Funding Research and Educating People in a Growth Model with Increasing Population *

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Abstract

In this paper we specify an interdependence between a R&D sector that produces knowledge and a human capital production process that educates individuals. We treat the question of research funding in a model without including any intermediate goods production sector. We assume that the users of discoveries reward directly innovators that own a patent on their innovations. We construct two equilibria. The first one is a benchmark: there is perfect competition on private goods markets and because of the non-convexity of technologies using a public good (knowledge) as a productive factor, the payments of innovations are subsidized by the government. In the second one, we assume imperfect competition and firms that use innovations are not subsidized.

Keywords: Knowledge, Human Capital, Non-convexity, Imperfect Competition.

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1 Introduction

The modern growth literature considers human capital accumulation and research and development (R&D) activity as two important engines of economic growth. Lucas (1988) shows how accumulation of skills may explain long-term economic growth; the R&D-based growth literature inspired by Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) points out the importance of knowledge or accumulation of ideas in the development of the economy.[1] In an interesting study on the source of the economic growth in United-States (US), Jones (2002) confirms these views. He explains that 80 percent of US economic growth is due to increases in human capital investment rates and research intensity while population growth accounts only for 20 percent.

An important point about human capital is that it appears as a key ingredient to make research. For instance, Nelson and Phelps (1966) explain that education facilitates adoption and implementation of new technologies. In his seminal paper, Romer (1990) distinguishes clearly unskilled labor from human capital and he insists on the key role of educated people to produce innovations. The analysis of Benhabib and Spiegel (1994), based on the framework of Nelson and Phelps, is an empirical support to this view. They explain that differences in growth rates between two economies are essentially due to the gap in the available stocks of human capital in these economies but not to the difference between the accumulation rates of human capital, as it was suggested by Lucas (1988). Other empirical studies like Bartel and Lichtenberg (1987), Goldin and Katz (1998) show a large degree of complementarity between the technical progress and human capital. They find that a greater rate of technological progress increases the demand of skilled workers relatively to unskilled ones. This observation deserves some attention to the study of the interaction between the production processes of human capital and of knowledge.

In this paper, we consider an economy in which knowledge is produced using human capital (as suggested above), but also in which human capital is itself produced using the knowledge developed in the economy. That is to say, we specify an interdependence between research and education activities. Moreover, the decisions to acquire skills and to innovate are endogenous. The main reason to specify an interdependence between the two production processes comes from the difficulty to dissociate the way to acquire skills from the way to produce knowledge. If skilled workers benefit from a comparative advantage, relatively to unskilled workers, to produce new technologies, it is at first because they have learned the knowledge developed by scientists

[1] Observe that the second wave of endogenous growth theory started with Judd (1985).

or engineers. For example, we benefit today from the theory of Einstein on relativity. Since the physician is dead, in order to improve his theory or to find a new one, it is necessary to produce at first new physicians. Before to innovate, the future scientist must at first learn the theories developed by Einstein (and others) to know what the state of knowledge is. Once the theory is embodied in their brains (i.e. known), they may try and find how to improve it.

Our analysis can be related to a large literature developed for instance by Ziesemer (1990, 1991), Eicher (1996), Redding (1996), Arnold (1998), Blackburn, Hung and Pozzolo (2000), Funke and Strulik (2000), Jones (1996, 2002). Ziesemer (1990, 1991) uses models in which public factors (knowledge) are a key ingredient of the human capital formation. However, in his 1990 article, he assumes that knowledge is provided by the government i.e. there not a specific R&D sector. In the 1991 paper, he considers a model with endogenous technological progress, but he assumes the simultaneous contribution of the total quantity of human capital to the production of the output and to the production of new units of knowledge. In contrast, in our paper, the available quantity of human capital is endogenously shared between two sectors: one part is used to innovate, the other one is devoted to the production of the consumption good.

Eicher (1996) uses an overlapping generation model in which the production of knowledge is a by-product of education. Human capital is induced by requirement of absorbing new type of technology, whereas new technologies needs human capital to be produced. In this context, the interaction between human capital and knowledge both promote growth. However, the assumption that innovations are by-product of education suppresses the incentives for research activities.

Arnold (1998), Blackburn, Hung and Pozzolo (2000), Funke and Strulik (2000) extend the analysis of Romer (1990) whereas Redding (1996) extends the one of Aghion and Howitt (1992). These authors consider models in which both the decisions to acquire skills and to innovate are endogenous. However, they do not account for any interaction between the knowledge and the human capital production processes as it is done here. They analyze economies in which innovations take place through a R&D activity using human capital and eventually the existing stock of knowledge. But individuals increase their level of skill through a human capital accumulation process as in the model of Lucas (1988). That is to say, human capital is a necessary input to produce educated individuals but knowledge is not a production factor of the education process. A common result of these authors is that human capital is essential to sustain long-term growth but other knowledge is not. Indeed, long-term growth is possible even if knowledge is not produced.

Jones (1996) uses a similar framework to the previous ones, but he allows for a possible interaction between the production of knowledge and the way to acquire skills. The author writes: "the linearity of the human capital equation generates endogenous growth as in Lucas (1988). However, the linearity of the human capital accumulation equation is then somewhat arbitrary, and the endogenous growth arises from human capital accumulation, not from research" (see pp. 10). The linearity of the production process of human capital is precisely the one studied by Redding (1996), Arnold (1998), Blackburn, Hung and Pozzolo (2000), Funke and Strulik (2000). Finally, Jones does not characterize neither the optimal allocation nor the decentralized equilibrium one. He just makes an empirical analysis of the model.

Jones (2002) uses a model in which human capital is an input of the research activity but in which knowledge is not necessary to educate people. Moreover, due to the chosen specifications human capital does not appear as a key ingredient to sustain long-run per-capita growth.

Using the production technology for human capital developed by Ziesemer (1990, 1991), our approach can be seen as an extension of Jones (2002) for two reasons: first, population growth is a necessary condition to sustain per-capita long-term economic growth;[2] second like Jones (2002), we use a model without any intermediate goods production sector. This approach simplifies greatly the analysis but at the time it asks the question of the funding of discoveries. The author writes about his framework: "This can be viewed as a precursor to the richer analysis that comes from adding markets to the model and analyzing equilibrium conditions as well as technologies" (p. 223). Then, he suggests to characterize an equilibrium since he has just performed an empirical analysis of his model. That is one of the main purposes of the present paper.

Because of the public good nature of innovations, several problems arise. The first ones are standard in economics literature: they are relative to the possibility to verify which agent uses a discovery; they are linked to the possibility to exclude any agent that does not pay to access to an innovation; they concern the problems of information about the willingness to pay of agents to use an innovation.

The second type of problem comes from the non convexity of technologies using ideas as productive factors. On this point, the replication argument states that there are constant returns to scale with respect to private inputs and increasing returns to scale with respect to both private and public inputs. As in a competitive market the payment of private factors fully

[2] Jones (1995a,), Kortum (1997), Segerstrom (1998), Howitt (1999) share also this property. We will discuss about it later.

exhaust revenue, firms are unable to pay for the public good they use.[3] Thus, an equilibrium with perfect competition on private goods markets does not exist if firms that pay to access to innovations are not subsidized; this induces that imperfect competition is a necessary condition to have an equilibrium in which firms, that pay to use innovations, are not subsidized.

We make the following assumptions: innovators can verify if a firm uses an innovation; they can exclude it, if it does not pay to use the discovery; moreover, they have the information about the willingness to pay to use the innovation. In other words, verifiability, excludability and information are not problems in our model. On the other hand, we concentrate on the problem raised by the non-convexity of technologies using knowledge as an input. In the standard R&D-based literature, the assumption of perfect competition on all markets (except the one of intermediate goods) is sustainable because the models induce equilibria with incomplete markets. The goods sold to firms are intermediate goods which are private and divisible, embodying ideas. However, innovations which are public, indivisible and durable goods are not priced.

In our economy, as there is not intermediate goods production sector, we must find an other way to fund research. Jones (2003) proposes one particular type of decentralized equilibrium in such a framework. However, he does not define any market nor any price for innovations. The consequence of this drawback is that we do not know the price of an innovation in the decentralized economy.[4]

Our approach is different and can be seen as a formalization of ideas already expressed by economists like for instance Arrow (1962), Scotchmer (1991), Dasgupta et-al (1996). We construct two equilibria with complete markets. That is to say, we define a market and we compute the relevant prices for each good produced in the economy, and in particular for innovations. In both equilibria, we assume that innovations are protected by intellectual property rights and that innovators rent the discoveries they have produced to any potential user.

In the first equilibrium that we call the benchmark, we maintain the perfectly competitive assumption on private goods markets. To avoid the problem of potential negative profits explained above, firms that use knowledge as a productive factor are subsidized by the government. In that case, we show how to implement the optimum. In the second equilibrium, the government does not intervene and knowledge is privately funded. To deal with the non-convexity of

[3] See for instance Manning et al (1985), Feehan (1989), Romer (1990), Jones (2003) for more details on this point.

[4] An other drawback is that the shares of labor allocated to the different sectors of the economy are exogenously given. In contrast, in the present analysis, they are endogenous. We compute their exact values as a functions of the parameters of the model.

production processes, we assume imperfect competition on markets using knowledge.

The remainder of the paper is organized as follows: Section 2 describes the model; Sections 3 characterizes the benchmark equilibrium; we account for an imperfect competitive equilibrium in Section 4, and Section 5 concludes. Computations are provided in the Appendix, in Section 6.

2 The model

We consider a model in continuous time. The members or individuals of a representative household own unskilled labor (L_t) as an initial endowment and there are three production sectors. An output sector produces a consumption good, Y_t. A human capital production sector educates people producing human capital, H_t. Throughout the paper, human capital is interpreted as the general level of skill of individuals. And a R&D sector produces ideas or innovations whose total stock, N_t, is interpreted as the available knowledge developed in the economy. The technologies and the preferences of the agents are described in the following sub-sections.

2.1 Consumption good

The production function for the consumption good is given by

$$Y_t = A\left(N_t\right)^{\beta}\left(L_{Yt}\right)^{1-\alpha}\left(H_{Yt}\right)^{\alpha}, \tag{1}$$

where $A > 0$, $\alpha \in (0, 1)$, and $\beta > 0$ are constant parameters; H_{Yt} and L_{Yt} are respectively the quantity of skilled and unskilled labor employed; N_t is the available range of ideas developed in the economy.[5]

The production function (1) displays constant returns to scale with respect to private inputs and increasing returns with respect to all productive factors. Thus, for a constant stock of knowledge N_t, if the quantities of skilled and unskilled labor are doubled the quantity of output produced is doubled. The replication argument applies because ideas are non-rival goods: for each scale of production private inputs are still combined with the same amount of knowledge.

[5] It is possible to incorporate physical capital in the production process of the final good. In that case the technology would be given by $Y_t = A\left(N_t\right)^{\beta}\left(K_t\right)^{\gamma}\left(H_{Yt}\right)^{\alpha}\left(L_{Yt}\right)^{1-\gamma-\alpha}$. Nevertheless the main results remain unchanged despite this modification.

2.2 Human capital production process

The representative school or university uses unskilled labor, L_{Ht}, and all the stock of knowledge, N_t, to produce skilled labor, H_t through the technology given by

$$H_t = \phi L_{Ht} (N_t)^{\psi}, \tag{2}$$

where $\phi > 0$, and $\psi > 0$ are constant parameters.

The technology (2) displays constant returns to scale with respect to unskilled labor and increasing returns with respect to both unskilled labor and knowledge. As in the case of the output sector, the replication argument applies because knowledge is a public good: doubling the quantity of unskilled labor allocated to the educational sector doubles the quantity of human capital produced in the economy.

The technology (2) is similar to the one used by Ziesemer (1990, 1991). The educative activity (2) is the mean of transmission of knowledge across time. Indeed, it allows to transform non educated people into skilled ones. Each time, the level of skills of individuals is updated by latest discoveries. Once knowledge is embodied in people's brains, they can try to innovate (see equation (3)). Following Ziesemer (1990, 1991), human capital is treated as a flow to capture the fact that skills are more difficult to make over than knowledge. Indeed, knowledge is a durable good which can be inherited for example in written forms while the level of skill of an individual is lost if he retires or dies. Then, complete depreciation of human capital is the simplest way to account for this feature.

Our last remark concerns the parameter ψ which plays an important role in the specification (2): we assume that a strictly positive value of ψ is required to educate people. Indeed, let us consider the special case $\psi = 0$ and $\phi = 1$: the educational sector transforms one unit of unskilled labor into one unit of human capital, i.e. it simply relabels raw labor.[6] In this case, knowledge is not learned, which is not in accordance with the definition of human capital. Indeed, the human capital of a person is composed of two distinct parts. The first one is the time or the quantity of unskilled labor owned by the individual. By its nature, this component has a finite value. The second one is the knowledge learned by the individual and which makes him skilled. Then, when an individual supplies one unit of human capital, he supplies simultaneously raw labor and knowledge.[7] This is the knowledge embodied in the brain of the scientist, which makes him

[6] We thank one referee for this remark.

[7] Note that this is the reason why there is not any problem of appropriability with this good. In contrast with knowledge, this is a private good. When a scientist works on a task, he cannot simultaneously work on an other

skilled and which gives him the comparative advantage to innovate compared to an unskilled worker.

2.3 Development of ideas

The R&D sector is composed of J firms. The number of discoveries achieved per unit of time by the R&D firm j $(j = 1, ..., J)$, is given by

$$\overset{\bullet}{N}_{jt} = \xi_t \left(H_{jt}\right) \left(N_t\right)^{\gamma}, \tag{3}$$

where $\gamma < 1$, $\overset{\bullet}{N}_{jt}$ is the arrival's rate of ideas per unit of time and H_{jt} is the quantity of human capital used to discover new ideas. The parameter ξ_t is taken as given by each firm. It measures external effects in research and verifies $\xi_t = \delta \left(H_{Rt}\right)^{\theta-1}$; $\delta > 0$ is a constant and exogenous technological parameter; $H_{Rt} = \sum_{j=1}^{J} H_{jt}$ is the total quantity of human capital devoted to research, and we assume $\theta > 0$.[8] One can note that if $\theta = 1$, there is not any external effect in research.

Summing over j, we see from (3) that the aggregate production function of knowledge in the economy is given by

$$\overset{\bullet}{N}_t = \delta \left(H_{Rt}\right)^{\theta} \left(N_t\right)^{\gamma}. \tag{4}$$

According to the technologies (3) and (4) human capital is necessary to produce ideas but unskilled labor is not. This specification is made to capture the fact that only educated individuals (scientists and researchers) can innovate.

The stock of knowledge, N_t, is a continuum of ideas, i, which take, for example, the form of scientific reports. It can be the medical formula of a vaccine, the plan for a new car, ... In the two equilibria we construct, the patented and priced goods are innovations, $i \in [0, N_t]$, which are public, indivisible and durable goods (see later).

one.

[8] Authors like Jones (1995a, b), Arnold (1998), Blackburn, Hung and Pozzolo (2000), Kortum (1993) among others assume that $\theta \in [0, 1]$. Kortum estimates that θ belongs to the set $[0.1, 0.6]$ supporting the assumption of decreasing returns due to duplicative research. However, other authors indicate that the social rate of return to R&D is larger than the private one, indicating positive spillovers. In the model, this induces that $\theta > 1$. We thank one referee for this remark.

2.4 Preferences and endowment

The economy is composed of an infinitely lived representative household whose members are identical. Their number grows over-time at the exogenous rate $g_L > 0$. Each individual is initially endowed with one unit of unskilled labor, so that the total supply at time t is given by $L_t = L_0 e^{g_L t}$, where L_0 represents the initial number of members at date 0. Preferences are represented by the discounted utility function defined by

$$U(c_t) = \int_0^\infty L_t e^{-\rho t} \frac{(c_t)^{1-\varepsilon} - 1}{1-\varepsilon} dt, \tag{5}$$

where c_t is per-capita consumption at time t, $\varepsilon > 0$ is the inverse of the elasticity of substitution and $\rho > 0$ is the rate of time preferences.

At each time, unskilled labor (L_t) is allocated between the human capital production sector (L_{Ht}) in which individuals acquire their skills and the output sector (L_{Yt}). Then, the human capital is devoted to the output sector (H_{Yt}) and to the R&D sector (H_{Rt}). The aggregate constraints for unskilled labor and human capital are

$$L_t = L_{Yt} + L_{Ht} \tag{6}$$

$$\text{and} \quad H_t = H_{Yt} + H_{Rt} \tag{7}$$

Remark: one can verify that R&D, human capital and population growth are three necessary ingredients for per-capita long-term growth (see Appendix 6.1).

3 Equilibrium: the benchmark

The objective of this section is to construct an equilibrium in which the only distortions with respect to the first best optimum comes from the externalities inside the R&D sector. We recall that if $\theta = 1$, these externalities disappear: then, the benchmark equilibrium is the first best optimum.

In our economy, there are three private goods (the consumption good, Y_t, unskilled labor, L_t, human capital, H_t), and a continuum of public goods, $i \in [0, N_t]$, which are differentiated and symmetric: the marginal productivities of two innovations are equal in a given sector where they are used (see equations (1), (2), (3), (4)).

To construct this equilibrium, we assume that the markets of private goods are perfectly competitive. In order to obtain an optimal production of innovations, we make the following two assumptions: first, innovators protect the discoveries they produce by an infinitely lived intellectual property right (a patent); second, they are able to extract the willingness to pay from each agent using innovations they have produced. Note that this practice implies that different users pay different prices for the same good.

We can interpret such equilibrium as a formalization of ideas already expressed in the literature, for instance by Arrow (1962), Scotchmer (1991) and Dasgupta et al (1996). Arrow (1962) and Scotchmer (1991) suggest that any user of a discovery must pay to use it. Arrow (1962) writes: "Suppose, as the result of elaborate tests, some metal is discovered to have a desirable property, say resistance to high heat. Then of course every use of the metal for which this property is relevant would also use this information, and the user would be made to pay for it. But, even more, if another inventor is stimulated to examine chemically related metals for heat resistance, he is using the information already discovered and should pay for it in some measure; and any beneficiary of his discoveries should also pay" (pp. 150). Scotchmer (1991) explains that "a system of property rights that might seem natural would be to protect the first innovator so broadly that licensing is required from all second generation innovators who use the initial technology, whether in research or in production". (pp. 32). Dasgupta et al (1996) propose a strategy to construct such equilibrium. They write: "A possible scheme is for society to grant intellectual property rights to private producers for their discoveries, and permit them to charge (possibly differential) fees for their use by others. This creates private markets for knowledge. Patent and copyright protections are means of enforcing intellectual property rights. It is as well to note here that, in this scheme the producer (or owner) of a piece of information should ideally set different prices for different buyers, because different buyers typically value the information differently. In economics, these variegated prices are called Lindahl prices, in honor of the person who provided the first articulation of this scheme" (pp. 10).[9]

There are two types of limits to this equilibrium. First, there are problems of identification, exclusion and information that we do not treat in the present paper. Second, there are problems of non-convexity. Indeed, the technologies using knowledge as a productive factor, display constant returns to scale with respect to private factors and increasing returns to scale with respect to both private and public inputs. As the markets for private goods are perfectly competitive, their payment completely exhaust firms' revenue. Thus, this induces the necessity of a subsidy

[9] We could consider other types of equilibria. For example, we could assume that each sector (Y, H, R&D) is composed of several identical firms and there are three segmented and competitive markets for discoveries. The prices obtained are also Lindahl prices. This result comes from the property of symmetry of innovations.

(government intervention) to cover the willingnesses to pay of firms that use innovations. We will discuss on the direct and private funding of research in Section 4.

To remove externalities in research, we assume the intervention of the government through an economic policy tool, τ_t, charged on R&D firms. This economic policy, as the subsidies to the willingnesses to pay to use innovations, is funded through a lump-sum tax (or lump-sum transfer), T_t, charged on (or given to) the representative household (see below).

The price of the consumption good is normalized to one ($p_{Yt} = 1$), the wage per unit of unskilled labor, the wage of human capital and the rate of return on R&D investments are respectively noted w_t, q_t and r_t.

We denote by v_{Yt}, v_{Ht} and v_{jt} $(j = 1, ..., J)$ the willingnesses to pay of the consumption good sector, of the human capital sector and of research firms to use an innovation *at time* t. Thus, since there is perfect discrimination, v_{Yt}, v_{Ht} and v_{jt} are the rental prices paid by these sectors to use an innovation at date t. The total gain perceived by an innovator from the renting of an innovation at time t is $v_t = v_{Yt} + v_{Ht} + v_{Rt}$, where $v_{Rt} = \sum_{j=1}^{J} v_{jt}$.

Note that the value of an innovation at date t is $V_t = \int_t^{\infty} v_s.e^{-\int_t^s r_u du} ds$. Differentiating the expression of V_t with respect to time yields, $r_t = v_t/V_t + \dot{V}_t/V_t$. Now we proceed as follows: we give the definition of the benchmark equilibrium in the economy described in Section 2; we describe the behavior of each agent; we characterize the benchmark equilibrium.

Definition 1 *A benchmark equilibrium is a set of temporal profiles of quantities* ($\{Y_t\}$, $\{N_t\}$, $\{L_{Yt}\}$, $\{L_{Ht}\}$, $\{H_{Yt}\}$, $\{H_{jt}\}_{j=1,...,J}$) *and of prices* ($\{w_t\}$, $\{q_t\}$, $\{r_t\}$, $\{v_{Yt}\}$, $\{v_{Ht}\}$, $\{v_{jt}\}_{j=1,...,J}$), *for* $t \in [0, \infty[$, *such that:*

- *firms maximize their profits;*
- *the representative household maximizes its utility;*
- *markets of private goods* (Y, L, H) *clear at each date* t*;*
- v_{Yt}, v_{Ht} *and* v_{jt} *are respectively the willingnesses to pay (rental prices) of the output sector, of the educational sector and of R&D firms to use an innovation.*

3.1 Behavior of agents

a) The output sector chooses the quantities of skilled and unskilled labor that maximize its profit given by $\Pi_{Yt} = Y_t - q_t H_{Yt} - w_t L_{Yt}$, where $Y_t = A (N_t)^{\beta} (L_{Yt})^{1-\alpha} (H_{Yt})^{\alpha}$. Solving this problem yields the following first order conditions,

$$q_t = \alpha Y_t / H_{Yt}, \tag{8}$$

$$w_t = (1-\alpha) Y_t / L_{Yt}. \tag{9}$$

The willingness to pay to use an innovation at time t is

$$v_{Yt} = \partial \Pi_{Yt} / \partial N_t = \beta Y_t / N_t. \tag{10}$$

Remark: it is possible to disaggregate the sector in K firms, $(k = 1, ..., K)$, sharing the same technology $Y_{kt} = A(N_t)^{\beta}(L_{kt})^{1-\alpha}(H_{kt})^{\alpha}$. The profit of firm k is $\Pi_{kt} = Y_{kt} - q_t H_{kt} - w_t L_{kt}$ and the first order conditions are $q_t = \alpha Y_{kt}/H_{kt}$, and $w_t = \alpha Y_{kt}/L_{kt}$.

The willingness to pay of firm k to use an innovation at time t is $v_{kt} = \partial \Pi_{kt}/\partial N_t = \beta Y_{kt}/N_t$. Note that different firms (i.e. firms with different output Y_{kt}) pay different rental prices to use an innovation. However, the key point is that the total willingness to pay of the whole sector is unchanged. Indeed, we have $v_{Yt} = \sum_{k=1}^{K} v_{kt} = \beta Y_t/N_t$ (see equation (10)). This result justifies that we use a representative firm, despite the presence of public goods and increasing returns to scale.

b) The representative school of the educational sector chooses the quantity of unskilled labor that maximizes its profit $\Pi_{Ht} = q_t H_t - w_t L_{Ht}$, where $H_t = \phi L_{Ht}(N_t)^{\psi}$. The first order condition of this program is

$$w_t = q_t H_t / L_{Ht}. \tag{11}$$

The willingness to pay to use an innovation at time t is

$$v_{Ht} = \partial \Pi_{Ht} / \partial N_t = \psi q_t H_t / N_t. \tag{12}$$

Remark: as before, we could disaggregate this sector; the result would be the same.

c) The firm j maximizes the sum of the present values of its expected profits given by $\int_0^{\infty} [v_t N_{jt} - (1+\tau_t) q_t H_{jt}] e^{-\int_0^t r_u du} dt$, subject to the technology (3). Associating the costate variable ν_t to the law of motion of ideas in the firm j, the Hamiltonian of this problem is

$$\Gamma = [v_t N_{jt} - (1+\tau_t) q_t H_{jt}] e^{-\int_0^t r_u du} + \nu_t \delta (H_{Rt})^{\theta-1} H_{jt} (N_t)^{\gamma}$$

The first order conditions are: $\partial \Gamma / \partial H_{jt} = \nu_t \delta (H_{Rt})^{\theta-1} (N_t)^{\gamma} - (1+\tau_t) q_t e^{-\int_0^t r_u du} = 0$ (a), and $\partial \Gamma / \partial N_{jt} = -\dot{\nu}_t = v_t . e^{-\int_0^t r_u du}$ (b). Integrating (b) between t and infinity, we obtain

$\int_t^\infty -\overset{\bullet}{\nu}_s ds = \int_t^\infty v_s.e^{-\int_0^s r_u du} ds = \left(e^{-\int_0^t r_u du}\right).\int_t^\infty v_s.e^{-\int_t^s r_u du} ds$. Note that the transversality condition $\lim\limits_{t\to\infty} \nu_t N_{jt} = 0$ implies $\nu_\infty = 0$ because $N_{jt} > 0$ for all t. Therefore, we have $\nu_t = V_t.e^{-\int_0^t r_u du}$, where $V_t = \int_t^\infty v_s.e^{-\int_t^s r_u du} ds$ (see above). Using the expression of ν_t, (a) becomes $V_t \delta (H_{Rt})^{\theta-1} (N_t)^\gamma = (1+\tau_t) q_t$. Multiplying both sides by H_{Rt} and using (4), we get

$$\overset{\bullet}{N}_t V_t = (1+\tau_t) q_t H_{Rt}. \tag{13}$$

The willingness to pay at time 0 to use an innovation at time t is $\partial\Gamma/\partial N_t = \nu_t \gamma\delta (H_{Rt})^{\theta-1} H_{jt} (N_t)^{\gamma-1}$. Multiplying both sides by $e^{\int_0^t r_u du}$, one gets the willingness to pay at time t for an innovation used at t, that is to say $v_{jt} = V_t \gamma\delta (H_{Rt})^{\theta-1} H_{jt} (N_t)^{\gamma-1}$. Thus, the willingness to pay at time t to use an innovation at t by all firms of the R&D sector is

$$v_{Rt} = \sum_{j=1}^J v_{jt} = V_t \gamma\delta (H_{Rt})^\theta (N_t)^{\gamma-1}. \tag{14}$$

d) We assume that the budget constraint of the government is balanced at each time; it is given by

$$T_t = v_t N_t - \tau_t q_t H_{Rt}. \tag{15}$$

$v_t N_t$ is the total subsidy given to sectors that rent innovations at date t, and $\tau_t q_t H_{Rt}$ represents the tax charged (or the subsidy given) to research firms.

e) The representative household maximizes (5) subject to the budget constraint $\overset{\bullet}{b}_t = r_t b_t + L_t w_t - L_t c_t - T_t$, where $b_t = N_t V_t$.[10] Solving this program, we obtain the standard Keynes-Ramsey rule,

$$\varepsilon g_c + \rho = r_t. \tag{16}$$

3.2 Characterization of the benchmark equilibrium

Here we study the steady-state equilibrium paths and we analyze the effects of variations of the value of the economic policy tool on the equilibrium quantities of labor allocated to the different sectors. Since we focus only on a balanced growth path, the time subscript can be omitted in the policy tool τ, which must be constant in this case.

[10] In fact, this constraint is the reduced form of $\overset{\bullet}{b}_t = r_t b_t + w_t L_{Yt} + q_t H_t - (q_t H_t - w_t L_{Ht}) - L_t c_t - T_t$. In this expression, the household sells his raw labor, L_{Ht}, to the educational sector and buys the human capital, H_t, that he resells to the consumption good and R&D sectors: the first $q_t H_t$ is the revenue from these sectors; $(q_t H_t - w_t L_{Ht})$ is the net payment to the educational sector.

Given the agents' behavior, we can determine the growth rates of the variables in the economy and compute the shares of labor allocated to the different sectors as a function of the rate of tax (τ) charged on R&D firms. Proposition 1 summarizes the results and describes the equilibrium paths we obtain in the economy as a function of τ. Equilibrium values are denoted with a subscript "e". The growth rate of any variable x is noted g_x. The proof of the proposition is provided in Appendix 6.2.

Proposition 1 *The benchmark equilibrium balanced growth path with perfect competition on private goods markets and innovations rewarded at their Lindahl price levels, is characterized by constant growth rates verifying,*

$$\begin{aligned} g_H^e &= \frac{1-\gamma}{1-\gamma-\theta\psi} g_L > g_L, \\ g_N^e &= \frac{\theta}{1-\gamma-\theta\psi} g_L = \frac{\theta}{1-\gamma} g_H^e, \\ g_c^e &= g_Y^e - g_L = \frac{\theta(\beta+\alpha\psi)}{1-\gamma-\theta\psi} g_L = [\beta+\alpha\psi]\, g_N^e, \end{aligned}$$

constant shares of skilled labor allocated to the R&D sector and to the final sector,

$$\left(\frac{H_{Rt}}{H_t}\right)^e = (\beta+\alpha\psi)\left[\alpha(\varepsilon-1)(1+\tau_t)\frac{g_c}{g_N} + \frac{\alpha(\rho-g_L)(1+\tau_t)}{g_N} + \alpha\theta(1+\tau_t)\frac{g_H}{g_N} + \beta\right]^{-1},$$

$$\left(\frac{H_{Yt}}{H_t}\right)^e = 1 - \left(\frac{H_{Rt}}{H_t}\right)^e,$$

and constant shares of unskilled labor allocated to the educational sector and to the final sector,

$$\left(\frac{L_t}{L_{Ht}}\right)^e = \frac{1}{\alpha} - \frac{(1-\alpha)}{\alpha}\left(\frac{H_{Rt}}{H_t}\right)^e,$$

$$\left(\frac{L_{Yt}}{L_t}\right)^e = 1 - \left(\frac{L_{Ht}}{L_t}\right)^e.$$

The prices are given by: $q_t^e = \alpha Y_t/H_{Yt}$*;* $w_t^e = (1-\alpha)Y_t/L_{Yt}$*;* $r_t^e = \varepsilon g_c^e + \rho$*;* $v_{Yt}^e = \beta Y_t/N_t$*;* $v_{Ht}^e = \psi q_t H_t/N_t$*;* $v_{Rt}^e = \gamma g_N^e V_t^e$*, where* $V_t^e = \int_t^\infty v_s^e . e^{-\int_t^s r_u^e du} ds$*. Finally, the following equality must be satisfied:* $(w_t/q_t)^e = (1-\alpha) H_{Yt}/L_{Yt} = \alpha H_t/L_{Ht}$*.*

First of all, note that from Proposition 1 it is possible to compute the growth rates of prices. Moreover, the necessary condition for the existence of a balanced growth path is $1-\gamma-\theta\psi < 1$. Note that the term γ measures the degree of the knowledge spill-over effect in research; the term $\theta\psi$ reflects the interaction between knowledge and human capital. As discussed previously, human capital acts as a mean to transmit knowledge over-time. On the one hand, when a new innovation is produced, it helps researchers to produce new ones (see equation (3) and (4)). On the other hand, as N_t grows researchers become more productive because the quantity of knowledge embodied in their brain is higher (see equation (2)).

The condition $1 - \gamma - \theta\psi < 1$ means simply that the global marginal productivity of knowledge in research is decreasing. Indeed, the term $\gamma + \theta\psi$ of our framework corresponds to the parameter ϕ of the model of Jones (1995a, b) in which the aggregate production of ideas is given by, $\dot{N}_t = \delta (L_{Rt})^{\lambda} (N_t)^{\phi}$: L_{Rt} is the quantity of raw labor allocated to research; $\lambda \in]0, 1]$ measures the external effect in research; $\phi < 1$ allows past discoveries to either increase ($\phi > 0$) or decrease ($\phi < 0$) current research productivity. With our specification incorporating education of individuals, one can compute that the elasticity of production of new ideas with respect to the stock of knowledge is equal to $\gamma + \theta\psi$, while it is equal to ϕ in the paper of Jones.

The growth rate of population appears as a necessary condition to sustain a strictly positive per-capita long-run growth rate. This result may appear puzzling. The growth rate of the output per-capita in countries that have a high fertility rate is not necessarily higher than the one of the countries in which the growth rate of population is lower. Nevertheless, as pointed out by Jones (1995a), if we consider growth in the world economy, this result is plausible. In that case, the stock of knowledge must be interpreted as a stock of ideas that can be used everywhere around the world. Following Jones, we must think to this type of model as relating to the growth in the effective number of researchers, which does not necessarily requires population growth as whole.

Proposition 1 states the equilibrium is characterized by constant shares of skilled and unskilled labor allocated to the different sectors. Since population grows at exogenous rate $g_L > 0$, this means that skilled workers and thus the number of scientists in R&D grow at rate g_L. The main difference with Jones (1995a) is that individuals must be educated to conduct research projects. Each time, they increase their productivity. Indeed, through the educational process (2), the total level of skills in the economy grows over-time according to $g_H = g_L + \psi g_N > g_L$. Thus the total quantity of human capital in the economy grows because both the population and the stock of knowledge increase simultaneously. This result differs from Jones' (2002) model in which human capital grows at the same rate than population. In the present framework, for a given amount of knowledge a larger number of people educate themselves. In that sense, human capital increases at first because $g_L > 0$. Then, the level of skills increases because each time individuals update their productivity through learning. In that case, the growth rate of productivity of skilled workers is measured by the term ψg_N. To sum up, the economy we describe is characterized by an increasing number of better skilled scientists who conduct research projects.

Scientists are better skilled because an increasing amount of knowledge is embodied in their brain.

In a seminal paper, Jones (1995a) raises the question of scale effects' prediction of R&D-based models for which the level of the long-run growth rate is proportional to the number of scientists engaged in research: in these studies, a permanent increase of resources allocated to research induces a faster economic growth. Against this property, the author argues that the number of researchers in United-States has grown over-time from 200.000 in 1950 to nearly one million at the end of the 80's. At the same time either the per-capita growth rates remained approximately constant or their average declined. As a consequence, long-term economic growth has not accelerated despite the increasing level of employment in research.

Our model is consistent with these observations: as seen above, in our economy the number of scientists increases over-time; then, one can remark that growth rates are independent from any scale effects. As Jones points out, "scale effects are replaced by an intuitive dependence on the growth rate of the labor force rather than on its level" (see Jones 1995a, p. 768). Finally, one can remark that long-term growth rates depend on parameters considered as invariant to any economic policies. According to Jones (1995b), Arnold (1998), Blackburn, Hung and Pozzolo (2000) for instance, this result seems consistent with empirical regularities since subsidy to research do not seem to induce any long-run impact on growth rates of advanced economies.

Simultaneously to the dramatic increase in the number of scientists engaged in research, one can observe the fairly constant ratio R&D expenditures over gross domestic product, for instance at 2.5% in United-States. Let us show that this empirical fact is taken into account in our model at steady-state. The total expenditures in R&D, ($E_{R\&D}$), is given by $E_{R\&D} = q_t H_{Rt} (1 + \tau_t) + v_{Rt} N_t$. Using equations (13) and (14), one gets $E_{R\&D} = V_t N_t g_N (1 + \gamma)$. At steady-state, differentiating this expression with respect to time, yields $g_{E_{R\&D}} = g_V + g_N = g_Y$. This, implies that $E_{R\&D}/Y_t$ is constant over-time along a balanced growth path.

Now let us study the effects of variations of the tax rate on the equilibrium shares of labor (skilled and unskilled). We compute easily that $\partial (H_{Rt}/H_t)^e / \partial \tau < 0$ and $\partial (L_{Ht}/L_t)^e / \partial \tau < 0$. Therefore, as the value of the tax rate increases, R&D firms reduces the skilled labor's share they employ. Due to the interaction between knowledge and human capital, this behavior induces the decrease of the rate of innovation in the economy. As a consequence, it becomes less interesting for schools to employ unskilled labor, which explains the decline of L_{Ht}/L_t.

3.3 Optimal taxation

The aim of this sub-section is to compute the value of the economic policy tool charged on R&D firms that implements the optimum. Indeed, for each possible value of τ corresponds a particular equilibrium (see Proposition 1), and only one is optimal; this equilibrium is the first best optimum. Before to characterize it, it is necessary to compute the optimal balanced growth path. The problem of a benevolent and informed social planner is to maximize (5) subject to (1) to (4) and (6)-(7).

Solving the above problem, we obtain the optimal growth rates of the variables in the economy (which are identical to the decentralized equilibrium ones) and the optimal shares of labor allocated to the different sectors. The computations are gathered in Appendix 6.3. Then, comparing the results of the optimum and of the benchmark equilibrium we may obtain the proposition 2 given below. To distinguish the case in which the government intervenes to remove externalities in research (first best optimum) from the one in which it does not, we adopt the following notation: the sign "o" is used for optimal values while the sign "$*$" is used for the cases in which externalities remain in the benchmark equilibrium.

Proposition 2 *If the government chooses* $\tau^o = 1/\theta - 1$, *the benchmark equilibrium path is optimal.*

If there is not any externality in research (i.e. if $\theta = 1$), it is not necessary to use an economic policy tool to implement the optimal balanced growth path. The benchmark equilibrium coincides with the optimum; in this case, the value of the policy tool is zero, $\tau^o = 0$. In the other cases, the government must intervene. If $\theta < 1$ (resp. $\theta > 1$), the policy tool is a tax-rate (resp. a subsidy-rate) charged on R&D firms.

If $\tau = 0$ and $\theta < 1$ (resp. $\theta > 1$), one can compute that $(H_{Rt}/H_t)^* - (H_{Rt}/H_t)^o > 0$ (resp. $(H_{Rt}/H_t)^* - (H_{Rt}/H_t)^o < 0$) and $(L_{Ht}/L_t)^* - (L_{Ht}/L_t)^o > 0$ (resp. $(L_{Ht}/L_t)^* - (L_{Ht}/L_t)^o < 0$). The reason is the following: R&D firms do not account for the externalities in research; if $\theta < 1$ (resp. $\theta > 1$), without any intervention of the government, research firms are attempted to hire an excessive (resp. an insufficient) amount of human capital in order to innovate; this induces that the number of innovations produced per unit of time is higher (resp. lower) than in the case of the optimum; because of the interaction between the human capital and the R&D processes, it becomes more (resp. less) interesting for schools to educate individuals.

4 Equilibrium with imperfect competition

The aim of this section is to present an equilibrium in which research is privately funded. In other words, the government does not intervene to subsidy the willingnesses to pay for innovations. As discussed earlier, one difficulty to account for such equilibrium comes from the non-convexity of technologies using knowledge as an input. To deal with this property, we consider an equilibrium in which there is imperfect competition on markets that use ideas as productive factors. In the present paper this situation prevails on the markets of output, human capital and innovations. To construct the equilibrium, we assume that imperfect competition leads to two conditions. First, the profits including both the payment of private factors and the reward of innovators are nil. This condition can be interpreted as a free entry condition on the markets. Second, each firm uses the amounts of inputs so that the marginal rate of substitution between two productive factors equals the ratio of their market prices. This behavior underlines the fact that firms are price takers on the markets of inputs.

To simplify the analysis, we assume that the government does not intervene neither to remove the external effect in research nor to subsidy the willingness to pay of firms to use discoveries. After giving the definition of the equilibrium with imperfect competition, we proceed to its characterization. Notations are identical to the ones used in the previous section.

Definition 2 *An equilibrium with imperfect competition is a set of temporal profiles of quantities* ($\{Y_t\}$, $\{N_t\}$, $\{L_{Yt}\}$, $\{L_{Ht}\}$, $\{H_{Yt}\}$, $\{H_{jt}\}_{j=1,\ldots,J}$) *and of prices* ($\{w_t\}$, $\{q_t\}$, $\{r_t\}$, $\{v_{Yt}\}$, $\{v_{Ht}\}$, $\{v_{jt}\}_{j=1,\ldots,J}$), *for* $t \in [0, \infty[$, *such that:*

- *all firms minimize their costs (i.e. the marginal rates of substitution of two factors are equal to the corresponding prices ratios);*
- *all profits are nil;*
- *the representative household maximizes its utility;*
- *markets of private goods* (Y, L, H) *clear at each date* t*;*
- v_{Yt}, v_{Ht} *and* v_{jt} ($j = 1, ..., J$) *are respectively the rental prices paid by the output sector, the educational sector, and R&D firms to use an innovation.*

a) For the output sector, the program $Min.\ q_t H_{Yt} + w_t L_{Yt} + v_{Yt} N_t$ subject to $Y_t = A(N_t)^{\beta}(L_{Yt})^{1-\alpha}(H_{Yt})^{\alpha}$, yields

$$\begin{aligned}
\frac{(1-\alpha)}{\alpha}\frac{H_{Yt}}{L_{Yt}} &= \frac{w_t}{q_t}, && (17)\\
\frac{(1-\alpha)}{\beta}\frac{N_t}{L_{Yt}} &= \frac{w_t}{v_{Yt}}, && (18)\\
\frac{\alpha}{\beta}\frac{N_t}{H_{Yt}} &= \frac{q_t}{v_{Yt}}. && (19)
\end{aligned}$$

Moreover, we have $\Pi_{Yt} = A(N_t)^{\beta}(L_{Yt})^{1-\alpha}(H_{Yt})^{\alpha} - q_t H_{Yt} - w_t L_{Yt} - v_{Yt} N_t = 0$.

b) For the school of the educational sector, the relationship between the marginal rate of substitution of labor and knowledge with the prices ratio is given by

$$\frac{N_t}{\psi L_{Ht}} = \frac{w_t}{v_{Ht}}. \tag{20}$$

The zero profit condition is: $\Pi_{Ht} = q_t \phi L_{Ht}(N_t)^{\psi} - w_t L_{Ht} - v_{Ht} N_t = 0$.

c) Let us consider the research firm j. At each time t, the minimization of the cost $q_t H_{jt} + v_{jt} N_t$ subject to $\overset{\bullet}{N}_{jt} = \xi_t(H_{jt})(N_t)^{\gamma}$ leads to

$$\frac{N_t}{\gamma H_{jt}} = \frac{q_t}{v_{jt}}. \tag{21}$$

The free entry condition implies that, at each date t, the value of the stock of innovations is equal to the sum of the present values of the expected profits. That is to say, $\int_t^{\infty}[v_s N_{js} - q_s H_{js} - v_{js} N_s].e^{-\int_t^s r_u du}ds - V_t N_{jt} = 0$ for all t. We can give a simpler expression of this condition. Let us consider the integral $I = \int_t^{\infty} V_s \overset{\bullet}{N}_{js} e^{-\int_t^s r_u du}ds$, where $V_s = \int_s^{\infty} v_x.e^{-\int_s^x r_u du}dx$ (see above). One gets $I = \int_t^{\infty} \overset{\bullet}{N}_{js}\left[\int_s^{\infty} v_x.e^{-\int_t^x r_u du}dx\right]ds$. Integrating by parts yields $I = \left[N_{js}\int_s^{\infty} v_x.e^{-\int_t^x r_u du}dx\right]_t^{\infty} + \int_t^{\infty} N_{js} v_s.e^{-\int_t^s r_u du}ds$. By assuming $\lim_{t\to\infty} N_{jt} V_t = 0$ (see the corresponding transversality condition in the case of the benchmark equilibrium), one gets $I = -N_{jt} V_t + \int_t^{\infty} N_{js} v_s.e^{-\int_t^s r_u du}ds$. Therefore, one has

$$\begin{aligned}
&\int_t^{\infty}[v_s N_{js} - q_s H_{js} - v_{js} N_s]\,e^{-\int_t^s r_u du}ds - V_t N_{jt}\\
=\ &\int_t^{\infty}\left(\overset{\bullet}{N}_{js} V_s - q_s H_{js} - v_{js} N_s\right).e^{-\int_t^s r_u du}ds = 0, \text{ for all } t,
\end{aligned}$$

that implies $V_t \overset{\bullet}{N}_{jt} - q_t H_{jt} - v_{jt} N_t = 0$, for all t.

d) The behavior of the household is the same than in the previous section.

Now, we characterize the steady-state. Given the new equilibrium conditions for the consumption good sector, for the educational sector, and for research firms, we can determine the growth rates of the variables in the economy and compute the shares of labor allocated to the different sectors. Proposition 3 summarizes the results obtained. Values are denoted with a subscript "ic". The proof of the proposition is gathered in Appendix 6.4.

Proposition 3 *An equilibrium balanced growth path with imperfect competition is characterized by constant growth rates identical to the ones of the benchmark equilibrium (see proposition 1), constant shares of skilled labor allocated to the R&D sector and to the final sector,*

$$\left(\frac{H_{Rt}}{H_t}\right)^{ic} = \frac{(1+\gamma)(1+\psi)}{\beta(1+\psi)+\alpha\psi}\left[\alpha(\varepsilon-1)\frac{g_c}{g_N}+\frac{\alpha(\rho-g_L)}{g_N}+\alpha\theta\frac{g_H}{g_N}+\frac{\alpha\gamma^2}{(1+\gamma)}+\frac{\beta}{(1+\gamma)}\right]^{-1},$$

$$\left(\frac{H_{Yt}}{H_t}\right)^{ic} = 1-\left(\frac{H_{Rt}}{H_t}\right)^{ic},$$

and constant shares of unskilled labor allocated to the educational sector and to the final sector,

$$\left(\frac{L_t}{L_{Ht}}\right)^{ic} = \frac{(1-\alpha)(1+\psi)}{\alpha}\left[1-\left(\frac{H_{Rt}}{H_t}\right)^{ic}\right]+1,$$

$$\left(\frac{L_{Yt}}{L_t}\right)^{ic} = 1-\left(\frac{L_{Ht}}{L_t}\right)^{ic}.$$

The prices are given by: $q_t^{ic} = \alpha Y_t/\left[(1+\beta)H_{Yt}\right]$; $w_t^{ic} = (1-\alpha)Y_t/\left[(1+\beta)L_{Yt}\right]$; $r_t^{ic} = \varepsilon g_c + \rho$, *where* g_c *is given in Proposition 1;* $v_{Yt}^{ic} = \beta Y_t/\left[(1+\beta)N_t\right]$; $v_{Ht}^{ic} = \psi q_t H_t/\left[(1+\psi)N_t\right]$; $v_{jt}^{ic} = \gamma V_t^{ic}\dot{N}_{jt}/\left[(1+\gamma)N_t\right]$, *where* $V_t^{ic} = \int_t^{\infty} v_s^{ic}.e^{-\int_t^s r_u^{ic}du}ds$. *Finally, the following equality must be satisfied* $(w_t/q_t)^{ic} = (1-\alpha)H_{Yt}/\left[\alpha L_{Yt}\right] = \psi H_t/\left[(1+\psi)L_{Ht}\right]$.

As previously in proposition 1 one can compute the growth rates of prices and one can find that the values are identical in the two types of equilibria. However, except for the rate of return on R&D investment, we observe that for any given Y_t, H_{Yt}, L_{Yt}, H_{Rt}, L_{Ht}, and N_t, imperfect competition leads to a decrease in the level of prices. The level of wage of unskilled labor is proportionally more affected than the wage of human capital since $(w_t/q_t)^{ic} = \psi H_t/\left[(1+\psi)L_{Ht}\right] < (w_t/q_t)^* = H_t/L_{Ht}$. Note that the prices of factors (unskilled labor, skilled labor and ideas) are lower than their marginal productivities: that explains why profits are not negative despite the properties of increasing returns to scale.

Concerning the shares of skilled and unskilled labor allocated to the final sector, to research and to education along the imperfect competitive equilibrium path, we can remark that they differ slightly comparing to the benchmark equilibrium. If we compare $(H_{Rt}/H_t)^{ic}$ with $(H_{Rt}/H_t)^{o}$

and $(L_{Ht}/L_t)^{ic}$ with $(L_{Ht}/L_t)^o$, we remark that imperfect competition can lead either to an excessive or to an insufficient allocation of resources in research and in education. This result is closed to the one of Jones and Williams (2000) who use a model incorporating intermediate goods.

5 Conclusion

This paper incorporates a R&D sector and a human capital production process so that both activity appears equally essential to sustain the per-capita long-run economic growth. In our economy, only scientists and engineers are able to produce innovations which are used at school to educate individuals. This analysis is consistent with several empirical regularities discussed in the relevant literature on endogenous growth: the per-capita growth rate does not exhibit scale effects and it is unaffected in the long run by economic policies. In the economy we describe, there is not any intermediate goods production sector. This specification simplifies the model but it raises the question of the funding of research. As an answer, we construct two equilibria in which the users of discoveries reward directly innovators; these equilibria can be interpreted as a formalization of concepts expressed by Arrow (1962), Dasgupta et al (1996) and Scotchmer (1991). In the first one, there is perfect competition on the markets of private goods and innovators which own a patent on their discoveries, are able to extract the willingness to pay from each agent using their innovations.

agents who use them. That is to say, each firm pays the maximum rental price it is willing to pay to use an innovation. Due to the non-convexity of technologies using innovations as productive factors, the willingnesses to pay (rental prices) of firms are subsidized by the government. In this case, we show how to implement an optimal balanced growth path by using a single economic policy tool charged on R&D firms, to remove externalities in this sector.

In the second equilibrium we construct, research is privately funded. We treat the problem of increasing returns to scale by assuming imperfect competition on markets whose technologies use knowledge as a productive factor. In that case, we present a methodology to characterize this equilibrium. Except for the rate of return on R&D investment which keeps the same value in the two equilibria, we find that the level of prices is lower in the imperfectly competitive case. However, their growth rates remain unchanged.

6 Appendix

6.1 R&D, Human capital and population growth: three necessary ingredients for per-capita long-term growth

Case 1 *R&D is necessary. Let us assume that* $\delta = 0$ *in the production process (4). This implies that* $\overset{\bullet}{N}_t = 0$*, then* $g_N = 0$*. There is not any creation of new units of knowledge. In that case, according to the educational process (2) we get* $g_H = g_L$*. According to the technology (1) we have* $g_c + g_L = \beta g_N + \alpha g_H + (1-\alpha) g_L$*. As* $g_N = 0$ *and* $g_H = g_L$*, we have* $g_c = 0$*.*

Case 2 *Human capital is necessary. Let us assume that* $\phi = 0$ *in the production process (2). Then* $H_t = H_{Yt} = H_{Rt} = 0$*, which implies* $\overset{\bullet}{N}_t = 0$ *and* $g_N = 0$*. One can assume that* $L_{Ht} = 0$ *(i.e.* $L_{Yt} = L_t$*) and* $Y_t = A (N_t)^{\beta} L_t$ *(the replication argument with respect to the private good unskilled labor,* L_t*, still applies). In that case,* $g_Y = g_L$*, and since* $g_Y = g_c + g_L$*, the per-capita long-run growth rate is nil:* $g_c = 0$*.*

Case 3 *Population growth is necessary. From equation (4), at steady-state, one gets* $g_N = \delta (H_{Rt})^{\theta} (N_t)^{\gamma - 1}$*. Since* g_N *is constant, we obtain* $\theta g_{H_R} = (1-\gamma) g_N,$ *for all* t*. Let us assume that* $g_L = 0$*. Then, we have* $g_{L_Y} = g_{L_H} = g_L = 0$*, and equation (2) implies that* $g_H = \psi g_N$*. Using the fact that* $g_{H_R} = g_H$ *at steady-state, we have* $(1-\gamma-\theta\psi) g_N = 0$*. Since* $(1-\gamma-\theta\psi) < 0$ *is a necessary condition for a balanced growth path to exist, we deduce that* $g_N = 0$ *and* $g_H = 0$*. Therefore* $Y_t = A (N_t)^{\beta} (L_{Yt})^{1-\alpha} (H_{Yt})^{\alpha}$ *is constant over-time, and the per-capita long-term growth is nil:* $g_c = 0$*.*

6.2 Characterization of the benchmark equilibrium

We focus on balanced growth paths. Thus, the levels of the growth rates and of the policy tool are constant. Using the fact that $r_t = v_t/V_t + \overset{\bullet}{V}_t/V_t$ with (16) yields $\varepsilon g_c + \rho = v_t/V_t + \overset{\bullet}{V}_t/V_t$ (a). Equations (10), (12) and (14) yield:

$v_t/V_t = g_N \{\beta H_t / [\alpha (1+\tau_t) H_{Rt}] - \beta / [\alpha (1+\tau_t)] + \psi H_t / [(1+\tau_t) H_{Rt}] + \gamma\}$ (b), which implies $g_{H_Y} = g_{H_R} = g_H$ at steady-state. Differentiating (13) with respect to time yields: $\overset{\bullet}{V}_t/V_t = g_q + (1-\theta) g_H - \gamma g_N$. Differentiating (8) with respect to time yields: $g_q = g_Y - g_H$.Combining the two previous results with the fact that $g_Y = g_c + g_L$ yields $\overset{\bullet}{V}_t/V_t = g_c + g_L - \theta g_H - \gamma g_N$ (c). Using (a), (b) and (c) we get $\varepsilon g_c + \rho = g_N \{\beta H_t / [\alpha (1+\tau_t) H_{Rt}] - \beta / [\alpha (1+\tau_t)] + \psi H_t / [(1+\tau_t) H_{Rt}] + \gamma\} + g_c + g_L - \theta g_H - \gamma g_N$. Then, we can determine H_{Rt}/H_t as function of g_c, g_N and g_H. After computations, we get

$$\left(\frac{H_{Rt}}{H_t}\right)^e = (\beta+\alpha\psi)\left[\alpha(\varepsilon-1)(1+\tau_t)\frac{g_c}{g_N}+\frac{\alpha(\rho-g_L)(1+\tau_t)}{g_N}+\alpha\theta(1+\tau_t)\frac{g_H}{g_N}+\beta\right]^{-1}$$

Now we compute the growth rates of the variables of the economy. Using the technology (4) we get $g_N = \theta g_H/(1-\gamma)$. Using the technology (2) we obtain $g_H = g_L + \psi g_N$, therefore we have $g_N = \theta g_L/(1-\gamma-\theta\psi)$ and $g_H = (1-\gamma)g_L/(1-\gamma-\theta\psi)$. Using the technology (1), we get $g_c + g_L = \beta g_N + \alpha g_H + (1-\alpha)g_L$ which implies $g_c = \theta(\beta+\alpha\psi)g_L/(1-\gamma-\theta\psi) = (\beta+\alpha\psi)g_N$. A necessary condition for the existence of a balanced growth path to exist is $1-\gamma-\theta\psi > 0$. We can deduce easily the value of H_{Yt}/H_t. The shares of unskilled labor allocated to the educational sector and to the output sector are found by using (8), (9) and (11).

6.3 Characterization of the Optimum

The Hamiltonian of the social planner's problem is given by,

$$\begin{aligned}\Gamma &= L_t e^{-\rho t}\frac{(c_t)^{1-\varepsilon}-1}{1-\varepsilon} + \lambda_t\left[A(N_t)^{\beta}(L_{Yt})^{1-\alpha}(H_{Yt})^{\alpha} - L_t c_t\right] \\ &\quad + \mu_t\delta(H_{Rt})^{\theta}(N_t)^{\gamma} + \nu_t\left[\phi L_{Ht}(N_t)^{\psi} - H_{Yt} - H_{Rt}\right] + \xi_t\left[L_t - L_{Yt} - L_{Ht}\right]\end{aligned}$$

The first order conditions can be written as $\lambda_t = e^{-\rho t}c_t^{-\varepsilon}$ (a), $\nu_t = \lambda_t\alpha Y_t/H_{Yt}$ (b), $\xi_t = \lambda_t(1-\alpha)Y_t/L_{Yt}$ (c), $\nu_t = \mu_t\delta\theta(H_{Rt})^{\theta-1}(N_t)^{\gamma}$ (d), $\xi_t = \nu_t\phi(N_t)^{\psi}$ (e), $-\dot{\mu}_t = \lambda_t\beta Y_t/N_t + \nu_t\psi H_t/N_t + \mu_t\gamma g_N$ (f). Since $Y_t = L_t c_t$, we get immediately $g_c = g_Y - g_L$. Differentiating equations (a), (b) and (d) with respect to time, we can obtain, $(\varepsilon-1)g_c + \rho - g_L = (1-\theta)g_{H_R} - g_{H_Y} - \gamma g_N - \dot{\mu}_t/\mu_t$ (g). Differentiating the production function one can obtain $g_Y = g_c + g_L = \beta g_N + (1-\alpha)g_{L_Y} + \alpha g_{H_Y}$ (h). Then, using (b), (c), (d), (f) with the human capital constraint we get after computations, $-\dot{\mu}_t/\mu_t = \theta g_N\left[(\beta+\alpha\psi)H_t/(\alpha H_{Rt}) - \beta/\alpha + \gamma/\theta\right]$ (i). Equations (g) and (i) imply that along a balanced growth path $g_{H_Y} = g_{H_R} = g_H$. Thus it follows that $g_{L_Y} = g_{L_H} = g_L$. One can obtain the values of the growth rates of the variables in the economy which are exactly the same than in the case of the benchmark equilibrium. Combining (g), (h) and (i) we get after computations the optimal share of labor allocated to research:

$$\left(\frac{H_{Rt}}{H_t}\right)^o = (\beta+\alpha\psi)\left[\frac{\alpha(\varepsilon-1)}{\theta}\frac{g_c}{g_N}+\frac{\alpha(\rho-g_L)}{\theta g_N}+\frac{\alpha g_H}{g_N}+\beta\right]^{-1}$$

The other results can be deduced.

6.4 Characterization of the equilibrium with imperfect competition

For each agent of each sector, we combine the two types of conditions given in the text (zero profit condition and equality between the marginal rate of substitution between two factors with their corresponding prices ratio) to find: $Y_t/H_{Yt} = (1+\beta)\, q_t/\alpha$ (a), $Y_t/L_{Yt} = (1+\beta)\, w_t/(1-\alpha)$ (b), $Y_t/N_t = (1+\beta)\, v_{Yt}/\beta$ (c) for the output sector; $q_t H_t/L_{Ht} = (1+\psi)\, w_t$ (d), $q_t H_t/N_t = v_{Ht}(1+\psi)/\psi$ (e) for the school of the educational sector; $q_t = V_t g_N/(1+\gamma)$ (f) and $v_{Rt} = \gamma V_t g_N/(1+\gamma)$ (g) for the research sector.

Remark: the price of inputs in imperfect competition are lower than their marginal productivity.

One can remark that equations (a), (b), (c), (d), (e), (f), (g) resemble to equations (8), (9), (10), (11), (12), (13), (14) of the previous section. The method of resolution is exactly the same than in the previous section. One finds that $\varepsilon g_c + \rho = v_t/V_t + \dot{V}_t/V_t$ (h). Equations (a), (c), (e), (f) and (g) yield:

$v_t/V_t = g_N \{\beta H_{Yt}/[\alpha H_{Rt}] + \psi H_t/[(1+\psi)\, H_{Rt}] + \gamma\}/(1+\gamma)$ (i) which implies $g_{H_Y} = g_{H_R} = g_H$ at steady-state. Differentiating (f) with respect to time yields: $\dot{V}_t/V_t = g_q + (1-\theta)\, g_H - \gamma g_N$. Differentiating (a) with respect to time yields: $g_q = g_Y - g_H$. Combining the two previous results with the fact that $g_Y = g_c + g_L$ yields $\dot{V}_t/V_t = g_c + g_L - \theta g_H - \gamma g_N$ (j). Using (h), (i) and (j) we get $\varepsilon g_c + \rho = g_N \{\beta H_{Yt}/[\alpha H_{Rt}] + \psi H_t/[(1+\psi)\, H_{Rt}] + \gamma\}/(1+\gamma) + g_c + g_L - \theta g_H - \gamma g_N$. Then, we can determine H_{Rt}/H_t as function of g_c, g_N and g_H. After computations, we get

$$\left(\frac{H_{Rt}}{H_t}\right)^{ic} = \frac{(1+\gamma)(1+\psi)}{\beta(1+\psi)+\alpha\psi}\left[\alpha(\varepsilon-1)\frac{g_c}{g_N} + \frac{\alpha(\rho-g_L)}{g_N} + \alpha\theta\frac{g_H}{g_N} + \frac{\alpha\gamma^2}{(1+\gamma)} + \frac{\beta}{(1+\gamma)}\right]^{-1}$$

The growth rates are the same than in the benchmark equilibrium.

7 References

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