Natural resources, knowledge and efficiency : beyond the Hotelling rule ?

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Abstract

In the standard model with two stocks, natural resource and physical capital, the fundamental efficiency condition is the Hotelling rule. If we add a third stock, knowledge, as for instance in endogenous growth models, a new efficiency condition is obtained. This condition highlights the fundamental public good nature of knowledge. Moreover, we show that if suffices to use Lindhal prices in a competitive economy to implement optimum.

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1 Introduction

Since the Hotelling's (1931) seminal paper, the standard literature on non renewable resources, in particular the literature on growth, has studied a model with essentially two stocks : the (finite) stock of natural resource and the stock of physical capital.

In this type of framework, the fundamental efficiency condition, that is to say the necessary local condition that has to be verified along any efficient path, is called the Hotelling rule. It says that, at each time, the marginal productivity of capital has to be equal to the rate of growth of the marginal productivity of the resource (for more details, see for instance Withagen (1999)). This condition is the socially optimal solution of the arbitrage problem in which the social planner has to choose between keeping the resource in situ, or extracting and embodying it in physical capital. Moreover, if one considers a decentralized economy in which the resource sector is competitive, the maximization of the profit function in this sector leads to the "Hotelling rule at equilibrium", which says that the interest rate on the perfect financial market is equal to the rate of growth of the resource price. This condition implies the preceeding one : in other words, in a competitive equilibrium, the efficiency Hotelling rule is satisfied, that is in fact a direct implication of the first welfare theorem.

From the 1980s, some new stocks variables have been introduced in endogenous growth models. For instance, Lucas (1988) introduced human capital and, some years later, Romer (1990), Grossman-Helpman (1991) and Aghion-Howitt (1992) introduced knowledge as the main factors of growth. In the second part of the 1990s, some authors, like Schou (1996), Aghion-Howitt (1998) and Scholz-Ziemes (1999), have reconsidered the question of sustainability of growth by introducing non renewable resources in endogenous growth models with knowledge accumulation. With respect to the standard literature of the 1970s-1980s, this new analysis consider now three stocks variables : natural resources, physical capital, and knowledge. Then, new questions are raised that we try to answer in this paper.

The first question concerns the characterization of efficient paths. We know that the Hotelling rule is the efficiency condition concerning the arbitrage between keeping the resource in situ or enbodying it in physical capital. What is the new efficiency condition, and what is its economic interpretation, if we consider the arbitrage between natural resource and knowledge ? In order to answer this question, we have to take into account two main characteristics of the problem. First, it is generally assumed that knowledge, for instance new goods in Romer (1990) or new qualities in Aghion-Howitt (1992), is produced by using labor (without resource). In this case, it is not generally possible to directly embody resource in knowledge. Then, the mechanism that has to be considered is the following : if the social planner increases the flow of extraction, he releases labor from the final output sector. This labor can be transfered to the research sector, that allows to increase knowledge : this transfer of labor allows indirectly to embody resource in knowledge. Second, contrarily to the physical capital which appears in the standard Hotelling rule and which is a private good, knowledge is a public good, that is to say a non rival, or non depletable good (see for instance Mas-Colell, Whinston, Green (1995), chapter 11), which is simultaneously used by the firms producing the final output and by the firms of the research sector. Thus, some features of the new efficiency condition obtained here are close to the standard Bowen-Lindahl-Samuelson condition.

The second point that we study in this paper concerns the functionning of a decentralized economy, and the implementation of optimum. Contrarily to the case of the standard model where the Hotelling rule is satisfied in a competitive economy, there is here a problem of decentralization due to the public good nature of knowledge. In standard endogenous growth models, it is generally assumed that each new good is produced by a monopoly, that allows to finance ex ante the research activity. In this paper, we proceed in two steps. First, we compute the Lindahl prices which allow to implement the optimal path in a decentralized competitive economy. Second, taking again the standard assumption of monopolies on intermediate goods, we describe the general equilibrium, and we compute the exact values of the public tools which allow to implement the optimum.

Along all the paper, at optimum and at equilibrium, we present the different results in a rather general model, that is to say without particular specifications. However, at each step, we consider an example in which we use our general formulas to compute analytically the solutions, in particular the rates of growth of the different variables at the steady state. In fact, this example takes again the common model of Schou (1996), Aghion-Howitt (1998) and Scholz-Ziemes (1999), and their results are progressively recovered, at optimum, then at equilibrium. With respect to the example studied by theses authors, the new results presented here concern essentially the equilibrium. First we compute the Lindahl prices in the first type of equilibrium. Second, in the other type, which is studied by Schou, Aghion-Howitt, and Sholz-Ziemes, we compute the optimal tools, subsidy to research and subsidy to monopolies, which allow to implement the optimal path obtained in section 2.

The paper is organized as follows. In section 2, we present the model and we characterize the efficient paths. In particular, we derive the new efficiency condition concerning the arbitrage between natural resource and knowledge. In section 3, we consider a decentralized economy, and we study the two types of equilibrium mentionned above : the first one with Lindahl prices, the second one with monopolies on intermediate goods. Finally, we present our conclusions in section 4.

2 Efficiency in an economy with a natural resource, physical capital and knowledge

2.1 The model

We consider an economy where a final homogeneous good (Y) is produced by m firms ($i = 1, \ldots, m$). Each firm i has a production function

$$Y_t^i \quad = \quad F^i \left(L_t^i, R_t^i, \int_0^{n_t} f^i(x_t^i(j))dj \right) \tag{1}$$

where L_t^i and R_t^i are the quantities of labor and natural resource used at time t ; $x_t^i(j)$ is the quantity of intermediate good j, with $j \in [0, n_t]$: n_t is the measure of the space of intermediate goods (interpreted as the "number" of goods, namely knowledge). We assume that $f^i()$ is an increasing and strictly concave function of $x^i(j)$. We denote by $X_t^i = \int_0^{n_t} f^i(x_t^i(j))dj$ the index of intermediate goods, and by F_L^i, F_R^i and F_X^i the partial derivatives of the production function.

The final good is used for consumption (c_t) and investment ($\dot{K}_t$). Thus we have

$$Y_t \quad = \quad \sum_{i=1}^{m} Y_t^i = c_t + \dot{K}_t \tag{2}$$

Following for instance Grossman-Helpman (1991) and Romer (1990), we assume that, in the $R\&D$ sector, M firms ($h = 1, \ldots, M$) produce innovations along with

$$\dot{n}_t^h \quad = \quad q^h(n_t, \ell_t^h), q_n^h > 0, q_\ell^h > 0, \tag{3}$$

where $\dot{n}_t^h$ and ℓ_t^h are respectively the number of innovations produced at t and the labor used in research by firm h. As it is usual in this type of model, we assume that the total number of goods, n_t, is also an input. Note that, at each time t, the total number of innovations in the economy is

$$\dot{n}_t = \sum_{h=1}^{M} \dot{n}_t^h = \sum_{h=1}^{M} q^h(n_t, \ell_t^h).$$

Normalizing labor supply to one, we have at each time t

$$\sum_{i=1}^{m} L_t^i + \sum_{h=1}^{M} \ell_t^h = 1. \tag{4}$$

We assume that, once a new good is invented, it is produced by capital alone : one unit of capital is needed for each unit of intermediate good. Thus, at each date t, we have

$$x_t(j) \quad = \quad K_t(j), \quad \text{for all} \quad j \in [0, n_t] \tag{5}$$

where $x_t(j) = \sum_{i=1}^{m} x_t^i(j)$ is the total quantity of intermediate good j. Recall (see (2) above) that $K_t = \int_0^{n_t} K_t(j)dj$ is the total stock of capital.

If we denote by S_0 the initial stock of resource, the stock at t is given by

$$S_t \quad = \quad S_0 - \int_0^t R_\nu d_\nu \tag{6}$$

where $R_t = \sum_{i=1}^{m} R_t^i$ is the total flow extracted at t. We assume that there is no extraction cost.

2.2 Efficient paths

Our first objective is to characterize the efficient paths. We consider an interval of time (t_0, t_1). Let be $\{c_t\}_{t_0}^{t_1}$ a *given* profile of consumption on this interval, and let be K_{t_0} and K_{t_1} the given levels of capital at t_0 and t_1. Then the program of the social planner is to minimize the flow of extraction under the constraints (1) to (6). After reorganization, this program can be written

$$\min \int_{t_0}^{t_1} \left(\sum_{i=1}^{m} R_t^i \right) dt$$

subject to

$$\begin{aligned}
\dot{K}_t &= \sum_{i=1}^{m} F^i \left(L_t^i, R_t^i, \int_0^{n_t} f^i(x_t^i(j))dj \right) - c_t \\
\dot{S}_t &= -\sum_{i=1}^{m} R_t^i \\
\dot{n}_t &= \sum_{h=1}^{M} q^h(n_t, l_t^h) \qquad (7)\\
&\int_0^{n_t} \left(\sum_{i=1}^{m} x_t^i(j) \right) dj - K_t = 0 \\
&\sum_{i=1}^{m} L_t^i + \sum_{h=1}^{M} l_t^h = 1
\end{aligned}$$

Proposition 1 *An efficient path of the economy is characterized by the two following conditions :*

$$\frac{\dot{F}_R}{F_R} = F_x \tag{8}$$

where $F_R = F_R^i \ \forall i$, is the marginal productivity of the resource, and $F_x = F_X^i f^{i'}(x^i(j)) \ \forall i, j$, is the marginal productivity of any intermediate good j;

$$\frac{\dot{F}_R}{F_R} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_e}{q_\ell} + \sum_{h=1}^{M} q_n^h + \frac{q_\ell}{F_L} \left(\sum_{i=1}^{m} F_X^i f^i(x^i(n) \right) - F_x \sum_{i=1}^{m} x^i(n)), \tag{9}$$

where $F_L = F_L^i$, $\forall i$, and $q_\ell = q_\ell^h$, $\forall h$, are respectively the marginal productivities of labor in the final output sector and in the research sector.

Proof : see appendix A.

Observe that, since each unit of intermediate good is produced with one unit of capital, F_x can also be interpreted as the marginal productivity of capital. Then, condition (8) says that the marginal productivity of capital must be equal to the rate of growth of the marginal productivity of the natural resource : it is the *standard Hotelling rule* which concerns the arbitrage between resource (R) and physical capital (K).

Condition (9) is a *new efficiency condition* which appears in this type of endogenous growth model, and which concerns the arbitrage between resource (R) and knowledge (n). We can already observe that the fundamental public good nature of knowledge (n) appears in the right side of the equation where we see the symbols $\sum_{h=1}^{M}()$ and $\sum_{i=1}^{m}()$, as for instance in the classic optimality condition derived by Samuelson (1954 ; 1955).

Let us give an intuitive interpretation of these two conditions.

2.2.1 Natural resource and physical capital : the Hotelling rule

First, in order to interpret the Hotelling rule, we consider an elementary interval of time $(t, t+\Delta t)$. At t, the social planner faces the following arbitrage concerning any unit of natural resource : either he keeps it in situ at t and he extracts it at $(t+\Delta t)$ in order to increase the final output ; or he extracts it at t, he uses it to accumulate more capital, that allows to produce more of any intermediate good, and thus to produce more output. In the first case, the increase in output at $t+\Delta t$ is $\Delta Y^1_{t+\Delta t} = F_R(t+\Delta t) \simeq \dot{F}_R(t)\Delta t + F_R(t) = F_R(t)\left(\dfrac{\dot{F}_R(t)}{F_R(t)}\Delta t + 1\right)$. In the second one, the new capital at t is $\Delta K_t = F_R(t)$, and the new output at $t+\Delta t$ (new capital and new production) is $\Delta Y^2_{t+\Delta t} = \Delta K_t + F_x(t)\Delta K_t \Delta t = F_R(t) + F_x(t)F_R(t)\Delta t = F_R(t)(F_x(t)\Delta t + 1)$. It is clear that we have $\Delta Y^1_{t+\Delta t} = \Delta Y^2_{t+\Delta t}$ if and only if $\dot{F}_R(t)/F_R(t) = F_x(t)$: it is the Hotelling rule. If $\dot{F}_R(t)/F_R(t) < F_x(t)$, the social planner has to extract more today in order to accumulate more capital. If $\dot{F}_R(t)/F_R(t) > F_x(t)$, he has to extract less today.

2.2.2 Natural resource and knowledge : the new condition

Now, we interpret the second efficiency condition (a more formal interpretation is given in Appendix B). In the Hotelling rule, the arbitrage concerns the choice between keeping the natural resource in situ or embodying it in physical capital. In the new condition, it is between keeping it in situ or indirectly embodying it in knowledge. As we said above, in this type of model, the natural resource is only used in the final sector. Thus, it is not possible to directly embody it in knowledge. If the social planner increases the flow of extraction, he can transfer labor from the final sector to the research activity, that allows to increase knowledge accumulation. This is the reason why we speak of indirect embodying.

We again consider an interval of time $(t, t+\Delta t)$. As before, if one unit of resource is kept in situ at t and extracted at $t+\Delta t$, the increase in output at $t+\Delta t$ is $\Delta^1_{t+\Delta t} = F_R(t)\left(\frac{\dot{F}_R(t)}{F_R(t)}\Delta t + 1\right)$. Assume that this unit is extracted at t. This extraction allows to stimulate research by three channels.

First, assuming that the total output is unchanged, it allows to decrease the labor (L) used in the final sector by $F_R(t)/F_L(t)$. If this labor is devoted to research $(dl_t = -dL_t)$, knowledge (n) increases by $dn^1 = \frac{F_R(t)}{F_L(t)}q_\ell(t)$.

Second, the intertemporal cumulative effect of n on $\dot{n}$ (remember that $\dot{n}_t = \sum_{h=1}^M q^h(n_t, \ell^h_t)$) leads to an increase in knowledge,

$$dn^2 = \frac{F_R(t)}{F_L(t)}q_\ell(t)\sum_{h=1}^{M} q^h_n(t)\Delta t, \quad \text{on} \quad (t, t+\Delta t).$$

Third, the increase in knowledge leads to more final output, that allows to save more labor (L) and thus to produce more knowledge. From (1), the gross increase in output is $\frac{F_R(t)}{F_L(t)}q_\ell(t)\sum_{i=1}^{m} F^i_X f^i(x^i(n))\Delta t$. However, since intermediate goods are produced from capital, we have $K_t = \int_0^{n_t}\sum_{i=1}^m x^i_t(j)dj$.

Thus we need a quantity of capital given by $\frac{F_R(t)q_\ell(t)}{F_L(t)}\sum_{i=1}^{m} x^i(n)$. This quantity of capital can be obtained by decreasing the corresponding quantity of any intermediate good j, that yields a decrease in final output given by $F_x(t)\frac{F_R(t)q_\ell(t)}{F_L(t)}\sum_{i=1}^{m} x^i(n)\Delta t$. Finally, the net increase in output is

$$\frac{F_R(t)}{F_L(t)}q_\ell(t)\left(\sum_{i=1}^{m} F_X^i f^i(x^i(n)) - F_x\sum_{i=1}^{m} x^i(n)\right)\Delta t.$$

If the social planner uses this new output to save more labor (L) and to transfer it to the research sector, this yields to an increase in knowledge,

$$dn^3 = \frac{F_R(t)q_\ell(t)}{F_L(t)}\frac{q_\ell(t)}{F_L(t)}\left(\sum_{i=1}^{m} F_X^i f^i(x^i(n)) - F_x\sum_{i=1}^{m} x^i(n)\right)\Delta t.$$

Summing up dn^1, dn^2 and dn^3, we obtain the new knowledge produced after the extraction of one unit of resource :

$$\begin{aligned}&dn = dn^1 + dn^2 + dn^3 =\\ &\frac{F_R(t)q_\ell(t)}{F_L(t)}\left[1 + \left(\sum_{h=1}^{M} q_n^h(t) + \frac{q_\ell(t)}{F_L(t)}\left(\sum_{i=1}^{m} F_X^i f^i(x^i(n)) - F_x\sum_{i=1}^{m} x^i(n)\right)\right)\Delta t\right]\end{aligned}$$

This increase in knowledge allows to obtain an increase in the final output at $(t+\Delta t)$ given by $\Delta Y_{t+\Delta t}^2 \simeq dn\frac{F_L(t+\Delta t)}{q_\ell(t+\Delta t)}$, where $\frac{q_\ell(t)F_L(t+\Delta t)}{F_L(t)q_\ell(t+\Delta t)}$ is approximatively equal to $1 + \left(\frac{\dot F_L(t)}{F_L(t)} - \frac{\dot q_\ell(t)}{q_\ell(t)}\right)\Delta t$. Finally, we have

$$\begin{aligned}&\Delta Y_{t+\Delta t}^2 \simeq\\ &F_R(t)\left\{1 + \left[\frac{\dot F_L}{F_L} - \frac{\dot q_\ell}{q_\ell} + \sum_{h=1}^{M} q_n^h + \frac{q_\ell}{F_L}\left(\sum_{i=1}^{m} F_X^i f^i(x^i(n)) - F_x\sum_{i=1}^{m} x^i(n)\right)\right]\Delta t\right\},\end{aligned}$$

where we neglect the second-order terms.

Equalizing this quantity to $\Delta Y_{t+\Delta t}^1 = F_R\left(1 + \frac{\dot F_R}{F_R}\Delta t\right)$, we find again exactly the arbitrage condition (9), whose we have now an interpretation.

In brief, in the Hotelling rule, the social planner can extract more today, embody this resource in capital, that allows to produce more tomorrow and thus to economize resource. In the new efficiency condition, if he extracts more today, he can allocate the released labor to the research sector, that allows to accumulate more knowledge, and thus to produce more tomorrow and to economize resource. The main point here is that each increase in knowledge has a cumulative effect that comes from the specific form of the research sector technology.

If $\Delta Y^1_{t+\Delta t} < \Delta Y^2_{t+\Delta t}$, the social planner has to extract more today, in order to transfer labor from the final output sector to the research one, and thus in order to produce more innovations.

If $\Delta Y^1_{t+\Delta t} > \Delta Y^2_{t+\Delta t}$, the social planner has to extract less today, and thus to produce less innovations.

Remark 1 : consider the standard neoclassical model, where the final good (Y) is produced by using the natural resource and physical capital along with $Y_t = F(K_t, R_t)$ and characterize the efficient paths. In this case, the social planner minimizes the flow of extraction $\int_{t0}^{t1} R_t dt$, subject to the constraints $\dot{K} = F(K_t, R_t) - c_t$ and $\dot{S}_t = -R_t$. This program easily leads to the Hotelling rule : $\dot{F}_R / F_R = F_K$. Indeed, the Hamiltonian is

$$H = R + \lambda(F(K, R) - c) - \mu R$$

The first order conditions are $1 + \lambda F_R - \mu = 0, -\dot{\mu} = 0$ and $-\dot{\lambda} = \lambda F_K$. Differentiating the first one with respect to t and using the two others give immediately the Hotelling rule : $\dot{F}_R / F_R = F_K$.

Remark 2 : until now, we have only been interested by efficiency. Assume that we want characterize the *optimal paths.* More precisely, inside the efficient paths, we select those which maximize the intertemporal utility $\int_0^t u(c_t) e^{-\rho t} dt$. Then we obtain in addition the Ramsey-Keynes condition, which is $\rho - u''\dot{c}/u' = F_x$ in the first model (with knowledge) and $\rho - u''\dot{c}/u' = F_K$ in the standard neoclassical model.

Example 1 : we consider the model studied by Aghion-Howitt ((1998), chapter 5), Schou (1996) and Scholz-Ziemes (1999), which is a particular case of the above model.

The m firms in the final sector have the same technology :

$$Y_t^i = (L_t^i)^\beta (R_t^i)^\nu \int_0^{n_t} x_t^i(j)^\alpha dj,$$

with $\alpha + \beta + \nu = 1$. The M firms in the research sector have also the same technology : $\dot{n}_t^h = \delta n_t l_t^h$, with $\delta > 0$.

From the proof of proposition 1 (see appendix A), we have $x^i(j) = x^i$ for all i and all j, $x^i/R^i = \sum_i x^i / \sum_i R^i = x/R$ for all i, and $x^i/L^i = \sum_i x^i / \sum_i L^i = x/L$ for all i. Then, each individual production function can be written $Y^i = (L_t^i)^\beta (R_t^i)^\nu n(x^i)^\alpha = n(L/x)^\beta (R/x)^\nu x^i$. Finally, the aggregate production function is $Y = nL^\beta R^\nu x^\alpha = K^\alpha n^{1-\alpha} L^\beta R^\nu$, with $K = xn$: see Schou (equation (2.3)), Scholz-Ziemes (equation (2.11)), and Aghion-Howitt (p. 163).

The total number of innovations at t is

$$\dot{n}_t = \sum_{h=1}^{M} \dot{n}_t^h = \delta n_t \sum_{h=1}^{M} \ell_t^h = \delta n_t \ell_t \ \ :$$

see Schou (equation (2.5)), Scholz-Ziemes (equation (2.13)), and Aghion-Howitt (p. 163).

First, consider the Hotelling rule (equation (8)) : $\dot{F}_R/F_R = F_x$, where $F_R = F_R^i$ and $F_x = F_x^i$, for all i (see proposition 1). Differentiating the production function of firm i with respect to R^i, we get :

$$\begin{aligned} F_R^i &= \nu (L^i)^\beta (R^i)^{\nu-1} \textstyle\int_0^n x^i(j)^\alpha dj = \nu (L^i)^\beta (R^i)^{\nu-1} n (x^i)^\alpha \\ &= \nu n (R^i/L^i)^{\nu-1} (x^i/L^i)^\alpha = \nu n (R/L)^{\nu-1} (x/L)^\alpha = \nu n L^\beta R^{\nu-1} x^\alpha \\ &= \nu Y/R = \partial Y/\partial R, \end{aligned}$$

since $Y = nL^\beta R^\nu x^\alpha$. Thus, we have $\dot{F}_R/F_R = g_Y - g_R$ (where g_y is the rate of growth of any variable y).

Similary, differentiating this production function with respect to $x^i(j)$, we get : $\partial Y^i/\partial x^i(j) = F_x^i = \alpha(L^i)^\beta(R^i)^\nu(x^i)^{\alpha-1} = \alpha(R^i/L^i)^\nu(x^i/L^i)^{\alpha-1} = \alpha(R/L)^\nu(x/L)^{\alpha-1} = \alpha L^\beta R^\nu x^{\alpha-1}$.

Finally, the Hotelling rule is

$$g_Y - g_R = \alpha L^\beta R^\nu x^{\alpha-1} \tag{10}$$

Secondly, consider the second efficiency condition (equation (9). We know that $\dot{F}_R/F_R = g_Y - g_R$. Similary, it is easy to see that $\dot{F}_L/F_L = g_Y - g_L$. Moreover we have $-\dot{q}_\ell/q_\ell + \sum_{h=1}^M q_n^h = 0$, since $\sum_{h=1}^M q_n^h = \delta\sum_h \ell^h = \delta\ell$ and $q_\ell = \delta n$, that implies $\dot{q}_\ell/q_\ell = \dot{n}/n = \delta\ell$. We can also compute the last term :

$$\begin{aligned}
& \sum_i F_X^i f^i(x^i(n)) - F_x\sum_i x^i(n) = \sum_i (L^i)^\beta(R^i)^\nu(x^i)^\alpha - \alpha L^\beta R^\nu x^{\alpha-1}x \\
= \; & \sum_i L^i(R/L)^\nu(x/L)^\alpha - \alpha(R/L)^\nu(x/L)^{\alpha-1}x \\
= \; & L(R/L)^\nu(x/L)^\alpha - \alpha L(R/L)^\nu(x/L)^\alpha = L(1-\alpha)(R/L)^\nu(x/L)^\alpha
\end{aligned}$$

The term q_ℓ/F_L is equal to $\delta nL/\beta Y$. Replacing Y by $nL^\beta R^\nu x^\alpha$, the last term of equation (9) is equal to $\delta(1-\alpha)L)/\beta$. Finally, this efficiency condition becomes

$$-g_R = -g_L + \frac{\delta(1-\alpha)L}{\beta} \tag{11}$$

At steady state, all variables grow at constant rate. Thus, from (11), L is constant, that implies $g_R = -\delta(1-\alpha)L/\beta$. Frome (10), we obtain $\nu g_R + (\alpha-1)g_x = 0$. Then using the expression of g_R, we obtain $g_x = -\nu\delta L/\beta$. Since $K = nx$, we have $g_K = g_n + g_x = \delta(1-L) - \nu\delta L/\beta = \delta - \delta L(1+\nu/\beta)$, that is also the expression of g_Y, since $g_Y = g_K$. Finally, for a given level of labor L used in the final sector (and thus a given level of labor $\ell = 1-L$ used in research) the two efficiency conditions give the rates of growth of Y and R at steady state : $g_Y = \delta - \delta L(1+\nu/\beta)$ and $g_R = -\delta(1-\alpha)L/\beta$.

Assume that we want to characterize the steady state optimal growth path with an (isoelastic) instantaneaous utility function : $u(c) = (c^{1-\varepsilon} -$

$1)/(1-\varepsilon)$, $\varepsilon > 0$. Then the Keynes-Ramsey condition is $g_Y - g_R = \rho + \varepsilon g_c$. Since $g_c = g_Y$, we have $g_Y = (\rho + g_R)/(1-\varepsilon)$. Using this condition and the two efficiency conditions, we obtain finally :

$$L = \left(\frac{\rho}{1-\varepsilon} + \delta\right)\frac{\beta(1-\varepsilon)}{\delta\varepsilon(1-\alpha)}$$

$$g_R = -\frac{\rho}{\varepsilon} + \delta\frac{1-\varepsilon}{\varepsilon}$$: see Schou (equation (3.12)) and Aghion-Howitt (chapter 5, Appendix 2, p. 169)

and $$g_Y = \frac{\delta - \rho}{\varepsilon}$$: see Schou (equation (3.10)) and Aghion-Howitt (p. 169)

Growth is positive if $\delta - \rho > 0$, and the condition $g_R < 0$ imposes $\delta - \rho < \delta\varepsilon$. Thus a balanced optimal growth path with positive growth rate exists if $0 < \delta - \rho < \delta\varepsilon$: see Aghion-Howitt (p. 164).

3 Equilibria in a market economy

After the optimum characterization, we have to construct equilibria. We claim that the fundamental difficulty to implement the optimum in a decentralized economy is that knowledge is a public good which is simultaneously used in the final sector and in the research sector. In order to stress this point, we consider successively two types of equilibria.

First, we construct an equilibrium which is an analytical benchmark : we assume that all markets are competitive, and we compute the Lindhal prices with which the research is financed and that allow to implement the optimal path. However, we are conscious that this type of equilibrium is not totally convincing. In particular, as it is explained in many text-books, price-taking behavior on markets with personalized prices is unlikely to occur. That is probably why this type of equilibrium is not generally studied, in particular in endogenous growth models with innovations.

Second, we consider a more usual equilibrium. We assume that, once an innovation has occured, the investor of the new good retains a perpetually monopoly right over the production and the sale of this good. Then, the expected profits of the monopolist allow to finance the research. The new problem is that, by introducing these new institutions in the model, we also introduce new distorsions that have to be corrected if we want implement the optimal path. In particular, on each intermediate good market, the price is higher than the marginal cost, that requires for instance to subsidy the demand of these goods.

In this section, the price of good Y is normalized to one and w_t, p_t^R and r_t are respectively the wage, the price of the resource and the interest rate on a perfect financial market. Moreover, since the intermediate goods are all identical, they have the same price : $p_t(j) = p_t, j \in [0, n_t]$.

We assume that the market of the natural resource is competitive. Then, the maximization of the profit function

$$\int_t^{\infty} p_s^R R_s e^{-\int_t^s rudu} ds, \text{ for all } t,$$

subject to the constraint $\dot{S}_s = -R_s$, leads to the standard "Hotelling rule at equilibrium"

$$\frac{\dot{p}_t^R}{p_t^R} = r_t, \ \forall\, t. \tag{12}$$

This condition can be interpreted as the efficiency condition (8) (see 2.2.1 above). Consider an elementary interval of time $(t, t+\Delta t)$. If the owner firm extracts one unit of resource at t, sells it at price p_t^R, and invest p_t^R on the financial market, the return of this operation at $(t+\Delta t)$ is $p_t^R r_t \Delta t$. If the firm keeps the resource in situ, the return at $t+\Delta t$ is $(dp_t^R/dt)\Delta t = \dot{p}_t^R \Delta t$. The two returns have to be equal, that gives $p_t^R r_t \Delta t = \dot{p}_t^R \Delta t$, and thus $\dot{p}_t^R / p_t^R = r_t$.

A direct consequence of (12) it that, in the standard neoclassical model where the production function is $Y_t = F(K_t, R_t)$, if all markets are competitive, we have $F_K = r_t$ and $F_R = p_t^R$ (that implies $\dot{F}_R = \dot{p}_t^R$). Thus, (12) can be written $\dot{F}_R / F_R = F_K$, that is exactly the "Hotelling rule at optimum" : this result is a particular case of the first theorem of welfare, that holds here in particular because physical capital is a private good. As we have said above, our problem in the present model is that it is not the case for knowledge.

3.1 Lindhal equilibria

We first consider an equilibrium in which innovations are financed by Lindhal prices. More precisely, we denote by v_t^i and v_t^h the Lindhal prices paid at t by each firm i and each firm h for any innovation. When a firm uses one particular innovation, it pays the Lindhal price to the investor. Thus the total Lindhal prices paid at time t by a firm, for instance firm i, is equal to $n_t v_t^i$. Let us observe that the Lindhal price paid by a firm for a given innovation is independant of the quantity of the intermediate good corresponding to this innovation. In this type of equilibrium, contrarily to that is generally done in endogenous growth models, we distinguish innovations, which are financed by Lindhal prices, and the intermediate goods in which they are embodied, which are sold on competitive markets (and not by monopolists, as in sub-section 3.2).

3.1.1 Agents behaviors

a) In the *final sector*, at each time t, the profit of each firm i is

$$\pi_t^i = F^i(L_t^i, R_t^i, \int_0^{n_t} f^i(x_t^i(j))dj) - w_t L_t^i - p_t^R R_t^i - \int_0^{n_t} p_t x_t^i(j)dj - n_t v_t^i$$

As we said above, the two last terms correspond to the payments for intermediate goods and the payments (Lindahl prices) for innovations. Observe that, in this particular problem, each innovation is an indivisible public good : each firm i has to decide if it uses an intermediate good j, or if it doesn't. If it uses it, it pays v_t^i to the inventor and $p_t x_t^i(j)$ on the market. Differentiating with respect to the quantities of inputs, $L_t^i, R_t^i, x_t^i(j)$, and the number of innovations n_t, and equating to zero, give the following first-order conditions :

$$F_L^i - w_t = 0, \tag{13}$$

$$F_R^i - p_t^R = 0, \tag{14}$$

$$F_{x(j)}^i - p_t = 0, \quad j \in [0, n_t], \tag{15}$$

where $F_{x(j)}^i = F_X^i f^{i'}(x_t^i(j))$ is the marginal productivity of the intermediate good j,

$$F_X^i f^i(x_t^i(n_t)) - p_t x_t^i(n_t) - v_t^i = 0. \tag{16}$$

From (13) and (14), we see that the marginal productivities of labor and resource are independant of i (as at optimum) : we can write $F_L^i = F_L$ and $F_R^i = F_R$, for all i, these productivities. From (15), we have also $F_X^i f^{i'}(x_t^i(j)) = F_x$, for all i and all j : all the intermediate goods have the same productivity in all firms (see (8) for the same propriety at optimum). Moreover, we have $x_t^i(j) = x_t^i$ (each firm i uses the same quantity of intermediate goods), and thus $x_t(j) = \sum_i x_t^i(j) = \sum_i x_t^i = x_t$, for all j : at equilibrium, intermediate goods are produced in the same quantity. Finally, (16) gives the Lindahl price paid by each firm i for a marginal increase in knowledge, that is to say for any new innovation.

b) In the *intermediate goods sector*, the constant returns to scale technology (see (5)) and the perfect competition assumption leads to

$$p_t = r_t. \tag{17}$$

c) Now we consider the research sector. We define the value of an innovation j at t as the sum of the present values of all the expected prices paid for this innovation. This value is

$$H_t = \int_t^\infty v_s e^{-\int_t^s r du} ds, \tag{18}$$

where $v_s = \sum_i v_s^i + \sum_h v_s^h$ is the sum of the Lindhal prices paid at s by all firms in the final sector and in the research sector. Differentiating (18) with respect to t gives $\dot{H}_t = -v_t + r_t H_t$, and thus

$$r_t \quad = \quad \frac{\dot{H}_t}{H_t} + \frac{v_t}{H_t}. \tag{19}$$

The profit on one innovation by a firm h in the research sector is

$$\pi_t^h = q^h(n_t, \ell_t^h) H_t - w_t \ell_t^h - n_t v_t^h$$

Maximizing π_t^h with respect to ℓ_t^h and n_t gives the two following first order conditions :

$$\frac{\partial \pi_t^h}{\partial \ell_t^h} \quad = \quad q_\ell^h H_t - w_t = 0 \tag{20}$$

$$\frac{\partial \pi_t^h}{\partial n_t} \quad = \quad q_n^h H_t - v_t^h = 0 \tag{21}$$

From (20), we see that q_ℓ^h is independant of h : we write it $q_\ell^h = q_\ell$. Differentiating the equality $H_t = w_t / q_\ell$ with respect to t gives

$$\frac{\dot{H}_t}{H_t} \quad = \quad \frac{\dot{w}_t}{w_t} - \frac{\dot{q}_\ell}{q_\ell}. \tag{22}$$

d) Finally, the maximization of the intertemporal utility gives the standard condition :

$$\rho - \frac{u''(c_t)\dot{c}_t}{u'(c_t)} \quad = \quad r_t. \tag{23}$$

Remark : this paragraph has been written assuming that firms of the final sector and of the research sector directly pay the Lindhal prices. This is possible only if their technologies exhibit constant or decreasing returns to scale. If it is not the case, as for instance in standard endogenous growth models where there are increasing returns to scale, it is necessary to make other assumptions. The more simple is to assume that the Lindhal prices are financed by public funds. An other possibility would be to assume that there is imperfect competition in these sectors, but this assumption is beyond the scope of this paper.

3.1.2 Equilibrium and optimum

Our objective now is to show that the two characteristic efficiency conditions obtained in section 2 are verified in this decentralized economy.

First it is easy to show that, as in the standard neoclassical model, the Hotelling rule (8) is here also verified. From (15), $F_x = p_t$, and (17), $r_t = p_t$, we have $F_x = r_t$. From (12), $\dot{p}_t^R / p_t^R = r_t$, and (14), $F_R = p_t^R$, that gives $\dot{F}_R / F_R = \dot{p}_t^R / p_t^R$, we have $r_t = \dot{F}_R / F_R$. Finally, we otbain $F_x = \dot{F}_R / F_R$, that is the Hotelling rule.

Second, we can also show that the new efficiency condition (9) is verified, and we can compute the Lindhal prices paid by each agent.

Using (12), (14), (22), (13) (that gives $\dot{F}_L / F_L = \dot{w}_t / w_t$), and the condition $q_\ell H_t = w_t$, (19) can be written

$$\frac{\dot{F}_R}{F_R} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_\ell}{q_\ell} + v_t \frac{q_\ell}{F_L},$$

$$\text{where} \qquad v_t = \sum_{i=1}^{m} v_t^i + \sum_{h=1}^{M} v_t^h.$$

From (16), (17) and (15), we have $v_t^i = F_X^i f^i(x_t^i(n_t)) - F_x x_t^i(n_t)$, and from (20) and (21) we have $v_t^h = q_n^h F_L / q_\ell$. After substitution, the previous condition becomes

$$\frac{\dot{F}_R}{F_R} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_\ell}{q_\ell} + \sum_{h=1}^{M} q_n^h + \frac{q_\ell}{F_L} \left(\sum_{i=1}^{m} F_X^i f^i(x^i(n)) - F_x \sum_{i=1}^{m} x^i(n) \right),$$

that is exactly the efficiency condition (9). These results confirm our claim that the main difficulty to decentralize this economy comes from the fact that knowledge is a public good.

Remark : if we assume that a representative household maximizes the intertemporal utility $\int_0^\infty u(c_t) e^{-\rho t} dt$, it is easy to verify that the standard Ramsey-Keynes condition is also verified, since from (23), (17) and (15), we have $\rho - \dfrac{u'' \dot{c}}{u'} = F_x$.

Example 2 : Let us go back to the Aghion-Howitt, Schou and Scholz-Ziemes model used in the example 1. From this example, we know that the sum of the Lindhal prices paid by the final sector is $\sum_i v^i = \sum_i F_X^i f^i(x^i(n)) - F_x \sum_i x^i(n) = L((1-\alpha)(R/L)^\nu (x/L)^\alpha$. Similary, it is easy to obtain the sum of the Lindhal prices paid by the research sector : $\sum_h v^h = \beta(1-L)(R/L)^\nu (x/L)^\alpha$. Finally, we have the Lindhal price received by each innovator :

$$v = \sum_i v^i + \sum_h v^h = (R/L)^\nu (x/L)^\alpha (\beta + \nu L).$$

Let us note that all these Lindhal prices decrease at the same rate : $g_v = \nu g_R = \nu \left(\frac{-\rho}{\varepsilon} + \delta \frac{1-\varepsilon}{\varepsilon} \right) < 0$ (see example 1).

3.2 Equilibrium with patents and optimal public policies

We assume now that the markets of the final good (Y), labor (L) and the natural resource (R) are competitive. Concerning the intermediate goods sector we make the standard assumption that, once a new good is invented, it is produced by a monopoly. In order to implement the optimal path obtained in section 2, we use two tools : first, a subsidy (τ) for the demand of each intermediate good ; secondly, a subsidy (σ) to the research.

3.2.1 Agents behaviors

a) In the *final sector*, the firm i profit is

$$\pi_t^i = F^i(L_t^i, R_t^i \int_0^{n_t} f^i(x_t^i(j))dj) - w_t L_t^i - p_t^R R_t^i - \int_0^{n_t} p_t(1-\tau)x_t^i(j)dj.$$

Differentiating π_t^i with respect to L_t^i, R_t^i and $x_t^i(j)$ gives the following first order conditions :

$$F_L^i - w_t = 0, \tag{24}$$

$$F_R^i - p_t^R = 0, \tag{25}$$

$$F_{x(j)}^i - p_t(1-\tau) = 0, \tag{26}$$

where we always have $F_{x(j)}^i = F_X^i f^{i'}(x_t^i(j))$.

These conditions are exactly the conditions (13)-(14)- (15) above, except the term $(1-\tau)$ in (26). Thus, as in the first equilibrium, we have $F_L^i = F_L, F_R^i = F_R$ and $F_{x(j)}^i = F_x$, for all i and all j. Moreover, we have as above $x_t^i(j) = x_t^i$ and $x_t(j) = \sum_i x_t^i(j) = x_t$, for all j.

Observe that (26) implicitely defines the demand of good j by firm i, the slope of which is

$$\frac{\partial x_t^i(j)}{\partial p_t} = \frac{1-\tau}{F_{xx}^i}, \quad \text{for all} \quad j, \tag{27}$$

where F_{xx}^i is the second derivative of F^i with respect to any intermediate good.

b) In the *intermediate goods sector*, the profit at each time t of the monopolist which produces any good j is

$$\pi_t^m = (p_t - r_t)x_t = \left(\frac{F_x}{1-\tau} - r_t\right)x_t, \tag{28}$$

where $x_t = \sum_t x_t^i$ is the total quantity of good.

Using (27), the maximization of π_t^m leads to

$$x_t + \left(\frac{F_x}{1-\tau} - r_t\right)\sum_{i=1}^{m} \frac{1-\tau}{F_{xx}^i} = 0, \tag{29}$$

that gives the profit at its maximum level :

$$\pi_t^m = \frac{-x_t^2}{\sum_i ((1-\tau)/F_{xx}^i)} \tag{30}$$

The value of a firm at t is $V_t = \int_t^\infty \pi_s^m e^{-\int_t^s r u du} ds$. Differentiating with respect to t and rearranging gives

$$r_t = \frac{\dot{V}_t}{V_t} + \frac{\pi_t^m}{V_t} \tag{31}$$

c) In the *research sector*, the profit of a firm h is $\pi_t^h = q^h(n_t, \ell_t^h)V_t - w_t(1-\sigma)\ell_t^h$. The maximization of π_t^h with respect to ℓ_t^h gives $q_\ell^h V_t - w_t(1-\sigma) = 0$. Thus, q_ℓ^h is independant of h and we write it q_ℓ. The first-order condition becomes

$$V_t = \frac{(1-\sigma)w_t}{q_\ell} \tag{32}$$

d) Finally, the maximization of the *intertemporal utility* gives the condition (23) : $\rho - \dfrac{u'' \dot{c}_t}{u'} = r_t$.

3.2.2 Implementation of optimum

In section 2, we have characterized an efficient path by two conditions : the Hotelling rule (8), and the "new" condition (9). It is possible to obtain two similar conditions at equilibrium.

From (25) and (12), we have $\dot{F}_R/F_R = \dot{p}_t^R/p_t^R = r_t$. Simultaneously, from (29) we obtain

$$r_t = \frac{1}{1-\tau}\left(F_x + \frac{x_t}{\sum_i (1/F_{xx}^i)}\right).$$

Thus, we obtain a first condition similar to the Hotelling one :

$$\frac{\dot{F}_R}{F_R} \quad = \quad \frac{1}{1-\tau}\left(F_x + \frac{x_t}{\sum_i (1/F_{xx}^i)}\right). \tag{33}$$

Now we start from (31), $r_t = \dot{V}_t/V_t + \pi_t^m/V_t$ (remember that $r_t = \dot{F}_R/F_R$). From (32), $V_t = (1-\sigma)w_t/q_\ell$, we know that $\dot{V}_t/V_t = \dot{w}_t/w_t - \dot{q}_\ell/q_\ell = \dot{F}_L/F_L - \dot{q}_\ell/q_\ell$. Finally, using (30) and (32) that give π_t^m and V_t, we obtain a second condition similar to the second efficiency condition :

$$\frac{\dot{F}_R}{F_R} \quad = \quad \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_\ell}{q_\ell} - \frac{x_t^2 q_\ell}{(1-\tau)(1-\sigma)F_L \sum_i (1/F_{xx}^i)}. \tag{34}$$

It is now possible to write the system of two equations with which we can compute the two optimal tools, τ and σ. For that, it is necessary that (8) and (33) on one hand, and that (9) and (34) on the other hand, are identical. We obtain

$$\tau = \frac{-x_t}{F_x \sum_i (1/F_{xx}^i)} \tag{35}$$

and

$$\sum_{h=1}^{M} q_n^h + \frac{q_\ell}{F_L}\left(\sum_{i=1}^{m} F_X^i f^i(x_t^i) - x_t F_x\right) = \frac{-x_t q_\ell}{(1-\tau)(1-\sigma)F_L \sum_i (1/F_{xx}^i)} \tag{36}$$

These two equations allow to compute the two optimal tools, σ and τ, as it is shown in the following example.

Example 3 : we come back to the Aghion-Howitt, Schou and Scholz-Ziemes model, and we study the equilibrium balanced growth paths.

First, as at optimum (Example 1), we verify that we obtain their results in this particular case at equilibrium. At optimum, the Hotelling rule (8) became (10) in the example, and the second efficiency condition (9) became (11). Here, the two equilibrium conditions (33) and (34) become

$$g_Y - g_R = \frac{1}{1-\tau}(\alpha^2 L^\beta R^\nu x^{\alpha-1}) \tag{37}$$

$$\text{and} \qquad g_R = g_L + \delta(1-L) - \frac{\alpha(1-\alpha)\delta L}{(1-\tau)(1-\sigma)\beta} \tag{38}$$

From the resource sector behavior, we know that $\dot{F}_R/F_R = g_Y - g_R = r$. From the household behavior, we have also $\varepsilon g_Y + \rho = r$. Thus, $g_R = g_Y(1-\varepsilon) - \rho$.

From the production function $Y = K^\alpha n^{1-\alpha} L^\beta R^\nu$, we obtain $g_Y = \delta(1-L) + \nu g_R/(1-\alpha)$ (remember that $g_L = 0$ at steady state). Plugging these results in (38), we have after some calculations

$$g_Y = \frac{\alpha(1-\alpha)\delta - \alpha\rho\nu + (1-\sigma)(1-\tau)\beta\rho(1+\nu/(1-\alpha))}{\alpha(\beta+\nu\varepsilon) + (1-\sigma)(1-\tau)\beta(1-\nu(1-\varepsilon/\varepsilon(1-\alpha)))} \tag{39}$$

If $\sigma = \tau = 0$ (no public intervention), we obtain the same formula than Schou (formula (4.13)) and Scholz-Ziemes (formula 3.32). However, we can go further because we can now compute the tools which allow to implement the optimal path obtained in the Example 1.

Comparing (10) and (37), we immediately obtain the optimal rate of subsidy to the demand of intermediate goods :

$$\tau = 1 - \alpha \tag{40}$$

that is a standard result in this type of model (see for instance Barro-Sala-I-Martin (1995)). Now, since the optimal rate of growth is $(\delta - \rho)/\varepsilon$ (see example 1), we obtain from (39) the optimal rate of subsidy to the research

$$\sigma = \frac{\varepsilon\delta\nu + \beta(\delta - \rho)}{\varepsilon\delta(1-\alpha) + \nu(\varepsilon\delta + \rho - \delta)} \tag{41}$$

We can see that, under the condition $0 < \delta - \rho < \delta\varepsilon$ (see example 1), we have $0 < \sigma < 1$ (σ is a positive subsidy, and not a tax). In some sense, this result confirms the result of Schou saying that "the market growth rate is smaller than the optimal growth rate" (Appendix 6.3 of Schou's paper).

4 Conclusion

The first objective of this paper was to characterize the efficient paths in an economy including three stocks : a natural resource, physical capital and knowledge. We obtained two condition. The first one is the standard Hotelling rule ; it concerns the arbitrage between natural resource and physical capital. The second one, which concerns the arbitrage between natural resource and knowledge, highlights some characteristics of knowledge. First, knowledge is a public good. Second, it is generally assumed (for instance, in endogenous growth models with innovations) that new knowledge is produced by using several factors, among which labor and the existing stock of knowledge, but not natural resources. Then the new efficiency condition has specific features which bring it nearer the standard Lindhal-Bowen-Samuelson condition, and which takes into account the auto-accumulation of knowledge.

The second objective was to construct equilibria in a decentralized economy. We studied two equilibria. In the first one, we assume that the new knowledge, that is to say innovations, is financed by Lindhal prices. We show that the two characteristics efficiency conditions are satisfied. In some sense, this result confirms that the fundamental problem of decentralization in this economy is the public good nature of knowledge. In the second one, which corresponds to the standard theory, we assume that once a new good is invented, it is produced by a monopoly. Then we show that it suffices to use two tools, a subsidy to the demand of each intermediate good and a subsidy to the research activity, to implement the efficient path.

Along the paper, all the results are obtained in a rather general model. However, using the standard specifications of Schou (1996), Aghion-Howitt (1998) and Scholz-Ziemes (1999), we show that their results are found again. Moreover, we obtain some new results in their example : for instance, we compute the Lindhal prices in the first type of equilibrium, and we give the exact values of the two optimal subsidies in the second type.

Appendices

Appendix A : Efficient paths

The Hamiltonian of the program (7) is

$$\begin{aligned} H &= \sum_{i=1}^{m} R^i + \lambda\left(\sum_i F^i(L^i, R^i, \int_0^n f^i(x^i(j))dj) - c\right) - \mu \sum_{i=1}^{m} R^i + \nu \sum_{h=1}^{M} q^h(n, \ell^h) \\ &\quad + \theta\left[\int_0^n (\sum_{i=1}^{m} x^i(j))dj - K\right] + \eta\left(\sum_{i=1}^{m} L^i + \sum_{h=1}^{M} \ell^h - 1\right) \end{aligned}$$

The first order conditions $\partial H/\partial R^i = 0, \partial H/\partial x^i(j) = 0, \partial H/\partial L^i = 0$, and $\partial \mu/\partial \ell^h = 0$ yield :

$$1 + \lambda F_R^i - \mu = 0 \tag{A.1}$$

$$\lambda F_X^i f^{i'}(x^i(j)) + \theta = 0 \tag{A.2}$$

$$\lambda F_L^i + \eta = 0 \tag{A.3}$$

$$\nu q_\ell^h + \eta = 0 \tag{A.4}$$

Moreover, $\partial H/\partial S = -\dot{\mu}, \partial H/\partial K = -\dot{\lambda}$ and $\partial H/\partial n = \dot{\nu}$ yield

$$-\dot{\mu} = 0 \tag{A.5}$$

$$-\theta = -\dot{\lambda} \tag{A.6}$$

$$\lambda \textstyle\sum_i [F_X^i f^i(x^i(n))] + \nu \sum_h q_n^h + \theta \sum_i x^i(n) = -\dot{\nu} \tag{A.7}$$

A.1 and A.3 show that F_R^i and F_L^i are independant of i; thus we write them F_R and F_L. Similary, A.2 shows that $F_X^i f^{i'}(x^i(j))$ is independant of i and j; we write it F_x. Finally, A.4 shows that q_ℓ^h is independant of h; we write it q_ℓ.

Differentiating A.1 with respect to t, and using A.5, yield $\dot{F}_R/F_R = -\dot{\lambda}/\lambda$. From A.2 and A.6, we have $F_x = -\dot{\lambda}/\lambda$. Thus, we obtain a first condition, that is the standard Hotelling rule :

$$\frac{\dot{F}_R}{F_R} = F_x \tag{A.8}$$

This condition concerns the arbitrage between natural resource and physical capital.

Dividing the two sides of A.7 by λ, we can replace ν/λ by F_L/q_ℓ (from A.3 and A.4), and $\theta \sum x^i(n)/\lambda$ by $-F_x \sum_i x^i(n)$ (from A.2), that yields

$$\sum_i \left(F_X^i f^i(x^i(n)) - F_x x^i(n)\right) + \frac{F_L}{q_\ell} \sum_h q_n^h = -\frac{\dot{\nu}}{\lambda}$$

From A.3 and A.4, we have $\lambda F_L = \nu q_\ell$. Differentiating with respect to t gives $\lambda \dot{F}_L + \lambda \dot{F}_L = \dot{\nu} q_\ell + \nu \dot{q}_\ell$, and thus

$$-\frac{\dot{\lambda}}{\lambda} = \frac{\dot{F}_L}{F_L} - \frac{\dot{\nu} q_\ell}{\lambda F_L} - \frac{\nu \dot{q}_L}{\lambda F_L}$$

Combining these two equations (remember that $-\dot{\lambda}/\lambda = \dot{F}_R/F_R$ and $\nu/\lambda = F_L/q_\ell$) yield the second efficiency condition :

$$\frac{\dot{F}_R}{F_R} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_\ell}{q_\ell} + \sum_{h=1}^{M} q_n^h + \frac{q_\ell}{F_L}\left(\sum_{i=1}^{m} F_X^i f^i(x^i(n)) - F_x \sum_{i=1}^{m} x^i(n)\right) \quad \text{(A.9)}$$

This condition concerns the arbitrage between natural resource and knowledge.

Appendix B : Arbitrage between resource and knowledge

The main objective of this appendix is to give a formal interpretation of the efficiency condition (9) :

$$\frac{\dot{F}_R}{F_R} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_\ell}{q_\ell} + \sum_{h=1}^{M} q_n^h + \frac{q_\ell}{F_L}\left(\sum_{i=1}^{m} F_X^i f^i(x^i(n)) - F_x \sum_{i=1}^{m} x^i(n)\right)$$

We consider two dates, t and $t+\eta$, and two intervals of time, $(t-dt, t)$ and $(t+\eta, t+\eta+dt)$, with $dt \ll \eta$. The social planner faces initial profiles of the different variables, denoted by $\{Y_t\}, \{K_t\}, \{c_t\}, \{R_t\}, \{L_t\}, \{\ell_t\}, \{n_t\}$, and, $x_t(j)$ for $j \in [0, n_t]$. In order to characterize the efficient paths, he modifies the initial profiles in the following manner (see Figure 1).

On $(t-dt, t)$, he increases the labor used in research and he decreases the labor used in the final sector. To maintain constant the level of output, he has to extract more resource. However, the increase in labor devoted to research allows to increase knowledge.

On $(t, t+\eta)$, the first increase in knowledge leads to an acceleration of the auto-accumulation of this good, because the flow of new knowledge at each date depends on the existing stock (see equation (3)). Then, if the profile of final output is always kept unchanged, the increase in knowledge leads to a progressive release of labor that can be also used to accelerate the accumulation of knowledge.

On $(t+\eta, t+\eta+dt)$, the social planner brings back knowledge to its original trajectory, by decreasing the labor used in this sector. Transfering this labor to the final output sector, and keeping always constant this output, he is now able to decrease the flow of extraction.

Finally, an efficient path of the economy is obtained if the initial increase in the flow of extraction is equal to the final decrease in this flow.

a) On $(t-dt,t)$, the social planner increases the labor devoted to research and he decreases the labor used in the final sector. Formally, we have $(\Delta L_t)dt = (-\Delta \ell_t)dt < 0$. This transfer has two consequences.

First, in order to maintain constant the level of output (Y_t), it is necessary to increase the flow of extraction. We obtain

$$(\Delta R_t)dt \;\; = \;\; (\Delta \ell_t)dt\frac{F_L(t)}{F_R(t)} \tag{B.1}$$

Second, the trajectory of knowledge is modified. Since $\dot{n}_t = \sum_h q^h(n_t, \ell_t^h)$, we obtain the new trajectory

$$\tilde{n}_t = n_t + q_\ell(t)(\Delta \ell_t)dt, \tag{B.2}$$

where q_ℓ is the marginal productivity of labor in any firm of this sector.

b) On $(t, t+\eta)$, the social planner keeps unchanged all the initial profiles, except $\{n_t\}$ and $\{\ell_t\}$.

The problem is to calculate $\tilde{n}_{t+\eta}$, that is to say the new level of knowledge at the end of this sub-period.

From $\dot{n}_t = \sum_h q^h(n_t, \ell_t^h)$, we have $\tilde{n}_{t+\eta} = \tilde{n}_t + \int_t^{t+\eta} \sum_h q^h(\tilde{n}_\tau, \tilde{\ell}_\tau)d\tau$, where $\tilde{\ell}_\tau$ is the new trajectory of ℓ_t. This equality can be written

$$\tilde{n}_{t+\eta} = \tilde{n}_t + \sum_h \int_t^{t+\eta} q^h(n_\tau + \tilde{n}_\tau - n_\tau, \ell_\tau^h + \tilde{\ell}_\tau^h - \ell_\tau^h)d\tau$$

Neglecting the second-order terms, we can approximate this expression by

$$\begin{aligned}\tilde{n}_{t+\eta} \;\; &\simeq \;\; \tilde{n}_t \\ &+ \;\; \sum_h \int_t^{t+\eta} q^h(n_\tau, \ell_\tau^h)d\tau + \sum_h \int_t^{t+\eta} (\tilde{n}_\tau - n_t)q_n^h(n_\tau, \ell_\tau)d\tau \\ &+ \;\; \sum_h \int_t^{t+\eta} \tilde{\ell}_\tau^h - \ell_\tau^h)q_\ell^h(n_\tau, \ell_\tau)d\tau\end{aligned}$$

Since $q_\ell^h = q_\ell$ for all h, the last term can be written

$$\int_t^{t+\eta} q_\ell(n_\tau, \ell_\tau) \sum_h (\tilde{\ell}_\tau^h - \ell_\tau^h) d\tau = \int_t^{t+\eta} (\tilde{\ell}_\tau - \ell_\tau) q_\ell(n_\tau, \ell_\tau) d\tau$$

From (B.2), $\tilde{n}_t = n_t + q_\ell(t)(\Delta \ell_t)dt$, and since $n_t = n_{t+\eta} - \sum_h \int_t^{t+\eta} q^h(n_\tau, \ell_t^h) d_\tau$, we obtain finally

$$\begin{aligned} \tilde{n}_{t+\eta} = n_{t+\eta} \quad &+ \quad q_\ell(t)(\Delta \ell_t)dt + \sum_h \int_t^{t+\eta} (\tilde{n}_\tau - n_\tau) q_n^h(n_\tau, \ell_\tau) d\tau \\ &+ \quad \int_t^{t+\eta} (\tilde{\ell}_\tau - \ell_\tau) q_\ell(n_\tau, \ell_\tau) d\tau \end{aligned} \tag{B.3}$$

Our problem now is to calculate $(\tilde{\ell}_\tau - \ell_\tau)$. This can be done by differentiating the production function $Y_\tau^i = F^i(L_\tau^i, R_\tau^i, \int_0^{n_\tau} f^i(x_\tau^i(j)dj)$. We obtain

$$dY_\tau^i = F_L^i dL_\tau^i + F_R^i dR_\tau^i + F_X^i f^i(x_\tau^i(n_{\tau)})dn_\tau + F_X^i \int_0^{n_\tau} f^{i'}(x_\tau^i(j))dx_\tau^i(j)dj$$

Since $\{Y_t\}$ and $\{R_t\}$ are unchanged, we have $dY_\tau^i = 0$ and $dR_\tau^i = 0$. Moreover, we use the fact that F_L^i does not depend on i, and we denote it by F_L. Then we obtain the new labor devoted to research, $\tilde{\ell}_\tau - \ell_\tau = \sum_h (\tilde{\ell}_\tau^h - \ell_\tau^h) = -\sum_i dL_\tau^i$, given by

$$\tilde{\ell}_\tau - \ell_\tau = \frac{\sum_i F_X^i f^i(x_\tau^i(n_\tau)dn_\tau + \sum_i F_X^i \int_0^{n_\tau} f^{i'}(x_\tau^i(j))dx_\tau^i(j)dj}{F_L} \ ,$$

where we also have $dn_\tau = \tilde{n}_\tau - n_\tau$.

In order to calculate the second term in numerator, let us observe that, since $x_\tau^i(j) = x_\tau^i$ for all j, we have

$$\sum_i F_X^i \int_0^{n_\tau} f^{i'}(x_\tau^i(j))dj = \sum_i F_X^i f^{i'}(x_\tau^i) \int_0^{n_\tau} dx_\tau^i(j)dj = F_x \sum_i \int_0^{n_\tau} dx_\tau^i(j)dj,$$

where $F_X^i f^{i'}(x_\tau^i) = F_x$, for all i, is the marginal productivity of any intermediate good in the economy.

Now we remember that each unit of intermediate good is produced with one unit of capital (see (5) above), that gives $K_\tau = \int_0^{n_\tau} (\sum_i x_\tau^i(j))dj$. Differentiating this equality with respect to n_τ and $x_\tau^i(j)$, for all $j \in [0, n_\tau]$, and keeping K_τ constant, gives

$$\sum_i x_\tau^i(n_\tau)dn_\tau + \int_0^{n_\tau} \sum_i dx_\tau^i(j)dj = 0.$$

Then, the second term in numerator above, $F_x \sum_i \int_0^{n_\tau} dx_\tau^i(j)dj$, becomes $-F_x \sum_i x_\tau^i(n_\tau)dn_\tau$, where $dn_\tau = \tilde{n}_\tau - n_\tau$. The new labor devoted to research is now given by

$$\tilde{\ell}_\tau - \ell_\tau = \frac{(\tilde{n}_\tau - n_\tau)(\sum_i F_X^i f^i(x_\tau^i(n_\tau)) - F_x \sum_i x_\tau^i(n_\tau)}{F_L}$$

Plugging this expression in (B.3), we obtain

$$\begin{aligned} \tilde{n}_{t+\eta} &= \tilde{n}_{t+\eta} + q_\ell(t)(\Delta\ell_t)dt + \sum_h \int_t^{t+\eta} (\tilde{n}_\tau - n_\tau)q_n^h(n_\tau, \ell_\tau)d\tau \\ &+ \int_t^{t+\eta} \frac{q_\ell}{F_L}(\tilde{n}_\tau - n_\tau)(\sum_i F_X^i f^i(x_\tau^i(n_\tau)) - F_x \sum x_\tau^i(n_\tau))d\tau, \end{aligned}$$

and thus

$$\begin{aligned} &\tilde{n}_{\tau+\eta} = n_{\tau+\eta} + q_\ell(t)\Delta\ell_t dt \\ &+ \int_t^{t+\eta} (\tilde{n}_\tau - n_\tau)\left[\sum_h q_n^h + \frac{q_\ell}{F_L}(\sum_i F_X^i f^i(x_\tau^i(n_\tau)) - F_x \sum_i x_\tau^i(n_\tau))\right] d\tau \end{aligned} \tag{B.4}$$

c) On $(t + \eta, t + \eta + dt)$, n comes back to its original trajectory, that is to say from $\tilde{n}_{t+\eta}$ to $n_{\tau+\eta+dt}$. This decrease in n allows to release labor from the research sector. The released labor is given by

$$\Delta(\ell_{t+\eta})dt = \frac{n_{t+\eta+dt} - \tilde{n}_{t+\eta}}{q_\ell(t+\eta)},$$

where $n_{t+\eta+dt} - \tilde{n}_{t+\eta}$ is approximatively given by (B.4). This labor can be devoted to the final sector. In other words, we have $(\Delta L_{t+\eta}dt =$

$-(\Delta\ell_{t+\eta})dt$, that gives a decrease in the flow of extraction given by

$$(\Delta R_{t+\eta})dt = -(\Delta L_{t+\eta})dt \frac{F_L(t+\eta)}{F_R(t+\eta)}$$

$$= \frac{n_{t+\eta+dt} - \tilde{n}_{t+\eta}}{q_\ell(t+\eta)} \frac{F_L(t+\eta)}{F_R(t+\eta)} \tag{B.5}$$

Before comparing (B.1) and (B.5), we can give a simplified expression of (B.4). When η is little, we can approximate $\tilde{n}_\tau - n_\tau$ by $q_\ell(t)(\Delta\ell_t)dt$ (see (B.2)). Then from (B.4), we obtain

$$\tilde{n}_{\tau+\eta} - n_{\tau+\eta} \simeq$$
$$q_\ell(t)(\Delta\ell_t)dt \left[1 + \left(\sum_h q_n^h(t) + \frac{q_\ell(t)}{F_L(t)}\right)\left(\sum_i F_X^i f^i(x_t^i(n_t)) - F_x \sum_i x_t^i(n_t)\right)\eta\right].$$

Plugging this expression in (B.5), we write the arbitrage equation that has to be verified along any efficient path, and which says that the initial increase $(\Delta R_t)dt$, given by (B.1), has to be equal to the final decrease $(\Delta R_{t+\eta})dt$, given by (B.4). We obtain

$$\frac{F_L(t)}{F_R(t)} = \frac{q_\ell(t)}{q_\ell(t+\eta)} \left[1 + \Big(\sum_h q_n^h(t) + \frac{q_\ell(t)}{F_L(t)}(\sum_i F_X^i f^i(x_t^i(n_t))\right.$$
$$\left. -F_x \sum_i x_t^i(n_t)\Big)\eta\right] \frac{F_L(t+\eta)}{F_R(t+\eta)}.$$

Since η is little, the term $\left(\frac{F_R(t+\eta)}{F_R(t)} \frac{q_\ell(t+\eta)}{q_\ell(t)}\right) / \frac{F_L(t+\eta)}{F_L(t)}$ can be approximated by $1 + \left(\frac{\dot{F}_R(t)}{F_R(t)} + \frac{\dot{q}_\ell(t)}{q_\ell(t)} - \frac{\dot{F}_L(t)}{F_L(t)}\right)\eta$.

Finally, the arbitrage equation becomes

$$\frac{\dot{F}_R}{F_R} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_\ell}{q_\ell} + \sum_h q_n^h + \frac{q_\ell}{F_L}\left(\sum_i F_X^i f^i(x^i(n)) - F_x \sum_i x^i(n)\right),$$

that is exactly the efficiency condition (9) which concerns the arbitrage between resource and knowledge.

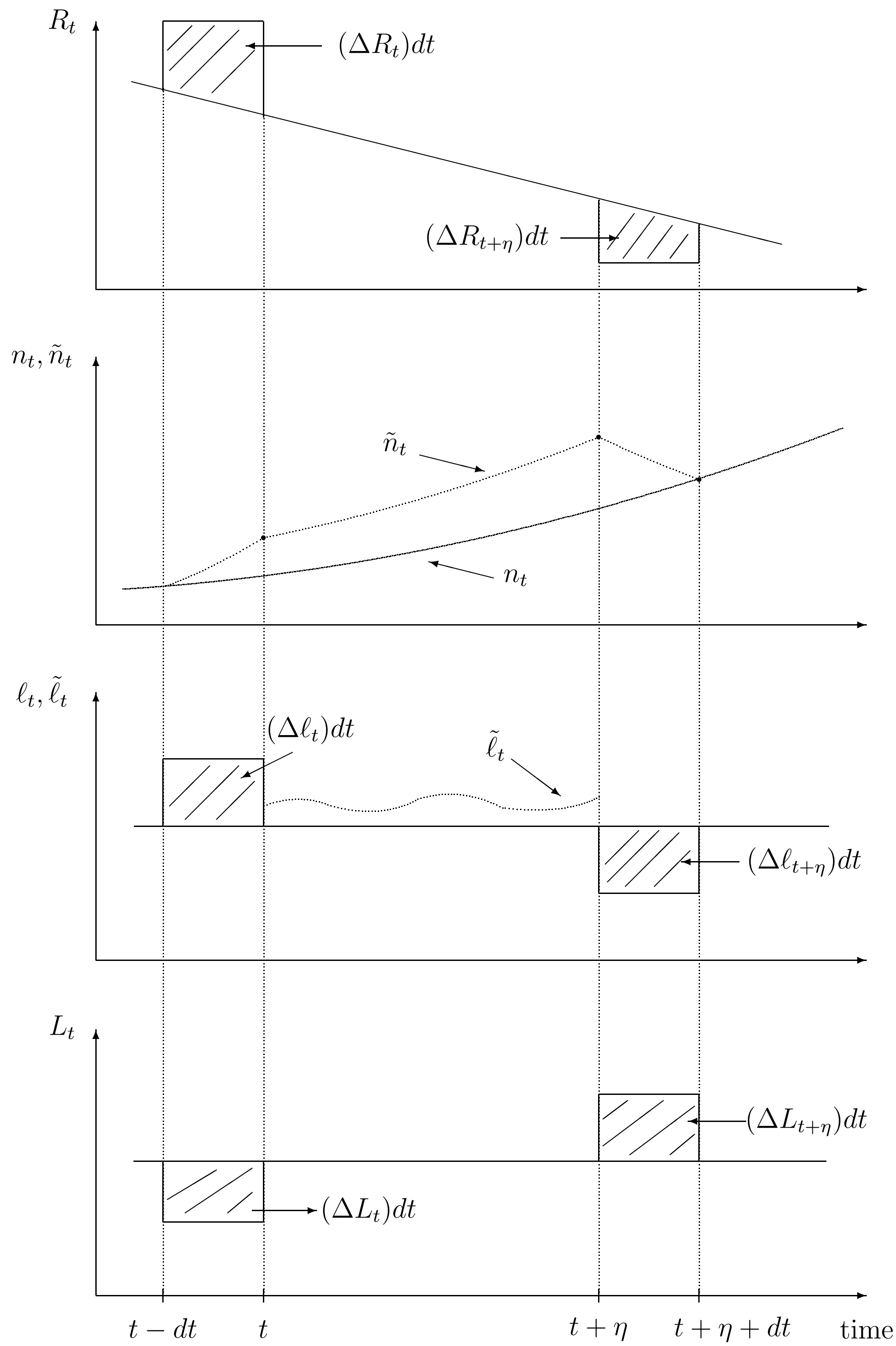

Figure 1 : Efficiency and arbitrage between resource and knowledge

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