A Logic for Collective Choice

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ABSTRACT

This paper presents a modal logic for modelling individual and collective choices over a set of feasible alternatives. The logic extends propositional logic with a binary modality so that a formula can express not only properties of alternatives but also priorities of individuals over the properties. More importantly, each formula of this logic determines a preference ordering over alternatives based on the priorities over properties that the formula expresses. In such a way, preferences of multiple agents can be represented by a set of formulas in the same logic. This allows us to treat the problem of collective choice in a multi-agent system as aggregation of logical formulas. We further use this language to express a few plausible collective choice rules. Similar to preference aggregation, we specify collective choice rules by Arrow's conditions. Interestingly, all Arrowian conditions are plausible under the new setting except Independence of Irrelevant Alternatives. This gives us a natural way to avoid Arrow's impossibility result. Finally, we develop a model checking algorithm to automatically generate individual and collective choices in the logic.

Keywords

Multi-agent systems; collective choice; modal logic; social choice

1. INTRODUCTION

Social Choice Theory deals with the problem of how to aggregate individual preferences into a social or collective preference so as to reach a collective decision [12, 13]. In the simplest setting when individual preferences are given by orderings over available alternatives, social choice is to select a rule that maps the set of individual preference orderings into a social preference ordering over the same alternatives and then make a collective choice from the alternatives in terms of the social preference ordering. However, in many situations, individual's preference may not be given in the form of an ordering over alternatives but the "reasons" that lead to the preference [8, 21]. For instance, when buying a property, we may express our preference in the way of specifying which locations we like the house to be, how many bedrooms we want it to have and which price range we can afford. These reasons not only induce a preference ordering over the set of alternatives, say a ranking over the houses available on the market, but also convey more information than the preference ordering, which is crucial for understanding rational choice of an agent, i.e., *reason-based choice* [6, 8, 21].

Representing reasons for a preference is more primary than representing the preference itself. There are a number of ways that the concept of a reason can be formalized. A simple way is to express a reason in a propositional formula [18]. For instance, if we want a four-bedroom house located in Mountain View, the reason can be represented as $Mountain\_View \wedge Four\_bedroom$. However, most often not all of the reasons can be satisfied, and we have to make some sort of compromises. One of the most natural and convenient methods is to sort reasons. If the best option is unavailable, the agent may then be satisfied by the second best option (or third, *etc*). For instance, we might want to express that we want to buy a four-bedroom house located in Mountain View most; if it is impossible, a four-bedroom house located in Menlo Park is also fine. There are some logical languages that we can use for describing the priorities among reasons [4, 17, 19, 21, 22]. For example, if we use the language of Brewka *et al.*'s qualitative choice logic [4], our reasons of house choice with the above-mentioned priority over locations can be represented as $(Mountain\_View \wedge Four\_bedroom)\vec{\times}(Menlo\_Park \wedge Four\_bedroom)$.

Logical representation of reason-based choices does not provide a solution to the problem of reason-based social choice. One idea would be that we convert the reason-based preference of each agent into a preference ordering over alternatives and then apply a conventional preference aggregation rule to deduce the social preference ordering. Unfortunately, this does not provide a solution to the problem because the outcome of the social preference is no longer reason-based, which does not give reasons for collective choice. In fact, we need a facility that is more like judgment aggregation with which we can aggregate individual reasons into collective reasons [7].

This paper aims to propose a logical formalism for representing and reasoning about individual and collective choices based on reasons. Firstly, we extend the language of propositional logic with a binary modality so that each formula in this language can express not only reasons for choices (i.e., properties of alternatives) but also priorities over the reasons. Each formula of this logic determines a preference ordering over alternatives on the basis of the priority

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over the reasons. Secondly, once we represent the preference ordering of each agent in a single formula, the problem of collective choice is reduced to how to aggregate a set of formulas into a single formula. We then use the same language to define a few plausible collective choice rules. Thus, not only preferences but also collective choice rules can be expressed in this logic. This allows us to employ the standard model checking techniques to generate individual and collective choices. Thirdly, similar to preference aggregation, we explore collective choice rules by Arrow's conditions. Interestingly, all Arrowian conditions are plausible under the new setting except Independence of Irrelevant Alternatives (IIA). This gives us a natural way to circumvent Arrow's impossibility result in this logic. We show a possibility result by replacing IIA with Monotonicity which is inspired by [11].

The rest of this paper is structured as follows. Section 2 establishes the syntax and semantics of our proposed logical formalism. Section 3 deals with representation of reason-based choice. Section 4 extends the logical formalism to the multi-agent case and investigates reason-based social choice rules as well as Arrowian conditions for specifying these rules. Section 5 studies the properties of the logic including its expressivity, compactness and model-checking complexity. Section 6 discusses the related work. Finally we conclude the paper with a discussion of possible applications of the logic and future work.

2. THE LOGICAL FORMALISM

In this section, we will establish a logical formalism for representing and reasoning about reason-based choices. In the rest of the paper, we call this logic *reason-based choice logic*, denoted by RCL.

2.1 Syntax

Consider a propositional modal language $\mathcal{L}$ that consists of: (i) a non-empty finite set Φ_0 of propositional variables, (ii) propositional connectives $\neg$ and $\wedge$ and (iii) a binary modality $\triangledown$, representing priority over reasons. Formulas in $\mathcal{L}$ are generated by the following BNF:

$$\varphi ::= A \mid \varphi \triangledown \varphi$$

where A is a standard propositional formula built as follows:

$$A ::= p \mid \neg A \mid A \wedge A$$

where $p \in \Phi_0$. Note that the formulas are in two levels. The lower level formulas are the standard propositional formulas, used for describing reasons or properties of alternatives. The other logical connectives, $\vee$, $\rightarrow$, $\leftrightarrow$ and the logical constants $\top$, $\bot$ can be introduced in the standard way. The higher level formulas are designed to express priorities over reasons. A formula $\varphi_1 \triangledown \varphi_2$ means "choose an alternative to make φ_1 true; if no alternatives make it true, choose one to make φ_2 true". In other words, the agent gives a higher priority to the reason φ_1 than to φ_2 in the decision making.

Note that we do not allow the prioritized connective being nested in any propositional connective. The reason is that once this is allowed, the intuition behind prioritized choice will be lost. For instance, it is unclear what the prior reasons are determined by the conjunction of two prioritized choices $\varphi_1 \triangledown \psi_1$ and $\varphi_2 \triangledown \psi_2$. There are many ways to merge prioritized choices. In fact, this is exactly a problem of choice aggregation, which is the main theme of this paper.

However, as the BNF shows, nesting prioritized choice formulas is allowed. For example, $(\varphi_1 \triangledown \varphi_2) \triangledown \varphi_3$ is a well-formed formula in $\mathcal{L}$. Since $\triangledown$ is associative as we will show in next section, the following abbreviation becomes meaningful:

$$\varphi_1 \triangledown \varphi_2 \triangledown \cdots \triangledown \varphi_m =_{def} ((\varphi_1 \triangledown \varphi_2) \triangledown \cdots) \triangledown \varphi_m$$

2.2 Semantics

The semantics for the classical propositional logic interprets each propositional variable with a truth value either true or false. Formally, an interpretation I is a function that maps Φ_0 to $\{\mathbf{true}, \mathbf{false}\}$. Alternatively, an interpretation can also be expressed as a set of literals[1], in which the positive literals represent the atomic propositions that are true under the interpretation while the negative literals represent the atomic propositions which are false.

Given a decision-making problem, let W be the set of alternatives from which an agent has to choose. Assume that each alternative is uniquely specified by its properties/attributes/characters expressed by atomic propositions. Take the restaurant menu as an example. Assume a restaurant offers a number of dishes $W = \{x, y, z, ...\}$ for us to choose. Each dish is characterised by its ingredients and styles. For instance, *Bouillabaisse Royale* is a French dish made up of fish fillets, prawns, scallops, scampi, mussels and cooked in a fish-and-tomato stock. If each character is expressed by a propositional variable in $\Phi_0 = \{p, q, r, ...\}$, a dish can be uniquely identified by an interpretation of Φ_0. In general, we can simply view each alternative in W as an interpretation over Φ_0 if we represent each property/attribute/character of the alternatives by an atomic proposition in Φ_0. Therefore, W becomes a set of interpretations over Φ_0, i.e., $W \subseteq 2^{\Phi_0}$. In the following, we assume any set W of alternatives is non-empty. Note that W is finite and does not have to contain all interpretations of Φ_0.

Next we consider how to interpret a propositional formula. Suppose that you want to eat seafood in a restaurant. You will then choose a dish from the restaurant menu that contains seafood, such as fish, prawn or others. Assume that, as we mentioned above, we represent the characters of food in propositional variables. Then the statement $seafood$ can be expressed in a propositional formula, such as $(fish \vee prawn) \wedge lemon$. You choose the dishes (represented as interpretations of the language) that can satisfy this formula. In general, an alternative w in W is a candidate of our choice if it satisfies our selection reason represented by A, i.e., $w \models A$.

Finally, we consider the interpretation of a prioritized formula. As we mentioned above, $\varphi \triangledown \psi$ means *"choose an alternative to meet φ; if it's impossible, choose one to meet ψ"*. Consider an alternative $w \in W$, if w satisfies φ, it certainly satisfies $\varphi \triangledown \psi$. However, if none of the alternatives in W satisfies φ, then an alternative w' satisfies $\varphi \triangledown \psi$ only if w' satisfies ψ.

Based on above intuitive discussion, we are now ready to define the truth conditions for any formula in our language.

DEFINITION 1 (TRUTH CONDITIONS). *Given a set, $W \subseteq 2^{\Phi_0}$, of alternatives, for any $w \in W$, a formula $\varphi \in \mathcal{L}$ is true at w in W, denoted by $W, w \models \varphi$, iff*

$W, w \models p$	*iff*	$p \in w$
$W, w \models \neg A$	*iff*	$w \not\models A$
$W, w \models A \wedge B$	*iff*	$w \models A$ *and* $w \models B$
$W, w \models \varphi_1 \triangledown \varphi_2$	*iff*	$W, w \models \varphi_1$, *or* ($W, w \models \varphi_2$ *and* $W, w' \not\models \varphi_1$ *for all* $w' \in W$).

We say φ is *valid* in W, denoted by $W \models \varphi$, if $W, w \models \varphi$ for every $w \in W$; φ is *valid*, denoted by $\models \varphi$, if $W \models \varphi$ for any $W \subseteq 2^{\Phi_0}$. Given any $\varphi, \psi \in \mathcal{L}$, φ is a logical consequence of ψ, denoted by $\varphi \models \psi$, iff for any W and any $w \in W$, if $W, w \models \varphi$, then $W, w \models \psi$.

[1] A literal is an atomic proposition or its negation.

The following result shows that the prioritized connective is associative.

PROPOSITION 1. *For any φ_1, φ_2 and $\varphi_3 \in \mathcal{L}$, for any W and for any $w \in W$,*

$$W, w \models \varphi_1 \triangledown (\varphi_2 \triangledown \varphi_3) \text{ iff } W, w \models (\varphi_1 \triangledown \varphi_2) \triangledown \varphi_3$$

3. EXPRESSING CHOICE

In this section, we first display how to use RCL for representing choice and then demonstrate with an example to show how to use the prioritized connective for making a choice. *A choice set* is a subset of alternatives that are selected by reasons specified by a formula $\varphi \in \mathcal{L}$. We now introduce the syntactical representation of a choice set.

DEFINITION 2. *Given a set W of alternatives and a formula $\varphi \in \mathcal{L}$,* the choice set *specified by φ in W, denoted by $C(W, \varphi)$, is defined as follows:*

$$C(W, \varphi) = \{w \in W : W, w \models \varphi\}$$

Intuitively, a choice set $C(W, \varphi)$ includes all the alternatives in W that satisfy φ.

To illustrate how to use prioritized connectives for making choices, let us consider the following example.

EXAMPLE 1. *Three friends Ann, Kate and Bill are going to watch a movie together. Ann is a super fan of cartoon comedies therefore is eager to find one. If nothing, other comedies are ok or a fiction as the least option. Kate also likes cartoons but only non-fiction cartoons. If nothing, she picks a comedy or a fiction if nothing else. Finally, Bill will surely go for a fiction and a non-cartoon is also ok. If nothing, any movie seems fine for him. They find three movies are on show: $Gravity$, $Flipped$ and $Frozen$. It is known that:*

- *$Gravity$ is a fiction but not a comedy or cartoon;*
- *$Flipped$ is a comedy but not a fiction or cartoon;*
- *$Frozen$ is a cartoon comedy but not a fiction.*

Let us first formalize this example. The set of atomic properties is $\Phi_0 = \{Fiction, Comedy, Cartoon\}$. The set of feasible alternatives is $W = \{Gravity, Flipped, Frozen\}$ where

$$\begin{aligned} Gravity &= \{Fiction, \neg Comedy, \neg Cartoon\} \\ Flipped &= \{\neg Fiction, \neg Cartoon, Comedy\} \\ Frozen &= \{\neg Fiction, Comedy, Cartoon\} \end{aligned}$$

The reasons with priorities of Ann, Kate and Bill are described as follows:

$$\begin{aligned} \varphi_{Ann} &= (Comedy \wedge Cartoon) \ \triangledown \ Comedy \ \triangledown \ Fiction \\ \varphi_{Kate} &= (Cartoon \wedge \neg Fiction) \ \triangledown \ Comedy \ \triangledown \ Fiction \\ \varphi_{Bill} &= Fiction \ \triangledown \ \neg Cartoon \ \triangledown \ \top \end{aligned}$$

According to their individual reasons, Ann and Kate both choose the movie $Frozen$, and Bill chooses the movie $Gravity$. This intuitive judgment is validated by the model, that is

- $W, Frozen \models \varphi_{Ann}$
- $W, Frozen \models \varphi_{Kate}$
- $W, Gravity \models \varphi_{Bill}$

Then it follows that

- $C(W, \varphi_{Ann}) = \{Frozen\}$.
- $C(W, \varphi_{Kate}) = \{Frozen\}$.
- $C(W, \varphi_{Bill}) = \{Gravity\}$.

As they would like to watch a movie together, then a natural question arises: which movie should they choose collectively? We will deal with this issue in the next section.

Before handling the collective dimension, we show that the approach to use prioritized connectives for making choices is rational, that is, this approach satisfies the two standard rationality conditions: *the contraction condition* and *the expansion condition* [13].

(Contraction Condition). Given two sets of alternatives W, W' with $W \subseteq W'$, for all $w \in W$ and for all $\varphi \in \mathcal{L}$, if $w \in C(W', \varphi)$, then $w \in C(W, \varphi)$. This condition requires that if you choose some alternative from a set of alternatives and this alternative remains available in a more restricted set, then you also choose it from the restricted one.

(Expansion Condition). Given two sets of alternatives W, W' with $W \subseteq W'$, for all alternatives $w, w' \in W$ and for any formula $\varphi \in \mathcal{L}$, if ($w \in C(W, \varphi)$ and $w' \in C(W, \varphi)$), then ($w \in C(W', \varphi)$ iff $w' \in C(W', \varphi)$). This condition requires that if you choose two alternatives from a set of alternatives, then you choose them or not choose them at the same time from a lager set.

PROPOSITION 2. *For any set W of alternatives and any formula $\varphi \in \mathcal{L}$, the choice set $C(W, \varphi)$ satisfies the contraction condition and the expansion condition.*

4. COLLECTIVE CHOICE

Let us now extend the logical formalism to the multi-agent case, explore the desirable conditions for collective choice under the reason-based setting, and further define plausible collective choice rules based on aggregation of reasons.

4.1 Setting

We consider a finite set of agents $N = \{1, 2, \cdots, n\}$. Each agent $i \in N$ has her own reasons which are specified by a formula $\varphi_i \in \mathcal{L}$ of the form $A_1^i \triangledown A_2^i \triangledown \cdots \triangledown A_m^i$ such that

(Individual Completeness) $W \models \bigvee_{k=1}^{m} A_k^i$.

Individual completeness means each individual takes all the alternatives in W into consideration, that is, her priority over these reasons induces a rank for every alternative in W, which corresponds to the completeness requirement in preference aggregation. This requirement guarantees that each individual has a non-empty choice set, i.e., $C(W, \varphi_i) \neq \emptyset$. In the following, we call a formula that satisfies individual completeness *individual choice*. Given each individual's choice φ_i, the vector $\langle \varphi_i \rangle_{i \in N}$ is called *a profile*.

Finally, a collective choice rule is a function f that assigns to each profile $\langle \varphi_i \rangle_{i \in N}$ of individual choices a single formula $\varphi \in \mathcal{L}$ of the form $A_1 \triangledown A_2 \triangledown \cdots \triangledown A_m$ such that

(Collective Completeness) $W \models \bigvee_{k=1}^{m} A_k$.

Similarly, this condition guarantees that the aggregate formula should always determinate a collective alternative, i.e., $C(W, \varphi) \neq \emptyset$, and induce a collective preference ordering over alternatives based on the priority over reasons. The set of admissible profiles is called the *domain* of f, denoted by $\text{Dom}(f)$.

Similar to preference aggregation [13], a collective choice rule must behave in a rational way and few constraints or conditions have to be set for enforcing this rationality. The very first requirement is that equivalent formulas should determine the same most preferable alternatives. However, the collective choice is not completely dependent on the set of individual most preferable alternatives. In fact, most of the time individuals have to make some compromises, and the second, the third, or even the last most preferred alternatives are taken into consideration. As the standard notions of logical consequence and equivalence are defined based on the most preferable alternatives, thus they are insufficient for handling collective choice and need to be strengthened.

DEFINITION 3. *Given a set W of alternatives, let $\varphi = A_1 \triangledown \cdots \triangledown A_m$ and $\psi = B_1 \triangledown \cdots \triangledown B_n$. Then ψ is a strong consequence of φ with respect to W, denoted by $\varphi \Vdash_W \psi$, iff*

1. *$m \geq n$, and*

2. *$W \models A_k \rightarrow \bigvee_{i=1}^{k} B_i$ for any $1 \leq k \leq n$.*

The intuitive meaning behind this notion is the following: first, the length of the priority over reasons specified by φ is at least as large as that by ψ. This means the number of ranks of alternatives generated by φ is at least as large as that given by ψ (Condition 1); secondly, for any alternative in W, its rank given by ψ is at least as high as the one given by φ (Condition 2). For instance, when $k = 1$, the most important reason in φ implies the most important reason in ψ. This means the most preferred alternative of φ if exists is also a most preferred alternative of ψ.

EXAMPLE 2. *Suppose three alternatives, $W = \{x, y, z\}$ s.t. $x = \{p, \neg q, \neg r\}$, $y = \{q, \neg p, \neg r\}$ and $z = \{r, \neg q, \neg p\}$. Let $\varphi = p \triangledown q \triangledown r$ and $\psi = (p \vee r) \triangledown q$. Then $\varphi \Vdash_W \psi$ as $W \models p \rightarrow (p \vee r)$ and $W \models q \rightarrow (p \vee r \vee q)$. Figure 1 illustrates the relation between the ranks for each alternative (the higher, the better).*

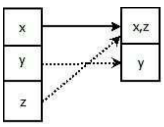

Figure 1: ψ (right) is a strong consequence of φ (left)

Furthermore, the strong notion of equivalence is defined as follows:

DEFINITION 4. *Given a set W of alternatives, and for any $\varphi, \psi \in \mathcal{L}$, φ is strongly equivalent to ψ with respect to W, denoted by $\Vdash_W \varphi \equiv \psi$, iff $\varphi \Vdash_W \psi$ and $\psi \Vdash_W \varphi$.*

This definition says that two strongly equivalent formulas have the same priority over reasons. In the other words, they assign the same rank to every alternatives in W. Let us continue Example 2. Consider another formula $\chi = \neg q \triangledown q$, then $\Vdash_W \psi \equiv \chi$, as $W \models \neg q \leftrightarrow (p \vee r)$ and $W \models q \leftrightarrow \neg(p \vee r) \wedge q$. Figure 2 illustrates this strong equivalence.

The following proposition restates the strong equivalence in terms of the standard logical equivalence. In fact, it also serves as an important lemma for proving our main result, e.g., Theorem 1.

PROPOSITION 3. *Given a set W of alternatives, let $\varphi = A_1 \triangledown \cdots \triangledown A_m$ and $\psi = B_1 \triangledown \cdots \triangledown B_n$. Then $\Vdash_W \varphi \equiv \psi$ iff*

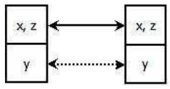

Figure 2: ψ (left) is strongly equivalent to χ (right)

1. *$m = n$,*

2. *$W \models A_1 \leftrightarrow B_1$, and*

3. *$W \models (\bigwedge_{i=1}^{k-1} \neg A_i \wedge A_k) \leftrightarrow (\bigwedge_{i=1}^{k-1} \neg B_i \wedge B_k)$ for any $2 \leq k \leq n$.*

Note that the strong consequence (equivalence) and the standard consequence (equivalence) are the same if φ is a propositional formula. However, once φ is a formula with prioritized connectives, these two versions become different. As we will see in the next section, the strong version is designed for specifying collective choice conditions.

4.2 Conditions on Collective Choice Rules

Now we are in the position to provide the conditions which a collective choice rule is expected to satisfy. Let f be a collective choice rule.

U (Unrestricted domain). *For any profile $\langle \varphi_i \rangle_{i \in N}$, $\langle \varphi_i \rangle_{i \in N} \in Dom(f)$.* The domain of the collective choice rule f includes all profiles of individual choices.

A (Anonymity). *For any profile $\langle \varphi_i \rangle_{i \in N}$, and any permutation $\sigma : N \mapsto N$, $\Vdash_W f(\langle \varphi_i \rangle_{i \in N}) \equiv f(\langle \varphi_{\sigma(i)} \rangle_{i \in N})$.* This requires that the ordering among the agents does not affect the collective result and the collective rule should treat each individual neutrally.

M (Monotonicity). *For any two profiles $\Phi = \langle \varphi_i \rangle_{i \in N}$, $\Phi' = \langle \varphi'_i \rangle_{i \in N}$, for all $i \in N$, if $\varphi_i \Vdash_W \varphi'_i$, then $f(\Phi) \Vdash_W f(\Phi')$.* This condition is the qualitative counterpart of monotonicity condition in [11]. It specifies that for any two preference profiles and any alternative if for each individual the rank of the alternative in one profile is at least as high as that in the other, then its collective rank of the former is at least as high as that of the later.

P (Pareto principle). *For any profile $\Phi = \langle \varphi_i \rangle_{i \in N}$ and for any formula $\varphi \in \mathcal{L}$, if $\Vdash_W \varphi_i \equiv \varphi$ for any $i \in N$, then $\Vdash_W f(\Phi) \equiv \varphi$.* Intuitively, if all individual priorities over reasons are the same, then this condition requires that the collective priority over reasons is the same as each individual's one.

The following proposition says Non-dictatorship can be derived from Anonymity.

PROPOSITION 4. *Every collective choice rule f satisfying **A** is non-dictatorial, i.e., there is no $i \in N$ such that $\Vdash_W \varphi_i \equiv f(\Phi)$ for any profile Φ.*

Under this reason-based setting Universal Domain, Non-dictatorship and Pareto principle correspond to their counterparts of Arrowian conditions in preference aggregation except Independence of Irrelevant alternatives which is replaced by Monotonicity. It turns out that different from Arrow's impossibility result, as we will show in the next section, there are collective choice rules that satisfying **UAMP**.

φ_1	$:=$	$A_1^1 \triangledown A_2^1 \triangledown \cdots \triangledown A_{m_1}^1$
φ_2	$:=$	$A_1^2 \triangledown A_2^2 \triangledown \cdots \triangledown A_{m_2}^2$
	$\cdots$	
φ_i	$:=$	$A_1^i \triangledown A_2^i \triangledown \cdots \triangledown A_{m_i}^i$
	$\cdots$	
φ_n	$:=$	$A_1^n \triangledown A_2^n \triangledown \cdots \triangledown A_{m_n}^n$

Table 1: The profile of individual choices

4.3 The Collective Choice Rules

We now show that RCL can not only represent individual preferences but also express collective choice rules.

Suppose a finite set $N = \{1, \cdots, n\}$ of agents and a profile of individual choices $\Phi = \langle \varphi_i \rangle_{i \in N}$ detailed by Table 1. Without loss of generalization, let $m_1 < m_2 < \cdots < m_n$.

We first define a naive rule F_{grd} called *the grounded rule*[2] as follows:

$$F_{grd}(\Phi) \quad = \quad (\bigvee_{i=1}^{n} A_1^i) \triangledown (\bigvee_{i=1}^{n} A_2^i) \triangledown \cdots \triangledown (\bigvee_{i=1}^{n} A_{m_1}^i)$$

The intuition behind the grounded rule is that *the collective choice should be a best choice for at least one of the agents*. The rule works in this way: first check whether there is any alternative satisfying one of individual's most important reasons, if yes, simply choose this alternative; Otherwise, go on and check whether there is any alternative satisfying one of the individual's second most important reasons. Continue this procedure until an alternative is found. This rule is simple and easy to be executed. However, it has drawbacks as it naively selects one of the individual's most preferred alternatives to be the collective choice without taking into account the ranks of this alternative in the others' preference orderings. Consider again, Example 1, then, according to F_{grd}, movie $Gravity$ is a possible output for collective choice as $Bill$ likes it most. However, this is counter-intuition as both Ann and Kate like $Gravity$ the least. Hence, the worst-off dimension should be more considered.

The next rule F_{max} called *the maximal rule* solves this problem. It is defined as follows:

$$\begin{aligned} F_{max}(\Phi) \quad = \quad & (\bigwedge_{i=1}^{n} A_1^i) \triangledown (\bigwedge_{i=1}^{n} (A_1^i \vee A_2^i)) \triangledown \cdots \\ & \triangledown (\bigwedge_{i=1}^{n} (\bigvee_{j=1}^{m_1} A_j^i)) \triangledown (\bigwedge_{i=2}^{n} (\bigvee_{j=1}^{m_2} A_j^i)) \triangledown \cdots \\ & \triangledown (\bigwedge_{i=n-1}^{n} (\bigvee_{j=1}^{m_{n-1}} A_j^i)) \triangledown (\bigvee_{j=1}^{m_n} A_j^n) \end{aligned}$$

The idea behind the maximal rule F_{max} is to *maximize the situation of the worst-off.* This rule guarantees that an alternative has to be collectively selected only if its lowest rank in all individual preference orderings is the highest among the lowest ranks of all the other alternatives. This specification rules out the possibility of movie $Gravity$ to a collective choice in Example 1. The maximal rule proceeds as follows:

1. Check whether or not there is some alternative satisfying the most important reasons of all individuals (the conjunction of the first prioritized disjunct of each formula). Yes, choose that alternative and halt; No, go to the next step.

2. Check whether or not there is some alternative satisfying either the first or the second important reasons of all individuals (the conjunction of all the disjunctions of the first two prioritized disjuncts of each individual formula). Yes, choose that alternative and halt; No, go to the next step.

3. Continue above procedure until there is k such that some alternative satisfies the disjunction of first k prioritized disjuncts of all individual formulas.

Since each individual formula is complete and its length is finite, so such k exists. Note that the last two lines take care of the case that individual formulas may have different lengths. This rule is the qualitative counterpart of *Fagin's Algorithm* in database system which is an efficient data aggregation algorithm with elegant mathematical properties [9].

Finally, we define a family of *uniform quota rules* [7]. Given a collective choice threshold $\tau \in \{1, 2, \cdots, n\}$, the corresponding uniform quota rule, denoted by F_τ, is defined as follows:

$$\begin{aligned} F_\tau(\Phi) \quad = \quad & (\bigvee_{C \subseteq N, |C| = \tau} \bigwedge_{i \in C} A_1^i) \triangledown \\ & (\bigvee_{C \subseteq N, |C| = \tau} \bigwedge_{i \in C} (A_1^i \vee A_2^i)) \triangledown \cdots \\ & (\bigvee_{C \subseteq N, |C| = \tau} \bigwedge_{i \in C} (\bigvee_{j=1}^{m_1} A_j^i)) \triangledown \\ & (\bigvee_{C \subseteq N \setminus \{1\}, |C| = \tau - 1} \bigwedge_{i \in C} (\bigvee_{j=1}^{m_2} A_j^i)) \triangledown \\ & (\bigvee_{C \subseteq N \setminus \{1,2\}, |C| = \tau - 2} \bigwedge_{i \in C} (\bigvee_{j=1}^{m_3} A_j^i)) \triangledown \cdots \\ & (\bigvee_{C \subseteq N \setminus \{1, \cdots, \tau - 1\}, |C| = 1} \bigwedge_{i \in C} (\bigvee_{j=1}^{m_\tau} A_j^i)) \end{aligned}$$

The rule proceeds in this way (the process is similar to F_{max}): it checks if there is any subset of agents $C \subseteq N$ such that (i) the size of this subset is equal to τ and (ii) the most important reasons of every agent belonging to C is satisfied. All possible subsets are evaluated (disjunction appearing in the first line). If no subset has been satisfied then we repeat the process except that we consider their first two most important reasons (Second line). The process continues until some alternative satisfies τ agents (lines 3 and further). The last three lines consider the case that individual formulas may have different lengths. This rule is a generalization of the majority rule as well as the other two rules.

- The simple majority rule, denoted by F_{maj}, can be encoded by setting[3] quota $\tau = \lceil \frac{n+1}{2} \rceil$.

- The grounded rule can be encoded by setting $\tau = 1$.

- The maximal rule can be encoded by setting $\tau = n$.

To illustrate how these rules work, let us back to Example 1.

Example 1 (*continued*) *The model of this example is given as follows:* $\Phi_0 = \{Fiction, Comedy, Cartoon\}$ *and* $N = \{Ann, Kate, Bill\}$. *The set of feasible alternatives* $W = \{Gravity, Flipped, Frozen\}$ *where*

- $Gravity = \{Fiction, \neg Comedy, \neg Cartoon\}$,

- $Flipped = \{\neg Fiction, \neg Cartoon, Comedy\}$,

- $Frozen = \{\neg Fiction, Comedy, Cartoon\}$.

The reason with priority of each agent is described as follows:

- $\varphi_{Ann} := (Comedy \wedge Cartoon) \triangledown Comedy \triangledown Fiction$

[2]The notion 'groundedness' is borrowed from [23].

[3]$\lceil \rho \rceil$ is defined as the smallest integer greater or equal to number ρ.

- $\varphi_{Kate} := (Cartoon \wedge \neg Fiction)\ \triangledown\ Comedy \triangledown Fiction$
- $\varphi_{Bill} := Fiction \triangledown \neg Cartoon \triangledown \top$

The collective formulas generated by the grounded rule, the maximal rule and the simple majority rule are calculated respectively as follows: let Φ denote the profile $\langle \varphi_{Ann}, \varphi_{Bill}, \varphi_{Jim}\rangle$, then

- $F_{grd}(\Phi) =$

  $(Fiction \vee Cartoon) \triangledown (Comedy \vee \neg Cartoon) \triangledown Fiction$

- $F_{max}(\Phi) = \bot \triangledown (Comedy \wedge (Fiction \vee \neg Cartoon))$ $\triangledown (Comedy \vee Fiction \vee Cartoon)$
- $F_{maj}(\Phi) = (Comedy \wedge Cartoon) \triangledown Comedy$ $\triangledown (Comedy \vee Fiction \vee Cartoon)$

It follows that the three aggregate formulas determine the following choice sets.

- $C(W, F_{grd}(\Phi)) = \{Gravity, Frozen\}$;
- $C(W, F_{max}(\Phi)) = \{Flipped\}$;
- $C(W, F_{maj}(\Phi)) = \{Frozen\}$.

This says that according to the grounded rule, the three friends are expected to choose either $Gravity$ or $Frozen$, and according to the maximal rule, $Flipped$ is the collective choice, while according to the simple majority rule, they should choose $Frozen$.

As stressed by the Example, the behavior of these rules are different, and thus deciding which rule to use is usually situation-dependent. Let us now provide the main result: uniform quota rules satisfy Universal Domain, Anonymity, Monotonicity and Pareto principle.

THEOREM 1. *The uniform quota rule F_τ satisfies **UAMP**.*

PROOF. (Sketch) It is straightforward that F_τ satisfies collective completeness and universal domain. F_τ satisfies Monotonicity by Definition 3, and F_τ satisfies Anonymity and Pareto principle by Proposition 3. □

COROLLARY 1. *The grounded rule F_{grd} and the maximal rule F_{max} both satisfy **UAMP**.*

Therefore, as we expect, different from Arrow's impossibility result, there are collective choice rules satisfying **UAMP**. At last, we would like to mention that the size of an aggregate formula can be significantly reduced via the following two ways. First, any aggregate formula represents a preference ordering over alternative, and thus its size is virtually determined only by the number of alternatives as well as their properties, and has nothing to do with the number of individuals. As shown in Example 1, prioritized disjuncts can be extremely simplified by equivalence laws of propositional logic. Secondly, there are many reasons that none of the alternatives in W satisfies, and if we remove such "dummy" reasons, the collective preference ordering over alternatives will not be changed, as they correspond to the empty set of alternatives. For instance, in Example 1, the normal form of the aggregate formula $F_{max}(\langle \varphi_{Ann}, \varphi_{Bill}, \varphi_{Jim}\rangle)$ is $(Comedy \wedge (Fiction \vee \neg Cartoon)) \triangledown (Comedy \vee Fiction \vee Cartoon)$ which represents the same preference ordering as itself, i.e., $Gravity$ and $Frozen$ are indifferent, and $Flipped$ is better than any of them. We leave the precise complexity result for future work.

4.4 Relation with Preference Aggregation

In this subsection, we discuss the relationship between the collective choice in RCL and the preference aggregation in social choice. In particular, we will compare our possibility result with Arrow's impossibility theorem.

We first show that the Condorcet's paradox [14] can be naturally avoided in RCL due to the character of its language. As we will see in Proposition 5, any formula generates a preference ordering over the alternatives on the basis of the priority over reasons. This guarantees that all collective rules in RCL always induce a preference ordering over alternatives. For example, given the set of atomic properties $\Phi_0 = \{p, q, r\}$, suppose we have three candidates $W = \{w_1, w_2, w_3\}$ where $w_1 = \{p, \neg q, \neg r\}$, $w_2 = \{\neg p, q, \neg r\}$ and $w_3 = \{\neg p, \neg q, r\}$. There are three voters 1, 2 and 3 with their voting reasons given as follows: $\varphi_1 = p \triangledown q \triangledown r$, $\varphi_2 = q \triangledown r \triangledown p$, and $\varphi_3 = r \triangledown p \triangledown q$. According to the simple majority rule ($\tau = 2$), we get the collective result as follows:

$F_{maj}(\langle \varphi_1, \varphi_2, \varphi_3\rangle) = [(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)] \triangledown [((p \vee q) \wedge (q \vee r)) \vee ((p \vee q) \wedge (p \vee r)) \vee ((q \vee r) \wedge (p \vee r))] \triangledown (p \vee q \vee r)$

This aggregate formula would result in a tie with three alternatives are indifferent, i.e., $C(W, F_2(\langle \varphi_1, \varphi_2, \varphi_3\rangle)) = \{w_1, w_2, w_3\}$ instead of a cyclic (intransitive) preference ordering.

On the other hand, all Arrowian conditions are plausible under the new setting except Independence of irrelevant alternatives (IIA). The main reason is that this condition requires for any two alternatives w and w', the collective preference between w and w' depends only upon individual preferences over that pair. Different from other Arrowian conditions, IIA is a specification about the order of two particular alternatives. Such a local property is inconsistent with the global property of RCL. As a RCL-formula always specifies a priority over reasons which further induces a preference ordering over whole alternatives, thus it is impossible for a RCL-formula to describe two particular alternatives. This gives us a natural way to circumvents Arrow's impossibility result in RCL.

In addition, we consider another condition Monotonicity which is the qualitative counterpart of the monotonicity in database aggregation algorithm [11]. This condition is an important property for rank aggregation rules. In fact, our proposed rules may be regarded as rank aggregation rules as they aggregate reasons according to their priorities layer by layer. It turns out that all other Arrowian conditions are consistent with Monotonicity.

5. PROPERTIES OF RCL

In this section, we first analyze the expressivity and succinctness of our language for preference representation, and then investigate the model-checking complexity of this logic.

5.1 Expressivity

We now consider how to extend the priority over reasons to a preference ordering over alternatives. A preference ordering over alternatives is a reflexive, total and transitive relation over alternatives. Hereafter, we show that the proposed language is simple, yet expressive enough to describe any preference ordering over alternatives.

Informally speaking, we generate a preference ordering over W in terms of a formula φ, denoted by $\preceq_\varphi$, in this way: an alternative w is preferred to an alternative w', denoted by $w \preceq_\varphi w'$, if the maximal important reason that w satisfies is given at least a similar priority as the maximal important reason that w' satisfies.

DEFINITION 5. *Given $\varphi \in \mathcal{L}$ of the form $A_1 \triangledown A_2 \triangledown \cdots \triangledown A_m$ and an alternative $w \in W$, let $h(w) = Min\{i \mid w \models A_i\}$ if $w \models$*

$\bigvee_{i=1}^{m} A_i$*; otherwise* $h(w) = m + 1$. *Then for any two alternatives* $w, w' \in W$, $w \preceq_\varphi w'$ *iff* $h(w) \leq h(w')$.

As usual, $w \prec_\varphi w' =_{def} w \preceq_\varphi w'$ and not $(w' \preceq_\varphi w)$, and $w \sim_\varphi w' =_{def} w \preceq_\varphi w'$ and $w' \preceq_\varphi w$. We next use a simple example to illustrate this definition.

EXAMPLE 3. *Given* $\Phi_0 = \{p, q, r, s\}$ *and* $\varphi = (p \wedge q) \nabla (r \to s)$*, let* $W = \{w_1, w_2, w_3, w_4\}$ *where* $w_1 = \{p, q, r, \neg s\}$, $w_2 = \{p, q, r, s\}$, $w_3 = \{\neg p, q, r, \neg s\}$ *and* $w_4 = \{p, \neg q, r, s\}$. *We have* $h(w_1) = h(w_2) = 1$, $h(w_3) = 3$ *and* $h(w_4) = 2$*, hence* $w_1 \sim_\varphi w_2 \prec_\varphi w_4 \prec_\varphi w_3$.

The following proposition says the language $\mathcal{L}$ can not only generate a preference ordering over W, but also express any preference ordering over W.

PROPOSITION 5. *Given a set* W *of alternatives,*

1. *for any formula* $\varphi \in \mathcal{L}$, $\preceq_\varphi$ *is a preference ordering over* W.
2. *for any preference ordering* $\preceq$ *over* W*, there is a formula* $\varphi \in \mathcal{L}$ *such that* $\preceq = \preceq_\varphi$.

5.2 Succinctness

We next investigate another property of this language: the relative space efficiency. In particular, we will show that one of the standard and compact languages called the goal-based language $R_{GB}^{bestout}$ in [5] can be translated to our language in polyminal-size, and vice versa.

DEFINITION 6. *A goal base* GB *is a tuple* $\langle\{G_1, \cdots, G_n\}, r\rangle$ *where*

- $\{G_1, \cdots, G_n\}$ *is a finite set of propositional formulas;*
- r *is an associated function from* $\mathbb{N}$ *to* $\mathbb{N}$.

If $r(i) = j$, then j is called the rank of the formula G_i. By convention, a lower rank means a higher priority. The priority on goals can be extended to a preference ordering over alternatives via *best-out ordering* [2].

DEFINITION 7. *Let* $r_{GB}(w) = Min\{r(i) \mid w \not\models G_i\}$*, then for any* $w, w' \in W$, $w \preceq_{GB}^{bo} w'$ *iff* $r_{GB}(w) \geq r_{GB}(w')$.

Then we have the following result saying that our language has the same degree of succinctness with $R_{GB}^{bestout}$.

PROPOSITION 6. *There is a polynominal-size translation from* $\mathcal{L}$ *to* $R_{GB}^{bestout}$*, and vice versa.*

This means RCL and $R_{GB}^{bestout}$ have the same space efficiency for preference representation, and the succinctness results of $R_{GB}^{bestout}$ in [5] hold for RCL as well.

5.3 Model Checking

One of the advantages of RCL is that the collective choice rules are built in the language, which allows us to use the model-checking techniques to automatically generate collective choices. The *model-checking problem* for RCL is the problem of determining: for a given RCL formula φ, a set W of alternatives and an alternative $w \in W$, whether $W, w \models \varphi$ or not.

PROPOSITION 7. *There is an algorithm that runs in time* $O(|W| \times \|\varphi\|)$ *to check, given any set* $W \subseteq 2^{\Phi_0}$*, any world* $w \in W$ *and any formula* $\varphi \in \mathcal{L}$*, whether* $W, w \models \varphi$*, where* $\|\varphi\|$ *denotes the size of* φ*, i.e., the number of symbols occurring in* φ.

Algorithm 1: computeTruth(W, φ)

```
input  : W ⊆ 2^{Φ0} and φ ∈ L
output: the set {w ∈ W : W, w ⊨ φ}
begin
    switch φ do
        case p, where p ∈ Φ0
            T ← {w ∈ W : p ∈ w};
            break;
        case ¬ψ, where ψ ∈ L
            T ← computeTruth(W, ψ);
            T ← W \ T;
            break;
        case ψ1 ∧ ψ2, where ψ1, ψ2 ∈ L
            T ← computeTruth(W, ψ1);
            T ← T ∩ computeTruth(W, ψ2);
            break;
        case ψ1 ∇ ψ2, where ψ1, ψ2 ∈ L
            T ← computeTruth(W, ψ1);
            if T = ∅ then
                T ← computeTruth(W, ψ2);
            break;
        otherwise
            T ← ∅;
    return T;
```

PROOF. (Sketch) It suffices to develop an algorithm that runs in time $O(|W| \times \|\varphi\|)$ to compute the set of worlds $w' \in W$ such that $W, w' \models \varphi$. We implement this computation by Algorithm 1. The general idea is to compute the desired world sets for all subformulas of φ recursively. Given any subformula ψ of φ, assuming that the mentioned world sets for all proper subformulas of ψ are available, the computation on ψ can be done in time $O(|W|)$. The number of subformulas of φ is clearly not greater than $\|\varphi\|$. Thus, Algorithm 1 must terminate in time $O(|W| \times \|\varphi\|)$. According to the semantics, it is also easy to verify the correctness of this algorithm. □

6. RELATED WORK

In recent years, many logical frameworks have been proposed for representing and reasoning about choices and preferences [3, 4, 17, 19, 21, 22, 24, 25, 26]. Most of the previous work on representing and reasoning about preferences takes preferences as fundamental and primitive concepts and typically treats them as modal operators [25, 26]. On the one hand, these preference logics mainly focus on investigating logical properties of preferences with little concern about how preferences are formed or where they come from. On the other hand, formulas in these logics are all interpreted by an arbitrary given (utilitarian or ordinal) preference relation and thus have no facility to represent different preference orderings.

Recent work [4, 19, 21, 22] takes a different angle and explores how preferences come into being.

[19] introduces *a priority base* that is ordered by importance and mainly discusses the ways to rationally derive preferences from it. However, the priority base is defined in the semantical level and the priority operator itself is not part of the language.

[21] and [22] focus on logical representations of reason-based preferences. Pedersen *et al* [22] develop a modal logic for reasoning about Dietrich and List's framework [8] and show how to use a standard modal logic for reasoning about reason-based preferences. Different from RCL, they use the standard modal language and just

encode the priority (weighing relation) over reasons into the semantical model. It is worth mentioning that they have generalized their work to multi-agent case mainly for modelling non-trivial properties such as disagreement, consensus in a multi-agent setting rather for aggregating preferences. Moreover, they assume that all agents share the same priority over reasons. This means all agents have the same preference ordering over alternatives, which is unsuitable for preference aggregation. Meanwhile, Osherson and Weinstein [21] develop a non-standard modal logic for reasoning about reason-based preferences. Different from our qualitative approach, their formalism is developed in the context of utility and each world can be evaluated according to various utility scales. They consider different ways to combine utilities induced by different reasons in the context of single agent without generalizing their work to multi-agent dimension.

One of the closest related work is probably Brewka *et al*'s framework [4] where an extension of propositional logic for representing qualitative choices, called *Qualitative Choice Logic* (QCL), is developed. The non-standard part of this logic is a new logical connective $\vec{\times}$ called ordered disjunction. It is worth mentioning that the intuition behind the binary operator $\triangledown$ is similar to that of the ordered disjunction. Also, the same idea, though applied to deontic reasoning, was independently developed in [15]. The semantics of QCL is based on the degree of satisfaction of a formula in a particular model. However, their motivation and approaches are different from ours. They propose a nonmonotonic formalism for representing reason-based choices, while we provide a modal logic not only for representing reason-based choices but also for representing and generating reason-based social choice.

Based on above analysis, we may notice that few logical formalisms can provide a logical language which is not only compact and expressive for representing preferences, but also equipped with efficient decision procedures so as to automatically generate individual and collective choices based on reasons. To our best of knowledge, [17] is the only one that deals with both aspects. It uses weighted logics for representing preferences, that is, each agent expresses her preferences by means of logical formulas weighted by importance degrees, and then generates the collective result by calculating the utility of each agent into a collective utility function. Compared to this work, the major difference is that we use a qualitative approach, and show that preferences as well as collective rules are expressed in a standard modal logic, and thus model-checking algorithm is developed for modal logics for generating individual choices and collective choices. The relationship to such formalism is left for future work.

7. CONCLUSION

In this paper, we have proposed a modal logic for representing and reasoning about individual and collective choices based on reasons. It has been shown that this language can not only describe reasons and the priorities over reasons, but also represent preference orderings over alternatives. We then use the same language to define a few collective choice rules. So, not only preferences but also the aggregation rules can be expressed with the same language. We further develop a model checking algorithm to generate individual and collective choices. Meanwhile, we have demonstrated with proposed collective choice rules that all Arrowian conditions are plausible under this new setting except Independence of Irrelevant Alternatives. These collective rules have been encoded in RCL in an elegant way and the specifications of these rules have shown how powerful the language is for representing and reasoning about collective choice problems.

We do believe that Reason-based Choice Logic is general enough for handling applications out of the classical scope of social choice of theory. Let us mention two typical problems where RCL is relevant.

- In database systems, prioritized queries can be used to describe suboptimal results [10]. For instance, one may start with a query like: find a second-hand house less than 5 years near subway, if subway is unavailable then near bus stop. A compound prioritized query which consists of at least two prioritized queries can be dealt with by collective rules. Moreover, the data aggregation algorithm combines information that may be provided by different agents so as to produce a top-k list. The combination of prioritized results issued from multiple agents can then be reduced to the aggregation of individual formulas. The proposed collective choice rules are likely to play a role in this process.

- In belief merging [16], the aim is to combine several pieces of information coming from different sources in a unique one. The aggregation procedure in belief merging faces problems close to those addressed in social choice theory. Key difference is that beliefs are flat and no priority are expressed among them. However, priorities are still somewhere: the output should be as close as possible from the individual set of beliefs. Closeness is usually represented through an appropriate notion of *distance* which is used to define distance-based merging operators. The notion of distance can be easily translated in RCL for representing what statements an agent prefers if her belief cannot be included in the output. Hence each flat set of belief can be transformed in a prioritized formula and our collective rules can be considered for combining them.

As RCL can encode preferences and collective choice rules at the same time, our long term goal is to enable intelligent agent to reason about collective choices. To this end, several extensions have to be considered as follows:

Firstly, to keep our logical formalism as simple and intuitive as possible, we currently do not allow the prioritized connective to interplay with the other classical connectives. Consequently, the context dependent preferences are beyond the expressivity power of this language. It would be interesting to remove some syntactical limitations of the language for being in position to represent more general preferences.

Secondly, in this paper we have assumed that one agent's choice is given without any influence from others. However, in many situations, beliefs about other agents' choices might affect the agent's own decision, and consequently change the collective choice. Thus, it's worth extending the language with epistemic operators so as to study the effect of beliefs on individual and collective choices. In particular, it's possible to introduce a hierarchy operator over formulas so as to express the hierarchy over the individual's preferences and study collective choice rules under hierarchic environment [1].

Finally, we want to investigate the representation results for the proposed collective choice rules. Though it is a difficult problem in social choice theory to provide characterization results for specific aggregation rules, based on some existing results [1, 7, 20], this is not impossible.

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