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David Bardey and Philippe De Donder

Toulouse School of Economics

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David Bardey[2] and Philippe De Donder[3]

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[2]University of Los Andes - Cede (Colombia) and Toulouse School of Economics (France). Email: d.bardey@uniandes.edu.co

[3]Toulouse School of Economics, CNRS, University of Toulouse Capitole, Toulouse, France. Email: philippe.dedonder@tse-fr.eu

Abstract

Personalized medicine is still in its infancy, with costly genetic tests providing little actionable information in terms of efficient prevention decisions. As a consequence, few people undertake these tests currently, and health insurance contracts pool all agents irrespective of their genetic background. Cheaper and especially more informative tests will induce more people to undertake these tests, potentially impacting not only the pricing but also the type of health insurance contracts. We develop a setting with endogenous observable prevention and adverse selection and we study which contract type (pooling or separating) emerges at equilibrium as a function of the proportion of agents undertaking the genetic test as well as of the informativeness of this test.

Starting from the current low take-up rate generating at equilibrium a pooling contract with no prevention effort, we show that an increase in the take-up rate may decrease welfare as long as the equilibrium remains pooling and is especially detrimental when the equilibrium becomes separating. Similarly, decreasing the prevention effort cost (a proxy for more informative tests) is detrimental to welfare when it changes the type of equilibrium from pooling to separating.

These results imply that the desirability of public policies encouraging genetic test taking or decreasing the cost of prevention effort varies according to the type of contracts observed in health insurance markets. Especially, such policies may not be advisable in the short run, as long as the equilibrium is pooling.

JEL Codes: D82, I13, I18.

Keywords: Adverse selection, consent law regulation, discrimination risk, personalized medicine, pooling and separating equilibria, Wilson anticipatory equilibrium.

1 Introduction

Personalized medicine can be defined as the development of ever more accurate diagnoses, prevention actions and therapies, based on the individual characteristics of the agents. This type of medicine is made possible by the development of genetic tests. Most often, such genetic tests do not currently provide much useful guidance for prevention.[1] As a consequence, few individuals currently undertake a genetic test to learn about their future probability of developing a disease, except in very specific instances. This situation may change in the near future, as genetic tests become cheaper and especially as they provide more actionable information about prevention strategies to decrease the likelihood of developing certain diseases (see the many examples in Snyder, 2016). Both trends should induce more individuals to undertake genetic tests. At first blush, this better and cheaper information, allowing for more tailored prevention efforts, should have a positive impact on individual utilities and aggregate welfare. Our claim in this paper is that it is also important to look at the consequences of these changes on health insurance markets, and that these insurance effects may be detrimental to individuals' utilities and to aggregate welfare, especially in the short run.

Since very few people undertake genetic tests for the moment, health insurers currently pool, within the same contracts, policyholders who are uninformed about their genetic propensity to develop specific diseases with the very few informed policyholders. There is then no discrimination risk caused by genetic testing. This may change when more individuals undergo genetic tests. The insurance pool will then consist of a higher proportion of informed types. Since it is difficult for health insurers to observe if their policyholders have done a genetic test, adverse selection will occur. Those who discover a deleterious genetic background will have an incentive to pretend that they are untested in order not to be identified as high risk (and thus costly) by the insurers. Those with good genetic news will have an incentive to signal to insurers that they have low probabilities of damage. At some point, these tensions may result in insurers providing separating contracts to attract the genetically informed, low-risk agents, rather than pooling contracts. The objective of this article is precisely to understand the impact of both a higher test take-up rate, and of more informative tests, on the type of equilibrium in health insurance contracts, and to assess the welfare consequences of these changes.

The economics literature dealing with genetic testing is silent on this evolution, for two reasons. First, to the best of our knowledge, most articles dealing with genetic testing assume that individuals are *ex ante* homogeneous in terms of testing costs and benefits, so that they all adopt the same decision regarding genetic tests and prevention

[1]See for instance the recent statement of the American College of Medical Genetics and Genomics (ACMG): "The use of DNA-based health screening to guide preventive care in the screened individual has long been discussed, but until recently has had limited applications." (Murray *et al.*, 2021, p. 989).

behaviors. It is then impossible to understand the impact of continuous increases in take-up rates of genetic tests when the equilibrium take-up rate is either zero or one. Second, the literature mainly tends to focus on separating equilibrium contracts à la Rothschild and Stiglitz (1976), although the currently observed contracts are most often pooling (see Hoy, 2006). Understanding the impact of cheaper and more informative genetic tests on insurance contracts requires that the type of contract, separating or pooling, be endogenously determined.

In this article, we consider a setting where agents have either a low or a high genetic probability of developing a disease. The proportion of each type of genetic background in the population is common knowledge. Agents who do not take a genetic test (hence called of type U for uninformed) only know their expected probability of developing the disease. An exogenous fraction of the population (called informed) has done a genetic test which reveals with certainty their genetic background.[2] This genetic information allows to tailor a costly prevention effort, which decreases the probability of developing the disease if the individual has a deleterious genetic background. This prevention effort is observable and contractible by a competitive fringe of insurers.[3]

Whether a test has been done or not, and its result if done, are private information of the agents, giving rise to adverse selection. More precisely, we assume that agents are allowed, but not required, to reveal to insurers their test results, a regulatory framework we discuss in section 3. Informed agents with low probabilities to develop the disease (denoted as type L) always have an incentive to reveal this information in order to obtain a cheaper contract with full coverage. Informed agents with high probabilities (type H) have an incentive to pretend that they did not do a test to mimic type U agents.

We make use of Wilson (1977)'s equilibrium concept, which allows us to be agnostic as to the equilibrium type, namely how informed agents with high damage probability (H) and uninformed agents (U) are treated. They can either be pooled (and offered the same contract), or be separated by insurers offering them a menu of contracts, subject to a self-selection (or incentive compatibility) constraint. Wilson equilibrium type (pooling or separating) is then the one preferred by the uninformed policyholders. We study how this contract's type is affected by two characteristics of personalized medicine: the share of agents who do the test, and the informativeness of the tests as proxied by the cost of the prevention effort which alleviates the consequences of having a deleterious genetic background. For instance, going back to the BRCA1/2 gene and the recommendation

[2] We leave the endogenization of the decision to test or not to future research.

[3] The best-known example of this setting is the recommendation for women testing positive to the BRCA1 or BRCA2 alleles, which increase the probability of developing breast cancer, to undertake a mastectomy. Snyder (2016) contains several other examples where the prevention effort consists in taking drugs or in modifying one's behavior. See section 3 for more details about the information structure considered.

to perform surgery when tested positive, one can only hope for the development of a less psychologically costly prevention technology in the future.

We now summarize our main results. We start with the current situation of a low fraction of informed agents and high prevention costs, resulting in a pooling (of types U and H) equilibrium with no prevention effort. Increasing exogenously the fraction of informed type worsens the average quality of the pool (with a higher fraction of H types pretending to be uninformed), raises its break-even price and thus decreases both types U's and H's utilities. At the same time, aggregate welfare (measured as the sum of individual utilities weighted by the equilibrium fraction of each type) is also affected by a *composition effect*, as more uninformed types (U) become informed about their individual risk and are thus replaced by types L or H. The genetic information allows agents to better tailor the prevention effort to their own need, so that this composition effect increases aggregate welfare. Whether the composition effect's (positive) impact on welfare is larger than the (negative) impact due to the worsening of the pool quality is analytically ambiguous. We provide a numerical example where aggregate welfare decreases with the fraction of informed agents, in pooling equilibria without effort.

When the fraction of informed agents reaches a threshold, the type of equilibrium moves from pooling to separating, because the utility of low risk (*i.e.*, type U) agents in the pool has decreased so much that they are now better off with a separating contract. Insurers then propose a pair of contracts to types U and H, devised such that type H has no incentive to pretend being uninformed. The separation occurs either by decreasing the coverage rate of the contract devised for type U, and/or by requiring a different prevention effort in both contracts. Type H agents' utility (as well as aggregate welfare) drops discontinuously as one moves from a pooling to a separating equilibrium, as they lose the benefits of the lower premium paid when pooled with the uninformed agents. Once the equilibrium becomes separating, further increases in the fraction of types tested does not affect the equilibrium contracts and utilities, but increases aggregate welfare thanks to the composition effect mentioned above.

We obtain similar results if we decrease the prevention effort cost (rather than increase the fraction of tested agents). Decreasing this cost has no impact on utilities or welfare as long as the equilibrium pooling contract prescribes no effort. Crossing a threshold cost level changes the equilibrium from pooling to separating (if the fraction of tested agents is large enough), with the same detrimental impact on type H's utility (and aggregate welfare) as described above. We then obtain the counter-intuitive result that a lower cost decreases aggregate welfare. The separating equilibrium calls for effort for type H but not for type U (since effort is useless for the latter if she has a favorable genetic background). A lower effort cost then increases type H's utility with her contract, relaxing the incentive constraint and thus allowing a larger coverage rate for type U, so that both types benefit from the lower effort cost, even though type U exerts no effort. Aggregate welfare then also increases. Once the effort cost is low enough, the equilibrium moves back to being pooling, but with a prevention effort. Type H's utility increases discontinuously with this move. Further decreases in effort cost then benefit both types of agents (and aggregate welfare) with such a pooling equilibrium.

The first main message of the paper is then that both an increase in the fraction of agents tested and a decrease in the prevention effort cost have detrimental effect in the short run–*i.e.*, as long as they keep the equilibrium unchanged as pooling without effort, or cross a threshold so that the equilibrium becomes separating. It is only in the longer run (after we have moved to a separating equilibrium) that these changes improve individual utilities and/or aggregate welfare. The second message of the paper is that the development of genetic tests is more susceptible to be beneficial (both for aggregate welfare and for those unlucky enough to have detrimental genetic backgrounds) when the larger fraction of tested agents is accompanied by a decrease in prevention effort cost (a proxy for the efficiency of the genetic test in terms of actionable health information) large enough that the equilibrium is pooling with prevention effort.

The structure of the paper runs as follows. Section 2 explains our contribution to the literature. Section 3 discusses the information structure of the model. Section 4 presents the model while section 5 defines a generic insurance contract in this setting. Section 6 analyzes the pooling contracts while section 7 studies the separating contracts.[4] Section 8 studies which kind of contract, separating or pooling, emerges at equilibrium as a function of the proportion of informed agents and of the level of the cost of the prevention effort. Section 9 performs a comparative static welfare analysis with endogenous contract type. Section 10 concludes. Most formal proofs are relegated to Appendices.

2 Related literature

Doherty and Thistle (1996) is the seminal article studying the incentive to gather information in insurance markets in the context of adverse selection. They first show that individuals do not have an incentive to acquire information (as in taking a costless genetic test for instance) when the informational status of the agent (*i.e.*, whether he has done a test or not, irrespective of its result) is observable by the insurer. They then obtain that individuals acquire information at equilibrium only if insurers cannot observe consumers' informational status. They characterize the separating contracts offered to agents under different configurations of information costs and benefits.

[4]We start with the pooling contract both because it corresponds to the current situation, and also for pedagogical reasons as it it is easier to describe since it does not rely on self-selection constraints.

The subsequent literature has built on Doherty and Thistle (1996) mainly by adding a prevention effort which either decreases the probability that the damage occurs (primary prevention, as in Hoel and Iversen [2002], Peter *et al.* [2017] and Bardey and De Donder [2013]) or decreases the extent of the damage when it occurs (secondary prevention, as in Crainich [2017] and Barigozzi and Henriet [2011]), and by studying different regulatory settings. Note that the existence of a prevention effort, which can be tailored to the test results, tends to increase the value of the information generated by these tests.

To the best of our knowledge, most of this literature has kept two assumptions made in Doherty and Thistle (1996). First, all individuals are *ex ante* identical (in both the individual costs and benefits of the test) and thus, at equilibrium, they either all choose to test, or no one tests. Second, they focus on separating equilibria à la Rothschild-Stiglitz. One exception to the first point is Hoel et *al.* (2006) who study the consequences for the testing decisions of introducing heterogeneity in psychological preferences (repulsion from chance). They provide an equilibrium analysis in a setting with separating equilibria. The exceptions to the second point are Hoy (2006), Hoy *et al.* (2003) and Crainich (2017), which consider more realistic settings that include a pooling equilibrium. Strohmenger and Wambach (2000) also study the impact of genetic tests in a large set of equilibrium contracts. However, they do not tackle the transition from pooling to separating equilibrium that may arise endogenously as, for instance, the test take-up rate increases. Finally, Bardey *et al.* (2019) run an experiment based on a theoretical set-up where individuals are heterogeneous and do not take the same decision with respect to genetic testing. They assume that agents who claim to be uninformed about their type are offered a pooling contract.

As explained above, we use here a more reduced form by assuming that an exogenous fraction of individuals have been tested while the rest of the population have not. Thus, we have individuals with different informational statuses at equilibrium and we provide comparative static analysis results with respect to the fraction of informed individuals and the prevention cost. Our analysis encompasses pooling and separating equilibria in a set-up à la Wilson.

3 The information structure: Adverse Selection without Moral Hazard

Before developing analytically the model, we come back to the justification of its information structure that, on the one hand, considers a problem of adverse selection caused by the policyholders' informational status related to genetic test, and, on the other hand, assumes that health insurers can observe their policyholders' prevention effort (*i.e.* implying the absence of *ex ante* moral hazard).

We study a context of adverse selection, where policyholders may choose whether to disclose or not that they have undertaken a genetic test, and its results. This setting corresponds to the *Consent Law* regulation (used in the Netherlands and in Switzerland, for instance) where individuals are allowed, but not required, to divulge this information to private health insurers. Other regulations exist, such as *Strict Prohibition* (where insurers cannot require applicants to provide existing tests results, and cannot use any genetic information in underwriting and rating) and *Disclosure Duty* (where applicants are obliged to reveal whether they did a test, and its results, to the insurers).

We claim that adverse selection is very difficult to avoid with genetic tests, even when a regulation different from Consent Law is implemented. Observe first that Strict Prohibition also generates adverse selection, since nothing prevents the insurers from offering a menu of contracts in order for the different types (informed or not) to self-select. In other words, Strict Prohibition is not collusion proof, since low risk agents would like to reveal their type, and insurers may screen those low risk types without regulators' knowledge.[5] The enforcement of a Disclosure Duty regulation seems questionable too given that a genetic test can be done abroad (for instance by mailing a biological sample and retrieving the genetic information via Internet). In such a context it is unlikely that patients' genetic information will be transmitted to their health insurer if they prefer it to remain private information. Hence, our point that adverse selection is a first-order concern when dealing with genetic testing.

We model Consent Law for its simplicity, as it allows us to concentrate on the adverse selection problem between truly uninformed agents and those who have received bad genetic news, and wish to hide them. Modeling Strict Prohibition would require to deal with an adverse selection problem with 3 types (uninformed, low risk and high risk informed agents), making the already complex analysis even more so. We acknowledge this limitation of our approach, and come back to why we surmise that the general message of this paper would also apply with other regulations generating adverse selection in the concluding section.

Second, we assume that the prevention effort is observable and contractible by insurers. Lifting this assumption would generate a moral hazard problem which, together with the adverse selection problem mentioned above, would very significantly complicate the analysis without commensurate gains in intuitions. Moreover, *ex ante* moral hazard does not seem to be a first-order problem in health insurance as pointed out by Bardey and Lesur (2005) who show in a theoretical model that a deductible is not necessary when policyholders internalize sufficiently the benefits of their prevention behavior. Chen *et al.* (2023) and Courbage and Coulon (2004) corroborate empirically the absence of correlation between health insurance coverage rates and prevention behaviors.

[5]For instance by using proprietary artificial intelligence: see "A.I. is changing Insurance" by S. Jeong, New York Times, April 10, 2019.

The assumption of observable effort holds well in many examples where the effort can take the form of surgery (mastectomy in the presence of BRCA1/2), weight loss, abstention from smoking or drinking (which can be assessed for instance by a blood test), or physical activity (with the rise of wearable devices). Insurers are indeed paying more attention to the use of these technologies. For instance, Soliño-Fernandez *et al.* (2019) mention that "wearables are gaining momentum in employer based group insurance. For example, health insurance companies such as United Health Group, Humana, Cigna, and Highmark have established programs that foster the use of wearable devices at the workplace. In general terms, health insurance companies use wearable devices to promote wellness and prevention at the workplace and keep the progress accountable; and in exchange for healthy behaviors, employers receive economic incentives such as lower group premiums in their health insurance policy" (p.2). The same study shows that "2 out of 3 Americans would be willing to adopt health insurance wellness programs based on wearable devices, particularly if they have benefits related to health promotion and disease prevention, and particularly with financial incentives "(p.1). These observations are in line with our assumption that observable prevention effort may be included more often in health insurance contracts as personalized medicine progresses.

4 The model

The economy is composed of a unitary mass of individuals. Each individual develops a disease with some probability, with sickness modeled as the occurrence of a monetary damage of amount m. A fraction $\lambda \in]0,1[$ of individuals is of type H and has a high probability of incurring the damage, while the remaining fraction $1-\lambda$ is of type L and has a lower probability.

Individuals choose to exert or not some primary prevention effort. We assume that the prevention decision is binary and that prevention has no effect for a low probability individual, while it decreases the disease probability of type H individuals.[6] We then denote by p_L the probability of developing the disease for a type L individual (whether he exerts the effort or not), and by p_H^0 (respectively, p_H^1) the probability of developing the disease of a type H agent who does not (resp., does) exert a prevention effort. We assume that $p_H^0 > p_H^1 > p_L$. We capture the prevention efficiency through $\Delta = p_H^0 - p_H^1$,

[6]It is now well established in the medical literature that "it is a combination of the genes that you have inherited and the environment that you live in that determines the outcome" (Collins 2010), so that prevention is more efficient with type H agents. For instance, for macular degeneration, "it became clear that almost 80 percent of the risk could be inferred from a combination of (...) two genetic risk factors, combined with just two environmental risk factors (smoking and obesity)" (Collins, 2010). This normalization to zero effort for type L is done for simplicity and without loss of generality, as the model could allow for a positive effort for type L, and concentrate on the additional effort provided if type H, without affecting the results.

with $0 < \Delta < \bar{\Delta}$, where $\bar{\Delta} = p_H^0 - p_L$.

An exogenous fraction k of individuals has done a genetic test and is thus informed about its type (L or H), while the remaining fraction $1-k$ is not informed. We denote an uninformed agent as having type U, with a probability of developing the disease equal to

$$p_U^i = \lambda p_H^i + (1-\lambda)p_L,$$

with $i \in \{0,1\}$ denoting whether the agent exerts ($i=1$) or not ($i=0$) the prevention effort. Note that $p_U^i < p_H^i$ since $\Delta < \bar{\Delta}$ and $\lambda < 1$.

We consider a setting where individuals, whether informed or not about their type, buy health insurance from a competitive fringe of insurers. An insurance contract is composed of a premium to be paid to the insurer, and of an indemnity from the insurer to the insured in the case the disease occurs. We further assume that the prevention effort is observable, so that there is no moral hazard in our setting and insurance contracts state whether this effort is required or not.

We assume that agents are not required by law to reveal their type, but may do so if they wish to, in which case insurers may use this information. This setting results in adverse selection: while agents of type L always show their test results in order to secure a low premium, agents who have been revealed to be of type H have an incentive to pretend that they are untested/uninformed about their risk.

The timing of the model is then as follows. A fringe of profit-maximizing health insurers offer a set of insurance contracts to agents who are exogenously informed (or not) about their individual probability of incurring the health damage. Agents then buy one insurance contract, and exert the prevention effort if the latter is required by the contract. Finally, the disease occurs or not, and the payoffs are realized.

We now describe the contracts offered by the insurers.

5 Generic insurance contract

A generic insurance contract is denoted by (π, I, i) where π denotes the premium in case of health, I the indemnity (net of the premium) in case of sickness, and where $i \in \{0,1\}$ denotes whether the contract prescribes the prevention effort or not. The premium is computed as $\pi = \alpha pm$, where α denotes the fraction of the damage m reimbursed in case it occurs, and where p is the probability that the agents buying this contract incur the damage, given whether the prevention effort is required or not. Competition forces insurers to offer actuarially fair contracts, so that the indemnity is $I = \alpha(1-p)m$. The expected utility of the agent buying this contract (π, I, i) is

$$pv(d) + (1-p)v(b) - \phi^i,$$

where $v(.)$ is a classical Bernouilli utility function ($v'(.) > 0$, $v''(.) < 0$), common to all agents, and where

$$d = y - m + I$$

is the consumption level if the damage occurs while

$$b = y - \pi$$

is the consumption level when the damage does not occur, with y the individual's exogenous income. We assume that $\phi^1 = \phi$ while $\phi^0 = 0$, where the effort cost (normalized to zero if no effort is undertaken) ϕ is measured in utility terms. The assumption of a utility (rather than monetary) cost is innocuous in our binary setting. All agents have the same utility function $v(.)$, income y and potential damage m, and differ only in their probability of damage p. In the case of complete coverage ($\alpha = 1$), we have

$$c \equiv d = b = y - pm.$$

It is straightforward that contracts offered to type L agents are not affected by adverse selection, since they are allowed to prove their type, and have an incentive to do so in order to benefit from the low premium reflecting their low disease probability p_L. By assumption, prevention has costs but no benefit when the individual is of a low type, so that the only contracts offered to type L agents entail no prevention effort, with the consumption level, denoted by c_L, given by

$$c_L = y - p_L m,$$

with the corresponding utility level $V_L = v(c_L)$.

The following assumption holds throughout the rest of the paper. It is made in order to simplify some (already long and convoluted) proofs.

Assumption 1 *The utility function $v(.)$ exhibits constant absolute risk aversion (CARA).*

We follow Wilson (1977)'s approach in which the equilibrium can be either pooling or separating (see Hoy (2006) and Seog (2010, section 7.3)). While Rothschild and Stiglitz (1976) (RS hereafter) apply a traditional Nash reasoning (*i.e.* insurers conjecture that the other insurers will not react to the introduction of a new contract), Wilson (1977) uses a more sophisticated conjecture, which is that other insurers will withdraw their contracts that become unprofitable as a result of the introduction a new contract. This conjecture provides an insurer with lower incentives to offer new contracts than under the RS conjecture. As a consequence, a RS equilibrium is also a Wilson equilibrium. But unlike in RS, a pooling equilibrium can emerge since a contract attracting only uninformed policyholders is not necessarily profitable under the Wilson conjecture. Whether these uninformed policyholders reach a higher utility with the RS (separating) contract or with a pooling contract determines whether the Wilson equilibrium is separating or pooling.

To identify the Wilson equilibrium, we first construct the equilibrium contract assuming that it is pooling (section 6) or that it is separating (section 7). Section 8 then assesses which of the two types of contracts is a Wilson equilibrium.

6 Pooling insurance contracts

6.1 Characterization

Recall first that the pooling contract does not concern type L, who has both the legal right and the incentive to reveal their type to the insurer in order to obtain an actuarially fair contract (with the low price of p_L) with full coverage.

A unique contract is offered to the pool of agents who claim to be uninformed about their type. By definition, in a pooling contract all agents must look alike, so that they all either undertake the (observable) prevention effort ($i = 1$, which we call the P^1 case or contract) or do not make this effort ($i = 0$, corresponding to the P^0 case/contract). This pool is composed of a mass of $1 - k$ agents who are truly uninformed (since they have not taken the test), and a mass $k\lambda$ of agents whose test has revealed them as type H. The competition among insurers results in a unit price of insurance p_P^i reflecting the average risk among this pool:

$$p_P^i = \frac{1-k}{1-k(1-\lambda)} p_U^i + \frac{k\lambda}{1-k(1-\lambda)} p_H^i, \ \ \forall i \in \{0, 1\}.$$

The pooling price p_P^i is lower than the actuarially fair price p_H^i for type H agents, who would then wish to buy full insurance. The coverage rate offered at equilibrium is then the one most-preferred by type U agents, and is lower than one since the pooling price is larger than p_U^i. We denote by α_P^i the equilibrium coverage rate of the pooling contract, which is obtained as

$$\alpha_P^i = \arg\max_{\alpha} p_U^i v(y + \alpha(1 - p_P^i)m - m) + (1 - p_U^i) v(y - \alpha p_P^i m) - \phi^i.$$

The first-order condition for the equilibrium pooling coverage rate is given by:

$$p_U^i (1 - p_P^i) v'(d_P^i) - (1 - p_U^i) p_P^i v'(b_P^i) = 0, \tag{1}$$

with

$$\begin{aligned} d_P^i &= y + \alpha_P^i (1 - p_P^i) m - m, \\ b_P^i &= y - \alpha_P^i p_P^i m, \end{aligned}$$

respectively, the consumption levels of (type U and H) agents who buy the pooling contract when the damage does (resp., does not) occur. It is obvious from (1) that α_P^i does not depend on the value of ϕ.

We denote the utility level attained by type U in the pooling contract as

$$V_{UP}^i = p_U^i v(d_P^i) + (1 - p_U^i) v(b_P^i) - \phi^i, i \in \{0, 1\}$$

and the one attained by type H as

$$V_{HP}^i = p_H^i v(d_P^i) + (1 - p_H^i) v(b_P^i) - \phi^i, i \in \{0, 1\}.$$

What determines whether effort is prescribed or not at equilibrium for the pooling contract is the comparison of V_{UP}^0 and V_{UP}^1. Insurers want to attract the least risky type (*i.e.*, U and not H) and competition among insurers ensures that the contract offering the highest utility to type U is offered at equilibrium. We then obtain the following proposition, which summarizes all the results obtained regarding the characterization of the pooling contracts.

Proposition 1 *(i) In all pooling equilibria, type L receives an actuarially fair contract with full coverage and never exerts the prevention effort.*
(ii) The type of pooling contract depends on both k and ϕ. There exists a unique value of ϕ, denoted by $\tilde{\phi}_P(k)$, so that:

1. *If $\phi < \tilde{\phi}_P(k)$, we have a P^1 equilibrium contract, where both types U and H make the prevention effort, and where the coverage rate of the pooling contract bought by both types is given by α_P^1.*
2. *If $\phi \geq \tilde{\phi}_P(k)$, we have a P^0 equilibrium contract, where neither type U nor H makes the prevention effort, and where the coverage rate of the pooling contract bought by both types is given by α_P^0.*

(iii) Insurance coverage rate α_P^i decreases with k, with $\alpha_P^1 \geq \alpha_P^0$.

Proof. See Appendix A.1. ■

It is intuitive that the prevention effort is made only if its cost is sufficiently low. Assumption 1 implies that the demand for insurance decreases with its unit price. A larger fraction of informed (H) agents in the pool increases the break-even premium and thus decreases the amount of coverage bought. Prevention, by decreasing the health risk, decreases the actuarially fair insurance premium, so that agents buy more insurance. Note for future reference that the threshold $\tilde{\phi}_P(k)$ depends on k in a non trivial way.

6.2 Comparative statics welfare analysis

We take as welfare function the utilitarian one where we use as weight for each type its proportion in the population of agents:[7]

$$\begin{aligned} W_P^i &= (1-k)V_{UP}^i + k\lambda V_{HP}^i + k(1-\lambda)V_L \\ &= k\left[(1-\lambda)V_L + \lambda V_{HP}^i - V_{UP}^i\right] + V_{UP}^i. \end{aligned} \quad (2)$$

We focus on the impact of decreasing ϕ (resp., increasing k), since it is likely to correspond to the empirically relevant case in a near future.

Proposition 2 *(i) Decreasing ϕ (a) has no impact on utilities nor on aggregate welfare in P^0, (b) increases types U's and H's utilities, as well as aggregate welfare, in P^1.*
(ii) Increasing k (a) decreases the utilities of both types U and H, but (b) has an ambiguous impact on aggregate welfare due to a composition effect.

Proof. See Appendix A.2. ■

The effort cost does not affect any contract in P^0, is paid by no one, and has no impact on the composition of the pool of insured agents, so that aggregate welfare is not affected by ϕ. Aggregate welfare decreases with ϕ in P^1, as a more costly effort does not affect the coverage rate but decreases the utility of types U and H who both pay this cost.

Increasing k has two countervailing effects on aggregate welfare in both P^0 and P^1. First, a larger k increases the price of the pooling contract and thus reduces the utilities of types U and especially of types H. Second, increasing k creates a *composition effect*, increasing the proportion of informed types L and H at the expense of uninformed ones, U. As can be seen from (2), the derivative of welfare with respect to k compares the expected payoff of knowing one's type with the payoff of remaining uninformed. The seminal paper by Hirshleifer (1971) has established that this composition effect (defined there as the "value of the information brought by a test") is negative when damage probabilities are exogenous and when individuals face a discrimination risk. We obtain here a positive composition effect, as we depart from Hirshleifer (1971)'s setting in two directions. First, becoming informed allows either to obtain full insurance (if revealed to be of type L) or to enjoy a better-than-fair insurance price (if revealed to be of type H). Second, the prevention decision may change with the informational status.

[7]See Hoy (2006) for a discussion of the equivalence between this welfare function and the expected utility of an individual behind the veil of ignorance, and for a welfare analysis that relies on the construction of Lorenz curves of the income distributions generated by the insurance contracts. The addition of a prevention cost in utility terms prevents us from using this approach relying on income distributions.

More precisely, in Case P^1, knowing one's type allows to save on the prevention cost of effort in case one is revealed to be of type L, generating another positive impact on the composition effect.

The overall sign of the impact of k on aggregate welfare is thus analytically ambiguous. Using a numerical example (see Appendix B), we obtain that $\partial W_P^0/\partial k$ is negative, so that the direct negative impact of a higher k on utilities is larger than the positive composition impact. Note that testing allows to save on the prevention effort cost (if revealed to be of type L), so that a larger value of ϕ increases the composition effect. Using the numerical example, we obtain that $\partial W_P^1/\partial k$ is negative for small values of ϕ, and positive for larger values. We come back to these impacts in section 9.2.

We now move to the description of the separating contracts.

7 Separating insurance contracts

7.1 Characterization

In a separating equilibrium, the competitive fringe of insurers offer a menu of two contracts (in addition to the contract offered to type L as described in section 5), one intended for type U and the other for type H. Formally, we define as $S^{ij}, \{i,j\} \in \{0,1\}^2$, the pair of (separating) contracts offered to types U and H, with effort required from type U (resp., H) if $i = 1$ (resp,. $j = 1$).[8]

We denote by V_{HS}^j the utility level obtained by type H with the separating contract devised for their type, depending on whether effort is prescribed in this contract or not ($j \in \{0,1\}$). Perfect competition ensures both that the contract offered to type H is actuarially fair with full coverage,

$$V_{HS}^j = v\left(y - p_H^j m\right) - j * \phi, \tag{3}$$

and that this contract includes the prevention effort if and only if type H's utility is higher with than without effort. This is the case if

$$\phi \leq \phi_{\max}^S \equiv v\left(y - p_H^1 m\right) - v\left(y - p_H^0 m\right). \tag{4}$$

The other contract is intended for type U agents and is also actuarially fair, thanks to competition. In order to prevent type H agents from buying this second, cheaper contract, the following incentive constraints have to be satisfied for $\{i,j\} \in \{0,1\}^2$:

$$V_{HS}^j \geq p_H^i v\left(d_{US}^{ij}\right) + \left(1 - p_H^i\right) v\left(b_{US}^{ij}\right) - i * \phi, \tag{5}$$

[8] We slightly abuse notation in sometimes calling S^{ij} the contract offered to type U, rather than the pair of contracts offered to U and H.

with

$$
\begin{aligned}
d_{US}^{ij} &= y + \alpha_S^{ij}(1 - p_U^i)m - m, \\
b_{US}^{ij} &= y - \alpha_S^{ij} p_U^i m,
\end{aligned}
$$

the consumption levels of agents who buy the separating contract devised for type U when, respectively, the damage does (resp., does not) occur. The utility attained by type U agents who buy the S^{ij} contract devised for their type is denoted by[9]

$$
V_{US}^{ij} = p_U^i v\left(d_{US}^{ij}\right) + \left(1 - p_U^i\right) v\left(b_{US}^{ij}\right) - i * \phi. \tag{6}
$$

To satisfy these incentive compatibility constraints, insurers can play on two dimensions in the contract devised for type U, namely the coverage rates α_S^{ij} and the prevention efforts ($i \in \{0, 1\}$). Perfect competition ensures that the effort is required from type U only if their utility is higher with than without effort in the contract offered to them which satisfies the incentive constraints (5). In the case where effort is required from type H (*i.e.*, if $\phi < \phi_{\max}^S$), this is the case if

$$
\phi \leq \phi_{\min}^S \equiv p_U^1 v\left(d_{US}^{11}\right) + \left(1 - p_U^1\right) v\left(b_{US}^{11}\right) - \left(p_U^0 v\left(d_{US}^{01}\right) + \left(1 - p_U^0\right) v\left(b_{US}^{01}\right)\right).
$$

The following proposition summarizes the main characteristics of the equilibrium separating contracts as a function of the prevention effort cost, ϕ, showing which of the two dimensions (*i.e.* coverage rate and effort in the contract devised for type U) are used to separate the two types, and, on the other hand, how the coverage rate offered to type U varies with the effort cost.

Proposition 3 *(i) When $\phi < \phi_{\min}^S$, the competitive fringe offers to type U (a) a S^{11} contract requiring effort with (b) a partial coverage α_S^{11} constant with the cost of effort.*
(ii) When $\phi_{\min}^S < \phi \leq \phi_{\max}^S$, the competitive fringe offers to type U (a) a S^{01} contract requiring no effort with (b) a partial coverage α_S^{01} (c) decreasing with the effort cost.
(iii) At $\phi = \phi_{\min}^S$, the coverage rate offered with the contract requiring prevention is strictly lower than the one forgoing prevention (i.e., $\alpha_S^{11} < \alpha_S^{01}$).
(iv) When $\phi > \phi_{\max}^S$, the competitive fringe offers to type U a S^{00} contract requiring no effort, with a partial coverage rate α_S^{00} constant with the cost of effort.

Proof. See Appendix A.3. ■

In all separating equilibria, types L and H receive an actuarially fair contract with full coverage, and type L never exerts the prevention effort as it is useless. The type of

[9] Unlike for type H, the utility level attained by type U with the contract devised for their own type depends on whether effort is included in the other contract (the one devised for type H) because of the incentive compatibility constraints.

separating contract does not depend on the proportion of informed agents but depends on the effort cost. Intuitively, agents perform the prevention effort if its cost is low enough, up to a threshold which is lower for type U ($\phi^S_{\min}$) than for type H ($\phi^S_{\max}$) because the expected pay-off of effort is lower for type U, who may not benefit at all from effort with probability $1-\lambda$.

We then obtain that the equilibrium separating contracts prescribe the same effort level for types U and H both when the effort cost is low ($\phi < \phi^S_{\min}$, effort required) and high ($\phi > \phi^S_{\max}$, effort not done), in which cases the separation between types only occurs in the usual, Rothschild-Stiglitz way of partial coverage to the least risky type (*i.e.*, type U here), with the coverage rate independent of the effort cost (parts (i) and (iv) of Proposition 3). When the effort cost is intermediate ($\phi^S_{\min} < \phi < \phi^S_{\max}$), the separation between types occurs both with partial coverage for type U and different effort levels for both types (S^{01} contracts, see part (ii)). Preventing effort in the contract designed for type U makes this contract less attractive to type H, enabling insurers to increase the coverage offered to type U, explaining the discontinuity in coverage rates when moving from contract S^{11} to S^{01} (part (iii)). At the same time, the coverage rate provided to U in S^{01} decreases with the cost of effort, because the latter increases the incentive for H to mimic U (since this cost is paid with the contract devised for H, but not for U) requiring to degrade the contract offered to U by providing less coverage.

Proposition 3 extends Barigozzi and Henriet (2011)'s Lemma 2 (itself an extension of Doherty and Thistle (1996)'s Proposition 2 to the case of secondary prevention) to our setting with both informed and uninformed types, so that agents who do not prove that they have a low genetic risk (*i.e.*, types U and H) have to be separated by the insurers.

7.2 Comparative statics welfare analysis

We now summarize the comparative statics results on welfare, defined as previously as the weighted sum of utilities, using as weight for each type its proportion in the population of agents.[10] We obtain the following proposition.

Proposition 4 *(i) Decreasing the prevention effort cost ϕ (a) has no impact on welfare in S^{00}, (b) increases welfare, in S^{11} and in S^{01}.*
(ii) Increasing the proportion of informed agents k increases welfare thanks to a composition effect.

Proof. See Appendix A.4. ■

[10] See Appendix A.4 for the precise mathematical statement.

The prevention effort has obviously no impact on utilities and welfare when no one does the effort (S^{00}) since the coverage rate offered to U is independent of ϕ (see Proposition 3 (iv)). A lower value of ϕ increases the utility of both U and H in all other cases–*i.e.*, when they both make the effort (S^{11}) but also in S^{01} because U's coverage rate increases as ϕ decreases, as explained in Proposition 3(ii)(c) above. In both cases, aggregate welfare then decreases with ϕ.

In a separating equilibrium, policyholders' utilities do not depend on k. However, the variation of k affects the welfare function through the *composition effect* already encountered in section 6, namely the increase in the proportion of informed types L and H at the expense of the uninformed type U. This composition effect increases welfare, as for pooling contracts, but for slightly different reasons.

First, remaining uninformed entails buying a contract with partial coverage, while being informed of one's type allows to buy full coverage, generating a positive impact on welfare. Second, the prevention decision may change with the informational status. With contracts S^{11}, knowing one's type allows to save on the prevention cost of effort in case one is revealed to be of type L, generating another positive impact on welfare. With contracts S^{01}, uninformed agents do not exert the prevention effort, so that knowing one's type now means incurring an effort cost ϕ if revealed to be of type H, with a negative impact on welfare. In both cases, the composition effect's net impact on welfare is positive.

8 Equilibrium contracts: separating or pooling?

The objective of this section is to understand what type of Wilson equilibrium, separating or pooling, emerges as a function of k and of ϕ. What determines whether the equilibrium is pooling or separating is the utility attained by type U policyholders under both contracts. Start with the pooling contract. It can only be defeated if insurers can attract the lower risk (type U) while offering them a separating contract. Hence, we have a pooling equilibrium if type U has a larger utility level with the pooling contract than with the separating contract, and *vice-versa*.

The following assumption is a sufficient (but not necessary) condition for type U to prefer the separating to the pooling contract when k tends toward 1. This assumption guarantees that with full adverse selection, a situation occurring when almost all policyholders have done a genetic test, the cross-subsidies contained in the pooling contract are so large that uninformed policyholders are better-off with a separating contract.

Assumption 2 Δ *is not too large:*

$$\frac{p_U^1\left(1-p_H^1\right)}{\left(1-p_U^1\right)p_H^1} < \frac{v'(y)}{v'(y-m)}. \tag{7}$$

This assumption is supposed to hold throughout the rest of the analysis.[11] We then obtain the following proposition.

Proposition 5 *For any given value of ϕ, there is a unique (strictly positive) threshold value of k, denoted by $\tilde{k}(\phi)$, such that a pooling equilibrium emerges if $k < \tilde{k}(\phi)$, and a separating equilibrium emerges if $k > \tilde{k}(\phi)$.*

Proof. See Appendix A.5. ■

Type U's utility decreases with k with the pooling contracts (since a larger k increases the contract's price) but is not affected by k with separating contracts. We then obtain that U prefers a pooling contract if k is lower than the threshold $\tilde{k}(\phi)$, and a separating contract if k is larger.

Observe that there are at most 6 possible comparisons of utility levels for type U, for any value of (k, ϕ), since there are 2 types of pooling (P^0 and P^1) and 3 types of separating (S^{00}, S^{01} and S^{11}) contracts.[12] It would be too cumbersome to try and solve all six cases. We then restrict ourselves to the configuration we observe in the numerical example described in Appendix B and depicted in Figure 1.

Insert Figure 1 here

Figure 1: In (k, ϕ) space, separation between P^1, P^0, S^{11}, S^{01} and S^{00} contracts.

Figure 1 can be understood with the help of Propositions 1, 3 and 5. The Wilson equilibrium is pooling if k is low enough (*i.e.* if $k < \tilde{k}(\phi)$) and separating otherwise (see Proposition 5). In the case of a pooling equilibrium, effort is prescribed if its cost is low enough ($\phi < \tilde{\phi}_P(k)$) and not otherwise (see Proposition 1). In the case of a separating equilibrium, contracts prescribe no effort to anyone if the effort cost is large ($\phi > \phi^S_{\max}$), effort only for type H if the effort cost is intermediate ($\phi^S_{\min} < \phi < \phi^S_{\max}$), and for both types U and H if it is low enough ($\phi < \phi^S_{\min}$) (see Proposition 3).[13]

[11] The inequality does not involve any endogenous variables, but only exogenous ones. Its left-hand-side is increasing in Δ, implying that Δ is not too large.

[12] Proposition 3 shows that there is no S^{10} equilibrium, since effort has a lower expected benefit for type U than for type H.

[13] Note that this configuration only excludes the two extreme comparisons of contracts among the potential six mentioned above: there is no value of (k, ϕ) where the pooling contract is P^1 and the separating contract S^{00} (because this would require to consider $\phi^S_{\max} < \phi < \tilde{\phi}_P(k)$) and where the pooling contract is P^0 and the separating contract S^{11} (because this would require to consider $\tilde{\phi}_P(k) < \phi < \phi^S_{\min}$). These two excluded situations anyway seem to be very unlikely to emerge in an equilibrium. Take for instance the comparison between P^1 and S^{00}. This supposes that, for the same parameters (k, ϕ), insurers who offer separating contracts would require of both types U and H not to do the prevention effort, while if insurers were to offer a pooling contract, they would ask both types to do this same prevention effort. The other four comparisons of contracts (P^0 and S^{00}, P^0 and S^{01}, P^1 and S^{01}, and P^1 and S^{11}) all exist for some parameter values (k, ϕ) in our example.

The following proposition assesses how $\tilde{k}(\phi)$ is affected by ϕ for the four possible configurations exemplified in Figure 1.

Proposition 6 *(a) For values of (k, ϕ) such that the potential equilibria are either P^0 and S^{00}, or P^1 and S^{11}, the threshold $\tilde{k}(\phi)$ is not affected by the value of ϕ.*
(b) For values of (k, ϕ) such that the potential equilibria are P^0 and S^{01}, the threshold $\tilde{k}(\phi)$ is increasing in the value of ϕ.
(c) For values of (k, ϕ) such that the potential equilibria are P^1 and S^{01}, the threshold $\tilde{k}(\phi)$ is decreasing in the value of ϕ.

Proof. See Appendix A.6. ■

Proposition 6 first shows that $\tilde{k}(\phi)$ is a constant (unaffected by ϕ) when both the pooling and the separating equilibria prescribe the same effort level to both types U and H, since the same effort cost is borne (or not) in both types of contracts. Part (b) compares a pooling contract without effort with a separating one where H exerts the effort and obtains that type U's utility decreases with the effort cost in the latter case (because the coverage rate decreases with k in S^{01}, as shown in Proposition 3 (ii)), but not in the former. A higher value of the effort cost then makes the pooling contract more attractive relative to the separating one, increasing the threshold value $\tilde{k}(\phi)$. Finally, when comparing a pooling contract with effort with the separating one S^{01}, the threshold $\tilde{k}(\phi)$ decreases with the effort cost because the latter decreases the utility with the pooling contract more than with the separating one.

In the next section, we study how utilities and aggregate welfare in the Wilson equilibrium are affected by changes in the prevention effort cost and in the proportion of tested agents.

9 Comparative static welfare analysis with endogenous contract type

We first vary the prevention effort cost and the fraction of informed agents separately, before looking at the impact of a simultaneous decrease in cost and increase in test take-up rate on utilities and welfare.

9.1 Welfare impact of the prevention effort cost ϕ

Compared to sections 6.2 and 7.2, we incorporate here how the equilibrium type (S^{11}, S^{01}, S^{00}, P^0 or P^1) and its corresponding health insurance contracts change endogenously as ϕ decreases. Focusing on the typology of cases that emerges from the numerical example (see Figure 1), we obtain four different situations, depending on the value of k. The Wilson equilibrium is always pooling when k is low enough (and moves from P^0 to P^1 as we decrease ϕ). For larger values of k, it moves from pooling to separating and then back to pooling (from P^0 to S^{01} to P^1) as ϕ decreases. For still larger values of k, the Wilson equilibrium changes from separating to pooling (S^{00} to S^{01} to P^1) as ϕ decreases. Finally, for very large value of k, the equilibrium is always separating (but changes from S^{00} to S^{01} to S^{11} as ϕ decreases).

We introduce the following assumption, whose role we discuss after Proposition 7, in footnote 14.

Assumption 3 *Δ is low enough that $p_P^1 > p_U^0$ for the value of k such that type U is indifferent between P^1 and S^{01}.*

This assumption implies that $p_H^1 > p_U^0$. We then obtain the following proposition.

Proposition 7 *With the typology of cases obtained in Figure 1, and under Assumption 3, a decrease in ϕ weakly increases both types U's and H's utility, and aggregate welfare, except when the combination of effort cost ϕ and test take-up rate k is such that the equilibrium changes from P^0 to S^{01}, in which case we have a downward discontinuity in H's utility and in aggregate welfare.*

Proof. See Appendix A.7 ■

It is intuitive that a lower effort cost benefits the types (U and/or H) exerting effort at equilibrium, whether the contract is pooling or separating. We then focus on the counter-intuitive case where a lower effort cost decreases utilities and welfare, which happens when the equilibrium e from pooling (P^0) to separating (S^{01}). Type U's utility is continuous in ϕ at this point because the equilibrium type is determined by type U's preferences. Type U is then indifferent when trading-off the higher price in P^0 (since $p_P^0 > p_U^0$) with the larger coverage ($\alpha_P^0 > \alpha_S^{01}$). As type H agents value more the higher insurance coverage in P^0 (since they have a larger damage probability, $p_H^0 > p_U^0$), their utility decreases discontinuously when moving from P^0 to S^{01}. A similar intuition applies to the other two discontinuities we observe (from P^0 to P^1, and from S^{01} to P^1).[14]

We refer the reader to Appendix B for figures showing how the utility of types U and H, and aggregate welfare, vary with ϕ for k intermediate in our numerical example.

[14]There is generically a discontinuity in H's utility (and aggregate welfare) when moving from S^{01} to P^1. Assumption 3 is sufficient (although not necessary) to ensure that the discontinuity is positive. Note that, if the discontinuity were negative, this would further reinforce our result that a lower value of ϕ does not always increase aggregate welfare.

9.2 With respect to test prevalence k

Focusing on the typology of cases that emerges from the numerical example (see Figure 1), we obtain that all transitions (as we increase k) have in common that we move at some point from a pooling to a separating Wilson equilibrium. But the specifics of the contracts (namely, whether they require effort or not) change with the value of ϕ. We then obtain the following proposition.

Proposition 8 *With the typology of cases obtained in Figure 1, and under Assumption 3, an increase in k has the following impact on utilities and welfare:*
(a) When k is low, the equilibrium is pooling where a higher k decreases both types U's and H's utilities, but has an ambiguous impact on aggregate welfare.
(b) For some intermediate values of ϕ, increasing further k then changes the equilibrium from P^0 to P^1, which is associated with an upward discontinuity in type H's utility and in aggregate welfare.
(c) Increasing k further then changes the equilibrium from pooling to separating, resulting in a downward discontinuity in both H's utility and aggregate welfare.
(d) Further increases in k do not change the equilibrium type (separating) and do not affect agent's utilities, but increase aggregate welfare by a composition effect.

Proof. See Appendix A.8. ■

When the test take-up rate k is low, the Wilson equilibrium is pooling (see Proposition 5). Increasing k then decreases the utility of both types (U and H) in the pool (because its quality degrades, with a larger fraction of risky types claiming to be uninformed, and so that the actuarially fair price of the pooling contract increases), while the impact on welfare is ambiguous, due to a composition effect (see Proposition 2). This is true whether the pooling contract prescribes effort (P^1) or not (P^0). When k attains a threshold level, the Wilson equilibrium becomes separating.[15] At this point, type U is indifferent in the trade-off between the smaller price but lower coverage with the separating contract. Type H, on the other hand, suffers more from the disappearance of the cross-subsidies when moving to a separating contract, and incurs a drop in utility, so that welfare also decreases discontinuously (same intuition as in Proposition 7 when moving from pooling to separating contracts). Finally, further increases in the proportion of informed individuals do not affect individuals' utilities, but increase welfare by the composition effect (see Proposition 4).

[15] For the intermediate values of ϕ studied in part (b), the equilibrium first moves from P^0 to P^1, with a discontinuous increase in H's utility. At this point, while U is indifferent between both contracts, H strictly prefers contract P^1 because it offers more coverage than P^0.

We obtain with our numerical example that welfare is slightly decreasing with k in the P^0 equilibrium. As already discussed at the end of section 6.2, welfare is increasing in k in the P^1 equilibrium when ϕ is large enough but is decreasing in k when ϕ is low enough. We refer the reader to Appendix B for additional figures depicting the utility of types U and H, and aggregate welfare, as a function of k when ϕ is such that part (b) of Proposition 8 applies.

9.3 When both ϕ and k vary simultaneously

In this section, we show numerically what could happen in the near future, as more people choose to do the genetic test (higher k) and as the informational content of those tests increases (which we proxy by a decrease in the prevention effort cost ϕ) simultaneously. We build on the numerical example presented in Appendix B, and assume a linear relationship between ϕ and k ($\phi(k) = 0.29 - 0.8k$). We then start in the upper left corner of Figure 1, in the (k, ϕ) space, with a high value of ϕ and a low value of k, corresponding to the situation currently observed in reality, and to a P^0 contract. As k increases and ϕ decreases, we move in the south-east direction on a straight line whose slope is such that we first cross to the S^{01} equilibrium contract, and then to the P^1 one.

Figures 2 and 3 depict respectively types U's and H's utility, and aggregate welfare, as we increase k/decrease ϕ simultaneously.

Figure 2: Types U's and H's utility as a function of k when $\phi(k) = 0.29 - 0.8k$.

Figure 3: Aggregate welfare as a function of k when $\phi(k) = 0.29 - 0.8k$.

As long as we remain in the P^0 equilibrium, types U's and H's utilities are not affected by ϕ but decrease with k, which makes the pooling contract more expensive. Proposition 2 has shown that the impact of k on aggregate welfare is ambiguous (because of a positive composition effect), but we obtain on Figure 3 that welfare decreases with k (so that the negative price effect is larger than the positive composition effect). The move from pooling to separating is associated with a drop in H's utility, who suffers from the disappearance of cross-subsidies, as seen in the previous subsection.

As long as we remain in the S^{01} equilibrium, types U's and H's utilities are not affected by k, but decrease with ϕ. We thus obtain an unambiguous increase in both types' utilities (and especially in H's utility, who is actually doing the effort),[16] as we

[16] Type U's utility increases thanks to the increase in coverage rate made possible by the lower effort cost borne by H–see Proposition 4.

increase k and decrease ϕ simultaneously, as exemplified in Figure 2. Aggregate welfare decreases with ϕ and increases with k (thanks to the composition effect, see Proposition 4) in S^{01}. We then obtain that aggregate welfare unambiguously increases as we increase k and decrease ϕ simultaneously, as exemplified in Figure 3. Moving from the separating to the pooling equilibrium increases discontinuously type H's utility (and thus welfare as well), who benefits from a return of the cross-subsidies.

As long as we remain in the P^1 equilibrium, types U's and H's utilities decrease with both k and ϕ (see Proposition 2). We obtain in our numerical example that the impact of a smaller ϕ supersedes the impact of a larger k, with both types' utilities increasing as we increase k and decrease ϕ simultaneously on Figure 2. Aggregate welfare decreases with ϕ, but the impact of a larger value of k is analytically ambiguous (see Proposition 2). We obtain on Figure 3 that aggregate welfare increases when we increase k and decrease ϕ simultaneously.

To summarize, we obtain that a larger value of k combined with a smaller value of ϕ is detrimental for types U and H and for society as long as we remain in a P^0 equilibrium, and when we move from the P^0 to the S^{01} equilibrium. The impact then becomes positive (both for U, for H and for aggregate welfare) in both the S^{01} and P^1 equilibria, and also when one moves from the former to the latter. The lowest level of utility (for H and for U) and of aggregate welfare corresponds to the combination of values of k and of ϕ that generates a change from the P^0 to the S^{01} equilibrium.

10 Conclusion

This article has studied the welfare implications on the health insurance market of the development of personalized medicine, as measured by the increase in the take-up rate of genetic tests providing more efficient and actionable prevention actions. Starting from the current low take-up rate generating at equilibrium a pooling contract with no prevention effort, we obtain that an increase in the take-up rate has first an ambiguous impact on welfare, and then unambiguously decreases welfare as one moves from a pooling to a separating equilibrium. It is only once the take-up rate is large enough that the equilibrium is separating that any further increase in take-up rate increases aggregate welfare, by a composition effect.

We also study the impact of a decrease in the prevention effort cost, taken as a proxy for the effectiveness of the genetic tests in terms of actionable health information. We obtain that decreasing this cost, starting from its current high level, moves us from the current pooling equilibrium without prevention to another pooling equilibrium with effort, with the possibility of having a separating equilibrium for intermediate values of the effort cost. Once more, the move from pooling to separating equilibrium is especially detrimental to those unlucky enough to get informed of their detrimental genetic background.

The fact that welfare is not monotone with respect to the genetic tests' take-up rate implies that policymakers must pay attention to the type of contracts offered in health insurance markets to determine the desirability of higher test take-up rates. More precisely, as long as uninformed policyholders and high risk policyholders remain pooled together within health insurance contracts, small increases in genetic tests take-up rates should be discouraged. On the contrary, once health insurance equilibrium contracts have switched from pooling to separating, policymakers should adopt policies that increase genetic test uses by policyholders. Decreasing prevention effort costs (for instance by subsidizing them) or trying to improve the effectiveness of these prevention efforts (for instance by subsidizing medical research) would be especially helpful in the long run to attain a pooling equilibrium with effort, which corresponds to the highest levels of utilitarian welfare in our setting.

Observe that we have used a simple utilitarian welfare function, weighting individual types by their share in the insured population at equilibrium. Moving to a welfare criterion that puts more weight on the least well-off (type H in our setting) would reinforce our conclusion that encouraging individuals to undertake a genetic test may result in short run welfare losses, as long as the equilibrium is not separating.

Our analysis considers a Consent Law setting where agents are allowed, but not required, to reveal to insurers whether they did the genetic test, and its results. While this setting limits the applicability of our analysis, we claim that the mechanisms at play here also apply to other regulations generating adverse selection, such as the Strict Prohibition legislation. In the latter case, informed low risk agents cannot signal their type, so that the adverse selection problem concerns the three types, L, U and H. Contracts can then be pooling (the 3 types), separating (the 3 types), or a mix of pooling (2 types) and separating (for the remaining type). Adopting this regulation would thoroughly complicate the resolution of the model, but the forces that we have identified here would also be at play. For instance, a higher fraction of tested agents would degrade the pool and the utility of its members, pushing the lowest risk agents in the pool to switch to a separating equilibrium, at the expanse of the utility of the higher risk agents inside the pool. Also, the separation between types would also occur through a combination of underprovision of insurance and/or different prevention effort levels. While recognizing the limits of our approach, we thus feel confident that the main message of the paper would hold through with other, more complex, regulations incorporating adverse selection.

Our model could be extended in several directions. First, one would like to endogenize the testing decisions. Second, it would be interesting to introduce insurers' market power, the limit case being the monopoly situation. Stiglitz (1977) shows that, even in a monopoly situation, a pooling contract cannot be supplied at equilibrium and we conjecture this result would apply here as well. It would be worth studying if a pooling contract can emerge in our genetic test context when one moves away from the pure monopoly situation. Third, in our model we assume that the fraction of agents who do the test is the same irrespective of their genetic background. One could surmise that it is actually higher for agents who have a detrimental genetic background, for instance because their family medical history constitutes a signal of this background, or because they are advised to get a test by their doctor. This would change the composition of the pooling contract (increasing the fraction of tested H agents) and the incentives faced by type H and U agents. We leave this promising extension to future research.

References

[1] Bardey D. and Ph. De Donder. 2013. "Genetic Testing with Primary Prevention and Moral Hazard". *Journal of Health Economics*, 32 (5), 768-779 & 1007-1012.

[2] Bardey D, De Donder Ph. and C. Mantilla, 2019. "How is the Trade-off between Adverse Selection and Discrimination Risk Affected by Genetic Testing? Theory and Experiment", *Journal of Health Economics*, 68. Article 102223.

[3] Bardey, D., and R. Lesur, 2005. "Optimal health insurance contract: Is a deductible useful?" *Economics Letters*, 87(3), 313-317.

[4] Barigozzi F. and D. Henriet. 2011, "Genetic information: comparing alternative regulatory approaches when prevention matters". *Journal of Public Economic Theory*, 13(1), p. 23-46.

[5] Chen C., G. G. Liu, T. Wang and J. Tan, 2023, "Ex-ante moral hazard and health insurance: Evidence from China's urban residence basic medical insurance scheme", *Health Economics*, 32(11), 2516-2534.

[6] Collins F., 2010, The Language of Life: DNA and the Revolution in Personalized Medicine, kindle edition, HarperCollins Publishers.

[7] Crainich, D. 2017. "Self-Insurance With Genetic Testing Tools", *Journal of Risk and Insurance*, 84(1), 73-94.

[8] Courbage, C. and Coulon, A.D., 2004. "Prevention and private health insurance in the UK", *The Geneva Papers on Risk and Insurance-Issues and Practice*, 29, pp.719-727.

[9] Doherty, N and P. Thistle, 1996. "Adverse Selection with Endogenous Information in Insurance Markets", *Journal of Public Economics*, 63, 83-102.

[10] Hirshleifer J. 1971. “The Private and Social Value of Information and the Reward to Incentive Activity”. *The American Economic Review*, 61, p. 561-74.

[11] Hoel M. and T. Iversen, 2002, ”Genetic testing when there is a mix of compulsory and voluntary health insurance” *Journal of Health Economics*, 21(2), p. 253-270.

[12] Hoel M. and Iversen T, Nilssen T. and Vislie J. 2006. “Genetic testing in competitive insurance markets with repulsion from chance: A welfare analysis”. *Journal of Health Economics*, 25, p. 847-860.

[13] Hoy, M., 2006. “Risk Classification and Social Welfare”. *The Geneva Papers on Risk and In-surance – Issues and Practice*, 31(2): 245-269.

[14] Hoy M. and Orsi F, Eisinger F. and Moatti JP, 2003. “The Impact of Genetic Testing on Healthcare Insurance”. *The Geneva Papers on Risk and Insurance - Issues and Practice*, 28(2), p. 203-221.

[15] Murray, M.F., Giovanni, M.A., Doyle, D.L., Harrison, S.M., Lyon, E., Manickam, K., Monaghan, K.G., Rasmussen, S.A., Scheuner, M.T., Palomaki, G.E. and Watson, M.S., 2021. “DNA-based screening and population health: a points to consider statement for programs and sponsoring organizations from the American College of Medical Genetics and Genomics (ACMG)”. *Genetics in Medicine*, 23(6), pp.989-995.

[16] Peter, R., Richter A. and P. Thistle. 2017. “Endogenous Information, Adverse Selection, and Prevention: Implications for Genetic Testing Policy”. *Journal of Health Economics*, 55: 95-107.

[17] Rothschild, M., and Stiglitz, J. E., 1976. “Equilibrium in competitive insurance markets: An essay on the economics of imperfect information”. *The Quarterly Journal of Economics*, 90 (4): 629-649.

[18] Schlesinger H., 2000, The Theory of Insurance, in Handbook of Insurance, (p131-150) Edited by Georges Dionne, Boston: Kluwer Academic Publishers

[19] Seog H., 2010, The Economics of Risk and Insurance. Wiley-Blackwell edition.

[20] Snyder M., 2016, Genomics and Personalized Medicine. Oxford University Press.

[21] Soliño-Fernandez D., Ding A., Bayro-Kaiser E and E. Ding. 2019. “Willingness to adopt wearable devices with behavioral and economic incentives by health insurance wellness programs: results of a US cross-sectional survey with multiple consumer health vignettes”. *BMC Public Health*, Vol. 19:1649.

[22] Stiglitz J., 1977. "Monopoly, Non-linear Pricing and Imperfect Information: The Insurance Market". *Review of Economic Studies*, Vol. 44, Issue 3, p. 407–430.

[23] Strohmenger R. and A. Wambach. 2000. "Adverse selection and categorical discrimination in the health insurance markets: the effects of genetic tests". *Journal of Health Economics*, 19, p.197-218.

[24] Wilson, C. 1977. "A model of insurance markets with incomplete information". *Journal of Economic Theory*, 16(2), p. 167-207.

Appendix A

Appendix A.1.: Proof of Proposition 1

(ii): Observe first from (1) that α_P^0 and α_P^1 do not depend on the effort cost ϕ. We then have that V_{UP}^0 is independent of ϕ, while V_{UP}^1 decreases linearly with ϕ. When $\phi = 0$, it is obvious that $V_{UP}^1 > V_{UP}^0$. Finally, we have

$$\lim_{\phi\to\infty} V_{UP}^1 = -\infty < \lim_{\phi\to\infty} V_{UP}^0.$$

(iii) It is well known that CARA preferences generate a downward sloping insurance demand function: see for instance Schlesinger (2000, p.137). We now compare the coverage rates with and without prevention.
Let us rewrite the first order condition for an effort j. We have:

$$\Upsilon(\alpha,\Delta) \equiv p_U^j\left(1-p_P^j\right)v'(d_P^j) - \left(1-p_U^j\right)p_P^j v'(b_P^j) = 0.$$

The implicit function theorem gives:

$$\frac{d\alpha}{d\Delta} = -\frac{\partial\Upsilon(\alpha,\Delta)/\partial\Delta}{\partial\Upsilon(\alpha,\Delta)/\partial\alpha}.$$

Note that $\alpha_P^1 \geq \alpha_P^0$ if and only if $d\alpha/d\Delta \geq 0$. The sign of the denominator is negative due to the second-order condition. Thus,

$$\frac{d\alpha}{d\Delta} \geq 0 \Leftrightarrow \frac{\partial\Upsilon(\alpha,\Delta)}{\partial\Delta} \geq 0$$

$$\Leftrightarrow$$

$$\begin{aligned}
&\left(-\lambda\left(1-p_P^1\right) + \left(\frac{(1-k)\lambda+k}{1-k(1-\lambda)}\right)p_U^1\right)v'(d_P^1) \\
&-\left(p_P^1\lambda - \left(1-p_U^1\right)\left(\frac{(1-k)\lambda+k}{1-k(1-\lambda)}\right)\right)v'(b_P^1) \\
&+\left(1-p_P^1\right)p_U^1 v''(d_P^1)\alpha m\left(\frac{(1-k)\lambda+k}{1-k(1-\lambda)}\right) \\
&-\left(1-p_U^1\right)p_P^1 v''(b_P^1)\alpha m\left(\frac{(1-k)\lambda+k}{1-k(1-\lambda)}\right) \geq 0
\end{aligned}$$

$$\Leftrightarrow$$

$$\begin{aligned}
&\left(\frac{p_U^1}{1-k(1-\lambda)} - \lambda\left(1-p_P^1\right)\right)v'(d_P^1) - \left(p_P^1\lambda - \left(\frac{1-p_U^1}{1-k(1-\lambda)}\right)\right)v'(b_P^1) \\
&\geq \frac{\alpha m}{1-k(1-\lambda)}\left[p_U^1\left(1-p_P^1\right)v''(d_P^1) - \left(1-p_U^1\right)p_P^1 v''(b_P^1)\right].
\end{aligned}$$

First, let us work on the RHS. We have:

$$RHS \equiv \frac{\alpha m \left(1-p_U^1\right) p_P^1}{1-k\left(1-\lambda\right)} \left[\frac{p_U^1 \left(1-p_P^1\right)}{\left(1-p_U^1\right) p_P^1} v''(d_P^1) - v''(b_P^1)\right].$$

Using the first order condition, we have:

$$RHS = \frac{\alpha m \left(1-p_U^1\right) p_P^1}{1-k\left(1-\lambda\right)} v'(d_P^1) \left[\frac{v''(d_P^1)}{v'(d_P^1)} - \frac{v''(b_P^1)}{v'(b_P^1)}\right].$$

Assumption 1 ensures that $RHS = 0$. Now, let us focus on the LHS. Since $v(.)$ is concave, we have $v'(d_P^1) \geq v'(b_P^1)$. Thus, a sufficient condition to ensure that $LHS > 0$ is:

$$\begin{aligned}
\frac{p_U^1}{1-k\left(1-\lambda\right)} - \lambda\left(1-p_P^1\right) &> p_P^1 \lambda - \left(\frac{1-p_U^1}{1-k\left(1-\lambda\right)}\right) \\
&\Leftrightarrow \\
p_U^1 - \lambda\left(1-p_P^1\right)\left(1-k\left(1-\lambda\right)\right) &> p_P^1 \lambda \left(1-k\left(1-\lambda\right) - \left(1-p_U^1\right)\right) \\
&\Leftrightarrow \\
1 &> \lambda\left(1-k\left(1-\lambda\right)\right),
\end{aligned}$$

which is true. Q.E.D.

Appendix A.2.: Proof of Proposition 2

(a) ϕ plays no role in P^0, and affects neither individual utilities nor aggregate welfare. We have

$$\frac{\partial V_{UP}^1}{\partial \phi} = \frac{\partial V_{HP}^1}{\partial \phi} = -1$$

so that

$$\frac{\partial W_P^1}{\partial \phi} = -\left[1-k\left(1-\lambda\right)\right] < 0.$$

(b)

$$\begin{aligned}
\frac{\partial W_P^i}{\partial k} &= \lambda V_{HP}^i + (1-\lambda) U_L - V_{UP}^i \\
&\quad + (1-k)\frac{\partial V_{UP}^i}{\partial k} + k\lambda \frac{\partial V_{HP}^i}{\partial k},
\end{aligned}$$

where the first line is the "composition effect" whose sign is positive,

$$\begin{aligned}
& \lambda V_{HP}^{i}+(1-\lambda)U_{L}-V_{UP}^{i} \\
= & \lambda\left[p_{H}^{i}v(d_{P}^{i})+(1-p_{H}^{i})v(b_{P}^{i})\right]+(1-\lambda)v(y-p_{L}m)-\left[p_{U}^{i}v(d_{P}^{i})+(1-p_{U}^{i})v(b_{P}^{i})\right]+(1-\lambda)\,\phi^{i} \\
= & v(d_{P}^{i})(\lambda p_{H}^{i}-p_{U}^{i})+v(b_{P}^{i})(\lambda(1-p_{H}^{i})-(1-p_{U}^{i}))+(1-\lambda)v(y-p_{L}m)+(1-\lambda)\,\phi^{i} \\
= & (1-\lambda)\left[v(y-p_{L}m+\phi^{i})-\left(p_{L}v(d_{P}^{i})+(1-p_{L})v(b_{P}^{i})\right)\right]>0,
\end{aligned}$$

while the second line is negative, since

$$\begin{aligned}
\frac{\partial V_{UP}^{i}}{\partial k} &= -\alpha_{P}^{i}\frac{\partial p_{P}^{i}}{\partial k}m\left(p_{U}^{i}v'(d_{P}^{i})+\left(1-p_{U}^{i}\right)v'(b_{P}^{i})\right) \\
&= -\alpha_{P}^{i}\frac{\lambda\left(p_{H}^{i}-p_{L}\right)}{\left(1-k\left(1-\lambda\right)\right)^{2}}m\left(p_{U}^{i}v'(d_{P}^{i})+\left(1-p_{U}^{i}\right)v'(b_{P}^{i})\right)<0,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial V_{HP}^{i}}{\partial k} &= -\alpha_{P}^{i}\frac{\lambda\left(p_{H}^{i}-p_{L}\right)}{\left(1-k\left(1-\lambda\right)\right)^{2}}m\left(p_{H}^{i}v'(d_{P}^{i})+\left(1-p_{H}^{i}\right)v'(b_{P}^{i})\right) \\
+ & \quad m\frac{\partial\alpha_{P}^{i}}{\partial k}\left[p_{U}^{i}(1-p_{P}^{i})v'(d_{P}^{i})-\left(1-p_{U}^{i}\right)p_{P}^{i}v'(b_{P}^{i})\right]<0,
\end{aligned}$$

resulting in an ambiguity as to the overall sign of $\partial W_{P}^{i}/\partial k$.

Appendix A.3.: Proof of Proposition 3

We denote by IC^{ij} for $(i,j)\in\{0,1\}^{2}$ the incentive compatibility constraint (5) when it holds with equality.

We first prove the following lemma, which describes how the coverage rate devised for U varies with the effort cost, ϕ, in S^{11} and S^{01}, and compares the coverage rates in the two cases.

Lemma 1 *(a) $\alpha_{S}^{11}<1$, (b) α_{S}^{11} is constant with ϕ, (c) α_{S}^{01} decreases with ϕ when $\alpha_{S}^{01}<1$, and (d) $\alpha_{S}^{01}<\alpha_{S}^{11}$ for $\phi=\phi_{\max}^{S}$.*

Proof. *(a) $\alpha_{S}^{11}<1$ is a necessary condition for (5 with $i=j=1$ to hold since $p_{H}^{1}>p_{U}^{1}$.*
(b) ϕ cancels out in both the LHS and RHS of (5) with $i=j=1$.
(c) If $\alpha_{S}^{01}<1$, applying the implicit function theory to (5) with $(i=0,j=1)$, we obtain that

$$\frac{\partial\alpha_{S}^{01}}{\partial\phi}=\frac{-1}{m\left[p_{H}^{0}\left(1-p_{U}^{0}\right)v'(d_{US}^{01})-\left(1-p_{H}^{0}\right)p_{U}^{0}v'(b_{US}^{01})\right]},\tag{8}$$

whose sign is negative since $v'(d_{US}^{01}) > v'(b_{US}^{01})$ *and* $p_H^0\left(1-p_U^0\right) > p_U^0\left(1-p_H^0\right)$.
(d) IC^{11} *yields*

$$v(c_H^1) = p_H^1 v(d_{US}^{11}) + (1-p_H^1)v(b_{US}^{11}), \tag{9}$$

while IC^{01}, *when measured at* $\phi = \phi_{\max}^S$, *yields*

$$v(c_H^0) = p_H^0 v(d_{US}^{01}) + (1-p_H^0)v(b_{US}^{01}), \tag{10}$$

where

$$c_H^i = y - p_H^i m.$$

Let us consider the following function $G(\alpha,\delta)$ *such that*

$$\begin{aligned} G(\alpha,\delta) \;\; = \;\; & v(y-(p_H^0-\delta)m) \\ & -\left(p_H^0-\delta\right) v\left(y+\alpha\left(1-\left(\left(\lambda\left(p_H^0-\delta\right)+(1-\lambda)p_L\right)\right)\right) m - m\right) \\ & -\left(1-\left(p_H^0-\delta\right)\right) v\left(y-\alpha\left(\left(\lambda\left(p_H^0-\delta\right)+(1-\lambda)p_L\right)\right) m\right). \end{aligned}$$

Observe that (9) corresponds to $G(\alpha_S^{11},\Delta)=0$ *while (10) corresponds to* $G(\alpha_S^{01},0)=0$. *Using the implicit function theorem, we have that*

$$\frac{d\alpha}{d\delta} = -\frac{\partial G/\partial\delta}{\partial G/\partial\alpha}.$$

We obtain:

$$-\frac{dG/\partial\delta}{dG/\partial\alpha} = \frac{mv'(c_H^1) - \left[v(b_{US}^{11}) - v(d_{US}^{11}) + \lambda\alpha_S^{11} m\left[p_H^1 v'(d_{US}^{11}) + (1-p_H^1)v'(b_{US}^{11})\right]\right]}{m\left[p_H^1(1-p_U^1)v'(d_{US}^{11}) - (1-p_H^1)p_U^1 v'(b_{US}^{11})\right]}.$$

As $p_H^1 \geq p_U^1$, $d_{US}^{11} \leq b_{US}^{11}$ *and* $v(.)$ *is an increasing and concave function, the denominator is positive. Then, we have* $\alpha_S^{01} < \alpha_S^{11}$, *if and only if:*

$$mv'(c_H^1) \geq \left[v(b_{US}^{11}) - v(d_{US}^{11}) + \lambda\alpha_S^{11} m\left[p_H^1 v'(d_{US}^{11}) + (1-p_H^1)v'(b_{US}^{11})\right]\right].$$

Moreover, from IC^{11}, *we know that:*

$$v(b_{US}^{11}) - v(c_H^1) = p_H^1\left[v(b_{US}^{11}) - v(d_{US}^{11})\right].$$

Introducing this last expression in the previous inequality yields:

$$m\left[v'(c_H^1)-\lambda\alpha_{US}^{11}v'(b_{US}^{11})\right]\geq v(b_{US}^{11})-v(d_{US}^{11})+\lambda\alpha_{US}^{11}dp_H^1\left[v'(d_{US}^{11})-v'(b_{US}^{11})\right]$$
$$\Leftrightarrow$$
$$mv'(c_H^1)\left[1-\lambda\alpha_{US}^{11}\right]+\lambda\alpha_S^{11}m\left[v'(c_H^1)-v'(b_{US}^{11})\right]$$
$$\geq\left[v(b_{US}^{11})-v(c_H^1)\right]\left[\frac{1}{p_H^1}-\lambda\alpha_{US}^{11}m\left[\frac{v'(d_{US}^{11})-v'(b_{US}^{11})}{v(d_{US}^{11})-v(b_{US}^{11})}\right]\right]$$
$$\Leftrightarrow$$
$$\frac{mv'(c_H^1)\left[1-\lambda\alpha_{US}^{11}\right]}{v'(c_H^1)-v'(b_{US}^{11})}+\lambda\alpha_{US}^{11}m$$
$$\geq\frac{v(b_{US}^{11})-v(c_H^1)}{v'(c_H^1)-v'(b_{US}^{11})}\left[\frac{1}{p_H^1}-\lambda\alpha_{US}^{11}m\left[\frac{v'(d_{US}^{11})-v'(b_{US}^{11})}{v(d_{US}^{11})-v(b_{US}^{11})}\right]\right].$$

The generalized mean value theorem implies that:

$$\frac{v(b_{US}^{11})-v(c_H^1)}{v'(c_H^1)-v'(b_{US}^{11})}=-\frac{v'(\gamma)}{v''(\gamma)}$$

and

$$\frac{v'(d_{US}^{11})-v'(b_{US}^{11})}{v(d_{US}^{11})-v(b_{US}^{11})}=-\frac{v''(\beta)}{v(\beta)},$$

with $\gamma\in\left[c_H^1,b_{US}^{11}\right]$ *and* $\beta\in\left[d_{US}^{11},b_{US}^{11}\right]$. *Assuming that* $v(.)$ *is a CARA function, we obtain that*

$$-\frac{v'(\gamma)}{v''(\gamma)}=-\frac{1}{\frac{v''(\beta)}{v(\beta)}}=K.$$

Then, the previous inequality can be rewritten:

$$\begin{array}{rcl}\frac{mv'(c_H^1)\left[1-\lambda\alpha_{US}^{11}\right]}{v'(c_H^1)-v'(b_{US}^{11})}+\lambda\alpha_{US}^{11}m & \geq & K\left[\frac{1}{p_H^1}+\lambda\alpha_{US}^{11}m\frac{1}{K}\right]\\ & \Leftrightarrow & \\ \frac{mv'(c_H^1)\left[1-\lambda\alpha_{US}^{11}\right]}{v'(c_H^1)-v'(b_{US}^{11})} & \geq & \frac{K}{p_H^1}.\end{array}$$

Again, the mean value theorem implies that

$$v'(c_H^1)-v'(b_{US}^{11})=v''(\zeta)\left(p_H^1-\alpha_S^{11}p_U^1\right)m,$$

with $\zeta \in \left[c_H^1, b_{US}^{11}\right]$. *Due to the concavity of* $v(.)$, *we have* $v'(c_H^1) \geq v'(\zeta)$. *Then, a sufficient to ensure the previous inequality is:*

$$
\begin{aligned}
-\frac{mv'(\zeta)\left[1-\lambda\alpha_{US}^{11}\right]}{v''(\zeta)\left(p_H^1-\alpha_S^{11}p_U^1\right)m} &\geq \frac{K}{p_H^1} \\
&\Leftrightarrow \\
p_H^1\left[1-\lambda\alpha_S^{11}v'(b_{US}^{11})\right] &\geq \left(p_H^1-\alpha_S^{11}p_U^1\right). \\
&\Leftrightarrow \\
\lambda p_H^1 &\leq p_U^1
\end{aligned}
$$

which is always satisfied. ■

We are now in a position to prove formally **Proposition 3**. We first prove that the equilibrium is S^{11} for $\phi < \phi_{\min}^S$ and S^{01} for $\phi_{\min}^S < \phi < \phi_{\max}^S$ (**Parts (i)(a) and (ii)(a) of Proposition 3**). We proceed in three steps: (1) Type U has larger utility with contract with effort (as determined by IC^{11}) than without (as determined by IC^{01}) when $\phi = 0$; (2) Both utilities continuously decrease with ϕ, but the utility with effort decreases faster; (3) Type U has a larger utility without effort (IC^{01}) than with (IC^{11}) when $\phi = \phi_{\max}^S$.

1) **Type U has larger utility with contract with effort (as determined by IC^{11}) than without (as determined by IC^{01}) when $\phi = 0$.**

Taken at $\phi = 0$, IC^{11} and IC^{00} imply that

$$
\begin{aligned}
& p_H^1 v\left(y+\alpha_S^{11}\left(1-p_U^1\right)m-m\right)+\left(1-p_H^1\right)v\left(y-\alpha_S^{11}p_U^1 m\right) \\
= \; & p_H^0 v\left(y+\alpha_S^{01}\left(1-p_U^0\right)m-m\right)+\left(1-p_H^0\right)v\left(y-\alpha_S^{01}p_U^0 m\right),
\end{aligned}
$$

which is equivalent to

$$
\begin{aligned}
& p_H^1\left[v(d_{US}^{11})-v(d_{US}^{01})\right]+\left(p_H^1-p_H^0\right)v(d_{US}^{01}) \\
= \; & v(b_{US}^{01})-v(b_{US}^{11})+p_H^1\left[v(b_{US}^{11})-v(b_{US}^{01})\right]+v(b_{US}^{01})\left(p_H^1-p_H^0\right) \\
\Leftrightarrow & \\
& p_H^1\left[v(d_{US}^{11})-v(d_{US}^{01})\right]+\left(1-p_H^1\right)\left[v(b_{US}^{11})-v(b_{US}^{01})\right] \\
= \; & \left(p_H^1-p_H^0\right)\left[v(b_{US}^{01})-v(d_{US}^{01})\right]. \qquad (11)
\end{aligned}
$$

We need to show that

$$
p_U^1 v(d_{US}^{11})+(1-p_U^1)v(b_{US}^{11}) \geq p_U^0 v(d_{US}^{01})+(1-p_U^0)v(b_{US}^{01}).
$$

This inequality is equivalent to

$$\begin{aligned} & p_U^1 \left[v(d_{US}^{11}) - v(d_{US}^{01})\right] + \lambda \left(p_H^1 - p_H^0\right) v(d_{US}^{01}) \\ \geq\ & v(b_{US}^{01}) - v(b_{US}^{11}) + p_U^1 \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] + \lambda \left(p_H^1 - p_H^0\right) v(b_{US}^{01}) \\ \Leftrightarrow & \\ & \lambda p_H^1 \left[v(d_{US}^{11}) - v(d_{US}^{01})\right] + \left(1 - p_U^1\right) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] \\ \geq\ & \lambda \left(p_H^1 - p_H^0\right) \left[v(b_{US}^{01}) - v(d_{US}^{01})\right]. \end{aligned}$$

Multiplying both sides of (11) by λ, we obtain

$$\begin{aligned} & \lambda p_H^1 \left[v(d_{US}^{11}) - v(d_{US}^{01})\right] - \lambda \left(p_H^1 - p_H^0\right) \left[v(b_{US}^{01}) - v(d_{US}^{01})\right] \\ =\ & \lambda(1 - p_H^1) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right]. \end{aligned}$$

Then, the previous inequality can be rewritten:

$$\begin{aligned} \lambda(1 - p_H^1) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] + \left(1 - p_U^1\right) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] & \geq 0 \\ & \Leftrightarrow \\ (1 - \lambda) (1 - p_L) \left[v(b_{US}^{11}) - v(b_{US}^{01})\right] & \geq 0, \end{aligned}$$

which is true. Q.E.D.

2) **Both utilities continuously decrease with ϕ, but the utility with effort decreases faster.**

By definition, $V_{US}^{11} = V_{US}^{01}$ when $\phi = \phi_{\min}^S$. We now show that

$$\frac{\partial V_{US}^{11}}{\partial \phi} < \frac{\partial V_{US}^{01}}{\partial \phi}, \tag{12}$$

so that $V_{US}^{11} > V_{US}^{01}$ for $\phi < \phi_{\min}^S$ and $V_{US}^{11} < V_{US}^{01}$ for $\phi > \phi_{\min}^S$.

As α_S^{11} is constant with respect to ϕ, the inequality (12) is equivalent to:

$$\begin{aligned} -m \frac{\partial \alpha_S^{01}}{\partial \phi} p_U^0 \left(1 - p_U^0\right) \left[v'(d_{US}^{01}) - v'(b_{US}^{01})\right] & \leq 1 \\ & \Longleftrightarrow \\ \frac{m p_U^0 \left(1 - p_U^0\right) \left[v'(d_{US}^{01}) - v'(b_{US}^{01})\right]}{m \left[p_H^0 \left(1 - p_U^0\right) v'(d_{US}^{01}) - \left(1 - p_H^0\right) p_U^0 v'(b_{US}^{01})\right]} & \leq 1 \\ & \Longleftrightarrow \\ \left[p_U^0 - p_H^0\right] p_U^0 v'(b_{US}^{01}) & \leq v'(d_{US}^{01}) \left(1 - p_U^0\right) \left[p_H^0 - p_U^0\right], \end{aligned}$$

which is always true.

To prove that $\phi_{\min}^S$ exists and is such that $0 < \phi_{\min}^S < \phi_{\max}^S$, we must show that $V_{US}^{11} > V_{US}^{01}$ for $\phi = 0$ while $V_{US}^{11} < V_{US}^{01}$ for $\phi = \phi_{\max}^S$. Q.E.D.

3) **Type U has a larger utility without effort (IC^{01}) than with (IC^{11}) when $\phi = \phi_{\max}^S$.**

Formally, we need to show that for $\phi = \phi_{\max}^S$:

$$p_U^1 v(d_{US}^{11}) + (1 - p_U^1) v(b_{US}^{11}) - \phi \leq p_U^0 v(d_{US}^{01}) + (1 - p_U^0) v(b_{US}^{01}).$$

Since $\phi = \phi_{\max}^S$, IC^{01} can be rewritten

$$v(c_H^0) = p_H^0 v(d_{US}^{01}) + (1 - p_H^0) v(b_{US}^{01}).$$

Then, taken at $\phi = \phi_{\max}^S$, the previous inequality yields:

$$p_U^1 v(d_{US}^{11}) + (1 - p_U^1) v(b_{US}^{11}) - v(c_H^1) \leq p_U^0 v(d_{US}^{01}) + (1 - p_U^0) v(b_{US}^{01}) - v(c_H^0)$$
$$\Longleftrightarrow$$
$$\begin{aligned} & p_U^1 v(d_{US}^{11}) + (1 - p_U^1) v(b_{US}^{11}) - \left[p_H^1 v(d_{US}^{11}) + (1 - p_H^1) v(b_{US}^{11})\right] \\ \leq\; & p_U^0 v(d_{US}^{01}) + (1 - p_U^0) v(b_{US}^{01}) - \left[p_H^0 v(d_{US}^{01}) + (1 - p_H^0) v(b_{US}^{01})\right] \end{aligned}$$
$$\Longleftrightarrow$$
$$\left(p_H^1 - p_L\right)\left[v(b_{US}^{11}) - v(d_{US}^{11})\right] \leq \left(p_H^0 - p_L\right)\left[v(b_{US}^{01}) - v(d_{US}^{01})\right].$$

As $p_H^0 > p_H^1$, $\alpha_S^{01} \leq \alpha_S^{11}$ (see Lemma 1 (d)) so that the previous inequality holds. Q.E.D.

Part (i) (b) of Proposition 3 is proved in Lemma 1 (a) and (b).

Part (ii) (b) of Proposition 3

We need to prove that $\alpha_S^{01} < 1$ for $\phi = \phi_{\min}^S$. The incentive condition that determines α_S^{01} is:

$$v(y - p_H^1 m) - \phi = p_H^0 v(d_S^{01}) + (1 - p_H^0) v(b_S^{01}).$$

Substituting ϕ by $\phi_{\min}^S$ yields:

$$\begin{aligned} & v(y - p_H^1 m) - \left[p_U^1 v(d_S^{11}) + (1 - p_U^1) v(b_S^{11}) - \left(p_H^0 v(d_S^{01}) + (1 - p_H^0) v(b_S^{01})\right)\right] \\ =\; & p_H^0 v(d_S^{01}) + (1 - p_H^0) v(b_S^{01}). \end{aligned}$$

Introducing IC^{11} gives:

$$\left(p_H^1 - p_L\right)\left[v(d_S^{11}) - v(b_S^{11})\right] = \left(p_H^0 - p_L\right)\left[v(d_S^{01}) - v(b_S^{01})\right].$$

Since $\alpha_S^{11} \leq 1$, we also have that $\alpha_S^{01} \leq 1$.

Part (ii) (c) of Proposition 3 is proved in Lemma 1 (c).

Part (iii) of Proposition 3: α_S^{11} and α_S^{01} are respectively determined by IC^{11} and IC^{01}, respectively. Combining these two conditions yields:

$$\begin{aligned} & p_H^0 v\left(d_{US}^{01}\right) + \left(1 - p_H^0\right) v\left(b_{US}^{01}\right) \\ = \; & p_H^1 v(d_{US}^{11}) + \left(1 - p_H^1\right) v\left(b_{US}^{11}\right) - \phi. \end{aligned}$$

Using $\phi = \phi_{\min}^S$, we obtain:

$$\begin{aligned} & p_H^0 v\left(d_{US}^{01}\right) + \left(1 - p_H^0\right) v\left(b_{US}^{01}\right) \\ & - \left[p_U^0 v\left(d_{US}^{01}\right) + \left(1 - p_U^0\right) v\left(b_{US}^{01}\right)\right] \\ = \; & p_H^1 v\left(d_{US}^{11}\right) + \left(1 - p_H^1\right) v\left(b_{US}^{11}\right) \\ & - \left(p_U^1 v\left(d_{US}^{11}\right) + \left(1 - p_U^1\right) v\left(b_{US}^{11}\right)\right). \end{aligned} \tag{13}$$

Let us consider the following function:

$$\begin{aligned} \Gamma(\alpha, \delta) \;=\; & \left(p_H^0 - \delta\right) v\left(y + \alpha\left(1 - \left(\left(\lambda\left(p_H^0 - \delta\right) + (1-\lambda)p_L\right)\right)\right) d - d\right) \\ & + \left(1 - \left(p_H^0 - \delta\right)\right) v\left(y - \alpha\left(\left(\lambda\left(p_H^0 - \delta\right) + (1-\lambda)p_L\right)\right) d\right) \\ & - \left(\lambda\left(p_H^0 - \delta\right) + (1-\lambda)p_L\right) v\left(y + \alpha\left(1 - \left(\left(\lambda\left(p_H^0 - \delta\right) + (1-\lambda)p_L\right)\right)\right) d - d\right) \\ & - \left(\left(1 - \left(\left(\lambda\left(p_H^0 - \delta\right) + (1-\lambda)p_L\right)\right)\right) v\left(y - \alpha\left(\left(\lambda\left(p_H^0 - \delta\right) + (1-\lambda)p_L\right)\right) d\right)\right). \end{aligned}$$

The equation (13) can then be rewritten as

$$\Gamma\left(\alpha_S^{01}, 0\right) = \Gamma\left(\alpha_S^{11}, \Delta\right).$$

Now, let us apply the implicit function theorem. We obtain:

$$\begin{aligned} \frac{d\alpha}{d\delta} \;=\; & -\frac{\partial\Gamma/\partial\delta}{\partial\Gamma/\partial\alpha} \\ =\; & -\frac{v(b_{US}^{11}) - v(d_{US}^{11}) + \alpha\lambda d\left(p_H^1 v'(d_{US}^{11}) + \left(1 - p_H^1\right) v'(b_{US}^{11})\right)}{d\left[p_H^1\left(1 - p_U^1\right) v'\left(d_{US}^{11}\right) - \left(1 - p_H^1\right) p_U^1 v'\left(b_{US}^{11}\right) - \left[p_U^1\left(1 - p_U^1\right)\left(v'\left(d_{US}^{11}\right) - v'\left(b_{US}^{11}\right)\right)\right]\right]} \\ & + \frac{\left[\lambda\left(v(b_{US}^{11}) - v(d_{US}^{11})\right)\right] + \alpha\lambda d\left(p_U^1 v'(d_{US}^{11}) - \left(1 - p_U^1\right) v'(b_{US}^{11})\right)}{d\left[p_H^1\left(1 - p_U^1\right) v'\left(d_{US}^{11}\right) - \left(1 - p_H^1\right) p_U^1 v'\left(b_{US}^{11}\right) - \left[p_U^1\left(1 - p_U^1\right)\left(v'\left(d_{US}^{11}\right) - v'\left(b_{US}^{11}\right)\right)\right]\right]} \\ =\; & -\frac{\left[v(b_{US}^{11}) - v(d_{US}^{11})\right](1 - \lambda) + \alpha\lambda d\left(p_H^1 - p_U^1\right)\left[\left(v'(d_{US}^{11}) - v'(b_{US}^{11})\right)\right]}{d\left(p_H^1 - p_U^1\right)\left[\left(1 - p_U^1\right) v'\left(d_{US}^{11}\right) + p_U^1 v'\left(b_{US}^{11}\right)\right]} < 0. \end{aligned}$$

Consequently, we have $\alpha_S^{11} < \alpha_S^{01}$ for $\phi = \phi_{\min}^S$.

Part (iv) of Proposition 3

IC^{00} is identical to IC^{01} when $\phi = \phi^S_{\max}$, and from Lemma 1 (d) IC^{00} binds with equality with $\alpha^{00}_S < 1$, where α^{00}_S takes the same value as α^{01}_S when $\phi = \phi^S_{\max}$. The coverage rate α^{00}_S is constant with $\phi \geq \phi^S_{\max}$ since ϕ does not appear in IC^{00}. Hence, V^{00}_{US} is constant with ϕ as long as $\phi \geq \phi^S_{\max}$.

IC^{10} is identical to IC^{11} when $\phi = \phi^S_{\max}$, and from Lemma 1 (d) IC^{10} binds with equality with $\alpha^{10}_S < 1$, where α^{10}_S takes the same value as α^{11}_S when $\phi = \phi^S_{\max}$. We know from the proof of part (ii) (a) above that $V^{00}_{US} > V^{10}_{US}$ when $\phi = \phi^S_{\max}$. We now prove that this inequality remains true for any $\phi > \phi^S_{\max}$, because V^{10}_{US} decreases with ϕ.
Using the implicit function theorem on IC^{10}, we obtain that α^{10}_S increases with ϕ. As ϕ increases, the first term in the RHS of IC^{10} then increases with ϕ, while the second term decreases. Comparing the RHS of IC^{10} with the definition of V^{10}_{US} (see equation (6)), we see that they only differ in the weight put on the two first terms. As $p^1_U < p^1_H$, it is easy to see that V^{10}_{US} decreases with ϕ.
Note that, when ϕ becomes large enough, we may obtain that IC^{10} holds with a strict inequality even with $\alpha^{10}_S = 1$. In that case, increasing further ϕ has no impact on α^{10}_S and thus also decreases V^{10}_{US}. Q.E.D.

Appendix A.4. Proof of Proposition 4

We first prove that, when H exerts an effort at equilibrium (*i.e.*, with both S^{01} and S^{11}), welfare decreases with ϕ and increases with k.

When H makes an effort, the (utilitarian) welfare function is given by:

$$\begin{aligned} W^{i1}_S &= (1-k)V^{i1}_{US} + k\lambda V^1_{HS} + k(1-\lambda)V_L \\ &= k\left[(1-\lambda)V_L + \lambda V^1_{HS} - V^{i1}_{US}\right] + V^{i1}_{US}, \end{aligned} \quad (14)$$

$i = \{0, 1\}$, where utility levels for types L, U and H are given, respectively, by V_L (see equation (5)), V^{ij}_{US} (see equation (6)), and V^j_{HS} (see equation (3)).

With S^{11}, the derivatives with respect to ϕ and k are respectively:

$$\begin{aligned} \frac{\partial W^{11}_S}{\partial \phi} &= -[1 - k(1-\lambda)] < 0, \\ \frac{\partial W^{11}_S}{\partial k} &= \lambda V^1_{HS} + (1-\lambda)V_L - V^{11}_{US}. \end{aligned}$$

Using the definition of α_S^{11}, we have:

$$\begin{aligned}
&\frac{\partial W_S^{11}}{\partial k} \\
=\ & \lambda\left[p_H^1 v(d_{US}^{11}) + (1-p_H^1)v(b_{US}^{11})\right] + (1-\lambda)v(y-p_Lm) - \left[p_U^1 v(d_{US}^{11}) + (1-p_U^1)v(b_{US}^{11})\right] + (1-\lambda)\,\phi \\
=\ & v(d_{US}^{11})(\lambda p_H^1 - p_U^1) + v(b_{US}^{11})(\lambda(1-p_H^1) - (1-p_U^1)) + (1-\lambda)v(y-p_Lm) + (1-\lambda)\,\phi \\
=\ & (1-\lambda)\left(v(y-p_Lm)+\phi\right) - (1-\lambda)p_L v(d_{US}^{11}) - (1-\lambda)(1-p_L)v(b_{US}^{11}) \\
=\ & (1-\lambda)\left[v(y-p_Lm) + \phi - \left(p_L v(d_{US}^{11}) + (1-p_L)v(b_{US}^{11})\right)\right] > 0.
\end{aligned}$$

With S^{01}, the derivatives with respect to ϕ and k are respectively:

$$\begin{aligned}
\frac{\partial W_S^{01}}{\partial \phi} &= -k\lambda + (1-k)p_U^0(1-p_U^0)m\frac{d\alpha_S^{01}}{d\phi}\left(v'(d_{US}^{01}) - v'(b_{US}^{01})\right) < 0, \\
\frac{\partial W_S^{01}}{\partial k} &= \lambda V_{HS}^1 + (1-\lambda)V_L - V_{US}^{01} - \lambda\phi > 0.
\end{aligned}$$

Using the definition of α_S^{01}, we have:

$$\begin{aligned}
\frac{\partial W_S^{01}}{\partial k} &= \lambda\left[p_H^0 v(d_{US}^{01}) + (1-p_H^0)v(b_{US}^{01})\right] + (1-\lambda)v(y-p_Lm) - \left[p_U^0 v(d_{US}^{01}) + (1-p_U^0)v(b_{US}^{01})\right] \\
&= v(d_U^0)(\lambda p_H^0 - p_U^0) + v(b_{US}^{01})(\lambda(1-p_H^0) - (1-p_U^0)) + (1-\lambda)v(y-p_Lm) \\
&= (1-\lambda)v(y-p_Lm) - (1-\lambda)p_L v(d_{US}^{01}) - (1-\lambda)(1-p_L)v(b_{US}^{01}) \\
&= (1-\lambda)\left[v(y-p_Lm) - \left(p_L v(d_{US}^{01}) + (1-p_L)v(b_{US}^{01})\right)\right] > 0.
\end{aligned}$$

We now prove that, when H makes no effort (*i.e.*, with equilibrium contracts S^{00}), welfare is not affected by ϕ, and increases with k.

With S^{00}, utility levels for types U and H are given, respectively, by V_{US}^{00} (see equation (6)) and V_{HS}^0 (see equation (3)). Utilities as well as the coverage level α_S^{00} do not depend on ϕ nor on k. The welfare function becomes:

$$W_S^{00} = (1-k)V_{US}^{00} + k\lambda V_{HS}^0 + k(1-\lambda)V_L,$$

and we obtain:

$$\frac{\partial W_S^{00}}{\partial k} = \lambda V_{HS}^0 + (1-\lambda)V_L - V_{US}^{00}.$$

Using the definition of α_S^{00}, we have

$$\frac{\partial W_S^{00}}{\partial k} = \lambda\left[p_H^0 v(d_{US}^{00}) + (1-p_H^0)v(b_{US}^{00})\right] + (1-\lambda)v(y-p_Lm) - \left[p_U^0 v(d_{US}^{00}) + (1-p_U^0)v(b_{US}^{00})\right],$$

so that the rest of the proof is identical to the proof for S^{01}, with an appropriate change of indices.

Appendix A.5.: Proof of Proposition 5

The following lemma will prove helpful in proving Proposition 5.

Lemma 2 *Under Assumption 2, $\alpha_P^1 = 0$ for k close enough to one.*

Proof. The coverage rate of the pooling contract P^1 is determined by

$$p_U^1 \left(1 - p_P^1\right) v'(d_P^1) - \left(1 - p_U^1\right) p_P^1 v'(b_P^1) \leq 0.$$

When $k = 1$, this condition becomes:

$$p_U^1 \left(1 - p_H^1\right) v'(d_P^1) - \left(1 - p_U^1\right) p_H^1 v'(b_P^1) \leq 0,$$

which is satisfied with a strict inequality for $\alpha_P^1 = 0$ under Assumption 2, so that, given Proposition 1 (v), $\alpha_P^1 \to 0$ for $k \to 1$. ■

We now prove Proposition 5 in 5 steps.

(1) Whatever the value of ϕ, the equilibrium contract is either the pooling one preferred by U (either P^0 or P^1) or the separating one preferred by U (S^{00}, S^{01} or S^{11}).

(2) The utility of U with a pooling equilibrium is strictly decreasing in k, while its utility with any separating equilibrium is independent of k.

(3) When k tends towards 0, the pooling equilibrium tends to full coverage with actuarially fair price. This is the highest utility type U can get. At the same time, the separating contract proposes either partial coverage and/or a distortion of the prevention decision. Hence, the utility of U is higher with pooling.

(4) As k tends towards 1, the price of the pooling contract increases and reflects type H's risk, which is higher than the price of the separating contract. Lemma 2 and Proposition 1 (iii) establish that, when k is large enough, we have that $\alpha_P^1 = \alpha_P^0 = 0$. We then have that U prefers the separating contract with some coverage to (the pooling contract with) no insurance.

(5) Given (2), we have the unique, strictly positive, threshold $\tilde{k}(\phi)$.

Appendix A.6.: Proof of Proposition 6

Proof. (a) The utility of U under P^0 and under S^{00} is not affected by ϕ, which then plays no role in the comparison of utilities. The utility of U under P^1 and under S^{11} is affected linearly by the value of ϕ (because the coverage rate is not affected by ϕ in P^1 nor in S^{11}), so that the ϕ term cancels out when comparing the two utilities, and ϕ plays no role in determining the value of $\tilde{k}(\phi)$.

(b) The utility of U under P^0 is not affected by ϕ, while its utility under S^{01} decreases

with ϕ (because the coverage rate decreases with ϕ). Using the implicit function theorem, we obtain that $\tilde{k}(\phi)$ increases with ϕ.
(c) The threshold $\tilde{k}$ is determined by the equality $V_{UP}^{1}(\tilde{k},\phi) = V_{US}^{01}(\phi)$, which corresponds to

$$\begin{aligned} & p_U^1 v(d_P^1) + (1-p_U^1)v(b_P^1) - \phi \\ = \; & p_U^0 v\left(y + \alpha_S^{01}\left(1-p_U^0\right)m - m\right) + \left(1-p_U^0\right)v\left(y - \alpha_S^{01} p_U^0 m\right), \end{aligned}$$

with α_P^1 and α_S^{01} respectively given by:

$$p_U^1(1-p_P^1)v'(d_P^1) - (1-p_U^1)p_P^1 v'(b_P^1) = 0$$

and

$$v(y - p_H^1 m) - \phi = p_U^0 v\left(y + \alpha_S^{01}\left(1-p_U^0\right)m - m\right) + \left(1-p_U^0\right)v\left(y - \alpha_S^{01} p_U^0 m\right).$$

Static comparatives yield:

$$\begin{aligned} \frac{d\alpha_S^{01}}{d\phi} &= \frac{-1}{m\left[p_H^0\left(1-p_U^0\right)v'(d_{US}^{01}) - \left(1-p_H^0\right)p_U^0 v'(b_{US}^{01})\right]} < 0, \\ \frac{d\alpha_P^1}{dk} &= \frac{(1-\lambda)\lambda\left(p_H^1 - p_L\right)}{1-k(1-\lambda)} \frac{p_U^1 v'(d_P^1) + (1-p_U^1)v'(b_P^1)}{m\left[p_U^1(1-p_P^1)^2 v''(d_P^1) + (1-p_U^1)\left(p_P^1\right)^2 v''(b_P^1)\right]}. \end{aligned}$$

Consider the following implicit function:

$$\begin{aligned} \Gamma\left(\tilde{k},\phi\right) &= p_U^1 v(d_P^1) + (1-p_U^1)v(b_P^1) - \phi \\ &\quad - \left[p_U^0 v\left(y + \alpha_S^{01}\left(1-p_U^0\right)m - m\right) + \left(1-p_U^0\right)v\left(y - \alpha_S^{01} p_U^0 m\right)\right]. \end{aligned}$$

The implicit function theorem yields:

$$\frac{d\tilde{k}}{d\phi} = -\frac{\partial\Gamma/\partial\phi}{\partial\Gamma/\partial\tilde{k}}.$$

The numerator gives:

$$\frac{\partial\Gamma}{\partial\phi} = -1 + \frac{p_U^0\left(1-p_U^0\right)\left(v'(d_{US}^{01}) - v'(b_{US}^{01})\right)}{p_H^0\left(1-p_U^0\right)v'(d_{US}^{01}) - \left(1-p_H^0\right)p_U^0 v'(b_{US}^{01})} < 0.$$

Regarding the denominator, we have:

$$\frac{\partial\Gamma}{\partial\tilde{k}} = \frac{-\alpha_P^1\lambda\left(p_H^1 - p_L\right)}{1-k(1-\lambda)}\left[p_U^1 v'(d_P^1) + (1-p_U^1)v'(b_P^1)\right] < 0.$$

We then obtain that $d\tilde{k}/d\phi < 0$. ■

Appendix A.7.: Proof of Proposition 7

Before proving the proposition, we introduce additional notation that will be helpful in this Appendix and the next. We denote by (k^*, ϕ^*) the unique (in Figure 1) intersection between $\tilde{\phi}_P(k)$ and $\tilde{k}(\phi)$. We denote by $\tilde{k}_{00}$ (resp., $\tilde{k}_{11}$) the value of $\tilde{k}(\phi)$ when $\phi > \phi^S_{\max}$, (resp., when $\phi < \phi^S_{\min}$). Both $\tilde{k}_{00}$ and $\tilde{k}_{11}$ are independent of ϕ (provided of course that ϕ is in the relevant range) as shown in Proposition 6(a). From Figure 1, we obtain that $k^* < \tilde{k}_{00} < \tilde{k}_{11}$.

We now prove Proposition 7.

Proof. We know from Propositions 2 and 4 that a lower value of ϕ either has no impact on types U's and H's utilities, and thus on aggregate welfare (in cases P^0 and S^{00}), or that it increases types U's and H's utilities, and thus aggregate welfare (in cases P^1, S^{01} and S^{11}).

Note that, by definition of an equilibrium (which maximizes the utility of type U, given the relevant constraints), the utility of type U is continuous with ϕ as one moves from one equilibrium type to another. We then concentrate on how type H's utility (and aggregate welfare) is impacted by ϕ as one moves from one equilibrium contract type to another.

We first establish that H's utility remains continuous as one moves from one separating equilibrium to another: there is no discontinuity in type H's utility as (a) one moves from a S^{01} to a S^{00} equilibrium contract, by definition of $\phi^S_{\max}$ (see equation (4)), and (b) as one moves from S^{11} to S^{01}, since H's utility does not depend on the effort level required from type U (see equation (3)). Hence, aggregate welfare is also continuous for these moves.

We have discontinuous increases in H's utility as one moves from P^0 to P^1 (see Result 1 in Appendix A.8.1.), from S^{01} to P^0 (Result 2 in Appendix A.8.2.) and from S^{01} to P^1 (Result 3 in Appendix A.8.3.). In these cases, we also have a discontinuous increase in aggregate welfare. ■

Appendix A.8.: Proof of Proposition 8

We know from Proposition 5 that a low value of k is associated with a pooling equilibrium, and from Proposition 2 that types U's and H's utilities decrease with k, while the impact on aggregate welfare is ambiguous. As k reaches $\tilde{k}(\phi)$, we move from pooling to separating equilibrium, each time resulting in a downward discontinuity in H's utility (and thus in aggregate welfare): (i) from P^0 to S^{00} when $\phi > \phi^S_{\max}$ (see Result 4 in the Appendix A.8.4. below), from P^0 to S^{01} (when $\phi^* < \phi < \phi^S_{\max}$) (see Result 2 in Appendix A.8.2.), from P^1 to S^{01} (when $\phi^S_{\min} < \phi < \tilde{\phi}_P(0)$ and when Assumption 3 holds) (see Result 3 in Appendix A.8.3) and from P^1 to S^{11} (when $\phi < \phi^S_{\min}$) (see Result

5 in Appendix A.8.5).[17] In the case where $\tilde{\phi}_P(0) < \phi < \phi^*$, the equilibrium first changes from P^0 to P^1 (with an upward jump in H's utility and in aggregate welfare, see Result 1 in Appendix A.8.1) and then from P^1 to S^{01}. Finally, as k further increases, we remain in a separating equilibrium (see Proposition 5), where utilities are not affected by k, but where a larger value of k increases aggregate welfare thanks to a composition effect (see Proposition 4).

Appendix A.8.1.: Result 1

Result 1 *There is an upward discontinuity in H's utility from P^0 to P^1 when U is indifferent between the two.*

Proof. We have to prove that

$$V^0_{UP} = V^1_{UP} \Rightarrow V^0_{HP} < V^1_{HP}.$$

We have:

$$\begin{aligned} V^0_{UP} &= V^1_{UP} \\ &\Leftrightarrow p^1_U v(d^1_P) + (1-p^1_U)v(b^1_P) - \phi = p^0_U v(d^0_P) + (1-p^0_U)v(b^0_P) \\ &\Leftrightarrow \phi = p^1_U v(d^1_P) - p^0_U v(d^0_P) + (1-p^1_U)v(b^1_P) - (1-p^0_U)v(b^0_P). \end{aligned}$$

Proving that

$$V^0_{HP} < V^1_{HP}$$

is then equivalent to proving that

$$\begin{aligned} p^0_H v(d^0_P) + (1-p^0_H)v(b^0_P) \quad < \quad & p^1_H v(d^1_P) + (1-p^1_H)v(b^1_P) \\ & -p^1_U v(d^1_P) - (1-p^1_U)v(b^1_P) \\ & +p^0_U v(d^0_P) + (1-p^0_U)v(b^0_P). \end{aligned}$$

Regrouping terms, this is equivalent to

$$\left(p^0_H - p^0_U\right)\left[v(b^0_P) - v(d^0_P)\right] > \left(p^1_H - p^1_U\right)\left[v(b^1_P) - v(d^1_P)\right],$$

which holds if

$$v(b^0_P) - v(d^0_P) > v(b^1_P) - v(d^1_P), \tag{15}$$

since

$$p^0_H - p^0_U = (1-\lambda)(p^0_H - p_L) > p^1_H - p^1_U = (1-\lambda)(p^1_H - p_L).$$

[17] As in Proposition 7, Assumption 3 is a sufficient condition to sign the discontinuity in H's utility (and thus in aggregate welfare) when moving from P^1 to S^{01}.

Note that, thanks to the concavity of $v(.)$, inequality (15) holds if $d_P^0 > d_P^1$ together with $b_P^0 - d_P^0 > b_P^1 - d_P^1$. We have that

$$b_P^0 - d_P^0 = d\left(1 - \alpha_P^0 m\right) > b_P^1 - d_P^1 = m\left(1 - \alpha_P^1 m\right),$$

since $\alpha_P^0 < \alpha_P^1$ (see Proposition 1 (iii)), and that $d_P^0 > d_P^1$ since $p_P^1 < p_P^0$. ■

Appendix A.8.2.: Result 2

We first have to prove the following lemma.

Lemma 3 *We have that $\alpha_P^0 > \alpha_S^{01}$ when $V_{UP}^0 = V_{US}^{01}$ when Assumption 3 holds.*

Proof. *Note that α_S^{01} is not affected by k, while α_P^0 decreases with k, starting from $\alpha_P^0 = 1$ when $k = 0$ (since in that case $p_P^0 = p_U^0$). Assumption 3 implies that $p_H^1 > p_U^0$, which in turn implies that $\alpha_S^{01} < 1$ (since IC^{01} is violated when $\alpha_S^{01} = 1$). We know that V_{UP}^0 is decreasing in k while V_{US}^{01} is not affected, so that there is a unique value of k such that $V_{UP}^0 = V_{US}^{01}$. It is easy to see that $\alpha_P^0 = \alpha_S^{01}$ implies that $V_{UP}^0 < V_{US}^{01}$ (since $p_P^0 > p_U^0$ implies that $d_P^0 < d_S^{01}$ and that $b_P^0 < b_S^{01}$). We then have that $V_{UP}^0 = V_{US}^{01}$ implies that $\alpha_P^0 > \alpha_S^{01}$.*

■

Result 2 *Under Assumption 3, there is a downward discontinuity in H's utility from P^0 to S^{01} when U is indifferent between the two.*

Proof. To prove Result 2, we have to prove that

$$V_{UP}^0 = V_{US}^{01} \Rightarrow V_{HP}^0 > V_{HS}^1.$$

We have

$$\begin{aligned} V_{UP}^0 &= V_{US}^{01} \\ &\Leftrightarrow p_U^0 v(d_P^0) + (1 - p_U^0)v(b_P^0) = p_U^0 v(d_{US}^{01}) + (1 - p_U^0)v(b_{US}^{01}) \\ &\Leftrightarrow p_U^0 \left[v(d_P^0) - v(d_{US}^{01})\right] + (1 - p_U^0)\left[v(b_P^0) - v(b_{US}^{01})\right] = 0. \end{aligned} \tag{16}$$

Proving that

$$V_{HP}^0 > V_{HS}^1$$

is then equivalent to proving that

$$\begin{aligned} & p_H^0 v(d_P^0) + (1 - p_H^0)v(b_P^0) > p_H^0 v(d_{US}^{01}) + (1 - p_H^0)v(b_{US}^{01}) \\ \Leftrightarrow\ & p_H^0 \left[v(d_P^0) - v(d_{US}^{01})\right] + (1 - p_H^0)\left[v(b_P^0) - v(b_{US}^{01})\right] > 0. \end{aligned} \tag{17}$$

We know that $p_P^0 > p_U^0 > p_U^1$ and we know from Lemma 3 that $\alpha_P^0 > \alpha_S^{01}$ when $V_{UP}^0 = V_{US}^{01}$, which together imply that $b_P^0 < b_{US}^{01}$. From (16), we obtain that $d_P^0 > d_{US}^{01}$, so that (17) holds since $p_H^0 > p_U^0$. ■

Appendix A.8.3.: Result 3

We first prove the following lemma.

Lemma 4 *We have that $\alpha_P^1 > \alpha_S^{01}$ when $V_{UP}^1 = V_{US}^{01}$ when Assumption 3 holds.*

Proof. *By Assumption 3, the P^1 contract is more expensive than $S^{01}$for type U, and moreover entails that type U pays the effort cost in P^1 but not in S^{01}. For U to be indifferent, it must then be the case that P^1 offers more coverage than S^{01} (recall that the coverage level in P^1 is the most-preferred one of U, while U is rationed in S^{01} because of IC^{01}).* ■

Result 3 *Under Assumption 3, there is an upward discontinuity in H's utility from S^{01} to P^1 when U is indifferent between the two.*

Proof. We now have to prove that

$$V_{UP}^1 = V_{US}^{01} \Rightarrow V_{HP}^1 > V_{HS}^1.$$

We have

$$\begin{aligned} V_{UP}^1 &= V_{US}^{01} \\ &\Leftrightarrow \; p_U^1 v(d_P^1) + (1-p_U^1)v(b_P^1) - \phi = p_U^0 v(d_{US}^{01}) + (1-p_U^0)v(b_{US}^{01}) \qquad (18)\\ &\Leftrightarrow \; \phi = p_U^1 v(d_P^1) - p_U^0 v(d_{US}^{01}) + (1-p_U^1)v(b_P^1) - (1-p_U^0)v(b_{US}^{01}). \end{aligned}$$

Proving that

$$V_{HP}^1 > V_{HS}^1$$

is then equivalent to proving that

$$\begin{aligned} p_H^0 v(d_{US}^{01}) + (1-p_H^0)v(b_{US}^{01}) \;<\; & p_H^1 v(d_P^1) + (1-p_H^1)v(b_P^1) \\ & -p_U^1 v(d_P^1) - (1-p_U^1)v(b_P^1) \\ & +p_U^0 v(d_{US}^{01}) + (1-p_U^0)v(b_{US}^{01}). \end{aligned}$$

Regrouping terms, this is equivalent to

$$\begin{aligned} & \left(p_H^0 - p_U^0\right)\left[v(b_{US}^{01}) - v(d_{US}^{01})\right] > \left(p_H^1 - p_U^1\right)\left[v(b_P^1) - v(d_P^1)\right] \\ \Leftrightarrow \; & \left(p_H^0 - p_L\right)\left[v(b_{US}^{01}) - v(d_{US}^{01})\right] > \left(p_H^1 - p_L\right)\left[v(b_P^1) - v(d_P^1)\right] \qquad (19) \end{aligned}$$

since

$$p_H^0 - p_U^0 = (1-\lambda)(p_H^0 - p_L) > p_H^1 - p_U^1 = (1-\lambda)(p_H^1 - p_L).$$

Note that $\alpha_P^1 > \alpha_S^{01}$ together with $p_P^1 > p_U^0$ imply that $b_P^1 < b_{US}^{01}$. If $d_P^1 > d_{US}^{01}$, then (19) is satisfied.
We now show that (19) is also satisfied in the case where $d_P^1 < d_{US}^{01}$. Note that, if (18) is satisfied for some $\phi > 0$, then we have that

$$\begin{aligned}
& p_U^1 v(d_P^1) + (1 - p_U^1) v(b_P^1) > p_U^0 v(d_{US}^{01}) + (1 - p_U^0) v(b_{US}^{01}) \\
\Leftrightarrow \quad & \left(\lambda p_H^1 + (1-\lambda) p_L\right) v(d_P^1) + \left(1 - \left(\lambda p_H^1 + (1-\lambda) p_L\right)\right) v(b_P^1) > \\
& \left(\lambda p_H^0 + (1-\lambda) p_L\right) v(d_{US}^{01}) + \left(1 - \left(\lambda p_H^0 + (1-\lambda) p_L\right)\right) v(b_{US}^{01}) \\
\Leftrightarrow \quad & p_L v(d_P^1) + \lambda \left(p_H^1 - p_L\right) v(d_P^1) + (1 - p_L) v(b_P^1) - \lambda \left(p_H^1 - p_L\right) v(b_P^1) > \\
& p_L v(d_{US}^{01}) + \lambda \left(p_H^0 - p_L\right) v(d_{US}^{01}) + (1 - p_L) v(b_{US}^{01}) - \lambda \left(p_H^0 - p_L\right) v(b_{US}^{01}) \\
\Leftrightarrow \quad & \lambda \left[\left(p_H^1 - p_L\right)\left(v(b_P^1) - v(d_P^1)\right) - \left(p_H^0 - p_L\right)\left(v(b_{US}^{01}) - v(d_{US}^{01})\right)\right] < \qquad (20) \\
& p_L v(d_P^1) + (1 - p_L) v(b_P^1) - \left[p_L v(d_{US}^{01}) + (1 - p_L) v(b_{US}^{01})\right].
\end{aligned}$$

If $d_P^1 < d_{US}^{01}$, then the RHS of (20) is negative, and so is its LHS, so that (19) is also satisfied. Q.E.D. ■

Appendix A.8.4.: Result 4

We first prove the following lemma:

Lemma 5 *We have that $\alpha_P^0 > \alpha_S^{00}$ when $V_{UP}^0 = V_{US}^{00}$.*

Proof. *Observe first that $\alpha_S^{00} < 1$ (this is already in Proposition 3(iv), and is easy to establish since the effort level is the same –nil– for H and U in that case, so that the only way to prevent H from mimicking U is by under-providing insurance to U) while $\alpha_P^0 = 1$ when $k = 0$ (since in that case $p_P^0 = p_U^0$). We know that α_P^0 is decreasing in k while α_S^{00} is not affected by k, and that V_{UP}^0 is decreasing in k while V_{US}^{00} is not affected, so that there is a unique value of k such that $V_{UP}^0 = V_{US}^{00}$. It is easy to see that $\alpha_P^0 = \alpha_S^{00}$ implies that $V_{UP}^0 < V_{US}^{00}$ (since $p_P^0 < p_U^0$ implies that $d_P^0 < d_S^{00}$ and that $b_P^0 < b_S^{00}$). We then have that $V_U^P = V_{US}^{00}$ implies that $\alpha_P^0 > \alpha_S^{00}$.* ■

We now prove the following:

Result 4 *There is a downward discontinuity in H's utility from P^0 to S^{00} when U is indifferent between the two.*

Proof. We have to prove that

$$V_{UP}^0 = V_{US}^{00} \Rightarrow V_{HP}^0 > V_{HS}^0.$$

We have

$$\begin{aligned} V_{UP}^{0} &= V_{US}^{00} \\ &\Leftrightarrow p_U^0 v(d_P^0) + (1-p_U^0)v(b_P^0) = p_U^0 v(d_S^{00}) + (1-p_U^0)v(b_S^{00}) \\ &\Leftrightarrow p_U^0\left[v(d_P^0) - v(d_S^{00})\right] + (1-p_U^0)\left[v(b_P^0) - v(b_S^{00})\right] = 0. \end{aligned} \tag{21}$$

We have that

$$\begin{aligned} & V_{HP}^{0} > V_{HS}^{0} \\ \Leftrightarrow\ & p_H^0 v(d_P^0) + (1-p_H^0)v(b_P^0) > v(y-p_H^0)m = p_H^0 v(d_S^{00}) + (1-p_H^0)v(b_S^{00}) \end{aligned} \tag{22}$$

where the equality comes from IC^{00} (equation 5). The inequality (22) can be reformulated as

$$p_H^0\left[v(d_P^0) - v(d_S^{00})\right] + (1-p_H^0)\left[v(b_P^0) - v(b_S^{00})\right] > 0.$$

We know from Lemma 5 that $\alpha_P^0 > a_S^{00}$ when $V_{UP}^0 = V_{US}^{00}$ which, together with $p_P^0 > p_U^0$, implies that $b_P^0 - b_S^{00} < 0$. We then obtain from (21) that $d_P^0 - d_S^{00} > 0$, and thus the inequality (22) is satisfied since $p_H^0 > p_U^0$. ■

Appendix A.8.5.: Result 5

We first prove the following lemma:

Lemma 6 *We have that* $\alpha_P^1 > \alpha_S^{11}$ *when* $V_{UP}^1 = V_{US}^{11}$.
Proof. *Same as Proof of Lemma 5, changing all superscripts 0 by 1.* ■

We are now in a position to prove:

Result 5 *There is a downward discontinuity in H's utility from* P^1 *to* S^{11} *when U is indifferent between the two.*

Proof. We have to prove that

$$V_{UP}^1 = V_{US}^{11} \Rightarrow V_{HP}^1 > V_{HS}^1.$$

We have

$$\begin{aligned} & V_{UP}^{1} = V_{US}^{11} \\ \Leftrightarrow\ & p_U^1 v(d_P^1) + (1-p_U^1)v(b_P^1) - \phi = p_U^1 v(d_{US}^{11}) + (1-p_U^1)v(b_{US}^{11}) - \phi \\ \Leftrightarrow\ & p_U^1\left[v(d_P^1) - v(d_{US}^{11})\right] + (1-p_U^1)\left[v(b_P^1) - v(b_{US}^{11})\right] = 0. \end{aligned} \tag{23}$$

We have that

$$\begin{array}{rl} & V^1_{HP} > V^1_{HS} \\ \Leftrightarrow & p^1_H v(d^1_P) + (1 - p^1_H) v(b^1_P) - \phi \\ > & v(y - p^1_H) m - \phi = p^1_H v(d^{11}_{US}) + (1 - p^1_H) v(b^{11}_{US}) - \phi, \end{array} \tag{24}$$

where the equality comes from IC^{11} (equation 5). The inequality (24) can be reformulated as

$$p^1_H \left[v(d^1_P) - v(d^{11}_{US}) \right] + (1 - p^1_H) \left[v(b^1_P) - v(b^{11}_{US}) \right] > 0.$$

We know from Lemma 6 that $\alpha^1_P > a^{11}_S$ when $V^1_{UP} = V^{11}_{US}$ which, together with $p^1_P > p^1_U$, implies that $b^1_P - b^{11}_{US} < 0$. We then obtain from (23) that $d^1_P - d^{11}_{US} > 0$, and thus the inequality (24) is satisfied since $p^1_H > p^1_U$. ■

Appendix B: Numerical example

This example is based on the following parameter values: $p_L = 0.1$, $p^0_H = 0.6$, $\lambda = 0.3$ (so that $p^0_U = 0.25$), $\Delta = 0.25$ (so that $p^1_H = 0.35$ and $p^1_U = 0.175$), $y = 5$, $m = 3$, and $v(x) = \sqrt{x}$. With these parameters, we obtain that

$$\begin{aligned} k^* &= 0.077 < k_{00} = 0.331 < k_{11} = 0.376, \\ \phi^S_{\min} &= 0.044 < \tilde{\phi}_P(0) = 0.054 < \phi^* = 0.057 < \phi^S_{\max} = 0.199. \end{aligned}$$

Figures B.1 to B.3 exemplify what happens, respectively, to the utility of types U and H, and to aggregate welfare, when we vary ϕ while $k^* < k < \tilde{k}_{00}$. Starting from large values of ϕ, we obtain a P^0 equilibrium, where utilities are not affected by ϕ. We then switch to the S^{01} case, with a downward discontinuity in both H's utility and aggregate welfare. As long as we remain in S^{01}, utilities and welfare increase as ϕ decreases. We then switch to the P^1 contract, with an upward discontinuity in both H's utility and aggregate welfare. Utilities and aggregate welfare further increase as we decrease ϕ while remaining in the P^1 case.

Figure B.1: Type U's utility as a function of ϕ, for $k^* < k < \tilde{k}_{00}$

Figure B.2: Type H's utility as a function of ϕ, for $k^* < k < \tilde{k}_{00}$

Figure B.3: Welfare as a function of ϕ, for $k^* < k < \tilde{k}_{00}$

We show in Figures B.4 to B.6, respectively, the utility of types U and H, and aggregate welfare, as a function of k when $\tilde{\phi}_P(0) < \phi < \phi^*$.

Figure B.4: Type U's utility as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

Figure B.5: Type H's utility as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

Figure B.6: Aggregate welfare as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

Welfare in that case is highly non monotone in k: it first decreases with k in the P^0 contract (because of the price effect), then increases discontinuously with k when one moves from P^0 to P^1 (thanks to the increase in H's utility), then increases with k in the P^1 contract (thanks to the composition effect), decreases discontinuously with k when one moves from P^1 to S^{01} (because of the decrease in H's utility), and finally increases with k in the S^{01} contract (thanks to the composition effect).

Figure 1: Separation between P^0, P^1, S^{00}, S^{01} and S^{11} contracts in (k, ϕ) space

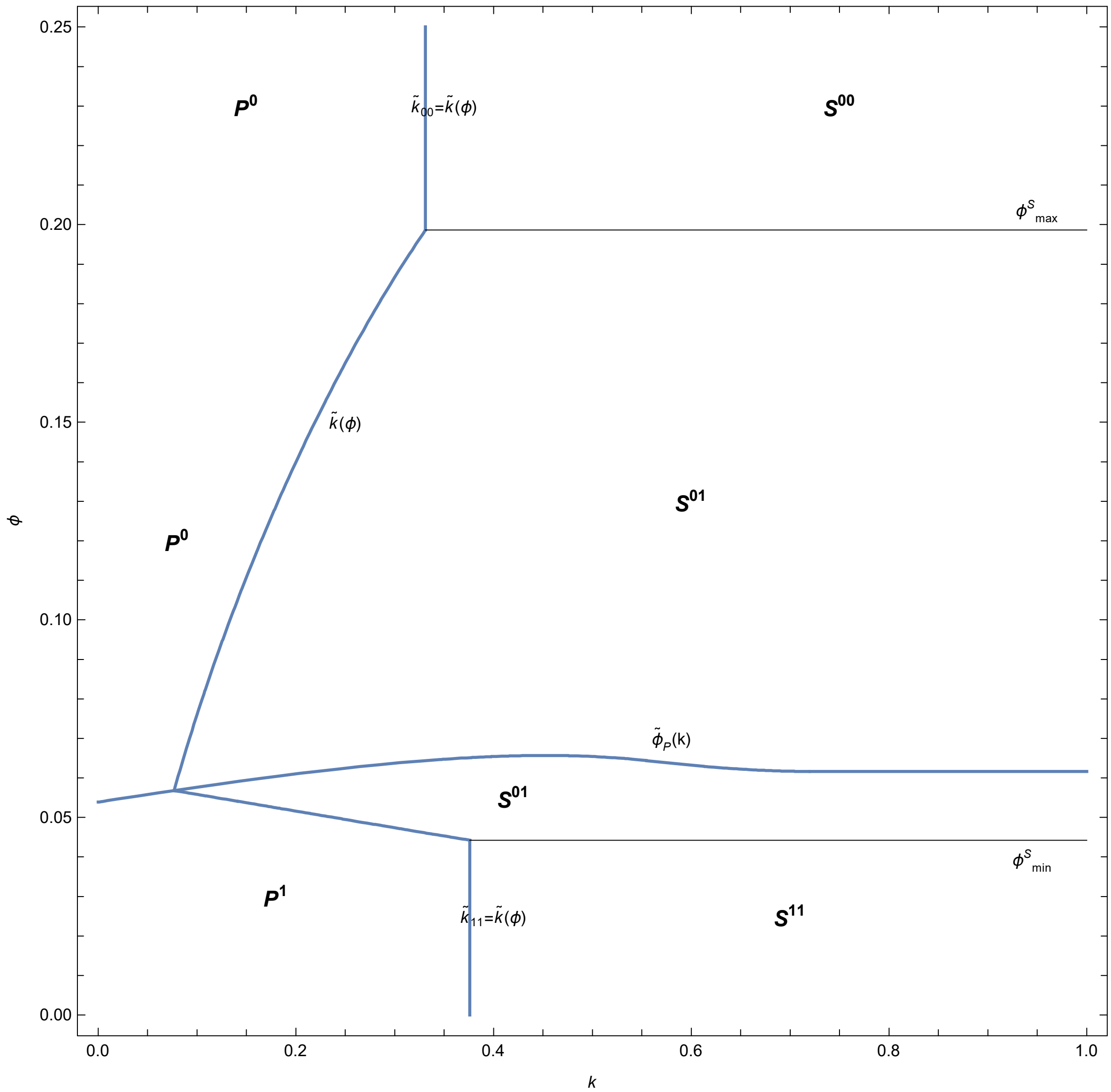

Figure 2 : Utility of U (above) and H (below) as a function of k when ϕ[k] = 0.29 - 0.8 k

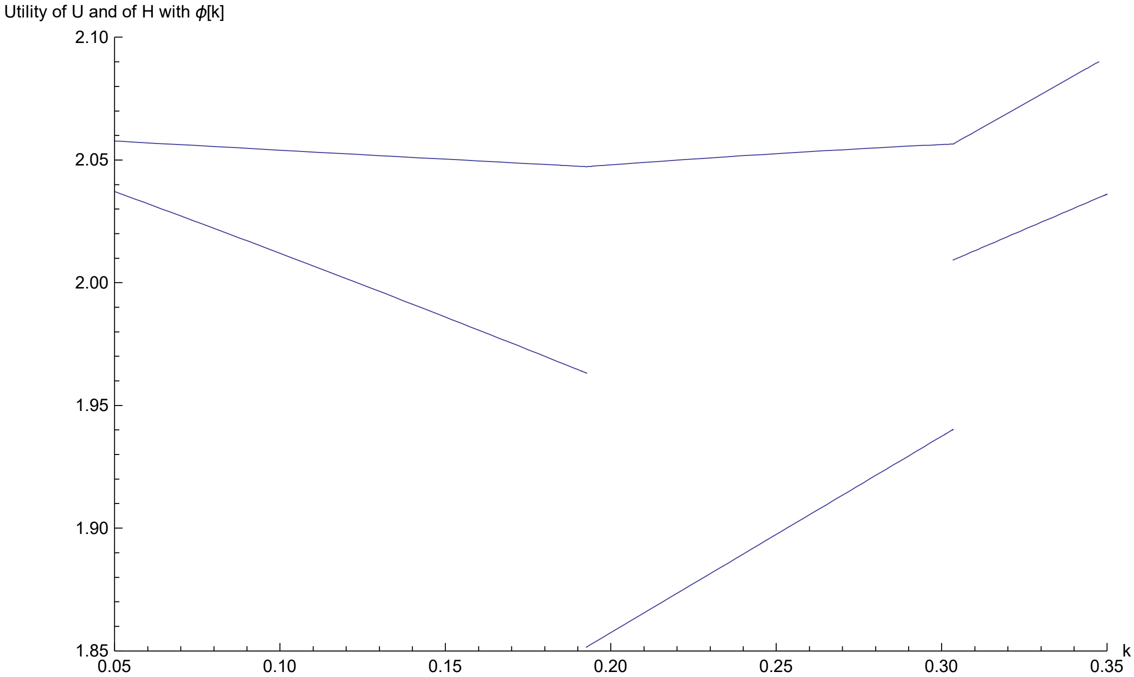

Figure 3 : Aggregate Welfare as a function of k when ϕ[k] = 0.29 - 0.8 k

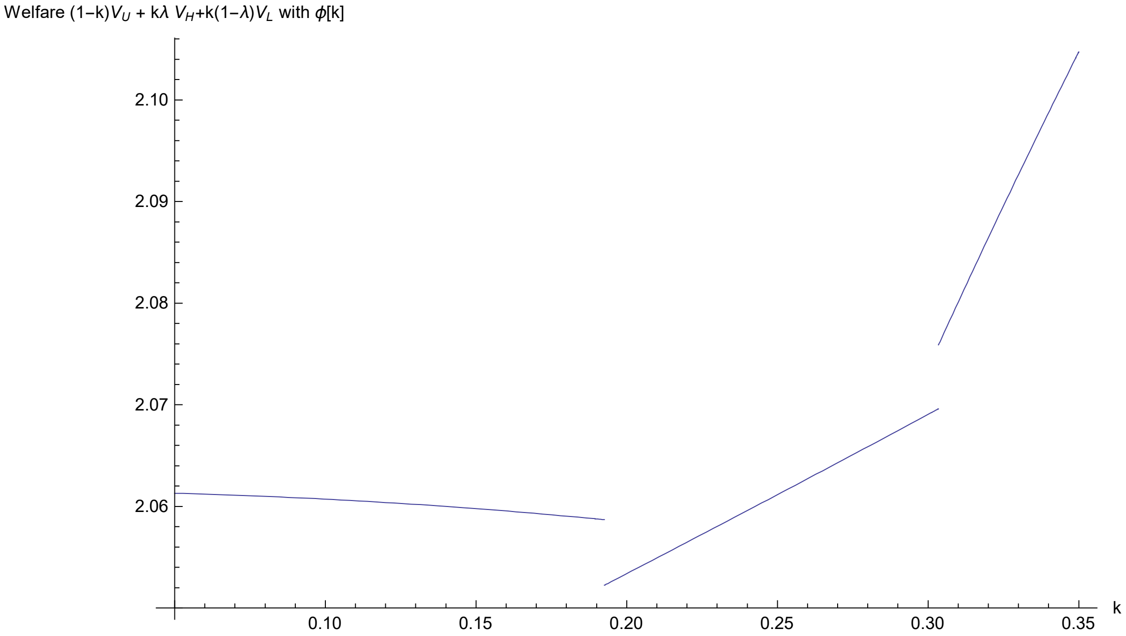

Figure B .1 : Type U ' s utility as a function of φ, for $k^* < k < k_{00}$

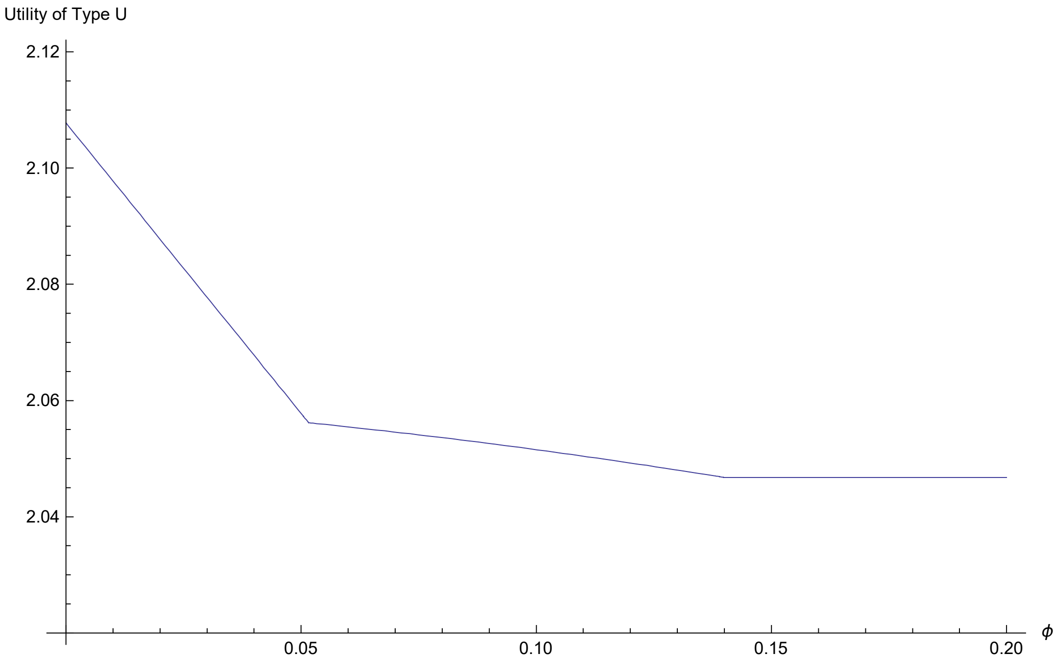

Figure B .2 : Type H ' s utility as a function of φ, for $k^* < k < k_{00}$

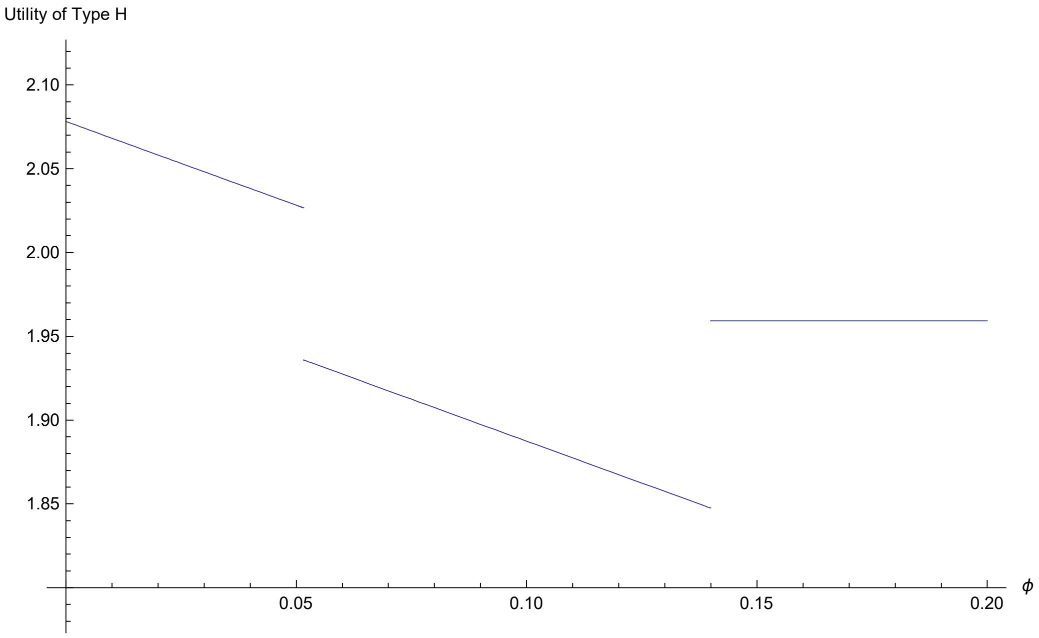

Figure B .3 : Aggregate welfare as a function of φ, for $k^* < k < k_{00}$

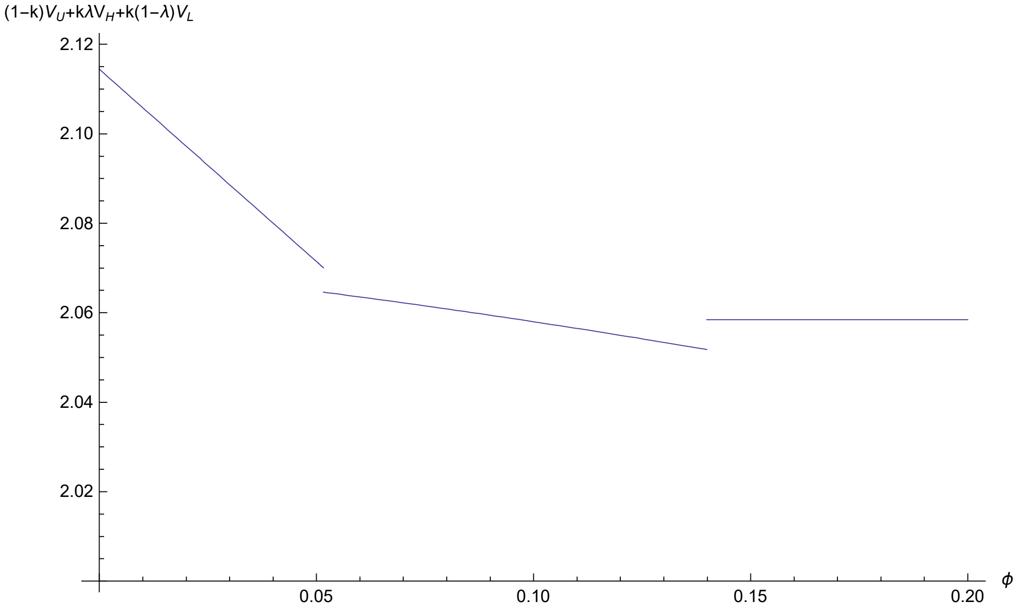

Figure B .4 : Type U ' s utility as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

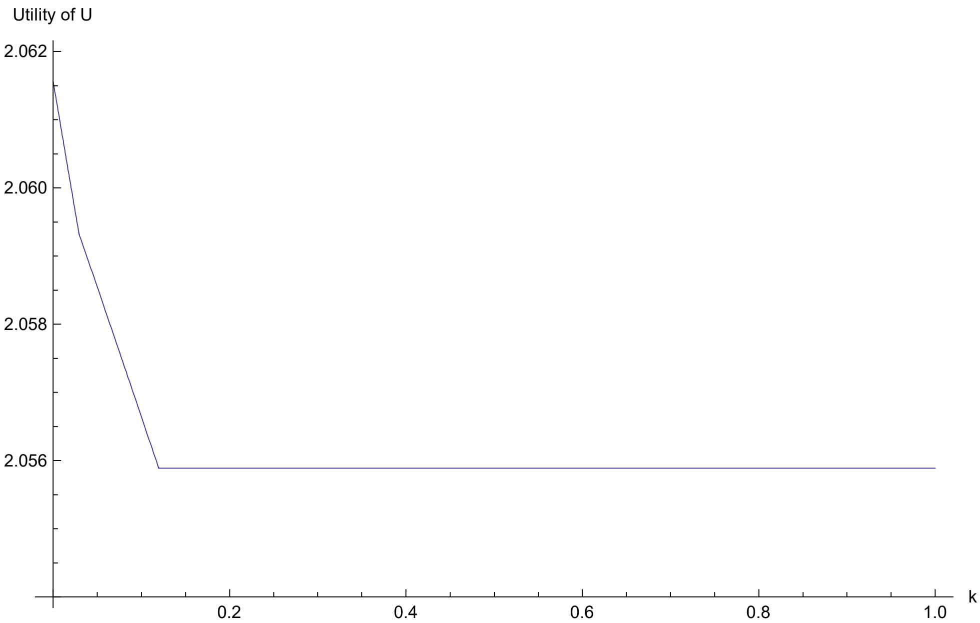

Figure B.5 : Type H's utility as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$

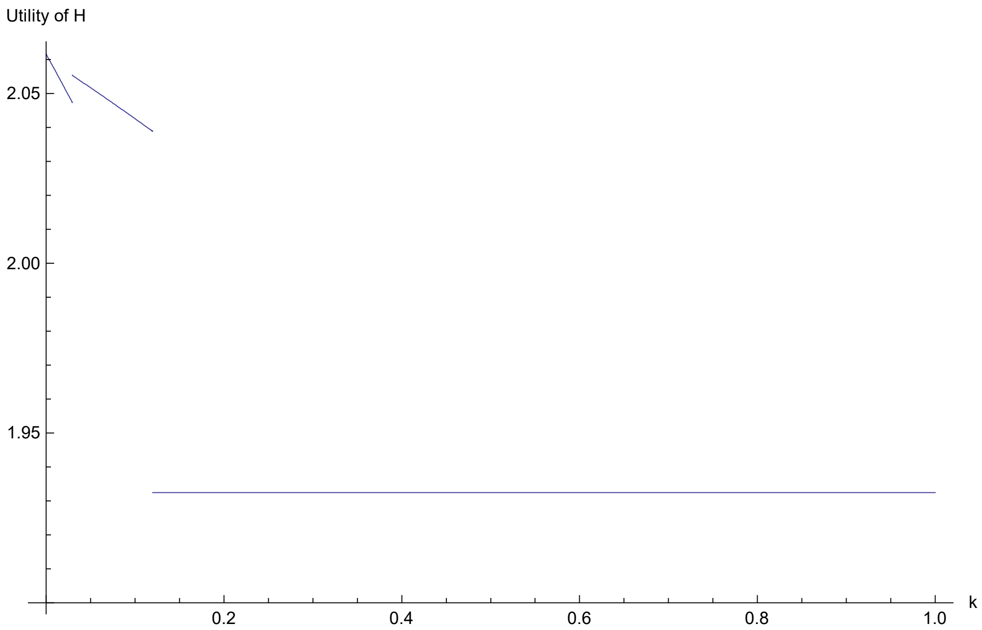

Figure B.6 : Aggregate Welfare as a function of k, for $\tilde{\phi}_P(0) < \phi < \phi^*$