Competing Land Uses and Fossil Fuel, Optimal Energy Conversion Rates During the Transition Toward a Green Economy Under a Pollution Stock Constraint

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Abstract

We study the transition to a carbon-free economy in a model with a polluting non-renewable resource and a clean renewable resource. Transforming primary energy into ready-to-use energy services is costly and more efficient energy transformation rates are more costly to achieve. Renewable energy competes with food production for land and the food productivity of land can be improved at some cost. To avoid catastrophic climate damages, the pollution stock is mandated to stay below a given cap. When the economy is not constrained by the cap, the efficiency of energy transformation increases steadily until the transition toward the ultimate green economy; when renewable energy is exploited, its land use rises at the expense of food production; food productivity increases together with the land rent but food production drops; the food and energy prices increase and renewables substitute for non-renewable energy. During the constrained phase, the economy follows a constant path of prices, quantities, efficiency rates, food productivity and land rent, a phenomenon we call the 'ceiling efficiency paradox'.

Keywords: energy efficiency; carbon pollution; non-renewable resources; renewable resources; land uses.

JEL classifications: Q00, Q32, Q43, Q54.

1 Introduction

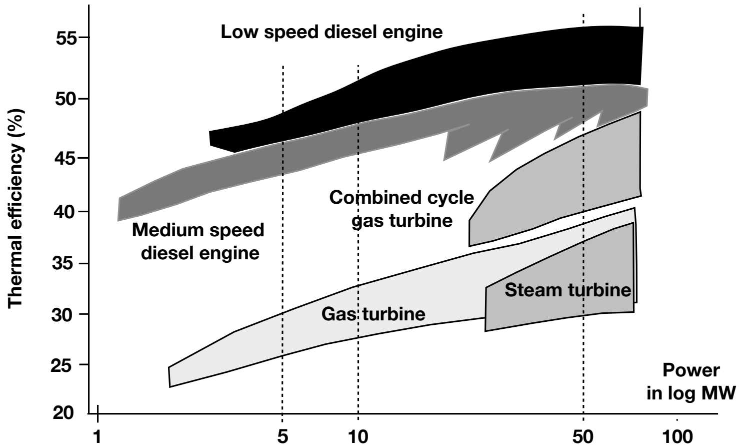
The objective of this paper is to examine how should be simultaneously determined the optimal dynamics of the conversion rates of primary energy resources into useful energy together with the energy mix and the allocation of land amongst food and energy production while taking into account that the atmospheric pollution stock cannot exceed some catastrophic triggering level. The transition toward a green economy requires significant improvements of the conversion rates of primary energies into useful energy, what Fouquet (2008) calls 'energy services', and a progressive substitution of carbon free energy sources for polluting fossil fuels. Developing the use of green energy implies also a competition for the land uses. The deployment of green energy consists in allocating a larger part of the solar radiation to the production of useful energy and a smaller part to the production of food because for capturing the solar primary resource, in most cases, land is needed.[1]

Furthermore, given that the atmospheric pollution stock must be kept under some ceiling to avoid catastrophic damages, the urgency of the substitution and/or the conversion rate improvements to carry out evolves through time. We show that along the optimal paths, all the substitutions and conversion rate improvements must be stopped within the phase during which the maximum pollution stock constraint is binding. The efforts must be made before the arrival at the ceiling, to delay the arrival time of the constraint while beginning the transition, and after the phase at the ceiling to complete the transition.

Dynamics of the conversion rates

It is well known from the pathbreaking work of Carnot (1824) that any conversion of some form of energy into another one implies a loss, the so-called Second Law of Thermodynamics. The increases of the conversion rates of primary energy into useful one, that is the increases of the thermal efficiency of the energy converters, have been spectacular from the beginning of the eighteenth century. The Figure 1 illustrates the evolution of the thermal efficiency of the succession of the different kinds of engines.

[1] Admittedly a small amount of green energy can be produced by windmills in shallows.



The power ratings and conversion efficiencies of steam engines, Otto-cycle gasoline engines, steam turbines, combined cycle gas turbines, and medium speed and low-speed diesel engines (based on a figure in Aabo 2007).

Figure 1: **Trends of Thermal Energy Conversion.**
Source: Smil V. (2010), p 210.

Similar increases can be recorded fo other energy converters in lightning and heating. Amongst the other converters are the converters of the solar radiation into useful energy and/or food.[2] Some records are short but significant, for example the increase of the solar cells efficiency or the decrease of the surfaces required by the successive generations of bio-fuels (Dutta *et al.* (2014)). For the conversion rates into agricultural products, themselves to be transformed in food and/or other form of useful energy, the record is more documented. Because the average solar radiation received per unit of land surface can be roughly taken as constant in a not too long run historical perspective, an index of the thermal efficiency of the transformation is the output per surface unit (see Figure 2).

Technical progress is generally seen as the main driving force of these thermal efficiency rate improvements.[3] Although it would not be appropriate to deny the role of cumulative scientific knowledge and technical mastery, the

[2]Food is nothing but some other form of energy services once cooked, what requires additional energy.

[3]The literature on the role of technical progress is immense. For a recent contribution see Acemoglu *et al.* (2016).

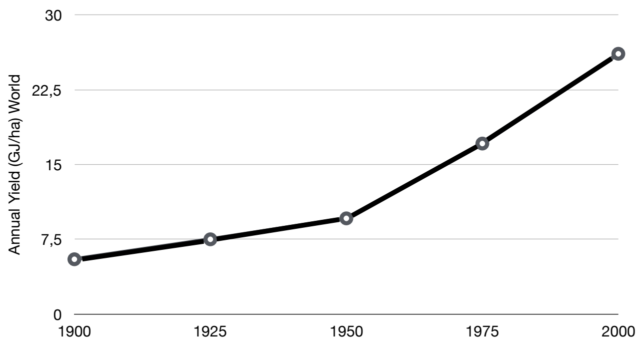


Figure 2: **Trends of Agricultural Yields (World Scale).**
Source: Smil V. (2008), p 396, compiled by the author from UN FAO statistical yearbooks.

choice of an efficiency rate is also an economic choice resulting from substitution decisions amongst inputs.[4] More thermally efficient energy converters are also generally more costly. Here we aim at determining the dynamics of thermal efficiency rates resulting from the substitutions absent any technical progress.[5]

Land uses and fossil fuel competition

[4]However note that all the improvements of the eighteenth century and the first part of the nineteenth century were made without strong theory of the thermodynamic laws, but with a deep empirical understanding of the necessity of heat saving. The first edition of the Carnot memoir is dated from 1824 and had a rather confidential reception. The mainly referred edition is the 1844 one, even today.

[5]A large part of the literature assumes given and fixed conversion rates for different energy sources or introduces exogenous trends of conversion improvements. On the theoretical side in this vein, main early contributions are Farzin (1986), Farzin and Tahvonen (1996), Tahvonen and Withagen (1996), Toman and Withagen (2000), Withagen (1994). For more recent contributions see André and Smulders(2014), Golosov *et al.* (2014), Van der Ploeg and Withagen (2014). The applied literature has intensively used integrated assessment models to assess climate policy options. Prominent contributions are Gerlagh and Van der Zwaan (2006), Nordhaus (2014) and Stern (2007). The other great tradition issued from Dasgupta and Heal (1974) on the contrary insists on the possible substitutions mainly between energy and capital.

The choices of the energy conversion rates are clearly entangled within the choice of using fossil fuels rather than renewable primary sources requiring land surface to be deployed, and land surfaces can be allocated either to the production of food or the production of other kinds of useful energy.[6] Thus the long way toward a green economy is also a path of land reallocation amongst food production and useful energy production together with a reallocation of useful energy production amongst non renewable primary resources and renewables ones.

The issue has raised many contributions. Empirical and theoretical analysis point out a positive effect on food prices of the expansion of bio-fuels production.[7] [8] Furthermore food and energy productions have also direct impacts on the deforestation and the value of forest lands, and the induced wood production contributes temporarily to the renewable energy supply.[9]

Atmospheric carbon pollution

The adoption of more thermally efficient techniques in the transformation of fossil fuels is in competition with the substitution with clean renewable resources to reduce the atmospheric CO_2 concentration. Burning fossil fuels to produce useful energy releases polluting emissions in the atmosphere. To prevent excessive climate damages the atmospheric carbon concentration must be maintained below some cap, or ceiling, as in Chakravorty *et al.* (2006). We consider the empirically relevant situation in which fossil fuels are so abundant that the economy will be eventually constrained by the atmospheric carbon cap. In contrast with climate models in the so-called 'carbon budget' style, we introduce explicitly the natural transfer of carbon from the atmosphere into the oceans and soils as a self-regenerating capacity of the environment determining together with the emission flow the dynamics of the pollution stock. We aim to assess not only the qualitative evolutions of the economic variables of the model, but also the optimal timing of the energy transition and the optimal period during which the economy should

[6]Mohr and Russan (2011), Sengers *et al.* (2010)

[7]Bahel *et al* (2011), Chakravorty *et al.* (2017), Diermeier and Schmidt (2014), Grafton *et al.* (2012), Gronwald *et al.* (2017), Hertel *et al.* (2010), Khanna *et al.* (2016), Popp *et al.* (2014), Rajcaniova *et al.* (2014), Rosegrant *et al.* (2008), Somerville and Long (2015).

[8]This interlinkage between the food and the energy prices extends to the transmission of price volatility between markets.

[9]Andrade de Sà *et al.* (2013), Hoel and Sletten (2016), Lundberg *et al.* (2015) and Zilberman *et al.* (2017).

face the carbon cap constraint.

To keep the model more easily tractable we aggregate the energy conversion activities by households and industries into one energy transformation sector using renewable and non renewable primary energy inputs to be converted into ready-to-use energy services. For each primary source the transformation sector adapts to the dynamics of the environmental regulation and to the economic conditions in primary resources provision by optimizing its conversion techniques. The fossil fuel price and the social cost of carbon are determined endogenously together with the optimal dynamics of the thermal efficiencies. We postulate stock-dependent extraction costs of the fossil resource and show how is also endogenously determined the part of the fossil endowment left forever underground, what we call the un-burned fossil resource stock.[10]

Main results

Our findings are the following. The energy transition is a sequence of three time phases. During a first phase, the economy accumulates carbon until it faces the cap constraint. Then the cap binds, implying a constant rate of fossil fuel extraction. The fossil resource being exhaustible, there must exist some time when even without a carbon constraint, the economy would choose to extract fossil resources at a lower rate than what is mandated by the cap. Thus the economy escapes from the cap and enters an unconstrained phase until the end of fossil fuel exploitation, that is the ultimate green economic regime when the society produces only renewable energy and food.[11] Before the constrained period, the shadow cost of carbon rises, next decreases during the constrained phase, down to zero at the end of the phase and forever. This sequence of time phases and the time profile of the carbon price are similar to the findings of the literature using carbon concentration mandates.

During the unconstrained phases, the useful energy price rises together with the conversion rate of crude fossil energy into useful energy. When renewable energy is produced jointly with non-renewable energy, its conversion

[10]Mc Glade and Ekins (2014), Resai and Van der Ploeg (2013, 2017).

[11]The actual possibility of a 100 % renewable energy economy is debated since long, see for example Mathiesen *et al.*, 2011, for the energy systems and Bryngelsson and Lindgren, 2013, for bio-energy. See also Sinn (2017) for a recent skeptical appraisal of the German program.

rate rises also and it takes an increasing share in the energy mix. Furthermore the land use devoted to renewable energy expands at the expense of the land use for food production and the land rent increases. The food sector reacts to this trend by increasing its productivity, although insufficiently to prevent the decline of the food production rate and thus the rise of the food price. Such findings are in line with the existing studies of the land use competition between food and bio-fuels production. Our contribution strengthens their conclusions by showing that even if the food sector can improve its productivity, food supply should still decrease.

In our framework the optimal carbon price path fully internalizes the potentially adverse impacts of the climate regulation on the food delivery conditions inside a dynamic setting, a novel feature of our model as far as we know. However full internalization does not alter qualitatively the conclusion of a positive effect of carbon regulation on food prices. A high degree of substituability between bio-fuels and fossils magnifies this impact, a point we emphasize in the concluding remarks of the paper by showing that with imperfect substitution possibilities, the impact of carbon pricing on food production could be lower.

During the constrained phase, the exploitation rate of fossil fuels must stay constant. This induces a constant price of useful energy and constant energy conversion rates. Implied by this constancy, the land sharing between renewable energy production and food production remains constant, together with the land rent level. The food production sector then maintains a constant productivity and constant levels of food production rate, thus of the food price. We call this phenomenon the 'ceiling efficiency paradox' since, paradoxically, in the period in which the environmental constraint is most pressing, the economy optimally refrains from further efforts to mitigate the effects of the constraint through better fossil energy conversion or more renewable energy production.

This rather striking conclusion is a direct product of our distinctive approach of energy efficiency gains. In our model the energy production sectors do not benefit from technical progress in the form of cost reductions per unit of useful energy but can improve their energy conversion performance through costly efforts. We show that when faced with a climate regulation policy aiming at stabilizing the carbon concentration in the atmosphere, the today policy goal of the governments, the energy industry determines its energy conversion strategy by solving a time independent static optimization problem, resulting in time independent, that is stationary energy conversion rates. This time stationarity extends to the land sharing between food production and renewable energy production, in turn stabilizing the food delivery conditions.

The paper is organized as follows. The next section presents the model. In section 3 we formulate the optimality problem and characterize the main features of the variables dynamics. The optimal paths are described in section 4. The last section 5 concludes.

2 The model

The model extends Chakravorty *et al.* (2008) and Bahel *et al.* (2013). Alternatively it may be seen as another example of how a renewable resource, the services from land, can be used to produce several kinds of final goods, one of which could be also produced by a nonrenewable resource, as in Gaudet *et al.* (2006). We consider an economy producing both food and energy services.[12] Food production requires both land and other inputs. Energy services can be produced from either the exploitation of a polluting non-renewable resource (oil) or the exploitation of land (solar) both with other inputs.[13]

Food, energy needs and gross surplus

Let us denote respectively by q_e and q_f the instantaneous consumption rates of energy and food. To simplify we first assume that the gross surplus generated by any pair of instantaneous consumption rates, $u(q_e, q_f)$, is additively separable and may be written as $u(q_e, q_f) = u_e(q_e) + u_f(q_f)$, each function u_i, $i = e, f$, satisfying the standard following assumption A.1.

[12]From a pure energy perspective, food is an *energy service*, and a vital one. However according to the *Guide Michelin* other characteristics than the pure energy content of food should be taken into account. We neglect the qualitative characteristics of food given the problem at stake.

[13]We subsume as 'solar energy', different renewable energy sources requiring space and/or sun energy like PV cells, wind energy or biofuels.

Assumption A. 1 *For any* $i = e, f$, $u_i : \mathbb{R}_+ \to \mathbb{R}$ *is twice continuously differentiable, strictly increasing,* $u_i' \equiv du_i/dq_i > 0$, *strictly concave,* $u_i''(q_i) \equiv d^2u_i/dq_i^2 < 0$, *and satisfies the basic Inada condition:* $\lim_{q_i \downarrow 0} u_i'(q_i) = +\infty$.

Assumption A.1 is not innocuous, asserting that a smaller food diet may be compensated by a larger energy consumption.[14] We denote by $p_i(q_i)$ the marginal gross surplus function, or inverse demand function, $p_i(q_i) \equiv u_i'(q_i)$.

The transformation sectors

The transformation sectors convert primary inputs into final goods, food or energy services. The solar and food sectors convert the solar radiation and space (land) into useful energy and food. The oil sector produces useful energy from crude oil. This sector includes two industries: the extractive, or mining industry, produces extracted oil from the underground resource and the oil transformation industry produces energy services from extracted oil.

Let $X(t)$ denote the underground stock of oil at time t measured in energy units, X^0 the initial endowment, $X(0) = X^0$, and $x(t)$ the instantaneous extraction rate or oil production: $\dot{X}(t) = -x(t)$. The unitary extraction cost depends on the grade under exploitation, as in Heal (1976) or Lehvari and Liviatan (1977). Let $a(X)$ denote this unitary cost. The function $a(.)$ satisfies the following standard assumptions.[15]

Assumption A. 2 $a : (0, X^0] \to \mathbb{R}_+$ *is twice continuously differentiable on* $(0, X^0)$, *strictly decreasing,* $a'(X) \equiv da(X)/dX < 0$, *strictly convex,* $a''(X) \equiv d^2a(X)/dX^2 > 0$, *with* $a(0^+) = +\infty$ *and* $a'(0^+) = +\infty$.

Under $a'(0^+) = +\infty$ and the below assumptions on the solar energy costs, some part of the oil endowment is left underground.

Let us denote by η_x the useful energy obtained by the oil transformation industry from one unit of extracted oil energy, what we call also the efficiency

[14] We discuss the implications of alternative assumptions in the sub-section 2.1.

[15] For any function $f(x)$ defined on $X \subseteq \mathbb{R}$, and for any $\bar{x} \in X$, we denote by $f(\bar{x}^-)$ and $f(\bar{x}^+)$ respectively, the limits $\lim_{x \uparrow \bar{x}} f(x)$ and $\lim_{x \downarrow \bar{x}} f(x)$ when such limits exist.

rate of the industry, and by $q_x \equiv \eta_x x$, the useful energy production rate of the oil sector. Similarly, we denote by η_y the fraction of intercepted solar energy transformed into useful energy. As for the food sector, we denote by η_f the food productivity of land.

For the sector i, $i = x, y, f$, let $\hat{\eta}_i$ be the upper bound on η_i, attainable only at prohibitive costs. Thermodynamic principles exclude the possibility of a one-to-one transformation of primary energy into useful energy, thus $\eta_i \leq \hat{\eta}_i < 1$ for the energy transformation sectors x and y. For the food sector, plant and animal physiology constrain η_f to be lower than some finite limit $\hat{\eta}_f$.

The solar energy sector uses some part L_y of the available land $\bar{L}$ to produce useful energy. This sector includes all the activities required to bring ready-to-use energy to the final users. Thus it may include agricultural activities when, for example, ethanol is produced from sugar cane, together with all the industrial processes necessary to transform the sugar cane into ethanol.[16] The available land is assumed to be homogeneous and to receive y^m units of solar energy per acreage unit. The problem of the solar energy sector is to convert y^m into useful energy. The useful energy produced by the solar sector, denoted by q_y, amounts to $\eta_y y^m L_y$.

The food sector includes not only the farming sector but also all the industrial activities necessary to bring ready-to-eat food to the consumers. Let L_f be the acreage of land devoted to the production of food so that the total food production amounts to $\eta_f L_f$.

Choosing higher conversion rates implies to put into operation more elaborate techniques in the energy transformation sectors, but also more costly ones. Let us denote by $b_x(\eta_x)$ the conversion cost per unit of extracted oil in the oil transformation industry, hence a total cost $b_x(\eta_x)x$. Similarly let $b_y(\eta_y)$ be the conversion cost per unit of solar energy in the solar transformation industry, hence a total cost $b_y(\eta_y)y^m L_y$. Then the average production costs per unit of useful energy amounts to $b_i(\eta_i)/\eta_i$, $i = x, y$, which is equal to its marginal cost. We assume that these average costs are increasing in both useful energy production sectors, implying that $b_i'(\eta_i)$ is also increasing.

[16] On the area required according to the type of power generation, see for example Cheng and Hammond (2017).

Let $b_f(\eta_f)$ be the food production cost per unit of land, hence an average cost $b_f(\eta_f)/\eta_f$ per unit of food, also equal to the marginal cost of food. Increasing food productivity requires more advanced techniques, thus more costly ones, so that the unitary cost per unit of food increases with the productivity of operated production process, together with $b_f'(\eta_f)$.[17]

Summing up the assumptions on the production cost structure in the three sectors:

Assumption A. 3 *$b_i : [0, \hat{\eta}_i) \to \mathbb{R}_+$, $i = x, y, f$, is twice continuously differentiable on $(0, \hat{\eta}_i)$, strictly increasing, $b_i'(\eta_i) > 0$, strictly convex, $b_i''(\eta_i) = d^2 b_i(\eta_i)/d\eta_i^2 > 0$, with $b_i(0^+) = 0$, $b_i'(0^+) > 0$, $b(\hat{\eta}_i^-) = +\infty$ and $b'(\hat{\eta}_i^-) = +\infty$. The average production cost of the final good, energy or food, and so the marginal cost, is a strictly increasing function of η_i: $b_i'(\eta_i) > b_i(\eta_i)/\eta_i$ and $\lim_{\eta_i \downarrow 0} b_i(\eta_i)/\eta_i > 0$.*[18]

Producing useful energy or food requires other costly inputs than the oil resource, land or solar radiation, hence a strictly positive marginal cost at 0^+: $\lim_{\eta_i \downarrow 0} b_i(\eta_i)/\eta_i > 0$. The assumptions $b_i(\hat{\eta}_i^-) = +\infty$ and $b_i'(\hat{\eta}_i^-) = +\infty$ mean that a complete conversion of crude energy or the attainment of the physiological limit to food productivity are not feasible.

The increasing pattern of production costs with the energy conversion performance, or the food productivity enhancement, can also be explained by the need to use more inputs that are costly to boost performance (see Appendix A.1 for a formal presentation).

Carbon pollution

Burning oil to produce useful energy generates a pollution flow proportional to the flow of the extracted oil input used in the transformation in-

[17]Differentiating the average production cost $b_i(\eta_i)/\eta_i$, $i = x, y, f$, yields:

$$\frac{d}{d\eta_i}\frac{b(\eta_i)}{\eta_i} = \frac{1}{\eta_i}\left[b_i'(\eta_i) - \frac{b_i(\eta_i)}{\eta_i}\right].$$

Hence $b_i'(\eta_i) > 0$ is necessary for $d\left(b_i(\eta_i)/\eta_i\right)/d\eta_i > 0$.

[18]Note that $b_i'(\eta_i) > b(\eta_i)/\eta_i$, $\eta_i \in (0, \hat{\eta}_i)$, implies that $b_i'(0^+) \geq \lim_{\eta_i \downarrow 0} b_i(\eta_i)/\eta_i$, hence $b_i'(0^+) > 0$.

dustry. Let ζ be the unitary pollution content of oil, hence a pollution flow $\zeta x(t)$ at time t, feeding the atmospheric pollution stock. Denote by $Z(t)$ the size of this pollution stock at time t and by Z^0 the stock inherited from the past, $Z(0) = Z^0$. The pollution stock self-depletes at a proportional rate α, assumed constant to simplify. Hence the dynamics of $Z(t)$ is given by $\dot{Z}(t) = \zeta x(t) - \alpha Z(t)$.

The atmospheric pollution concentration is constrained to be kept at most equal to some cap, or ceiling, $\bar{Z}$, to prevent excessive climate damages, as in Chakravorty *et al.* (2006). As far as the average earth temperature level is an increasing function of the atmospheric carbon concentration, such a cap may be seen as another formulation of the $+2^0C$ target assuming that $+2^0C$ is an effective constraint and not a mere wish. In order that the model makes sense we must assume that $Z^0 < \bar{Z}$. When the ceiling constraint binds, then there is a cap on the oil input in the transformation industry, a cap we denote by $\bar{x}$: $\bar{x} = \alpha\bar{Z}/\zeta$.

2.1 The optimal land use and the land rent

We assume that the allocation of land to the production of energy and food can be adjusted freely and instantaneously. For any acreage of land devoted to energy production, L_y^*, the optimal management of the food sector is this pair (η_f, L_f) solving the following food sector problem $(F.S.P)$:[19]

$$\begin{array}{lll} (F.S.P) & \max_{\eta_f, L_f} & u_f(\eta_f L_f) - b_f(\eta_f) L_f \\ & \text{s.t.} & \bar{L} - L_y^* - L_f \geq 0 \ . \end{array}$$

Denote by π the multiplier associated to the land availability constraint, equivalently the land rent. A strong implication of the assumption A.3 is the following proposition whose proof is given in Appendix A.2.

Proposition P. 1 *Under the assumptions A.1 and A.3, whatever $L_y^* \in [0, \bar{L})$, the food land is scarce, that is the solution L_f of the $(F.S.P.)$ problem is equal to $\bar{L} - L_y^*$ and the land rent π is strictly positive.*

[19] We neglect the non-negativity constraints on η_f, L_f and the upper bound constraint on η_f, all of which are satisfied.

The intuition behind this result is quite clear. Assumption A.3 means that extensive agricultures are less costly than intensive ones. Assume that a quantity of food q_f is produced with an acreage L_f and a productivity η_f: $q_f = \eta_f L_f$. Assume also that the land constraint is slack. Then the same quantity of food can be produced on a larger acreage with a lower productivity. Since a more extensive land exploitation is less costly, so is the total cost of q_f and (η_f, L_f) cannot be optimal.

It is worth pointing out that this result does not arise in Chakravorty *et al.* (2008). In their model, the food productivity level is fixed and cannot be chosen by the food sector. Given this level, it is possible that the food sector should cultivate only a fraction of the available land, the land not used for energy or food production remaining fallow. In the present case, the food sector can practice an extensive (and thus cheap) agriculture on the whole land which is not used for energy production.

The other assumption having strong implications on the agricultural sector is the additive separability of the gross surplus function.

Proposition P. 2 *Under the assumption of additive separability of the gross surplus function and the assumption A.3, the productivity in the food sector, the food production rate, hence the food price, are all constant when the land acreage devoted to the energy production is constant.*

Under the separability assumption, the surplus generated in the food sector is given by $u_f(\eta_f L_f)$. Thus when L_y is constant and thus $L_f = \bar{L} - L_y$ is also constant, the maximization of $u_f(\eta_f(t)L_f) - b_f(\eta_f(t))L_f$ does not depend upon t. Without the separability assumption, the objective function becomes $u(q_e(t), \eta_f(t)L_f) - b_f(\eta_f(t))L_f$ and $\eta_f(t)$ changes with $q_e(t)$ (see Appendix A.2 for further details).

An implication of the Proposition 2 is that, when the energy needs are fed only by oil, the food consumption rate, the food price level and the land rent should be constant for two reasons. On the one hand, $L_y = 0$ implies that $L_f(t) = \bar{L}$, a constant, and on the other hand, the food productivity rate, η_f, should also be constant under the separability assumption, hence $q_f = \eta_f \bar{L}$ is constant together with $p_f = u_f'(q_f)$ and $\pi = p_f \eta_f - b_f(\eta_f)$, independently of the possible time evolutions of the oil extraction rate, $x(t)$, the useful energy production rate, $q_e(t) = q_x(t)$, and the energy price, $p_e(t)$.

2.2 The ultimate green economy

Under the assumption A.2, the economy will reach, either in finite time or asymptotically in infinite time, the pure renewable energy regime, what we call the ultimate green economy.[20] We now describe the main characteristics of this final regime.

It will be shown later that the economy is no more facing the pollution stock constraint after some finite time $\bar{t}_Z$. The land allocation being freely adjustable, the renewable energy transformation rate, the food productivity level and the land allocation to renewable energy and food production converge toward a vector of constant levels, $(\tilde{\eta}_y, \tilde{\eta}_f, \tilde{L}_y, \tilde{L}_f)$. This vector solves the following static green economy problem $(G.E.P)$:[21]

$$(G.E.P) \quad \max_{\eta_y,\eta_f,L_y,L_f} \quad u_e(\eta_y y^m L_y) + u_f(\eta_f L_f) - b_y(\eta_y) y^m L_y - b_f(\eta_f) L_f$$
$$\text{s.t.} \quad \bar{L} - L_y - L_f \geq 0 \ .$$

Keeping the notation π for the multiplier associated to the land availability constraint yields among the f.o.c's:

$$u_e'(\eta_y y^m L_y)\eta_y y^m = b_y(\eta_y) y^m + \pi \tag{2.1}$$
$$u_f'(\eta_f L_f)\eta_f = b_f(\eta_f) + \pi \ , \tag{2.2}$$

together with the usual complementary slackness condition associated to the land availability constraint.

The conditions (2.1) and (2.2) together state the arbitrage condition in land allocation. The land rent π must be the same in both food and energy sectors:

$$\left[u_e'\left(\eta_y y^m L_y\right)\eta_y - b_y(\eta_y)\right] y^m = \pi = u_f'\left(\eta_f L_f\right) - b_f(\eta_f) \ . \tag{2.3}$$

[20]See Salant *et al.*, 1983, for a proof that in an exhaustible resource model of the present type, the economy could end the extraction process only asymptotically.

[21]We omit the constraints $0 \leq \eta_y \leq \hat{\eta}_y$, $0 \leq \eta_f \leq \hat{\eta}_f$, $0 \leq L_y$ and $0 \leq L_f$, which are satisfied as strict inequalities under the assumptions A.1 and A3.

The $l.h.s.$ of the equality is the marginal net surplus resulting from the allocation of an additional unit of land to energy production and the $r.h.s.$ is the net marginal surplus of allocating this land unit to food production. Under our assumptions, the program $(G.E.P)$ admits a unique strictly positive solution. For further reference, we denote by $\tilde{p}_i$, $i = e, f$, the corresponding shadow prices of energy and food and by $\tilde{\pi}$ the land rent.

3 The social planner problem

3.1 The problem

The social planner determines the paths of oil extraction, $x(t)$, of the energy transformation rates $\eta_x(t)$ and $\eta_y(t)$, of the productivity of land for food production, $\eta_f(t)$, and of the land allocation, $L_y(t)$ and $L_f(t)$, which maximize the social welfare. Let ρ be the social discount rate, assumed positive and constant. The planner solves the following $(S.P.)$ problem:[22]

[22]We omit the constraints $0 \leq X(t)$, $\eta_x(t) \leq \hat{\eta}_x$, $\eta_y(t) \leq \hat{\eta}_y$, $0 \leq \eta_f(t) < \hat{\eta}_f$ and $0 \leq L_f(t)$ which are all satisfied as strict inequalities under the assumptions A.1 to A.3. Note also that under costless and instantaneously adjustable land allocation, the land allocation at time $t = 0$ is endogenously determined and not inherited from the past, contrarily to the initial oil and pollution stocks.

$$
\begin{array}{ll}
(S.P.) & \\
\max\limits_{\substack{\{x(t),\eta_x(t),\eta_y(t)\\ \eta_f(t),L_y(t),L_f(t)\}_0^\infty}} & \displaystyle\int_0^\infty \{u_e\left(\eta_x(t)x(t)+\eta_y(t)y^m L_y(t)\right)+u_f\left(\eta_f(t)L_f(t)\right) \\
& -a(X(t))x(t)-b_x(\eta_x(t))x(t) \\
& -b_y(\eta_y(t))y^m L_y(t)-b_f(\eta_f(t))L_f(t)\}\, e^{-\rho t}dt \\
& \dot{X}(t)=-x(t)\ ,\ X(0)=X^0 \text{ given} \\
\text{s.t.} & \bar{L}-L_y(t)-L_f(t)\geq 0\ , L_f(t)\geq 0\ , L_y(t)\geq 0 \\
& \dot{Z}(t)=\zeta x(t)-\alpha Z(t)\ ,\ Z(0)=Z^0<\bar{Z} \text{ given} \\
& \text{and } \bar{Z}-Z(t)\geq 0 \\
& x(t)\geq 0\ ,\ \eta_x(t)\geq 0 \text{ and } \eta_y(t)\geq 0\ \ .
\end{array}
$$

Let λ and $-\mu$ be the co-state variables associated to X and Z respectively.[23] The current value Hamiltonian of the $(S.P.)$ problem, denoted by $\mathcal{H}$, reads:[24]

$$
\begin{aligned}
\mathcal{H} \;=\;& u_e\left(\eta_x x+\eta_y y^m L_y\right)+u_f\left(\eta_f L_f\right)-a(X)x \\
& -b_x(\eta_x)x-b_y(\eta_y)y^m L_y-b_f(\eta_f)L_f-\lambda x-\mu[\zeta x-\alpha Z]\ .
\end{aligned}
$$

Denote by π the Lagrange multiplier associated to the land availability constraint, like in the green economy problem, by γ_{Ly} the multiplier associated to the non-negativity constraint on the land acreage devoted to energy production, by ν the multiplier associated to the cap constraint on the pollution stock and by γ_x, $\gamma_{\eta x}$, and $\gamma_{\eta y}$ the multipliers associated to the non-negativity constraints on x, η_x and η_y respectively. Let $\mathcal{L}$ be the current value Lagrangian of the problem:

$$
\begin{aligned}
\mathcal{L} \;=\;& \mathcal{H}+\pi[\bar{L}-L_y-L_f]+\gamma_{Ly}L_y+\nu[\bar{Z}-Z] \\
& +\gamma_x x+\gamma_{\eta x}\eta_x+\gamma_{\eta y}\eta_y\ .
\end{aligned}
$$

[23] By choosing $-\mu$ as the co-state variable of Z, we may interpret μ as the shadow marginal cost of the pollution stock.

[24] We omit the time index when this causes no confusion.

The f.o.c's are:

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \implies u_e'(\eta_x x + \eta_y y^m L_y)\eta_x = a(X) + \lambda + b_x(\eta_x) + \zeta\mu - \gamma_x \tag{3.1}$$

$$\frac{\partial \mathcal{L}}{\partial \eta_x} = 0 \implies u_e'(\eta_x x + \eta_y y^m L_y)x = b_x'(\eta_x)x - \gamma_{\eta x} \tag{3.2}$$

$$\frac{\partial \mathcal{L}}{\partial L_y} = 0 \implies u_e'(\eta_x x + \eta_y y^m L_y)\eta_y y^m = b_y(\eta_y)y^m + \pi - \gamma_{Ly} \tag{3.3}$$

$$\frac{\partial \mathcal{L}}{\partial \eta_y} = 0 \implies u_e'(\eta_x x + \eta_y y^m L_y)y^m L_y = b_y'(\eta_y)y^m L_y - \gamma_{\eta y} \tag{3.4}$$

$$\frac{\partial \mathcal{L}}{\partial L_f} = 0 \implies u_f'(\eta_f L_f)\eta_f = b_f(\eta_f) + \pi \tag{3.5}$$

$$\frac{\partial \mathcal{L}}{\partial \eta_f} = 0 \implies u_f'(\eta_f L_f) = b_f'(\eta_f)\ , \tag{3.6}$$

together with the usual complementary slackness conditions.

The dynamics of the co-state variables must satisfy when time differentiable:

$$\dot{\lambda} = \rho\lambda - \frac{\partial \mathcal{L}}{\partial X} \implies \dot{\lambda}(t) = \rho\lambda(t) + a'\left(X(t)\right)x(t) \tag{3.7}$$

$$\dot{\mu} = \rho\mu + \frac{\partial \mathcal{L}}{\partial Z} \implies \dot{\mu}(t) = (\rho + \alpha)\mu(t) - \nu(t) \tag{3.8}$$

$$\nu(t) \geq 0, \bar{Z} - Z(t) \geq 0 \quad \text{and} \quad \nu(t)\left[\bar{Z} - Z(t)\right] = 0\ . \tag{3.9}$$

The transversality condition at infinity is:

$$\lim_{t\uparrow\infty}\left[\lambda(t)X(t) + \mu(t)Z(t)\right]e^{-\rho t} = 0\ . \tag{3.10}$$

3.2 General properties of the optimal paths

We first present general properties of the optimum. The next subsections will describe the characteristics of the optimal energy transition toward the green economy.

Cost minimization in the transformation industries

Consider first the transformation of oil into useful energy. One unit of oil costs $a + \lambda$ and burning it to produce useful energy has an opportunity cost $\zeta\mu$. The industry can produce η_x units of useful energy with this oil unit, at an additional cost $b_x(\eta_x)$. Thus the average cost of the useful energy amounts to $[a + \lambda + \zeta\mu + b_x(\eta_x)]/\eta_x$ and is U-shaped provided that $b_x''(\eta_x) > 0$. This unitary cost is minimized for η_x such that the marginal cost $b'(\eta_x)$ is equal to the average cost. This is the meaning of (3.1)-(3.2).

For the solar industry, one unit of land costs π and gives access to y^m units of solar energy allowing to produce $\eta_y y^m$ units of useful energy at the additional cost $b_y(\eta_y)y^m$, hence an average cost $[\pi + b_y(\eta_y)y^m]/\eta_y y^m$. This average cost is U-shaped provided that $b_y''(\eta_y) > 0$. The average cost is minimized for η_y such that the marginal cost $b_y'(\eta_y)/y^m$ is equal to the unit cost. This is the meaning of (3.3)-(3.4) when $L_y > 0$.

The same argument shows that (3.5)-(3.6) means that the minimum average cost of food $[\pi + b_f(\eta_f)]/\eta_f$ is equal to its marginal cost $b_f'(\eta_f)$.

Land use and land rent

The land allocation is determined by the sub-system (3.3)-(3.6). The link between the two alternative uses of the land is given by the land rent, π, which appears in (3.3) for the energy production and in (3.5) for the food production. The link between the land allocation problem and the competitiveness of the oil sector is given by the shadow price of the useful energy, $p_e = u_e'$, which appears in (3.3) and (3.4) for the solar energy sector and in (3.1) and (3.2) for the oil sector.

If $L_f > 0$, (3.6) defines the food productivity as an increasing function of the shadow price of food. Denote by $\eta_f^p(p_f)$ this relationship and by $v_f(p_f) \equiv p_f\eta_f^p(p_f) - b_f(\eta_f^p(p_f))$, the gross margin on food production per cultivated land unit as a function of the food price. The food gross margin is an increasing function of the food price.[25] On the other hand, an increase of p_f implies a decrease of $q_f = \eta_f L_f$. When the food price is higher, the food productivity is increased and L_f should decrease. Then (3.5): $v_f(p_f) = \pi$ shows that the food price will be raised by an increase of π. The optimal reaction of the food sector to an increase of the land rent is thus an increase

[25] Taking (3.6) into account, $v_f'(p_f) = \eta_f^p(p_f) + (p_f - b_f')d\eta_f^p/dp_f = \eta_f^p(p_f) > 0$.

of the food price, a rise of the food land productivity and a drop of the food land acreage together with the food supply.

On the other hand, (3.4) defines similarly $\eta_y^p(p_e)$, the optimal conversion rate of solar energy, as an increasing function of the shadow price of energy. Let $v_y(p_e) \equiv \left[p_e \eta_y^p(p_e) - b_y(\eta_y^p(p_e))\right] y^m$, denote the gross margin on solar energy production per land unit, an increasing function of the energy price. (3.3) : $v_y(p_e) = \pi$ shows that an increase of the useful energy price induces an increase of the land rent.

In order that the land be optimally allocated between energy and food production, an arbitrage condition similar to (2.3) must hold. The land rent must be the same for food land and energy land, (3.3) and (3.5) implying that:

$$v_y(p_e) = \pi = v_f(p_f) \ . \tag{3.11}$$

Hence an increase of the useful energy price induces an increase of the food price, and thus a rise of the land rent, of the food productivity rate together with a fall of L_f, the acreage of cultivated land for food production. The whole available land, $\bar{L}$, having to be exploited, a decrease of L_f means an increase of L_y, the land acreage cultivated for energy production.

The optimal reaction of the solar energy sector to an increase of the useful energy price is to increase its conversion performance while expanding its land occupation. This induces an increase of the land rent. The food sector reacts to this increase by raising its productivity, the productivity rise failing to compensate for the loss of food acreage. Hence the food supply decreases and the food price increases.

Competitiveness of solar energy exploitation

In order that the renewable energy sector be active, the energy price must be at least equal to the lowest unit cost of this energy, that is $p_e \geq \lim_{\eta_y \downarrow 0} b_y(\eta_y)/\eta_y$. Although necessary, this condition is not sufficient because it does not take into account the opportunity cost of diverting land from food production. This opportunity cost is the value of the land rent when all the land, $\bar{L}$, is allocated to the food sector.

If $L_f = \bar{L}$, the optimal productivity in the food sector, $\underline{\eta}_f$ solves (3.6): $u_f'(\underline{\eta}_f \bar{L}) = b_f'(\underline{\eta}_f)$. The food price is $\underline{p}_f = u_f'(\underline{\eta}_f \bar{L})$ and the land rent amounts to $\underline{\pi} = \underline{p}_f \underline{\eta}_f - b_f(\underline{\eta}_f)$. Hence the energy price triggering the production of solar energy, p_e^y, is the price level solving:

$$[p_e \eta_y - b_y(\eta_y)]\, y^m = \underline{\pi} \quad \text{where } \eta_y \text{ solves (3.3)} : p_e = b_y'(\eta_y)\ .$$

For energy prices $p_e > p_e^y$ some acreage must be allocated to energy production and (3.11) holds. When the production energy starts, the initial transformation η_y^f is this rate solution of $p_e^y = b_y'(\eta_y)$.[26]

Mining rent and full marginal cost of oil exploitation

According to (3.7) the dynamics of the mining rent is not necessarily monotonous since $a'(X) < 0$. However, the full marginal cost of the extracted oil, that is the marginal cost augmented by the mining rent, is increasing throughout time. Let us denote by $\theta(t)$ this full marginal cost: $\theta(t) \equiv a(X(t)) + \lambda(t)$. Time differentiating and substituting (3.7) for $\dot{\lambda}$ yields:

$$\dot{\theta}(t) \quad = \quad -a'(X(t))x(t) + \rho\lambda(t) + a'(X(t))x(t) \quad = \quad \rho\lambda(t) > 0\ . \tag{3.12}$$

Along the optimal path, the behavior of the full marginal cost of extracted oil is dominated by the increase of its marginal extraction cost component: $sign\, \dot{\theta}(t) = sign\, \dot{a}(X(t)) = -sign\, a'(X(t))x(t) > 0$, $\theta(t)$ being strictly time increasing until the transition to the green economy.[27]

Shadow marginal cost of pollution

[26]Note that although η_y, the transformation rate of renewable energy jumps from 0 up to η_y^f when the production of renewable energy starts, the production of renewable energy starts smoothly because the land allocated to this production does not jump but increases progressively.

[27]However the discounted mining rent, $\lambda(t)e^{-\rho t}$, is unambiguously decreasing. From (3.7) we obtain:

$$\frac{d}{dt}\left\{\lambda(t)e^{-\rho t}\right\} \quad = \quad a'\left(X(t)\right)x(t)e^{-\rho t} < 0\ .$$

Let us denote by $\underline{t}_Z$ and $\bar{t}_Z$ the times at which respectively begins and ends the period during which the constraint $\bar{Z} - Z(t) \geq 0$ binds. Since initially the pollution stock is smaller than the cap, $Z^0 < \bar{Z}$, there must exist some pre-ceiling period $[0, \underline{t}_Z)$, $0 < \underline{t}_Z$, during which the constraint does not bind, hence $\nu(t) = 0$, $t < \underline{t}_Z$, and $\dot{\mu} = (\rho + \alpha)\mu$, so that:

$$\mu(t) \quad = \quad \mu^0 e^{(\rho+\alpha)t} \ , \ t \in [0, \underline{t}_Z) \ , \ \mu^0 \equiv \mu(0) \ . \tag{3.13}$$

After $\bar{t}_Z$, since the constraint will be never anymore active, then:

$$\mu(t) = 0 \quad , \quad t \ \in \ [\bar{t}_Z, \infty) \ . \tag{3.14}$$

It remains to determine the path of $\mu(t)$ when the ceiling constraint binds. We show in the Subsection 3.4 that $\mu(t)$ decreases during the period at the ceiling so that $\mu(t)$ is single-peaked and the peak occurs at the precise time at which the constraint begins binding.

Necessity of a post-ceiling period preceding the ultimate green period

If oil exploitation lasts at infinity, it is physically impossible to sustain the mandated oil exploitation rate when at the ceiling, $\bar{x}$, with a finite fossil resource endowment. If oil exploitation is resumed in finite time, the oil exploitation rate should fall abruptly from $\bar{x}$ to 0 at the end of the oil extraction period if the ceiling period lasts until the complete transition to the ultimate green period. Trying to compensate completely or partially this fall by an abrupt increase of the renewable energy production will induce in all cases either an abrupt fall of the energy surplus, either a fall of the food surplus or either a joint fall of both surpluses, which cannot be optimal. Hence the economy cannot switch directly from the period at the ceiling to the green economy regime.

Proposition P. 3 *Assume that along the optimal path, there exists a time period at the ceiling, $[\underline{t}_Z, \bar{t}_Z]$, $\underline{t}_Z < \bar{t}_Z$, then there must also exist a post-ceiling period of either infinite duration, $(\bar{t}_Z, \infty)$, or either finite duration, $(\bar{t}_Z, \bar{t}_x)$, $\bar{t}_Z < \bar{t}_x < \infty$, during which oil is exploited before the ultimate green period.*

Proof: If the economy never stops using oil while remaining constrained by the pollution ceiling, then $x(t) = \bar{x}$, $t \in [\underline{t}_Z, \infty)$, which is physically impossible with a finite oil endowment. If oil exploitation ends at some finite date, $\bar{t}_x$, and the post-ceiling period does not exist, then $\bar{t}_Z = \bar{t}_x \equiv \bar{t}$. Since the optimal path of the shadow price of useful energy, $p_e(t)$, must be time continuous at $\bar{t}$, then $u_e'(\eta_x(\bar{t}^-))\bar{x} + \eta_y(\bar{t}^-)y^m L_y(\bar{t}^-)) = u_e'(\tilde{\eta}_y y^m \tilde{L}_y)$, hence $\tilde{\eta}_y \tilde{L}_y - \eta_y(\bar{t}^-)L_y(\bar{t}^-) = \eta_x(\bar{t}^-)\bar{x}/y^m > 0$, so that either $\tilde{L}_y - L_y(\bar{t}^-) > 0$, or $\tilde{\eta}_y - \eta_y(\bar{t}^-) > 0$, or both, so that the case $\tilde{L}_y - L_y(\bar{t}^-) < 0$ together with $\tilde{\eta}_y - \eta_y(\bar{t}^-) < 0$ must be excluded.

Assume first that $\tilde{L}_y - L_y(\bar{t}^-) > 0$, equivalently that $L_f(\bar{t}^-) > \tilde{L}_f$. Then from (3.6): $\eta_f(\bar{t}^-) < \tilde{\eta}_f$ and $b_f'(\tilde{\eta}_f) > b_f'(\eta_f(\bar{t}^-))$ as illustrated in Figure 1, so that u_f' would jump upward at $t = \bar{t}$, contradicting the time continuity of the optimal path of the food price, $p_f(t)$.

Assume now that $\tilde{\eta}_y - \eta_y(\bar{t}^-) > 0$ and $\tilde{L}_y - L_y(\bar{t}^-) > 0$. Thus $L_f(\bar{t}^-) > \tilde{L}_f$. Hence from (3.6), $\eta_f(\bar{t}^-) > \tilde{\eta}_f$ and $b_f'(\tilde{\eta}_f) < b_f'(\eta_f(\bar{t}^-))$ so that u_f' would jump downward, contradicting once again the time continuity of $p_f(t)$. ■

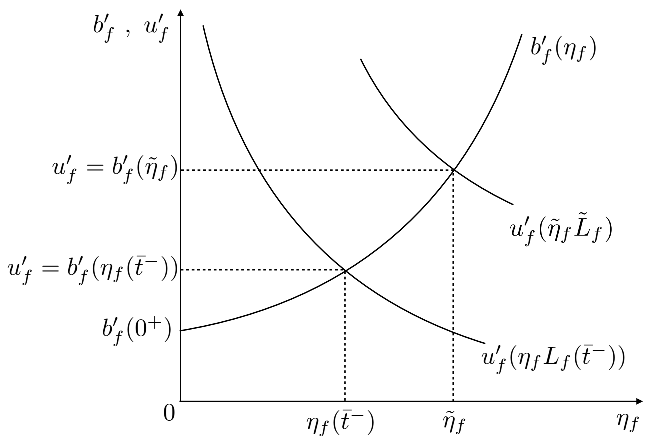


Figure 3: **Jump of u_f' at time $\bar{t}$. The case $L_f(\bar{t}^-) > \tilde{L}_f$.**

Un-burned oil

The Proposition 3 allows to determine the stock of un-burned oil, this part, $\tilde{X}$, of the initial oil endowment too costly to exploit given the energy price $\tilde{p}_e$, attained at the end of the transition phase to the green economy. The determination of this stranded oil stock $\tilde{X}$ is illustrated in Figure 4.

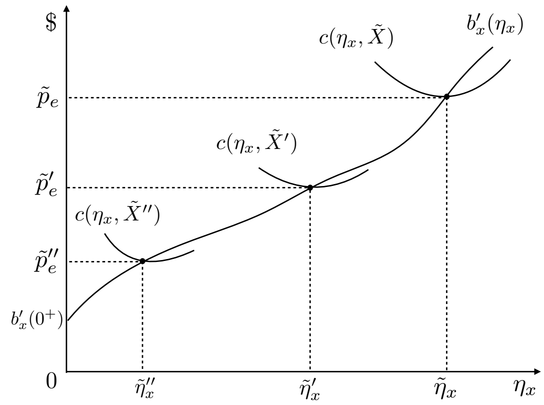


Figure 4: **Determination of the un-burned oil stock.**

In Figure 4 the curves $c(\eta_x, \tilde{X})$ are the average cost curves of the oil transformation industry at the end of the transition phase. The curves are drawn as functions of η_x for given hypothetical levels $\tilde{X}$, $\tilde{X}'$ and $\tilde{X}''$ of the stranded stock. Note that the mining rent on the last oil grade $\tilde{X}$ which is exploited must be nil. If not it would be worth exploiting an additional grade $\tilde{X} - dX$, $dX > 0$ and sufficiently small, at the energy price $\tilde{p}_e$. Also since the last phase of oil exploitation is unconstrained by the pollution ceiling, then $\mu = 0$ when oil exploitation ends. Thus the full average cost of oil transformation into useful energy is given by $c(\eta_x, \tilde{X}) = \left[a(\tilde{X}) + b_x(\eta_x)\right] / \eta_x$.

As shown at the beginning of the present sub-section the average cost curves are U-shaped and cost minimization implies that the marginal cost $b'_x(\eta_x)$ be equal to the minimum average cost. Hence the locus of the minima of the average curves is the curve $b'_x(\eta_x)$. Clearly the larger is $\tilde{X}$ the lower is located the curve $c(\eta_x, \tilde{X})$ since $a(X)$ is a decreasing function of X. Also the larger is $\tilde{X}$ the lower is the transformation rate $\tilde{\eta}_x$ at which $c(\eta_x, \tilde{X})$ is minimum since this minimum is located on the increasing curve $b'_x(\eta_x)$. In the Figure 2 the curve $c(\eta_x, \tilde{X}'')$ is located under the curve $c(\eta_x, \tilde{X}')$, itself located under the curve $c(\eta_x, \tilde{X})$, because $\tilde{X} < \tilde{X}' < \tilde{X}''$. Thus the minimum of $c(\eta_x, \tilde{X}'')$ is smaller than the minimum of $c(\eta_x, \tilde{X}')$ itself smaller than the minimum of $c(\eta_x, \tilde{X})$, and $\tilde{\eta}_x'' < \tilde{\eta}_x' < \tilde{\eta}_x$ is the ranking of η_x at which the minima are attained.

Last the energy price at the end of oil exploitation is the price $\tilde{p}_e$ of the green economy because the price path is continuous, and the marginal cost of the oil transformation industry must be equal to this price. Hence $\tilde{\eta}_x$ is given as the solution of $\tilde{p}_e = b_x'(\eta_x)$. The stranded stock $\tilde{X}$ is this stock for which the average cost $c(\eta_x, \tilde{X})$ is minimum at $\eta_x = \tilde{\eta}_x$. Hence $\tilde{X}$ is given as the solution of $\tilde{p}_e = [a(X) + b_x(\tilde{\eta}_x)] / \tilde{\eta}_x$. In Figure 2, to the hypothetical green energy prices $\tilde{p}_e$, $\tilde{p}_e'$ and $\tilde{p}_e''$, $\tilde{p}_e'' < \tilde{p}_e' < \tilde{p}_e$, correspond the respective transformation rates $\tilde{\eta}_x$, $\tilde{\eta}_x'$ and $\tilde{\eta}_x''$, $\tilde{\eta}_x'' < \tilde{\eta}_x' < \tilde{\eta}_x$ and the un-burned oil stocks $\tilde{X}$, $\tilde{X}'$ and $\tilde{X}''$, $\tilde{X}'' < \tilde{X}' < \tilde{X}$.

It must be noted that the un-burned stock does not depend upon the pollution ceiling $\bar{Z}$ but only upon the conditions prevailing in the ultimate green economy.[28] Note also that the determination of $\tilde{X}$ follows the same logic whether the economy converges in finite time toward the green economy regime or only asymptotically.

3.3 Periods of unconstrained oil exploitation

To characterize the dynamics of the different variables during the transition, it is useful to distinguish the periods of unconstrained oil exploitation from the period of constrained oil exploitation. During the unconstrained periods, the path of the shadow cost of pollution is characterized by (3.13) and (3.14). During the constrained period, only the oil extraction rate is known, $x(t) = \bar{x}$.

The following proposition presents what happens when oil exploitation is not constrained by the cap on the pollution stock.

Proposition P. 4 *Along the optimal path, during the periods of unconstrained exploitation, $t \notin (\underline{t}_Z, \bar{t}_Z)$:*

[28]However with a technical progress modifying the functions $a(.)$ or $b_i(.)$, $i = x, y, f$, different pollution ceilings would have ambiguous effects on $\tilde{X}$.

a. Irrespective of oil is the only exploited resource or both oil and solar energy are exploited:

a.i. The production of useful energy decreases and its price increases;

a.ii. The production of useful oil energy decreases and the transformation rate of extracted oil into useful energy increases, hence the production of extracted oil decreases.

b. If both oil and solar energies are exploited:

b.i. The useful solar energy production increases due to both the increase of the land acreage allocated to the production of this energy and the increase of the transformation rate of solar energy into useful energy;

b.ii. The land acreage allocated to food production decreases and the food productivity of land increases but not sufficiently to compensate for the acreage decrease so that the food production decreases and its price increases;

b.iii. The land rent increases.

A formal proof of the above claims is given in Appendix A.3, we now sketch the rationale of the Proposition 4. The f.o.c (3.2): $p_e = b'_x(\eta_x)$ defines $\eta^p_x(p_e)$, the optimal transformation rate of oil, as an increasing function of p_e. Let $v_x(p_e) \equiv p_e\eta^p_x(p_e) - b_x(\eta^p_x(p_e))$ denote the gross margin on oil energy transformation per unit of processed oil, an increasing function of p_e.

Since $\theta(t)$ increases throughout the whole oil exploitation phase (c.f. (3.12)) and $\mu(t)$ is either nil after $\bar{t}_Z$, or time increasing before $\underline{t}_Z$, (c.f. (3.13) and (3.14)), the full shadow cost of oil energy strictly increases during any period of unconstrained oil exploitation. Optimality requires the equalization of the unitary gross margin on oil transformation to the full shadow cost of oil: $v_x(p_e) = \theta + \zeta\mu$ (c.f. (3.1)). This implies that the useful energy price should increase during unconstrained periods. If oil is the only exploited energy source, the increasing trend of the useful energy price means both an increasing trend of the conversion rate of oil energy and a decreasing trend of useful energy production, thus the oil consumption rate decreases.

When both energy sources are exploited, we have shown previously that an increase of the useful energy price induces an increase of the solar energy production rate, an increase resulting from the increase of the solar energy conversion performance and the expansion of the land acreage devoted to this activity. Thus the solar sector should increase its energy supply, implying a fall of the energy production rate from the oil sector. Optimality requires a smooth substitution between oil energy and solar energy, the solar sector benefiting from an increasing competitiveness advantage with respect to the oil sector because of the increasing cost trend of oil extraction and the increasing trend of the carbon pollution shadow cost before the constrained period.

The energy conversion performance of the oil sector being increasing thank to the energy price increase, the fall of the oil useful energy supply implies that the oil consumption rate should decrease. Last, the solar land acreage increase implies a progressive decline of the land acreage devoted to food production, an increase of the land rent and thus a food price increase, together with an increasing trend of the productivity of food production.

3.4 Periods of constrained oil exploitation

During the constrained period, the dynamics of μ is not yet known because $\dot{\mu} = (\rho + \alpha)\mu - \nu$ and $\nu > 0$ since the constraint $\bar{Z} - Z(t) \geq 0$ binds. At this stage, the only qualitative information that we have is that $\dot{\theta}(t) > 0$ (c.f. (3.12)) and $\dot{x}(t) = 0$ since $x(t) = \bar{x}$.

The following Proposition 5 shows that although the oil use as an input in the useful energy production is constrained by the pollution stock upper bound, it is optimal to undertake no additional effort to push away the limits of the energy consumption and relax the constraint, either directly by improving the conversion rates in the energy transformations sectors, or indirectly, by improving the productivity rate in the food sector which would allow allocating more land to the renewable energy production. We call this rather counterintuitive result the *Ceiling Efficiency Paradox*.[29]

[29] The same kind of paradox holds when the pollution flow can be abated. The fraction of the abated pollution flow is constant within the period at the ceiling. See Amigues and Moreaux, 2016-b.

Proposition P. 5 Ceiling Efficiency Paradox.

Along the optimal path, whether oil is the only exploited energy or both oil and solar energies are simultaneously exploited, the economy moves along a stationary trajectory during the period at the ceiling: Efficiency rates, land productivity, production levels of extracted oil, useful energies and food, and the land allocation are all constant. The only changes come from the increase of the full marginal cost of extracted oil, exactly balanced by the decrease of the shadow marginal cost of the pollution stock, so that the full marginal cost of the useful oil energy is constant. The other costs, the marginal cost of the solar energy when exploited and the marginal cost of food, the prices of the useful energy and food, and the land rent are all constant.

An immediate implication of the Proposition 5 is the following corollary:

Corollary 1 *Along the optimal path the solar energy begins to be exploited either before the arrival at the ceiling or after the end of the period at the ceiling, but never within the period.*

A formal proof of the Proposition 5 is given in Appendix, we sketch now its main rationale. A cap on the admissible atmospheric carbon concentration implies a constant rate of carbon emissions and thus a constant rate of exploitation of fossil fuels, absent any abatement option. This is a general property of ceiling models not resulting from our simplifying assumption of a constant self-regeneration rate of carbon in the atmosphere. Any law of motion of the atmospheric carbon concentration of the form $\dot{Z} = G(\zeta x, Z)$ should admit under mild assumptions a unique root, $\bar{x}$, of the equation $G(\zeta x, \bar{Z}) = 0$ when the economy faces the carbon cap. There thus exists an equivalence between a constraint on carbon concentrations and a global constraint on carbon emissions.

However such an equivalence holds only during the time period at the ceiling $[\underline{t}_Z, \bar{t}_Z)$. Clearly the pollution flow, $\zeta x(t)$, must be larger than $\alpha \bar{Z}$ before $\underline{t}_Z$. To obtain the path solution of the problem $(S.P.)$ as the solution of a problem framed in terms of a constraint on the pollution flow, not only the cap on the flow must be specified, here $\alpha \bar{Z}$, but also the time interval during which the pollution constraint binds, here $[\underline{t}_Z, \bar{t}_Z)$.[30]

The main implication of the constancy of the admissible pollution flow, and thus of the fossil fuel consumption rate, during the constrained period is that the optimal levels of the variables are solution of a time independent static optimization problem. Hence these optimal levels are constant during the constrained phase, what we call the 'ceiling efficiency paradox'.

To get an explanation of the paradox, skip the renewable energy sector and assume that the economy uses only oil to produce useful energy. Then we may reformulate the optimal choices of the oil sector at any time t of the constrained period as a static problem in which first, the average full cost of the oil input, $\theta(t)$, is given and is independent of x, and second the amount of oil transformed is bounded from above by $\bar{x}$, the equivalent of the ceiling constraint $\bar{Z} - Z \geq 0$, a state variable constraint translated into a command variable or flow constraint variable. Hence the following problem:

$$\begin{array}{ll} \max\limits_{\eta_x, x} & u_e(\eta_x x) - \theta(t)x - b_x(\eta_x)x \\ s.t. & \bar{x} - x \geq 0 \;\text{ and }\; x \geq 0 \;. \end{array}$$

Let $\bar{\gamma}_x(t)$ and $\underline{\gamma}_x(t)$ be the multipliers associated to the maximum and minimum oil use constraints. The f.o.c's of the problem read:

$$\begin{aligned} w.r.t.\ x \quad &: \quad u_e'(\eta_x x)\eta_x = \theta(t) + b_x(\eta_x) + \bar{\gamma}_x(t) - \underline{\gamma}_x(t) \;; \\ &\qquad \bar{\gamma}_x(t)(\bar{x} - x) = \underline{\gamma}_x(t)x = 0 \;;\; \bar{\gamma}_x(t), \underline{\gamma}_x(t) \geq 0 \qquad (3.15) \\ w.r.t.\ \eta_x \quad &: \quad u_e'(\eta_x x)x = b_x'(\eta_x)x - \gamma_{\eta x}(t) \;;\; \gamma_{\eta x}(t)\eta_x = 0 \;. \qquad (3.16) \end{aligned}$$

Assuming that the ceiling constraint is tight means that $x = \bar{x}$, hence only one command variable has to be determined, the efficiency rate η_x. For $x = \bar{x}$ and $\gamma_{\eta x}(t) = 0$ because $\eta_x > 0$, the f.o.c (3.16) relative to η_x reduces to the following one variable equation:

$$u_e'(\eta_x \bar{x}) \;=\; b_x'(\eta_x) \;.$$

As illustrated in Figure 5, this equation has a unique solution $\bar{\eta}_x$.

[30] Other kinds of constraints and their effects on the pollution path have been explored by Saltari and Travaglino, 2016. In their partial equilibrium model, the planning horizon is finite, the pollution stock at the end of the planning period is given and there is a cap on the growth rate of the pollution flow of the form $\dot{Z}(t) \leq rZ(t)$, $0 < r < 1$, in our notations. However, they do not model explicitly the use of different types of energies.

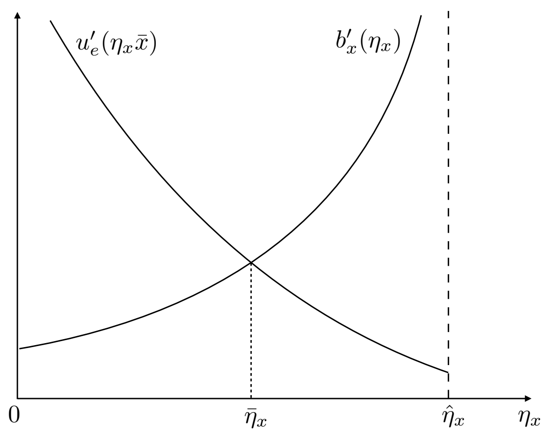


Figure 5: **Determination of the constant efficiency rate $\bar{\eta}_x$ of the oil transformation industry during the period at the ceiling.**

Thus during the constrained period the choice of the optimal transformation rate η_x depends upon $\bar{x}$, equivalently upon $\bar{Z}$, but not upon $\theta(t)$ the only dynamic parameter of the problem. Next fixing x and η_x in the f.o.c (3.15) relative to x, we get:

$$u'_e(\bar{\eta}_x \bar{x})\bar{\eta}_x = \theta(t) + b_x(\bar{\eta}_x) + \bar{\gamma}_x(t) \implies \frac{d\bar{\gamma}_x}{dt} = -\dot{\theta}(t) \ . \tag{3.17}$$

Here $\theta(t) + \bar{\gamma}_x(t)$ is the equivalent of $a\,(X(t)) + \lambda(t) + \zeta\mu(t) = \theta(t) + \zeta\mu(t)$ in the f.o.c (3.1) relative to x, of the $(S.P)$ problem, that is $\bar{\gamma}_x(t) = \zeta\mu(t)$. Since $\dot{\theta}(t) = \rho\lambda(t)$ (c.f. (3.12)) then:

$$\dot{\mu}(t) = \frac{1}{\zeta}\frac{d\bar{\gamma}(t)}{dt} = -\frac{\rho}{\zeta}\lambda(t) < 0 \ , \ t \in (\underline{t}_Z, \bar{t}_Z) \ .$$

If the ceiling constraint is not tight the two above f.o.c. equations must be solved simultaneously, $\bar{\gamma}_x(t) = 0$, and now their solution depends upon $\theta(t)$.

As time goes on, before the arrival at the ceiling, $\mu(t)$ increases because the date at which the constraint will become active is closer and closer. Once at the ceiling $\mu(t)$ decreases because now it is the date at which the constraint will end to be effective that becomes closer and closer.

Entering the solar energy inside the picture does not change fundamentally the problem. Assume now that the land acreage devoted to the production of useful energy must be at most equal to some given area $\bar{L} - L_f$. The energy problem becomes:

$$\max_{\substack{\eta_x, x \\ \eta_y, L_y}} \quad u_e\left(\eta_x x + \eta_y y^m L_y\right) - \theta(t)x - b_x(\eta_x)x - b_y(\eta_y)y^m L_y$$
$$s.t. \quad \bar{x} - x \geq 0 \;,\; x \geq 0 \;,\; \left(\bar{L} - L_f\right) - L_y \geq 0 \text{ and } L_y \geq 0 \;.$$

Denote by $\bar{\gamma}_y(t)$ and $\underline{\gamma}_y(t)$ the multipliers associated to the constraints on L_y. The f.o.c's read now:

$$\begin{aligned}
w.r.t.\ x &: u_e'(\eta_x x + \eta_y y^m L_y)\eta_x = \theta(t) + b(\eta_x) + \bar{\gamma}_x(t) - \underline{\gamma}_x(t) && (3.18)\\
w.r.t.\ \eta_x &: u_e'(\eta_x x + \eta_y y^m L_y)x = b_x'(\eta_x)x - \gamma_{\eta x}(t) && (3.19)\\
w.r.t.\ L_y &: u_e'(\eta_x x + \eta_y y^m L_y)\eta_y = b_y(\eta_y)\eta_y + \bar{\gamma}_y(t) - \underline{\gamma}_y(t) && (3.20)\\
w.r.t.\ \eta_y &: u_e'(\eta_x x + \eta_y y^m L_y)y^m L_y = b_y'(\eta_y)y^m L_y - \gamma_{\eta y}(t) \;. && (3.21)
\end{aligned}$$

When both the ceiling constraint and the maximum land surface constraint are active, (3.19) and (3.21) reduce to:

$$\begin{aligned}
u_e'\left(\eta_x \bar{x} + \eta_y y^m(\bar{L} - L_f)\right) &= b_x'(\eta_x) && (3.22)\\
u_e'\left(\eta_x \bar{x} + \eta_y y^m(\bar{L} - L_f)\right) &= b_y'(\eta_y) \;. && (3.23)
\end{aligned}$$

Although now η_x and η_y are simultaneously determined they do not depend upon $\theta(t)$. Denoting by $(\bar{\eta}_x, \bar{\eta}_y)$ the solution of (3.22)-(3.23), we get the equivalent of (3.17) in the present context:

$$u_e'\left(\bar{\eta}_x \bar{x} + \bar{\eta}_y y^m(\bar{L} - L_f)\right) = \theta(t) + b_x(\bar{\eta}_x) + \bar{\gamma}_x(t) \implies \frac{d\bar{\gamma}_x(t)}{dt} = -\dot{\theta}(t) \;. \tag{3.24}$$

To close this section, the following Proposition 6 describes the consequences of a stricter environmental constraint on the model variables during the ceiling period.

Proposition P. 6 *A more stringent climate constraint, that is a lower $\bar{Z}$, or equivalently a stricter carbon emission allowance during the constrained period, $d\bar{x} < 0$, has the following effects:*

. *If the economy consumes only oil energy when the constraint binds, it increases the conversion rate of fossil energy,* $d\bar{\eta}_x > 0$*, raises the energy price,* $d\bar{p}_e > 0$ *and reduces the energy consumption rate,* $d\bar{q}_e = d\bar{q}_x < 0$*. The food sector is unaffected by a change of the environmental constraint.*

. *If the economy uses both energy sources when the constraint binds, a lower mandated emission rate,* $d\bar{x} < 0$*:*

- *Increases the conversion rates of both fossil and renewable energies,* $d\bar{\eta}_x > 0$ *and* $d\bar{\eta}_y > 0$*.*
- *Increases the energy price,* $d\bar{p}_e > 0$*, and reduces the energy supply* $dq_e < 0$*. A stricter environmental constraint favours the energy transition from fossil energy to renewable energy. The energy consumption from the fossil source decreases,* $d\bar{q}_x < 0$*, while the renewable energy consumption increases,* $d\bar{q}_y > 0$*.*
- *Increases the land acreage allocated to renewable energy production,* $d\bar{L}_y > 0$*, the food land acreage being reduced by the same amount,* $d\bar{L}_f = -d\bar{L}_y < 0$*.*
- *Increases the land rent,* $d\bar{\pi} > 0$*, inducing a rise of the food productivity,* $d\eta_f > 0$*, together with a higher food price,* $d\bar{p}_f > 0$*, and a lower food consumption rate,* $d\bar{q}_f < 0$*.*

When the economy consumes only fossil energy, the proof of the claims of the Proposition 6 is straightforward. Differentiating (3.2): $u'_e(\eta_x\bar{x}) = b'(\eta_x)$ yields:

$$\frac{d\eta_x}{d\bar{x}} = \frac{\eta_x u''_e(q_e)}{b''(\eta_x) - \bar{x}u''_e(q_e)} < 0 \ .$$

Thus a tighter constraint, $d\bar{x} < 0$, induces a more efficient transformation, $d\eta_x > 0$, however not sufficient to compensate the input reduction, since by (3.2) again, $dp_e/d\bar{x} = b''_x(\eta_x)d\eta_x/d\bar{x} > 0$, hence $dq_e/d\bar{x} < 0$, the energy price is increased and the energy consumption is decreased.

When the economy uses both energies, a stricter environmental constraint requires to adjust all productivities in the energy and the food sector together with reallocating the land between energy and food production. We provide a formal proof of the proposition in Appendix A.5.

4 Optimal paths

All the optimal paths share a common strong qualitative structure determined by the increasing scarcity of the non-renewable resource and the ceiling constraint when this constraint is effective. Putting together the results of the Propositions 1 to 5 with the time continuity of the shadow price paths, a necessary condition for optimality in the present context, Proposition 7 characterizes the optimal paths.

Proposition P. 7 *Assuming a sufficiently large initial oil endowment and/or a sufficiently low atmospheric pollution ceiling, any optimal path is a sequence of at most four main periods:*

- *A first period of decreasing oil extraction and decreasing energy consumption, increasing efficiency of the oil transformation, increasing atmospheric pollution and increasing shadow cost of carbon;*
- *A second period of constant oil extraction and energy consumption and constant efficiency of the oil transformation industry, a constant atmospheric pollution stock at the ceiling and a decreasing pollution shadow cost;*
- *A third period similar to the first one excepted that now the pollution stock, although not necessarily monotonically decreasing, never comes back to its ceiling level, hence a nil carbon shadow cost;*
- *A fourth and ultimate green period of infinite duration. This regime can be attained only asymptotically if oil exploitation lasts forever.*
- *Depending on the cost functions, the production of renewable energy may begin either within the first period, possibly from $t = 0$, or within the third one, but never during the second period of binding pollution constraint.*

Once the production of renewable energy starts, the land devoted to its production and the efficiency of the transformation increase hence also its production rate till the ultimate green economy. If the renewable energy production begins during the first phase, the acreage allocated to energy production and the transformation efficiency rate are kept constant during the ceiling period, hence also the renewable energy production. Correlatively when the land allocated to food production decreases and although the efficiency of the food sector increases, the food production decreases.

The different components of an optimal path along which the renewable energy begins to be exploited within the first period are illustrated in Figures 6, 7, and 8 for the case of a finite period of oil exploitation.[31]

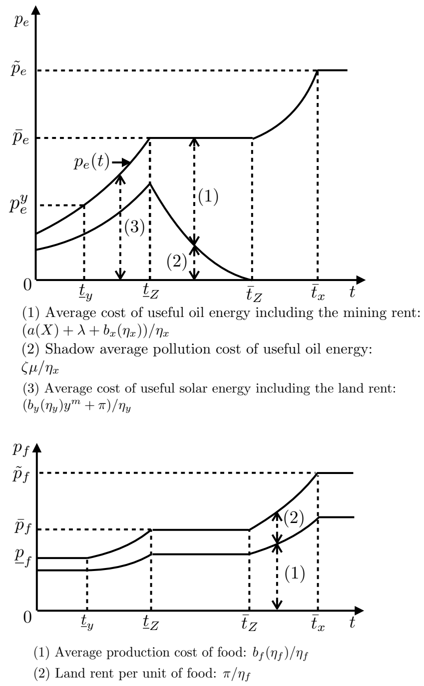


Figure 6: **Optimal price paths:**
Top: Useful energy price
Bottom: Food price.

[31]Note that, as shown in the Figure 8, the land rent per unit of food, π/η_f, being equal to $u' - b_f/\eta_f = b_f' - b_f/\eta_f$, through (3.5) and (3.6), is an increasing function of η_f and thus of time during the unconstrained exploitation periods.

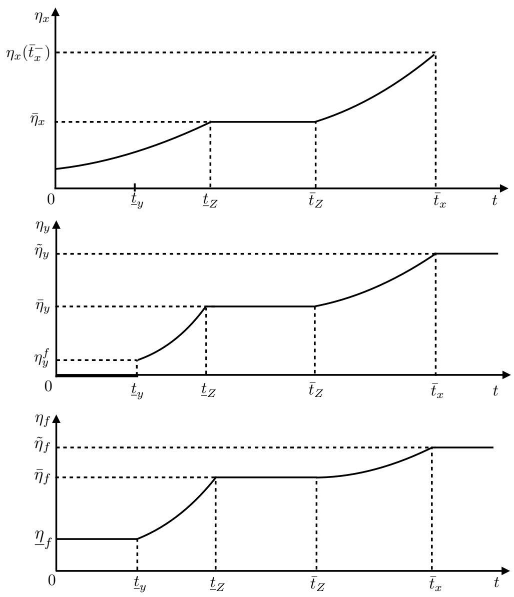


Figure 7: **Paths of efficiency and productivity rates:**
Top: Efficiency rate in the oil transformation industry
Middle: Efficiency rate in the solar energy sector
Bottom: Productivity rate in the food sector.

5 Concluding remarks and the robustness of the ceiling paradox

Facing increasingly costly fossil fuels and the more and more stringent pollution problems raised by their exploitation, the economy should improve the efficiency of useful energy production from any primary source, fossils or renewables. However, efficiency gains come at a cost. This does not prevent renewables to take progressively a larger share in the energy mix while expanding their land use. Under standard assumptions on costs and demands, the development of renewables is both an intensive process, through continuous efficiency gains, and an extensive one, through a larger occupation of space. Being in competition for land access with renewables, the food production sector intensifies its activity on less land but not sufficiently to counterbalance the acreage decrease. Hence the food output decreases and the food prices increase.

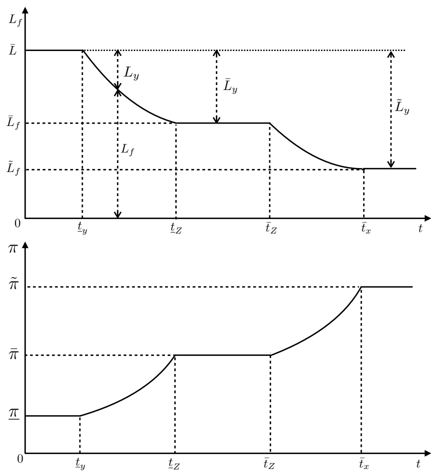


Figure 8: **Paths of land allocation and land rent:**
Top: Land allocation
Bottom: Land rent.

The above dynamics of the energy and food sectors stops when the economy is actually constrained by the cap on the atmospheric carbon concentration. The cap implies a constant rate of exploitation of fossil fuels, absent any abatement option. Facing a constant supply of extracted fossils, the transformation industry makes no more any effort to improve its conversion efficiency performance and delivers useful energy at a constant rate. The energy price being now constant, the renewables sector also stops making efficiency gains and increasing its land occupation.

In a decentralized economy in which the pollution problem is the only externality, the first best can be reached through a carbon pricing scheme that adds the shadow cost of carbon to the useful energy price. The analysis shows that the carbon price should rise before the attainment of the ceiling. However, during the ceiling period, the carbon price must decline in order to keep constant the net surplus from fossil fuels exploitation, just balancing the rise of the full marginal cost of extracted fossil fuels, the sum of their extraction cost and mining rent.

The energy price and the food price are positively correlated because of the competition for land. Introducing a carbon price will raise the food price and reduce the food supply when both oil and solar energy are supplied.

The carbon price increasing the competitiveness of solar energy with respect to fossil fuels, this competitiveness improvement increases the comparative advantage of solar energy production over food production in terms of land valuation.

The ceiling paradox is robust to alternative structures of the surplus function. We have assumed first that food and energy are substitutes and that the surplus function is additively separable and second, that oil and solar energy are perfect substitutes. Although the food-energy substituability assumption is empirically the most pertinent one, the separability assumption could be judged as excessively strong. Let us assume a surplus function $u(q_e, q_f)$ and a perfect substitution between oil and solar useful energies, hence $q_e = \eta_x + \eta_y y^m L_y$ and $q_f = \eta_f L_f$. Then the f.o.c's (3.1)-(3.4) would have to be rewritten with $\partial u\left(\eta_x x + \eta_y y^m L_y, \eta_f L_f\right)/\partial q_e$ substituted for $u'_e(\eta_x x + \eta_y y^m L_y)$ and the f.o.c's (3.5)-(3.6) with $\partial u\left(\eta_x x + \eta_y y^m L_y, \eta_f L_f\right)$ substituted for $u'_f(\eta_f L_f)$. For $x = \bar{x}$ the new conditions (3.2)-(3.6) together with the constraint $\bar{L} - L_y - L_f \geq 0$ determine some $\bar{\eta}_x$, $\bar{\eta}_y$, $\bar{\eta}_f$, $\bar{L}_f$, $\bar{L}_y$ and $\bar{\pi}$, satisfying all the f.o.c's and the land availability constraint. The new condition (3.1) would read:

$$\frac{\partial u\left(\bar{\eta}_x \bar{x} + \bar{\eta}_y y^m \bar{L}_y, \bar{\eta}_y \bar{L}_f\right)}{\partial q_e}\bar{\eta}_x \quad = \quad \theta(t) + \zeta\mu(t) + b_x(\bar{\eta}_x)\ .$$

This condition is satisfied provided that $\theta(t) + \zeta\mu(t)$ be constant and equal to $(\partial u/\partial q_e)\bar{\eta}_x - b_x(\bar{\eta}_x)$, hence $\dot{\mu}(t) = -\dot{\theta}(t)/\zeta < 0$, as in the case of separability. Again the carbon price is maximum at the beginning of the period at the ceiling and during this time period, the energy transformation rates η_x, η_y and η_f must be kept constant together with the land allocation, L_y and L_f, between energy and food production. However during the unconstrained phases the dynamics is much more complex depending upon the strength of the food energy substituability, captured by the absolute value of $\partial^2 u/\partial q_l \partial q_f < 0$.

The other problem is the probably imperfect substituability between oil and solar useful energies. The case in which each type of useful energy generates independent surpluses is an extreme example of such imperfect substitutability. To keep the matter as simple as possible let us also assume that energy and food generate independent surpluses, so that the gross surplus function may be written as $u(q_x, q_y, q_f) = u_{ex}(q_x) + u_{ey}(q_y) + u_f(q_f)$ the functions u_{ex}, u_{ey} and u_f all satisfying the assumption $A.1$. The direct link between the valuation of the two energy sources is now broken. Denoting $p_{ex} \equiv u'_{ex}(q_x)$ and $p_{ey} \equiv u'_{ey}(q_y)$, the equivalents of (3.1)-(3.4) during a time period of simultaneous exploitation of the two energies read:

$$\begin{aligned} p_{ex}\eta_x &= \theta + \zeta\mu + b_x(\eta_x) \quad \text{and} \quad p_{ex} = b'_x(\eta_x) \\ p_{ey}\eta_y y^m &= b_y(\eta_y)y^m + \pi \quad \text{and} \quad p_{ey} = b'_y(\eta_y) \ . \end{aligned}$$

The two first relations determine p_{ex}, η_x and thus q_x and x as functions of the full shadow cost of oil energy, $\theta + \zeta\mu$ alone. On the other hand, the two last relations together with the optimality conditions in the food sector determine some land allocation between food and solar energy production. During unconstrained time periods, the oil sector will improve its energy conversion efficiency, and reduce the supply of oil energy. During constrained periods, the fixed supply of oil will induce a constant energy conversion rate and thus a constant useful oil energy supply. In contrast, the solar energy sector will produce at a forever constant rate during any type of time periods, with a constant transformation rate, inducing a constant food price and a constant food supply and food productivity. Imperfect substitution between fossil and renewable energy at the end-users stage attenuates the positive impact of carbon pricing on the food price under land access competition.[32]

Concerning the energy supply side of the model, the assumption of a cost free reallocation of land to food and energy production may appear as an extreme assumption. Converting land from food to energy production is generally costly, the conversion cost rising with the speed of conversion, equivalently we mean that the instantaneous conversion cost is an increasing and convex function of the conversion rate $\dot{L}_y(t) = -\dot{L}_f(t)$. The result should be a dampening of the speed of the conversion rate during the unconstrained phases. However taking care that the derivative of the adjustment cost function should be strictly positive at $\dot{L} = 0^+$, because converting any piece of land, however small, requires costly inputs, it is not clear that the conversion process should be stopped once at the ceiling and not be restarted, if closed, before the end of the phase at the ceiling (see Amigues *et al.*, (2015) for such effects in another context).

Thus the potential supply of renewable resource $y^m L_y$ could not be constant during the phase at the ceiling because the conversion process has to

[32]See Long, 2013, for a thorough study of the imperfect substitution between energies issue.

be smoothed. An extreme case of such an inherently dampening process is the minimum time-to-build case explored by Kydland and Prescott (1982). We leave these problems for further research.

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Appendix

A.1 A production structure interpretation of the assumption A.3.

Let f_i be the production function of the transformation sector i:

$$q_i = f_i(h_i, m_{i,1}, \cdots, m_{i,j}, \cdots, m_{i,J}) \qquad i = x, y, f \ ,$$

where $h_x = x$ for the oil transformation sector, $h_y = L_y y^m$ for the solar transformation sector, $h_f = L_f$ for the food sector and $m_{i,j}$, $i = x, y, f$, $j = 1, \cdots, J$, are the other production factors. Assume that f_i is homogenous of degree 1, hence may be rewritten as:

$$q_i = h_i f_i(1, z_{i,1}, \cdots, z_{i,j}, \cdots, z_{i,J}) \qquad i = x, y, f \ ,$$

where $z_{i,j} = m_{i,j}/h_i$ and denote by $\eta_i(z_{i,1}, \cdots, z_{i,J})$ the function $f_i(1, z_{i,1}, \cdots, z_{i,J})$. The function $\eta_i(.)$ is assumed twice continuously differentiable with $\eta_i(0) = 0$, $\eta_i \leq \hat{\eta}_i$, $\partial\eta_i/\partial z_j > 0$ and strictly concave. Let $(p_1, \cdots, p_j, \cdots, p_J)$ be the input price vector assumed constant throughout time for simplicity. Then $b_i(\eta_i)$ is the cost function resulting from the expenditure minimization problem:

$$\begin{aligned} \min \quad & \sum_{j=1}^{J} p_j z_{i,j} \\ s.t. \quad & \eta_i(z_{i,1}, \cdots, z_{i,j}, \cdots, z_{i,J}) - \eta_i \geq 0 \\ & z_{i,j} \geq 0, \quad j = 1, \cdots, J \ . \end{aligned}$$

The assumption A.3 results from the assumptions on the $\eta_i(.)$ functions, $i = x, y, f$.

A.2 Proofs of the Propositions 1 and 2

Let L_y^*, $L_y^* \in [0, \bar{L})$, be the optimal acreage of land devoted to energy production, then the optimal management of the food sector is this pair (η_f, L_f)

solving the following food sector problem $(F.S.P)$:[33]

$$(F.S.P) \quad \max_{\eta_f, L_f} \quad u_f\left(\eta_f L_f\right) - b_f(\eta_f) L_f \qquad \text{s.t.} \quad \bar{L} - L_y^* - L_f \geq 0 \ .$$

Denote by π the multiplier associated to the land availability constraint and by $\mathcal{L}_f$ the Lagrangian:

$$\mathcal{L}_f \;=\; u_f(\eta_f L_f) - b_f(\eta_f) L_f + \pi \left[\bar{L} - L_y^* - L_f\right] \ .$$

The f.o.c's are:

$$\frac{\partial \mathcal{L}_f}{\partial \eta_f} = 0 \implies u_f'\left(\eta_f L_f\right) = b_f'(\eta_f) \ , \tag{A.2.1}$$

$$\frac{\partial \mathcal{L}_f}{\partial L_f} = 0 \implies u_f'\left(\eta_f L_f\right) \eta_f = b_f(\eta_f) + \pi \ , \tag{A.2.2}$$

together with the complementary slackness condition:

$$\pi \geq 0 \ , \ \bar{L} - L_y^* - L_f \geq 0 \quad \text{and} \quad \pi\left[\bar{L} - L_y^* - L_f\right] = 0 \ . \tag{A.2.3}$$

Let (η_f^*, L_f^*, π^*) be a solution of the system (A.2.1)-(A.2.3) such that $\pi^* = 0$ so that (A.2.2) may be simplified to get $u_f'\left(\eta_f^* L_f^*\right) = b_f(\eta_f^*)/\eta_f^*$, hence by (A.2.1): $b_f'(\eta_f^*) = b_f(\eta_f^*)/\eta_f^*$. However according to A.3: $b_f'(\eta_f) > b_f(\eta_f)/\eta_f$ for all $\eta_f \in (0, \hat{\eta}_f)$, hence for η_f^*, a contradiction. ■

Without the separability assumption, it must be pointed out that the problem $(F.S.P)$ would have to be written as :

$$(F.S.P) \quad \max_{\eta_f, L_f} \quad u\left(q_e(t), \eta_f L_f\right) - b_f(\eta_f) L_f \qquad \text{s.t.} \quad \bar{L} - L_y^* - L_f \geq 0 \ .$$

The f.o.c's would be now:

$$\frac{\partial u\left(q_e(t), q_f(t)\right)}{\partial q_f(t)} \;=\; b_f'(\eta_f(t)) \tag{A.2.4}$$

$$\frac{\partial u\left(q_e(t), q_f(t)\right)}{\partial q_f(t)} \eta_f(t) \;=\; b_f(\eta_f(t)) + \pi(t) \ . \tag{A.2.5}$$

Without the separability assumption, $\partial^2 u / \partial q_e \partial q_f \neq 0$, hence (A.2.4) and (A.2.5) cannot be reduced to (A.2.1) and (A.2.2). With L_y held constant, but not $q_e(t)$, L_f would be constant because Proposition 1 still holds, but the productivity in the food sector would be no more constant.

[33] We neglect the non-negativity constraints on η_f, L_f and the upper bound constraint on η_f, all of which are satisfied.

A.3 Proof of the Proposition 4

Let us start from the f.o.c's (3.1) and (3.2) written as follows:

$$\begin{aligned} u_e'(q_e)\eta_x &= \theta + b_x(\eta_x) + \zeta\mu && \text{(A.3.1)} \\ u_e'(q_e) &= b_x'(\eta_x) \ . && \text{(A.3.2)} \end{aligned}$$

Time differentiating (A.3.1), using (3.12), (3.13) and (A.3.2), we obtain

$$\dot{q}_e(t) = \frac{\rho\lambda(t) + \zeta(\rho+\alpha)\mu(t)}{u_e''(q_e(t))\eta_x(t)} \ < \ 0 \qquad \text{with } \mu(t) > 0 \text{ if } t \leq \underline{t}_Z$$
$$\text{and } \mu(t) = 0 \text{ if } t \geq \bar{t}_Z \ . \quad \text{(A.3.3)}$$

Time differentiating (A.3.2) and making use of (A.3.3), yield

$$\dot{\eta}_x(t) \ = \ \frac{u_e''(q_e(t))}{b_x''(\eta_x(t))}\dot{q}_e(t) \ > \ 0 \ , \ t \notin (\underline{t}_Z, \bar{t}_Z) \ . \qquad \text{(A.3.4)}$$

Only oil is exploited

In this case: $q_e = \eta_x x$, hence

$$\dot{x}(t) \ = \ \frac{1}{\eta_x(t)}\left(\dot{q}_e(t) - \dot{\eta}_x(t)x(t)\right) < 0 \ . \qquad \text{(A.3.5)}$$

Both energy resources are exploited

Since now $L_y > 0$ and $\eta_y > 0$, then we may write (3.4) as follows: $u_e'(q_e) = b_y'(\eta_y)$. Time differentiating and making use of (A.3.3), we obtain

$$\dot{\eta}_y(t) \ = \ \frac{u_e''(q_e(t))}{b_y''(\eta_y(t))}\dot{q}_e(t) \ > \ 0 \ , \ t \notin (\underline{t}_Z, \bar{t}_Z) \ . \qquad \text{(A.3.6)}$$

Since $L_y > 0$, then (3.3) may be written as: $(u_e'(q_e)\eta_y - b_y(\eta_y))\, y^m = \pi$. Again time differentiating, then by (A.3.3) and (3.4), $u_e' - b_y' = 0$, we get

$$\dot{\pi}(t) \ = \ u_e''(q_e(t))\eta_y(t)y^m\dot{q}_e(t) > 0 \ , \ t \notin (\underline{t}_Z, \bar{t}_Z) \ . \qquad \text{(A.3.7)}$$

Last, consider the food sub-system (3.5)-(3.6):

$$\begin{aligned} u_f'(\eta_f L_f)\eta_f &= b_f(\eta_f) + \pi \\ u_f'(\eta_f L_f) &= b_f'(\eta_f)\ . \end{aligned}$$

Time differentiating and using (3.6), $u_f' - b_f' = 0$, we can express $\dot{\eta}_f$ and $\dot{L}_f$ as the following functions of $\dot{\pi}$

$$\begin{bmatrix} u_f''\eta_f L_f & u_f''\eta_f^2 \\ & \\ u_f''L_f - b_f'' & u_f''\eta_f \end{bmatrix} \begin{bmatrix} \dot{\eta}_f \\ \\ \dot{L}_f \end{bmatrix} = \begin{bmatrix} \dot{\pi} \\ \\ 0 \end{bmatrix} .$$

Hence, for $t \notin (\underline{t}_Z, \bar{t}_Z)$

$$\dot{\eta}_f(t) = \frac{1}{b_f''(\eta_f(t))\eta_f(t)}\dot{\pi}(t) > 0\ , \tag{A.3.8}$$

$$\dot{L}_f(t) = -\frac{u_f''(q_f(t))L_f(t) - b_f''(\eta_f(t))}{u_f''(q_f(t))b_f''(\eta_f(t))\eta_f^2(t)}\dot{\pi}(t) < 0\ , \tag{A.3.9}$$

$$\dot{q}_f(t) = \frac{1}{u_f''(q_f(t))\eta_f(t)}\dot{\pi}(t) < 0\ , \tag{A.3.10}$$

$$\dot{L}_y(t) > 0 \text{ and } \dot{q}_y(t) = \eta_y(t)\dot{L}_y(t) + \dot{\eta}_y(t)L_y(t) > 0\ , \tag{A.3.11}$$

$$\dot{q}_x(t) = \dot{q}_e(t) - \dot{q}_y(t) < 0 \quad \text{and} \quad \dot{x}(t) = \frac{1}{\eta_x(t)}\left[\dot{q}_x(t) - \dot{\eta}_x(t)x(t)\right] < 0\ , \tag{A.3.12}$$

which complete the proof of the claims of the Proposition 4. ■

A.4 Proof of the Proposition 5

To characterize the constrained period, we express the yet unknown dynamics as functions of the only known dynamics, $\dot{\theta} > 0$, taking care that $\dot{x} = 0$. We may have two types of constrained oil exploitation periods according to solar energy is simultaneously exploited with oil or not.

A.4.1 Exclusive exploitation of oil: $q_e = q_x = \eta_x \bar{x}$

Let us start from the f.o.c's (3.1) and (3.2) written now as follows:

$$\begin{aligned} u'_e(\eta_x \bar{x})\eta_x &= \theta + b_x(\eta_x) + \zeta\mu & \text{(A.4.1)} \\ u'_e(\eta_x \bar{x}) &= b'_x(\eta_x) \ . & \text{(A.4.2)} \end{aligned}$$

Time differentiating and using $u'_e - b'_x = 0$, we obtain the following system:

$$\begin{bmatrix} u''_e \bar{x} \eta_x & -\zeta \\ u''_e \bar{x} - b''_x & 0 \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ 0 \end{bmatrix} .$$

Hence

$$\dot{\eta}_x(t) = 0 \quad \text{and} \quad \dot{\mu}(t) = -\frac{1}{\zeta}\dot{\theta}(t) < 0 \ , \ t \in (\underline{t}_Z, \bar{t}_Z) \ . \qquad \text{(A.4.3)}$$

Let us denote by $\bar{\eta}_x$ the constant level of the efficiency rate η_x. Then q_e is constant, $q_e(t) = q_x(t) = \bar{\eta}_x \bar{x}$, and also the price of useful energy: $p_e(t) = \bar{p}_e = u'_e(\bar{\eta}_x \bar{x})$.

The increase of the full marginal cost of the extracted oil is exactly balanced by the decrease of the shadow marginal cost of the pollution stock: $\dot{\theta}(t) + \zeta\dot{\mu}(t) = 0$. The full marginal cost of useful energy, the input cost of the oil transformation industry, is constant and equal to $b'_x(\bar{\eta}_x)$, itself equal to $\bar{p}_e$.

A.4.2 Simultaneous exploitation of both oil and solar energies

Writing L_f as $\bar{L} - L_y$, and using the same methodology we determine now $\dot{\eta}_x$, $\dot{\eta}_y$, $\dot{L}_y$, $\dot{\mu}$, $\dot{\eta}_f$ and $\dot{\pi}$ as functions of $\dot{\theta}$.

Let us consider the system of the six f.o.c's obtained after some simplifications and substitution of $\bar{L} - L_y$ for L_f:

$$\begin{aligned} u'_e\left(\eta_x\bar{x} + \eta_y y^m L_y\right)\eta_x &= \theta + b_x(\eta_x) + \zeta\mu && \text{(A.4.4)} \\ u'_e\left(\eta_x\bar{x} + \eta_y y^m L_y\right) &= b'_x(\eta_x) && \text{(A.4.5)} \\ u'_e\left(\eta_x\bar{x} + \eta_y y^m L_y\right) &= b'_y(\eta_y) && \text{(A.4.6)} \\ u'_e\left(\eta_x\bar{x} + \eta_y y^m L_y\right)\eta_y y^m &= b_y(\eta_y)y^m + \pi && \text{(A.4.7)} \\ u'_f\left(\eta_f\left[\bar{L} - L_y\right]\right) &= b'_f(\eta_f) && \text{(A.4.8)} \\ u'_f\left(\eta_f\left[\bar{L} - L_y\right]\right)\eta_f &= b_f(\eta_f) + \pi\ . && \text{(A.4.9)} \end{aligned}$$

Time differentiating results in the following system:

$$\begin{bmatrix} & ; & -\zeta \\ & ; & 0 \\ & ; & 0 \\ M & ; & 0 \\ & ; & 0 \\ & ; & 0 \end{bmatrix} \begin{bmatrix} \dot{\eta}_x \\ \dot{\eta}_y \\ \dot{L}_y \\ \dot{\eta}_f \\ \dot{\pi} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \text{(A.4.10)}$$

where M is a 5×6 sub-matrix the details of which are given in the below subsection.

Let us denote by Δ the determinant of the system and by $\Delta_{(k,l)}$ the sub-determinant obtained by deleting the k^{th} line and the l^{th} column. Thus $\Delta = \zeta\Delta_{(1,6)}$. Note that $\Delta_{(1,l)} = 0$, for $l = 1, \cdots, 5$, so that

$$\begin{aligned} &\dot{\eta}_x(t) = \dot{\eta}_y(t) = \dot{L}_y(t) = \dot{\eta}_f(t) = \dot{\pi}(t) = 0 \\ &\text{and } \dot{\mu}(t) = -\frac{1}{\zeta}\dot{\theta}(t)\ ,\ t \in (\underline{t}_Z, \bar{t}_Z)\ , \end{aligned} \qquad \text{(A.4.11)}$$

showing the claims of the Proposition 5. ■

A.4.3 Details of the matrix M of equation (A.4.10)

$$
\mathrm{M} = \begin{bmatrix}
u''_e \bar{x} \eta_x & u''_e \eta_x y^m L_y & u''_e \eta_x \eta_y y^m & 0 & 0 \\
u''_e \bar{x} - b''_x & u''_e y^m L_y & u''_e \eta_y y^m & 0 & 0 \\
u''_e \bar{x} - b''_y & u''_e y^m L_y & u''_e \eta_y y^m & 0 & 0 \\
u''_e \bar{x} \eta_y y^m & u''_e \eta_y (y^m)^2 & u''_e (\eta_y y^m)^2 & 0 & -1 \\
0 & 0 & -u''_f \eta_f & u''_f \left[\bar{L} - L_y\right] - b''_f & 0 \\
0 & 0 & -u''_f \eta_f^2 & u'_f + u''_f q_f - b''_f & -1
\end{bmatrix}
$$

A.5 Proof of the Proposition 6

We focus on the case of simultaneous exploitation of both energies. Through (3.2) and (3.4): $b'_x(\eta_x) = b'_y(\eta_y)$ we get by differentiating:

$$
\frac{d\eta_y}{d\eta_x} \;=\; \frac{b''_x(\eta_x)}{b''_y(\eta_y)} > 0 \; . \tag{A.5.1}
$$

Next, we identify the sensitivity of the energy land acreage, L_y, to the fossil energy transformation rate by the following argument. Through (3.2) once again, we get $dp_e/d\eta_x = b_x''(\eta_x) > 0$, thus (3.11) yields

$$v_y(p_e) = \pi = v_f(p_f) \implies \frac{d\pi}{d\eta_x} = \eta_y b''(\eta_x) > 0 \text{ and } \frac{dp_f}{d\eta_x} = \frac{\eta_y}{\eta_f} b_x''(\eta_x) > 0 \ . \tag{A.5.2}$$

Taking (3.6) into account: $dp_f = b_f''(\eta_f)d\eta_f$ and $dp_f = u_f''(q_f)dq_f$ imply that:

$$\frac{d\eta_f}{d\eta_x} = \frac{1}{b_f''(\eta_f)}\frac{dp_f}{d\eta_x} = \frac{\eta_y b_x''(\eta_x)}{\eta_f b_f''(\eta_f)} > 0 \tag{A.5.3}$$

$$\frac{dq_f}{d\eta_x} = \frac{1}{u_f''(q_f)}\frac{dp_f}{d\eta_x} = \frac{\eta_y b_x''(\eta_x)}{\eta_f u_f''(q_f) b_x''(\eta_x)} < 0 \tag{A.5.4}$$

Thus:

$$\begin{aligned} & \frac{dq_f}{d\eta_x} = \frac{d\eta_f}{d\eta_x}L_f + \eta_f \frac{dL_f}{d\eta_x} \\ \implies & \frac{dL_y}{d\eta_x} = -\frac{dL_f}{d\eta_x} = -\frac{1}{\eta_f}\left[\frac{dq_f}{d\eta_x} - \frac{d\eta_f}{d\eta_x}L_f\right] \\ & = -\frac{1}{\eta_f}\left[\frac{\eta_y b_x''(\eta_x)}{\eta_f u_f''(q_f) b_x''(\eta_x)} - \frac{\eta_y b_x''(\eta_x)}{\eta_f b_f''(\eta_f)}L_f\right] > 0 \end{aligned} \tag{A.5.5}$$

Making use of (A.5.1), (A.5.5) and (3.2):

$$\begin{aligned} & u_e'(\eta_x \bar{x} + \eta_y L_y y^m) = b_x'(\eta_x) \\ \implies & u_e''(q_e)\left[d\eta_x \bar{x} + \eta_x d\bar{x} + d\eta_y L_y y^m + \eta_y dL_y y^m\right] = b_x''(\eta_x)d\eta_x \\ \iff & u_e''(q_e)\eta_x d\bar{x} = \left\{b_x''(\eta_x) - u_e''(q_e)\left[\bar{x} + \frac{d\eta_y}{d\eta_x}L_y y^m + \eta_y \frac{dL_y}{d\eta_x} y^m\right]\right\} d\eta_x \\ \iff & \frac{d\eta_x}{d\bar{x}} = \frac{\eta_x \overbrace{u_e''(q_e)}^{(<0)}}{\underbrace{\left\{b_x''(\eta_x) - \underbrace{u_e''(q_e)}_{(<0)}\left[\bar{x} + \underbrace{\frac{d\eta_y}{d\eta_x}}_{(>0)} L_y y^m + \eta_y \underbrace{\frac{dL_y}{d\eta_x}}_{(>0)} y^m\right]\right\}}_{(>0)}} < 0 \ . \end{aligned}$$

Taking (3.2), (A.5.1), (A.5.2), (A.5.3), (A.5.4) and (A.5.5) into account yields:

$$\frac{d\eta_y}{d\bar{x}} < 0 \ ; \ \frac{dL_y}{d\bar{x}} < 0 \ ; \ \frac{dp_e}{d\bar{x}} < 0 \ ; \ \frac{dq_e}{d\bar{x}} > 0 \ ; \ \frac{dq_x}{d\bar{x}} > 0 \ ; \ \frac{dq_y}{d\bar{x}} < 0$$
$$\frac{d\eta_f}{d\bar{x}} < 0 \ ; \ \frac{dL_f}{d\bar{x}} > 0 \ ; \ \frac{dp_f}{d\bar{x}} < 0 \ ; \ \frac{dq_f}{d\bar{x}} > 0 \ .$$

Considering a tightening of the carbon emissions constraint, $d\bar{x} < 0$, the claims of the Proposition 6 follow. ■