Department of Transportation 2016).

[2]For example, according to OECD (2017), Hungary still implements a flat income tax system, whereas other OECD countries implement a progressive tax system.

statics analysis of the effect of taxation system imperfections on the optimal level of public safety.

In our analysis, we proceed as follows. We compare the optimal level of public safety selected by a utilitarian social planner under three exogenous benchmark types of the taxation system: individual lump-sum tax (first-best), uniform lump-sum tax (uniform tax) and uniform flat tax (income tax).[3] We consider in turn two types of individual heterogeneity, namely wealth and mortality risk heterogeneity, and we also consider distortionary taxation. Our primary results are the following. Under wealth heterogeneity, compared with the first-best level of public safety, we show that the optimal level of public safety provision is usually lower under uniform taxation, but that it can be greater under income taxation. Under mortality risk heterogeneity, the comparison is generally ambiguous, as it typically depends on whether the heterogeneity concerns the baseline risk or the reduction in risk. Finally, we show that under reasonable assumptions on labor supply and the shape of the utility function, public safety under first-best is generally higher compared to distortionary taxation.

From this theoretical analysis, we conclude that the imperfection of the taxation system cannot generically justify more or less public safety provision. The basic intuition is simple. Take the wealth heterogeneity case for example. Under perfect taxation, the rich are taxed more than the poor. Imperfect taxation shifts some of the tax burden from the rich to the poor. Thus, the rich are relatively richer and would prefer more public safety, whereas the poor are relatively poorer and would prefer less public safety. Depending on the shape of the utility function, the demand for safety of the rich may, or may not, over shadow that of the poor, so that more or less safety should be provided. Hence, the answer about which effect dominates is essentially empirical.

At the end of the paper, we discuss some policy implications. In particular, we develop some simulation exercises based on the calibration of distributional weights using data from the U.S. population. These simulations indicate that the VSL, and in turn the optimal level of public safety, should be significantly adjusted downwards because of imperfect taxation. For instance, in our illustrative analysis of the COVID-19 early prevention policy, the induced weighted VSL should be reduced by about one-third under

[3] In the optimal taxation literature, endogenous taxation is typically studied to account for the issue of imperfect information (Mirrlees 1971). In our setting, we assume for simplicity that the tax system is exogenously given.

uniform taxation compared to the first-best case.

Our paper builds on two strands of literature: the VSL and the optimal provision of public good literature. First, the VSL represents the individual's marginal willingness to pay for a small reduction in mortality risk (Drèze 1962; Jones-Lee 1974). The VSL literature has examined both theoretically and empirically how VSL varies with the characteristics of individuals or of the decision-making environment (Andersson and Treich 2011; Viscusi and Aldy 2003). However, the vast majority of this literature has ignored the issue of imperfect taxation, with two notable exceptions. Pratt and Zeckhauser (1996) study the optimal allocation of safety among heterogeneous individuals under uniform taxation.[4] Armantier and Treich (2004) examine the bias induced by the standard VSL approach under uniform taxation when individuals are heterogeneous in wealth and mortality risk. However, these two papers do not compare the impact of various taxation systems. Moreover, they do not consider labor supply distortions.

Second, in the public good provision literature, a standard reference is the Pigou conjecture. Pigou (1947) states that, under distortionary taxation, the marginal benefit of the public good should be greater than the marginal production cost, implying a lower provision of the public good.[5] This conjecture led to the development of the marginal cost of public funds (MCPF) concept, which was first incorporated into Samuelson's rule for the optimal public good provision by Stiglitz and Dasgupta (1971), Diamond and Mirrlees (1971) and Atkinson and Stern (1974). If Pigou's conjecture holds, the value of MCPF should be greater than 1. However, the literature has shown that this conjecture holds only under specific settings and that the value of the MCPF depends on the relationship between the public good, labor supply, and the taxed activities (Atkinson and Stern 1974; Ballard and Fullerton 1992; Stiglitz and Dasgupta 1971). Gaube (2000) shows for instance that with heterogeneous households, equity considerations may increase public expenditure in the second-best. In practice, BCA typically recommends using an MCPF larger than one to account for imperfect taxation, which seems questionable given the

[4] Pratt and Zeckhauser (1996) focus on the collectively purchased risk reduction that can be targeted at different individuals. Under uniform taxation, they study the optimal individual safety level, whereas we study the optimal public safety (i.e. individuals consume the same amount of safety).

[5] Pigou (1947, p.33-34) noted: "Where there is indirect damage, it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure. It follows that, in general, expenditure ought not to be carried so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hands of the representative citizen."

lack of consensus in the literature. Although the MCPF has been extensively studied, we are not aware of any specific application to public safety.

Moreover, most of the literature in public economics examines the level and properties of optimal public good provision under a specific taxation system, but does not compare various systems. Thus, we contribute to this literature by considering a public safety setting, and by developing a comparative statics analysis of different taxation systems.

Before proceeding further with the model, we want to stress two strong limitations. The first is that the model is most directly applicable to government-funded public safety programs that are financed by taxes. We have in mind for instance public clean-up programs (such as those concerning sites contaminated with hazardous substances), infrastructure investments (such as those improving transport safety or reducing flood risks), water sanitation, R&D research in health, or nuclear wastes management. However, in reality, many public safety programs are based on regulations that impose costs on private firms and households. Our model is not directly relevant to this latter form of regulation that would typically require a precise modeling of the polluting industry market structure and of the households' demand for the goods produced by that industry.[6] The second important limitation is that the model is static. However, many mortality risks are long term risks which affect the dynamics of private decisions such as self-protection, insurance demand, borrowing and savings (Shepard and Zeckhauser 1984). Moreover, the optimal fiscal policy is better conceived in a dynamic setting (Motta and Rossi 2019; Werning 2007). Here, we abstract from the various complexities induced by a dynamic model with a sequence of public intervention and individual decisions, including that of time-inconsistency.

# 2 The Model

In this section, we set up the benchmark first-best model of optimal public safety provision. We consider a single period economy with $H$ individuals that differ only in wealth $w_i$ and mortality risk $1 - p_i$ $(i = 1, ..., H)$. We assume that the utility function is uniform across individuals and the bequest motive is normalized to zero. Following the VSL

[6] However, we note that we include a numerical exploration of the COVID-19 prevention policy (see 7.2), which may be somehow informative about the impact of a safety program that imposes large costs on firms and households. This numerical exercise was made possible because we could rely on existing papers that estimate the cost of public safety in terms of the global loss in GDP.

literature, the individual $i$'s expected utility is given by

$$EU_i = p_i(G)u(c_i, l_i) \tag{2.1}$$

Here $p_i(G)$ denotes the probability of survival given the level of public expenditure on safety $G$. $u(\cdot)$ is the individual's survival utility as a function of her consumption level $c_i$ and labor supply $l_i$. Under an exogenous wage rate $\omega_i$, individual $i$ has wealth $w_i \equiv \omega_i l_i$ and when the individual faces a tax rate $t_i$, the consumption level is $c_i = \omega_i l_i - t_i$. We assume that the tax is collected ex ante when everyone is alive and before knowing who will die.[7]

In this model, the utility function is assumed to be strictly positive ($u > 0$), since the bequest motive is normalized to zero and survival is assumed to be strictly preferred to death.[8] The utility function is increasing and concave in the consumption level ($u_c > 0$, $u_{cc} < 0$), and decreasing and concave in labor supply ($u_l < 0$, $u_{ll} < 0$). The survival function is positive, increasing, and weakly concave ($p_i(\cdot) > 0$, $p_i'(\cdot) > 0$, $p_i''(\cdot) \leq 0$), and $p_i(G) < 1$ for all $i$s.

In the first-best, the utilitarian social planner chooses the optimal level of public safety $G$ and the lump-sum tax rate $t_i$ (a subsidy is a negative tax) for each individual $i$ by maximizing social welfare and taking into account the individual optimal labor supply response for a given level of $t_i$. As the tax levied on each individual is lump-sum, the individual's labor supply is not distorted and the optimal decision $c_i^*(t_i)$, $l_i^*(t_i)$ satisfies: $-\frac{u_l^*}{u_c^*} = \omega_i$, where $u_c^* \equiv u_c(c_i^*(t_i), l_i^*(t_i))$ and $u_l^* \equiv u_l(c_i^*(t_i), l_i^*(t_i))$.[9] Therefore, the social

[7] This assumption is a shortcut for a more complex model, where all living individuals today finance public safety expenditures, but these individuals will die at different times in the future. Here, in our one-period model, there are only two possibilities, either the individual survives the period and can enjoy the consumption of his income, or he dies (and his utility is normalized to zero and thus independent of income).

[8] In a special case, the possibility of a bequest motive $v(c_i, l_i)$ can also be considered, with $EU_i = p_i(G)u(c_i, l_i) + (1 - p_i(G)v(c_i, l_i)$. As is common in the literature, assume $v(c_i, l_i) = ku(c_i, l_i)$ for some $k$ ($k \in [0, 1)$ for $u > 0$ and $k > 1$ for $u < 0$) (Kaplow 2005; Viscusi and Evans 1990). This means that the utility in the death state is proportionally lower than the survival utility. Therefore, for each individual, we can write $\pi_i(G) = k + (1 - k)p_i(G)$, $\pi_i(\cdot) > 0$, $\pi_i'(\cdot) > 0$, $\pi_i''(\cdot) \leq 0$, and $EU_i = \pi_i(G)u(c_i, l_i)$. It is straightforward that all results of the paper carry out under this particular case.

[9] This can be obtained by solving $\max_{c_i, l_i} u(c_i, l_i) \quad \text{s.t.} \quad c_i = w_i l_i - t_i$ for all $i$.

planner solves the following welfare maximization problem:

$$\begin{aligned}
\max_{G,\{t_i\}_{i\in\{1,\dots H\}}} \quad & \sum_{i=1}^{H} p_i(G)u(c_i^*(t_i), l_i^*(t_i)) \\
\text{s.t.} \quad & G \leq \sum_{i=1}^{H} t_i \\
& c_i^*(t_i) = \omega_i l_i^*(t_i) - t_i \quad \forall i
\end{aligned} \tag{2.2}$$

Setting the Lagrangian:

$$\mathcal{L} = \sum_{i=1}^{H} p_i(G)u(c_i^*(t_i), l_i^*(t_i)) + \mu(\sum_{i=1}^{H} t_i - G) \tag{2.3}$$

the first order conditions (focs) with respect to $t_i$ and $G$ give

$$\frac{\partial \mathcal{L}}{\partial t_i} = p_i(G^*)u_c(c_i^*(t_i), l_i^*(t_i)) - \mu = 0, \quad \forall i \tag{2.4}$$

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_{i=1}^{H} p_i'(G^*)u(c_i^*(t_i), l_i^*(t_i)) - \mu = 0, \tag{2.5}$$

where $\mu$ denotes the shadow price of one additional unit of public safety.[10] Assuming interior solutions, the system of focs has a unique set of solutions denoted by $t_i^*$ and $G^*$. The focs indicate that the social planner equalizes the expected marginal utility of consumption across individuals.

Replacing $\mu$ in equation 2.5 and rearranging, we get

$$\sum_{i=1}^{H} p_i'(G^*)VSL_i = 1 \tag{2.6}$$

$$\text{where} \quad VSL_i \equiv \frac{u^*}{p_i(G^*)u_c^*}$$

with $u^* \equiv u(c_i^*(t_i^*), l_i^*(t_i^*))$ (and similarly for $u_c^*$) and $G^* = \sum_{i=1}^{H} t_i^*$. $VSL_i$ is the VSL of individual $i$, which describes the marginal rate of substitution (MRS) between wealth and survival probability. The VSL term exhibits two standard effects, namely the dead-anyway effect and the wealth effect. The dead-anyway effect states that VSL decreases

[10] We will assume throughout that the second order conditions hold globally. See appendix A.1 for more details.

in the survival probability $p_i$, i.e., the individual facing higher risks has the incentive to increase his spending on risk reduction (Pratt and Zeckhauser 1996). The wealth effect states that VSL increases in the individual's disposable wealth $c_i$.

Equation 2.6 characterizes the efficiency condition to achieve the optimal level of public safety provision. It corresponds to Samuelson's condition of equalizing social marginal benefit to the social marginal cost of providing for the public good (Samuelson 1954).

In the following, we relax the assumption of perfect taxation. Moreover, we want to the compare optimal public safety level under first and second-best taxation. Because this comparison is very difficult in a general setting, we consider two simple alternative taxation schemes, uniform and income taxation. Moreover, we carry out the analyses with one variation at a time.[11] We examine in section 3 and 4 the case of imperfect redistribution between heterogeneous individuals with exogenous labor supply. In that case, we denote without loss of generality that $u(c_i) \equiv u(c_i, l_i)$. To illustrate the analysis, two common utility forms are used, namely constant relative risk aversion (CRRA) utility and constant absolute risk aversion (CARA) utility: with CRRA utility, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \gamma \in (0,1)$; with CARA utility, $u(c) = \frac{1-e^{-\alpha c}}{\alpha}, \alpha > 0$.[12] Two analytically important coefficients are relative risk aversion $R(c) = -c\frac{u''(c)}{u'(c)}$ and the fear of ruin $FR(c) = \frac{u(c)}{u'(c)}$ (Foncel and Treich 2005). The only class of utility function that has linear fear of ruin is CRRA, which also has $R(c) = \gamma$. In section 5, we study the case of labor effort distortion with income taxes on identical individuals with utilities $u(c, l)$.

# 3 Wealth Heterogeneity

In this section, we examine the case of individual heterogeneity in wealth only. We assume that the labor supply is exogenously determined (or consider wealth in the form of an endowment) and the survival probabilities are homogeneous across individuals. In the first-best, tax $t_i$ is levied on individual $i$. Deviating from the first-best, we consider two cases: uniform tax $t_U$ and income tax $\tau w_i$. Thus, the social planner solves the following

[11] We thus do not explore theoretically the global impact of all individual heterogeneities combined. Although we partially address this in the simulations, we do not explore the impact of possible correlations among wealth and risk. It is well documented for instance that wealthier individuals are usually less exposed to risks. We leave this for future research.

[12] Note that because we assume $u > 0$, we impose $\gamma < 1$ and $\alpha > 0$. This assumption restricts the class of CRRA and CARA utility functions that we consider in this paper.

maximization problems under the three tax systems:

First-best:

$$\begin{aligned} \max_{G_F, t_i} \quad & p(G_F) \sum_{i=1}^{H} u(w_i - t_i) \\ \text{s.t.} \quad & G_F = \sum_{i=1}^{H} t_i \end{aligned} \tag{3.1}$$

Uniform Tax:

$$\begin{aligned} \max_{G_U, t_U} \quad & p(G_U) \sum_{i=1}^{H} u(w_i - t_U) \\ \text{s.t.} \quad & G_U = H t_U \end{aligned} \tag{3.2}$$

Income Tax:

$$\begin{aligned} \max_{G_I, \tau} \quad & p(G_I) \sum_{i=1}^{H} u(w_i(1 - \tau)) \\ \text{s.t.} \quad & G_I = \tau \sum_{i=1}^{H} w_i \end{aligned} \tag{3.3}$$

Rearranging the focs, we can easily get the following equations:

$$\text{First-best:} \qquad \frac{p(G_F^*)}{p'(G_F^*)} = \sum_{i=1}^{H} \frac{u(w_i - t_i^*)}{u'(w_i - t_i^*)}, \quad \forall i \tag{3.4}$$

$$\text{Uniform Tax:} \qquad \frac{p(G_U^*)}{p'(G_U^*)} = H \frac{\sum_{i=1}^{H} u(w_i - t_U^*)}{\sum_{i=1}^{H} u'(w_i - t_U^*)}, \quad \forall i \tag{3.5}$$

$$\text{Income Tax:} \qquad \frac{p(G_I^*)}{p'(G_I^*)} = \frac{\sum_{i=1}^{H} w_i \sum_{i=1}^{H} u(w_i(1 - \tau^*))}{\sum_{i=1}^{H} w_i u'(w_i(1 - \tau^*))}, \quad \forall i \tag{3.6}$$

The focs 3.4 imply $w_i - t_i^* = w_j - t_j^* \; \forall i, j \in \{1, .., H\}$. Assuming $w_i > w_j$, we can infer that $t_i^* > t_j^*$. Thus under wealth heterogeneity, the first-best requires a higher tax on the wealthier individual. In the remainder of this section, we separately compare first-best $G_F^*$ with uniform tax $G_U^*$ and with income tax $G_I^*$.

## 3.1 First-best and uniform tax comparison

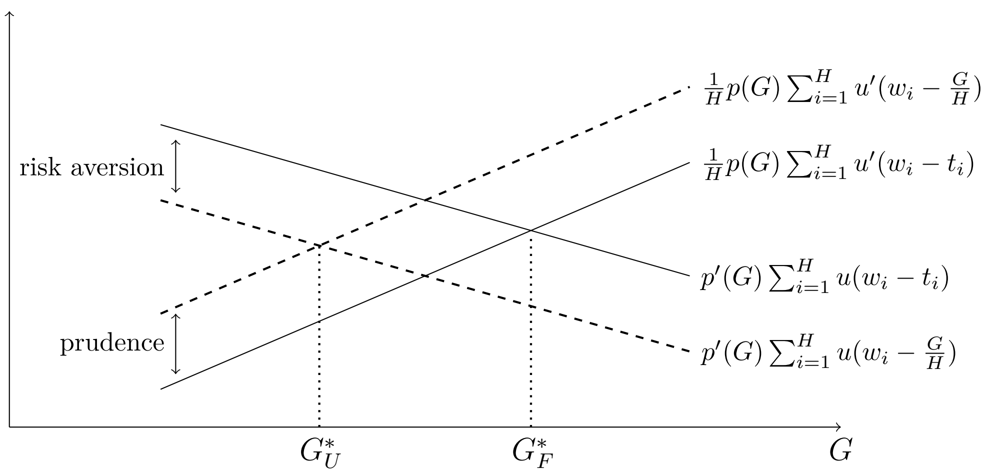
**Proposition 1.** *Under wealth heterogeneity with homogeneous risk and exogenous labor supply, with $u'''(x) \geq 0$, the optimal level of public safety in the first-best is higher than that with uniform taxation ($G_F^* > G_U^*$).*

*Proof.* See Appendix A.2. □

Proposition 1 shows that, under the common assumption of prudence (Kimball 1990), where the marginal utility of consumption is decreasing at a diminishing rate, the optimal level of public safety in the first-best is higher than that under uniform tax.

The intuition for this result is illustrated in Figure 1. In the first-best, the social planner equalizes the individual marginal utilities. When perfect taxation is not possible, the marginal utility cannot be equalized which affects both the marginal benefit and marginal cost of safety provision. Comparing to the first-best, uniform taxation decreases the marginal benefit of safety for any given level of safety due to the unequal distribution of after-tax wealth under risk aversion. In other words, saving a life has less value on average because imperfect taxation lowers the average utility in the society. Uniform taxation also increases the marginal cost of safety provision because the average marginal utility of consumption is higher (under prudence) due to sub-optimal financing. This is consistent with the Pigou conjecture. Combining the two effects, less safety is provided under uniform taxation than in the first-best.

**Figure 1** Illustration of the comparison between first-best and uniform tax




To illustrate the result with a specific and extreme example, consider two individuals with wealth 1000 and 10 respectively. They both have the same CRRA utility $u(c) = \frac{c^{0.5}}{0.5}$

and survival function $p(G) = a + (1-a)(1 - e^{-(1-a)G})$ with $a = 0.8$. In the first best, the rich is taxed 505.9, and the poor is given a subsidy of 484.1. Therefore, the total investment in public safety is 21.8. Under uniform taxation, each is taxed 5.9, and the total investment on public safety is now 11.8, which is about one-half of the level of safety in the first-best.

## 3.2 First-best and income tax comparison

**Remark 1.** *Under wealth heterogeneity with homogeneous risk and exogenous labor supply, the optimal level of public safety in the first-best could be above, below or equal to the level under income tax.*

We illustrate Remark 1 with the case of two individuals. Table 1 presents simulations of the optimal public safety provision under three specific cases with CRRA and CARA utility. With CRRA utility, the optimal level is the same under first-best and income tax. With CARA utility, the level of provision may be higher or lower in first-best than with income tax given the degree of risk aversion (parameter $\alpha$ in the utility function).

**Table 1** Simulations of the optimal public safety under wealth heterogeneity

| Utility | | CRRA | CARA | |
|---|---|---|---|---|
| Parameter value | | $\gamma = 0.5$ | $\alpha = 0.02$ | $\alpha = 0.001$ |
| Tax Rate | $t_1$ | 794.6 | 1047.5 | 794.1 |
| | $t_2$ | -705.3 | -452.6 | -705.9 |
| | $\tau(w_1 + w_2)$ | 89.3 | 306.0 | 91.5 |
| | | $G_F^* = G_I^*$ | $G_F^* > G_I^*$ | $G_F^* < G_I^*$ |

Note: Simulated in Mathematica. $p(G) = a + (1-a)(1 - e^{-0.3(1-a)G})$ with $a = 0.9$, CRRA utility $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, CARA utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $w_1 = 2000$ and $w_2 = 500$.

In the following, we further study the case of CRRA utility.

**Remark 2.** *Under wealth heterogeneity with homogeneous risk and exogenous labor supply, if the utility function satisfies CRRA, then the optimal level of public safety in the first-best is always the same as that with income taxation ($G_F^* = G_I^*$).*

*Proof.* See Appendix A.3. □

The linear fear of ruin property of CRRA utility is instrumental to this result. Notice that with CRRA utility, the RHS of equation 3.6 becomes $\frac{1-\tau^*}{1-\gamma} \sum_{i=1}^{H} w_i$. This indicates that only the sum of wealth matters for determining the optimal level of public safety. Therefore, if the sum of wealth remains the same, optimal level of public safety would always coincide in the first-best and in income tax, regardless of how the wealth is distributed.

# 4 Mortality Risk Heterogeneity

In this section, we consider individual heterogeneity on survival probability $p_i(G)$ (or risk heterogeneity $1 - p_i(G)$), with homogeneous wealth and exogenous labor supply. As we will see, the comparison depends on the specification of the survival probability function, and the results are thus not generic. We therefore present results as "remarks" in this section, and restrict to the case of $H = 2$ to facilitate the exposition. Without loss of generality, we assume $p_1(G) > p_2(G)$. The social planner's problems can be written as follows:

First-Best:

$$\begin{aligned} \max_{G_F, t_1, t_2} \quad & p_1(G_F)u(w - t_1) + p_2(G_F)u(w - t_2) \\ \text{s.t.} \quad & G_F = t_1 + t_2 \end{aligned} \tag{4.1}$$

Uniform Tax:

$$\begin{aligned} \max_{G_U, t_U} \quad & (p_1(G_U) + p_2(G_U))u(w - t_U) \\ \text{s.t.} \quad & G_U = 2t_U \end{aligned} \tag{4.2}$$

Income Tax:

$$\begin{aligned} \max_{G_I, \tau} \quad & (p_1(G_I) + p_2(G_I))u(w(1 - \tau)) \\ \text{s.t.} \quad & G_I = 2\tau w \end{aligned} \tag{4.3}$$

Note that income tax is equivalent to uniform tax in this scenario as there is no heterogeneity in wealth. Indeed, one can always set $\tau = \frac{t_U}{w}$ to have $w(1 - \tau) = w - t_U$ and obtain $G_I = G_U$. Therefore, we focus the analysis on the uniform tax case.

We show in Appendix A.4 that the ranking of $G_F^*$ and $G_U^*$ depends on the shape of the fear of ruin $(\frac{u}{u'})$ and the relationship between $\frac{p_1'(G_F^*)}{p_1(G_F^*)}$ and $\frac{p_2'(G_F^*)}{p_2(G_F^*)}$. To determine this relationship, it is important to identify where the risk heterogeneity arises. The risk heterogeneity may come from two sources: baseline risk and risk reduction. More specifically, baseline risk refers to the individual risk prior to the implementation of the public safety project, and risk reduction refers to the individual benefit from the project. In the remainder of this section, we first separately analyze heterogeneous baseline risk and heterogeneous risk reduction, and then we consider a proportional risk reduction which implies that the two sources are correlated.

**Heterogeneous Baseline Risk**

With heterogeneous baseline risk, agents have different baseline survival probability $p_i$, but receive the same level of benefit from the public safety project $\varepsilon(G)$. The survival function could be expressed as $p_i(G) = p_i + \varepsilon(G)$, with $\varepsilon(\cdot) < 1 - \max\{p_1, p_2\}$, $\varepsilon(\cdot) > 0$, $\varepsilon'(\cdot) > 0$, and $\varepsilon''(\cdot) \leq 0$.

**Remark 3.** *Under heterogeneous baseline risk ($p_i(G) = p_i + \varepsilon(G)$) with homogeneous wealth and exogenous labor supply, if fear of ruin is weakly concave, the optimal level of public safety is lower in the first-best than with uniform or income tax ($G_F^* < G_U^* = G_I^*$).*

Remark 3 can be shown directly by assuming $p_1 > p_2$. Then $\frac{p_1'(G_F^*)}{p_1(G_F^*)} = \frac{\varepsilon'(G_F^*)}{p_1+\varepsilon(G_F^*)} < \frac{\varepsilon'(G_F^*)}{p_2+\varepsilon(G_F^*)} = \frac{p_2'(G_F^*)}{p_2(G_F^*)}$. We show in Appendix A.4 that with fear of ruin weakly concave, e.g., CRRA utility, $\frac{p_1'(G_F^*)}{p_1(G_F^*)} < \frac{p_2'(G_F^*)}{p_2(G_F^*)}$ implies $G_F^* < G_U^* = G_I^*$.

We show in Table 2 that if utility is CARA, for which fear of ruin is convex, the optimal level of public safety can be greater or lower in the first-best than under uniform or income tax.

**Table 2** Simulation for heterogeneous baseline risk with CARA utility

| Utility | | CARA | |
|---|---|---|---|
| Wealth level | | $w = 1000$ | $w = 200$ |
| Tax Rate | $t_1$ | -1.3 | -116.7 |
| | $t_2$ | 245.3 | 179.9 |
| | $t_U$ | 121.9 | 32.7 |
| | | $G_F^* > G_U^* = G_I^*$ | $G_F^* < G_U^* = G_I^*$ |

Note: Simulated in Mathematica. $p(G) = p_i + \frac{0.01G}{1+0.02G}$, $p_1 = 0.5$, $p_2 = 0.3$. CARA utility $U(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $\alpha = 0.001$.

The result displayed in Remark 3 goes in the opposite direction of the Pigou conjecture. First-best equalizes the expected marginal utility of individuals by imposing a lower tax on the less-exposed individual, i.e., one with higher baseline survival probability, and a higher tax on the more-exposed individual, i.e., one with lower baseline survival probability. This is in line with the dead-anyway effect (Pratt and Zeckhauser 1996). Under uniform taxation and weakly concave fear of ruin, uniform taxation may exacerbate this effect, which results in a higher public safety level.

**Heterogeneous Risk Reduction**

With heterogeneous risk reduction, agents have the same baseline survival probability $p$, but have different degrees of benefit $\delta_i$ from the safety project. The survival function is assumed to be linear in the public safety level, thus $p_i(G) = p + \delta_i G$, $\delta_i < \frac{1-p}{G}$ for any $G$.

**Remark 4.** *Under heterogeneous linear risk reduction ($p_i(G) = p + \delta_i G$) with homogeneous wealth and exogenous labor supply, if fear of ruin is weakly convex, the optimal level of safety provision in the first-best is higher than that with uniform or income tax ($G_F^* > G_U^* = G_I^*$).*

Similarly, assuming $\delta_1 > \delta_2$, we have $\frac{p_1'(G_F)}{p_1(G_F)} = \frac{\delta_1}{p+\delta_1 G_F} > \frac{\delta_2}{p+\delta_2 G_F} = \frac{p_2'(G_F)}{p_2(G_F)}$. Thus, if fear of ruin is weakly convex, $G_F^* > G_U^* = G_I^*$.

**Proportional risk reduction**

The third case we look at is that of a proportional risk reduction, which implies a correlation between the baseline risk and the risk change. It is well documented in the environmental health literature that individuals with underlying health conditions are often more susceptible to mortality risks from air pollution and thus may benefit more from a regulatory intervention that reduces those risks (Goldberg et al. 2013; Pope III et al. 2015). Here we consider a standard proportional hazards model of risk wherein exposure to a pollutant would increase health risks in proportion to an individual's baseline risk. The survival function could be expressed as $p_i(G) = p_i + (1 - p_i)\varepsilon(G)$, with $0 \leq \varepsilon(\cdot) < 1$, $\varepsilon'(\cdot) > 0$, and $\varepsilon''(\cdot) < 0$.[13]

**Remark 5.** *Under proportional risk reduction ($p_i(G) = p_i + (1 - p_i)\varepsilon(G)$) with homogeneous wealth and exogenous labor supply, if fear of ruin is weakly concave, the optimal level of public safety is lower in the first-best than with uniform or income tax ($G_F^* < G_U^* = G_I^*$).*

[13] To see that this corresponds to a proportional risk reduction, consider the initial mortality risk exposure $x_i = 1 - p_i$. Then $p_i(G) = 1 - x_i(1 - \varepsilon(G))$, where $\varepsilon(G)$ indeed represents the reduction in proportion of the risk exposure.

Remark 5 follows by showing $\frac{p_1'(G_F^*)}{p_1(G_F^*)} = \frac{\varepsilon'(G_F^*)(1-p_1)}{p_1+(1-p_1)\varepsilon(G_F^*)} < \frac{\varepsilon'(G_F^*)(1-p_2)}{p_2+(1-p_2)\varepsilon(G_F^*)} = \frac{p_2'(G_F^*)}{p_2(G_F^*)}$ if $p_1 > p_2$.

The results in this section are closely related to those in Armantier and Treich (2004). Most specifically, Armantier and Treich show in their Proposition 1 that, under uniform taxation, there is over-provision of public safety under baseline risk heterogeneity while there is under-provision under heterogeneous changes in risk. There are some differences though with Armantier and Treich which explain why their results hold for all probability and utility functions while our results can go either way depending on these functions. First, Armantier and Treich only consider first-order approximations while our results hold in the large. Second, the comparative statics exercise is slightly different. Indeed, Armantier and Treich do not compare as we do here optimal public safety in the first-best optimum and second-best optimum. Instead, they examine whether the net benefit of an indivisible public safety project financed by uniform taxation, but evaluated by the aggregate VSL method (as if the economy was at the first best optimum), is overestimated or underestimated.

# 5 Distortionary Taxation

In this section, we focus on the distortionary aspect of imperfect taxation. The main result of this section is an application of the general results in previous studies in public economics.[14] We assume that individuals are identical (so that we drop the individual $i$'s index) and that they maximize their utility by choosing the consumption $c$ and labor supply $l$, subject to the tax rate. Because individuals are small compared to the size of the economy, they do not take into account the feedback effect of taxation.[15] With identical individuals, the first-best is equivalent to uniform lump-sum taxation. Here, income tax corresponds to the imperfect taxation case.

The social planner's problems under first-best and income tax are respectively:

[14]Extensive research has been done on the issue of public good provision with distortionary tax (Atkinson and Stern 1974; Atkinson and Stiglitz 1980; Gaube 2000, 2005).

[15]Individuals take the level of public safety as given. That is, they do not take into consideration that their labor supply level could influence the total amount of safety.

First-best:

$$\begin{aligned} \max_{G_F,t} \quad & Hp(G_F)u\Big(c_t^*(t), l_t^*(t)\Big) \\ \text{s.t.} \quad & G_F = Ht \end{aligned} \tag{5.1}$$

Income tax:

$$\begin{aligned} \max_{G_I,\tau} \quad & Hp(G_I)u\Big(c_\tau^*(\tau), l_\tau^*(\tau)\Big) \\ \text{s.t.} \quad & G_I = H\omega l_\tau^*(\tau)\tau \end{aligned} \tag{5.2}$$

$\{c_t^*(t), l_t^*(t)\}$ and $\{c_\tau^*(\tau), l_\tau^*(\tau)\}$ are the individual's optimal choice bundles given the tax rate $t$ and $\tau$. By solving the individual utility maximization problem, we know that $-\frac{u_l(c_t^*(t),l_t^*(t))}{u_c(c_t^*(t),l_t^*(t))} = \omega$ and $-\frac{u_l(c_\tau^*(\tau),l_\tau^*(\tau))}{u_c(c_\tau^*(\tau),l_\tau^*(\tau))} = \omega(1-\tau)$. We can easily obtain from the focs:
First-best:

$$Hp'(G_F^*)u\Big(c_t^*(t^*), l_t^*(t^*)\Big) = p(G_F^*)u_c\Big(c_t^*(t^*), l_t^*(t^*)\Big) \tag{5.3}$$

Income tax:

$$Hp'(G_I^*)u\Big(c_\tau^*(\tau^*), l_\tau^*(\tau^*)\Big) = p(G_I^*)u_c\Big(c_\tau^*(\tau^*), l_\tau^*(\tau^*)\Big)\frac{1}{1+\varepsilon_{l\tau^*}} \tag{5.4}$$

where $G_F^* = Ht^*$, $G_I^* = \tau^* w l_\tau^*(\tau^*)$, and $\varepsilon_{l\tau^*} = \frac{\partial l}{\partial \tau}/\frac{l}{\tau}$ denotes the labor supply elasticity of income tax.

Comparing equation 5.3 and 5.4, we can see that the ranking of $G_F^*$ and $G_I^*$ depends on the properties of the utility function $u(\cdot)$, the individual's optimal choice bundles given $t$ and $\tau$, and the elasticity term $\varepsilon_{l\tau^*}$.

**Proposition 2.** *Under distortionary tax with identical individuals, assume labor supply is an inferior good, then a sufficient condition for the optimal level of public safety in the first-best to be greater than that under income tax ($G_F^* = G_U^* > G_I^*$) is to have a weakly negative labor supply elasticity of income tax ($\varepsilon_{l\tau} \leq 0$).*

*Proof.* See Appendix A.5. □

Proposition 2 is very similar to Gaube (2000, Proposition 2). The meta-analysis on labor supply elasticity conducted by Bargain and Peichl (2016) supports the assumptions made in Proposition 2. They summarize over 90 studies that estimate labor supply elasticity in Europe and US. The majority of studies estimate a positive uncompensated labor supply elasticity of wage rate, which corresponds to $\varepsilon_{l\tau} < 0$. Moreover, in these studies, the income elasticity of labor generally has a negative sign, which is consistent with the assumption that labor is an inferior good.

**Corollary 1.** *Let $e(l) > 0$ denote the labor effort with $e'(l) > 0$ and $e''(l) > 0$. Then $G_F^* > G_I^*$ if*

1. *the labor effort is commensurable with consumption $u(c, l) = v(c - e(l))$;*
2. *or the labor effort is separable from consumption $u(c, l) = v(c) - e(l)$, and the relative risk aversion of $v(c)$ is less than 1.*

*Proof.* See Appendix A.6. □

Corollary 1 specifies two benchmark utility functional forms inducing a downward sloping labor supply. This is when the substitution effect between consumption and leisure always dominates the income effect, and less labor is provided when there is a higher tax rate. Consequently, following Proposition 2, the first-best level of public safety is always greater than under distortionary tax under these specific utility functions.

The results in this section also have implications on the marginal cost of public fund (MCPF). From equations 5.3 and 5.4, we can define the $MCPF = \frac{1}{1+\varepsilon_{l\tau^*}}$, which denotes the distortion of income tax on labor supply.[16] Depending on the sign of the labor supply elasticity, the MCPF may be greater or lower than unity.

Proposition 2 shows that if the value of MCPF is greater than one, then the second-best level of public safety is always lower than the first-best level. This is in line with the Pigou conjecture. However, the proposition is not conclusive for the case where the MCPF is lower than one. We can in fact show that this condition is not sufficient to induce more public safety in the second-best. Consider a utility function $u(c, l) = (1 - l)e^{-e^{\frac{c}{1-l}}}$ and a survival function $p(G) = a + (1 - a)(1 - e^{-bG})$, where $a = 0.8$ and $b = 0.01$.[17] Assume there are $H = 100$ identical individuals. Given these specific functional forms, $G_F^* = 297.6$ and $G_I^* = 287.8$. In this case, the $MCPF = 0.65$ with $\varepsilon_{l\tau} = 0.538$.

[16] The expression of MCPF comes from rearranging equation 5.4 to fit the modified Samuelson's rule for distortionary taxation, where the sum of the marginal rate of substitution between the public good and private consumption $\frac{Hp'(G)u}{p(G)u_c}$ equals to the marginal rate of transformation 1 multiplied by a term usually denoted as MCPF.

[17] This utility function has the property of backward bending labor supply as shown in Hanoch (1965).

# 6 Inequalities and Public Safety

As the World Inequality Report 2018 documents, wealth and income inequalities within world regions vary greatly and have been increasing in nearly all countries since 1980 (Alvaredo et al. 2018). Moreover, several studies in the public health sector document significant level of inequality in mortality risks caused by differences in both socioeconomic status and health behaviors (Laaksonen et al. 2007; Mackenbach et al. 2008). In this section, we ask: How do wealth and risk inequalities affect optimal public safety provision? And to what extent does the relationship between inequality and optimal public safety vary with tax system imperfections?

## 6.1 Wealth inequality

Here we consider the model in section 3, and we assume $H = 2$ where $w_1 = (1 + \eta)\bar{w}$, $w_2 = (1 - \eta)\bar{w}$, with $\bar{w}$ denoting the average wealth. Here $\eta \in [0, 1)$ measures wealth inequality with $\eta = 0$ indicating perfect equality.

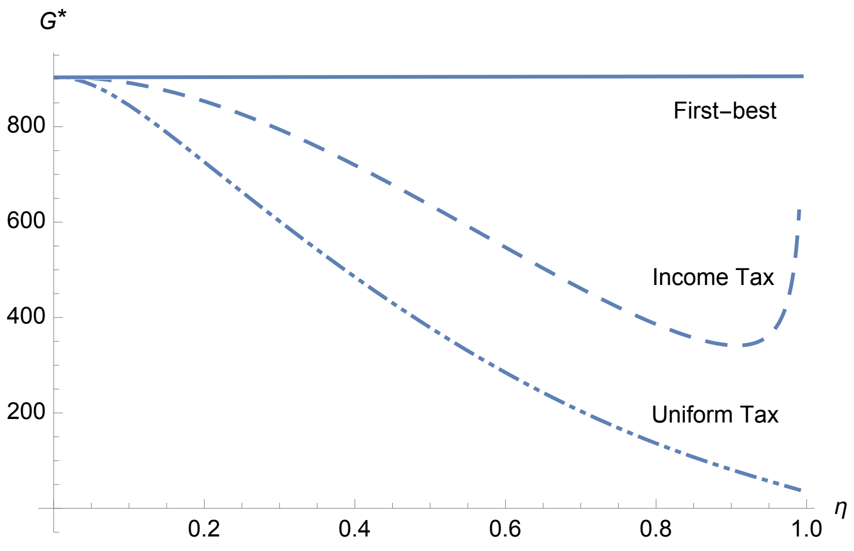
**Remark 6.** *Under homogeneous risk and exogenous labor supply, an increase in wealth inequality does not affect the first-best optimal level of public safety, but reduces the optimal safety level under uniform taxation if* $u''' \geq 0$.

*Proof.* See Appendix A.7. □

Figure 2 illustrates Remark 6: in the first-best, increasing inequality does not affect the optimal level of safety; under uniform tax, the level is monotonically decreasing with increasing inequality. The first part of Remark 6 is analogous to the well known result of private provision of public good: wealth redistribution among contributors does not change the equilibrium supply of public good (Bergstrom et al. 1986).

Figure 2 also shows that with income taxation, increasing wealth inequality may not monotonically change the optimal level of public safety. For example, with a specific CARA utility function, increasing wealth inequality first decreases and then increases the optimal level of public safety. If utility satisfies CRRA, given the result from Remark 2, the optimal level of public safety remains unchanged regardless of the degree of inequality.

**Figure 2** Effect of wealth inequality on the optimal level of public safety under the three tax systems




Note: Simulated in Mathematica. $p(x) = 0.2 + \frac{0.02x}{1+0.04x}$, CARA utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $\alpha = 0.01$ and $w = 1000$.

## 6.2 Risk inequality

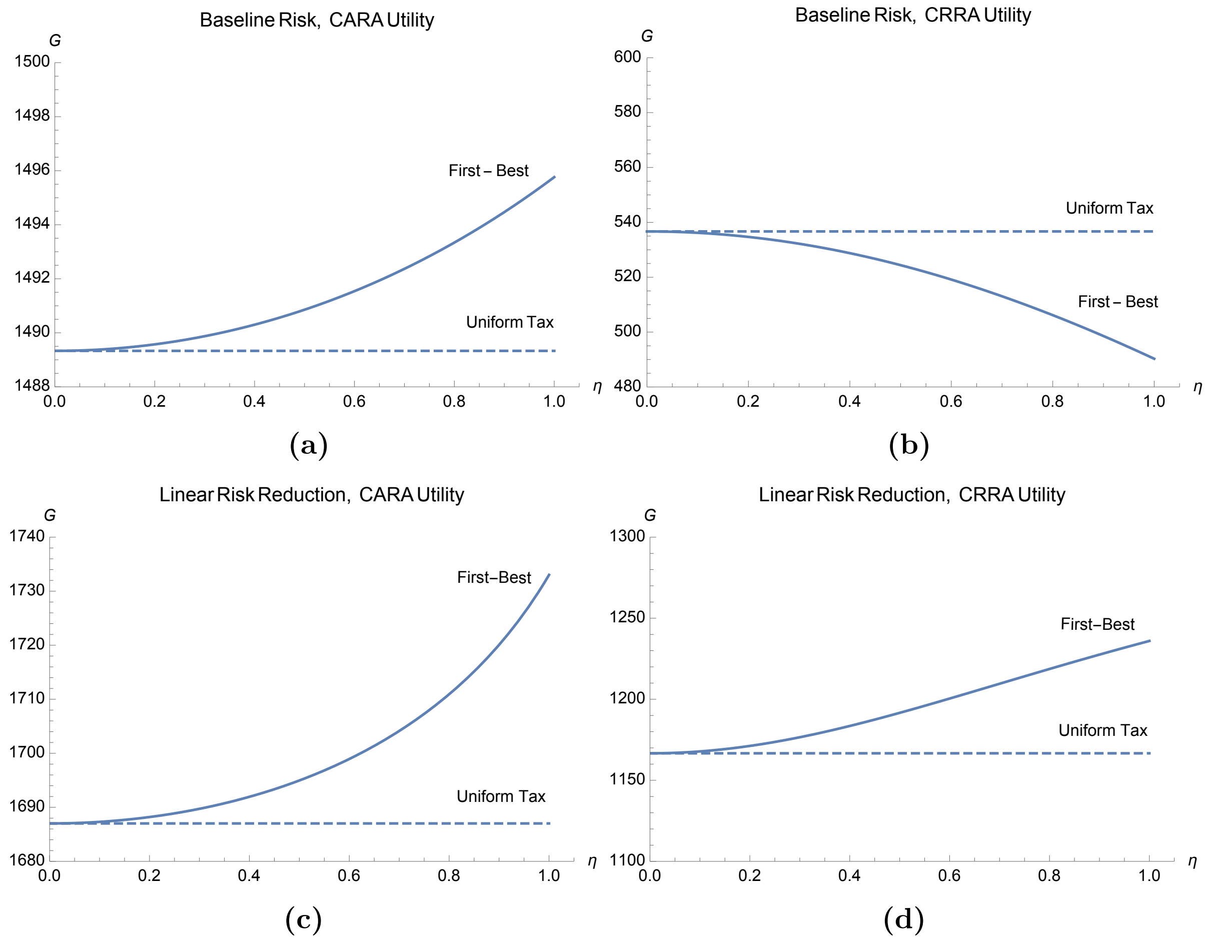
Here we consider the model in section 4, and we separately analyze the effect of baseline risk inequality and risk reduction inequality. For baseline risk inequality, we set $p_1^b(G) = (1+\eta)\bar{p} + \varepsilon(G)$ and $p_2^b(G) = (1-\eta)\bar{p} + \varepsilon(G)$. For risk reduction inequality, we set $p_1^r(G) = p + (1+\eta)\bar{\delta}G$ and $p_2^r(G) = p + (1-\eta)\bar{\delta}G$. As before, $\eta$ denotes the degree of inequality and $\eta \in [0, 1)$.

**Remark 7.** *Under homogeneous wealth and exogenous labor supply, an increase in risk inequality (both baseline risk and risk reduction) does not affect the optimal level of public safety under uniform and income tax.*

*Proof.* See Appendix A.8. $\square$

The simulations show that in the first-best, however, increasing risk inequality affects the optimal public safety level and magnifies the gap between the level in the first-best and under uniform taxation. Figure 3 shows the optimal safety level with respect to the baseline risk inequality and risk reduction inequality.

**Figure 3** Effect of risk inequality on the optimal safety level




Note: Simulated in Mathematica. $p_i(x) = p_i + \frac{0.02x}{1+0.04x}$, $p_1 = (1+\eta)\bar{p}$, $p_2 = (1-\eta)\bar{p}$, $\bar{p} = 0.25$, CARA utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $\alpha = 0.02$, CRRA utility $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, $\gamma = 0.5$ and $w = 1000$.

# 7 Policy Implications

In this section, we discuss some implications for policies and for their economic evaluation. We discuss how to adjust BCA, and thus the VSL approach, to account for imperfect taxation. Specifically, we show that imperfect taxation can be accounted for in BCA by applying distributional weights to VSLs, and we illustrate the impact of this adjustment with an evaluation of the COVID-19 prevention policy. We also discuss how to adjust the VSL transfer method and the use of the MCPF concept.

## 7.1 VSL and distributional weights under imperfect taxation

In practice, several policy-making agencies that implement public safety projects, e.g. the U.S. EPA and the U.S. DOT, commonly use the VSL to monetize mortality risk reduction benefit. The recommended VSL is usually obtained from meta-analysis of VSL estimates from stated or revealed preferences studies (U.S. Environmental Protection Agency 2016b), and is often interpreted as a population average of individual VSLs.

Recall from section 2 that the efficient condition to achieve optimality in public safety provision in the first-best is $\sum_{i=1}^{H} p_i'(G^*)VSL_i = 1$, where $VSL_i = \frac{u(w_i - t_i^*)}{p_i(G^*)u'(w_i - t_i^*)}$. Observe that when $p_i'(G^*)$ is independent from $VSL_i$, the efficiency condition can be rewritten as $\frac{1}{n}\sum_{i=1}^{H} VSL_i = \frac{1}{\sum_{i=1}^{H} p_i'(G^*)}$, which equates the average VSL to the social marginal cost of saving a life. Therefore, average VSL can determine the optimal level of public safety if taxation is perfect and $p_i'(G^*)$ is independent of $VSL_i$. In the absence of either conditions, however, the average VSL can lead to an over- or under-valuation of the social value of public safety. One way to address this concern in practice is to incorporate "distributional weights" into BCA (Adler 2016).

Currently, the official guidelines for BCA in the UK recommend using distributional weights that can be expressed as the marginal utility of each quintile as a percentage of average marginal utility (Her Majesty's Treasury 2003). However, this weighting scheme only accounts for income inequalities and does not consider other inequalities, such as risk inequality. Moreover, it does not explicitly address the question of imperfect taxation. In the following, we present a simple exercise of re-expressing the optimality conditions under imperfect taxation in terms of a weighted VSL and discuss the weighting rules.

We rewrite the optimality conditions for any taxation scheme in the following form

$$\sum_{i=1}^{H} \lambda_i p_i'(G)VSL_i = 1, \tag{7.1}$$

where $\lambda_i$ is the corresponding weight assigned to each individual. The weights vary with the tax systems and sources of heterogeneity. Table 3 shows the weights in each case.[18]

The Table shows that there is not one set of weights that can be applied to all taxation cases. In the first-best, no weight is needed as redistribution is fully taken care of through

[18] The distributional weights for the wealth heterogeneity case are obtained by rearranging equations 3.5 and 3.6. The weights for the risk heterogeneity case are obtained by rearranging the focs of 4.2 for the $H$ individuals case. These computations are straightforward and left to the readers.

**Table 3** VSL weights under different taxation systems and heterogeneities

| | Wealth Heterogeneity | Risk Heterogeneity |
|---|---|---|
| First-best | 1 | 1 |
| Uniform tax | $\frac{u'(w_i - t_U^*)}{\frac{1}{H}\sum_j u'(w_j - t_U^*)}$ | $\frac{p_i(G_U^*)}{\frac{1}{H}\sum_j p_j(G_U^*)}$ |
| Income tax | $\frac{u'(w_i(1-\tau^*))}{\sum_k \frac{w_k}{\sum_j w_j} u'(w_k(1-\tau^*))}$ | $\frac{p_i(G_I^*)}{\frac{1}{H}\sum_j p_j(G_I^*)}$ |

taxation. In the case of wealth heterogeneity, under uniform tax, the weights are similar to the recommended distributional weights in the UK ($\lambda_i = \frac{u'(w_i - t_U^*)}{\frac{1}{H}\sum_j u'(w_j - t_U^*)}$). Under income tax, the weights can be expressed as the individual marginal utility over the sum of wealth weighted ($\frac{w_k}{\sum_j w_j}$) marginal utilities. Moreover, under risk heterogeneity with uniform and income taxation, the weights should be the individual survival probability as a percentage of the population average survival probability ($\lambda_i = \frac{p_i(G_U^*)}{\frac{1}{H}\sum_j p_j(G_U^*)}$).

To illustrate how VSL changes when using distributional weights, we calibrate the weights under uniform taxation for both wealth and risk heterogeneity using U.S. data. In turn, we compare the average and the weighted level of VSLs. Table 4 exhibits the parameters used for the calibration exercise. We take the average income in the top and bottom quartile of the US population in the age group of 40 to construct a "rich" and a "poor" group. Similarly, we take the all inclusive mortality risk in the age group 40-44 to construct a "risky" and a "safe" group.[19]

[19] According to the National Vital Statistics Reports of U.S. CDC, within the age group of 40-44, the Non-hispanic American or Alaska Native has the highest number of death per 10 thousand people. The Non-hispanic Asian or Pacific Islander has the lowest number of death per 10 thousand people. We take the death rate of these two groups as the high and low mortality risk, respectively.

**Table 4** Parameters used to calibrate the distributional weights

| Parameters | Description | Source | Value |
|---|---|---|---|
| $y_{\text{rich}}$ | average annual income for people, age 40, top 25% | U.S. Census Bureau | \$80,400 |
| $y_{\text{poor}}$ | average annual income for people, age 40, bottom 25% | U.S. Census Bureau | \$27,509 |
| $y_{\text{median}}$ | median annual income for people, age 40 | U.S. Census Bureau | \$47,529 |
| $r_{\text{median}}$ | average all inclusive mortality risk, age 40-44 | U.S. CDC | 0.0022 |
| $r_{\text{risky}}$ | average all inclusive mortality risk, age 40-44, high risk | U.S. CDC | 0.0047 |
| $r_{\text{safe}}$ | average all inclusive mortality risk, age 40-44, low risk | U.S. CDC | 0.0008 |
| $T$ | average life expectancy for people of age 40 | U.S. CDC | 40 |
| $i$ | interest rate | | 1% |
| $\overline{VSL}$ | population average VSL | U.S. EPA | \$9,700,000 |

Note: The mortality risk data from the U.S. Centers for Disease Control and Prevention (U.S. CDC) is found in Kochanek et al. (2019).

We assume that individuals have CRRA utility of the form $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$. Since we do not observe the level of risk aversion, we calibrate the parameter $\gamma$ such that the average population VSL matches that of the EPA recommended level of 9.7 million dollars. The pre-tax wealth of individuals are the net present value of their life-time income $w = \sum_{t=1}^{T} \frac{y_t}{(1+i)^{t-1}}$. The calibrated degree of relative risk aversion is shown in Table 5.

**Table 5** Calibrated degree of relative risk aversion

| $\gamma$ | Description | Value |
|---|---|---|
| $\gamma_{\text{wealth}}$ | for wealth heterogeneity | 0.8114 |
| $\gamma_{\text{risk}}$ | for risk heterogeneity | 0.8338 |

Note: $\gamma_{\text{wealth}}$ is calibrated using the rich and poor life time discounted wealth and the median mortality risk. $\gamma_{\text{risk}}$ is calibrated using the median life time discounted wealth with the risky and safe mortality risk.

We then calculate the pre-tax distributional weights based on the formulas in Table 3 and compare the averages and weighted level of VSL.[20] Table 6 displays the result. The top half of the table shows the wealth heterogeneity case. We can see that under uniform taxation, the poor are assigned a higher weight than the rich. Thus, the weighted

[20] The weights are calibrated only for the uniform tax case. Indeed, with CRRA utility, the optimal safety level under income tax and the first-best are always the same under wealth heterogeneity, as demonstrated in Remark 2. For risk heterogeneity, income tax is equivalent to uniform tax.

VSL under uniform tax is lower than the unweighted level, which indicates that, if the underlying public finance for a safety project was a uniform tax, using the average VSL would result in over-provision of public safety.

Similarly, the bottom half of Table 6 shows the risk heterogeneity case. Under uniform taxation, the safe are assigned a higher weight. The weighted VSL is lower than the average VSL, although to a very small extent due to the relatively small difference between the initial risk level of the two groups.

**Table 6** Calibrated distributional weights and VSLs

| | VSL | Distributional Weights ($\lambda$) |
|---|---|---|
| Rich | \$14,455,492 | 0.590 |
| Poor | \$494,597 | 1.410 |
| Average (weighted) VSL | \$9,700,731 | \$7,753,248 |
| Risky | \$9,721,552 | 0.998 |
| Safe | \$9,691,367 | 1.002 |
| Average (weighted) VSL | \$9,706,459 | \$9,706,436 |

## 7.2 Illustrative example – COVID-19

We now illustrate the analysis of the optimal level of public safety provision with the example of policy interventions in the COVID-19 pandemic. Due to the pandemic, the mortality risk increased significantly in 2020 and led to the implementation of social distancing and lock-down rules to contain the spread of the virus. Here we simulate how stringent the social distancing rules should be, depending on how the economic cost is shared between different population cohorts.

Individuals face a general mortality risk $r_{0i}$ and a COVID-19 specific mortality risk $r_{ci}$. We assume that social distancing rules decrease $r_{ci}$ but do not affect $r_{0i}$. Imposing social distancing rules $G$ would incur an economic cost measured by percentage of GDP. The survival function $p_i(G)$ is concave in $G$.We take the example of the US. Following Adler (2020), we divide the individuals into seven age groups ranging from 20 to above 80. Within each age group there are four income groups divided by quartiles. Altogether there are 28 cohorts of individuals. The income and risk levels for each cohort are shown in the tables below.

**Table 7** Annual income by age group and income quartile

| Income ($) | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|
| 1st | 16005 | 26023 | 28488 | 29021 | 29277 | 28378 | 27002 |
| 2nd | 27757 | 44756 | 49645 | 50334 | 50193 | 48404 | 47170 |
| 3rd | 43458 | 71927 | 81126 | 84393 | 85651 | 85440 | 79041 |
| 4th | 63262 | 110391 | 131646 | 140525 | 142882 | 157759 | 129139 |

Data source: U.S. Census Bureau

**Table 8** Lifetime income by age group and income quartile

| Life time income (million $) | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|
| 1st | 1.151 | 1.058 | 0.892 | 0.700 | 0.341 | 0.341 | 0.258 |
| 2nd | 1.976 | 1.825 | 1.531 | 1.201 | 0.588 | 0.588 | 0.451 |
| 3rd | 3.294 | 3.077 | 2.611 | 2.071 | 1.020 | 1.020 | 0.756 |
| 4th | 5.416 | 5.149 | 4.447 | 3.574 | 1.742 | 1.742 | 1.235 |

Note: The lifetime income is computed using $w_i = \sum_{k=0}^{T_i - t_i} \beta^k y_{i,t_i+k}$, where $T_i$ is the life expectancy, $t_i$ is the current age, $\beta = \frac{1}{1+r}$ is the discount factor, $y_{i,t_i+k}$ is the income level at age $t_i + k$. We assume that individuals remain in their income quartile throughout their life and the discount rate $r$ is taken at 1%. The life expectancy data is taken from U.S. CDC.

**Table 9** General and COVID-19 risk by age group

| Age group risk (%) | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|
| General risk | 0.1087 | 0.1594 | 0.2674 | 0.6140 | 1.2437 | 2.7377 | 9.8789 |
| COVID-19 risk | 0.0240 | 0.0650 | 0.1220 | 0.4860 | 1.7820 | 4.1310 | 7.5330 |

Data source: General risk data are obtained from U.S. CDC; COVID-19 risk data are taken from Adler (2020)'s Table 2. The COVID-19 mortality risk is defined as the mortality risk in the absence of any government intervention, which would result in 80% of the population contracting the virus.

We calibrate the utility function and survival function using the above mentioned data. The specific functional forms are shown in Table 10.

**Table 10** Calibrated function and parameter values

| | Functional form | Parameter Value |
|---|---|---|
| Utility function | $u(c_i) = \frac{c_i^{1-\gamma}}{1-\gamma}$ | $\gamma = 0.7902$ |
| Survival function | $p_i(G) = 1 - r_{0i} - r_{ci}e^{-\alpha G}$ | $\alpha = -3.27171$ |

Note: $\gamma$ is calibrated using the life time income level and general mortality risk, such that the population average VSL equals to the recommended VSL of EPA, which amounts to 9.7 million USD. Following Acemoglu et al. (2020), we calibrate a concave survival function such that a 37.5% (resp. 10%) decrease in GDP reduces the average mortality risk from COVID-19 to 0.2% (resp. 1.05%). The calibration is done using least square method to fit the exponential survival probability function to the data points. Following Adler (2020), GDP is measured by the population weighted sum of individual income in the current year. The population weight is obtained from U.S. Census Bureau, Current Population Survey, "Age and Sex Tables," Table 1.

We simulate the optimal cost incurred by social distancing given different cost sharing schemes. To be consistent with the theoretical modelling, we consider three cases, optimal cost sharing (first-best), uniform cost sharing (uniform), and proportional cost sharing (income). Table 11 shows the optimal level of social distancing costs given different cost sharing schemes as well as the corresponding distributional weighted VSL.

**Table 11** Optimal social distancing rules given different cost sharing schemes

| | First-best | Uniform | Income |
|---|---|---|---|
| Optimal policy (% of GDP reduction) | 51.65% | 33.41% | 47.61% |
| Weighted VSL ($) | 9,608,700 | 6,056,900 | 9,606,300 |

Note: The weighted VSL is average VSL with cohort specific distributional weights. The distributional weights are defined by equation 7.1 and we consider post-policy weights. In the uniform case, $\lambda_i$ can be expressed as $\lambda_{i,uni} = \frac{p_i(G_U)u'(y_i - t_U)}{\sum_j \theta_j p_j(G_U)u'(y_j - t_U)}$. In the income case, $\lambda_{i,income} = \sum_k \theta_k y_k \frac{p_i(G_I)u'(y_i(1-\tau))}{\sum_j \theta_j y_j p_j(G_I)u'(y_j(1-\tau))}$.

The results show that if the economic cost of social distancing is shared evenly across individuals (i.e., in the uniform case), then the social distancing rules should be relaxed by about one-third compared to the first-best cost sharing case. Similarly, the distributional weighted VSL in the uniform case is also reduced by about one-third compared to the population VSL in the first-best. Note finally that there is evidence that the more economically vulnerable workers are more likely to be affected by social distancing policies (Mongey et al. 2020). Hence, although the uniform case can be viewed in general as an extreme form of imperfect taxation, it may not be a bad approximation for the cost sharing rule of the COVID-19 early prevention policies.

## 7.3 VSL transfer

VSL is used in BCA for a variety of policy evaluations. However, it is costly to conduct case-specific VSL studies. Thus, a common practice is to take the VSL value in some case studies and quantitatively adjust the value to fit the policy context, known as "benefit transfers" (U.S. Environmental Protection Agency 2011). A common practice is to adjust VSL by the income elasticity of the populations under study (Hammitt and Robinson 2011). Our analysis suggests that, in addition to income elasticity, which accounts for the wealth differences across populations, the population wealth and risk inequalities as well as tax system imperfections also need to be considered.

Meta-analyses of wage-risk studies have shown that the VSL estimates of developed countries (e.g. U.S., UK) can be more than ten times the estimates of middle-income countries (e.g. China) (Viscusi and Aldy 2003). Moreover, the extrapolated VSL under income adjustment for low-income countries (e.g. Kenya, Ethiopia) could be 50 times lower than that of the U.S. (Hammitt and Robinson 2011). Although these values already raise controversy, we argue there are two reasons to even further adjust these estimates: the inequality of wealth and risk as well as the imperfection in the taxation systems. In particular, Proposition 1 indicates that imperfect taxation justifies lower public safety. Remarks 6 and 7 show that a higher degree of wealth or mortality risk inequalities may also call for a lower safety provision.

## 7.4 The marginal cost of public funds

The marginal cost of public funds (MCPF) measures the loss incurred by raising additional revenues to finance government spending. However, no consensus has yet been reached on the value of MCPF (Dahlby 2008). In practice, agencies adopt different values of MCPF in their guidelines for BCA, but they are usually greater than unity. For example, the U.S. Office of Management and Budget (OMB) recommends using an MCPF of 1.25 (Office of Management and Budget 1992, article 11), the European Union uses a default MCPF of 1 in the absence of national guidelines (Florio et al. 2008) while the French government recommends using a median value of 1.2 (Quinet 2013).[21]

We show in section 5 that the MCPF can theoretically be greater or lower than unity depending on the labor supply elasticity. Empirical evidence suggests that the MCPF is larger than unity.[22] However, the BCA practice of accounting for tax distortions only through MCPF on the cost side appears insufficient to determine the optimal level of public safety provision. This is because distortionary taxation also affects the marginal benefit of public safety (see the LHS of equation 5.4).[23] Even if the MCPF is lower than unity, which implies a lower marginal cost of providing public safety, the optimal public safety in the second-best may still be lower than in the first-best due to the impact of distortionary taxation on the marginal benefit.

# 8 Conclusion

It is well known that BCA focuses on efficiency. It rests on the Kaldor-Hicks concept, which measures the (unweighted) sum of individuals' willingness to pay for a project. However, it is also well known that a project that does pass the BCA test may fail to increase social welfare if its financing is sub-optimal. In the practice of policy evaluation, a "common belief" is that the imperfections in the taxation system should decrease the social value of a costly project. This belief is reminiscent of Pigou's famous conjecture that distortionary taxation should induce a lower provision of the public good. Our main objective in this paper is to examine formally this common belief in the context of public safety provision.

A central result in our paper is Proposition 1. Confirming the common belief, this result shows that an (imperfect) uniform tax reduces the level of optimal public safety

[21]There are two competing approaches to the MCPF, namely the Dasgupta-Stiglitz-Atkinson-Stern (DSAS) tradition, and the Pigou-Harberger-Browning (PHB) tradition (Dahlby 2008). Our analysis follows the DSAS approach, where the social planner's budget is balanced.

[22]Although it is a theoretical possibility in accordance with Atkinson and Stern (1974) that the income effect may dominate the substitution effect resulting in a positive labor supply elasticity, there is little empirical evidence of such occurrence (Meghir and Phillips 2010). However, Manski (2014) argues that the consensus in the empirical literature may be an artifact of the strong assumptions made in the models. He states that without the knowledge of income-leisure preference, one cannot predict how labor effort may change with the tax rate (Manski 2014, p.147) .

[23]This is due to the non-separability between public safety and private consumption. If the public good and private consumption were separable, distortionary tax would have no effect on the marginal benefit of public good. See (e.g.) Atkinson and Stern (1974) for the analysis of the separable case.

compared to a first best lump-sum tax. The intuition is the following. First, the marginal cost of safety provision is higher under imperfect taxation because the tax burden on the poor is greater. Second, the marginal benefit is lower because imperfect taxation lowers the average utility in the society, and thus lowers the social value of saving a life. However, the rest of the theoretical analysis presents a much more complex picture. Indeed, we show that imperfect taxation may in fact increase, and not decrease, safety provision if imperfect taxation takes the form of an income tax or if the heterogeneity concerns individuals' mortality risks. Therefore, we must add a word of caution, and recognize that public safety need not decrease in general under imperfect taxation.

Moreover, the paper develops some preliminary numerical analyses. When we calibrate the distributional weights for imperfect taxation, we find that the weighted VSL should be lower by between 0% to 20%. In our illustrative analysis of the COVID-19 early prevention policy, the induced weighted VSL should be reduced by about one-third under uniform cost sharing compared to the first-best case. Our simulations also show that wealth inequality supports less safety provision under imperfect taxation. Furthermore, when we study the impact of distortionary taxation, we find that public safety should be reduced under a negative labor supply elasticity of the income tax, an assumption which is relatively well supported empirically. Overall, we thus suggest that the VSL should in general be adjusted downwards because of imperfect taxation. Further empirical studies are needed however to estimate more precisely the size of this adjustment.

# A Appendix

## A.1 Second order conditions

For the general framework, we assume that the second order conditions (socs) are satisfied.

For the wealth heterogeneity and distortionary taxation case, the socs of the social planner's problems (3.1, 3.2, 3.3, 5.3, and 5.4) are always satisfied under the assumptions made.

For the risk heterogeneity case, the socs of the uniform and income tax problem (4.2, 4.3) are always satisfied. For the first-best (4.1), in order to have the socs satisfied, the Hessian of 4.1 need to be negative definite. Denoting $f(t_1^*, t_2^*) \equiv p_1(t_1^* + t_2^*)u(w - t_1^*) + p_2(t_1^* + t_2^*)u(w - t_2^*)$, this would require that $\frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_1^2} < 0$, $\frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_2^2} < 0$ and $\begin{vmatrix} \frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_1^2} & \frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_1 \partial t_2} \\ \frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_1 \partial t_2} & \frac{\partial^2 f(t_1^*, t_2^*)}{\partial t_2^2} \end{vmatrix} > 0$. The first two conditions are easy to show. For the last condition, denote: $A_1 = p_1'' u_1, A_2 = p_2'' u_2, B_1 = p_1' u_1', B_2 = p_2' u_2', C_1 = p_1 u_1'', C_2 = p_2 u_2''$. If $(A_1 + A_2)(C_1 + C_2) - (B_1 - B_2)^2 - 2B_1C_2 - 2B_2C_1 + C_1C_2 > 0$, then the soc is satisfied globally.

## A.2 Proof of Proposition 1

**Proposition 1.** *Under wealth heterogeneity with homogeneous risk and exogenous labor supply, with $u'''(x) \geq 0$, the optimal level of public safety in the first-best is higher than that with uniform taxation ($G_F^* > G_U^*$).*

*Proof.* The focs of the first-best and uniform tax maximization problems equalize the marginal benefit of public safety to its marginal cost of provision. Thus equations 3.4 and 3.5 can be rewritten as

$$p'(G_F^*) \sum_{i=1}^{H} u(w_i - t_i^*) = \frac{1}{H} p(G_F^*) \sum_{i=1}^{H} u'(w_i - t_i^*) \tag{A.1}$$

$$p'(G_U^*) \sum_{i=1}^{H} u(w_i - \frac{G_U^*}{H}) = \frac{1}{H} p(G_U^*) \sum_{i=1}^{H} u'(w_i - \frac{G_U^*}{H}) \tag{A.2}$$

The left-hand side (LHS) for both equations A.1 and A.2 corresponds to the marginal benefit and the right-hand side (RHS) corresponds to the marginal cost. Observe that for any given level of $G = \sum_{i=1}^{H} t_i$ such that $w_i - t_i = w_j - t_j$, $i, j \in \{1, ..., H\}$, risk aversion implies that

$$\sum_{i=1}^{H} u(w_i - t_i) > \sum_{i=1}^{H} u(w_i - \frac{G}{H}),$$

and under prudence $u''' \geq 0$, we have

$$\sum_{i=1}^{H} u'(w_i - t_i) \leq \sum_{i=1}^{H} u'(w_i - \frac{G}{H}).$$

Therefore, for the same level of $G$, the LHS of equation A.1 is greater than that of A.2 and the RHS of A.1 is lower than that of A.2. As Figure 1 illustrates, under risk aversion and prudence, we must have $G_F^* > G_U^*$ at the optimum. □

## A.3 Proof of Remark 2

**Remark 2.** *Under wealth heterogeneity with homogeneous risk and exogenous labor supply, if the utility function satisfies CRRA, then the optimal level of public safety in the first-best is always the same as that with income taxation ($G_F^* = G_I^*$).*

*Proof.* Under CRRA, $u'(c) = c^{-\gamma}$ and the fear of ruin coefficient $FR(c) = \frac{u(c)}{u'(c)} = \frac{c}{1-\gamma}$ is linear in $c$.

Substituting the utility function into equation 3.4 and 3.6, we get

First-Best:

$$\frac{p(G_F^*)}{p'(G_F^*)} = \frac{\sum_{i=1}^{H} w_i - \sum_{i=1}^{H} t_i^*}{1-\gamma} \tag{A.3}$$

Income Tax:

$$\frac{p(G_I^*)}{p'(G_I^*)} = \frac{\sum_{i=1}^{H} w_i - \tau^* \sum_{i=1}^{H} w_i}{1-\gamma} \tag{A.4}$$

The result is then immediate because $\tau^* \sum_{i=1}^{H} w_i = \sum_{i=1}^{H} t_i^*$. □

## A.4 Risk heterogeneity

Rearranging the focs of problems 4.1 and 4.2 we get:

First-Best:

$$\frac{p_1(G_F^*)+p_2(G_F^*)}{p_1'(G_F^*)+p_2'(G_F^*)} = \underbrace{\frac{u(w-t_1^*)}{u'(w-t_1^*)}+\frac{u(w-t_2^*)}{u'(w-t_2^*)}}_{U_{FB}^*} + \underbrace{\frac{p_2(G_F^*)}{p_1'(G_F^*)+p_2'(G_F^*)}}_{A} \times \underbrace{\frac{u(w-t_1^*)-u(w-t_2^*)}{u'(w-t_1^*)}}_{B} \underbrace{\left(\frac{p_1'(G_F^*)}{p_1(G_F^*)}-\frac{p_2'(G_F^*)}{p_2(G_F^*)}\right)}_{C} \tag{A.5}$$

$$p_1(G_F^*)u'(w-t_1^*) = p_2(G_F^*)u'(w-t_2^*) \tag{A.6}$$

Uniform Tax:

$$\frac{p_1(G_U^*)+p_2(G_U^*)}{p_1'(G_U^*)+p_2'(G_U^*)} = \underbrace{\frac{2u(w-t_U^*)}{u'(w-t_U^*)}}_{U_{Uni}^*} \tag{A.7}$$

Again, we are interested in comparing $G_F^*$ and $G_U^*$. As the LHS of equations A.5 and A.7 are of the same form and are increasing functions of $G$, we only need to examine the RHS of the equations.

Denote $\hat{t_U}$ and $\hat{U}_{Uni}$ such that $\hat{t_U} = \frac{t_1^*+t_2^*}{2}$ and $\hat{U}_{Uni} = \frac{2u(w-\hat{t_U})}{u'(w-\hat{t_U})}$. If fear of ruin ($\frac{u}{u'}$) is weakly convex, $U_{FB}^* \geq \hat{U}_{Uni}$. Given our assumptions on the functional forms, we know that $A > 0$ and $B > 0$. Thus the ranking of $G_F^*$ and $G_U^*$ depends on the sign of $C$. If $C \geq 0$, that is $\frac{p_1'(G_F^*)}{p_1(G_F^*)} \geq \frac{p_2'(G_F^*)}{p_2(G_F^*)}$, the RHS of A.5 is greater than the RHS of A.7 when $t_U^* = \hat{t_U}$. Therefore, it must be that $t_U^* < \hat{t_U}$ and $G_U^* < G_F^*$. If $C < 0$, i.e., $\frac{p_1'(G_F^*)}{p_1(G_F^*)} < \frac{p_2'(G_F^*)}{p_2(G_F^*)}$, a sufficient condition for the RHS of A.5 to be lower than A.7 is fear of ruin weakly concave ($U_{FB}^* \leq \hat{U}_{Uni}$). In this case, $t_U^* > \hat{t_U}$ and $G_U^* > G_F^*$.

## A.5 Proof of Proposition 2

**Proposition 2.** *Under distortionary tax with identical individuals, assume labor supply is an inferior good, then a sufficient condition for the optimal level of public safety in the first-best to be greater than that under income tax ($G_F^* = G_U^* > G_I^*$) is to have a weakly negative labor supply elasticity of income tax ($\varepsilon_{l\tau} \leq 0$).*

*Proof.* Equation 5.3 gives the implicit value of $G_F^*$. Take $\hat{G}_I = G_F^*$, we can construct a $\hat{\tau}$ such that $\hat{\tau}\omega l_\tau^*(\hat{\tau}) = \hat{G}_I$. Given equation 5.4, if we determine whether

$\frac{Hp'(\hat{G}_I)u(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))}{p(\hat{G}_I)u_c(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))}(1+\varepsilon_{l\hat{\tau}}) \gtreqless 1$, then we can conclude about the ranking between $G^*_F$ and $G^*_I$. We get the following equation by substituting $\frac{Hp'(\hat{G}_I)}{p(\hat{G}_I)}$ using equation 5.3.

$$\frac{Hp'(\hat{G}_I)u(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))}{p(\hat{G}_I)u_c(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))}(1+\varepsilon_{l\hat{\tau}}) = \frac{u_c(c^*_t(t^*),l^*_t(t^*))}{u(c^*_t(t^*),l^*_t(t^*))}\frac{u(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))}{u_c(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))}(1+\varepsilon_{l\hat{\tau}}) \tag{A.8}$$

Therefore,

$$G^*_F \begin{cases} > G^*_I & \text{if } \varepsilon_{l\hat{\tau}} < \frac{FR_t}{FR_\tau} - 1 \\ \leq G^*_I & \text{if } \varepsilon_{l\hat{\tau}} \geq \frac{FR_t}{FR_\tau} - 1 \end{cases}$$

where $FR_t = \frac{u(c^*_t(t^*),l^*_t(t^*))}{u_c(c^*_t(t^*),l^*_t(t^*))}$ and $FR_\tau = \frac{u(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))}{u_c(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))}$.

We show that for a lump-sum tax and an income tax that obtains the same level of tax revenue, the fear of ruin under the lump-sum tax is strictly higher than under the income tax, $FR_t > FR_\tau$, if labor is an inferior good. To compare $FR_t$ and $FR_\tau$, we show that $u(c^*_t(t^*),l^*_t(t^*)) > u(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))$ and $u_c(c^*_t(t^*),l^*_t(t^*)) < u_c(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))$. Given the same level of tax revenue, it is obvious that the lump-sum tax can achieve a strictly higher utility level than the income tax ($u(c^*_t(t^*),l^*_t(t^*)) > u(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))$). It is easy to show that for the same level of tax revenue, $c^*_t(t^*) > c^*_\tau(\hat{\tau})$ and $l^*_t(t^*) > l^*_\tau(\hat{\tau})$. Taking the full derivative of $u_c$ we get

$$\mathrm{d}u_c = u_{cc}\mathrm{d}c + u_{cl}\mathrm{d}l = (wu_{cc} + u_{cl})\mathrm{d}l \tag{A.9}$$

The last equality is obtained by substituting $\mathrm{d}c = \omega \mathrm{d}l$ from the budget constraint. Therefore, $u_c(c^*_t(t^*),l^*_t(t^*)) < u_c(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))$ iff $u_{cl} + \omega u_{cc} < 0$. From the foc of the individual utility maximization in the first-best, we have $F(l,t) = u_c(c^*_t,l^*_t)\omega + u_l(c^*_t,l^*_t) = 0$. Using the implicit function theorem we get

$$\frac{\partial l}{\partial t} = -\frac{\frac{\partial F(l,t)}{\partial t}}{\frac{\partial F(l,t)}{\partial l}} = \frac{u_{cl} + \omega u_{cc}}{u_{cc}\omega\frac{\partial c}{\partial l} + u_{cl}\omega + u_{cl}\frac{\partial c}{\partial l} + u_{ll}} \tag{A.10}$$

We know that the denominator must be negative so that the second-order condition is satisfied. Thus, if labor is an inferior (normal) good, the numerator is negative (positive). Under the assumption that labor is an inferior good, then $u_{cl} + \omega u_{cc} < 0$, $u_c(c^*_t(t^*),l^*_t(t^*)) < u_c(c^*_\tau(\hat{\tau}),l^*_\tau(\hat{\tau}))$, and $FR_t > FR_\tau$.

Thus, $\frac{FR_t}{FR_\tau} - 1 > 0$. A sufficient condition for $G^*_F > G^*_I$ is to have $\varepsilon_{l\tau} \leq 0$. □

## A.6 Proof of Corollary 1

**Corollary 1.** *Let $e(l) > 0$ denote the labor effort with $e'(l) > 0$ and $e''(l) > 0$. Then $G_F^* > G_I^*$ if*

1. *the labor effort is commensurable with consumption $u(c, l) = v(c - e(l))$;*

2. *or the labor effort is separable from consumption $u(c, l) = v(c) - e(l)$, and the relative risk aversion of $v(c)$ is less than 1.*

*Proof.* As $\varepsilon_{l\tau^*} = \frac{\partial l}{\partial \tau} / \frac{l}{\tau}$ and $\frac{l_t^*(\tau)}{\tau^*} > 0$, to obtain the sign of $\varepsilon_{l\tau^*}$, we just need to sign $\frac{\partial l^*(\tau)}{\partial \tau}$.

1. With commensurable labor effort, $u(c, l) = v(c - e(l))$, $c = wl(1 - \tau)$, we get

$$\frac{\partial l^*(\tau)}{\partial \tau^*} = -\frac{w}{e''(l^*(\tau))} \tag{A.11}$$

   By assumption, $e''(l) > 0$, then $\frac{\partial l^*(\tau)}{\partial \tau^*} < 0$ which implies $\varepsilon_{l\tau^*} < 0$.

   We can also check that labor is an inferior good under commensurable labor effort: $u_{cl} + wu_{cc} = v'' \times (w - e'(l)) < 0$, since $e'(l) = w(1 - \tau)$ and $v''(\cdot) < 0$.

2. With separable labor effort, $u(c, l) = v(c) - e(l)$, $c = wl(1 - \tau)$, we get

$$\frac{\partial l^*(\tau)}{\partial \tau^*} = \frac{v''(c^*(\tau))w^2 l^*(1 - \tau) + v'(c^*(\tau))w}{v''(c^*(\tau))w^2(1 - \tau)^2 - e''(l^*(\tau))} \tag{A.12}$$

   By assumption, the denominator is negative. In this case, if the relative risk aversion coefficient $R(c^*(\tau)) = -c^*(\tau)\frac{v''(c^*(\tau))}{v'(c^*(\tau))} < 1$, then the numerator of the RHS of equation A.12 is positive, which implies $\varepsilon_{l\tau^*} < 0$.

   Labor is indeed an inferior good under separable labor effort: $u_{cl} + wu_{cc} = wv''(c) < 0$.

We can easily conclude with Proposition 2 that $G_F^* > G_I^*$ under the assumptions of 1 and 2. □

## A.7 Proof of Remark 6

**Remark 6.** *Under homogeneous risk and exogenous labor supply, an increase in wealth inequality does not affect the first-best optimal level of public safety, but reduces the optimal safety level under uniform taxation if $u''' \geq 0$.*

*Proof.* In the first-best, the optimal level of taxation is characterized by $(1+\eta)\bar{w} - t_1^* = (1-\eta)\bar{w} - t_2^*$ from equation 3.4 when $H = 2$. A change in wealth inequality can be expressed as $\eta' = \eta + \Delta\eta$. The optimality condition gives $(1+\eta+\Delta\eta)\bar{w} - T_1^* = (1-\eta-\Delta\eta)\bar{w} - T_2^*$. Thus, it is straightforward that $t_1^* = T_1^* - \Delta\eta\bar{w}$ and $t_2^* = T_2^* + \Delta\eta\bar{w}$. It follows that $T_1^* + T_2^* = t_1^* + t_2^*$.

For uniform taxation, we can rewrite equation 3.5 as a function of $\eta$:

$$F(t_U^*, \eta) \equiv p(2t_U^*)(u_1' + u_2') - 2p'(2t_U^*)(u_1 + u_2) = 0 \tag{A.13}$$

where $u_1 = u((1+\eta)\bar{w} - t_U^*)$ and $u_2 = u((1-\eta)\bar{w} - t_U^*)$. Applying the implicit function theorem, it is easy to obtain that $\frac{\mathrm{d}t_U^*}{\mathrm{d}\eta} = -\frac{\frac{\partial F(t_U^*,\eta)}{\partial\eta}}{\frac{\partial F(t_U^*,\eta)}{\partial t_U^*}} < 0$ if $u''' \geq 0$. Therefore, assuming prudence, $t_U^*$ decreases in $\eta$. □

## A.8 Proof of Remark 7

**Remark 7.** *Under homogeneous wealth and exogenous labor supply, an increase in risk inequality (both baseline risk and risk reduction) does not affect the optimal level of public safety under uniform and income tax.*

*Proof.* For baseline risk inequality, equation A.7 can be rewritten as:

$$\frac{\bar{p} + \varepsilon(G_U^*)}{\varepsilon'(G_U^*)} = \frac{2u(w - t_U^*)}{u'(w - t_U^*)}.$$

As the foc of uniform tax is independent of $\eta$, $G_U$ (and equivalently $G_I$) remains constant regardless of $\eta$.

For risk reduction inequality, the LHS of A.7 can be written as $\frac{p+\bar{\delta}G_U^*}{\bar{\delta}}$, which is also independent of $\eta$. □

# References


Acemoglu, D., Chernozhukov, V., Werning, I., & Whinston, M. D. (2020). Optimal targeted lockdowns in a multi-group sir model. *NBER Working Paper*, *27102*.

Adler, M. D. (2008). Risk equity: A new proposal. *Harvard Environmental Law Review*, *32*(1), 1–47.

Adler, M. D. (2016). Cost-benefit analysis and distributional weights: An overview. *Review of Environmental Economics & Policy*, *10*(2), 264–285.

Adler, M. D. (2020). A critique of the value of statistical life. *COVID Economics*, *33*.

Alvaredo, F., Chancel, L., Piketty, T., Saez, E., & Zucman, G. (2018). *World Inequality Report 2018*. World Inequality Lab. Retrieved from http://wir2018.wid.world/files/download/wir2018-summary-english.pdf

Andersson, H., & Treich, N. (2011). The value of a statistical life. In A. De Palma, R. Lindsey, E. Quinet, & R. Vickerman (Eds.), *The Handbook of Transport Economics*. Edward Elgar Publishing.

Armantier, O., & Treich, N. (2004). Social willingness to pay, mortality risks and contingent valuation. *Journal of Risk and Uncertainty*, *29*(1), 7–19.

Atkinson, A. B., & Stern, N. H. (1974). Pigou, taxation and public goods. *The Review of Economic Studies*, *41*(1), 119–128.

Atkinson, A. B., & Stiglitz, J. E. (1980). *Lectures on Public Economics*. McGraw-Hill Book.

Ballard, C., & Fullerton, D. (1992). Distortionary taxes and the provision of public goods. *Journal of Economic Perspectives*, *6*(3), 117–131.

Bargain, O., & Peichl, A. (2016). Own-wage labor supply elasticities: Variation across time and estimation methods. *IZA Journal of Labor Economics*, *5*(1), 10.

Bergstrom, T., Blume, L., & Varian, H. (1986). On the private provision of public goods. *Journal of Public Economics*, *29*(1), 25–49.

Dahlby, B. (2008). *The Marginal Cost of Public Funds: Theory and Applications*. MIT Press.

Diamond, P. A., & Mirrlees, J. A. (1971). Optimal taxation and public production I: Production efficiency. *The American Economic Review*, *61*(1), 8–27.

Drèze, J. (1962). L'utilité sociale d'une vie humaine. *Revue Française de Recherche Opérationelle*, *6*(93), 93–118.

European Commission. (2009). *Impact Assessment Guidelines*. European Commission. Retrieved from http://ec.europa.eu/smart-regulation/impact/commission_guidelines/docs/iag_2009_en.pdf

Florio, M., Maffii, S., Atkinson, G., De Rus, G., Evans, D., Ponti, M., … Vignetti, S. (2008). *Guide to cost-benefit analysis of Investment Projects*. European Commission Directorate General Regional Policy. Retrieved from http://ec.europa.eu/regional_policy/sources/docgener/guides/cost/guide2008_en.pdf

Foncel, J., & Treich, N. (2005). Fear of ruin. *Journal of Risk and Uncertainty*, *31*(3), 289–300.

Gaube, T. (2000). When do distortionary taxes reduce the optimal supply of public goods? *Journal of Public Economics*, *76*(2), 151–180.

Gaube, T. (2005). Financing public goods with income taxation: Provision rules vs. provision level. *International Tax and Public Finance*, *12*(3), 319–334.

Goldberg, M. S., Burnett, R. T., Stieb, D. M., Brophy, J. M., Daskalopoulou, S. S., Valois, M.-F., & Brook, J. R. (2013). Associations between ambient air pollution and daily mortality among elderly persons in montreal, quebec. *Science of the Total Environment*, *463*, 931–942.

Hammitt, J. K., & Robinson, L. A. (2011). The income elasticity of the value per statistical life: Transferring estimates between high and low income populations. *Journal of Benefit-Cost Analysis*, *2*(1), 1–29.

Hanoch, G. (1965). The" backward-bending" supply of labor. *Journal of Political Economy*, *73*(6), 636–642.

Her Majesty's Treasury. (2003). *The Green Book: Appraisal and Evaluation in Central Government*. Her Majesty's Treasury.

Jones-Lee, M. W. (1974). The value of changes in the probability of death or injury. *Journal of Political Economy*, *82*, 835–849.

Kaplow, L. (2005). The value of a statistical life and the coefficient of relative risk aversion. *Journal of Risk and Uncertainty*, *31*(1), 23–34.

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica*, *58*(1), 53–73.

Kochanek, K. D., Murphy, S. L., Xu, J., & Arias, E. (2019). Deaths: Final data for 2017. *National Vital Statistics Reports*, *68*(9).

Laaksonen, M., Talala, K., Martelin, T., Rahkonen, O., Roos, E., Helakorpi, S., … Prättälä, R. (2007). Health behaviours as explanations for educational level differences in cardiovascular and all-cause mortality: A follow-up of 60 000 men and women over 23 years. *European Journal of Public Health*, *18*(1), 38–43.

Mackenbach, J. P., Stirbu, I., Roskam, A.-J. R., Schaap, M. M., Menvielle, G., Leinsalu, M., & Kunst, A. E. (2008). Socioeconomic inequalities in health in 22 European countries. *New England Journal of Medicine*, *358*(23), 2468–2481.

Manski, C. F. (2014). Identification of income–leisure preferences and evaluation of income tax policy. *Quantitative Economics*, *5*(1), 145–174.

Meghir, C., & Phillips, D. (2010). Labour supply and taxes. *Dimensions of Tax Design: The Mirrlees Review*, 202–274.

Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *The Review of Economic Studies*, *38*(2), 175–208.

Mongey, S., Pilossoph, L., & Weinberg, A. (2020). Which workers bear the burden of social distancing policies? (w27085). *National Bureau of Economic Research*.

Motta, G., & Rossi, R. (2019). Optimal fiscal policy with consumption taxation. *Journal of Money, Credit and Banking*, *51*(1), 139–161.

OECD. (2017). *Taxing Wages 2017*. Paris, France: OECD Publishing. Retrieved from http://dx.doi.org/10.1787/tax_wages-2017-en

Office of Management and Budget. (1992). *Circular A-94: Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs*. Office of Management and Budget. Retrieved from https://www.whitehouse.gov/sites/whitehouse.gov/files/omb/assets/a94/a094.pdf

Pigou, A. C. (1947). *A Study in Public Finance* (3rd). London: Macmillan and co., limited.

Pope III, C. A., Turner, M. C., Burnett, R. T., Jerrett, M., Gapstur, S. M., Diver, W. R., … Brook, R. D. (2015). Relationships between fine particulate air pollution, cardiometabolic disorders, and cardiovascular mortality. *Circulation Research*, *116*(1), 108–115.

Pratt, J. W., & Zeckhauser, R. J. (1996). Willingness to pay and the distribution of risk and wealth. *Journal of Political Economy*, *104*(4), 747–763.

Quinet, É. (2013). *L'Évaluation Socioéconomique des Investissements Publics*. Commissariat Général à la Stratégie et à la Prospective.

Samuelson, P. A. (1954). The pure theory of public expenditure. *The Review of Economics and Statistics*, *36*(4), 387–389.

Shepard, D. S., & Zeckhauser, R. J. (1984). Survival versus consumption. *Management Science*, *30*(4), 423–439.

Stiglitz, J. E., & Dasgupta, P. (1971). Differential taxation, public goods, and economic efficiency. *The Review of Economic Studies*, *38*(2), 151–174.

Tanzi, V., & Zee, H. H. (2001). *Tax Policy for Developing Countries*. International Monetary Fund.

U.S. Department of Transportation. (2016). *Guidance on Treatment of the Economic Value of a Statistical Life (VSL) in the U.S. Department of Transportation Analyses*. United States Department of Transportation. Retrieved from https://www.transportation.gov/sites/dot.gov/files/docs/2016%20Revised%20Value%20of%20a%20Statistical%20Life%20Guidance.pdf

U.S. Environmental Protection Agency. (2010). *The Guidelines for Preparing Economic Analyses*. United States Environmental Protection Agency. Retrieved from https://www.epa.gov/sites/production/files/2017-08/documents/ee-0568-50.pdf

U.S. Environmental Protection Agency. (2011). *The Benefits and Costs of the Clean Air Act from 1990 to 2020*. U.S. Environmental Protection Agency Office of Air and Radiation. Retrieved from https://www.epa.gov/sites/production/files/2015-07/documents/summaryreport.pdf

U.S. Environmental Protection Agency. (2016a). *Technical Guidance for Assessing Environmental Justice in Regulatory Analysis*. United States Environmental Protection Agency. Retrieved from https://www.epa.gov/sites/production/files/2016-06/documents/ejtg_5_6_16_v5.1.pdf

U.S. Environmental Protection Agency. (2016b). *Valuing Mortality Risk Reductions for Policy: A Meta-analytic Approach*. United States Environmental Protection Agency. Retrieved from https://yosemite.epa.gov/sab/sabproduct.nsf/0/0CA9E925C9A702F285257F3800 $File/VSL%20white%20paper_final_020516.pdf

Viscusi, W. K., & Aldy, J. E. (2003). The value of a statistical life: A critical review of market estimates throughout the world. *Journal of Risk and Uncertainty*, *27*(1), 5–76.

Viscusi, W. K., & Evans, W. N. (1990). Utility functions that depend on health status: Estimates and economic implications. *The American Economic Review*, *80*(3), 353–374.

Werning, I. (2007). Optimal fiscal policy with redistribution. *The Quarterly Journal of Economics*, *122*(3), 925–967.