Liquidity, Contagion and Financial Crisis*

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ABSTRACT

We develop a theoretical model where a redistribution of bank capital (e.g., due to reckless trading and/or faulty risk management) leads to a "freeze" of the interbank market. The fire-sale market plays a central role in spreading the crisis to the real economy. In crisis, credit rationing and liquidity hoarding appear simultaneously; endogenous levels of collateral (or margin requirements) are affected by both low fire-sale prices and high lending rates. Relative to previous analysis, this dual channel generates a stronger price and output effect. The main focus is on the policy analysis. We show that i) non-discriminating equity injections are more effective than liquidity injections, but in both the welfare effect is an order-of-magnitude lower than the price effect; ii) a discriminating policy that bails out only distressed banks is feasible but will be limited by incentive-compatibility constraints; iii) a restriction on international capital flows has an ambiguous effect on welfare.

Keywords: Debt deflation, bailout, liquidity injection

JEL Classifications: G21, G28, G33

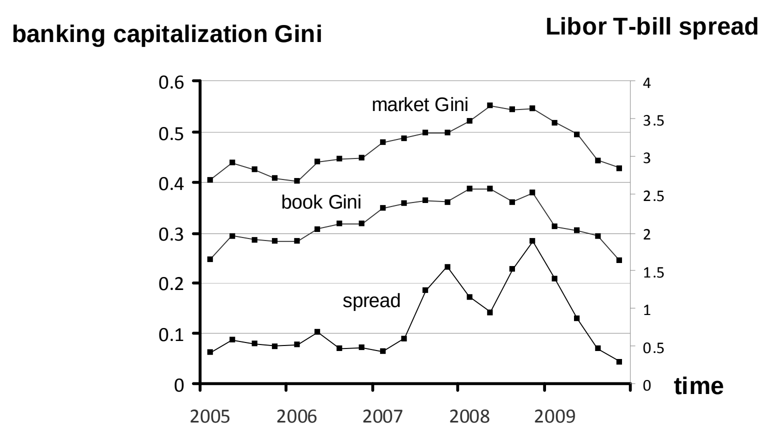
I. Introduction

The "freeze" of the interbank market following the collapse of Lehman Brothers was one of the defining moments of the recent financial crisis. First, a "dry up" of such dramatic proportions was unprecedented. Second, at that point the crisis became systemic, spreading across the entire financial industry and from there to the rest of the economy. Soon after, "haircuts" widened in the repo market, commercial banks started hoarding liquidity instead of lending it out while funding in some credit lines (e.g. home mortgages) virtually vanished. Suddenly, the possibility of a global recession became very real; see Brunnermeier, 2009, and Adrian and Shin (2009).

A growing body of theoretical literature has already produced some important insights into this episode, mainly about the mechanism through which financial distress may become contagious (for a recent survey see Babus, Carletti and Allen, 2009). Most of this literature exploits the Diamond-Dybvig (1983) conceptual framework, often identifying the triggering shock as a high level of deposit withdrawals by banks' clients. In this paper we approach the problem from a different direction. We argue that the basic shock was a redistribution of capital across financial intermediaries, probably due to reckless trading and faulty risk management. Since the trading loss of one intermediary is the gain of another, the redistribution has no direct aggregate effect. At the same time, it tends to create disparity in capitalization across intermediaries and put the interbank and other liquidity-sensitive markets under stress. At some point the market may break down and push the entire economy into recession.[1]

Figure 1

The evolution of spreads, book and market banking-capitalization Gini coefficients in the US. We use the Gini coefficient to measure differences in capitalization across percentiles of bank liabilities. The Gini is increasing in differences in capitalization. For more detail see Appendix 1.



To illustrate our approach, we investigate whether a capital redistribution actually took place in the run-up to the crisis. We calculate the degree of inequality in capitalization across US banks from 2005.Q1 to 2009.Q4 using the Gini coefficient. Conventionally, the Gini coefficient is used in order to measure inequality in income across percentiles of population;

[1]Although in the recent crisis the value of subprime mortgages had to be written down in aggregate, the amounts involved were modest in comparison to the $8 trillion loss in stock market capitalization between October 2007 and October 2008 (see Brunnermeier, 2009).

here we use it in order to measure differences in capitalization across percentiles of bank liabilities. Figure 1 shows that, Gini coefficients based on both book and market equity have increased towards the crisis,[2] even before the LIBOR-T-bill spread indicated any sign of weakness in the market. Although this empirical observation falls short of a comprehensive analysis, it is consistent with the idea that the redistribution "caused" the crisis.

The main objective of our paper is to present a tractable model that captures the salient features of financial crisis, notably liquidity dry-ups and contagion across intermediaries with a spill-over to the real sector of the economy. The emphasis here is on tractability so as to (i) allow us to assess whether a purely redistributive shock can plausibly explain the real effects of actual crises, and (ii) provide a precise welfare comparison of policy choices. The latter analysis is executed within the standard conceptual framework of Welfare Economics, but it has the attractive characteristic that under our assumption the welfare measure is reduced to expected GNP, where expectations are taken over both the boom and the bust phase of the business cycle.

Our model has the following structure, elements of which are shared with other papers (discussed in more detail below). There are speculators, such as hedge funds who can hold liquidity to buy assets cheaply in times of crisis or inject capital in the lending market. Although the banking industry has enough funding on aggregate, the redistributive shock gives rise to inter-bank lending. The terms of lending in that market are endogenously determined, and under the optimal contract the creditor has a right to repossess a fraction of assets upon default and sell them off in the fire-sale market. Repossession may be interpreted as either insolvency or a breach of margin requirements. The fire-sale price of repossessed assets is affected by the liquidity available in the market, as in Allen and Gale's (1994) model of "cash-in-the-market" pricing. The amount of liquidity speculators stock up is endogenously determined *ex ante*. We show that in equilibrium, a liquidity shortage and a financial crisis will appear *ex post* with a strictly positive probability (as for example in Gorton and Huang, 2004, or Acharya, Shin and Yorulmazer, 2009b).

This mechanism in itself gives rise to the familiar financial amplifier, whereby more fire-sales depress prices, trigger further "margin calls", increasing the amount of fire sales even more (see Allen and Gale, 2005, or Krishnamurthy, 2009 for a discussion). We go beyond the above by identifying an additional channel through which liquidity shortages spill over into the real economy, implying stronger amplification than previously thought. In particular, we show that the interbank lending rate increases during times of crisis because the rate of return on the alternative use of funds – the purchase of fire-sale assets with hoarded liquidity

[2] As might be expected from a regulated industry, the series based on market values lies above the one based on book values.

– is higher, which increases the collateral requirements even further.

This insight has several important implications. First, because the tight linkage between the lending and the fire-sales markets, some banks are credit rationed in a crisis, while others hoard liquidity. Second, the dual channel from liquidity shortage to collateral requirements (through both liquidation values and lending rates) implies that the magnitude of price drops in crisis is higher than previous models have suggested. Using plausible parameters, our model closely matches actual magnitudes (as reported by Reinhart and Rogoff (2008, 2009)) of price and GNP changes during crisis as well as their observed frequency. For example, the model's probability of a financial crisis is 7.7% while fire-sale price and GNP drop 39.3% and 3.9%, respectively, in crisis. In comparison, Reinhart and Rogoff's (2009) empirical work estimates the probability of a crisis at 7.2%, while asset prices are estimated to drop 35.5% - 55.9% and GNP drops are around 3.5% - 8.7%. To our knowledge our analysis is the first to show that redistributive shocks can have such a sizeable effect on equilibrium outcomes.

Competitive equilibrium is generically inefficient in our economy, which calls for a comparative analysis of alternative policies. During the 2008 crisis, two policy measures have been widely used: liquidity and equity injections. The former was implemented mainly through the direct purchase of mortgage-backed securities by the US Fed. Although economists have a rudimentary understanding that supporting the price of assets in the fire-sales market can alleviate a credit crunch, little is known about the order of magnitude of a liquidity injection needed or how much welfare improvement it generates. We show that by injecting large amounts of liquidity (2.6% of GNP in our numerical example) a relatively small welfare improvement can be achieved, around 0.3%. This is because a government liquidity injection crowds out the private supply of liquidity, which makes the policy neutral up to the point of full crowding out. Moreover, the welfare improvement is small, because liquidity is costly to stock up (through public borrowing) but stands idle out of crisis, which is most of the time.

The US government also provided an equity infusion so as to strengthen banks' balance sheets. Since the government cannot observe financial distress, we assume that it infuses equal amounts to all banks, distressed and non-distressed alike. Surprisingly, in spite of this "waste" we show that an equity infusion is a more effective policy than a liquidity injection because equity that is injected to non-distressed banks trickles down to the interbank market, increasing the supply of liquidity. At the same time, equity that is injected into distressed banks would improve their balance sheets and moderate fire sales. Hence, an equity infusion has both a liquidity and a balance-sheet effect, while liquidity injection lacks the latter.

An important question raised by the above discussion is whether the government can design a policy that uses less public funds by designing a policy that targets only distressed banks. Given that the government cannot directly observe distress, such a policy has to incentivize only distressed banks to apply for a bailout. This imposes an additional constraint on the designed policy. We show that forcing a bank to liquidate a certain fraction of its assets and bailing out the rest provides the right incentive. Although such a policy hits incentive constraints at a somewhat low level of implementation, it can still generate some value. This policy is thus relevant for countries that have reached high levels of debt and tax distortion at the margin (not modelled explicitly).

Finally, we investigate the role of international capital mobility in spreading financial crisis globally. We execute the analysis by fragmenting our benchmark economy into separate countries, requiring that some liquidity is earmarked for domestic use only, while allowing private speculators to trade across countries. Since contagion results from speculators withdrawing liquidity from one country (affected by a low-level shock) in order to participate in the market of another (affected by crisis and low fire-sale prices due to a high-level shock), restrictions on capital flows may mitigate contagion. But the policy also has a downside: in some cases the flow of liquidity in the direction above may prevent a crisis in the latter country without tipping the former into crisis. Hence, when some capital is not mobile liquidity may sit idly in one country while another is affected by crisis. As a result, the effect of a policy that restricts capital flows is ambiguous, improving welfare in some cases and decreasing it in others. This means that detailed information on the economy's structural parameters is needed in order to fine-tune the policy.

Related literature

There are three major approaches to the analysis of financial crisis. From McKay's (1841) "extraordinary popular delusions" to Shiller's (2000) "irrational exuberance" there is a view that financial crisis results from a black-out of human reason. We shall not consider this view here, if only because it is hard to square with a proper welfare analysis, which is one of our main objectives. Then, there is a view, initially formulated by Diamond and Dybvig (1983), followed by Morris and Shin (2004) and many others, that crises result from a coordination failure, usually associated with a (defective) financial structure that has a built-in first-mover advantage, like demand deposits, or loss limits. Lastly, there is Fisher's (1933) debt-deflation theory that builds on "two dominant factors, namely *over indebtedness* to begin with and *deflation following* soon after" (emphasis in the original text). Fisher's own focus on the role of "distressed selling" hints that the main issue is not the nominal rigidity of debt contracts per se, but rather the drop (during crisis) in the relative price of capital goods and industrial output relative to the value of corporate debt, which drains away companies' equity.[3, 4]

In the autumn of 1998, the US government managed, by sponsoring the rescue of LTCM, to stop the Asian-Russian financial crisis from spreading to the developed economies, with hardly any cost to tax-payers. That episode strengthened the view that financial crises may be a pure coordination failure. Unfortunately, such an optimistic scenario failed to materialize during the recent crisis. Governments have committed large amounts of resources with the intention of stabilizing the markets, but the effectiveness of the policy is still debated. This strongly suggests that coordination failures alone cannot provide a complete explanation of financial crises. Rather, Fisherian elements such as high levels of indebtedness, fire sales and depressed prices of capital goods seem to be important in understanding financial crisis. At the same time, it seems that multiple equilibria should play a role in a theory of financial crisis, perhaps for cases where the magnitude of the shock is relatively mild. Our model captures both of these aspects.

There are two main branches of research that are relevant to our paper. Firstly, research that identifies a propagation mechanism, whereby shocks affect the price of a production factor, which in turn affects a firm's access to external finance. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) construct such propagation models that can be interpreted as intertemporal contagion. A transitory technological shock propagates for several periods and dies out eventually.[5] A number of papers have extended this type of analysis to allow for a richer set of frictions within the canonical macro model that is then used for calibration (see for example Gertler and Kiyotaki, 2009 and references therein). To some extent, the current paper internalizes these dynamic effects into a single period, where the welfare analysis yields sharper results.[6]

Secondly, there is a rich literature that develops the role of liquidity in banking crisis and contagion via interbank lending (see Bhattacharya and Gale, 1987 or Allen and Gale, 2000, among others). A lot of this research starts from the premise that banking crises are caused by deposit withdrawals, either because there is a run, maybe following news about

[3] According to Fisher (see Chart V), from 1929 to 1933 the price level fell by 40%, nominal debt by 20% while nominal national wealth fell by 59%; so the price of capital goods must have fallen significantly more than 40%.

[4] In addition there is a literature that focuses on the role of asymmetric information in financial crisis (see Bolton, Santos and Scheinkman, 2008, 2009, and Heiderer, Hoerova, Holthausen, 2009). Since these papers are less concerned with the real effects of crisis we will not discuss them here in detail.

[5] Suarez and Sussman (1997, 2007) use a similar setting in order to generate endogenous, rational expectations, financially driven cycles that may never die out. Fire sales of assets play a pivotal role in generating the necessary price effects: start-ups buy capital goods on the second-hand market from financially distressed companies of the previous generation.

[6] There is a further trade-off in working with the canonical, infinite-horizon, representative-agent model. In order to retain tractability the latter type of model has to impose strong assumptions in one way or another.

poor future fundamentals (Allen and Gale, 1998) or there is an unexpectedly large amount of withdrawals, with welfare effects due to imperfect risk sharing.[7] Our approach differs along two dimensions. First, the triggering shock in our model is a redistribution of equity capital across banks. We believe this type of shock was more relevant in the recent crisis than a shock to consumer preferences. Second, in our model welfare effects are driven by the deadweight loss of liquidating assets and not by imperfect risk sharing. We believe that the main policy concern in the current crisis are real effects of financial frictions, and not primarily risk sharing. Our modelling approach is thus closer to Shleifer and Vishny (1992), Kiyotaki and Moore (1997), and Holmstrom and Tirole (1998), where liquidity plays a role for corporate investment and a lack of liquidity leads to a direct loss in output. Gorton and Huang (2004) summarize the two key ingredients that are necessary for the modeling of liquidity: "first, there must be a need to trade... [and] second, there is a restriction needed, namely, not all assets can be used to purchase all other assets. Buyers must be restricted to making purchases only with certain assets, "liquid" assets. This restriction is akin to a "cash-in-advance constraint".

Several authors have already done some work on integrating these two lines of research. One basic finding is that when productive assets change hands at an interim date through a spot market (rather than delivered at an ex-ante contracted price that captures some insurance opportunities), an ex-ante externality arises that leads to inefficient levels of borrowing or investment in liquidity. This type of externality has been explored in numerous papers, among others by Caballero and Krishnamurthy (2001, 2003, 2004) in the context of international finance, in a banking context by Bhattacharya and Gale (1987), Freixas, Martin and Skeie (2009), and Acharya, Shin and Yorulmazer (2009a), and in a corporate finance context by Gorton and Huang (2004), Lorenzoni (2008), Korinek (2009) and Acharya, Shin and Yorulmazer (2009b).[8]

Our paper differs from the above in several respects. Firstly, in the above papers that focus on non-banking applications, the shock that triggers the propagation mechanism is to cash flows generated by investment. In this sense it is akin to a technology shock in the Real-Business-Cycle literature. It is widely accepted, however, that price movements during crisis are far too dramatic to be explained by any technological news. Hence, we took the modeling decision to ignore any technological uncertainty. Secondly, unlike most of the papers on financial amplification, we fully endogenize the "margin requirements" that are at the heart

[7] See, however, Goodhart, Tsomocos and Sudirand (2004, 2006) who analyze financial fragility in a general equilibrium setting. There, a liquidity need is modelled through a cash-in-advance constraint, rather than shocks to consumption preferences.

[8] See also Fostel and Geanakoplos (2008a, b) for a more general analysis of efficiency properties of equilibrium in a related setting.

of the feedback between fire-sales prices and the volume of distressed selling. In an interesting paper Brunnermeier and Pedersen (2009) argue that margin requirements based on value-at-risk worsen precisely during times of crisis. We present a similar finding, although for very different reasons. In Brunnermeier and Pedersen (2009) margin requirements respond to increases in volatility. In our setting they increase because the real lending rate increases during times of crisis.[9] Finally, endogenizing contracts in a tractable way allows us to conduct a precise welfare analysis of some of the key policies actually employed during the recent crisis. This allows us to rank policies in terms of welfare improvements per dollar of taxpayer money spent, and provide a basic quantitative assessment of their likely welfare impact.

The remainder of the paper proceeds as follows. Section II sets out the model. The financial contract is solved for in Section III and Section IV characterizes the economy's equilibrium. We explore welfare and policy implications in Section V. Section VI contains an extension in which analyze the implications of market fragmentation and liberalization. Section VII concludes.

II. The Model

We start with a technical description of the model and postpone the discussion of the economic significance of the assumptions to a subsection below. Consider a small, open economy with four periods $t = 0, ..., 3$. There are two types of agents: banks, of which there is a continuum of measure one, and speculators. All agents are risk neutral and maximize total consumption (there is no discounting).

Speculators can decide at the initial date $t = 0$ to invest their funds across two assets. There is an illiquid asset that yields a riskless gross return $\rho_0 \geq 1$ at $t = 3$. Exiting from the illiquid technology prior to $t = 3$ yields a zero payoff.[10] Moreover, we assume that the payoff from that investment is non-verifiable and thus non-pledgeable, so that speculators cannot borrow against the illiquid positions. In addition there is a storage technology that yields a gross return of one every period. The storage technology corresponds to a liquid asset. We denote by F the aggregate amount of funds invested in the liquid asset and assume that speculators have sufficient funds such that there is no technological upper bound on F. The supply of liquidity at $t = 0$ is therefore perfectly elastic. Because of our assumptions on the illiquid technology, the $t = 1$ supply of liquidity is F completely inelastic.

Each bank is owned and managed by a single owner-manager. At $t = 1$ each bank gains

[9] In a recent paper Diamond and Rajan (2009) also point out that the lending rate to the real sector may increase during times of crisis, because of the high returns that can be earned in the fire-sales market. In their paper, however, there is no feedback effect from this rate increase to the "margin requirements" in the interbank market.

[10] This assumption is standard in the banking literature; see Allen and Gale (1994) among others.

access to one long-term investment project and receives an endowment of consumption goods, their capital, which they use in order to fund the project. To operate, the project requires one unit of investment good. At this stage banks may convert one unit of consumption good to one unit of investment good at no extra cost. A random fraction, θ, of banks have an endowment of $\underline{w} < 1$ (fixed deterministically), and need external funding in order to operate their projects. All other banks have a capital slack with an endowment of w^n (a random variable) that is always sufficient to fund them internally. The aggregate endowment W of the economy is fixed (deterministically), namely

$$W \equiv \theta \underline{w} + (1 - \theta) w^n = 1. \tag{1}$$

Notice that equation (1) determines w^n as a function of θ so that $w^n \geq 1$ for any realization of θ. Let $h(\theta)$ be the density function of θ with a cumulative distribution function $H(\theta)$. Assume that $H(\theta)$ is continuous and strictly increasing on its entire support $\left[0, \overline{\theta}\right]$. We assume that endowments are *non-verifiable*. As a result, no insurance scheme against capital shortage can operate.[11]

The banks' lending activity is economically viable: *if* carried to maturity each bank's project generates the same earning flows, $2y$, of consumption goods. That amount is evenly distributed over the life-cycle of the loans: y units of consumption goods at $t = 2$ and y units at $t = 3$. Some banks' projects may, however, experience a liquidity shock such that they generate zero at $t = 2$ and $2y$ at $t = 3$. We assume $y > \rho_0$. As will become evident later, this is a necessary condition for speculators to want to invest in the liquid asset. Liquidity shortages are idiosyncratic to each bank and occur with a probability of $1 - \pi$. By the law of large numbers $1 - \pi$ is thus also the fraction of banks subject to a liquidity shock. Notice the distinction between $t = 1$ capital shortage and $t = 2$ liquidity shortage.

The market for external funding operates at $t = 1$ after θ has been realized and publicly observed. We assume that earning flows from the projects are non-verifiable and cannot be pledged to a third party. As a result, the only incentive-compatible form of external funding for banks is a standard debt contract as in Hart and Moore (1998); see contract section below. We denote the fraction of the investment that a bank has to pledge as collateral by β, which the lender has the right to repossess at $t = 2$ if the contracted repayment, R, is in default. This set-up is a convenient way of capturing the important real-world feature that banks fund their long-term lending activity (the project) through short-term debt. The risk-free gross lending rate (on loans payable at $t = 2$) is endogenously determined and denoted

[11] This is in line with the assumption made in models of adverse selection in the interbank market, where banks cannot insure up front against changes in the value of their assets (see for example Heiderer, Hoerova and Holthausen, 2009 or Bolton, Santos and Scheinkman, 2008). For a detailed analysis of the role of insurance against liquidity shocks see Holmstrom and Tirole (2008).

by ρ_1. By our assumption that $W = 1$ the banking sector is self sufficient so that the lending market can be thought of as an interbank market.

We make two further assumptions about the way in which lending and collateral repossessions work. First, debt contracts cannot be settled by transferring some or all of the bank's project to the creditor. Rather, collateral needs to be auctioned off, using the proceeds to satisfy lenders. This generates a role for a fire-sale market. Thus, a market for repossessed capital goods opens at $t = 2$. We denote the equilibrium spot price by q. Additionally, bidders need to submit orders and the necessary liquidity to execute them *before* the $t = 2$ production process is completed. As a result, $t = 2$ manufactured consumption goods are excluded from fire-sale auction and only liquidity that was stocked up $t = 1$ can participate. Second, repossessions destroy value. We assume that once some or all of a loan portfolio is detached from its originating bank, it only yields $\delta \leq 1$ units of non-verifiable consumption goods at $t = 3$. This captures the idea that banks have some local monitoring expertise. When another bank (or speculator) takes over the project, his monitoring will be less effective, reducing project value (or, alternatively, he will have to incur a higher monitoring cost to preserve full project value).

Notice that the $t = 2$ capital-goods price, q, can be perfectly anticipated at $t = 1$ when θ is realized and observed. Banks with spare funds can therefore earn a gross rate of return of $\frac{\delta}{q}$ from hoarding funds for the future purchase of fire-sales assets. By arbitrage, the return ρ_1 from lending on the interbank market must thus equal the return from participating in the fire-sale market, i.e.,

$$\rho_1 = \frac{\delta}{q}. \tag{2}$$

A. Discussion of the assumptions

The motivation for assuming redistributive shocks is already explained in the introduction. We attempt to capture a pure financial shock caused by "reckless trading and faulty risk management", which by itself has no effect on real productive capacity. In the absence of financial frictions, the shock has no effect on aggregate output. The challenge facing theory is to explain how, in a world with financial frictions (here in the form of the non-pledgeability of cash flows), financial shocks destroy real value in equilibrium. Notice that equation (1) implies not only that aggregate wealth, W, is deterministic but also that it equals the economy's total investment opportunities, namely 1. That has the convenient implication that for any realization of θ, w^n exceeds 1 so that banks are cleanly separated to capital short and capital slack – potential lenders and borrowers in the interbank market. In the same spirit, we abstract from productivity shocks on a project level: potentially, all projects are economically viable unless disrupted by a shortage of liquidity that leads to repossession. By that assumption we capture the dominant policy concern during financial crisis: that economically viable projects are "inefficiently" liquidated due to a temporary shortage of liquidity.

The assumption that banks operate projects directly is obviously an abstraction, albeit one that has become common in the literature following Diamond and Dybvig (1983). We prefer to think about these projects as real activities rather than financial securities so as to allow us to explore the link between financial markets and the real economy; see the discussion of the model's quantitative implications in Section A below. The effect on aggregate real output is due to the deadweight loss of repossessions. In this respect our approach differs from most papers in the banking literature which focus on the insurance role of banks and do not model repossessions. Our approach is thus closer to the one taken in the corporate finance literature (e.g., Kiyotaki and Moore, 1997).

As for the interpretation of β, we are aware that lending in some interbank markets, e.g. LIBOR, is unsecured. Others, like the repo market are secured. Hence, the proper interpretation of β is the share of borrowing that is directed towards the secured segment of the market. From a modelling point of view one could think of a creditor as providing two loans, one unsecured and one secured. If the borrowing bank does not honor its payments on the unsecured loan, the creditor would cease his collateral on the secured loan and stop lending. β can then be thought of as the fraction of secured lending needed to support unsecured lending on the remaining fraction $1-\beta$ of debt. Notice also that the terminology differs across markets and that "collateral" may be relabeled as "margin requirement".

The restriction on settlement in kind of debt contracts is standard in the banking literature.[12] It is plausible since, lenders are in practice often unable to repossess collateral directly. Instead, the trustee in a bankruptcy procedure may have to auction off the assets and distribute the proceeds among the creditors according to their seniority. We do not model this argument explicitly. Clearly, the price determined in this auction is affected by the availability of liquidity, which is where the assumption that bids need to be submitted prior to the completion of the $t = 2$ production process, binds. This assumption is made mainly for its quantitative plausibility and will be discussed in Section A below.

[12] The assumption is also standard in the macroeconomics literature based on cash-in-advance constraints, which derives from the observation that "money buys goods ... but goods don't buy goods" (in our setting, liquidity is defined in terms of a numeraire consumption good rather than fiat money).

III. The contract

Consider the case of a bank in need of external finance ($\underline{w} < 1$). Due to non-verifiability, a cash-contingent contract is not implementable. Instead, Hart and Moore (1998) show that the optimal contract allows the providers of external funding to collect payment under threat of repossession. Namely, the borrower commits to a certain repayment, R, and the lenders are given default-contingent repossession rights on a fraction β of the project. (Since, by the very nature of the interbank market, both sides to the deal are banks, we conduct the analysis in terms of lenders and borrowers.) In short, the optimal contract is a standard, secured, debt contract.

To see why, consider, first, a liquid borrower (earning y in both $t = 2, 3$). Clearly, payments cannot be deferred to $t = 3$, for then the assets have already depleted and the threat of repossession is empty. Even at $t = 2$, repayments are subject to the constraint that the borrower cannot renegotiate a lower payment. Renegotiations are modelled as a Nash bargaining game: with a probability λ the lender can make a take-it-or-leave-it offer; if it is rejected he can exercise his rights and repossess the investment. Hence, the lender's equilibrium offer equals the opportunity cost of the assets to the borrower, βy. With probability $1 - \lambda$ the borrower makes a take-it-or-leave-it offer, which (by a symmetric argument) would be as low as the liquidation value $q\beta$. It follows that the repayment, R, has to satisfy an incentive compatibility constraint (IC)

$$R \leq \beta\left[(1-\lambda)\,q + \lambda y\right]. \tag{IC}$$

Subject to the IC, liquid borrowers repay their debt, leaving repossession and contract renegotiation off the equilibrium path.

Consider now a financially-distressed borrower (earning zero at $t = 2$ and $2y$ at $t = 3$). Clearly such a bank cannot defer payment to $t = 3$, and has no liquidity to settle at $t = 2$. At the same time the lender has no incentive to forgive payment, so he will exercise his rights and repossess the collateral. It follows that the lender's participation constraint (PC) is

$$\pi R + (1-\pi)\,\beta q = \rho_1\,(1-\underline{w})\,, \tag{PC}$$

where $\rho_1 = \frac{\delta}{q}$ (see equation (2)). We assume that the PC holds with equality due to the competition among lenders. The contract also needs to satisfy the feasibility constraints

$$R \leq y, \quad \text{and} \quad \beta \in [0,1]\ . \tag{FC}$$

Now, the contract problem is to maximize the borrower's value (at $t = 3$) subject to the various constraints above:

$$\max_{R,\beta} \pi\left(2y-R\right)+\left(1-\pi\right)\left(1-\beta\right)2y, \qquad (3)$$
$$s.t.\ (IC),\quad (PC),\quad (FC)\ .$$

(We deal with the participation constraint of the borrower below.) Substituting (PC) into the objective function and re-arranging, we express the final value of a capital short bank in terms of β alone:

$$V|_{\underline{w}} = 2y - \rho_1\left(1-\underline{w}\right) - \left(1-\pi\right)\beta(2y-q). \qquad (4)$$

The bank's value is the project payoff $2y$, minus the cost of external funding, minus the dead-weight loss of external funding, which is the probability of distress, $(1-\pi)$, times the collateralized fraction of the investment, β, times the deadweight loss per unit of capital repossessed $(2y-q)$. Clearly, the smaller β, the higher is the borrower's value.

LEMMA 1 *Let*

$$b = \frac{\rho_1\left(1-\underline{w}\right)}{q\left(1-\lambda\pi\right)+\lambda\pi y}. \qquad (5)$$

If $b \leq 1$, *the optimal contract is* $\beta = b$; *if* $b > 1$, *a capital short bank cannot obtain external funding.*

Now substitute the arbitrage condition (2) into equation (5) to obtain b as a function of q alone: $b\left(q\right)$. (Remember that q is perfectly anticipated at $t=1$.) Clearly, banks need to pledge a greater fraction of their project as collateral when the fire-sale price, q, drops. On the PC side, when distressed assets sell at a lower price, more needs to be sold so that the lender can break even. At the same time, the opportunity cost of funding, ρ_1, increases because lenders can earn a higher rate of return by holding their funds to participate in next period's fire-sale market. On the IC side, with a lower fire-sale price, the threat of repossession is less effective, which supports a lower repayment. Both ways, collateral requirements increase. Since all pledged assets of distressed banks are repossessed and then sold off, it follows that the *volume* of liquidations, $\theta\left(1-\pi\right)\beta$, increases when fire-sale prices drop. Moreover, substituting (2) into (5), it is clear that the *value* of repossessed assets, $q \times \theta\left(1-\pi\right)\beta$, also increases when the fire-sale price drops. Hence, the demand for liquidity is downwards sloping in q, which plays an important role in the equilibrium analysis below.

Let $\underline{q}$ be the positive root of $b\left(q\right)=1$:

$$\underline{q} = \frac{-\lambda\pi y + \sqrt{\left(\lambda\pi y\right)^2 + 4\left(1-\lambda\pi\right)\delta\left(1-\underline{w}\right)}}{2\left(1-\lambda\pi\right)}. \qquad (6)$$

If prices ever drop that low, a credit-rationing equilibrium occurs (generically). Clearly, prices cannot get lower than that. For then, lenders cannot recover a market return even at the highest feasible β. The only way to avoid this outcome is to deny credit to some banks. Let μ be the probability of obtaining credit. Clearly, if $q > \underline{q}$, $\mu = 1$; if $q = \underline{q}$, $\mu \in [0, 1]$, to be determined endogenously in equilibrium. Notice that our assumptions do not prevent credit-rationed banks from lending their capital to others on the interbank market.

Consider next the case of a capital slack bank with $w^n > 1$, so that the project is internally funded ($\beta = 0$); its final value is $2y + (w^n - 1)\,\rho_1$. Using the arbitrage condition in (2) we can express the expected maturity values for any endowment $w \in \{\underline{w}, w^n\}$ as:

$$V = w\frac{\delta}{q} + \mu S, \tag{7}$$

$$S = \left(2y - \frac{\delta}{q}\right) - (1-\pi)\,\frac{\frac{\delta}{q}\left[1 - \min\left(1, w\right)\right]}{q\left(1-\lambda\pi\right) + \lambda\pi y}(2y - q). \tag{8}$$

It is convenient to think of the final value as the sum of endowment wealth (including interest), $w\rho_1$, plus the rent, S, generated by the bank's access to a project, provided that it can get funding. The rent equals the value of the technology, net of the opportunity cost of capital, less the deadweight loss of external funding (if used). Crucially, in a model with financial frictions the value of the rent is not just a function of technology but also of the bank's initial wealth and of the fire-sale price, q.

Now that we have expressed the lowest conceivable equilibrium price, $\underline{q}$, in terms of the model's structural parameters, we can use it in order to lay down two additional parametric assumptions that guarantee the existence of equilibrium. Firstly, speculators will not participate in the fire-sale market if prices are above δ, for then the return on holding liquidity at date zero is higher than the highest possible return on liquidity. At the same time, the contract requires that the fire-sale price is (weakly) above $\underline{q}$. We therefore assume that

$$\frac{\delta}{\underline{q}} > \rho_0. \tag{A1}$$

Since $\rho_0 > 1$ assumption (A1) guarantees that $\underline{q} < \delta$ so that the set of feasible equilibrium prices, $\left[\underline{q}, \delta\right]$, is non-empty.

Secondly, we still have to check that it is in the best interest of banks to operate their projects rather than invest their capital at the market rate. Given expressions (7) and (8) it is clear that participation requires that the rent for project access, S, is positive for both $\underline{w}$ and w^n for any price in $\left[\underline{q}, \delta\right]$.

Lemma 2 *$\pi 2y - \frac{\delta}{\underline{q}} \geq 0$ is a sufficient condition for $S > 0$, at any feasible equilibrium price.*

We thus assume that

$$\pi 2y - \frac{\delta}{\underline{q}} \geq 0. \tag{A2}$$

Notice the intuitive interpretation of assumption (A2): in a perfect market a positive NPV is sufficient in order to guarantee project acceptance (remember that $\rho_1 = \frac{\delta}{\underline{q}}$). In our setting, a stronger condition is required in order to account for the potential loss of value due to financial distress.

A. Model parameterization

We carry along the paper a numerical example that helps to illustrate the functioning of the model and allows us to make a few points that are hard to make analytically. Some of the parameters in Table 1 seem plausible on their own and some are hard to pin down due to the stylized nature of the model. As we shall see, the model matches a few important statistic of financial crisis, yet we would not claim this is to be, in any way, a test of the model.

As noted above, we think of "projects" as real production facilities. As a result, we think of the time horizon of our model, from $t = 0$ to $t = 3$ as a five-year period. Needless to say, the time index , t, indicates decision and trading events rather than equally-spaced points within a five-year interval; for example, the production period $t = 1$ to $t = 2$ is probably much lengthier than, say, the time between $t = 0$ and $t = 1$. We then assume that $y = 1.25$ which implies an internal rate of return (IRR) of around 20% *per annum*. This is way above, say, 8% that would be consistent with macroeconomic data. One reason for using such a high number is the view that young, productive and innovative ventures are harder hit during financial crisis. Another motivation is to include in y "private benefits" of stakeholders that are not explicitly modelled, predominantly labor. We assume that the probability that a project will suffer from cash shortage is 5% per annum or 25% over the 5 year period, which yields the parameterization for π. This is consistent with US banking statistics of around 1% write-off rate [13] assuming recovery rate of about 80% on average (so that $1\% = (1 - 0.8) * 5\%$).[14]

Perhaps the most difficult parameter to quantify is external funding, $1 - \underline{w}$. Under our assumption capital-short banks raise all that amount externally. Since debt is the only source of external finance, $1 - \underline{w}$ is the leverage of capital-short banks. Clearly, 40% is very low for banks. However, if "projects" are to be interpreted as production facilities, our model's "banks" should be interpreted as a consolidation of banks and their clients. That makes some economic sense: the financial stability of banks depends, at least to some

[13] See "loan and lease charge off" against the balance-sheet value of loan and lease in FDIC statistics since the early 1980's.

[14] See Franks and Sussman (2005).

extent, on the capitalization of their clients. From that point of view a 40% leverage seems more reasonable.[15] We also assume that up to 40% of banks may be short of capital. Our parameterization of δ and ρ_0 is standard; for λ and the distribution function $h(\theta)$ we have no supporting data.

In light of these assumptions, it is worth pointing out another motivation for the previous section's assumption that consumption goods produced at $t = 2$ are not allowed to participate in the fire sale market. According to our parameterization, no more than 10% (namely $\overline{\theta}(1-\pi)$) of capital stock is actually repossessed at $t = 2$. At the same time, 37.5% (namely $(1-\pi)/2$) of the five-year potential output has already been produced, and is available in the form of consumption goods. Obviously, it would be unrealistic to assume that all that amount is available to bid for fire-sale assets. Rather, we assume that part of that production is consumed and part may find its way to the liquidity market but only with a lag. As a result, liquidity needs to be earmarked for that purpose one period ahead of market opening.

Table 1

Structural parameters for the numerical examples below.

Description	Model's notation	Parameterization
Project parameters		
IRR	$2y-1$	150%
prob. liquidity shortage	$1-\pi$	25%
capital shortage	$1-\underline{w}$	40%
depreciation	δ	50%
bargaining power	λ	50%
Market parameters		
ex-ante riskless rate	$\rho_0 - 1$	5%
worse-case incidence of shortfall	$\overline{\theta}$	40%
distribution of θ	h	uniform

B. Discussion and numerical example

Using the parameters above, we plot in Figure 2 rent, S, that bank's owner derives from being able to access projects, as a function of the fire-sale price q for both types of banks in the model. As mentioned, the graph of a capital-short bank lies below the graph of a capital-slack bank. Clearly, the model's parameterization does not violate assumptions (A1)-(A2). The vertical segment corresponds to different values of μ. At $\mu = 0$ a capital-short bank gets

[15] Rajan and Zingales (1995) report mean book (market) leverage (debt to *net* assets) for listed companies of 38% (27%) in the United States and 25% (19%) in Germany.

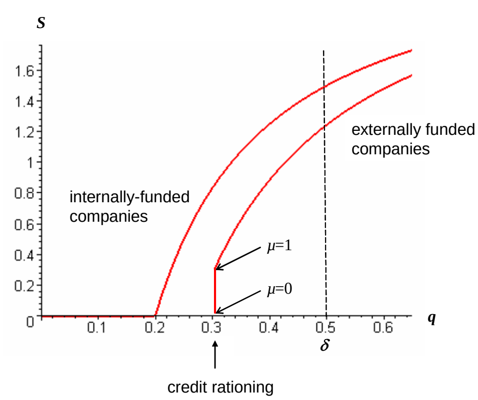
and has rent of zero.

A basic intuition for the workings of the model can already be acquired at this point. Consider a redistribution of \$1 from a capital-short bank to a capital-slack bank, holding the liquidation price at its highest possible level: $q = \delta = 0.5$. As a result, aggregate rent would fall by ¢64. This is because the extra wealth does not generate any rent (above market return) in the hands of the capital-slack bank; however, once a bank becomes capital short, it needs to fund externally and pledge a fraction β of its investment as collateral. That fraction will be repossessed in case of liquidity shortage, destroying value.

At a lower q, rents are lower for both types because the opportunity cost of capital, ρ_1, is higher (see Figure 1). Yet the gap between the two increases as well. As a result, a redistribution of \$1 from a capital-short bank to a capital-slacked bank at a fire-sale price of $q = \underline{q}$ would result in a loss of output of \$1.37.

Figure 2

Rent obtained from access to project, S, for capital-slacked capital short banks. For structural parameters, see Table 1.



IV. Competitive equilibrium

We take two steps in characterizing the competitive equilibrium. First, we analyze the ex-post market for liquidity, where F is predetermined and θ is already realized. Second, we analyze the ex-ante determination of F at $t = 0$. We then discuss the economic significance of the results and analyze the quantitative equilibrium implications of the numerical example above.

A. The ex-post market for liquidity

The ex-post equilibrium condition combines two spot markets. First, at $t = 1$, capital-short banks are funded. Then, at $t = 2$, fire sales are absorbed by market liquidity. Since the prices in these markets are linked via the arbitrage condition (2), equilibrium is determined by a single market-clearing condition:

$$F + (w^n - 1)(1 - \theta) + \theta(1 - \mu)\underline{w} - \theta\mu(1 - \underline{w}) - \theta\mu q(1 - \pi)b(q) \geq 0. \tag{9}$$

On the supply side we have speculators, capital-slack banks supplying $(w^n - 1)$, and a fraction, $(1 - \mu)$, of the capital-short banks who happen to be credit-rationed and are thus willing to lend their capital endowment at the market rate. On the demand side, there are the capital-short banks who are not credit rationed. The last term in equation (9) comes from the fire-sale market. There are θ capital-short banks, of which a fraction μ actually gets funded. Of these, a fraction $(1 - \pi)$ will be short of liquidity and thus default, in which case lenders would repossess a fraction $b(q)$ of the investment. The amount of liquidity that is required in order to absorb it is a product of the above times the fire-sale price, q. Since there might be more liquidity available than demand, the clearing condition might hold with inequality, where the supply of liquidity is greater then the demand.

Using (1) we can rewrite the clearing condition (9) as

$$F + W \geq (1 - \theta) + \theta\mu[1 + q(1 - \pi)b(q)], \tag{10}$$

where the left- and right-hand side of the inequality can be thought of as liquidity supply and demand, respectively.

PROPOSITION 1 *There exists an ex-post equilibrium in the market for liquidity, with three possible regimes:*

- *If* $\theta < \frac{F+W-1}{\underline{q}(1-\pi)}$ *then there is a unique equilibrium with a slack of liquidity:* $q = \delta, \mu = 1$.
- *If* $\frac{F+W-1}{\underline{q}(1-\pi)} \leq \theta \leq \frac{F+W-1}{\delta(1-\pi)b(\delta)}$ *there is a multiplicity of equilibria as follows: (i)* $q = \delta$ *and* $\mu = 1$, *(ii)* $q \in (\underline{q}, \delta)$ *and* $\mu = 1$, *(iii)* $q = \underline{q}, \mu < 1$.
- *If* $\theta > \frac{F+W-1}{\delta(1-\pi)b(\delta)}$ *there is a unique credit-rationing equilibrium:* $q = \underline{q}, \mu < 1$.

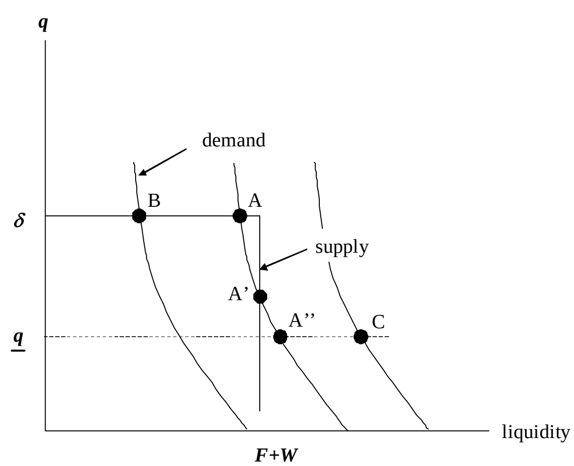
In a credit rationing equilibrium (either the second or the third regime) the amount of credit rationing is:

$$\mu = \frac{F + W - (1 - \theta)}{\theta\left[1 + \underline{q}(1 - \pi)\right]}. \tag{11}$$

Figure 3 provides a diagrammatic exposition of the equilibrium and the existence argument in Proposition (1). The supply of liquidity is the inverted L-shaped graph, δ horizontally and $F+W$ vertically. As noted the demand for liquidity (the right-hand side of (10)) is decreasing in the fire-sale price, q. Hence, both supply and demand for liquidity are (weakly) decreasing in q. Three possible realizations of θ are plotted: for the low and high realizations there is a unique equilibrium at points B and C, respectively, while for the interim realization there are multiple equilibria at points A, A' and A'', one with $q = \delta$ and a slack of liquidity, one $q \in \left[\delta, \underline{q}\right]$ and market clearing, and one with $q = \underline{q}$ and credit rationing, respectively.[16]

Figure 3

The ex-post market for liquidity



Since asset repossession destroys value, the three equilibria in Figure 3 are Pareto ranked: point A dominating points A' and A''. Hence, the government should try to coordinate expectations towards point A. We thus show:

PROPOSITION 2 *In case of multiple equilibria, a policy that guarantees a fire-sale price of δ eliminates the Pareto-dominated equilibrium points at a zero fiscal cost to the government.*

Clearly, when supply and demand are both (weakly) decreasing, multiple equilibria may arise. Unlike in Diamond and Dybvig (1983), our model does not rely on any "defective" financial instrument – demand deposits with their built-in first-mover advantage. However,

[16]Since C and A'' denote rationing equilibria, one cannot read equilibrium quantities straight from the figure.

like in Diamond and Dybvig (1983), the government can coordinate expectations towards one of the equilibrium points and improve welfare at zero fiscal cost (in equilibrium). The underlying policy instruments are also similar: deposit insurance in Diamond and Dybvig and credit guarantees in our model.[17] Nevertheless, there are some interesting differences: in our model the elimination of dominated equilibria will not restore the first best; even without multiplicity markets do fail and value is destroyed (see next section). Another important implication is that in our setting multiple equilibria appear at intermediate levels of the liquidity shock, θ, but vanish as the shock intensifies. Hence, the issue is not so much which theory is correct; rather, at what intensity of the shock policy might have to move from a phase where value can be created just by coordinating expectations to one where value can be created only by active market support.

From this point onwards we shall assume that the government always implements a zero-cost policy that eliminates Pareto dominated equilibria via coordination of expectations. As a result, there is a unique critical point,

$$\theta^* = \frac{F + W - 1}{\delta (1 - \pi) b(\delta)} \tag{12}$$

such that a credit-rationing equilibrium appears when $\theta > \theta^*$.

Lastly, and in order to improve the macro interpretation of the model we calculate the Gross Domestic Product (GDP), Y, as it should be measured in our model:

$$Y \equiv (1 - \theta) 2y + \theta\mu [2y - (1 - \pi) b(q)(2y - \delta)] + \theta (1 - \mu).$$

It is "gross" in the sense that depreciation is netted out, and "domestic" in the sense that interest payments to foreigners (speculators) are excluded.

B. The ex-ante equilibrium: speculators' choice

Speculators allocate their funds at $t = 0$ so as to maximize their expected $t = 3$ payoff. We then show the following.

PROPOSITION 3 *Given assumption (A1) and the implementation of policies to coordinate expectations as suggested by Proposition 2, there exists a unique competitive equilibrium. The amount of liquidity held by the market, F, satisfies*

$$H(\theta^*) + [1 - H(\theta^*)] \frac{\delta}{\underline{q}} = \rho_0, \tag{13}$$

[17] Bebchuk and Goldstein (2009) discuss the role of government guarantees when banks' lending decisions may be subject to coordination failure. In their paper complementarities arise from the production technology, while in ours they arise from price effects.

and there is a strictly positive probability of a financial crisis.

C. Discussion and numerical example

Many metaphors have been used in order to describe financial crisis: price spirals, liquidity black holes (see Morris and Shin, 2004), a break-down or freeze of the interbank market, a crash, contagion etc. Our analysis suggests that they can all be reduced to one simple property of the market for liquidity: that both supply and demand are downwards sloping. As a result, prices may be highly sensitive to changes in the demand for liquidity, particularly in the neighborhood of the critical point θ^*. As prices fall, lenders to capital-short banks demand more collateral, which will be repossessed and sold off in case the bank is also short of liquidity, which will increase the demand for liquidity even further. In that respect, the crisis is contagious and feeds itself: a slight increase in the number of capital-short banks can generate a credit crunch that affects all other capital-short banks.

Liquidity hoarding deserves special emphasis. In crisis, the clearing condition (9) holds with equality, which means that *all* the liquidity that was stocked up at $t = 0$ is used, either to fund capital-short banks or to absorb fire sales. Although the funding and the fire-sale markets are aggregated into a single market-clearing condition, the two markets actually operate sequentially. As a result, banks hoard liquidity at $t = 1$ in order to participate in the fire-sale market at $t = 2$. Moreover, in times of crisis liquidity hoarding coexists with credit rationing in the interbank lending market: Some capital-short banks are *denied* credit at the market rate (which is exceptionally high) while capital-slack banks "sit" on resources that are not utilized in the short run.[18] Hence the impression of a "freeze": liquidity is apparently available, but the market no longer succeeds in transferring it from providers to users.

In spite of these seemingly anomalous results, our modeling remains solidly neoclassical. The equilibrium concept is Walrasian, with only few additional restrictions. Importantly, the "margin requirements" that many models impose so as to trigger asset sales upon the decline in market value are fully endogenized. Besides elegance this addition opens the way to the welfare analysis below: models with exogenous margin requirements cannot address normative questions of welfare and government policy since they exclude from the analysis the frictions that necessitate the margin requirements in the first place. Hence, they ignore by construction how government policy affects the resolution of the underlying frictions. Less common in the literature is the introduction of rationing equilibria. Beyond adding realism this aspect completes the modelling in an important manner: when both supply and demand

[18] Other papers that feature liquidity hoarding include Diamond and Rajan (2009) and Acharya, Shin and Yorulmazer (2009a). Simultaneous credit rationing, however, does not incur in those models.

are downwards sloping the curves may not intersect, so a price floor needs to be introduced. In our modelling, this floor is also endogenously determined through the same contractual mechanism that endogenizes the margin requirements.

In Table 2 we present equilibrium magnitudes computed for the parameterization from Table 1. Where possible we list the actual data based on the most comprehensive survey of financial crisis to date: Reinhart and Rogoff (2008, 2009). Stylized as it is, the model is rich enough to generate most of the statistics that one uses to describe financial crisis: at the contagion point, θ^*, the price of repossessed capital goods drops by almost 40% and GDP falls by 3.9%.[19] The number might seem small given the massive loss of value at the project level upon liquidation. Notice however that although up to 40% of banks can be short of capital, only 25% of them become financially distressed (remember that internally-financed banks avoid financial distress even if short of liquidity). The crisis is accompanied by a severe "credit crunch": in normal times capital-short banks can borrow ¢78 on a dollar of collateral but during a crisis they can borrow only ¢40 on a dollar of collateral. Credit rationing will appear during financial crisis, but the magnitude is not substantial, between 1.1% and 1.6%, depending on θ, which implies a relatively steep demand for liquidity (see Figure 3). Perhaps most importantly, the crisis probability, 7.7%, is remarkably close to the actual magnitude.

Table 2

Competitive equilibrium, numerical example. For structural parameters see Table 1. Source for actual data is Reinhart and Rogoff (RR), from top to bottom: (a) peak-to-trough house and equity price decline, RR (2009) Figures 1 and 2; (b) calculations based on depth and duration of GDP decline, annualized (left) or stretched over a five-year period relative to a growth trend of 2% per annum (right), RR (2009) Figure 4; (c) share of years in banking crisis, RR (2008) Table 5a.

Description	Model	Values	Actual data
Competitive equilibrium			
price drop in crisis	$\frac{q}{\delta} - 1$	-39.3%	-35.5%, $-55.9\%^{(a)}$
loan to security	$\frac{(1-w)}{b(\delta)}, \frac{(1-w)}{b(\underline{q})}$	78%, 40%	–
output drop at θ^*	$Y\rvert_{q=\underline{q}} \big/ Y\rvert_{q=\delta} - 1$	-3.9%	-3.5%, $-8.8\%^{(b)}$
credit rationing at θ^*	$[1-\mu(\theta^*)]$	1.1%	–
probability of a crisis	$1 - H(\theta^*)$	7.7%	$7.2\%^{(c)}$

In order to get some idea of the robustness of the numerical results with respect to underlying parameters, we conduct a sensitivity analysis. The upshot of it is that the

[19] The relatively-wide range in the actual numbers reflects that several mappings from Reinhart and Rogoff's evidence into the frequency of our data are admissible.

orders of magnitude of our results are not affected by small modifications of the parametric assumptions. Details of this analysis are provided in Appendix 3.

V. Policy and welfare analysis

During the recent crisis policies towards the restoration "financial stability" have been implemented on an unprecedented scale. Governments and central banks all over the world were given permission to inject liquidity into the open market by purchasing "troubled assets" that are "off limits" during normal times. The rationale for this policy is that public liquidity supports prices in secondary markets and helps distressed banks from further deterioration. In other cases, governments have "infused equity" directly into the balance sheet of troubled institutions through ownership stakes and, in some extreme cases, de facto nationalization. By their very nature, decisions of that sort are taken in great hurry and with very little reliable information about the true state of the beneficiaries' balance sheets. Hence the two basic questions that we address in this section: i) which policy generates more value per "dollar of taxpayer money", liquidity injection or equity infusion; ii) can a policy be designed in such a way that it provides an incentive for troubled institutions to reveal themselves truthfully to the authorities, preventing others that are not distressed from applying for help. In addition to answering these questions, we provide a detailed analysis of how such policies work, including a preliminary quantitative assessment of the value the policy creates. A major concern that we explore is the "crowding out" of private liquidity by public liquidity.

The conceptual framework of the analysis is that of standard welfare economics, but the model has the attractive property that social welfare, SW, is measured as expected GNP. Since the banking industry has a measure of one, the welfare index also equals to the ex-ante terminal value of an owner-manager of a bank. All policies involve public-sector borrowing, ex ante at a gross interest rate of ρ_0, i.e., the opportunity cost of liquidity provision is the same for the public and the private sector. Since we assume that national debt is paid via lump-sum taxes, the government can increase the scale of the intervention with no substantial cost. In theory, it can tax and redistribute away the entire shock. Since this is not a realistic assumption we introduce the above-mentioned test of policy effectiveness measured by the value per dollar of taxpayers' money.

A. Liquidity injection

We start by analyzing the effect of total liquidity, X, on welfare SW (from this point onwards, the symbol F denotes private liquidity). We analyze the socially-optimal level of liquidity in the economy, which the government (or the central bank) would implement had it been the sole supplier. Later on we re-introduce private speculators and discuss "the division of labour" between private and public action. Clearly, the government will "make losses" on its liquidity position, X, off crisis (when liquidity is held idly) and will "make a profits" in crisis (when liquidity is used to buy capital goods cheaply); both profits and losses are redistributed back the economy via lump-sum taxes or transfers.

A bank may turn out to be either short of capital (SC) or slack/long (LC), in normal market conditions (N) or in financial crisis (C). In the latter case, a SC bank may be either credit rationed (CR) or not (NR). Table 3 lists the unconditional expected masses of banks and their payoffs conditional on the realization of θ, before any distribution of taxes or transfers:

Table 3

Unconditional expected masses and payoffs for various realizations: banks may be capital-short (SC) or capital slacked (LC), in normal (N) market conditions or in crisis (C). SC companies can be either credit rationed (CR) or not (NR). Payoffs do not include taxes or lump-sum transfer of the government's "trading profits".

Realization	Expected mass	Payoff
N, LC	$\int_0^{\theta^*}(1-\theta)\,h(\theta)\,d\theta$	$w^n+(2y-1)$
N, SC	$\int_0^{\theta^*}\theta h(\theta)\,d\theta$	$\underline{w}+(2y-1)-(1-\pi)\,b(\delta)\,(2y-\delta)$
C, LC	$\int_{\theta^*}^{\overline{\theta}}(1-\theta)\,h(\theta)\,d\theta$	$w^n\frac{\delta}{\underline{q}}+\left(2y-\frac{\delta}{\underline{q}}\right)$
C, SC, NR	$\int_{\theta^*}^{\overline{\theta}}\theta\mu(\theta)\,h(\theta)\,d\theta$	$\underline{w}\frac{\delta}{\underline{q}}+\left(2y-\frac{\delta}{\underline{q}}\right)-(1-\pi)\left(2y-\underline{q}\right)$
C, SC, CR	$\int_{\theta^*}^{\overline{\theta}}\theta\,[1-\mu(\theta)]\,h(\theta)\,d\theta$	$\underline{w}\frac{\delta}{\underline{q}}$

Using the assumption that $W=1$ we cancel out the redistributive effect of the shock, θ. Hence, the SW function includes the expected level of wealth (plus interest), profits (net of interest charges), with liquidations accounted for in market prices. From that we subtract the government's debt repayment, $\rho_0 X$, and add the maturity value of the government's portfolio, X in no-crisis states and $\frac{\delta}{\underline{q}}X$ in crisis states, which is lump-sum transferred back to the economy:

$$\begin{aligned} SW &= \int_0^{\theta^*}\left[1+(2y-1)-\theta\,(1-\pi)\,b(\delta)\,(2y-\delta)\right]h(\theta)\,d\theta \\ &\quad +\int_{\theta^*}^{\overline{\theta}}\left\{\frac{\delta}{\underline{q}}+\left[1-\theta\,(1-\mu)\right]\left(2y-\frac{\delta}{\underline{q}}\right)-\theta\mu\,(1-\pi)\left(2y-\underline{q}\right)\right\}h(\theta)\,d\theta \\ &\quad -(\rho_0-1)\,X+(1-H)\left(\frac{\delta}{\underline{q}}-1\right)X. \end{aligned} \tag{14}$$

Unfortunately, the SW function lacks the usual regularity properties (say, concavity) and is thus difficult to analyze. The following proposition reflects this difficulty.

PROPOSITION 4 *i) For any probability density function h, the social-welfare function is locally increasing in X at the competitive-equilibrium point. ii) For any density function h, there exist a $\rho_0 > 1$ sufficiently low so that the social-welfare function is increasing monotonically over $\left[0, \overline{\theta}\right]$ and the optimal liquidity-injection policy involves a zero-probability of crisis. iii) When θ is uniformly distributed, i.e. $h \equiv 1/\overline{\theta}$, the policy-maker's problem always has a corner solution with a zero probability of a crisis.*

The proposition shows that the amount of liquidity supplied in a competitive equilibrium is inefficiently low. Welfare can therefore be improved by increasing total liquidity. The intuition is straightforward: low fire-sales prices increase the amount of collateral in lending and thereby the amount of liquidations. Liquidations are inefficient, but the speculators who supply liquidity do not internalize this inefficiency. The price at which they buy does not reflect the productivity loss of the assets, but rather the scarcity of available liquidity.[20]

Could the government implement the optimal X calculated in Proposition (4) above by "topping up" private liquidity, F, with its own injected liquidity, I, such that $F + I = X$? The following lemma shows that the answer to this question is no.

LEMMA 3 *(Crowding out) As long as speculators participate in the market, namely $F > 0$, injected public liquidity, I, crowds out private liquidity one-for-one, with no effect on its aggregate supply, X. Hence, if the government wants to increase the supply of liquidity beyond the competitive level, it will have to supply the entire market, not just the increment above the competitive level.*

Proof. As long as $F > 0$ the aggregate supply of liquidity, X, needs to satisfy conditions (12) and (13), otherwise there is no private incentive to participate in the market. Hence, X is uniquely given and an increase in I crowds out F to leave X unchanged. $\blacksquare$

Hence, in order to have any effect, a liquidity-injection policy will have to "nationalize" the liquidity market. This suggests that governments will need to take on a large debt burden so as to raise the resources necessary to take over the entire provision of liquidity to the economy. Subsection D develops a numerical example to illustrate the quantitative implications of this result.

[20] This result is related to the pecuniary externality of banks' decisions to invest in liquid assets that operates via the interim spot market for illiquid assets (see Bhattacharya and Gale, 1987).

B. Equity injection

The standard interpretation of the assumption that cash flows are non-verifiable in Hart and Moore (1998) is that the parties to the contract can observe outcomes (hence, contract renegotiation is modelled with bargaining under symmetric information) while third parties – particularly the courts – cannot (which makes outcomes non contractible). It is natural to extend the third-party assumption to the government. Hence, we turn next to the analysis of non-discriminating equity injections – a bailout (perhaps partial) of the entire banking industry.

Suppose that the government borrows an amount E and distributes it equally across all the banks in the economy. Having their resources increased to $\underline{w} + E$ will have two effects on capital-short banks. First, during normal times, they can fund a greater fraction of the investment internally and need to pledge less as collateral:

$$\frac{db(\delta)}{dE} \equiv -\frac{1}{\delta(1-\lambda\pi)+\lambda\pi y} < 0.$$

During financial crisis the equilibrium amount of collateral b will still equal 1, but being better capitalized, distressed banks bear lower prices before they hit credit rationing:

$$\frac{d\underline{q}}{dE} = -\frac{\delta}{\sqrt{(\lambda\pi y)^2 + 4(1-\lambda\pi)\delta(1-\underline{w})}} < 0.$$

An important observation is that the supply of liquidity is:

$$F + (1-\theta)(w^n + E - 1) + \theta(1-\mu)(\underline{w} + E),$$

where F still denotes the private supply of liquidity (perhaps zero). Hence, all the extra equity that is injected into capital-slack banks will find its way into the liquidity market. By the same logic, the demand for liquidity is:

$$\mu\theta(1-\underline{w}-E) + \theta\mu(1-\pi)\beta q.$$

Hence, the equity that is injected into capital-short companies will directly decrease their demand for external funding. We can thus calculate:

$$\begin{aligned} \theta^* &= \frac{F+E}{\delta(1-\pi)b(\delta)}, \\ \mu &= \frac{F+E+\theta}{\theta\left[1+\underline{q}(1-\pi)\right]}, \end{aligned} \tag{15}$$

where

$$b(\delta) = \frac{1-\underline{w}-E}{\delta(1-\lambda\pi)+\lambda\pi y}.$$

A crucial observation is immediately apparent: equity injections have the same direct effect on the market as liquidity injections but they also have a wealth effect. Going over the same steps as in the previous sub-section, we can see that the social-welfare function is almost identical to equation (20), which is a simplified version of the welfare function SW in (14).

$$\begin{aligned} SW &= \int_0^{\theta^*} \left[2y - \theta\left(1-\pi\right) b\left(\delta\right)\left(2y-\delta\right)\right] h\left(\theta\right) d\theta \\ &+ \int_{\theta^*}^{\overline{\theta}} \left[2y - \theta\left(1-\mu\right) 2y + \theta\left(1-\mu\right) - \theta\mu\left(1-\pi\right)\left(2y-\delta\right)\right] h\left(\theta\right) d\theta \\ &- \left(\rho_0 - 1\right)\left(F+E\right). \end{aligned} \tag{16}$$

PROPOSITION 5 *For the three cases identified in Proposition 4, an equity injection dominates a liquidity injection. That includes the case where private speculators are active; namely, the one-to-one crowding-out result is no longer valid for equity injections.*

Although other papers have suggested that liquidity or equity injections can mitigate a financial crisis (e.g., Krishnamurty, 2009, or Gertler and Kiyotaki, 2009), we are, to the best of our knowledge, the first to provide a clear welfare comparison between the two policies.

C. Bailouts

Next, we enrich the policy analysis by examining the government's ability to design a targeted policy that bails out only distressed banks. Conventionally, a "bailout" means a government program that buys the liquidation rights of *distressed* borrowers from their respective creditors and writes them off. We assume that distress is not directly observable by the government (in line with our assumptions so far), but that banks can declare to be distressed and request a bailout. The government can then buy liquidation rights from the distressed bank's lender and cancel them. We assume that the transaction is done at market prices, and that the lender cannot bargain a higher price with the government.

As described above, even a non-distressed bank would enroll in a bailout program, cancel as much debt as possible and then settle the rest. To prevent that from happening, the program will have to include some additional terms such that the bailout is only incentive compatible for the distressed banks. Specifically, we assume that liquidations are observed by the government and that the transfer of the government's bailout money can be made contingent on observing that some pre-specified amount of collateral is actually repossessed by the lender, i.e., liquidated.

Let

$$\gamma \equiv \frac{\text{units bailed out}}{\text{units of collateral}},$$

where "units of collateral" refers to the units of capital goods that can be seized by the lender under the debt contract. γ is then the fraction of the collateral that the government buys from the lender and leaves with the borrower if he participates in the bailout programme.

We assume that the government commits to a certain γ (after the realization of θ), which neither the debtor nor the creditor can re-negotiate. Notice that we do make the strong assumption here that "liquidation" is an observable action. Moreover, that this liquidation is *irreversible* so that the borrower and the lender cannot collude to fake a liquidation or undo it once it has been taken. To some extent, the strength of this assumption justifies the analysis of the previous sub-section.

In case of repossession the payoffs to the debtor and to the creditor are $(1-\beta^c)\,y+\gamma\beta^c y$ and $\beta^c q$, respectively, where β^c is the *contracted* collateral, the basis on which the bailout is calculated; notice that β^c can exceed one. It is thus more economically meaningful to express the problem in terms of the effective collateral, $\beta=(1-\gamma)\,\beta^c$. Written that way, the feasibility constraint remains $\beta \leq 1$, as before. Defining

$$\sigma=\frac{\gamma}{1-\gamma},$$

we can then write

$$\beta^c q=(1+\sigma)\,\beta q$$

and interpret σ as a subsidy paid to creditors on top of the liquidation value βq.

Consider debt renegotiation when the borrower does not suffer from a liquidity shock, i.e., his date 2 cash flow is y. Like before, the borrower and lender are each granted the opportunity to make a take-it or leave-it offer R^D and R^C to the other party, with probabilities $(1-\lambda)$ and λ, respectively. *If* the borrower wants his offer to be accepted by the lender he will have to set it at no less than $R^D=(1+\sigma)\,\beta q$, leaving him with $y-R^D$. Clearly, the higher the subsidy, the higher the payment that the lender can extract from the debtor. If the borrower makes an offer that is not accepted by the lender, the former loses a fraction $1-\gamma$ of contracted collateral. He thus receives $(1-\beta)\,y$. The government therefore needs to keep the subsidy down to levels that still leave a non-distressed borrower with an incentive to repay rather than opt for default. Comparing the two payoffs, it is easy to see that an incentive-compatible bailout policy has to satisfy the following constraint:

$$\sigma \leq \frac{y-q}{q}. \tag{17}$$

The same constraint applies to the case where the lender gives the borrower an offer R^C. Intuitively, the constraint implies that the government should keep the after-subsidy fire-sale price below the fundamental value.

We can now complete the specification of the contract problem in an environment where the government has already announced a policy that satisfies the constraint (17). The optimal contract (R, β) can be calculated, like before, from the borrower's incentive compatibility constraint on the amount of debt repayment R and lender's participation constraint. These are given by

$$\begin{aligned} R &\leq \beta\left[(1-\lambda)(1+\sigma)q+\lambda y\right], \\ \pi R+(1-\pi)(1+\sigma)\beta q &= \rho_1(1-\underline{w}), \end{aligned} \tag{18}$$

respectively, and the solution to the contract problem is

$$b=\frac{\rho_1(1-\underline{w})}{q(1+\sigma)(1-\lambda\pi)+\lambda\pi y}. \tag{19}$$

Clearly, at a given fire-sale price, collateral requirements are decreasing in the subsidy. Notice that once the subsidy hits the policy constraint (17), the denominator in equation (19) reduces to y.

Consider, first, an equilibrium with a slack of liquidity ($\rho_1 = 1$). In that case the subsidy can go all the way up to the limit defined by the policy constraint (17) which yields $b = (1-\underline{w})/y < 1$. Next, consider the case of a crisis equilibrium. Could the subsidy be increased to the point where it hits the (17) constraint? Using (19) we can calculate the equilibrium fire-sale price that would result, i.e., $\underline{q}$ that sets $b = 1$. This allows us to calculate $\rho_1 = y/(1-\underline{w})$, which under the parametric assumptions defined in Table 1 would violate assumption (A2). Namely, low fire-sale prices would push the rate of return ρ_1 to a level that would strip banks of any incentive to operate their projects; the government would have to set the subsidy at a lower rate. Regardless of whether liquidity is slack or in short supply, the fiscal cost of an individual bailout is $\sigma(1-\pi)\beta q$ and the cost of the entire program is $\theta(1-\pi)\sigma\beta q$.

Proposition 6 *Consider an economy with a "small" equity injection program, E, where E is constant across all θ's, implemented alongside a competitive liquidity market where speculators supply an amount $F^* > 0$, the probability of a crisis is $(1-H^*)$ and equilibrium fire-sale prices are $q \in \{\delta, q^*\}$. A bailout program can achieve the same level of collateral requirement (and thus liquidation) at less than a $\overline{\theta}\rho_1$ fraction of the national debt, albeit with a possible increase in the cost of aggregate (public plus private) liquidity.*

The above proposition shows that a policy that targets distressed banks alone can achieve the same welfare improvement as an equity injection while incurring less national debt. Hence, its value added per dollar of taxpayer money is higher. The effectiveness of such a policy is, however, limited by the two policy constraints described above (incentive compatibility for a non-distressed borrower to repay, and incentive for banks to invest in a project). This raises the important question what the quantitative importance of this limitation is. We will discuss quantitative relevance in the next section.

D. Discussion and numerical example

The market for liquidity is inherently inefficient. Normally, markets "create value" by allocating commodities from low-value users to high-value users. The opposite happens when investment goods are sold on the fire-sale market. Here, commodities are allocated from high-value users to low-value users, driven not by a "fundamental" trading motive but rather by verifiability constraints. The stark difference between equation (13), which determines F in a competitive equilibrium, and condition (21) which the government uses in order to calculate socially-desired X provides a technical substance to this observation. Evidently, the difference between the fundamental value of repossessed investment goods and their fire-sale price (namely, the loss of value due to liquidation) does not appear in the former expression but does appear in the latter. Hence, by ex-post price-manipulation the government can diminish potential destruction of value and decrease the probability of crisis.

Although policy in our model has a first-order effect on welfare, our numerical example traces only modest quantitative effects: a policy of liquidity-injection that brings the probability of crisis down to zero would increase welfare by only 0.3% relative to the competitive equilibrium; see Table 4. This, surprisingly small number might seem like an artefact of the model. We believe that this is not the case: the ex ante loss of welfare due to financial crises is, roughly, the loss of output in a crisis, times the probability of crisis. Using the actual Reinhart-Rogoff data (a probability of 7.2% times a GDP decline of 8.8%, see Table 2) will leave us with a similar order of magnitude. Hence, the more general conclusion that seems to arise is that welfare effects might be an order of magnitude below the output effect. This conclusion does not require that we interpret our numbers as a calibration of the model: any parameterization that would fit the Reinhart-Rogoff data is likely to reach similar results.

Ultimately, our conclusions are driven by the observation that the government's ability to inject liquidity, *conditional on a crisis* requires it to have the option of accessing the corresponding funds *unconditionally.* This option is costly. In our model the cost is simply the difference in the rate of return between investment in a liquid or an illiquid asset. Other authors (e.g., Holmstrom and Tirole, 1997) allow governments to "produce" liquidity ex post, i.e., after a crisis has occurred. Such superior access of governments to liquidity, compared to the technology of private agents is argued to stem from the government's ability to issue debt against its future tax base. We do not adopt this modelling approach for the following reason.

While governments have a unique ability to tax, they also have the unique weakness of not being able to commit contractually to repay their debt. This limits their debt capacity.[21] In order to be able to produce liquidity ex post, governments thus need to leave some unused debt capacity ex ante (and thus invest less in illiquid technologies like the education system or infrastructure). Our assumption that the government's liquidity needs to be set aside ex ante (just like private speculators') thus can be viewed as capturing the cost of unutilized debt capacity in term of the lost output from an illiquid technology.

Table 4

A comparison of welfare and national debt (ND) under competitive equilibrium (CE), liquidity injection (LI), equity injection (EI) and bailouts (BO). EI is implemented at two levels: either bringing the probability of crisis down to zero (ZC), or at ND $= 1\%$. BO is implemented at a level that would achieve the same β as the second EI policy (for any realization of θ), leaving the government with a slack of liquidity, generically. ND is expressed as a percentage of full-capacity (i.e. no rationing) capital stock, namely one unit. For structural parameters see Table 1.

Description	Model	Value
Liquidity injection		
welfare gain at ZC	$SW^{LI-ZC}/SW^{CE}-1$	0.3%
ND under LI-ZC	$\overline{\theta}\delta\left(1-\pi\right)\beta$	2.6%
Equity Injection		
welfare gain at ZC	$SW^{EI-ZC}/SW^{CE}-1$	0.4%
ND under EI-ZC	$\overline{\theta}\delta\left(1-\pi\right)\beta^{EI}$	2.4%
welfare gain at 1% injection	$SW^{EI-1\%}/SW^{CE}-1$	0.06%
ND		1%
prob. of crisis	$1-H\left(\theta^{EI-1\%}\right)$	7.4%
Bailouts equivalent to 1% EI		
ND	$\overline{\theta}\delta\left(1-\pi\right)\beta^{BO}$	0.3%
government's liquidity slack		0 to 0.3%

In line with the theoretical results, Table 4 also shows the dominance of equity injections

[21] See Guembel and Sussman (2009) for a model of a government's debt capacity when a sovereign default is costless.

over liquidity injections: to bring the probability of crisis down to zero it takes an equity injection of only 2.4%, with a welfare gain of 0.4% (compared with 2.6% and 0.3% under liquidity injection, respectively). This is surprising since indiscriminate transfers to distressed and non-distressed banks alike seems like a "waste of money". This logic does not apply since all the "wasteful" equity injections trickle down to the equity market and thus support fire-sale prices. To put it differently, equity injections have both a balance-sheet and a price effect while liquidity injections will have only the latter effect.

Table 4 also shows that a discriminating bail-out policy will give taxpayers more value for their money: an equity injection of 1% would decrease the probability of financial crisis down from 7.7% in the competitive case (see Table 2) to 7.4%. A bailout policy would achieve the same level collateral requirement and liquidation (under the worst case of $\overline{\theta}$) with national debt of only 0.3%, still leaving the government with some slack of liquidity in all other realizations of θ.

VI. Globalization, capital-flows and international contagion

It has been argued that the globalized financial system that was created by the gradual deregulation of capital flows during the last twenty years or so has facilitated cross-country contagion and has contributed to financial instability. For example, Calvo (1998) claims that the 1998 Asian-Russian crisis was caused by a "sudden stop" where capital inflows were reversed and market liquidity escaped abroad. In this section we examine this claim by splitting the model economy above – now used as a benchmark – into two separate countries and then analyze its performance with and without restrictions on the free capital flows.

Suppose that banks in each country are exposed to a redistributive shock, but the sum of the two country-shocks has the same density function as the redistributive shock in the benchmark economy. Since all other parameters are the same, the two-country model reduces to the benchmark model once the regulation on capital flows across countries is removed. We make a distinction between two types of liquidity: domestic and international.[22] The former is prevented by regulation from leaving the country (once in), the latter is unregulated and is allowed to flow freely across countries. As we shall see, there is no private incentive to hold domestic liquidity. It should thus be interpreted either as liquidity injection by the domestic authorities (perhaps in the form of central bank reserves) or as a capital / liquidity requirement imposed on market makers and investment banks in exchange of a

[22] A similar distinction is made in Acharya, Shin and Yorulmazer (2009) who, like us, point out that international contagion may result when countries access a common pool of liquidity. In their paper, however, restrictions on capital flows are technological. In ours they are a choice variable, which allows us to conduct a corresponding policy and welfare analysis.

license to maintain another activity, domestically. In line with the assumptions made so far, we do not allow the regulation to be contingent on the realization of the shock, giving it an interpretation of a long-term structural parameter of the economy rather than a short-term policy instrument.

More specifically, assume two countries, A and B. Let the measure of banks in each be one half. Denote the mass of capital-short banks by θ_A and θ_B with a joint density $h(\theta_A, \theta_B) = \frac{1}{\overline{\theta}(\theta_A+\theta_B)}$ on $[0, \overline{\theta}] \times [0, \overline{\theta}]$ with $\theta_A + \theta_B \leq \overline{\theta} < \frac{1}{2}$. It follows that $\theta \equiv \theta_A + \theta_B$ is uniformly distributed on $\left[0, \overline{\theta}\right]$. The shock is redistributive: country A, say, has $\frac{1}{2} - \theta_A$ capital-slacked banks, w^n is a function of θ_A so that the aggregate country endowment is fixed deterministically at $\frac{W}{2}$. At $t = 0$ speculators may set aside "international" liquidity, F, that can flow from one country to another upon the realization of the country shocks. At the same time, $t = 0$, speculators can also allocate resources towards country-specific "domestic" liquidity L_A and L_B, that is restricted by regulation from flowing to the other country upon the realization of uncertainty. As in the benchmark model, liquidity is used either to fund capital-short banks or participate in the fire-sale market. All other assumptions remain the same implying that the special case $L_A = L_B = 0$ reduces the benchmark model with a uniform distribution of θ.

Using (10), we can define the following variables:

$$\begin{aligned}
\widehat{\theta}_A &= \min\left\{\overline{\theta}, \frac{L_A}{\delta(1-\pi)\, b(\delta)}\right\}, \\
\theta_A^* &= \min\left\{\overline{\theta}, \frac{F + L_A}{\delta(1-\pi)\, b(\delta)}\right\}, \\
\theta_{\max} &= \min\left\{\overline{\theta}, \frac{F + L_A + L_B}{\delta(1-\pi)\, b(\delta)}\right\}.
\end{aligned}$$

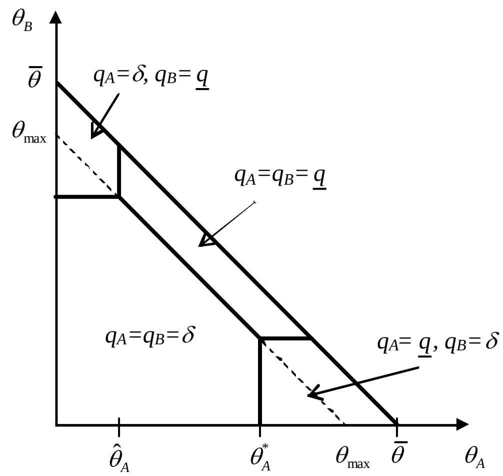
We think of $\widehat{\theta}_A$ as the largest country-A shock such that the country avoids crisis while using its domestic liquidity, L_A, alone. (Like before, crisis means a shortage of liquidity, low fire-sale prices and credit rationing.) Similarly, θ_A^* denotes the largest shock such that country A avoids crisis while using both its domestic liquidity, L_A, and the international liquidity, F. For country B , $\widehat{\theta}_B$ and θ_B^* are defined in the same manner.

It is clear that international liquidity will flow to the country with the higher return. Hence, if one country is in crisis and the other is not, all international liquidity will flow to the former. This allows us to characterize, in Figure 4, combinations of θ_A and θ_B in which crises occurs.[23]

[23] Like before, we focus only the Pareto dominant equilibrium.

Figure 4

The equilibrium level of liquidation prices q_A and q_B in countries A and B, depending on the realizations of θ_A and θ_B.



Consider country A. If $\theta_A \leq \widehat{\theta}_A$, then the country's own liquidity L_A is sufficient to avoid crisis, no matter how much liquidity is required in country B. When $\widehat{\theta}_A < \theta_A \leq \theta^*_A$ then country A needs to draw on the international liquidity in order to avoid crisis. To what extent it can do so depends on the shock realized in country B. If θ_B is sufficiently low (namely $\theta_B \leq \theta_{\max} - \theta_A$) both countries would avoid crisis. However, as θ_B increases, it draws in more international liquidity, up to the point where the international liquidity is not sufficient to avoid a simultaneous or a *systemic* crisis in both countries. This happens when $\theta_B > \theta_{\max} - \theta_A$. At this point, if one country is in crisis (say θ_B just tips across the point $\theta_A + \theta_B = \theta_{\max}$) then the other country is also in crisis. This follows immediately from the downwards sloping demand curves for liquidity.

Hence, crisis is contagious across countries: holding θ_A constant at some level smaller than θ^*_A and increasing the level of the shock in country B, there is a point at which the shock spills over and creates crisis in country A. It is thus clear that domestic liquidity is useful: it shields one country from the other's shock. But there is a downside as well: if $\theta_A > \theta^*_A$, country A is in crisis while it uses all domestic as well as international liquidity, while country B is not in crisis and has a slack of domestic liquidity. Moreover, below the dotted line, both countries could avoid crisis if only the domestic liquidity of country B could be deployed to country A. Evidently, compartmentalizing the world economy into liquidity-enclosed countries is potentially ex-post inefficient in its use of liquidity.

Next, we calculate the probability of crisis. Denote by ρ_F and ρ_A the realized rate of return on international liquidity, F, and domestic liquidity L_A, respectively. Obviously, $\rho_F = 1$ if there is a slack of liquidity in both countries and $\rho_F = \frac{\delta}{\underline{q}}$ otherwise. Similarly,

$\rho_A = 1$ if there is a slack of liquidity in country A and $\rho_A = \frac{\delta}{q}$ otherwise. We can calculate the probabilities of high / low returns on domestic or international liquidity by integrating the density function $h(\theta_A, \theta_B)$ over the appropriate intervals depicted in Figure 4. This yields:

$$\begin{aligned}
\Pr(\rho_F = 1) &= \frac{\theta_B^*}{\overline{\theta}}(\ln\theta_{\max} - \ln(\theta_B^*)) + \frac{\theta_A^*}{\overline{\theta}}(\ln\theta_{\max} - \ln\theta_A^*) + \frac{\theta_A^* - \widehat{\theta}_A}{\overline{\theta}}, \\
\Pr(\rho_A = 1) &= \frac{1}{\overline{\theta}}\left(\theta_A^*(1 + \ln\theta_{\max} - \ln\theta_A^*) + \widehat{\theta}_A\left(\ln\overline{\theta} - \ln\theta_{\max}\right)\right).
\end{aligned}$$

But would profit-oriented speculators ever wish to invest in L_A or L_B, instead of F? The following lemma provides a negative answer.

LEMMA 4 *In a competitive equilibrium the amount of domestic liquidity (L_A, L_B) is zero while international liquidity is*

$$F = \overline{\theta}\delta(1-\pi)\,b(\delta)\,\frac{\frac{\delta}{q} - \rho_0}{\frac{\delta}{q} - 1}.$$

A crisis occurs with probability $\frac{\rho_0 - 1}{\frac{\delta}{q} - 1}$.

It follows that strictly positive amounts of domestic liquidity can be obtained only through public policy: either directly through domestic liquidity injection (by the government or the central bank) or indirectly by requiring that foreign speculators in some lines of domestic business have to hold domestic liquidity. Since the rate of return on domestic liquidity is low relative to international liquidity, the regulation will have to compensate them in some other way, perhaps by allowing them to collect rents on these other lines of business (which might explain why restrictions on international capital flows often go hand in hand with oligopolistic financial markets).

The next question is whether such policy adds value. Consider the following events: (i) there is a crisis in country A alone, (ii) there is a crisis in country B alone, (iii) there is a systemic crisis in both countries. Denote the probabilities of these events by p_A, p_B, and $p_{A\curlywedge B}$, respectively. Only if one of the three events occurs will $\rho_F > 1$; hence $\Pr(\rho_F > 1) = p_A + p_B + p_{A\curlywedge B}$. The following proposition computes the equilibrium probabilities of these events, taking into consideration the crowding-out effect on international liquidity.

PROPOSITION 7 *Suppose L_A and L_B are sufficiently small such that the competitive liquidity supply $F > 0$ and $\theta_{\max} < \overline{\theta}$. In that case an increase in L_A reduces the probability $p_{A\curlywedge B}$ of a systemic crisis, but it does not affect the probability $p_{A\curlywedge B} + p_A + p_B$ that at least one country experiences a crisis.*

There are two points worth noting. First, domestic liquidity injection leads to crowding out of private liquidity so that the overall probability of a crisis remains unaffected. Second, although market compartmentalization does not affect the overall probability of a crisis, it dampens contagion: there are now realizations of θ_A, for which crisis does not occur in country A when one would have occurred there in absence of domestic liquidity L_A. Take values $\theta_A < \widehat{\theta}_A$, where domestic liquidity avoids a crisis in country A for any value of θ_B. If markets were fully integrated ($L_A = L_B = 0$), a crisis in A would have occurred whenever $\theta_B > \frac{F}{\delta(1-\pi)b(\delta)} - \theta_A$. It follows that a crisis could be contained domestically when it would have spilled over to the other country in a market without domestic liquidity.

A. Discussion and numerical example

It follows from the above discussion that domestic liquidity has an ambiguous effect on the economy: it avoids contagion but at a cost of idle liquidity. Hence, the correct way to evaluate the policy is by calculating ex-ante social welfare, SW, which is measured (like before) in terms of expected GDP in both countries, equivalent to expected terminal value per owner-manager of a bank. The calculation is similar to that in Section V, taking expectations over banks' short / slacked capital, when none of the countries, one or both of them going through financial crisis. See Appendix 3 for more detail. Unfortunately, at this point the model becomes too complicated to handle analytically, so we carry on with our numerical example. We consider symmetric regulation, namely $L = L_A = L_B$.

Figure 4 plots SW as a function of domestic liquidity for two cases: with and without restrictions on capital-flows – dashed and bold lines, respectively. In the former case domestic liquidity is just L as defined above. The latter case reduces to benchmark model with the combined domestic liquidity in both countries playing the same role as injected liquidity in Section A above, generating numerically identical results to those derived there. The flat segment is due to the one-to-one crowding-out effect discussed in Lemma 3. The steeply increasing segment is where private liquidity is fully crowded out and the policy of liquidity injections starts being effective. Once there is enough liquidity to prevent crisis completely, liquidity stands idle for any realization of θ and SW is falling in liquidity (now only incurring the opportunity cost).

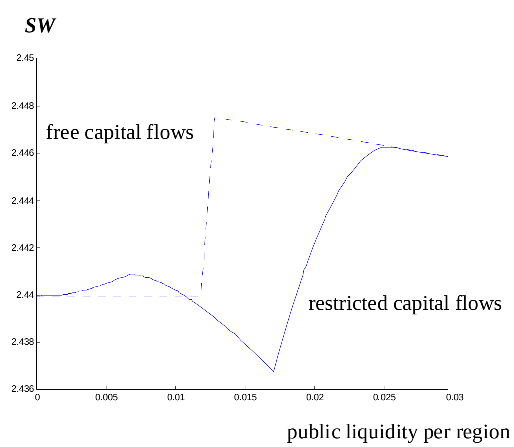
In the case where capital flows are restricted, private liquidity, F, is also allowed to respond endogenously to the changes in domestic liquidity. However, the policy is not neutral as implied by Proposition 7. Initially, more domestic liquidity decreases the probability of systemic crisis and increases SW. At some point, however, systemic crisis vanishes and the effect of idle liquidity dominates, so the curve starts sloping downwards. Then, the SW function reaches a minimum at the point where private liquidity, F, vanishes completely.

From there on SW is increasing again in L because it reduces the probability of domestic crisis. Crucially, the curve is way below the free-capital-mobility curve because of idle liquidity. The reason is obvious: in each country, the shock may be as high as $\overline{\theta}$, the same as the combined shock for both countries. Hence, to avoid a country crisis with certainty, each country has to hold enough liquidity to meet a $\overline{\theta}$ shock, the same as an integrated market needs to hold. Hence, by integrating their liquidity markets, the two countries can decrease by half the amount of liquidity they need to hold in order to avoid a crisis. To conclude, restrictions on capital flows can add value when liquidity is scarce and the governments can afford to inject only small amounts, but the non-monotonic nature of the SW function would make it hard to fine-tune a policy of that nature.

Figure 5

Social welfare as a function of domestic country liquidity,

with and without restrictions on capital mobility.



VII. Conclusions

This paper provides an analytical framework for the study of liquidity crises. It is couched firmly in a neoclassical framework with contracting frictions. The destabilizing effect of the market for liquidity derives from one simple peculiarity: demand and supply are sloping in the same direction. This observation allows us to capture the phenomenon of a crisis, which may occur both as a multiplicity of equilibria or as the unique equilibrium, depending on the magnitude of an aggregate shock.

Although many of the ingredients of our model have been used elsewhere, our framework unifies them and allows a simple and realistic characterization of crises. This allows us to generate a simple and observable measure of welfare and thus lends credibility to our policy analysis. In it, we find that the welfare gains from liquidity injections or bailouts by the government improve welfare, but only by a small amount. This is surprising, given the dramatic price effects of a crisis.

An interesting avenue for further research would be to link the current analysis more closely to macroeconomic questions of monetary and fiscal policy. Some of that literature uses concepts that are closely related to our, more micro founded analysis, for example cash-in-advance constraints or a liquidity trap. A full integration of the macro and microeconomic research in this fields remains an open topic for research.

Appendix 1: Data construction for Figure 1

Let $e_{i,t}$ and $l_{i,t}$ be the equity (either book or market) and total book liabilities of banking holding-company i at end-of-quarter t. For the period $Q1-2005$ to $Q4-2009$, all COMPUSTAT commercial banks (SIC codes 6021-6022) are surveyed, of which only the listed ones are kept, and matched with CRSP market equity. The number of banks in the data set varies from 257 to 350. Each period, the data are sorted by an increasing order of capitalization, so that the most highly levered bank is indexed one.

For each period, the Lorenz Curve is defined as $\left(\bar{l}_{j,t}, \bar{e}_{j,t}\right)$ where

$$\begin{aligned}\bar{l}_{j,t} &= \sum_{i=1}^{j} \frac{l_{i,t}}{\sum_{i=1}^{50} l_{i,t}},\\ \bar{e}_{j,t} &= \sum_{i=1}^{j} \frac{e_{i,t}}{\sum_{i=1}^{50} e_{i,t}}.\end{aligned}$$

By construction, the Lorenz Curve is increasing, concave, and straddles from $(0,0)$ to $(1,1)$. The Gini coefficient is the ratio between two areas: i) the one between the Lorenz Curve and the diagonal of the $(0,0)$-$(1,1)$ box, and ii) the triangle bellow that diagonal. Since the Lorenz Curve is always below the above diagonal, the Gini coefficient is between zero and one. If all banks are equally-well capitalized, the Gini would be zero; otherwise, the bigger are the differences in cross-banks capitalization, the higher is the Gini. Market (book) Gini is defined for market (book) equity, $e_{i,t}$.

The spread series measures the yield differential between LIBOR and government bonds, both of 6-months maturity denominated in US dollars.

Appendix 2: Sensitivity analysis

To check the robustness of our numerical example, we calculate below the elasticity of the key endogenous variables with respect to the main structural parameters. As key endogenous variables we consider the three variables in Table 2, for which there is actual data, namely those that are used in order to check the model's quantitative fit. These variables are: price-drop in crisis, output-drop in crisis and the probability of a crisis. We describe the results of the exercise in Table A1 below. To remove doubt, the result should be red as follows: a -2.92 elasticity of the probability of crisis with respect to the wealth endowment of capital-short companies means that when the latter increases from 0.6 to 0.606, the former falls from 7.7% to 7.5%. Obviously, the calculations are valid for a single point in the parameter space, namely that which is characterized by Table 1. Since all the main results are based on the model's non-linearity, it is important to check that this point in the parameter space is not knife-edged, namely that the sensitivity of the results to the parameters is not explosive. Indeed, the analysis seems to confirm that the model is (locally) stable in that respect: the highest elasticity is just below 3 and many below 1.

Table A1

Sensitivity analysis: the elasticity of key endogenous variables with respect to some of the structural parameters: price drop in crisis, output drop in crisis and the probability of crisis.

	$\frac{\underline{q}}{\delta} - 1$	$Y\vert_{q=\underline{q}} \big/ Y\vert_{q=\delta} - 1$	$1 - H(\theta^*)$
parameters			
δ	0.34	0.26	-0.57
y	0.85	0.97	-1.39
π	0.65	$-2,65$	-1.06
$\underline{w}$	1.81	1.73	-2.92

Appendix 3: Proofs

Proof of Lemma 1. : It follows from the discussion of (4) that the solution of the contracting problem is the minimal β within the problem's feasible set. The graph of (PC) is a downwards-sloping line in the (R, β) space; (IC) is represented by the area above an upwards-sloping line in the same space. Hence, the minimal point that satisfies both (IC) and (PC) is the intersection point between the two graphs (when the former holds with equality). That point is defined by equation (5). If $b \leq 1$, then (5) also defines the optimal contract; if $b > 1$ the feasible set is empty and the bank cannot obtain any funding. ■

Proof of Lemma 2. : It follows from equation (8) that S is increasing in q and that the rent for capital-slacked companies is greater than the rent for capital-short companies.

Hence, a sufficient condition for a positive rent at any feasible equilibrium price is that the rent for capital short companies at $q = \underline{q}$ is positive. Substituting $w = \underline{w}$ and $b\left(\underline{q}\right) = 1$ into equation (8) we derive the condition above. ■

Proof of Proposition 1. : When there is a slack of liquidity in the market speculators bid-up the fire-sale price to δ and still there is enough funding for all banks, so $\mu = 1$. Clearly

$$F + W - 1 - \theta\left[1 + \delta\left(1 - \pi\right) b\left(\delta\right)\right] \geqq 0.$$

When this condition fails, and since $qb\left(q\right)$ is decreasing in q – see equation (5) – the market-clearing condition cannot hold with $\mu = 1$ for any other price in $\left[\underline{q}, \delta\right]$. Hence the critical point, $\frac{F+W-1}{\delta(1-\pi)b(\delta)}$, so that for any θ above, credit rationing is the unique equilibrium.

In a credit-rationing equilibrium the fire-sale price drops to $\underline{q}$ and

$$F + W - 1 - \theta\left[1 + \underline{q}\left(1 - \pi\right)\right] \leqq 0;$$

remember that, by construction, $b\left(\underline{q}\right) = 1$. Consider the point where that condition holds with equality so that credit-rationing just appears. Since $qb\left(q\right)$ is decreasing in q, there must also be a liquidity-slack equilibrium. Hence the critical point $\frac{F+W-1}{\underline{q}(1-\pi)}$ so that for any θ below, the liquidity slack is the unique equilibrium.

Due to assumption (A1) the second critical point is greater than the first. It follows that in between there are multiple equilibria with either credit rationing, liquidity slack or a fire sale price in $\left(\underline{q}, \delta\right)$ where the market-clearing condition (10) holds with equality for $\mu = 1$.

Substituting $q = \underline{q}$ into the market-clearing condition (10) we can solve for the amount of credit rationing, or the unconditional probability of being credit rationed. ■

Proof of Proposition 2. : Let q^ebe the expected fire-sale price. Then, equilibrium is determined by substituting

$$b_{GR} = \frac{\frac{\delta}{q^e}\left(1 - \underline{w}\right)}{q^e\left(1 - \lambda\pi\right) + \lambda\pi y}$$

into the equilibrium condition (10). Clearly, the demand for liquidity is no longer sensitive to the actual market price, q. Hence, if for a certain realization of θ, δ was one of a few equilibrium prices, it is the unique equilibrium if $q^e = \delta$. Since there was enough liquidity to fund capital-short banks and absorb fire sales at a price of δ when expectations were not fixed, δ will be the actual fire-sale price when $q^e = \delta$. In such a case, if the government guarantees a price of δ, the holders of the guarantees will not exercise their option and the fiscal cost of the policy to the government is zero. Hence, the policy is credible and, indeed, $q^e = \delta$. Notice that this argument does not hold for θs where $\underline{q}$ is the unique equilibrium price (without price guarantees). ■

Proof of Proposition 3. : First, notice that the expected return on investing in liquidity is given by $E(\rho_1) = H(\theta^*) + [1 - H(\theta^*)] \frac{\delta}{\underline{q}}$, which is strictly decreasing in F and is equal to $\frac{\delta}{\underline{q}}$ for $F = 0$. If F is small (large) so that $E(\rho_1) > \rho_0$ $(E(\rho_1) < \rho_0)$ this cannot be an equilibrium since more (less) speculators would invest in the liquid asset. Hence, θ^* is uniquely given by the solution to (13) and F is determined by equation (12). Notice that the probability of a financial crisis is $(\rho_0 - 1) / \left(\frac{\delta}{\underline{q}} - 1\right)$, independently of h. ■

Proof of Proposition 4. : Since the government's trading profits (during crisis) are redistributive, they can be cancelled out using

$$\theta\mu(\theta)\underline{q}(1-\pi) = X + [1-\mu(\theta)]\theta,$$

which is derived from (11), so as to simplify the social-welfare function

$$\begin{aligned} SW &= \textstyle\int_0^{\theta^*} [2y - \theta(1-\pi)b(\delta)(2y-\delta)]h(\theta)\,d\theta \\ &+ \textstyle\int_{\theta^*}^{\overline{\theta}} [2y - \theta(1-\mu)(2y-1) - \theta\mu(1-\pi)(2y-\delta)]h(\theta)\,d\theta \\ &- (\rho_0 - 1)X. \end{aligned} \tag{20}$$

Taking a derivative with respect to X and denoting $\mu^* \equiv \mu(\theta^*)$ we get

$$\begin{aligned} \frac{d}{dX}SW &= [2y - \theta^*(1-\pi)b(\delta)(2y-\delta)]h(\theta^*)\frac{d\theta^*}{dF} \\ &- [2y - \theta^*(1-\mu^*)(2y-1) - \theta^*\mu^*(1-\pi)(2y-\delta)]h(\theta^*)\frac{d\theta^*}{dX} \\ &+ [(2y-1) - (1-\pi)(2y-\delta)][1-H(\theta^*)]\frac{\partial}{\partial X}\theta^*\mu^* \\ &- (\rho_0 - 1), \end{aligned}$$

which can be simplified to

$$\begin{aligned} \frac{d}{dF}SW &= [A_b\theta^* - A\theta^*\mu^*]h(\theta^*)\frac{d\theta^*}{dX} \\ &+ A[1-H(\theta^*)]\frac{\partial(\theta^*\mu^*)}{\partial X} \\ &- (\rho_0 - 1), \end{aligned} \tag{21}$$

where

$$\begin{aligned} A &= (2y-1) - (1-\pi)(2y-\delta), \\ A_b &= (2y-1) - b(\delta)(1-\pi)(2y-\delta), \\ \frac{d\theta^*}{dX} &= \frac{1}{\delta(1-\pi)b(\delta)}, \\ \frac{\partial(\theta^*\mu^*)}{\partial X} &= \frac{1}{1+\underline{q}(1-\pi)}. \end{aligned}$$

i) Since $A_b > A\mu^*$ and $\frac{d\theta^*}{dX} > 0$ we focus on the second and third lines of equation (21). We rewrite condition (13) as:

$$[1 - H(\theta^*)]\left(\frac{\delta}{\underline{q}} - 1\right) = (\rho_0 - 1)$$

and using Lemma 2 we derive

$$\begin{aligned} & A\frac{1 - H(\theta^*)}{1 + \underline{q}(1 - \pi)} - (\rho_0 - 1) \\ = \quad & \frac{1 - H(\theta^*)}{1 + \underline{q}(1 - \pi)}\left[\left(2y - \frac{\delta}{\underline{q}}\right) - (1 - \pi)\left(2y - \underline{q}\right)\right] > 0. \end{aligned}$$

ii) We show, first, that $A > 0$ as the left-hand side of the three inequalities below add up to A

$$\begin{aligned} \left(2y - \frac{\delta}{\underline{q}}\right) - (1 - \pi)\left(2y - \underline{q}\right) &> 0, \\ -(1 - \pi)\left(\underline{q} - \delta\right) &> 0, \\ \frac{\delta}{\underline{q}} - 1 &> 0. \end{aligned}$$

Since $A_b > A$ the first line of equation (21) is positive for any $\mu \in [0, 1]$; the whole expression is thus positive for a sufficiently low ρ_0.

iii) We show, first, that the social-welfare function is convex when θ is uniformly-distributed:

$$\begin{aligned} \frac{d^2}{dX}SW &= \frac{1}{\overline{\theta}}\frac{d\theta^*}{dX}\left[A_b\frac{d\theta^*}{dX} - 2A\frac{d(\theta^*\mu^*)}{dX}\right] \\ &= \frac{1}{\overline{\theta}}\left(\frac{d\theta^*}{dX}\right)^2\frac{d(\theta^*\mu^*)}{dX}\left\{[A_b - A\delta(1 - \pi)b(\delta)] + (1 - \pi)\left[A_b\underline{q} - A\delta b(\delta)\right]\right\} > 0 \end{aligned}$$

where $\underline{q} > \delta b(\delta)$ is implied by the downward sloping demand for liquidity. Since we have already shown that social welfare is increasing at the competitive X (where $\theta^* < \overline{\theta}$) that implies a corner solution (at the point where $\theta^* = \overline{\theta}$). ■

Proof of Proposition 5. : Differentiating (16) with respect to E we get:

$$\begin{aligned} & \frac{d}{dE}SW = [2y - \theta^*(1 - \pi)b(\delta)(2y - \delta)]\,h(\theta^*)\frac{d\theta^*}{dE} \\ & -\frac{db(\delta)}{dE}(1 - \pi)(2y - \delta)\textstyle\int_0^{\theta^*}\theta h(\theta)\,d\theta \\ & -[2y - \theta^*(1 - \mu^*)(2y - 1) - \theta^*\mu^*(1 - \pi)(2y - \delta)]\,h(\theta^*)\frac{d\theta^*}{dE} \\ & +[(2y - 1) - (1 - \pi)(2y - \delta)][1 - H(\theta^*)]\left[\frac{d}{dE}\frac{F(E) + E}{1 + \underline{q}(1 - \pi)}\right] \\ & -(\rho_0 - 1), \end{aligned}$$

The notation $F(E)$ is used in order to account for the case where where the private speculators are still active. In this case

$$\begin{aligned}\frac{d\theta^*}{dE} &= -\frac{\left[1-H(\theta^*)\right]\frac{\delta}{\underline{q}^2}\frac{dq}{dE}}{h(\theta^*)\left(\frac{\delta}{\underline{q}}-1\right)} > 0, \\ \frac{d}{dE}\frac{F(E)+E}{1+\underline{q}(1-\pi)} &= \frac{\delta(1-\pi)b(\delta)\frac{d\theta^*}{dE}-(F+E)\frac{dq}{dE}}{\left[1+\underline{q}(1-\pi)\right]^2}.\end{aligned}$$

When the private speculators are no longer active,

$$\begin{aligned}\frac{d\theta^*}{dE} &= \frac{1}{\delta(1-\pi)b(\delta)}\left[1-\frac{E}{b(\delta)}\cdot\frac{db(\delta)}{dE}\right], \\ \frac{d}{dE}\frac{E}{1+\underline{q}(1-\pi)} &= \frac{1}{1+\underline{q}(1-\pi)}\left[1-\frac{(1-\pi)}{1+\underline{q}(1-\pi)}\frac{d\underline{q}}{dE}\right].\end{aligned}$$

Both ways, for the three cases identified in Proposition 4, by comparing the derivative above to equation (21) one can establish that the effect of equity injection is greater than the effect of liquidity injection. Clearly, the full crowding-out result for the former case is no longer valid. ■

Proof of Proposition 6. An equity-injection equilibrium satisfies a market-clearing condition and collateral requirement

$$\begin{aligned}W+F^*+E-(1-\theta) &\geq \mu\theta\left[1+(1-\pi)\beta^I q\right], \\ \beta^I &= \frac{(1-\underline{w}-E)\rho_1}{q(1-\lambda\pi)+\lambda\pi y},\end{aligned}$$

respectively. Now consider a subsidy (contingent on the realized θ) of $\sigma = \frac{E\rho_1}{(1-\pi)\beta^I q}$; using (19) one can calculate

$$\overline{\beta}^\sigma = \frac{\left(1-\underline{w}-\frac{1-\lambda\pi}{1-\pi}E\right)\rho_1}{q(1-\lambda\pi)+\lambda\pi y} < \beta^I$$

(because $\frac{1-\lambda\pi}{1-\pi} > 1$). Since β, as defined in equation (19) is decreasing in σ, $\overline{\beta}^\sigma$ is the upper bound on the subsidy that is required in order to implement the β^I achieved by the equity-injection for any realization of θ. The cost of this program, in the worst possible case where $\theta = \overline{\theta}$, is bounded by $E\rho_1\overline{\theta}$. If the government wishes to implement the duplicating subsidy policy in any state of nature then $E\rho_1\overline{\theta}$ is the liquid reserve – and the national debt – that it needs to create. In all states of nature other than the worse case, the government would have excess liquidity that it can use in order to generate additional welfare.

Notice that under the subsidy, the term E would vanish from the market-clearing condition, but would be substituted by private liquidity according to the crowding-out argument.

The aggregate cost of liquidity to the economy, $\left[F + \overline{\theta}\left(1-\pi\right)\sigma\beta q\right]\left(\rho_0 - 1\right)$, would thus increase although the cost of the public component would decrease.

The original equity-injection program should be small in the sense that the effect of E can be duplicated by the subsidy without violating the (17), namely $E \leq \left(y-\delta\right)\left(1-\pi\right)\beta^I\big|_{q=\delta}$. Also, the equity-injection problem should be small in the sense that $q\left(\rho\right)$ will stay sufficiently high (low) so that $\overline{\theta}\rho_1 \leq 1$. ■

Proof of Lemma 4. It is obvious from Figure 4 that $\Pr\left(\rho_A = 1\right) > \Pr\left(\rho_F = 1\right)$. It follows that the expected return on domestic liquidity is strictly lower than on international liquidity.

F is determined by the condition

$$\Pr\left(\rho_F = 1\right) + \frac{\delta}{\underline{q}}\left(1 - \Pr\left(\rho_F = 1\right)\right) = \rho_0. \tag{22}$$

Using the fact that $L_A = L_B = 0$ and substituting into $\Pr\left(\rho_F = 1\right)$ yields the result. Moreover, $\Pr\left(\rho_F > 1\right) = \frac{\rho_0 - 1}{\frac{\delta}{\underline{q}} - 1}$ follows directly from the equilibrium condition (22). ■

Proof of Proposition 7. We know that if L_A and L_B are sufficiently low, the equilibrium condition (22) has an interior solution with $F > 0$. It follows that in this case F is such that $p_A + p_B + p_{A\curlywedge B} = \frac{\rho_0 - 1}{\frac{\delta}{\underline{q}} - 1}$, i.e., a change in L_A does not affect the overall probability of a crisis somewhere.

We can calculate

$$\begin{aligned} p_{A\curlywedge B} &= \int_{\widehat{\theta}_A}^{\theta_A^*}\int_{\theta_{\max}-\theta_A}^{\overline{\theta}-\theta_A}\frac{1}{\overline{\theta}\left(\theta_A+\theta_B\right)}d\theta_B d\theta_A \\ &\quad + \int_{\theta_A^*}^{\theta_A^*+\overline{\theta}-\theta_{\max}}\int_{\theta_{\max}-\theta_A^*}^{\overline{\theta}-\theta_A}\frac{1}{\overline{\theta}\left(\theta_A+\theta_B\right)}d\theta_B d\theta_A \end{aligned}$$

This can be rewritten as

$$\begin{aligned} p_{A\curlywedge B} &= \frac{1}{\overline{\theta}}\left(\theta_A^* - \widehat{\theta}_A\right)\left(\ln\overline{\theta} - \ln\theta_{\max}\right) \\ &\quad + \frac{1}{\overline{\theta}}\left(\overline{\theta} - \theta_{\max}\left(1 + \ln\overline{\theta} - \ln\theta_{\max}\right)\right). \end{aligned}$$

Note that when L_A and L_B are so large that $\theta_{\max} = \overline{\theta}$, then $p_{A\curlywedge B} = 0$. When $\theta_{\max} < \overline{\theta}$ it is clear that the first term is decreasing in L_A. Taking the derivative of the second term with respect to L_A yields $-\frac{\ln\overline{\theta} - \ln\theta_{\max}}{\overline{\theta}\delta(1-\pi)b(\delta)} < 0$, so $p_{A\curlywedge B}$ is decreasing in L_A. ■

A. Social Welfare (section VI)

Social welfare can be calculated as follows. First we take expected payoffs for the parameter region where neither region experiences a crisis. For region A this is given by

$$\int_0^{\widehat{\theta}_A}\int_0^{\overline{\theta}-\theta_A}\frac{y-\theta_A B_{NC}}{\overline{\theta}\left(\theta_A+\theta_B\right)}d\theta_B d\theta_A+\int_{\widehat{\theta}_A}^{\theta_A^*}\int_0^{\theta_{\max}-\theta_A}\frac{y-\theta_A B_{NC}}{\overline{\theta}\left(\theta_A+\theta_B\right)}d\theta_B d\theta_A,$$

and similarly for region B, where

$$B_{NC}=(1-\pi)\,b\,(\delta)\,(2y-\delta)\,.$$

We can then calculate the expected payoffs when θ_A and θ_B are such that only region A, but not B is in a crisis.

$$\int_{\theta_A^*}^{\overline{\theta}-\left(\theta_{\max}-\theta_A^*\right)}\int_0^{\theta_{\max}-\theta_A^*}\frac{A_{C,A}-\theta_A B_C}{\overline{\theta}\left(\theta_A+\theta_B\right)}d\theta_B d\theta_A+\int_{\overline{\theta}-\left(\theta_{\max}-\theta_A^*\right)}^{\overline{\theta}}\int_0^{\overline{\theta}-\theta_A}\frac{A_{C,A}-\theta_A B_C}{\overline{\theta}\left(\theta_A+\theta_B\right)}d\theta_B d\theta_A,$$

and similarly for region B, where for $j\in\{A,B\}$

$$\begin{aligned}A_{C,j} &= y+\frac{F+L_j}{1+\underline{q}\,(1-\pi)}\left(2y-\frac{\delta}{\underline{q}}-(1-\pi)\left(2y-\underline{q}\right)\right),\\ B_C &= 2y-\frac{\delta}{\underline{q}}-\frac{2y-\frac{\delta}{\underline{q}}-(1-\pi)\left(2y-\underline{q}\right)}{1+\underline{q}\,(1-\pi)}.\end{aligned}$$

Finally, we can calculate the expected payoffs when both regions experience a crisis. Here we calculate the sum of the payoffs across both regions directly:

$$\int_{\widehat{\theta}_A}^{\theta_A^*}\int_{\theta_{\max}-\theta_A}^{\overline{\theta}-\theta_A}\frac{A_{CC}-\left(\theta_A+\theta_B\right)B_C}{\overline{\theta}\left(\theta_A+\theta_B\right)}d\theta_B d\theta_A+\int_{\theta_A^*}^{\overline{\theta}-\left(\theta_{\max}-\theta_A^*\right)}\int_{\theta_{\max}-\theta_A^*}^{\overline{\theta}-\theta_A}\frac{A_{CC}-\left(\theta_A+\theta_B\right)B_C}{\overline{\theta}\left(\theta_A+\theta_B\right)}d\theta_B d\theta_A,$$

where

$$A_{CC}=2y+\frac{F+L_A+L_B}{1+\underline{q}\,(1-\pi)}\left(2y-\frac{\delta}{\underline{q}}-(1-\pi)\left(2y-\underline{q}\right)\right).$$

From these payoffs, we need to subtract the opportunity cost of local, and mobile liquidity.

Solving the integrals and adding up over all possible regimes, we obtain total welfare as a function of F, where F is given by the solution to (22) (or zero, if (22) cannot be satisfied at a positive F).

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