Note on the Schou's Paper : "When Environmental Policy is Superfluous : Growth and Polluting Resources"

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Abstract

In a growth model with polluting resources, Schou shows that the pollution externality does not distort the decisions of the market economy, so that a specific environmental policy is superfluous. We show that this result is not generally true, and that it stems from the particular specifications chosen by Schou.

1 Introduction

In a recent paper, Schou (2002) studies an endogenous growth model in which a non renewable resource gives rise to pollution. He obtains a very stimulating result along with the pollution externality does not distort the decisions of the market economy, so that a specific environmental policy is superfluous.

The purpose of the present paper is to show that this result is not generally true, and that is stems from the particular specifications of the model chosen by Schou.

To obtain more general results, we study a non specified version of the Schou's model in which we characterize the optimal paths and the equilibrium ones. Comparing the two shows that an environmental policy is generally necessary to implement the optimum.

The non specified model is studied in section 2. In section 3, we consider two specified models : the model of Schou and a model with a separable utility function.

2 The non specified model

The homogenous good is produced according to the technology (for convenience, the time index is suppressed)

$$Y = F(\int_0^A f(x_i)di, L_Y, R). \tag{1}$$

L_Y is the amount of labour, and R is the flow of non-renewable resource ; x_i is the amount of intermediate good i, A is the number of existing types of intermediate goods (the stock of knowledge) and $X = \int_0^A f(x_i)di$ is an index in which one assumes $f' > 0$ and $f'' < 0$. The partial derivatives F_X, F_L and F_R are positive. In the R&D sector, knowledge is produced along with

$$\dot{A} = q(L_A, A), \quad q_L > 0, \quad q_A > 0, \tag{2}$$

and $L_A + L_Y = 1$ (normalized labor force).

The natural resource is extracted without cost from an initial stock So, and we have the standard resource law of motion :

$$\dot{S} = -R. \tag{3}$$

Pollution is generated by the use of the natural resource within the production process

$$P = h(R) \ , \ h' > 0. \tag{4}$$

The homogenous good is used both for consumtion, C, and investment in intermediate goods ; one unit of homogenous good can be transformed into one unit of intermediate good, that gives

$$Y = C + \int_0^A x_i di. \tag{5}$$

Finally, the utility of the representative household is

$$\int_0^\infty U(C,P)e^{-\rho t}dt,\ U_C > 0,\ U_P < 0,\ \rho > 0. \tag{6}$$

2.1 Welfare

The social planner maximizes $\int_0^\infty U(C,P)e^{-\rho t}$, subject to (1), (2), (3), (4) and (5). After elimination of the costate variables, the first-order conditions reduce to the three following characteristic conditions (see Appendix A) :

$$F_{x_i} = F_X f'(x_i) = 1, \text{ for all } i, \tag{7}$$

that implies $x_i = x\ \forall\ i$, and that allows to note $F_x = F_{x_i},\ \forall\ i$.

$$\rho - \frac{\dot{U}_C}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{q_L}{F_L}(F_X f(x) - x) + q_A. \tag{8}$$

$$\frac{\dot{F}_R}{F_R} - \frac{U_P h'}{U_C F_R}\left(\rho - \frac{\dot{\overline{U_P h'}}}{U_P h'}\right) = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{q_L}{F_L}(F_X f(x) - x) + q_A. \tag{9}$$

Condition (7) (which is the "static efficiency" condition (3) of Schou) says that the marginal productivities of intermediate goods are equal to their marginal cost, that is to say to one. Equations (8) and (9) are respectively the conditions of Ramsey-Keynes and of Hotelling. In these conditions, the RHS (let us denote it H) replaces the marginal productivity of physical capital ("F_K") of the standard neo-classical model. In order to interpret (9), we multiply the two sides by F_R and we consider an interval of time $(t, t + \Delta t)$.

The term $\dot{F}_R(t)\Delta t$ is the increase in production (Y) on $(t, t + \Delta t)$ if one unit of resource is kept in the ground at t and if it is extracted at $t + \Delta t$.

The term $F_R(t)H(t)\Delta t$ is the increase in Y if one unit of resource is extracted at t and if it is (indirectly) embodied in the R&D sector to produce more Y. We use the expression "indirectly" because extracting more resource allows to liberate labor from the final good sector (L_Y decreases), then to increase labor in R&D (L_A increases), and finally to increase A and Y.

If there is no pollution, the standard Hotelling condition is $\dot{F}_R = F_R H$. In the present model, extracting means also polluting, and extracting at t or at $t + \Delta t$ means also polluting at t or at $t + \Delta t$.

The term $[U_P h'(\rho - \dot{U}_P h'/U_P h')/U_C]\Delta t$ is the quantity of consumption good that has to be given to the household to compensate him for the transfer of pollution of one unit of resource from t to $t + \Delta t$. Thus the difference $\dot{F}_R \Delta t - [U_P h'(\rho - \dot{U}_P h'/U_P h')/U_C]\Delta t$ is the *net* increase of good (after compensation of households) resulting from the transfer of one unit of extraction from t to $t+\Delta t$. At optimum, it is equal to $F_R H \Delta t$, that explains the Hotelling condition (9).

2.2 Equilibrium

At equilibrium, there are three market failures (see Schou, p.613). Thus we use three tools : a subsidy to research (σ), a demand subsidy for the intermediate goods (s), and a tax on the resource (θ) (since P and R are linked by a functional relation, it is equivalent to tax P or R). The price of good Y is normalized to one, and $p = p_i \; \forall \, i, w, p_R$ and r are, respectively, the price of intermediate goods, the wage, the resource price, and the interest rate.

In the homogenous good sector, the profit is

$$\Pi_Y = F(\int_0^A f(x_i)di, L_Y, R) - \int_0^A p(1-s)x_i di - wL_Y - p_R(1+\theta)R.$$

The first order conditions of the maximization are

$$F_{x_i} - p(1-s) = 0, \quad \text{where} \quad F_{x_i} = F_X f'(x_i), \text{ for all } i \tag{10}$$

$$F_R - \tau p_R = 0, \quad \text{where} \quad \tau = 1 + \theta, \tag{11}$$

$$F_L - w = 0 \, . \tag{12}$$

From (10), one gets $x_i = x$, for all i. Moreover, differentiating (10) with respect to x_i gives the slope of the demand of any intermediate good i :

$$\frac{\partial x_i}{\partial p_i} = \frac{1-s}{F_{xx}}, \quad \text{where} \quad F_{xx} = \frac{\partial^2 F}{\partial x_i^2}, \; \forall \, i. \tag{13}$$

The profit of the monopolist which produces $x_i = x$, $\forall \, i$, is

$$\Pi^m = (p-1)x = (\frac{F_x}{1-s} - 1)x.$$

Using (13), the maximization of Π^m leads to

$$\frac{F_{xx}}{1-s}x + \frac{F_x}{1-s} - 1 = 0, \tag{14}$$

that gives the profit at its maximum level,

$$\Pi^m = \frac{-x^2 F_{xx}}{1-s}. \tag{15}$$

The value of the firm at t is $V_t = \int_t^\infty \Pi_s^m e^{-\int_t^s r du} ds$. Differentiating with respect to t and rearranging gives $r_t = \dot{V}_t/V_t + \Pi_t^m/V_t$.

In the research sector, the profit is $Vq(L_A, A) - w(1-\sigma)L_A$. The maximization with respect to L_A gives

$$V = \frac{1-\sigma}{q_L} w. \tag{16}$$

In the resource sector, we have the standard equilibrium "Hotelling rule" :

$$\frac{\dot{p}_R}{p_R} = r. \tag{17}$$

Finally, the maximization of the intertemporal utility gives

$$\rho - \frac{\dot{U}_C}{U_C} = r. \tag{18}$$

Now we eliminate prices in order to obtain conditions which can be compared to (7)-(8)-(9).

From (14), one gets

$$F_x = (1-s) - xF_{xx} \tag{19}$$

which can be compared to (7).

From (12) and (16), one gets $V = (1-\sigma)F_L/q_L$, that gives $\dot{V}/V = \dot{F}_L/F_L - \dot{q}_L/q_L$. Using (15) and (16), we have $r = \dot{V}/V + \Pi^m/V = \dot{F}_L/F_L - \dot{q}_L/q_L - x^2 F_{xx} q_L/(1-s)(1-\sigma)F_L$.

Finally, using (18), one gets

$$\rho - \frac{\dot{U}_C}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} - \frac{x^2 F_{xx} q_L}{(1-s)(1-\sigma)F_L}, \tag{20}$$

which can be compared to (8).

From (11) and (17), we have $\dot{F}_R/F_R = \dot{\tau}/\tau + r$. Then, using (18) and (20), one gets

$$\frac{\dot{F}_R}{F_R} - \frac{\dot{\tau}}{\tau} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} - \frac{x^2 F_{xx} q_L}{(1-s)(1-\sigma)F_L}, \tag{21}$$

which can be compared to (9).

2.3 Optimal policies

Putting together (7)-(8)-(9) and (19)-(20)-(21) allows to compute the optimal tools.

From (7) and (19), one gets the demand subsidy for intermediate goods :

$$s = -xF_{xx}. \tag{22}$$

From (8) and (20), one gets

$$\frac{q_L}{F_L}(F_X f(x) - x) - q_A = \frac{x^2 F_{xx} q_L}{(1 + xF_{xx})(1 - \sigma)F_L}, \tag{23}$$

which allows to obtain σ, the subsidy to research.

Finally, from (9) and (21), one gets the rate of growth of the tax τ :

$$\frac{\dot{\tau}}{\tau} = \frac{U_P h'}{U_C F_R}(\rho - \frac{\overset{\bullet}{\overline{U_P h'}}}{U_P h'}). \tag{24}$$

There is no reason to think that the expression between brackets in (24) is nil. This implies that it is generally necessary to conduct an environmental policy to implement the optimal path. Let us note that this policy concerns only the rate of growth of the tax, and not its level. We come back on this point below.

3 Specified models

Here we consider two different specified models : first, the model of Schou ; second, a model with an additive utility function.

3.1 The Schou's model

We consider the following specifications :

$$Y = B(\int_0^A x_i^\alpha di) L_Y^\beta R^\gamma \ \ (B > 0, \alpha > 0, \beta > 0, \gamma > 0, \alpha + \beta + \gamma = 1),$$

$$\overset{\bullet}{A} = DL_A A \ \ (D > 0, A(0) = A_0 > 0), P = \zeta R^\delta \ \ (\delta > 0, \zeta > 0)$$

and $\quad U(C, P) = [(CP^{-\lambda})^{1-\varepsilon} - 1]/(1 - \varepsilon) \ \ (\varepsilon > 0, \varepsilon \neq 0, \rho > 0, \lambda > 0).$

Using the three conditions (7)-(8)-(9) which characterize any optimal path, it is easy to find again the solution obtained by Schou (see Appendix B) :

$$g_R = \frac{D(1 - \varepsilon) - \rho}{\lambda\delta(1 - \varepsilon) + \varepsilon} \tag{25}$$

which corresponds to (13) in Schou,

$$g_Y = \frac{D[\delta\lambda(1-\varepsilon)+1]-\rho}{\lambda\delta(1-\varepsilon)+\varepsilon} \tag{26}$$

which corresponds to (14) in Schou,

$$\text{and} \qquad g_A = \frac{D[(\beta+\gamma)\delta\gamma(1-\varepsilon)+\beta+\gamma\varepsilon]-\beta\rho}{(\gamma+\beta)[\delta\lambda(1-\varepsilon)+\varepsilon]}, \tag{27}$$

which corresponds to (12) in Schou.

Then, from (22)-(23)-(24), one gets the optimal tools :

$$s = 1-\alpha \tag{28}$$

$$\sigma = \frac{(\beta+\gamma)[\delta\lambda(1-\varepsilon)+\varepsilon]-\beta[\rho/D-(1-\varepsilon)]}{(\beta+\gamma[\delta\lambda(1-\varepsilon)+\varepsilon]+\gamma[\rho/D-(1-\varepsilon)]} \tag{29}$$

$$\text{and} \qquad \frac{\dot{\tau}}{\tau} = 0. \tag{30}$$

(30) confirms the result of Schou. In this particular case, the term between brackets of (24), $\rho - (\overset{\bullet}{U_P} h'/U_P h')$, is nil, that explains why it is unnecessary to conduct environmental policy.

3.2 A model with a separable utility

Now we consider an example that it is exactly the same than Schou, except for the utility function which is separable. We retain the specification of Aghion and Howitt (1998) : $U(C,P) = C^{1-\varepsilon}/(1-\varepsilon) - P^{1+\omega}/(1+\omega), \varepsilon > 0, \omega > 0$ (see Appendix C).

In this case, the term between brackets in (24) is

$$\rho - \frac{\overset{\bullet}{U_P} h'}{U_P h'} = \rho - \frac{(\delta(\omega+1)-1)(D(1-\alpha)-\beta\rho)(1-\varepsilon)}{\delta(\omega+1)(\beta(\varepsilon-1)+1-\alpha)-\gamma(1-\varepsilon)},$$

which generally is not nil. This implies that the rate of growth of the optimal tax is different of zero.

We show in Appendix C that the rate of growth of the optimal tax, $\dot{\tau}/\tau$, can be positive or negative. Note that in the particular case $\delta = 1$, that is to say if pollution is a linear function of the flow of extracted resource ($P = \zeta R$), we have $\dot{\tau}/\tau < 0$.

A second basic result is that the tax level has no effect on the equilibrium, except that it leads to a rent transfer from the resource sector to the government. To prove this result, let us consider a steady state where the tax is $\tau_t = \tau_0 e^{g_\tau t}$: τ_0 is the level of the tax and g_τ its rate of growth. From (11), one gets $\tau_t p_{R_t} = F_R = \gamma B A_t x_t^\alpha (L_Y)^\beta R_t^{\gamma-1}$. Thus an increase in τ_0 leads to a decrease in p_{R_t}, such that the product $\tau_t p_{R_t}$ is unchanged.

4 Conclusion

In a model which is exactly the model of Schou, but without particular specifications, we have characterized the optimal and the equilibrium paths. Then we have given the general expression of the rate of growth of the optimal tax on the natural resource. We have verified that, with the specifications of Schou, this rate of growth is nil. Finally, in the case of a non separable utility, we have shown that the level of the tax does not matter and that its rate of growth can be positive or negative.

Appendix A : Welfare

The hamiltonian of the social planner's program is :

$$H = U\{F[\int_0^A f(x_i)di, L_Y R] - \int_0^A x_i di, h(R)\} + \mu q(A, L - L_Y) - \nu R.$$

The first-order conditions $\partial H/\partial x_i = 0, \partial H/\partial L_Y = 0$, and $\partial H/\partial R = 0$ yield

$$U_C(F_X f'(x_i) - 1) = 0, \ \forall \ i \tag{A.1}$$

$$U_C F_L - \mu q_L + 0 \tag{A.2}$$

$$U_C F_R + U_P h'(R) - \nu = 0. \tag{A.3}$$

Moreover, $\partial H/\partial A = \rho\mu - \dot{\mu}$ and $\partial H/\partial S = \rho\nu - \dot{\nu}$ yield

$$U_C[F_X f(x) - x] + \mu q_A = \rho\mu - \dot{\mu} \tag{A.4}$$

$$\rho\nu - \dot{\nu} = 0. \tag{A.5}$$

From (A.1), one gets $x_i = x \ \forall i$ and $F_X f'(x) = 1$, that is the static efficiency condition (3) obtained by Schou. Log-differentiating (A.2) and using (A.4) gives the Keynes-Ramsey condition :

$$\rho - \frac{\dot{U}_C}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{q_L}{q_L}(F_X f(x) - x) + q_A.$$

Log-differentiating (A.3) and using (A.5) gives after some calculations the Hotelling condition :

$$\rho - \frac{\dot{U}_C}{U_C} = \frac{\dot{F}_R}{F_R} - \frac{U_P h'}{U_C F_R}(\rho - \frac{\dot{\overline{U_P h'}}}{U_P h'}).$$

Appendix B : The Schou's model

Equations (1), (2), and (4) are now $Y = B(\int_0^A x_i^\alpha di)L_Y^\beta R^\gamma$, $\dot{A} = DL_A A$ and $P = \zeta R^\delta$, that gives at steady state : $g_Y = g_A + \alpha g_x + \gamma g_R$, $g_A = DL_A$, and $g_P = \delta g_R$.

Moreover, from the utility function $U(C,P) = [(CP^{-\lambda})^{1-\varepsilon} - 1]/(1-\varepsilon)$, one gets $\dot{U}_C/U_C = -\varepsilon g_c - \lambda(1-\varepsilon)g_P$.

Then, by eliminating g_x, the efficiency condition (7) becomes

$$g_Y = g_A + \gamma g_R/(1-\alpha).$$

The Keynes-Ramsey condition (8) is

$$\rho + \varepsilon g_Y + \lambda\delta(1-\varepsilon)g_R = g_Y + D(1-\alpha)g_A/\beta.$$

Finally, the Hotelling condition (9) is

$$\rho + \varepsilon g_Y + \lambda\delta(1-\varepsilon)g_R = g_Y - g_R + \frac{\lambda\delta(1-\alpha)}{\gamma}[\rho - (1-\varepsilon)g_Y + (\delta\lambda(1-\varepsilon)+1)g_R].$$

These three equations allow to compute the three rates of growth, g_Y, g_A and g_R, given by (25)-(26)-(27), which are the ones obtained by Schou.

Then, from (22)-(23)-(24), one gets the optimal policies (28)-(29)-(30).

Appendix C : Separable utility function

Now the utility function is $U(C,P) = C^{1-\varepsilon}/(1-\varepsilon) - P^{1+\omega}/(1+\omega)$, that gives $\dot{U}_C/U_C = -\varepsilon g_c$.

As previously, the efficiency condition is $g_Y = g_A + \gamma g_R/(1-\alpha)$.

The Keynes-Ramsey condition is

$$\rho + \varepsilon g_Y = g_Y + D(1-\alpha)/\beta + (1-\alpha)g_A/\beta.$$

Finally, the Hotelling condition is

$$\rho + \varepsilon g_Y = g_Y - g_R + \frac{\delta(1-\alpha)(\zeta R^\delta)^{\omega+1}}{\gamma C^{1-\varepsilon}}[\rho - (\delta(\omega+1)-1)g_R],$$

that implies at steady state $\delta(\omega+1)g_R = (1-\varepsilon)g_Y$.

As in the previous case, one obtains the three rates of growth g_Y, g_R and g_A :

$$g_Y = \frac{\delta(\omega+1)[D(1-\alpha)-\beta\rho]}{\delta(\omega+1)(\beta(\varepsilon-1)+(1-\alpha))-\gamma(1-\varepsilon)},$$

$$g_R = \frac{(1-\varepsilon)g_Y}{\delta(\omega+1)}, \quad \text{and} \quad g_A = \frac{(1-\alpha)\delta(\omega+1)-\gamma(1-\varepsilon)}{\delta(1-\alpha)(\omega+1)}g_Y.$$

Since $U_P h' = -\delta\zeta^{\omega+1}R^{\delta(\omega+1)-1}$, we have $\dot{\overline{U_P h'}}/U_P h' = [\delta(\omega+1)-1]g_R$.

After substitution, one gets

$$\frac{\dot{\overline{U_P h'}}}{U_P h'} = \frac{(\delta(\omega+1)-1)(1-\varepsilon)(D(1-\alpha)-\beta\rho)}{\delta(\omega+1)(\beta(\varepsilon-1)+1-\alpha)-\gamma(1-\varepsilon)},$$

which is different of ρ.

If $D(1-\alpha)-\beta\rho > 0$ and $\varepsilon > 1$, one gets $g_Y > 0$ and $g_R < 0$. Then the sign of $\dot{\overline{U_P h'}}/U_P h'$ is given by the sign of $1-\delta(\omega+1)$.

If $\delta > 1/(\omega+1)$, we have $\dot{\overline{U_P h'}}/U_P h' < 0$. In this case, the difference $\rho - \dot{\overline{U_P h'}}/U_P h'$ is positive, that implies $\dot{\tau}/\tau < 0$.

If $\delta < 1/(\omega+1)$, $\dot{\overline{U_P h'}}/U_P h'$ is positive and two cases can occur. If $\rho - (\delta(\omega+1)-1)g_R$ is positive, one gets $\dot{\tau}/\tau < 0$. In the other case, we have $\dot{\tau}/\tau > 0$.

Finally, we can observe that if $\delta = 1$, that is to say if $P = \zeta R$ (pollution is a linear function of the flow of extraction), the optimal tax decreases along time : $\dot{\tau}/\tau < 0$.

References

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