Polluting Non-Renewable Resources, Innovation and Growth: Welfare and Environmental Policy

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Abstract

We analyze the impact of the pollution generated by the use of non-renewable resources on the standard results of growth models. In this context, we obtain a Hotelling rule which is not a pure efficiency condition any longer. Subsequently, we show that some of the optimal paths' standard properties change: in particular, an increase in the households' psychological discount rate leads to a slower extraction of the resource. Moreover, we present a simple endogenous growth model that allows us to study the effects of an environmental policy aimed at correcting the distortion introduced at the equilibrium. We show that the tax level does not matter, and that a decreasing tax on the resource use yields the optimum.

Keywords: non-renewable resources, pollution, innovation, growth, extraction path, environmental policy.

JEL classification: O32, O41, Q20, Q32

1 Introduction

When considering the standard analysis of economic growth in the presence of non-renewable natural resources, the main questions that arise are the following. Is positive long term growth possible despite the fact that one of the production process' inputs is only available in a finite quantity? What is the optimal growth path (namely, what is the optimal rate of resource extraction)? What is the equilibrium path? What are its properties, in particular, is it optimal?

The literature has answered these questions through two main types of models: the Ramsey growth model (where we can quote Stiglitz (1974), Garg and Sweeney (1978) or Dasgupta and Heal (1979)) and the endogenous growth models (Schou (1996), Aghion and Howitt (1998), Scholz and Ziemes (1999), Barbier (1999) or Grimaud and Rouge (2003)). Both kinds of models show that long-term positive growth is possible under certain technological conditions. In the former, the key optimality point is given by Hotelling's rule, an efficiency condition that characterizes the optimal resource extraction rate. It says that the marginal productivity of capital must equal the growth rate of the resource's marginal productivity. At market equilibrium, one of the conditions implies that the growth rate of the resource's price must equal the interest rate. As competition is assumed to be perfect in all markets, the unit prices of capital and of the resource must be respectively equal to their marginal productivities. Therefore, Hotelling's rule is verified at the equilibrium, and the equilibrium and optimal paths are identical.

In the second type of models, whether we consider those with horizontal (Romer) or vertical (Aghion-Howitt) differentiation, the optimal paths are in general not followed by the market equilibrium -a feature essentially due to intertemporal externalities that arise from the fact that knowledge is a public good, and to the mark-up imposed by monopolies that sell the intermediate goods.

A common feature of the papers cited above is that they neglect a key characteristic of nearly all non renewable resources that are presently used (fossil fuels / mineral resources). Indeed, the combustion of petroleum, coal and, to a lesser extent, natural gas is responsible for an important part of CO2, the main greenhouse gas (and also SOx and NOx causes of acid rain) -and thus responsible for an eventual climatic change. A part of the literature has already incorporated this aspect when considering the problem of non-renewable resources. Kolstad and Krautkraemer (1993) realise a survey of this literature. More recently, we can quote for example Tahvonen's (1997 and 2001) or Schou's (2000 and 2002) contributions. The latter uses Uzawa-Lucas and Romer-type growth models to consider the problem ; we will refer to his results further below.

When polluting emissions are a by-product of the use of non-renewable resources, several questions arise. One can first ask whether the emissions thus generated constitute a stock or a flow. Several elements may lead to believe that pollution must be treated as a stock variable. Indeed, though carbon dioxide emissions are retained by the atmosphere, a part of these is also seized by oceans (and forests) and trapped for long periods of time. It therefore appears that a dynamic analysis of their effects is required. However, the fact that this can sensibly complicate calculations must also be taken into account. We will see further on that the growth model we develop initially incorporates two state variables (the stocks of knowledge and resource), and that the analysis of the stock of pollution requires us to incorporate a third one. Yet, as Kolstad and Krautkraemer (1993) put it: "in general, it is difficult or impossible to characterize the qualitative features of a model with three state variables without restrictive assumptions about the functional forms of important relationships". For the preceding reason we will treat pollution uniquely as a flow, as was done for example in Schou (2000), keeping in mind that it is only a first step in the analysis.

Another major question concerns the effects generated by this pollution. In fact, we can assume that it affects two targets in particular: the production process and households' utility. In the first case, for example, its effects could be an increased depreciation of capital due to air pollution. Schou (2000) focuses on this case. Here, we will only consider the effects on the second target (pollution reduces household's utility), because it appears to be more straightforward, and because it is the subject of a great part of the literature (see for example Smulders and Gradus (1993) or Aghion and Howitt (1998)). Note, however, that the simultaneous analysis of both effects is a line of research to be considered in the future.

We therefore introduce an endogenous growth model with Romer horizontal differentiation in the presence of non-renewable resources whose use in the production process generates a flow of pollution that negatively affects households' utility. It is then clear how our problem differs from the problem posed by the standard growth models with non-renewable natural resources presented above. Indeed, the question before was: how should a finite stock of resources be distributed among an infinite set of generations? Now, a supplementary question is added to the problem: how to distribute the associated finite stock of pollution among an infinite set of generations? By introducing an externality linked to the use of the resource, we have added an additional dimension to the problem: the choice of an extraction path now implies the simultaneous choice of a pollution path. We can thus imagine that the resource's optimal extraction rate will be modified ; we will see, for example, that the disutility created by pollution appears in the optimality conditions, and we will obtain a non-standard version of the Hotelling rule. We then go on to specify functional forms and we characterize the steady state solutions. In particular, we obtain that higher household preferences for the present imply a slower resource extraction rate, a result contrary to that obtained by the standard literature (see Stiglitz (1974) for example).

Next we adopt a particular type of economic decentralization. Given that our main objective is to concentrate on the consequences of introducing pollution into the standard theory of non-renewable resources, we eliminate all other distortions in order to isolate its effects. In so doing, we also wish to avoid some of the technical difficulties that are usually associated with this kind of models. Our endogenous growth model therefore has no intermediate goods, and we assume that innovations are indivisible public goods directly financed by patents. Naturally, such a characterization of decentralization cannot be considered as a reference equilibrium (we will come to this later), but we believe it is interesting for the reasons discussed above. We then show the differences between equilibrium and optimal conditions and we interpret this non-optimality. Next, we characterize the equilibrium steady state and, namely, we show that resources are extracted too rapidly (with respect to the optimal rate) and that the equilibrium level of R&D is under optimal.

Finally, we implement an environmental policy aimed at correcting the model's sole distortion, which arises from the polluting emissions generated by the use of the resource. The policy is a unit tax on resource purchases. Contrary to Schou (2002), we show that such an instrument is necessary in order to correct equilibrium conditions and bring them closer to optimality. Moreover, our analysis indicates that it is the tax's growth rate, and not its level, which allows the distortion to be corrected. We then analyze the effects of this policy on the model's variables and we define the tax's optimal growth rate, that is, the rate which makes the equilibrium path become an optimal one.

In the section that follows, we present the model and the welfare analysis. We define the economy's market equilibrium and present its properties in section 3. Then, in section 4, we study the effects of the environmental policy and we characterize the optimal policy. Finally, we give concluding remarks in section 5.

2 Presentation of the model and welfare analysis

2.1 The model

We first present a non specified version of the model in order to get general characteristic conditions at optimum (section 2.2) and equilibrium (3.1), and to compare them.

At each time t, a quantity Y_t of homogeneous good is produced according to the following technology:

$$Y_t = F(L_{Yt}, A_t, R_t), \tag{1}$$

where L_{Yt} is the amount of labour devoted to production, A_t is the stock of knowledge and R_t is the flow of non-renewable resource used within the production process. We will denote by F_L, F_A and F_R the marginal productivities.

We consider that each innovation is a public, indivisible and infinitely durable good, which is simultaneously used by the homogeneous good sector and the R&D sector [1]. Formally, it is a point of the segment $[0, A_t]$. At each time t, the stock of knowledge

[1] For instance, we think of a scientific report in which is described a new theory which can be used within the production process.

evolves as follows:

$$\dot{A}_t = q(A_t, L_{RDt}), \tag{2}$$

where L_{RDt} is the amount of labour devoted to R&D. We will denote by q_A and q_L the marginal productivities. Note that the flow of knowledge depends on the pre-existing stock.

Remark: In this model, contrarily to what is done in the standard endogenous growth literature (see for instance Aghion-Howitt (1992), Grossman-Helpman (1991), or Romer (1990)), knowledge is not embodied inside intermediate goods.

We suppose that the technologies $F()$ and $q()$ exhibit constant returns to scale in private inputs, that is, $F(\lambda L_Y, A, \lambda R) = \lambda F(L_Y, A, R)$ and $q(A, \lambda L_{RD}) = \lambda q(A, L_{RD})$. This assumption is justified by the standard replication argument: see for instance Romer (1990) or Jones (2001).

Population is assumed constant, normalized to one, and each individual is endowed with one unit of labour. Thus we have:

$$1 = L_{Yt} + L_{RDt}. \tag{3}$$

The resource is extracted from a initial finite stock S_0, and we have the standard resource stock law of motion:

$$\dot{S}_t = -R_t. \tag{4}$$

The whole production of homogeneous good is consumed by the representative household:

$$Y_t = C_t. \tag{5}$$

The household instantaneous utility function depends both on consumption C_t and on the flow of pollution P_t. We write the intertemporal utility function as follows:

$$U_0 = \int_0^{+\infty} u\left(C_t, P_t\right) e^{-\rho t} dt, \tag{6}$$

where the partial derivatives of the utility function with respect to consumption and pollution are denoted by U_C and U_P and are positive and negative respectively.

Pollution is generated by the use of the non-renewable natural resource within the production process:

$$P_t = h(R_t), \text{ with } h' > 0. \tag{7}$$

2.2 Welfare analysis

The social planner maximizes $\int_0^{+\infty} u\left(C_t, P_t\right) e^{-\rho t} dt$ subject to (1), (2), (3), (4), (5) and (7).

After elimination of the co-state variables, the first order conditions reduce to the two following characteristic conditions (we drop time subscripts for notational convenience):

$$\rho - \frac{\dot{U}_C}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A \tag{8}$$

and

$$\frac{\dot{F}_R}{F_R} + \left(\frac{\overset{\bullet}{U_P h'} - \rho U_P h'}{U_C F_R}\right) = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A. \tag{9}$$

Proof. See Appendix A. ■

Equations (8) and (9) are versions of the Ramsey-Keynes and Hotelling rules respectively[2]. There are three differences with the standard formulations. First, contrary to the standard neo-classical growth model, we do not have physical capital but a stock A_t of knowledge: this explains that the marginal productivity of capital is replaced by the RHS in (8) and (9). The relative complexity of this expression comes from the fact that a variation of C_t is compensated by a transfer of labor between the consumption good

[2] A complete economic interpretation of these two conditions is available from the authors upon request.

sector (L_Y) and the research sector (L_A) which modifies the whole trajectory of variable A_t. The second difference is that in (8), $\dot{U}_C$ is equal to $U_{CC}\dot{C} + U_{CP}\dot{P}$ (U_{CP} is nil in the case of a non-separable utility function). Third, the Hotelling rule is no longer strictly a pure efficiency condition as in the standard model without pollution (see Withagen (1999), p 52). Indeed, extracting one unit of resource allows to produce more (this is standard), but this also entails an increase in pollution, which creates a disutility. This explains the term between brackets in (9).

2.3 Specification and steady-state analysis

2.3.1 Steady-state optimum: characterization and existence

In order to go further in the analysis, and later to study the effects of a green tax, we consider the following standard specifications. As mentioned above, production and $R\&D$ technologies exhibit constant returns to scale in private inputs:

$$Y_t = L_{Yt}^{\alpha} R_t^{1-\alpha} A_t^{\nu}, \text{ with } 0 < \alpha < 1 \text{ and } \nu > 0, \tag{10}$$

$$\dot{A}_t = \delta L_{RDt} A_t, \text{ with } \delta > 0. \tag{11}$$

We assume that emissions are a linear function of extraction flows:

$$h\left(R_t\right) = \gamma R_t, \text{ with } \gamma > 0. \tag{12}$$

Finally we take the separable instantaneous utility function used by Aghion and Howitt (Chapter 5, 1998) for example:

$$\text{and } u(C_t, P_t) = \frac{C_t^{1-\varepsilon}}{1-\varepsilon} - \frac{P_t^{1+\omega}}{1+\omega}, \text{ with } \varepsilon > 0 \text{ and } \omega > 0. \tag{13}$$

Characterization:

We focus now on the study of steady-state paths, i.e., on paths along which the growth rate of any variable is constant.

Proposition 1 *At the steady-state optimum, the quantities and growth rates take the following values (we denote g_z the growth rate of any variable z, we use upper-script ^{o} for optimum, and we drop time subscripts for notational convenience):*

$$L_{RD}^o = \frac{(\delta\nu - \rho\alpha)(\varepsilon(1-\alpha) + \alpha + \omega)}{\delta\nu(\varepsilon + \omega(1-\alpha+\varepsilon\alpha))}, \tag{14}$$

$$L_Y^o = 1 - L_{RD}^o, \tag{15}$$

$$g_A^o = \delta L_{RD}^o, \tag{16}$$

$$g_R^o = g_P^o = g_S^o = \frac{(\delta\nu - \rho\alpha)(1-\varepsilon)}{\varepsilon + \omega(1-\alpha+\varepsilon\alpha)}, \tag{17}$$

$$g_C^o = g_Y^o = \frac{(\delta\nu - \rho\alpha)(1+\omega)}{\varepsilon + \omega(1-\alpha+\varepsilon\alpha)}. \tag{18}$$

Remark: At the steady-state optimum, given S_0 and A_0, the initial quantities are $Y_0^o = C_0^o = (L_Y^o)^\alpha(-S_0 g_R^o)^{1-\alpha}(A_0)^\nu$, $R_0^o = P_0^o/\gamma = -S_0 g_R^o$ (dividing (4) by S_t yields this result), and at each time t we have $x_t^o = x_0^o e^{g_x^o t}$ for any variable x.

Proof. Applying the specifications (10), (11), (12) and (13) to (8) and (9), and considering that all rates of growth are constant, the result follows. ■

Existence of interior optimum:

The two transversality conditions of the social planner's program are respectively $\lim_{t\to+\infty} \mu_t A_t^o e^{-\rho t} = 0$ and $\lim_{t\to+\infty} \nu_t S_t^o e^{-\rho t} = 0$, and they imply $L_{RD}^o < 1$ and $g_R^o < 0$ respectively (see Appendix C for details). Moreover, we need $L_{RD}^o > 0$ to have a solution. The three conditions are together fulfilled if $\varepsilon > 1$ and $\rho < \delta\nu/\alpha$ (see equations (14), (15) and (17))[3]. Note that on this set of parameters values, we have $g_Y^o > 0$ (see (18)). These results are summarized in Figure 1.

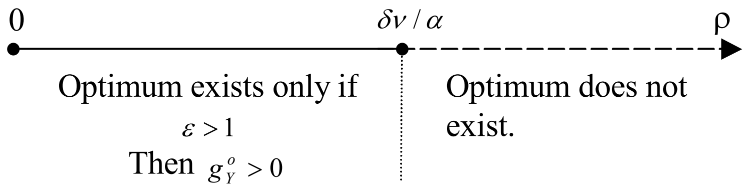

Figure 1: Existence of interior optimum

2.3.2 Properties of the optimal path

We now perform some comparative statics so as to describe the main properties of the path we just defined, and to compare it with what has been obtained in more standard growth models. Letters S and GR indicate that similar results have been obtained by Stiglitz (1974) and Grimaud-Rouge (2003), respectively in a "à la Ramsey" and a "à la Aghion-Howitt" standard models with a non-renewable resource (which does not pollute).

[3] Similar conditions are obtained by Aghion-Howitt (1998) in a "Schumpeterian Approach to Pollution", p 161.

	$\xi = \delta$	$\xi = \varepsilon$	$\xi = \rho$	$\xi = \nu$	$\xi = \omega$
$\frac{\partial L^o_{RD}}{\partial \xi}$	> 0 GR	< 0 GR	< 0 GR	> 0	< 0
$\frac{\partial g^o_A}{\partial \xi}$	> 0 GR	< 0 GR	< 0 GR	> 0	< 0
$\frac{\partial g^o_R}{\partial \xi}$	< 0 S, GR	< 0 S, GR	> 0	< 0	> 0
$\frac{\partial g^o_Y}{\partial \xi}$	> 0 S, GR	< 0 S, GR	< 0 S, GR	> 0	> 0

Table 1: Properties of the optimal path

- The effects of a change in δ or ε on the variables of the model are quite standard. In our model, δ is a parameter which characterizes the effectiveness of the $R\&D$ sector, and it can be seen as the exogenous part of technological progress: it corresponds to η in Stiglitz. A higher δ means that devoting labour to research is better, hence labour in research (L^o_{RD}) and the growth rate of knowledge (g^o_A) grow. This will increase the output growth rate, despite a lower resource extraction growth as in Stiglitz and Grimaud-Rouge. On the reverse, an increase in the elasticity of marginal utility, ε, means that more utility is derived from uniform consumption paths. That is why a social planner will invest less in $R\&D$ (L^o_{RD} and g^o_A decrease). Moreover, this planner will lower consumption growth and therefore choose a lower resource extraction growth rate g^o_R (we have the same result in Stiglitz and Grimaud-Rouge).
- Here, taking into account the pollution emitted by the use of the non-renewable resource modifies the impact of a change in ρ (the psychological discount rate) on g^o_R, with respect with the standard literature.

An increase in ρ means that utility derived from current times are more valued relative to utility derived from future times. In the literature mentioned above, this implies that the representative household values more current consumption relative to future consumption. Thus the steady-state growth rate of the output will decrease, which corresponds to more output today and less tomorrow (relative to the situation before the increase in ρ). In order to produce more today, the social planner will increase the amount L_Y^o of labour and the flow R^o of resource both used within the production process. An increase in the "production-labour" today means a decrease in the "$R\&D$-labour" today, thus a decrease in the growth rate of knowledge (see (2)). An increase in the flow of resource used today implies a decrease in the flow used tomorrow: g_R^o decreases.

In our framework, the fundamental difference comes from the fact that valuing more utilities derived from current times does not only mean valuing more current consumptions. This also means valuing more the current states of the environment (see (6)): improving the first generations's welfare means also improving their environment. Thus P_0^o (i.e. R_0^o) must decrease. So, as above, the planner will increase L_Y^o today, that is, decrease L_{RD}^o and thus decrease g_A^o; but at the same time, the planner will decrease the level of pollution today, and increase this level tomorrow (recall that, in our framework, the stock of pollution is finite): that is why, contrarily to what is obtained in models without pollution, g_P^o (that is, g_R^o) increases.

- A higher ω corresponds to an increase in the household's care for the cleanliness of the environment. One fundamental feature of our model, as we just recalled, is that pollution is created by the use of a non-renewable resource. That is why the stock of pollution is finite; for this reason, at the steady-state, polluting more (resp. less) today means polluting less (resp. more) in the future. Thus, for a given time preference, an increase in ω implies less pollution today and more tomorrow. Hence, at the steady-state, the optimal growth rate of the resource extraction will increase: less resource is consumed today (i.e. less pollution is emitted) and more tomorrow (i.e. more pollution tomorrow). To compensate the fall in R_0^o, the planner will have to increase L_Y^o today, and thus to decrease L_{RD}^o; that is why the growth rate of knowledge decreases. From (10), we have $g_Y^o = (1-\alpha)g_R^o + \nu g_A^o$ at the steady-state. It turns out that the combination of both effects (a rise in g_R^o and a fall in g_A^o) results in a domination of the former: g_Y^o increases.

So, the more households will value their environment, the slowlier the resource will be extracted, and the more rapidly output will grow.

Remark: With a non-separable utility function (as in Schou (2002)), one could not disentangle the effects of ω and ρ. For instance, a higher ρ would lead to a slower extraction provided that households care enough for their environment.

3 Decentralized equilibrium

3.1 Behaviour of agents and equilibrium conditions

Let us consider the non-specified version of the model presented in section 2.1. The price of good Y is normalized to one, and w_t, p_t^R and r_t are, respectively, the wage, the resource price, and the interest rate on a perfect financial market. In order to eliminate the market failure arising from the fact that firms do not take into account the negative externality due to the use of the non-renewable resource within the production process , i.e., pollution, we use a policy tool: a tax on the demand of resource.

Remark: Here we do not tax P_t but R_t. Indeed, since P_t and R_t are linked by a functional relation, we can implement the optimum by only taxing the resource use. Moreover, it is generally more convenient to observe the quantity of resource and thus to use such a policy design.

Homogeneous good sector:

At each time t, the profit of the firm is

$$\pi_t^Y = F(L_{Yt}, A_t, R_t) - w_t L_{Yt} - p_{Rt}(1+\sigma_t)R_t \tag{19}$$

where σ_t is the unit tax on the resource use. From now on, we will note $\tau_t = 1 + \sigma_t$ for computational convenience.

Differenciating π_t^Y with respect to L_{Yt} and R_t, and equating to zero, gives the two following first order conditions:

$$F_L = w_t, \tag{20}$$

$$\text{and } F_R = \tau_t p_{Rt}. \tag{21}$$

R&D sector:

As we mentioned above, knowledge is not embodied inside intermediate goods, thus it cannot be financed by the sale of these goods. We suppose here that knowledge is directly financed. Nonetheless, as the sectors using knowledge exhibit constant returns to scale in the private goods, their profits are nil; thus we assume that once an innovation has occured, the government pays to the innovator a sum equal to the willingnesses to pay of both sectors.

Remark: This type of decentralization defines an equilibrium that is a benchmark: in the absence of any pollution, this equilibrium is a first best optimum (as we will show it below). This is a formalization of ideas already expressed by Arrow (1962), Dasgupta et al. (1996) or Scotchmer (1991 and 1999). The main interest of designing such an equilibrium is that it allows us to focus on one single distorsion: the one caused by pollution. Hence we avoid the distorsions (monopolies rents, intertemporal spillovers...) inherent to the equilibria used in the standard endogenous growth models. Indeed, these distorsions entail a useless complexity of the model. Note that, as seen above, considering such an equilibrium leads to assume that the governement finances entirely $R\&D$. However, it is possible, assuming that the two sectors using innovations (final sector and $R\&D$ sector) are imperfectly competitive, to construct an equilibrium in which $R\&D$ is entirely and privately financed by these sectors (we find a similar equilibrium path in this case).

At each time t, the value of an innovation is:

$$V_t = \int_0^{+\infty} v_s e^{-\int_t^s r_u du} ds, \tag{22}$$

where v_s is the sum of the willingnesses to pay of the homogeneous good sector (v_s^Y) and the R&D sector (v_s^{RD}): $v_s = v_s^Y + v_s^{RD}$.

The profit on innovations produced at t is:

$$\pi_t^{RD} = q(A_t, L_{RDt})V_t - w_t L_{RDt} \tag{23}$$

The maximization of this profit function with respect to L_{Rt} leads to the following first order condition:

$$q_L V_t = w_t \tag{24}$$

From (19) and (23), we have:

$$v_t^Y = \frac{\partial \pi_t^Y}{\partial A_t} = F_A$$

$$\text{and } v_t^{RD} = \frac{\partial \pi_t^{RD}}{\partial A_t} = q_A V_t,$$

which gives

$$v_t = F_A + q_A V_t. \tag{25}$$

Representative household:

At each time t, the representative household maximizes the utility function $\int_0^{+\infty} u\left(C_t, P_t\right) e^{-\rho t} dt$ subject to $\dot{B}_t = w_t + r_t B_t + p_{Rt} R_t - T_t - C_t$, where B_t is the stock of bonds at t, and T_t is a lump-sum tax levied by the government to finance research. This maximization leads to the following condition:

$$\rho - \frac{U_{CC}\dot{C} + U_{CP}\dot{P}}{U_C} = r_t. \tag{26}$$

Resource sector:

On the competitive natural resource market, the maximization of the profit function $\int_t^{+\infty} p_{Rs}R_s e^{-\int_t^s r_u du} ds$, subject to $\dot{S}_s = -R_s$, $S_s \geq 0$, $R_s \geq 0$, $s \geq t$, yields the standard equilibrium "Hotelling rule":

$$\frac{\dot{p}_{Rt}}{p_{Rt}} = r_t, \text{ for all t.} \tag{27}$$

Observe that, since $p_{Rt} = F_R$ in absence of environmental policy (see (21) above), it is clear that here, the decentralized economy does not implement the optimum Hotelling rule given by (9) (we come back to this point later in the text).

Government:

The government's budget constraint is[4]

$$T_t + \sigma_t p_{Rt} R_t = v_t^Y + v_t^{RD}, \text{ for all } t.$$

Our objective is now to exhibit the two characteristic equilibrium conditions that can be compared to the optimum ones ((8) and (9)). We obtain:

$$\rho - \frac{U_{CC}\dot{C} + U_{CP}\dot{P}}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A \tag{28}$$

and

$$\frac{\dot{F}_R}{F_R} - \frac{\dot{\tau}_t}{\tau_t} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A \tag{29}$$

[4] We assume here that the government possesses all the information and thus can perfectly determine v^Y and v^{RD}. Of course this is just a benchmark and informational asymmetries could be introduced.

Proof. Differentiating (22) and (24) with respect to time gives $\frac{\dot{w}_t}{w_t} = r_t - \frac{v_t}{V_t} + \frac{\dot{q}_L}{q_L}$. Using (20), (24) and (25), we get: $r_t = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A$. This equation, together with (26) gives (28).

The same equation together with (21) and (27) leads to (29). ■

First of all, we can observe that (8) and (28) are identical: in other words, at the equilibrium, the optimum Ramsey-Keynes rule is verified.

However, we can see (compare (9) and (29)) that in absence of any environmental policy, the optimum Hotelling rule is not verified. This comes from the fact that the social cost of pollution is not reflected in the resource price. We analyse this point in section 4 and we show that it implies that the resource is extracted too fast at the equilibrium. Nonetheless, if pollution has no impact on the utility of households ($U_P = 0$), equations (9) and (29) are identical and the first best optimum is reached, as mentioned above (see the remark after (21)).

3.2 Specification and steady-state analysis

3.2.1 Steady-state equilibrium: characterization and existence

Now we use the specifications (10), (11), (12) and (13) presented in section 2.3.1, and here also we focus on steady-state paths.

Characterization:

Proposition 2 *At the steady-state equilibrium, the quantities and rates of growth take the following values (the upper-script ^{e} is used for equilibrium):*

$$L_{RD}^e = \frac{(\varepsilon - \alpha\varepsilon + \alpha)\delta\nu - \alpha\rho + \alpha(\varepsilon - 1)(1-\alpha)g_\tau}{\varepsilon\delta\nu}, \tag{30}$$

$$L_Y^e = 1 - L_{RD}^e, \tag{31}$$

$$g_A^e = \delta L_{RD}^e, \tag{32}$$

$$g_R^e = g_P^e = g_S^e = \frac{\delta\nu\,(1-\varepsilon)-\rho}{\varepsilon} + \frac{\alpha-1-\alpha\varepsilon}{\varepsilon} g_\tau, \tag{33}$$

$$g_C^e = g_Y^e = \frac{\delta\nu-\rho}{\varepsilon} - \frac{1-\alpha}{\varepsilon} g_\tau. \tag{34}$$

Remark: At the steady-state equilibrium, given S_0 and A_0, the initial quantities are $Y_0^e = C_0^e = (L_Y^e)^\alpha(-S_0 g_R^e)^{1-\alpha}(A_0)^\nu$, $R_0^e = P_0^e/\gamma = -S_0 g_R^e$, and at each time t we have $x_t^e = x_0^e e^{g_x^e t}$ for any variable x.

Proof. Applying the specifications (10), (11), (12) and (13) to (28) and (29), and considering that all rates of growth are constant, the result follows. ■

At the equilibrium steady-state, prices are:

$$w_t = \alpha(L_Y^e)^{\alpha-1}(R_t^e)^{1-\alpha}(A_t^e)^\nu, \tag{35}$$

$$p_{Rt} = (1-\alpha)(L_Y^e)^\alpha(R_t^e)^{-\alpha}(A_t^e)^\nu(\tau_0 e^{g_\tau t})^{-1}, \tag{36}$$

and $r = \delta\nu-(1-\alpha)g_\tau$, $v_t^Y = \nu(L_Y^e)^\alpha(R_t^e)^{1-\alpha}(A_t^e)^{\nu-1}$, $v_t^{RD} = L_{RD}^e\alpha(L_Y^e)^{\alpha-1}(R_t^e)^{1-\alpha}(A_t^e)^{\nu-1}$, $V_t = (\alpha/\delta)(L_Y^e)^{\alpha-1}(R_t^e)^{1-\alpha}(A_t^e)^{\nu-1}$, where $R_t^e = -S_0^e g_R^e e^{g_R^e t}$ and $A_t^e = A_0^e e^{g_A^e t}$.

Existence of interior equilibrium:

Within this analysis, we suppose that there is no environmental policy, that is, $\tau_t = 1$ (or equivalently, $\sigma_t = 0$).

As we did for the optimal path, we are looking for the set of parameter values in which $0 < L_{RD}^e < 1$, and $g_R^e < 0$. To do so, we use equations (30) and (33). The set is described in Figure 2:

3.2.2 Equilibrium properties

As it is done for the optimum, we now study the impact of different exogenous parameters variations on L_{RD}^e, g_A^e, g_R^e and g_Y^e. The results are depicted in Table 2.

Letters GR indicate that similar results have been obtained by Grimaud-Rouge (2003)

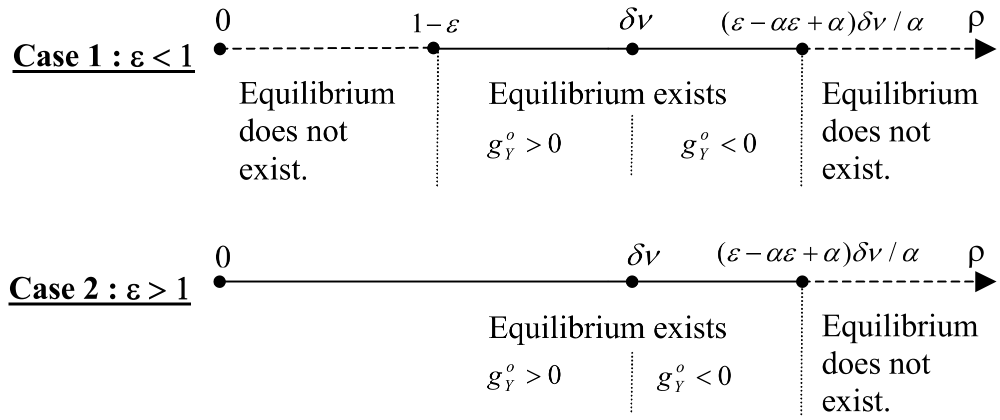

Figure 2: Existence of interior equilibrium

in a model "à la Aghion-Howitt" with a non-polluting non-renewable resource.

	$\xi=\delta$	$\xi=\varepsilon$	$\xi=\rho$	$\xi=\nu$	$\xi=\omega$
$\frac{\partial L_{RD}^e}{\partial\xi}$	>0 GR	<0 GR	<0 GR	>0	$=0$
$\frac{\partial g_A^e}{\partial\xi}$	>0 GR	<0 GR	<0 GR	>0	$=0$
$\frac{\partial g_R^e}{\partial\xi}$	<0 if $\varepsilon>1$ GR	<0 if $g_Y^e>0$ GR	<0 GR	<0 if $\varepsilon>1$	$=0$
$\frac{\partial g_Y^e}{\partial\xi}$	>0 GR	<0 if $g_Y^e>0$ GR	<0 GR	>0	$=0$

Table 2: Properties of the equilibrium path

The effects of the parameters variations presented in Table 2 result from market mechanisms (in Table 1, they resulted from the planner's decisions). We can see that the properties of the equilibrium path are very similar to the properties of the optimal one. There are only two differences. First, g_R^e is an decreasing function of ρ. Indeed, if ρ gets higher, households derive more utility from current consumption, thus want to consume more today: g_C^e decreases. Hence firms will produce more today and less tomorrow. In order to increase the output today, they will use more inputs: more labour and more resource. Indeed, contrarily to the social planner, firms do not take into account the effects of the resource use on the utility of consumers. As more resource is extracted today, the stock being finite, less will be extracted tomorrow: g_R^e decreases. The second difference with the properties of the optimal path is that ω has no effect on the equilibrium variables. Indeed this parameter represents the taste of households for their environment (or equivalently their vulnerability towards pollution); however, in the absence of any environmental policy, pollution is not priced in this model, thus ω affects no market.

4 Impact of the environmental policy and implementation of the optimum

4.1 Optimum versus equilibrium

First of all, Figure 1 and Figure 2 show that both the optimal path and the equilibrium path are defined together if $\varepsilon > 1$ and $\rho < \delta\nu/\alpha$. Thus, as we want to compare these paths and to analyse the effects of an environmental policy, one of the main goals of which is the implementation of the optimum, this conditions will hold throughout the remainder of this section.

Proposition 3 *In absence of any environmental policy, we have:*

$$L_{RD}^e > L_{RD}^o, \tag{37}$$

$$g_R^e < g_R^o, \tag{38}$$

$$g_Y^e < g_Y^o. \tag{39}$$

Proof. In the set of parameter values described above, we compare L_{RD}^e (see (30)) and L_{RD}^o (see (14)), g_R^e (see (33)) and g_R^o (see (17)), and g_Y^e (see (34)) and g_Y^o (see (18)). The results follow. ■

In this model, there is one single distorsion: the pollution emitted by the resource use. If there is no pollution (or equivalently if consumers do not value their environment), the decentralized equilibrium is a first best optimum (see above). If there is pollution, but no tax, the equilibrium remains unchanged because firms do not take into account this pollution: this equilibrium remains equivalent to the first best without pollution.

Hence, comparing the equilibrium (without any public intervention) and the optimal paths is exactly the same as comparing the optimal path in the absence of pollution and the optimal path when the resource use yields a flow of pollution that affects negatively consumers'utility.

In section 2.3.2, we showed that the more consumers value their environment, the slowlier the resource is extracted: g_R^o increases in order to stand less pollution today and more tomorrow (i.e. exhausting the finite stock of pollution less rapidly). Thus the limit case corresponding to an indifference towards environment (P_t does not enter the utility function) will be characterized by a lower g_R^o (i.e. a lower g_P^o), because pollution is not taken into account by the planner. Market mechanisms yield the same result: at the decentralized equilibrium, the resource is extracted more rapidly because pollution is not priced (see Figure 3). We also proved in section 2.3.2 that a higher care for the environment leads lower the amount of labour devoted to $R\&D$ so as to compensate the increase in g_R^o. Hence, for the same reasons, L_{RD}^e is over-optimal: there is too much $R\&D$ at the equilibrium.

4.2 Impact of the environmental policy

Here we want to study the effects of the environmental policy we set up (a tax on the use of the non-renewable resource) on the main relevant variables of the model. At the steady-state, the tax can be written: $\tau_t = \tau_0 e^{g_\tau t}$.

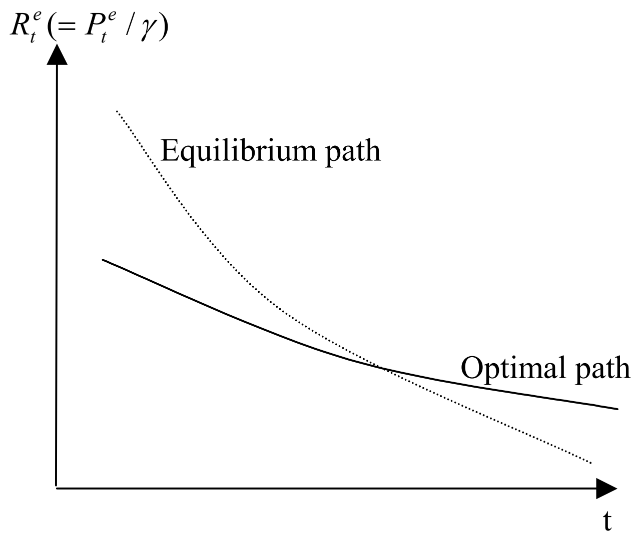

Figure 3: Optimum vs. equilibrium without any public intervention *(t: time, R_t^e: resource extraction, P_t^e: flow of pollution)*

Proposition 4 *a) A change in the tax level has no effect on the equilibrium but a rent transfer from the resource sector to the government.*

b) A change in the tax's growth rate affects all prices, quantities and growth rates (as summarized in table 3).

This proposition contrasts with most of the results obtained in the standard literature where the tax level generally has an impact. First of all, note that an increase (resp. a decrease) in τ_0 has one unique effect on the equilibrium defined in section 3.2.1: a decrease (resp. an increase) in p_{Rt} (see (36)), such that the total unit cost of the resource, i.e. $\tau_t p_{Rt}$, is unchanged for the final sector firm. In other words, a modification in the tax level (for a given growth rate g_τ) has only one effect on the economy: a modification in the opposite sense of the resource price level, that is to say a rent transfer from the resource sector to the government. The other prices, and the quantities and rates of growth are not changed. We obtain in this framework a result corresponding to an idea already mentioned in Schou (2000, 2002).

On the reverse, the entire path is modified by a change in g_τ. Moreover, proposition 5 below proves that the optimal g_τ (the one that allows the implementation of the optimal path) is negative. So, let us first consider a constant τ, and then start to increase it during the first times and decrease it during future times: g_τ becomes negative. Using equations (30)-(34), we obtain the results summarized in table 3.

	$\xi = L^e_{RD}$	$\xi = g^e_A$	$\xi = g^e_R$	$\xi = g^e_Y$	$\xi = r$
$\dfrac{\partial \xi}{\partial g_\tau}$	>0 if $\varepsilon > 1$	>0 if $\varepsilon > 1$	<0	<0	<0

Table 3: Effects of the environmental policy

First of all, note that the price ratio $\tau_t p_{Rt}/w_t$ is a decreasing function of g_τ (see (36) and (35)). Thus at steady-state, the lower g_τ is, the higher $\tau_t p_{Rt}$ (the price paid by the firm for the resource) is relative to the wage. For this reason the firm producing the homogeneous good will use more labour: L_Y increases.

Moreover, at steady-state, $g_{\tau_t p_{Rt}/w_t}$ is equal to $g_\tau + g_{p_R} - g_w$. Thus, equations (27) and (35) allow us to write $g_{\tau_t p_{Rt}/w_t} = g_\tau + r - g^e_Y$, which is an increasing function of g_τ. Thus whereas a lower g_τ makes the resource price higher relative to the wage, this effect diminishes along time. Hence the firm will choose to buy less resource today and more tomorrow relative to the path it chose with the initial g_τ (see Figure 4): g^e_R increases,.the resource is extracted at a lower path.as it is shown in Table 3.

4.3 Implementation of the optimum

Proposition 5 *If* $g_\tau = -(\frac{\dot{U_P h'} - \rho U_P h'}{U_C F_R})$, *which is equal to* $-\frac{\delta\nu\omega(\varepsilon-1)+\rho(\varepsilon+\omega)}{\varepsilon+\omega(1-\alpha+\alpha\varepsilon)}$ *in the specified case at the steady-state, then the equilibrium path is optimal.*

Note that the optimal g_τ is negative, which means that along the equilibrium growth path, as the use of the resource decreases (g^e_R is negative), the tax level decreases also.

Proof. We are searching g_τ such as (8) and (9) are equivalent to (28) and (29). Clearly, (8) and (28) are identical, and if $g_\tau = -(\frac{\dot{U_P h'} - \rho U_P h'}{U_C F_R})$, then (9) and (29) are identical also. In the specified case, simple computations allow us to find g_τ.such that L^e_{RD} (see (30)) and L^o_{RD} (see (14)) be equal. ■

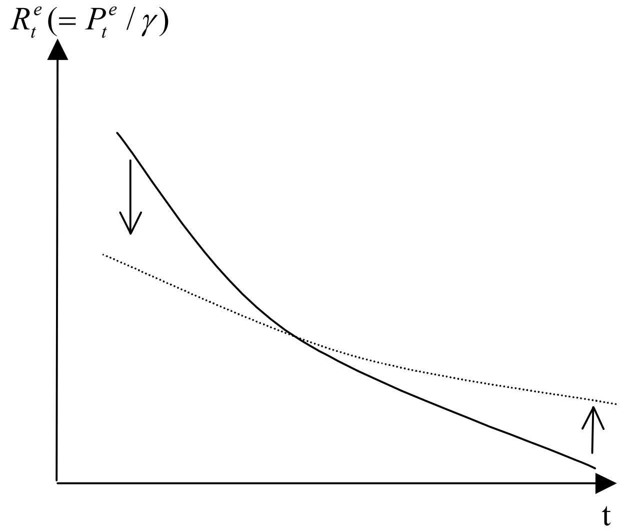

Figure 4: Impact of the environmental policy consisting in a decrease in g_τ *(t: time,* R_t^e*: resource extraction,* P_t^e*: flow of pollution)*

5 Conclusion

In this paper we considered the following problem: how are the standard results of non-renewable resources (growth) theory modified when we take into account the presence of polluting emissions arising from the use of those resources?

Firstly, we were able to characterize optimality conditions in a very general manner (that is, without specifying functional forms) since we avoided the technical complexity inherent to certain hypothesis of the standard endogenous growth models. We thus defined a Hotelling rule that was modified in two senses: on one hand, it is no longer strictly an efficiency condition. Since pollution affects households' preferences, these are accounted for in the new Hotelling rule. On the other hand, it is now the accumulation of intellectual capital (knowledge), instead of the accumulation of physical capital, which delays the depletion of the resource.

Next, we studied the steady-state's optimal paths and we described their properties. Namely, we found once more that increases in households' impatience (or in the egoism of present generations), or in their preference for environmental quality, result in a slower extraction of the resource.

We went on to present a decentralization of the economy, such that pollution is the source of the model's only distortion. There, the violation of the Hotelling rule at the market equilibrium -when no environmental policy has been applied- is clearly identified. Agents fail to incorporate the negative externality caused by the use of the resource: there is no market for pollution. We also characterized the steady-state's equilibrium path and showed that the growth rate at which the resource is extracted is under-optimal: the resource is depleted too rapidly. Moreover, contrarily to that observed at the optimum, an increase in households' impatience makes the economy deplete the resource even faster.

Finally, the fact that there is only one distortion in the model allows us to implement a simple economic policy: a unit tax on the demand for the resource. We then showed that, contrary to what is found in a large part of the literature, a change in the tax level at the steady-state has no effect on the real variables of the economy and does not allow us to correct the equilibrium path (in order to make it optimal). Indeed, such a policy only entails a rent transfer from the enterprise that extracts the resource to the government (who levies the tax). Conversely, a change in the tax's growth rate affects all prices, quantities and growth rates. We calculate the exact level of the latter growth rate (negative) which sends the appropriate signal to markets and for which the optimum is attained at equilibrium.

Certain elements -such as considering pollution as a stock and not a flow- could certainly add to our results. Moreover, including in our problem the possibility of substituting the polluting non-renewable resource with a non-polluting but less productive renewable resource (such as solar energy or hydrogen fuel cells) remains a very interesting line of future research.

Appendix

A. Non specified optimality conditions

The current value Hamiltonian of the social planner's program is

$$H = U[F(1 - L_{RDt}, A_t, R_t), h(R_t)] + \mu_t q(A_t, L_{RDt}) - \nu_t R_t.$$

where μ_t and ν_t are the costate variables.

The first order conditions $\partial H/\partial L_{RDt} = 0$ and $\partial H/\partial R_t = 0$ yield

$$\mu_t = \frac{U_C F_L}{q_L} \tag{40}$$

$$\nu_t = U_C F_R + U_P h'. \tag{41}$$

Moreover, $\partial H/\partial A_t = \rho\mu_t - \dot{\mu}_t$ and $\partial H/\partial S_t = \rho\nu_t - \dot{\nu}_t$ yield

$$\frac{\dot{\mu}_t}{\mu_t} = \rho - \frac{F_A q_L}{F_L} - q_A \tag{42}$$

$$\text{and} \quad \frac{\dot{\nu}_t}{\nu_t} = \rho. \tag{43}$$

Differentiating (40) with respect to time, we have

$$\frac{\dot{\mu}_t}{\mu_t} = \frac{U_{CC}\dot{C} + U_{CP}\dot{P}}{U_C} + \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L}. \tag{44}$$

(44), together with (42) gives

$$\rho - \frac{U_{CC}\dot{C} + U_{CP}\dot{P}}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A, \tag{45}$$

that is, the Ramsey-Keynes condition.

Differentiating (41) with respect to time, we have

$$\frac{\dot{\nu}_t}{\nu_t} = \frac{U_C F_R(\frac{U_{CC}\dot{C}+U_{CP}\dot{P}}{U_C} + \frac{\dot{F}_R}{F_R}) + U_P h'(\frac{U_{PC}\dot{C}+U_{PP}\dot{P}}{U_P} + \frac{\dot{h'}}{h'})}{U_C F_R + U_P h'}. \tag{46}$$

(46) together with (43) gives

$$\rho - \frac{U_{CC}\dot{C} + U_{CP}\dot{P}}{U_C} = \frac{\dot{F}_R}{F_R} + \frac{U_P h'}{U_C F_R}(\frac{U_{PC}\dot{C} + U_{PP}\dot{P}}{U_P} + \frac{\dot{h'}}{h'} - \rho). \tag{47}$$

Plugging (45) into (47) yields:

$$\frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A = \frac{\dot{F}_R}{F_R} + \frac{U_P h'}{U_C F_R}(\frac{U_{PC}\dot{C} + U_{PP}\dot{P}}{U_P} + \frac{\dot{h'}}{h'} - \rho),$$

where $\frac{U_P h'}{U_C F_R}(\frac{U_{PC}\dot{C}+U_{PP}\dot{P}}{U_P} + \frac{\dot{h'}}{h'} - \rho) = \frac{U_P h'}{U_C F_R}(\frac{\dot{U_P} h' + \dot{h'} U_P}{U_P h'} - \rho) = \frac{U_P h'}{U_C F_R}(\frac{\dot{(U_P h')}}{U_P h'} - \rho)$.

One gets finally

$$\frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A = \frac{\dot{F}_R}{F_R} + (\frac{\dot{(U_P h')} - \rho U_P h'}{U_C F_R}), \tag{48}$$

that is the Hotelling condition.

B. Transversality conditions at the steady-state optimum

The first transversality condition is $\lim_{t\to+\infty} \mu_t A_t^o e^{-\rho t} = 0$, which is equivalent to $\lim_{t\to+\infty} \mu_0 e^{g_\mu t} A_0^o e^{g_A^o t} e^{-\rho t} = 0$ at the steady state. Moreover, with the specifications presented in section 2.3.1. within in the main text, equation (42) in Appendix A gives us $g_\mu = \rho - \frac{\delta\nu}{\alpha} + \delta L_{RD}^o(\frac{\nu-\alpha}{\alpha})$. Thus we can see that $g_\mu + g_A^o - \rho$ is negative if and only if L_{RD}^o is lower than one.

The second transversality condition is $\lim_{t\to+\infty} \nu_t S_t^o e^{-\rho t} = 0$, which is equivalent to $\lim_{t\to+\infty} \nu_0 e^{g_\nu t} S_0^o e^{g_S^o t} e^{-\rho t} = 0$ at the steady-state. Equation (43) in Appendix A allows us to find that $g_\nu + g_S^o - \rho$ is negative if and only if $g_S^o (= g_R^o)$ is negative.

References

Aghion, P. and P. Howitt, 1992, A Model of Growth Through Creative Destruction, Econometrica 60, 323–351.

Aghion, P. and P. Howitt, 1998, Endogenous Growth Theory (The MIT Press).

Arrow, K.J., 1962, The Economic Implications of Learning-by-Doing, Review of Economic Studies, 29(1), 155-173.

Barbier, E.B., 1999, Endogenous Growth and Natural Resource Scarcity, Environmental and Resource Economics, 14, 51–74.

Dasgupta, P.S. and G.M. Heal, 1979, Economic Theory and Exhaustible Resources, UK (Oxford University Press).

Dasgupta, P.S., K.G. Mäler, G.B. Navaretti and D. Siniscalo, 1996, On Institutions that Produce and Dissipate Knowledge, Mimeo.

Garg, P.C. and J.L. Sweeney, 1978, Optimal Growth with Depletable Resources, Resources and Energy 1, 43–56.

Gradus, R. and S. Smulders, 1993, The Trade-off between Environmental Care and Long-term Growth; pollution in three proto-type growth models, Journal of Economics, 58, 25-51.

Grimaud, A. and L. Rouge, 2003, Non-Renewable Resources and Growth with Vertical Innovations: Optimum, Equilibrium and Economic Policies, Journal of Environmental Economics and Management, 45, 433-453.

Grossman, G. and E. Helpman, 1991, Innovation and Growth in the Global Economy, Cambridge MA (MIT Press).

Hotelling, H., 1931, The Economics of Exhaustible Resources, Journal of Political Economy, 39, 137–175.

Jones, C.I., 2001, Population and Ideas: a Theory of Endogenous Growth, U.C. Berkeley mimeo.

Kolstad, C.D. and J.A. Krautkraemer, 1993, Natural Resource Use and the Environment, in: A.V. Kneese and J.L. Sweeney, eds., Handbook of Natural Resources and Energy Economics, vol. III (Elsevier Science Publishers), 1219-1265.

Romer, P., 1990, Endogenous Technological Change, Journal of Political Economy, 98, 71–102.

Scholz, C.M. and G. Ziemes, 1999, Exhaustible Resources, Monopolistic Competition and Endogeneous Growth, Environmental and Resource Economics, 13, 169–185.

Schou, P., 1996, A Growth Model with Technological Progress and Non-renewable Resources, Mimeo, University of Copenhagen.

Schou, P., 2000, Polluting Non-Renewable Resources and Growth, Environmental and Resource Economics, 16, 211-227.

Schou, P., 2002, When Environmental Policy is Superfluous: Growth and Polluting Resources, Mimeo, University of Copenhagen.

Scotchmer, S., 1991, Standing on the Shoulders of Giants: Cumulative Research and the Patent Law, Journal of Economic Perspective, Symposium on Intellectual Property Rights.

Scotchmer, S., 1999, Cumulative Innovations in Theory and Practice, Working Paper, U.C. Berkeley.

Stiglitz, J., 1974, Growth with Exhaustible Natural Resources : I) Efficient and Optimal Growth, II) The Competitive Economy, Review of Economic Studies Symposium, 41, 123–152.

Tahvonen, O., 1997, Fossil Fuels, Stock Externalities, and Backstop Technology, Canadian Journal of Economics, XXX, 855-874.

Tahvonen, O. and S. Salo, 2001, Economic Growth and Transition between Renewable and Nonrenewable Energy Resources, European Economic Review, 45, 1379-1398.

Withagen, C., 1999, Optimal Extraction of Non-Renewable Resources, in J.C.J.M. van den Bergh (ed), Handbook of Environmental and Resource Economics, Edward Elgar, 49-58.

	$\xi = \delta$	$\xi = \varepsilon$	$\xi = \rho$	$\xi = \nu$	$\xi = \omega$
$\frac{\partial L^o_{RD}}{\partial \xi}$	> 0 GR	< 0 GR	< 0 GR	> 0	< 0
$\frac{\partial g^o_A}{\partial \xi}$	> 0 GR	< 0 GR	< 0 GR	> 0	< 0
$\frac{\partial g^o_R}{\partial \xi}$	< 0 S, GR	< 0 S, GR	> 0	< 0	> 0
$\frac{\partial g^o_Y}{\partial \xi}$	> 0 S, GR	< 0 S, GR	< 0 S, GR	> 0	> 0

Table 1: Properties of the optimal path

	$\xi = \delta$	$\xi = \varepsilon$	$\xi = \rho$	$\xi = \nu$	$\xi = \omega$
$\frac{\partial L_{RD}^e}{\partial \xi}$	> 0 GR	< 0 GR	< 0 GR	> 0	$= 0$
$\frac{\partial g_A^e}{\partial \xi}$	> 0 GR	< 0 GR	< 0 GR	> 0	$= 0$
$\frac{\partial g_R^e}{\partial \xi}$	< 0 if $\varepsilon > 1$ GR	< 0 if $g_Y^e > 0$ GR	< 0 GR	< 0 if $\varepsilon > 1$	$= 0$
$\frac{\partial g_Y^e}{\partial \xi}$	> 0 GR	< 0 if $g_Y^e > 0$ GR	< 0 GR	> 0	$= 0$

Table 2: Properties of the equilibrium path

	$\xi = L^e_{RD}$	$\xi = g^e_A$	$\xi = g^e_R$	$\xi = g^e_Y$	$\xi = r$
$\frac{\partial \xi}{\partial g_\tau}$	> 0 if $\varepsilon > 1$	> 0 if $\varepsilon > 1$	< 0	< 0	< 0

Table 3: Effects of the environmental policy