Funding Directly Research in Growth Models without Intermediate Goods*

André Grimaud† Frédéric Tournemaine‡

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Abstract

In this paper, we treat the question of the funding of research in an endogenous growth model that does not incorporate any production sector of intermediate goods. Following the suggestions of several authors like Arrow (1962), Dasgupta et-al (1996), Scotchmer (1991, 1999), Lerner and Tirole (2002) we define specific markets and specific prices for the discoveries produced by researchers. We construct two types of equilibria. In the first one, research is publicly funded. We characterize the first best equilibrium and we compute the system of prices that implement it. In the second one funding is coming from private sources.

Keywords: Ideas, Non-Convexity, Funding Research, Imperfect Competition.

JEL Classification: O3

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†Université de Toulouse I (GREMAQ, IDEI and LEERNA), 21 allée de Brienne, 31000 Toulouse, France, and ESCT. E-mail: Grimaud@cict.fr

‡Université de Toulouse I (GREMAQ), 21 allée de Brienne, 31000 Toulouse, France. E-mail: frederic.tournemaine@univ-tlse1.fr

1 Introduction

It is now well recognized that the research and development (R&D) activity which allows to discover new technologies or to improve them is essential in the development of an economy. A fundamental question which may be set concerns the different possible schemes to fund research. From the seminal papers of Romer (1990), Grossman and Helpman (1991a), Aghion and Howitt (1992), the way to answer this problem is usually the same in endogenous growth literature. Once an idea is produced by a scientist, it is associated to a particular intermediate good. Then, any intermediate producer benefits from a patent, purchased from the R&D sector, to produce and to sell his good. The rents earned from the sales of intermediate goods produced around innovations allow to reward scientists and therefore to keep incentives to conduct research.

In this paper, we look at the question of the funding of research in R&D-based models without any intermediate goods production sector. Basically, the main characteristics of such models are the following. The final output is produced along with $Y_t = F(A_t, private\ inputs)$ and the R&D technology is given by $\overset{\bullet}{A}_t = G\left(A_t, private\ inputs\right)$, where A_t is knowledge and there is not any intermediate goods embodying ideas inside private inputs. This approach simplifies greatly the analysis, in particular the welfare one. But at the same time, it raises the problem of the way to give incentives to invest in research. Indeed, it is now impossible to consider the standard equilibrium used in the whole endogenous growth literature.

Jones (2001, 2002, 2003) exposes several examples of models that do not incorporate any intermediate goods production sector. Jones (2001) proposes one particular type of decentralized equilibrium in such a framework. However, he does not define any market and any price for innovations. The consequence of this drawback is that we do not know neither the instantaneous price nor the current expected value of an innovation in the decentralized economy.[1]

To make our analysis, we use the model developed by Jones (2002). There are two main reasons for this choice. First, this framework is a very good illustration since it is complete both on a theoretical and on an empirical point of view. The author performs an interesting study on the source of the economic growth in United-States (US). He uses a model incorporating a R&D activity, physical capital accumulation, education of individuals and population growth. He finds that resources allocated to research have increased because in addition to population growth both the level of education and the share of labor allocated to research have grown; he explains that 80 percent of US economic growth is due to increases in human capital investment rates and research intensity while population growth accounts only for 20 percent. Second, we give an answer to a question asked by the author himself in his paper. He writes about his framework: "This can be viewed as a precursor to the richer analysis that comes from adding markets to the model and analyzing equilibrium conditions as well as technologies" (p. 223). Then, the author suggests to characterize an equilibrium since he has just

[1] An other drawback is that the shares of labor allocated to the different sectors of the economy are exogenously given. In contrast, in the present analysis, they are endogenous. We compute their exact values as a functions of the parameters of the model.

performed an empirical analysis of his model without accounting for goods' markets. In that sense, our analysis completes Jones' (2002) one by adding markets and by computing the relevant prices of goods produced in the economy.

The research activity cannot be funded like in the standard R&D-based literature. Then, we try to formalize ideas expressed by economists like Arrow (1962), Dasgupta et-al (1996), Scotchmer (1991, 1999), Lerner and Tirole (2002). Because of the public good nature of innovations, several problems arise. The first ones are standard in economics literature. First, they are relative to the possibility to verify which agent uses a discovery. Second they are linked to the possibility to exclude any agent that does not pay to access to an innovation. Third, they concern the problems of informations about the willingness to pay of agents.

The second types of problem are due to the non convexity of technologies using ideas as productive factors. Indeed, most of economists agree with the replication argument. According to Feehan (1989), Manning et-al (1985), Sandmo (1972), Kaizuka (1965), and more recently according to Romer (1990), and Jones (2003) among others, technologies display constant returns to scale with respect to private inputs and increasing returns with respect to both private and public inputs. As in a competitive market the payment of private factors fully exhausts revenue, firms are unable to pay for the public good they use. As pointed out by Jones (2003), this fundamental property leads to several problems, in particular on the type of equilibrium considered to fund research. For instance, a perfectly competitive equilibrium does not exist, except if research is publicly funded. Thus, imperfect competition appears as a necessary condition to give private incentives to invest in research.

In the present framework, we construct two possible equilibria. Following Arrow (1962) who points out that "the property rights may be in the information itself, through patents and similar legal devices" (p. 149), we assume that ideas are protected by property rights. Like Lerner and Tirole (2002), we define a specific market and a price for the ideas produced by the scientists. Then, we assume that scientist keep the property rights and license their ideas to any user of discoveries. That is, any agent using innovations must directly reward researchers who have produced them. In that way, we are consistent with Scotchmer (1991) who writes: "A system of property rights that might seem natural would be to protect the first innovator so broadly that licensing is required from all second generations innovators who use the initial technology, whether in research or in production" (p. 32).

In the first equilibrium considered, there is perfect competition on all private goods markets. To implement the first best optimum, we assume that each agent using a discovery pays its maximum willingness to pay. As mentioned above, this practice requires the possibility to verify, which agent uses an innovation and when the discovery is used. It is necessary to be able to exclude any agent that does not pay to access to a discovery. Moreover, it requires a complete information about the willingness to pay of each agent. To avoid potential negative profits due to increasing returns to scale, we assume that firms using knowledge as a productive factor are subsidized by the government. In that case, research is publicly funded which may, perhaps, appear unrealistic. However, this equilibrium must be seen as a benchmark. In the second equilibrium considered, research is privately funded. To deal with the non-convexity of production processes, we assume that there is imperfect competition on the markets whose technologies use knowledge as a productive factor. Therefore, we are consistent with Jones (2003) who argues that the non-rival property of ideas prevents the perfect competition to prevail (see p. 2).

One must note that the equilibria described in our paper and, as said above, suggested for instance by Arrow (1962), Dasgupta et-al (1996), Scotchmer (1991, 1999), Lerner and Tirole (2002), diverge greatly from the ones studied in the standard R&D-based literature. Indeed, in addition to the fact that we do not specify any intermediate goods production sector, the equilibria we characterize display complete markets. A strange aspect of the standard R&D-based models comes from the fact that the discoveries produced by scientists and which are public, indivisible and infinitely durable goods, have not any specific price. In these models, the goods sold are the private intermediate goods embodying ideas, but the ideas themselves are not. In our knowledge, neither in the literature on endogenous growth nor in the well-known books of Grossman and Helpman (1991b) Barro and Sala-I-Martin (1995), Aghion and Howitt (1998), any author defines a market and a price for discoveries. Thus, these models induce equilibria with incomplete markets. Note that this property allows to keep the perfectly competitive assumption in all markets (except the one of intermediate goods). However, the non-convexity problem raised by the non-rival property of ideas is not solved by the existence of imperfect competition. It is solved by the incompleteness of markets. In these models, imperfect competition on the market of intermediate goods is just a mean to fund indirectly research.

The remainder of the paper is organized as follows. We describe the model in Section 2. We characterize the perfectly competitive equilibrium of the model in Section 3. We construct an imperfect competition equilibrium in Section 4. We conclude in Section 5. The Appendix is provided in Section 6.

2 The model

We consider the same model than Jones (2002) with identical notations. Nevertheless, in contrast with the author who takes the rate of savings as exogenously given, here we assume that it is endogenous. We show how to compute equilibria with an exogenous rate of savings in Appendix 6.4.

Time is continuous and three kinds of goods are produced in the economy: a consumption-capital good ("output"), ideas and human capital. Total output Y_t produced at time t is given by,

$$Y_t = (A_t)^{\sigma} (K_t)^{\alpha} (H_{Yt})^{1-\alpha} \tag{1}$$

where $\alpha \in (0, 1)$, $\sigma > 0$, K_t is physical capital, H_{Yt} is the total quantity of human capital employed to produce output and A_t is the total stock of ideas available in the economy. Physical capital is accumulated through the process,

$$\dot{K}_t = Y_t - C_t - dK_t \tag{2}$$

where $d > 0$ is the exogenous constant rate of depreciation and C_t is aggregate consumption.

Ideas are produced by researchers through the technology,

$$\dot{A}_t = \delta \left(H_{At}\right)^{\lambda} \left(A_t\right)^{\phi} \tag{3}$$

where $\delta > 0$ is a constant productivity parameter, $\dot{A}_t$ is the total number of ideas produced per unit of time and H_{At} is the total quantity of human capital devoted to research. The possibility of duplication effect or redundancy in research is captured by $\lambda \in (0, 1]$, and $\phi < 1$ allows past discoveries to either increase ($\phi > 0$) or decrease ($\phi < 0$) current research productivity. To highlight the duplication effect in research, we assume that innovations are produced by a large number S ($s = 1, ..., S$) of firms whose technologies are identical and given by $\dot{A}_t^s = \zeta_t H_{At}^s \left(A_t\right)^{\phi}$, where $\dot{A}_t^s$ is the number of innovations produced by a typical firm s, H_{At}^s represents the amount of human capital employed in this firm, and ζ_t is a productivity factor in research which is external to each firm and which verifies $\zeta_t = \left(H_{At}\right)^{\lambda-1}$.[2] The stock of knowledge, A_t, is formed by ideas or innovations, i, which take for example the form of scientific reports. Formally, each innovation is a particular point of the set $[0, A_t]$ which expands over-time. In the two equilibria we construct, innovations are patented and priced goods.

The quantities of human capital employed producing output and ideas are respectively given by,

$$H_{Yt} = h_t L_{Yt} \tag{4}$$

and

$$H_{At} = \left(h_t\right)^{\theta} L_{At} \tag{5}$$

where L_{Yt} is the total amount of raw labor employed producing output, L_{At} represents the number of scientists in the economy, h_t is human capital per person and $\theta \geq 0$ is a constant parameter. The individual's human capital is produced forgoing labor in the labor force through the process

$$h_t = e^{\psi L_{Ht}/N_t} \tag{6}$$

where $\psi > 0$ is a constant parameter, L_{Ht} is the quantity of labor devoted to education and N_t is the total number of individuals in the economy at date t.

Finally, the economy is composed of an infinitely lived representative household whose members are identical and grow over-time at the exogenous rate, $n > 0$. At each time, their number is $N_t = N_0 e^{nt}$ where $N_0 > 0$ is the initial number of members of the household at time 0. The preferences of the

[2] Jones (2002) does not disaggregate the research sector. However, our specification is still compatible with the motion of ideas of his model given in the present note by equation (3).

household are represented by the discounted utility function

$$U = \int_0^\infty \frac{(c_t)^{1-\varepsilon} - 1}{1-\varepsilon} N_t e^{-\rho t} dt \tag{7}$$

where $c_t \equiv C_t/N_t$ is per-capita consumption at time t, $\varepsilon > 0$ is the inverse of the elasticity of substitution and $\rho > 0$ is the rate of time preferences. Each individual is endowed with one unit of labor and divides this unit among producing goods, producing ideas, and producing human capital. Thus, the aggregate resource constraint is given by,

$$N_t = L_{Yt} + L_{At} + L_{Ht} \tag{8}$$

3 Perfectly competitive equilibrium

Our objective is to compute an equilibrium with complete and perfectly competitive markets. Concerning the financing of innovations, we make two assumptions. Firstly, R&D firms protect their innovations by an infinitely-lived patent giving them the possibility to exclude any agent that does not pay ideas they produce. Secondly, they are able to practice a first degree price discrimination among agents: any firm using knowledge as a productive factor pays the maximum price it is willing to pay to access to scientific reports. In that way, we follow Dasgupta et-al (1996) who write: "A possible scheme is for society to grant intellectual property rights to private producers for their discoveries, and permit them to charge (possibly differential) fees for their use by others. This creates private markets for knowledge. Patent and copyright protections are means of enforcing intellectual property rights. It is as well to note here that, in this scheme the producer (or owner) of a piece of information should ideally set different prices for different buyers, because different buyers typically value the information differently. In economics, these variegated prices are called Lindahl prices, in honor of the person who provided the first articulation of this scheme" (p. 10). Since technologies using knowledge display constant returns to scale with respect to private factors inducing that their payment completely exhaust firms' revenue, we assume that the willingness to pay of firms are subsidized by the government.

To justify this approach, we assume that the R&D sector keeps its infinitely-lived patents and licenses them to potential users. In that way, any agent using a patented innovation rewards directly the researcher or the scientist who has produced it. Then, the price of an idea is defined by the value of the license. Note that we are consistent with Arrow (1962) who explains: "Suppose, as the result of elaborate tests, some metal is discovered to have a desirable property, say resistance to high heat. Then of course every use of the metal for which this property is relevant would also use this information, and the user would be made to pay for it. But, even more, if another inventor is stimulated to examine chemically related metals for heat resistance, he is using the information already discovered and should pay for it in some measure; and any beneficiary of his discoveries should also pay" (p. 150). Formally, in our economy, two types of agents will pay (or buy a license) to use innovations: first, the R&D sector (see equation (3)); second, the final sector (see equation (1)).

In this context, the duplication effect in research remains the only external effect. Thus, in order to implement an optimal balanced growth path, we assume that the government intervenes by the mean of a tax rate τ_t charged on R&D firms. The two economic policies of the government (subsidies for the use of discoveries and the tax charged on R&D firms) are financed through a lump-sum tax , T_t, charged on the representative household.

First, we characterize the behavior of the agents. Second, we derive the solution of the decentralized equilibrium and we show how the government may implement the optimal balanced growth path. We assume that the representative household buys physical capital and patents from the R&D sector and rents them to firms. The price of the final homogenous good is normalized to one. The unit price of human capital employed in the final sector, the one of human capital employed in research, the rental price of capital and the rate of return on R&D investment are respectively noted q_{Yt}, q_{At}, r_{Kt} and r_{At}. We denote by v_{Yt} and v_{At}^s the prices paid by the final sector's firm and any R&D firm s to use an innovation. Finally, the growth rate of any variable x is noted g_x.

3.1 Behavior of agents

a) The final sector's firm maximizes its profit given by $\Pi_{Yt} = (A_t)^\sigma (K_t)^\alpha (H_{Yt})^{1-\alpha} - r_{Kt}K_t - q_{Yt}H_{Yt}$, which gives,

$$r_{Kt} = \alpha \frac{Y_t}{K_t} \tag{9}$$

$$q_{Yt} = (1-\alpha)\frac{Y_t}{H_{Yt}} \tag{10}$$

Moreover, the willingness to pay, v_{Yt}, of the representative firm to use an innovation is given by, $v_{Yt} = \partial \Pi_{Yt}/\partial A_t = \sigma Y_t/A_t$.

b) In the R&D sector the profit of a firm s is $\Pi_{At}^s = V_t \zeta_t H_{At}^s (A_t)^\phi - q_{At}(1+\tau_t) H_{At}^s$, where V_t is the value of an innovation (see below). The free-entry condition is,

$$V_t \zeta_t (A_t)^\phi = (1+\tau_t) q_{At} \tag{11}$$

In addition, the willingness to pay of a research firm s to have access to the stock of knowledge is given by, $v_{At}^s = \partial \Pi_{At}^s / \partial A_t = \phi V_t \zeta_t H_{At}^s (A_t)^{\phi-1}$. The value of an innovation is measured by the flow of gains earned from the date at which the researcher has discovered a new idea and has patented it, until the infinity. Thus, we can write $V_t = \int_t^\infty v_u e^{-\int_t^u r_A(s)ds} du$, where v_u is the instantaneous gain from the sale of an innovation. It is equal to the sum of the willingnesses to pay of the final sector's firm and of the research firms: it verifies $v_t = v_{Yt} + v_{At}$ where $v_{At} = \sum_{s=1}^{S} v_{At}^s$. Note that differentiating the expression of V_t with respect to time yields,

$$r_{At} = \frac{v_t}{V_t} + \frac{\dot{V}_t}{V_t} \tag{12}$$

c) Concerning the government, we assume that his budget constraint is balanced at each time. It is given by $T_t = v_{Yt} + \sum_{s=1}^{S} v_{At}^s - \tau_t q_{At} H_{At}$, where $\tau_t q_{At} H_{At}$ represents the tax on human capital charged on research firms. To implement an optimal equilibrium path, at each time he chooses τ_t that maximizes the total welfare.

d) Finally, the representative household chooses the per-capita consumption path and the quantities of labor allocated to the production of output, of ideas and of his level of skill.[3] Solving his problem (whose proof is provided in the Appendix 6.1) one can find the usual Keynes-Ramsey rule,

$$\varepsilon g_c + \rho = r_{At} = r_{Kt} - d \tag{13}$$

and

$$\frac{L_{Yt}}{N_t} = \frac{1}{\psi} - \theta \frac{L_{At}}{N_t}. \tag{14}$$

3.2 Characterization of the perfectly competitive equilibrium

In this section we focus on the balanced growth path. The time subscript is skipped from the policy tool, τ, which must be constant in that case. Given the agents' behavior, we can determine the growth rates of the variables in the economy and we can compute the shares of labor allocated to the different sectors as a function of the tax, τ, charged on R&D firms. Proposition 1 summarizes the results obtained in this economy. Equilibrium values are denoted with a subscript "$*$". The proof of the proposition is gathered in Appendix 6.1.

Proposition 1 *An equilibrium balanced growth path with perfect competition on private goods markets and research funded at Lindahl prices levels is characterized by constant growth rates,*

$$\begin{aligned} g_{H_Y}^* &= g_{H_A}^* = n, \\ g_A^* &= \frac{\lambda n}{(1-\phi)}, \\ g_c^* &= g_Y^* - n = \frac{\sigma}{(1-\alpha)} g_A^*, \end{aligned}$$

[3] It is possible to decentralize the human capital production process. In that case, we can compute additional prices corresponding to the individual's level of human capital and to the level of wage of raw labor. The way to decentralize the educational process is gathered in Appendix 6.4.

and constant shares of labor allocated to research, to the final sector and to education,

$$\left(\frac{L_{At}}{N_t}\right)^* = \frac{1}{(1+\tau)\,\psi}\left[(\varepsilon-1)+\frac{(\rho-n)\,(1-\alpha)\,(1-\phi)}{\sigma\lambda n}+\frac{(1-\phi)\,(1-\alpha)+\frac{\theta\sigma}{(1+\tau)}}{\sigma}\right]^{-1},$$

$$\left(\frac{L_{Yt}}{N_t}\right)^* = \frac{1}{\psi}-\theta\left(\frac{L_{At}}{N_t}\right)^*,$$

$$\left(\frac{L_{Ht}}{N_t}\right)^* = 1-\left(\frac{L_{Yt}}{N_t}\right)^*-\left(\frac{L_{At}}{N_t}\right)^*.$$

In addition the prices are given by, $q^*_{Yt} = (1-\alpha)\,Y_t/H_{Yt}$, $r^*_{Kt} = \alpha Y_t/K_t$, $v^*_{Yt} = \sigma Y_t/A_t$, $q^*_{At} = V^*_t A_t g_A/\left[(1+\tau)\,H_{At}\right]$, $v^*_{At} = \phi g_A V^*_t$, $V^*_t = (1+\tau)\,q^*_{At}\,(H_{At})^{1-\lambda}/\,(A_t)^{\phi}$, $r^*_A = r^*_{Kt} - d = \varepsilon g^*_c + \rho$.

From proposition 1 it is possible to compute the growth rates of prices which complete the statistical analysis of Jones (2002). Moreover, for each possible value of the tax rate, τ, there is an associated equilibrium, and only one of these is optimal. Indeed, if we compare the steady-state optimum which is computed in Appendix 6.2 with the characterization of the steady-state equilibrium given in proposition 1, we obtain the following proposition:

Proposition 2 *If the government chooses* $\tau^* = 1/\lambda - 1 > 0$, *the steady-state equilibrium is optimal.*

Proposition 2 implies that without any intervention, a research firm is attempted to hire too much skilled labor to innovate. The reason is that it does not account for the duplication effect in research. Thus, it results an equilibrium characterized by an excessive share of human capital devoted to research: indeed, if $\tau = 0$ one can compute easily that $(L_{At}/N_t)^* - (L_{At}/N_t)^o > 0$ where the subscript "o" denotes an optimal value.

Nevertheless, we can notice that $\partial\tau^*/\partial\lambda < 0$. Thus, the higher is λ (i.e. the lower is the duplication effect measured by this parameter), the lower will be the tax imposed on research firms to implement the optimum. And, if there is not any duplication effect in research, ($\lambda = 1$), it is not necessary to tax human capital in R&D firms. In that case $\tau^* = 0$ and the decentralized equilibrium path coincides with the optimal one. This result proves that the equilibrium with complete markets considered here is the benchmark of the model presented in Section 2.

4 Imperfect competition equilibrium

The aim of this section is to present an equilibrium in which research is privately funded. In other words, the government does not intervene to reubsidy the willingness to rpay for inn discussed earlier, one difficulty to account for reuch equilibrium comes from the non-convexity of technologies using knowledge as an inut. To deal with this di fficulty, we assume imperfect competition on marketre that use ideas as productrive factors. rThus, in our economy, trhis srituation prfenvalls in the sector (1) and in the R&D sector (see (3)). In addition, we assume that the imperfect competition leads to two conditions. Firstly, the profits of firms including both the payment of private factors and the reward of innovators are nil. This condition can be interpreted as a free entry condition on these markets. Secondly, firms choose amounts of inputs so that the marginal rate of substitution between two productive factors equals their prices ratio. This behavior may underline the fact that firms are price takers on the markets of inputs.

Formally, for the final sector's firm, we have $\Pi_{Yt} = (A_t)^{\sigma}(K_t)^{\alpha}(H_{Yt})^{1-\alpha} - r_{Kt}K_t - q_{Yt}H_{Yt} - v_{Yt}A_t = 0$. Moreover, equating the marginal rate of substitution between two factors with their corresponding prices ratio, we obtain the following equations,

$$\frac{\alpha}{1-\alpha}\frac{H_{Yt}}{K_t} = \frac{r_{Kt}}{q_{Yt}} \tag{15}$$

$$\frac{\alpha}{\sigma}\frac{A_t}{K_t} = \frac{r_{Kt}}{v_{Yt}} \tag{16}$$

$$\frac{1-\alpha}{\sigma}\frac{A_t}{H_{Yt}} = \frac{q_{Yt}}{v_{Yt}} \tag{17}$$

The method for the R&D firms is the same. The zero profit condition for a firm is, $V_t\dot{A}_t^s - q_{At}H_{At}^s - v_{At}^s A_t = 0$. The relationship between the marginal rate of substitution of human capital and knowledge with the prices ratio is given by,

$$\frac{\phi H_{At}^s}{A_t} = \frac{v_{At}^s}{q_{At}} \tag{18}$$

On the other markets the behavior of the agents is identical to the one of the perfect competitive equilibrium except for the government that does not intervene neither to subsidize the willingness to pay of firms to use innovations, nor to remove the duplication effect.

Now, we can characterize the steady-state. Proposition 3 summarizes the results obtained. Values are denoted with a subscript "*ic*" and the proof of the proposition is gathered in Appendix 6.3.

Proposition 3 *An equilibrium balanced growth path with imperfect competition is characterized by constant growth rates identical to those of the perfect competitive equilibrium and constant shares of raw labor allocated to research, to the final sector and to education,*

$$\left(\frac{L_{At}}{N_t}\right)^{ic} = \frac{1}{\psi}\left[(\varepsilon-1)(1+\phi) + \frac{(\rho-n)(1+\phi)(1-\phi)(1-\alpha)}{\lambda\sigma n} + \frac{(1-\alpha)+\theta\sigma}{\sigma}\right]^{-1}$$

$$\left(\frac{L_{Yt}}{N_t}\right)^{ic} = \frac{1}{\psi} - \theta\left(\frac{L_{At}}{N_t}\right)^{ic}$$

$$\left(\frac{L_{Ht}}{N_t}\right)^{ic} = 1 - \left(\frac{L_{Yt}}{N_t}\right)^{ic} - \left(\frac{L_{At}}{N_t}\right)^{ic}$$

In addition the prices are given by, $q^{ic}_{Yt} = (1 -$

could be used to study different types of problems in a simpler way. For instance, it could allow to study sustainable development, interaction between education and research, agency problems, etc...

6 Appendix

6.1 Characterization of the equilibrium with perfect competition

We begin by solving the household's problem. At each time, he maximizes (7) subject to the budget constraint given by $\dot{a}_t = r_t a_t + q_{Yt} h_t L_{Yt} + q_{At} (h_t)^{\theta} L_{At} - N_t c_t - T_t$, the education process $h_t = e^{\psi L_{Ht}/N_t}$, and the aggregate labor constraint $N_t = L_{Yt} + L_{At} + L_{Ht}$. In this program a_t represents the stock of wealth of the household and r_t is the rate of return of his portfolio: the household distributes assets between physical capital and patents investment. Thus, his stock of wealth is equal to the sum of the physical capital stock and the value of patents he possesses, $a_t = K_t + A_t V_t$. Since both type of investment must have the same return, we can write $r_t = r_{Kt} - d = r_{At}$. Then, writing the Hamiltonian of the household's problem, and solving it, we find easily equations (13) and (14) given in the text.

Given the behavior of the household, we are now able to characterize the decentralized equilibrium path. As mentioned in the text, the rate of tax must be constant at the steady-state. Thus we can write $(1 + \dot{\tau}_t) = 0$. Then, equations (12) and (13) yield $\varepsilon g_c + \rho = v_t/V_t + \dot{V}_t/V_t$. Using (10) and (11) we obtain $\dot{V}_t/V_t = g_Y - g_A + g_{L_A} - g_{L_Y}$. Using (9), (10), (11) and the willingness to pay of agents to use an innovation , we obtain after computations $v_t/V_t = g_A \sigma L_{Yt}/\left\{L_{At}\left[(1+\tau)(1-\alpha)\right]\right\} + g_A \phi$. We deduce that L_{Yt}/L_{At} is constant and it follows that $g_{L_Y} = g_{L_A} = g_{L_H} = n$. Thus, we can deduce the growth rates of the variables in the economy. After some computations we get the equilibrium value of (L_{At}/N_t) as a function of the rate of tax τ and of the parameters of the model. The others shares of labor are easily deduced.

6.2 Characterization of the optimal path

The social planner's problem is to maximize (7) subject to (1) to (5) and (8). After substitutions, the Hamiltonian is given by,

$$\begin{aligned} \Gamma &= N_t \frac{(c_t)^{1-\varepsilon} - 1}{1-\varepsilon} e^{-\rho t} + \mu_t \left((A_t)^{\sigma} (K_t)^{\alpha} \left(L_{Yt} e^{\psi L_{Ht}/N_t} \right)^{1-\alpha} - N_t c_t - d K_t \right) \\ &\quad + \nu_t \delta \left(L_{At} e^{\theta \psi L_{Ht}/N_t} \right)^{\lambda} (A_t)^{\phi} + \xi_t \left(N_t - L_{Yt} - L_{At} - L_{Ht} \right) \end{aligned}$$

The first order conditions are $(c_t)^{-\varepsilon} e^{-\rho t} = \mu_t$ (a), $\mu (1-\alpha) Y_t / L_{Yt} = \xi_t$ (b), $\nu_t \lambda \dot{A}_t / L_{At} = \xi_t$ (c), $\mu_t \psi (1-\alpha) Y_t / N_t + \nu_t \theta \psi \lambda \dot{A}_t / N_t = \xi_t$ (d), $\alpha Y_t / K_t - d = -\dot{\mu}_t / \mu_t$ (e), $\mu_t \sigma Y_t / A_t + \nu_t \phi \dot{A}_t / A_t = -\dot{\nu}_t$ (f).

Differentiating (a) yields $\varepsilon g_c + \rho = -\dot{\mu}_t / \mu_t$ and combining equations (b), (c), (e) yields $L_{Yt}/N_t = 1/\psi - \theta L_{At}/N_t$ which is identical to (14). Equations (b), (c), (f) yields $g_A \left(\sigma \lambda L_{Yt} / \left[(1-\alpha) L_{At}\right] + \phi \right) = -\dot{\nu}_t / \nu_t$. Then, equations (b), (c) yields $-g_{\mu} = -g_{\nu} + g_Y - g_A - g_{L_Y} + g_{L_A}$. Therefore, after some additional algebra one can find, $\varepsilon g_c + \rho = g_Y - g_A - g_{L_Y} + g_{L_A} + g_A \sigma \lambda L_{Yt} / [L_{At} (1-\alpha)] + g_A \phi$. We deduce that the ratio L_{Yt}/L_{At} is constant at the steady-state. As in the preceding sub-section, we deduce the value of the growth rates of the variables. Then, we compute the optimal share of labor allocated to research. The shares of labor allocated to the final sector and to the accumulation of human capital are easily deduced.

6.3 Characterization of the equilibrium with imperfect competition

We use the same method than in the perfectly competitive equilibrium. After some computations we obtain, $\dot{V}_t/V_t = g_Y - g_A$ and $v_t/V_t = g_A (\sigma L_{Yt} / [L_{At} (1-\alpha)] + \phi) / (1+\phi)$. Some additional algebra yields the results given in Proposition 3. The prices are computed by combining the zero profit condition with (15), (16), (17) for the output sector and with (18) for the R&D sector.

6.4 Exogenous rate of savings and decentralized schools

In this section we characterize an imperfect competition equilibrium in which the rate of savings is exogenous like in the original paper of Jones (2002). In that case, equation (2) becomes $\dot{K}_t = s_K Y_t - dK_t$ (2') where s_K is the exogenous rate of savings. In addition, we propose a way to decentralize the educational process.[4]

Concerning the human capital, let us consider three types of schools. The first, that we call the individuals' school, produces people's level of skill, h_t through the technology (6). The two others, namely respectively the final sector school and the R&D sector school, produce the available amount of human capital used in the final sector, (H_{Yt}), and in the R&D sector, (H_{At}) with technologies (4) and (5). Notice that these technologies display constant returns to scale with respect to raw labor and increasing returns with both raw labor and individual's human capital. Since h_t is used simultaneously in the two last schools it can be considered as a non-rival good. Thus, we may apply the methodologies described in Sections 3 and 4 to the schools of the final sector and of the R&D sector. In this part, we follow the methodology used in section 4; that is to say, we restrict our attention to imperfect competition on human capital's markets[5]

To deal with the technology of h_t given by (6), we assume that there is free entry so that the profit of an individual's school equals zero. We denote respectively by w_t, z_{Yt} and z_{At} the level of wage of raw labor and the prices paid by the final sector school and the R&D sector school to use h_t. Other notations are the same. Since the equilibrium conditions of the final sector and of the R&D sector are given in section 4, here, we just describe the ones of the educational sector.

For the school producing H_{Yt}, we write $\Pi_{H_{Yt}} = q_{Yt} h_t L_{Yt} - w_t L_{Yt} - z_{Yt} h_t = 0$. Then, equating the marginal rate of substitution between raw labor and individual's human capital with the price ratio

[4] Notice that the methodology used to decentralize the human capital production process can also be applied to the case in which the rate of savings is endogenous.

[5] One could assume that schools behave on a perfect competitive market. In that case, the governement must intervene to subsidy the willingness to pay of schools to use h_t.

yields $L_{Yt}/h_t = z_{Yt}/w_t$. For the school producing H_{At}, we write $\Pi_{H_{At}} = q_{At}(h_t)^{\theta} L_{At} - w_t L_{At} - z_{At} h_t = 0$. Then, equating the marginal rate of substitution between raw labor and individual's human capital with the price ratio yields, $\theta L_{At}/h_t = z_{At}/w_t$. Concerning the school producing h_t, we write simply $\Pi_{ht} = (z_{Yt} + z_{At}) h_t - w_t L_{Ht} = 0$.

Given the equilibrium conditions for schools we can compute the following prices: $w_t = q_{Yt} h_t / 2 = q_{At}(h_t)^{\theta} / (1+\theta)$, $z_{Yt} = q_{Yt} L_{Yt}/2$, $z_{At} = \theta q_{At} (h_t)^{\theta-1} L_{At} / (1+\theta)$. Since, the equilibrium conditions for the final sector and for the R&D sector are identical to those of section 4, we can write: $q_{Yt} = (1-\alpha) Y_t / [(1+\sigma) H_{Yt}]$, $r_{Kt} = \alpha Y_t / [(1+\sigma) K_t]$, $v_{Yt} = \sigma Y_t / [(1+\sigma) A_t]$, $v_{At} = V_t g_A \phi / (1+\phi)$, $q_{At} = V_t A_t g_A / [(1+\phi) H_{At}]$, $V_t = q_{At} (H_{At})^{1-\lambda} / (A_t)^{\phi}$, $r_{At} = \dot{V}_t / V_t + v_t / V_t$. Then, the arbitrage condition for investors imply that $r_{At} = r_{Kt} - d$. From (2') we obtain $g_K = s_K Y_t / K_t - d$. Therefore we can compute that $r_{Kt} = \alpha (g_K + d) / [s_K (1+\sigma)]$.

Now, we have to compute the growth rates and the allocation of labor between the different sectors. For the growth rates, we get immediately that they are identical to those given in the text. For the allocation of labor, we combine the equilibrium conditions of schools and we use the value of prices defined above. After computations, one can obtain $L_{Yt} + \theta L_{At} = L_{Ht}$ which is similar to (14). This equation, with (8) and the value r_{At} define a system of three equations with three unknowns that allow to determine the shares of labor allocated to the different sector.

7 References

Aghion, P., Howitt, P., 1992. A model of growth through creative destruction, Econometrica 60, 323-351.

Aghion, P., Howitt, P., 1998. Endogenous Growth Theory, Cambridge, MA: MIT Press.

Arrow, K.J. 1962. Economic Welfare and the Allocation of Resources for Inventions, in Richard R. Nelson ed. *The Rate and Direction of Inventive Activity,* Princeton University Press and NBER.

Barro, R., J., Sala-I-Martin, X., 1995. Economic Growth. McGraw-Hill, New-York.

Dasgupta, P., Mäler, K.G., Navaretti, G.B., Siniscalco, D., 1996. On Institutions that Produce and Disseminate Knowledge, Mimeo.

Feehan, J.P., 1989, Pareto-efficency with Three Varieties of Public Inputs, Public Finance, 2, 237-248.

Grossman, G.M., Helpman, E., 1991a. Quality ladders in the theory of growth, Review of economic studies, 58, 557-586.

Grossman, G.M., Helpman, E., 1991b. Innovation and Growth in the Global Economy, Cambridge, MA: MIT Press.

Jones, C. I., 2001., Population and Ideas: A Theory of Endogenous Growth, Working Paper U.C., Berkeley and NBER.

Jones, C. I., 2002., Source of U.S. Economic Growth in a World of Ideas, American Economic Review, 92, 220-239.

Jones, C., I., 2003. Growth and ideas, Working paper, U.C. Berkeley and NBER.

Kaizuka, K., 1965. Public Goods and Decentralization of Production, Review of Economics and Statistics, 47, 118-120.

Lerner, J., Tirole, J., 2002. Efficient Patent Pools, Working paper, Harvard University and NBER, and IDEI.

Manning, R., Markusen, J.R., Mc Millan, J., 1985, Paying for Public Inputs,American Economic Review, 75, 235-238.

Romer, P., 1990. Endogenous Technological Change, Journal of Political Economy, 98, S71-S102.

Sandmo, A., 1972, Optimality Rules for the Provision of Collective Factors of Production, Journal of Public Economics, 1, 149-157.

Scotchmer, S., 1991. Standing on the Schoulders of Giants: Cumulative Research and the Patent Law, Journal of Economic Perspective, Symposium on Intellectual Property Rights.

Scotchmer, S., 1999. Cumulative Innovations in Theory and Practice, Working paper, U.C., Berkeley.

Shy, O., 1995, Industrial Organization: Theory and Applications, Cambridge, MA: MIT Press.