Soda tax incidence and design under market power[1]

Helmuth Cremer
Toulouse School of Economics, University of Toulouse Capitole, France

Catarina Goulão
Toulouse School of Economics, INRAE, University of Toulouse Capitole, France

Jean-Marie Lozachmeur
Toulouse School of Economics, CNRS, University of Toulouse Capitole, France

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Abstract

Health damages of an unhealthy good, such as a sugar-sweetened beverage, are misperceived by consumers. Market power affects both output and sugar content and these effects have to be balanced against Pigouvian considerations. Under "pseudo" perfect competition, a Pigouvian tax proportional to sugar content is sufficient to achieve a first best solution. Under monopoly, a specific tax on output achieves an efficient solution, but it must be an *affine* function of the sugar content. The calibrations of the French and US markets illustrate that both the total tax as well as its sugar component can be positive or negative.

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1 Introduction

In 2016, 40% of the world's population was overweight. More than 60% of the American and European populations were overweight, with 28% and 23% obese, respectively.[1] Being overweight or obese constitutes a major risk factor for non-communicable chronic diseases, such as diabetes, cancer or cardiovascular conditions, which represent the major cause of death worldwide; WHO (2018).

A possible explanation for the excess consumption of unhealthy goods is that individuals misperceive the adverse health effects of these goods in the long run. Alternatively, individuals may be subject to problems of self-control in their consumption (see, among others, Gruber and Koszegi, 2004; O'Donoghue and Rabin, 2006; Kotakorpi, 2008; Haavio and Kotakorpi, 2011; Cremer et al., 2012 and Cremer et al., 2016). In either case, individuals make their consumption decisions according to a "perceived" or short-run utility function, which does not fully account for the long-run harmful impact of these goods on their health. Consequently, the *laissez-faire* solution will be "inefficient" in the sense that consumers' long-run utility would be larger if these effects were properly accounted for in their consumption decisions. This may call for policy intervention aimed at reducing the consumption of unhealthy goods and/or their fat or sugar content.

Increasing the price of unhealthy foods and beverages through taxes is a potential policy measure by which to discourage over-consumption. Soda taxes imposed on sugar-sweetened beverages (SSB) are already used in several countries, and are under consideration in many others. Preliminary evidence from existing taxes on food and beverages suggests that soda taxes have been effective in reducing purchases but the long-run effects on health are the subject of debate; see Cornelsen and Carreido (2015). Yet it is striking to note how much these taxes on SSB vary in form and level. Many countries, such as France, Mexico and some US cities, adopt specific taxes per liter or ounce. The United Arab Emirates, on the other hand, apply an *ad valorem* tax of 50% on SSB, and recently the UK opted for a two-level specific tax depending on the sugar content.

The effectiveness of a tax in reducing the consumption of unhealthy foods and beverages depends on its design. Should the tax be imposed on sodas *per se* or, alternatively, on their sugar content? In either case, what would be the appropriate level of this tax?

[1] Overweight and obesity prevalence rates available from the World Health Organization (WHO), Global Health Observatory data repository, http://apps.who.int/gho/data/node.main.NCDMBMIOVERWEIGHTC?lang=en, accessed on November 8, 2018.

One concern in the debate about the effectiveness of fat or soda taxes is that retailers might absorb the tax rather than passing it on to customers, thereby obscuring the signal that governments are trying to send. This in turn is an issue of tax incidence, which is well understood under perfect competition, where it is essentially a matter of demand and supply elasticities. However, most markets for SSBs are imperfectly competitive. Furthermore, taxes will and should affect not only consumption but also the nature of the good, and specifically its sugar or fat content; see, for instance, Hamilton and Requillart (2017). In this context, the issue of tax incidence is more complex and the appropriate design of taxes has to be amended accordingly.

We first illustrate this point by considering the simplest form of imperfect competition namely a monopoly. For simplicity of exposition, we concentrate on SSBs and their sugar content, but our analysis also applies to fatty goods, for instance. As a reference, we first study the solution under "pseudo" perfect competition (generalized to account for the endogenous choice of sugar content). We show that in that case, a simple Pigouvian tax of a constant amount per unit of output but proportional to the sugar content is sufficient to achieve a first best solution. Then we consider a monopoly for which the tax design is more complex. Market power affects both output and sugar content, possibly in opposite directions, and these effects have to be balanced against Pigouvian considerations. We show that, nevertheless, a tax per unit of output achieves an efficient solution, but it must be an *affine* function of the sugar content; taxing "grams of sugar" is no longer sufficient. Interestingly, both the total tax as well as its sugar component can be positive as well as negative.

We then provide an example which illustrates our theoretical results. The setting is a standard Mussa-Rosen type of vertical product differentiation with some customizing to fit our problem. In particular, we consider a health cost which may be misperceived so that a consumers willingness to pay is not necessarily increasing in the sugar content. We solve this model analytically and determine the monopoly and the first-best solutions as well as the implementation of the first-best. The general model has shown the there is some ambiguity as for the sign of the tax instruments. The examples show that the degree of misperception plays a crucial role in that context. Most significantly the example allows us to provide numerical solution for the optimal policy for calibrated versions of the model, which provide a stylized representation of the demand side of the US and the French SSB markets. In reality neither of these markets is of course a monopoly. However, our calibration allows us to simulate the monopoly scenarios for empirically relevant demand conditions. Furthermore, since the monopoly setting is admittedly extreme we also simulate several duopoly scenarios. These show how the results are affected when market power is mitigated. In the process we can examine how policies targeted to high and low sugar content variants should be designed.

2 The model

Individuals consume x units of an unhealthy good (soda) of sugar content s and a numeraire good y. Their exogenous income is equal to m. Their true utility function is given by:

$$U = u(x, s) + y - h(xs), \tag{1}$$

where $u(x, s)$ reflects the short-term utility from the consumption of the unhealthy good. Assume that $\partial u/\partial x > 0$, $\partial u/\partial s > 0$. Observe that one can simply think of s as "quality" except that unlike in traditional models, utility is not monotonic in "quality".

In the long run, higher consumption of the unhealthy good causes overweight or obesity, along with associated health problems. These negative effects of the unhealthy good consumption are captured by the harm function $h(S) = h(xs)$ with $\partial h(S)/\partial x > 0$, $\partial h(S)/\partial s > 0$ and $\partial^2 h(S)/\partial s^2 > 0$. The health effects of unhealthy eating tend to occur later in life and the full understanding of their occurrence is difficult. Thus individuals may not perceive fully these negative health effects (see Devaux et al., 2011, among others). Their perceived harm function is given by $\beta h(xs)$, where $\beta \in]0, 1]$ so that their perceived utility is given by:

$$\hat{U} = u(x, s) + y - \beta h(xs). \tag{2}$$

Consumers make their consumption decisions according to their perceived utility function. For given s and a consumer price q, consumers maximize:

$$\max_x \hat{U} = u(x, s) + m - qx - \beta h(xs),$$

which yields the FOC:

$$q(x, s; \beta) = \frac{\partial u(x, s)}{\partial x} - \beta \frac{\partial h(S)}{\partial S} s. \tag{3}$$

Equation (3) defines the inverse demand function $q(x, s; \beta)$ in the traditional way. For future reference, note that by adding and subtracting $\partial h(S)/\partial S s$, this can be decomposed in an inverse demand with fully perceived health effects, plus a term that accounts for misperception:

$$q(x, s; \beta) = q(x, s; 1) + (1 - \beta) \frac{\partial h(S)}{\partial S} s. \tag{4}$$

3 First best

With quasi-linear preferences, Pareto-efficiency requires surplus maximization based on consumers' true utility. The efficient allocation (x^*, s^*) is thus obtained by solving:

$$\max_{x,s} W = u(x,s) + m - C(x,s) - h(xs).$$

First-order conditions are given by:

$$\frac{\partial u(x^*, s^*)}{\partial x} - \frac{\partial h(S^*)}{\partial S} s^* = \frac{\partial C(x^*, s^*)}{\partial x}, \tag{5}$$
$$\frac{\partial u(x^*, s^*)}{\partial s} - \frac{\partial h(S^*)}{\partial S} x^* = \frac{\partial C(x^*, s^*)}{\partial s}, \tag{6}$$

where $S^* = s^* x^*$ is the efficient total consumption of sugar.

4 Pseudo perfect competition

To obtain a perfect competition benchmark in a world where a characteristic of the product is endogenous, assume that each potential variant of the product – characterized by its sugar content s – is sold in a competitive market at price $\widetilde{p}(s)$. This equilibrium is inspired by the "competitive solution" presented by Mussa and Rossen (1978). Both producers and consumers are price-takers. Consequently, the competitive allocation (x^c, s^c) supported by the price system $\widetilde{p}(s)$ must satisfy, for all s:

$$\widetilde{p}(s^c) = \frac{\partial C(x^c, s^c)}{\partial x}, \tag{7}$$
$$x^c \frac{\partial \widetilde{p}(s^c)}{\partial s} = \frac{\partial C(x^c, s^c)}{\partial s}. \tag{8}$$

In words, both the price and the implicit price for quality must equal the respective marginal costs of x and s.

On the consumer side the market equilibrium requires that (x^c, s^c) solves:

$$\max_{x,s} \hat{U} = u(x,s) + m - qx - \beta h(xs).$$

The FOC are given by:

$$\frac{\partial u(x^c, s^c)}{\partial x} - \widetilde{q}(s^c) - \beta \frac{\partial h(S^c)}{\partial S} s^c = 0, \tag{9}$$
$$\frac{\partial u(x^c, s^c)}{\partial s} - x^c \frac{\partial \widetilde{q}(s^c)}{\partial s} - \beta \frac{\partial h(S^c)}{\partial S} x^c = 0. \tag{10}$$

The decentralization of the first best solution requires that (7)–(8) and (9)–(10) are satisfied by (x^*, s^*) as defined by (5)–(6). This can be attained with a per unit tax on x, which is proportional, at rate τ, to the sugar content s. The consumer's expenditures for the good are then given by:

$$\widetilde{q}(s)x = [\widetilde{p}(s) + \tau s]x. \tag{11}$$

Departing from (9) and using (5), (7), and (11), decentralization then requires:

$$\widetilde{p}(s^*) + \frac{\partial h(S^*)}{\partial S}s^* - [\widetilde{p}(s^*) + \tau s^*] - \beta\frac{\partial h(S^*)}{\partial S}s^* = 0,$$

and solving for τ yields:

$$\tau^P = (1-\beta)\frac{\partial h(S^*)}{\partial S}.$$

Note that (8) and (10) are simultaneously satisfied for τ^P. Intuitively, τ^P reflects the wedge between social (true) utility and individual (perceived) utility, which determines demand. With marginal cost pricing, this Pigouvian tax on the sugar content imposed per unit of output ensures that the consumer price $\widetilde{q}(s)$ corresponds to the true marginal cost of x, which in turn implies that quantity is at its optimal level.

Therefore, under pseudo-perfect competition, a single instrument – namely a Pigouvian specific tax on the sugar content – is sufficient to achieve an optimal choice of both quantity x and sugar content s. If individuals perceive perfectly the health effects ($\beta = 1$), the tax vanishes to zero.

5 Monopoly

5.1 Profit maximizing solution

Consider now a monopoly producing the unhealthy good x with a given content of sugar s. It solves:

$$\max_{x,s} p(x, s; \beta)x - C(x, s),$$

where the producer price $p(x, s; \beta)$ is given by the inverse demand function defined by (4) adjusted by any applicable taxes or subsidies.

Now assume that the good is subject to a (positive or negative) per unit tax which is an affine function of the sugar content and given by $t = \bar{t} + \tau s$. We then have:

$$q(x, s; \beta)x = \left[p(x, s; \beta) + \bar{t} + \tau s\right] x. \tag{12}$$

Using it in the monopoly maximization problem, the FOCs, defining the monopoly solution (x^m, s^m) are given by:

$$q(x^m, s^m; \beta) - \bar{t} - \tau s + \frac{\partial q(x^m, s^m; \beta)}{\partial x} x^m = \frac{\partial C(x^m, s^m)}{\partial x}, \tag{13}$$

$$\left(\frac{\partial q(x^m, s^m; \beta)}{\partial s} - \tau \right) x^m = \frac{\partial C(x^m, s^m)}{\partial s}. \tag{14}$$

5.2 Decentralization of the first best

The decentralization of the first best solution requires that equations (13)–(14) and the consumer FOCs, are satisfied by (x^*, s^*). Departing from (5) and (6), and using (13) and (14), we get:

$$\frac{\partial u(x^*, s^*)}{\partial x} - \frac{\partial h(S^*)}{\partial S} s^* = q(x^*, s^*; \beta) - \bar{t} - \tau s + \frac{\partial q(x^*, s^*; \beta)}{\partial x} x^*,$$

$$\frac{\partial u(x^*, s^*)}{\partial s} - \frac{\partial h(S^*)}{\partial S} x^* = \left(\frac{\partial q(x^*, s^*; \beta)}{\partial s} - \tau \right) x^*.$$

Rearranging and solving for $t = \bar{t} + \tau s$ and τ leads to:

$$t = \bar{t} + \tau s = q(x^*, s^*; \beta) + \frac{\partial q(x^*, s^*)}{\partial x} x^* - \frac{\partial u(x^*, s^*)}{\partial x} + \frac{\partial h(S^*)}{\partial S}, \tag{15}$$

$$\tau = \frac{\partial q(x^*, s^*; \beta)}{\partial s} - \left[\frac{\frac{\partial u(x^*, s^*)}{\partial s}}{x^*} - \frac{\partial h(S^*)}{\partial S} \right]. \tag{16}$$

Differentiating inverse demand defined by (4) with respect to s shows that:

$$\frac{\partial q(x, s; \beta)}{\partial s} = \frac{\partial q(x, s; 1)}{\partial s} + (1 - \beta) \left[\frac{\partial h(S)}{\partial S} + \frac{\partial h^2(S)}{\partial S^2} s x \right].$$

Finally, substituting $q(x^*, s^*; \beta)$ and $\partial q(x^*, s^*; \beta)/\partial s$ in (15)–(16), respectively, yields:

$$t = \bar{t} + \tau s^* = \frac{\partial q(x^*, s^*; \beta)}{\partial x} x^* + (1 - \beta) \frac{\partial h(S^*)}{\partial S}, \tag{17}$$

$$\tau = \left\{ \frac{\partial q(x^*, s^*; 1)}{\partial s} - \frac{\left[\frac{\partial u(x^*, s^*)}{\partial s} - x^* \frac{\partial h(S^*)}{\partial S} \right]}{x^*} \right\} + (1 - \beta) \left[\frac{\partial h(S^*)}{\partial S} + \frac{\partial h^2(S^*)}{\partial S^2} s^* x^* \right]. \tag{18}$$

Expression (17) specifies the per unit tax $t = \bar{t} + \tau s$, which is designed to achieve the optimal level of output. To interpret this, suppose first that individuals perceive correctly the health effects of soda and sugar consumption ($\beta = 1$). In this case, it is sufficient to correct for the monopoly power to achieve the first best. It follows that $(t + \tau s) < 0$, and we have the standard subsidization of demand to induce a higher quantity supplied by the monopoly. When there is misperception ($\beta < 1$), this effect is mitigated or outweighed by the positive Pigouvian term, which is the same as under perfect competition and has the same interpretation as in Section 4.

Consequently, the net sign of t is ambiguous. Intuitively, absent of misperception the monopoly output is lower than efficient; this can be corrected by a subsidy. However, because of the misperception, the competitive output would be too large, which in turn calls for a tax. The sign of t is then determined by trading off the market power and the Pigouvian effect. This is in line with the classical result, whereby absent of any taxation or regulation a monopoly may produce a more efficient output level of a polluting than a competitive market.

Turning to (18), recall that absent of misperception one can simply think of s as "quality", except that unlike in traditional models our utility is not monotonic in "quality". The first term in curly brackets is the standard term measuring the sign of the quality (s) distortion in a monopoly. We know from Spence (1975) that the monopoly level of quality may be smaller or larger than efficient. This depends on the shape of the demand curve, and $s^m > s^*$ obtains when the marginal valuation of sugar content s compares with the average valuation of s. To sum up, the first term is positive if $s^m > s^*$ (so that it is desirable to reduce s from the monopoly level), and negative otherwise. Note, however, that in a two dimensional setting (both x and s are endogenous), local comparisons have to be interpreted with care. The comparison between s^m and s^* *may* depend on the level of output.[2] The relevant comparison for our purpose is the one given the first best level of output x^*. This point is illustrated by the example presented in Section 6. The second term is the Pigouvian term which vanishes when $\beta = 1$. Otherwise it is positive.

Expressions (17) and (18) determine the "total" per unit tax t and tax on the sugar content τ. The constant in the affine per unit tax function is then simply determined as a residual with $\bar{t} = t - \tau s^*$, which can be positive or negative depending on whether the sugar component of the tax is larger or smaller than the required per unit tax on quantity. It is not in general equal to zero, which explains why under monopoly the affine function is necessary; unlike under perfect competition a simple linear function is not sufficient.

6 Example

We now provide an example which illustrates our theoretical results. Most significantly it allows us to provide numerical solutions for the optimal policy for calibrated versions of the model, which provide a stylized representation of the demand side of the US and the French SSB markets. In reality neither of these markets is of course a monopoly. However, our calibration allows

[2] In other words, the sign first term in (18) may depend on the level of x. This is determined by the properties of the demand function.

us to simulate the monopoly scenarios for empirically relevant demand conditions. Furthermore, since the monopoly setting is admittedly extreme we also simulate several duopoly scenarios. These show how the results are affected when market power is mitigated. In the process we can examine how policies targeted to high and low sugar content variants should be designed.

We use a product differentiation model inspired by Mussa and Rosen (1978) which has by now become a classical setup used in the IO literature. Consumers may buy a unit of soda or none at all. Their perceived utility (2) is given by:

$$\hat{U} = \theta s + m - q - \beta h(s), \tag{19}$$

when consuming one unit of soda with sugar content s and $\hat{U} = m$ otherwise. The true utility is $U = \theta s + m - q - h(s)$ when consuming one unit of soda and $U = m$ otherwise. Consumers differ in θ, which is uniformly distributed over the interval $[0, b]$, with $b \in \mathbb{R}^+$. For the sake of illustration we assume $h(s) = \gamma\,(1/2)\,s^2$. Let θ_p denote the marginal consumer indifferent between consuming one unit or none at all. It is defined by the level of θ_p that satisfies:

$$\theta_p(q, s; \beta) = \frac{q + \beta h(s)}{s}. \tag{20}$$

Demand for the unhealthy good is then given by:

$$x\,(q, s; \beta) = b - \max[0, \theta_p(q, s; \beta)]. \tag{21}$$

Note that our example uses a setting where individual demand is discrete (0 or 1), but where consumers are heterogenous. Since preferences are quasi-linear this yields an aggregate demand (and welfare) which is perfectly consistent with the representative consumer approach used in the previous sections.[3] It implies of course additional assumptions but is better suited to calibrate demand behavior, particularly in the case where two variants of the product are supplied which becomes relevant when we study the duopoly scenarios below.

For future reference note that

$$\frac{\partial x}{\partial s} = -\frac{\partial \theta_p}{\partial s} = -\frac{1}{2}\beta\gamma + \frac{q}{s^2}. \tag{22}$$

Note that (22) is decreasing in β. Consequently a higher degree of misperception (a lower β) implies that demand becomes more responsive to sugar content. Additionally, demand decreases in s, for $s > \sqrt{2q/\gamma\beta}$, and increases for $0 < s < \sqrt{2q/\gamma\beta}$. Also note that the threshold is

[3] See for instance Varian (1992), Section 9.4.

	s	q	x	
Monopoly	$\frac{2(b-c)}{3\gamma\beta}$	$\frac{2(b-c)(b+2c)}{9\gamma\beta}$	$\frac{b-c}{3}$	
First Best	$\frac{2(b-c)}{3\gamma}$	$-$	$\frac{2(b-c)}{3}$	

Table 1: Monopolistic and first best solutions.

decreasing in β, that is, higher misperception (lower β) increases the interval where demand increases with sugar content.

For the time being, assume that the good is supplied by a monopoly with a linear technology; per unit production costs are constant in quantity and given by $c(s) = cs$, with $0 < c < b$. The monopoly price p is given by the consumer price q adjusted by taxes or subsidies. From section 5.2 we know that the first best can be decentralized via an affine per unit tax $t = \bar{t} + \tau s$. Consequently, we already incorporated it in the expression for the profit function to which we now turn. Using demand $x(q, s; \beta)$ as given by (21), the monopoly solves:

$$\{q^m, s^m\} = \arg\max \{(q - \bar{t} - \tau s - c(s))\, x(q, s; \beta)\}. \quad (23)$$

To determine the first best solution, we set the consumer price equal to marginal cost (with respect to quantity) and substitute q by $c(s)$ in (20). The first best level of sugar is then given by:

$$s^* = \arg\max_s W = \int_0^{\theta_p(c(s),s;1)} m d\theta + \int_{\theta_p(c(s),s;1)}^{b} (m - c(s) + \theta s - h(s))\, d\theta. \quad (24)$$

Solving (23) and (44) for $h(s) = \gamma(1/2)s^2$ and $c(s) = cs$, using (20) and (21) and rearranging, yields the values for sugar content, consumers prices and demand in the monopoly solution without taxes and in the first best presented in Table 1.

Comparing the no tax monopoly solution to the first best solution highlights the effects already identified in the general model. We have $x^m < x^* < b$, and $s^* < s^m$. As expected, the first best solution implies a larger output, but the monopoly is choosing an excessive sugar content.

The first best can be decentralized through the affine per unit tax $t = \bar{t} + \tau s$, specified in Table 2. As the monopoly is under-providing quantity, the consumer price has to decrease (negative per unit tax) so that demand increases for all levels of β. More interestingly, we obtain that even though the monopoly chooses a sugar content that is too high ($s^* < s^m$), it may be optimal to subsidize the sugar content. More precisely, we have $\tau < 0$ if and only if $\beta > 1/2$, that

	$\bar{t}$	τ	$t = \bar{t} + \tau s^*$
Monopoly	$-\frac{2(b-c)^2(2-\beta)}{9\gamma}$	$\frac{(b-c)(1-2\beta)}{3}$	$-\frac{2(b-c)^2(1+\beta)}{9\gamma}$

Table 2: Taxes implementing the first best.

is when misperception is sufficiently small. To understand this result, consider a hypothetical scenario where s can be controlled directly and set to s^*. To implement the FB level of x one then can rely on a tax on output only, and simply set $\bar{t}$ to the level of t specified in the table. The relevant question is then if starting from this solution, the monopoly finds it profitable to increase or to decrease s. This in turn depends on the sign of

$$\frac{\partial \pi(x^*, s^*)}{\partial s} = (q^* - cs^* - t)\frac{\partial x}{\partial s} - cx^*$$

which using expression (22) can be rewritten as

$$(q^* - cs^* - t)\left(-\frac{1}{2}\beta\gamma + \frac{q^*}{s^{*2}}\right) - cx^*, \tag{25}$$

where $(q^* - cs^* - t) > 0$ because by FB implementation requires that the monopoly's profits are positive. When this expression is negative, the monopoly can increase profits by lowering s. To make this unattractive to the firm and to implement s^* (without controlling it directly) we then have to subsidize s. When, by contrast, the expression is positive the monopoly wants to set s above s^* and a tax on sugar is necessary. This explain why the optimal tax mix may imply a subsidy or a tax on s.

To understand the role played by β in determining the relevant case note that the first term in (25) represents the demand effect while the second term is the cost effect. When β is large misperception is small and consumers are sufficiently aware of the health effects so that an increase in s has a small (or possibly even negative) effect on demand. Consequently the cost effect dominates. On the other hand, when β is sufficiently small the demand effect is large an outweighs the cost effect. The numerical examples below confirm this intuition. In particular we find that while for the considered parameter values $\partial x/\partial s$ is always positive (at the considered point), expression (25) is indeed negative for $\beta = 1$ or $\beta = 0.87$ (the benchmark case) while is is negative for $\beta = 0.4$; see Appendix A.

7 Numerical illustrations

In this section we use a calibrated version of the analytical model just presented to provide estimates of the taxes that would implement the first best in France. As the data used for the calibration show, these two countries illustrate two contrasting markets of SSB and can be seen as benchmark references. Specifically, demand for SSB is much higher and less elastic in the US than in France.

We first consider the extreme case of a monopoly supplying a single variant of SSB characterized by its sugar content. This provides a numerical illustration of the analytical example presented in Section 6. We then depart from the monopoly by considering a duopoly, with firms competing in sugar content and in price. Following the product differentiation literature we consider a sequential game. In the first stage the firms simultaneously choose the sugar content of their variant and in the second stage they simultaneously set prices. We determine the subgame perfect equilibrium of this game. As a duopoly with two products is not immediately comparable with a monopoly, we also consider a monopoly supplying two variants of SSBs. Note that this solution can also be interpreted as a duopoly in which the firms collude and maximize joint profits. Adding this extra scenario allows us to disentangle market power and product variety effects. We provide numerical solutions for the optimal taxes for these various scenarios which represent different degrees of market power.

Details on the calibration of the parameters are provided in Appendix B. As explained there, the empirical literature has shown that demand for SSB in the US is higher and less elastic than in France. Specifically the US and French demand elasticities are set at $\varepsilon_{xq} = -1.44$ and $\varepsilon_{xq} = -3.52$ respectively while the upper bounds of the distribution of θ's (which determine the levels of demand) are given by $b = 0.0430$ and $b = 0.0343$ respectively.

Following the estimates of Wilde et al., (2019), we set the health harm at 2.6 €/ℓ of SSB. This value represents mainly the monetary valuation of healthy years of life lost due to SSB consumption. It also includes health care costs but these represent a relatively insignificant of total harm. Following Allcott et al., (2019b)'s estimations we calibrate the level of misperception at $\beta = 0.87$ for both countries in the baseline scenario, but we also consider scenarios with larger as well as with smaller degrees of misperception ($\beta = 0.4$ and $\beta = 1$).

Tables 3 and 4 report the results for the baseline scenario with $\beta = 0.87$. The first two columns provide the first best and the monopoly solution. The bottom part of the table specifies the taxes that would implement the first best. Consider first Table 3, the French market. As anticipated from the analytical example, monopoly output would be too small ($x_F^m = 0.11$, $x_F^* = 0.21$) but with an excessive sugar content ($s_F^m = 14.30$, $s_F^* = 12.44$, where we use the subscripts F and US to denote France and the US, respectively). Achieving a FB output level requires a total subsidy of -0.0869 €/ℓ. Since misperception is rather low ($\beta = 0.87$), we are in the case where the cost effect in expression (25) dominates so that a subsidy on sugar ($\tau = -0.0028$) is necessary to avoid that the monopoly reduces the sugar content below its optimal level.

The remaining columns in Table 3 consider scenarios with two variants. They present the FB and the subgame perfect equilibrium of the duopoly when firm set sugar levels and then prices. The last column represents the multiproduct monopoly (duopoly with collusion). The marginal consumer θ_{p1} is indifferent between the low sugar content variant and not consuming the product. Consumer θ_{p2} is indifferent between the two variants. Demand for the low sugar variant is thus given by $(\theta_{p2} - \theta_{p1})/b$, while that for the high sugar variant is given by $(b - \theta_{p2})/b$. This solution yields higher sugar contents and lower output levels than in the FB for both variants. We have $s^m_{1F} = 8.58$ and $s^m_{2F} = 17.16$ each of which is higher than their FB counterparts $s^*_{1F} = 7.47$ and $s^*_{2F} = 14.93$. Each variant is covering 0.065 of the market as opposed to 0.31 in the FB.

When the two variants are produced by competing firms, the degree of product differentiation increases albeit slightly: sugar content decreases for the low-sugar content SSB, and increases for the high-sugar content SSB ($s^d_{1F} = 8.55$ and $s^d_{2F} = 17.58$). This is in line with standard results obtained in the IO literature; competing firms tend to increase the degree of product differentiation in order to mitigate price competition intensity. Still since market power decreases so do prices and demand levels increase: total market coverage is now at 0.20.

The interesting results from our perspective is the appropriate policy design. This was not discussed in the analytical part because even for the example the expressions are complex and not very telling, particularly for the duopoly case. The numerical results are presented in the lower part of the last two columns. In both scenarios, the FB with two products can be implemented with *product specific* per unit taxes which are an affine function of sugar contents $t_i = \bar{t}_i + \tau_i s_i$. As with a single product, sugar contents and final products are subsidized the multiproduct monopoly subsidies on sugar content and subsidies in the final price of the two products. However in a duopoly the low sugar content SSB faces a tax on sugar $\tau^d_{1F} = 2.01$ while there is a subsidy $\tau^d_{2F} = -0.02$ for the high sugar content variant. Intuitively these results arise for the same reasons as under a single product monopoly. When the FB output is with a subsidy on output levels firms may have an incentive to choose a sugar content different from the FB level. As explained in the discussion of expression (25) deviating from the FB levels has demand and a cost effects. While the cost effect is similar here, demand effects are more complex, especially in the duopoly, where the change in s induces changes in the second stage equilibrium prices. However, the basic intuition continues to hold: when starting from an implementation with an output tax only, the firm finds it profitable to increase sugar content (that is when the demand effect outweighs the cost effect), then there should be a tax on sugar. In the opposite case, a subsidy on sugar content is again optimal. The numerical results show that the latter case applies for both variants in the monopoly case but only for the high sugar variant in a duopoly where the low sugar content variant now faces a tax on sugar.

Table 4 presents the results for the US. While the specific numbers differ most qualitative results are the same as those discussed for France. The only exception is that in the duopoly none of the firms faces a tax on sugar so that the firm supplying the lowest sugar content also faces a subsidy on sugar content.

The calibrations of the French and US SSB markets and the estimations of the taxes implementing the first best illustrate the findings of our analytical model and show that the main message remains valid for a wider range of market structures. Misperception of health care cost associated with SSB consumption coupled with inefficient market structures call for more complex tax instruments. More precisely, on their own, neither a per unit tax per liter of SSB nor a tax per grams of sugar content are enough to implement the first best. Yet, using both instruments in the form of an affine per unit tax $t = \bar{t} + \tau s$ renders it possible. Note that a simple taxation of sugar would exacerbate the problem that due to market power prices are too large so that market coverage is too small. Furthermore, even when the market equilibrium implies an excessive sugar content, its reduction through a tax policy may call for a subsidy in sugar content $\tau < 0$ combined with a reduction in total price $t < 0$. We have illustrated this effect through the estimations provided in the Tables 3 and 4. Table 1 has already shown this results analytically for a monopoly where τ is negative when $\beta > 0.5$ a condition which holds for our baseline calibration. Numerical illustrations reinforce the message by showing that the policy is even more complex in a duopoly then when two variants are supplied by a single firm.

We now depart from out baseline calibration by considering alternative levels of misperception. Since our analytical results suggest that this parameter has a crucial impact on the design of the tax rule, it is important to consider scenarios with alternative assumptions.

We first consider a case with a larger degree of misperception namely $\beta = 0.4$. We already know from Table 1 that in this case the monopoly case calls for a tax on sugar content. Tables 5 and 6 present the different solutions for this level of misperception. Consistently sugar content for the monopoly would be taxed both in France and in the US, respectively at 0.0007 and 0.0013, but the output price is subsidized, which also in consistent with Table 1. In the duopoly case, a richer pattern of results emerges. Many more effects are now at work because of the strategic interaction and the two-stage nature of the game. Consequently, the country-specific parameters have crucial impact on the balance of the various effects. A tax on sugar content is now applied only to the low-level sugar content SSB in France and to the high-sugar content SSB in the US. Furthermore, in both countries the final price of the high-sugar content good is taxed $t_{2_F} = 0.0067$ and $t_{2_{US}} = 0.0211$. This reflects the fact that market power is reduced under duopoly so that there is no longer a need for a subsidy on all products to correct for this market power.

Finally, Tables 7 and 8 present the results considering no misperception ($\beta = 1$). Two facts should be highlighted. First, monopoly and collusion supply the first best levels of sugar content but underprovide quantity. Correction in quantities supplied calls for a decrease in the market price. However absent of mispersception, demand decreases with sugar content so that both effects in equation (25) go in the same direction and suppliers would tend to decrease SSB's sugar content below their optimal levels. Therefore a subsidy in sugar content needs to follow a decrease in price, and τ_1 and τ_2 are negative for a monopoly with a single and with two variants, both in France and in the US. Second, in a duopoly sugar content is taxed for each good and τ_1 and τ_2 are both positive, for both France and the US. This shows once again that the market structure has a drastic impact on the results. Results under single and multi product monopoly differ already significantly form those under perfect competition. And a duopoly involving strategic interactions represents an even more drastic departure with many more effects at work. Consequently, the country-specific parameters have crucial impact on the balance of the various effects.

8 Conclusion

The main lesson that emerges from this paper is that the design of the appropriate tax policy for real world SSB makets is a complex issue. In a textbook scenario with perfect competition, a Pigouvian tax on sugar content is sufficient to achieve both the optimal sugar content and output levels. Its rate corresponds to the mispercieved marginal health cost. This rule is fairly simple especially when, as we assume, the degree of misperception is the same for all. When the degree of misperception differs accross individuals a linear tax equal to the average degree

	First best	Monopoly	First Best	Duopoly	Collusion
s_1	12.44	14.30	7.47	8.55	8.58
s_2			14.93	17.58	17.16
θ_{p1}	0.0268	0.0306	0.0253	0.027	0.030
θ_{p2}			0.0298	0.031	0.032
% mkt_1	0.22	0.11	0.31	0.11	0.065
% mkt_2			0.31	0.09	0.065
q_1		0.38		0.21	0.24
q_2				0.43	0.45
Taxes implementing the first-best solutions					
$\bar{t}_1$		-0.0525		-14.98	-0.02
τ_1		-0.0028		2.01	-0.006
$t_1 = \bar{t}_1 + \tau_1 s_1^*$		-0.0869		-0.01	-0.06
q_1		0.29		0.17	0.17
$\bar{t}_2$				0.31	-0.08
τ_2				-0.02	-0.001
$t_2 = \bar{t}_2 + \tau_2 s_2^*$				-0.02	-0.09
q_2				0.35	0.35

Table 3: Taxes calibration implementing the first best for France. Parameters estimated at $\beta = 0.87$, $\gamma = 0.0006$, $\varepsilon_{xq} = -3.52$, $c = 0.0231$, $b = 0.0343$. Sugar content is in g/ℓ, prices and taxes are in €$/\ell$.

	First best	Monopoly	First Best	Duopoly	Collusion
s_1	22.11	25.42	13.27	15.20	15.25
s_2			26.53	31.24	30.50
θ_{p1}	0.0297	0.0364	0.0271	0.0306	0.035
θ_{p2}			0.0350	0.0374	0.039
% mkt_1	0.31	0.15	0.31	0.009	0.093
% mkt_2			0.31	0.012	0.093
q_1		0.76		0.40	0.47
q_2				0.81	0.89
Taxes implementing the first-best solutions					
$\bar{t}_1$		-0.165		0.4562	-0.0597
τ_1		-0.0049		-0.0379	-0.0109
$t_1 = \bar{t}_1 + \tau_1 s_1^*$		-0.274		-0.0459	-0.2043
q_1		0.53		0.31	0.31
$\bar{t}_2$				2.3452	-0.2387
τ_2				-0.0913	-0.0019
$t_2 = \bar{t}_2 + \tau_2 s_2^*$				-0.0781	-0.289
q_2				0.64	0.64

Table 4: Taxes calibration implementing the first best for the US. Parameters estimated at $\beta = 0.87$, $\gamma = 0.0006$, $\varepsilon_{xq} = -1.44$, $c = 0.0231$, $b = 0.0430$. Sugar content is in g/ℓ, prices and taxes are in €$/\ell$.

	First best	Monopoly	First Best	Duopoly	Collusion
s_1	12.44	31.11	7.47	18.61	18.67
s_2			14.93	38.24	37.33
θ_{p1}	0.0268	0.0306	0.0253	0.027	0.030
θ_{p2}			0.0298	0.031	0.032
% mkt_1	0.22	0.11	0.31	0.11	0.065
% mkt_2			0.31	0.09	0.065
q_1		0.83		0.47	0.51
q_2				0.94	0.99
Taxes implementing the first-best solutions					
$\bar{t}_1$		-0.0743		-2.3	-0.03
τ_1		0.0007		0.3071	-0.0040
$t_1 = \bar{t}_1 + \tau_1 s_1^*$		-0.0650		-0.0067	-0.0569
q_1		0.32		0.18	0.18
$\bar{t}_2$				0.125	-0.11
τ_2				-0.0079	0.0031
$t_2 = \bar{t}_2 + \tau_2 s_2^*$				0.0067	-0.060
q_2				0.39	0.39

Table 5: Taxes calibration implementing the first best for France with misperception set at $\beta = 0.4$, $\gamma = 0.0006$, $c = 0.0231$, $b = 0.0343$. Sugar content is in g/ℓ, prices and taxes are in €$/\ell$.

	First best	Monopoly	First Best	Duopoly	Collusion
s_1	22.11	55.28	13.27	33.06	33.17
s_2			26.53	67.95	66.33
θ_{p1}	0.0297	0.0364	0.0271	0.031	0.035
θ_{p2}			0.0350	0.039	0.039
% mkt_1	0.31	0.15	0.31	0.16	0.093
% mkt_2			0.31	0.13	0.093
q_1		1.64		0.88	1.03
q_2				1.76	1.92
Taxes implementing the first-best solutions					
$\bar{t}_1$		-0.2347		0.7546	-0.0845
τ_1		0.0013		-0.0585	-0.0072
$t_1 = \bar{t}_1 + \tau_1 s_1^*$		-0.2053		-0.0211	-0.1795
q_1		0.60		0.34	0.34
$\bar{t}_2$				-2.266	-0.338
τ_2				0.0862	0.0056
$t_2 = \bar{t}_2 + \tau_2 s_2^*$				0.0211	-0.1901
q_2				0.74	0.74

Table 6: Taxes calibration implementing the first best for the US with misperception set at $\beta = 0.4$, and for $\gamma = 0.0006$, $c = 0.0231$, $b = 0.0430$. Sugar content is in g/ℓ, prices and taxes are in €$/\ell$.

	First best	Monopoly	First Best	Duopoly	Collusion
s_1	12.44	12.44	7.47	7.44	7.47
s_2			14.93	15.30	14.93
θ_{p1}	0.0268	0.0306	0.0253	0.027	0.029
θ_{p2}			0.0298	0.031	0.032
% mkt_1	0.22	0.11	0.31	0.11	0.065
% mkt_2			0.31	0.09	0.065
q_1		0.33		0.19	0.21
q_2				0.373	0.40
Taxes implementing the first-best solutions					
$\bar{t}_1$		-0.0464		-17.56	-0.02
τ_1		-0.0037		2.350	-0.007
$t_1 = \bar{t}_1 + \tau_1 s_1^*$		-0.0929		-0.017	-0.07
q_1		0.29		0.17	0.17
$\bar{t}_2$				-0.875	-0.07
τ_2				0.056	-0.002
$t_2 = \bar{t}_2 + \tau_2 s_2^*$				-0.033	-0.10
q_2				0.34	0.34

Table 7: Taxes calibration implementing the first best for France if there is no misperception: $\beta = 1$, $\gamma = 0.0006$, $c = 0.0231$, $b = 0.0343$. Sugar content is in g/ℓ, prices and taxes are in €$/\ell$.

of misperception achieves a second-best solution; see Cremer et al. (2016). But the tax base continues to be solely the sugar content.[4]

But this results is valid in the textbook world only. We show that taxing sugar is indeed *necessary* but may not be *sufficient* to restore optimality in non-competitive settings. Under imperfect competition with endogenous product characteristic, the appropriate tax rule is more complex. We have illustrated this point by considering, first, the simplest form of imperfect competition, namely a monopoly supplying one and then two variants of SSB. Two sources of inefficiency have to be considered. First, market power leads to inefficient output levels and sugar content, even in the absence of misperception. Second, output level and sugar content are suboptimal because of misperception. A per unit tax proportional to sugar content is no longer sufficient. We show that, nevertheless, a per unit tax continues to be sufficient, but it must be an affine function of the sugar content. In other words, the per unit tax specification contains a constant which, for practical purposes, means that "liters of soda" must also be taxed, and that this tax is in part independent of the sugar content.

[4]See also see Allcott et al., (2019 a).

	First best	Monopoly	First Best	Duopoly	Collusion
s_1	22.11	22.11	13.27	13.22	13.27
s_2			26.53	27.18	26.53
θ_{p1}	0.0297	0.0364	0.0271	0.0306	0.035
θ_{p2}			0.0350	0.0374	0.039
% mkt_1	0.31	0.15	0.31	0.159	0.093
% mkt_2			0.31	0.129	0.093
q_1		0.66		0.35	0.41
q_2				0.71	0.77
Taxes implementing the first-best solutions					
$\bar{t}_1$		-0.1467		-0.781	-0.0528
τ_1		-0.0066		0.0549	-0.0119
$t_1 = \bar{t}_1 + \tau_1 s_1^*$		-0.293		-0.053	-0.211
q_1		0.51		0.31	0.31
$\bar{t}_2$				-1.774	-0.2112
τ_2				0.0629	-0.0040
$t_2 = \bar{t}_2 + \tau_2 s_2^*$				-0.106	-0.3168
q_2				0.61	0.61

Table 8: Taxes calibration implementing the first best for the US if there is no misperception: $\beta = 1$, and for $\gamma = 0.0006$, $c = 0.0231$, $b = 0.0430$. Sugar content is in g/ℓ, prices and taxes are in €$/\ell$.

We provide calibrations of such taxes if the SSB market in the US and in France were monopolies or duopolies. Our estimations show that trading off inefficient output levels and sugar content could call for subsidizing sugar content even if in the end a per unit tax is imposed on SSB, and if the laissez-faire is characterized by too much sugar content.

Perfect competition and monopoly are extreme forms of market structures. Most real world markets are oligopolies, which are "in between" these extremes but also raise different challenges because they involve strategic interaction. Even if we provide estimations for the case of a duopoly, soda tax design under oligopoly is still an open question. This paper represents only a first step, which, however, is already sufficient to show that the simple "tax grams of sugar" only recommendation is not a robust result. To account for the interaction between market power and misperception more instruments will be needed, and their appropriate use will depend on the specific characteristics of the considered market.

We have used the market of SSB and their sugar content as an application, however other markets would also be as suitable illustrations; for instance, the market of breakfast cereals and their sugar content, or, alternatively, the market of processed foods and their salt content. Furthermore, whatever the considered application, one has to keep in mind that nutritional policies should account for the global effects on the whole diet, accounting for substitution and complementary effects across final and intermediate goods. From that perspective, concentrating on a single harmful substance represents only a first step which has to be supplemented by a more comprehensive approach. These issues are left for future research.

Appendix A

When implementing FB while directly setting s^* the monopoly profit is given by

$$\pi = (q^* - c(s^*) - t)x$$

at that point, using (22)

$$\frac{\partial \pi}{\partial s} = (q^* - cs^* - t)\frac{\partial x}{\partial s} - cx^*$$
$$(q^* - cs^* - t)\left(-\frac{1}{2}\beta\gamma + \frac{q^*}{s^{*2}}\right) - cx^*$$

Now, simply set t to the level of $\bar{t}$ that descentralizes first best. Computing for our parameter values as calibrated in Appendix B we get the values of the derivative reported in Table 9.

	France	US
$\beta = 0.4$	0.0002	0.0004
$\beta = 0.87$	-0.0006	-0.0015
$\beta = 1$	-0.0008	-0.0020

Table 9: Value of the partial derivative of monopoly profit with respect to s, at (x^*, s^*), if the value of the tax was independent of s.

Appendix B

Obtaining numerical solutions for the optimal taxes requires the calibration of the parameters β, γ, c, and b. Starting with β, we use Allcott et al. (2019 b) who define the money metrics for consumer bias as the compensated price change that produces the same effect on demand as the bias does. The authors estimate that the average marginal bias across all the American households is \$1.13 cents/oz or €0.35/ℓ. In our model this corresponds to the value of $(1-\beta)h(s)$. Therefore, knowing $h(s)$ allow us to calibrate β.

The function $h(s) = \gamma s^2/2$ represents the health impact of SSB consumption (per liter) on individuals' utility, and it can be estimated from Wilde et al. (2019). They use a synthetic population of 1 million adults aged 35–80 years and micro-simulate the impact of a 0.01/oz tax on SSB, implying a reduction in SSB consumption, on lifetime health outcomes and cardiovascular diseases-related health care costs. Using the reduction of SSB consumption and the associated health gains we can determine the average impact of a liter of SSB on health, that is the function $h(s)$.

Starting with the health gains, Wilde et al. (2019) predict that a 0.01/oz tax on SSB implies a saving of 3.40 million quality-adjusted-life-years (QALYs) as well as \$45 million saved in health care costs (Table 4 in Wilde et al., 2019). We use the most conservative valuation of a QALY of the UK National Institut for Health and Care Excellence (NICE), £20 000, to estimate that approximately €76 200 million would be saved in both QALYs and health care costs.

We turn to estimate the decrease in consumption due to the tax. Using Wilde et al. (2019)'s average price after tax, the total taxes collected and the total SSB market value after taxes we infer a total of 233 068 million liters consumed after tax. The authors suppose a price-elasticity of -0.66 (based on the meta-analysis of Afshin et al., 2017) and that taxes are 100% passed through consumers. Consequently, the 16.9% price increase due to the \$0.01/oz tax implies a decrease of 29 355 million liters of SSB consumption. Averaging, we are thus able to estimate the impact of a liter of SSB on health and health care costs. This is simply €76 200 million/29

355 million/ℓ= €2.6/ℓ. Therefore the average health impact of a 1 liter of SSB is $h(s) = \gamma s^2/2 = 2.6$€. We suppose it to be the same across populations (consumers and countries) which is obviously a simplification but it represents a benchmark. Finally, we can set value of the parameter measuring perception of health effects β. Using both $(1-\beta)h(s) =$€$0.35/\ell$, and $h(s) = \gamma s^2/2 = 2.6$€, we get $\beta = 0.87$.

Turning to the parameter γ, we get it from $h(s) = \gamma/2s^2 = 2.6$, using the average sugar content of SSB in the French market ($s = 92.5g/\ell$, see Table 1 in Bonnet and Réquillart, 2013). Therefore $\gamma = 0.0006$.

To calibrate the parameter c in the cost function, we use Bonnet and Réquillart, (2013) observed average sugar content of $92.5g/\ell$ and the international market value of sugar of 0.25€/kg. We get $c = 0.0231$ €/ℓ. Bonnet and Réquillart, (2013)'s observed average sugar content relates to SSB traded in the French market. We assume it to be the same in the US.

Finally, the parameter b represents the upper limit of consumers' taste distribution over sugar content. Using (21) we compute elasticity of demand where θ_p is given by (20)

$$\varepsilon_{xq} = -\frac{\partial \theta_p}{\partial q}\frac{q}{b-\theta_p}, \tag{26}$$

and solving for b gives

$$b = -\frac{1}{s}\frac{q}{\varepsilon_{xq}} + \frac{q+\beta h(s)}{s}. \tag{27}$$

For both markets we use the observed average sugar content of $s = 92.5g/\ell$, reported by Bonnet and Réquillart, (2013). We allow b to be country specific. For the US market we use Allcott et al. (2019 b)'s estimated elasticity of -1.44, and average SSB price of 1.02 €/ℓ (1.12 USD/ℓ, Table 1 in Allcott et al., 2019 b), and get the value of $b = 0.0430$. As to France we use Bonnet and Réquillart, (2013)'s estimated elasticity of -3.52 and average price of 0.72€/ℓ and obtain $b = 0.0343$, slightly lower than in the US. The French SSB market is thus more reactive to price changes and has a lower intensity of taste for sugar.

Appendix C

8.1 Market supplying two varieties of SSB

Assume that two varieties of SSB are offered with $s_1 < s_2$. Each consumer with preferences given by (19) either consumes one unit of SSB, choosing between variants 1 and 2, or none at all. Let $\theta_{p_1}(q_1, s_1; \beta)$ denote the marginal consumer indifferent between consuming one unit of variant 1 or none at all; and $\theta_{p_2}(q_1, q_2, s_1, s_2; \beta)$ denote the marginal consumer indifferent between consuming one unit of variant 1 or one unit of variant 2. Respectively,

$$\theta_{p_1}(q_1, s_1; \beta) = \frac{q_1 + \beta h(s_1)}{s_1}, \tag{28}$$

$$\theta_{p_2}(q_1, q_2, s_1, s_2; \beta) = \frac{(q_2 - q_1) + \beta(h(s_2) - h(s_1))}{s_2 - s_1}. \tag{29}$$

Demand functions for each variety off SSB are thus:

$$x_1\,(q_1, q_2, s_1, s_2; \beta) = \frac{\theta_{p_2}(q_1, q_2, s_1, s_2; \beta) - \theta_{p_1}(q_1, s_1; \beta)}{b} \tag{30}$$

$$x_2\,(q_1, q_2, s_1, s_2; \beta) = \frac{b - \theta_{p_2}(q_1, q_2, s_1, s_2; \beta)}{b}, \tag{31}$$

And note that

$$\frac{\partial x_1\,(q_1, q_2, s_1, s_2; \beta)}{\partial q_i} = \frac{1}{b}\left(\frac{\partial \theta_{p_2}}{\partial q_i} - \frac{\partial \theta_{p_1}}{\partial q_i}\right) \tag{32}$$

$$\frac{\partial x_2\,(q_1, q_2, s_1, s_2; \beta)}{\partial q_i} = -\frac{1}{b}\frac{\partial \theta_{p_2}}{\partial q_i} \tag{33}$$

$$\frac{\partial x_1\,(q_1, q_2, s_1, s_2; \beta)}{\partial s_i} = \frac{1}{b}\left(\frac{\partial \theta_{p_2}}{\partial s_i} - \frac{\partial \theta_{p_1}}{\partial s_i}\right) \tag{34}$$

$$\frac{\partial x_2\,(q_1, q_2, s_1, s_2; \beta)}{\partial s_i} = -\frac{1}{b}\frac{\partial \theta_{p_2}}{\partial s_i}, \tag{35}$$

using the specified $h(s)$ and $c(s)$ functions, with,

$$\frac{\partial \theta_{p_1}}{\partial s_1} = -\frac{q_1}{s_1^2} + \frac{\beta\gamma}{2} \tag{36}$$

$$\frac{\partial \theta_{p_1}}{\partial q_1} = \frac{1}{s_1} > 0 \tag{37}$$

$$\frac{\partial \theta_{p_1}}{\partial s_2} = 0 \tag{38}$$

$$\frac{\partial \theta_{p_1}}{\partial q_2} = 0 \tag{39}$$

$$\frac{\partial \theta_{p_2}}{\partial s_1} = \frac{q_2 - q_1}{(s_2 - s_1)^2} + \frac{\beta\gamma}{2} > 0 \tag{40}$$

$$\frac{\partial \theta_{p_2}}{\partial q_1} = \frac{-1}{s_2 - s_1} < 0 \tag{41}$$

$$\frac{\partial \theta_{p_2}}{\partial s_2} = -\frac{q_2 - q_1}{(s_2 - s_1)^2} + \frac{\beta\gamma}{2} \tag{42}$$

$$\frac{\partial \theta_{p_2}}{\partial q_2} = \frac{1}{s_2 - s_1} > 0. \tag{43}$$

Two remarks are in order. Price effects go on the expected direction with demand for each good decreasing with own price and increasing in the other good's price. The effect of own or other good's sugar content on demand it is not so straightforward and depends on the level of misperception. Take for instances $\partial x_1/\partial s_1$. Since s_1 increases the marginal consumer θ_{p_2} but may as well increase θ_{p_1}, the overall effect is unclear; for high level of misperception (low β) demand for variant 1 increases.

8.2 First best

As for the one variant SSB case, to determine the first-best solution, we set the each variant consumer price equal to marginal cost (with respect to quantity) and substitute q_i by $c(s_i)$ in (28)-(29). The first best levels of sugar are then given by:

$$\{s_1^*, s_2^*\} = \arg\max \left\{ \int_0^{\theta_{p_1}(c(s_1),s_1;1)} m d\theta + \int_{\theta_{p_1}(c(s_1),s_1;1)}^{\theta_{p_2}(c(s_1),c(s_2),s_1,s_2;1)} (m - c(s_1) + \theta s_1 - h(s_1)) d\theta \right.$$

$$\left. + \int_{\theta_{p_2}(c(s_1),c(s_2),s_1,s_2;1)}^{b} (m - c(s_2) + \theta s_2 - h(s_2)) d\theta \right\} \tag{44}$$

In a market with two variants of SSB, the first best levels of sugar content can be implemented with variant specific affine per unit taxes $t_i = \bar{t}_i + \tau_i s_i^*$. These tax functions depend on the market structure, mainly in whether there is a monopoly supplying two variants of SSB or a duopoly in which each firm supplies one variant of SSB. These taxes are specified below for each market structure. For now note the first best solution for the specified functions $h(s)$ and $c(s)$ presented in Table 10.

8.3 Monopoly

A monopoly supplying two variants of SSB solves:

$$\{q_1^m, q_2^m, s_1^m, s_2^m\} = \arg\max \{x_1(q_1, q_2, s_1, s_2; \beta)(q_1 - \bar{t_1} - \tau_1 s_1 - c s_1) \tag{45}$$

$$+ x_2(q_1, q_2, s_1, s_2; \beta)(q_2 - \bar{t_2} - \tau_2 s_2 - c s_2)\} \tag{46}$$

The monopoly solution is presented in Table 10 and the taxes implementing the first best are presented in Table 11.

8.4 Duopoly

Consider two firms each supplying a variant of SSB, with firm 1 supplying the SSB with the lowest sugar content (s_1). Following the product differentiation literature we consider a sequential

	s_1	q_1	x_1	s_2	q_2	x_2
Monopoly	$\frac{2(b-c)}{5\gamma\beta}$	$\frac{2(b-c)(2b+3c)}{25\gamma\beta}$	$\frac{(b-c)}{5b}$	$\frac{4(b-c)}{5\gamma\beta}$	$\frac{2(b-c)(3b+7c)}{25\gamma\beta}$	$\frac{(b-c)}{5b}$
First best	$\frac{2(b-c)}{5\gamma}$	–	$\frac{2(b-c)}{5b}$	$\frac{4(b-c)}{5\gamma}$	-	$\frac{2(b-c)}{5b}$

Table 10: Monopolistic and first best solutions with two variants of SSB.

$\bar{t_1}$	$\frac{2(b-c)^2(\beta-2)}{25\gamma}$	$\bar{t_2}$	$\frac{8(b-c)^2(\beta-2)}{25\gamma}$
τ_1	$-\frac{(b-c)(1+2\beta)}{5}$	τ_2	$\frac{(b-c)(3-4\beta)}{5}$
$t_1 = \bar{t_1} + \tau_1 s_1^*$	$\frac{-2(a-b)^2(3+\beta)}{25\gamma}$	$t_2 = \bar{t_2} + \tau_2 s_2^*$	$\frac{-4(a-b)^2(1+2\beta)}{25\gamma}$

Table 11: Taxes implementing the first best for a monopoly with two variants of SSB.

game. In the first stage the firms simultaneously choose the sugar content of their variant and in the second stage they simultaneously set prices. We determine the subgame perfect equilibrium of this game.

The profit functions are:

$$\pi_1 = x_1(q_1, q_2, s_1, s_2; \beta)(q_1 - \bar{t_1} - \tau_1 s_1 - cs_1) \tag{47}$$

$$\pi_2 = x_2(q_1, q_2, s_1, s_2; \beta)(q_2 - \bar{t_2} - \tau_2 s_2 - cs_2), \tag{48}$$

with demand functions given by (30) and (31). We solve the game backwards.

8.4.1 Second stage: price competition.

Firms set prices taking as given sugar choices of the first stage and anticipating demand functions. Using the specified harm function $h(s)$ and cost function $c(s)$ first order conditions with respect to prices give

$$\frac{\partial \pi_1(.)}{\partial q_1} = \frac{1}{b}\left(\frac{\partial \theta_{p_2}}{\partial q_1} - \frac{\partial \theta_{p_1}}{\partial q_1}\right)(q_1 - \bar{t_1} - \tau_1 s_1 - cs_1) + \frac{1}{b}(\theta_{p_2} - \theta_{p_1}) = 0 \tag{49}$$

$$\frac{\partial \pi_2(.)}{\partial q_2} = \frac{1}{b}\left(-\frac{\partial \theta_{p_2}}{\partial q_2}\right)(q_2 - \bar{t_2} - \tau_2 s_2 - cs_2) + \frac{1}{b}(b - \theta_{p_2}) = 0 \tag{50}$$

Solving for consumer prices, we get the following equilibrium prices $q_1^d(s_1, s_2; T)$ and $q_2^d(s_1, s_2; T)$, where $T = \{\bar{t_1}, \bar{t_2}, \tau_1, \tau_2\}$:

$$q_1^d(s_1, s_2; T) = \frac{6cs_1s_2 + 2bs_1(s_2 - s_1) + 2(2\bar{t_1}s_2 + \bar{t_2}s_1) + 2s_1s_2(2\tau_1 + \tau_2) + \beta\gamma s_1(s_1^2 + s_2(1 - 2s_1))}{2(4s_2 - s_1)}$$

$$q_2^d(s_1, s_2; T) = \frac{s_2\left(2c(s_1 + 2s_2) + 4b(s_2 - s_1) + 2(\bar{t_1} + 2\bar{t_2}) + \beta\gamma(s_1^2 + s_2(s_1 - 2s_2)) + 2(s_1\tau_1 + 2s_2\tau_2)\right)}{2(4s_2 - s_1)}$$

8.4.2 First stage: choice of sugar content

At this first stage, firms chose simultaneously sugar content of their own variant, anticipating the equilibrium in prices of the second stage:

$$\pi_1 = x_1\left(q_1^d(s_1,s_2;T), q_2^d(s_1,s_2;T), s_1, s_2; \beta\right)(q_1^d(s_1,s_2;T) - \bar{t_1} - \tau_1 s_1 - cs_1) \tag{51}$$

$$\pi_2 = x_2\left(q_1^d(s_1,s_2;T), q_2^d(s_1,s_2;T), s_1, s_2; \beta\right)(q_2^d(s_1,s_2;T) - \bar{t_2} - \tau_2 s_2 - cs_2), \tag{52}$$

Focs are:

$$\frac{\partial \pi_1}{\partial s_1} = \frac{1}{b}\left(\frac{\partial \theta_{p_2}}{\partial s_1} + \frac{\partial \theta_{p_2}}{\partial q_1^d}\frac{\partial q_1^d}{\partial s_1} + \frac{\partial \theta_{p_2}}{\partial q_2^d}\frac{\partial q_2^d}{\partial s_1} - \left(\frac{\partial \theta_{p_1}}{\partial s_1} + \frac{\partial \theta_{p_1}}{\partial q_1^d}\frac{\partial q_1^d}{\partial s_1}\right)\right)(q_1^d - \bar{t_1} - \tau_1 s_1 - cs_1)$$
$$+ \frac{1}{b}(\theta_{p_2} - \theta_{p_1})\left(\frac{\partial q_1^d}{\partial s_1} - \tau_1 - c\right) = 0$$

$$\frac{\partial \pi_2}{\partial s_2} = \frac{1}{b}\left(-\frac{\partial \theta_{p_2}}{\partial s_2} - \frac{\partial \theta_{p_2}}{\partial q_1^d}\frac{\partial q_1^d}{\partial s_2} - \frac{\partial \theta_{p_2}}{\partial q_2^d}\frac{\partial q_2^d}{\partial s_2}\right)(q_2^d - \bar{t_2} - \tau_2 s_2 - cs_2)$$
$$+ \frac{1}{b}(b - \theta_{p_2})\left(\frac{\partial q_2^d}{\partial s_2} - \tau_2 - c\right) = 0$$

Equilibrium is given by the levels of sugar (s_1^d, s_2^d) that satisfy the above FOCs, and the prices $q_1^d(s_1^d, s_2^d; T)$, $q_2^d(s_1^d, s_2^d; T)$. Setting taxes to zero, we get the numerical solutions of the duopoly presented in Tables 3-8 for the different scenarios considered.

8.4.3 Tax solution

The government implements the first best level of sugar content for each product. Taxes are set so that firms' price and sugar content choices' satisfy first order conditions. We solve the following system:

$$\theta_{p_1}^d(q_1^d(s_1^*, s_2^*; T), s_1^*; \beta) = \theta_{p_1}^*(s_1^*)$$
$$\theta_{p_2}^d(q_1^d(s_1^*, s_2^*, T), q_2^d(s_1^*, s_2^*, T), s_1^*, s_2^*; \beta) = \theta_{p_2}^*(s_1^*, s_2^*)$$
$$\frac{\partial \pi_1}{\partial q_1}\left(\theta_{p_1}^d(q_1^d(s_1^*, s_2^*; T), s_1^*; \beta), \theta_{p_2}^d(q_1^d(s_1^*, s_2^*, T), q_2^d(s_1^*, s_2^*, T), s_1^*, s_2^*; \beta)\right) = 0$$
$$\frac{\partial \pi_2}{\partial q_2}\left(\theta_{p_1}^d(q_1^d(s_1^*, s_2^*; T), s_1^*; \beta), \theta_{p_2}^d(q_1^d(s_1^*, s_2^*, T), q_2^d(s_1^*, s_2^*, T), s_1^*, s_2^*; \beta)\right) = 0$$

Solving for $t_1, t_2, \tau 1, \tau 2$ we get the numerical solutions in the Tables 3-8.

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