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“Demand Shocks, Learning-by-Doing and Exclusion"

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Abstract

This note examines how an exogenous industry-wide demand shock, such as the one resulting from the use of governmental subsidies, affects the exclusionary potential of learning-by-doing. We develop a two-period duopoly model in which an increase in a firm's first-period output leads to a decrease in its second-period marginal cost, and apply it to two special scenarios: one in which demand and learning technologies are linear and one in which firms are infinitely impatient. In the first scenario, we establish that a positive demand shock amplifies the exclusionary effect of learning-by-doing if and only if firms are *sufficiently* asymmetric in their learning abilities. In the second scenario, we emphasize the key role of the demand curvature as a determinant of the effect of a demand shock on the exclusionary potential of learning-by-doing.

Keywords: Demand shocks, learning-by-doing, market structure, exit.
JEL codes: D11, L13, Q42

1 Introduction

This note investigates the effects of industry-wide demand shocks on the well-documented exclusionary potential of learning-by-doing (see e.g., Dasgupta and Stiglitz, 1988; Agliardi, 1990; Cabral and Riordan, 1994, 1997; Petrakis et al., 1997). While the existing literature has investigated thoroughly the effects of learning-by-doing on market structure, it did not explore the way these effects can be altered by demand shocks such as those resulting from governmental subsidies. We believe that it is useful to fill this gap for two reasons. First, the existence of learning-by-doing in some industries is often presented as a justification for the

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desirability of subsidies in those industries. Second, the use of subsidies in such industries may have a more complex impact on market structure than in industries without learning-by-doing.

To illustrate these two reasons, consider the extensive use by several European countries (e.g., Germany, France, Italy and Spain) of generous feed-in-tariffs (FITs) for renewable energy as a way to boost the demand for photovoltaic panels and wind turbines. The rationale behind this policy has been to take advantage of learning effects to drive renewables' costs to the level of fossil fuel energy. In addition to this reason, European governments have also bet on an industrial side-effect: the promotion of national champions, or at least European champions. However, this is not what happened: while a number of photovoltaic panel manufacturers were European before the use of FITs, all major European firms have now exited the market. One potential explanation is that FITs, which benefit both European and non-European firms in the current free-trade environment, may have amplified the differences in efficiency between European and Asian manufacturers, in a way that drove the former out of the market.[1]

The model we develop in this note examines whether (and when) the exclusionary potential of learning-by-doing is amplified by an industry-wide demand shock such as the one induced by the use of FITs. We consider two (potentially asymmetric) firms competing *à la* Cournot over two periods. Each firm's second-period marginal cost depends on its first-period output. We examine whether a positive demand shock will increase or decrease the likelihood that one of the two firms exits in the second period. We first develop a general framework that allows to identify the key variables affecting the way demand shocks alter the exclusionary effect of learning-by-doing. Then, we apply this framework to two special scenarios: one in which demand and learning technologies are linear and one in which firms are infinitely impatient.

The first scenario allows us to shed light on how the effect of a positive demand shock on the exclusionary potential of learning-by-doing depends on the (potential) asymmetry between firms. More specifically, under that scenario, we show that a positive demand shock amplifies the exclusionary potential of learning-by-doing if and only if firms are *sufficiently* asymmetric in terms of their learning abilities. To grasp the intuition behind this finding, consider the special case in which firms have identical first-period marginal costs but differ in their learning abilities. Then, learning-by-doing leads to different second-period marginal costs. The difference between these marginal costs is systematically amplified by a positive demand shock but this does not always lead to an increase in the likelihood that one of the firms exits the market. The reason for this is the following: it is not the difference between the marginal costs of the two firms that matters for the market to become a monopoly in the second period but the difference between the monopoly price of the more efficient firm and the marginal cost of the less efficient firm.

The second scenario allows to abstract from the intertemporal strategic effects and focus on

[1]As shown by the photovoltaic barometer of EurObserv'ER (https://www.eurobserv-er.org/photovoltaic-barometer-2017/) only one European manufacturer (a German-Korean firm) is still ranked in the top 10 firms, which includes six Chinese firms, while there were 6 European firms in the top 10 in 2008.

how the demand shape can affect the way (passive) learning-by-doing is altered by a demand shock. We show that when the difference in the curvatures of the firms' perceived first-period demands is sufficiently large then a positive demand shock will increase the output of one of the firms and decrease the output of its rival. In that case, the shock has an unambiguous effect on the exclusionary potential of learning-by-doing: it amplifies (mitigates) the exclusionary potential of learning-by-doing by the firm whose demand curvature is lower (higher). However, if the difference between the curvatures is relatively small then a positive demand shock leads to an increase in both firms' outputs and, therefore, the sign of the net effect on the exclusionary potential of learning-by-doing becomes ambiguous. We show that this sign depends on the curvature of a monopolist's second period demand and the relative learning abilities of the two firms.

2 General framework

2.1 Setup

Consider two firms A and B producing a homogeneous good and competing over two periods $t = 1, 2$. Denote δ the firms' (common) discount factor and $c_{i,t}$ the (constant) marginal cost of firm $i \in \{A, B\}$ in period $t \in \{1, 2\}$. Suppose that firm i's learning-by-doing curve is given by

$$c_{i,2} = \max(\underline{c}_i, f_i(c_{i,1}, q_{i,1}))$$

for $i = A, B$, where $f_i(c_{i,1}, q_{i,1})$ is differentiable and decreasing in $q_{i,1}$, and $\underline{c}_i$ is firm i's incompressible marginal cost.[2]

Assume that the (inverse) demand is given by $p_1 = P_1(Q_1) + S$ in period 1 and $p_2 = P_2(Q_2)$ in period 2, where $Q_1 = q_{A,1} + q_{B,1}$ and $Q_2 = q_{A,2} + q_{B,2}$. An increase in the parameter S can be interpreted as a (first-period) positive demand shock, which can be induced for instance by the use of demand-side subsidies. Suppose that $P_t(.)$ satisfies the following conditions:

A1 $P_t(.)$ is twice continuously differentiable and decreasing whenever $P_t(Q_t) > 0$.

A2 $P_t(Q_t) \longrightarrow 0$ when $Q_t \longrightarrow +\infty$

A3 $Q_t P_t''(Q_t) + P_t'(Q_t) < 0$ for all $Q_t \geq 0$ such that $P_t(Q_t) > 0$.

These standard conditions ensure in particular the existence and uniqueness of the second-period subgame equilibrium for any level of second-period marginal costs (and therefore, for any level of first-period productions).[3] Denote $R_{i,1}(q_{-i,1}, S)$ the best-response function of firm $i = A, B$ in the first stage of the game when firm i correctly anticipates the second-

[2] This specification of the learning-by-doing process assumes away any spillovers between competitors. However, such effects may exist, in particular in countries where intellectual property right protection is relatively weak.

[3] See for instance Novshek (1985).

period equilibrium, and assume that it is uniquely defined. Moreover, suppose that the two-stage competition game has a unique subgame-perfect equilibrium and denote $q^*_{i,t}(S)$ firm i's equilibrium output in period t. Finally, for the sake of simplicity, assume that the parameters of the model are such that neither firm completes its learning process at the equilibrium (i.e., its marginal cost remains strictly above its incompressible cost).

Let us now examine the way a positive demand shock affects the exclusionary effect of a firm's learning-by-doing. More precisely, we seek to understand whether an increase in the demand parameter S makes it more or less likely that a firm exits the market in period 2.

2.2 Effect of a demand shock on the likelihood of exclusion

Let us first determine under which condition a firm excludes its rival from the market in period 2. We focus hereafter on whether firm B gets excluded or not. The analysis is clearly symmetric for firm A.

Denote $p_2^m(c) = P_2(Q_2^m(c))$ where

$$Q_2^m(c) = \arg\max_{Q_2 \geq 0} \left[P_2(Q_2) - c\right] Q_2$$

which can be shown to be uniquely defined under assumptions **A1-A3**.

Firm B is active in the second period's equilibrium if and only if its marginal cost in that period is below the monopoly price associated to firm A's second-period marginal cost, i.e.,

$$p_2^m(c^*_{A,2}(S)) - c^*_{B,2}(S) > 0 \tag{1}$$

where

$$c^*_{i,2}(S) = f_i\left(c_{i,1}, q^*_{i,1}(S)\right).$$

To determine the effect of a demand shock on the exclusionary potential of learning-by-doing we need to study the effect of a variation in S on the left-hand side (LHS) of (1). If an increase in S leads to an increase (decrease) in the LHS, then (1) becomes less (more) stringent, which implies that the set of parameters for which firm B is driven out of the market in the second period shrinks (expands). This means that the exclusionary potential of firm A's learning-by-doing is mitigated (amplified) by a positive demand shock.

The derivative of the LHS of (1) can be rewritten as

$$\frac{d}{dS}\left(p_2^m(c^*_{A,2}(S)) - c^*_{B,2}(S)\right) = \frac{dp_2^m}{dc_{A,2}}\frac{\partial f_A}{\partial q_{A,1}}\frac{dq^*_{A,1}}{dS} - \frac{\partial f_B}{\partial q_{B,1}}\frac{dq^*_{B,1}}{dS}.$$

A marginal increase in S mitigates the exclusionary potential of firm A's learning-by-doing if the derivative above is positive and amplifies it if it is negative. This, combined with the fact that $\frac{\partial f_B}{\partial q_{B,1}} < 0$, leads to the following lemma.

Lemma 1 *Denote*

$$g_A(S) = -\Omega \frac{dq^*_{A,1}}{dS} + \frac{dq^*_{B,1}}{dS}$$

where

$$\Omega = \frac{dp_2^m}{dc_{A,2}} \frac{\frac{\partial f_A}{\partial q_{A,1}}}{\frac{\partial f_B}{\partial q_{B,1}}}.$$

Then, a positive demand shock, i.e., a marginal increase in S, amplifies (mitigates) the exclusionary potential of firm A's learning-by-doing if $g_A(S) < (>)\ 0$.

The parameter Ω can be interpreted as the learning speed of firm A relative to firm B adjusted by the (second-period) pass-through rate. Lemma 1 shows that the effect of a demand shock on the exclusionary potential of firm A's learning-by-doing depends on a weighted difference between the effect of the shock on the outputs of the two firms, with Ω being the relative weight on firm A's output.

Let us now derive the expressions of Ω, $\frac{dq^*_{A,1}}{dS}$ and $\frac{dq^*_{B,1}}{dS}$, which will allow us to identify the key primitives that affect the impact of a demand shock on the exclusionary potential of learning-by-doing. In the Appendix, we show that

$$\Omega = \frac{1}{2 + E_2^m} \frac{\frac{\partial f_A}{\partial q_{A,1}}}{\frac{\partial f_B}{\partial q_{B,1}}}, \tag{2}$$

where $E_2^m = P_2''\left(Q_2^m(c_{A,2})\right) Q_2^m(c_{A,2}) / P_2'\left(Q^m(c_{A,2})\right)$ is the curvature of the second-period (inverse) demand at the monopoly output.

This shows that the parameter Ω is related to the supply side of the industry through the firm's relative performances in terms of learning-by-doing, and to the demand side through the curvature of the demand. Note that this parameter is constant in the special case of linear demand and learning curves.

We also establish in the Appendix that the effect of a marginal increase in S is given by

$$\frac{dq^*_{i,1}}{dS} = \frac{\frac{\partial R_{i,1}}{\partial S} + \frac{\partial R_{i,1}}{\partial q_{-i,1}} \frac{\partial R_{-i,1}}{\partial S}}{1 - \frac{\partial R_{i,1}}{\partial q_{-i,1}} \frac{\partial R_{-i,1}}{\partial q_{i,1}}}. \tag{3}$$

The term $\frac{\partial R_{i,1}}{\partial S}$ captures the direct effect of an increase in S on firm i's output, while the term $\frac{\partial R_{i,1}}{\partial q_{-i,1}} \frac{\partial R_{-i,1}}{\partial S}$ captures a strategic effect resulting from the reaction of firm $-i$ to an increase in S. Moreover, the formal analysis in the Appendix shows that the term $\frac{\partial R_{i,1}}{\partial q_{-i,1}} \frac{\partial R_{-i,1}}{\partial q_{i,1}}$ in the denominator captures a feedback equilibrium effect.

Finally, we show in the Appendix that the direct effect of an increase in S on firm i's output is

$$\frac{\partial R_{i,1}}{\partial S} = -\frac{1}{P_1'} \frac{1}{E_{i,1}^* + 2 + \frac{\delta}{P_1'} \left[\frac{\partial^2 \pi_{i,2}^*}{\partial c_{i,2}^2} \left(\frac{\partial f_i}{\partial q_{i,1}} \right)^2 + \frac{\partial \pi_{i,2}^*}{\partial c_{i,2}} \frac{\partial^2 f_i}{\partial q_{i,1}^2} \right]} \tag{4}$$

and that the slope of firm i's reaction function, which is a determinant of the strategic effect discussed above, is given by

$$\frac{\partial R_{i,1}}{\partial q_{-i,1}} = -\frac{E_{i,1}^* + 1 + \frac{\delta}{P_1'} \frac{\partial^2 \pi_{i,2}^*}{\partial c_{i,2} \partial c_{-i,2}} \frac{\partial f_i}{\partial q_{i,1}} \frac{\partial f_{-i}}{\partial q_{-i,1}}}{E_{i,1}^* + 2 + \frac{\delta}{P_1'} \left[\frac{\partial^2 \pi_{i,2}^*}{\partial c_{i,2}^2} \left(\frac{\partial f_i}{\partial q_{i,1}} \right)^2 + \frac{\partial \pi_{i,2}^*}{\partial c_{i,2}} \frac{\partial^2 f_i}{\partial q_{i,1}^2} \right]} \tag{5}$$

where $\pi_{i,2}^* (c_{i,2}, c_{-i,2})$ is firm i's equilibrium profit in period 2 when it produces at marginal cost $c_{i,2}$ and its rival produces at marginal cost $c_{-i,2}$, and $E_{i,1}^* = P_1'' \left(Q_{i,1}^* \right) q_{i,1}^* / P_1' \left(Q_{i,1}^* \right)$ is the curvature of the (inverse) demand perceived by firm i at the equilibrium.

Expressions (4) and (5) provide the necessary blocks to compute the effect of the variation in S on the first-period equilibrium quantities using (3). These and expression (2) show which information is needed to compute the sign of $g_A(S)$ in Lemma 1 and, therefore, the (sign of the) effect of a positive demand shock on the exclusionary potential of learning-by-doing. Given the difficulty in deriving further insights within the current general framework, we now consider two special scenarios: one in which demand and learning curves are linear and one in which firms are infinitely impatient.

3 Applications

3.1 Linear demand and learning functions

Assume that both the demand and learning functions are linear:

$$P_t(Q_t) = a_t - Q_t \quad \text{and} \quad f_i(c_{i,1}, q_{i,1}) = c_{i,1} - \lambda_i q_{i,1}$$

where a_t and λ_i are positive parameters. Under this specification, it can be easily checked that the (second-period) pass-through rate is given by $\frac{dp_2^m}{dc_{A,2}} = \frac{1}{2}$, which implies that

$$g_A(S) = -\frac{\lambda_A}{2\lambda_B} \frac{dq_{A,1}^*}{dS} + \frac{dq_{B,1}^*}{dS}.$$

Denote

$$\mu_i \equiv \frac{2}{3} \sqrt{\delta} \lambda_i$$

for $i = A, B$, and assume that $\mu_i < 1$ to ensure the concavity of firms' maximization program. The parameter μ_i is a combined measure of a firm's learning ability and patience and, as shown by the subsequent analysis, turns out to be a sufficient statistic for the determination of the sign of $g_A(S)$.

Expression (4) simplifies in the current scenario to

$$\frac{\partial R_{i,1}}{\partial S} = \frac{1}{2\left(1-\mu_i^2\right)}$$

which implies that the direct effect of a positive demand shock is positive, and (5) becomes

$$\frac{\partial R_{i,1}}{\partial q_{-i,1}} = -\frac{1+\mu_i\mu_{-i}}{2\left(1-\mu_i^2\right)}$$

which implies that quantities remain strategic substitutes in the (linear) Cournot game with (linear) learning-by-doing. In the Appendix, we show that

$$g_A(S) < 0 \Longleftrightarrow \mu_B < \frac{\mu_A}{2-3\mu_A^2} \equiv w(\mu_A) \tag{6}$$

in the region where the equilibrium is stable, i.e., the parameter space defined by $2\max\left(\mu_A^2, \mu_B^2\right) + \mu_A\mu_B < 1$.[4] Note that under this condition,

$$w(\mu_A) < \mu_A \tag{7}$$

for any $\mu_A \neq 0$.[5] Also, notice that comparing μ_A and μ_B amounts to comparing λ_A and λ_B. Therefore, condition (6) and inequality (7) imply that a positive demand shock can only mitigate the exclusionary potential of a firm's learning by-doing if the two firms are symmetric or mildly asymmetric in terms of their learning abilities.[6] The firms need to be *sufficiently* asymmetric regarding their learning abilities for subsidies to amplify the exclusionary effect of its learning-by-doing. This is summarized in the following proposition.

Proposition 1 *Assume that the demand function and the learning-by-doing technology are linear:* $P_t(Q_t) = a_t - Q_t$ *and* $f_i(c_{i,1}, q_{i,1}) = c_{i,1} - \lambda_i q_{i,1}$. *Moreover, suppose that* $2\max\left(\mu_A^2, \mu_B^2\right) + \mu_A\mu_B < 1$ *where* $\mu_i = 2/3\sqrt{\delta}\lambda_i$.

Then, a positive demand shock mitigates (amplifies) the exclusionary potential of firm A's learning-by-doing if $\mu_B > (<) w(\mu_A)$ *where* $w(\mu_A) = \frac{\mu_A}{2-3\mu_A^2} \leq \mu_A$.

[4] This inequality implies that learning-by-doing is not too fast (relatively low λ_i) and/or firms are not too patient (relatively low δ_i).

[5] Applying the inequality $2\max\left(\mu_A^2, \mu_B^2\right) + \mu_A\mu_B < 1$ to $\mu_A = \mu_B$ yields $3\mu_A^2 < 1$, which implies that $w(\mu_A) < \mu_A$ (for $\mu_A \neq 0$).

[6] In this linear setting, this holds regardless of the firms' initial marginal costs.

3.2 Infinitely impatient firms

Suppose in this section that firms are infinitely impatient, i.e., $\delta = 0$. Then, from (3) and (5) it follows that

$$\frac{dq^*_{i,1}}{dS} = -\frac{1}{P'_1}\frac{1}{\left(2+E^*_{i,1}\right)^2}\frac{1}{1-\frac{\partial R_{i,1}}{\partial q_{-i,1}}\frac{\partial R_{-i,1}}{\partial q_{i,1}}}\left(1+E^*_{-i,1}-E^*_{i,1}\right)$$

where

$$\frac{\partial R_{i,1}}{\partial q_{-i,1}} = -\frac{1+E^*_{i,1}}{2+E^*_{i,1}}.$$

Note that $\frac{\partial R_{i,1}}{\partial q_{-i,1}} \in (-1,0)$ under assumptions **A1-A3**,[7] which implies that quantities are strategic substitutes and the equilibrium is stable. Therefore, $\frac{dq^*_{i,1}}{dS}$ has the same sign as

$$1+E^*_{-i,1}-E^*_{i,1}.$$

First, if $E^*_{A,1}-E^*_{B,1} < -1$, then the asymmetry between the firms (in favor of firm A) is so strong that the effect of a positive demand shock on firm A's output is positive while the effect on firm B's output is negative. In this case, a positive demand shock clearly amplifies the exclusionary potential of firm A's learning-by-doing.

Second, if $E^*_{A,1}-E^*_{B,1} > 1$, then the asymmetry between the firms (in favor of firm B) is so strong that the effect of a positive demand shock on firm A's output is negative while the effect on firm B's output is positive. In this case, a positive demand shock mitigates the exclusionary potential of firm A's learning-by-doing.

Finally, if $E^*_{A,1}-E^*_{B,1} \in (-1,1)$ then the asymmetry between firms A and B is sufficiently moderate for the effect of a positive demand shock to be qualitatively the same for both firms (the outputs of both firms increase). In this case, whether the demand shock amplifies or mitigates the exclusionary potential of firm A's learning-by-doing depends on the comparison between the learning-adjusted pass-through rate Ω and the relative increase in firm B's output with respect to firm A's output, which is given by

$$\frac{1-\left(E^*_{B,1}-E^*_{A,1}\right)}{1-\left(E^*_{A,1}-E^*_{B,1}\right)}.$$

These findings can be summarized in the following proposition.

[7] This follows from the fact that $E^*_{i,1} > -1$ or, equivalently, $P''_1\left(Q^*_{i,1}\right)q^*_{i,1}+P'_1\left(Q^*_{i,1}\right) < 0$, under assumptions **A1-A3**. To see why, note first that if $P''_1\left(Q^*_{i,1}\right) \geq 0$, then $P''_1\left(Q^*_{i,1}\right)q^*_{i,1}+P'_1\left(Q^*_{i,1}\right) \leq P''_1\left(Q^*_{i,1}\right)Q^*_{i,1}+P'_1\left(Q^*_{i,1}\right) < 0$ where the latter inequality follows from **A3**. Moreover, if $P''_1\left(Q^*_{i,1}\right) < 0$ then $P''_1\left(Q^*_{i,1}\right)q^*_{i,1}+P'_1\left(Q^*_{i,1}\right) < P'_1\left(Q^*_{i,1}\right) < 0$ by **A1**. Therefore, in both cases, $E^*_{i,1} > -1$.

Proposition 2 *Assume that firms are infinitely impatient (i.e., $\delta_i = 0$ for $i = A, B$). Then the effect of a positive demand shock on the exclusionary potential of firm A's learning-by-doing depends on the relative learning abilities and the curvatures of the demand functions as follows:*

1. *If $E^*_{A,1} - E^*_{B,1} < -1$, then a positive demand shock amplifies the exclusionary potential of firm A's learning-by-doing.*

2. *If $E^*_{A,1} - E^*_{B,1} > 1$, then a positive demand shock mitigates the exclusionary potential of firm A's learning-by-doing.*

3. *If $E^*_{A,1} - E^*_{B,1} \in (-1, 1)$, then a positive demand shock amplifies (mitigates) the exclusionary potential of firm A's learning-by-doing if the learning-adjusted pass-through effect dominates (is dominated by) the relative increase of firm A's output with respect to firm B's output, i.e.,*

$$\Omega > (<) \frac{1 - \left(E^*_{B,1} - E^*_{A,1}\right)}{1 - \left(E^*_{A,1} - E^*_{B,1}\right)}.$$

4 Conclusion

We develop a general framework to study the effect of demand shocks on the exclusionary potential of learning-by-doing. Applying this framework to two special scenarios provides us with two key insights. First, even if firms are asymmetric in their learning abilities, a positive demand shock can mitigate the exclusionary effect of learning-by-doing. Second, relaxing the assumption that the demand function is linear (a standard assumption in the literature on learning-by-doing) allows us to uncover the key role of the demand curvature as a determinant of the effect of a demand shock on the exclusionary potential of learning-by-doing. Finally, note that demand shocks in our model can be reinterpreted as (unit) cost shocks such as supply-side subsidies or corporate taxes. Our findings suggest in particular that taxation can have counterintuitive effects on market structure in the presence of learning-by-doing.

5 Appendix

Computation of $\frac{dq^*_{i,1}}{dS}$. Consider first the term Ω. For a given marginal cost $c_{A,2}$, the monopoly quantity $Q^m_2(c_{A,2})$ satisfies the FOC

$$P'_2\left(Q^m_2(c_{A,2})\right) Q^m_2(c_{A,2}) + P_2\left(Q^m_2(c_{A,2})\right) - c_{A,2} = 0.$$

Differentiating this with respect to $c_{A,2}$ leads to

$$\frac{dQ^m_2}{dc_{A,2}} = \frac{1}{P''_2\left(Q^m_2(c_{A,2})\right) Q^m_2(c_{A,2}) + 2P'_2\left(Q^m(c_{A,2})\right)}.$$

From $p_2^m(c_{A,2}) = P_2(Q^m(c_{A,2}))$ it then follows that

$$\frac{dp_2^m}{dc_{A,2}} = P_2'(Q_2^m(c_{A,2}))\frac{dQ_2^m}{dc_{A,2}} = \frac{P_2'\left(Q_2^m(c_{A,2})\right)}{P_2''\left(Q_2^m(c_{A,2})\right)Q_2^m(c_{A,2}) + 2P_2'\left(Q^m(c_{A,2})\right)}$$

which yields $\frac{dp_2^m}{dc_{A,2}} = \frac{1}{2+E_2^m}$ where $E_2^m = \frac{P_2''\left(Q_2^m(c_{A,2})\right)Q_2^m(c_{A,2})}{P_2'\left(Q^m(c_{A,2})\right)}$. Thus, we get that $\Omega = \frac{1}{2+E_2^m}\frac{\frac{\partial f_A}{\partial q_{A,1}}}{\frac{\partial f_B}{\partial q_{B,1}}}$.

Let us now examine the terms $\frac{dq_{A,1}^*}{dS}$ and $\frac{dq_{B,1}^*}{dS}$. Differentiating $q_{i,1}^*(S) = R_{i,1}\left(q_{-i,1}^*(S), S\right)$ with respect to S yields

$$\frac{dq_{i,1}^*}{dS} = \frac{\partial R_{i,1}}{\partial S} + \frac{\partial R_{i,1}}{\partial q_{-i,1}}\frac{dq_{-i,1}^*}{dS}. \tag{8}$$

Applying (8) to i and $-i$ leads to

$$\frac{dq_{i,1}^*}{dS} = \frac{\partial R_{i,1}}{\partial S} + \frac{\partial R_{i,1}}{\partial q_{-i,1}}\left(\frac{\partial R_{-i,1}}{\partial S} + \frac{\partial R_{-i,1}}{\partial q_{i,1}}\frac{dq_{i,1}^*}{dS}\right)$$

which yields

$$\frac{dq_{i,1}^*}{dS} = \frac{\frac{\partial R_{i,1}}{\partial S} + \frac{\partial R_{i,1}}{\partial q_{-i,1}}\frac{\partial R_{-i,1}}{\partial S}}{1 - \frac{\partial R_{i,1}}{\partial q_{-i,1}}\frac{\partial R_{-i,1}}{\partial q_{i,1}}}.$$

Computation of $\frac{\partial R_{i,1}}{\partial S}$ and $\frac{\partial R_{i,1}}{\partial q_{-i,1}}$. Firm i's total discounted profit is given by

$$\pi_i\left(q_{i,1}, q_{-i,1}, S\right) = \left(P_1\left(q_{i,1} + q_{-i,1}\right) + S - c_{i,1}\right)q_{i,1} + \delta\pi_{i,2}^*\left(f_i\left(c_{i,1}, q_{i,1}\right)\right), f_{-i}\left(c_{-i,1}, q_{-i,1}\right))$$

The FOC defining $R_{i,1}(q_{-i,1}, S)$ is therefore given by

$$\begin{aligned}&P_1'\left(q_{-i,1} + R_{i,1}(q_{-i,1}, S)\right)R_{i,1}(q_{-i,1}, S) + P_1\left(q_{-i,1} + R_{i,1}(q_{-i,1}, S)\right) + S - c_{i,1} + \\ &\delta\frac{\partial \pi_{i,2}^*}{\partial c_{i,2}}\left(f_i\left(c_{i,1}, R_{i,1}(q_{-i,1}, S)\right), f_{-i}\left(c_{-i,1}, q_{-i,1}\right)\right)\frac{\partial f_i}{\partial q_{i,1}}\left(c_{i,1}, R_{i,1}(q_{-i,1}, S)\right) = 0,\end{aligned} \tag{9}$$

where $\pi_{i,2}^*\left(c_{i,2}, c_{-i,2}\right)$ is firm i's equilibrium profit in period 2 when it produces at marginal cost $c_{i,2}$ and its rival produces at marginal cost $c_{-i,2}$. Differentiating (9) with respect to S leads to

$$\frac{\partial R_{i,1}}{\partial S} = -\frac{1}{P_1''R_{i,1} + 2P_1' + \delta\left[\frac{\partial^2\pi_{i,2}^*}{\partial c_{i,2}^2}\left(\frac{\partial f_i}{\partial q_{i,1}}\right)^2 + \frac{\partial \pi_{i,2}^*}{\partial c_{i,2}}\frac{\partial^2 f_i}{\partial q_{i,1}^2}\right]}$$

Therefore, at the equilibrium we have

$$\frac{\partial R_{i,1}}{\partial S} = -\frac{1}{P_1'}\frac{1}{E_{i,1}^* + 2 + \frac{\delta}{P_1'}\left[\frac{\partial^2\pi_{i,2}^*}{\partial c_{i,2}^2}\left(\frac{\partial f_i}{\partial q_{i,1}}\right)^2 + \frac{\partial \pi_{i,2}^*}{\partial c_{i,2}}\frac{\partial^2 f_i}{\partial q_{i,1}^2}\right]}.$$

where $E^*_{i,1} = \frac{P''_1(Q^*_{i,1})q^*_{i,1}}{P'_1(Q^*_{i,1})}$ is the curvature of the (inverse) demand perceived by firm i at the equilibrium.
Differentiating (9) with respect to $q_{-i,1}$ yields

$$\frac{\partial R_{i,1}}{\partial q_{-i,1}} = -\frac{P''_1 R_{i,1} + P'_1 + \delta_i \frac{\partial^2 \pi^*_{i,2}}{\partial c_{i,2}\partial c_{-i,2}} \frac{\partial f_{-i}}{\partial q_{-i,1}} \frac{\partial f_i}{\partial q_{i,1}}}{P''_1 R_{i,1} + 2P'_1 + \delta \left[\frac{\partial^2 \pi^*_{i,2}}{\partial c^2_{i,2}} \left(\frac{\partial f_i}{\partial q_{i,1}} \right)^2 + \frac{\partial \pi^*_{i,2}}{\partial c_{i,2}} \frac{\partial^2 f_i}{\partial q^2_{i,1}} \right]}.$$

Then, at the equilibrium,

$$\frac{\partial R_{i,1}}{\partial q_{-i,1}} = -\frac{E^*_{i,1} + 1 + \frac{\delta}{P'_1} \frac{\partial^2 \pi^*_{i,2}}{\partial c_{i,2}\partial c_{-i,2}} \frac{\partial f_i}{\partial q_{i,1}} \frac{\partial f_{-i}}{\partial q_{-i,1}}}{E^*_{i,1} + 2 + \frac{\delta}{P'_1} \left[\frac{\partial^2 \pi^*_{i,2}}{\partial c^2_{i,2}} \left(\frac{\partial f_i}{\partial q_{i,1}} \right)^2 + \frac{\partial \pi^*_{i,2}}{\partial c_{i,2}} \frac{\partial^2 f_i}{\partial q^2_{i,1}} \right]}.$$

Sign of $g_A(S)$. From (3) it follows that

$$\frac{dq^*_{i,1}}{dS} = \frac{\frac{1}{2(1-\mu_i^2)} - \frac{1+\mu_i\mu_{-i}}{2(1-\mu_i^2)} \times \frac{1}{2(1-\mu_{-i}^2)}}{1 - \frac{(1+\mu_i\mu_{-i})^2}{4(1-\mu_i^2)(1-\mu_{-i}^2)}} = \frac{1 - 2\mu_{-i}^2 - \mu_i\mu_{-i}}{4\left(1-\mu_i^2\right)\left(1-\mu_{-i}^2\right) - \left(1+\mu_i\mu_{-i}\right)^2}$$

For the equilibrium to be stable we need the slopes of the reaction curves to be less than 1 in absolute value. This requires that $\frac{1+\mu_i\mu_{-i}}{2(1-\mu_i^2)} < 1$ for $i = A, B$, which can be rewritten as

$$2\max\left(\mu_A^2, \mu_B^2\right) + \mu_A\mu_B < 1 \tag{10}$$

This assumption implies that $\frac{dq^*_{i,1}}{dS} > 0$. Therefore, the sign of $g_A(S)$ is the same as the sign of

$$\frac{\frac{dq^*_{B,1}}{dS}}{\frac{dq^*_{A,1}}{dS}} - \frac{\lambda_A}{2\lambda_B} = \frac{1 - 2\mu_A^2 - \mu_A\mu_B}{1 - 2\mu_B^2 - \mu_A\mu_B} - \frac{\mu_A}{2\mu_B} = \frac{2\mu_B - \mu_A - 3\mu_A^2\mu_B}{2\mu_B\left(1 - 2\mu_B^2 - \mu_A\mu_B\right)}.$$

Thus, using again (10) we get that

$$g_A(S) < 0 \Longleftrightarrow \mu_B < \frac{\mu_A}{2 - 3\mu_A^2} \equiv w(\mu_A).$$

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