

Regional Market Power and Transport Capacity in the Gas Industry

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Abstract

This paper analyzes the role of gas pipeline capacity in mitigating regional market power. We characterize the policy prescriptions of various control schemes used by a network operator, and focus on transport capacity. We start from a situation where the network operator has the ability to use transfers, price, and capacity. We then restrict the set of available control instruments. We first consider the case where transfers cannot be used when setting price and capacity. Then, we examine the no-transfer no-price-control case. Our main finding is that restricting the set of control instruments does not necessarily lead to higher-capacity pipelines.

JEL-code: L51, L95

Key words: Market power, Transport capacity, Natural gas.

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1 Introduction

The opening of gas markets is underway in most of the industrialized world, particularly so within the European Union which is lagging behind the United States in terms of the degree of introduction of market forces in the gas industry and integration of their national networks. In parallel to the launching of vast programs of structural reforms aimed at enhancing gas-to-gas competition, many European countries are heavily investing in the development of their pipeline networks. Such investments are often justified by the need to anticipate growth of demand, but some observers have come to question the idea that the large investments made in capacity expansion are all that needed.¹ Given the high degree of concentration of the gas industry in Europe, the issue of the impact of transport capacity on market structure and market power certainly deserves some attention.

The impact of transport capacity on market structure and market power has been documented in both the institutional/empirical and the theoretical literature on energy. On the empirical front, for the case of the US gas industry, a large stream of the literature has examined the impact of interconnecting sub-networks on the degree of market integration and competition (see, e.g., Doane and Spulber, 1994, and De Vany and Walls, 1994). From a more theoretical perspective, for the case of electricity, one line of literature has directly examined the impact of transmission capacity on local market power (e.g., Borenstein et al., 2000, Lautier, 2001) leading to the tentative

¹See, e.g., the article by Jill Junola in www.energyintel.com entitled "Spain Building More Gas Import Capacity Than It Needs," February 19, 2003.

conclusion that transmission link expansion, even if additional links are not actually used (Borenstein et al., 2000), is an effective policy instrument for promoting competition.²

In an exploratory theoretical piece on the gas industry, Cremer and Laffont (2002) examine the possibility of building "excess" capacity for the purpose of mitigating local market power. They obtain results that are not unambiguous although their main focus is on cases where excess capacity arises. Our paper generalizes Cremer and Laffont's by enlarging the set of instruments that can be used to control regional power. We thus carry further the analysis of the issue of over- versus under- capacity by assuming that the latter is but one of a set of possible weapons that are available for combating regional market power. The analysis is done within a simple theoretical framework the basic ingredients of which are presented in the next section.

In section 3 we characterize the policy prescriptions of a control scheme of a gas supplier by the network owner/operator that is based on transfers between consumers and the firm, price or equivalently output, and capacity. Then, we restrain the set of available control instruments. Section 4 assumes that transfers are not allowed and section 5 assumes that the network operator does not even have the ability to control price. In section 6, we investigate the issue of excess capacity building in a more systematic way. We provide

²Borenstein et al. (2000) show that if the capacity of the transmission line linking two regional markets is sufficiently large, both markets would be driven to the competitive price. Interestingly, they also construct an example where the competitive impact of the line on any of the two regional markets is stronger when the other market is a monopoly than when it is competitive. The intuition is that an aggressive output strategy is more effective in deterring imports when there is monopoly power at the other end of the line.

and analyze in detail two of three examples considered in the paper that allow us to highlight the role of capacity in market-power control. Section 7 summarizes our results on capacity ranking and discusses some simulations. The last section concludes the paper and the appendix presents the results obtained in the third example.

2 A simple two-market network configuration

Consider a regional natural gas market, market M , dominated by a single supplier, firm m , producing output q_m at cost $C^m(q_m)$. Gas is also supplied at marginal cost c in a perfectly competitive market, market C_p , which is geographically distinct from market M but linked to it by means of a pipeline of capacity K built at cost $C(K)$ (see figure 1). In this simple two-market configuration, gas that flows from market C_p into market M should exert competitive pressure on the regional monopoly and hence mitigate the exercise of market power by firm m in its local market.³

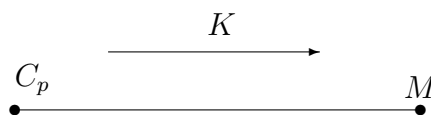


Figure 1: A simple two-market network

The analysis conducted in this paper rests on the presumption that the transport line linking these two markets is built by the network owner/operator for the purpose of allowing imports of gas from market C_p into market M .

³Throughout, we will assume that the price in the regional monopoly market is always above the marginal cost in the competitive market. A sufficient condition is that the monopolist marginal cost is above the marginal cost in the competitive market.

Letting $Q_M(\cdot)$ represent the demand function in market M , if a quantity of gas corresponding to full capacity of the pipeline, K , is shipped from the competitive market into market M , the supplier in this market would be a monopoly on the residual demand $Q_M(p_m) - K$ where p_m is price.

We now proceed to characterize the prescriptions of various policies that are used to control market power in the regional market M .⁴ It is worthwhile noting that, given the structure of the model, any pricing policy that is implemented in the regional monopoly market wouldn't affect welfare in the competitive market since price in the latter is at the first-best (marginal-cost) level. Hence, without loss of generality, we can ignore welfare of consumers and firms in this competitive market.⁵

We start from a situation where the network operator has the ability to control the gas supplier's market power by means of three regulatory instruments, namely, (possibly two-way) transfers between consumers and the firm, price (or equivalently output), and transport capacity of the network (section 3). We then restrict the set of regulatory instruments available to the network operator. We first consider the case where the operator may not use transfers when he sets the price and capacity levels (section 4). Then, we examine the situation where besides the fact that transfers are not allowed, the network operator loses the ability to control price (section 5).

⁴In this paper we assume that control of the monopoly is exercised under complete information. In a paper in progress (Gasmi et al., 2003), we introduce asymmetric information on the technology of gas production.

⁵Another factor that is also neglected in the analysis without affecting its main qualitative results is the marginal cost of transport. Alternatively, if marginal cost of transport is constant it can be included in the constant c , i.e., we may write $c = c_p + c_t$ where c_p is now the marginal cost of production in the competitive market and c_t is the marginal cost of transport.

3 Control of price and capacity when transfers are allowed

In this section we assume that the network operator, whose objectives coincide with those of the government, may use public funds raised through taxes to operate transfers between consumers and the firm. The economic distortions (the deadweight loss) generated by taxation are captured through a nonzero social cost of public funds λ . That is, if the network operator makes a monetary transfer T to the firm, this transfer costs consumers $(1 + \lambda)T$.

Let $S(\cdot)$ represent the utility function of consumers in market M . Total supply of gas $Q_M(p_m)$ in this market, composed of K units imported from the competitive market and q_m units produced locally by the firm, brings consumers an aggregate net welfare, V ,

$$V = S(Q_M(p_m)) - p_m Q_M(p_m) + (1 + \lambda) [(p_m - c)K - C(K)] - (1 + \lambda)T \quad (1)$$

This consumers' net welfare is composed of three terms: the net surplus of consumers, the social valuation of profits generated by the K units of gas provided competitively, and the social cost of the transfer made to the firm.

As to the regional monopoly welfare, denoted U , it is given by the sum of its profits and the transfer it receives from the network operator:

$$U = p_m [Q_M(p_m) - K] - C^m(Q_M(p_m) - K) + T \quad (2)$$

For the firm to be willing to supply gas at all, the network operator ought to guarantee it a given level of utility. This participation constraint is nor-

malized to

$$U \geq 0 \tag{3}$$

The utilitarian social welfare function, W , is defined as the unweighted sum of aggregate consumers' net welfare V and firm's utility U :

$$W = V + U \tag{4}$$

Substituting for V from (1) and for T from (2) yields social welfare

$$\begin{aligned} W = & S(Q_M(p_m)) + \lambda p_m Q_M(p_m) \\ & - (1 + \lambda) [cK + C(K) + C^m(Q_M(p_m) - K)] - \lambda U \end{aligned} \tag{5}$$

as the social valuation of the total production, minus its social cost, minus the social opportunity cost of the firm's utility. From this expression of social welfare we see that reducing the monopoly's utility is a socially desirable objective, for this utility includes a transfer of funds collected through distortive taxation (see (2)). Relatedly, we see from (5) that the social valuation of total production explicitly includes the fiscal value of the revenues that it generates.⁶

The regulatory program consists in maximizing social welfare W given by (5) with respect to p_m and K , under the participation constraint (3).⁷ From the expression of social welfare, we immediately see that the participation constraint is binding. Hence, the availability of transfers allows the network

⁶Indeed, these revenues allow the government to lessen the need to rely on distortive taxation.

⁷To be somewhat more realistic, one can assume a timing of decisions such that the network operator sets up first the capacity level and then the price level. Solving this problem by backward induction and using the envelope theorem yields the solution that is characterized in Proposition 1.

operator to extract the firm's profit through taxation. Substituting for $U = 0$ in (5) and using the fact that $\partial S(Q_M)/\partial Q_M = p_m$, we obtain the following first-order conditions:⁸

$$p_m Q'_M + \lambda [Q_M + p_m Q'_M] - (1 + \lambda) \left[\frac{\partial C^m}{\partial Q_M} \right] Q'_M = 0 \quad (6)$$

$$-(1 + \lambda) \left[c + C'(K) - \frac{\partial C^m}{\partial Q_M} \right] = 0 \quad (7)$$

Letting $\eta(Q_M) \equiv -Q'_M p_m / Q_M$, represent the price-elasticity of demand in market M , these conditions can be rewritten as in the following proposition:

Proposition 1 *When price (or equivalently output) and capacity are both controlled by the network operator and the latter can use public funds to make transfers between consumers and the firm, the following conditions are satisfied at the optimum:*

$$\frac{p_m - \frac{\partial C^m}{\partial Q_M}}{p_m} = \frac{\lambda}{1 + \lambda \eta(Q_M)} \quad (8)$$

$$(1 + \lambda) \left[\frac{\partial C^m}{\partial Q_M} \right] = (1 + \lambda) [c + C'(K)] \quad (9)$$

From equation (8) we see that pricing obeys a standard Ramsey principle according to which the price markup is inversely proportional to the price-elasticity of demand.⁹ Equation (9) says that the social cost of having an additional unit of gas produced locally by the firm just equals the social cost of having this unit shipped in from the competitive market.

⁸To minimize notation, the arguments of some of the demand and cost functions will be dropped in the presentation.

⁹A difference with the standard Ramsey-Boiteux rule though is that here the cost of public funds λ is exogenous.

4 Control of price and capacity when transfers are not allowed

We now assume that the network operator sets the capacity and price levels without the additional possibility of making any transfer between consumers and the firm.¹⁰ If one assumes that this controlled price is both a ceiling and a floor for the firm, this method of price control is of a price-cap type.¹¹

Social welfare W is now expressed as

$$\begin{aligned} W = & S(Q_M(p_m)) - p_m Q_M(p_m) \\ & + (1 + \lambda) [(p_m - c)K - C(K)] \\ & + p_m [Q_M(p_m) - K] - C^m(Q_M(p_m) - K) \end{aligned} \quad (10)$$

that is, as the sum of the net consumer surplus, the social value of the profits generated by the K units imported under competitive conditions, and the profits of the firm that now are not taxed. Gathering terms, we obtain

$$\begin{aligned} W = & S(Q_M(p_m)) + \lambda p_m K \\ & - (1 + \lambda) [cK + C(K)] - C^m(Q_M(p_m) - K) \end{aligned} \quad (11)$$

Cross-examining (5) and (11), we see that as he now cannot use transfers to collect firm's profits, the network operator assigns a fiscal value only to the revenue (and the cost) of the K units that are provided competitively.

¹⁰In the next section, we consider the case where, in addition to not being able to use transfers, the network operator loses the ability to control price.

¹¹While this method of price control is of a price-cap type, in this paper it is implemented under complete information. In our ongoing research that further explores the topic of this paper (Gasmi et al., 2003), we consider an asymmetric information version of this price-cap regulatory mechanism.

Maximizing social welfare given in (11) with respect to price and capacity yields the following first-order conditions:

$$p_m Q'_M + \lambda K - \left[\frac{\partial C^m}{\partial Q_M} \right] Q'_M = 0 \quad (12)$$

$$\lambda p_m - (1 + \lambda) [c + C'(K)] + \frac{\partial C^m}{\partial Q_M} = 0 \quad (13)$$

The next proposition rewrites these conditions in a form that is comparable with those of Proposition 1.

Proposition 2 *When price (or equivalently output) and capacity are both controlled by the network operator and the latter cannot use public funds to make transfers between consumers and the firm, the following conditions are satisfied at the optimum:*

$$\frac{p_m - \frac{\partial C^m}{\partial Q_M}}{p_m} = \lambda \frac{K}{Q_M} \frac{1}{\eta(Q_M)} \quad (14)$$

$$(1 + \lambda) \left[\frac{\partial C^m}{\partial Q_M} \right] + \lambda \left[p_m - \frac{\partial C^m}{\partial Q_M} \right] = (1 + \lambda) [c + C'(K)] \quad (15)$$

From equation (14) we see that pricing is still such that the price markup is inversely proportional to the demand elasticity, but is now proportional to the share of imports from the competitive market in the total consumption of gas in the monopoly market.¹² Provided the standard assumption of concavity of demand, we see from equation (13) that this latter property is due to the fact that as the price increases, the social marginal valuation of capacity increases.

¹²Note also that, as in the Ramsey formula established for the case with transfers, the price markup is proportional to the shadow cost of public funds λ , but it is now more sensitive to it.

The capacity equation (15) says that, at the optimum, the social cost of the marginal unit of gas shipped from the competitive market just equals the social cost of having this unit produced locally plus the social opportunity cost of its profitability for the firm. Compared to the case with transfers, this additional term comes from the fact that monopoly profits generated by additional units produced locally can no longer be collected by the network operator who now lacks the instrument that would allow him to do so.¹³

5 The no-transfer no-price-control case

In this section we assume that the network operator lacks an additional instrument of control of the regional monopoly activity, namely, pricing and can only use the level of capacity of the pipeline to counter its market power. In practice though, we model this case as if the network operator still continues to set the price level, but now this price has to fall within a constrained set of values. Let us be more specific.

For a given volume of gas K imported from the competitive market, the firm remains a monopoly in its local market on the residual demand $Q_M(p_m) - K$ where p_m is price. Given this demand, the firm sets price so as to maximize its profit π^m given by

$$\pi^m = p_m [Q_M(p_m) - K] - C^m(Q_M(p_m) - K) \quad (16)$$

¹³Note that this opportunity cost is for a given price p_m that satisfies (14). To see that marginal units produced locally generate a social opportunity cost, merely rewrite social welfare as $W = S + \lambda p_m Q_M - (1 + \lambda)[C^m + cK + C(K)] - \lambda \pi^m$ where $\pi^m = p_m(Q_M - K) - C^m$ is firm's profit.

The first-order condition of this profit-maximization problem is

$$\left[p_m - \frac{\partial C^m}{\partial Q_M} \right] Q'_M + Q_M - K = 0 \quad (17)$$

while the second-order condition that ensures that we are indeed at a maximum is

$$\left[p_m - \frac{\partial C^m}{\partial Q_M} \right] Q''_M + \left[1 - \frac{\partial^2 C^m}{\partial Q_M^2} Q'_M \right] Q'_M + Q'_M < 0 \quad (18)$$

Given that transfers are not allowed, it is clear enough that the form of the social welfare function for this case is analogous to the one described in the previous section which we recall here:

$$W = S(Q_M(p_m)) + \lambda p_m K - (1 + \lambda)[cK + C(K)] - C^m(Q_M(p_m) - K) \quad (19)$$

The optimization program that corresponds to this no-price control case requires then maximizing (19) with respect to p_m and K , under the constraint (17).¹⁴

Letting μ designate the Lagrange multiplier associated with the firm's profit-maximization constraint, we obtain the following first-order conditions:

$$p_m Q'_M + \lambda K - \left[\frac{\partial C^m}{\partial Q_M} \right] Q'_M - \mu \left\{ \left[p_m - \frac{\partial C^m}{\partial Q_M} \right] Q''_M + \left[1 - \frac{\partial^2 C^m}{\partial Q_M^2} Q'_M \right] Q'_M + Q'_M \right\} = 0 \quad (20)$$

$$\lambda p_m - (1 + \lambda)[c + C'(K)] + \frac{\partial C^m}{\partial Q_M} + \mu \left[1 - \frac{\partial^2 C^m}{\partial Q_M^2} Q'_M \right] = 0 \quad (21)$$

to which the constraint (17) is appended. Rewriting these first-order conditions allows us to state the following proposition:

¹⁴Strictly speaking, (18) should also be taken as a constraint. The standard way to deal with this issue, however, is to check ex post that this second-order condition is satisfied by the solution to the optimization problem.

Proposition 3 *When capacity only is controlled by the network operator, the following conditions are satisfied at the optimum:*

$$\frac{p_m - \frac{\partial C^m}{\partial Q_M}}{p_m} = \lambda \frac{K}{Q_M} \frac{1}{\eta(Q_M)} - \frac{\mu}{\eta(Q_M)} \left\{ \frac{\left[p_m - \frac{\partial C^m}{\partial Q_M} \right] Q_M'' + \left[1 - \frac{\partial^2 C^m}{\partial Q_M^2} Q_M' \right] Q_M' + Q_M'}{Q_M} \right\} \quad (22)$$

$$(1 + \lambda) \left[\frac{\partial C^m}{\partial Q_M} \right] + \lambda \left[p_m - \frac{\partial C^m}{\partial Q_M} \right] = (1 + \lambda)[c + C'(K)] + \mu \left[\frac{\partial^2 C^m}{\partial Q_M^2} Q_M' - 1 \right] \quad (23)$$

$$\left[p_m - \frac{\partial C^m}{\partial Q_M} \right] Q_M' + Q_M - K = 0 \quad (24)$$

A few comments on these conditions are in order.

First, note that in this no-transfer scheme the price markup is again proportional to the cost of public funds in a more sensitive way than in the case where the network operator could use transfers (section 3). Second, in contrast to the scheme analyzed in the previous section that gave the network operator more freedom in pricing, we see from (21) that increasing price will also have an (ambiguous) effect on the cost of violating the profit-maximization constraint (17) induced by imports from the competitive market.

Third, observe that, with respect to (14), equation (22) has an extra term the sign of which, assuming that the second-order condition of the firm (18) holds, is the same as that of μ , the shadow cost of the profit maximization constraint. Finally, we see from (23) that, at the optimum, the social cost

of having an extra unit produced locally by the monopoly, plus the social opportunity cost of the profit this unit generates for the firm, should be balanced against the social cost of importing this unit from the competitive region plus the cost of the violation of the profit-maximization constraint this imported unit induces.

As we might conclude from our discussion so far, this scheme reveals a number of rather complex price and capacity effects that turn out to make its comparison with the two previously analyzed schemes an interesting exercise. Let us now turn to the comparison of these three alternative ways of combating regional monopoly market power while paying a particular attention to the role played by network transport capacity.

6 Market power and transport capacity

This section compares the three schemes analyzed in the previous section in terms of the levels of network transport capacity they prescribe. We first discuss the general case and then give results obtained for some special cases.

6.1 Discussion of the general case

For clarity of exposition we refer to the schemes described in section 3 (control of price and capacity with transfers), section 4 (control of price and capacity without transfers), and section 5 (control of capacity only) as schemes A , B , and C respectively. Let K^A , K^B , and K^C designate the associated optimal levels of network capacity.

For the purpose of comparing the capacity levels obtained under schemes A and B , let us examine the first-order conditions (9) and (15) that, respectively, characterize these two schemes. Observe that the left-hand side of equation (15) includes an extra term that represents the social opportunity cost of the profitability of a marginal unit produced by the firm. Since from (14) this term is positive, assuming that the (variable) cost function of capacity building exhibits constant to decreasing returns, it is straightforward to see that $K^A < K^B$, i.e., that non-availability of transfers in scheme B leads to excess capacity relative to scheme A that allows for transfers.

Using a similar approach for comparing the capacities associated with schemes B and C (see equations (15) and (23)), on the one hand, and A and C (see equations (9) and (23)), on the other hand, we see that the ordering of these capacities is not unambiguous. Indeed, it turns out that $K^B < K^C$ if the shadow cost of the profit-maximization constraint (17), μ , is positive, and $K^B > K^C$ otherwise.¹⁵ As to schemes A and C , we obtain that $K^A < K^C$ if the cost of violating the profit-maximization constraint by importing an extra unit from the competitive market is lower than the social opportunity cost of the profitability of this unit if it is produced locally by the monopoly, and $K^A > K^C$ otherwise.

Given the crucial role played in these capacity comparisons by μ , the (endogenous) shadow cost of the firm's profit-maximization constraint under scheme C , it is useful to further investigate this issue under some specific functional forms for the demand and cost functions. In the remainder of this

¹⁵This result holds under the standard assumptions of downward-sloping demand and constant to decreasing returns firm's technology.

paper, we consider three commonly assumed configurations of demand and costs. Two of these configurations are analyzed in the next two subsections. The third one is presented in the appendix.

6.2 Case 1: Linear demand, constant returns in gas supply, decreasing returns in capacity building

In this subsection, we consider the case where demand is linear, and the technology of gas supply exhibits constant returns to scale while that of capacity building has decreasing returns to scale. More specifically, let

$$Q_M(p_m) = \gamma - p_m, C^m(q_m) = \theta q_m, C(K) = \frac{\omega}{2} K^2; \gamma, \theta, \omega > 0 \quad (25)$$

Solving for the endogenous variables of the three control schemes A , B , and C , and comparing the associated capacity levels, the following theorem can be stated:

Theorem 1 *Let demand and costs be described by (25) and K^A , K^B , and K^C designate optimal capacity levels achieved under schemes A , B , and C respectively. The following provides first, pairwise orderings of the capacity levels, and then a ranking of the three capacity levels.*

i) Assume that the following conditions hold:

$$\frac{\lambda}{1 + \lambda} < \frac{\omega}{\lambda} \quad (26)$$

$$c < \theta < \gamma \quad (27)$$

$$(1 + \lambda) [c + \omega(\gamma - \theta)] > (1 + \lambda)\theta + \lambda [(1 + \lambda)(\theta - c) + \lambda(\gamma - \theta)] \quad (28)$$

Then, the pair of schemes $\{A, B\}$ is "feasible" in the sense that each scheme of this pair yields a positive price, quantity, and capacity level.¹⁶ Furthermore, $K^A < K^B$.

ii) Assume that the same conditions as those given in i) above hold. Then, the pair of schemes $\{B, C\}$ is feasible. Moreover, if

$$(1 + \lambda) [c + \omega(\gamma - \theta)] > (1 + \lambda)\theta + \lambda [(1 + \lambda)(\theta - c)] \\ + \lambda [(1 + \lambda)(\theta - c) + \lambda(\gamma - \theta)] \quad (29)$$

then $K^B < K^C$. Otherwise, i.e., if

$$(1 + \lambda)\theta + \lambda [(1 + \lambda)(\theta - c) + \lambda(\gamma - \theta)] < \\ (1 + \lambda) [c + \omega(\gamma - \theta)] < \\ (1 + \lambda)\theta + \lambda [(1 + \lambda)(\theta - c)] + \lambda [(1 + \lambda)(\theta - c) + \lambda(\gamma - \theta)] \quad (30)$$

then $K^B > K^C$.¹⁷

iii) Assume that conditions (26), (27), and

$$(1 + \lambda) [c + \omega(\gamma - \theta)] > (1 + \lambda)\theta + \lambda(\theta - c) \quad (31)$$

hold. Then, the pair of schemes $\{A, C\}$ is feasible. Moreover, if

$$(1 + \lambda) [c + \omega(\gamma - \theta)] > (1 + \lambda)\theta + 2\lambda(\theta - c) + \lambda [\theta - (c + \omega(\gamma - \theta))] \quad (32)$$

$K^A < K^C$. Otherwise, i.e., if

$$(1 + \lambda)\theta + \lambda(\theta - c) < (1 + \lambda) [c + \omega(\gamma - \theta)] <$$

¹⁶Note that our definition of feasibility rules out shutting down any of the two sources of natural gas (markets C_p and M).

¹⁷Our use of the term "otherwise" here (and in what follows) carries some degree of abuse of language. Indeed, the reader should realize that, for instance, condition (30) encompasses both the reverse inequality of condition (29) and the feasibility conditions (26)-(28).

$$(1 + \lambda)\theta + 2\lambda(\theta - c) + \lambda[\theta - (c + \omega(\gamma - \theta))] \quad (33)$$

then $K^A > K^C$.

iv) Assume conditions (26)-(28) given in i) hold. Then, the triple of schemes $\{A, B, C\}$ is feasible. Moreover, if condition (29) holds, then $K^A < K^B < K^C$. If (29) does not hold and (32) holds, namely,

$$\begin{aligned} (1 + \lambda)\theta + 2\lambda(\theta - c) + \lambda[\theta - (c + \omega(\gamma - \theta))] &< \\ (1 + \lambda)[c + \omega(\gamma - \theta)] &< \\ (1 + \lambda)\theta + \lambda[(1 + \lambda)(\theta - c)] + \lambda[(1 + \lambda)(\theta - c) + \lambda(\gamma - \theta)] &\quad (34) \end{aligned}$$

then $K^A < K^C < K^B$. Otherwise, if both (29) and (32) do not hold, i.e.,

$$\begin{aligned} (1 + \lambda)\theta + \lambda[(1 + \lambda)(\theta - c) + \lambda(\gamma - \theta)] &< (1 + \lambda)[c + \omega(\gamma - \theta)] < \\ (1 + \lambda)\theta + 2\lambda(\theta - c) + \lambda[\theta - (c + \omega(\gamma - \theta))] &\quad (35) \end{aligned}$$

then $K^C < K^A < K^B$.¹⁸

Proof 1 Solving the systems of equations composed of the first-order conditions associated with each of the three schemes A, B, and C, namely (6)-(7), (12)-(13), and (20)-(21) and (17) respectively, yields the following closed-form solutions:

$$p_m^A = \theta + \left[\frac{\lambda}{1 + 2\lambda} \right] (\gamma - \theta) \quad (36)$$

$$K^A = \frac{(\theta - c)}{\omega} \quad (37)$$

$$q_m^A = \left[\frac{1 + \lambda}{1 + 2\lambda} \right] (\gamma - \theta) - \frac{(\theta - c)}{\omega} \quad (38)$$

¹⁸We should indicate that nonemptiness of the interval defined by (35) requires $\lambda(\gamma - c) < 2(\theta - c) - \omega(\gamma - \theta)$ (see footnote 26).

$$p_m^B = \theta + \left[\frac{\lambda(1+\lambda)}{\omega(1+\lambda) - \lambda^2} \right] (\theta - c) \quad (39)$$

$$K^B = \left[\frac{1+\lambda}{\omega(1+\lambda) - \lambda^2} \right] (\theta - c) \quad (40)$$

$$q_m^B = (\gamma - \theta) - \left[\frac{(1+\lambda)^2}{\omega(1+\lambda) - \lambda^2} \right] (\theta - c) \quad (41)$$

$$p_m^C = \theta + \frac{[\lambda + 2\omega(1+\lambda)](\gamma - \theta) - 2(1+\lambda)(\theta - c)}{(1+2\lambda)(1+2\omega) + 2(\lambda + \omega)} \quad (42)$$

$$K^C = \frac{(1+2\lambda)(\gamma - \theta) + 4(1+\lambda)(\theta - c)}{(1+2\lambda)(1+2\omega) + 2(\lambda + \omega)} \quad (43)$$

$$q_m^C = \frac{[\lambda + 2\omega(1+\lambda)](\gamma - \theta) - 2(1+\lambda)(\theta - c)}{(1+2\lambda)(1+2\omega) + 2(\lambda + \omega)} \quad (44)$$

$$\mu = \frac{[\omega(1+\lambda) - \lambda^2](\gamma - \theta) - (1+\lambda)(1+2\lambda)(\theta - c)}{(1+2\lambda)(1+2\omega) + 2(\lambda + \omega)} \quad (45)$$

As to the second-order conditions (the Hessian matrix being negative definite) which guarantee that the above solutions are local maximizers, they are always satisfied for scheme A whereas for schemes B and C they require $\omega(1+\lambda) - \lambda^2 > 0$, which is equivalent to $\lambda/(1+\lambda) < \omega/\lambda$, i.e., condition (26) stated in the theorem.

i) Examining (37) and (40), we see that pipeline capacity levels under A and B will be positive if (26) holds and $\theta > c$. On the other hand, (26) and $\gamma > \theta$ allow us to conclude that if

$$(\gamma - \theta) > \left[\frac{(1+\lambda)^2}{\omega(1+\lambda) - \lambda^2} \right] (\theta - c) \quad (46)$$

rewritten as (28), then $q_m^B > 0$, and the latter implies that $q_m^A > 0$, as can be seen from (38) and (41).¹⁹ Next, we derive the capacity gap between schemes B and A, $K^B - K^A$, and see that it is equal to $[\lambda^2/(\omega(1+\lambda) - \lambda^2)]$

¹⁹Note that (26) and (27) guarantee that prices under schemes A and B are strictly positive.

$[(\theta - c)/\omega]$. Finally, we observe that this capacity gap is positive within the domain of feasibility of these two schemes.

ii) From (40) and (43), we see that if (26) and (27) hold, then K^B and K^C are both positive. As to locally produced output, we see from (41) and (44) that whenever $q_m^B > 0$, it will be the case that $q_m^C > 0$. Thus, (26), (27) and (46), the latter being rewritten as (28), guarantee that the pair of schemes $\{B, C\}$ is feasible.²⁰ Now, we calculate $K^C - K^B$ and see that it is positive if

$$(\gamma - \theta) > \left[\frac{(1 + \lambda)(1 + 2\lambda)}{\omega(1 + \lambda) - \lambda^2} \right] (\theta - c) \quad (47)$$

which can be rewritten as (29), and negative if

$$\left[\frac{(1 + \lambda)^2}{\omega(1 + \lambda) - \lambda^2} \right] (\theta - c) < (\gamma - \theta) < \left[\frac{(1 + \lambda)(1 + 2\lambda)}{\omega(1 + \lambda) - \lambda^2} \right] (\theta - c) \quad (48)$$

which can be rewritten as (30).²¹ As alluded to in our discussion of the general case (see subsection 6.1), these conditions reflect the sign of μ (given by (45)), the shadow cost of the profit-maximization constraint (17) accounted for in scheme C. More specifically, (47) is equivalent to $\mu > 0$ and (48) combines $\mu < 0$ with the conditions that ensure that the pair $\{B, C\}$ is feasible.

iii) From (37) and (43), we see that if (26) and (27) hold, then $K^A > 0$ and $K^B > 0$. Furthermore, if

$$(\gamma - \theta) > \left[\frac{1 + 2\lambda}{1 + \lambda} \right] \frac{(\theta - c)}{\omega} \quad (49)$$

which can be rewritten as (31), holds, then $q_m^A > 0$ and the pair of schemes

²⁰Note that (27) $\Rightarrow p_m^B > 0$ and $q_m^C > 0 \Rightarrow p_m^C > 0$.

²¹Observe that under (27), the interval defined by (30) is always nonempty, which says that $K^C < K^B$ might indeed occur.

$\{A, C\}$ is feasible.²² Finally, we calculate $K^C - K^A$ and observe that it is positive if

$$(\gamma - \theta) > \left[\frac{1 + 4\lambda}{1 + 2\lambda} \right] \frac{(\theta - c)}{\omega} \quad (50)$$

and negative otherwise. Inequality (50) and its reverse can be rewritten as (32) and (33) respectively.²³ As in ii), a crucial factor related to these conditions is the sign of μ .

iv) If (26), (27) and (46), the latter being equivalent to (28), hold, then we obtain joint feasibility of schemes A, B, and C.²⁴ Next, we form $K^C - K^A$, $K^C - K^B$, and $K^B - K^A$, using (37), (40), and (43). If condition (47), which can be rewritten as (29), holds, we obtain $K^A < K^B < K^C$. Now suppose that (29) does not hold, i.e., (30) holds, and thus by ii) $K^B > K^C$. If, in addition to (30), (32) holds, i.e.,

$$\left[\frac{1 + 4\lambda}{1 + 2\lambda} \right] \frac{(\theta - c)}{\omega} < (\gamma - \theta) < \left[\frac{(1 + \lambda)(1 + 2\lambda)}{\omega(1 + \lambda) - \lambda^2} \right] (\theta - c) \quad (51)$$

rewritten as (34), we obtain that $K^A < K^C < K^B$.²⁵ Finally, suppose that both (29) and (32) do not hold, i.e.,

$$\left[\frac{(1 + \lambda)^2}{\omega(1 + \lambda) - \lambda^2} \right] (\theta - c) < (\gamma - \theta) < \left[\frac{1 + 4\lambda}{1 + 2\lambda} \right] \frac{(\theta - c)}{\omega} \quad (52)$$

rewritten as (35), holds, then we obtain that $K^C < K^A < K^B$.²⁶

²²Positivity of prices is here established by noting that (27) $\Rightarrow p_m^A > 0$, and $q_m^A > 0 \Rightarrow q_m^C > 0 \Rightarrow p_m^C > 0$.

²³Again, to see that (33) defines a nonempty interval (i.e., that $K^C < K^A$ may arise), note that the inequality $(1 + \lambda)\theta + \lambda(\theta - c) < (1 + \lambda)\theta + 2\lambda(\theta - c) + \lambda[\theta - (c + \omega(\gamma - \theta))]$ is equivalent to $(\theta - c)/\omega > (\gamma - \theta)/2$, which indeed is compatible with $K^C = K^A$ and hence for all capacity levels such that $K^C < K^A$.

²⁴At this point it is clear that prices are strictly positive.

²⁵Again, to see that $K^A < K^C < K^B$ may arise, note that nonemptiness of the interval defined by (34) is equivalent to $\lambda(\gamma - c) > (1 - \lambda)(\theta - c) - \omega(\gamma - \theta)$, which can be shown to be compatible with capacity levels such that $K^C \leq K^B$.

²⁶To guarantee that $K^C < K^A < K^B$ may indeed arise, note that the interval defined by

This completes the proof of Theorem 1.

Let us further discuss the intuition behind some of the statements in Theorem 1. Scheme B being merely scheme A without the possibility of using transfers, the result in Theorem 1-(i) says that higher capacity (in B) compensates for the lack of transfers. Thus, the comparison of these two schemes reveals that capacity and transfers are substitutes when it comes to controlling monopoly power. Recall that for each comparison performed, we first make sure that the appropriate control schemes are feasible, i.e., that at the optimum they prescribe strictly positive prices, quantities and capacities. For the case of schemes A and B , as indicated in the proof, it turns out that feasibility is achieved if merely optimal monopoly production under scheme B is strictly positive.

The reason why $q_m^B > 0$ is socially optimal can be analyzed by means of condition (28). Indeed, observe that $\lambda[(1 + \lambda)(\theta - c) + \lambda(\gamma - \theta)]$ is the highest level of social marginal opportunity cost of capacity under scheme B , namely, $\lambda[\sup\{\partial W^B/\partial K\}] = \lambda[\partial W^B/\partial K|_{q_m^B, K^B=0}]$.²⁷ Condition (28) then says that it is optimal to allow the local monopoly to produce under this scheme, if this social opportunity cost of capacity plus the social marginal cost of production by the monopoly, $(1 + \lambda)\theta$, is less than the highest level of social marginal cost of imports, namely, $(1 + \lambda)[c + \omega(\gamma - \theta)]$.

(35) being nonempty is equivalent to $(1 - \lambda)(\theta - c) - \omega(\gamma - \theta) < \lambda(\gamma - c) < 2(\theta - c) - \omega(\gamma - \theta)$ or, since the left part of this inequality is always compatible with $K^C \leq K^A$, $\lambda(\gamma - c) < 2(\theta - c) - \omega(\gamma - \theta)$.

²⁷Social welfare under scheme B , denoted W^B , can be computed by substituting (25) into (11).

For the case of B versus C (Theorem 1-(ii)), again feasibility of these two schemes is achieved if $q_m^B > 0$, which is implied by (28). The reason for this is that if it is socially optimal to let a monopoly produce when the constraint of its profit maximization is not taken into account (scheme B), then it will necessarily be the case under a scheme that explicitly accounts for this constraint (scheme C).

To see the behavior of capacity under these two alternative schemes, let us make more explicit the relationship between condition (29) and μ , the shadow cost of the profit-maximization constraint under scheme C . We have already mentioned in subsection 6.1 where we discussed the general case and shown in the proof of statement 1-(ii), that the condition for $K^C - K^B > 0$ is identical to the one for $\mu > 0$. Now, it is easy to show that given the functional forms considered in this theorem, (25), $\partial\mathcal{L}^C/\partial K = \partial W^B/\partial K + \mu$, where \mathcal{L}^C is the Lagrangian associated with the optimization program under scheme C . By the envelope theorem $\partial\mathcal{L}^C/\partial K|_{K=K^B} = \mu$, where K^B is the optimal capacity under scheme B given by (40). Accordingly, when $\mu > 0$, the optimal capacity under scheme C , K^C , must be greater than under scheme B .

Now, note that this social marginal utility of capacity is positive if (highest) achievable social marginal cost of imports is greater than social marginal cost of the local monopoly, plus the social opportunity cost (in terms of capacity) of distorting price away from marginal cost of the monopoly, plus the highest level of social marginal opportunity cost of capacity under scheme

B .²⁸ Given that (28) has to be satisfied, if (29) does not hold, we obtain (30) which merely says that $\mu < 0$, or equivalently, $\partial\mathcal{L}^C/\partial K|_{K=K^B} < 0$, and thus $K^C < K^B$.

Condition (31) in Theorem 1-(iii) ensures that q_m^A is positive. It says that social marginal cost of the local monopoly plus social opportunity cost of the foregone revenues due to monopoly production of an additional unit (evaluated at $K^A = 0$), $\theta - c = (p_m^A - c) - (p_m^A - \theta)$, should be less than highest achievable social marginal cost of imports.²⁹ Again, to understand the ranking of optimal capacity levels under schemes A and C , we examine the relationship between the capacity gap and μ . From (6), (21), and (25), we obtain $\partial\mathcal{L}^C/\partial K = \partial W^A/\partial K + \lambda(p_m - \theta) + \mu$. By the envelope theorem $\partial\mathcal{L}^C/\partial K|_{K=K^A} = \lambda(p_m - \theta) + \mu$, where K^A is the optimal capacity under scheme A given by (37). Therefore, optimal capacity under C must be greater than under A if $\mu > -\lambda(p_m^c - \theta)$.³⁰ On the other hand, given that (31) has to be satisfied, if (32) does not hold, we obtain (33) which is equivalent to $\partial\mathcal{L}^C/\partial K|_{K=K^A} < 0$, leading to $K^C < K^A$.

The social marginal utility of capacity under scheme C (evaluated at $K = K^A$) will be positive if (highest) achievable social marginal cost of imports is greater than social marginal cost of the local monopoly, plus twice the social opportunity cost of foregone revenues due to production by the

²⁸Note that since capacity allows to ship "good" gas, it makes sense to use it in the social measurement of opportunity costs.

²⁹Both $(p_m^A - c)$ and $(p_m^A - \theta)$ entail a social opportunity cost under scheme A because these revenues are recoverable under this scheme that allows for transfers.

³⁰Note that this condition, which has been discussed in the general case (see subsection 6.1), is a weaker condition than the one that implies $K^C - K^B > 0$. This suggests that the outcome K^C to the left of K^B is "more likely" than K^C to the left of K^A .

monopoly of an extra unit (evaluated at $K^A = 0$), plus this social opportunity cost evaluated at maximum feasible capacity $K^A = \gamma - \theta$.

We conclude this discussion with an illustration of the argument underlying the results in Theorem 1-(iv). In order to compare schemes A , B , and C , we first make sure that the scheme triple $\{A, B, C\}$ is feasible. We do this by relying on the already studied feasibility of the pairs $\{A, B\}$, $\{B, C\}$, and $\{A, C\}$. It turns out that all that is needed is feasibility of the pair $\{B, C\}$, which is ensured by conditions (26)-(28). Since it is always the case that $K^A < K^B$, we focus on the behavior of capacity levels only within the pairs $\{A, C\}$ and $\{B, C\}$. Figure 2 below visualizes the argument in the space of values of $(1 + \lambda)[c + \omega(\gamma - \theta)]$, the highest achievable social marginal cost of imports already mentioned above.

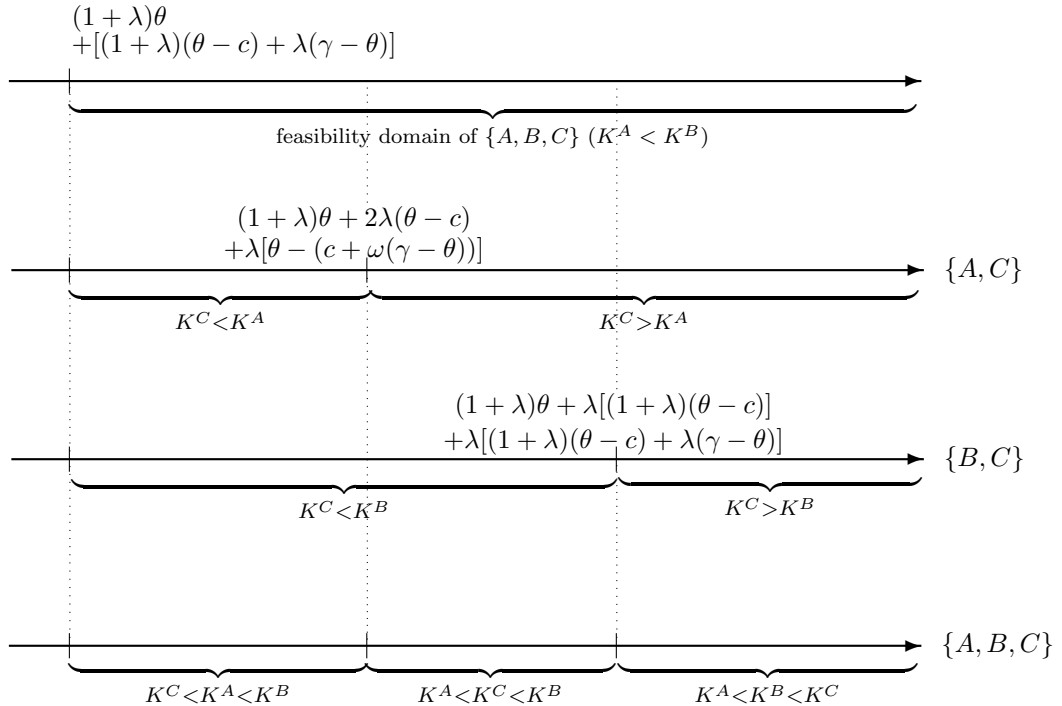


Figure 2: Comparison of capacity levels under schemes A , B , and C

The upper-line of this figure characterizes the feasibility region of the scheme triple $\{A, B, C\}$. The two intermediate lines show the regions that determine the sign of the pairwise capacity gaps associated with the pairs $\{A, C\}$ and $\{B, C\}$. The regions shown in the bottom-line are merely found as the intersection of those defined in the lines above it. Given that the outcome $K^C < K^A < K^B$ is in some sense the most surprising, we examine a bit more closely the likelihood of its occurrence. Recall that we have shown that an additional inequality should hold for this outcome to arise (see footnote 26). We simulated the behavior of this inequality, and the results suggest that the ranking $K^C < K^A < K^B$ would correspond to low-enough values of the cost of public funds so that social opportunity costs from allowing the local monopoly to produce would be of small magnitude.³¹

6.3 Case 2: Linear demand, decreasing returns in gas supply, constant returns in capacity building

In this subsection, we consider a second case that combines linear demand with decreasing returns to scale in gas supply and constant returns to scale in capacity building. Let us assume that

$$Q_M(p_m) = \gamma - p_m, C^m(q_m) = \frac{\theta}{2}q_m^2, C(K) = \omega K; \gamma, \theta, \omega > 0 \quad (53)$$

The following theorem can then be stated:

Theorem 2 *Under the assumptions described in (53), the optimal capacity*

³¹For values of the slope of marginal cost of capacity building $\omega = 1/4, 1/2, 1, 3/2$, we get that the highest values of the social cost of public funds compatible with this inequality, are approximately $\lambda = 0.13, 0.20, 0.27, 0.34$ respectively.

levels K^A , K^B , and K^C achieved under, respectively, schemes A , B , and C are ordered as follows:

i) Assume that the following conditions hold

$$\frac{\lambda}{1+\lambda} < \frac{\theta}{\lambda} \left[\frac{1+2\lambda}{1+\lambda} \right] \quad (54)$$

$$c + \omega < \gamma \quad (55)$$

$$(1+\lambda)(c+\omega) > \lambda[\lambda(\gamma - (c+\omega)) - (c+\omega)] \quad (56)$$

$$\lambda(1+\lambda) [\theta(\gamma - (c+\omega)) - (c+\omega)] > \lambda^2(c+\omega) \quad (57)$$

Then, the pair of schemes $\{A, B\}$ is feasible, and $K^A < K^B$.

ii) Assume that conditions (54)-(56) exhibited in i) and

$$\lambda(1+\lambda) [\theta(\gamma - (c+\omega)) - (c+\omega)] > 0 \quad (58)$$

hold. Then, the scheme pair $\{B, C\}$ is feasible. Moreover, if

$$0 < \lambda(1+\lambda) [\theta(\gamma - (c+\omega)) - (c+\omega)] < (1+\lambda)^2(c+\omega) - \lambda^2\gamma \quad (59)$$

then $K^B < K^C$. Otherwise, i.e., if

$$\begin{aligned} 0 < (1+\lambda)(c+\omega) - \lambda[\lambda(\gamma - (c+\omega)) - (c+\omega)] < \\ \lambda(1+\lambda) [\theta(\gamma - (c+\omega)) - (c+\omega)] \end{aligned} \quad (60)$$

then, $K^B > K^C$.

iii) Suppose that conditions (54), (55), and (57) hold. Then, the pair $\{A, C\}$ is feasible. Moreover, if

$$\begin{aligned} \lambda^2(c+\omega) < \lambda(1+\lambda) [\theta(\gamma - (c+\omega)) - (c+\omega)] < \\ (1+\lambda) [(1+\lambda) + \lambda(1+2\lambda)(2+\theta)^2] (c+\omega) \end{aligned} \quad (61)$$

then $K^A < K^C$. Otherwise, i.e., if

$$\lambda[\theta(\gamma - (c + \omega)) - (c + \omega)] > [(1 + \lambda) + \lambda(1 + 2\lambda)(2 + \theta)^2](c + \omega) \quad (62)$$

then $K^A > K^C$.

iv) Assume that conditions (54)-(57) stated in i) hold. Then, the scheme triple $\{A, B, C\}$ is feasible. Moreover, if

$$\begin{aligned} \lambda^2(c + \omega) < \lambda(1 + \lambda)[\theta(\gamma - (c + \omega)) - (c + \omega)] < \\ (1 + \lambda)^2(c + \omega) - \lambda^2\gamma \end{aligned} \quad (63)$$

then $K^A < K^B < K^C$. If (63) is not satisfied but (61) is, or equivalently, if

$$\begin{aligned} (1 + \lambda)(c + \omega) - \lambda[\lambda(\gamma - (c + \omega)) - (c + \omega)] < \\ \lambda(1 + \lambda)[\theta(\gamma - (c + \omega)) - (c + \omega)] < \\ (1 + \lambda)[(1 + \lambda) + \lambda(1 + 2\lambda)(2 + \theta)^2](c + \omega) \end{aligned} \quad (64)$$

holds, then $K^A < K^C < K^B$. Otherwise, if both (61) and (63) are not satisfied, or equivalently, condition (62) is, then $K^C < K^A < K^B$.

Proof 2 Using (53) and solving the first-order conditions corresponding to schemes A, B, and C, yields the following closed-form solutions:

$$p_m^A = \left[\frac{1 + \lambda}{1 + 2\lambda} \right] (c + \omega) + \left[\frac{\lambda}{1 + 2\lambda} \right] \gamma \quad (65)$$

$$K^A = \left[\frac{1 + \lambda}{1 + 2\lambda} \right] [\gamma - (c + \omega)] - \frac{c + \omega}{\theta} \quad (66)$$

$$q_m^A = \frac{c + \omega}{\theta} \quad (67)$$

$$p_m^B = (c + \omega) + \left[\frac{\lambda}{(1 + 2\lambda)\theta - \lambda^2} \right] [\theta(\gamma - (c + \omega)) - (c + \omega)] \quad (68)$$

$$K^B = \left[\frac{1 + \lambda}{(1 + 2\lambda)\theta - \lambda^2} \right] [\theta(\gamma - (c + \omega)) - (c + \omega)] \quad (69)$$

$$q_m^B = \frac{(1 + 2\lambda)(c + \omega) - \lambda^2[\gamma - (c + \omega)]}{(1 + 2\lambda)\theta - \lambda^2} \quad (70)$$

$$p_m^C = (c + \omega) + \frac{\lambda(1 + \theta)^2[\gamma - (c + \omega)] + [1 - \lambda(1 + \theta)](c + \omega)}{(1 + 2\lambda)(1 + \theta)(2 + \theta) - 1} \quad (71)$$

$$K^C = \frac{(1 + \lambda)(2 + \theta)^2[\gamma - (c + \omega)] - [1 + (1 + \lambda)(2 + \theta)]\gamma}{(1 + 2\lambda)(1 + \theta)(2 + \theta) - 1} \quad (72)$$

$$q_m^C = \frac{(1 + \lambda)(2 + \theta)(c + \omega) + \lambda(1 + \theta)\gamma}{(1 + 2\lambda)(1 + \theta)(2 + \theta) - 1} \quad (73)$$

$$\mu = \frac{(1 + \lambda)[1 + \lambda(2 + \theta)](c + \omega) - \lambda[\lambda + (1 + \lambda)\theta]\gamma}{(1 + 2\lambda)(1 + \theta)(2 + \theta) - 1} \quad (74)$$

The second-order conditions are always satisfied for scheme A, whereas for schemes B and C they require $\theta(1 + 2\lambda) - \lambda^2 > 0$ which is equivalent to condition (54) given in the theorem.

i) Given (54) and (55), from an examination of (68) and (69) we can see that $K^B > 0 \Rightarrow p_m^B > 0$, as the former is equivalent to

$$(\gamma - (c + \omega)) > \frac{c + \omega}{\theta} \quad (75)$$

whereas the latter is equivalent to

$$(\gamma - (c + \omega)) > \frac{(c + \omega)}{\theta} \left[\frac{\lambda^2 - \theta(1 + 2\lambda) + \lambda}{\lambda} \right] \quad (76)$$

and, from (54), the term in brackets in (76) is less than one. Furthermore, looking at (66) and (69) we have $K^A > 0 \Rightarrow K^B > 0$.³² Therefore, feasibility of the scheme pair $\{A, B\}$ is achieved if $q_m^B, K^A > 0$, which is guaranteed by conditions (56) and (57) given in the theorem. Now, we calculate the capacity

³²Note that q_m^A and p_m^A are strictly positive.

gap between schemes B and A , $K^B - K^A$, and we observe that it is positive if

$$\lambda(1 + \lambda)[\theta(\gamma - (c + \omega)) - (c + \omega)] > (1 + 2\lambda)(\lambda - \theta)(c + \omega) \quad (77)$$

In order to conclude that $K^B - K^A$ is always positive within the feasibility domain of the pair $\{A, B\}$, it suffices to show that any of the two feasibility conditions, (56) and (57), implies (77), and hence that the latter can be ignored. That (57) implies (77) is obvious from the expressions themselves. To see that (77) is also implied by (56), merely rewrite (77) as

$$\begin{aligned} (1 + \lambda)\theta(c + \omega) &> \lambda\theta[\lambda(\gamma - (c + \omega)) - (c + \omega)] \\ &\quad - \lambda(1 + 2\lambda)[\theta(\gamma - (c + \omega)) - (c + \omega)] \end{aligned} \quad (78)$$

ii) Given (54) and (55), examining (69) and (72) allows us to conclude that $K^B > 0 \Rightarrow K^C > 0$, as the first inequality amounts to (75) while the second is equivalent to

$$(\gamma - (c + \omega)) > \frac{(c + \omega)}{\theta} \left[\frac{\theta(1 + (1 + \lambda)(2 + \theta))}{(1 + \lambda)(1 + \theta)(2 + \theta) - 1} \right] \quad (79)$$

and the term in brackets in (79) is less than one.³³ Hence, feasibility of the pair $\{B, C\}$ is obtained if $q_m^B > 0$ and $K^B > 0$, which are guaranteed by conditions (56) and (58) given in the theorem. Next, we form $K^C - K^B$ which turns out to be positive if

$$(\gamma - (c + \omega)) < \frac{(c + \omega)}{\theta} \left[\frac{\theta(\lambda + (1 + \lambda)^2)}{\lambda(\lambda + \theta(1 + \lambda))} \right] \quad (80)$$

rewritten as (59) when combined with feasibility conditions, and negative if

³³Note that $K^B > 0 \Rightarrow p_m^B > 0$, and p_m^C and q_m^C are strictly positive.

(80) does not hold.³⁴ In the case where (80) does not hold, meeting the conditions that guarantee feasibility yields condition (60) given in the theorem.³⁵

iii) Given (54) and (55), recall from, respectively, parts i) and ii) of this proof that $K^A > 0 \Rightarrow K^B > 0$ and $K^B > 0 \Rightarrow K^C > 0$.³⁶ Hence, $K^A > 0$, ensured by (57), guarantees that the pair composed of schemes A and C is feasible. Next, making use of (66) and (72), we calculate the capacity differential between schemes A and C and see that it is positive if

$$(\gamma - (c + \omega)) < \frac{(c + \omega)}{\theta} \left[\frac{(1 + 2\lambda)(1 + \lambda(2 + \theta)^2)}{\lambda} \right] \quad (81)$$

and negative otherwise. It is then easy to show that the inequality exhibited in (81) and its reverse can be rewritten as (61) and (62) respectively.³⁷

iv) From the previous parts of this proof, it is straightforward to see that conditions (54)-(57) ensure feasibility of the scheme triple $\{A, B, C\}$. Next, using (66), (69), and (72), we form $K^C - K^A$, $K^C - K^B$, and $K^B - K^A$. First, suppose condition (63) holds. Then, since (63) \Rightarrow (59) we have from ii) above that $K^B < K^C$. Combining this with $K^A < K^B$, which is always true within the feasibility domain, we conclude that $K^A < K^B < K^C$.³⁸ Now, assume that condition (63) does not hold. Thus, we have either $K^A < K^C < K^B$ or $K^C < K^A < K^B$. If we further assume that (61) holds, the only possibility left then is $K^A < K^C < K^B$ since (61) \Rightarrow $K^A < K^C$. It is easy to show

³⁴We should point out that (56) ensures that the interval defined by the inequality (59) is nonempty. Indeed, the reader can easily check that $(1 + \lambda)^2(c + \omega) - \lambda^2\gamma > 0 \Leftrightarrow$ (56).

³⁵Note that nonemptiness of the interval defined by (60) is implied by (58).

³⁶Note that $p_m^A, q_m^A, p_m^C, q_m^C > 0$.

³⁷It is also easy to show that the interval defined by the inequality (61) is nonempty.

³⁸Using the same approach as in the proof of Theorem 1-(iv), the reader might verify that the interval defined by condition (63) is nonempty by checking that the inequality $(1 + \lambda)^2(c + \omega) > \lambda^2[\gamma - (c + \omega)]$ is compatible with $K^C \geq K^B$.

that condition (63) not holding and (61) holding is equivalent to (64) given in the theorem.³⁹ Finally, assume that both conditions (61) and (63) are not satisfied (thus $K^C < K^A$ and $K^C < K^B$), or equivalently condition (62) is, we find that $K^C < K^A < K^B$.⁴⁰

This completes the proof of Theorem 2.

7 Summary of results and simulations

In the previous section we have provided, first at a general level and then under some specific demand and cost conditions, a characterization of the ordering of the capacity levels across the three control schemes A , B and C .⁴¹ Table 1 below recapitulates the ordering obtained under each of the three economic scenarios considered, labelled Cases 1, 2 and 3, and the conditions under which these orderings are obtained. We see from this table that while K^A is consistently smaller than K^B , K^C may be greater or smaller than both K^A and K^B .

Cremer and Laffont (2002) have directed attention to this ambiguity issue, although they have mainly focused on the "excess" capacity case by providing examples where the lack of price control leads to an over-sizing of the pipeline network. In this paper, we have further investigated this issue of network sizing by uncovering cases where network capacity and alternative

³⁹It is straightforward to check that the interval defined by the inequality in condition (64) is always nonempty.

⁴⁰The reader can easily verify that the interval defined by (62) is strictly contained in the feasibility domain.

⁴¹Cases 1 and 2 are analyzed in the previous section, whereas Case 3 is presented in the appendix.

instruments of market power control are both complements and substitutes. In contrast to Cremer and Laffont's study, our analysis allows us to identify situations of both over- and under-sizing of the network.

It is easy to see that Cremer and Laffont's comparison exercise is a special case of the comparisons we have performed in this paper. Indeed, setting $\lambda = 0$ in our modeling framework, we obtain scheme A as the first-best, and B and C reduce to the schemes considered by the authors in their comparison, namely, with and without price control.⁴² Moreover, a straight application of Theorems 1-3 of this paper yields the unambiguous "excess" capacity result discussed by these authors, i.e., $K^C > K^B$ in all of the three cases we have analyzed.

⁴²To be more precise, the specific case considered by Cremer and Laffont (2002) in their footnote 7 corresponds to our Case 2, provided that $\lambda = 0$.

Table 1: Ordering of capacity levels

Available schemes	Case	Conditions	Implied ranking
{A, B}	(1)	(26)-(28)	$K^A < K^B$
	(2)	(54)-(57)	$K^A < K^B$
	(3)	(A.2)-(A.5)	$K^A < K^B$
{B, C}	(1)	$\left\{ \begin{array}{l} (26)-(29) \\ (26)-(28), (30) \end{array} \right.$	$K^B < K^C$ $K^C < K^B$
	(2)	$\left\{ \begin{array}{l} (54)-(56), (58), (59) \\ (54)-(56), (58), (60) \end{array} \right.$	$K^B < K^C$ $K^C < K^B$
	(3)	$\left\{ \begin{array}{l} (A.2)-(A.4), (A.6), (A.7) \\ (A.2)-(A.4), (A.6), (A.8) \end{array} \right.$	$K^B < K^C$ $K^C < K^B$
{A, C}	(1)	$\left\{ \begin{array}{l} (26), (27), (31), (32) \\ (26), (27), (31), (33) \end{array} \right.$	$K^A < K^C$ $K^C < K^A$
	(2)	$\left\{ \begin{array}{l} (54), (55), (57), (61) \\ (54), (55), (57), (62) \end{array} \right.$	$K^A < K^C$ $K^C < K^A$
	(3)	$\left\{ \begin{array}{l} (A.2), (A.3), (A.5), (A.9) \\ (A.2), (A.3), (A.5), (A.10) \end{array} \right.$	$K^A < K^C$ $K^C < K^A$
{A, B, C}	(1)	$\left\{ \begin{array}{l} (26)-(29) \\ (26)-(28), (34) \\ (26)-(28), (35) \end{array} \right.$	$K^A < K^B < K^C$ $K^A < K^C < K^B$ $K^C < K^A < K^B$
	(2)	$\left\{ \begin{array}{l} (54)-(57), (63) \\ (54)-(57), (64) \\ (54)-(57), (62) \end{array} \right.$	$K^A < K^B < K^C$ $K^A < K^C < K^B$ $K^C < K^A < K^B$
	(3)	$\left\{ \begin{array}{l} (A.2)-(A.5), (A.11) \\ (A.2)-(A.5), (A.12) \\ (A.2)-(A.5), (A.10) \end{array} \right.$	$K^A < K^B < K^C$ $K^A < K^C < K^B$ $K^C < K^A < K^B$

While capacity is an important variable to analyze from both a theoretical and institutional standpoint, it is also instructive to examine the behavior of the other endogenous variables, namely, natural gas price and output levels in the regional market. In the remainder of this section, we summarize some

simulations that we have performed, by taking a particular specification of Case 1 (see (25)) with $\omega = 1$ and $\gamma - \theta = 5$. The motivation for taking a specific value (equal to 5) for the maximum price-cost margin of the local monopoly, $\gamma - \theta$, stems from noting that the closed-form solutions of the welfare-maximization programs corresponding to schemes *A*, *B*, and *C* (see (36)-(44)) are expressed in terms of this price-cost margin, the monopoly's marginal cost, θ , and the gap between the marginal cost of the monopoly and the competitive market, $\theta - c$.⁴³ Setting the maximum price-cost margin equal to 5 yields the expressions for capacity, local output, and price levels exhibited in Table 2 below.

Table 2: Capacity, local output, and price levels

	Scheme A	Scheme B	Scheme C
K	$(\theta - c)$	$\left[\frac{1+\lambda}{1+\lambda-\lambda^2} \right] (\theta - c)$	$\frac{5(1+\lambda)}{5+8\lambda} + \left[\frac{4(1+\lambda)}{5+8\lambda} \right] (\theta - c)$
q_m	$\frac{5(1+\lambda)}{1+2\lambda} - (\theta - c)$	$5 - \left[\frac{(1+\lambda)^2}{1+\lambda-\lambda^2} \right] (\theta - c)$	$\frac{5(2+3\lambda)}{5+8\lambda} - \left[\frac{2(1+\lambda)}{5+8\lambda} \right] (\theta - c)$
p_m	$\theta + \frac{5\lambda}{1+2\lambda}$	$\theta + \left[\frac{\lambda(1+\lambda)}{1+\lambda-\lambda^2} \right] (\theta - c)$	$\theta + \frac{5(2+3\lambda)}{5+8\lambda} - \left[\frac{2(1+\lambda)}{5+8\lambda} \right] (\theta - c)$

This table shows some quite interesting comparative statics effects. Indeed, letting $\Delta \equiv \theta - c$, under scheme *A*, we see that an increase in Δ leads to an increase in capacity and a decrease in monopoly output of the same magnitude, and hence price in the regional market remains unchanged. Under scheme *B*, the increase in capacity is smaller than the decrease in monopoly output, leading to an increase in price. Finally, under scheme *C*, the implied

⁴³The marginal-cost differential, $\theta - c$, is at the heart of the policy that is the subject of this paper, consisting of controlling market power with gas imports.

capacity increase more than offsets the local output decrease with a resulting net effect of decreasing the local market price.

For the purpose of exploring the cross-effect of the marginal-cost gap (Δ) and the shadow cost of public funds (λ), we perform some simulations using the grid $\lambda = 1/5, 1/4, 1/3, 1/2, 1$.⁴⁴ Tables 3, 4, and 5 present, respectively, the levels of capacity, local monopoly output, and price in terms of Δ for values of the cost of public funds in this grid.

Table 3: Optimal capacity levels

	Scheme A	Scheme B	Scheme C
$\lambda = \frac{1}{5}$	Δ	$\frac{30}{29}\Delta$	$\frac{1}{33}(35 + 24\Delta)$
$\lambda = \frac{1}{4}$	Δ	$\frac{20}{19}\Delta$	$\frac{5}{14}(3 + 2\Delta)$
$\lambda = \frac{1}{3}$	Δ	$\frac{12}{11}\Delta$	$\frac{1}{23}(25 + 16\Delta)$
$\lambda = \frac{1}{2}$	Δ	$\frac{6}{5}\Delta$	$\frac{2}{9}(5 + 3\Delta)$
$\lambda = 1$	Δ	2Δ	$\frac{1}{13}(15 + 8\Delta)$

⁴⁴For simplicity of the presentation, we set $\theta = 5$.

Table 4: Optimal local monopoly output

	Scheme A	Scheme B	Scheme C
$\lambda = \frac{1}{5}$	$\frac{30}{7} - \Delta$	$5 - \frac{36}{29}\Delta$	$\frac{1}{33} (65 - 12\Delta)$
$\lambda = \frac{1}{4}$	$\frac{25}{6} - \Delta$	$5 - \frac{25}{19}\Delta$	$\frac{5}{28} (11 - 2\Delta)$
$\lambda = \frac{1}{3}$	$4 - \Delta$	$5 - \frac{16}{11}\Delta$	$\frac{1}{23} (45 - 8\Delta)$
$\lambda = \frac{1}{2}$	$\frac{15}{4} - \Delta$	$5 - \frac{9}{5}\Delta$	$\frac{1}{18} (35 - 6\Delta)$
$\lambda = 1$	$\frac{10}{3} - \Delta$	$5 - 4\Delta$	$\frac{1}{13} (25 - 4\Delta)$

Table 5: Optimal price levels

	Scheme A	Scheme B	Scheme C
$\lambda = \frac{1}{5}$	$\frac{40}{7}$	$5 + \frac{6}{29}\Delta$	$\frac{2}{33} (115 - 6\Delta)$
$\lambda = \frac{1}{4}$	$\frac{35}{6}$	$5 + \frac{5}{19}\Delta$	$\frac{85}{28} (39 - 2\Delta)$
$\lambda = \frac{1}{3}$	6	$5 + \frac{4}{11}\Delta$	$\frac{8}{23} (20 - \Delta)$
$\lambda = \frac{1}{2}$	$\frac{25}{4}$	$5 + \frac{3}{5}\Delta$	$\frac{1}{18} (125 - 6\Delta)$
$\lambda = 1$	$\frac{20}{3}$	$5 + 2\Delta$	$\frac{1}{13} (90 - 4\Delta)$

As far as schemes *A* and *B* are concerned, Tables 3-5 confirm what is already conveyed by Table 2. First, we see that optimal capacity (local output) level is monotonically increasing (decreasing) in the cost of public funds. However, since the local output effect dominates the capacity effect, this results in a monotonically increasing regional market price. In contrast, under scheme *C*, this monotonicity property does not hold. Indeed, from

Table 2 we find that $\partial K^C/\partial\lambda = (10 - 12\Delta)/(5 + 8\lambda)^2$ and $\partial q_m^C/\partial\lambda = (-5 + 6\Delta)/(5 + 8\lambda)^2$, implying, as can be checked in Tables 3 and 4, that capacity (local output) is increasing (decreasing) in λ for $\Delta < 5/6$, and decreasing (increasing) for $\Delta > 5/6$. The output effect being twice as large as the capacity effect in magnitude, thanks to our assumption of unitary-slope linear demand, we obtain a behavioral pattern for the optimal price analogous to that of the local output.⁴⁵

While the main focus of this paper is on the ranking of capacity across alternative control schemes, our knowledge of the behavior of the two other endogenous variables, monopoly output and market price, allows us to examine their ranking as well. Tables 6-8 give the ordering of, respectively, capacity, local output, and price in terms of Δ . A striking feature of the data contained in these tables is that while capacity (as found in Theorems 1-3) is ranked in three different manners, monopoly output is ranked in four manners and price in two (see the columns of Tables 6-8). Figure 3 below displays these different rankings in the Δ -line, for any value of λ . Figures 4-6 illustrate these orderings for the case where $\lambda = 1/4$. These figures also highlight the role played by the sign of μ , the Lagrange multiplier associated with the profit-maximization constraint under scheme C .

⁴⁵This lack of monotonicity of the endogenous variables with respect to the cost of public funds under scheme C confirms the capacity rankings documented in the previous section.

Table 6: Ranking of optimal capacity levels

	$K^A < K^B < K^C$	$K^A < K^C < K^B$	$K^C < K^A < K^B$
$\lambda = \frac{1}{5}$	$0 \leq \Delta \leq \frac{145}{42}$	$\frac{145}{42} < \Delta \leq \frac{35}{9}$	$\frac{35}{9} < \Delta \leq \frac{145}{36}$
$\lambda = \frac{1}{4}$	$0 \leq \Delta \leq \frac{19}{6}$	$\frac{19}{6} < \Delta \leq \frac{15}{4}$	$\frac{15}{4} < \Delta \leq \frac{19}{5}$
$\lambda = \frac{1}{3}$	$0 \leq \Delta \leq \frac{11}{4}$	$\frac{11}{4} < \Delta \leq \frac{55}{16}$	<i>NF*</i>
$\lambda = \frac{1}{2}$	$0 \leq \Delta \leq \frac{25}{12}$	$\frac{25}{12} < \Delta \leq \frac{25}{9}$	<i>NF</i>
$\lambda = 1$	$0 \leq \Delta \leq \frac{5}{6}$	$\frac{5}{6} < \Delta \leq \frac{5}{4}$	<i>NF</i>

* Nonfeasible.

Table 7: Ranking of optimal local monopoly output

	$q_m^C < q_m^A < q_m^B$	$q_m^C < q_m^B < q_m^A$	$q_m^B < q_m^C < q_m^A$	$q_m^B < q_m^A < q_m^C$
$\lambda = \frac{1}{5}$	$0 \leq \Delta \leq \frac{145}{49}$	$\frac{145}{49} < \Delta \leq \frac{145}{42}$	$\frac{145}{42} < \Delta \leq \frac{535}{147}$	$\frac{535}{147} < \Delta \leq \frac{145}{36}$
$\lambda = \frac{1}{4}$	$0 \leq \Delta \leq \frac{95}{36}$	$\frac{95}{36} < \Delta \leq \frac{19}{6}$	$\frac{19}{6} < \Delta \leq \frac{185}{54}$	$\frac{185}{54} < \Delta \leq \frac{19}{5}$
$\lambda = \frac{1}{3}$	$0 \leq \Delta \leq \frac{11}{5}$	$\frac{11}{5} < \Delta \leq \frac{11}{4}$	$\frac{11}{4} < \Delta \leq \frac{47}{15}$	$\frac{47}{15} < \Delta \leq \frac{55}{16}$
$\lambda = \frac{1}{2}$	$0 \leq \Delta \leq \frac{25}{16}$	$\frac{25}{16} < \Delta \leq \frac{25}{12}$	$\frac{25}{12} < \Delta \leq \frac{65}{24}$	$\frac{65}{24} < \Delta \leq \frac{25}{9}$
$\lambda = 1$	$0 \leq \Delta \leq \frac{5}{9}$	$\frac{5}{9} < \Delta \leq \frac{5}{6}$	$\frac{5}{6} < \Delta \leq \frac{5}{4}$	<i>NF</i>

Table 8: Ranking of optimal price levels

	$p_m^B < p_m^A < p_m^C$	$p_m^C < p_m^A < p_m^B$
$\lambda = \frac{1}{5}$	$0 \leq \Delta \leq \frac{145}{42}$	$\frac{145}{42} < \Delta \leq \theta$
$\lambda = \frac{1}{4}$	$0 \leq \Delta \leq \frac{19}{6}$	$\frac{19}{6} < \Delta \leq \theta$
$\lambda = \frac{1}{3}$	$0 \leq \Delta \leq \frac{11}{4}$	$\frac{11}{4} < \Delta \leq \theta$
$\lambda = \frac{1}{2}$	$0 \leq \Delta \leq \frac{25}{12}$	$\frac{25}{12} < \Delta \leq \theta$
$\lambda = 1$	$0 \leq \Delta \leq \frac{5}{6}$	$\frac{5}{6} < \Delta \leq \theta$

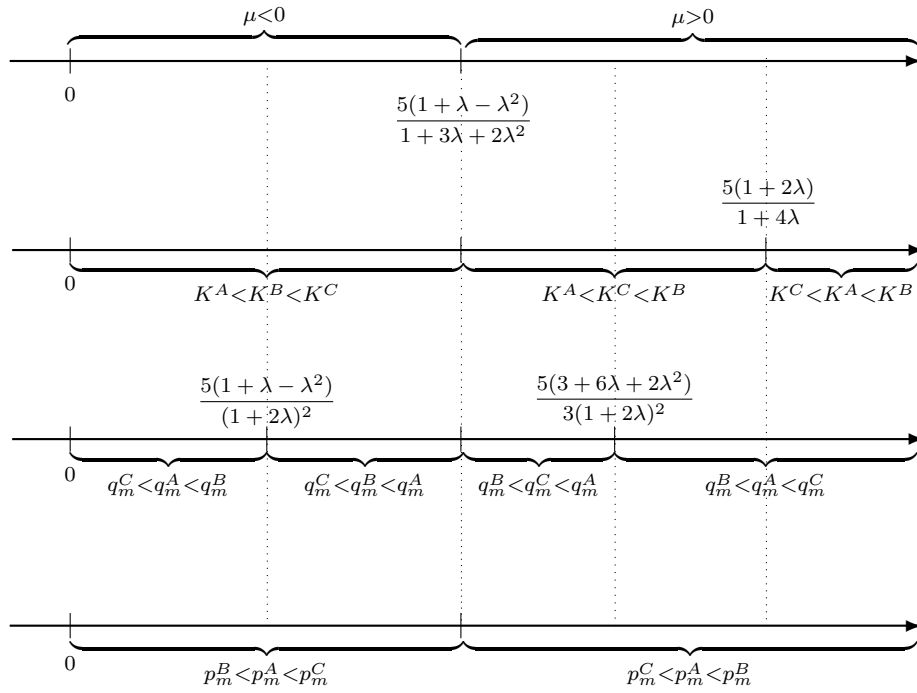


Figure 3: Ranking of capacity, output, and price for any λ

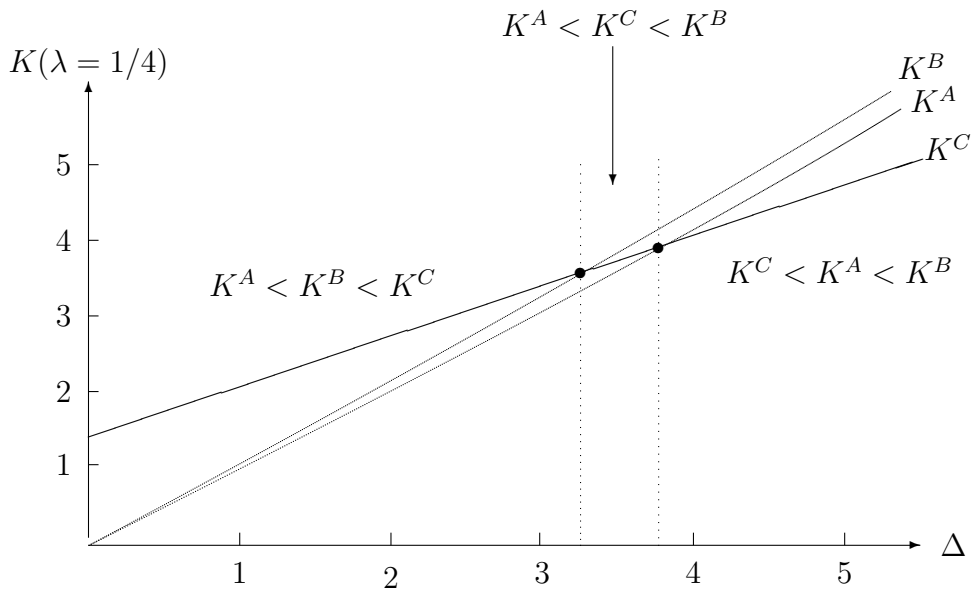


Figure 4: Ranking of capacity for $\lambda = 1/4$

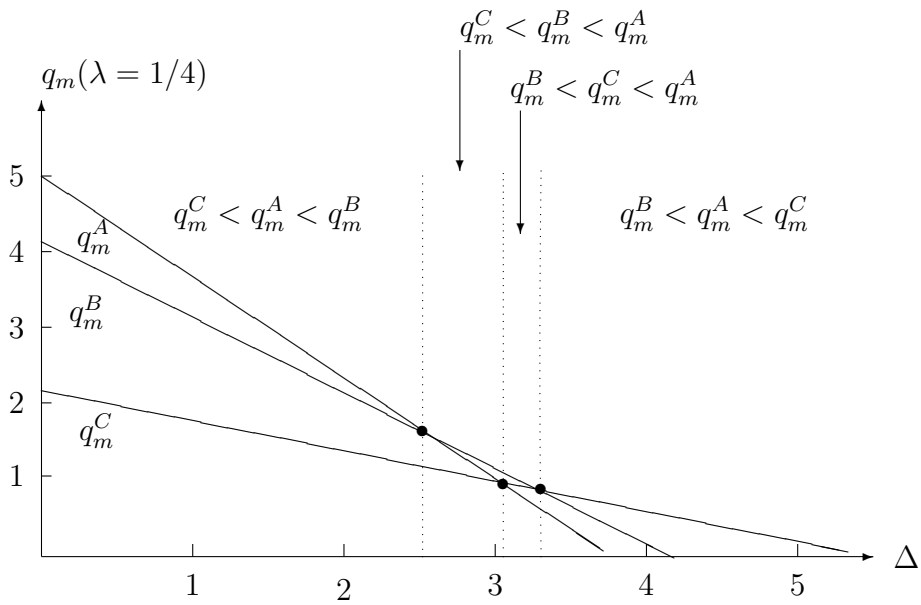


Figure 5: Ranking of output for $\lambda = 1/4$

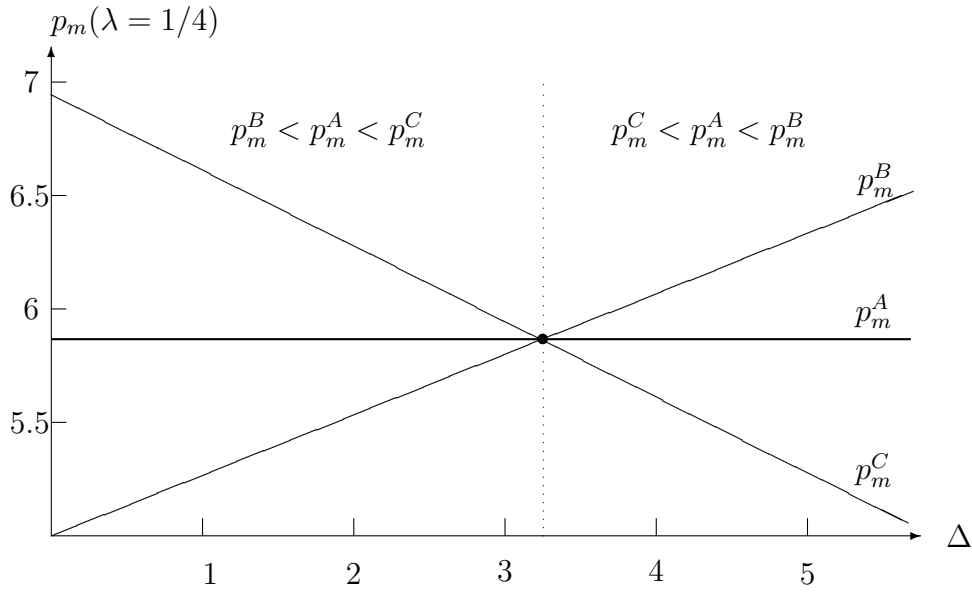


Figure 6: Ranking of price for $\lambda = 1/4$

8 Conclusion

The gas industry, throughout the world, and particularly so, in Europe, has been facing an important question that is common to most of the network industries. In a context where some segments of the industry are increasingly open to competition, how to make sure that monopoly power, inherited from the historical market structure that prevailed prior to the reforms, is not going to be exercised. This paper has paved the road for a thorough analysis of the means to mitigate this type of monopoly behavior in the gas industry by analyzing the extent to which network sizing is an effective weapon to control market power.

As a starting point, we have considered a situation where a planner makes transfers between consumers and a regional monopoly, controls the firm's

output/price, and sets the capacity of a pipeline that is used to bring in some competitive gas into the regional market. We then have examined the effect on this pipeline capacity of preventing the planner from using transfers and controlling price. The analysis sheds light on the question of whether these various means of mitigating regional monopoly power in the gas industry are complements or substitutes. Our main finding is that restricting the set of control instruments does not always result in an "over"-sized network.

Control of monopoly power is to a large extent the subject of regulatory economics. The purpose of this paper was to initiate a process aimed at better understanding the interaction among regulatory tools under the admittedly strong assumption of complete information. A necessary step in our future research agenda is to introduce asymmetric information, a milestone of the new view of regulation (see Laffont and Tirole, 1993).⁴⁶ Our conjecture is that the introduction of information incompleteness will affect in some important ways the effectiveness of the regulatory instruments introduced in this paper, but how and to what degree remains to be seen.

⁴⁶In some work progress of ours (Gasmi et al., 2003), we have begun exploring this avenue of research.

References

- Borenstein, S., J. Bushnell and S. Stoft, 2000, "The Competitive Effects of Transmission Capacity in a Deregulated Electricity Industry," *RAND Journal of Economics*, 31(2): 294-325.
- Cremer, H. and J.J. Laffont, 2002, "Competition in Gas Markets," *European Economic Review*, 46: 928-935.
- De Vany, A. and W. Walls, 1994, "Open Access and the Emergence of a Competitive Natural Gas Market," *Contemporary Economic Policy*, 12: 77-93.
- Doane, M. and D. Spulber, 1994, "Open Access and the Evolution of the US Spot Market for Natural Gas," *Journal of Law and Economics*, 37(2): 477-515.
- Gasmi, F., J.J. Laffont and J.D. Oviedo, 2003, "Regulation of Regional Gas Monopolies," mimeo, Université de Toulouse (IDEI).
- Laffont, J.J. and J. Tirole, 1993, *A Theory of Incentives in Procurement and Regulation*, MIT Press.
- Lautier, T.O., 2001, "Transmission Constraints and Imperfect Markets for Power," *Journal of Regulatory Economics*, 19(1): 27-54.

Appendix

Case 3: Linear demand, decreasing returns in both gas supply and capacity building

For the purpose of confirming the lack of stability of the capacity ranking of scheme C , in this appendix we consider an economic environment where demand is linear, and both gas supply and capacity building exhibit decreasing returns. We then consider that

$$Q_M(p_m) = \gamma - p_m, C^m(q_m) = \frac{\theta}{2}q_m^2, C(K) = \frac{\omega}{2}K^2; \gamma, \theta, \omega > 0 \quad (A.1)$$

from which we derive the results stated in the next theorem.⁴⁷

Theorem 3 *Under the assumptions described in (A.1), the optimal capacity levels K^A , K^B , and K^C achieved under, respectively, schemes A , B , and C are ordered as follows:*

i) Assume that the following conditions are met

$$\frac{\lambda}{1 + \lambda} < \frac{\omega}{\lambda} + \theta \left[\frac{\lambda + (1 + \lambda)(1 + \omega)}{\lambda(1 + \lambda)} \right] \quad (A.2)$$

$$c < \gamma \quad (A.3)$$

$$(1 + \lambda)(c + \omega\gamma) > \lambda[\lambda(\gamma - c) - c] \quad (A.4)$$

$$\lambda(1 + \lambda) [\theta(\gamma - c) - c] > \lambda^2 c \quad (A.5)$$

Then, the pair of control schemes $\{A, B\}$ is feasible, and $K^A < K^B$.

⁴⁷Because the proof of this theorem is very similar to that of Theorems 1 and 2 given in the text, it is only sketched in this appendix.

ii) Assume that conditions (A.2)-(A.4), and

$$\lambda(1 + \lambda) [\theta(\gamma - c) - c] > 0 \quad (\text{A.6})$$

hold. Then, the scheme pair $\{B, C\}$ is feasible. Moreover, if

$$0 < \lambda(1 + \lambda) [\theta(\gamma - c) - c] < (1 + \lambda)(c + \omega\gamma) - \lambda[\lambda(\gamma - c) - c] \quad (\text{A.7})$$

then $K^B < K^C$. Otherwise, i.e., if

$$0 < (1 + \lambda)(c + \omega\gamma) - \lambda[\lambda(\gamma - c) - c] < \lambda(1 + \lambda) [\theta(\gamma - c) - c] \quad (\text{A.8})$$

then $K^B > K^C$.

iii) Suppose that conditions (A.2), (A.3), and (A.5) hold. Then, schemes A and C are feasible. Moreover, if

$$\begin{aligned} \lambda^2 c < \lambda(1 + \lambda) [\theta(\gamma - c) - c] < (1 + \lambda) [1 + \lambda(1 + \lambda)(2 + \theta)^2] (c + \omega\gamma) \\ + \lambda [1 + \lambda(1 + \lambda)(2 + \theta)^2] c + \lambda^2 c \end{aligned} \quad (\text{A.9})$$

then $K^A < K^C$. Otherwise, i.e., if

$$\begin{aligned} \lambda(1 + \lambda) [\theta(\gamma - c) - c] > (1 + \lambda) [1 + \lambda(1 + \lambda)(2 + \theta)^2] (c + \omega\gamma) \\ + \lambda [1 + \lambda(1 + \lambda)(2 + \theta)^2] c + \lambda^2 c \end{aligned} \quad (\text{A.10})$$

then $K^A > K^C$.

iv) Suppose that the same conditions as those stated in i) above hold. Then, the scheme triple $\{A, B, C\}$ is feasible. Moreover, if

$$\lambda^2 c < \lambda(1 + \lambda) [\theta(\gamma - c) - c] < (1 + \lambda)(c + \omega\gamma) - \lambda[\lambda(\gamma - c) - c] \quad (\text{A.11})$$

holds, then $K^A < K^B < K^C$. If (A.11) does not hold and (A.9) holds, i.e.,

$$\begin{aligned}
(1 + \lambda)(c + \omega\gamma) - \lambda[\lambda(\gamma - c) - c] &< \lambda(1 + \lambda)[\theta(\gamma - c) - c] < \\
(1 + \lambda)[1 + \lambda(1 + \lambda)(2 + \theta)^2](c + \omega\gamma) & \tag{A.12} \\
+ \lambda[1 + \lambda(1 + \lambda)(2 + \theta)^2]c + \lambda^2c &
\end{aligned}$$

holds, then $K^A < K^C < K^B$. Finally, if both conditions (A.11) and (A.9) do not hold, or equivalently, (A.10) is satisfied, then $K^C < K^A < K^B$.

Proof 3 Using (A.1) and solving the first-order conditions associated with each of the three schemes A, B, and C, yields the following closed-form solutions:

$$p_m^A = \frac{\lambda\gamma(\theta + \omega) + (1 + \lambda)\theta(c + \omega\gamma)}{(1 + 2\lambda)(\theta + \omega) + (1 + \lambda)\theta\omega} \tag{A.13}$$

$$K^A = \frac{(1 + \lambda)[\theta(\gamma - c) - c] - \lambda c}{(1 + 2\lambda)(\theta + \omega) + (1 + \lambda)\theta\omega} \tag{A.14}$$

$$q_m^A = \frac{(1 + \lambda)(c + \omega\gamma) + \lambda c}{(1 + 2\lambda)(\theta + \omega) + (1 + \lambda)\theta\omega} \tag{A.15}$$

$$p_m^B = \frac{(1 + \lambda)\theta(c + \omega\gamma) + \lambda[\theta(\gamma - c) - c] + \lambda c(\theta - \lambda)}{\omega(1 + \lambda) - \lambda^2 + \theta[1 + 2\lambda + \omega(1 + \lambda)]} \tag{A.16}$$

$$K^B = \frac{(1 + \lambda)[\theta(\gamma - c) - c]}{\omega(1 + \lambda) - \lambda^2 + \theta[1 + 2\lambda + \omega(1 + \lambda)]} \tag{A.17}$$

$$q_m^B = \frac{(1 + \lambda)(c + \omega\gamma) - \lambda[\lambda(\gamma - c) - c]}{\omega(1 + \lambda) - \lambda^2 + \theta[1 + 2\lambda + \omega(1 + \lambda)]} \tag{A.18}$$

$$p_m^C = \frac{(1 + \lambda)(1 + \theta)(2 + \theta)(c + \omega\gamma) + \lambda(1 + \theta)^2\gamma}{\theta + (1 + \theta)^2 + (1 + \lambda)\omega(2 + \theta)^2 + 2\lambda(1 + \theta)(2 + \theta)} \tag{A.19}$$

$$K^C = \frac{(1 + \lambda)(2 + \theta)^2(\gamma - c) - [1 + (1 + \lambda)(2 + \theta)]\gamma}{\theta + (1 + \theta)^2 + (1 + \lambda)\omega(2 + \theta)^2 + 2\lambda(1 + \theta)(2 + \theta)} \tag{A.20}$$

$$q_m^C = \frac{(1 + \lambda)(2 + \theta)(c + \omega\gamma) + \lambda(1 + \theta)\gamma}{\theta + (1 + \theta)^2 + (1 + \lambda)\omega(2 + \theta)^2 + 2\lambda(1 + \theta)(2 + \theta)} \tag{A.21}$$

$$\mu = \frac{(1 + \lambda)(c + \omega\gamma) - \lambda(1 + \lambda)[\theta(\gamma - c) - c] - \lambda[\lambda(\gamma - c) - c]}{\theta + (1 + \theta)^2 + (1 + \lambda)\omega(2 + \theta)^2 + 2\lambda(1 + \theta)(2 + \theta)} \tag{A.22}$$

The second-order conditions are always satisfied for scheme A. For schemes B and C they require

$$\omega(1 + \lambda) - \lambda^2 + \theta[1 + 2\lambda + \omega(1 + \lambda)] > 0 \quad (\text{A.23})$$

which is equivalent to condition (A.2) given in the theorem.

i) In order to compare schemes A and B, we first make sure that they are feasible, i.e., that they prescribe positive prices, capacity, and output levels given by (A.13)-(A.18). Given (A.2) and (A.3), we have $K^B > 0 \Rightarrow p_m^B > 0$, as the former is equivalent to

$$(\gamma - c) > \frac{c}{\theta} \quad (\text{A.24})$$

whereas the latter amounts to

$$(\gamma - c) > \frac{c}{\theta} \left[\frac{\lambda(1 + \lambda) - \theta(1 + 2\lambda + \omega(1 + \lambda))}{\lambda + \omega(1 + \lambda)} \right] \quad (\text{A.25})$$

and, from (A.2), the term in brackets in (A.25) is less than one. Furthermore, we have $K^A > 0 \Rightarrow K^B > 0$, as can be seen from (A.14) and (A.17).⁴⁸ Therefore, feasibility of schemes A and B is achieved if $q_m^B > 0$ and $K^A > 0$, which, respectively, are ensured by conditions (A.4) and (A.5). Now, we calculate the capacity gap $K^B - K^A$ and find that it is positive if

$$\lambda(1 + \lambda)[\theta(\gamma - c) - c] + (1 + \lambda)\theta(c + \omega\gamma) + \lambda\theta c > \lambda^2 c \quad (\text{A.26})$$

a condition that can be ignored given feasibility of the scheme pair $\{A, B\}$.⁴⁹

⁴⁸Note that q_m^A and p_m^A are strictly positive.

⁴⁹The argument that justifies why this condition should be ignored is similar to the one provided in the proof of Theorem 2-(i).

ii) Given (A.2) and (A.3), we have $K^B > 0 \Rightarrow K^C > 0$, as the first inequality is equivalent to (A.24) while the second amounts to

$$(\gamma - c) > \frac{c}{\theta} \left[\frac{\theta[(1 + \lambda)(2 + \theta) + 1]}{\theta[(1 + \lambda)(2 + \theta) + 1] + 1 + \lambda(2 + \theta)} \right] \quad (\text{A.27})$$

and the term in brackets in (A.27) is less than one.⁵⁰ Thus, feasibility of schemes B and C is obtained if $q_m^B > 0$ and $K^B > 0$, which, respectively, hold if conditions (A.4) and (A.6) are satisfied. Now, making use of (A.17) and (A.20), we express the capacity gap $K^C - K^B$ and see that it is positive if

$$(1 + \lambda)[c(1 + \lambda(2 + \theta)) + \omega\gamma] > \lambda[\lambda + (1 + \lambda)\theta]\gamma, \quad (\text{A.28})$$

which, combined with feasibility conditions, can be rewritten as (A.7), and negative if (A.28) does not hold. When (A.28) does not hold, meeting the conditions that ensure feasibility, we obtain (A.8).⁵¹

iii) Given (A.2) and (A.3), recall from, respectively, parts i) and ii) of this proof that $K^A > 0 \Rightarrow K^B > 0$, and, $K^B > 0 \Rightarrow K^C > 0$.⁵² Next, making use of (A.14) and (A.20), we calculate the capacity gap $K^C - K^A$ and see that it is positive if

$$[1 + \lambda(1 + \lambda)(2 + \theta)^2]\omega\gamma + [1 + \lambda(2 + \theta) + \lambda(1 + 2\lambda)(2 + \theta)^2]c > \lambda\gamma\theta \quad (\text{A.29})$$

and negative otherwise. Inequality (A.29) and its reverse, merged with the conditions that ensure feasibility can be respectively rewritten as (A.9) and (A.10) given in the theorem.⁵³

⁵⁰Note that $K^B > 0 \Rightarrow p_m^B > 0$, and p_m^C and q_m^C are strictly positive.

⁵¹Nonemptiness of the intervals defined by (A.7) and (A.8) is guaranteed by, respectively, (A.4) and (A.6).

⁵²Note that p_m^A , q_m^A , p_m^C , and q_m^C are strictly positive.

⁵³It can be easily shown that, within the feasibility domain, the interval defined by (A.9) is always nonempty.

iv) Clearly, conditions (A.2)-(A.5) imply feasibility of schemes A, B and C. Next, we form $K^C - K^A$, $K^C - K^B$, and $K^B - K^A$, using (A.14), (A.17), and (A.20). First, suppose condition (A.11) holds. Then, since (A.11) \Rightarrow (A.7) we have from ii) above that $K^B < K^C$. Combining this with $K^A < K^B$, which is always true within the feasibility domain, we conclude that $K^A < K^B < K^C$.⁵⁴ Now, assume that condition (A.11) is not satisfied. Then, two cases are possible: $K^A < K^C < K^B$ and $K^C < K^A < K^B$. If we further assume that (A.9) holds, we are left only with the case $K^A < K^C < K^B$ since (A.9) $\Rightarrow K^A < K^C$. It is easy to show that condition (A.9) not holding and (A.11) holding is equivalent to (A.12).⁵⁵ Finally, assume that both conditions (A.9) and (A.11) do not hold (thus $K^C < K^A$ and $K^C < K^B$), or equivalently condition (A.10) does, we find that $K^C < K^A < K^B$.⁵⁶

This completes the proof of Theorem 3.

⁵⁴The interval defined by (A.11) can be shown, in the same way as in Theorem 1-(iv), to be nonempty.

⁵⁵A casual look at inequality (A.12) reveals that the interval it spans is nonempty.

⁵⁶The reader can easily verify that the interval defined by (A.10) is strictly contained in the feasibility domain.