The Value of Endogenous above Exogenous Information in Irreversible Environmental Decisions*

Giuseppe ATTANASI (Toulouse School of Economics, LERNA) †

Aldo MONTESANO (Bocconi University, Milan) ‡

November 2011

*We thank Philippe Mahenc, François Salanié, Nicolas Treich, participants at the 2006 Meeting on Game Theory and Practice dedicated to development, natural resources and the environment in Zaragoza, at the FUR 2006 in Rome, at the AFSE 2008 in Toulouse and at a seminar at University of Padua, as well as three anonymous referees for their useful comments and suggestions. Financial support by the Chair "Georges Meyer" in Mathematical Economics at Jean-Jacques Laffont Foundation (TSE) and by the French National Research Agency (ANR) through the project on "Fair Environmental Policies" is gratefully acknowledged.

†Corresponding author: Chair "Georges Meyer" in Mathematical Economics, Toulouse School of Economics, 21 allée de Brienne, 31000 Toulouse. Phone: +33(0)561125776. Fax: +33(0)561128520. Email: attanasi@cict.fr

‡Department of Economics, Bocconi University, via Roentgen, 1 20136 Milan. Email: aldo.montesano@unibocconi.it

The Value of Endogenous above Exogenous Information in Irreversible Environmental Decisions

Abstract This paper introduces the concept of the *Testing Value* into the analysis of environmental decisions under uncertainty and irreversibility. It is defined as the value of endogenous above exogenous information. We start from a situation where information concerning future economic benefits and costs of resource preservation is exogenous. We show that if information can also depend on the level of development carried out (i.e., it may be acquired also endogenously) the Testing Value could push a risk-neutral decision maker to preserve more in the present and eventually in the future. Although its existence stems from endogenous information, surprisingly enough, the Testing Value can be positively related to the probability of acquiring information exogenously.

JEL references: C61; D81; Q32.

Keywords: Environmental Preservation; Irreversibility; Exogenous and Endogenous Information; Value of Information; Testing Value.

1 Introduction

This paper extends the dynamic environmental preservation under uncertainty and irreversibility model of Arrow and Fisher (1974) in order to incorporate the possibility of learning both by exogenous information and by developing the environmental resource (endogenous information). The aim of the paper is to examine the implications of adding endogenous information in the standard setting in which exogenous information is available.

The issue of irreversibility and uncertainty in environmental decisions has been broadly analyzed by economic theorists. Since the first definition of the quasi-option value given by Arrow and Fisher (1974), the key concept has been developed in several articles, including Henry (1974), Freixas and Laffont (1984), Hanemann (1989) and Fisher (2000).

The concept of quasi-option value has been introduced by Arrow and Fisher (1974) in a two-period model of the choice of the optimal preservation level of a natural resource. Development can take place either in the current or in the future period. However, once undertaken, the resource cannot be restored to its original state of preservation. The future benefits of preservation and development are uncertain. The expected net benefits of preservation in the future period are conditional upon the current choice. Two crucial assumptions are risk neutrality of the Decision Maker (DM henceforth) and the ability to learn without developing the environmental resource (exogenous information). The latter implies that the DM can receive information about the future benefits of her current choice independently of this choice.[1] In this framework preserving the whole environmental resource in the current period "preserves" flexibility in the future, and the quasi-option value is the value of such flexibility. This value is a "quasi"-option because it vanishes when the resource is completely developed.

The quasi-option value strain of the literature has been especially rife with confusion regarding key terms and equations.[2] Indeed, there is a long history of referring to such

[1] In particular, information can emerge with the passage of time (e.g., as the second period approaches, one can make a more accurate assessment of the social value of wilderness preservation in that period) or as the result of a separate research program.

[2] See Hanemann (1989), who corrects Conrad (1980), about the interpretation of the Arrow-Fisher quasi-option value, and Mensink and Requate (2005), who correct Fisher (2000), about the relation between the Arrow-Fisher quasi-option value and the real option value of Dixit and Pindyck (1994).

measures as the value of information (e.g., Raiffa and Schlaifer, 1961). Following this approach, in this paper we will use the notion of value of information instead of quasi-option value when analyzing the Arrow-Fisher decision problem.

As anticipated above, the conclusions drawn by Arrow and Fisher (1974) and the related literature on the optimality of complete preservation of an environmental resource due to the irreversibility effect are derived under the assumptions of *linear benefit functions* and *only exogenous information*. In this case, the DM's maximization problem generically leads to corner solutions, i.e. either completely developing or completely preserving the environmental asset. As is well known (Kolstad, 1996; Ulph and Ulph, 1997; Gollier, Jullien and Treich, 2000; Fisher and Narain, 2003), with only exogenous information, a partial resource preservation can be optimal if the benefit function is non-linear. Moreover, according to the form of the benefit function assumed, the direction of the irreversibility effect can lead to more or less preservation with respect to a scenario with no information.[3] However, in this paper we maintain the benefit function linearity assumption, which allows us to obtain the explicit solution of the maximization problem and to easily combine quantitative analysis with economic intuition.

We generalize the Arrow-Fisher exogenous information scenario by introducing a more complex technology for information production, which also includes the possibility of obtaining information by developing the environmental resource (endogenous information). The aim of this paper is to underline the potential role of additional endogenous information in terms of environmental protection in the Arrow-Fisher decision problem. In this framework, few papers have analyzed the role of endogenous information (e.g. Miller and Lad, 1984), some of them introducing information production functions which are too specific (e.g. Freeman, 1984; Fisher and Hanemann, 1987).[4]

In our framework, we focus on an information scenario where both exogenous and en-

[3]In particular, Epstein (1980) states a set of sufficient conditions on the expected net benefits function so that the Arrow and Fisher (1974) irreversibility effect appears. Graham-Tomasi (1995), Mäler and Fisher (2005) and Salanié and Treich (2009) further clarify this specific issue.

[4]Freeman (1984) assumed that information can be acquired with certainty by developing any portion of the environmental resource. In this endogenous information scenario, there are only corner solutions for the current level of development, in the sense that the DM either develops fully now, or engages in an infinitesimal amount $\varepsilon > 0$ of development. Accordingly, Fisher and Hanemann (1987) have defined the quasi-option value of the minimum feasible development ($\varepsilon-$development).

dogenous information are available ((*exo*+*endo*) henceforth). We interpret this scenario as an alteration of the Arrow-Fisher (only) exogenous information scenario through the addition of the possibility of endogenous learning. This framework admits three particular cases: only exogenous information (*exo*), only endogenous information (*endo*), and no information (*no*). The (*exo*) information case is an extension of the Arrow-Fisher framework by allowing information to arrive (before the future period) with a varying degree of uncertainty, while in the Arrow-Fisher framework it arrives with certainty. In the (*endo*) information case we assume that the probability of obtaining information (before the future period) depends positively on the amount of resource developed in the current period.

We define the *value of information* as the difference between the expected value of the optimal choice in each information scenario and the expected value of the optimal choice in the (*no*) information scenario. Accordingly, we obtain the value of information for the (*exo*+*endo*), (*exo*) and (*endo*) information scenarios. Furthermore, we define as Testing Value the *value of endogenous above exogenous information*, i.e. the difference between the value of information in the (*exo*+*endo*) scenario and the value of information in the (*exo*) scenario. The Testing Value is the additional value gained by access to endogenous information, additional with respect to information arriving exogenously.

Our theoretical analysis produces some key results, which can be used to investigate crucial environmental policy issues. First of all, we show that there exists a non-negligible subset of net benefits of preservation in the current and in the future period such that the possibility of acquiring information endogenously (added to the possibility of acquiring it exogenously) leads the DM to preserve more in the current period (and often also in the future) compared to the case in which only exogenous information is potentially available. Furthermore, the environmental resource saving due to additional endogenous information is increasing in the probability of exogenous information. Finally, when this environmental resource saving is positive, the Testing Value is also increasing in the probability of exogenous information. This last result is evidence of a form of complementarity between endogenous and exogenous information.

There is a large number of environmental problems to which our theoretical insights could apply. Consider the problem of extraction and use of shale gas, a natural gas produced from shale. In the last ten years, the importance of shale gas as a source of natural gas has been steadily increasing in the United States, and interest in potential gas shales is spreading to the rest of the world. However, there is uncertainty about both its total supply (economic uncertainty) and the environmental damages caused by its extraction and use (ecological uncertainty). Concerning the former, since shales ordinarily have insufficient permeability to allow significant fluid flow to a wellbore, most shales are not commercial sources of natural gas. As for the latter, there is growing exogenous evidence that the extraction of shale gas leads to direct land damages due to hydraulic fracturing[5] and that its use results in the release of more greenhouse gases than conventional natural gas. Consider another more abstract example: an untouched piece of land with a beautiful landscape can be developed into a holiday village through an economic project which involves building infrastructures and facilities. There are two values attached to this project: an environmental one (preservation of the beauty and biodiversity of the land) and an economic one (the aim is to attract tourists who want to enjoy activities like skiing, fishing and walking). Uncertain benefits of preservation concern local people's preferences about their land. Uncertain benefits of development concern the land's touristic attraction. Both these uncertainties can be exogenously resolved: researchers can interview local people and analyze potential tourists' tastes about the landscape. However, the broader the land development project, the faster these two uncertainties are resolved.[6] In both examples, resource development in the current period speeds up the acquisition of knowledge of the true state of the world (i.e., the sooner the state of the world is learned when more resource is developed). Under this scenario, our theoretical analysis suggests that, if exogenous research programs are already in place, exploiting the possibility of learning by developing could ultimately lead to more resource preservation.

[5]Hydraulic fracturing is the propagation of fractures in a rock layer caused by the presence of a pressurized fluid. The practice of hydraulic fracturing has come under scrutiny internationally due to concerns about the environment, health and safety, and has been suspended or banned in some countries.

[6]For example, the higher the number of new buildings, the faster biodiversity reduction and the more is understood about the value local people attach to their land and about whether and how they agree with the development project (new infrastructures might or might not fit in well with the surroundings). From the other side, one learns by attracting people. Therefore, the broader the holiday village, the higher the number of tourists attracted, the faster the estimation of their willingness to pay.

If this is the case, the resource saving due to additional endogenous information is higher as much exogenous research on the resource has already been carried out.

Notice that the topic of the role of endogenous *above* exogenous information in environmental decision making has not been analyzed elsewhere, although there are articles where the distinction between exogenous and endogenous information is addressed. For example, there exists a literature on adaptive management in which *passive* adaptive management and *active* adaptive management (Walters and Hilborn, 1978) are close to the ideas of exogenous and of endogenous information examined here, while the words "passive"and "exogenous"do not exactly overlap.[7] Besides that, the active adaptive management approach is generally analyzed as an alternative to the passive adaptive one and not as integrated with the latter.

Much relevant work about the presence of exogenous and endogenous information has been done in the real options literature. For example, Martzoukos (2003) provides the valuation of real investment options in three different information scenarios, namely with only exogenous learning, with only endogenous learning, and with both exogenous and endogenous learning (although their definitions of exogenous and endogenous differ somewhat from our own).[8] Closer to our analysis is the one of Marwah and Zhao (2010). Although they focus on a more specific environmental problem with irreversibility,[9] they compare the optimal decisions of three types of DMs that correspond respectively to our exogenous, endogenous and no information scenarios. However, they do not analyze the case of both exogenous and endogenous information simultaneously arriving. Therefore, the additional role of endogenous information over the exogenous one, which is central to our paper, is not considered at all.

[7] In Walters and Hilborn (1978) the passive adaptive management approach includes, as possible ways of producing exogenous information about a specific resource, (only) general-processes studies and previous experience with "similar"resources. Therefore, non-environmentally-costly research about the specific resource, which gives exogenous information in our framework, is not included in the passive adaptive approach.

[8] As in Walters and Hilborn (1978), R&D programs not affecting environmental preservation are not framed as exogenous learning. Rather, they are classified as "pure learning controls" in the endogenous learning scenario, where, contrarily to our framework, it is possible to actively learn without depleting the environmental resource. In our framework endogenous learning implies resource depletion. Moreover, in Martzoukos (2003) the issue of irreversibility is not treated at all.

[9] They consider a DM who can convert land between agricultural use and serving as a wildlife habitat for a certain species. In this problem, they take into account two-sided irreversibilities, where species loss is irreversible and land preservation efforts involve sunk costs.

The remaining part of the paper is structured as follows. In section 2 we present our two-period model of environmental choice under uncertainty and irreversibility. In particular, we discuss the main features of our information structure. In section 3 we analyze the DM's maximization problem and optimal environmental choices in the four information scenarios, namely (*exo*+*endo*), (*exo*), (*endo*), and (*no*) information. For each information scenario, we provide a graphical representation of the regions of net benefits for which a specific optimal preservation path emerges. By referring to these graphs, in section 4 we calculate the value of information in the four information scenarios as functions of the parameters of the environmental decision problem. In the second part of the section, we introduce the value of endogenous above exogenous information and we study in depth how it depends on the probability of endogenous information and on the probability of exogenous information. Section 5 concludes the paper by discussing some policy implications of our theoretical insights.

2 The Model

2.1 Assumptions and notation

Consider a two-period model of environmental decision. The DM chooses the amount of environmental resource to preserve at two subsequent *times* ($\tau = 1, 2$). We call *period* 1 the time period between $\tau = 1$ and $\tau = 2$ and *period* 2 the time period after $\tau = 2$. At $\tau = 1$ the DM chooses the amount of environmental resource to be preserved in *period* 1, i.e. until $\tau = 2$. At $\tau = 2$ she chooses the amount of resource to be preserved in *period* 2. Given the assumption that development is irreversible, the DM's options at $\tau = 2$ are constrained by the decision taken at $\tau = 1$. Normalizing the level of the environmental resource to 1, $c_1 \in [0, 1]$ denotes the amount preserved at $\tau = 1$. By irreversibility, the amount preserved at $\tau = 2$ cannot be greater than c_1.

We define the two-period expected net benefits adopting the same separable and linear functional form used by Arrow and Fisher (1974). Let the net benefit be directly proportional to the amount of preserved resource, with b_1 representing the net benefit per unit of resource preserved in period 1.[10] We assume that the current net benefit from

[10] In Dasgupta and Heal (1979) and Chichilnisky and Heal (1993), b_τ represents the benefit of preser-

preservation is known to the DM at $\tau = 1$ and is negative, i.e. $b_1 < 0$, because it is net of the opportunity cost from forgone development. Thus, the unique incentive to choose $c_1 \neq 0$ at $\tau = 1$ is given by the possibility to obtain a positive future net benefit from preservation in period 2.[11]

This future net benefit is uncertain, depending on two possible states of the world. We indicate with b_2^j the net benefit per unit of resource still preserved in period 2, when the state of the world is s^j, with $j = u, f$. The future net benefit from preservation is negative if the state of the world is s^u (*u*nfavorable state), and positive if the state of the world is s^f (*f*avorable state), i.e. $b_2^u < 0$, $b_2^f > 0$. We indicate with $p \in [0, 1]$ the probability of the unfavorable state s^u.

With probability π, the state is revealed to the DM in period 1, i.e. *before* she takes her decision at $\tau = 2$. With probability $1 - \pi$, the DM does not know the state of the world when she takes her decision at $\tau = 2$: this state will be revealed in period 2, *after* this decision has been taken.

We indicate with c_2 the amount of environmental resource preserved at $\tau = 2$ when the state of the world has not been revealed in period 1 and with c_2^j the amount of environmental resource preserved at $\tau = 2$ when the DM knows the revealed state of the world is s^j. The structure of the decision problem is represented in Fig. 1, where squares indicate decision nodes and circles represent moves of nature. The decision problem can be summarized as follows:

- $\tau = 1$: the DM chooses the amount of the resource to be preserved in *period* 1;
- *period* 1: the state of the world is either revealed or not;
- $\tau = 2$: the DM chooses the amount of the resource to be preserved in *period* 2;
- *period* 2: the state of the world is revealed if it was not revealed in *period* 1.

Therefore, when the DM receives information in period 1, at $\tau = 2$ she is in the upper part of the decision tree and she chooses the optimal preservation level at $\tau = 2$, $(c_2^u)^*$ or $(c_2^f)^*$, knowing the state of the world, s^u or s^f, respectively. Otherwise, at $\tau = 2$ she is in

vation in period τ, with $\tau = 1, 2$. We interpret it as the difference between the benefit of preservation and the benefit of development in period τ.

[11]We choose not to consider the case $b_1 = 0$ in the analysis, since it makes the choice of c_1 irrelevant concerning the net benefit in period 1.

the lower part of the decision tree and the optimal choice at $\tau = 2$, c_2^*, is independent of the state of the world, as it will not be revealed until period 2.

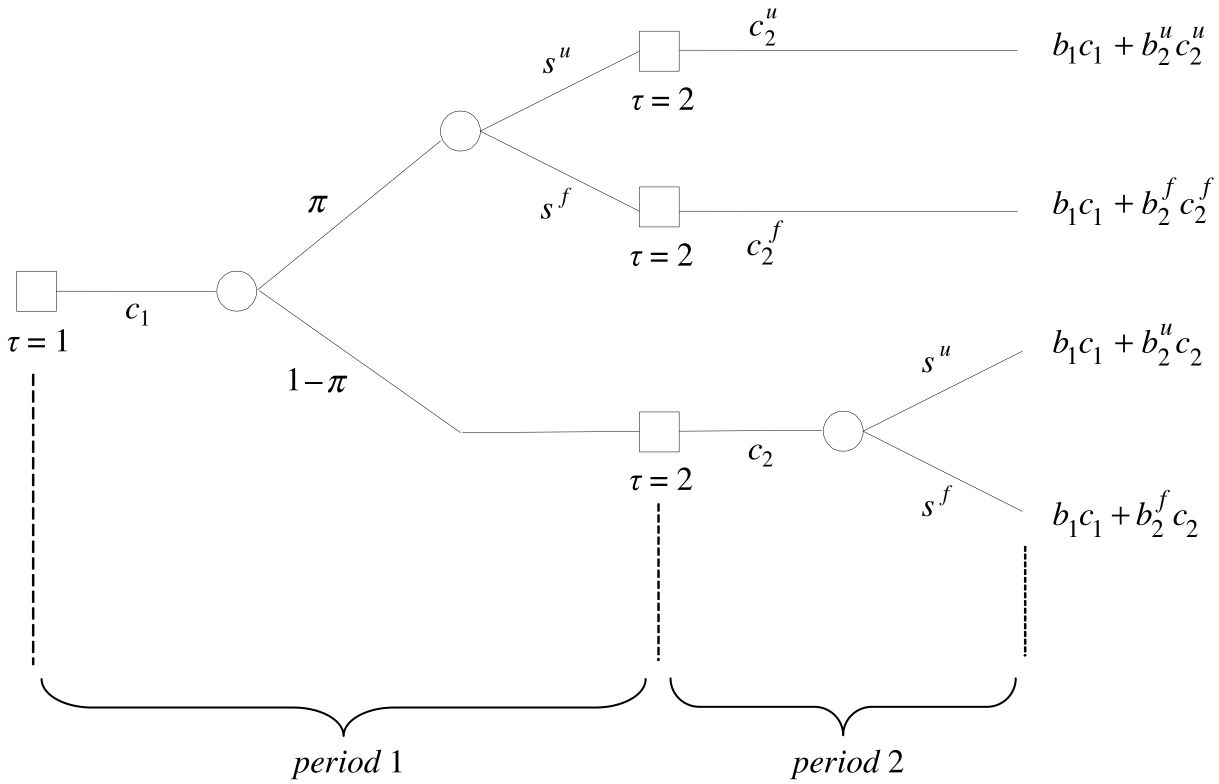

Figure 1: The decision tree.

Notice that in our model we have two events: one concerns *whether* the state of the world is unfavorable (with probability p) or favorable to preservation; the other one indicates *when* the disclosure of the state of the world takes place, i.e. before (with probability π) or after $\tau = 2$. If the true state of the world is not revealed before $\tau = 2$, the DM cannot get any additional information about its probability. Indeed, at $\tau = 2$ only two cases are possible: either the DM knows the true state of the world (because of information disclosure before $\tau = 2$), or she has the same information that she had when choosing at $\tau = 1$ (i.e., $\Pr(s^u) = p$ both in the upper part of the decision tree in period 1 and in the lower part of the decision tree in period 2) and information disclosure will take place after $\tau = 2$. Given that if new information arrives in period 1 it fully resolves uncertainty about the state of the world, throughout this paper the expression "information arrives in period 1" means "the state of the world is revealed in period 1".

2.2 Information structure

In our framework, when choosing the amount of the resource to preserve at $\tau = 1$, the DM does not know whether she will know the state of the world or not when she comes to choose the amount to preserve at $\tau = 2$. The key parameter is $\pi \in [0, 1]$, the probability that the state will be revealed in period 1, i.e. the probability that information will arrive before $\tau = 2$.

The parameters of which π is a function can be used to distinguish different "kinds" of information. Information can be (only) exogenous, (only) endogenous, or both. In the first case, π does not depend on $(1 - c_1)$, the amount of environmental resource developed at $\tau = 1$. In the second case, π depends only on $(1 - c_1)$. Here, as discussed in section 1, we assume that in the case of endogenous learning the probability of information arrival in period 1 depends positively on the level of development carried out at $\tau = 1$. In the third case, π depends both on an exogenous parameter and on the amount of the resource developed at $\tau = 1$, i.e. $(1 - c_1)$.

Therefore, in the general case in which both *exo*genous and *endo*genous information may occur (henceforth referred to as the (*exo*+*endo*) scenario), the probability that information will arrive in period 1 is an increasing function of $q \in [0, 1]$, the probability of acquiring exogenous information (i.e., the probability of information arrival when $c_1 = 1$), and a non-increasing function of c_1, the level of preservation of the environmental resource at $\tau = 1$. More precisely, $\pi = f(q, c_1)$, with $f : [0, 1] \times [0, 1] \to [0, 1]$. We assume that the information production function f is continuous and differentiable, with $\partial f/\partial q > 0$ and $\partial f/\partial c_1 \leq 0$. Two other reasonable assumptions characterize our information production function. First, if the DM completely preserves the environmental resource at $\tau = 1$, then (endogenous learning is not possible at all and) the probability that the state will be revealed in period 1 coincides with the probability of information exogenously arriving, i.e. $f(q, 1) = q$. Furthermore, as in Arrow and Fisher (1974), if $q = 1$ then the state of the world is exogenously revealed in period 1 whatever the level of preservation/development carried out at $\tau = 1$, i.e. $f(1, c_1) = 1$.

We can express π explicitly through the Taylor expansion of $f(q, c_1)$ starting from

$(0, 1)$ up to the second order:

$$\pi = \alpha q + \beta(1 - c_1) + \gamma q^2 + \theta(1 - c_1)^2 + \eta q(1 - c_1) \tag{1}$$

Condition $f(q, 1) = q$ leads to $\alpha = 1$ and $\gamma = 0$. Condition $f(1, c_1) = 1$ leads to $\theta = 0$, $\alpha + \gamma = 1$ and $\beta + \eta = 0$; the two conditions together give $\gamma = 0$ and $\eta = -\beta$. Putting these values into (1), we obtain

$$\pi = q + \beta(1 - q)(1 - c_1) \tag{2}$$

Recall that $f(q, c_1) \in [0, 1]$ for each $q \in [0, 1]$ and $c_1 \in [0, 1]$. This means that in (2) we have $\beta \in [0, 1]$. Defining $\lambda := \beta(1 - q)$, the probability that the state of the world is revealed in period 1, stopping at the second order of the Taylor expansion of $f(q, c_1)$, can be rewritten as

$$\pi = q + \lambda(1 - c_1), \qquad \text{for} \quad c_1 \in [0, 1], \quad \text{with} \quad q \in [0, 1], \quad \lambda \in [0, 1 - q], \tag{3}$$

where $\lambda(1 - c_1)$ represents the probability of information endogenously arriving. In this sense, λ coincides with the upper bound of the probability of endogenous information, i.e. with $c_1 = 0$. The other relevant information scenarios are derived by imposing specific restrictions on the key parameters in (3):

(*exo*) only *exo*genous information: $\lambda = 0$ for $c_1 \in [0, 1]$;

(*no*) *no* information: $\lambda = q = 0$ for $c_1 \in [0, 1]$;

(*endo*) only *endo*genous information: $q = 0$ for $c_1 \in [0, 1]$.

Notice that in subcase (*exo*) the DM is certain that she will know the state of the world in period 1 if and only if $q = 1$.[12] Conversely, in subcase (*endo*), complete resource development at $\tau = 1$ (i.e., $c_1 = 0$) is only a necessary condition for the state of the world to be revealed with certainty in period 1; it should be assumed that $\lambda = 1$ too.[13] In the (*exo*+*endo*) scenario, this last condition corresponds to setting $\lambda = 1 - q$: if this is the case, by choosing $c_1 = 0$ the DM is certain that she will know the state of the world in period 1 even if $q < 1$.

[12] This is the case analyzed in Arrow and Fisher (1974), in Hanemann (1989) and in related papers on environmental option values where information is only exogenous.

[13] This demonstrates how our information structure crucially differs from the Freeman (1984) endogenous information scenario, where Fisher and Hanemann (1987) define the quasi-option value of the minimum feasible development (ε-development). In these papers, the assumption that information is provided with certainty by any amount of development would imply in our endogenous information setting that $\pi = 1$ for every $c_1 \neq 0$.

3 Optimal preservation choices

3.1 Second-period optimal behavior

In this section, for each of the four information structures described in section 2.2, we find the risk-neutral DM's optimal preservation levels at $\tau = 1$ and at $\tau = 2$. We follow a backward induction procedure, by first analyzing the DM's decision problem at $\tau = 2$, then solving backward her expected value maximization problem at $\tau = 1$.

In this subsection we state two basic results about the optimal preservation levels at $\tau = 2$: when the state of the world is revealed in period 1 (i.e., $(c_2^j)^*$, for $j = u, f$) and when it is not revealed (i.e., c_2^*). Both results hold independent of the kind of information structure we deal with, i.e. independent of the values of q and λ in (3). Referring to Fig. 1:

Lemma 1. Suppose that the state of the world is revealed in period 1 (upper part of the decision tree at $\tau = 2$). Then $(c_2^u)^* = 0$ and $(c_2^f)^* = c_1$.

Lemma 2. Suppose that the state of the world is not revealed in period 1 (lower part of the decision tree at $\tau = 2$). If the expected second-period net benefit of preservation (i.e., $pb_2^u + (1-p)b_2^f$) is negative, then ${c_2}^* = 0$; if it is null, then ${c_2}^* \in [0, c_1]$; if it is positive, then ${c_2}^* = c_1$.

Therefore, if the DM knows the state of the world when choosing at $\tau = 2$, she develops all the resource if the unfavorable state of the world has been realized, and preserves everything otherwise. If instead she has to choose at $\tau = 2$ under the same uncertainty about the state of the world she faced at $\tau = 1$, the optimal second-period preservation level depends on the sign of the difference between the two relative weights $-b_2^u/b_2^f$ and $(1-p)/p$. The former is the ratio between the possible loss and the possible gain from second-period preservation, while the latter accounts for the inverse of their probabilities.

In the next subsection, we rely on Lemma 1 and Lemma 2 in order to solve backward for the optimal preservation level at $\tau = 1$, which crucially depends on the specific information structure the DM faces in period 1, as expressed by (3).

3.2 General case (*exo+endo*): both exogenous and endogenous information

Let us write and solve the DM's utility maximization problem in the general case, in which both exogenous and endogenous information are available with some probability (respectively, with $q \in [0,1]$ and $\lambda(1-c_1) \in [0,(1-q)(1-c_1)]$) in period 1. Given Lemma 1, the realized payoffs are as indicated in Fig. 2.

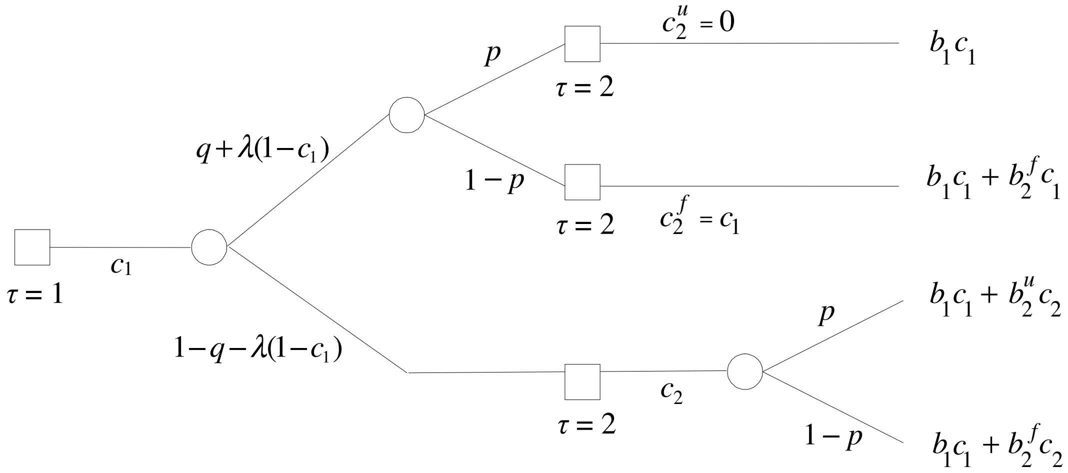

Figure 2: The decision problem in the (*exo* + *endo*) scenario.

The DM's expected value of net benefits of preservation in both periods is:

$$EV_{exo+endo}(c_1, c_2 |\ c_2^u = 0, c_2^f = c_1) = [q + \lambda(1-c_1)] \left[b_1 + (1-p)b_2^f\right] c_1 + \{1 - [q + \lambda(1-c_1)]\} \left[b_1c_1 + pb_2^uc_2 + (1-p)b_2^fc_2\right] \tag{4}$$

Let us normalize the net benefits in terms of b_2^f by defining $x := -b_1/b_2^f$, $y := -b_2^u/b_2^f$. Notice that both x and y are positive. We call x the *relative first-period loss from preservation*: it is the ratio between the loss from preservation in period 1 and the gain in the favorable state of the world in period 2. Accordingly, we call y the *relative second-period loss from preservation*: it is the ratio between the loss from preservation in period 2 in the unfavorable state and the gain in the favorable one. We can rewrite the objective function (4) normalized with respect to b_2^f (which is consequently set equal to 1):

$$EV_{exo+endo}(c_1, c_2 |\ c_2^u = 0, c_2^f = c_1) = (q + \lambda - \lambda c_1)(1-p-x)c_1 + (1 - q - \lambda + \lambda c_1)\left[(1-p-py)c_2 - xc_1\right] \tag{5}$$

By Lemma 2, we can make (5) explicit in terms of expected second-period net benefit:

$$EV_{exo+endo} = \begin{cases} [(1-p)(q+\lambda) - x]\, c_1 - (1-p)\lambda\, (c_1)^2 & \text{if } y \in ((1-p)/p, +\infty) \\ [1-p-x-(1-q-\lambda)\, py]\, c_1 - \lambda py\, (c_1)^2 & \text{if } y \in (0, (1-p)/p] \end{cases} \quad (6)$$

Notice that the objective function is quadratic and concave in c_1, so that its maximization with respect to c_1 in $[0,1]$ leads to (both boundary and) internal solutions for c_1. Considering the solutions for c_2 as stated in Lemma 2, the optimal preservation levels at $\tau = 1, 2$ as a function of the parameters of the problem (p, q, λ, x, y) are:

$$(c_1^*, c_2^*)_{exo+endo} = \begin{cases} (0,0) & \text{if} \quad (x,y) \in \boldsymbol{DD}(p,q,\lambda) \\ \frac{(1-p)(q+\lambda)-x}{2(1-p)\lambda} \times (1,0) & \text{if} \quad (x,y) \in \boldsymbol{LD}(p,q,\lambda) \\ (1,0) & \text{if} \quad (x,y) \in \boldsymbol{PD}(p,q,\lambda) \\ \frac{(1-p-x)-(1-q-\lambda)py}{2\lambda py} \times (1,1) & \text{if} \quad (x,y) \in \boldsymbol{LL}(p,q,\lambda) \\ (1,1) & \text{if} \quad (x,y) \in \boldsymbol{PP}(p,q,\lambda) \end{cases} \quad (7)$$

where $\boldsymbol{kj}(p,q,\lambda)$, for $\boldsymbol{k}, \boldsymbol{j} = \boldsymbol{D}, \boldsymbol{L}, \boldsymbol{P}$, indicates the regions of pairs of relative losses from preservation $(x,y) \in \mathbb{R}^2_{++}$ leading to a specific profile of optimal preservation levels at $\tau = 1$, and at $\tau = 2$ (when the state of the world is not revealed in period 1), for given probabilities $p, q \in (0,1)$ and $\lambda \in (0, 1-q)$.

In Fig. 3 (p. 16) the five different regions of optimal values of $(c_1^*, c_2^*)_{exo+endo}$ in (7) are represented for given values of p, q and λ, for the three cases $q > \lambda$ (Fig. 3.1), $q = \lambda$ (Fig. 3.2) and $q < \lambda$ (Fig. 3.3). Fig. 3 shows that moving from the top-right to the bottom-left corner of the set of possible pairs of relative losses from preservation (x,y), the optimal level of environmental resource preservation (is constant or) increases. Notice that the darker the color of a region, the greater the optimal amount of preservation at $\tau = 1$ for the pairs of relative losses from preservation belonging to that region. In particular:

- Region $\boldsymbol{DD}(p,q,\lambda)$, which represents complete **D**evelopment at $\tau = 1$ and so at $\tau = 2$, emerges both for $x \in (1-p-(1-q-\lambda)py, +\infty)$ and $y \in (0, (1-p)/p)$, and for $x \in [(q+\lambda)(1-p), +\infty)$ and $y \in ((1-p)/p, +\infty)$. If the relative first-period loss from preservation (x) is higher than the maximum expected relative second-period net benefit from preservation (i.e., $1-p$),[14] then the optimal choice is to completely develop the

[14]The second-period net benefit from preservation is maximal when the net benefit from preservation in the unfavorable state of the world is null, i.e. $y = 0$.

environmental resource at $\tau = 1$ and, by irreversibility, also at $\tau = 2$ (bottom-right corner of Fig. 3). The same also happens for lower values of x when the relative second-period loss from preservation, y, is greater than zero (right-hand side of Fig. 3). In particular (at the top of Fig. 3), if the expected second-period net benefit of preservation is negative (i.e., $py > 1 - p$), then, by Lemma 2, in the case of information not arriving in period 1, at $\tau = 2$ complete resource development is optimal. Consequently, if $py > 1 - p$, the only opportunity cost that should matter in case of complete development at $\tau = 1$ is the second-period gain from preservation if the state of the world is revealed in period 1 (with probability $(q + \lambda)$, given that $c_1 = 0$) and it is the favorable one (with probability $1 - p$). If this opportunity cost is lower than the first-period loss from preservation (i.e. $(q + \lambda)(1 - p) < x$), then $(c_1^*)_{exo+endo} = 0$ is the optimal choice (top-right corner of Fig. 3). Notice that, due to irreversibility, in this region complete resource development takes place at $\tau = 2$ regardless of whether information arrives or not in period 1, i.e. $(c_2^u)^* = (c_2^f)^* = (c_2^*)_{exo+endo} = 0$.

- Region $\boldsymbol{LD}(p, q, \lambda)$, which represents $\boldsymbol{L}$imited preservation at $\tau = 1$ and complete $\boldsymbol{D}$evelopment at $\tau = 2$ (i.e., $(c_1^*)_{exo+endo} \in (0, 1)$ and $(c_2^*)_{exo+endo} = 0$), emerges for $x \in (\max\{0, (q - \lambda)(1 - p)\}, (q + \lambda)(1 - p)]$ and $y \in ((1 - p)/p, +\infty)$. Since the expected second-period net benefit of preservation is negative, complete development of the resource takes place at $\tau = 2$ in the case of information not arriving in period 1 (at the top of Fig. 3). However, due to the fact that the relative first-period loss from preservation (x) is lower than $(q + \lambda)(1 - p)$, choosing $(c_1)_{exo+endo} = 0$ is not optimal. The amount of the resource that it is optimal to preserve at $\tau = 1$ does not depend on y, depends negatively on x and p, and depends positively on q. The dependence on λ is ambiguous. If $x = q(1 - p)$, then $(c_1^*)_{exo+endo} = 1/2$, regardless of λ. If $x \in (q(1 - p), (q + \lambda)(1 - p))$, then $(c_1^*)_{exo+endo} \in (0, 1/2)$, and increases with λ: we are in the sub-region $\boldsymbol{L_\ell D}$, where the subscript $\boldsymbol{\ell}$ stands for a preservation level $\boldsymbol{\ell}$ower than 1/2. If $x \in (\max\{0, (q - \lambda)(1 - p)\}, q(1 - p))$, then $(c_1^*)_{exo+endo} \in (1/2, 1)$, and decreases with λ: we are in the sub-region $\boldsymbol{L_h D}$, where the subscript $\boldsymbol{h}$ stands for a preservation level $\boldsymbol{h}$igher than 1/2.

- Region $\boldsymbol{PD}(p, q, \lambda)$, which represents complete $\boldsymbol{P}$reservation at $\tau = 1$ and complete $\boldsymbol{D}$evelopment at $\tau = 2$, emerges for $q > \lambda$, $x \in (0, (q - \lambda)(1 - p)]$ and $y \in$

$((1-p)/p,+\infty)$. Given that y is high, by Lemma 2 we are again in a situation where it is optimal to develop all the resource at $\tau=2$ in the case of information not arriving in period 1 (at the top of Fig. 3). However, x is so low that it is optimal to completely preserve the environmental resource at $\tau=1$ (top-left corner of Fig. 3.1). Notice that this region emerges only when $q>\lambda$: a necessary condition is that the probability of acquiring exogenous information is higher than the maximum (i.e. with $c_1=0$) probability of information endogenously arriving.

- Region $\boldsymbol{LL}(p,q,\lambda)$, which represents $\boldsymbol{L}$imited preservation at $\tau=1$ and at $\tau=2$ (i.e., $(c_1^*)_{exo+endo}=(c_2^*)_{exo+endo}\in(0,1)$), emerges for $x\in[\max\{0,1-p-(1-q+\lambda)py\},1-p-(1-q-\lambda)py]$ and $y\in(0,(1-p)/p)$. Given that y is low, by Lemma 2, in the case of information not arriving in period 1, at $\tau=2$ it is optimal to preserve all the resource that was preserved at $\tau=1$. In turn, the amount of the resource preserved at $\tau=1$ depends negatively on x,y and p, and depends positively on q. The dependence on λ is ambiguous. If $x=1-p-(1-q)py$, then $(c_1^*)_{exo+endo}=1/2$ regardless of λ. If $x\in(1-p-(1-q)py,1-p-(1-q-\lambda)py)$, then $(c_1^*)_{exo+endo}\in(0,1/2)$ and increases with λ: we are in the sub-region $\boldsymbol{L_\ell L_\ell}$. If $x\in(\max\{0,1-p-(1-q+\lambda)py\},1-p-(1-q)py)$, then $(c_1^*)_{exo+endo}\in(1/2,1)$, and decreases with λ: we are in the sub-region $\boldsymbol{L_h L_h}$.

- Region $\boldsymbol{PP}(p,q,\lambda)$, which represents complete $\boldsymbol{P}$reservation at $\tau=1$ and at $\tau=2$, emerges for $x\in(0,1-p-(1-q+\lambda)py)$ and $y\in(0,\min\{1,1/(1-q+\lambda)\}(1-p)/p)$. Both the relative first-period loss from preservation and the second-period one are so low that it is optimal to completely preserve the environmental resource in both periods (bottom-left corner of Fig. 3). In general, notice that when the expected second-period net benefit of preservation is positive, there could be complete preservation at $\tau=1$ even if $q\leq\lambda$, while we have seen that in the opposite case $(c_1^*)_{exo+endo}=1$ only if $q>\lambda$ (region $\boldsymbol{PD}$). When $q>\lambda$ (Fig. 3.1), for $x<(q-\lambda)(1-p)$, we have complete preservation in both periods when the expected second-period net benefit of preservation is positive, and if negative we only have complete preservation at $\tau=1$. If instead $q\leq\lambda$ (Fig. 3.2 and Fig. 3.3), we have complete preservation in both periods only if the expected relative second-period net benefit of preservation (i.e., $1-p-py$) is greater than $(\lambda-q)py$. If this condition is not satisfied then complete preservation at $\tau=1$ is not optimal at all.

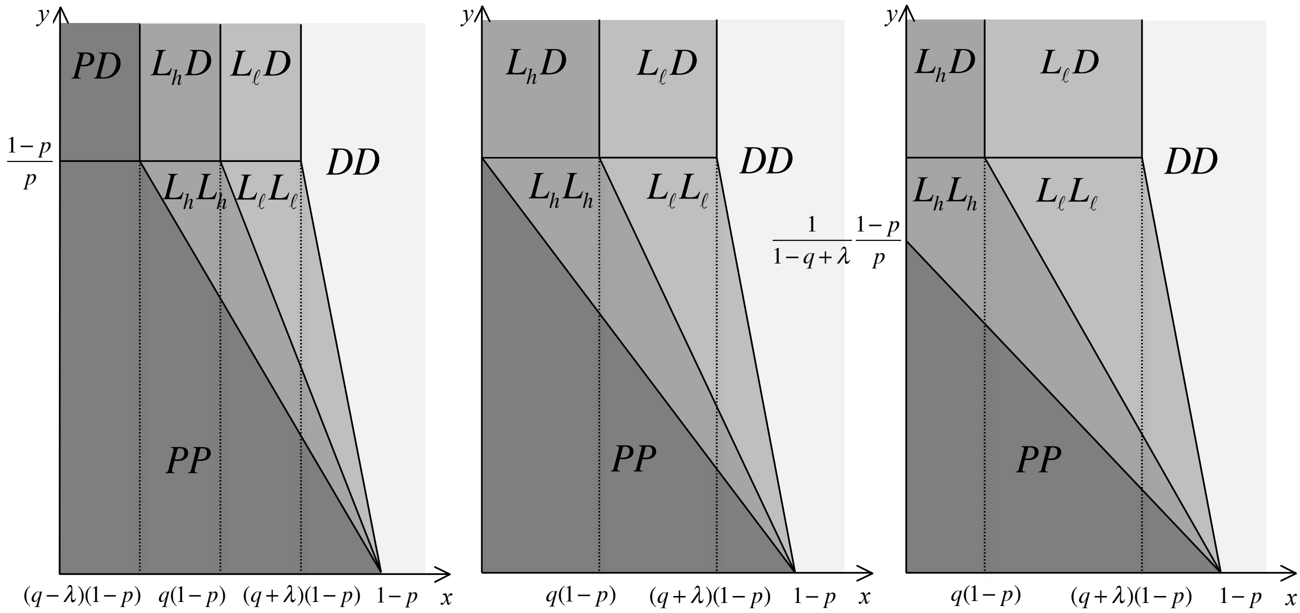

Figure 3.1: $q > \lambda > 0$.

Figure 3.2: $q = \lambda > 0$.

Figure 3.3: $\lambda > q > 0$.

Figure 3: Optimal choices in the (*exo*+*endo*) scenario [$p = 0.8$; in Fig. 3.1, $q = 0.5, \lambda = 0.25$; in Fig. 3.2, $q = \lambda = 0.375$; in Fig. 3.3, $q = 0.25, \lambda = 0.5$].

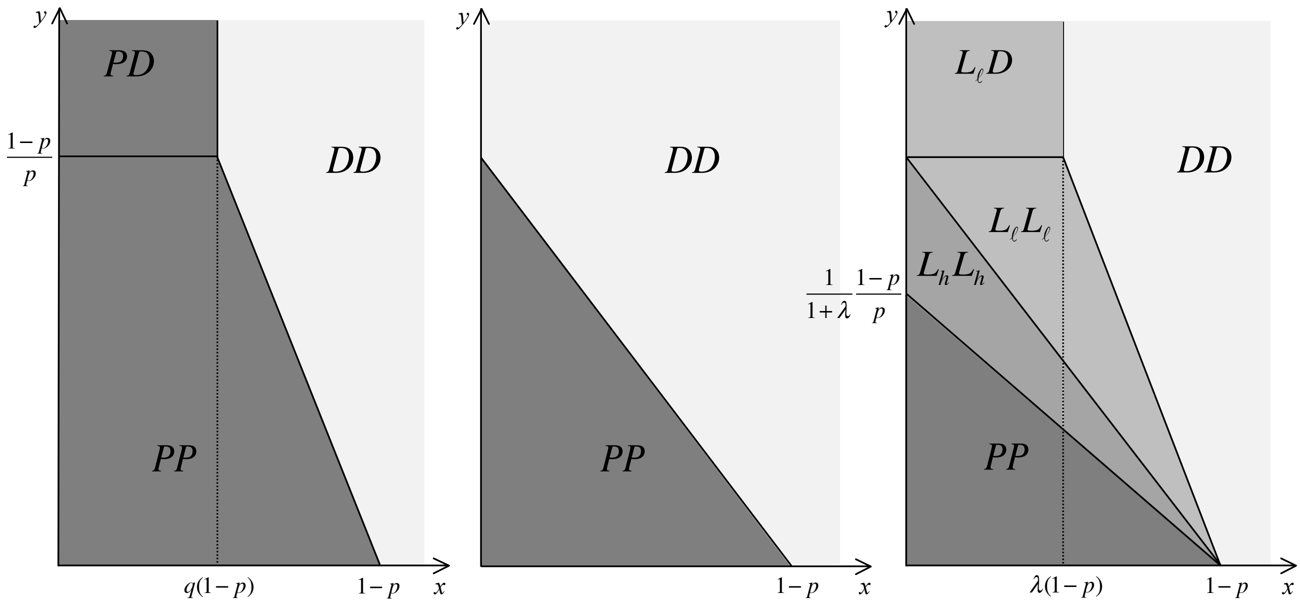

Figure 4.1: $q > \lambda = 0$.

Figure 4.2: $q = \lambda = 0$.

Figure 4.3: $\lambda > q = 0$.

Figure 4: Optimal choices in the (*exo*), (*no*), (*endo*) information scenario [$p = 0.8$; in Fig. 4.1, $q = 0.5, \lambda = 0$; in Fig. 4.2, $q = \lambda = 0$; in Fig. 4.3, $q = 0, \lambda = 0.5$].

3.3 Specific information scenarios

Let us specify the previous results in the three subcases introduced in section 2.2. They are represented in Fig. 4.

Subcase (*exo*): only exogenous information. Since $\lambda = 0$, the DM at $\tau = 1$ knows that, independent of the preservation level chosen at $\tau = 1$, with probability $q \in [0, 1]$ the second-period state of the world is revealed before choosing at $\tau = 2$. The optimal levels of preservation $(c_1^*, c_2^*)_{exo}$ are easily obtainable by substituting $\lambda = 0$ into (7). Fig. 4.1 represents the regions $\boldsymbol{kj}(p, q, 0)$, for $\boldsymbol{k}, \boldsymbol{j} = \boldsymbol{D}, \boldsymbol{P}$, of pairs of relative losses $(x, y) \in \mathbb{R}^2_{++}$ leading to a specific profile of optimal preservation levels at $\tau = 1$, and at $\tau = 2$ (when the state of the world is not revealed in period 1) if information can arrive only exogenously. Optimal behavior in the (*exo*) scenario in Fig. 4.1 has been represented for the same values of p and q of the (*exo*+*endo*) scenario in Fig. 3.1, by imposing $\lambda = 0$. Notice that complete preservation at $\tau = 1$ (region $\boldsymbol{PD}(p, q, 0)$) is possible even if the expected value of the second-period net benefit is negative ($y > (1 - p)/p$). If the relative second-period loss from preservation is so high, a necessary condition for complete preservation at $\tau = 1$ is that the relative first-period loss from preservation is smaller than $q(1 - p)$,[15] this condition being less stringent with respect to the general case, which requires $x < (q - \lambda)(1 - p)$. If the expected value of the second-period net benefit is positive, preservation at $\tau = 1$, and at $\tau = 2$ in the case of information not arriving in period 1 (region $\boldsymbol{PP}(p, q, 0)$) can be optimal for a relative first-period loss from preservation lower than $(1 - p)$, as in the general case. Because of the linearity of the expected benefit function with respect to c_1 and c_2, in our optimization problem only corner solutions for preservation at $\tau = 1, 2$ are possible when information is only exogenous.[16] Therefore, in every case where it is not optimal for the DM to completely preserve the environmental resource, she develops it completely (region $\boldsymbol{DD}(p, q, 0)$).

Subcase (*no*): no information. Since $\pi = 0$, the DM cannot obtain information in period 1. Hence, in the backward induction procedure only the lower part of the

[15] Recall that $(1 - p)$ is the relative second-period net benefit from preservation in the favorable state of the world multiplied by its probability.

[16] In section 4.3, we show that our main results continue to hold even when we relax this assumption.

decision tree in Fig. 1 matters. The optimal levels of preservation $(c_1^*, c_2^*)_{no}$, which can be easily obtained by substituting $q = \lambda = 0$ into (7), are shown in Fig. 4.2. This represents the regions $\boldsymbol{kj}(p, 0, 0)$, for $\boldsymbol{k}, \boldsymbol{j} = \boldsymbol{D}, \boldsymbol{P}$, of pairs of relative losses from preservation $(x, y) \in \mathbb{R}^2_{++}$ leading to a specific profile of optimal preservation levels at $\tau = 1$ and at $\tau = 2$, given that no information can arrive in period 1. Fig. 4.2 can be interpreted as a particular case of Fig. 3.2, given that in both cases we have $q = \lambda$: in the latter, $q = \lambda > 0$ (with $q + \lambda < 1$); in the former, $q = \lambda = 0$. Given that there is no possibility of new information arriving before $\tau = 2$, for the DM the optimization problem reduces to choosing c_1 and c_2 simultaneously at $\tau = 1$ (i.e., we always find $(c_1^*)_{no} = (c_2^*)_{no}$). Therefore, she preserves everything in both periods (region $\boldsymbol{PP}(p, 0, 0)$) only if the expected value of the second-period net benefit of preservation is positive and greater than the absolute value of the first-period net benefit (i.e. $1 - p - py > x$); if this is not the case, she develops everything (region $\boldsymbol{DD}(p, 0, 0)$).

Subcase (*endo*): only endogenous information. Since $q = 0$, the DM can obtain information in period 1 with probability $\lambda(1 - c_1)$, with $\lambda \in [0, 1]$, only if at $\tau = 1$ she develops a portion of the environmental resource, i.e. $(c_1)_{endo} \neq 1$. The probability of information arriving in period 1 depends negatively on c_1. The optimal levels of preservation $(c_1^*, c_2^*)_{endo}$ are easily obtained by substituting $q = 0$ into (7). Fig. 4.3 represents the regions $\boldsymbol{kj}(p, 0, \lambda)$, for $\boldsymbol{k}, \boldsymbol{j} = \boldsymbol{D}, \boldsymbol{L}, \boldsymbol{P}$, of pairs of relative net benefits $(x, y) \in \mathbb{R}^2_{++}$ leading to a specific profile of optimal preservation levels at $\tau = 1$ and at $\tau = 2$ (when the state of the world is not revealed in period 1) if information can arrive only endogenously. Optimal behavior in the (*endo*) scenario in Fig. 4.3 has been represented for the same values of p and λ of the (*exo*+*endo*) scenario in Fig. 3.3, by imposing $q = 0$. Notice that when the expected value of the second-period net benefit of preservation is negative $(y > (1 - p)/p)$, it is never optimal to preserve completely the environmental resource at $\tau = 1$ (regions $\boldsymbol{L_\ell D}(p, 0, \lambda)$ or $\boldsymbol{DD}(p, 0, \lambda)$). In particular, the highest possible amount of the resource that it is optimal to preserve at $\tau = 1$ is limited by 1/2. Conversely, when the expected value of the second-period net benefit of preservation is positive, preservation of the whole amount of the resource at $\tau = 1$ (and at $\tau = 2$ in the case of information not arriving) can still occur (region $\boldsymbol{PP}(p, 0, \lambda)$), which means the DM gives up the chance to receive information in period 1. However, the expected value of the second-period net benefit of preservation being greater than the absolute value of the first-period net benefit (i.e. $1 - p - py > x$) is not sufficient to guarantee no development at all, as was the case in the (*no*) information scenario. If the positive difference between the two values $(1-p-py-x)$ is lower than λpy, than the DM prefers partially preserving the resource at $\tau = 1$ and at $\tau = 2$ in the case of information not arriving in period 1 (region $\boldsymbol{LL}(p, 0, \lambda)$). This is because the expected second-period net benefit of preservation, while being greater than the first-period loss from preservation, is not big enough to compensate the loss from not exploiting the information endogenously arriving, this loss (when the revealed state of the world is the unfavorable one) being represented by λpy. Therefore, region $\boldsymbol{LL}(p, 0, \lambda)$ is more likely to emerge the higher the probability of information endogenously arriving for whatever $c_1 > 0$, the higher the probability that the state of the world is unfavorable to preservation at $\tau = 2$, and the greater the relative second-period loss from preservation. By comparing Fig. 4.3 (*endo*) to Fig. 4.1 (*exo*), it is easy to notice that, given p, if the maximum probability of information endogenously arriving (λ) is equal to the probability of information exogenously arriving (q), then region $\boldsymbol{DD}(p, 0, \lambda)$ emerges for the same values of (x, y) for which $\boldsymbol{DD}(p, q, 0)$ emerges.

4 Analysis of environmental values of information

4.1 Optimal expected value function and Value of information

In this section we compare the DM's behavior in the different information scenarios in order to measure the value of exogenous and of endogenous information. In particular, we state case by case whether and why the possibility that a specific "kind" of information (exogenous, endogenous, or both exogenous and endogenous) arrives in period 1 leads to more or less preservation - at $\tau = 1$, and at $\tau = 2$ in the case of information not arriving in period 1 - with respect to a scenario in which this possibility is totally absent. In each information scenario (*exo*), (*endo*), and (*exo*+*endo*), we first define the value of the specific kind of information as a function of the decision problem parameters. Then, we describe its main features, and its effects on the DM's optimal preservation behavior.

In order to compute the value of information through comparison of two different information scenarios, we first need to calculate the optimal expected net benefit from preservation in each of the four information scenarios introduced in section 2.2. Let us consider the general case (*exo*+*endo*). By substituting the optimal preservation levels $(c_1^*, c_2^*)_{exo+endo}$ as in (7) into the expression of the expected net benefit (5), we obtain the *optimal expected value function* in the (*exo*+*endo*) scenario:

$$EV^*_{exo+endo}(x,y) = \begin{cases} 0 & \text{if } (x,y) \in \boldsymbol{DD}(p,q,\lambda) \\ \frac{[(1-p)(q+\lambda)-x]^2}{4(1-p)\lambda} & \text{if } (x,y) \in \boldsymbol{LD}(p,q,\lambda) \\ (1-p)q-x & \text{if } (x,y) \in \boldsymbol{PD}(p,q,\lambda) \\ \frac{[(1-p-x)-(1-q-\lambda)py]^2}{4\lambda py} & \text{if } (x,y) \in \boldsymbol{LL}(p,q,\lambda) \\ (1-p-x)-(1-q)py & \text{if } (x,y) \in \boldsymbol{PP}(p,q,\lambda) \end{cases} \tag{8}$$

The optimal expected value function in each of the three other information scenarios can be easily obtained by substituting the specific values for q and λ characterizing the particular scenario into (8), i.e. $EV^*_{exo}(x,y) = EV^*_{exo+endo}(x;y;\lambda = 0)$, $EV^*_{no}(x,y) = EV^*_{exo+endo}(x;y;q = \lambda = 0)$, and $EV^*_{endo}(x,y) = EV^*_{exo+endo}(x;y;q = 0)$.

Following Raiffa and Schlaifer (1961), we define the *value of information* (*VOI* henceforth) of a given information scenario as the optimal expected value function in that scenario minus the expected value function in the (*no*) information scenario, i.e. with respect to a situation in which the DM knows that no information about the true state of the world will arrive in period 1. In the alternative information scenario, information can arrive in period 1 with probability π, this information having one of three possible natures: only exogenous, only endogenous, or both exogenous and endogenous. The fact that the prospect of future information is fully recognized and explicitly incorporated in the decision at $\tau = 1$ gives a value to this new information, that is measured as

$$VOI_{info}(x,y) = EV^*_{info}(x,y) - EV^*_{no}(x,y) \tag{9}$$

with $info = exo, endo, exo+endo$.

As anticipated in section 1, our main analytical interest is about the role that *additional* information *endogenously* arriving has on the DM's optimal environmental preservation behavior. In section 4.2 we investigate the value of additional information with respect to a situation in which information cannot arrive at all. We first compare the (*exo*) scenario to the (*no*) information one, thus defining the value of exogenous information ($VOI_{exo}(x, y)$). We make this comparison because it has been frequently analyzed in the environmental option values literature: it enables us to show how our framework (in which exogenous information can arrive with some probability) generalizes the quasi-option value analysis of Arrow and Fisher (1974) and in related papers. Then, the comparison is made between the (*endo*) scenario and the (*no*) information one, defining the value of endogenous information ($VOI_{endo}(x, y)$). This comparison is crucial in order to assess how the arrival of endogenous information can lead to more/less preservation in a situation in which exogenous learning is not possible. Finally, we generalize the analysis by comparing the (*exo+endo*) scenario to the (*no*) information one, hence defining the value of both exogenous and endogenous information ($VOI_{exo+endo}(x, y)$). This comparison is important both to understand the interplay of exogenous and endogenous learning and to gradually move to the more relevant case of endogenous above exogenous information (section 4.3), that represents the main focus of the paper.

In section 4.3 we investigate the value of additional endogenous information with respect to a situation in which information can arrive only exogenously: the (*exo+endo*) scenario is compared to the (*exo*) one. This comparative statics analysis will allow us to introduce the value of endogenous above exogenous information, which we call the "Testing Value".

4.2 The value of exogenous and of endogenous information

4.2.1 The value of exogenous information

Let us first focus on the case where the only way in which the true state of the world can be revealed in period 1 is exogenously. Therefore, using (8), we calculate the difference between the optimal expected value of net benefits of preservation in the (*exo*) scenario (Fig. 4.1) and in the (*no*) scenario (Fig. 4.2). In the regions in which this difference is positive, the *value of exogenous information* is equal to

$$VOI_{exo}(x,y) = \begin{cases} (1-p)q - x & \text{if} \quad (x,y) \in \boldsymbol{PD}(p,q,0) \cap \boldsymbol{DD}(p,0,0) \\ (1-p-x) - (1-q)py & \text{if} \quad (x,y) \in \boldsymbol{PP}(p,q,0) \cap \boldsymbol{DD}(p,0,0) \\ qpy & \text{if} \quad (x,y) \in \boldsymbol{PP}(p,q,0) \cap \boldsymbol{PP}(p,0,0) \end{cases} \tag{10}$$

and equal to zero otherwise. In particular, the VOI_{exo} is null only when despite the possibility of receiving exogenous information in period 1, the DM's optimal choice at $\tau = 1$ is $(c_1^*)_{exo} = 0$ (region $\boldsymbol{DD}(p,q,0)$ in Fig. 4.1). Therefore, the VOI_{exo} is positive when $(c_1^*)_{exo}$ is positive.

Let us analyze how the VOI_{exo} and $(c_1^*)_{exo}$ depend on the parameters of the environmental decision problem. Both quantities are decreasing (or constant) with the relative first-period loss from preservation x: the higher this opportunity cost, the less profitable is waiting (not developing) at $\tau = 1$. The VOI_{exo} decreases with the relative second-period loss from preservation y only in the region where it is $(c_2^*)_{exo} > (c_2^*)_{no}$. This is the only region where a higher y, by diminishing the expected second-period net benefit of preservation, decreases the "advantage" of the optimal preservation path in the (exo) information scenario ($(c_1^*)_{exo} = (c_2^*)_{exo} = 1$) over the one in the ($no$) information scenario ($(c_1^*)_{no} = (c_2^*)_{no} = 0$). As for the probability p of the unfavorable state, the VOI_{exo} is increasing in it only in the region where it is optimal to preserve even in the (no) information scenario ($\boldsymbol{PP}(p,0,0)$); notice, however, that this region shrinks as p increases. In the regions $\boldsymbol{PP}(p,q,0) \cap \boldsymbol{DD}(p,0,0)$ and $\boldsymbol{PD}(p,q,0) \cap \boldsymbol{DD}(p,0,0)$, the optimal preservation behavior diverges in the two information scenarios and the VOI_{exo} is decreasing in p: the higher the probability that the second-period state of the world will be unfavorable to preservation, the lower the expected benefits from preservation at $\tau = 1$ (and at $\tau = 2$), this choice being incentivized by the possibility of exogenous information. Finally, both the VOI_{exo} and $(c_1^*)_{exo}$ are increasing with the probability of receiving information exogenously in period 1. Indeed, the higher q, the more profitable is waiting before eventually developing the resource.

Let us analyze more deeply the features of VOI_{exo} in the region $\boldsymbol{PP}(p,q,0) \cap \boldsymbol{PP}(p,0,0)$: in the case of information not arriving in period 1, the DM's optimal behavior at $\tau = 2$ is the same in both information scenarios. However, the VOI_{exo} is positive even in this region. This is because, when exogenous information arrives (with probability q) in period 1 and the unfavorable state (whose probability is p) is revealed, the DM develops completely the whole resource at $\tau = 2$, i.e. $(c_2^u)^* = 0$, hence obtaining the net benefit y, so that the VOI_{exo} is qpy. This advantage occurs in $\boldsymbol{PP}(p, 0, 0)$ and disappears when x and/or y becomes so large that we fall into $\boldsymbol{PP}(p, q, 0) \cap \boldsymbol{DD}(p, 0, 0)$. In this case, if information does not arrive in period 1 and the unfavorable state occurs in period 2, the decision to preserve at $\tau = 2$ taken in the (*exo*) scenario is disadvantageous: it would have been better to develop the resource at $\tau = 2$, as done by the DM in the (*no*) scenario. Consequently, the VOI_{exo} is reduced by py, thus becoming $(1 - p - x) + qpy - py$.

Generally speaking, the VOI_{exo} reflects the main conclusion of Arrow and Fisher (1974): with information exogenously arriving, the VOI_{exo} leads the DM to choose a higher level of preservation of the environmental area for every $\tau = 1, 2$. We extend this result to the case where exogenous information does not arrive with certainty, hence it holds for every $q \in [0, 1]$. In fact, we have $({c_1}^*)_{exo} \geq ({c_1}^*)_{no}$ and $({c_2}^*)_{exo} \geq ({c_2}^*)_{no}$, independent of $q \in [0, 1]$. This result can be observed by comparing Fig. 4.2 to Fig. 4.1: there exists no $(x, y) \in \boldsymbol{PP}(p, 0, 0)$ for which $(x, y) \notin \boldsymbol{PP}(p, q, 0)$.

Lastly, a simple mathematical relation can be established between the VOI_{exo} and the quasi-option value à la Arrow-Fisher. The latter is the value of "certain" ($q = 1$) exogenous information (compared to no information), conditional on having chosen to preserve the whole environmental area at $\tau = 1$, i.e. conditional on $(c_1^*)_{exo} = (c_1^*)_{no} = 1$. Recall that the VOI_{exo} is non-decreasing with respect to q. However, even for $q = 1$, the VOI_{exo} is never larger than the quasi-option value à la Arrow-Fisher. The two values coincide only in region $\boldsymbol{PP}(p, 0, 0)$, when both x and y are so low that it is optimal to preserve everything at each $\tau = 1, 2$ even in the (*no*) scenario. Therefore, the quasi-option value à la Arrow-Fisher can be interpreted as an upper bound of the VOI_{exo}.

4.2.2 The value of endogenous information

Let us now focus on the case where the only way in which the true state of the world can be revealed in period 1 is endogenously, i.e. according to the level of development carried out at $\tau = 1$. Therefore, using (8), we calculate the difference between the optimal expected value of net benefits of preservation in the (*endo*) scenario (Fig. 4.3) and in the (*no*) scenario (Fig. 4.2). In the regions in which this difference is positive, the *value of endogenous information* is equal to

$$VOI_{endo}(x,y) = \begin{cases} \frac{[(1-p)\lambda - x]^2}{4(1-p)\lambda} & \text{if} \quad (x,y) \in \boldsymbol{L_\ell D}(p,0,\lambda) \cap \boldsymbol{DD}(p,0,0) \\ \frac{[(1-p-x)-(1-\lambda)py]^2}{4\lambda py} & \text{if} \quad (x,y) \in \boldsymbol{L_\ell L_\ell}(p,0,\lambda) \cap \boldsymbol{DD}(p,0,0) \\ \frac{[(1+\lambda)py-(1-p-x)]^2}{4\lambda py} & \text{if} \quad (x,y) \in \boldsymbol{L_h L_h}(p,0,\lambda) \cap \boldsymbol{PP}(p,0,0) \end{cases} \quad (11)$$

and equal to zero otherwise.[17] In particular, the VOI_{endo} is null both in $\boldsymbol{DD}(p,0,\lambda)$ and in $\boldsymbol{PP}(p,0,\lambda)$ in Fig. 4.3. In the former region, the option of receiving endogenous information in period 1 (being the most likely possible, given that $(c_1^*)_{endo} = 0$) is not exploited because of the irreversibility of development: the DM develops completely the environmental resource at $\tau = 1$, which precludes the possibility to preserve it at $\tau = 2$ if the revealed state of the world is the favorable one. In the latter region, despite the possibility of receiving endogenous information in period 1 (by choosing $c_1 \neq 1$), the DM's optimal choice at $\tau = 1$ is $(c_1^*)_{endo} = 1$, which leads to no information arriving at all in period 1. Therefore, the VOI_{endo} is positive only when $(c_1^*)_{no}$ is positive and lower than 1, i.e. when there is a limited preservation of the environmental resource $\tau = 1$. This choice allows the DM to potentially obtain information endogenously in period 1, at the same time maintaining the option to exploit this information when choosing at $\tau = 2$. That is why the VOI_{endo} is positive even when it leads to less preservation compared to the case in which information in period 1 is not available at all (region $\boldsymbol{L_h L_h}(p,0,\lambda)$).

Let us analyze how VOI_{endo} and $(c_1^*)_{endo}$, the optimal first-period choice in this information scenario, depend on the parameters of the environmental decision problem. Despite the fact that $(c_1^*)_{endo}$ is always decreasing (or non-increasing) with both the relative first-period loss from preservation (x) and with the probability of the unfavorable state of the world (p), the VOI_{endo} has the same trend only in the regions of net benefits where preservation is greater than in the (*no*) information scenario. Conversely, in

[17] Notice that the Fisher and Hanemann (1987) quasi-option value of the minimum feasible development ($\varepsilon-$development) is a particular VOI_{endo} that emerges when the following three conditions are simultaneously satisfied: the level of information arriving endogenously in period 1 is the same for every $c_1 \in [0,1)$; information arrives with certainty in period 1 for every $c_1 \in [0,1)$; the optimal choice at $\tau = 1$ both in the (*endo*) and in the (*no*) information scenario should be $(c_1^*)_{endo} = (c_1^*)_{no} = 1 - \varepsilon$. However, because of linearity of net benefits in the (*no*) information scenario, it can never be $(c_1^*)_{no} = 1 - \varepsilon$.

region $\boldsymbol{L_h L_h}(p,0,\lambda)$, where the possibility of endogenous information leads to the DM optimally preserving less resource than in the (*no*) information scenario, the VOI_{endo} depends positively on both x and p: a higher value of these parameters makes developing at $\tau = 1$ - a prerequisite to obtain information endogenously - more profitable. Notice that $\boldsymbol{L_h L_h}(p,0,\lambda)$ is also the only region where the VOI_{endo} depends positively on y: a higher y, by diminishing the expected second-period net benefit of preservation, increases the "advantage" of the optimal preservation path in the (*endo*) information scenario ($(c_1^*)_{endo} = (c_2^*)_{endo} \in (1/2, 1)$) over the one in the (*no*) information scenario ($(c_1^*)_{no} = (c_2^*)_{no} = 1$). Although the VOI_{endo} is always increasing with λ (the upper bound of the probability of receiving information endogenously in period 1), $(c_1^*)_{endo}$ increases with λ when it is greater than $(c_1^*)_{no}$ and decreases otherwise (see relation (7)). Indeed, in regions $\boldsymbol{L_\ell D}(p,0,\lambda)$ and $\boldsymbol{L_\ell L_\ell}(p,0,\lambda)$ where the possibility of endogenous information leads to more preservation with respect to the (*no*) information scenario, a higher λ leads to a greater $(c_1^*)_{endo}$. In region $\boldsymbol{L_h L_h}(p,0,\lambda)$, where the optimal preservation path is lower than in the (*no*) information scenario, a higher λ leads to a smaller $(c_1^*)_{endo}$.

Intuitively, the potential emergence of endogenous information creates a trade-off in the first-period choice: relative to the (*no*) information scenario, at $\tau = 1$ the DM should preserve less in order to get information in period 1, whereas she should preserve more in order to exploit this information at $\tau = 2$ by having a greater choice set. That is why when x and y are so low that it is optimal to completely preserve the resource in the (*no*) information scenario, the possibility of endogenous information can only lead to the DM preserving less, in order to increase the probability that the state of the world is revealed in period 1. For higher relative first-period and second-period losses from preservation, the potential emergence of endogenous information can only lead to more preservation with respect to the (*no*) information scenario. Notice, however, that the increase in preservation in both periods involves at most half of the resource. In fact, in regions $\boldsymbol{L_\ell D}(p,0,\lambda)$ and $\boldsymbol{L_\ell L_\ell}(p,0,\lambda)$ we have $(c_1^*)_{endo} \in (0, 1/2)$.

It is interesting to see what happens in the case where the arrival of endogenous information also brings exogenous information. This means comparing the optimal preservation behavior in the (*exo+endo*) scenario (Fig. 3) and in the (*no*) scenario (Fig. 4.2).

Suppose that developing the resource stimulates a stream of scientific research that is independent of the one accompanying the development project. A nontrivial question would be: does the emergence of exogenous information together with that which arrives endogenously always lead to more preservation? Intuitively, the simultaneous emergence of exogenous information should be an incentive to hold down the level of development needed at $\tau = 1$ to endogenously get information in period 1. Our framework substantiates this intuition: by comparing the three (*exo*+*endo*) scenarios in Fig. 3 to the (*endo*) scenario in Fig. 4.3 it is worth noticing that: regions $\boldsymbol{L_\ell D}(p, q, \lambda)$ and $\boldsymbol{L_\ell L_\ell}(p, q, \lambda)$ are shifts to the right (by $q(1-p)$) of regions $\boldsymbol{L_\ell D}(p, 0, \lambda)$ and $\boldsymbol{L_\ell L_\ell}(p, 0, \lambda)$ respectively; for all $q > 0$, region $\boldsymbol{L_h D}(p, q, \lambda)$ emerges, where the increase in preservation at $\tau = 1$ with respect to the (*no*) information scenario can cover more than half of the resource; for all $q > 0$, region $\boldsymbol{L_h L_h}(p, q, \lambda)$ is a shift to the right (by $q(1-p)$) of region $\boldsymbol{L_h L_h}(p, 0, \lambda)$: in $\boldsymbol{L_h L_h}(p, q, \lambda)$, the increase in preservation at each $\tau = 1, 2$ with respect to the (*no*) information scenario can cover more than half of the resource.

However, the emergence of exogenous information coupled with endogenous information definitely leads to a greater or equal preservation than in the (*no*) information scenario only if $q \geq \lambda$ (notice that $(c_1^*)_{exo+endo} \geq (c_1^*)_{no}$ in every region of Fig. 3.1 and Fig. 3.2 compared to Fig. 4.2). Conversely, when the probability of exogenous information is lower than the upper bound of the probability of endogenous information, the possibility of information arrival in period 1 can be detrimental to environmental preservation. Indeed, comparing Fig. 3.3. to Fig. 4.2, one can see that in the subregion $\boldsymbol{L_h L_h}(p, q, \lambda) \cap \boldsymbol{PP}(p, 0, 0)$ preservation in both periods is greater in the (*no*) information scenario. This subregion shrinks as q increases and vanishes for $q \to \lambda$. Notice, however, that in both the (*exo*+*endo*) and the (*endo*) information scenarios, the greatest possible amount of the resource wasted with respect to the (*no*) information scenario is limited by 1/2. The main conclusions of the analysis above are summarized by Result 1.

Result 1. Despite the possibility of learning by developing the resource, there is always a non-negligible region of net benefits of preservation over which first-period and second-period preservations in the (*endo*) information scenario are greater than in the (*no*) information scenario. This region widens when it is possible that exogenous information is added to the endogenous one. However, the possibility of both exogenous and endogenous information can be detrimental to environmental preservation. The (*exo+endo*) scenario definitely leads to a greater or equal preservation with respect to the (*no*) information scenario only if $q \geq \lambda$.

Finally, notice that the *value of both exogenous and endogenous information*, $VOI_{exo+endo}$, can be easily calculated from (8) as the difference between the optimal expected value of net benefits of preservation in the (*exo+endo*) scenario (Fig. 3) and in the (*no*) information scenario (Fig. 4.2). It is positive in each region of net benefits of preservation apart from in $\boldsymbol{DD}(p, q, \lambda)$, where it is null, since the arrival of new information is not exploited at $\tau = 2$ because of irreversibility. In particular, it coincides with $EV^*_{exo+endo}(x, y)$ for all $(x, y) \notin \boldsymbol{PP}(p, 0, 0)$ and with $VOI_{exo}(x; y)$ for all $(x, y) \in \boldsymbol{PP}(p, q, \lambda) \cap \boldsymbol{PP}(p, 0, 0)$.

4.3 The value of endogenous above exogenous information: the Testing Value

In the previous section, we defined the value of information (*VOI*) taking as a reference point the (*no*) information scenario, i.e. a situation in which the DM is certain that no new information about the true state of the world will arrive in period 1.

In this section, we use a different reference point: the (*exo*) information scenario, i.e. a situation in which the DM knows that with probability q the true state of the world will be exogenously revealed in period 1. The alternative information scenario is the (*exo+endo*) one, here interpreted as a situation in which it is possible to acquire information endogenously, *additionally* with respect to information arriving exogenously.

By interpreting the (*exo+endo*) information scenario as an alteration of the (*exo*) information scenario through the addition of the possibility of endogenous learning, we define the *value of endogenous above exogenous information* as the additional value attached to endogenous information, additional with respect to information arriving exogenously. We refer to it as the *Testing Value* (TV_{endo} henceforth), relying on the intuition that the additional presence of endogenous information should incentivize the DM to "test" the environmental resource (by developing a portion of it) at $\tau = 1$, whereas the possibility of only exogenous information should influence her to "wait" (by preserving the resource) and to eventually develop it at $\tau = 2$ according to the revealed state of the world.

Therefore, the TV_{endo} can be calculated as the difference between the optimal expected value of net benefits of preservation in the (*exo*+*endo*) scenario and in the (*exo*) scenario. This value is necessarily non-negative, since the level of preservation which is optimal in the (*exo*) information scenario is available to the DM in the (*exo*+*endo*) too. The TV_{endo} is equal to

$$TV_{endo}(x,y) = \begin{cases} \frac{[(1-p)(q+\lambda)-x]^2}{4(1-p)\lambda} & \text{if} \quad (x,y) \in \boldsymbol{L_\ell D}(p,q,\lambda) \cap \boldsymbol{DD}(p,q,0) \\ \frac{[x-(1-p)(q-\lambda)]^2}{4(1-p)\lambda} & \text{if} \quad (x,y) \in \boldsymbol{L_h D}(p,q,\lambda) \cap \boldsymbol{PD}(p,q,0) \\ \frac{[(1-p-x)-(1-q-\lambda)py]^2}{4\lambda py} & \text{if} \quad (x,y) \in \boldsymbol{L_\ell L_\ell}(p,q,\lambda) \cap \boldsymbol{DD}(p,q,0) \\ \frac{[(1-q+\lambda)py-(1-p-x)]^2}{4\lambda py} & \text{if} \quad (x,y) \in \boldsymbol{L_h L_h}(p,q,\lambda) \cap \boldsymbol{PP}(p,q,0) \end{cases} \quad (12)$$

and equal to zero otherwise. Let us define the *environmental resource saving due to additional endogenous information* as the difference between the optimal levels of preservation at $\tau = 1$ in the two scenarios as $\Delta_{endo}((c_1^*)_{exo}) := (c_1^*)_{exo+endo} - (c_1^*)_{exo}$. Notice that the TV_{endo} is positive only in those regions of net benefits (x,y) where $\Delta_{endo}((c_1^*)_{exo}) \neq 0$, i.e. when with additional endogenous information a limited preservation of the environmental resource is optimal. In this case, "testing" the resource instead of completely preserving or completely developing it is the best choice.[18]

In particular, in the regions $\boldsymbol{L_h D}(p,q,\lambda) \cap \boldsymbol{PD}(p,q,0)$ and $\boldsymbol{L_h L_h}(p,q,\lambda) \cap \boldsymbol{PP}(p,q,0)$ the values for (x,y) are such that the level of preservation is lower in the (*exo*+*endo*) scenario than in the (*exo*) scenario, respectively at $\tau = 1$, and also at $\tau = 2$ in the case of information not arriving in period 1: we have $\Delta_{endo}((c_1^*)_{exo}) \in (-1/2, 0)$. Conversely, in the regions $\boldsymbol{L_\ell D}(p,q,\lambda) \cap \boldsymbol{DD}(p,q,0)$ and $\boldsymbol{L_\ell L_\ell}(p,q,\lambda) \cap \boldsymbol{DD}(p,q,0)$ the values for (x,y) are such that the level of preservation is higher in the (*exo*+*endo*) scenario than in the (*exo*) scenario, respectively at $\tau = 1$, and also at $\tau = 2$ in the case of information not arriving in period 1: we have $\Delta_{endo}((c_1^*)_{exo}) \in (0, 1/2)$. A comparison of the emergence of regions $\boldsymbol{L_\ell D}(p,q,\lambda)$ and $\boldsymbol{L_\ell L_\ell}(p,q,\lambda)$ in Fig. 3 with that of region $\boldsymbol{DD}(p,q,0)$ in Fig. 4.1 proves that the possibility of acquiring information both endogenously and ex-

[18]Obviously, if $(c_1^*)_{exo\&endo} = 1$, there is only exogenous information, so $EV^*_{exo+endo} \equiv EV^*_{exo}$, $(c_1^*)_{exo+endo} = (c_1^*)_{exo}$ and $TV_{endo} = 0$. Attanasi and Montesano (2008) show that if there is strategic interaction between two DMs, the TV_{endo} can be positive even for a DM choosing $(c_1^*)_{exo+endo} = 1$.

ogenously could push the DM towards a higher level of preservation compared to the case where information arrives only exogenously. This happens whatever the sign of the expected second-period net benefit of preservation. If this is negative (region $\boldsymbol{L_\ell D}(p,q,\lambda)$), the possibility of additional endogenous information leads to more preservation if and only if the relative first-period loss from preservation (x) is higher than $q(1-p)$ but lower than $(q+\lambda)(1-p)$.[19] Further, notice that regions $\boldsymbol{L_\ell D}(p,q,\lambda) \cap \boldsymbol{DD}(p,q,0)$ and $\boldsymbol{L_\ell L_\ell}(p,q,\lambda) \cap \boldsymbol{DD}(p,q,0)$ emerge whatever $q \gtreqless \lambda$. When the vector of relative net benefits of preservation belongs to one of these regions, the TV_{endo} pushes the risk-neutral DM towards a higher level of preservation of the environmental resource. The main conclusions of the analysis above are formally stated in Result 2.

Result 2. There is a non-negligible subset of net benefits of preservation over which current (and eventually future) preservation in the (*exo*+*endo*) scenario is greater than in the (*exo*) scenario. This subset exists whatever the relative size of exogenous and endogenous information and whatever the expected second-period net benefit of preservation. When this benefit is negative, then additional endogenous information leads to more preservation if the difference between the opportunity cost of current preservation and the probability of the favorable state of the world being exogenously revealed is positive (i.e. $x - q(1-p) > 0$) and this positive difference is lower than the upper bound of the probability of the favorable state of the world being endogenously revealed (i.e. $x - q(1-p) < \lambda(1-p)$).

An economic interpretation of the previous conclusion follows. In both $\boldsymbol{L_\ell D}(p,q,\lambda)$ and $\boldsymbol{L_\ell L_\ell}(p,q,\lambda)$, given that the opportunity cost of preservation at $\tau = 1$ is higher than the potential positive benefit brought by exogenous information, with only exogenous information it is optimal to completely develop the environmental resource at $\tau = 1$ (hence also at $\tau = 2$). However, with additional endogenous information a higher opportunity cost of preservation makes acquiring information through developing at $\tau = 1$ less costly

[19] The quantity $\lambda(1-p)$ represents the highest possible additional incentive for preservation brought by endogenous information. In the case where the expected second-period net benefit of preservation is positive (region $\boldsymbol{L_\ell L_\ell}(p,q,\lambda)$), developing a portion of the resource in the (*exo*+*endo*) scenario versus completely developing it in the (*exo*) scenario can be optimal even for $x > (q+\lambda)(1-p)$. This is because with $y < (1-p)/p$, preserving at $\tau = 2$ is optimal in the case of information not arriving in period 1. This creates an additional incentive to preserve at $\tau = 1$ which counterbalances x, the opportunity cost.

for the DM. Thus she faces a trade-off between preserving and obtaining endogenous information. As long as the opportunity cost of preserving at $\tau = 1$ is not higher than the probability of the state of the world being favorable to preservation, it can be optimal to "test" the environmental resource at $\tau = 1$: the DM develops only portion of it, allowing herself to choose again (being potentially informed) whether to develop or preserve at $\tau = 2$ everything she has not "tested" at $\tau = 1$. If information does not emerge in period 1, at $\tau = 2$ she develops everything if the expected second-period net benefit of preservation is negative (region $\boldsymbol{L_\ell D}(p, q, \lambda)$) and she preserves everything she has preserved at $\tau = 1$ otherwise (region $\boldsymbol{L_\ell L_\ell}(p, q, \lambda)$).

A policy-relevant aspect to study in depth is the set of conditions leading to the highest saving of environmental resource when additional endogenous learning is possible. This requires analyzing how $\Delta_{endo}((c_1^*)_{exo})$, the environmental resource saving due to additional endogenous information, depends on the parameters of the environmental decision problem. By calculating $\Delta_{endo}((c_1^*)_{exo})$ from (7) in the regions in which it is non-null and differentiating it with respect to the parameters of the problem, it is easy to see that, whatever the sign of $\Delta_{endo}((c_1^*)_{exo})$, it is always decreasing in p, decreasing in x and non-increasing in y. In particular, if $\Delta_{endo}((c_1^*)_{exo}) > 0$, with respect to x it is maximized on the left frontier of regions $\boldsymbol{L_\ell D}(p, q, \lambda)$ and $\boldsymbol{L_\ell L_\ell}(p, q, \lambda)$. In correspondence of the left frontier of the region $\boldsymbol{L_\ell D}(p, q, \lambda)$ we also have the maximum $TV_{endo}(x, y)$, which is equal to $\lambda(1 - p)/4$.

Moreover, whatever the sign of $\Delta_{endo}((c_1^*)_{exo})$, its absolute value depends positively on λ. This means that λ amplifies the difference between $(c_1^*)_{exo+endo}$ and $(c_1^*)_{exo}$. This result is intuitive when this difference is positive: if additional endogenous information leads to more preservation, the higher the upper bound of the probability of information endogenously arriving the lower the amount of development needed to get the same likelihood that information arrives in period 1. The intuition is not so immediate when $(c_1^*)_{exo+endo} < (c_1^*)_{exo}$: if additional endogenous information leads to less preservation, a higher λ makes development more profitable, thus stimulating the DM to preserve even less (i.e., $\lambda(1 - c_1)$ definitely increases).

Finally, $\Delta_{endo}((c_1^*)_{exo})$ depends positively on q. Consider the case $\Delta_{endo}((c_1^*)_{exo}) < 0$:

when additional endogenous information leads to less preservation, an increase in q limits the greater resource development. In other words, given that information can be more easily obtained exogenously, the DM can test less. A similar interpretation can be provided for the case $\Delta_{endo}((c_1^*)_{exo}) > 0$: when additional endogenous information leads to more preservation, the positive effect on preservation is greater the higher the probability of exogenous information. This result suggests a complementarity between exogenous and endogenous information in limiting the resource development. Suppose that with only exogenous information complete resource development is optimal, while with additional endogenous information the resource would be only partially developed (regions $\boldsymbol{L_\ell D}(p, q, \lambda)$ and $\boldsymbol{L_\ell L_\ell}(p, q, \lambda)$). Then, a higher probability of exogenous information, while not changing the optimal choice of complete development in the (*exo*) scenario, would instead decrease the amount of the resource tested in the (*exo+endo*) scenario.

The main conclusions about the dependence of $\Delta_{endo}((c_1^*)_{exo})$ with respect to λ and q are formally stated in Result 3.

Result 3. If additional endogenous information leads to more preservation in the current period (i.e. $(c_1^*)_{exo+endo} - (c_1^*)_{exo} > 0$), then this difference is greater the higher the upper bound of the probability of endogenous information and the higher the probability of exogenous information. In the opposite case (i.e. $(c_1^*)_{exo+endo} - (c_1^*)_{exo} < 0$), the absolute value of this difference is greater the higher the (upper bound of the) probability of endogenous information and the lower the probability of exogenous information.

In order to better clarify the complementarity between exogenous and endogenous information in our model, let us analyze how the TV_{endo} depends on λ and on q. The signs of these relations are formally indicated in Result 4.

Result 4. The TV_{endo} is always increasing in λ, the upper bound of the probability of acquiring endogenous information. The TV_{endo} may be either increasing or decreasing in q, the probability of receiving exogenous information. It is increasing in q for those regions of net benefits of preservation where preservation in the (*exo+endo*) scenario is greater than in the (*exo*) scenario, and it is decreasing in the opposite case. [20]

[20]**Proof.** Let us first study the relationship between TV_{endo} and λ. Define $\nu = \frac{(1-p)(q+\lambda)-x}{(1-p)\lambda}$. In the

The first part of Result 4 confirms the intuition that the TV_{endo}, arising from additional endogenous information, should be increasing in λ. The second part of Result 4 is less intuitive: although its existence stems from endogenous information, surprisingly enough, the TV_{endo} can be positively related to the probability of acquiring information exogenously. Indeed, the TV_{endo} is increasing in q only in those regions of net benefits where additional endogenous information leads to more preservation.

The dependence of $\Delta_{endo}((c_1^*)_{exo})$ on λ and q (Result 3) and of TV_{endo} on λ and q (Result 4) is summarized in Table 1, for those regions of net benefits of preservation where $\Delta_{endo}((c_1^*)_{exo}) \neq 0$ and $TV_{endo} > 0$ (see (12)).

Region of net benefits (x, y)	Effect of *endo* and *exo* info			
	$\frac{\partial(\Delta_{endo}((c_1^*)_{exo}))}{\partial\lambda}$	$\frac{\partial TV_{endo}}{\partial\lambda}$	$\frac{\partial(\Delta_{endo}((c_1^*)_{exo}))}{\partial q}$	$\frac{\partial TV_{endo}}{\partial q}$
$\boldsymbol{L_\ell D}(p,q,\lambda) \cap \boldsymbol{DD}(p,q,0)$	positive	positive	positive	positive
$\boldsymbol{L_h D}(p,q,\lambda) \cap \boldsymbol{PD}(p,q,0)$	negative	positive	positive	negative
$\boldsymbol{L_\ell L_\ell}(p,q,\lambda) \cap \boldsymbol{DD}(p,q,0)$	positive	positive	positive	positive
$\boldsymbol{L_h L_h}(p,q,\lambda) \cap \boldsymbol{PP}(p,q,0)$	negative	positive	positive	negative

(Table 1. Effects of λ and q on $\Delta_{endo}((c_1^*)_{exo})$ and on TV_{endo}.)

subregion $\boldsymbol{L_\ell D}(p,q,\lambda)$, we have $\nu \in (0,1]$ and, from (12), the testing value is $TV_{endo}(x,y) = \nu^2 \frac{(1-p)\lambda}{4}$. By differentiating this expression with respect to λ (and recalling that ν is a function of λ), we find $\frac{\partial TV_{endo}}{\partial\lambda}\Big|_{\boldsymbol{L_\ell D}(p,q,\lambda)} = \nu(2-\nu)\frac{1-p}{4} > 0$, given that $\nu \in (0,1]$ in this subregion. In the subregion $\boldsymbol{L_h D}(p,q,\lambda)$, we have $\nu \in [1,2)$ and, from (12), the testing value is $TV_{endo}(x,y) = (\nu-2)^2\frac{(1-p)\lambda}{4}$. By differentiating this expression with respect to λ (and recalling that ν is a function of λ), we find $\frac{\partial TV_{endo}}{\partial\lambda}\Big|_{\boldsymbol{L_h D}(p,q,\lambda)} = -\nu(\nu-2)\frac{1-p}{4} > 0$, given that $\nu \in [1,2)$ in this subregion. Now define $\xi = \frac{1-p-x-(1-q-\lambda)py}{\lambda py}$. In the subregion $\boldsymbol{L_\ell L_\ell}(p,q,\lambda)$, we have $\xi \in (0,1]$ and, from (12), the testing value is $TV_{endo}(x,y) = \xi^2\frac{\lambda py}{4}$. By differentiating this expression with respect to λ (and recalling that ξ is a function of λ), we find $\frac{\partial TV_{endo}}{\partial\lambda}\Big|_{\boldsymbol{L_\ell L_\ell}(p,q,\lambda)} = \xi(2-\xi)\frac{py}{4} > 0$, given that $\xi \in (0,1]$ in this subregion. In the subregion $\boldsymbol{L_h L_h}(p,q,\lambda)$, we have $\xi \in [1,2)$ and, from (12), the testing value is $TV_{endo}(x,y) = (\xi-2)^2\frac{\lambda py}{4}$. By differentiating this expression with respect to λ (and recalling that ξ is a function of λ), we find $\frac{\partial TV_{endo}}{\partial\lambda}\Big|_{\boldsymbol{L_h L_h}(p,q,\lambda)} = -\xi(\xi-2)\frac{py}{4} > 0$, given that $\xi \in [1,2)$ in this subregion. Let us now study the relationship between TV_{endo} and q. Recall that $\nu := \frac{(1-p)(q+\lambda)-x}{(1-p)\lambda}$ and $\xi := \frac{1-p-x-(1-q-\lambda)py}{\lambda py}$. In the subregion $\boldsymbol{L_\ell D}(p,q,\lambda)$, we have $\nu \in (0,1]$ and $\frac{\partial TV_{endo}}{\partial q}\Big|_{\boldsymbol{L_\ell D}(p,q,\lambda)} = \nu\frac{1-p}{2} > 0$. In the subregion $\boldsymbol{L_h D}(p,q,\lambda)$, we have $\nu \in [1,2)$ and $\frac{\partial TV_{endo}}{\partial q}\Big|_{\boldsymbol{L_h D}(p,q,\lambda)} = (\nu-2)\frac{1-p}{2} < 0$. In the subregion $\boldsymbol{L_\ell L_\ell}(p,q,\lambda)$, we have $\xi \in (0,1]$ and $\frac{\partial TV_{endo}}{\partial q}\Big|_{\boldsymbol{L_\ell L_\ell}(p,q,\lambda)} = \xi\frac{py}{2} > 0$. In the subregion $\boldsymbol{L_h L_h}(p,q,\lambda)$, we have $\xi \in [1,2)$ and $\frac{\partial TV_{endo}}{\partial q}\Big|_{\boldsymbol{L_h L_h}(p,q,\lambda)} = (\xi-2)\frac{py}{2} < 0$.

An intuitive explanation of Results 2, 3 and 4 can be provided by looking at the VOI_{exo} (as defined in section 4.2.1) and at the TV_{endo}. Consider the (*exo*) information scenario in Fig. 4.1. There exists a threshold value of q, namely $\hat{q}$, such that for each $q < \hat{q}$ we have $(c_1^*)_{exo} = 0$, $VOI_{exo} = 0$ and for each $q > \hat{q}$ we have $(c_1^*)_{exo} = 1$, $VOI_{exo} > 0$.[21]

Suppose that in the (*exo*) scenario we have $q < \hat{q}$, i.e. the probability of (only) exogenous information is too low to persuade the DM to preserve the resource. Thus, completely developing the environmental resource at $\tau = 1$ is the optimal choice in this case. Suppose that the DM considers the possibility of endogenous information, in addition to information which arrives exogenously. Then, there exists a threshold value of λ, namely $\hat{\lambda}$, such that for each $\lambda < \hat{\lambda}$ we still have $(c_1^*)_{exo+endo} = (c_1^*)_{exo} = 0$, $TV_{endo} = 0$ and for each $\lambda > \hat{\lambda}$ we have $\Delta_{endo}((c_1^*)_{exo}) \in (0, 1/2)$ and $TV_{endo} > 0$.[22] The condition $\lambda < \hat{\lambda}$ means that the possibility of additional endogenous information is not sufficient to move the DM's choice from completely developing the resource to only testing it. If instead we have $\lambda > \hat{\lambda}$, in the (*exo*+*endo*) information scenario preserving a portion of the resource is optimal, while in the (*exo*) information scenario the optimal choice is complete development (given that $q < \hat{q}$). This is what happens in regions $\boldsymbol{L_\ell D}(p, q, \lambda)$ and $\boldsymbol{L_\ell L_\ell}(p, q, \lambda)$ of Fig. 3, where $VOI_{exo} = 0$, since we are in region $\boldsymbol{DD}(p, q, 0)$ in Fig. 4.1. In fact, from (8), we know that for every (x, y) in region $\boldsymbol{DD}(p, q, 0)$, $EV_{exo}^*(x, y) = EV_{no}^*(x, y) = 0$. Therefore, from (12) we have that $TV_{endo} \equiv EV_{exo+endo}^*$, which in regions $\boldsymbol{L_\ell D}(p, q, \lambda)$ and $\boldsymbol{L_\ell L_\ell}(p, q, \lambda)$ is a positive function of both λ and q. In this case what matters for environmental resource saving due to additional endogenous information is the sum $\lambda + q$. Therefore exogenous and endogenous information act together in the direction of environmental resource preservation. In this case, as it appears in Table 1, both $\Delta_{endo}((c_1^*)_{exo})$ and TV_{endo} depend positively on both λ and q. In addition, $\frac{\partial^2 TV_{endo}}{\partial\lambda\partial q} > 0$, i.e. exogenous information increases the influence of additional endogenous information on the TV_{endo}.

Suppose now that in the (*exo*) scenario we have $q > \hat{q}$, so that $(c_1^*)_{exo} = 1$. If the DM considers the possibility of additional endogenous information, then for any $\lambda > 0$

[21] This threshold value is $\hat{q} = \frac{x}{1-p}$ if $y > (1-p)/p$ and $\hat{q} = \frac{x-(1-p-py)}{py}$ if $y < (1-p)/p$.

[22] This threshold value is $\hat{\lambda} = -q + \frac{x}{1-p}$ if $y > (1-p)/p$ and $\hat{\lambda} = -q + \frac{x-(1-p-py)}{py}$ if $y < (1-p)/p$. That is, in both cases, $\hat{\lambda} = -q + \hat{q}$.

the optimal level of preservation in the (*exo*+*endo*) scenario cannot increase. For λ high enough, we are in regions $\boldsymbol{L_h D}(p,q,\lambda)$ or $\boldsymbol{L_h L_h}(p,q,\lambda)$, where $(c_1^*)_{exo+endo} \in (1/2,1)$. From (7) it results that in these regions $(c_1^*)_{exo+endo}$ depends negatively on λ. Moreover, it depends positively on q, since testing is less profitable if exogenous information is more likely.[23] Therefore, in this case λ and q have opposite effects on resource preservation in the (*exo*+*endo*) scenario: the former leads to less preservation, the latter to more. However, both have a positive effect on $VOI_{exo+endo}$, the value of both exogenous and endogenous information.[24] Finally, given that in these regions $EV_{no}^*(x,y) = 0$, and so $VOI_{exo+endo} \equiv EV_{exo+endo}^*$, we have $\frac{\partial^2 EV_{exo+endo}^*}{\partial\lambda\partial q} < 0$: a higher q leads to an increase in VOI_{exo}, at the same time decreasing the influence of λ on the $EV_{exo+endo}^*$, so that $\frac{\partial TV_{endo}}{\partial q} < 0$ and $\frac{\partial^2 TV_{endo}}{\partial\lambda\partial q} < 0$.

The importance of exogenous information for the nature of the TV_{endo} is even more clear if we compare the TV_{endo} to the VOI_{endo} (as defined in section 4.2.2), the latter emerging in a setting without exogenous information, i.e. in the (*endo*) scenario, and so being independent of q. By comparing Fig. 3 to Fig. 4.3, it is easy to see that for every pair of relative losses from preservation (x,y) we always have $(c_\tau^*)_{exo+endo} \geq (c_\tau^*)_{endo}$ for $\tau = 1,2$. Furthermore, notice that the value of both exogenous and endogenous information (coming out from (9) with *info* =*exo*+*endo*) can be alternatively calculated as $VOI_{exo+endo}(x,y) = VOI_{exo}(x,y) + TV_{endo}(x,y)$. Therefore, given that $EV_{exo+endo}^* \geq EV_{endo}^*$ for every pair of net benefits (x,y), we always have $TV_{endo} \geq VOI_{endo} - VOI_{exo}$.

We conclude this section with a clarification of the generalizability of our theoretical results. One can claim that Results 1-4 strongly rely on the assumption of linearity of the expected benefit function in our model. Indeed, this linearity assumption ensures that the irreversibility effect always goes in the direction of no less preservation in the (*exo*)

[23] From a mathematical point of view, it is easy to see the opposite effect of λ and q on $(c_1^*)_{exo+endo}$ in regions $\boldsymbol{L_\ell D}(p,q,\lambda)$ and $\boldsymbol{L_\ell L_\ell}(p,q,\lambda)$, by writing the relation between λ and q such that $(c_1^*)_{exo+endo} = \bar{c}_1$ constant whatever the values of λ and q. We would obtain $q - \lambda(2\bar{c}_1 - 1) = \frac{x}{1-p}$ if $y > (1-p)/p$ and $q - \lambda(2\bar{c}_1 - 1) = \frac{x-(1-p-py)}{py}$ if $y < (1-p)/p$. In both cases, if λ increases, then q has to decrease in order to keep the same $\bar{c}_1$.

[24] Notice that, despite the TV_{endo} can be decreasing in q, the $VOI_{exo+endo}(x,y)$ is always increasing in the probability of acquiring exogenous information. In fact, from (9) we have $VOI_{exo+endo}(x,y) = EV_{exo+endo}^*(x,y) - EV_{no}^*(x,y)$ and, given that $EV_{no}^*(x,y)$ is independent of q, we have $\frac{\partial VOI_{exo+endo}^*}{\partial q} = \frac{\partial EV_{exo+endo}^*}{\partial q}$. From (8) it is easy to see that $\frac{\partial EV_{exo+endo}^*}{\partial q} \geq 0$ in each region of net benefits of preservation.

information scenario compared to the (no) information one. In addition, it allows the DM's maximization problem in all scenarios without endogenous information (i.e., in the (no) and in the (exo) scenario) to generically lead to corner solutions for any $q \in [0, 1]$, i.e. to develop completely or preserve completely the environmental asset.[25] Therefore, in our model internal solutions are possible only in the ($endo$) and in the (exo+$endo$) information scenarios.

Nonetheless, it is easy to show that our main results also hold when the expected benefit function is non-linear. As an example, suppose that, as in the model in section 2, both the current net benefit of preservation and the future net benefit of preservation in the unfavorable state of the world are linear, such that $b_1(c_1) = -c_1$ and $b_2^u(c_2) = -4c_2$. In addition, consider the future net benefit of preservation in the favorable state of the world being $b_2^f(c_2) = 6c_2 - 3(c_2)^2$, i.e. non-linear in the level of preservation at $\tau = 2$. In this case, under $p = q = \lambda = 1/2$, the optimal preservation levels in the four information scenarios are $(c_1^*)_{no} = (c_2^*)_{no} = 0$, $(c_1^*)_{exo} = (c_2^*)_{exo} \simeq 0.33$, $(c_1^*)_{endo} = (c_2^*)_{endo} = 0.20$, and $(c_1^*)_{exo+endo} \simeq 0.41 > (c_2^*)_{exo+endo} \simeq 0.33$. This example shows that Result 1 and Result 2 also hold when the benefit function is non-linear.

5 Conclusions

In this paper, we have extended the Arrow and Fisher (1974) two-period model in order to analyze and compare the effects of exogenous information and of endogenous information on the optimal preservation choices of a risk-neutral decision maker.

We focus on an information scenario where both exogenous and endogenous information are available. We interpret this scenario as an alteration of the exogenous information scenario through the addition of the possibility of endogenous learning. Thus, we define the *value of endogenous above exogenous information* as the additional value attached to endogenous information, where "additional" is with respect to information arriving exogenously. We call this the *Testing Value* (TV_{endo}), relying on the intuition that the additional presence of endogenous information should incentivize the DM to "test" the

[25]However, there are singular cases in which we can have indeterminate choices $(c_1^*)_{info} \in [0, 1]$ and/or $(c_2^*)_{info} \in \left[0, (c_1^*)_{info}\right]$, for $info = no, exo$.

environmental resource (by developing a portion of it) in the current period, whereas the possibility of only exogenous information should induce her to “wait” (by preserving the resource) and eventually develop it in the future according to the information potentially arriving.

Nevertheless, we show the unexpected result that the possibility of acquiring information both endogenously and exogenously could push the decision maker towards a higher level of preservation compared to the case where information arrives only exogenously. Indeed, we prove that there exists a subset of net benefits of preservation in the current and in the future period such that the possibility of acquiring information endogenously (added to the possibility of acquiring it exogenously) leads the decision maker to preserve more in the current and in the future period compared to the case in which only exogenous information is potentially available.

Although its existence stems from endogenous information, surprisingly enough, the TV_{endo} can be positively related to the probability of acquiring information exogenously. In fact, if additional endogenous information leads to more preservation, the TV_{endo} is increasing in the probability of acquiring information exogenously. This result is evidence of a form of complementarity between endogenous and exogenous information. When endogenous information is available, the possibility of also acquiring information exogenously holds back the DM from developing too much in order to acquire information endogenously. Therefore, when both exogenous and endogenous information are available, their complementarity seems to counterbalance their substitutability, hence reinforcing the effect of the TV_{endo} on environmental preservation, through moderating its intrinsic incentive to develop the resource in order to get (endogenous) information.

Our theoretical insights about the importance of additional endogenous information have interesting policy implications. Suppose that both the current and the future benefits of preservation are so low that if information can only be obtained exogenously it is optimal to completely develop the environmental resource in the present (and, because of irreversibility, in the future). In this case, disregarding the possibility of endogenous above exogenous information would mean underestimating the potential beneficial role of endogenous learning in terms of both present and future preservation. This role is stronger the higher the probability of exogenous information, which leads to less testing of the resource. This leads to another policy-relevant conclusion: the possibility of endogenous learning should be taken more into account as a means of environmental resource preservation especially in those situations where much exogenous research has already been carried out. When the possibility of learning exogenously is high, a lower amount of the resource has to be developed to obtain a given level of information. Thus, by developing only a small part of the resource in the present, it is possible to save a large part of it in the future, because of the production of additional endogenous information.

The results of this paper, while obtained in a simple two-period model with linear benefit functions, also hold when weakening some of our key assumptions, such as, for example, the risk neutrality of the decision maker. Despite its simple framework, our paper suggests a wider story about environmental option values which is worth exploring in more general contexts.

References

[1] Arrow, K. J. and A. C. Fisher (1974), "Environmental Preservation, Uncertainty and Irreversibility", *Quarterly Journal of Economics*, 89, 312-319.

[2] Attanasi, G. and A. Montesano (2008), "Competing for Endogenous Information in an Irreversible Environmental Resource Problem: a Game-Theoretic Analysis", *International Game Theory Review*, 10, 229-243.

[3] Chichilnisky, G. and G. Heal (1993), "Global Environmental Risks", *Journal of Economic Perspectives*, 7, 65-86.

[4] Conrad, J. M. (1980), "Quasi-Option Value and the Expected Value of Information", *Quarterly Journal of Economics*, 95, 812-820.

[5] Dasgupta, P. S. and G. M. Heal (1979), "Economic Theory and Exhaustible Resources", Cambridge University Press, Cambridge, UK.

[6] Dixit, A. and R. Pindyck (1994), "Investment under Uncertainty", Princeton University Press, Princeton, NJ.

[7] Epstein, L. G. (1980), "Decision Making and the Temporal Resolution of Uncertainty", *International Economic Review*, 21, 269-283.

[8] Fisher, A. C. (2000), "Investment under Uncertainty and Option Value in Environmental Economics", *Resource and Energy Economics*, 22, 197-204.

[9] Fisher A. C. and W. M. Hanemann (1987), "Quasi-Option Value: Some Misconceptions Dispelled", *Journal of Environmental Economics and Management*, 14, 183-190.

[10] Fisher, U. A. and U. Narain (2003), "Global Warming, Endogenous Risk, and Irreversibility", *Environmental and Resource Economics*, 25, 395-416.

[11] Freeman, A. M. (1984), "The Quasi-Option Value of Irreversible Development", *Journal of Environmental Economics and Management*, 11, 292-295.

[12] Freixas, X. and J. J. Laffont (1984), "On the Irreversibility Effect", in *Bayesian Models in Economic Theory*, Boyer, M. and R. Kihlstrom (eds.), North-Holland, Amsterdam, 105-113.

[13] Gollier, C., Jullien, B. and N. Treich (2000), "Scientific progress and irreversibility: an economic interpretation of the 'Precautionary Principle'", *Journal of Public Economics*, 5, 229-253.

[14] Graham-Tomasi, T. (1995), "Quasi-Option Value", in *Handbook of Environmental Economics*, Bromley, D. W. (ed.), Blackwell, London, 594-614.

[15] Hanemann, W. M. (1989), "Information and the Concept of Option Value", *Journal of Environmental Economics and Management*, 16, 23-37.

[16] Henry, C. (1974), "Investment Decisions under Uncertainty: The 'Irreversibility Effect'", *American Economic Review*, 64, 1006-1012.

[17] Kolstad, C. D. (1996), "Fundamental Irreversibilities in Stock Externalities"", *Journal of Public Economics*, 60, 221-233.

[18] Mäler, K. G. and A. C. Fisher (2005), "Environment, Uncertainty and Option Values", in *Handbook of Environmental Economics*, 2, Mäler, K. G. and J. R. Vincent (eds.), Elsevier, Amsterdam, 571-620.

[19] Miller, J. R. and F. Lad (1984), "Flexibility, Learning, and Irreversibility in Environmental Decisions: A Bayesian Approach", *Journal of Environmental Economics and Management*, 11, 161-172.

[20] Martzoukos, S. H. (2003), "Real R&D options with endogenous and exogenous learning", *Real R&D options*, 111-129.

[21] Marwah, S. and Zhao, J. (2010), "Double Irreversibilities and Endogenous Learning in Land Conversion Decisions", Michigan State University, Working Paper.

[22] Mensink, P. and T. Requate (2005), "The Dixit-Pindyck and the Arrow-Fisher-Hanemann-Henry option values are not equivalent: a note on Fisher (2000)", *Resource and Energy Economics*, 27, 83-88.

[23] Raiffa, H. and R. O. Schlaifer (1961), "Applied Statistical Decision Theory", MIT Press Cambridge, MA.

[24] Salanié, F. and N. Treich (2009), "Option Value and Flexibility: A General Theorem with Applications", LERNA, University of Toulouse, W.P. no. 09.12.288.

[25] Ulph, A. and D. Ulph (1997), "Global Warming, Irreversibility and Learning", *Economic Journal*, 107, 636-650.

[26] Walters, C. J., and R. Hilborn (1978), "Ecological optimization and adaptive management", *Annual Review of Ecology and Systematics*, 9, 157–188.