Multiple Lenders, Strategic Default and Covenants[*]

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Abstract

We study capital markets subject to moral hazard when investors cannot prevent side trading, thereby facing an externality if firms raise funds from multiple sources. We analyze whether investors' ability to design financial covenants that may include exclusivity clauses mitigates this externality. Following covenant violations, investors can accelerate the repayment of their loan, adjust its size, or increase interest rates. Enlarging contracting opportunities generates a severe market failure: with covenants, equilibria are indeterminate and Pareto ranked. We show that an investors-financed subsidy scheme to entrepreneurs alleviates the incentive to overborrow and sustains the competitive allocation as the unique equilibrium one.

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1   Introduction

When they provide capital to firms, investors do not always control all the financial transactions firms enter into. To the extent that outside financing affects firms' behavior and their ability to meet contractual obligations, the possibility of side trading creates an externality for investors: if an entrepreneur borrows from multiple sources, she may have less incentives to improve financial performance, which in turn affects each lender's profit. Whether this externality can be internalized by letting agents design appropriate financial contracts is a central question for the regulation of modern capital markets. Yet, it remains largely unanswered. As a matter of fact, the financial literature has either evaded the issue by postulating that agents enforce exclusive relationships from the outset, or magnified the problem by considering an extreme form of nonexclusivity whereby few, if any, contractual instruments can limit side trading. Capital markets probably fall between these two extremes. On the one hand, no legal system imposes that entrepreneurs must raise funds from a single investor. On the other hand, financial transactions are seldom secret and investors can use the available information to design financial covenants in an attempt to curb firms' financing policy.

This paper combines these two elements to study the functioning of capital markets subject to moral hazard. Our fundamental insight is that competition over sophisticated contracts, that potentially include exclusivity clauses, leads to a severe market failure. The result challenges the conventional view that a system of contracts fully contingent on additional financing is able to enforce exclusive trading and foster market efficiency. As a consequence, to circumvent the externality induced by side trading, a mere increase in the transparency of trades may be more of a problem than a solution. Financial regulation should explicitly take into account the investors' strategic use of information.

Investors' contract design under the threat of competition is a relevant issue in corporate finance. In practice, firms can enter into multiple financial relationships and investors extensively use financial covenants in their contracts to define thresholds of financial ratios that firms must comply with.[1] Since violating such covenants triggers penalties, this is a way for investors to con-

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[1]Rauh and Sufi (2010) document that most large rated firms simultaneously use different types and sources of corporate debt. According to Roberts and Sufi (2009), almost 97% of credit agreements contain financial covenants, 90% of which limit implicitly or explicitly firms' ability to raise additional debt (see also Demiroglu and James (2010)).

trol firms' financing policy. Intuitively, the ability to design covenants should enhance competition by limiting firms' opportunity to raise additional funds, thereby eliminating the source of the externality. In a strategic context, however, covenants can act as a barrier to entry to protect incumbents' rents against the threat of new offers.[2]

These effects are analyzed in the following model. A representative entrepreneur seeks funds to invest in a project subject to moral hazard: she has to forgo a private benefit and exert effort to increase the project's profitability. A finite number of investors compete by simultaneously offering menus of financial contracts. If the entrepreneur's *aggregate* investment is observable, contracts can include financial covenants. The entrepreneur selects one contract in each menu, possibly raising funds from several investors, and chooses an effort level. If, in a given state of nature, the sum of payments promised to investors exceeds the firm's cash flow, then, given her limited liability, the entrepreneur *strategically* defaults.[3] All the cash flow is then seized by investors, but the entrepreneur's private benefit is inalienable.

Our benchmark is a fully nonexclusive setting: investors cannot write financial covenants, i.e. they are restricted to trade bilateral contracts with the entrepreneur. In this context, we provide a complete characterization of equilibrium allocations. Despite the fact that investors earn a positive profit at equilibrium, all aggregate allocations are constrained efficient. The result is robust, in the sense that it is established by allowing investors to post arbitrary menus of contracts.

We next introduce financial covenants. Covenants prescribe punishments to the entrepreneur when she departs from a target financing policy. To the extent that the latter is determined by the entire profile of investors' offers, this introduces a degree of "contractibility of contracts" and allows us to explore the investors' ability to write exclusive contracts in a strategic setting. In our analysis, the set of punishments available to investors depends on when the information on outside financing is obtained, and it is sufficiently large to include the penalties commonly observed following a covenant violation: acceleration of the loan repayment, adjustment of the initial loan size, or raise in the interest rate.

We first consider that investors only observe the firm's financing policy *after* the project is un-

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[2]The possibility that exclusive contracts play an anticompetitive role has been suggested by Aghion and Bolton (1987) in a monopolistic setting with no side trading externality.

[3]Default is denoted strategic because it is determined by the contracts chosen by the entrepreneur.

dertaken. Covenants then allow them to modify interest rates according to the entrepreneur's level of outside debt. The entrepreneur's limited liability sets an upper bound on the monetary penalties that any investor can impose when she trades with others. As a consequence, financial covenants do not have the power to simply forbid multiple financial relationships. They do however introduce new strategic interactions compared to the benchmark case of fully unobservable side trades. Each deviating investor can exploit the protection of covenants to serve the market alone by punishing, up to the realized cash flows, the entrepreneur's attempt to raise funds from multiple financiers. At the same time, any incumbent investor can write covenants to prevent his competitors from providing additional investment and complementing existing offers. We show that the combination of these two effects leads to an indeterminacy: a large number of Pareto ranked allocations are sustained at equilibrium.

We next modify the information structure and allow investors to observe outside financing *before* production takes place. This enlarges de facto the class of available financial covenants: investors can adjust their contractual terms according to the entrepreneur's intended financial policy. In particular, they can include loan acceleration clauses if they observe excessive debt issuance, or propose additional financing if they observe insufficient borrowing. This second feature can be interpreted as a contingent line of credit. It turns out that the strategic provision of lines of credit discourages more competitive offers backed by a loan acceleration clause. This reinforces the idea that enlarging investors' contractual opportunities does not enhance competition. Overall, the same indeterminacy result holds.

On the normative side, indeterminacy of market equilibria provides a natural setting to study how a proper design of the market fosters investors' incentives to supply capital. We construct a system of subsidies from investors to the entrepreneur, that grants her a transfer if her investment is too low. This system enables one investor to exploit gains from trade with a more competitive offer by alleviating the entrepreneur's resulting incentive to default. Investors do not choose the size of the subsidies but have the option to finance them or not. The decision results from comparing the cost of the subsidy for each investor and his gain from avoiding default when the transfer is provided to the entrepreneur. We show that a subsidy scheme can be designed so that all gains from trade are exploited, and only the competitive allocation is sustained at equilibrium. This result provides a possible foundation for regulatory schemes that make banks or financial intermediaries

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financially liable whenever their strategic interactions exacerbate the risk of default. Examples of related market institutions include the guarantee funds of central clearinghouses in derivative markets or the recent Single Resolution Fund (SRF) of the European banking union.

Relationship to the literature

Our paper is linked to the general issue of side trading in financial markets,[4] and more directly speaks to the literature on nonexclusive competition that endogenizes side trading opportunities as part of the strategies of competing investors. Bizer and DeMarzo (1992) and Kahn and Mookherjee (1998) are the first to establish that nonexclusivity creates an externality in financial markets subject to moral hazard. Parlour and Rajan (2001) and Bisin and Guaitoli (2004) explicitly consider strategic competition among lenders. They show that, compared to a hypothetical exclusive competition benchmark, nonexclusivity typically reduces trades and generates an extra profit for investors.[5] In these models, financial contracts are bilateral in essence: no supplier can condition his offers on the customer's entire profile of trades. Bilateral contracting does not fit well with the practice of modern capital markets in which investors set punishments based on entrepreneurs' financial policies. We therefore study financial relationships when investors write general contracts that may include exclusive clauses. We show that the use of such contracts generates a severe market failure. Multiple Pareto ranked allocations are sustained at equilibrium.

Equilibrium indeterminacy is a major result of the competing mechanism literature in which principals compete through mechanisms in face of several privately informed agents. Yamashita (2010), Peters and Troncoso-Valverde (2013), Peters (2015), and Han (2014) provide different versions of a Folk Theorem: if no restriction is put on the set of available mechanisms, then a very large number of allocations can be supported at equilibrium. These results crucially hinge on having multiple agents. A contribution of our analysis is hence to present a standard economic setting in which indeterminacy arises with a single agent thanks to the strategic role of covenants.[6]

Strategic default is also modeled as the entrepreneur's ability to divert cash instead of repaying

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[4]See for instance Allen and Gale (2004) or Farhi et al. (2009).

[5]A form of market breakdown also arises in adverse selection settings, where nonexclusivity often leads to Akerlof (1970)-like results (Attar et al. (2011) and Attar et al. (2014)).

[6]Similar insights arise in the game-theoretic analysis of Szentes (2015) who characterizes equilibria of games in which several principals can write potentially sophisticated contractible contracts.

investors in multiple lending relationships: Bolton and Scharfstein (1996) offer the view that having multiple lenders weakens the incentive to default by affecting ex post renegotiation between parties. Bolton and Jeanne (2009) apply this analysis to sovereign lending, showing that governments prefer to issue different types of debt. In these models, the incentive to default arises from the impossibility to write contracts contingent on cash flows. In contrast, we allow for contingent contracts, and we emphasize that the possibility to have multiple financiers is at the origin of the entrepreneur's willingness to default.[7]

The paper is also linked to the literature on covenants in financial contracts. The conventional approach is to study how covenants can mitigate agency conflicts between shareholders and debtholders (Smith and Warner (1979), Myers (1977)). Much attention has been devoted to explain how covenants affect renegotiation between parties (Gorton and Kahn (2000), Gârleanu and Zwiebel (2009)). We take a different view and investigate how the design of covenants affects competition between debtholders.

Several regulatory mechanisms have been suggested to mitigate the damaging implications of side trading. Some rely on enhancing the observability of trades. Acharya and Bisin (2014) construct a centralized clearing mechanism that sustains efficiency of market equilibria. Bennardo et al. (2015) study the role of credit bureaus in a fully nonexclusive setting and show that information sharing systems decrease investors' rents, although they can worsen credit rationing. Both mechanisms fail to take into account the strategic design of contracts by individual investors. We show that this possibility creates barriers to entry that resist regulations enhancing the observability of trades. In our analysis, financial covenants are responsible for market inefficiencies whenever investors can observe outside debt. Others point out the role of repayment rules in case of default. In particular, Bisin and Rampini (2006) put forward the idea that the institution of bankruptcy can improve on nonexclusive contractual relationships but is not a perfect substitute for exclusivity. In our setting, market failures arise even if a bankruptcy procedure imposes repayment priority or captures part of the entrepreneur's private benefit. In both cases, the threat of overborrowing is sufficiently strong to impede perfect competition. A third class of mechanisms involves transfers to distressed agents in order to affect their willingness to trade. Tirole (2012) and Philippon and Skreta (2012) stress the fact that a regulator can repurchase assets, reducing adverse selection

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[7]We argue in Section 4 that our results extend to the case in which the entrepreneur can also divert funds.

and restoring agents' participation on the private market. Our subsidy scheme exhibits a similar feature in a moral hazard setting. From the viewpoint of an investor, providing a subsidy modifies the entrepreneur's incentive to default which in turn affects the investor's willingness to pay for it. An important difference with Tirole (2012) and Philippon and Skreta (2012) is that in our case the subsidy is entirely financed by the lending sector without resorting to an external costly source of funds. We therefore provide a rationale for the private regulation of counterparty risk through a collective transfer system, in the spirit of the guarantor function performed by central clearinghouses.

The remainder of the paper is organized as follows. The model is exposed in Section 2. The benchmark case in which investors offer bilateral contracts is analyzed in Section 3, while Section 4 studies the impact of covenants on equilibrium outcomes. Section 5 is devoted to market design. Proofs are provided either in the Appendix or in the additional Appendix A. The additional Appendices B, C and D provide robustness results.

2 The model

We build on the standard capital market model of Holmstrom and Tirole (1997, 1998).

Agents, technology and preferences. We consider a production economy populated by a single representative entrepreneur and a finite number N of investors. At date 0, the entrepreneur owns a variable size project that generates at date 1 a random output over two verifiable states: an investment of $I \geq 0$ yields a cash flow GI with $G > 0$ if the project succeeds, and a cash flow of 0 if it fails. The probability distribution over states depends on an unobservable effort $e = \{L, H\}$ chosen by the entrepreneur. Let $(\pi_e, 1 - \pi_e)$ be the distribution induced by effort e, where π_e is the probability of success. Denote $e = H$ the high level of effort, and assume that $\pi_H > \pi_L$. If the entrepreneur selects $e = L$, she receives a private benefit $B \geq 0$ per unit invested in the project. As in Holmstrom and Tirole (1997, 1998), the investment project has a positive net present value if and only if the entrepreneur selects $e = H$, that is:

$$\pi_H G > 1 > \pi_L G + B. \tag{1}$$

The entrepreneur is risk-neutral and protected by limited liability. She has a verifiable endowment of $A > 0$ and can raise additional funds by trading financial contracts issued by competing investors. If she raises I units of funds, invests her endowment A, pays back R in case of success, and 0 in case of failure, her net payoff is $\pi_H\big(G(I + A) - R\big) - A$ if she chooses $e = H$, and $\pi_L\big(G(I + A) - R\big) + B(I + A) - A$ if she chooses $e = L$. The expected profit of investor i when he lends I_i to the entrepreneur and obtains R_i in case of success and 0 in case of failure, is $\pi_e R_i - I_i$ for a given effort $e \in \{L, H\}$.

Contracts, default, and equilibrium. Investors compete by offering arbitrary menus of financial contracts to the entrepreneur.[8] The entrepreneur can simultaneously trade with any subset of investors, optimally choosing the aggregate investment and the corresponding repayment by combining available contracts. The nature of a financial contract depends on the variables that we assume to be observable. In most of our analysis, a contract is an individual investment and a function that associates a repayment to each amount of aggregate outside financing. Given the entrepreneur's limited liability, the repayment is always set equal to zero if the project fails. More formally, a contract proposed by investor i is an array $C_i = (I_i, R_i(.))$, where I_i is the investment he supplies, and $R_i(.) : \mathbb{R}_+ \to \mathbb{R}$ is the function specifying the repayment he requires when the project succeeds, for each I. In Section 4.2, we allow investors to also observe the aggregate financing raised by the entrepreneur *before* production takes place and to modify their initial offer $(I_i, R_i(.))$ accordingly. The following timing of events describes our competition game:

1. Each investor i offers a menu M_i, that is a set of contracts which includes the null one $(0, 0)$.[9]

2. Given these offers, the entrepreneur chooses one contract in each menu and an effort level.

3. The uncertainty is realized and payments are made.

Being protected by limited liability, the entrepreneur can trade contracts which involve conflicting prescriptions. In these situations, the outside financing $I = \sum_i I_i$ is such that the aggregate *contractual* repayment $R = \sum_i R_i(I) > G(I + A)$, and we say that strategic default takes place.

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[8]Investors can therefore include contracts that are not traded at equilibrium. Menus are sufficiently general to reproduce any bilateral communication between investors and the entrepreneur (Martimort and Stole (2002), Peters (2001)).

[9]This is done to incorporate agents' participation decisions in a simple way.

The entrepreneur's payoff can hence be written as

$$U(I, R, e) = \begin{cases} \pi_H \big(G(I+A) - R \big)^+ - A & \text{if } e = H \\ \pi_L \big(G(I+A) - R \big)^+ + B(I+A) - A & \text{if } e = L, \end{cases} \qquad (2)$$

and her reservation utility is $U(0) \equiv (\pi_H G - 1)A$, which is strictly positive given (1).

Under strategic default, the entrepreneur chooses $e = L$ to obtain her private benefit. Investors cannot be repaid according to contractual terms, and a pro rata rule applies: each investor receives a share of the cash flow proportional to his investment.[10] The expected profit of investor i is

$$V_i(I_i, R_i(I), I, R, e) = \pi_e \Big[G\big(I+A\big) \frac{I_i}{I} \mathbb{1}_{\{R > G(I+A)\}} + R_i(I) \mathbb{1}_{\{R \leq G(I+A)\}} \Big] - I_i. \qquad (3)$$

A pure strategy for the entrepreneur is a mapping that associates to each profile of investors' menus $(M_1, M_2, ..., M_N)$ a vector of contracts $(C_1, C_2, ..., C_N)$ and an effort choice $e \in \{L, H\}$. A pure strategy for investor i is a menu M_i.[11] Throughout the paper, we restrict attention to pure strategy subgame perfect equilibria. Observe that the entrepreneur necessarily selects $e = H$ at equilibrium: given (1), investors' aggregate profit is negative if $e = L$, which implies that at least one active investor would strictly prefer not to participate.

Feasible and second-best allocations. We denote $\mathcal{H} = \{(I, R) \in \mathbb{R}_+^2 : U(I, R, H) \geq U(I, R, L)\}$ the set of (I, R) pairs inducing $e = H$ as an optimal choice, and $\mathcal{L} = \{(I, R) \in \mathbb{R}_+^2 : U(I, R, H) \leq U(I, R, L)\}$ those inducing $e = L$. For a given profile of menus, the entrepreneur's choices determine agents' final allocations, that is, investments and *actual* repayments. If there is no strategic default, investors are repaid according to contractual terms, in which case we refer to (I, R) as an aggregate allocation.

We say that an allocation $\{(I_1, R_1(I)), ..., (I_N, R_N(I)), (I, R)\}$ is feasible if $(I, R) \in \mathcal{H}$, and if it guarantees to any agent at least his reservation utility. The corresponding set $\mathcal{F}$ of *aggregate*

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[10]We discuss in the additional Appendix C the case in which one investor can issue a senior claim.

[11]We make no specific assumption on the menus available to investors. We only require that they are compact sets so that the entrepreneur's choice problem of maximizing (2) over contracts and effort admits a solution for every profile $(M_1, M_2, ..., M_N)$. A direct way to ensure this is to assume for instance that each I_i belongs to a closed interval and each $R_i(.)$ is a bounded function.

Figure 1: Set $\mathcal{F}$ of aggregate feasible allocations.

feasible allocations is depicted by the grey area in Figure 1. All such (I, R) allocations lie to the right of the indifference curve relative to the entrepreneur's reservation utility $U(0)$, and to the left of the line (O, C^c), which represents the aggregate zero profit condition for investors when $e = H$. Condition (1) ensures that $\mathcal{F}$ is non-empty: when $e = H$, the entrepreneur's marginal rate of substitution G is greater than the investors' one $\dfrac{1}{\pi_H}$. Last, to ensure that $e = H$ is optimal for the entrepreneur, any aggregate feasible allocation must lie below the line that corresponds to $U(I, R, H) = U(I, R, L)$, which has a slope $G - \dfrac{B}{\Delta \pi}$. In line with Holmstrom and Tirole (1998), we also assume that

$$G - \frac{B}{\Delta \pi} < \frac{1}{\pi_H}, \tag{4}$$

where $\Delta \pi = \pi_H - \pi_L$, which ensures that $\mathcal{F}$ is closed. The frontier $\Psi \equiv \{(I, R) \in \mathcal{F} : U(I, R, H) = U(I, R, L)\}$ in Figure 1 represents the set of second-best aggregate allocations: a social planner who cannot control the entrepreneur's effort but who can control final trades finds optimal to select an aggregate allocation on Ψ. Indeed, (1) and (4) guarantee that starting from any $(I, R) \in \Psi$, it is not possible to simultaneously increase the entrepreneur's payoff and the profit of all investors. Conversely, and using the same inequalities, one gets that for any allocation

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$\{(I_1, R_1(I)), ..., (I_N, R_N(I)), (I, R)\}$ such that $(I, R) \notin \Psi$, there is another feasible allocation which Pareto-dominates it. Two relevant second-best aggregate allocations are $C^c = (I^c, R^c)$, which maximizes the entrepreneur's payoff, and $C^m = (I^m, R^m)$, which maximizes the investors' joint profit.

3   Market equilibria with bilateral contracts

We first assume that investors post menus of contracts contingent on the success or failure state, but not on outside financing or on the final cash flow. This setting constitutes our benchmark and corresponds to the standard representation of nonexclusive competition under moral hazard. The credit card industry, in which credit card issuers cannot observe their customers' portfolio of credit cards, or the pure OTC markets, in which assets are traded outside of organized exchanges, are good examples of this environment. A contract is then a pair $(I_i, R_i) \in \mathbb{R}_+^2$, where I_i is an investment level, and R_i is the repayment required by investor i in the success state.

3.1   Equilibrium analysis

We derive below necessary conditions for an aggregate allocation $(I^*, R^*) \in \mathcal{F}$ to be supported at equilibrium.

For a given profile of menus $(M_1, M_2, ..., M_N)$, the entrepreneur determines her credit demand (I, R) by optimally combining investors' offers. Since for each $e \in \{H, L\}$, her preferences are linear in I and R, the corresponding choice problem is simple. As an illustration, suppose that $M_i = \{(I_i, R_i); (0, 0)\}$, with $I_i > 0$. If strategic default does not take place, the entrepreneur is willing to trade (I_i, R_i) if its price $\dfrac{R_i}{I_i}$ is smaller than her corresponding marginal rate of substitution, irrespective of the other offered menus.[12] If the entrepreneur chooses to default, she selects the highest investment contract in each menu in order to maximize her private benefit.

The analysis of investors' behavior is more involved. A crucial feature of nonexclusive competition is that every investor may gain by offering contracts that the entrepreneur accepts in combination with other offers. This makes the set of unilateral deviations very large compared to the

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[12]This idea is formalized in Lemma 2 in the Appendix, which is key to the proof of our results.

case in which exclusive relationships are assumed, and potentially difficult to characterize. Since there are gains from trade when the entrepreneur chooses $e = H$, there is always room for deviations that increase aggregate investment provided that they do not trigger $e = L$. The following proposition builds on this intuition and provides a central result of this section.

Proposition 1 *If (I^*, R^*) is an equilibrium aggregate allocation, then*

$$U(I^*, R^*, H) = \max_{\left((I_i, R_i) \in M_i\right)_{i=1,\ldots,N}} U(I, R, L). \tag{5}$$

Since the right-hand side of (5) cannot be greater than the equilibrium payoff $U(I^*, R^*, H)$, Proposition 1 is straightforward if $(I^*, R^*) \in \Psi$. If $(I^*, R^*) \notin \Psi$, the intuition for the proposition can easily be understood in a free entry equilibrium.[13] Suppose that an entrant deviates by offering, together with the null contract $(0, 0)$, the additional contract $\left(x, (\frac{1}{\pi_H} + \varepsilon)x\right)$, of price $\frac{1}{\pi_H} + \varepsilon$, with x and ε strictly positive. Choose ε small enough to guarantee that the price of this contract is strictly less than the entrepreneur's marginal rate of substitution G when $e = H$. Clearly, she picks this contract at the deviation stage. For (I^*, R^*) to be supported at equilibrium, this choice must induce $e = L$. Given that both x and ε can be arbitrarily small, investors' equilibrium menus must be such that (5) holds. In this nonexclusive context, (5) can be interpreted as an *aggregate* incentive compatibility constraint. The allocations satisfying (5) are such that the threat of overborrowing prevents any investor from profitably complementing existing offers. The welfare implications of aggregate incentive compatibility are derived in the next corollary.

Corollary 1 *If (I^*, R^*) is an equilibrium aggregate allocation, then $(I^*, R^*) \in \Psi$, i.e. equilibrium allocations are second-best efficient.*

Corollary 1 exploits a direct implication of Proposition 1: each contract traded at equilibrium, combined with her preferred options in all other menus, must yield the entrepreneur the same payoff whether she selects $e = H$ or $e = L$. If this *individual* incentive compatibility condition is violated, a single investor can straightforwardly deviate, even if his rivals post arbitrary menus. The proof of Corollary 1 shows that individual incentive contracts are not compatible with the aggregate condition (5) if (I^*, R^*) does not belong to the frontier Ψ.

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[13]The proof of Proposition 1 is established for any fixed number of investors.

Therefore, with bilateral contracts, investors successfully exploit the opportunity to complement their rivals' offers, exhausting thereby all gains from trade. This efficiency result sheds light on the welfare properties of nonexclusive markets under moral hazard. Indeed, while Kahn and Mookherjee (1998) and Bisin and Guaitoli (2004) obtain similar findings,[14] they only provide partial characterizations of market equilibria. In contrast, our result is robust in that it does not rely on exogenous restrictions on investors' menus or on the structure of equilibrium strategies.

3.2 Equilibrium aggregate allocations

We now provide a full characterization of equilibrium aggregate allocations in terms of the private benefit B. To this end, we say that moral hazard is mild if $B \leq \pi_H G - 1$, and that it is strong if $B > \pi_H G - 1$. If moral hazard is mild, the entrepreneur's marginal private benefit is lower than the marginal return of the project with $e = H$. The severity of moral hazard affects the set of profitable deviations and the resulting equilibrium outcomes, as formally stated below.

Proposition 2 *The following holds:*

1. *If $B \leq \pi_H G - 1$, any aggregate allocation $(I^*, R^*) \in \Psi$ can be supported at equilibrium.*

2. *If $B > \pi_H G - 1$, (I^m, R^m) is the unique equilibrium aggregate allocation.*

Irrespective of the severity of moral hazard, there is always an equilibrium in which investors earn a monopolistic profit. Suppose that one investor, say investor 1, offers $M_1 = \{C^m = (I^m, R^m), (0, 0)\}$, and all other investors $i \neq 1$ propose $M_i = \{(0, 0)\}$. The proof of Proposition 2 shows that no investor can profitably deviate by inducing the entrepreneur to select $e = H$. This is illustrated in Figure 2. Consider any investor $i \neq 1$: a deviation $C_i' = (I_i', R_i')$ is profitable only if $\frac{R_i'}{I_i'} > \frac{1}{\pi_H}$. However, following any such deviation, it is optimal for the entrepreneur to trade C_i' together with C^m and to select $e = L$, achieving $U(C_i' + C^m, L) > U(C_i', H)$. Therefore, the opportunity to trade the monopolistic allocation together with a deviating contract makes the low effort's threat fully credible.

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[14]Specifically, Proposition 5 in Kahn and Mookherjee (1998) and Proposition 4 in Bisin and Guaitoli (2004) provide the following constrained efficiency result: even when equilibria fail to be second-best efficient, they are typically efficient from the viewpoint of a planner who cannot control final trades (third-best efficiency). In our setting, all equilibrium aggregate allocations belong to the second-best frontier Ψ, which therefore coincides with the third-best one.
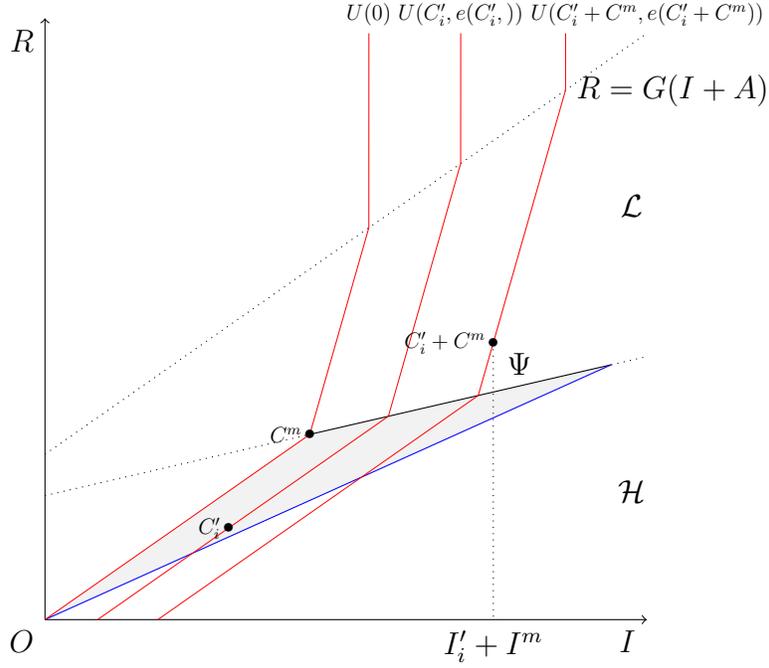
Figure 2: The monopolistic allocation equilibrium.

Proposition 2 also states that, if moral hazard is strong, the monopolistic allocation is the only one supported at equilibrium. The result is established by contradiction. Consider any aggregate allocation on the frontier Ψ, and any investor who provides funding at equilibrium. Linearity of the entrepreneur's preferences guarantees that this investor is indispensable. This implies that if he deviates by offering a higher-price, smaller-investment contract, the entrepreneur's payoff is reduced (irrespective of her effort choice). When B is high, her private benefit sharply decreases and she chooses $e = H$ which makes the deviation profitable. Monopoly then arises as the only equilibrium allocation.

When B is small, and moral hazard is mild, any aggregate allocation on Ψ can be sustained at equilibrium. The reason is that the above deviations induce $e = L$ as the private benefit decreases less than the entrepreneur's payoff under $e = H$. In this case, only one investor is active and there is at least another investor offering a positive investment contract. This contract is *latent*: despite not being traded it needs to be issued for the equilibrium to exist. The latent contract serves the role of a threat, that is, it ensures that the entrepreneur selects $e = L$ following any investor's deviation.

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Our benchmark incorporates the main insights of the literature on nonexclusive competition under moral hazard. As in the credit economy of Parlour and Rajan (2001), monopoly is sustained at equilibrium. As in the insurance settings of Kahn and Mookherjee (1998), Bisin and Guaitoli (2004) or Attar and Chassagnon (2009), latent contracts are used to sustain credit rationing and positive profit at equilibrium.[15] At the same time, we contribute to this literature by providing a complete equilibrium characterization and a full strategic analysis of a financial market in which no information is available on aggregate trades.[16]

4 Market equilibria with covenants

Corporate financing relationships are not well-described by bilateral contracts. Most financial contracts include covenants aimed at monitoring firms' financial decisions by specifying target debt ratios. In case of violations, creditors can accelerate the payment of their outstanding debt. Empirical evidence suggests that they can also increase interest rates or reduce future lending.[17]

Building on these insights, we allow financial contracts to be contingent on outside financing. To provide a thorough analysis of the strategic role of financial covenants, we distinguish two informational structures. First, we assume that outside financing is observed only after production takes place. This enables investors to require higher interest rates when undesired levels of debt are observed. We next allow investors to also observe the funds raised before they are invested. Investors can exploit this information to accelerate the repayment of their loan, or to modify their initial investment. We therefore consider the most favorable setting to enforce exclusive contracting: investors observe aggregate trades, and can use a very large set of punishments to deter departures from their desired financing policy.

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[15]A traditional criticism of latent contracts is that, to be attractive for the entrepreneur, they might be issued at a loss-making price for investors. In our context, credit rationing and positive profits are either supported by such latent contracts, or by the active C^m contract which is sufficient to sustain the monopolistic allocation equilibrium.

[16]Parlour and Rajan (2001) and Kahn and Mookherjee (1998) restrict investors to post single offers, while Bisin and Guaitoli (2004) also consider piecewise linear menus. Allowing investors to compete over arbitrary menus may sustain additional equilibria, as it is the case in the complete information setting of Martimort and Stole (2003) or in the adverse selection one of Attar et al. (2011).

[17]Roberts and Sufi (2009) report that when creditors respond to covenant violations, they increase interest rates on their loans in about half of the cases and that covenant violators exhibit a decline in their net debt issuing activity. Relatedly, Chava and Roberts (2008) find that covenant violations induce a 13% to 20% decline in the level of investment.

4.1 Covenant violation after investment

We explore here the situation in which outside funding is only observed ex post. Enlarging investors' contracting opportunities to include such debt-based covenants introduces new strategic interactions. On the one hand, they have access to an additional class of deviations. Any investor can set punishments to circumvent the entrepreneur's threat to overborrow. In particular, they can stipulate a raise in interest rates to eliminate the entrepreneur's rent when she chooses $e = L$ and repays all investors. To illustrate this procompetitive effect, consider the monopolistic equilibrium arising with bilateral contracts. With covenants, the simple offer (I^m, R^m) is not robust to entry. Any inactive investor can undercut it by proposing a small investment at a unit price slightly above $\frac{1}{\pi_H}$, writing covenants that trigger default if additional funding is raised. By construction, the entrepreneur is better off trading this deviating contract and choosing $e = H$, which makes the deviation profitable.

On the other hand, any investor can use covenants to prevent his competitors from proposing contracts to be traded in addition to his own. Under bilateral contracting, such deviations are key to establish Corollary 1 and to show that all gains from trade are exploited at equilibrium. Covenants could then undermine the constrained efficiency result. To isolate this latter anticompetitive effect, we first present a stripped-down version of our model in which $\pi_L = 0$, that is, strategic default always occurs when $e = L$ is chosen. This case corresponds to the standard framework of Parlour and Rajan (2001) which is extended to allow investors to use covenants contingent on outside financing.

Proposition 3 *If $\pi_L = 0$, any aggregate allocation $(I^*, R^*) \in \mathcal{F}$ such that $I^* \geq I^m$ is sustained at equilibrium.*

Proposition 3 states that the use of covenants leads to an indeterminacy of equilibrium allocations. To provide an intuition for this result it is useful to describe equilibrium strategies. Every investor proposes the same menu, which includes two non degenerate contracts on top of the null one. The entrepreneur chooses the same contract in each menu, i.e. every investor is active. Equilibrium contracts incorporate the following financial covenants. If the entrepreneur raises a large amount of debt from other investors, covenants impose large penalties which can be interpreted as an increase in interest rate. Given these covenants, the entrepreneur strategically defaults whenever

an investor tries to gain by complementing existing offers. Symmetrically, if the entrepreneur does not raise enough debt, covenants specify a reduction in the interest rate. This lower interest rate is designed to guarantee that no investor is indispensable at equilibrium. This in turn blocks any deviation of an investor who wishes to increase the price of his loan.

Equilibrium menus also include an additional high price, large investment contract. Although investors anticipate that it will not be traded at equilibrium, this second contract is used as a threat against the deviations of any investor trying to expand his market share. The contract is therefore strategically issued by active investors to protect their rents and to induce collusive outcomes at equilibrium.[18]

To clarify how inefficient allocations can be sustained at equilibrium, consider any $(I^*, R^*) \notin \Psi$. In contrast with bilateral contracts, the covenants issued by his competitors prevent a deviating investor from gaining by complementing existing offers. He must therefore endeavor to replace some (or all) of his competitors' offers. Such deviations entail proposing a substantial amount of investment. Since moral hazard is strong when $\pi_L = 0$,[19] the entrepreneur's private benefit from investing this amount together with the large loans issued by the others is greater than the payoff of trading with the deviator, provided that $I^* \geq I^m$. This shows that any (I^*, R^*) such that $I^* \geq I^m$ is supported at equilibrium: equilibrium allocations are therefore Pareto-ranked. Although all investors are active at equilibrium and each of them typically earns a strictly positive profit, the corresponding outcome is entry-proof: given equilibrium covenants, no potential entrant can do better than offering the null contract.[20]

If $\pi_L > 0$, a deviating investor can fully exploit the procompetitive power of covenants to undercut competing offers. The following proposition shows that indeterminacy and inefficiency of equilibrium allocations still arise for any value of π_L.

Proposition 4 *There exists a threshold $\underline{N}$ and a sequence of payoffs $(U_N)_{N > \underline{N}}$ with $U_N < U(I^c, R^c, H)$*

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[18]The entrepreneur is indifferent between trading equilibrium contracts and defaulting on these large investment contracts. In the latter case, investors lose money in the aggregate, which leads to question their rationality to post such contracts in the first place. The fact that all investors are active and make a positive profit at equilibrium ensures that issuing them can be optimal against the entrepreneur's trembling behaviors.

[19]If $\pi_L = 0$, then, given (4), we get $B > \pi_H G - 1$.

[20]Our equilibria can therefore exhibit an arbitrary number of inactive investors: the proof of Proposition 3 suggests that equilibrium allocations may alternatively be supported with two active investors who equally share the market and $N - 2$ inactive ones who only offer the null contract $(0, 0)$.

such that any aggregate allocation $(I^*, R^*) \in \mathcal{F}$ *satisfying*

$$U(I^*, R^*, H) \geq U_N \tag{6}$$

is sustained at equilibrium. When $B > \pi_H G - 1$, $\lim_{N \to \infty} U_N = U(0)$.

From (6), the set of equilibrium allocations is non-empty whenever $N > \underline{N}$. These allocations are characterized in terms of a threshold payoff U_N, which role can be understood as follows. For each $N > \underline{N}$, any inefficient equilibrium is in principle vulnerable to the undercutting of an investor who offers a contract including covenants designed to enforce exclusivity. When moral hazard is strong, as suggested in the discussion of Proposition 3, these offers are more likely to induce default as the amount of funds offered by the deviator gets larger, and can easily be circumvented. When moral hazard is mild, the opposite holds and deviations that entail a sufficiently large investment can in principle induce $e = H$. The threshold U_N is constructed to limit the maximal amount of funds that a deviator can propose to exploit this opportunity, thereby deterring such deviations.

A striking result of Proposition 4 is that, when moral hazard is strong, U_N tends to $U(0)$ as N goes to infinity, that is, the entrepreneur can achieve any payoff above $U(0)$. This stands in sharp contrast with standard insights that oligopolistic rents are dissipated in the limit. The reason is that when N is large, each investor's contract has a negligible impact on the entrepreneur's threat of default. In this case, the threat of default is sufficient to prevent even those deviations that provide a payoff to the entrepreneur slightly above $U(0)$. We then get a Folk Theorem-like result: every feasible allocation can be sustained at equilibrium. Therefore in the limit the level of B determines the set of equilibrium aggregate allocations, as in the bilateral contracting setting of Section 3.

In contrast with that of Proposition 3, the proof of Proposition 4 relies on asymmetric equilibrium strategies. A subset of investors are active and equally share the market by offering contracts with covenants similar to those exhibited in the case $\pi_L = 0$. The remaining investors post latent contracts in order to render the threat of default effective even when moral hazard is mild.[21] The investment available in each latent contract is decreasing with the total number of investors. As

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[21]Observe that, since Proposition 4 also holds for $\pi_L = 0$, Propositions 3 and 4 together imply that some aggregate allocations can be sustained with different equilibrium strategies. Therefore indeterminacy also prevails regarding investors' equilibrium menus.

N goes to infinity, the investment included in each latent contract tends to zero, and no investor is indispensable to provide the threat of default. The aggregate available funds can then be seen as a barrier to entry and the equilibrium interpreted as entry-proof.

The minimal number of investors $\underline{N}$ is finite, but it can in principle be large. We provide however in the additional Appendix B an alternative profile of investors' strategies guaranteeing that indeterminacy arises with $N = 2$ for the general case $\pi_L \geq 0$ if the project's profitability is sufficiently high.

4.2 Covenant violation before investment

So far, our modeling only allows covenants to specify contractual penalties that can be enforced *after* investment is undertaken. In practice, covenant violations also entitle investors to accelerate the repayment of their loan or to renegotiate credit facilities. We extend below our setting to introduce the possibility for investors to observe all funds raised by the entrepreneur *before* investment takes place. Investors can then demand the early repayment of all or part of their loan or alternatively provide additional liquidity. We interpret these features as loan acceleration clauses and contingent lines of credit, respectively.

To fix ideas, suppose that before investment takes place, any investor i can observe the aggregate initial financing, denoted I_0, raised by the entrepreneur. He can therefore commit to modify his initial arrangement according to I_0. In particular, he can withdraw any fraction of his initial offer in order to protect himself against the threat of overborrowing.[22] This could in principle foster competition. We show below that these procompetitive effects of loan acceleration clauses are undermined by the simultaneous issuance of lines of credit. As a consequence, equilibrium allocations remain indeterminate.

More formally, define a contract for investor i as $\left(I_i, R_i(.), I_i^+(.), R_i^+(.)\right)$ with $I_i \geq 0$, $I_i^+(I_0) \geq -I_i$ and $R_i(.)$ and $R_i^+(.)$ functions of the aggregate initial debt I_0 and of the final outside financing I. In this setting, a loan acceleration clause following an undesired level of debt I_0 is written: $I_i^+(I_0) = -I_i$ and $R_i^+(I_0, I) = -R_i(I_0, I)$ $\forall I$. Similarly, a line of credit can be written: $I_i^+(I_0) > 0$, and $R_i^+(I_0, I) = pI_i^+(I_0)$, with $I_i^+(I_0)$ being the credit limit available at a unit price p. The

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[22]This is indeed the most common rationale for the use of loan acceleration clauses in debt contracts.

following proposition summarizes our results.

Proposition 5 *Propositions 3 and 4 extend to the case in which investors can write covenants contingent on the initial debt I_0.*

When the initial debt is observable, an investor can try to enforce exclusivity by asking to be reimbursed if he observes additional debt issuance. Such a deviation is however blocked by investors' ability to offer lines of credit contingent on the initial debt. At equilibrium, investors do not need to ask for an acceleration of their loan, and the entrepreneur does not draw on any line of credit. Yet the presence of this additional source of liquidity undermines the threat of asking for an early loan repayment. In general terms, Proposition 5 shows that the joint issuance of ex post and ex ante debt-based covenants constitutes a barrier against any attempt of investors to enter the market using sophisticated contracts.[23]

It is interesting to discuss to what extent these financial covenants are consistent with some empirical observations on firms' credit relationships. To start with, the equilibrium strategies of Proposition 5 are such that investors do not ask for any acceleration of their loans. They do however specify changes in interest rates following the violation of financial covenants resulting from firms' multiple borrowing. This is in line with the observation that creditors do not ask for the early repayment of their loan following a covenant violation but rather use this right to modify other features of the firm's financial policy, like the interest rate (Roberts and Sufi (2009)).

Next, at equilibrium, lines of credit are issued, and are not drawn upon, unless the borrower modifies her investment policy. At the same time, investment is typically inefficiently low, and firms are credit-rationed. Relatedly, Sufi (2009) points out that lines of credit are not fully committed and that their access is contingent on firms' financial performance. This illustrates the idea that lines of credit are not a perfect substitute for liquidity, and are strategically offered by banks, as also emphasized by Acharya et al. (2014).

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[23]The distinction between raised and invested funds creates a possibility for the entrepreneur to divert funds, which can constitute an additional source of moral hazard. To illustrate its impact, suppose that the entrepreneur can divert a fraction $\gamma \in (0,1)$ of the investment $I + A$. The relevant incentive constraint is then $\pi_H\big(G(I + A) - R\big) \geq \pi_L\big(G(1 - \gamma)(I + A) - R\big) + \big(\gamma + B\big)(I + A)$, since diverting is optimal when $e = L$, but not when $e = H$. The constraint is binding whenever $R = \big(G - \dfrac{B + \gamma(1 - \pi_L G)}{\Delta\pi}\big)(I + A)$. If γ is not too large, conditions (1) and (4) are satisfied, and the possibility of fund diversion is hence equivalent to an increase in B, which reinforces our results.

More fundamentally, in our analysis, the strategic role of covenants is incorporated in investors' out-of-equilibrium offers. The *entire* equilibrium capital supply therefore matters to determine firms' investment policy. The recent paper of Degryse et al. (2015) is the first to investigate the effect of multiple lending on the supply of credit of a given bank. Their main result is that the bank's willingness to lend decreases when its customers contract loans with other lenders, unless the bank loan is fully secured. Our work, which explicitly derives investors' equilibrium best responses, provides further insights. Both the willingness to lend and the contracts issued by a bank depend on those covenants included in its competitors' loans, which should therefore be taken into account to identify banks' credit supply. When competitors do not use covenants, a bank can issue covenants that reduce the threat of dilution, leaving its capital supply unaffected. Facing such covenants, a bank might be unable to secure its loan, which induces a reduction in investment.

5   Market design under the threat of default

The protection offered by covenants turns out to be a double-edged sword. It encourages investors to offer more competitive contracts, but it also prevents entry of potential competitors. The latter effect dominates and a large number of low investment, inefficient allocations are sustained at equilibrium. On the normative side, this calls for a proper design of the market architecture.

At the root of market inefficiency is the investors' individual inability to prevent overborrowing and strategic default. It is therefore natural to ask whether this threat can be mitigated by appropriately designed bankruptcy rules. We show however in the additional Appendices C and D that our results survive the introduction of standard procedures. Precisely, Proposition C.1 analyzes the case in which one investor has repayment priority over the others, and Proposition D.1 that in which part of the entrepreneur's private benefit is seized under bankruptcy. In both cases, a large number of inefficient allocations are sustained at equilibrium.

We therefore consider an alternative way to circumvent overborrowing based on a market mechanism in which investors can contribute to the provision of a subsidy to the entrepreneur. This mechanism induces the entrepreneur *not* to default and sustains the competitive allocation as the unique equilibrium one. We introduce in the next subsection a subsidy mechanism and discuss

its empirical relevance in Subsection 5.2.

5.1 The subsidy game

Consider a subsidy scheme, conceived, say, by a market designer, such that for every observed outside financing I, investor i is asked to pay a transfer $T_i(I)$, with $T_i(0) = 0$. After the cash flow realization, the entrepreneur receives a (possibly null) subsidy equal to the sum of investors' transfers. This mechanism induces the following game:

1. Given $(T_1(.), T_2(.), ..., T_N(.))$, each investor i offers a menu M_i of contracts that can include any class of financial covenants.[24]

2. After observing all offers, each investor decides whether to pay his transfer or not.

3. The entrepreneur chooses a contract in each menu and makes her effort choice.

4. The uncertainty is realized, and payoffs are distributed according to financial contracts and to the subsidy scheme.

We focus on the case $\pi_L = 0$ which exacerbates the anticompetitive effect of covenants, and we consider pure strategy subgame perfect symmetric equilibria. The following proposition highlights the role of the subsidy in disciplining investors and resolving indeterminacy.

Proposition 6 *If $\pi_L = 0$, there is a profile of transfers $(T_1(.), T_2(.), ..., T_N(.))$ such that (I^c, R^c) is the unique aggregate allocation supported in a symmetric equilibrium of the subsidy game.*

The intuition for Proposition 6 is the following. Suppose that an allocation different from the competitive one is supported at equilibrium. In the absence of a subsidy scheme, we know from the previous section that the entrepreneur's threat to default hinders deviations by an investor who tries to exploit gains from trade. We show that there exist individual transfers that guarantee a profitable deviation for a well-chosen investor. The design of the subsidy scheme involves the following trade-off. On the one hand, the subsidy provided to the entrepreneur has to be sufficiently large to alleviate the threat of default and induce $e = H$ at the deviation stage. On the other

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[24]That is, the analysis applies to both the informational structures of Subsections 4.1 and 4.2.

hand, investors must be willing to pay their individual transfers. To ensure their participation, the mechanism is such that the entrepreneur's subsidy, that is, the sum of individual transfers, is strictly positive *only* if *all* investors accept to pay. This implies that the entrepreneur defaults in the absence of the subsidy. Also, given that investors observe all offers when deciding whether to pay or not, they can anticipate the entrepreneur's behavior. Each investor prefers to contribute whenever the cost to induce $e = H$ is smaller than his loss following default. We construct a subsidy scheme satisfying both requirements. Given this mechanism, no aggregate allocation different from (I^c, R^c) can be supported in a symmetric equilibrium. The corresponding transfers are simple: investors' payments are linear and increasing in the entrepreneur's investment unless $I = I^c$, in which case they are null.[25] This leaves (I^c, R^c) as the unique equilibrium aggregate allocation.

To implement the above mechanism, the market designer only needs to observe investors' participation decisions on top of the realized investment I. The subsidy scheme therefore relies on similar instruments compared to those available to investors.[26] Also, in contrast with standard taxation schemes, the mechanism cannot exclude non-participating agents from entering the market: it relies on a *voluntary* contribution of investors. The question therefore arises of whether the same subsidy scheme can be otherwise enforced by competing investors who individually design their transfer schedules. Intuition suggests that this is not the case. Allowing investor i to pay a transfer according to I is equivalent to letting him free to set a possibly negative repayment $R_i(.)$. Since we already allow repayments to take negative values in the general model of Section 2, the equilibrium characterization of Proposition 3 would not be affected. We therefore stress the need for a centralized entity with the power to commit to collect and redistribute resources according to the volume of trades.

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[25]To ensure equilibrium uniqueness in the investors' participation subgame, we design transfers to guarantee that participating is a dominant strategy. Precisely, each contributor is granted a reward if he participates and at least one of the others does not.

[26]Indeed, financial covenants contingent on realized investment allow de facto to control the entrepreneur's participation decision with every investor.

5.2 Guarantee funds as subsidy mechanisms

The history of financial regulation offers many examples of privately financed mechanisms that perform functions similar to those of our subsidy scheme. In modern futures markets, central clearinghouses set up and manage guarantee funds to provide insurance against counterparty default (Kroszner (1999)).[27] Credit markets also offer different types of rescue mechanisms supported by banks. During the Comptoir d'Escompte crisis in 1889 in Paris, the Banque de France granted a loan to the insolvent Comptoir d'Escompte, asking other banks to provide a guarantee to absorb losses on this loan (Hautcoeur et al. (2014)). A similar role was performed by the New York Clearing House Association (NYCHA) before the creation of the Federal Reserve System in 1913. During a crisis, the NYCHA could allow its members to issue loan certificates, the payment of which was guaranteed by the clearinghouse itself, that is, by the whole banking industry (Gorton (1985)). More recently, the emerging European banking union has decided the creation of a Single Resolution Fund that will be financed by European banks and that will help to restructure distressed banks.[28] Our analysis offers a rationale for market-based regulatory schemes that make banks or financial intermediaries liable for the negative externalities stemming from their strategic interactions. In the subsidy game, the threat to provide a subsidy has a disciplining effect on credit markets, enhancing competition and decreasing rents. While it is hard to assess to what extent actual guarantee mechanisms cope with counterparty externalities, some authors argue that they do have a disciplinary role. In his analysis of the 1987 financial markets crash, Bernanke (1990) points at the effectiveness of the clearinghouse institution as an insurance company. Riva and White (2011) find evidence that during the nineteenth century, the number of brokers' defaults at the Paris Bourse decreases as the size of the guarantee fund relative to the volume of transactions increases. Whether these insurance systems also affect the volume of trades and the size of the market remains an open question.

Our mechanism shares important features with the above institutions. In particular, the subsidy

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[27]OTC derivative markets can also organize some form of mutualization of losses across traders upon default of one party. For instance, the stock brokers on the forward market at the Paris Bourse in the nineteenth century organized a Common Fund to rescue defaulting brokers (Riva and White (2011)).

[28]Vice-President Michel Barnier, responsible for Internal Market and Services recently said: "To respond to the financial crisis, we have worked hard to improve the financial system so that banks pay for themselves if they have problems, and not the taxpayers. The detailed rules on resolution funds financed by the banking sector, adopted today, are an important step to making that a reality." European Commission Press Release, October 21, 2014.

scheme we suggest does not have the power to shut markets down: investors can refuse to subscribe to the system of transfers, and still offer financial contracts. Relatedly, the European banking union lets non-Euro-area banks free to participate to the Single Resolution Mechanism. Also, the financial contributions of banks increase in the volume of the available investment. In practice, the guarantee funds of clearinghouses are financed by members' contributions that depend on the volume of trades they generate, and therefore on their risk exposure.[29] An important difference with actual mechanisms is that our subsidy scheme does not involve a direct transfer from some banks to others. It is therefore immune from the criticism that such bailouts create moral hazard and induce the insured agents (banks or traders) to take excessive risk. Indeed, subsidizing the entrepreneur when she invests too little does not provide incentives for banks to lend excessively.

6   Conclusion

This paper shows that investors' competition over financial contracts incorporating exclusivity clauses does not necessarily lead to efficient outcomes in the presence of side trading. That is, letting investors free to design financial covenants may exacerbate the negative effects of the counterparty externalities. Our results suggest new ways to empirically identify investors' behaviors and the associated market outcomes. In particular, to measure the impact of side trading externalities in capital markets, one should look into the entire profile of investors' offers. Observing financial contracts that, although not traded, discourage the entry of rivals, along with covenants contingent on outside financing would provide (indirect) evidence of such externalities.

On the theoretical side, our analysis calls for further research in two directions. The first one relates to the degree of observability of outside financing. We focus on the two extreme cases of either fully unobservable or fully observable outside debt. However, it might be easier for investors to observe cash flows rather than total debt. For instance, some liabilities may be hidden in off-balance sheet items. This raises the question of whether cash flow-based covenants introduce new strategic effects. Intuitively, it could be difficult for an investor to detect multiple trades if the

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[29]These are recurrent features in the history of common funds: at the Paris Bourse in the nineteenth century, most of the Common Fund's revenue came from a stamp tax on the paper used by brokers for their operations (White (2007)). In the US, the Board of Trade Clearing Corporation created in 1925 built its reserve fund from the clearing fees charged to its members (Kroszner (1999)).

observed cash flow only provides an imperfect signal of outside debt, which might undermine the anticompetitive role of covenants.[30] More generally, since accounting principles determine the informativeness of financial variables, this analysis could provide novel insights on the economic implications of accounting standards.

The other interesting extension would be to explicitly consider repeated lending relationships in the presence of strategic default. In such contexts, covenants can threaten to reduce borrowers' future financing capacity. Reinforcing this coercitive role over a long horizon could thereby enhance their procompetitive effect. At the same time, the opportunities to side-trade extend to future periods, which could generate a novel source of market failure. Exploring this tradeoff is relevant to our understanding of the relationship between debt constraints and the performance of dynamic economies.

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[30]This effect cannot be captured in our model, in which both variables are informationally equivalent.

Appendix

We first state Lemma 1 which is used in most of our proofs.

Lemma 1 *The following holds:*

1. *Let $(M_1, M_2, ..., M_N)$ be a profile of arbitrary menus. Any entrepreneur's best response leading to strategic default induces a strictly negative profit to each investor $i = 1, 2, ..., N$.*

2. *Let $(M_1, M_2, ..., M_N)$ be a profile of menus of bilateral contracts. Any entrepreneur's best response leading to $e = L$ induces a strictly negative profit to each investor $i = 1, 2, ..., N$.*

PROOF

1. Suppose that $(M_1, M_2, ..., M_N)$ is a profile of arbitrary menus, and let $(I_i, R_i(.))$ be the contract chosen by the entrepreneur in M_i in a given best response. If the entrepreneur strategically defaults, the profit to investor i is $V_i = \pi_L G(\sum_i I_i + A)\dfrac{I_i}{\sum_i I_i} - I_i = I_i\big(\pi_L G - 1 + \dfrac{\pi_L G A}{\sum_i I_i}\big)$. It is immediate to check that V_i has the same sign as $V = \sum_i V_i = I\big(\pi_L G - 1\big) + \pi_L G A$. Observe also that since the entrepreneur finds optimal to strategically default, then $B(I + A) \geq \pi_H G A > A$, otherwise she would have not invested. We therefore have: $V < I\big(\pi_L G - 1\big) + \pi_L G A + B(I + A) - A = (B + \pi_L G - 1)(I + A) < 0$. It follows that $V_i < 0$ for each $i = 1, 2, ..., N$.

2. Suppose next that $(M_1, M_2, ..., M_N)$ are menus of bilateral contracts. Let $(I_i, R_i)_{i=1,2,...,N}$ be the array of entrepreneur's optimal choices in each menu M_i in a given best response. By assumption, $(I, R) = \big(\sum_i I_i, \sum_i R_i\big) \in \mathcal{L}$, which implies that $U(I, R, L) \geq U(I - I_i, R - R_i, L)$ for each $i = 1, 2, ..., N$. To complete the proof, consider the case in which the entrepreneur's best response entails no strategic default. In this case, the former inequality is equivalent to $\dfrac{R_i}{I_i} \leq \dfrac{B}{\pi_L} + G$. The profit to each investor $i = 1, 2, ..., N$ is $V_i = \pi_L R_i - I_i \leq I_i(\pi_L(\dfrac{B}{\pi_L} + G) - 1) = I_i(\pi_L G + B - 1) < 0$, where the last inequality follows from $I_i \geq 0$, and from (1). ∎

The following lemma characterizes the entrepreneur's optimal choices in the bilateral contracting setting of Section 3.

Lemma 2 *Consider the bilateral contracting game of Section 3 and assume that the entrepreneur chooses $e = H$. Then, for each given menu M_i one has:*

1. *If choosing $(0,0)$ in M_i is optimal for the entrepreneur, then each $(I_i, R_i) \in M_i\backslash(0,0)$ is such that $p_i \geq G$.*

2. *If each $(I_i, R_i) \in M_i\backslash(0,0)$ is such that $p_i > G$, then $(0,0)$ is the unique optimal choice in M_i for the entrepreneur.*

PROOF: Consider a given menu M_i, and let $p_i = \dfrac{R_i}{I_i}$ be the price of every contract $(I_i, R_i) \in M_i\backslash(0,0)$. The entrepreneur's marginal rate of substitution is equal to G when $e = H$.

1. Assume that $(0,0)$ is an entrepreneur's optimal choice in M_i. If there is a contract $(I_i, R_i) \in M_i$ such that $p_i = \dfrac{R_i}{I_i} < G$, then

$$U(\sum_{j\neq i}I_j + I_i, \sum_{j\neq i}R_j + R_i, H) = \pi_H\big(G(\sum_{j\neq i}I_j + I_i + A) - (\sum_{j\neq i}R_j + R_i)\big)^+ - A$$

$$= U\big(\sum_{j\neq i}I_j, \sum_{j\neq i}R_j, H\big) + \pi_H(GI_i - R_i) > U\big(\sum_{j\neq i}I_j, \sum_{j\neq i}R_j, H\big),$$

for each $(\sum_{j\neq i}I_j, \sum_{j\neq i}R_j)$. This contradicts the assumption that $(0,0)$ is optimal in M_i.

2. Assume that each contract in $M_i\backslash(0,0)$ is such that $p_i > G$, and let $(I_i, R_i) \neq (0,0)$ be an optimal choice in M_i. Then

$$U(\sum_{j\neq i}I_j + I_i, \sum_{j\neq i}R_j + R_i, H) = \pi_H\big(G(\sum_{j\neq i}I_j + I_i + A) - (\sum_{j\neq i}R_j + R_i)\big)^+ - A$$

$$= U\big(\sum_{j\neq i}I_j, \sum_{j\neq i}R_j, H\big) + \pi_H(GI_i - R_i) < U\big(\sum_{j\neq i}I_j, \sum_{j\neq i}R_j, H\big),$$

for each $(\sum_{j\neq i}I_j, \sum_{j\neq i}R_j)$. This contradicts the assumption that (I_i, R_i) is optimal in M_i. ∎

PROOF OF PROPOSITION 1 : We introduce the indirect utility functions

$$\bar{U}_{-i}(I_i, R_i, e) \equiv \max\left\{U\big(I_i + \sum_{j\neq i}I_j, R_i + \sum_{j\neq i}R_j, e\big) : (I_j, R_j) \in M_j \text{ for all } j \neq i\right\},$$

for $i = 1, 2, ..., N$. For a fixed profile of menus $(M_1, M_2, ..., M_N)$ and for a given effort e, the function $\bar{U}_{-i}(I_i, R_i, e)$ denotes the maximum payoff of the entrepreneur when she trades (I_i, R_i) with investor i and optimally chooses one contract in each of the menus offered by his rivals. Thus,

if (I^*, R^*) is an equilibrium allocation, we have $U(I^*, R^*, H) = \bar{U}_{-i}(I_i^*, R_i^*, H)$ for $i = 1, 2, ..., N$. Observe that each $\bar{U}_{-i}$ function is continuous in (I_i, R_i) by Berge's maximum theorem.

We now develop the proof, letting $(M_1^*, M_2^*, ..., M_N^*)$ be the equilibrium menus. If $(I^*, R^*) \in \Psi$, Proposition 1 is straightforward. Consider the case $(I^*, R^*) \notin \Psi$. If $(I^*, R^*) \neq (0, 0)$ there exists at least one investor i such that the entrepreneur chooses a contract $(I_i^*, R_i^*) \neq (0, 0)$ in M_i^*. It follows from Lemma 2 that $\dfrac{R_i^*}{I_i^*} \leq G$. Assume by contradiction that $U(I^*, R^*, H) > \max\limits_{((I_i, R_i) \in M_i)_{i=1,...,N}} U(I, R, L)$. Then, since $\max\limits_{((I_i, R_i) \in M_i)_{i=1,...,N}} U(I, R, L) \geq \bar{U}_{-i}(I_i^*, R_i^*, L)$, we get

$$U(I^*, R^*, H) = \bar{U}_{-i}(I_i^*, R_i^*, H) > \bar{U}_{-i}(I_i^*, R_i^*, L). \tag{7}$$

Suppose now that investor i deviates to the menu $M_i^\varepsilon = \{(0, 0), (I_i^* + \varepsilon, R_i^* + \dfrac{\varepsilon}{\pi_H} + \varepsilon^2)\}$ with $\varepsilon > 0$. Observe that $\dfrac{R_i^* + \dfrac{\varepsilon}{\pi_H} + \varepsilon^2}{I_i^* + \varepsilon} < \max\left\{\dfrac{R_i^*}{I_i^*}, \dfrac{\dfrac{\varepsilon}{\pi_H} + \varepsilon^2}{\varepsilon}\right\} \leq G$ for ε small. It follows from Lemma 2 that the entrepreneur chooses $(I_i^* + \varepsilon, R_i^* + \dfrac{\varepsilon}{\pi_H} + \varepsilon^2)$ in M_i^ε. Moreover, given (7) and the continuity of $\bar{U}_{-i}$, we have $\bar{U}_{-i}(I_i^* + \varepsilon, R_i^* + \dfrac{\varepsilon}{\pi_H} + \varepsilon^2, H) > \bar{U}_{-i}(I_i^* + \varepsilon, R_i^* + \dfrac{\varepsilon}{\pi_H} + \varepsilon^2, L)$ which guarantees that the entrepreneur chooses $e = H$ at the deviation stage. Since $\pi_H(R_i^* + \dfrac{\varepsilon}{\pi_H} + \varepsilon^2) - I_i^* - \varepsilon > \pi_H R_i^* - I_i^*$ the deviation is profitable, which contradicts the assumption that (I^*, R^*) is an equilibrium aggregate allocation. If $(I^*, R^*) = (0, 0)$, the same reasoning applies: letting any investor i deviate to $M_i^\varepsilon = \{(0, 0), (\varepsilon, \dfrac{\varepsilon}{\pi_H} + \varepsilon^2)\}$, it is immediate to check that $e = H$ is chosen and that the deviation is profitable. In particular, the proof establishes the stronger result

$$\forall i = 1, 2, \ldots, N \quad : \quad \bar{U}_{-i}(I_i^*, R_i^*, H) = \bar{U}_{-i}(I_i^*, R_i^*, L) = \max\limits_{((I_i, R_i) \in M_i)_{i=1,...,N}} U(I, R, L). \tag{8}$$

∎

PROOF OF COROLLARY 1: Consider the equilibrium menus $(M_1^*, M_2^*, ..., M_N^*)$. Let $z_{ji} \in M_j^*$ be an optimal choice in M_j^*, given that the entrepreneur has chosen (I_i^*, R_i^*) in M_i^*, and selects $e = L$. In particular, we denote $z_{ii} = (I_i^*, R_i^*)$, and we let $Z_i = \sum\limits_j z_{ji} \in \mathcal{L}$ be the corresponding aggregate allocation. Consider now the vector $d_{ji} = (z_{ji} - (I_j^*, R_j^*)) \equiv (\alpha_{ji}, \beta_{ji})$. Given the definition of Z_i, we have $Z_i = (I^*, R^*) + \sum\limits_j d_{ji}$. The system of equations in (8) can hence be rewritten as

$$\forall i = 1, 2, \ldots, N \quad : \quad U(I^*, R^*, H) = U(Z_i, L) = U\big((I^*, R^*) + \sum_j d_{ji}, L\big). \qquad (9)$$

Suppose, by contradiction, that $(I^*, R^*) \notin \Psi$. Then, for (9) to be satisfied, it should be that $\sum_j d_{ji} \in \mathbb{R}_{++}^2$ for each i. Take any $(i, j) \in \{1, 2, .., N\}^2$. See that each allocation $Z_j + d_{ji} = Z_j + (z_{ji} - (I_j^*, R_j^*))$ is available: the entrepreneur can achieve it by trading z_{ij} in each menu M_i^*, with $i \neq j$, and z_{ji} instead of (I_j^*, R_j^*) in M_j^*. In a similar way, each allocation $Z_i - d_{ji}$ can be achieved by letting the entrepreneur trade the same contracts needed to obtain Z_i, but choosing (I_j^*, R_j^*) instead of z_{ji} in M_j^*. Condition (9) therefore implies that

$$\forall (i, j) \in \{1, 2, .., N\}^2 : \qquad U(Z_j + d_{ji}, L) \leq U(Z_j, L), \qquad (10)$$

$$\forall (i, j) \in \{1, 2, .., N\}^2 : \qquad U(Z_i - d_{ji}, L) \leq U(Z_i, L). \qquad (11)$$

We now show that, given (10) and (11), each d_{ji} vector is such that

$$\forall (i, j) \in \{1, 2, .., N\}^2 : \quad \beta_{ji} \geq (G + \frac{B}{\pi_L})\alpha_{ji}. \qquad (12)$$

To establish the result, it is useful to denote $\mathcal{D} = \{(I, R) \in \mathbb{R}_+^2 : R > G(I + A)\}$. Consider any (i, j) pair. Suppose first that $\alpha_{ji} > 0$. In this case, (10) implies that $Z_j \notin \mathcal{D}$ and that $\beta_{ji} \geq (G + \frac{B}{\pi_L})\alpha_{ji}$, otherwise the payoff associated to $Z_j + d_{ji}$ would be greater than that corresponding to Z_j, and Z_j would fail to be optimal. Suppose next that $\alpha_{ji} < 0$. The same reasoning (starting by considering $Z_j - d_{ji}$) guarantees that (11) implies that $Z_i \notin \mathcal{D}$ and that $\beta_{ji} \geq (G + \frac{B}{\pi_L})\alpha_{ji}$. Suppose finally that $\alpha_{ji} = 0$. Then, by definition, $z_{ji} = (I_j^*, R_j^* + \beta_{ji}) \in M_j^*$. Since (I_j^*, R_j^*) is optimal in M_j^*, (10) implies that $\beta_{ji} \geq 0$ at equilibrium. This in turn establishes (12).

Taking the sum of all the inequalities (12), we get $\sum_j \beta_{ji} \geq (G + \frac{B}{\pi_L})\sum_j \alpha_{ji}$. That is, the vector $\sum_j d_{ji}$ has a slope greater than or equal to $G + \frac{B}{\pi_L}$. It hence follows from (9) that if $(I^*, R^*) \notin \Psi$, then for each $i \in \{1, 2, ..., N\}$, $Z_i \in \mathcal{D}$. Take now any i. Since $\sum_j d_{ji} \in \mathbb{R}_{++}^2$ whenever $(I^*, R^*) \notin \Psi$, there exists an associated j such that $\alpha_{ji} > 0$. Condition (10) then implies that $Z_j \notin \mathcal{D}$, which contradicts the assumption that for each $i \in \{1, 2, ..., N\}$ $Z_i \in \mathcal{D}$, and completes the proof. ∎

PROOF OF PROPOSITION 2:

Mild moral hazard : $B \leq \pi_H G - 1$. We show that any aggregate allocation $(I^*, R^*) \in \Psi$ is supported at equilibrium by the following strategies. Investor 1 offers $M_1^* = \{(I^*, R^*), (0, 0)\}$, investor 2 offers $M_2^* = \{(\bar{I}, \bar{R}), (0, 0)\}$. The investment level $\bar{I} = \dfrac{\pi_L}{\Delta \pi}(I^* + A)$ is such that

$$U(I^*, R^*, H) = U(I^*, R^*, L) = B(I^* + \bar{I} + A) - A, \tag{13}$$

and $\bar{R} = G(I^* + \bar{I} + A)$. Each other investor $i = 3, 4, ..., N$ offers $M_i^* = \{(0, 0)\}$.

Investing only $\bar{I}$ is not an optimal choice for the entrepreneur. This is because, given (1), we have $\dfrac{\bar{R}}{\bar{I}} = G + \dfrac{G \Delta \pi}{\pi_L} > G + \dfrac{B}{\pi_L}$ implying that, unless she defaults, the entrepreneur gets a payoff smaller than the reservation one. Observe also that, given $\bar{R}$, if $I^* + \bar{I}$ is invested the entrepreneur defaults and gets a payoff $B(I^* + \bar{I} + A) - A$. From (13), choosing (I^*, R^*) in M_1^*, $(0, 0)$ in M_2^* and selecting $e = H$ is an optimal choice for the entrepreneur. We now prove that no investor has a unilateral profitable deviation.

Deviations of investor 1. Without loss of generality, any deviation by investor 1 can be represented by a menu $M_1' = \{(I_1', R_1'), (0, 0)\}$. Given Lemma 1, any profitable deviation should induce the entrepreneur to choose $e = H$. In this case, the deviation is profitable only if

$$\pi_H R_1' - I_1' > \pi_H R^* - I^*. \tag{14}$$

Since $(I^*, R^*) \in \Psi$, (4) implies that (14) is satisfied only if $I_1' < I^*$ and $R_1' < R^*$. We therefore write $(I_1', R_1') = (I^* - \varepsilon, R^* - \eta \varepsilon)$, with $\eta \in (G - \dfrac{B}{\Delta \pi}, \dfrac{1}{\pi_H})$ and $\varepsilon \in (0, \dfrac{GI^* - R^*}{G - \eta})$. The lower bound on η ensures that, if she only trades the deviating contract, the entrepreneur selects $e = H$, while the upper bound ensures that (14) is satisfied. The upper bound on ε ensures that, if she only trades the deviating contract, the entrepreneur's payoff is above her reservation one. It follows from Lemma 2 that when she chooses $e = H$, the entrepreneur selects (I_1', R_1') in M_1', and $(0, 0)$ in M_2^*, since $\dfrac{\bar{R}}{\bar{I}} > G$. Given η and ε, choosing only (I_1', R_1') optimally induces $e = H$. Once again, when choosing $e = L$, she prefers not to invest rather than investing only $\bar{I}$. She therefore strategically defaults, since $\pi_L \big(G(I' + \bar{I} + A) - (R' + \bar{R}) \big) < 0$. Comparing her respective payoff under $e = H$ and $e = L$, one gets

$$\pi_H \big(G(I^* - \varepsilon + A) - (R^* - \eta \, \varepsilon) \big) = B(I^* + \bar{I} + A) - \pi_H (G - \eta) \varepsilon < B(I^* - \varepsilon + \bar{I} + A),$$

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where the last inequality obtains since the assumption of mild moral hazard together with $\eta < \dfrac{1}{\pi_H}$ and $\varepsilon > 0$ imply that $B \leq \pi_H(G - 1/\pi_H) < \pi_H(G - \eta)$. Thus, the entrepreneur finds optimal to default, contradicting the assumption that $e = H$ is an optimal choice.

Deviations of investor 2. Let $M_2' = \{(I_2', R_2'), (0, 0)\}$ be a deviating menu of investor 2. Once again, we restrict attention to deviations inducing the entrepreneur to choose $e = H$. In that case, it follows from Lemma 2 that for (I_2', R_2') to be traded by the entrepreneur, we must have $\dfrac{R_2'}{I_2'} \leq G$. Since $(I^*, R^*) \in \mathcal{F}$, we also have $\dfrac{R^*}{I^*} \leq G$. It follows that

$$U(I^* + I_2', R^* + R_2', H) \geq \max\{U(I_2', R_2', H), U(I^*, R^*, H)\}.$$

For (I_2', R_2') to be a profitable deviation, it must be $\dfrac{R_2'}{I_2'} > \dfrac{1}{\pi_H}$. It then follows from (4) that $\dfrac{R_2'}{I_2'} > G - \dfrac{B}{\Delta\pi}$ and $U(I^* + I_2', R^* + R_2', L) > U(I^* + I_2', R^* + R_2', H)$. This implies that the entrepreneur chooses $e = L$ following the deviation to M_2', which constitutes a contradiction. The same reasoning yields that none of the inactive investors $i = 3, ..., N$ has a unilateral deviation.

Strong moral hazard: $B > \pi_H G - 1$. First, we show that no aggregate allocation different from (I^m, R^m) can be supported at equilibrium. Suppose that $(I^*, R^*) \in \Psi \setminus \{(I^m, R^m)\}$ is an aggregate equilibrium allocation. There must hence be at least one traded contract, say (I_i^*, R_i^*), of price $p_i < G$. We then show that investor i can profitably deviate to $M_i' = \{(0, 0), (I_i^* - \varepsilon, R_i^* - \eta\varepsilon)\}$, with $\eta \in (G - \dfrac{B}{\Delta\pi}, \dfrac{1}{\pi_H})$ and ε small enough. If the entrepreneur selects $e = H$, then, by Lemma 1, the contract $(I_i^* - \varepsilon, R_i^* - \eta\varepsilon)$ is chosen in the deviating menu. In that case, the deviation is profitable by construction. The maximum payoff available at the deviation stage when $e = H$ is selected is

$$\bar{U}_{-i}(I_i^* - \varepsilon, R_i^* - \eta\varepsilon, H) = \varepsilon\,\pi_H(\eta - G) + \bar{U}_{-i}(I_i^*, R_i^*, H),$$

and the maximum payoff available at the deviation stage if $e = L$ is chosen is

$$\bar{U}_{-i}(I_i^* - \varepsilon, R_i^* - \eta\varepsilon, L) = \begin{cases} \varepsilon\,\pi_L(\eta - G) - B\varepsilon + \bar{U}_{-i}(I_i^*, R_i^*, L) & \text{if no default,} \\ -B\varepsilon + \bar{U}_{-i}(I_i^*, R_i^*, L) & \text{if default.} \end{cases}$$

In case of no default, following (8), we have that $\bar{U}_{-i}(I_i^* - \varepsilon, R_i^* - \eta\varepsilon, H) > \bar{U}_{-i}(I_i^* - \varepsilon, R_i^* - \eta\varepsilon, L) \Leftrightarrow \eta - G - \dfrac{B}{\Delta\pi} > 0$, which is satisfied by definition of strong moral hazard. Similarly, in

case of default, $\bar{U}_{-i}(I_i^* - \varepsilon, R_i^* - \eta\varepsilon, H) > \bar{U}_{-i}(I_i^* - \varepsilon, R_i^* - \eta\varepsilon, L) \Leftrightarrow \pi_H(\eta - G) + B > 0$, which is satisfied by construction whenever $B > \pi_H G - 1$. Thus, $(I^*, R^*) \in \Psi \setminus \{(I^m, R^m)\}$ cannot be an equilibrium allocation.

Second, we show that (I^m, R^m) is supported at equilibrium. Consider the following strategies. Investor 1 offers $M_1 = \{(I^m, R^m), (0, 0)\}$ and each other investor $i = 2, ..., N$ offers $M_i = \{(0, 0)\}$. Then, it is a best reply for the entrepreneur to select the contract (I^m, R^m) in M_1 and to choose $e = H$. Since investor 1 earns the monopolistic profit, only deviations of the inactive investors must be considered. The same reasoning developed for the deviations of investor 2 in the mild moral hazard case guarantees that no investor has a profitable deviation. ∎

PROOF OF PROPOSITION 3: Take any $(I^*, R^*) \in \mathcal{F}$ such that $I^* \geq I^m$ and consider the following profile of strategies. Each investor $i = 1, 2, ..., N$ offers $M^* = \{(0, 0); (\frac{I^*}{N}, R^*(.)); (\frac{\hat{I}}{N}, \hat{R}(.))\}$, with $R^*(I) = G(I + A)$ for $I \notin \{I^*, \frac{I^*}{N}\}$, and

$$R^*(I) = \begin{cases} \dfrac{R^*}{N} & \text{if } I = I^*, \\ R^* - \dfrac{N-1}{N}GI^* & \text{if } I = \dfrac{I^*}{N}. \end{cases}$$

The investment level $\hat{I}$ is such that

$$U(I^*, R^*, H) = B(\hat{I} + A) - A, \tag{15}$$

which guarantees that $\hat{I} \geq I^* \geq 0,$[31] and $\hat{R}(I) = G(I + A) \; \forall I \in \mathbb{R}_+$. It is immediate to verify that choosing the investment $\frac{I^*}{N}$ in each menu and selecting $e = H$ is an optimal choice for the entrepreneur. Observe also that the repayment $R^*(\frac{I^*}{N})$ guarantees that the entrepreneur achieves her equilibrium payoff by trading with only one investor. This is turn implies that $U(I^*, R^*, H)$ is available to the entrepreneur if any of the investors withdraws his offer. Consider now investors' deviations. Without loss of generality, any unilateral deviation can be represented by a menu $M' = \{(\frac{I^*}{N} + I', \frac{R^*}{N} + R'(.)), (0, 0)\}$ with $I' \in [-\frac{I^*}{N}, I^c - \frac{I^*}{N}]$. Since $\pi_L = 0$, it follows from Lemma 1 that any profitable deviation necessarily induces $e = H$. We then have that

$$R'(I) > \frac{1}{\pi_H}I', \tag{16}$$

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[31] As $U(I^*, R^*, H) \geq B(I^* + A) - A$.

with I being the aggregate investment traded at the deviation stage. If, following the deviation, the entrepreneur chooses $e = L$, then, as $\pi_L = 0$, and given (15), she gets

$$U_{sd} = U(I^*, R^*, H) + B\big(I' + \frac{I^*}{N} - \frac{\hat{I}}{N}\big).$$

Consider the entrepreneur's payoff when she chooses $e = H$. Assume first that several contracts are traded at the deviation stage. Given equilibrium covenants, $e = H$ is an optimal choice only if the corresponding investment is $I = I^*$. The deviating contract should hence be such that $I' = k\frac{I^*}{N}$, with $k \in \{1, ..., N-2\}$ and $N - k - 1$ being the number of non deviating lenders who actively trade at the deviation stage.[32] The corresponding entrepreneur's payoff is

$$\pi_H\big(G(I^* + A) - \frac{N-k}{N}R^* - R'(I^*)\big)^+ - A < U(I^*, R^*, H) + \frac{k}{N}(\pi_H R^* - I^*),$$

where the inequality follows from (16). Observe that, since $\pi_H R^* - I^* = (\pi_H G - 1)I^* - (U(I^*, R^*, H) - U(0))$, the entrepreneur strategically defaults if $\frac{k}{N}\big(U(I^*, R^*, H) - U(0)\big) + \frac{B}{N}(I^* - \hat{I}) \geq \frac{k}{N}(\pi_H G - 1 - B)I^*$ or, equivalently, if

$$U(I^*, R^*, H) - U(0) - \frac{B}{k}(\hat{I} - I^*) \geq (\pi_H G - 1 - B)I^*, \tag{17}$$

which right-hand side is negative because $I^* \geq 0$, and $B > \pi_H G - 1$.[33] Since, for any $(I^*, R^*) \in \mathcal{F}$, the left-hand side of (17) is increasing in k, and the right-hand side is decreasing in I^*, a sufficient condition for the inequality to hold for all (k, I^*) is that it is satisfied for $k = 1$ when the right-hand side equal to zero. That is, we have to check that

$$U(I^*, R^*, H) - U(0) - B(\hat{I} - I^*) \geq 0, \tag{18}$$

which, given (15), is equivalent to $B(I^* + A) - A \geq U(0) \Leftrightarrow I^* \geq I^m$ since $\pi_L = 0$.

Suppose now that the entrepreneur only trades with the deviating investor, that is $I = \frac{I^*}{N} + I'$. In this case, choosing $e = H$ yields the payoff

$$\pi_H\big(G(\frac{I^*}{N} + I' + A) - \frac{R^*}{N} - R'(\frac{I^*}{N} + I')\big)^+ - A < \frac{U(0)(N-1) + U(I^*, R^*, H)}{N} + (\pi_H G - 1)I'.$$

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[32]Observe that one cannot have $N - 1$ non deviating lenders who actively trade. In that case, I' is equal to zero and the deviating lender who offers $\frac{I^*}{N}$ cannot increase his profit since $U(I^*, R^*, H)$ is available to the entrepreneur at the deviation stage.

[33]The last inequality follows from (4) when $\pi_L = 0$.

Since the entrepreneur's equilibrium utility remains available at the deviation stage, the right-hand side of the last inequality must a fortiori be strictly greater than $U(I^*, R^*, H)$, which implies that $I' > 0$. The entrepreneur therefore finds optimal to strategically default if

$$\frac{N-1}{N}\big(U(I^*, R^*, H) - U(0)\big) - \frac{B}{N}(\hat{I} - I^*) \geq (\pi_H G - 1 - B)I'. \tag{19}$$

Since $N \geq 2$, and because $\pi_H G - 1 - B < 0$ when $\pi_L = 0$, any (I^*, R^*) which satisfies (18) also satisfies (19). This implies that each $(I^*, R^*) \in \mathcal{F}$ such that $I^* \geq I^m$ is supported at equilibrium. ∎

PROOF OF PROPOSITIONS 4 and 5: See additional Appendix A.

PROOF OF PROPOSITION 6: We design the schedules $T_1(.), ..., T_k(.), ..., T_N(.)$, which identify the transfers to be paid by investors if the project succeeds contingent on the array of investors' participation decisions. Fix an arbitrary $k \in \{1, \ldots, N\}$. For each investor $i \neq k$, we let

$$T_i(I) = \begin{cases} \dfrac{1}{\pi_H}\dfrac{1-\delta}{N}I & \text{if } I \in [0, I^c), \\ 0 & \text{if } I \geq I^c, \end{cases} \tag{20}$$

with $\delta \in (0, 1 - B)$, if all investors agree to participate. In addition, we let

$$T_i(I) = \begin{cases} -\nu & \text{if } I \in [0, I^c), \\ 0 & \text{if } I \geq I^c, \end{cases} \tag{21}$$

for some $\nu > 0$, if investor i agrees to participate and at least one of the others does not. In this case, investor i receives a positive reward. Finally, $T_i(I) = 0$ for all $I \geq 0$ if investor i does not participate. Consider then investor k. We have $T_k(I) = 0 \; \forall I \geq 0$ if all remaining $N-1$ investors pay for the transfer, and $T_k(I) = K\nu \; \forall I \geq 0$ if only K of them decide to do so. In this last case, investor k effectively rewards his competitors, which guarantees feasibility of the mechanism. The subsidy to the entrepreneur is therefore

$$T(I) = \begin{cases} \dfrac{1}{\pi_H}\dfrac{N-1}{N}(1-\delta)I & \text{if all investors participate,} \\ 0 & \text{if at least one investor does not participate.} \end{cases} \tag{22}$$

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The proof is by contradiction. Consider any equilibrium allocation $(I^*, R^*) \neq (I^c, R^c)$ supported by investors' menus $M_1^* = \ldots = M_N^*$. In a symmetric equilibrium, $(I_i^*, R_i^*) = (\frac{I^*}{N}, \frac{R^*}{N})$ for each i and each investor earns the same profit. Thus, given that the transfer schedules are not symmetric, no transfer is provided at equilibrium and we have $U(I^*, R^*, H) \geq B(\hat{I} + A) - A$, with $\hat{I} = \sum_i \hat{I}_i$, and $\hat{I}_i = \frac{\hat{I}}{N}$ being the highest investment level in M_i^*. We show that, given the schedules $(T_1(.), T_2(.), \ldots, T_N(.))$, investor k has a profitable deviation. Suppose that he deviates to the menu $M_k' = ((0, 0), (I_k' + \varepsilon, R_k'(.)))$ with $I_k' = I^* - \frac{N-1}{N} \frac{\pi_H R^* - I^*}{\pi_H G - 1}$ and $\varepsilon > 0$. The function $R_k'(.)$ is such that

$$R_k'(I) = \begin{cases} \frac{R^*}{N} + \frac{1}{\pi_H}\left(I_k' - \frac{I^*}{N} + \varepsilon\right) + \varepsilon^2 & \text{if } I = I_k' + \varepsilon, \\ G(I + A) & \text{if } I \neq I_k' + \varepsilon. \end{cases}$$

For $\varepsilon = 0$, the pair $(I_k' + \varepsilon, R_k'(I_k' + \varepsilon))$ is such that $U(I_k', R_k'(I_k'), H) = U(I^*, R^*, H)$ and $\pi_H R_k'(I_k') - I_k' = \pi_H \frac{R^*}{N} - \frac{I^*}{N}$. That is, $(I_k' + \varepsilon, R_k'(I_k' + \varepsilon))$ lies at the intersection of the entrepreneur's equilibrium indifference curve with the investor k's equilibrium isoprofit line. In addition, since $(I^*, R^*) \neq (I^c, R^c)$, $(I_k' + \varepsilon, R_k'(I_k' + \varepsilon)) \in int(\mathcal{F})$ for ε small enough. The deviation is designed to induce the entrepreneur to trade $(I_k' + \varepsilon, R_k'(I_k' + \varepsilon))$ and to choose $e = H$. It is immediate to see that, given M_k', if the entrepreneur chooses $e = H$, she can get a payoff above the equilibrium one by choosing $(I_k' + \varepsilon, R_k'(I_k' + \varepsilon))$. In this case, however, given $R_k'(.)$, she finds optimal to trade with investor k alone. It follows that, as long as $\varepsilon > 0$, M_k' is a profitable deviation for investor k whenever $e = H$ is chosen. We now show that if investors agree to finance the subsidy to the entrepreneur, she strictly prefers to choose $e = H$ following this deviation. This is indeed the case if

$$U(I^*, R^*, H) + (\pi_H G - 1 - \varepsilon)\varepsilon + \frac{N-1}{N}(1 - \delta)(I_k' + \varepsilon) > B(\hat{I} + A) - A + B\left(I_k' + \varepsilon - \frac{\hat{I}}{N}\right),$$

where the left-hand side is equal to $U(I_k' + \varepsilon, R_k'(I_k' + \varepsilon), H)$. Choose ε small enough to ensure $(\pi_H G - 1 - \varepsilon) > 0$; since $\frac{I_k'}{N} \leq \frac{I^*}{N} \leq \frac{\hat{I}}{N}$, a sufficient condition to verify the above inequality is

$$U(I^*, R^*, H) - (B(\hat{I} + A) - A) + \frac{N-1}{N}(1 - \delta)I_k' > B\frac{N-1}{N}I_k',$$

which is satisfied because $U(I^*, R^*, H) \geq B(\hat{I} + A) - A$, and $\delta < 1 - B$.

Consider then the investors' participation subgame. Given (20) and (21), it is a dominant strategy for each nondeviating investor $i \neq k$ to pay for the transfer. In particular, paying is an optimal choice when investor i anticipates that all other investors pay if $\frac{1-\delta}{N}(I'_k + \varepsilon) < \frac{\hat{I}}{N}$ which, given that $\hat{I} > 0$, $\delta < 1$, and $I'_k \leq I^* \leq \hat{I}$, is satisfied for ε sufficiently small. It follows that providing the subsidy to the entrepreneur is the unique equilibrium outcome of the continuation game. This in turn guarantees that M'_k induces the entrepreneur to choose $e = H$, and constitutes a profitable deviation for investor k.

To complete the proof, see that given the schedules $(T_1(.), T_2(.), ..., T_N(.))$, there exists a profile of symmetric menus $(M_1^*, ..., M_N^*)$ which supports (I^c, R^c) as an equilibrium allocation. This follows straightforwardly from the fact that $T(I) = 0$ for $I \geq I^c$ and that (I^c, R^c) can be supported at equilibrium with the symmetric strategies exhibited in Proposition 3.

The proof extends to the case in which investors can write covenants contingent on the initial debt I_0 (see additional Appendix A). ∎

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PROOF OF PROPOSITION 4: Define

$$\gamma \equiv \max\{\frac{G}{B(G - \frac{B}{\Delta\pi})}, \frac{G}{(\pi_H G - 1)(G - \frac{B}{\Delta\pi})}, \left(\frac{\pi_H G - B}{B}\right)^2 \frac{1}{\pi_H(G - \frac{B}{\Delta\pi})}\} > 0,$$

and

$$U_N = U_B + \frac{2\gamma}{\sqrt{N} - 1}(U(I^c, R^c, H) - U_B) \text{ with } U_B = \max\{U(0), U(I^c, R^c, H) - BI^c\}.$$

Let us consider a number of investors N satisfying $N > \underline{N} = (2\gamma + 1)^2$. Observe that $U_N < U(I^c, R^c, H)$ for any $N > \underline{N}$. Note also that, when $B > \pi_H G - 1$, we have $U_B = U(0)$ so that $\lim_{N \longrightarrow \infty} U_N = U(0)$ as stated in Proposition 4.

We exhibit a profile of investors' strategies that supports any allocation (I^*, R^*) satisfying (6) at equilibrium. Each investor $i = \{1, 2, ..., K\}$, with $K \equiv \lfloor\sqrt{N}\rfloor$, offers $M_i^* = \{(0, 0); (\frac{I^*}{K}, R_i^*(.))\}$, with $R_i^*(I) = G(I + A)$ for $I \notin \{I^*, \frac{I^*}{K}\}$, and

$$\begin{cases} R_i^*(I^*) = \frac{R^*}{K}, \\ R_i^*(\frac{I^*}{K}) = (R^* - \frac{K-1}{K}GI^*). \end{cases}$$

Each investor $j = \{K + 1, ..., N\}$ offers $M_j^* = \{(0, 0); (\frac{\bar{I}}{N - K}, \bar{R}_j(.))\}$ with $\bar{R}_j(I) = G(I + A) \ \forall I \in \mathbb{R}_+$. The investment level $\bar{I}$ is such that

$$U(I^*, R^*, H) = B(I^* + \bar{I} + A) - A. \tag{A.1}$$

Choosing $\frac{I^*}{K}$ in each menu and selecting $e = H$ is an optimal choice for the entrepreneur. Observe also that the repayment $R_i^*(\frac{I^*}{K})$ guarantees that $U(I^*, R^*, H)$ is available to the entrepreneur if any of the investors withdraws his offer.

Now consider the investors' deviations. We first remark that every profitable deviation must induce the entrepreneur to choose $e = H$. Given Lemma 1, a single investor may achieve a positive profit by inducing $e = L$ only if the entrepreneur trades several contracts out of equilibrium and does not default. Given the equilibrium covenants, this is possible only if she takes up an aggregate loan of I^* and chooses $e = L$, which yields the entrepreneur a payoff smaller than the available equilibrium one, and cannot be an optimal choice.

We next show that there are not unilaterally profitable deviations for investors, which sustains the allocation (I^*, R^*) at equilibrium. In particular, a deviation of investor $i \in \{1, \cdots, K\}$ can be characterized, without loss of generality, by the menu $M_i' = \{(\frac{I^*}{K} + I_i', \frac{R^*}{K} + R_i'(.)), (0,0)\}$, with $I_i' \in \left[-\frac{I^*}{K}, I^c - \frac{I^*}{K}\right]$. Similarly, a deviation of any investor $j \in \{K+1, \cdots, N\}$ can be characterized by the menu $M_j' = \{(\frac{\bar{I}}{N-K} + I_j', R_j'(.)), (0,0)\}$, with $I_j' \in \left[-\frac{\bar{I}}{N-K}, I^c - \frac{\bar{I}}{N-K}\right]$. Observe that for any such deviation to be profitable when the entrepreneur chooses the aggregate investment I and the effort $e = H$, one needs that

$$R_i'(I) > \frac{1}{\pi_H} I_i', \quad R_j'(I) > \frac{1}{\pi_H}\left(I_j' + \frac{\bar{I}}{N-K}\right), \tag{A.2}$$

for $i \in \{1, \cdots, K\}, j \in \{K+1, \cdots, N\}$. If, following any such deviation, the entrepreneur defaults, her payoff is

$$U_{sd} = U(I^*, R^*, H) + BI_h',$$

with $h = i, j$. We now evaluate the entrepreneur's payoff when $e = H$ is chosen.

First, consider a deviation of an active investor i. If the entrepreneur only trades with investor i at the deviation stage, her payoff is

$$\pi_H\left(G(\frac{I^*}{K} + I_i' + A) - \frac{R^*}{K} - R_i'(I_i' + \frac{I^*}{K})\right)^+ - A < \frac{K-1}{K}U(0) + \frac{U(I^*, R^*, H)}{K} + I_i'(\pi_H G - 1), \tag{A.3}$$

where the inequality follows from (A.2). Since the equilibrium utility is available at the deviation stage, the right-hand side of (A.3) must be strictly larger than $U(I^*, R^*, H)$, which implies that $I_i' > 0$. Thus, if U_{sd} is greater than the right-hand side of (A.3), the entrepreneur finds it optimal to strategically default. This is the case whenever

$$\frac{K-1}{K}\left(U(I^*, R^*, H) - U(0)\right) \geq (\pi_H G - 1 - B)I_i'. \tag{A.4}$$

We show that each $(I^*, R^*) \in \mathcal{F}$ satisfying (6) also satisfies (A.4). We consider two cases:
1. If $B > \pi_H G - 1$, then since the right-hand side of (A.4) is negative for each $I_i' > 0$, (A.4) is satisfied by any $(I^*, R^*) \in \mathcal{F}$.
2. If $B \leq \pi_H G - 1$, a sufficient condition for (A.4) to hold is

$$U(I^*, R^*, H) - U_B \geq \frac{1}{K-1}(U_B - U(0)), \tag{A.5}$$

in which I_i' is replaced by I^c in (A.4). Note that, as $K - 1 \geq \sqrt{N} - 1$, we have that $\frac{2}{\sqrt{N} - 1} \geq \frac{1}{K-1}$. Moreover, as

$$\gamma \geq \frac{G}{B(G - \frac{B}{\Delta\pi})} > \frac{\pi_H G}{B} > \frac{\pi_H G - 1 - B}{B} = \frac{U_B - U(0)}{U(I^c, R^c, H) - U_B},$$

condition (6) implies (A.5) thus the result.

If several contracts are traded at the deviation stage, then, given the equilibrium covenants, $e = H$ is an optimal choice only if the aggregate investment is I^*. In this case, we necessarily have $I'_i = k\frac{I^*}{K}$, with $k \in \{1, ..., K - 2\}$. The entrepreneur's payoff when $e = H$ is chosen is then

$$\pi_H\big(G(I^* + A) - \frac{[K - (k+1)] + 1}{K}R^* - R'_i(I^*)\big) - A < U(I^*, R^*, H) + \frac{k}{K}(\pi_H R^* - I^*), \quad \text{(A.6)}$$

where the inequality follows from (A.2). The entrepreneur strategically defaults if U_{sd} is greater than the value in the right-hand side of (A.6) which, since $\pi_H R^* - I^* = (\pi_H G - 1)I^* - (U(I^*, R^*, H) - U(0))$, yields

$$\frac{k}{K}\big(U(I^*, R^*, H) - U(0)\big) \geq \frac{k}{K}(\pi_H G - 1 - B)I^*. \quad \text{(A.7)}$$

Since each $(I^*, R^*) \in \mathcal{F}$ satisfying (6) is such that $U(I^*, R^*, H) \geq U_B$, any of these allocations also satisfies (A.7).

Suppose now that an inactive investor j deviates. If the entrepreneur only trades with investor j at the deviation stage, given (A.2), her payoff when $e = H$ is chosen is bounded above by $U(0) + (\pi_H G - 1)\big(\frac{\bar{I}}{N - K} + I'_j\big)$. The entrepreneur finds therefore optimal to strategically default if

$$U(I^*, R^*, H) - U(0) \geq I'_j(\pi_H G - 1 - B) + \frac{\bar{I}}{N - K}(\pi_H G - 1). \quad \text{(A.8)}$$

From (A.1), we get

$$B\bar{I} \leq (\pi_H G - 1 - B)I^* + U(0) + (1 - B)A. \quad \text{(A.9)}$$

Given (A.9), a sufficient condition for (A.8) is then

$$U(I^*, R^*, H) - U(0) \geq (\pi_H G - 1 - B)I'_j$$
$$+ \frac{1}{N - K}\frac{\pi_H G - 1}{B}\big(U(0) + (1 - B)A + (\pi_H G - 1 - B)I^*\big).$$
$$\text{(A.10)}$$

We show that each $(I^*, R^*) \in \mathcal{F}$ satisfying (6) also satisfies (A.10). We consider two cases:

1. If $B > \pi_H G - 1$, we remark that the inequality $U(I^*, R^*, H) - U(0) \geq (-\frac{\bar{I}}{N-K})(\pi_H G - 1 - B) + \frac{\bar{I}}{N-K}(\pi_H G - 1) = B\frac{\bar{I}}{N-K}$ is weaker than (A.8). Indeed, we obtain the right-hand side by replacing in (A.8) I'_j by its lower bound $-\frac{\bar{I}}{N-K}$. Given (A.9), and since $\pi_H G - 1 - B < 0$, (A.8) is a fortiori weaker than $U(I^*, R^*, H) - U(0) \geq \frac{1}{N-K}(U(0) + (1-B)A)$, which in turn implies (A.10). To show that this inequality holds for each $(I^*, R^*) \in \mathcal{F}$, observe that, since $N \geq 3$ by construction, we get $\frac{2}{\sqrt{N}-1} \geq \frac{1}{N-\sqrt{N}-1} \geq \frac{1}{N-K}$. In addition, using $\pi_H G - 1 < B < 1$ we have

$$\gamma \geq \frac{G}{(\pi_H G - 1)(G - \frac{B}{\Delta\pi})} > \frac{(\pi_H G - B)\left(\frac{1}{\pi_H} - (G - \frac{B}{\Delta\pi})\right)}{(\pi_H G - 1)(G - \frac{B}{\Delta\pi})} = \frac{(\pi_H G - B)A}{(\pi_H G - 1)I^c} = \frac{U(0) + (1-B)A}{U(I^c, R^c, H) - U(0)}.$$

Condition (6) implies then that

$$U(I^*, R^*, H) - U(0) \geq \frac{1}{N-K}\frac{U(0) + (1-B)A}{U(I^c, R^c, H) - U(0)}(U(I^c, R^c, H) - U(0)) = \frac{1}{N-K}(U(0) + (1-B)A),$$

thus the result.

2. If $B \leq \pi_H G - 1$, replacing I'_j and I^* by I^c, a sufficient condition for (A.10) is

$$U(I^*, R^*, H) - U(0) \geq (U_B - U(0)) + \frac{1}{N-K}\frac{\pi_H G - 1}{B}\left(U_B + (1-B)A\right).$$

As in the former case, $\frac{2}{\sqrt{N}-1} \geq \frac{1}{N-K}$. Moreover,

$$\gamma \geq \left(\frac{\pi_H G - B}{B}\right)^2 \frac{1}{\pi_H(G - \frac{B}{\Delta\pi})} \geq \left(\frac{\pi_H G - 1}{B}\right)\left(\frac{\pi_H G - B}{B}\right)\left(1 + \frac{\frac{1}{\pi_H} - (G - \frac{B}{\Delta\pi})}{(G - \frac{B}{\Delta\pi})}\right)$$

$$\geq \left(\frac{\pi_H G - 1}{B}\right)\left(\frac{\pi_H G - B - 1}{B} + \frac{\pi_H G - B}{B}\frac{A}{I^c}\right)$$

$$\geq \frac{\pi_H G - 1}{B}\frac{(\pi_H G - 1 - B)I^c + A(\pi_H G - B)}{BI^c} = \frac{\pi_H G - 1}{B}\frac{U_B + (1-B)A}{U(I^c, R^c, H) - U_B}. \qquad (A.11)$$

Then, (6) implies that

$$U(I^*, R^*, H) - U_B \geq \frac{1}{N-K}\frac{\pi_H G - 1}{B}\frac{U_B + (1-B)A}{U(I^c, R^c, H) - U_B}(U(I^c, R^c, H) - U_B),$$

thus the result.

If several contracts are traded at the deviation stage, then, given the equilibrium covenants, $e = H$ is an optimal choice only if the aggregate investment is I^*. In this case, we necessarily have $I'_j = k\frac{I^*}{K} - \frac{\bar{I}}{N-K}$, with $k \in \{1, ..., K-1\}$. The entrepreneur's payoff when $e = H$ is chosen is then

$$\pi_H\big(G(I^* + A) - \frac{K-k}{K}R^* - R'_j(I^*)\big) - A \leq U(I^*, R^*, H) + \frac{k}{K}(\pi_H R^* - I^*), \quad \text{(A.12)}$$

where the inequality follows from (A.2). The entrepreneur finds optimal to strategically default if U_{sd} is greater than the value in the right-hand side of (A.12), which is the case if

$$B\big(\frac{k}{K}I^* - \frac{\bar{I}}{N-K}\big) \geq \frac{k}{K}(\pi_H R^* - I^*).$$

Using (A.9) and $\pi_H R^* - I^* = (\pi_H G - 1)I^* - (U(I^*, R^*, H) - U(0))$, a sufficient condition for (A.12) is

$$\frac{k}{K}\big((1+B-\pi_H G)I^* + U(I^*, R^*, H) - U(0)\big) \geq \frac{1}{N-K}\big((\pi_H G - 1 - B)I^* + U(0) + (1-B)A\big).$$
$$\text{(A.13)}$$

The left-hand side of (A.13) is increasing in k. This is straightforward if $B > \pi_H G - 1$. In the case $B \leq \pi_H G - 1$, the result follows from $U(I^*, R^*, H) - U(0) \geq U_B - U(0) = (\pi_H G - 1 - B)I^c$ and $(I^c - I^*) \geq 0$. It is hence enough to verify (A.13) for $k = 1$, that is,

$$U(I^*, R^*, H) - U(0) \geq \frac{K}{N-K}\big(U(0) + (1-B)A\big) + \frac{N}{N-K}(\pi_H G - 1 - B)I^*. \quad \text{(A.14)}$$

We show that each $(I^*, R^*) \in \mathcal{F}$ satisfying (6) also satisfies (A.14). We consider two cases:

1. If $B > \pi_H G - 1$, we show that $U(I^*, R^*, H) - U(0) \geq \frac{K}{N-K}(U(0) + A)$, which is stronger than (A.14), is satisfied. Note that $(\sqrt{N} - 1)^2 \geq 0$, or equivalently, $2N - 2\sqrt{N} \geq N - 1$. This implies that $\frac{2}{\sqrt{N}-1} \geq \frac{\sqrt{N}+1}{N-\sqrt{N}} \geq \frac{K}{N-K}$. In addition,

$$\gamma \geq \frac{G}{(\pi_H G - 1)(G - \frac{B}{\Delta\pi})} = \frac{\pi_H G}{\pi_H G - 1}\frac{1/\pi_H}{G - \frac{B}{\Delta\pi}} \geq \frac{\pi_H G}{\pi_H G - 1}\frac{A}{I^c} = \frac{U(0) + A}{U(I^c, R^c, H) - U(0)}.$$

To conclude, observe that (6) implies

$$U(I^*, R^*, H) - U(0) \geq \frac{K}{N-K}\frac{U(0) + A}{U(I^c, R^c, H) - U(0)}(U(I^c, R^c, H) - U(0)).$$

2. If $B \leq \pi_H G - 1$, we remark that

$$\frac{K}{N-K}\left(U(0)+(1-B)A\right) + N(\pi_H G - 1 - B)I^*$$

$$\leq \frac{K}{N-K}\left(U(0)+(1-B)A\right) + N(U_B - U(0)) = U_B - U(0) + \frac{K}{N-K}\left(U_B + (1-B)A\right).$$

We show that $U(I^*, R^*, H) - U_B \geq \frac{K}{N-K}(U_B + (1-B)A)$, which is stronger than (A.14). As in the former case, we have $\frac{2}{\sqrt{N}-1} \geq \frac{K}{N-K}$. Then, using (A.11), condition (6) implies that

$$U(I^*, R^*, H) - U_B \geq \frac{K}{N-K}\frac{U_B + (1-B)A}{U(I^c, R^c, H) - U_B}(U(I^c, R^c, H) - U_B) = \frac{K}{N-K}(U_B + (1-B)A).$$

Thus, any allocation $(I^*, R^*) \in \mathcal{F}$ satisfying (6) also satisfies (A.4), (A.7), (A.10), and (A.14). This proves that any such allocation is supported at equilibrium by the investors' strategies $M_1^*, ..., M_N^*$.

∎

PROOF OF PROPOSITION 5: We extend the results of Proposition 3 and Proposition 4 to the case in which covenants can be contingent on the initial debt I_0. Assume $\pi_L = 0$, take any $(I^*, R^*) \in \mathcal{F}$ such that $I^* \geq I^m$ and consider the following profile of strategies. Each investor $i = 1, 2, ..., N$ offers the same menu $M^* = \{(0,0,0,0), (\frac{I^*}{N}, R^*(.), I^+(.), R^+(.)), (0, 0, \hat{I}^+(.), \hat{R}^+(.))\}$. We shall refer to $(0,0,0,0)$ as the null contract, to $(\frac{I^*}{N}, R^*(.), I^+(.), R^+(.))$ as the equilibrium contract, and to $(0, 0, \hat{I}^+(.), \hat{R}^+(.))$ as the latent contract. In each equilibrium contract, $R^*(.)$ is such that

$$R^*(I_0, I^F(I_0)) = \begin{cases} R^* - \dfrac{N-1}{N}GI^* & \text{if } I_0 = I^F(I_0) = \dfrac{I^*}{N}, \\ \dfrac{R^*}{N} & \text{otherwise,} \end{cases}$$

where $I^F(I_0)$ is the amount ultimately invested for a given initial I_0. The additional offer $(I^+(.), R^+(.))$ is such that

$$I^+(I_0) = \begin{cases} 0 & \text{if } I_0 = k\dfrac{I^*}{N}, \text{ for } k = 1, 2, ..., N, \\ I^{CL} & \text{otherwise,} \end{cases}$$

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$$
R^+(I_0, I^F(I_0)) = \begin{cases} 0 & \text{if } I_0 = I^F(I_0) = I^*, \text{or } I_0 = I^F(I_0) = \dfrac{I^*}{N}, \\ G(I^F(I_0) + A) & \text{if } I_0 = I^*, I^F(I_0) \neq I^*, \text{or } I_0 = \dfrac{I^*}{N}, I^F(I_0) \neq \dfrac{I^*}{N}, \\ p^* I^{CL} & \text{otherwise,} \end{cases}
$$

with $p^* = \dfrac{R^*}{I^*}$. The investment I^{CL} is the additional credit line which any investor stands ready to provide against any competitors' threat of asking for an accelerated repayment. It is such that

$$
U(\frac{I^*}{N} + I^{CL}, p^*(\frac{I^*}{N} + I^{CL}), L) = U(I^c, R^c, H).
$$

In each latent contract, the additional offer $(\hat{I}^+(.), \hat{R}^+(.))$ is such that

$$
\hat{I}^+(I_0) = \begin{cases} \dfrac{\hat{I}}{N} & \text{if } I_0 = 0, \\ \dfrac{1}{N-1}(\hat{I} - \dfrac{I^*}{N}) & \text{if } I_0 = \dfrac{I^*}{N}, \\ 0 & \text{otherwise,} \end{cases}
$$

and $\hat{R}^+(I_0, I^F(I_0)) = G(I^F(I_0) + A)$ for all I_0 and $I^F(I_0)$. As in the proof of Proposition 3, the investment $\hat{I}$ is characterized by $U(I^*, R^*, H) = B(\hat{I} + A) - A$.

Given these offers, the entrepreneur cannot obtain a payoff higher than $U(I^*, R^*, H)$. At equilibrium, she achieves $U(I^*, R^*, H)$ by trading the same equilibrium contract with each of the investors, receiving thereby no additional funds at the second stage, and selecting $e = H$. As in the proof of Proposition 3, none of the investors is indispensable to provide the equilibrium payoff: the entrepreneur can get $U(I^*, R^*, H)$ by trading the equilibrium contract with only one investor.[34]

Consider then investors' deviations. Without loss of generality, any unilateral deviation can be represented by a menu $M' = \{(0,0,0,0), \left(I', R'(.), I'^+(.), R'^+(.)\right)\}$. By Lemma 1, and given that $\pi_L = 0$, each profitable deviation must induce the entrepreneur to choose $e = H$. Hence, it must be that $I' \in \{0, \dfrac{I^*}{N}\}$ in any profitable deviation. Indeed, if $I' \notin \{0, \dfrac{I^*}{N}\}$, the entrepreneur can combine the deviating contract with (at least) one equilibrium contract, and obtain the line of credit $(I^{CL}, p^* I^{CL})$. This guarantees her (at least) the payoff $U(\dfrac{I^*}{N} + I^{CL}, p^*(\dfrac{I^*}{N} + I^{CL}), L) = U(I^c, R^c, H)$. The entrepreneur's strategy can therefore be constructed so that she strategically defaults when trading I' with the deviating investor. In addition, if the entrepreneur chooses $e = H$, then the aggregate initial financing induced by the deviation to M' must be $I_0 \in \{0, \dfrac{I^*}{N}, I^*\}$.

---

[34] Alternatively, she can also get $U(I^*, R^*, H)$ by trading all latent contracts and defaulting.

Indeed, the repayment $R^+(.)$ is such that, if she trades any equilibrium contract together with the deviating one, the entrepreneur strategically defaults unless her initial outside financing is $\frac{I^*}{N}$ or I^*.[35]

Assuming that $e = H$ is chosen, we distinguish two cases, depending on whether the borrower trades at least one of the equilibrium contracts, or she only trades with the deviating investor. In the first case, given $R^+(.)$, it must be that $I_0 = I^F(I_0) \leq I^*$ and $I'^+(I_0) = I^+(I_0) = 0$, otherwise she would default. It follows that, since $I' \in \{0, \frac{I^*}{N}\}$, we must have $I' = \frac{I^*}{N}$ for the deviation to be profitable. Thus, the corresponding entrepreneur's payoff is

$$
\begin{aligned}
& \pi_H(G(\frac{I^*}{N} + k\frac{I^*}{N} + A) - R'(I_0, I^F(I_0)) - R'^+(I_0, I^F(I_0)) - k\frac{R^*}{N}) - A \\
< \ & \pi_H G(I^* + A) - I' + \frac{I^*}{N} - \pi_H R^* - A = U(I^*, R^*, H) + (\frac{I^*}{N} - I'),
\end{aligned}
\tag{A.15}
$$

where $k \in \{1, ..., N-1\}$ is any number of equilibrium contracts optimally traded by the entrepreneur when $e = H$. The latter inequality obtains since $\pi_H(R'(I_0, I^F) + R'^+(I_0, I^F)) - I' > (\pi_H \frac{R^*}{N} - \frac{I^*}{N})$, which guarantees that the deviation is profitable, and by observing that $G\frac{I^*}{N} - \frac{R^*}{N} > 0$ by construction. Thus, (A.15) implies that, following the deviation, the payoff achieved by the entrepreneur when choosing $e = H$ is strictly below $U(I^*, R^*, H)$, which contradicts the fact that no investor is indispensable to provide the equilibrium payoff under $e = H$.

We next consider the case in which, when choosing $e = H$, the entrepreneur only trades with the deviating investor, which implies that $I_0 = I' \in \{0, \frac{I^*}{N}\}$. Her corresponding payoff is

$$
\begin{aligned}
& \pi_H(G(I' + I'^+(I') + A) - R'(I', I^F(I')) - R'^+(I', I^F(I'))) - A \\
< \ & \pi_H(G(I' + A) - \frac{R^*}{N}) - A + (\pi_H G - 1)I'^+(I') + (\frac{I^*}{N} - I') \tag{A.16} \\
< \ & U(I^*, R^*, H) + (\pi_H G - 1)I'^+(I'). \tag{A.17}
\end{aligned}
$$

Inequality (A.16) follows from

$$
\pi_H(R'(I', I^F(I') + R'^+(I', I^F(I'))) > I' + I'^+(I') + (\pi_H \frac{R^*}{N} - \frac{I^*}{N}),
$$

which guarantees that the deviation is profitable. Inequality (A.17) obtains because $I' \leq \frac{I^*}{N}$ and $\pi_H G - 1 > 0$. Since $U(I^*, R^*, H)$ is available to the entrepreneur at the deviation stage, (A.17) implies that $I'^+(I') > 0$. We now prove that, by strategically defaulting, the entrepreneur gets

---

[35] $\hat{R}^+(.)$ is such that trading any of the latent contracts straightforwardly leads to strategic default.

a payoff which is above the upper bound in (A.16). Suppose that, together with the deviating contract, she takes $N-1$ latent contracts at the deviation stage. Given $\hat{R}^+(.)$, she then finds optimal to default. Her corresponding payoff is

$$U_{sd} = B(I' + I'^+(I') + (N-1)\bar{I}^+(I') + A) - A. \tag{A.18}$$

If $I_0 = I' = \dfrac{I^*}{N}$, (A.18) yields $U_{sd} = U(I^*, R^*, H) + BI'^+(I') > U(I^*, R^*, H) + (\pi_H G - 1)I'^+(I')$, as $I'^+(I') > 0$, and $B > \pi_H G - 1$ since $\pi_L = 0$. If $I_0 = I' = 0$, (A.16) together with the fact that $U(I^*, R^*, H)$ remains available at the deviation stage imply $\pi_H(G(I'^+(0) + A) - I'^+(0) - (\pi_H \dfrac{R^*}{N} - \dfrac{I^*}{N}) > U^*(I^*, R^*, H)$. Since $U^*(I^*, R^*, H) - U(0) = (\pi_H G - 1)I^* - (\pi_H R^* - I^*)$, we get $I'^+(0) > I^*$. Thus, without loss of generality, we can write $I'^+(0) = \dfrac{I^*}{N} + I''$ with $I'' > \dfrac{N-1}{N}I^* > 0$. Then, (A.16) implies that the entrepreneur's payoff is bounded by $\dfrac{U(0)(N-1) + U(I^*, R^*, H)}{N} + (\pi_H G - 1)I''$, and (A.18) can be rewritten as

$$U_{sd} = B\Big(\frac{N-1}{N}\hat{I} + \frac{I^*}{N} + I'' + A\Big) - A = U(I^*, R^*, H) + B\Big(I'' + \frac{I^*}{N} - \frac{\hat{I}}{N}\Big).$$

As shown in the proof of Proposition 3, we have $U_{sd} \geq \dfrac{U(0)(N-1) + U(I^*, R^*, H)}{N} + (\pi_H G - 1)I''$ for each $I'' > 0$. This guarantees that the entrepreneur strategically defaults and reestablishes that any aggregate allocation $(I^*, R^*) \in \mathcal{F}$ satisfying $I^* \geq I^m$ is sustained at equilibrium.

We now extend the result of Proposition 4. We exhibit a profile of investors' strategies that supports at equilibrium any allocation (I^*, R^*) satisfying (6). Each investor $i = \{1, 2, ..., K\}$, with $K \equiv \lfloor \sqrt{N} \rfloor$, offers

$$M_i^* = \{(0, 0, 0, 0); \big(\frac{I^*}{K}, R_i^*(.), I_i^+(.), R_i^+(.)\big), \big(0, 0, \bar{I}_i^+(.), \bar{R}_i^+(.)\big)\},$$

and each investor $j = \{K+1, ..., N\}$ offers

$$M_j^* = \{(0, 0, 0, 0); \big(\frac{\bar{I}}{N-K}, \bar{R}_j(.), 0, 0\big); \big(0, 0, \tilde{I}_j^+(.), \tilde{R}_j^+(.)\big)\}.$$

We shall refer to $(0, 0, 0, 0)$ as the null contract, to $(\dfrac{I^*}{K}, R_i^*(.), I_i^+(.), R_i^+(.))$ as the equilibrium contract, to $(\dfrac{\bar{I}}{N-K}, \bar{R}_j(.), 0, 0)$ as the type-1 latent contract, to $(0, 0, \bar{I}_i^+(.), \bar{R}_i^+(.))$ as the type-2 latent contract, and, to $(0, 0, \tilde{I}_j^+(.), \tilde{R}_j^+(.))$ as the type-3 latent contract.

In each equilibrium contract, $R_i^*(.)$ is such that

$$
\begin{cases}
R_i^*(I_0) = (R^* - \dfrac{K-1}{K}GI^*) & \text{if } I_0 = I^F(I_0) = \dfrac{I^*}{K}, \\
R_i^*(I_0) = \dfrac{R^*}{K}, & \text{otherwise.}
\end{cases}
$$

The additional offer $(I_i^+(.), R_i^+(.))$ is such that:

$$
I_i^+(I_0) =
\begin{cases}
0 & \text{if } I_0 = k\dfrac{I^*}{K} + l\dfrac{\bar{I}}{N-K}, \text{ for } k = 1, 2, ..., K \text{ and } l = 0, ..., N-K, \\
I^{CL} & \text{otherwise,}
\end{cases}
$$

and

$$
R_i^+(I_0, I^F(I_0)) =
\begin{cases}
0 & \text{if } I_0 = I^*, I^F(I_0) = I^*, \text{ or } I_0 = \dfrac{I^*}{K}, I^F(I_0) = \dfrac{I^*}{K}, \\
G(I^F(I_0) + A) & \text{if } I_0 = I^*, I^F(I_0) \neq I^*, \text{ or } I_0 = \dfrac{I^*}{K}, I^F(I_0) \neq \dfrac{I^*}{K}, \\
& \text{or } I_0 = k\dfrac{I^*}{K} + l\dfrac{\bar{I}}{N-K}, \ \forall I^F(I_0) \\
& \text{for } k = 1, 2, ..., K \text{ and } l = 0, ..., N-K, \text{with } (k,l) \notin \{(K,0), (1,0)\}, \\
G(I^{CL} + A) & \text{otherwise,}
\end{cases}
$$

where I^{CL} is such that $B(\dfrac{I^*}{K} + I^{CL} + A) - A = U(I^c, R^c, H)$ and $\bar{I}$ is such that $U(I^*, R^*, H) = B(I^* + \bar{I} + A) - A$. The additional offer $(\bar{I}_i^+(.), \bar{R}_i^+(.))$ is such that

$$
\bar{I}_i^+(I_0) =
\begin{cases}
\dfrac{I^*}{K} & \text{if } I_0 = 0 \text{ or, } I_0 = \dfrac{I^*}{K} \text{ or, } I_0 = \dfrac{\bar{I}}{N-K}, \\
0 & \text{otherwise,}
\end{cases}
$$

and $\bar{R}_i^+(I_0, I^F(I_0)) = G(I^F(I_0) + A)$ for all I_0 and $I^F(I_0)$.

For any $j \in \{K+1, 2, ..., N\}$, the repayment $\bar{R}_j(.)$ satisfies $\bar{R}_j^+(I_0, I^F(I_0)) = G(I^F(I_0) + A)$ for all I_0 and $I^F(I_0)$ and the offers $(\tilde{I}_j^+(.), \tilde{R}_j^+(.))$ are such that

$$
\tilde{I}_j^+(I_0) =
\begin{cases}
\dfrac{\bar{I}}{N-K} & \text{if } I_0 = \dfrac{I^*}{K} \text{ or, } I_0 = 0, \\
0 & \text{otherwise,}
\end{cases}
$$

and $\tilde{R}_j^+(I_0, I^F(I_0)) = G(I^F(I_0) + A)$ for all I_0 and $I^F(I_0)$.

Given these offers, the entrepreneur cannot obtain a payoff higher than $U(I^*, R^*, H)$. At equilibrium, she achieves $U(I^*, R^*, H)$ by trading the equilibrium contract with each lender $i \in \{1, 2, ..., K\}$, the null contract with each lender $j \in \{K + 1, 2, ..., N\}$, obtains no additional funds in the second stage and selects $e = H$. As before, none of the investors is indispensable: the entrepreneur can get $U(I^*, R^*, H)$ by trading the equilibrium contract with only one investor.[36]

Consider then investors' deviations. Without loss of generality, a deviation by any investor can be represented by a menu $M' = \{(0, 0, 0, 0), (I', R'(.), I'^+(.), R'^+(.))\}$. Every profitable deviation must induce the entrepreneur to choose $e = H$. Indeed, given Lemma 1, a single investor may achieve a positive profit by inducing $e = L$ only if the entrepreneur trades several contracts out of equilibrium and does not default. Given the equilibrium contracts, this is possible only if she takes up an aggregate initial financing $I_0 = I^*$, invests ultimately $I^F(I^*) = I^*$ and chooses $e = L$, which yields the entrepreneur a payoff smaller than the available equilibrium one, and thus cannot be an optimal choice.

Suppose an investor $i \in \{1, ..., K\}$ deviates. Any profitable deviation must be such that $I' \in \{0, \frac{I^*}{K}\}$. Indeed, if $I' \notin \{0, \frac{I^*}{K}\}$, then the entrepreneur can combine the deviating contract with (at least) one equilibrium contract, and get access to the line of credit $(I^{CL}, G(I^{CL} + A))$. which ensures her a payoff at least equal to $B(\frac{I^*}{K} + I^{CL} + A) - A = U(I^c, R^c, H)$. This shows that in any profitable deviation the borrower defaults, which constitutes a contradiction. In addition, at the deviation stage, the initial investment I_0 belongs to the set $\{0, \frac{I^*}{K}, I^*\}$. Indeed, $R_i^+(.)$ is such that, if she trades any equilibrium contract together with the deviating one, the entrepreneur necessarily defaults if $I_0 \notin \{\frac{I^*}{K}, I^*\}$. Assuming that $e = H$ is chosen, we distinguish below two cases, depending on whether the entrepreneur trades at least one of the equilibrium contracts, or she only trades with the deviating investor.

Consider that the entrepreneur trades equilibrium contracts at the deviation stage. Given $R_i^+(.)$, we have $I_0 = I^F(I_0)$ which implies $I'^+(I_0) = I^+(I_0) = 0$, otherwise the entrepreneur would default. It follows that, since $I' \in \{0, \frac{I^*}{K}\}$, it must be that $I' = \frac{I^*}{K}$ for the deviation to be profitable. Thus, the entrepreneur's payoff is

$$\pi_H(G(\frac{I^*}{K} + k\frac{I^*}{K} + A) - R'(I_0, I^F(I_0)) - R'^+(I_0, I^F(I_0)) - k\frac{R^*}{K}) - A < U(I^*, R^*, H), \ (A.19)$$

where $k \in \{1, ..., N - 1\}$ is any number of equilibrium contracts optimally traded by the entrepreneur when $e = H$. The latter inequality obtains since $\pi_H(R'(I_0, I^F) + R'^+(I_0, I^F)) - \frac{I^*}{K} >$

---

[36]She may also get $U(I^*, R^*, H)$ by trading the type-1 latent contract with each investor $j \in \{K + 1, 2, ..., N\}$.

$(\pi_H \frac{R^*}{K} - \frac{I^*}{K})$, which guarantees that the deviation is profitable, and by observing that $G\frac{I^*}{K} - \frac{R^*}{K} > 0$ by construction. Thus, (A.19) implies that, following the deviation, the payoff achieved by the entrepreneur when choosing $e = H$ is strictly below $U(I^*, R^*, H)$, which contradicts the fact that no investor is indispensable to provide the equilibrium payoff under $e = H$.

Consider the case where the entrepreneur trades the null contract with all non deviating lenders. This implies that $I_0 = I' \in \{0, \frac{I^*}{K}\}$ and her payoff is

$$\pi_H\big(G(I' + I'^+(I') + A) - R'(I', I^F(I')) - R'^+(I', I^F(I'))\big) - A$$

$$< \pi_H(G(I' + A) - \frac{R^*}{K}) - A + (\pi_H G - 1)I'^+(I') + (\frac{I^*}{K} - I') \tag{A.20}$$

$$< U(I^*, R^*, H) + (\pi_H G - 1)I'^+(I'), \tag{A.21}$$

where (A.20) obtains since

$$\pi_H(R'(I', I^F(I')) + R'^+(I', I^F(I'))) > I' + I'^+(I') + (\pi_H \frac{R^*}{K} - \frac{I^*}{K}) \tag{A.22}$$

for the deviation to be profitable. The second one obtains because $I' \leq \frac{I^*}{K}$ and $\pi_H G - 1 > 0$. Since the payoff $U(I^*, R^*, H)$ is available to the entrepreneur at the deviation stage, (A.21) implies that $I'^+(\frac{I^*}{K}) > 0$.

We prove that the upper-bound (A.20) of the entrepreneur's payoff is less than what she gets if she strategically defaults. Indeed, if, following the deviation, the entrepreneur strategically defaults, she can select the contract $(0, 0, \bar{I}_i^+(.), \bar{R}_i^+(.))$ in the menu of each non-deviating investor $i \in \{1, ..., K\}$ and the contract $(0, 0, \tilde{I}_j^+(.), \tilde{R}_j^+(.))$ in the menu of each lender $j \in \{K+1, ..., N\}$. By doing that, she obtains

$$U_{sd} = B\big(I' + I'^+(I') + (K-1)\bar{I}_i^+(I') + (N-K)\tilde{I}_j^+(I') + A)\big) - A. \tag{A.23}$$

If $I' = \frac{I^*}{K}$, then $U_{sd} = U(I^*, R^*, H) + BI'^+(\frac{I^*}{K})$. Thus, using (A.21), a sufficient condition for the entrepreneur to strategically default is $\frac{K-1}{K}(U(I^*, R^*, H) - U(0)) \geq (\pi_H G - 1 - B)I'^+(\frac{I^*}{K})$. This corresponds to (A.4), which holds from the proof of Proposition 4. If $I' = 0$, we deduce from (A.22) that

$$\pi_H(R'(0, I^F(0)) + R'^+(0, I^F(0))) > I'^+(0) + (\pi_H \frac{R^*}{K} - \frac{I^*}{K}), \tag{A.24}$$

from which it follows that $I'^+(0) > I^*$. Thus, without loss of generality, we write $I'^+(0) = \frac{I^*}{K} + I''$ with $I'' > \frac{K-1}{K} I^* > 0$. Then, using again (A.20) and (A.24), we get an upper bound for the entrepreneur's payoff:

$$\pi_H\big(G(I'^+(0)+A)-R'(0,I^F(0))-R'^+(0,I^F(0))\big)-A < \frac{U(0)(K-1)+U(I^*,R^*,H)}{K}+(\pi_H G-1)I''.$$

But (A.23) becomes $U_{sd} = B\big(\frac{K-1}{K}I^*+\frac{I^*}{K}+I''+\bar{I}+A\big)-A = U(I^*,R^*,H)+BI''$. Thus, a sufficient condition for the entrepreneur to default is $\frac{K-1}{K}(U(I^*,R^*,H)-U(0)) \geq (\pi_H G-1-B)I''$ with $I'' > 0$. Again, we have established this relation in the proof of Proposition 4.

Suppose an investor $j \in \{K+1,...,N\}$ deviates. Any profitable deviation must be such that $I' \in \{0, \frac{\bar{I}}{N-K}\}$. Indeed, if $I' \notin \{0, \frac{\bar{I}}{N-K}\}$, then the entrepreneur can combine equilibrium contracts and/or latent contracts of type-1, get access to the line of credit, and earn at least the payoff $B(\frac{I^*}{K} + I^{CL} + A) - A = U(I^c, R^c, H)$. This shows that, in any profitable deviation, the entrepreneur defaults, which constitutes a contradiction. Below, we consider the two cases $I' = \frac{\bar{I}}{N-K}$ and $I' = 0$ and we show that in each case the entrepreneur strategically defaults following the deviation.

First, consider the case $I' = \frac{\bar{I}}{N-K}$. Because $e = H$ is chosen at the deviation stage, the entrepreneur does not trade latent contracts. Furthermore, given the additional offers $(I_i^+(.), R_i^+(.))$ she trades null contracts with each lender $i \in \{1,...,K\}$. Thus, following the deviation, the entrepreneur trades the null contract with each non-deviating investor, which implies that $I_0 = \frac{\bar{I}}{N-K}$. When the entrepreneur chooses $e = H$, her payoff is bounded above by $U(0) + (\pi_H G - 1)(\frac{\bar{I}}{N-K} + I'^+(\frac{\bar{I}}{N-K}))$, where $I'^+(\frac{\bar{I}}{N-K}) \geq 0$. Under default, the entrepreneur's payoff is bounded below by

$$\begin{aligned} U_{sd} &= B\big(\frac{\bar{I}}{N-K}+I'^+(\frac{\bar{I}}{N-K})+\frac{N-K-1}{N-K}\bar{I}+K\bar{I}_i^+(\frac{\bar{I}}{N-K})-A\big)-A \\ &= U^*(I^*,R^*,H)+BI'^+(\frac{\bar{I}}{N-K}). \end{aligned}$$

Thus, a sufficient condition for the entrepreneur to default is

$$U^*(I^*,R^*,H)-U(0) \geq (\pi_H G-1-B)I'^+(\frac{\bar{I}}{N-K})+(\pi_H G-1)\frac{\bar{I}}{N-K},$$

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which corresponds to (A.8) established in the proof of Proposition 4.

Second, consider the case $I' = 0$. Again, because $e = H$ is chosen at the deviation stage, the entrepreneur does not trade latent contracts together with the deviating contract. Remark that we must have $I'^{+}(I_0) > 0$ for the deviation to be profitable. Furthermore, given the additional offers $(I_i^{+}(.), R_i^{+}(.))$, the entrepreneur raises the amount $I_0 = \dfrac{I^*}{K}$, or the amount $I_0 = I^*$.

If the entrepreneur chooses $I_0 = \dfrac{I^*}{K}$ together with $e = H$, she must trade one equilibrium contract at the deviating stage. The inequality $I'^{+}(\dfrac{I^*}{K}) > 0$ implies that $I^{F}(\dfrac{I^*}{K}) > \dfrac{I^*}{K}$. Thus, given $R_i^{+}(.)$, the entrepreneur cannot get more than her reservation utility. Thus, when choosing $e = H$, she prefers not to trade the deviating contract.

The same logic applies if the entrepreneur chooses $I_0 = I^*$ together with $e = H$. In that case she must trade an equilibrium contract with each investor $i \in \{1, ..., K\}$. The inequality $I'^{+}(I^*) > 0$ implies that $I^{F}(I^*) > I^*$, and, given $R_i^{+}(.)$, the entrepreneur cannot get more than her reservation utility. ∎

PROOF OF PROPOSITION 6 (continued): The proof extends to the case in which investors can write covenants contingent on the initial debt I_0. Take any aggregate allocation $(I^*, R^*) \neq (I^c, R^c)$ supported in a symmetric equilibrium. Let $\left(\dfrac{I}{N}, \dfrac{R(I^*)}{N}, \dfrac{I^{+}(I)}{N}, \dfrac{R^{+}(I, I^*)}{N} \right)$ be the equilibrium trades of each lender, with $I + I^{+}(I) = I^*$.

Let investor k deviate to the menu $M'_k = ((0, 0, 0, 0), (\dfrac{I}{N}, \dfrac{R(I^*)}{N}, I_k^{+'}(.) + \varepsilon, R_k^{+'}(.)))$ with $\varepsilon > 0$. The additional offer $(I_k^{+'}(.), R_k^{+'}(.))$ is such that

$$
I_k^{+'}(I_0) = \begin{cases} I^* - \dfrac{I}{N} - \dfrac{N-1}{N} \dfrac{\pi_H R^* - I^*}{\pi_H G - 1} & \text{if } I_0 = \dfrac{I}{N}, \\ 0 & \text{if } I_0 \neq \dfrac{I}{N}, \end{cases}
$$

$$
R_k^{+'}(I_0, I^F(I_0)) = \begin{cases} \dfrac{R^{+}(I, I^*)}{N} + \dfrac{1}{\pi_H}(I'^{+}(\dfrac{I}{N}) - \dfrac{I}{N} + \varepsilon) + \varepsilon^2 & \text{if } I_0 = \dfrac{I}{N} \text{ and } I^F(I_0) = \dfrac{I}{N} + I_k^{+'}(\dfrac{I}{N}) + \varepsilon, \\ G(I^F(I_0) + A) & \text{otherwise.} \end{cases}
$$

The logic developed in the first part of the proof applies to this more general case. If $\varepsilon = 0$, the pair $(\dfrac{I}{N} + I_k^{+'}(\dfrac{I}{N}) + \varepsilon, \dfrac{R(I^*)}{N} + R_k^{+'}(\dfrac{I}{N}, \dfrac{I}{N} + I_k^{+'}(\dfrac{I}{N}) + \varepsilon))$ is such that

$$
U(\dfrac{I}{N} + I_k^{+'}(\dfrac{I}{N}), \dfrac{R(I^*)}{N} + R_k^{+'}(\dfrac{I}{N}, \dfrac{I}{N} + I_k^{+'}(\dfrac{I}{N})), H) = U(I^*, R^*, H)
$$

and $\pi_H R_k^{+'}(\frac{I}{N}, \frac{I}{N} + I_k^{+'}(\frac{I}{N})) - I_k^{+'}(\frac{I}{N}) = \pi_H \frac{1}{N} R^+(I, I^*) - \frac{1}{N} I^+(I)$. Since $(I^*, R^*) \neq (I^c, R^c)$, $(\frac{I}{N} + I_k^{+'}(\frac{I}{N}) + \varepsilon, \frac{R(I^*)}{N} + R_k^{+'}(\frac{I}{N}, \frac{I}{N} + I_k^{+'}(\frac{I}{N}) + \varepsilon)) \in int(\mathcal{F})$ for ε small enough. The deviation is designed to induce the entrepreneur to trade $(\frac{I}{N} + I_k^{+'}(\frac{I}{N}) + \varepsilon, \frac{R(I^*)}{N} + R_k^{+'}(\frac{I}{N}, \frac{I}{N} + I_k^{+'}(\frac{I}{N}) + \varepsilon))$, and to choose $e = H$. Given M_k', if the entrepreneur chooses $e = H$, she can get a payoff above the equilibrium one by choosing $(\frac{I}{N} + I_k^{+'}(\frac{I}{N}) + \varepsilon, \frac{R(I^*)}{N} + R_k^{+'}(\frac{I}{N}, \frac{I}{N} + I_k^{+'}(\frac{I}{N}) + \varepsilon))$ in the deviating menu. In this case, however, given $R_k'(.)$, she finds optimal to trade with investor k alone. Thus, as long as $\varepsilon > 0$, M_k' is a profitable deviation for investor k whenever $e = H$ is chosen. When the entrepreneur chooses $e = L$, the only possibility to get a payoff above the equilibrium one is to choose $I_0 = \frac{I}{N}$. In this case, she trades the aggregate amount $\frac{I}{N} + I_k^{+'}(\frac{I}{N}) + \varepsilon + \frac{N-1}{N} \hat{I}^+(\frac{I}{N})$, where $\frac{1}{N} \hat{I}^+(\frac{I}{N})$ denotes the largest additional investment offered in any equilibrium menu for $I_0 = \frac{I}{N}$. Given the equilibrium schedules in (22), the deviation is hence profitable if

$$U(I^*, R^*, H) + (\pi_H G - 1 - \varepsilon)\varepsilon + \frac{N-1}{N}(1-\delta)(\frac{I}{N} + I_k^{+'}(\frac{I}{N}) + \varepsilon) >$$
$$B(\frac{I}{N} + I_k^{+'}(\frac{I}{N}) + \varepsilon + \frac{N-1}{N} \hat{I}^+(\frac{I}{N}) + A) - A. \qquad (A.25)$$

As in the first part of the proof, (A.25) is satisfied, since $U(I^*, R^*, H) \geq B(\frac{I}{N} + \frac{N-1}{N} \hat{I}^+(\frac{I}{N}) + A) - A$ by construction, $\pi_H G - 1 - \varepsilon > 0$ for ε small enough, and $\delta < 1 - B$. The rest of the proof is a straightforward adaptation of the reasoning developed when I is only observed ex post.

∎

ADDITIONAL APPENDIX B: Indeterminacy with $N \geq 2$

Proposition B.1 *Take any $N \geq 2$. If $\pi_H G - 1 > 2\sqrt{\dfrac{\pi_L}{\pi_H}}$, there exists an investment level $\underline{I} < I^c$ such that any aggregate allocation $(I^*, R^*) \in \mathcal{F}$ with $I^* \geq \underline{I}$ can be supported at equilibrium.*

PROOF: Assume

$$\pi_H G - 1 > 2\sqrt{\frac{\pi_L}{\pi_H}}. \tag{B.1}$$

We first establish a set of relationships that will be used throughout the proof. First, from (1) we get

$$G - \frac{B}{\Delta \pi} > G + \frac{\pi_L G - 1}{\Delta \pi} = \frac{\pi_H G - 1}{\Delta \pi} > \frac{2}{\Delta \pi}\sqrt{\frac{\pi_L}{\pi_H}}, \tag{B.2}$$

where the last inequality follows from (B.1).

Second, given (B.1), (1) and (4) together imply that[37]

$$1 - \frac{\pi_H}{\Delta \pi} B > 2\sqrt{\frac{\pi_L}{\pi_H}\frac{\pi_L}{\Delta \pi}} \quad \text{and} \quad \frac{\pi_H}{\Delta \pi} B > 2\sqrt{\frac{\pi_L}{\pi_H}}.$$

Adding the two conditions, one gets

$$1 > 2\frac{\sqrt{\pi_L \pi_H}}{\Delta \pi} \;\Leftrightarrow\; \sqrt{\frac{\pi_H}{\pi_L}} - \sqrt{\frac{\pi_L}{\pi_H}} > 2 \;\Leftrightarrow\; \left(\sqrt{\frac{\pi_H}{\pi_L}} - 1\right)^2 > 2,$$

which yields

$$\sqrt{\frac{\pi_H}{\pi_L}} > 1 + \sqrt{2}, \qquad \sqrt{\frac{\pi_L}{\pi_H}} < \sqrt{2} - 1 \qquad \text{and} \qquad \pi_H > (3 + 2\sqrt{2})\pi_L. \tag{B.3}$$

We now turn to the proof of Proposition B.1. It is useful to characterize equilibrium allocations in terms of two parameters, which we denote ε and η. Precisely, let $(\varepsilon, \eta) \in [0, \underline{\varepsilon}] \times \left[\pi_H \left(G - \frac{B}{\Delta \pi}\right), 1\right]$, with

$$\underline{\varepsilon} = \frac{1}{N}\min\left(\frac{\Delta \pi}{B}(G - \frac{1}{\pi_H}), \; \frac{\pi_H G - 1}{B} - \frac{\pi_L}{\pi_H \Delta \pi}\frac{1}{G - \frac{B}{\Delta \pi}}, \; \left(1 - \frac{\pi_L}{\Delta \pi}\frac{1/\pi_H}{G - \frac{B}{\Delta \pi}}\right)\frac{B}{2B + (1 - \pi_H G)}\right).$$

Observe that (1) and (4) imply that $\frac{\Delta \pi}{B}(G - \frac{1}{\pi_H}) \in (0, 1)$, and, given (B.2), both the second and third terms are strictly positive. Thus, we get $0 < N\underline{\varepsilon} < 1$. Consider now the aggregate allocation $(I^\varepsilon, R^\varepsilon_\eta) = \left(I^c(1 - \varepsilon), \frac{1}{\pi_H}I^c(1 - \eta\varepsilon)\right)$. It is immediate to check that, if $e = H$, the aggregate profit

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[37]It is useful to rewrite (1) as $\pi_H G - 1 < \frac{\Delta \pi}{\pi_L}(1 - \frac{\pi_H}{\Delta \pi} B)$, and (4) as $\pi_H G - 1 < \frac{\pi_H}{\Delta \pi} B$.

$\pi_H R_\eta^\varepsilon - I^\varepsilon$ is strictly decreasing in η. See also that $\forall \varepsilon \in [0, \underline{\varepsilon}]$, $I^\varepsilon > I^m$.[38] Letting $\underline{I} = I^{\underline{\varepsilon}} > I^m$, and considering all $(\varepsilon, \eta) \in [0, \underline{\varepsilon}] \times \left[\pi_H \left(G - \frac{B}{\Delta\pi} \right), 1 \right]$, one can hence generate a closed subset of $\mathcal{F}$ including all aggregate allocations $(I^\varepsilon, R_\eta^\varepsilon) \in \mathcal{F}$ such that $I^\varepsilon \geq \underline{I}$. The subset is represented in the dashed area in Figure 3.
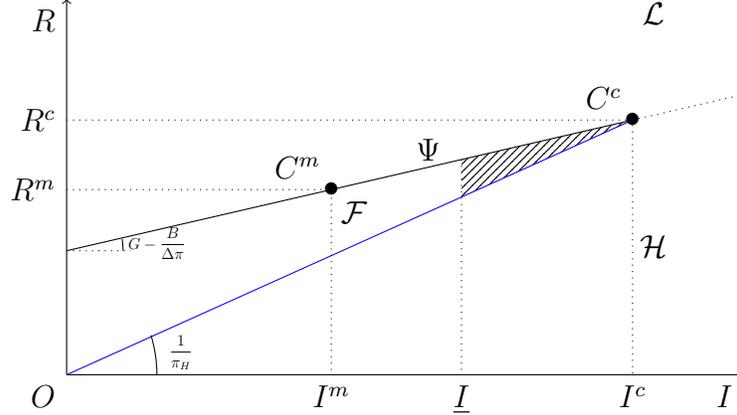

Figure 3: Set of aggregate allocations $(I^\varepsilon, R_\eta^\varepsilon)$

Consider now any allocation $(I^\varepsilon, R_\eta^\varepsilon)$ such that $I^\varepsilon \geq \underline{I}$, and denote it (I^*, R^*). We show that it is supported at equilibrium by the following strategies, which are similar to those used in the proof of Proposition 3. Each investor $i = 1, 2, ..., N$ offers the same menu $M^* = \{(0, 0), (\frac{I^*}{N}, R^*(.)), (\frac{\hat{I}}{N}, \hat{R}(.))\}$, with $R^*(I) = G(I + A)$ for $I \notin \{I^*, \frac{I^*}{N}\}$, and

$$R^*(I) = \begin{cases} \dfrac{R^*}{N} & \text{if } I = I^*, \\ R^* - \dfrac{N-1}{N}GI^* & \text{if } I = \dfrac{I^*}{N}. \end{cases} \tag{B.4}$$

The investment level $\hat{I}$ is such that

$$B(\hat{I} + A) - A = U(I^*, R^*, H) = U^c - \varepsilon I^c(\pi_H G - \eta), \tag{B.5}$$

which guarantees, given that $U(I^*, R^*, H) \geq B(I^* + A) - A$ whenever $(I^*, R^*) \in \mathcal{F}$, that $\hat{I} > I^* \geq 0$.

As in the proof of Proposition 3, one can check that choosing the investment $\dfrac{I^*}{N}$ in each menu and selecting $e = H$ is an optimal choice for the entrepreneur. Consider now investors' deviations.

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[38]Indeed, $I^c \left(1 - \dfrac{\Delta\pi}{B} \left(G - \dfrac{1}{\pi_H} \right) \right) = \dfrac{\Delta\pi}{B} \left(G - \dfrac{B}{\Delta\pi} \right) A = I^m$ and $\underline{\varepsilon} < \dfrac{\Delta\pi}{B} \left(G - \dfrac{1}{\pi_H} \right)$.

Without loss of generality, any unilateral deviation can be represented by a menu $M' = \{(\frac{I^*}{N} + I', \frac{R^*}{N} + R'(.)), (0,0)\}$. A profitable deviation must necessarily induces $e = H$,[39] and one must have that

$$R'(I) > \frac{1}{\pi_H} I',$$

with I being the aggregate investment traded at the deviation stage, and that $I' \leq I^c - \frac{I^*}{N}$. If, following the deviation, the entrepreneur chooses $e = L$ and strategically defaults, then, given (B.5), she gets

$$U_{sd} = U(I^*, R^*, H) + B\left(I' + \frac{I^*}{N} - \frac{\hat{I}}{N}\right).$$

Assume first that the entrepreneur only trades with the deviating investor. In this case, choosing $e = H$ yields her the payoff

$$\pi_H\left(G(\frac{I^*}{N} + I' + A) - \frac{R^*}{N} - R'(\frac{I^*}{N} + I')\right)^+ - A < \frac{U(0)(N-1) + U(I^*, R^*, H)}{N} + (\pi_H G - 1)I'.$$

Since the entrepreneur's equilibrium utility remains available at the deviation stage, the right-hand side of the last inequality must be strictly greater than $U(I^*, R^*, H)$, which implies that $I' > 0$. The entrepreneur therefore finds optimal to strategically default if

$$\frac{N-1}{N}\left(U(I^*, R^*, H) - U(0)\right) - \frac{B}{N}(\hat{I} - I^*) \geq (\pi_H G - 1 - B)I'. \tag{B.6}$$

The left-hand side being increasing with N, a sufficient condition for (B.6) obtains with $N = 2$. That is, after rearranging: $B(I^* + A) - \pi_H G A \geq 2(\pi_H G - 1 - B)I'$ or $B(I^c + A) - \pi_H G A \geq B\varepsilon I^c + 2(\pi_H G - 1 - B)I'$. See that $B(I^c + A) - \pi_H G A = -\frac{\pi_L}{\Delta\pi}B(I^c + A) + (\pi_H G - 1)I^c$ which leads to the condition:

$$\pi_H G - 1 \geq f(I', \varepsilon), \tag{B.7}$$

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[39]Indeed, given Lemma 1, a deviating investor may achieve a positive profit by inducing $e = L$ only if the entrepreneur trades several contracts out of equilibrium and does not default. Given equilibrium menus, this is only possible if the entrepreneur invests $I = I^*$ and selects $e = L$; this in turn provides her a payoff smaller than the equilibrium one, which guarantees that this is not an optimal choice.

with $f(I', \varepsilon) = B \left(\dfrac{\pi_L}{\Delta\pi} \dfrac{I^c + A}{I^c} + \varepsilon \right) + 2(\pi_H G - 1 - B)\dfrac{I'}{I^c}$. Given the linearity of f, we have that

$$f(I', \varepsilon) = f\left(\frac{I'}{I^c}(I^c, 0) + \left(1 - \frac{I'}{I^c}\right)\left(0, \frac{\varepsilon}{1 - \frac{I'}{I^c}}\right) \right) = \frac{I'}{I^c}f(I^c, 0) + \left(1 - \frac{I'}{I^c}\right)f\left(0, \frac{\varepsilon}{1 - \frac{I'}{I^c}}\right).$$

To prove (B.7), using $0 < I' \le \frac{N-1}{N}I^c < I^c$, we simply need to show that

$$\pi_H G - 1 \ \ge \ f(I^c, 0), \tag{B.8}$$

$$\pi_H G - 1 \ \ge \ f\left(0, \frac{\varepsilon}{1 - \frac{I'}{I^c}}\right). \tag{B.9}$$

Condition (B.8) is equivalent to $B\left(2 - \dfrac{\pi_L}{\Delta\pi} \dfrac{1/\pi_H}{G - \frac{B}{\Delta\pi}} \right) \ge \pi_H G - 1$, which holds since $B\left(2 - \dfrac{\pi_L}{\Delta\pi} \dfrac{1/\pi_H}{G - \frac{B}{\Delta\pi}} \right) >$
$B\left(2 - \dfrac{1}{2}\sqrt{\dfrac{\pi_L}{\pi_H}} \right) > B(2 - \dfrac{1}{2}(\sqrt{2} - 1)) > \dfrac{B}{1 - (3 - 2\sqrt{2})} > \dfrac{\pi_H}{\Delta\pi}B \ge \pi_H G - 1$. The first in-
equality comes from (B.2), the second and the fourth from (B.3) and the last one is (4). To prove
(B.9), first remark that (B.7) holds for any couple $(0, \varepsilon)$ with $\varepsilon \le N\underline{\varepsilon}$. Indeed, by definition of $\underline{\varepsilon}$,
we have $\varepsilon \le \dfrac{\pi_H G - 1}{B} - \dfrac{\pi_L}{\Delta\pi} \dfrac{1/\pi_H}{G - \frac{B}{\Delta\pi}}$ for any $\varepsilon \le N\underline{\varepsilon}$. To complete the proof of (B.9), observe
that, since $I' \le \frac{N-1}{N}I^c$ and $\varepsilon \le \underline{\varepsilon}$, we have

$$\frac{\varepsilon}{1 - \frac{I'}{I^c}} \le \frac{1}{N}\frac{N\underline{\varepsilon}}{1 - \frac{I'}{I^c}} \le \frac{1}{N}\frac{N\underline{\varepsilon}}{1 - \frac{N-1}{N}} = N\underline{\varepsilon}.$$

Assume next that several contracts are traded at the deviation stage. Getting back to the proof
of Proposition 3, following any unilateral deviation, the entrepreneur strategically defaults if (17)
holds. Again, the left-hand side of (17) is increasing in k. A sufficient condition for (17) is
therefore, using (B.5) and $I^* = I^c(1 - \varepsilon)$

$$BI^c\left(1 - \frac{\pi_L}{\Delta\pi} \frac{1/\pi_H}{G - \frac{B}{\Delta\pi}} \right) \ge \varepsilon I^c(2B + 1 - \pi_H G),$$

which is equivalent to $\varepsilon \le \dfrac{B}{2B + (1 - \pi_H G)}\left(1 - \dfrac{\pi_L}{\Delta\pi} \dfrac{1/\pi_H}{G - \frac{B}{\Delta\pi}} \right)$. The inequality holds by defi-
nition of $\underline{\varepsilon}$, which concludes the proof that (I^*, R^*) is supported at equilibrium. $\blacksquare$

ADDITIONAL APPENDIX C: Seniority under bankruptcy

Proposition C.1 *For sufficiently small values of π_L, if one investor is repaid first in case of default, then every allocation characterized in Proposition 4 can be supported at equilibrium.*

PROOF: Fix N such that the set of allocations satisfying (6) is non-empty. Consider any strategy profile introduced in the proof of Proposition 4 with the equilibrium allocation (I^*, R^*) and let j be the investor whose claim is senior in case of default. We show that if π_L is sufficiently small, this investor cannot profitably deviate inducing strategic default. Let $M_j' = \{(0,0), (I_j', R_j'(.))\}$ be a deviating menu with $R_j'(I) = G(I + A)\ \forall I$ the repayment that maximizes investor j's profit in case of default. Suppose first that $j \in \{1, \cdots, K\}$. Given equilibrium covenants, $(I_j', R_j'(.))$ induces the entrepreneur to strategically default only if

$$U(I^*, R^*, H) = B(\bar{I} + I^* + A) - A < B(I_j' + \frac{K-1}{K}I^* + \bar{I} + A) - A,$$

which corresponds to

$$I_j' > \frac{I^*}{K}. \tag{C.1}$$

A sufficient condition for the deviation not to be profitable is that it yields a strictly negative profit, that is

$$(\pi_L G - 1)I_j' + \pi_L G(\bar{I} + \frac{K-1}{K}I^* + A) < 0. \tag{C.2}$$

For a given N, it follows from Proposition 4 that $\dfrac{I^*}{K} > 0$ at equilibrium. See that the strict inequalities (C.1) and (C.2) are satisfied for $\pi_L = 0$. By continuity, one can then construct an open neighborhood of $\pi_L^* = 0$ such that, for each π_L in this interval, any I_j' that satisfies (C.1) also satisfies (C.2).

The same reasoning applies to the case $j \in \{K + 1, \cdots, N\}$. $\blacksquare$

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ADDITIONAL APPENDIX D: Reducing B under bankruptcy

We consider the following bankruptcy mechanism. Before the game is played, an authority commits to reduce the entrepreneur's private benefit to some $B' = \lambda B$ with $\lambda \in (0, 1)$ after observing $R > G(I + A)$. We then have the following proposition.

Proposition D.1 *Assume $\pi_L = 0$ and consider an economy with a bankruptcy mechanism of parameter λ. If*

$$\lambda > \max \left(\frac{\pi_H G - 1}{B}, 1 - (\pi_H G - B)\frac{\pi_H G - 1}{B} \right), \tag{D.1}$$

there exists an investment level $\underline{I}^\lambda$ such that any aggregate allocation $(I^*, R^*) \in \mathcal{F}$ with $I \geq \underline{I}^\lambda$ can be supported at equilibrium.

PROOF: Consider first any array of parameters (π_H, π_L, G, B) such that $\pi_L = 0$ and (1) and (4) are satisfied. Observe that, in this case, (4) can be written $\frac{\pi_H G - 1}{B} < 1$, which implies $1 - (\pi_H G - B)\frac{\pi_H G - 1}{B} > 0$. Construct then a modified array of parameters, by replacing B with any $B' = \lambda B$, with λ satisfying (D.1). We refer to these parameters as the economy $\mathcal{E}^\lambda$. We denote $\mathcal{F}^\lambda$ its feasibility set and I^m_λ (I^c_λ) the corresponding monopoly (competitive) investment level. Observe that since $G - \frac{\lambda B}{\Delta \pi} > G - \frac{B}{\Delta \pi}$, we have

$$\mathcal{F} \subset \mathcal{F}^\lambda, \qquad I^c \leq I^c_\lambda. \tag{D.2}$$

We know from Proposition 3 that any aggregate allocation $(I^*, R^*) \in \mathcal{F}$ satisfying $I^* \geq I^m$ can be sustained at equilibrium. This implies that any $(I^*, R^*) \in \mathcal{F}^\lambda$ with $I^* \geq I^m_\lambda$ can be sustained at equilibrium in $\mathcal{E}^\lambda$ with the symmetric menus $M^\lambda = \{(0, 0), \left(\frac{I^*}{N}, R^*(.)\right), \left(\frac{\hat{I}}{N}, \hat{R}(.)\right)\}$, where $R^*(.)$ and $\hat{R}(.)$ are defined as in the proof of Proposition 3, and

$$\lambda B(\hat{I} + A) - A = U(I^*, R^*, H). \tag{D.3}$$

Simple manipulations lead to $I^m_\lambda + A = \frac{\pi_H G}{\lambda B} A$ and $I^c + A = \frac{1/\pi_H}{\frac{1}{\pi_H} - (G - \frac{B}{\pi_H})} A$. The inequality (D.1) therefore implies that

$$I^m_\lambda < I^c. \tag{D.4}$$

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Define $\underline{I}^\lambda = I_\lambda^m$ until the end of the proof.

We now turn to the original economy, identified by the parameters array $(\pi_H, \pi_L = 0, G, B)$ and suppose that a bankruptcy mechanism is introduced. Consider any $(I^*, R^*) \in \mathcal{F}$ such that $I^* \geq \underline{I}^\lambda$. We show that the menu M^λ defined above supports (I^*, R^*) at equilibrium even in the presence of a bankruptcy mechanism.

Observe first that when the entrepreneur chooses an investment different from I^* or $\dfrac{I^*}{N}$ at equilibrium, covenants ensure that she strategically defaults obtaining $B'(I+A)$.[40] As in the proof of Proposition 3, choosing $\dfrac{I^*}{N}$ in each menu and selecting $e = H$ is an optimal choice for the entrepreneur by (D.3).[41]

Consider now investors' deviations. As in the proof of Proposition 3, any unilateral deviation can be represented by a menu $M' = \{(\dfrac{I^*}{N} + I', \dfrac{R^*}{N} + R'(.)), (0,0)\}$ and must induce the entrepreneur to choose $e = H$ to be profitable. See that by (D.4) and (D.2), any allocation $(I^*, R^*) \in \mathcal{F}$ with $I^* \geq \underline{I}^\lambda$ is also supported at equilibrium with the menus M^λ in the economy $\mathcal{E}^\lambda$. This establishes that the same allocation is supported at equilibrium in the original economy when a bankruptcy mechanism is in place. ∎

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[40]That is, she cannot select $e = L$ without triggering the bankruptcy mechanism.

[41]If the entrepreneur accepts $\dfrac{I^*}{N}$ in each menu and chooses $e = L$, we necessary have $B(I^* + A) - A \leq U(I^*, R^*, H)$ since $(I^*, R^*) \in \mathcal{F}$.