Cross-Licensing and Competition*

Doh-Shin Jeon[†] Yassine Lefouili[‡]

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Abstract

We analyze the competitive effects of bilateral cross-licensing agreements in a setting with many competing firms. We show that firms can sustain the monopoly outcome if they can sign unconstrained bilateral cross-licensing contracts. This result is robust to increasing the number of firms who can enter into a cross-licensing agreement. We also investigate the scenario in which a cross-licensing contract cannot involve the payment of a royalty by a licensee who decides ex post not to use the licensed technology. Finally, policy implications regarding the antitrust treatment of cross-licensing agreements are derived.

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†Toulouse School of Economics, University of Toulouse Capitole and CEPR. E-mail: dohshin.jeon@gmail.com

‡Toulouse School of Economics, University of Toulouse Capitole. E-mail: yassine.lefouili@tse-fr.eu

1 Introduction

A cross-license is an agreement between two firms that allows each to use the other's patents (Shapiro, 2001; Régibeau and Rockett, 2011). Cross-licensing has long been a common practice. For instance, Taylor and Silberston (1973) report that cross-licensing accounts for a significant share of all licensing arrangements in many industries: 50% in the telecommunications and broadcasting industry, 25% in the electronic components industry, 23% in the pharmaceutical industry, etc.[1] Cross-licensing is therefore likely to have an impact on competition in a large number of sectors.

Cross-licensing agreements involve both technological and monetary transfers. Technological transfers are generally perceived as pro-competitive: they can result in goods being produced at lower costs by potentially more firms. These transfers are particularly useful in Information Technology (IT) industries, such as the semiconductor and mobile phone industries, where the intellectual property rights necessary to market a product are typically held by a large number of parties, a situation known as a *patent thicket* (Shapiro, 2001; DOJ and FTC, 2007; Galasso and Schankerman, 2010).[2] Monetary transfers, however, can be anticompetitive. More specifically, high per-unit royalties can allow firms to sustain high prices.

The following natural question arises: do cross-licensing partners have incentives to agree on high royalties? The existing literature provides an answer to this question in a duopoly setting: in that case, two firms can sign a cross-licensing agreement that specifies royalties high enough to replicate the monopoly profit (Katz and Shapiro, 1985; Fershtman and Kamien, 1992). This monopolization result can be generalized in a straightforward way to a setting with more than two firms signing a multilateral agreement involving all of them (see Section 2.2).

However, in practice, we often see *bilateral* cross-licensing in industries with more than two firms. In this setting, would any pair of firms agree on high royalties that might weaken their competitive positions *vis-à-vis* their rivals? We build a model to investigate whether bilateral cross-licensing agreements can still allow firms to sustain the monopoly outcome in this scenario.

[1] In particular, cross-licensing in the semiconductor industry has received much attention in the literature (Grindley and Teece, 1997; Hall and Ziedonis, 2001; Galasso, 2012).

[2] According to FTC (2011, pp.55-56), "The IT patent landscape involves products containing a multitude of components, each covered by numerous patents. ... This contrasts with the relationship between products and patents in the pharmaceutical and biotech industries where innovation is generally directed at producing a discrete product covered by a small number of patents." Patent thickets raise many concerns and are considered as one of the most crucial intellectual property issues of the day (Shapiro, 2007; Régibeau and Rockett, 2011).

We consider $N(> 2)$ competing firms owning one patent each. Firms can get access to the technologies covered by their rivals' patents through cross-licensing agreements, before competing in a product market. We suppose that the larger the set of patented technologies a firm has access to, the lower its marginal cost.[3]

We assume that cross-licensing contracts are private, i.e., their terms are observable only to the parties signing them, and focus on *bilaterally efficient* agreements. A set of cross-licensing agreements is said to be bilaterally efficient if each agreement maximizes the joint profit of the pair of firms who sign it, given all other agreements.[4] Note that a firm's overall profit is composed of the profit it makes from selling its product and the revenues generated by the licensing of its technology.

In Section 2, we analyze our baseline model in which firms are symmetric and engage in Cournot competition. We focus on symmetric equilibria where any two distinct firms sign a cross-licensing contract and every firm pays the same royalty to any other firm. Two firms in a given coalition can indirectly affect their joint output through the royalties they charge each other. When deciding these royalties, they take into account two opposite effects: the *coordination effect*, which captures the idea that the two firms have joint incentives to restrict their joint output below its non-cooperative equilibrium level, and the *royalty-saving effect* which refers to the idea that the coalition's marginal cost is lower than each of its member's marginal cost because the royalties the two firms charge each other are internal transfers within the coalition. The royalty-saving effect provides incentives to reduce the royalties charged by each firm to the other one, whereas the coordination effect provides incentives to increase these royalties. We show that these two effects cancel out when the (symmetric) per-unit royalty is equal to the one that *maximizes the industry profit*. This implies that the monopoly outcome can be sustained through bilaterally efficient cross-licensing agreements.

We show that this monopolization result extends to an environment in which cross-licensing agreements can be signed by coalitions of any size (Section 3.1). We also establish in Section 3.3 that this finding holds in a general two-stage game that applies to any situation in which firms that have interactions in a product market sell inputs to each other through bilateral agreements. Examples include not only cross-licensing of patents but also two-way access pricing in telecommunications (Armstrong, 1998; Laffont, Rey, Tirole, 1998a,b), interconnection among Internet backbone companies (Crémer, Rey and Tirole, 2000) and interbank payments for the use of ATMs (Donze and Dubec, 2006). Section 3.4 further extends the general two-stage game to two overlapping networks of bilateral agreements.

[3] Via a change in variables, there is a way to interpret the model so that, rather than being cost reducing, the patented technology enhances consumers' valuation for the product being sold. See the Appendix for details.

[4] A more precise statement is provided in Definition 1.

In Section 3.2, we provide an extension of the baseline model which incorporates *ex post usage constraints*. In the baseline model, firm i pays to firm j the royalty specified in their cross-licensing agreement regardless of whether the former uses the latter's patented technology. This can lead to royaties that are higher than the cost reduction derived from the use of a given licensed technology. However, competition authorities usually prohibit the use of royalties that are disproportionate with respect to the market value of the license. Therefore, we introduce an ex post usage constraint such that the royalty is paid only if firm i uses firm j's patented technology. We show that our previous result extends naturally in the sense that there exists an equilibrium in which every patented technology is licensed at a symmetric royalty and every firm uses all the patented technologies. The symmetric royalty is equal to the minimum between the monopoly royalty and the highest royalty that satisfies the ex post usage constraint for all patented technologies. However, the equilibrium symmetric royalty becomes smaller as patents become more substitutable. In particular, for (almost) perfectly substitutable patents, the symmetric equilibrium leads (almost) to the most competitive outcome. For this reason, we also study whether bilateral cross-licensing can lead to the exit of some firm(s) in a setting where $N = 3$ hold almost perfectly substitutable patents. We find that the monopoly outcome cannot be sustained but the duopoly outcome can be sustained through bilateral cross-licensing agreements.

We lay out the policy implications of our findings in Section 4. Both American and European competition authorities grant antitrust safety zone to (cross-) licensing agreements signed by firms whose combined market share is below a certain threshold.[5] These policies are partly based on the presumption that market forces can discipline cross-licensing partners regarding the level of royalties they agree on: firms with relatively low market power are expected to find it unprofitable to charge each other high per-unit royalties. Our analysis shows that such disciplining effect does not exist when firms can engage in multiple cross-licensing agreements. Therefore, it does not support the use of antitrust automatic exemptions for bilateral cross-licensing agreements based *only* on a market-share criterion. Moreover, our findings suggest that cross-licensing contracts that require licensees to pay per-unit royalties regardless of the actual use of the licensed technology should not be exempted as they allow firms to sustain the monopoly outcome through bilateral cross-licensing agreements. Finally, our analysis shows that alleviating the collusive potential of bilateral cross-licensing

[5] For instance, Article 10 of the EC Technology Transfer Block Exemption Regulation provides antitrust exemption to bilateral licensing agreements between competitors if their combined market share does not exceed 20%. Similarly, according to the US guidelines (DOJ and FTC, 1995, p.22), "... the Agencies will not challenge a restraint in an intellectual property licensing arrangement if (1) the restraint is not facially anticompetitive and (2) the licensor and its licensees collectively account for no more than twenty percent of each relevant market significantly affected by the restraint."

agreements may come at the cost of increasing their exclusionary potential.

All proofs are relegated to the Appendix.

2 Baseline Model

2.1 Setting

Consider an industry consisting of $N \geq 3$ symmetric firms producing a homogeneous good. Each firm owns one patent covering a cost-reducing technology and can get access to its rivals' patented technologies through cross-licensing agreements. We assume that the patents are symmetric in the sense that the marginal cost of a firm only depends on the number of patented technologies it has access to. Let $c(n)$ be a firm's marginal cost when it has access to a number $n \in \{1, ..., N\}$ of technologies with $c(N)(\equiv \underline{c}) \leq c(N-1) \leq ... \leq c(1)(\equiv \bar{c})$.

We consider a two-stage game in which, prior to engaging in Cournot competition, each pair of firms can sign a cross-licensing agreement whereby each party gets access to the patented technology of the other one. More precisely, the two-stage game is described as follows:

- Stage 1: *Cross-licensing*

Distinct firms i and j decide whether to sign a cross-licensing contract and determine the terms of the contract if any. We assume that a bilateral cross-licensing contract between firm i and firm j specifies a pair of royalties $(r_{i\to j}, r_{j\to i}) \in [0, +\infty) \times [0, +\infty)$, and a lump-sum transfer $F_{i\to j} \in \mathbf{R}$,[6] where the notation $i \to j$ indicates that firm i is paying firm j. Note that $F_{i\to j} = F < 0$ is equivalent to $F_{j\to i} = |F| > 0$ (i.e., a payment from j to i).[7] All bilateral negotiations occur simultaneously.

- Stage 2: *Competition in the product market*

[6] According to Shapiro (2001), "[c]ross licenses may or may not involve fixed fees or running royalties; running royalties can in principle run in one direction or both (Shapiro, 2001, p. 127)." Moreover, the FTC found that "... nearly half of the Wireless Manufacturer licenses included a running-royalty, and nearly a third included running-royalty and lump-sum payments (FTC, 2016, p.118)."

[7] The lump-sum transfers make it possible to separate internal distribution of profit from joint profit maximization, which justifies the solution concept we use later (see Definition 1). However, the result and the analysis in Section 2 hold even if we assume that a cross-licensing contract can use per-unit royalties only. This is because we focus on symmetric equilibria, and Lemma 1 shows that, without loss of generality, we can focus on deviations by two-firm coalitions that involve the payment of the same royalty by each firm of the coalition to the other one. However, when bilateral cross-licensing induces some firms not to be active either because firms have asymmetric costs (in Section 3.2) or because of the presence of ex post usage constraints (in Section 3.3), joint profit maximization may require asymmetric royalties and then a fixed fee would be necessary to share the surplus.

Firms compete *à la* Cournot with the cost structure inherited from stage 1. We assume that cross-licensing agreements are private: the terms of the agreement between firms i and j are known only to these two firms and, therefore, each firm $k \neq i, j$ forms beliefs about those terms before competing with its rivals in the product market.

We assume that the firms face an inverse demand function $P(\cdot)$ satisfying the following standard conditions (Novshek, 1985):

A1 $P(\cdot)$ is twice continuously differentiable and $P'(\cdot) < 0$ whenever $P(\cdot) > 0$.

A2 $P(0) > \bar{c} > \underline{c} > P(Q)$ for Q sufficiently large.

A3 $P'(Q) + QP''(Q) < 0$ for all $Q \geq 0$ with $P(Q) > 0$.

These mild assumptions ensure the existence and uniqueness of a Cournot equilibrium $(q_i^*)_{i=1,\dots,n}$ satisfying the following (intuitive) comparative statics properties, where c_i denotes firm i's marginal cost (see e.g., Amir, Encaoua and Lefouili, 2014):

i) $\frac{\partial q_i^*}{\partial c_i} < 0$ and $\frac{\partial q_i^*}{\partial c_j} > 0$ for any $j \neq i$; $\frac{\partial Q^*}{\partial c_i} < 0$ for any i, where $Q^* = \sum_i q_i^*$ is the total equilibrium output;

ii) $\frac{\partial \pi_i^*}{\partial c_i} < 0$ and $\frac{\partial \pi_i^*}{\partial c_j} > 0$ for any $j \neq i$, where π_i^* is firm i's equilibrium profit.

2.2 Benchmark: multilateral cross-licensing agreement

We consider here as a benchmark the case of a multilateral licensing agreement among all firms. This corresponds to a *closed* patent pool (Lerner and Tirole, 2004), i.e., a patent pool whose only customers are its contributors. We focus on a symmetric outcome where all firms pay the same royalty r to each other.

Let $P^m(\underline{c})$ be the monopoly price when each firm's marginal cost is $\underline{c}$. It is characterized by

$$\frac{P^m(\underline{c}) - \underline{c}}{P^m(\underline{c})} = \frac{1}{\varepsilon(P^m(\underline{c}))}, \tag{1}$$

where $\varepsilon(.)$ is the elasticity of demand.

Given a symmetric royalty r, each firm's marginal cost is $\underline{c} + (N-1)r$. The firms will agree on a royalty to achieve the monopoly price. Given a symmetric royalty r, firm i chooses its output q_i in the second stage to maximize $[P(Q_{-i} + q_i) - \underline{c} - (N-1)r]\, q_i + rQ_{-i}$ where $Q_{-i} \equiv Q - q_i$ is the quantity chosen by all other firms. Let r^m be the royalty that leads to the monopoly price $P^m(\underline{c})$. Then, from the first-order condition associated with firm i's maximization program, we have

$$\frac{P^m(\underline{c}) - \underline{c} - (N-1)r^m}{P^m(\underline{c})} = \frac{1}{\varepsilon(P^m(\underline{c}))N}. \tag{2}$$

From (1) and (2), r^m is determined by

$$\frac{P^m(\underline{c}) - \underline{c}}{N} = r^m. \tag{3}$$

Proposition 1 *(Multilateral cross-licensing). Suppose that all N firms in an industry jointly agree on a symmetric royalty. Then the royalty to which they would agree, r^m, is $1/N$th of the monopoly markup (i.e., $r^m = (P^m(\underline{c}) - \underline{c})/N$). The ensuing equilibrium price is the monopoly price, $P^m(\underline{c})$.*

2.3 Bilateral cross-licensing agreements

We first define our solution concept.

Definition 1 *(Bilateral efficiency) A set of cross-licensing agreements is bilaterally efficient if, for any $i, j \in \{1, 2, ..., N\}$ such that $i \neq j$, firms i and j cannot increase their joint profit by changing the agreement between them, holding constant all the other agreements, and the beliefs of all firms about the agreements they are not involved in.*

As any bilateral agreement can include the payment of a fixed fee, it is reasonable to assume that a bilateral agreement signed between a pair of firms maximizes their joint profit.[8] Given the private nature of the agreements, a deviation by a two-firm coalition in the cross-licensing stage is not observed by its rivals who keep the same beliefs about the agreements made by their competitors. Moreover, when a coalition of two firms deviates by changing the terms of the agreement between them, each of these two firms maintains the same beliefs about all other agreements. This assumption is the counterpart in our setting of the usual passive-belief assumption in the literature on vertical contracting (Hart and Tirole, 1990; McAfee and Schwartz, 1994).

Notice first that any given pair of firms finds it (jointly) optimal to sell a license to each other. To see why, assume that, initially, firm i does not license its patent to firm j. These two firms can (weakly) increase their joint profit if firm i licenses its patent to firm j by specifying the payment of a per-unit royalty $r_{j\to i}$ equal to the reduction $\Delta_{j\to i}$ in marginal cost generated by firm j's use of the technology covered by firm i's patent.[9] Such licensing agreement would not affect the firms' marginal costs of production but would allow them to

[8] Our solution concept is similar to the concept of *contract equilibrium* (Crémer and Riordan, 1987; O'Brien and Shaffer, 1992) and *pairwise-proof contracts* (McAfee and Schwartz, 1994). It is also closely related to the concept of *Nash equilibrium in Nash bargains* used in the bilateral monopoly/oligopoly literature (Horn and Wolinsky, 1988; Inderst and Wey, 2003; Collard-Wexler, Gowrisankaran and Lee, 2017).

[9] In any pure-strategy equilibrium (including the symmetric equilibrium we focus on), firm i anticipates correctly the number of technologies that firm j has access to and, therefore, can compute $\Delta_{j\to i}$.

save (jointly) $\Delta_{j\to i}$ per unit of output produced by firm j. It will therefore (weakly) increase their joint profit.[10]

In what follows, we consider a symmetric situation where every pair of firms signs a bilateral cross-licensing agreement specifying the same royalty. Let r denote the per-unit royalty paid by any firm $i \in \{1, ..., N\}$ to have access to the technology of a firm $j \in \{1, ..., N\} \setminus \{i\}$, and $S(r, N)$ denote the corresponding set of symmetric cross-licensing agreements. We below study the incentives of a two-firm coalition to deviate from the symmetric royalty r under the assumption that all firms are active (i.e., produce a positive output) no matter what royalties the deviating coalition chooses.[11]

The next lemma shows that it is sufficient to focus on deviations such that the firms in a deviating coalition pay the same royalty to each other. Indeed, the joint payoff from any asymmetric deviation can be replicated by a symmetric one because the joint payoff depends on the royalties paid by each firm to the other one only through their *sum*.

Lemma 1 *Consider a symmetric set of cross-licensing agreements $S(r, N)$. The joint payoff a coalition $\{i, j\}$ gets from a deviation to a cross-licensing agreement with royalties $(\hat{r}_{i\to j}, \hat{r}_{j\to i})$ depends on these royalties only through their sum $\hat{r}_{i\to j} + \hat{r}_{j\to i}$.*

Denote $\{1, 2\}$ the coalition formed by firms 1 and 2 and Q^*_{-12} the joint equilibrium output of the firms outside this coalition when all firms charge each other a royalty r. Consider a deviation by coalition $\{1, 2\}$ and let $\hat{r}$ be the royalty that firms 1 and 2 charge each other. Because agreements are private, the total equilibrium output of the other firms is still $Q^*_{-12}(r)$. Therefore, the equilibrium joint output $Q^*_{12}(r, \hat{r})$ of the deviating pair is such that $\frac{Q^*_{12}(r,\hat{r})}{2}$ is the individual equilibrium output in a symmetric duopoly with inverse demand function $\tilde{P}(Q_{12}) = P\left(Q^*_{-12}(r) + Q_{12}\right)$. In other words, $Q^*_{12}(r, \hat{r})$ is given by the following first-order condition:

$$P'\left(Q^*(r,\hat{r})\right)\frac{Q^*_{12}(r,\hat{r})}{2} + P\left(Q^*(r,\hat{r})\right) - \left(\underline{c} + (N-2)\,r + \hat{r}\right) = 0, \tag{4}$$

where $Q^*(r, \hat{r}) \equiv Q^*_{12}(r, \hat{r}) + Q^*_{-12}(r)$ denotes the total industry output in the (second-stage) Cournot equilibrium. Then, the considered set of symmetric agreements is bilaterally efficient if and only if:

$$r \in \arg\max_{\hat{r}\geq 0}\ \pi_{12}(r,\hat{r})$$

where

$$\pi_{12}(r,\hat{r}) \equiv \left[P\left(Q^*(r,\hat{r})\right) - \left(\underline{c} + (N-2)\,r\right)\right] Q^*_{12}(r,\hat{r})$$

[10] The joint profit increases strictly whenever $\Delta_{j\to i} > 0$ and firm j initially produces a positive quantity.

[11] We study equilibria in which some firm(s) are not active when we consider asymmetric costs/technologies in Section 3.2 and when we introduce an ex post usage constraint in Section 3.3.

is the profit of the deviating coalition. This leads to the following equivalence result:

Lemma 2 *A symmetric set of cross-licensing agreements $S(r,N)$ is bilaterally efficient if and only if*

$$Q_{12}^*(r,r) \in \underset{Q_{12}\in\left[0,Q_{12}^*(r,0)\right]}{\arg\max} \left[P\left(Q_{-12}^*(r)+Q_{12}\right)-\left(\underline{c}+(N-2)\,r\right)\right]Q_{12}+2rQ_{-12}^*(r).$$

Note that in Lemma 2, $Q_{12}^*(r,\hat{r})$ increases from 0 to $Q_{12}^*(r,0)$ as $\hat{r}$ decreases from $+\infty$ to 0. The lemma means that a set of cross-licensing agreement is bilaterally efficient if and only if the joint output of a two-firm coalition in the Cournot equilibrium maximizes the coalition's profit.

2.4 Incentives to deviate

We now study the incentives of coalition $\{1,2\}$ to marginally expand or contract its output with respect to $Q_{12}^*(r,r)$. Note that the coalition's marginal cost at the cross-licensing stage is $\underline{c}+(N-2)\,r$ whereas each of its member's marginal cost at the Cournot competition stage is $\underline{c}+(N-1)\,r$. The difference between the two has to do with the royalty payment between firms 1 and 2. In what follows, we call rQ_{12} the *royalty saving* of the coalition (as compared to a single firm producing the same quantity Q_{12}).

The coalition's profit can be rewritten as

$$\pi_{12}\left(Q_{12},r\right)=\underbrace{\left[P\left(Q_{12}+Q_{-12}^*(r)\right)-\left(\underline{c}+(N-1)\,r\right)\right]Q_{12}}_{\pi_{12}^P(Q_{12},r)}+\underbrace{r\left[Q_{12}+2Q_{-12}^*(r)\right]}_{\pi_{12}^T(Q_{12},r)}. \quad (5)$$

The term $\pi_{12}^P\left(Q_{12},r\right)$ represents the coalition's profit in the product market.[12] The term $\pi_{12}^T(Q_{12},r)$ represents the coalition's profit in the technology (licensing) market, which is composed of the royalty saving and the licensing revenues received from all firms outside the coalition. We below study the effect of a (local) variation of Q_{12} on each of the two sources of profit.

- Effect on the profit in the product market

The partial derivative of $\pi_{12}^P\left(Q_{12},r\right)$ with respect to Q_{12}, when evaluated at $Q_{12}^*(r,r)$, is given by

[12] The profit in the product market is defined with respect to the individual marginal cost of each member of the coalition at Stage 2. This facilitates our analysis because we can use each firm's first-order condition at the Cournot competition stage (i.e., condition (4) where $\hat{r}$ is set at r).

$$\frac{\partial \pi_{12}^P}{\partial Q_{12}}(Q_{12}^*(r,r),r) = P'\left(Q^*(r,r)\right) Q_{12}^*(r,r) + \left[P\left(Q^*(r,r)\right) - \left(\underline{c} + (N-1)\, r\right)\right]. \tag{6}$$

This derivative represents the marginal profit of the coalition in the product market and captures a *coordination effect*: the coalition has an incentive to reduce output below the Cournot level $Q_{12}^*(r,r)$ because the joint output of the coalition when each member chooses its quantity in a non-cooperative way is too large with respect to what maximizes its joint profit in the product market. Indeed, using (4) evaluated at $\hat{r} = r$, we find that

$$\frac{\partial \pi_{12}^P}{\partial Q_{12}}(Q_{12}^*(r,r),r) = -\left[P\left(Q^*(r,r)\right) - \left(\underline{c} + (N-1)\, r\right)\right] < 0. \tag{7}$$

- Effect on the profit in the technology market

Let us now turn to the effect of a local variation in Q_{12} on the coalition's profit in the technology market $\pi_{12}^T(Q_{12})$. We have:

$$\frac{\partial \pi_{12}^T}{\partial Q_{12}}(Q_{12}^*(r,r),r) = r \geq 0. \tag{8}$$

A marginal increase in Q_{12} results in a royalty saving equal to r. We call this the *royalty-saving effect*, which is strictly positive for any $r > 0$.

It follows from (7) and (8) that for any $r > 0$, a marginal increase in Q_{12} has two opposite effects on the coalition's overall profit: the profit in the product market decreases whereas the profit in the technology market increases.

By summing up (7) and (8), the total effect of a marginal increase in Q_{12} on the coalition's profit can be written as:

$$\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}^*(r,r),r) = \left[\underline{c} + Nr - P\left(Q^*(r,r)\right)\right]. \tag{9}$$

The coordination effect dominates the royalty-saving effect if $f(r,N) \equiv \underline{c} + Nr - P\left(Q^*(r,r)\right) < 0$. For instance, when $r = 0$, the coordination effect dominates because the profit in the technology market is zero and we have $f(0,N) = \underline{c} - P\left(Q^*(0,0)\right) < 0$. As r increases, the royalty saving effect becomes more important: we show below that $\frac{\partial f}{\partial r}(r,N) = N - \frac{dQ^*}{dr} P'\left(Q^*\right)$ is strictly positive.

Summing the first-order conditions for each firm i' s maximization program from $i = 1$

to $i = N$ yields

$$P'(Q^*)\,Q^* + NP(Q^*) - N(\underline{c} + (N-1)r) = 0. \tag{10}$$

Differentiating (10) with respect to r leads to

$$\frac{dQ^*}{dr}\left[P'(Q^*) + P''(Q^*)\,Q^*\right] + N\left[P'(Q^*)\,\frac{dQ^*}{dr} - (N-1)\right] = 0.$$

From $P'(Q^*)+P''(Q^*)\,Q^* < 0$ (by **A3**) and $\frac{dQ^*}{dr} < 0$, it follows that $P'(Q^*)\,\frac{dQ^*}{dr} - (N-1) < 0$, which implies that $\frac{\partial f}{\partial r}(r, N) > 0$ for any $N \geq 3$. As $f(r, N)$ strictly increases with r, the solution in r to $f(r, N) = 0$ is unique whenever it exists.

Surprisingly, it turns out that the unique royalty r for which $f(r, N) = 0$ is the fully cooperative royalty r^m, defined in (3). At $r = r^m$, we have $P(Q^*(r^m, r^m)) = P^m(\underline{c})$ and, therefore, $\underline{c} + Nr^m - P(Q^*(r^m, r^m)) = 0$. Thus, the coordination effect dominates the royalty-saving effect for $r < r^m$ whereas the reverse holds for $r > r^m$.

2.5 Bilaterally efficient royalties

From the previous analysis of local deviations, we know that there are two possible cases depending on whether the coordination effect dominates, or is dominated by, the royalty-saving effect. The local analysis above allowed us to identify r^m a the unique candidate for a symmetric bilaterally efficient royalty.[13] A global analysis (i.e., looking at global, rather than only local, deviations) confirms that this candidate is indeed bilaterally efficient. This follows from the fact that assumptions **A1** and **A3** ensure that $\pi_{12}(Q_{12}, r)$ is concave in Q_{12} (see the proof of Proposition 2 in the Appendix). Therefore, we obtain the following result:

Proposition 2 *(Bilaterally efficient agreements) In the baseline model, a set $S(r, N)$ of symmetric cross-licensing agreements is bilaterally efficient if and only if $r = r^m$.*

This proposition shows that the monopoly outcome is always sustainable through bilaterally efficient cross-licensing agreements. Let us now provide the intuition for this finding. The monopoly output Q^m is defined by the following first-order condition:

$$P'(Q^m)Q^m + P(Q^m) = \underline{c}. \tag{11}$$

[13] We say that a symmetric royalty r is bilaterally efficient if the set of cross-licensing agreements $S(r, N)$ is bilaterally efficient.

Moreover, because r^m leads to the monopoly outcome when firms compete in quantity in the second stage, the first-order condition with respect to a single firm's output, q_i, is given by:

$$P'(Q^m)\frac{Q^m}{N} + P(Q^m) = \underline{c} + (N-1)\, r^m. \tag{12}$$

Therefore, an increase in a firm's marginal cost of production by $(N-1)\, r^m$ makes it act as if it were internalizing the effects of its decision on its $(N-1)$ rivals. This implies that the payment of a per-unit royalty r^m to each rival has the same effect as internalizing the impact of a price reduction on that rival. Formally, this amounts to writing

$$-P'(Q^m)\frac{Q^m}{N} = r^m, \tag{13}$$

which follows immediately from (11) and (12).

Suppose now that two distinct firms i and j contemplate a joint deviation in the cross-licensing stage. By agreeing on some royalties $(r_{i\to j}, r_{j\to i})$, they can choose a joint output $q_i + q_j$ different from $2Q^m/N$. However, it turns out that the first-order condition for the coalition's maximization program is satisfied exactly at $q_i + q_j = 2Q^m/N$:

$$P'(Q^m)\frac{2Q^m}{N} + P(Q^m) = \underline{c} + (N-2)\, r^m, \tag{14}$$

which is easily derived from (12) by adding $P'(Q^m)\frac{Q^m}{N}$ to its L.H.S. and substracting r^m from its R.H.S. The intuition for this result is as follows. Consider two firms jointly deciding the royalties they charge each other. On the one hand, they internalize the competitive externalities they exert on each other in the competition stage. This is the coordination effect. On the other hand, a two-firm coalition's marginal cost is lower than a single firm's marginal cost by r^m, which gives the coalition an incentive to increase its output. This is the royalty-saving effect. These two effects cancel out because the payment of r^m is exactly equivalent to internalizing the effect of price reduction on one rival firm.

3 Extensions

In this section, we provide several extensions of the baseline model to assess the robustness of our main result.

3.1 k-efficient agreements

In this extension, we investigate cross-licensing agreements that are *k-efficient* in the sense that no coalition of size $k \in \{3, ..., N-1\}$ finds it optimal to change the terms of the cross-licensing agreements among its members. Note that the case of $k = 2$ corresponds to the bilateral efficiency criterion whereas $k = N$ corresponds to industry-profit maximization.

Let r be the symmetric royalty that every firm pays to each other firm as part of their cross-licensing agreement. Consider the deviation of a coalition composed of $K = \{1, ..., k\}$ in the first stage. Lemma 1 continues to hold in the case of a coalition of size k. Therefore, without loss of generality, we can restrict attention to deviations involving a symmetric royalty $\hat{r}$. For given $(r, \hat{r})$, let $Q_K^*(r, \hat{r})$ denote the sum of the outputs of the firms in the coalition in the (second-stage) Cournot equilibrium. Let $Q_{-K}^*(r)$ denote the joint equilibrium output of all firms outside the coalition when they expect all firms to charge each other a royalty r.

Then, the coalition's profit can be rewritten as

$$\pi_K(Q_K, r) = \underbrace{\left[P\left(Q_K + Q_{-K}^*(r)\right) - \left(\underline{c} + (N-1)\,r\right)\right] Q_K}_{\pi_K^P(Q_k, r)} + \underbrace{r\left[(k-1)Q_K + Q_{-K}^*(r)\right]}_{\pi_K^T(Q_k, r)}.$$

Consider the coalition's incentives to expand or contract its output Q_k with respect to $Q_K^*(r, r)$. We have

$$\frac{\partial \pi_K^P}{\partial Q_K}(Q_K^*(r, r), r) = -\left[P\left(Q^*(r, r)\right) - \left(\underline{c} + (N-1)\,r\right)\right](k-1)$$

where $Q^*(r, \hat{r}) = Q_K^*(r, \hat{r}) + Q_{-K}^*(r)$, and

$$\frac{\partial \pi_K^T}{\partial Q_K}(Q_K^*(r, r), r) = r\,(k-1)\,.$$

Summing up the two terms leads to

$$\frac{\partial \pi_K}{\partial Q_K}(Q_K^*(r, r), r) = (k-1)\left[\underline{c} + Nr - P\left(Q^*(r, r)\right)\right]. \tag{15}$$

The important point is that at $r = r^m$, the bracket term in the R.H.S. of (15) is zero *regardless of the coalition size*: $\underline{c} + Nr^m - P\left(Q^*(r^m, r^m)\right) = 0$. This, combined with the fact that $\pi_K(Q_K, r)$ is concave in Q_K,[14] leads to the following result:

[14] The proof for the concavity of $\pi_K(Q_K, r)$ with respect to Q_K is similar to the proof for the concavity of $\pi_{12}(Q_{12}, r)$ with respect to Q_{12} (see the proof of Proposition 2).

Proposition 3 *(k-efficient agreements) In the baseline model, there exists a unique k-efficient set of symmetric agreements, in which all firms charge each other the royalty r^m.*

The intuition for Proposition 3 is similar to that for Proposition 2. A coalition of size k internalizes the effect of price reduction on $k-1$ more firms than a single firm does but the marginal cost of the coalition is lower by $(k-1)r^m$ than that of a single firm. Therefore, the two opposite effects cancel out for any k because the payment of r^m is equivalent to internalizing the effect of price reduction on one additional firm.

3.2 Ex post usage constraint

In our baseline model we assume that after firm i signs a cross-licensing contract with firm j, the former pays the per-unit royalty $r_{i\to j}$ regardless of whether it uses or not the latter's patented technology. Such an agreement can raise the suspicion of antitrust authorities for two reasons. First, if a royalty is paid by a licensee that does not use the licensed technology, then the corresponding licensing contract is a sham contract whose only effect is to increase artificially marginal costs in order to sustain higher prices. Second, even when the technology is actually used by the licensee (in equilibrium), the mere fact that the royalty would be paid even if the technology were not used makes it possible for firms to sustain royalties that are *above* the cost reduction resulting from the use of the technology. This could trigger an antitrust investigation because competition authorities usually prohibit the use of royalties that are disproportionate with respect to the market value of the licensed patents.[15]

In this extension of the baseline model, we study the scenario in which a per-unit royalty is paid to the licensor only if the licensed technology is actually used by the licensee: in other words, a royalty is paid to the licensor only if it satisfies the licensee's *ex post usage constraint.*[16] To account for this constraint, we modify the game as follows:

- Stage 1: *Cross-licensing*

[15] Note that prohibiting licensing contracts whereby a per-unit royalty is paid by the licensee regardless of the actual use of the technology is an indirect way of enforcing an antitrust rule prohibiting the payment of royalties that are above the cost reduction resulting from the use of the licensed technology.

[16] Formally, a firm i who has signed cross-licensing agreements with a set L_i of rivals will choose to use the technologies of a subset $\tilde{L}_i \subseteq L_i$ of these firms in order to minimize its marginal cost

$$c\left(1 + card\left(\tilde{L}_i\right)\right) + \sum_{j\in\tilde{L}_i} r_{i\to j},$$

where $card\left(\tilde{L}_i\right)$ denotes the number of firms in $\tilde{L}_i$. Note that the optimal subset $\tilde{L}_i$ may be empty and that it may not be unique.

Distinct firms i and j decide whether to sign a cross-licensing contract and determine the terms of the contract if any. A bilateral cross-licensing agreement between firm i and firm j specifies a pair of royalties $(r_{i\to j}, r_{j\to i}) \in \mathbf{R}^2_+$ and a lump-sum transfer $F_{i\to j} \in \mathbf{R}$. All bilateral negotiations occur simultaneously. The lump-sum transfer $F_{i\to j}$ is paid regardless of whether the patented technology of j (respectively, of i) is used by i (respectively, by j).

- Stage 2: *Competition in the product market*

All firms simultaneously select the technologies they use among those they have access to and choose their output. We assume that at the end of this stage, after all productions occurred, any firm j can observe whether or not its patented technology has been used by any firm i $(\neq j)$. The per-unit royalty $r_{i\to j}$ is paid only if firm j's patented technology is used by firm i.

We continue to assume that bilateral cross-licensing agreements are private. In this subsection, we assume that the marginal benefit from an additional technology is (weakly) decreasing in the number of technologies a given firm has access to:

$$c(1) - c(2) \geq c(2) - c(3) \geq ... \geq c(N-1) - c(N). \tag{16}$$

In what follows, we first study a symmetric equilibrium in which any distinct firms i and j use each other's technology and pay each other a symmetric royalty r. Consider the benchmark scenario where firms sign a multilateral cross-licensing agreement involving all of them. Suppose that the agreement induces all firms to pay the same royalty to each other and to use each other's patented technology. Then, it is straightforward to show that it is optimal for the firms to agree on a symmetric royalty equal to $\min\{r^m, c(N-1) - c(N)\}$.[17] In the case of bilateral cross-licensing, we have the following result:

Proposition 4 *(Ex post usage constraint and no exclusion) Assume that condition (16) holds and that the ex post usage constraint must be satisfied for a royalty to be paid. Then, a set $S(r, N)$ of symmetric cross-licensing agreements in which every firm uses all the technologies is bilaterally efficient if and only if $r = \min\{r^m, c(N-1) - c(N)\}$.*

This proposition again establishes the equivalence between multilateral cross-licensing and bilateral cross-licensing conditional on every patented technology being licensed to and used by all firms.

[17] This is optimal conditional on every firm using all the technologies. Of course, if $c(k) - c(k-1)$ decreases very quickly with k, it may be optimal for the firms to design the multilateral agreement such that not all firms are active. We discuss this later in this subsection.

However, in the benchmark scenario of multilateral cross-licensing, firms may prefer to sign an agreement which limits the access of some firm(s) to the available patented technologies. To see why, consider the extreme case in which patents are perfectly substituable in the sense that $c(N-1) - c(N) = 0$. In this case, the only multilateral agreement in which all technologies are used by all firms is the one which specifies a royalty $r = 0$. Such an agreement would lead to the most competitive outcome because all firms compete with a marginal cost equal to $\underline{c}$. Firms may be able to achieve a higher industry profit by designing a multilateral agreement such that some of them do not use all technologies. In particular, if $P^m(\underline{c}) < \bar{c}$, the multilateral agreement that specifies

$$r_{1\to j} = 0 \text{ and } r_{j\to 1} = r_{i\to j} = c(1) - c(2) \text{ for } i, j \neq 1, i \neq j.$$

generates a cost structure $(c_1, c_2, ..., c_N) = (\underline{c}, \bar{c}, ..., \bar{c})$ and, consequently, induces the exit of all firms $j \neq 1$ and leads to the highest industry profit (i.e., the monopoly profit at the lowest possible cost $\underline{c}$).

Therefore, it is interesting to study whether bilateral cross-licensing agreements can lead to the exit of some firms when the condition $P^m(\underline{c}) < \bar{c}$ holds. We address this question below in a simple setting where the number of firms is $N = 3$ and $c(1) - c(2) > c(2) - c(3) = \varepsilon > 0$, and in which ε is sufficiently small such that all three firms are active for any $(c_1, c_2, c_3) \in [\underline{c}, \underline{c} + 2\varepsilon]^3$,[18] and $P^m(\underline{c} + \varepsilon) < \bar{c}$. The following proposition summarizes our results.

Proposition 5 *(Ex post usage constraint and exclusion) Suppose that $N = 3$ and $c(1) - c(2) > c(2) - c(3) = \varepsilon > 0$ where ε is sufficiently small such that all three firms are active for any $(c_1, c_2, c_3) \in [\underline{c}, \underline{c} + 2\varepsilon]^3$ and $P^m(\underline{c} + \varepsilon) < \bar{c}$. Moreover, assume that the ex post usage constraint must be satisfied for a royalty to be paid.*

(i) No set of bilaterally efficient cross-licensing agreements can sustain an outcome in which only one firm is active.

(ii) There exists a set of bilaterally efficient cross-licensing agreements which leads to an outcome in which only two firms are active. More specifically, the set of cross-licensing agreements characterized by

$$\begin{aligned} r_{1\to 2} &= r_{2\to 1} = \varepsilon, r_{1\to 3} = r_{2\to 3} = 0, r_{3\to 1} = r_{3\to 2} = c(1) - c(2), \\ F_{1\to 2} &= F_{1\to 3} = F_{2\to 3} = 0, \end{aligned}$$

is bilaterally efficient and leads to an outcome where only firms 1 and 2 are active.

[18] This assumption implies $P^m(\underline{c}) > \underline{c} + 2\varepsilon$.

This proposition shows that the monopoly outcome cannot be sustained by a set of bilaterally efficient agreements. To explain why, consider a candidate equilibrium in which firm 1 is the only firm which is active in stage 2. Firms 2 and 3 have joint incentives to change the cross-licensing agreement between them so that they become active in stage 2: they will still get the same licensing revenues from firm 1 (who will keep producing the same output as before because the agreements are private) and, in addition, they will make positive market profits. Moreover, firms 2 and 3 have the joint *ability* to become active: by signing a cross-licensing agreement such that $r_{2\to 3} = r_{3\to 2} = 0$, they have marginal costs that do not exceed $\underline{c} + \varepsilon$, which allows them to be active in stage 2.

However, the duopoly outcome can be sustained. The bilateral cross-licensing agreements described in Proposition 5(ii) generates the following cost structure $(c_1, c_2, c_3) = (\underline{c}+\varepsilon, \underline{c}+\varepsilon, \overline{c})$, which implies that firm 3 is not active. Surprisingly, firms 1 and 2 can exclude firm 3 without making any compensation to it. The agreements are bilaterally efficient as no coalition has an incentive to deviate. First, the coalition $\{1, 2\}$ has no incentive to deviate. To see this, note that $r_{i\to 3} = 0$ for $i = 1, 2$ implies that no matter the agreement between firm 1 and 2, firm i is active as its marginal cost cannot be higher than $\underline{c} + \varepsilon$. Given this, bilateral efficiency requires firm 1 and 2 to use $(r_{1\to 2}, r_{2\to 1})$ to reduce the most their joint output while satisfying the ex post usage constraints, which is achieved by setting $r_{1\to 2} = r_{2\to 1} = \varepsilon$. Second, the coalition of firms 1 and 3 has no incentive to deviate either. No matter the agreement between firms 1 and 3, the lowest marginal cost of the coalition is $\underline{c} + \varepsilon = c_1$ and the joint profit maximization requires to produce the best response output at this marginal cost with respect to $q_2 = q^D(\underline{c}+\varepsilon, \underline{c}+\varepsilon)$, the output chosen by firm 2 in the duopoly structure. This best-response output is exactly what firm 1 produces in the equilibrium candidate.

The following general message can be obtained from Proposition 4 and 5. If the multilateral agreement that maximizes the industry profit induces all firms to be active and to use all technologies, this outcome can also be achieved by bilateral agreements. This situation arises in particular when the ex post usage constraint is not binding for the equilibrium royalty. In contrast, if the ex post usage constraint limits severely the royalties that firms can charge each other, industry profit maximization under a multilateral agreement may require some firms not to be active. In particular, when $P^m(\underline{c}) < \overline{c}$, a multilateral agreement making only one firm use all technologies at zero per-unit royalty and no other firm use any of its competitors' technologies leads to the monopoly outcome (associated to the lowest possible marginal cost $\underline{c}$). In this case, there is a (significant) difference between a multilateral agreement and a set of bilateral agreements: the most exclusionary outcome, i.e., the monopoly outcome, may not be sustainable by bilateral agreements although some firm(s) may be excluded under bilateral agreements.

3.3 A general class of games

We now develop a general model that can also be applied to situations different from the cross-licensing of patents, and show that our main result extends to that general setting. Consider the following two-stage game played by $N(>2)$ number of firms:

- Stage 1 (bilateral agreements in the input market): Every pair of distinct firms (i, j) signs a bilateral agreement which specifies a pair of input prices $(r_{i\to j}, r_{j\to i})$ as well as a pair of fixed transfers $(F_{i\to j}, F_{j\to i})$. All bilateral negotiations occur simultaneously.

- Stage 2 (competition in the product market): Firms choose non-cooperatively and simultaneously their actions x_i. At this stage, each firm i only knows the input prices $(r_{i\to j}, r_{j\to i})$ and the fixed transfers $(F_{i\to j}, F_{j\to i})$ involving it.

Let $\boldsymbol{r} \equiv ((r_{i\to j}, r_{j\to i}))_{1\le i<j<N}$, $\boldsymbol{F} \equiv ((F_{i\to j}, F_{j\to i}))_{i\neq j}$, $\mathbf{x} = (x_i)_i$ and denote $\Pi_i(\mathbf{x},\boldsymbol{r},\boldsymbol{F})$ player i's payoff function. Moreover, let $\mathbf{x}_{-ij}$ denote the vector obtained from vector $\mathbf{x}$ by removing x_i and x_j and $\boldsymbol{r}_{-ij}$ the vector obtained from $\boldsymbol{r}$ by removing $r_{i\to j}$ and $r_{j\to i}$.

We set the following assumptions regarding the effects of transfers on payoffs:

G1 For any i, there exists a function π_i such that, for any $(\mathbf{x},\boldsymbol{r},\boldsymbol{F})$, $\Pi_i(\mathbf{x},\boldsymbol{r},\boldsymbol{F}) = \pi_i(\mathbf{x},\boldsymbol{r}) + \sum\limits_{j\neq i}(F_{j\to i} - F_{i\to j})$

G2 For any distinct i and j and any $\mathbf{x}$, $\pi_i(\mathbf{x},\boldsymbol{r}) + \pi_j(\mathbf{x},\boldsymbol{r})$ does not depend on $r_{i\to j}$.

G3 For any distinct i, j, k and any $\mathbf{x}$, $\pi_k(\mathbf{x},\boldsymbol{r})$ does not depend on $r_{i\to j}$.

We also make the following technical assumptions:

G4 For any $\boldsymbol{r}$, there exists a unique Nash equilibrium $\mathbf{x}^*(\boldsymbol{r})$ to the second-stage subgame.

G5 For any $\boldsymbol{r}$, any distinct i and j and any $\left(r'_{i\to j}, r'_{j\to i}\right)$, the two-player game (played by firms i and j) derived from the second-stage subgame by fixing the action of each player $k \notin \{i, j\}$ to $x^*_k(\boldsymbol{r})$ has a unique Nash equilibrium $\left(\tilde{x}^*_i\left(r'_{i\to j}, r'_{j\to i}, \boldsymbol{r}_{-ij}, \mathbf{x}^*_{-ij}(\boldsymbol{r})\right), \tilde{x}^*_j\left(r'_{i\to j}, r'_{j\to i}, \boldsymbol{r}_{-ij}, \mathbf{x}^*_{-ij}(\boldsymbol{r})\right)\right)$.

G6 There exists a unique vector $\mathbf{x}^{\mathbf{m}}$ of second-stage actions that maximizes the joint payoff of all players; moreover the joint payoff function is differentiable at $\mathbf{x}^{\mathbf{m}}$ and the latter is the unique solution to the corresponding system of F.O.Cs.

This general model can be applied to many economic situations, including:

- *Cross-licensing:* $r_{i\to j}$ is a per-unit royalty paid by patent holder i to patent holder j and x_i is a price or a quantity chosen by i. Note that the general model applies not only to the case in which cross-licensing partners produce substitutable goods but also to the case in which they produce complementary goods.

- *Two-way access pricing in telecommunication networks:* $r_{i \to j}$ is the access charge paid by network i to network j and x_i is the linear retail price charged by network i to its customers (see Armstrong (1998) and Laffont, Rey and Tirole (1998a, 1998b) for a duopolistic setting).

- *Interconnection among Internet backbone companies*: $r_{i \to j}$ is the access charge paid by backbone company i to j in a transit agreement and x_i is the capacity choice made by i (see Crémer, Rey and Tirole (2000)).

- *Interbank payments for the use of ATMs:* $r_{i \to j}$ is the interchange fee paid by bank i to bank j and x_i is the number of ATMs deployed by bank i (see Donze and Dubec (2006) for a setting with multilateral negotiation of the interchange fee).

Note that the general model introduced above is not a generalization of our baseline model of cross-licensing in a strict sense. First, in contrast to the cross-licensing model, input prices can take positive as well as negative values. This rules out non-interior equilibria, which simplifies the analysis by making it possible to rely on first-order conditions. Second, the first stage of our general model is slightly different from that of the cross-licensing model: in the latter, we assumed that firms can decide not to sign an agreement in the first stage whereas in the former, it is implicitly assumed that each pair of firms signs an agreement (the only decision variable is the terms of their agreement). However, this restriction does not entail any loss of generality when firms' incentives are such that each pair of firms finds it jointly profitable to sign a bilateral agreement. This is in particular the case if we apply the general model to cross-licensing as the argument we provided between Definition 1 and Lemma 1 still applies. Moreover, the assumption that all pairs of firms sign an agreement is also satisfied in the scenario where a regulator makes it mandatory for firms to agree with each other regarding access to particular inputs. This is, for instance, typically the case with interconnection among telecommunication companies.

We now introduce the following definitions which generalize those adopted in our cross-licensing model:

Definition 2 *A vector* $\mathbf{r}$ *of input prices is fully cooperative if*

$$\boldsymbol{r} \in Arg \max_{\boldsymbol{r}'} \sum_{i=1}^{N} \pi_i \left(\mathbf{x}^* \left(\boldsymbol{r}' \right), \boldsymbol{r}' \right).$$

Definition 3 *A vector* $\mathbf{r}$ *of input prices is bilaterally efficient if for any* (i,j) *with* $i \neq j$*, the following holds:*

$$\left(r_{i\to j},r_{j\to i}\right)\in Arg\max_{\left(r'_{i\to j},r'_{j\to i}\right)}\left[\begin{array}{c}\pi_i\left(\tilde{x}_i^*\left(r'_{i\to j},r'_{j\to i},\boldsymbol{r}_{-ij},\mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right)\right),\tilde{x}_j^*\left(r'_{i\to j},r'_{j\to i},\boldsymbol{r}_{-ij},\mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right)\right),\mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right),\boldsymbol{r}_{-ij}\right)\\ +\pi_j\left(\tilde{x}_i^*\left(r'_{i\to j},r'_{j\to i},\boldsymbol{r}_{-ij},\mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right)\right),\tilde{x}_j^*\left(r'_{i\to j},r'_{j\to i},\boldsymbol{r}_{-ij},\mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right)\right),\mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right),\boldsymbol{r}_{-ij}\right)\end{array}\right]$$

Let $\mathbf{D}$ denote the set of vectors $\mathbf{r}$ of input prices such that for any (i,j), $x_j^*(.)$ and $\tilde{x}_j^*(.)$ are differentiable with respect to all their arguments at $\boldsymbol{r}$ and $\pi_i(.,\boldsymbol{r})$ is differentiable with respect to all its arguments at $\mathbf{x}^*(\boldsymbol{r})$.[19] The following lemma provides a sufficient condition for a vector $\boldsymbol{r}\in\mathbf{D}$ of input prices to be fully cooperative. This condition also ensures that a multilateral agreement in the input market involving all firms leads to the monopoly outcome in the product market.

Lemma 3 *A sufficient condition for a vector of input prices $\boldsymbol{r}\in\mathbf{D}$ to be fully cooperative is that for any $j\in\{1,...,N\}$,*

$$\sum_{i=1}^{N}\frac{\partial\pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)=0. \tag{17}$$

Moreover, when this condition is met, the fully cooperative agreements in the input market leads to the fully cooperative outcome in the product market.

We now provide a necessary condition for a vector of input prices in $\mathbf{D}$ to be bilaterally efficient.

Lemma 4 *Assume that, for any $\boldsymbol{r}\in\mathbf{D}$ and any $(i,j)\in\{1,...,N\}^2$ with $i\neq j$, we have*

$$\begin{vmatrix}\frac{\partial\tilde{x}_i^*}{\partial r_{i\to j}} & \frac{\partial\tilde{x}_j^*}{\partial r_{i\to j}}\\ \frac{\partial\tilde{x}_i^*}{\partial r_{j\to i}} & \frac{\partial\tilde{x}_j^*}{\partial r_{j\to i}}\end{vmatrix}\neq 0, \tag{18}$$

where the argument $\left(r_{i\to j},r_{j\to i},\mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right)\right)$ is omitted. Then, a necessary condition for a vector of input prices $\boldsymbol{r}\in\mathbf{D}$ to be bilaterally efficient is that

$$\frac{\partial\pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)=0$$

for any $(i,j)\in\{1,...,N\}^2$ such that $i\neq j$.

The rank condition (18) means that $r_{i\to j}$ and $r_{j\to i}$ are *independent* instruments in the sense that any local deviation in the product market can be obtained through a local deviation

[19] Note that in usual quantity/price competition games, the subset of $\mathbf{r}\notin\mathbf{D}$ is typically of zero measure under standard regularity assumptions.

in the input market. This condition ensures that the set of instruments in the input market is rich enough to implement any desired actions in the product market. Let us show that it is satisfied, for instance, in the simple context of the previous cross-licensing model with a Cournot oligopoly featuring (potentially asymmetric) linear costs and linear (inverse) demand $p = a - Q$. Then we have

$$\begin{vmatrix} \frac{\partial \tilde{x}_i^*}{\partial r_{i \to j}} & \frac{\partial \tilde{x}_j^*}{\partial r_{i \to j}} \\ \frac{\partial \tilde{x}_i^*}{\partial r_{j \to i}} & \frac{\partial \tilde{x}_j^*}{\partial r_{j \to i}} \end{vmatrix} = \begin{vmatrix} \frac{\partial \tilde{x}_i^*}{\partial c_i} & \frac{\partial \tilde{x}_j^*}{\partial c_i} \\ \frac{\partial \tilde{x}_i^*}{\partial c_j} & \frac{\partial \tilde{x}_j^*}{\partial c_j} \end{vmatrix} = \frac{1}{9} > 0.$$

Note that in environments in which the second stage takes the form of a Cournot game and the input prices affect only the marginal cost of production (such as our cross-licensing example), Condition (18) means that own-cost effects (on output) are not equal to cross-cost effects. In fact, in imperfect competition models, the property that own-cost effects strictly dominate cross-cost effects is quite standard (see e.g., Vives, 1999).

Using the previous two lemmas, it is straightforward to get the following result about the cooperative potential of bilateral agreements in the input market.

Proposition 6 *Assume that Condition (18) holds for any $\boldsymbol{r} \in \mathbf{D}$ and any distinct $i, j \in \{1, ..., N\}$. Then a bilaterally efficient vector of input prices $\boldsymbol{r} \in \mathbf{D}$ is necessarily fully cooperative.*

In contrast to the baseline cross-licensing model, we do not establish the existence of a bilaterally efficient vector of input prices in the current general framework.[20] We however show that whenever a bilaterally efficient vector of input prices exists, it will maximize the N firms' joint profit.

3.4 Overlapping networks

So far we have considered a single network of bilateral agreements: we assumed that any pair of firms can sign a bilateral agreement. We depart now from that setting and consider instead two potentially overlapping networks.[21] More specifically, consider the general model in Section 3.4 and assume that there are two subsets S^1 and S^2 of firms such that two distinct firms i and j sign a bilateral agreement in stage 1 if and only if $(i, j) \in S^1 \times S^1$ or $(i, j) \in S^2 \times S^2$. Moreover, assume that these two subsets cover the whole set of firms S, i.e.,

[20] This would essentially amount to showing that a given system of $N(N-1)$ first-order equations with $N(N-1)$ unknowns has at least one solution. This turns out to be complicated because of the (fully) asymmetric nature of the equations.

[21] We consider a very simple network structure in this section. See Ballester, Calvó-Armengol and Zenou (2006) for a general structure.

$S^1 \cup S^2 = S = \{1, ..., N\}$. When $S^1 \cap S^2 \neq \varnothing$, the two networks of bilateral agreements are overlapping, whereas they are not in the special case of $S^1 \cap S^2 = \varnothing$. A firm in $S^1 \cap S^2$, if any, is connected through a bilateral agreement with any other firm in S, whereas a firm in $S \setminus S^1 \cap S^2$ is only connected to the firms in the subset (S^1 or S^2) it belongs to. Finally, assume that stage 2 remains unchanged: all firms choose non-cooperatively and simultaneously their actions.[22]

Define $\boldsymbol{r}^1 \equiv ((r_{i\to j}, r_{j\to i}))_{(i,j)\in S^1\times S^1, i<j}$, $\boldsymbol{r}^2 \equiv ((r_{i\to j}, r_{j\to i}))_{(i,j)\in S^2\times S^2, i<j}$ and $\boldsymbol{r}^{1\cap 2} \equiv ((r_{i\to j}, r_{j\to i}))_{(i,j)\in S^1\cap S^2\times S^1\cap S^2, i<j}$ as the vectors of input prices paid to each other by the firms in S^1, S^2 and $S^1 \cap S^2$ respectively. Moreover, define $\boldsymbol{r}^1_{-1\cap 2} \equiv ((r_{i\to j}, r_{j\to i}))_{(i,j)\in (S^1\times S^1)\setminus(S^1\cap S^2\times S^1\cap S^2), i<j}$ as the vector of input prices paid to each other by the firms in S^1 excluding the input prices paid to each other by the firms in $S^1 \cap S^2$, and define $\boldsymbol{r}^2_{-1\cap 2}$ in a similar way. Finally, denote $\boldsymbol{r}^{12} \equiv \left(\boldsymbol{r}^1, \boldsymbol{r}^2_{-1\cap 2}\right) = \left(\boldsymbol{r}^1_{-1\cap 2}, \boldsymbol{r}^2\right)$ the vector of input prices paid by all firms in S to each other, and $\boldsymbol{r}^{12}_{-ij}$ the vector obtained from $\boldsymbol{r}^{12}$ by removing $(r_{i\to j}, r_{j\to i})$. Define in a similar way $\boldsymbol{F}^1$, $\boldsymbol{F}^2$, $\boldsymbol{F}^{1\cap 2}$, $\boldsymbol{F}^1_{-1\cap 2}$, $\boldsymbol{F}^2_{-1\cap 2}$, and $\boldsymbol{F}^{12} \equiv \left(\boldsymbol{F}^1, \boldsymbol{F}^2_{-1\cap 2}\right) = \left(\boldsymbol{F}^1_{-1\cap 2}, \boldsymbol{F}^2\right)$.

Also, let $\mathbf{x}^1$, $\mathbf{x}^2$, $\mathbf{x}^{1\setminus 2}$, $\mathbf{x}^{2\setminus 1}$ and $\mathbf{x}^{1\cap 2}$ denote the (vector of) actions of the firms in S^1, S^2, $S^1 \setminus S^2$, $S^2 \setminus S^1$, $S^1 \cap S^2$ respectively. Recall that $\mathbf{x}$ denotes the (vector of) actions of all firms and $\mathbf{x}_{-ij}$ the vector obtained from $\mathbf{x}$ by removing x_i and x_j.

We suppose that the following counterparts to assumptions **G1-G6** hold:

G1' For any i, there exists a function π_i such that, for any $\left(\mathbf{x}, \boldsymbol{r}^{12}, \boldsymbol{F}^{12}\right)$, $\Pi_i\left(\mathbf{x}, \boldsymbol{r}^{12}, \boldsymbol{F}^{12}\right) = \pi_i\left(\mathbf{x}, \boldsymbol{r}^{12}\right) + \sum_{j\in C_i} \left(F_{j\to i} - F_{i\to j}\right)$, where $C_i = S^1 \setminus \{i\}$ for $i \in S^1 \setminus S^2$, $C_i = S^2 \setminus \{i\}$ for $i \in S^2 \setminus S^1$, and $C_i = S \setminus \{i\}$ for $i \in S^1 \cap S^2$.

G2' For any distinct i and j and any $\mathbf{x}$, $\pi_i\left(\mathbf{x}, \boldsymbol{r}^{12}\right) + \pi_j\left(\mathbf{x}, \boldsymbol{r}^{12}\right)$ does not depend on $r_{i\to j}$.

G3' For any distinct i, j, k such that $(i,j) \in S^1 \times S^1 \cup\, S^2 \times S^2$ and $k \in S$, and any $\mathbf{x}$, $\pi_k\left(\mathbf{x}, \boldsymbol{r}^{12}\right)$ does not depend on $r_{i\to j}$.

G4' For any $\boldsymbol{r}^{12}$, there exists a unique Nash equilibrium $\mathbf{x}^*\left(\boldsymbol{r}^{12}\right)$ to the second-stage subgame.

G5' For any $\boldsymbol{r}^{12}$, any $(i,j) \in S^1 \times S^1 \cup\, S^2 \times S^2$ and any $\left(r'_{i\to j}, r'_{j\to i}\right)$, the two-player game (played by firms i and j) derived from the second-stage subgame by fixing the action of each player $k \notin \{i,j\}$ to $x^*_k\left(\boldsymbol{r}^{12}\right)$ has a unique Nash equilibrium which we denote by $\left(\tilde{x}^*_i\left(r'_{i\to j}, r'_{j\to i}, \boldsymbol{r}^{12}_{-ij}, \mathbf{x}^*_{-ij}\left(\boldsymbol{r}^{12}\right)\right), \tilde{x}^*_j\left(r'_{i\to j}, r'_{j\to i}, \boldsymbol{r}^{12}_{-ij}, , \mathbf{x}^*_{-ij}\left(\boldsymbol{r}^{12}\right)\right)\right)$.

G5" For any given $\boldsymbol{r}^{12} = (\boldsymbol{r}^1, \boldsymbol{r}^2_{-1\cap 2})$ and $\check{\boldsymbol{r}}^1$, the game (played by the firms in S_1) derived from the second-stage subgame by fixing the action of each player in $S^2 \setminus S^1$ to $\mathbf{x}^{2\setminus 1 *}\left(\boldsymbol{r}^{12}\right)$ has

[22] Again, at this stage, each firm i only knows the input prices $(r_{i\to j}, r_{j\to i})$ and the fixed transfers $(F_{i\to j}, F_{j\to i})$ involving it.

a unique Nash equilibrium $\hat{\mathbf{x}}^{1*}\left(\breve{\boldsymbol{r}}^1, \boldsymbol{r}^2_{-1\cap 2}, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right)$.[23] Similarly, for any $\boldsymbol{r}^{12} = \left(\boldsymbol{r}^1_{-1\cap 2}, \boldsymbol{r}^2\right)$ and any $\breve{\boldsymbol{r}}^2$, the game (played by the firms in S_2) derived from the second-stage subgame by fixing the actions of each player in $S^1\backslash S^2$ to $\mathbf{x}^{1\backslash 2*}\left(\boldsymbol{r}^{12}\right)$ has a unique Nash equilibrium $\hat{\mathbf{x}}^{2*}\left(\breve{\boldsymbol{r}}^2, \boldsymbol{r}^1_{-1\cap 2}, \mathbf{x}^{1\backslash 2*}\left(\boldsymbol{r}^{12}\right)\right)$.

G6' For any given actions $\mathbf{x}^{2\backslash 1}$ of the firms $S^2\backslash S^1$ and any given $\boldsymbol{r}^2_{-1\cap 2}$, there exists a unique vector $\tilde{\mathbf{x}}^1\left(\boldsymbol{r}^2_{-1\cap 2}, \mathbf{x}^{2\backslash 1}\right)$ that maximizes the joint payoff of all the firms in S^1,[24] and this vector is characterized by the corresponding system of first-order conditions.[25] Similarly, for any given actions $\mathbf{x}^{1\backslash 2}$ of the firms $S^1\backslash S^2$ and any given $\boldsymbol{r}^1_{-1\cap 2}$ there exists a unique vector $\tilde{\mathbf{x}}^2\left(\boldsymbol{r}^1_{-1\cap 2}, \mathbf{x}^{1\backslash 2}\right)$ that maximizes the joint payoff of all the firms in S^2, and this vector is characterized by the corresponding system of first-order conditions.

We now adapt the concept of fully cooperative vectors of input prices to the current context as follows:

Definition 4 *A vector* $\boldsymbol{r}^{12} = \left(\boldsymbol{r}^1, \boldsymbol{r}^2_{-1\cap 2}\right) = \left(\boldsymbol{r}^1_{-1\cap 2}, \boldsymbol{r}^2\right)$ *of input prices is intra-group fully cooperative if*

$$\boldsymbol{r}^1 \in Arg\max_{\breve{\boldsymbol{r}}^1}\sum_{i\in S^1}\pi_i\left(\hat{\mathbf{x}}^{1*}\left(\breve{\boldsymbol{r}}^1, \boldsymbol{r}^2_{-1\cap 2}, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right), \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right), \breve{\boldsymbol{r}}^1, \boldsymbol{r}^2_{-1\cap 2}\right)$$

and

$$\boldsymbol{r}^2 \in Arg\max_{\breve{\boldsymbol{r}}^2}\sum_{j\in S^2}\pi_j\left(\hat{\mathbf{x}}^{2*}\left(\breve{\boldsymbol{r}}^2, \boldsymbol{r}^1_{-1\cap 2}, \mathbf{x}^{1\backslash 2*}\left(\boldsymbol{r}^{12}\right)\right), \mathbf{x}^{1\backslash 2*}\left(\boldsymbol{r}^{12}\right), \boldsymbol{r}^1_{-1\cap 2}, \breve{\boldsymbol{r}}^2\right)$$

Note that the joint payoff of all the firms in S^1 (resp. S^2) depends only indirectly on $\boldsymbol{r}^1$ (resp. $\boldsymbol{r}^2$) through its effect on firms' equilibrium actions, but depends directly on $\boldsymbol{r}^2_{-1\cap 2}$ (resp. $\boldsymbol{r}^1_{-1\cap 2}$) through the royalties that the firms in $S^1\cap S^2$ pay to the firms in $S^2\backslash S^1$ (resp. $S^1\backslash S^2$). A vector of input prices is intra-group fully cooperative if the input prices charged to each other by the firms belonging to the same subset maximizes their joint profit given all the terms of the agreements involving at least one firm outside that subset.[26]

Let $\mathbf{D}^{12}$ denote the set of vectors $\boldsymbol{r}^{12}$ of input prices such that for any $i \in S$, $x^*_i(.)$ and $\tilde{x}^*_i(.)$ are differentiable with respect to all their arguments at $\boldsymbol{r}^{12}$ and $\pi_i\left(., \boldsymbol{r}^{12}\right)$ is differentiable with respect to all its arguments at $\mathbf{x}^*\left(\boldsymbol{r}^{12}\right)$.

[23] Note that $\hat{\mathbf{x}}^{1*}\left(\breve{\boldsymbol{r}}^1, \boldsymbol{r}^2_{-1\cap 2}, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right)$ depends on its second argument $\boldsymbol{r}^2_{-1\cap 2}$ only through the input prices $r_{i\to j}$ such that $i \in S_1\cap S_2$ and $j\in S_2\backslash S_1$.

[24] We omit the arguments of $\tilde{\mathbf{x}}^1$ for the sake of brevity.

[25] Recall that $\boldsymbol{r}^1$ does not affect directly the joint payoff of all the firms in S^1.

[26] In other words, a vector of input prices is intra-group fully cooperative if the coalition made of all firms within a given subset (S^1 or S^2) does not find it optimal to deviate by changing (some of) the input prices charged to each other.

The following proposition shows that a bilaterally efficient vector of input prices is necessarily intra-group fully cooperative if the rank condition (18) is satisfied for all i, j such that $i \neq j$ and $(i, j) \in S^1 \times S^1 \cup S^2 \times S^2$.

Proposition 7 *Assume that Condition (18) holds for any* $\boldsymbol{r}^{12} \in \mathbf{D}^{12}$ *and any* $(i, j) \in S^1 \times S^1 \cup S^2 \times S^2$ *with* $i \neq j$. *Then, a bilaterally efficient vector of input prices* $\boldsymbol{r}^{12} \in \mathbf{D}^{12}$ *is necessarily intra-group fully cooperative*

This result generalizes Proposition 6 in the following sense: it shows that bilaterally efficient agreements lead to the same outcome as the one resulting from two separate multilateral agreements each involving all the firms in each subset. In other words, each multilateral agreement maximizing the joint payoff of all firms within each subset can still be "decentralized" through a (complete) network of efficient bilateral agreements within the subset.

To explore further the properties of bilaterally efficient vectors of input prices in the current environment, we introduce the following definition:

Definition 5 *A vector* $\boldsymbol{r}^{12}$ *of input prices induces fully cooperative actions of the firms in* $S^1 \cap S^2$ *if*

$$\mathbf{x}^{1\cap 2*}\left(\boldsymbol{r}^{12}\right) \in Arg\max_{\breve{\boldsymbol{x}}^{1\cap 2}} \sum_{i=1}^{N} \pi_i\left(\breve{\boldsymbol{x}}^{1\cap 2}, \mathbf{x}^{1\backslash 2*}\left(\boldsymbol{r}^{12}\right), \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right), \boldsymbol{r}^{12}\right)$$

Recall that, for any given vector of actions $\mathbf{x}$, the industry profit does not depend on $\boldsymbol{r}^{12}$. A vector of input prices induces *fully cooperative* actions of the firms in $S^1 \cap S^2$ if their actions maximize the industry profit.[27]

Proposition 8 *Assume that Condition (18) holds for any* $\boldsymbol{r}^{12} \in \mathbf{D}^{12}$ *and any* $(i, j) \in S^1 \times S^1 \cup S^2 \times S^2$ *with* $i \neq j$. *Then a bilaterally efficient vector of input prices* $\boldsymbol{r}^{12} \in \mathbf{D}^{12}$ *necessarily induces fully cooperative actions of the firms in* $S^1 \cap S^2$.

Thus, bilateral efficiency implies that those firms which sign bilateral agreements with all other firms in the industry, i.e., the firms in $S^1 \cap S^2$, choose industry-profit-maximizing actions. Hence, as the set $S^1 \cap S^2$ expands, more and more firms take industry-profit-maximizing actions such that in the limit case where $S^1 = S^2 = S$, we obtain the monopoly outcome as in Proposition 6.

[27] In other words, a vector of input prices induces fully cooperative actions of the firms in $S^1 \cap S^2$ if it makes these firms internalize fully the effects of their (second-stage) decisions on all other firms in the industry.

4 Policy implications

We now discuss the policy implications of our results regarding the antitrust treatment of bilateral cross-licensing agreements between competitors.

Competition authorities usually prohibit the use of royalties that are disproportionate with respect to the market value of the license. For instance, according to the Guidelines on the application of Article 101 of the EC Treaty to technology transfer agreements (European Commission, 2014), "… Article 101(1) may be applicable where competitors cross license and impose running royalties that are clearly disproportionate compared to the market value of the licence and where such royalties have a significant impact on market prices." However, the Technology Transfer Block Exemption Regulation (TTBER) of the European Commission grants antitrust exemption to bilateral cross-licensing agreements between competitors if their joint market share does not exceed 20%. In a similar vein, the competition authorities in the U.S. grant a safe harbor to cross-licensing agreements (not necessarily bilateral) among partners whose joint market share is below 20% (DOJ and FTC, 1995, p.22).

Our analysis does not support an antitrust exemption to bilateral cross-licensing agreements based *only* on the joint market shares of the firms involved in those agreements. Consider for instance the specific example of an industry comprised of ten symmetric firms. In such setting, any bilateral cross-licensing agreements would satisfy the joint market share criterion used by American and European antitrust authorities. However, our findings show that, absent any other legal restriction, such agreements can be used by firms to sustain the monopoly outcome.

This conclusion is in line with antitrust law in the U.S. where bilateral cross-licensing agreements cannot benefit from a safe harbor if they are "facially anticompetitive" (U.S. DOJ and FTC, 1995, p.22). Similarly, in the EU, an agreement is exempted from the benefit of the Technology Transfer Block Exemption Regulation (TTBER) if it involves "hardcore restrictions". Our analysis sheds light on the importance of enforcing the rules that exclude some cross-licensing agreements from an automatic exemption regime based on market shares.[28] It also suggests that one such rule should be the prohibition of cross-licensing contracts that require (per-unit) royalties to be paid regardless of whether the licensed technology is actually used. In the case of cost-reducing technologies, under this rule, firms' post-licensing marginal costs do not exceed their pre-licensing marginal costs, which implies that consumers cannot be negatively affected by cross-licensing.

Finally, our analysis shows that constraining the royalties that cross-licensing partners

[28] In other words, antitrust authorities should not rely on market forces to discipline firms with low market shares regarding the (per-unit) royalties they charge each other as part of a cross-licensing agreement.

can agree upon may lead to the exclusion of some firms from the market. This suggests that mitigating the collusive effect of bilateral cross-licensing agreements may come at the cost of increasing their exclusionary potential.

5 Concluding remarks

Our general message is that under a wide range of circumstances, bilateral agreements in an input market among all firms competing in a product market can lead to the same outcome as under full cooperation in the input market. This result has been shown to hold independently of the number of firms and the nature of interactions in the product market and regardless of whether firms are symmetric or not. This finding does not necessarily imply that bilateral agreements in the input market reduce social welfare. First, if firms produce complements rather than substitutes, full cooperation in the input market is socially desirable, and so are bilateral agreements. Second, even if firms produce substitutable products, the outcome of full cooperation in the input market can be superior to the outcome of no agreement at all. For instance, cross-licensing of patents can lower firms' marginal costs and lead to a lower final price than without cross-licensing. Third, in the case of cross-licensing of patents, one should also take into account how cross-licensing affects firms' incentives to invest in innovation.

Our setting can be extended to study other policy issues related to cross-licensing. First, we can introduce, in addition to incumbent firms, entrants with no (or weak) patent portfolios. This would allow us to study whether cross-licensing can be used to raise barriers to entry (DOJ and FTC, 2007). Second, we can include in the set of players non-operating entities which do not compete in the product market. This would allow us to study the conditions under which non-operating entities weaken competition and (when these conditions are met) to isolate the anticompetitive effects generated by non-operating entities from the effects resulting from cross-licensing in their absence.[29] Note that non-operating entities and entrants involve completely opposite asymmetries. The former are present in the input market of patent licensing but are absent in the product market whereas the second are absent (or have very weak presence) in the input market but are present in the product market.

[29] The issue of how NPEs affect competition and innovation is of substantial current interest to policy makers (Scott Morton and Shapiro, 2014; FTC, 2016).

6 Appendix

6.1 Alternative interpretation: value-increasing innovations

Instead of assuming that access to more patents reduces a firm's marginal cost, we can assume that access to more patents increases the value of the product produced by the firm. We below show that our model of cost-reducing innovations can be equivalently interpreted as a model of value-increasing innovations.

We consider a constant symmetric marginal cost c for all firms. Each firm has one patent. Let $v(n)$ represent the value of the product produced by a firm when the firm has access to $n \in \{1, ..., N\}$ number of distinct patents with $v(N) \geq v(N-1) \geq ... \geq v(1)(\equiv \underline{v})$. Let $\mathbf{v} \equiv (v_1, ..., v_N)$ be the vector representing the value of each firm's product after the licensing stage.

We define Cournot competition for given $\mathbf{v} \equiv (v_1, ..., v_N)$ as follows. Each firm i simultaneously chooses its quantity q_i. Given $\mathbf{v} \equiv (v_1, ..., v_N)$, $\mathbf{q} \equiv (q_1, ..., q_N)$ and $Q = q_1 + ... + q_N$, the quality-adjusted equilibrium prices are determined by the following two conditions:

- an indifference condition:

$$v_i - p_i = v_j - p_j \qquad \text{for all } (i, j) \in \{1, ..., N\}^2 ;$$

- a market-clearing condition:

$$Q = D(p) \text{ where } p_i = p + v_i - \underline{v}.$$

In other words, p is the price for the product of a firm which has access to its own patent only. The market clearing condition means that this price is adjusted to make the total supply equal to the demand. The indifference condition implies that the price each firm charges is adjusted such that all consumers who buy any product are indifferent among all products. A micro-foundation of this setup can be provided as follows. There is a mass one of consumers. Each consumer has a unit demand and hence buys at most one unit among all products. A consumer's gross utility from having a unit of product of firm i is given by $u+v_i$: u is specific to the consumer while v_i is common to all consumers. Let $F(u)$ represent the cumulative distribution function of u. Then, by construction of quality-adjusted prices, any consumer is indifferent among all products and the marginal consumer indifferent between buying any product and not buying is characterized by $u + \underline{v} - p = 0$, implying

$$D(p) = 1 - F(p - \underline{v}).$$

In equilibrium, p is adjusted such that $1 - F(p - \underline{v}) = Q$. Let $P(Q)$ be the inverse demand function. In equilibrium, a firm's profit is given by

$$\pi_i = \left(P(Q) + v_i - \underline{v} - c - \sum_{j \neq i} r_{i \to j} \right) q_i + \sum_{j \neq i} r_{j \to i} q_j.$$

After making the following change of variables

$$c - (v_i - \underline{v}) = c_i,$$

the profit can be equivalently written as

$$\pi_i = \left(P(Q) - c_i - \sum_{j \neq i} r_{i \to j} \right) q_i + \sum_{j \neq i} r_{j \to i} q_j,$$

which is the profit expression in our original model of cost-reducing patents. Therefore, our model of cost-reducing innovations can be equivalently interpreted as a model of value-increasing innovations.

6.2 Proofs

Proof of Lemma 1

Assume, without loss of generality, that $(i, j) = (1, 2)$. The joint payoff that firms 1 and 2 derive from a deviation to a cross-licensing agreement involving the payment of $(\hat{r}_{1 \to 2}, \hat{r}_{2 \to 1})$ is:

$$\begin{aligned} \hat{\pi}_1 + \hat{\pi}_2 &= \left[P\left(Q^*_{-12} + \hat{q}_1 + \hat{q}_2 \right) - \underline{c} - \hat{r}_{1 \to 2} - (N-2)\, r \right] \hat{q}_1 + \hat{r}_{2 \to 1} \hat{q}_2 + r Q^*_{-12} \\ &\quad + \left[P\left(Q^*_{-12} + \hat{q}_1 + \hat{q}_2 \right) - \underline{c} - \hat{r}_{2 \to 1} - (N-2)\, r \right] \hat{q}_2 + \hat{r}_{1 \to 2} \hat{q}_1 + r Q^*_{-12} \\ &= \left[P\left(Q^*_{-12} + \hat{q}_1 + \hat{q}_2 \right) - \underline{c} - (N-2)\, r \right] (\hat{q}_1 + \hat{q}_2) + 2 r Q^*_{-12}. \end{aligned}$$

where $\hat{q}_1$ and $\hat{q}_2$ satisfy the following F.O.Cs:

$$P\left(Q^*_{-12} + \hat{q}_1 + \hat{q}_2 \right) - \left[\underline{c} + \hat{r}_{1 \to 2} + (N-2)\, r \right] + \hat{q}_1 P'\left(Q^*_{-12} + \hat{q}_1 + \hat{q}_2 \right) = 0,$$

$$P\left(Q^*_{-12} + \hat{q}_1 + \hat{q}_2 \right) - \left[\underline{c} + r_{2 \to 1} + (N-2)\, r \right] + \hat{q}_2 P'\left(Q^*_{-12} + \hat{q}_1 + \hat{q}_2 \right) = 0.$$

Summing these F.O.Cs yields

$$2P\left(Q^*_{-12}+\hat{q}_1+\hat{q}_2\right)-\left[\underline{c}+\hat{r}_{1\to 2}+\hat{r}_{2\to 1}+(N-2)\,r\right]+\left(\hat{q}_1+\hat{q}_2\right)P'\left(Q^*_{-12}+\hat{q}_1+\hat{q}_2\right)=0,$$

which shows that $\hat{q}_1+\hat{q}_2$ depends on $(\hat{r}_{1\to 2},\hat{r}_{2\to 1})$ only through $\hat{r}_{1\to 2}+\hat{r}_{2\to 1}$. Combining this with the fact that

$$\hat{\pi}_1+\hat{\pi}_2=\left[P\left(Q^*_{-12}+\hat{q}_1+\hat{q}_2\right)-\underline{c}-(N-2)\,r\right]\left(\hat{q}_1+\hat{q}_2\right)+2rQ^*_{-12} \tag{19}$$

implies that the coalition's deviation payoff $\hat{\pi}_1+\hat{\pi}_2$ depends on $(\hat{r}_{1\to 2},\hat{r}_{2\to 1})$ only through $\hat{r}_{1\to 2}+\hat{r}_{2\to 1}$.

Proof of Proposition 2

We below prove that

$$\frac{\partial^2\pi_{12}}{\partial Q_{12}^2}(Q_{12},r)=P''\left(Q_{12}+Q^*_{-12}(r)\right)Q_{12}+2P'\left(Q_{12}+Q^*_{-12}(r)\right)<0.$$

If $P''\left(Q_{12}+Q^*_{-12}(r)\right)\leq 0$, the result follows from $P'(Q_{12}+Q^*_{-12}(r))<0$. Suppose now that $P''(Q_{12}+Q^*_{-12}(r))>0$. Then, we have

$$\begin{aligned}0 &> P''\left(Q_{12}+Q^*_{-12}(r)\right)\left[Q_{12}+Q^*_{-12}(r)\right]+2P'\left(Q_{12}+Q^*_{-12}(r)\right)\\ &> P''\left(Q_{12}+Q^*_{-12}(r)\right)Q_{12}+2P'\left(Q_{12}+Q^*_{-12}(r)\right)\end{aligned}$$

where the first inequality follows from **A1** and **A3** and the second from $P''(Q_{12}+Q^*_{-12}(r))>0>0$. This proves that in both cases $\partial^2\pi_{12}/\partial Q_{12}^2<0$.

Proof of Proposition 4

Suppose first that $r^m\leq c(N-1)-c(N)$. Then, the ex post usage constraint is not binding and hence, from Proposition 2, we obtain the result that the unique bilaterally efficient symmetric royalty is $r=r^m$.

Suppose now that $r^m>c(N-1)-c(N)$. All firms using all the technologies in the presence of the ex post usage constraint implies that the symmetric royalty r cannot be larger than $c(N-1)-c(N)$: if $r>c(N-1)-c(N)$, at least one technology will not be used. Therefore, suppose that any distinct firms i and j agree on $r\leq c(N-1)-c(N)$ and consider the deviation of firms 1 and 2 in stage 1.

We first show that any $r_{1\to 2} > c(N-1) - c(N)$ is strictly dominated by $r_{1\to 2} = c(N-1) - c(N)$. If $r_{1\to 2} > c(N-1) - c(N)$, firm 1 will not use firm 2's technology in stage 2 and, therefore, will have a marginal cost $c_1 = c(N-1) + (N-2)r$, which is the same as its marginal cost from agreeing on $r_{1\to 2} = c(N) - c(N-1)$ and using firm 2's technology. Given that both royalties lead to the same marginal cost of firm 1, inducing firm 1 to use firm 2's technology generates a higher joint profit than not using firm 2's technology. As the same argument applies to $r_{2\to 1}$, we can conclude that the coalition of $\{1,2\}$ will use royalties satisfying $r_{1\to 2} \leq c(N-1) - c(N)$ and $r_{2\to 1} \leq c(N-1) - c(N)$.

Lemma 1 still applies and therefore, without loss of generality, we can restrict attention to deviations to a symmetric royalty $r_{1\to 2} = r_{2\to 1} = \widehat{r} \leq c(N-1) - c(N)$.

From the analysis in Section 2.4, we have

$$\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}^{*}(r,r), r) = \underline{c} + Nr - P\left(Q^{*}(r,r)\right),$$

and

$$\underline{c} + Nr - P\left(Q^{*}(r,r)\right) < 0 \text{ for any } r < r^{m}.$$

Hence, when $r = c(N-1) - c(N)$, the coalition has no incentive to increase Q_{12} locally (starting from $Q_{12}^{*}(r,r)$); it has an incentive to reduce Q_{12} but it cannot do so because of the ex post usage constraint. Moreover, as we have shown that $\frac{\partial \pi_{12}}{\partial Q_{12}}$ is globally decreasing in Q_{12} (i.e., $\pi_{12}(Q_{12}, r)$ is globally concave in Q_{12}), we can state that there is no profitable deviation.

Proof of Proposition 5

For notational simplicity, let

$$c(1) \equiv \overline{c}, c(2) \equiv \widehat{c} = \underline{c} + \varepsilon, c(3) \equiv \underline{c}.$$

Proof of (i). Suppose that in the equilibrium candidate, firm 1 is the only firm active in stage 2. Let $F_{1\to i}$ be the net fixed fee that firm i (= 2 or 3) receives from firm 1. Without loss of generality, we assume that $r_{1\to 2} \leq r_{1\to 3}$. Let c_1 denote firm 1's marginal cost in the equilibrium candidate. Then, in the equilibrium candidate, firms 2 and 3 jointly obtain

$$\sum_{i=2,3} \left[F_{1\to i} + r_{1\to i} q^{M}(c_1)\right].$$

The inequality $r_{i\to 1} > \varepsilon$ must hold for $i = 2, 3$ in the candidate equilibrium; otherwise, $c_i = \widehat{c} + r_{i\to 1} \leq \underline{c} + 2\varepsilon$ and, therefore, firm i would become active.

Note that for any deviation of the coalition $\{2,3\}$, each firm $i = 2,3$ receives $F_{1\to i} + r_{1\to i}q^M(c_1)$ from firm 1 as the latter keeps producing $q^M(c_1)$. Suppose now that firms 2 and 3 deviate by signing a cross-licensing agreement with $r_{2\to 3} = r_{3\to 2} = 0$. After the deviation, the marginal cost of each firm i is $c_i = \widehat{c}$ for $i = 2,3$ because each firm i finds it optimal to use only the technology of firm $j \in \{2,3\} \setminus \{i\}$. From

$$P^M(c_1) \geq P^M(\underline{c}) > \widehat{c},$$

it follows that firms 2 and 3 will be active: in stage 2, each firm i will produce q_i which is a best response to $q^M(c_1) + q_j$ for $i, j = 2,3$ and $i \neq j$ and make an extra positive profit, in addition to the licensing revenue $F_{1\to i} + r_{1\to i}q^M(c_1)$. Therefore, the considered deviation by the coalition $\{2,3\}$ is profitable.

Proof of (ii). Let us show that the set of bilateral cross-licensing agreements presented in (ii) is bilaterally efficient.

Note first that the inequality $\overline{c} > P^m(\underline{c} + \varepsilon)$ implies that $\overline{c} > P^d(\underline{c} + \varepsilon)$ where $P^d(\underline{c} + \varepsilon)$ is the duopoly price when both firms have the same marginal cost $\underline{c} + \varepsilon$.

Consider first the coalition of firms 1 and 2. Note first that even if firm 1 does not use firm 2's technology, its marginal cost will be $c_1 = \widehat{c}$. The same is true regarding c_2. This implies that, when firms 1 and 2 deviate by changing their cross-licensing agreement, they both remain active and firm 3 remains inactive, whatever the deviation. Given that, the cross-licensing agreement which maximizes the joint profits of firms 1 and 2 is such that they agree on the highest royalties consistent with the ex post usage constraint (in order to contract their joint output and make it closer to the monopoly output). This is achieved with $r_{1\to 2} = r_{2\to 1} = \varepsilon$, which implies that there is no profitable deviation by the coalition of firms 1 and 2.

Second, consider the coalition of firms 1 and 3. In the candidate equilibrium, firm 1's profit is $\pi^D(\widehat{c}, \widehat{c})/2$, i.e., half of the duopoly industry profit when $(c_1, c_2) = (\widehat{c}, \widehat{c})$, and firm 3's profit is zero. As long as firm 3 is inactive, the joint profit of firms 1 and 3 is maximized by reducing firm 1's marginal cost, which requires $r_{1\to 3} = 0$. Can firms 1 and 3 increase their joint profit by inducing firm 3 to be active? The answer is no. To see why, note first that given that $r_{3\to 2} = \overline{c} - \widehat{c}$, a lower bound of c_3 is given by $\widehat{c}$ which can be achieved by $r_{3\to 1} = 0$. This lower bound is the same as $c_1 = \widehat{c}$. Therefore, regardless of whether firm 3 is active or not, the lowest marginal cost from the coalition's point of view is $\widehat{c}$. Hence, what matters for the coalition's payoff is the joint output produced at the marginal cost $\widehat{c}$. More precisely, as firm 2 will keep producing $q_2 = q^D(\widehat{c}, \widehat{c})$, its quantity in the duopoly, the best response of the coalition of firms 1 and 3 is to produce $q^D(\widehat{c}, \widehat{c})$, which is what firm 1 does when firm 3 is not active. Therefore, there is no profitable deviation by the coalition of firms 1 and 3.

By symmetry, the coalition of firms 2 and 3 has no incentive to deviate.

Proof of Lemma 3

By **G1, G2** and **G3** it holds that $\sum_{i=1}^{N} \Pi_i (\mathbf{x}, \boldsymbol{r}, \boldsymbol{F}) = \sum_{i=1}^{N} \pi_i (\mathbf{x}, \mathbf{r})$ does not depend on $\mathbf{r}$ for any $\mathbf{x}$. By **G6**, $\mathbf{x^m}$ is then the unique solution to the system of N equations:

$$\sum_{i=1}^{N} \frac{\partial \pi_i}{\partial x_j} (\mathbf{x}, \mathbf{r}) = 0 \text{ for } j \in \{1, ..., N\},$$

for any $\mathbf{r}$. Therefore, if a vector $\mathbf{r} \in \mathbf{D}$ is such that $\sum_{i=1}^{N} \frac{\partial \pi_i}{\partial x_j} (\mathbf{x}^* (\mathbf{r}), \mathbf{r}) = 0$ for any $j \in \{1, ..., N\}$, then it must be that $\mathbf{x}^* (\mathbf{r}) = \mathbf{x^m}$, which implies that (i) $\sum_{i=1}^{N} \pi_i (\mathbf{x}^* (\mathbf{r}), \mathbf{r}) \geq \sum_{i=1}^{N} \pi_i (\mathbf{x}^* (\mathbf{r}'), \mathbf{r}')$ for any $\mathbf{r}'$, that is, $\mathbf{r}$ is fully cooperative, and (ii) the fully cooperative agreements in the input market lead to the fully cooperative outcome in the product market.

Proof of Lemma 4

Assume that $\boldsymbol{r} \in \mathbf{D}$ is bilaterally efficient. Then for any $(i, j) \in \{1, ..., N\}^2$ with $i \neq j$, it must hold that

$$\begin{aligned} & \frac{\partial}{\partial r_{i \to j}} \pi_i \left(\tilde{x}_i^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \tilde{x}_j^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \mathbf{x}_{-ij}^* (\boldsymbol{r}), \boldsymbol{r}_{-ij} \right) + \\ & \frac{\partial}{\partial r_{i \to j}} \pi_j \left(\tilde{x}_i^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \tilde{x}_j^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \mathbf{x}_{-ij}^* (\boldsymbol{r}), \boldsymbol{r}_{-ij} \right) \\ = \; & 0, \end{aligned}$$

which can be rewritten as

$$\begin{aligned} & \frac{\partial \tilde{x}_i^*}{\partial r_{i \to j}} \times \frac{\partial \pi_i}{\partial x_i} \left(\tilde{x}_i^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \tilde{x}_j^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \mathbf{x}_{-ij}^* (\boldsymbol{r}), \boldsymbol{r}_{-ij} \right) + \\ & \frac{\partial \tilde{x}_j^*}{\partial r_{i \to j}} \times \frac{\partial \pi_i}{\partial x_j} \left(\tilde{x}_i^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \tilde{x}_j^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \mathbf{x}_{-ij}^* (\boldsymbol{r}), \boldsymbol{r}_{-ij} \right) + \\ & \frac{\partial \tilde{x}_i^*}{\partial r_{i \to j}} \times \frac{\partial \pi_j}{\partial x_i} \left(\tilde{x}_i^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \tilde{x}_j^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \mathbf{x}_{-ij}^* (\boldsymbol{r}), \boldsymbol{r}_{-ij} \right) + \\ & \frac{\partial \tilde{x}_j^*}{\partial r_{i \to j}} \times \frac{\partial \pi_j}{\partial x_j} \left(\tilde{x}_i^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \tilde{x}_j^* \left(r_{i \to j}, r_{j \to i}, \boldsymbol{r}_{-ij}, \mathbf{x}_{-ij}^* (\boldsymbol{r}) \right), \mathbf{x}_{-ij}^* (\boldsymbol{r}), \boldsymbol{r}_{-ij} \right) \\ = \; & 0 \end{aligned}$$

where the arguments $\left(r_{i\to j}, r_{j\to i}, \boldsymbol{r}_{-ij}, \mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right)\right)$ of $\frac{\partial \tilde{x}^*_i}{\partial r_{i\to j}}$ and $\frac{\partial \tilde{x}^*_j}{\partial r_{i\to j}}$ are omitted.

Using the definition of a Nash equilibrium and the uniqueness of $x^*_i\left(\boldsymbol{r}\right)$ (by **G4**), it is straightforward to see that $\tilde{x}^*_i\left(r_{i\to j}, r_{j\to i}, \mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right)\right) = x^*_i\left(\boldsymbol{r}\right)$ and that $\tilde{x}^*_j\left(r_{i\to j}, r_{j\to i}, \mathbf{x}^*_{-ij}\left(\boldsymbol{r}\right)\right) = x^*_j\left(\boldsymbol{r}\right)$. Therefore, it holds that

$$\frac{\partial \tilde{x}^*_i}{\partial r_{i\to j}} \times \frac{\partial \pi_i}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) + \frac{\partial \tilde{x}^*_j}{\partial r_{i\to j}} \times \frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) +$$
$$\frac{\partial \tilde{x}^*_i}{\partial r_{i\to j}} \times \frac{\partial \pi_j}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) + \frac{\partial \tilde{x}^*_j}{\partial r_{i\to j}} \times \frac{\partial \pi_j}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) = 0.$$

By definition of the Nash equilibrium $\mathbf{x}^*\left(\mathbf{r}\right)$, it holds that

$$\frac{\partial \pi_i}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) = \frac{\partial \pi_j}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) = 0.$$

This yields

$$\frac{\partial \tilde{x}^*_j}{\partial r_{i\to j}} \times \frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) + \frac{\partial \tilde{x}^*_i}{\partial r_{i\to j}} \times \frac{\partial \pi_j}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) = 0.$$

By symmetry we also have

$$\frac{\partial \tilde{x}^*_i}{\partial r_{j\to i}} \times \frac{\partial \pi_j}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) + \frac{\partial \tilde{x}^*_j}{\partial r_{j\to i}} \times \frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) = 0.$$

Denoting $y_{ij} = \frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right)$ and $y_{ji} = \frac{\partial \pi_j}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right)$, the latter two equations can be rewritten as a two-equation *linear* system in y_{ji} and y_{ij}:

$$\begin{cases} \frac{\partial \tilde{x}^*_i}{\partial r_{i\to j}}.y_{ji} + \frac{\partial \tilde{x}^*_j}{\partial r_{i\to j}}.y_{ij} = 0; \\ \frac{\partial \tilde{x}^*_i}{\partial r_{j\to i}}.y_{ji} + \frac{\partial \tilde{x}^*_j}{\partial r_{j\to i}}.y_{ij} = 0. \end{cases}$$

If $\begin{vmatrix} \frac{\partial \tilde{x}^*_i}{\partial r_{i\to j}} & \frac{\partial \tilde{x}^*_j}{\partial r_{i\to j}} \\ \frac{\partial \tilde{x}^*_i}{\partial r_{j\to i}} & \frac{\partial \tilde{x}^*_j}{\partial r_{j\to i}} \end{vmatrix} \neq 0$, then the latter system has a unique solution, given by $y_{ji} = y_{ij} = 0$. Hence, we get the following: for any $(i, j) \in \{1, ..., N\}^2$ with $i \neq j$, the following equation must hold

$$\frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right), \boldsymbol{r}\right) = 0.$$

Proof of Proposition 7

Assume that the input price vector $\boldsymbol{r}^{12} \in \mathbf{D}^{12}$ is bilaterally efficient. Following the same steps as those of the proof of Lemma 4, we get that

$$\frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}^{12}\right), \boldsymbol{r}^{12}\right) = 0,$$

for any $(i,j) \in S^1 \times S^1 \cup S^2 \times S^2$ with $i \neq j$. This implies that

$$\sum_{i \in S^1} \frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}^{12}\right), \boldsymbol{r}^{12}\right) = 0, \tag{20}$$

for any $j \in S^1$, and, similarly

$$\sum_{i \in S^2} \frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}^{12}\right), \boldsymbol{r}^{12}\right) = 0, \tag{21}$$

for any $j \in S^2$. Therefore, by **G6'**, it must hold that $\mathbf{x}^{1*}\left(\boldsymbol{r}^{12}\right) = \tilde{\mathbf{x}}^1\left(\boldsymbol{r}^2_{-1\cap 2}, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right)$, which implies in particular that

$$\sum_{i \in S^1} \pi_i\left(\mathbf{x}^*\left(\boldsymbol{r}^{12}\right), \boldsymbol{r}^{12}\right) \geq \sum_{i \in S^1} \pi_i\left(\hat{\mathbf{x}}^{1*}\left(\breve{\boldsymbol{r}}^1, \boldsymbol{r}^2_{-1\cap 2}, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right), \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right), \breve{\boldsymbol{r}}^1, \boldsymbol{r}^2_{-1\cap 2}\right)$$

for any $\breve{\boldsymbol{r}}^1$ (recall that the joint payoff of all the firms in S^1 depends on $\breve{\boldsymbol{r}}^1$ only through $\hat{\mathbf{x}}^{1*}\left(\breve{\boldsymbol{r}}^1, \boldsymbol{r}^2, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right)$). Then, using the fact that $\hat{\mathbf{x}}^{1*}\left(\boldsymbol{r}^1, \boldsymbol{r}^2_{-1\cap 2}, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right) = \mathbf{x}^{1*}\left(\boldsymbol{r}^{12}\right)$ (which holds because $\hat{\mathbf{x}}^{1*}$ and $\mathbf{x}^{1*}$ are unique by **G4'** and **G5"**), we obtain that

$$\begin{aligned}&\sum_{i \in S^1} \pi_i\left(\hat{\mathbf{x}}^{1*}\left(\boldsymbol{r}^1, \boldsymbol{r}^2_{-1\cap 2}, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right), \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right), \boldsymbol{r}^1, \boldsymbol{r}^2_{-1\cap 2}\right) \geq \\ &\sum_{i \in S^1} \pi_i\left(\hat{\mathbf{x}}^{1*}\left(\breve{\boldsymbol{r}}^1, \boldsymbol{r}^2_{-1\cap 2}, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right), \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right), \breve{\boldsymbol{r}}^1, \boldsymbol{r}^2_{-1\cap 2}\right)\end{aligned}$$

for any $\breve{\boldsymbol{r}}^1$, which means that

$$\boldsymbol{r}^1 \in Arg\max_{\breve{\boldsymbol{r}}^1} \sum_{i \in S^1} \pi_i\left(\hat{\mathbf{x}}^{1*}\left(\breve{\boldsymbol{r}}^1, \boldsymbol{r}^2, \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right)\right), \mathbf{x}^{2\backslash 1*}\left(\boldsymbol{r}^{12}\right), \breve{\boldsymbol{r}}^1, \boldsymbol{r}^2_{-1\cap 2}\right)$$

We can show in a similar way that

$$\boldsymbol{r}^2 \in Arg\max_{\breve{\boldsymbol{r}}^2} \sum_{j \in S^2} \pi_j\left(\hat{\mathbf{x}}^{2*}\left(\breve{\boldsymbol{r}}^2, \mathbf{x}^{1\backslash 2*}\left(\boldsymbol{r}^{12}\right)\right), \mathbf{x}^{1\backslash 2*}\left(\boldsymbol{r}^{12}\right), \boldsymbol{r}^1_{-1\cap 2}, \breve{\boldsymbol{r}}^2\right)$$

Therefore, $\boldsymbol{r}^{12}$ is an intra-group fully cooperative vector of input prices.

Proof of Proposition 8

Consider a firm j in $S^1 \cap S^2$. Combining (20) and (21) together with firm j's F.O.C. with respect to x_j yields

$$\sum_{i \in S} \frac{\partial \pi_i}{\partial x_j} \left(\mathbf{x}^* \left(\boldsymbol{r}^{12} \right), \boldsymbol{r}^{12} \right) = 0,$$

which implies that firm j's action maximizes the industry profit.

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