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Three Essays in International Trade and Macroeconomics

Ph.D Thesis

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On top of my years in Toulouse, I have benefited from the extraordinary environment offered at the University of Minnesota and at the Federal Reserve Bank of Minneapolis. In Minneapolis in general, and in the Trade Workshop in particular, I witnessed a passion for intellectual challenges that was contagious. Encouraged by extremely helpful discussions with Tim Kehoe and Kei-Mu Yi, I felt welcomed to play my part and contribute, as an economic researcher, to a better understanding of the forces driving our economic system. This experience had a profound impact on my work and led me to firmly believe that the academic system as a whole can play a decisive part in improving our society.

This work benefited from fruitful discussions with my fellow PhD candidates and I wish to thank my former flat mates at Chateau Ozenne Jakob Hennig and Shekhar Tomar as well as Konrad Adler, Simon Fuchs, Maxime Liegey, Alban Moura and many others in Toulouse, but also Joao Ayres, Georgios Stefanidis, Guillaume Sublet and Daniela Viana Costa in Minneapolis.

I am grateful to my parents who supported me in the difficult choice of entering a PhD program and to my whole family and family-in-law for their support in the process. My friends, most of who are far away from the academic world, helped me keep contact with reality and not drawn in the intellectual complexity of academic research. Having family and friends making fun of me for still being a student at 30 was also a good reminder that this path is a very peculiar one.

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Abstract – English

This thesis contains three essays on international trade and macroeconomics, with a special focus on the role of input-output linkages.

In the first chapter, I study the relationship between international trade and business cycle synchronization. Using data from 40 OECD countries and major emerging markets, I find that trade in intermediate inputs plays a significant role in synchronizing GDP fluctuations across countries while trade in final goods is not significant. Motivated by this new fact, I build a model of international trade in intermediates that is able to replicate more than 85% of the empirical trade-comovement slope, offering the first quantitative solution for the "Trade Comovement Puzzle". The model relies on two key assumptions: (i) price distortions due to monopolistic competition and (ii) fluctuations in the mass of firms serving each country. The combination of those ingredients creates a link between domestic measured productivity and foreign shocks through trade linkages. Finally, I provide evidence for the importance of those elements in the link between foreign shocks and domestic GDP and test other predictions of the model.

In the second chapter, Guillaume Sublet and I propose a model of international trade with heterogeneous firms and global value chains. Firms access new varieties when breaking into any foreign market and jointly choose where to import from and to export to. The unit production cost depends on the importing/exporting strategy of each firm as well the strategy of a firm's direct and indirect suppliers, giving rise to complementarities across firms decisions. In general, the model admits multiple equilibria. In this context, we study the consequences of trade disruption between two countries on global trade flows and show that aggregate trade flows can be complements rather than substitutes. As a result, any trade disruption between two specific countries propagates to all other trading partners through the network of input-output linkages. Hence, imposing sanctions to one country leads to a reduction in trade between all other countries. We further test the empirical implication of our theoretical framework.

In the third chapter, Shekhar Tomar and I study the consequence of a technological improvement in one sector on employment in sectors located downstream in the supply-chain. On the one hand, if material and labor are gross substitute in the production function, the price decrease for the former tends to reduce labor demand for the latter per unit produced. On the other hand, the upstream positive technological shock also increases the number of unit produced through a decrease in the marginal cost. The net effect on employment simply depends on the ratio between the elasticity of substitution in the production function and the price elasticity of demand. We estimate those parameters at the sector level using detailed French data and show that employment sensitivity of sectors following a decrease in their material input price are very heterogeneous. Consequences for forecasting the effect of an increase in machine efficiency are discussed.

Abstract – French

Cette thèse est composée de trois chapitres traitant du commerce international et de macroéconomie avec une attention particulière donnée au rôle des chaines de productions.

Le premier chapitre étudie la relation entre le commerce international et la synchronisation des cycles économiques. A l'aide de données couvrant 40 pays de l'OCDE et des principaux pays émergeants, je montre que le commerce en bien intermédiaire joue un rôle significatif pour la synchronisation des fluctuations du PIB tandis que le commerce en bien final n'est pas significatif. Motive par ce fait nouveau, je construis ensuite un modèle de biens intermédiaires capable de reproduire plus de 85% de la relation empirique entre commerce et co-mouvement de PIB, offrant ainsi la première solution quantitative au « l'Anomalie Commerce Co-mouvement ». Le modèle s'appuie sur deux hypothèses : (i) des distorsions de prix liées a une compétition monopolistique et (ii) une fluctuation du nombre de firmes active dans chaque pays. La combinaison de ces deux éléments produit un lien endogène entre la productivité mesurée dans différents pays. Enfin, je présente des éléments empiriques supportant les hypothèses théoriques et je teste de prédictions du modèle.

Dans le second chapitre, Guillaume Sublet et moi proposons un modèle de commerce international avec des entreprises hétérogènes et des chaines de production globales. Les firmes gagnent accès à de nouvelles variétés de fournisseurs lorsqu'elles entrent dans un marché étranger et décident jointement d'importer et d'exporter. Le coût unitaire de production est fonction de la stratégie d'import et d'export de chaque firme mais aussi de ses fournisseurs directs et indirects, ce qui donne lieu à des complémentarités dans les décisions. En général, le modèle admet plusieurs équilibres. Dans ce contexte, nous étudions les conséquences d'une perturbation du commerce entre deux pays sur la réorganisation du commerce mondial et montrons que les flux commerciaux sont complémentaires et non substituts. Ainsi, imposer des sanctions entre deux pays donnes peut conduire à réduire les flux entre tous les pays du monde. Nous testons enfin de manière empirique les prédictions théoriques de notre structure.

Le troisième chapitre, écrit avec Shekhar Tomar, s'intéresse aux conséquences d'une innovation technologique dans un secteur sur le niveau d'emploi des autres secteurs situés plus loin dans la chaine de production. D'un cote, si le travail et les facteurs intermédiaires de production sont substituts, une réduction de prix de ces derniers tend à réduire la demande pour le travail pour chaque unité produite. D'un autre côté, une innovation technologique plus haut dans la chaine de production conduit à produire un plus grand nombre d'unités car le cout marginal de production décroît. L'effet net sur l'emploi dépend simplement du rapport entre l'élasticité de substitution de la fonction production et de l'élasticité de prix de la demande agrégée. Nous estimons ces deux paramètres pour un grand nombre de secteurs avec des données micro-économiques françaises. Les résultats montrent des sensibilités très différentes entre les secteurs.

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Chapter 1

Value Added and Productivity Linkages Across Countries

By François de Soyres[1]

Abstract

What is the relationship between international trade and business cycle synchronization? Using data from 40 OECD countries and major emerging markets, I find that trade in intermediate inputs plays a significant role in synchronizing GDP fluctuations across countries while trade in final goods is not significant. Motivated by this new fact, I build a model of international trade in intermediates that is able to replicate more than 85% of the empirical trade-comovement slope, offering the first quantitative solution for the "Trade Comovement Puzzle". The model relies on two key assumptions: (i) price distortions due to monopolistic competition and (ii) fluctuations in the mass of firms serving each country. The combination of those ingredients creates a link between domestic measured productivity and foreign shocks through trade linkages. Finally, I provide evidence for the importance of those elements in the link between foreign shocks and domestic GDP and test other predictions of the model.

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1.1 Introduction

The "Trade Comovement Puzzle", uncovered by Kose and Yi (2001 and 2006), refers to the inability of international business cycle models to quantitatively account for the high and robust empirical relationship between international trade and GDP comovement.[2] Using different versions of the workhorse international real business cycle (IRBC) model, several authors have succeeded to qualitatively replicate the positive link between trade and GDP comovement but fall short of the quantitative relationship by an order of magnitude.[3]

In this paper, I refine previous empirical investigations of the association between bilateral trade and GDP comovement and I propose a model that quantitatively accounts for this relationship. First, using data from OECD countries, I show that trade in intermediate inputs plays a significant role in synchronizing GDP fluctuations across countries while trade in final goods is found insignificant, uncovering the strong role of global value chains. Motivated by this new fact, I then propose a general equilibrium dynamic model of trade in inputs with monopolistic pricing and firms entry/exit. In the benchmark calibration, the model is able to replicate more than 85% of the trade-comovement slope, hence offering the first quantitative solution for the "Trade Comovement Puzzle". The model features a quantitatively important link between foreign shocks and domestic productivity through trade linkages suggesting that countries with input-output linkages should have correlated Solow residual, a prediction that I validate in the data. Finally, I provide evidence for the role of the two key ingredients generating the quantitative results, namely the importance of price distortions and of the fluctuations of the mass of firms serving every market.

Empirics Since the seminal paper by Frankel and Rose (1998), a large empirical literature has studied cross countries' GDP synchronization, showing that bilateral trade is an important and robust determinant of GDP correlation in the cross section. I update those findings using a panel of 40 countries and uncover a new fact, namely that business cycle synchronization is associated with trade in intermediate inputs while trade in final good is found insignificant.

[2] For empirical studies on this topic, among many others, see Frankel and Rose (1998), Clark and van Wincoop (2001), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2006), Calderon, Chong, and Stein (2007), Inklaar, Jong-A-Pin, and Haan (2008), di Giovanni and Levchenko (2010), Ng (2010), Liao and Santacreu (2015), di Giovanni, Levchenko and Mejean (2016) and Duval et al (2016)

[3] For quantitative studies, see Kose and Yi (2001, 2006), Burstein, Kurz and Tesar (2008), Johnson (2014) or Liao and Santacreu (2015)

First, I refine previous analysis by constructing a panel dataset consisting of four 10-years windows ranging from 1970 to 2009. Controlling for country pair fixed effects that can be correlated with bilateral trade, I show that the relationship between trade and comovement stays high and statistically significant, keeping the "Trade Comovement Puzzle" alive.

Furthermore, I make use of disaggregated trade data to disentangle the role of final good from intermediate inputs trade. Regressing GDP comovement on indexes of trade proximity in final and intermediate goods, I show that trade in intermediates captures all of the explanatory power. This new finding suggests that the rise in global value chains plays a particular role in the synchronization of GDP across countries.

Theory As discussed in Kehoe and Ruhl (2008) or Burstein and Cravino (2015), international production linkages alone are not sufficient to generate a strong link between domestic GDP and foreign shocks. The intuition is as follows: GDP is the sum of value added produced within a country and is computed by statistical agencies as the difference between final production and imports, measured using base prices. When imports are used in production, price taking firms choose a quantity of imported input that equalizes their marginal cost and their marginal revenue. Up to a first order approximation, any change in the quantity of imported input yields exactly as much benefit as it brings costs. Hence, foreign shocks have an impact on domestic value added *only* to the extent that they impact the supply of domestic production factors. In other words, foreign shocks have no impact on domestic *productivity.* This "negative result" is at the heart of the Trade-Comovement Puzzle. In this paper, I incorporate two ingredients that create an endogenous link between domestic productivity and foreign shock through trade linkages.

First, when firms chose their price (i.e. are not price takers), they do not equalize the marginal cost and the marginal revenue product of their inputs. As noted previously by Hall (1988) and discussed in Basu and Fernald (2002), Gopinath and Neiman (2014) or Llosa (2014), this wedge between marginal cost and marginal product of inputs implies that any change in intermediate input usage is associated with a first order change in value added, over and beyond adjustments due to basic production factors. Intuitively, the value added produced by a monopolistic firm includes not only the payment to domestic factors of production, but also the firm's profit. This last part is strongly size dependent: any change in the production scale of a firm translates into a variation of profit which is also a change in value added, even for fixed domestic factors of production. At the aggregate level, after a foreign shock, the first order variation of GDP for a country populated by price setting firms is not limited to changes in domestic factor supply.[4]

Second, fluctuations along the extensive margin have the potential to create an additional link between domestic productivity and foreign technology. With love of variety, a firm with more suppliers produces a higher level of output for the same level of inputs. Hence, any change in the quantity of imports that is accompanied by a variation in the mass of suppliers leads to a first order productivity change. Love for variety is a form of increasing return: a firm with more suppliers is more efficient at transforming inputs into output, which allows measured value added to react over and beyond variations in domestic factor supply.

Quantitative analysis Motivated by the discussion above, I propose a multi-country dynamic general equilibrium a model of international trade in inputs that relies on two key assumptions: (i) monopolistic competition and (ii) fluctuations in the mass of firms serving each country. Production is performed by a continuum of heterogeneous firms combining in a Cobb-Douglas fashion labor, capital and a nested CES aggregate of intermediate inputs bought from domestic and foreign firms. Based on their expected profit, firms choose the set of countries they serve (if any). In this context, firms' marginal cost depends on the number and on the productivity of their suppliers, giving rise to a strong interdependency in pricing and revenues as well as in the export decisions. Crucially, monopolistic competition and fluctuations in the mass of producing firms are key elements that allow domestic GDP to be affected by foreign shocks through trade linkages.

I calibrate the model to 14 countries and a composite "rest-of-the-world" and assess its ability to replicate the strong relationship between trade in inputs and GDP synchronization. The model is first calibrated to match GDP, trade flows and the *level* of GDP comovement across all country pairs between 1989 and 2008. Since my goal is to use within country-pair variations in order to perform a fixed-effect estimation of the effect of trade on GDP synchronization, I then recalibrate the model with different targets for trade proximity across countries, decreasing and

[4] Related to this point, Burstein and Cravino (2015) show that if all firms take prices as given, a change in trade costs can affect aggregate productivity only to the extent that it changes the production possibility frontier at constant prices. This can be interpreted as saying that shocks to the *foreign* trading technology have no impact on aggregate *domestic* TFP if all firms have constant returns to scale and take prices as given. Hence, any change in GDP is due to variations in the supply of domestic factors of production.

increasing the target by 10%. In all configurations, I feed the model with the same sequence of technological shocks, creating a panel dataset in which each country-pair appears three times with three different levels of trade, thus allowing me to estimate the trade comovement *slope*. Fixed effect regressions on this simulated dataset shows that the model is able to replicate more than 85% of the trade-comovement *slope* observed in the data, a significant improvement compared to previous studies.[5]

Decomposing the role of each ingredient, I show quantitatively that trade in intermediates alone is not sufficient to replicate the trade-comovement relationship. The addition of monopolistic pricing and extensive margin adjustments increase the simulated trade-comovement slope by a factor seven and allow the model to better fit the data.

Further empirical evidence In the last part of the paper, I provide evidence supporting the modeling assumptions. First, using the Price Cost Margin as a proxy for monopoly power and OECD data at the industry level, I find that countries with higher markups have a GDP that is more systematically negatively correlated with terms-of-trade movements, meaning that they experience a larger GDP decrease when the price of their imports rises.

Second, I construct the extensive and intensive margins of trade and regress country-pair GDP correlations on those indexes. A higher degree of business cycle synchronization is associated with an increase in the range of goods traded and is not associated with an increase in the quantity traded for a given set of goods. This is especially striking since the extensive margin accounts for only a fourth of the variability in total trade.[6]

Finally, I test the prediction that higher trade proximity is associated with larger measured TFP comovement. I compute and detrend the Solow Residual for 18 OECD countries and compute all pairwise correlations. Regressing measured TFP correlations on indexes of trade proximity shows that, controlling for country-pair fixed effects, a higher trade proximity is associated with larger measured TFP comovement, as predicted by the model.

Relationship to the literature Starting with Frankel and Rose (1998), a number of papers have studied and confirmed the positive association between trade and comovement in the cross-

[5]See papers cited in the footnote 2

[6]This result is in line with the analysis in Liao and Santacreu (2015) which emphasizes the role of the extensive margin. Compared to them, I am adding the panel dimension by performing fixed effect regression which allows me to control for country-pair fixed effects that can be correlated with trade intensity.

section.[7] The empirical part of this paper is mostly related to two recent contributions. First, Liao and Santacreu (2015) is the first to study the importance of the extensive margin for GDP and TFP synchronization. Second, di Giovanni et al (2016) uses a cross-section of French firms and presents evidence that international input-output linkages at the micro level are a important drivers of the value added comovement observed at the macro level. Their evidence is in line with the findings of this paper and support the mechanism I develop in the quantitative part but also add elements relative to multinational firms that I do not model explicitly.

If the empirical association between bilateral trade and GDP comovement has long been known, the underlying economic mechanism leading to this relationship is still unclear. Using the workhorse IRBC with three countries, Kose and Yi (2006) have shown that the model can explain at most 10% of the *slope* between trade and business cycle synchronization, leading to what they called the "Trade Comovement Puzzle". Since then, many papers have refined the puzzle, highlighting different ingredients that could bridge the gap between the data and the predictions of classic models.

Burstein, Kurz and Tesar (2008) show that allowing for production sharing among countries can deliver tighter business cycle synchronization if the elasticity of substitution between home and foreign intermediate inputs is extremely low.[8] Arkolakis & Ramanarayanan (2009) analyse the impact of vertical specialization on the relationship between trade and business cycle synchronization. In their Ricardian model with perfect competition, they do not generate significant dependence of business cycle synchronization on trade intensity, but show that the introduction of price distortions that react to foreign economic conditions allows their model to better fit the data. Incorporating trade in inputs in an otherwise standard IRBC, Johnson (2014) shows that the puzzle cannot be solved by adding the direct propagation due to the international segmentation of supply chains. Compare to those papers, I add firm entry and exit as well as monopolistic competition and argue that those are key ingredients for the model to deliver quantitative results in line with the data. Liao and Santacreu (2015) build on Ghironi & Melitz (2005) and Alessandria & Choi (2007) to develop a two-country IRBC model with trade in differentiated intermediates. They show that trade in intermediate varieties leads to an endogenous correlation of measured TFP[9] across trading partners. Compare to this paper,

[7] see papers cited in footnote 2.
[8] In their benchmark simulations, the authors take the value of 0.05 for this elasticity.
[9] Defined as the Solow residual at the country's level

I add multinational production with long supply chains which creates a strong interdependency in firms' pricing end export decisions. Furthermore, I extend the quantitative analysis to many countries and show the international propagation of shocks is affected by the whole network of input-output linkages.[10] Finally, a complementary approach has been developed by Drozd, Kolbin and Nosal (2014) which model the dynamics of trade elasticity. Building on Drozd and Nosal (2012), their model features customers accumulation with matching frictions between producers and retailers. Changes in relative marketing capital across country-specific goods create time variations in the trade elasticity which allows the model to better match the data. Similar to my paper, the setup gives rise to a wedge between the price of imports and their marginal product in final good production, but in their case it is driven by the Nash bargaining process over the surplus generated by matches between producers and retailers.

The consequences of input trade on firms efficiency has been studied by Gopinath and Neiman (2014). Focusing on the 2001-2002 Argentinian crisis, they show that trade disruption can cause a significant drop in aggregate productivity. Building a model with monopolistic pricing and an exogenous cost of changing the number of suppliers, they replicate the empirical relationship between trade disruption and productivity, showing the importance of within firms' dynamics to understand aggregate productivity. The role of firms heterogeneity in international business cycles has been pioneered by Ghironi & Melitz (2005) and the business cycle implication of firms' heterogeneity is studied in Fattal-Jaef & Lopez (2014).

Finally, in another area, Baqaee develops a model of input-output linkages where idiosyncratic shocks can lead to aggregate fluctuations. The importance of distortions as well as extensive margin adjustments in a network economy are related to the mechanism developed in this paper.

The rest of the paper is organized as follows: the second section studies empirically the relationship between trade and GDP synchronization and highlights the important role of trade in intermediate inputs. Section three presents a simple static model of small open economy that provides clear intuitions for the role of markups and entry/exit in generating a link between trade and GDP fluctuations. The fourth section proposes a quantitative model of international trade in intermediate goods with heterogeneous firms and monopolistic competition. In the fifth section, I present the calibration strategy and the quantitative results are presented in section

[10] In their model, no firm is both an importer and an exporter. While this assumption simplifies the resolution, it prevents any network effect.

six. Section seven provides further empirical evidence supporting the model while section eight concludes.

1.2 Empirical Evidence

In this section, I use a sample of 40 countries[11] during period stretching from 1970 to 2009 and update the initial Frankel and Rose (1998) analysis on the relationship between bilateral trade and GDP comovement as well as provide empirical support for the specific role of trade in intermediate inputs.

There are two main findings. First, the empirical association between business cycle synchronization and international trade is robust to country-pair fixed effects. Second, trade in intermediate goods play a significant role in explaining GDP comovement, while the explanatory power of trade in final good is found not significant. I first describe the data, then I explain the econometric strategy and finally I present the results in details.

I use annual data on real GDP from the Penn World Table which is transformed in two ways: (i) HP filter with smoothing parameter 6.25 to capture the business cycle frequencies and (ii) log first difference. Trade data come from Johnson and Noguera (2016) who combine data on trade, production, and input use to construct trade flows from 1970 to 2009 separating between trade in final good from the trade in intermediate inputs.

In a first set of regressions, I construct a symmetric measure of bilateral trade intensity between countries i and j using total trade flows as: $\text{Total}_{ij}=\frac{Total\ Trade_{ij}}{GDP_i+GDP_j}$, which measure the importance of the trade relationship relative to total GDP.[12]

In order to disentangle the influence of trade flows in inputs from the final goods, I further construct the indexes Final_{ij} and Intermediate_{ij} with the same formulation but taking into account only the trade flows in final and intermediate goods respectively.[13]

[11] The list of countries is: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Russia, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, United States and Vietnam

[12] In a supplemental appendix, I also used an index definedas $\text{Total}_{ij}=\max\left(\frac{Total\ Trade_{ij}}{GDP_i}, \frac{Total\ Trade_{ij}}{GDP_j}\right)$. This measure has the advantage to take a high value whenever one of the two countries depends heavily on the other for its imports or exports. The empirical and simulated results hold when I use this index.

[13] In appendix, I verify the robustness of my findings using an alternative dataset and method of separating intermediate from final goods. In the STAN database of the OECD, input-output tables have been used at the

The extent to which countries have correlated GDP can be influenced by many factors beyond international trade, including correlated shocks, financial linkages, monetary policies, etc... Because those other factors can themselves be correlated with the index of trade proximity in the cross section, using cross-section identification could yield biased results. In order to separate the effect of trade linkages from other elements, I construct a panel dataset by creating four periods of ten years each. In every time window, I compute GDP correlation for all country pairs as well as trade indexes as defined above. The trade index relative to a given time window is then constructed by taking the average of all yearly indexes. Using panel data allows me to control for time invariant country-pair specific factors that are not observables.

I estimate the following equations:

$$(1) \qquad \text{corr}(GDP_{it}^{filtered}, GDP_{jt}^{filtered}) = \alpha_1 + \beta_T \log(\text{Total}_{ijt}) + controls + \epsilon_{1,ijt}$$

$$(2) \qquad \text{corr}(GDP_{it}^{filtered}, GDP_{jt}^{filtered}) = \alpha_2 + \beta_I \log(\text{Intermediate}_{ijt}) + \beta_F \log(\text{Final}_{ijt}) + controls + \epsilon_{2,ijt}$$

In the rest of this section I present two facts that characterize the relationship between GDP synchronization and international trade. Results are gathered in tables 1.1 and 1.1

Finding 1: The trade-comovement slope is robust to country-pair fixed effect

The initial Frankel and Rose (1998) finding that bilateral trade correlates with business cycle synchronization does not answer the question of trade's role in transmitting shocks. Using cross-sectional variation shows that country-pairs with higher trade linkages experience more correlated GDP fluctuations but does not rule out omitted variable bias such as, for example, the fact that neighboring countries have at the same time more correlated shocks and larger trade flows. By constructing a panel dataset and controlling for country-pair fixed effects, this paper relates to recent studies that try to control for unobserved characteristics.[14]

Using only *within* country-pair variations and controlling for aggregate time windows fixed effects, the strong relationship between trade in GDP correlation still holds for HP filter and first differences as shown in columns (1) and (3) table 1.1. Those numbers imply that moving from

country level to disentangle trade flows in intermediate and final goods from 1995 to 2014. All results are robust using this categorization.

[14] Di Giovanni and Levchenko (2010) includes country pair fixed effects in a large cross-section of industry-level data with 55 countries from 1970 to 1999 in order to test for the relationship between sectoral trade and *output* (not *value-added*) comovement at the industry level. Duval et al (2016) includes country pair fixed effects in a panel of 63 countries from 1995 to 2013 and test the importance of *value added* trade in GDP comovement.

the 25th to the 75th percentile of trade proximity in my sample[15] is associated with an increase of GDP correlations between 0.05 and 0.062. These findings are also robust when using two time windows of 20 years each, as shown in table 1.2.

Finding 2: Trade in Intermediate inputs plays a strong role in GDP comovement

To investigate further the relationship between trade and GDP comovement at business cycle frequency, columns (2) and (4) of 1.1 disentangle the effect of trade in intermediate inputs from trade in final goods. The results highlight a specific role for trade in intermediate inputs, both in the cross section and in the panel dimensions.[16] With HP filter as well as log difference, the index of trade proximity in intermediate goods is high and significant. According to th point estimates in 1.1, moving from the 25th to the 75th percentile of trade proximity in intermediate inputs is associated with a GDP comovement increase between 0.1 (column (6)) and 0.12 (column (2)). Again, these findings are robust when using two time windows of 20 years each, as shown in table 1.2. These results strongly suggest that international supply chains are an important determinant of the degree of business cycle synchronization across countries.[17]

[15] In my sample, moving from the 25th to the 75th percentile of trade proximity means multiplying by 10 total trade over total GDP.

[16] Di Giovanni and Levchenko (2010) investigate the role of vertical linkages in *output* synchronization (not *value added*) using input-output matrices from the BEA. Their estimates imply that vertical production linkages account for some 30 percent of the total impact of bilateral trade on the business cycle correlation

[17] The results presented here used a fixed effect specification. One might also consider that the variation across country-pairs are assumed to be random and uncorrelated with trade proximity indexes, in which case a random effect treatment would be a better fit. To discriminate between fixed or random effects, I run a Hausman specification test where the null hypothesis is that the preferred model is random effects against the fixed effects. This tests whether the error terms ϵ_{ijt} are correlated with the regressors, with the null hypothesis being they are not. Results display a significant difference ($p < 0.001$), indicating that the two models are different enough to reject the null hypothesis, and hence to reject the random effects in favor of the fixed effect model.

	dependent variable: corr($GDP_i^{filtered}$,$GDP_j^{filtered}$)			
	HP filter		First Difference	
	(1)	(2)	(3)	(4)
log(Total)	0.022**		0.027**	
	(2.07)		(2.55)	
log(Intermediate)		0.053**		0.042*
		(2.18)		(1.87)
log(Final)		-0.030		-0.016
		(-1.25)		(-0.70)
Country-Pair FE	yes	yes	yes	yes
Time Window FE	yes	yes	yes	yes
R-squared (within)	0.153	0.155	0.141	0.142
R-squared (overall)	0.137	0.135	0.132	0.129
N	——————- 2900 ——————-			

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1.1: Trade and GDP correlation with 10 years time windows

	dependent variable: corr($GDP_i^{filtered}$,$GDP_j^{filtered}$)			
	HP filter			First Difference
	(1)	(2)	(3)	(4)
log(Total)	0.019*		0.017	
	(1.67)		(1.39)	
log(Intermediate)		0.073**		0.074**
		(2.20)		(2.40)
log(Final)		-0.053		-0.057*
		(-1.55)		(-1.85)
Country-Pair FE	yes	yes	yes	yes
Time Window FE	yes	yes	yes	yes
R-squared (within)	0.068	0.075	0.009	0.018
R-squared (overall)	0.115	0.091	0.117	0.050
N	——————- 1450 ——————-			

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1.2: Trade and GDP correlation with 20 years time windows

1.3 A simple model

In this section, I show in a simple framework why the inclusion of input-output linkages across countries is not sufficient for a model to generate a strong relationship between trade and GDP comovement, and how the inclusion of two new elements (monopolistic pricing and extensive margin adjustments) goes a long way toward generating a link between a shock in a trading partner's economy and domestic GDP. Section 1.4 will then present a quantitative general equilibrium model with many countries that is able to replicate 85% of the trade-comovement relationship observed in the data, hence proposing the first quantitative solution for the trade comovement puzzle.

For the sake of exposition, I consider here a static small open economy. In such a world, KR showed that a change in the price of imported inputs has no impact, up to a first order approximation, on measured productivity. This means that any change in GDP is due to variations in domestic factors supply. I start by briefly reviewing this important result.

1.3.1 The Kehoe and Ruhl (2008) negative result

The economy produces a final good y, used for consumption and exports, which is produced by combining imported inputs x and domestic factors of production ℓ (possibly a vector) according to:

$$y = F(\ell, x) \tag{1.1}$$

where $F(.,.)$ has constant returns to scale and is concave with respect to each of its argument. The final good producer chooses intermediate and imported inputs to maximize its profit taking as given all prices. Optimality requires that factors are paid their marginal product:

$$p_y F_\ell(\ell, x) = w \qquad \text{and} \qquad p_y F_x(\ell, x) = p_x \tag{1.2}$$

with p_y the final good price, p_x the price of imported inputs x and w the price of domestic factors. Gross Domestic Product is the sum of value added in the country, which is simply the value of final goods minus the value of imported inputs. Importantly, many statistical agencies (and in particular the OECD database used in the empirical analysis above) use *base period prices* when valuing estimated quantities in the construction of GDP.[18] Since prices are kept constant at their base value, we denote them with the superscript b to emphasize the fact that they are treated as parameter and not as endogenous objects:

$$GDP = p_y^b F(\ell, x) - p_x^b . x \tag{1.3}$$

[18]In the US, the Bureau of Economic Analysis uses a Fisher chain-weighted price index to construct GDP at time t relative to GDP at time $t-1$ according to:

$$\frac{GDP_t}{GDP_{t-1}} = \left(\frac{\sum_k p_{t-1}^k q_t^k}{\sum_k p_{t-1}^k q_{t-1}^k} \right)^{0.5} \left(\frac{\sum_k p_t^k q_t^k}{\sum_k p_t^k q_{t-1}^k} \right)^{0.5}$$

where k indexes all components of GDP. Intuitively, the Fisher index is a mix between two base period pricing methods where the base price is alternatively the price at $t-1$ and at t.

Let us now compute the first order change in GDP when the Terms-of-Trade ($\equiv p_x$) change:

$$\frac{dGDP}{dp_x} = \underbrace{p_y^b F_\ell(\ell, x)\frac{\partial \ell}{\partial p_x}}_{\text{Factor Supply Effect}} + \underbrace{\frac{\partial x}{\partial p_x}(p_y^b F_x(\ell, x) - p_x^b)}_{\text{Input-Output Effect}} \tag{1.4}$$

The first term in equation (1.4) captures the value added change due to variations in factor supply and depends heavily on the degree of substitutability or complementarity between foreign and domestic inputs[19] as well as on the elasticity of factor supply. The second term captures the direct impact that changes of imported input usage have on GDP. With perfect competition, profit maximization insures that $p_y F_x(\ell, x) = p_x$ when using current prices. When base period prices p_y^b and p_x^b are close to their current value,[20] this term disappears. In such a model, any first order change in GDP following a terms of trade shock is solely driven by variations in domestic factor supply. This is the negative result presented in KR: when firms take prices as given, profit maximization insures that the marginal benefit of using an additional unit of imported input x ($p_y F_x(\ell, x)$) is equal to its marginal cost (p_x). Hence, up to a first order approximation, domestic value added is affected by a foreign technological shock only through a change in factor supply. In other words, the measured productivity is not affected to foreign shocks.[21]

Equation (1.4) encapsulates in a simple way the reasons why even sophisticated RBC models cannot generate a quantitatively important link between trade linkages and GDP comovement. In models with perfect competition and constant returns to scale, the change in GDP after a foreign shock is solely driven by variations in domestic factors supply. Such a change, in turn, is disciplined by (i) the elasticity of labor supply and (ii) the complementarity between domestic and foreign inputs.[22]

1.3.2 Markups and Love for variety

Consider now a variant of the economy described above with an additional production step: inputs are imported by a continuum of intermediate producers with a linear production function

[19] The role of complementarity is discussed at length in Burstein et al (2008) or in Boehm et al (2015).

[20] With a Fisher chain-weighted price index in the construction GDP, base period prices are always close to current prices.

[21] Note that an important part of the reasoning rests upon the fact that GDP is constructed using constant base prices. If the prices used to value final goods and imported inputs were to change due to the shock, one would have an additional term in equation (1.4).

[22] If domestic and foreign inputs are strongly complements, any shock that increases foreign input usage also rises demand for domestic inputs, which increases GDP.

$m = x$. Critically, I now add two new elements: (1) a *price wedge* for intermediate producers $\mu \neq 1$ so that the price of intermediates m is given by $p_m = \mu \times p_x$, and (2) love for variety in the final good production technology in the form of a Dixit-Stiglitz aggregation of intermediates.[23] The production function in the final good sector is:

$$y = F(\ell, \mathcal{I}) \qquad \text{with } \mathcal{I} = \left(\int_0^{\mathcal{M}} m_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{1.5}$$

This production function displays love for variety in the following sense: for a given amount of total imports, the larger the mass of input suppliers $\mathcal{M}$, the higher the amount of final production obtainable.

For each variety m_i, there is a producer with a linear technology using imports only:

$$\forall \ i \ \in [0, \mathcal{M}], \quad m_i = x_i \tag{1.6}$$

All intermediate producers are completely symmetric and I denote by m their (common) production and by x their (common) import levels. The bundle $\mathcal{I}$ can then be simply expressed as $\mathcal{I} = \mathcal{M}^{\sigma/(\sigma-1)} m$ and the price index dual to the definition of the bundle is $\mathcal{P} = \mathcal{M}^{1/(1-\sigma)} p_m$, which is also equal to $F_{\mathcal{I}}(\ell, \mathcal{I})$, the marginal productivity of the input bundle in final good production. Finally, taking the derivative of GDP with respect to p_x while keeping prices constant at their base period value, we obtain:

$$\frac{dGDP}{dp_x} = \underbrace{p_y^b F_\ell(\ell, \mathcal{I}) \frac{\partial \ell}{\partial p_x}}_{\text{Factor Supply Effect}} + \underbrace{\left(\mathcal{M} \frac{\partial m}{\partial p_x^b} + \frac{\partial \mathcal{M}}{\partial p_x} m \right) . (\mu - 1) \, p_x}_{\text{Markup Effect}} + \underbrace{\frac{1}{\sigma - 1} p_m m \frac{\partial \mathcal{M}}{\partial p_x}}_{\text{Entry/Exit Effect}} \tag{1.7}$$

Equation (1.7) is the counterpart of (1.4) in a model with extensive margin adjustments and where importing firms are not price takers. Crucially, the introduction of those two elements create a link between a foreign shock and domestic value added variations, over and beyond any change in domestic factor supply.

First, the existence of a price wedge $\mu \neq 1$ means that the first term does not vanish. With

[23] In many models, the elasticity of substitution in the CES aggregation governs at the same time the markup charged by monopolistic competitors and the love degree of love for variety. In order to clearly differentiate the sheer effect of markup from the love for variety, I assume here that the markup μ can take any value, including the case where $\mu = \sigma/(\sigma - 1)$.

$m'(p_x) < 0$,[24] a decrease in the price of imported inputs leads to an increase in GDP. When firms are price setters and earn a positive profit, the marginal revenue generated by an additional unit of imported input x is larger than its marginal cost p_x. Hence, cheaper inputs means more sales, more profit and more value added.

Moreover, any change in the mass of firms $\mathcal{M}$ also impacts domestic value added. One can model many reasons why the mass of producing firms would change, including a free entry condition with initial sunk cost or any reason that changes the supply of potential entrepreneurs.[25] A change in the number of price setting firms gives a time varying element to the effect described above, triggering a greater reaction of GDP after a foreign shock. Note that this effect is not governed by the love for variety which is captured by the parameter σ. Overall, the key idea governing this first term can be expressed as follows: firms that charge a markup have a disconnect between the marginal cost and the marginal revenue product of their inputs. The difference between those two is accounted as value added in the form of profits. Any change in input usage leading to a change in profits triggers a change in value added produced.

Second, when $\sigma < +\infty$, another effect arises. When the production function exhibits love for variety, any change in the mass of firms implies an additional reaction for the input bundle $\mathcal{I}$. If the decrease of p_x is accompanied by an increase in the mass of producing firm,[26] the bundle $\mathcal{I}$ increases not only because each intermediate producer will tend to produce more, but also because an increase in the mass of firms mechanically increases $\mathcal{I}$ even for a fixed amount of intermediates.

With love for variety, a producer that has access to more suppliers can produce more output for the same level of input, and a change in the mass of firms impacts the marginal cost of producing final goods over and beyond the change in input prices. Another way of saying this is that the set of feasible combinations of output $\mathcal{I}$, and inputs $\int_0^{\mathcal{M}} m_i di = X$ is not independent of the mass of producers $\mathcal{M}$: a change of $\mathcal{M}$ has an effect on the production possibility frontier. Interestingly, this channel is at work independently of the price distortion channel discussed pre-

[24] This can be easily proved if assuming that $F(.)$ is a Cobb Douglas aggregation of domestic factors and intermediates.

[25] In an additional appendix available upon request, I model the free condition and show that indeed an increase of import price leads to a decrease in the mass of firms.

[26] If the mass of firms is pinned down by a free entry condition, the increase in profits of each intermediate producer when the price of imported input goes up leads to a increase in the mass of firms.

viously. Even in the absence of monopoly pricing, the sheer fluctuation in the mass of producing firms coupled with a love for variety in final good production creates a link between import price and GDP fluctuation even with fixed factor supply.

Finally, note that the introduction of markups and love for variety allows GDP to change over and beyond changes in the domestic factors of production. Using a "growth accounting" perspective, this means that the introduction of those two elements makes domestic productivity change after a foreign shock, even with a fixed technology. Two countries that have important trade flows in intermediate inputs should then have correlated measured TFP, a prediction I test in the data in section 1.7.

1.4 A model of International Trade in Inputs

1.4.1 Setup

In this section, I build a quantitative model of international trade in inputs with *monopolistic competition* and *firm entry/exit* and assess its ability to replicate the strong relationship between trade and business cycle synchronization.[27] The model is related to Ghironi and Melitz (2005) and Alessandria and Choi (2007), extended with multiple asymmetric countries and global value chains with intermediate goods crossing borders multiple times. The combination of international input-output linkages, price distortions and extensive margin adjustments allows the model to give a quantitative account of the relationship between trade and GDP movements.[28]

I consider an environment with N countries indexed by k. In each country, there is a representative agent with preferences over leisure and the set of available goods Ω_k described by

$$\begin{aligned} U_{k,0} &= \mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \beta^t \left(\log\left(C_{k,t}\right) - \psi_k \frac{L_{k,t}^{1+\nu}}{1+\nu} \right) \right] \\ \text{with} \qquad & C_t = \left(\int_{\Omega_k} q_{i,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where ψ_k is a scaling parameter, ν is the inverse of the Frisch elasticity of labor supply and

[27] In section 1.6, I present a decomposition of the role that each of the novel ingredients has on the quantitative results.

[28] Alternatively, the model presented here can be though of as an extension of the IRBC model presented in Johnson (2014) with two new elements: markups and extensive margin adjustments. Again, section 1.6 shows that those two ingredients are quantitatively essentials in generating a link between trade and GDP comovement.

σ the elasticity of substitution between different varieties of final goods. The agent chooses consumption, investment and labor in each period subject to the budget constraint:

$$P_{k,t}\left(C_{k,t} + K_{k,t+1} - (1-\delta)K_{k,t}\right) = w_{k,t}L_{k,t} + r_{k,t}K_{k,t}$$

Production is performed by a continuum of heterogeneous firms combining in a Cobb-Douglas fashion labor ℓ_k, capital k_k and intermediate inputs $I_{k,t}$ bought from other firms from their home country as well as from abroad. Firms' productivity is the product of an idiosyncratic part φ and a country specific part $Z_{k,t}$. Firms maximize their static profit taking as given all input prices. Omitting time indexes for now, the intermediate input index in country k, I_k is an Armington aggregation of country specific bundles $M_{k',k}$ for all k', with the Armington elasticity denoted ρ. In order to introduce a rationale for markups and for love for variety, each country specific bundle is itself a CES aggregation of many varieties, with the elasticity of substitution σ (which governs both the markup firms charge and the degree of love for variety). The production function is:

$$\begin{aligned} Q_k(\varphi) &= Z_k.\varphi\ .\ I_k(\varphi)^{1-\eta_k-\chi_k}\ .\ \ell_k(\varphi)^{\chi_k}\ .\ k_k(\varphi)^{\eta_k} \\ \text{with} \qquad & I_k(\varphi) = \left(\sum_{k'} \omega_k(k')^{\frac{1}{\rho}} M_{k',k}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \\ \text{and} \qquad & M_{k',k} = \left(\int_{\Omega_{k',k}} m_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where $\omega_k(k')$ is the share of country k' in the production process of country k with $\sum_{k'} \omega_k(k') = 1$ and $\Omega_{k',k}$ is the endogenous set of firms based in k' and exporting to k. For later use, I define notations for the ideal price indexes dual to the two layers of the nested CES aggregation. $\mathcal{P}_{k,k'}$ denotes the price of the country-pair specific bundle $M_{k',k}$ and IP_k the unit price of the intermediate input bundle I_k. The unit cost of the Cobb Douglas bundle aggregating I_k, k_k and ℓ_k (called the "input bundle") is PB_k and represents the price of the basic production factor in country k. The exact expressions of those objects are standard and can be found in the appendix.

The only stochastic elements of this model are the country specific technological shocks (Z_k) which follow an AR(1) process. In all countries, the distribution of productivity φ is Pareto with shape parameter γ and density $g(\varphi) = \gamma\varphi^{-\gamma-1}$. For simplicity and in line with the empirical results in section 2, I restrict trade to be only between firms which means that I consider only trade in intermediate inputs.

In order to be allowed to sell its variety to a country j, a firm from country i must pay a fixed cost f_{ij} (labeled in unit of the "input bundle") as well as a variable (iceberg) cost τ_{ij}. Firms choose which countries they enter (if any), affecting both the level of competition and the marginal cost of all firms in the country. As will be clear below, profits are strictly increasing with productivity φ so that equilibrium export decisions are defined by country-pair specific thresholds $\varphi_{k,k'}$ above which firms from k find it profitable to pay the fixed cost $f_{kk'}$ and serve country k'. Finally there is an overhead entry cost $f_{E,k}$, sunk at the production stage, to be paid before firms know their actual productivity. Based on their expected profit in all markets, firms enter the economy until the expected value of doing so equals the overhead entry cost. This process determines the mass of firms M_k actually drawing a productivity, some of which optimally decide to exit the market before production due to the presence of fixed costs.

1.4.2 Equilibrium

In this section, I present the key conditions that characterize the equilibrium of the model. Introducing X_k the aggregate consumers' revenue in k and S_k the total firms' spendings (including fixed costs payments) in country k respectively, total demand faced by firm φ is given by

$$q(\varphi) = \left(\frac{p_{k,k}(\varphi)}{\mathcal{P}_k}\right)^{-\sigma} \frac{X_k}{\mathcal{P}_k} + \sum_{k'} \left(\frac{p_{k,k'}(\varphi)}{\mathcal{P}_{k,k'}}\right)^{-\sigma} \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}}\right)^{-\rho} \frac{\omega_{k'}(k)(1-\eta_{k'}-\chi_{k'})S_{k'}}{IP_{k'}} \tag{1.8}$$

where $p_{k,k'}(\varphi)$ is the price charged by a firm from country k, with productivity φ, when selling in country k' and the summation is done over all markets that are served by a firm with productivity φ. Firms are monopolists within their variety and choose their price at a constant markup over marginal cost and the markup depends on the price elasticity of demand. In this case, the only elasticity that is relevant to firms' pricing is σ, capturing the fact that firms compete primarily with other firms coming from their home country since their individual pricing decision has no impact on the country-specific price index in every market.[29] The marginal cost of a firm with

[29] With a finite number of firms, both elasticities σ and ρ would appear in the pricing strategy. In such a case, every firm would take into account the fact that its own price has an impact on the unit cost of the corresponding country-specific bundle. Therefore, when decreasing its price a firm would attract more demand compare to firms from its own country but also increase the share of total demand that goes to every other firms from the its country.

productivity φ in country k is $PB_k/(Z_k\varphi)$ and its optimal price is given by:

$$p_{k,k'} = \tau_{k,k'} \frac{\sigma}{\sigma - 1} \frac{PB_k}{Z_k \varphi} \tag{1.9}$$

Unlike in the canonical Krugman (1980), Melitz (2003) or Ghironi and Melitz (2005) models, one cannot solve for prices for each firm independently. Through PB_k, the price charged by firm φ in country k depends on the prices charged by all firms supplying country k (both domestic and foreign) which in turn depend on the prices charged by their suppliers and so on and so forth. The input-output linkages across firms create a link between the pricing strategies of all firms and one needs to solve for all those prices at once. Doing so requires solving for all country-pair specific price indexes $\mathcal{P}_{k,k'}$.

The definitions of price indexes give rise to a simple relationship between the price of the country k specific bundle at home, $\mathcal{P}_{k,k}$, and its counterpart in country k', $\mathcal{P}_{k',k}$:

$$\mathcal{P}_{k,k'} = \tau_{kk'} \left(\frac{\varphi_{k,k'}}{\varphi_{k,k}} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \times \mathcal{P}_k \tag{1.10}$$

Intuitively, the ratio between the price of a country specific bundle in two different markets depends on the relative iceberg costs as well as the relative entry thresholds. Using this relation in the definition of price indexes in every country yields a system of N equations which jointly defines all price indexes:

$$\mathcal{P}_k^{1-\rho} = \mu_k \left(\sum_{k'} \omega_k(k') \left(\tau_{k'k} \left(\frac{\varphi_{k',k}}{\varphi_{k',k'}} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \mathcal{P}_{k'} \right)^{1-\rho} \right)^{1-\eta_k-\chi_k} \quad , \ k = 1, ..., N \tag{1.11}$$

with μ_k depending on entry thresholds, the mass of firms and parameters.[30] For given thresholds and mass of firms, this system admits a unique non negative solution.[31]

Turning now to the determination of export strategies, the productivity thresholds above which firms from country k optimally decide to pay the fixed cost and serve market k' are simply

[30] $\mu_k^{\frac{1-\sigma}{1-\rho}} = \frac{\gamma \varphi_{k,k}^{\sigma-\gamma-1}}{\gamma-(\sigma-1)} M_k \left(\frac{\sigma}{\sigma-1} \frac{w_k^{\chi_k} \times r_k^{\eta_k}}{\chi_k^{\chi_k} \times \eta_k^{\eta_k} \times (1-\eta_k-\chi_k)^{1-\eta_k-\chi_k}} \frac{1}{Z_k} \right)^{1-\sigma}$

[31] Following Kennan (2001) and denoting $G_k = \mathcal{P}_k^{1-\rho}$ and G the associated $N \times 1$ vector, it suffices to show that the system is of the form $G = f(G)$ with $f \ : \ \mathbb{R}^N \rightarrow \mathbb{R}^N$ a vector function which is strictly concave with respect to each argument, which is obvious as long as $0 < \eta_k + \chi_k < 1$. This argument stresses the importance of decreasing return to scale with respect to intermediate inputs in order to ensure unicity of the equilibrium.

given by:

$$\pi_{k,k'}(\varphi_{k,k'}) = \frac{PB_k}{Z_k}.f_{k,k'} \quad \text{for all } k \text{ and } k' \tag{1.12}$$

where $\pi_{k,k'}(\varphi)$ is the variable profit earned by a firm with productivity φ in market k'. I assume that the fixed cost $f_{k,k'}$ is paid in units of the basic production factor in country k deflated by the aggregate level of productivity, as is the case in Ghironi and Melitz (2005) for example.

The mass of firms deciding to enter the market in each period is finally determined by the free entry condition. With the assumption that $f_{E,k}$ is labeled in units of labor, the condition writes:

$$\Pi_k = M_k \frac{w_k}{Z_k}.f_{E,k} \quad \text{for all } k \tag{1.13}$$

where Π_k denotes aggregate profits of all firms in country k. An expression of Π_k can be found using a property first noted by Eaton and Kortum (2005) according to which total profit in country k are proportional to total revenues. Defining R_k the total sales of firms from country k made on all markets, we have the following result:

Lemma 1 : Total profit in country k are proportional to total revenues:

$$\Pi_k = \frac{\sigma - 1}{\gamma\sigma} R_k \tag{1.14}$$

Proof: see Appendix.

Closing the model involves market clearing conditions for capital, labor and goods. Labor can be used either for production (L_k^p) or for the entry cost (L_k^e) so that $L_k = L_k^p + L_k^e$. Classic properties of Cobb-Douglas production functions imply that total labor and capital payments for production are equal to a fraction $\eta_k + \chi_k$ of firms' variable spendings. Since total profit are used in the entry fixed cost payment, total consumer's spending is defined as $X_k = w_k L_k + r_k K_k = (\eta_k + \chi_k) S_k + \Pi_k$. Moreover, the investment Euler Equation (capital supply) is given by

$$\frac{1}{C_{k,t}} = \beta \mathbb{E}_t \left[\frac{1}{C_{k,t+1}} \times \left(\frac{r_{k,t+1}}{P_{k,t+1}} + (1 - \delta) \right) \right] \tag{1.15}$$

while labor supply is:

$$\psi_k L_{k,t}^{\nu} = \frac{w_{k,t}}{P_{k,t}} \frac{1}{C_{k,t}} \tag{1.16}$$

Finally, trade being allowed in intermediate goods only, revenues in foreign countries come from other firms' spending while domestic revenues also include consumers' spendings. Total revenues of all firms from country k are:

$$R_k = X_k + \left[\sum_{k'} \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}} \right)^{1-\rho} \omega_{k'}(k)(1-\eta_{k'}-\chi_{k'})S_{k'} \right] \tag{1.17}$$

This formula has a simple interpretation: firms in country k receive revenues from their final good sales to their home consumers (for a total amount of X_k) as well as from sales as intermediate goods on all markets. In every country k', firms allocate a constant fraction $1-\eta_k-\chi_k$ of their total spendings to intermediate inputs, which is then scaled by the weight $\omega_{k'}(k)$ representing the importance of country k in the production process of country k'. Finally, since country k specific bundle in k' is in competition with other country specific bundles in the input market, total revenues of k-firms when selling in k' also depend on the ratio of $\mathcal{P}_{k,k'}$ to $IP_{k'}$ to a power reflecting the relevant the price elasticity, in this case the macro (Armington) one ρ. For later use, it is useful to define total trade between k and k' as

$$T_{k\to k'} = \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}} \right)^{1-\rho} \omega_{k'}(k)(1-\eta_k-\chi_k)S_{k'}$$

Using $X_k = (\eta_k+\chi_k)S_k + \Pi_k$, the good market clearing condition can be written in compact form as

$$\left\{ \underbrace{\left(\mathcal{I}_N - \left(W^T \circ P\right)\right)}_{=M} \begin{pmatrix} (1-\eta_1-\chi_1).S_1 \\ \vdots \\ (1-\eta_N-\chi_N).S_N \end{pmatrix} = 0_{\mathbb{R}^N} \right. \tag{1.18}$$

where W the weighting matrix defined as $W_{ij} = \omega_i(j)$, P a matrix defined by $P_{ij} = \left(\frac{\mathcal{P}_{i,j}}{IP_i}\right)^{1-\rho}$ and $\circ$ is the element-wise (Hadamard) product. To gain intuitions, one can note that the matrix P scales the weights $\omega_{k'}(k)$ in order to account for the competition across country-specific bundles. If the Armington elasticity ρ is above unity (country specific bundles are substitutes) then a country i which is able to charge low prices in some market j (a low $\mathcal{P}_{i,j}$) will attract a higher share of total expenditures from all firms in this country. Classically, this effect completely disappears in the case of a Cobb-Douglas aggregation of country specific bundles, because in such a case the spending shares are fixed.

The solutions of this system form a one dimensional vector space.[32] Setting $w_1 = 1$, implying $S_1 = L_1^p/\chi_k$, provides a unique solution for all variables by solving together the price system (1.11), the threshold system (1.12), the Spending system (1.18), the Free Entry system (1.13) as well as the labor and capital market equilibrium conditions.

GDP definition An important feature of GDP construction in OECD data is the use of base prices and quantity estimates. In order to be as close as possible to the method used in the construction of the data used in the empirical analysis, I define GDP using steady state prices as base prices.[33] The GDP definition that is model-consistent is obtained by using welfare-based price indexes to deflate nominal spending, such that:

$$GDP_{k,t} = \underbrace{\mathcal{P}_k^{ss} \frac{X_{k,t}}{\mathcal{P}_{k,t}}}_{\text{Consumption + Investment}} + \underbrace{\sum_{k'} \mathcal{P}_{k,k'}^{ss} \frac{T_{k\to k',t}}{\mathcal{P}_{k,k',t}}}_{\text{Exports}} - \underbrace{\sum_{k'} \mathcal{P}_{k',k}^{ss} \frac{T_{k'\to k,t}}{\mathcal{P}_{k',k,t}}}_{\text{Imports}} \tag{1.19}$$

Note that the first term is also equal to the Gross National Income (GNI_k) since there is no trade in assets across countries.

However, since both consumers' utility and production functions have a CES component, it is well known that the associated price indexes can be decomposed into components reflecting average prices (captured by statistical agencies) and product variety (which is not taken into account in national statistics).[34] In order to be consistent with the way actual data are collected, I then define GDP using modified price indexes such that $\widehat{\mathcal{P}_{k,k'}} = \left(M_k.\varphi_{k,k'}^{-\gamma}\right)^{1/(\sigma-1)} \mathcal{P}_{k,k'}$. Using those statistical-consistent price indexes in the GDP definition yields $\widehat{GDP_k}$, a measure of GDP that can be compared to the actual data:

$$\widehat{GDP_k} = \underbrace{\widehat{\mathcal{P}_k^{ss}} \frac{X_k}{\widehat{\mathcal{P}_k}}}_{\text{Consumption + Investment}} + \underbrace{\sum_{k'} \widehat{\mathcal{P}_{k,k'}^{ss}} \frac{T_{k\to k',t}}{\widehat{\mathcal{P}_{k,k',t}}}}_{\text{Exports}} - \underbrace{\sum_{k'} \widehat{\mathcal{P}_{k',k}^{ss}} \frac{T_{k'\to k,t}}{\widehat{\mathcal{P}_{k',k,t}}}}_{\text{Imports}} \tag{1.20}$$

[32]One can easily show that the matrix M is non invertible: summing all rows results in a column of zero.

[33]In the data, GDP is defined using the Fisher ideal quantity index which is a geometric mean of the Laspeyres and Paasche indexes. Hence, for all periods t, the base period price is a geometric mean between period t and period $t + 1$.

[34]See Feenstra (1994) or Ghironi and Melitz (2005) for a discussion of this

1.4.3 GNI elasticity in a simplified case

In order to investigate the mechanics driving the propagation of shocks across countries in the model, let us study a special case with $\rho = 1$ and fixed labor, capital and mass of potential entrants.[35] The goal of this section is to compute the elasticity of GNI in any country i with respect to a technology shock in country 1:

$$\eta_{GNI_i,Z_1} = \frac{\partial \log(GNI_i)}{\partial \log(Z_1)}$$

Moreover, in order to understand the differences between using model-based and statistic-based price indexes, I also compute the elasticity of Gross National Income as computed in national statistics ($\widehat{GNI_k} = (w_k L_k + r_k K_k)/\widehat{\mathcal{P}_k}$):

$$\eta_{\widehat{GNI_i},Z_1} = \frac{\partial \log(\widehat{GNI_i})}{\partial \log(Z_1)}$$

Computing the elasticity of all endogenous variable with respect to technological shocks leads to the closed-form formula in lemma 2.

Lemma 2 : In the Cobb-Douglas ($\rho = 1$) case and fixing both labor and capital supply, the elasticity of model-based GNI and statistical GNI in all countries with respect to a technology shock in country 1 are given by:

$$\begin{pmatrix} \eta_{GNI_1,Z_1} \\ \vdots \\ \eta_{GNI_N,Z_1} \end{pmatrix} = (\mathcal{I}_N - \widehat{W} - T)^{-1} \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix} \tag{1.21}$$

and

$$\begin{pmatrix} \eta_{\widehat{GNI_1},Z_1} \\ \vdots \\ \eta_{\widehat{GNI_N},Z_1} \end{pmatrix} = \left(\frac{\gamma - (\sigma - 1)}{\sigma\gamma - (\sigma - 1)} \right) \cdot \begin{pmatrix} \eta_{GNI_1,Z_1} \\ \vdots \\ \eta_{GNI_N,Z_1} \end{pmatrix} \tag{1.22}$$

[35] Without capital supply, the model is completely static. A fixed mass of potential entrants does not mean a fixed mass of actual producers because entry thresholds $\varphi_{k,k}$ are not fixed.

with $\widetilde{W}_{i,j} = (1 - \eta_i - \chi_i)\omega_{i,j}$ the matrix of scaled weights $\omega_{i,j}$ representing the intensive margin adjustments and T a "Transmission" matrix[36], function of γ and σ, accounting for extensive margin movements.

Proof: see Appendix.

These expressions are reminiscent of what can be found in static Cobb-Douglas network models such as Acemoglu et al (2012) for example, with an additional effect coming from firm heterogeneity and the extensive margin adjustments captured by the matrix T. In this context, the international propagation pattern of country specific shocks runs through two channels. First, for fixed spending share, the matrix $\widetilde{W}$ records the input-output linkages if the export decision of firms are kept constant. Second, the change in prices and revenues in all markets triggers a change in the productivity thresholds $\varphi_{k,k'}$. This channel is characterized by the matrix T which is a function of σ and γ which govern the adjustments along the extensive margin. Note that the elasticities of model- and statistical agency-based GNIs are exactly proportional, with $\eta_{\widehat{GNI}_k,Z_1} < \eta_{GNI_k,Z_1}$ for all k. Not taking into account the love for variety effect in the computation of price indexes leads to a downward bias in the response of price indexes to technological shocks.

The computations leading to the expressions of the GNI elasticities in this lemma are greatly simplified by the assumption that factors of production (labor and capital) are fixed. In the general model, however, this constitute an important amplification channel through two effects. First, as in many macro models, a positive productivity shock in any country contributes to the decrease of prices all over the world and hence an increase in real wage. This triggers an increase in labor supply that amplifies the benefits of the shock in terms of output.[37] In addition, there is a second channel going through the change in the mass of active firms in every country. With the assumption that the mass of potential entrepreneurs is proportional to the labor size, an increase in labor supply results in a proportional increase in the mass of potential entrants. Whether the mass of actual producing firms goes up or down in any country k will also be determined

[36] $T = \Lambda \mathcal{I}_N$, with $\Lambda = \frac{1}{\sigma + \frac{(\sigma-1)^2}{\gamma-(\sigma-1)}}$

[37] This increase in labor supply is tempered by the wealth effect.

by the changes in the thresholds φ_{ik} for all i which in turns crucially depends on the value of the Armington elasticity ρ. In the Cobb-Douglas case where the expenditure shares are fixed, a positive technological shock will result in a decrease of all entry thresholds in every market. Putting pieces together, a positive shock triggers at the same time more potential entrepreneurs and a decrease of the entry threshold in every market. As a result, the new structure of input-output linkages amplifies the benefits of the shock.

1.5 Calibration

The goal of this section is to quantitatively assess the model's ability to match the strong empirical relationship between trade proximity in intermediate input and GDP synchronization. The model is calibrated to 14 countries and a composite rest-of-the-world for the time period 1989 to 2008. In what follows, I explain in detail the calibration strategy while the simulation results are gathered in the next section.

For a calibration with N countries, there are $3 \times N^2 + N + 6$ parameters to determine, on top of which one needs to set parameters relative to the technological shocks. For each country-pair (i, j), one needs values for the weights $\omega_i(j)$, the iceberg trade costs τ_{ij} and the fixed costs $f_{i,j}$, then for every country i one needs values for "value added" share in production $(\eta_k + \chi_k)$ and scaling parameter ψ_i. The set of common parameters is given by $\chi_k/(/chi_k + \eta_k)$ the labor share in value added, ν for the (inverse) elasticity of labor supply, γ for the distribution of productivity draws, σ for the within country (micro) elasticity of substitution across varieties and ρ for the (macro) elasticity of substitution of country-specific bundles. Finally, we will also need to set the volatility, covariance and auto-correlation of the TFP shocks in all countries, as discussed in detail below.

My calibration is a mixture of estimations from micro data (taken from the literature as well as re-estimated) and a matching of macro moments. The goal is to match exactly the relative GDP across all country pairs, the volatility, persistence and level of GDP co-movement as well as the trade proximity in intermediate goods in order to give a reasonable account of the ability of the model to generate a strong link between bilateral trade and GDP synchronization despite the fact that typical trade flows between two given countries are very low compare to their GDPs.

From micro data

The discount factor β is 0.99. The (inverse) elasticity of labor supply ν is 2/3 leading to a Frisch elasticity of 1.5. The sunk entry cost $f_{E,k}$ in each country is set in order to get a ratio of total number of *potential* (not actual) firms divided by the population of 10%, in line with US estimates taking into account that not all potential entrepreneurs enter the economy in equilibrium. The variable (iceberg) trade costs are taken from the ESCAP World Bank: "International Trade Costs Database"[38]. This database features symmetric bilateral trade costs in its wider sense, including not only international transport costs and tariffs but also other trade cost components discussed in Anderson and van Wincoop (2004).

As in di Giovanni and Levchenko (2013), fixed access costs are computed from the "Doing Business Indicators".[39] In particular, I measure the relative entry fixed costs in domestic markets by using the information on the amount of time required to set up a business in the country relative to the US.[40] If according to the Doing Business Indicators database, in country i it takes 10 times longer to register a business than in the U.S., then $f_{i,i} = 10 \times f_{US,US}$. I normalize the lowest entry fixed cost so that no entry threshold lies below the lower bound o the productivity distribution, which is taken to be one in every country. To measure the fixed costs associated with entry in a foreign market, I use the Trading Across Borders module of the Doing Business Indicators. I choose the number of days it takes to import to a specific country, using the same normalization as for the domestic entry cost.[41]

In the benchmark simulations, I choose the macro (Armington) elasticity ρ to be equal to unity while the micro elasticity σ is equal to 5. There are many papers estimating those elasticities for intermediate or final goods. Saito (2004) provides estimations from 0.24 to 3.5 for the Armington elasticity[42] and Anderson and van Wincoop (2004) report available estimates for the

[38]See at http://artnet.unescap.org/

[39]The World Bank's Doing Business Initiative collected data on regulations regarding obtaining licenses, registering property, hiring workers, getting credit, and more. See http://francais.doingbusiness.org/data/exploretopics/trading-across-borders and http://francais.doingbusiness.org/data/exploretopics/starting-a-business

[40]As argued in di Giovanni and Levchenko (2013), using the time taken to open a business is a good indicator because it measures entry costs either in dollars or in units of per capita income, because in the model $f_{i,i}$ is a quantity of inputs rather than value.

[41]This approach means that the fixed cost associated with trade from France to the US is the same as the one from Germany to the US. One must keep in mind, however, that the iceberg variable cost will differ.

[42]Feenstra et al (2014) studies the macro and micro elasticities for final goods and reports estimates between

micro elasticity in the range of 3 to 10. Following Bernard, Eaton, Jensen, and Kortum (2003), Ghironi and Melitz (2005) choose a micro elasticity of 3.8. Recently, papers such as Barrot and Sauvagnat (2015) or Boehm, Flaaen and Pandalai-Nayar (2015) argue that firms' ability to substitute between their suppliers can be very low. The choice of a value of $\sigma = 5$ leads to markups of 25%. The aggregate profit rate, however, is only of 17.4% since firms have to pay fixed cost in order to access any market. There is also a theoretical convenience to use $\rho = 1$, as it allows the model to take the same form as other network models such as Acemoglu (2012), Bigio and La'O (2015) and many others. Finally, the capital and labor shares in value added are fixed at 2/3 and 1/3 respectively and I set $\gamma = \sigma - 0.4$ as described in Fattal-Jaef and Lopez (2010).

Parameter	**Value**	**Counterpart**
β	0.99	Discount factor – Annual discount rate of 4%
ρ	1	Macro (Armington) Elasticity of substitution (from Literature)
σ	5	Micro Elasticity of substitution – 25% markup, average profit of 17.4%
ν	2/3	Labor Curvature – Frisch elasticity of 1.5
$f_{E,i}$	[1 - 10]	$M/L = 0.1$ – Mass of plants over working population
τ_{ij}	[1 - 3]	Iceberg trade cost – From ESCAP - World Bank
f_{ij}	[1 - 10]	Fixed trade cost – "Doing Business Indicators"
γ	4.6	Pareto shape – (Fattal-Jaef & Lopez (2014))
$\chi_k/(\chi_k + \eta_k)$	0.7	Labor share – 70% of value added.

Table 1.3: Parameters fixed using micro data

Matching of macro moments

For the remaining parameters, I use data on 14 countries from 1989 to 2008 and chose parameter values in order to match specific targets. More precisely, I jointly set the country size parameters $(\psi_i)_{i=1,...,N}$, the value added share $\chi_k + \eta_k$ as well as the spending weights $\omega_i(j)$ (the matrix W) in order to match all countries relative GDP and all relative trade flows in real terms. I normalize the real GDP of the composite rest-of-the-world to 100 and set all other real GDPs so that the ratio of their real GDP to the one of the rest-of-the-world in the simulated economy matches exactly its counterpart in the data for the time window 1989 to 2008. My targets are then N real GDP targets as well as N^2 directed trade flows (over GDP), to which one must add the constraint that spending shares $\omega_i(j)$ sum to one for each country, which leads to $(N^2 + 2N)$ equations

-0.29 and 4.08 for the Armington elasticity. They find that for half of goods the macro elasticity is significantly lower than the micro elasticity, even when they are estimated at the same level of disaggregation.

for an equal number of parameters to match. Since complete financial autarky is inconsistent with the trade balances observed in the data, I calibrate the model to match steady-state trade imbalances, and then hold those nominal imbalances constant. Note that in order to be as close as possible to the data used in the empirical analysis, I construct the quantity estimates by deflating the nominal spendings by the price index that do not take into account love for variety, as described in section 1.4.2.

Finally, I need to calibrate the persistence and the variance-covariance matrix for the country-level TFP shocks $(Z_i)_{i=1,\ldots,N}$. In order not to overestimate the impact of idiosyncratic shocks, I chose to set their volatility (the diagonal elements of the variance-covariance matrix) so that the model can replicate GDP volatility (de-trended using HP filtering) in every country. This allows me to generate fluctuations in the simulated economy that are similar to those observed in the data. Similarly, I set the off diagonal elements (the covariance terms) so that the average correlation of GDP in the model match the one observed in the data, which is 0.475 for the 1989-2008 time window. Recall that the goal of this exercise is not to explain the *level* of comovement across countries, but its *slope* when there is a change in trade. Hence, I set the *level* at the observed value and will vary parameters governing trade in order to evaluate the *slope.* Finally, I set a common value for auto-correlation of shocks so that the GDP series generated by the model is exactly 0.84 which is the value of GDP autocorrelation observed in the data.

1.6 Quantitative results

Trade Comovement Slope

The goal of this section is to assess the ability of the model to replicate the strong empirical relationship between trade proximity in intermediate inputs and GDP synchronization. The calibration procedure presented in the previous section yields values for all parameters so that the model economy matches the data for the period 1989 to 2008. With those values, I simulate a sequence of 5,000 shocks and record the correlation of HP-filtered GDP as well as the average index of trade proximity. Since my goal is to use within country-pair variations in order to perform a fixed-effect estimation of the effect of trade on GDP comovement, I then recalibrate the model with different targets for trade proximity across countries, decreasing and increasing the target by 10%. For each configuration, I feed the economy with the *exact same* sequence of 5,000 shocks and record the correlation of HP-filtered GDP as well as the average index of trade proximity. This gives rise to a panel dataset in which I have $14 \times 13/2 = 91$ observations for each of the 3 configurations, hence a total of 273 observations. I then perform fixed effect regressions on the simulated dataset and find that the model is able to explain more than 85% of the trade-comovement slope.

	dependent variable: corr(GDP_i^{HP},GDP_j^{HP})	
	Data	Model
log(Intermediate)	0.053***	0.047***

Decomposition - Role of the ingredients

In order to assess the role of each ingredient in the quantitative results, I then turn off one by one the key elements of the model. Results yield interesting insights. First, the sole addition of price distortions to an otherwise classic IRBC model with input-output linkages increases the trade comovement slope from 0.007 to 0.032. Finally, the amplification coming through the fluctuation in the mass of firms serving all markets increases the slope from 0.032 to 0.047, showing that adjustments along the extensive margin is a powerful way to generate quantitative results in line with the empirical link between trade in inputs and GDP comovement.

	Trade-Comovement Slope
I/O linkages + Markups + Extensive Margin	0.047***
I/O linkages + Markups	0.031***
I/O linkages	0.007***

Table 1.4: Decomposition of the result

Quantifying the Entry/Exit Margin

An important part of the quantitative results presented above come from the variation in the mass of firms serving every market. It is then necessary to understand if the entry/exit pattern predicted by the model is in line with what is observed in the data. Using French data from 1993 to 2008, I compute the number of products exported to many country.[43] After taking the logarithm to remove any level effect, I then apply the HP filter with smoothing parameter 6.25 to isolate the business cycle frequency fluctuations and compute the standard deviation across all years. Taking the average across all countries yields a value of 0.0086, meaning that on average the standard deviation of exported product represents 0.86 percent of the total number of product.

Computing the counterpart of this moment in the simulated dataset, I find a value of 0.0111 meaning that the model is roughly in line with the data on this respect, although it is slightly over-predicting the variance of the entry-exit pattern on foreign markets. Computing now the volatility of the number firms serving the domestic market (and not only export markets), using the universe of all French firms with at least one employee, the associated standard deviation is equal to 0.087, ten times larger than the value when considering only export markets. In the model, however, the value is 0.0114, meaning that the model *under*-predicts the entry/exit pattern in the domestic market.

Impulse Response functions

In order to give a better sense of the mechanics behind the model, I consider a simplified version with two countries (Home and Foreign) that are symmetric in the steady state. Keeping the value of all technological parameter as described above[44], I generate impulse response functions of Home GDP after a technological shock in Foreign. In order to have a sense of the trade comovement *slope*, I consider two calibrations of the W matrix: one that induces a low level of trade and the other with a high level of trade. By comparing the GDP responses in those two cases, one can understand the effect of increasing trade on GDP synchronization. Figure 1.1 presents the result of this exercise for three versions of the model. In the benchmark case with no markups (perfect competition within each variety) and no extensive margin (no fixed cost to enter any market and a fixed mass of firms), the GDP hardly moves. When introducing monopolistic pricing for all varieties, increasing trade between the two countries leads to a significant increase in the Home GDP reaction after a foreign technological shock. Finally, letting the mass

[43] Due to data availability, destination countries considered are Australia, Austria, Canada, Denmark, Germany, Ireland, Italy, Mexico, The Netherlands, Spain, United Kingdom and United States

[44] Except for the W matrix which is now symmetric and 2x2.

of firms and entry decisions be as described in the quantitative models further amplify the trade comovement slope, with an increase in trade inducing a very high increase in GDP reaction.

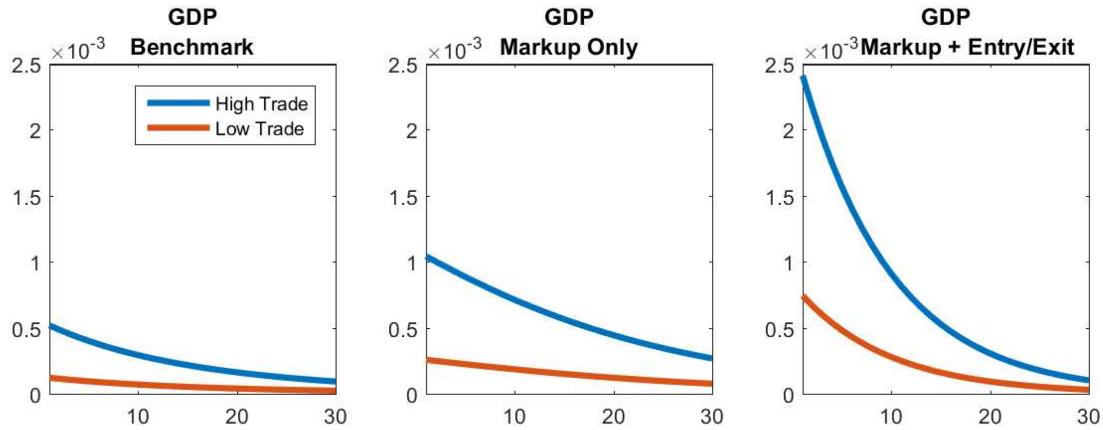

Figure 1.1: IRF of domestic GDP after a foreign shock

Before describing the role of each of those ingredients in the context of a simplified model in section 1.3, I further decompose the GDP reactions described above by performing a "growth accounting" exercise in which I decompose GDP fluctuations into labor and capital movements as well as the Solow residual that is usually referred to as the aggregate TFP.[45] In the benchmark case with no markups and no extensive margin, one can see that GDP fluctuation is due almost only to fluctuations in factor supply with TFP playing a insignificant role. This result is consistent with Kehoe and Ruhl (2008) or Cravino and Burstein (2015) which argue that foreign technological shocks have no effect on domestic productivity up to a first order approximation.

Interestingly, this result does not hold anymore when markups are introduced and measured TFP is affected by a foreign shock. As described more precisely in section 1.3, the reason stems from fact that in the presence of markups, the change in import due to the positive technological shock in the foreign country is smaller than the increase in final good production. As noted in Hall (1988) or Basu and Fernald (2002), when firms are price setters, the opportunity cost of using inputs is lower than their marginal revenue product. Note also that the TFP change

[45]Consistently with the theory, I used $\eta_k/(\eta_k + \chi_k)$ for the labor share and $\chi_k/(\eta_k + \chi_k)$ for the capital share to compute the solow residual

induces a larger reaction of domestic factors (labor and capital) which increases the GDP reaction after the foreign shock.

Finally, introducing fluctuations in the mass of firms serving all countries increases further the TFP reaction. This effect is due to the love for variety encompassed in the Dixit-Stiglitz aggregation of inputs. With love for variety, one can think of the mass of firms as being an input for production since an economy with a higher number of firms has the ability to produce more final output with the same amount of inputs. As suggested by the decomposition in table 1.4, the most important part of this mechanism is not due to the fixed cost associated to the access of any market but rather to the fluctuation in the mass of potential entrants, that is assumed to be proportional to the labor force. Indeed, any fluctuations along the labor supply margin is associated with a change in the mass of potential entrants. With love for varieties, the production technology frontier is affected by such a change in the number of producer, so that the final output reacts more than imported inputs. Moreover, since the Solow residual is computed using only Labor and Capital as domestic inputs and not controlling for the change in the mass of domestic firms, this increase in the production technology frontier is reflected in the TFP.

1.7 Further Empirical Evidence

In section 1.6, it has been shown that the combination of global value chains with price setting firms and extensive margin adjustments went a long way toward providing a quantitative solution for the trade comovement puzzle. While the empirical relevance of international input-output linkages as been uncovered in section 1.2, it is also interesting to test for the empirical relevance of markups and firms' entry/exit. In this section, I go back to the data and provide empirical support for the role of markups and entry/exit in creating a link between trade and GDP fluctuations.

First, following Liao and Santacreu (2015), I disentangle trade flows into their intensive and extensive margins and show that the empirical association between trade and business cycle synchronization is almost only driven by the extensive margin. Next, turning to the importance of price setting, I start by using sector level data to construct measures of markups that are then

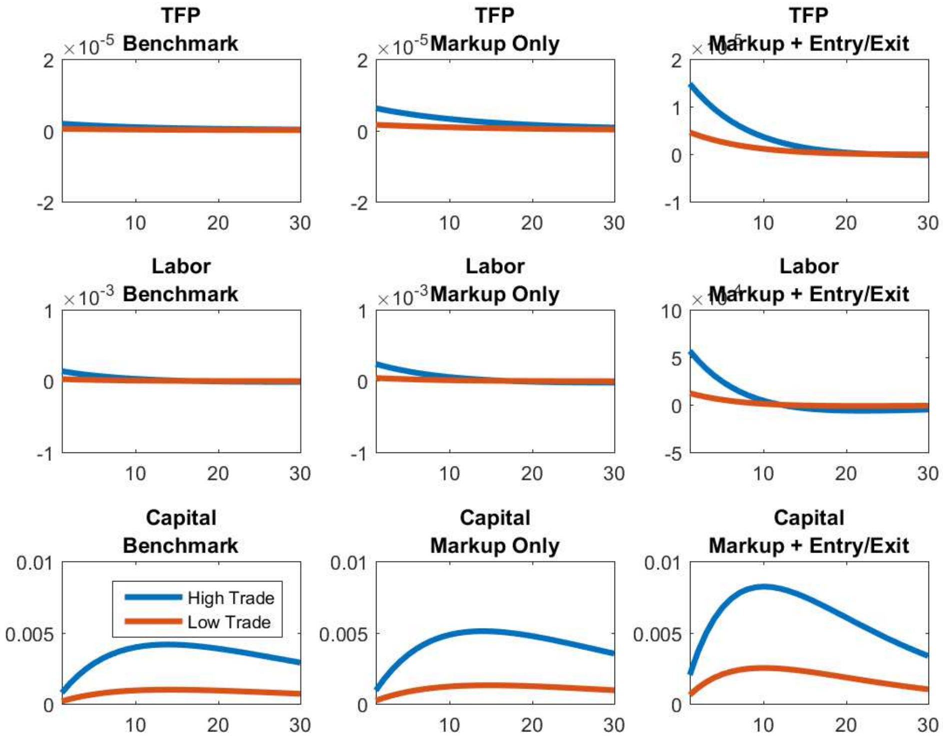

Figure 1.2: Growth Accounting Decomposition

aggregated at the country level. I then show that countries with larger markups are also more sensitive to terms of trade shocks.

1.7.1 The Role of Extensive Margin of Trade

Following Hummels & Klenow (2005) as well as Feenstra & Markusen (1994), I construct the Extensive and Intensive margins of trade between countries j and m using the Rest-of-the-World as a reference country k. The extensive margin (EM) is defined as a weighted count of varieties exported from j to m relative to those exported from k to m. If all categories are of equal importance and the reference country k exports all categories to m, then the extensive margin is simply the fraction of categories in which j exports to m. More generally, categories are weighted by their importance in k's exports to m. The corresponding intensive margin (IM) is defined as the ratio of nominal shipments from j to m and from k to m in a common set of goods. With this construction, the product of both margins of trade between j and m is equal to the ratio of total trade flows between j and m to total trade flows from the reference country k to m, which is usually denoted as OT. Formally, the margins are defined as:

$$\text{Extensive Margin} \quad EM_{jm} = \frac{\sum\limits_{i \in I_{jm}} p_{kmi} q_{kmi}}{\sum\limits_{i \in I} p_{kmi} q_{kmi}}$$

$$\text{Intensive Margin} \quad IM_{jm} = \frac{\sum\limits_{i \in I_{jm}} p_{jmi} q_{jmi}}{\sum\limits_{i \in I_{jm}} p_{kmi} q_{kmi}}$$

$$\text{Trade Ratio} \quad OT_{jm} = \frac{X_{j \to m}}{X_{k \to m}} = EM_{jm} \times IM_{jm}$$

Where I_{jm} is the set of observable categories in which j has a positive shipment to m, I is the set of all categories exported by the reference country which is supposed to be the universe of all categories and $X_{j \to m}$ is total trade flows from country j to country m. Since those measures are not symmetric within every country-pair, I define for a given country pair (i, j) as the sum of the margins from i to j and from j to i, which are then averaged over the time window.

Constructing four 10-years time window ranging from 1969Q1 to 2008Q4, I estimate the following equation

$$\text{corr}(Y_{it}^{HP}, Y_{jt}^{HP}) = \alpha + \beta_{EM} \log(\text{EM}_{ijt}) + \beta_{IM} \log(\text{IM}_{ijt}) + controls + \epsilon_{ijt} \tag{1.23}$$

Results are gathered in 1.5 and show that the extensive margin of trade is an important determinant of GDP comovement. This result is particularly striking given that most of the variation in trade is explained by variations along the intensive margin. Indeed, performing a Shapley value decomposition of OT on the intensive and extensive margins, one finds that only one fourth of the total variance in OT is explained by the variation of the extensive margin. Put simply: even though EM does not vary too much (compare to IM), its variations are strongly correlated with the variations of GDP comovement.[46]

[46]Those results are in line with the similar analysis in Liao and Santacreu (2015).

	dependent variable: corr(GDP_i^{HP},GDP_j^{HP})		
	(1)	(2)	(3)
log(EM)	0.249***	0.246***	0.104
	(8.91)	(6.27)	(1.91)
log(IM)	0.0111	0.120	0.023
	(1.08)	(0.45)	(1.08)
Country-Pair FE	no	yes	yes
Time FE	no	no	yes
N	760	760	760

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1.5: Strong Influence of the Extensive Margin of trade

1.7.2 Terms of Trade and GDP: the role of Markups

Using data from 22 countries from 1971 to 2010,[47] I assess the role of markups in generating a link between terms of trade and GDP fluctuations.

I test the following hypothesis: countries where markups are high experience a larger decrease in GDP when experiencing an increase in their terms-of-trade. In order to test this hypothesis, I compute the correlation of filtered GDP with the terms of trade and regress this correlation on markups estimates. Results show that markups have a significant impact on GDP-Terms of Trade correlation, with higher markups associated with lower correlation between GDP and the terms of trade.

Data on real GDP and terms of trade at the annual frequency are both taken from the OECD database and filtered according to two different procedure. I first apply the Hodrick and Prescott filter with a smoothing parameter of 6.25 which captures the business cycle frequencies. I also apply the Baxter and King band pass filter and keep fluctuations between 8 and 25 years in order to capture medium-term business cycles (Comin and Gertler (2006)). Using the detrended series, I compute the correlation between filtered GDP and filtered terms-of-trade for two 20-years time

[47] The list of countries is: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Iceland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, Portugal, Spain, Sweden, the United-Kingdom and the United-States

windows from 1971 to 2010, hence creating a panel dataset where each country appears two times.

I use Price Cost Margin (PCM) as an estimate of markups within each industry. Introduced by Collins and Preston (1969) and widely used in the literature, PCM is the difference between revenue and variable cost, i.e. the sum of labor and material expenditures, over revenue:

$$PCM = \frac{\text{Sales} - \text{Labor expenditure} - \text{Material expenditure}}{\text{Sales}} \tag{1.24}$$

Data at the industry level come from the OECD STAN database, an unbalanced panel covering 107 sectors for 34 countries between 1970 and 2010. Due to missing data for many countries in the earliest years, I restrict the analysis for 22 countries.[48] I compute PCM for each industry-country-year and then construct an average of PCM within each country-year by taking the sales-weighted average of PCM over each industry. Finally, the average PCM for a given time window is simply the mean of country-year PCM over all time periods. Results are presented in table 1.6.

	dependent variable: corr($GDP_i^{filtered}$,$ToT_i^{filtered}$)			
	HP-filter	BK-filter	HP-filter	BK-filter
Average PCM	-1.507***	-2.049***	-2.650***	-3.705***
	(-2.70)	(-3.11)	(-2.87)	(-4.10)
Country FE	no	no	yes	yes
Time FE	no	no	yes	yes
N	43			

Note: The dependent variable is the correlation of filtered GDP with ToT. t stat. in parentheses, *** means $p < 0.01$

Table 1.6: Markups and GDP-ToT correlation

The first two columns of table 1.6 show the results of pooled cross-section analysis where I do not use the panel dimension of the dataset. In the last two columns, I perform fixed effect regression and add time dummies to control for time specific factors that might affect the

[48] For Germany, data are available only from 1991 onward (after the reunification), which is why the total number of observation in the regressions is 43.

correlation of GDP and terms-of-trade. In each of those cases, regressions are performed using the two filtering methods and are found to be statistically significant, implying that countries with higher markups also experience a larger decrease in their GDP when the relative price of their import rises.

1.7.3 Trade and TFP comovement

The model predicts that in the presence of markups and extensive margin adjustment, a country's TFP is impacted by foreign shocks even when technology is fixed. As a result, trade proximity across countries should be positively related to TFP correlation. I test this prediction using 18 OECD countries. Computing the correlation of all pairwise filtered TFP wihtin four 10-years time window ranging from 1969Q1 to 2008Q4, I estimate the following equations:

$$(1) \quad \text{corr}(TFP_{it}^{filtered}, TFP_{jt}^{filtered}) = \alpha_1 + \beta_T \log(\text{Total}_{ijt}) + controls + \epsilon_{1,ijt}$$

$$(2) \quad \text{corr}(TFP_{it}^{filtered}, TFP_{jt}^{filtered}) = \alpha_2 + \beta_I \log(\text{Intermediate}_{ijt}) + \beta_F \log(\text{Final}_{ijt}) + controls + \epsilon_{2,ijt}$$

Results are presented in table 1.7 for the HP-filtered TFP, capturing the business cycle fluctuations and in table 1.8 for the BK-filtered TFP capturing medium run cycles. When using HP filter, total trade is positively associated with TFP correlation, with trade in intermediate input capturing all the statistical significance in columns (2) and (4) while neither trade in intermediate nor final good is found significant in column (6). The picture is clearer when studying the medium term fluctuation, as can be seen in table 1.8: trade in intermediate input captures all the statistical significance in columns (2), (4) and (6), leaving final good trade with no explanatory power. Overall, this analysis is more nuanced that when studying the relationship between trade and GDP comovement. Nevertheless, it suggests that international trade is linked to TFP synchronization across countries as predicted by the theory.

	dependent variable: corr(TFP_i^{HP},TFP_j^{HP})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.092***		0.272***		0.099**	
	(9.12)		(11.05)		(2.78)	
log(Intermediate)		0.99***		0.205***		0.049
		(4.47)		(7.53)		(1.45)
log(Final)		-0.14		0.018		0.044
		(-0.56)		(0.44)		(1.11)
Country-Pair FE	no	no	yes	yes	yes	yes
Time Trend	no	no	no	no	yes	yes
R-squared (within)			0.185	0.194	0.245	0.244
R-squared (overall)	0.118	0.128	0.120	0.130	0.213	0.217
N	612					

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1.7: Relationship between Trade and HP filtered TFP correlation

	dependent variable: corr(TFP_i^{BK},TFP_j^{BK})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.091***		0.296***		0.079	
	(6.97)		(9.58)		(1.63)	
log(Intermediate)		0.133***		0.290***		0.126**
		(4.68)		(8.55)		(2.56)
log(Final)		-0.53*		-0.081		-0.054
		(-1.66)		(-1.48)		(-1.00)
Country-Pair FE	no	no	yes	yes	yes	yes
Time Trend	no	no	no	no	yes	yes
R-squared (within)			0.140	0.172	0.201	0.207
R-squared (overall)	0.072	0.089	0.074	0.091	0.161	0.155
N	612					

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1.8: Relationship between Trade and BK filtered TFP correlation

1.8 Conclusion

This paper analyzes the relationship between international trade and business cycle synchronization across countries. I start by refining previous empirical studies and show that higher trade in intermediate input is associated with an increase in GDP comovement, while trade in final good is found insignificant.

Motivated by this new fact, I propose a model of trade in intermediates with two key ingredients: (1) monopolistic pricing and (2) firm entry/exit. Both elements are necessary in order for foreign shocks to have a first order impact on domestic productivity through trade linkages. The propagation of technological shocks across countries depends on the worldwide network of input-output linkages, which emphasize the importance of going beyond two-country models to understand international GDP comovement.

I calibrate this model to 14 countries and assess its ability to replicate the empirical findings. Overall, the quantitative exercise suggests that the model is able to replicate more than 85% of the trade comovement slope, offering the first quantitative solution for the "Trade Comovement Puzzle". Decomposing the role of each ingredient, I show that trade in intermediates alone is not sufficient to replicate the trade-comovement relationship. The addition of monopolistic pricing and extensive margin adjustments increase the simulated trade-comovement slope by a factor seven.

1.9 Empirical Appendix

1.9.1 Data description

I focus the empirical analysis on 40 OECD countries and major emerging markets, which account for around 90% of world GDP. Trade data comes from Johnson and Noguera (2016) who constructed bilateral trade flows separated between final and intermediate goods for 42 countries between 1970 and 2009[49]. According to their data appendix A.2, here is the method used for data construction: for bilateral goods trade, they use the NBER-UN Database [http://cid.econ.ucdavis.edu] for 1970-2000 and the CEPII BACI Database [http://www.cepii.fr] for 1995-2009. This data is reported on a commodity-basis. They assign commodities to end uses and industries using existing correspondences from the World Bank [http://wits.worldbank.org]. To assign commodities to end uses, they use correspondences between SITC (Revision 2) 4-digit or HS (1996 Revision) 6-digit commodities and the BEC end use classifications. To assign commodities to industries, they use correspondences between SITC and HS categories and ISIC (Revision 2) industries. GDP data comes from the Penn World Tables version 9.0 (http://www.rug.nl/ggdc/productivity/pwt/).

In Johnson and Noguera (2016)'s data for Russia starts only in 1990 while data for Estonia, Slovak Republic, Slovenia and Czech Republic start only in 1993. All country-pairs involving one of those five countries appears only for times in the case of 10 years time-windows and cannot be used at all in the caseof 20 years time-windows. In total, I have 630 country-pairs appearing 4 times and 190 pairs appearing 2 times (both in the case of 10 years time windows), leading to a dataset with a total of 2900 observations.

1.9.2 Robustness Checks and other results

Changing the Dataset

As a robustness check, I also use the STAN Bilateral Trade Database by Industry and End-Use data (BTDIxE).[50] BTDIxE consists of values of imports and exports of goods, broken down by end-use categories. Estimates are expressed in nominal terms, in current US dollars for all OECD member countries. The trade flows are divided into capital goods, intermediate inputs and consumption. For the sake of comparison with the results in the main text, I first group the

[49] I drop Romania and South Africa from their sample because of lack of GDP series in the Penn World Tables

[50] See at http://www.oecd.org/trade/bilateraltradeingoodsbyindustryandend-usecategory.htm.

capital and intermediate goods together and create the index of trade proximity as explained in the main text. Due to data availability, I use the data from 1995 to 2014 which allows me to create four time windows of 5 years each (tables 1.9 and 1.10). With 20 countries, the dataset contains 190 pairs, for a total of 760 observations with four time windows. The tables below present the robustness results using both the HP filter (for business cycle frequencies) and then the Baxter and King filter (for medium term frequencies).

	dependent variable: corr(GDP_i^{HP},GDP_j^{HP})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.064***		-0.009		0.103	
	(5.94)		(-0.14)		(1.53)	
log(Intermediate)		0.044*		0.146*		0.209***
		(1.88)		(1.77)		(2.59)
log(Final)		0.021		-0.152*		-0.107
		(1.06)		(-2.04)		(-1.39)
Country-Pair FE	no	no	yes	yes	yes	yes
Time Trend	no	no	no	no	yes	yes
N	760					

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1.9: Trade and HP-Filtered GDP - STAN database (1995 to 2014)

	dependent variable: corr(GDP_i^{BK},GDP_j^{BK})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.075***		0.433***		0.397***	
	(5.23)		(3.86)		(3.16)	
log(Intermediate)		0.115***		0.562***		0.538***
		(3.71)		(3.71)		(3.60)
log(Final)		-0.036		-0.106		-0.122
		(-1.32)		(-0.76)		(-0.83)
Country-Pair FE	no	no	yes	yes	yes	yes
Time Trend	no	no	no	no	yes	yes
N	760					

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1.10: Trade and BK-Filtered GDP - STAN database (1995 to 2014)

Separating Intermediate goods from Capital goods

In the OECD STAN database, one can separate intermediate goods from capital goods. In table 1.11 I use this categorization and perform the same empirical exercise as above.

	dependent variable: corr(GDP_i^{HP},GDP_j^{HP})		
	(1)	(2)	(3)
log(Intermediate)	0.044	0.073	0.143*
	(1.47)	(0.89)	(1.74)
log(Capital)	0.004	0.114*	0.094
	(0.15)	(1.70)	(1.47)
log(Final)	0.018	-0.18**	-0.129
	(0.84)	(-2.36)	(-1.62)
Country-Pair FE	no	yes	yes
Time Trend	no	no	yes
R-squared (within)		0.011	0.304
R-squared (overall)	0.044	0.000	0.391
N		760	

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1.11: Trade and HP-Filtered GDP - STAN database (1995 to 2014), further disaggregation

	dependent variable: corr(GDP_i^{BK},GDP_j^{BK})		
	(1)	(2)	(3)
log(Intermediate)	0.024	0.449***	0.420***
	(0.62)	(3.05)	(2.95)
log(Capital)	0.112***	0.150	0.158
	(2.95)	(1.37)	(1.42)
log(Final)	-0.053*	-0.132	-0.153
	(-1.88)	(-0.96)	(-1.04)
Country-Pair FE	no	yes	yes
Time Trend	no	no	yes
R-squared (within)		0.057	0.059
R-squared (overall)	0.045	0.042	0.044
N	——— 760 ———		

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1.12: Trade and BK-Filtered GDP - STAN database (1995 to 2014), further disaggregation

1.10 Theoretical Appendix

1.10.1 Equilibrium Conditions in the general CES case

Price Indexes and Pricing System

$$\mathcal{P}_{k,k'} = \left(\int\limits_{\Omega_{k,k'}} p_{k,k'}(\varphi)^{1-\sigma} g(\varphi) d\varphi \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad IP_k = \left(\sum_{k'=1,\ldots,N} \omega_k(k') \mathcal{P}_{k',k}^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

$$PB_k = \chi_k^{-\chi_k} \times \eta_k^{-\eta_k} \times (1 - \eta_k - \chi_k)^{(\eta_k + \chi_k - 1)} \times IP_k^{1-\eta_k-\chi_k} \times w_k^{\chi_k} \times r_k^{\eta_k}$$

Using the optimal pricing strategy $p_{k,k'} = \tau_{k,k'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k \varphi}$ with the definition of the price index relative to each country specific bundle, we have the pricing system:

$$\mathcal{P}_k^{1-\rho} = \mu_k \left(\sum_{k'} \omega_k(k') \left(\tau_{k'k} \left(\frac{\varphi_{k',k}}{\varphi_{k',k'}} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \mathcal{P}_{k'} \right)^{1-\rho} \right)^{1-\eta_k-\chi_k} \quad , \ k = 1, ..., N$$

with $\mu_k^{\frac{1-\sigma}{1-\rho}} = \frac{\gamma \varphi_{k,k}^{\sigma-\gamma-1}}{\gamma-(\sigma-1)} M_k \left(\frac{\sigma}{\sigma-1} \frac{w_k^{\chi_k} \times r_k^{\eta_k}}{\chi_k^{\chi_k} \times \eta_k^{\eta_k} \times (1-\eta_k-\chi_k)^{1-\eta_k-\chi_k}} \frac{1}{Z_k} \right)^{1-\sigma}$.

Entry Thresholds

In very market, entry occurs until the profit of the least productive firms is equal to the fixed cost of accessing the market. Denoting by X_k total final good spending by consumers ($X_k = P_k(C_k + I_k) = w_k L_k + r_k K_k + \Pi_k$), we get

- At Home

$$\pi_{k,k}(\varphi_{k,k}) = f_{k,k} \frac{PB_k}{Z_k}$$

$$\Leftrightarrow \quad \varphi_{k,k} = \left(\frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_k} \right) \times \left(\frac{\sigma f_{k,k} \frac{PB_k}{Z_k}}{X_k + \left(\frac{\mathcal{P}_k}{IP_k} \right)^{1-\rho} \omega_k(k)(1-\eta_k-\chi_k) S_k} \right)^{\frac{1}{\sigma-1}}$$

- Abroad

$$\pi_{k,k'}(\varphi_{k,k'}) = f_{k,k'} \frac{PB_k}{Z_k}$$

$$\Leftrightarrow \quad \varphi_{k,k'} = \left(\tau_{kk'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_{k,k'}} \right) \times \left(\frac{\sigma f_{k,k} \frac{PB_k}{Z_k}}{\left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}} \right)^{1-\rho} \omega_{k'}(k)(1-\eta_k-\chi_k) S_{k'}} \right)^{\frac{1}{\sigma-1}}$$

Replacing $\mathcal{P}_{k,k'}$ by its expression using $\mathcal{P}_k$, we also get

$$\varphi_{k,k'}^{1+\frac{(\gamma-(\sigma-1)).(\sigma-\rho)}{(\sigma-1)^2}} = \left(\tau_{kk'}^{\frac{\rho-1}{\sigma-1}} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k} IP_{k'}^{\frac{1-\rho}{\sigma-1}} \left(\varphi_{k,k}^{\frac{\sigma-\gamma-1}{\sigma-1}} . \mathcal{P}_k \right)^{\frac{\rho-\sigma}{\sigma-1}} \right) \times \left(\frac{\sigma f_{k,k'} \frac{PB_k}{Z_k}}{\omega_{k'}(k)(1-\eta_k-\chi_k) S_{k'}} \right)^{\frac{1}{\sigma-1}}$$

Spending System

Total revenue of all firms from country k can be written as

$$R_k = X_k + \left[\sum_{k'} \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}} \right)^{1-\rho} \omega_{k'}(k)(1 - \eta_k - \chi_k) S_{k'} \right]$$

Free entry insures that variable profits are exactly equal, on aggregate, to fixed costs and entry costs payment, implying that $R_k = S_k$. Capital and labor demand impose $r_k K_k + w_k L_k = (\eta_k + \chi_k) S_k + \Pi_k$. Finally, the spending system can be simply written as

$$\left\{ \underbrace{\left(\mathcal{I}_N - (W^T \circ P)\right)}_{=M} \begin{pmatrix} S_1 \\ \vdots \\ S_N \end{pmatrix} = 0_{\mathbb{R}^N} \right.$$

where W the weighting matrix defined as $W_{ij} = \omega_i(j)$, P a matrix defined by $P_{ij} = \left(\frac{\mathcal{P}_{i,j}}{IP_i}\right)^{1-\rho}$ and $\circ$ is the element-wise (Hadamard) product. One can easily show that the matrix M is non invertible[51] and is of rank exactly $N - 1$, meaning that the solutions of the system is a one dimensional space. This is reassuring because it means we can normalize one price to one. I then normalize $w_1 = 1$ and with the labor demand equation this results is

$$S_1 = \frac{L_1}{\chi_1}$$

Labor and Capital Market Equilibrium

Using the labor supply equation, L_k is simply

$$L_k^{\nu} = \frac{1}{\psi_k} \frac{w_k}{\mathcal{P}_k}$$

Equipped with S_k the total spending of all firms in k, wages w_k and rental rate r_k are defined simply by

$$w_k = \chi_k \frac{S_k}{L_k} \qquad \text{and} \qquad r_k = \eta_k \frac{S_k}{K_k}$$

[51] One can easily see that summing all rows results in a column of zero.

Free Entry

At each date, firms enter the model until total profits are equal to total sunk cost payment:

$$\Pi_k = M_k \frac{PB_k}{Z_k}.f_{E,k} \quad \text{for all } k$$

1.10.2 Proof of Lemma 1

Reminder of Lemma 1 : Total profit in country k are proportional to total revenues:

$$\Pi_k = \frac{\sigma - 1}{\gamma\sigma} R_k$$

Proof

First, since firms charge a constant markup $\sigma/(\sigma-1)$ over marginal cost, we know that variable profits are a fraction $1/\sigma$ of total revenues. Hence, total profits net of fixed costs for all firms in k are simply

$$\Pi_k = \frac{R_k}{\sigma} - \sum_{k'} FC_{k\to k'}$$

where $FC_{k\to k'}$ is the sum of fixed cost payment from all firms from country k serving market k'. Then, note that total fixed cost payment for all firms in country k is

$$\begin{aligned} FC_{k\to k'} &= M_k \int_{\varphi_{k,k'}}^{+\infty} f_{kk'} \times \frac{PB_k}{Z_k} \times \gamma\varphi^{-\gamma-1} \times d\varphi \\ &= M_k f_{kk'} \frac{PB_k}{Z_k} \times \varphi_{k,k'}^{-\gamma} \end{aligned}$$

- If $k \neq k'$, we can also express total revenues (sales) from k to k' as

$$\begin{aligned} R_{k,k'} &= M_k \int_{\varphi_{k,k'}}^{+\infty} \left(\tau_{kk'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_{k,k'}} \right)^{1-\sigma} \times \omega_{k'}(k) S_{k'} \varphi^{\sigma-1} g(\varphi) d\varphi \\ &= \frac{\gamma M_k}{\gamma - (\sigma-1)} \times \left(\tau_{kk'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_{k,k'}} \right)^{1-\sigma} \times \omega_{k'}(k) S_{k'} \varphi_{k,k'}^{\sigma-\gamma-1} \end{aligned}$$

  Next, using the expression for the threshold $\varphi_{k,k'}^{\sigma-1}$ derived above (as a function of $\mathcal{P}_{k,k'}$), we get

$$R_{k,k'} = \frac{\gamma M_k}{\gamma - (\sigma-1)} \times \sigma f_{k,k'} \frac{PB_k}{Z_k} \varphi_{k,k'}^{-\gamma}$$

And we recognize finally that

$$R_{k,k'} = \frac{\gamma}{\gamma - (\sigma - 1)} \times \sigma FC_{k \to k'}$$

- For domestic revenues, we can show using the same steps that

$$X_k + R_{k,k} = \frac{\gamma}{\gamma - (\sigma - 1)} \times \sigma FC_{k \to k}$$

Combining those expressions, we get

$$\begin{aligned} \sum_{k'} FC_{k \to k'} &= \frac{\gamma - (\sigma - 1)}{\gamma \sigma} \times \left(X_k + \sum_{k'} R_{k,k'} \right) \\ &= \frac{\gamma - (\sigma - 1)}{\gamma \sigma} \times R_k \end{aligned}$$

Using this expression of $\sum_{k'} FC_{k \to k'}$ in the definition of profits completes the proof.

1.10.3 Proof of Lemma 2

Reminder of Lemma 2 : In the Cobb-Douglas ($\rho = 1$) and fixed labor supply case, the elasticity of every GNI with respect to a technology shock in country 1 is given by:

$$\begin{pmatrix} \eta_{GNI_1, Z_1} \\ \vdots \\ \eta_{GNI_N, Z_1} \end{pmatrix} = (\mathcal{I}_N - (1 - \eta_k - \chi_k) W - T)^{-1} \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix}$$

with W the weighting matrix defined above and T a "Transmission" matrix function of γ and σ.

Proof:

In this simplified case ($\rho = 1$ and fixed labor and capital supply), the labor and capital demand schedules $w_k L_k = \chi_k S_k$ and $r_k K_k = \eta_k S_k$ provide a one to one mapping between total spendings S_k and the wages w_k and the interest rate r_k. Moreover, inspecting the spending system (1.18) when $\rho = 1$ reveals that once a choice of numeraire is done (that is, taking $w_1 = 1$ and hence fixing $S_1 = L_1/\chi_1$), the vector of spendings $(S_i)_{i=1,\ldots,N}$ is independent of the technology level. Using lemma 1 and the fact that labor and capital supply are fixed, we can then show that total consumers' spending X_i also independent of technology level. Thus, since $GNI_k = X_k/\mathcal{P}_k$ the GNI elasticity is simply the opposite of the elasticity of the country's consumers price index. Moreover, with fixed labor supply and the assumption that the mass of potential entrepreneurs is proportional to labor size, the mass of firms M_i is fixed for every country i. In the next sections, I compute elasticities of all endogenous variables step by step until I can solve for the price index elasticities.

Model-based Price Indexes

Home Price Index at home $\mathcal{P}_k$

Using the definitions of price indexes, we can easily show that

$$\frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)} = -1 + \frac{\partial \log(PB_k)}{\partial \log(Z_k)} + \left(\frac{\gamma - (\sigma - 1)}{\sigma - 1}\right) \frac{\partial \log(\varphi_{k,k})}{\partial \log(Z_k)}$$

We can see in this formula the direct effect of lowering all prices in country k plus two other indirect effects : the propagation going through the input-output linkages in the PB_k term as well as the extensive margin of entry of new firms through the $\varphi_{k,k}$ term.

Foreign Price Index "at their home" $\mathcal{P}_{k'}$

$$\frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)} = \frac{\partial \log(PB_{k'})}{\partial \log(Z_k)} + \left(\frac{\gamma - (\sigma - 1)}{\sigma - 1}\right) \frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)}$$

The foreign price index "at their home" is not affected directly but only through the effects going through the input output linkages as well as the entry of new firms.

Export Price indexes $\mathcal{P}_{i,j}$

The price index relative to varieties from i sold on j's market is affected by the shock according to:

$$\frac{\partial \log(\mathcal{P}_{i,j})}{\partial \log(Z_k)} = \frac{\partial \log(\mathcal{P}_i)}{\partial \log(Z_k)} + \left(\frac{\gamma - (\sigma - 1)}{\sigma - 1}\right) \left(\frac{\partial \log(\varphi_{i,j})}{\partial \log(Z_k)} - \frac{\partial \log(\varphi_{i,i})}{\partial \log(Z_k)}\right)$$

We can see that the effect of a technology shock on exporting price indexes depends on the widening in the range of exported goods, as measured by the second term, in the brackets.

Input Bundle Price $PB_{k'}$ Abroad

Using the fact that wages are not affected by technology shocks, I can compute the elasticity of the input bundle price with respect to a technology shock at home as follow:

$$\frac{\partial \log(PB_{k'})}{\partial \log(Z_k)} = (1-\eta_k-\chi_k)\sum_j \omega_{k'}(j)\left[\frac{\partial \log(\mathcal{P}_j)}{\partial \log(Z_k)} + \left(\frac{\gamma-(\sigma-1)}{\sigma-1}\right)\left(\frac{\partial \log(\varphi_{j,k'})}{\partial \log(Z_k)} - \frac{\partial \log(\varphi_{j,j})}{\partial \log(Z_k)}\right)\right]$$

Thresholds

Home Entry Threshold $\varphi_{k,k}$ at Home

Using the definition of the thresholds from above and replacing $\frac{\partial \log(PB_k)}{\partial \log(Z_k)} - 1$ by its expression in the expression of the elasticity of the Home price index at home, we get

$$\frac{\partial \log(\varphi_{k,k})}{\partial \log(Z_k)} = \frac{1}{\sigma-1+\kappa\sigma} \times \frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)}$$

The scaling factor ($\frac{1}{\sigma-1+\kappa\sigma}$) is positive while the second term is negative, meaning that a positive technology shock trigers the entry of more firms in the country, which amplifies the effect of the shock.

Export Entry Threshold $\varphi_{k,k'}$ for Home firms exporting to k'

Using the second definition of the export thresholds from above, we get

$$(\frac{\gamma}{\sigma-1})\frac{\partial \log(\varphi_{k,k'})}{\partial \log(Z_k)} = (1+\frac{1}{\sigma-1}) \times \left(\frac{\partial \log(PB_k)}{\partial \log(Z_k)} - 1\right) + \kappa\frac{\partial \log(\varphi_{k,k})}{\partial \log(Z_k)} - \frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)}$$

Moreover, replacing $\frac{\partial \log(PB_k)}{\partial \log(Z_k)} - 1$ by its expression we get and using the fact that $1+\kappa = \frac{\gamma}{\sigma-1}$, we get

$$\frac{\partial \log(\varphi_{k,k'})}{\partial \log(Z_k)} = \frac{1}{\sigma-1+\kappa\sigma} \times \frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)}$$

Home Entry Threshold $\varphi_{k',k'}$ Abroad

Using the definition of the thresholds from above and replacing $\frac{\partial \log(PB_{k'})}{\partial \log(Z_k)}$ by its expression, we get

$$\frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)} = \frac{1}{\sigma-1+\kappa\sigma} \times \frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)}$$

Export Entry Threshold $\varphi_{k',j}$ for Foreign firms exporting to j

With the "second" definition of the threshold and using the expression of $\eta_{\varphi_{k',k'},Z_k}$, one can show that the elasticity of the exporting threshold is proportional to the elasticity of the domestic entry threshold and that the scaling factor do not depend on the export market considered:

$$\frac{\partial \log(\varphi_{k',j})}{\partial \log(Z_k)} = \frac{1+\kappa}{\frac{\gamma}{\sigma-1}} \times \frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)}$$

Finally, using the expression for $1+\kappa$, we get

$$\frac{\partial \log(\varphi_{k',j})}{\partial \log(Z_k)} = \frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)} = \frac{1}{\sigma - 1 + \kappa\sigma} \times \frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)}$$

Price indexes as constructed by statistical agencies

Using results above together with the definition of $\widehat{\mathcal{P}_k}$, we get:

$$\frac{\partial \log(\widehat{\mathcal{P}_{k'}})}{\partial \log(Z_k)} = \frac{\gamma - (\sigma-1)}{\sigma\gamma - (\sigma-1)} \frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)}$$

Final Expression

Now that I have an expression for the elasticity of all thresholds as functions of the elasticities of price indexes, I can gather the results. Introducing $\Lambda = \frac{1}{\sigma + \frac{(\sigma-1)^2}{\gamma-(\sigma-1)}}$, I define a matrix T (for Transmission) as $T = \text{diag}(\Lambda, ..., \Lambda)$. This matrix characterizes the additional propagation mechanism due to the change in the mass of firms in all markets. Then, the price index elasticities are defined by

$$\begin{pmatrix} \eta_{\mathcal{P}_1,Z_1} \\ \vdots \\ \eta_{\mathcal{P}_N,Z_1} \end{pmatrix} = (\mathcal{I}_N - (1-\eta-\chi)W - T)^{-1} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Finally, noting that for all i, $\eta_{\mathcal{P}_i,Z_1} = -\eta_{GNI_i,Z_1}$ concludes the demonstration.

In order to gain intuition on this formula, a few comments are in order. First, note that in the case of complete autarky of all countries we have $W = \mathcal{I}_N$ to that the elasticity for country 1 is simply $\eta_{GNI_1,Z_1} = 1/(\eta_1 + \chi_1 - \Lambda)$ whereas all other elasticities are zero. This result is reminiscent of what is found in Jones (2011) with the additional propagation mechanism due to the adjustment along the extensive margin captured by Λ. Interestingly, this special case highlights the fact that we need $(1-\eta-\chi)+\Lambda<1$ in order to get a positive own-country elasticity. This condition is necessary for the validity of (1.21), since it corresponds to imposing that the reason of the geometric sequence is below one.[52] Secondly, noting that $\Lambda=\frac{1}{\sigma+\frac{(\sigma-1)^2}{\gamma-(\sigma-1)}}$, one can see that for a fixed σ, $\Lambda(\gamma)$ is a strictly increasing function. When $\gamma\to\sigma-1$, $\Lambda\to 0$ and when $\gamma\to+\infty$, $\Lambda\to 1/\sigma$. For a labor and capital share so that $\eta+\chi=0.7$ we can see that any value $\sigma>1.5$ is sufficient to insure the validity of the condition $(1-\eta-\chi)+\Lambda<1$ for any value of γ within the range of admissible values ($\gamma>\sigma-1$).

[52]The formula (1.21) is the matrix analogue of summing an infinite geometric sequence. $(1-\eta-\chi)W+T$ corresponds to the first order effect of the shock, $((1-\eta-\chi)W+T)^2$ is the second order effect, etc... The total effect can then be described by the matrix $(\mathcal{I}_N-(1-\eta-\chi)W-T)^{-1}$ if and only if the eigenvalues of the matrix $(1-\eta-\chi)W+T$ all lie within the unit circle. In the autarcy case, this condition is insured by $(1-\eta-\chi)+\Lambda<1$.

Chapter 2

Propagation of Trade Disruption

By François de Soyres and Guillaume Sublet[1]

Abstract

We propose a model of international trade with heterogeneous firms and global value chains. Firms access new varieties when breaking into any foreign market and jointly choose where to import from and to export to. The unit production cost depends on the importing/exporting strategy of each firm as well the strategy of a firm's direct and indirect suppliers, giving rise to complementarities across firms decisions. In general, the model admits multiple equilibria. In this context, we study the consequences of trade disruption between two countries on global trade flows and show that aggregate trade flows can be complements rather than substitutes. As a result, any trade disruption between two specific countries propagates to all other trading partners through the network of input-output linkages. Hence, imposing sanctions to one country leads to a reduction in trade between all other countries.

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2.1 Introduction

The rise of global value chains is a dominating feature of recent evolutions in the structure of international trade. In the OECD, the import content of exports increased by 63% between 1995 and 2011, reaching a value of 24.3% on average.[2] In this paper, we argue that such an internationalization of production creates interdependence across firms and countries and yields important consequences on the way policy makers think about trade agreements and trade sanctions. In particular, we show that global value chains have the potential to create complementarity between trade flows in the sense so that imposing sanctions to one particular country can reduce trade between all existing country pairs in the world. On the other hand, when two countries sign a free trade agreement, the increase in efficiency in the worldwide production process has the potential to benefit all countries. As a consequence, important debates such as the extend of trade diversion upon signing free trade agreements might well be overdue. Finally, our analysis yields potentially important insights for consequences of future trade disruptions such as the exit of the United-Kingdom from the European Union (Brexit) or the renegotiation of NAFTA.

We start by studying theoretically the consequences of trade disruption between two countries on the reorganization of trade flows all around the world. In our setup, firms use labor and intermediate inputs to produce their goods, which is then used as both final good by consumers and intermediate input by other firms. Upon paying a fixed cost, firms decide whether to import part of their inputs and export their good to other countries which gives rise to endogenous global value chains. With love of variety in the production function as a way to model the benefits of accessing foreign inputs, the incentives to pay the fixed cost associated with importing from a specific market depend on (i) the number of firms (varieties) that this country is actually exporting and (ii) the productivity of those exporting firms, which in turn depends on their own decision to outsource part of their production. The complementarity between firms' payoff in different countries create a non separability in the decision of importing and exporting for each market. Different from models of exporting, firms in our setup cannot consider each foreign market separately but must heuristically rank all combination of importing and exporting strategies in order to make an optimal decision. Such a situation, with complementarity and non convex

[2] For some countries such as Luxembourg or Belgium, imports make up more than a third of their exports. See data https://data.oecd.org/trade/import-content-of-exports.htm

decisions, create a coordination problem which naturally gives rise to multiple equilibria.

In models of trade in final good, the consequences of one country leaving a multi-country free trade area is a reduction of welfare in all countries but an increase in trade flows between those countries staying in the free trade area. However, with input-output linkages between countries, new forces gives rise to two types of interdependencies which have the potential to create aggregate complementarity between trade flows. First, from the revenue side, firms losing access to an export market will pass on this negative demand shock to their own suppliers, reducing the incentive for further international trade. Moreover, from the supply side, some firms losing access to their supplier have to use imperfect substitutes and might experience an increase in their marginal cost. Such a negative shock on the efficiency of production for some firms is transmitted to other firms through input linkages and reduces the overall efficiency of the economy. Ultimately, those two forces, through revenue and marginal cost, have to be compared to standard economic mechanism associated with trade flow substitutions such has demand diversion and the effect through the competition level in each market. Quantitatively, we show that a reasonable calibration of our model leads to aggregate complementarity of trade flows and highlights the importance of the worldwide network of input-output linkages when forecasting the consequences of trade disruption and trade agreements.

In the last part of this paper, we provide suggestive evidence for microfoundation of our theoretical framework as well as support for the model's prediction.

The rest of the paper is organized as follows: the second section presents the theoretical framework and discusses the equilibrium structure; the third section presents a numerical exercise, emphasizing the aggregate complementarity of trade flows and the consequences for the propagation of trade disruption or trade agreement. Section four presents preliminary empirical results while section five concludes.

2.2 Model

We consider an environment with three countries indexed by k with respective size L_k, and Aggregate Productivity level Z_k, $k \in \{A, B, C\}$.

2.2.1 Basics of the model

Production

In any country k, production is performed by a continuum of heterogeneous firms combining in a Cobb-Douglas fashion labor ℓ_k and intermediate inputs M_k bought from other firms. Firms' productivity is the product of an idiosyncratic part φ and a country specific part Z_k. The production function for a firm with productivity φ in country k writes:

$$Y_k(\varphi) = Z_k \times \varphi \times \ell_k^{\alpha}(\varphi) \times M_k^{1-\alpha}(\varphi)$$

where $M_k(\varphi)$ is a CES aggregate of all *intermediate goods* which is defined as

$$M_k(\varphi) = \left(\int_{j \in \mathcal{U}_{k,\varphi}} m_{k,\varphi j}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}$$

In the expression above, $m_{k,\varphi j}$ is the quantity of input sold by firm j to a firm with productivity φ located in country k and $\mathcal{U}_{k,\varphi}$ is the endogenous set of all firms that are *upstream* to firm φ. We describe this set more precisely below when we define the trade structure. Finally, firms draw their productivity φ from a Pareto distribution with shape parameter γ_k and pdf $g(\varphi) = \gamma_k \varphi^{-\gamma_k - 1}$.

Consumers

In every country, a representative consumer ranks different consumption bundles according to the utility function:

$$U_k = q_0^{1-\beta} \times \left(\int_{\Omega_k} q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1} \times \beta}$$

where Ω_k denotes the endogenous set of all varieties available for consumption in country k. As in Helpman, Melitz and Yeaple (2004), utility is defined over all varieties created by monopolistic firms as well as an homogeneous good q_0 which is freely traded and produced competitively with one unit of labor producing exactly Z_k units of the good. β is the share of spending attributed to differentiated goods (and $1 - \beta$ to the homogeneous good). As is usual in this framework, we assume that q_0 is freely traded and consider only equilibria in which this good is produced in all countries, which implies that wages are fixed at all time in all countries and are given by $w_k = Z_k$.

Trade Structure

As in Chaney (2008), Arkolakis, Demidova, Klenow and Rodriguez-Clare (2008) or Ossa (2011) we assume a fixed mass of firms in each country, proportional to the mass of consumers. In order to be allowed to sell its variety to any country j (including their own country), a firm from country i must pay a fixed cost f_{ij} in unit of the firm's "production bundle"[3] as well as a variable (iceberg) cost τ_{ij} per unit sold. Importantly, we assume that once the fixed cost associated with j is paid, a firm becomes *both* an exporter to and an importer from country j. Hence, paying the fixed entry cost to a given market entails not only the benefit of accessing new consumers, but also new suppliers, hence reducing the firm's unit production cost.[4]

Based on their expected profit, firms choose which countries they enter (if any), affecting both the level of competition and the marginal cost of all firms in the country. It is easy to show that profits are strictly increasing with productivity φ so that export decision in equilibrium is defined by a simple country-pair specific threshold $\varphi_{k,k'}$ above which firms from k find it profitable to pay the fix cost $f_{kk'}$ and serve country k'. However, because the decision to become exporter/importer also affects a firm's unit production cost, it also *benefits* on other firms along the global value chain, which opens the room for a coordination issue and hence to multiple equilibria. We will discuss in detail the interdependence in individual firms' decision and the appearance of payoff complementarity when we describe the equilibrium.

Finally, the assumption that the same fixed cost $f_{k,k'}$ allows firms to become both exporters and importers leads to a simple definition of the sets of upstream suppliers $\mathcal{U}_{k,\varphi}$ and downstream customers $\mathcal{D}_{k,\varphi}$. All producing firms, regardless of their international strategy, use every other firm in their home country as a supplier. Moreover, if we denote by $\Omega_{k,k'}$ the set of firms from k that export to and import from k', we can simply express the set $\mathcal{U}_{k,\varphi}$ and $\mathcal{D}_{k,\varphi}$ as a function of the exporting strategy $s(\varphi)$:

$$\mathcal{U}_{k,\varphi} = \mathcal{D}_{k,\varphi} = \times_{k' \in s(\varphi)} \Omega_{k,k'} \tag{2.1}$$

[3]The production bundle of any firm is the bundle of all its suppliers and hence depends on its importing strategy.

[4]Section four provides empirical justification for this simplifying assumption.

Finally, the threshold structure of the equilibrium leads to simple definitions of those sets and we have $\Omega_{k,k'} = \big[\varphi_{k,k'}, +\infty\big)$.

2.2.2 Equilibrium

In this section, we present the key conditions that characterize the equilibria of the model. Generically, the model admits multiple equilibria. This is due to the positive price and revenue interdependence that firms exert on their trading partner when they decide to enter a foreign market. We discuss this point in detail below, but the key intuition goes as follow: when a firm i decides to pay the fixed cost associated with the access to a foreign market, their are two ways in which this decision actually *benefits* other firms in the economy. First, by accessing new varieties, firm i reduces its unit production cost. This, in turn, reduces the unit production cost of all firms that are buying firm i's variety as an input, directly or through other suppliers. Secondly, when accessing a new market, firm i reaches out to new customers. This leads to an increase in firm i's demand for all its suppliers, which triggers entry of new firms in the economy which translates into new inputs. Both effects - through *marginal cost* and *demand* - reflect the strong interdependency across firms that arise in a world of input-output linkages. One can easily see that several equilibria can coexist in these conditions: either only few firms access foreign markets and the average unit production cost is high while average demand is low, or many firms access foreign markets and unit production costs are low while overall demand is high. Both situations could be equilibria in the sense that no firm has an individual incentive to deviate from the allocation.

Let us denote X_k total consumers' revenue in country k, which is the sum of labor payment and profits rebate ($X_k = w_k L_k + \Pi_k$). Consumers spend a fraction β of this spending on differentiated good so that **consumer's demand** for variety i is:

$$q_i = \left(\frac{p_{i,k}}{\mathcal{P}_k}\right)^{-\sigma} \frac{\beta X_k}{\mathcal{P}_k} \quad \text{with } \mathcal{P}_k = \left(\int_{s\in\Omega_k} p_{s,k}^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}}$$

Moreover, firm i's **demand for intermediates** j is given by m_{ij}:

$$m_{ij} = \left(\frac{p_{j,i}}{\mathcal{P}_{M_i}}\right)^{-\sigma} M_i \quad \text{with } \mathcal{P}_{M_i} = \left(\int_{j\in\mathcal{U}_i} p_{j,i}^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}}$$

Finally, total demand faced by a firm from country k with productivity φ is simply

$$Y_k(\varphi) = \sum_{k'} \left(\frac{p_{k,k'}(\varphi)}{\mathcal{P}_k}\right)^{-\sigma} \frac{\beta X_k}{\mathcal{P}_k} + \int\limits_{j \in \mathcal{D}_{k,\varphi}} \left(\frac{p_{k,k'(j)}(\varphi)}{\mathcal{P}_{M_j}}\right)^{-\sigma} M_j dj \tag{2.2}$$

where $p_{k,k'}(\varphi)$ is the price charged by a firm from k with productivity φ when selling in country k' and the integral is taken over all firms j that are downstream to a firm from k with productivity φ.

Pricing strategy

Firms are monopolists within their variety. Classically, they choose their price at a constant markup over marginal cost and the markup depends on the price elasticity of demand. The marginal cost of any firm depends on the price of its input bundle $M_k(\Phi)$. The corresponding price index is noted $\mathcal{P}_{M_k}(\varphi)$ and is defined as $\mathcal{P}_{M_k}(\varphi) = \left(\int\limits_{j \in \mathcal{U}_{k,\varphi}} p_{j,k}(\varphi(j))^{1-\sigma} dj \right)^{1/(1-\sigma)}$. The marginal cost of a firm with productivity φ in country k is then

$$MC_k(\varphi) = \frac{1}{Z_k \varphi} \times \frac{w_k^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \times \mathcal{P}_{M_k}(\varphi)^{1-\alpha} \tag{2.3}$$

The optimal pricing strategy is finally given by:

$$p_{k,k'}(\varphi) = \frac{\mu_k}{Z_k \varphi} \mathcal{P}_{M_k}(\varphi)^{1-\alpha} \tag{2.4}$$

with μ_k a constant depending on the elasticity of substitution σ, the wage w_k and the labor share α.[5] Unlike in the canonical Krugman (1980) or Melitz (2003) models of international trade, one cannot solve for prices independently for each firm. Through $\mathcal{P}_{M_k}$, the price charged by firm φ in country k depends on the price charged by all its suppliers, which in turn depend on the prices charged by their suppliers and so on and so forth. The input-output linkages across firms creates a link between the pricing strategies of all firms and one needs to solve for all those prices at the same time.

Let us denote by s an international strategy, with $s \in \mathbb{P}(\{A, B, C\})$ where $\mathbb{P}(\{S\})$ is the

[5]

$$\mu_k = \frac{\sigma}{\sigma - 1} \frac{w_k^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$$

set of all subset of S including S itself. Each strategy s is associated with a specific set of suppliers and hence with a specific price index dual to the bundle aggregate $M_{k,s}$. Solving for unit production cost and prices requires solving jointly for all those strategy-dependent price indexes in all countries. In turn, this requires the imposition of an equilibrium structure because the pricing system depends on the *pecking order* that firms consider when defining their strategy.

The equilibrium we consider obeys the following structure. In A, firms consider first exporting to B and then C so that they will compare and rank strategies $s \in \{A, AB, ABC\}$. In B, firms consider first exporting to A and then C and will rank strategies $s \in \{B, AB, ABC\}$. Finally, in C, firms consider first exporting to B and then A so that the set of international strategies they consider will be $s \in \{C, BC, ABC\}$. Such a structure corresponds to an equilibrium if and only if no firms has an incentive to deviate. In particular, we will have to insure that given the equilibrium structure, no firm in A has an incentive to deviate and consider the strategy $s = \{AC\}$, and equivalently for deviations in B and C.

Note that the incentive for a firm in k to deviate from the equilibrium structure depends on the number and on the productivity of potential input suppliers the firm can access in other countries and hence on the decision of other firms. Again, this complementarity in payoff coupled with discrete nature of firms' choice leads naturally to multiplicity. Indeed, we show in the next section that for some regions of the parameter space, multiple equilibria coexist and we show how one can construct several equilibrium by imposing the structure and pecking order of firms within each country.

let (P) be the system associated with this problem, which can be found in appendix. In order to gain intuition, let us simply write the sub-system (P_A) consisting of the definition of the three price indexes associated with the export strategies in A, namely (A), (AB) and (ABC):

$$
\begin{cases}
\mathcal{P}_{M_{A,A}}{}^{1-\sigma} = & \mathcal{M}_A Z_A^{\sigma-1} \times \left[\int\limits_{\varphi_{A,A}}^{\varphi_{A,AB}} \left(\frac{\mu_A}{\varphi} \mathcal{P}_{M_{A,A}}{}^{1-\alpha} \right)^{1-\sigma} + \int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} \left(\frac{\mu_A}{\varphi} \mathcal{P}_{M_{A,AB}}{}^{1-\alpha} \right)^{1-\sigma} + \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\frac{\mu_A}{\varphi} \mathcal{P}_{M_{A,ABC}}{}^{1-\alpha} \right)^{1-\sigma} \right] \\
\mathcal{P}_{M_{A,AB}}{}^{1-\sigma} = & \mathcal{P}_{M_{A,A}}{}^{1-\sigma} \\
& + \mathcal{M}_B Z_B^{\sigma-1} \times \left[\int\limits_{\varphi_{B,AB}}^{\varphi_{B,ABC}} \left(\tau_{BA} \frac{\mu_B}{\varphi} \mathcal{P}_{M_{B,AB}}{}^{1-\alpha} \right)^{1-\sigma} + \int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\tau_{BA} \frac{\mu_B}{\varphi} \mathcal{P}_{M_{B,ABC}}{}^{1-\alpha} \right)^{1-\sigma} \right] \\
\mathcal{P}_{M_{A,ABC}}{}^{1-\sigma} = & \mathcal{P}_{M_{A,AB}}{}^{1-\sigma} \\
& + \mathcal{M}_C Z_C^{\sigma-1} \times \left[\int\limits_{\varphi_{C,ABC}}^{+\infty} \left(\tau_{CA} \frac{\mu_C}{\varphi} \mathcal{P}_{M_{C,ABC}}{}^{1-\alpha} \right)^{1-\sigma} \right]
\end{cases}
\tag{PA}
$$

Lemma The system (P) (grouping the three sub-systems (PA), (PB) and (PC)) admits a unique positive solution, and the price index associated with the unit cost of the input bundle decreases when the number of market served increases.

Proof inspired by Kennan (2001) for existence and unicity. The ordering of price indexes is simply obtained by forming the difference across equations.

Interdependence through prices

The system (PA) shows the interdependency that arises from input-output linkages across firms. For fixed international strategies (i.e. given thresholds $\varphi_{k,s}$), the price index governing the unit cost of the intermediate bundle for a firm with the international strategy s depends directly on the price chosen by its suppliers which in turn depends on the price of their suppliers and so on and so forth. Moreover, because of the love for variety in the CES aggregation, the allocation of firms into different export strategies also plays an important role in the determination of the price indexes. This creates a form of complementarity in the choice of international strategies: if many firms in A decide to export abroad, they will access new inputs and increase their measured productivity (decrease their unit cost). As a consequence, all their suppliers benefit from their choice, further enhancing the benefits of the reduction in marginal cost due to the round-about production structure in the economy.

Using equation (2.3), one can plot the measured productivity defined as the inverse of the unit production cost of every firm in the economy for given thresholds. In figure 2.1, we plot the measured productivity as function of a transformation of the idiosyncratic productivity draws for all firms in A. Calling $s = i$ the export strategy consisting in serving i different markets and $C = Z_k/\mu_k$, the graphs shows that the relationship is linear in $\varphi^{\sigma-1}$ with discrete jumps at each thresholds. It is the existence of those jumps at endogenous locations $\varphi_{k,s}$ that drives the coexistence of multiple equilibria.

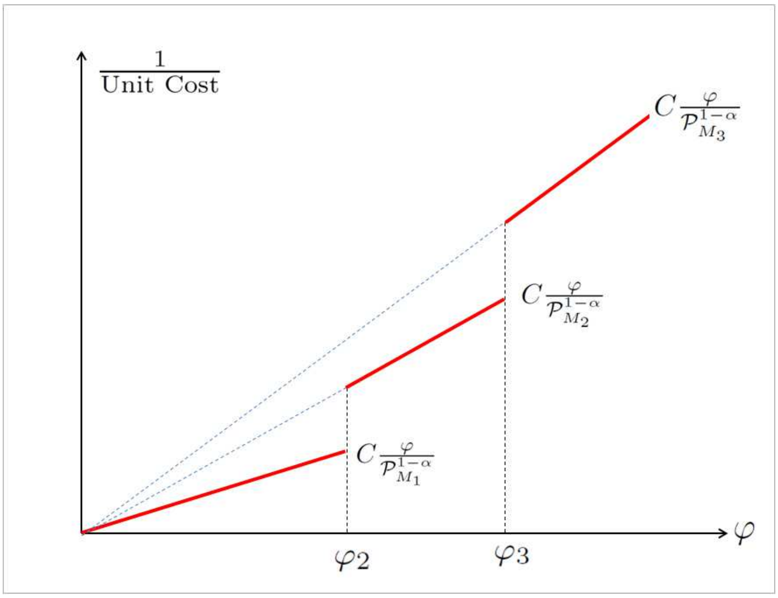

Figure 2.1: Unit Cost as a function of φ

Total Revenues and Profits

Let us call $r_{k,s}(\varphi)$ the variable revenues gross of fixed costs of a firm based in country k with productivity φ if it adopts the import/export strategy $s \in \mathbb{P}(\{A, B, C\})$. Those revenues are the sum of sales to consumers and firms in all markets within the import/export strategy s. Because revenues earned by one firm depends on the spending of its customers, all revenues must be solved jointly in all countries. The complete system describing revenues is given in appendix, and in what follows we describe the steps leading to its determination.

Recall that X_k represents total consumers' revenue in country k. Consumers spend a fraction β of this spending on differentiated good and the price index associated with competition in the final good market is the same as the one used by firms serving all three markets.[6] Hence, the revenues earned by a firm from k with export strategy s when selling to consumers in country k' is $\left(\frac{\tau_{k,k'}p_{k,s}(\varphi)}{\mathcal{P}_{M_{k',ABC}}}\right)^{1-\sigma}\beta X_{k'}$.

[6]Since international trade is done in *both* intermediate *and* final goods, consumers have access to all varieties that are exported to their country. This is not the case for firms, who can buy their inputs only from countries they are exporting to and for which they paid the associated fixed cost.

Aggregate profits

With a fixed mass of firms, firms make positive profits at the equilibrium. We assume those profits are redistributed to consumer from the same country. Calling R_k total revenues from all firms in country k and Π_k the corresponding aggregate profits, one can then prove that

$$\Pi_k = \frac{\sigma - 1}{\gamma\sigma} R_k \tag{2.5}$$

The proof of this formula can be found in Appendix and follow Eaton and Kortum (2005) as well as di Giovanni and Levchenko (2012).

We now turn to the determination of revenues made on the intermediate market. Cumulated demand from all firms in k' with strategy s' addressed to a firm from k with strategy s and productivity φ can be written:

$$\mathcal{M}_k \times \int_{\varphi_{k',s'}}^{\varphi_{k',s'+1}} \left(\frac{p_{k,s}(\varphi)}{\mathcal{P}_{M_{k',s'}}} \right)^{1-\sigma} \mathcal{P}_{M_{k',s'}} M_{k',s'}(u) g(u) du$$

where the meaning of $\varphi_{k',s'}$ and $\varphi_{k',s'+1}$ are natural. Using the fact that each firm charges a constant markup $\frac{\sigma}{\sigma-1}$ over marginal cost and spends a constant fraction $1-\alpha$ on intermediates inputs, it follows that total spending on intermediate inputs $\mathcal{P}_{M_{k,s}} M_{k,s}(\varphi)$ is a constant fraction of total sales. For every country k and export strategy s, we have

$$\mathcal{P}_{M_{k,s}} M_{k,s}(\varphi) = (1-\alpha)\frac{\sigma-1}{\sigma} r_{k,s}(\varphi)$$

Integrating this expression over all firms with strategy s in country k, we define the endogenous variable $\Lambda_{k,s}$ as

$$\Lambda_{k,s} = \int_{\varphi_{k,s}}^{\varphi_{k,s+1}} \mathcal{P}_{M_{k,s}} M_{k,s}(\varphi) g(\varphi) d\varphi = \mathcal{P}_{M_{k,s}}^{\sigma-1} (1-\alpha) \frac{\sigma-1}{\sigma} \int_{\varphi_{k,s}}^{\varphi_{k,s+1}} r_{k,s}(\varphi) g(\varphi) d\varphi$$

$\Lambda_{k,s}$ measures the demand addressed by a firm in k with export strategy s to all its suppliers. It naturally depends on the pricing associated with the international strategy. We can now write the revenues of a firm from country k with strategy s: it consists of the sum of revenues made on all markets, with the revenues on any market k' being the sum of sales to final consumers and sales to firms that for which country k belong to their international strategy (meaning firms with international strategies $s' \mid k \in s'$). The system R_k describes the revenues for any firm in k as:

$$\left\{ r_{k,s}(\varphi) = \sum_{k' \in s} \left(\left(\tau_{k,k'} \frac{p_{k,s}}{\mathcal{P}_{k'}} \right)^{1-\sigma} \beta X_{k'} + \mathcal{M}_{k'} \times \sum_{s' \mid k \in s'} \left(\frac{\tau_{k,k'} p_{k,s}}{\mathcal{P}_{M_{k',s'}}} \right)^{1-\sigma} \Lambda_{k',s'} \right) \right. \quad , \forall s \quad \text{(R(k))}$$

where the prices $p_{k,s}(\varphi)$ can be expressed using (2.4). In order to find a solution for revenues, one must first solve for the endogenous vector of $\{\Lambda_{k,s}\}_{k \in \{A,B,C\}, s \in \mathbb{P}(\{A,B,C\})}$. This can simply be done by integrating the revenues above for all productivities φ within the international strategy s. To clarify notations, we first introduce $C_{k,s}$, for every country k and strategy s, as follow:

$$C_{k,s} = (1-\alpha) \frac{\sigma - 1}{\sigma} \left(\frac{\mu_k \mathcal{P}_{M_{k,s}}^{-\alpha}}{Z_k} \right)^{1-\sigma} \times \frac{\gamma_k}{\gamma_k - (\sigma - 1)} \left(\varphi_{k,s}^{\sigma - \gamma_k - 1} - \varphi_{k,s+1}^{\sigma - \gamma_k - 1} \right)$$

Then, the systems (Λk), for $k \in \{A, B, C\}$ defining the objects $\Lambda_{k,s}$ can be written

$$\left\{ \Lambda_{k,s} = C_{k,s} \times \sum_{k' \in s} \tau_{k,k'}^{1-\sigma} \left[\beta X_{k'} \mathcal{P}_{M_{k',ABC}}^{\sigma-1} + \mathcal{M}_{k'} \times \sum_{s' \mid k \in s'} \Lambda_{k',s'} \right] \right. \quad , \forall s \qquad (\Lambda(\text{k}))$$

The system Λ, grouping (Λ(k)) for $k \in \{A, B, C\}$ can be found in appendix. It is a simple linear system that can be solved for. With the values for all Λs, one can use the (R(k)) systems to compute the revenues of all firms in the economy, for given thresholds $\varphi_{k,s}$.

Interdependence through revenues

When a firm decides to be active in a foreign country, it increases its measured productivity by gaining access to new inputs. This translates into higher sales in all markets (including the domestic one), which in turn increase the demand that the firm addresses to its suppliers. When solving for prices, we saw that the price of a firm impacts the price of its customers, which in turns affect its customer's customers etc. The mechanism at play for the revenues works in the other direction, but due to the round-about production structure of the economy, this propagate in the entire economy. Both mechanisms (through price and revenue) lead to a form of complementarity in firms' payoffs, which stems from their vertical integration. This complementarity, coupled with the non convex decisions to import and export opens the door for coordination issues and multiplicity of equilibria. Intuitively, if many firms from A decide to export/import to B, then firms in B have access to many inputs. This pushes up their measured productivity, increases their sales in all countries and rises the demand they address to their suppliers, hence validating the existence of the equilibrium by making it incentive compatible for many firms in A to pay the fixed cost associated with access to country B. On the other hand, if many firms decide to export to C, the same mechanism will be at play between A and C and not between A and B. Hence, two equilibria can coexist at the same time: firms in A can consider importing/exporting first to B and then, for the highest φ, also C. Or they can consider the reverse order. In the next section, we numerically show that several equilibria can co-exist in some part of the parameter space.

Thresholds Definition

In standard trade models with many countries, firms consider markets independently from one another and decide to break into market by simply comparing the profit associated with entering the country with the corresponding fixed cost. In the model presented in this paper, however, one cannot use the same condition because accessing a new market has an impact on the pricing strategy and the revenues earned in all other markets. Hence, as in many discrete choice problems, one has to successively compare all possible international strategies in order to determine a firm's optimal choice. In a case with N possible import/export strategies, this means that the determination of a firm's optimal choice requires the bilateral comparison of all possible export strategies, a set of cardinal 2^N. This is a potentially strong limitation of the this model if one wants to apply it to a large number of countries. In the present work, we limit ourselves to a world with three countries in order to show the complementarity of trade flows and the propagation of trade disruption in international networks.

With constant markup over marginal cost, variable profits are simply a fraction $1/\sigma$ of variable revenues. Moreover, the fixed cost associated with access to the market place k' from country k is paid in unit of the production bundle available to each firm, which is international-strategy-dependent, so that fixed cost payment for a firm from k accessing k' and using strategy s is equal to $f_{k,k'} \times \frac{PB_{k,s}}{Z_k}$ where $PB_{k,s} = \frac{w_k^{\alpha} \cdot \mathcal{P}_{M_{k,s}}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$. Hence, the pay-off associated with the international strategy s is simply:

$$\pi_{k,s}(\varphi) = \frac{1}{\sigma} r_{k,s}(\varphi) - \frac{PB_{k,s}}{Z_k} \sum_{k' \in s} f_{k,k'} \tag{2.6}$$

Looking at the system (R(k)), one can see that variable revenues are linear functions of $\varphi^{\sigma-1}$ with the slope depending on the export strategy used. Hence, total profits are simply affine functions of $\varphi^{\sigma-1}$ that can be represented in a graph as in figure 2.2. Thresholds are then simply defined as the intersection between the lines representing the payoff associated with different strategies and the actual equilibrium payoff are simply the upper envelope of all lines in the graph. In country A, given the structure of the equilibrium that we assumed and the associated pecking order, thresholds are implicitly defined by the three equations:

$$\begin{cases} \frac{1}{\sigma} S_{A,A} \times \varphi_{A,A}^{\sigma-1} - \frac{PB_{A,A}}{Z_A} f_{AA} = 0 \\ \frac{1}{\sigma} S_{A,AB} \times \varphi_{A,AB}^{\sigma-1} - \frac{PB_{A,AB}}{Z_A} \left(f_{AA} + f_{AB}\right) = \frac{1}{\sigma} S_{A,A} \times \varphi_{A,AB}^{\sigma-1} - \frac{PB_{A,A}}{Z_A} f_{AA} \\ \frac{1}{\sigma} S_{A,ABC} \times \varphi_{A,ABC}^{\sigma-1} - \frac{PB_{A,ABC}}{Z_A} \left(f_{AA} + f_{AB} + f_{AC}\right) = \frac{1}{\sigma} S_{A,AB} \times \varphi_{A,ABC}^{\sigma-1} - \frac{PB_{A,AB}}{Z_A} \left(f_{AA} + f_{AB}\right) \end{cases} \tag{2.7}$$

with $S_{k,s}$ the "slope" associated with the revenues of a firm from country k with international strategy s. More precisely, for each of the strategy considered under the structure imposed, we have

$$S_{k,s} = \left(\frac{\mu_k \mathcal{P}_{M_{k,s}}^{1-\alpha}}{Z_k} \right)^{1-\sigma} \times \sum_{k' \in s} \tau_{k,k'}^{1-\sigma} \left[\frac{\beta X_{k'}}{\mathcal{P}_{M_{k',ABC}}^{1-\sigma}} + \sum_{s' | k \in s'} \mathcal{M}_{k'} \Lambda_{k',s'} \right]$$

Then, solving for thresholds in system (2.7) yields to:

$$\left\{ \varphi_{k,s}^{\sigma-1} = \left[\frac{PB_{k,s}}{Z_k} \left(\sum_{k' \in s} f_{k,k'} \right) - \frac{PB_{k,s-1}}{Z_k} \left(\sum_{k' \in s-1} f_{k,k'} \right) \right] \frac{\sigma}{(S_{k,s} - S_{k,s-1})} \right. \tag{2.8}$$

In order to provide better intuition, we depict the logic underlying the construction of such an equilibrium in figure 2.2. Productivity thresholds separating the strategies are at the intersection of strategy-specific lines. From both the revenue and price systems, it is obvious that slopes are increasing in the number of market reached.[7]

Verifying the Incentive Compatibility

When constructing the equilibrium, we assumed a specific pecking order of import/export mar-

[7]The reason why serving more markets is associated with a higher slope is twofold. First, accessing an other country is associated with a marginal cost decrease which yields a higher market share in any market served. Second, accessing a new market increases the number of firms downstream.

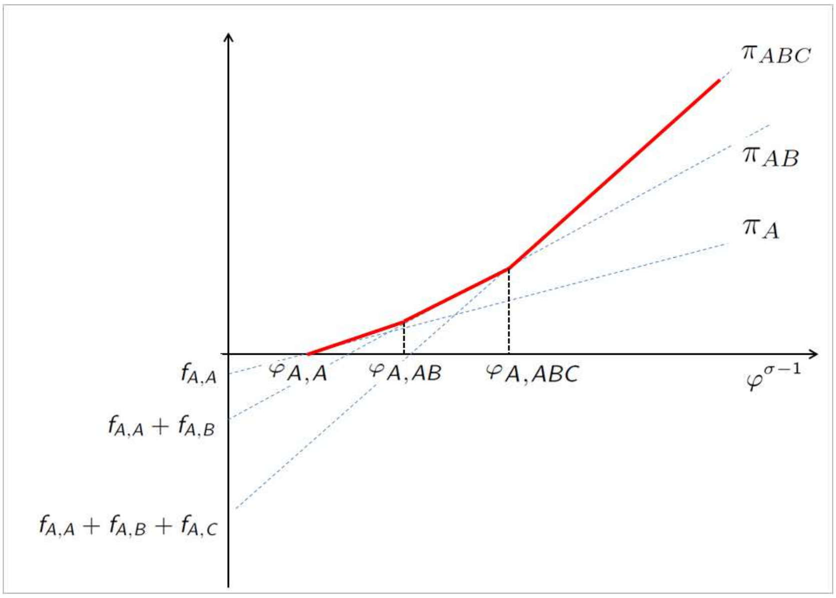

Figure 2.2: Thresholds Definition

kets in every country. In A, the ranking is $B > C$, in B the it is $A > C$ and finally in C the pecking order $B > A$. This structure yields the candidate equilibrium thresholds defined above, but it need not constitute an equilibrium since firms may still have an incentive to deviate. While the comparison of slopes insure that reverse pecking order is not possible, we still have a number of deviation to verify.

First, with the structure specified, it could be the case that no firm in A has an incentive to access country A and B only but are better of breaking into all markets A, B and C. In order to verify incentive compatibility along this deviation, one must insure that the numerical value found for thresholds respect the pecking order specified. If this is not the case, it means that the structure imposed does not constitute an equilibrium and the natural next step is to reformulate the structure with a pecking order in country A being $s \in \{A, ABC\}$. The same reasoning must then of course be applied to countries B and C individually.

Second, in country A, one must also make sure that no firm has an incentive to use the international strategy $s = \{AC\}$. Since the "productivity slope" associated with such a strategy can only be lower than the slope of strategy ABC, one must simply ensure that the firm with productivity $\varphi_{A,ABC}$ has no incentive to deviate, meaning that $\pi_{A,AB}(\varphi_{A,ABC}) > \pi_{A,AC}(\varphi_{A,ABC})$. In other words, one must verify that the π^{AC} locus is below the π^{AB} locus at the $\varphi_{A,ABC}$ point. Identically, it is necessary to also insure that the π^{BC} locus is below the π^{AB} locus at the $\varphi_{B,ABC}$

point ($\pi_{B,AB}(\varphi_{B,ABC}) > \pi_{B,BC}(\varphi_{B,ABC})$) and finally that the π^{AC} locus is below the π^{BC} locus at the $\varphi_{C,ABC}$ point ($\pi_{C,BC}(\varphi_{C,ABC}) > \pi_{C,AC}(\varphi_{C,ABC})$).

To this end, we first define the price $p_{A,AC}(\varphi)$ that a deviating firm in country A firm would charge:

$$p_{A,AC}(\varphi) = \frac{\mu_A}{Z_A}\mathcal{P}_{M_{A,AC}}^{1-\alpha}\frac{1}{\varphi}$$

where we have introduced the price index $\mathcal{P}_{M_{A,AC}}$ dual to the *off equilibrium* strategy $s = \{AC\}$:

$$\mathcal{P}_{M_{A,AC}}^{1-\sigma} = \mathcal{P}_{M_{A,A}}^{1-\sigma} + \lambda_C \tau_{CA}^{1-\sigma} \times \left[{\varphi_{C,ABC}}^{\sigma-\gamma_C-1} \mathcal{P}_{M_{C,ABC}}^{(1-\sigma).(1-\alpha)} \right]$$

Using these variables, the profit of a deviating firm with productivity $\varphi_{A,ABC}$ is defined by:

$$\begin{aligned}\pi_{A,AC}(\varphi_{A,ABC}) = \frac{1}{\sigma}\left(\frac{\mu_A \mathcal{P}_{M_{A,AC}}^{1-\alpha}}{Z_A}\right)^{1-\sigma} &\times \left[\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} + \mathcal{M}_A \times [\Lambda_{A,A} + \Lambda_{A,AB} + \Lambda_{A,ABC}]\right. \\ &\left. + \tau_{AC}^{1-\sigma}\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \tau_{AC}^{1-\sigma} \times \Lambda_{C,ABC}\right] \varphi_{A,ABC}^{\sigma-1} - f_{A,C}\end{aligned} \tag{2.9}$$

We can then numerically verify if indeed we have:

$$\pi_{A,AB}(\varphi_{A,ABC}) > \pi_{A,AC}(\varphi_{A,ABC}) \tag{2.10}$$

If this is the case, then no firm in A as any incentive to deviate from the equilibrium structure imposed *ex-ante*. Naturally, similar verifications must be also be done in country B and C as exposed in appendix.

This process of insuring incentive compatibility can be easily be represent in figure 2.3. When imposing our pecking order in country A, we ruled out the strategy AC. Using aggregate variables constructed at the equilibrium specified above, equation (2.10) is satisfied if and only if the line associated with strategy AC lies below the upper envelope of all lines represented in the graph.

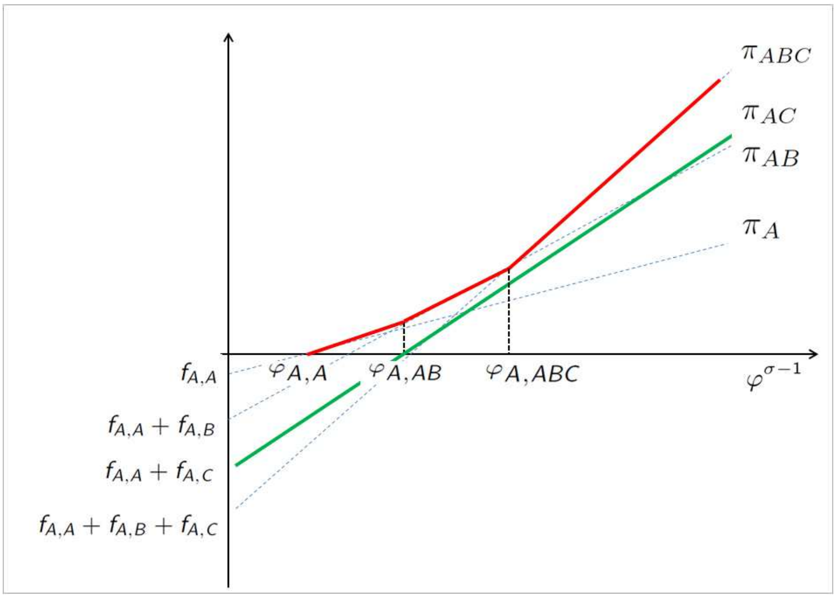

Figure 2.3: Verifying Individual Incentive Constraints

Defining export from A to B

Since the goal of this model is to analyze the extend to which trade flows can be gross *complements*, we define $X^{AB}_{A\to B}(\varphi)$ and $X^{ABC}_{A\to B}(\varphi)$ the sales realized by a firm from country A in the export market B when it is optimally choosing export strategy AB and ABC respectively.

$$\begin{aligned} X^{AB}_{A\to B}(\varphi) &= \left(\frac{\mu_A \mathcal{P}^{1-\alpha}_{M_{A,AB}}}{Z_A}\right)^{1-\sigma} \times \tau^{1-\sigma}_{AB}\left[\beta X_B \mathcal{P}^{\sigma-1}_{M_{B,ABC}} + \mathcal{M}_B \times (\Lambda_{B,AB}+\Lambda_{B,ABC})\right]\varphi^{\sigma-1} \\ X^{ABC}_{A\to B}(\varphi) &= \left(\frac{\mu_A \mathcal{P}^{1-\alpha}_{M_{A,ABC}}}{Z_A}\right)^{1-\sigma} \times \tau^{1-\sigma}_{AB}\left[\beta X_B \mathcal{P}^{\sigma-1}_{M_{B,ABC}} + \mathcal{M}_B \times (\Lambda_{B,AB}+\Lambda_{B,ABC})\right]\varphi^{\sigma-1} \end{aligned}$$

Then, total export from A to B can be expressed as

$$X_{A\to B} = \mathcal{M}_A \left(\int_{\varphi_{A,AB}}^{\varphi_{A,ABC}} X^{AB}_{A\to B}(\varphi) g(\varphi) d\varphi + \int_{\varphi_{A,ABC}}^{+\infty} X^{ABC}_{A\to B}(\varphi) g(\varphi) d\varphi \right) \tag{2.11}$$

2.3 Numerical Exercise

The goal of this section is to numerically illustrate the consequences of trade disruption on the reorganization of trade flows. In order to do so, we compute the equilibrium under the pre-specified structure and vary the trade barriers between two countries A and B. By following the endogenous behavior of trade flows with a third country C, we show that our model with input-output linkages across countries generate aggregate complementarities in trade flows.

We consider three countries of equal size and chose $L_A = L_B = L_C = 100$. Since wages are determined one-for-one by aggregate productivity, it is equivalent to chose a value for Z_k and w_k and we normalize all wages to one so that countries all share the same productivity and the same final demand.

Two parameters playing a decisive role in the emergence of trade flow complementarity are (i) the share of intermediate input in production and (ii) the elasticity of substitution across varieties which the key parameter determining the productivity advantage of breaking into an import market. We chose a share of intermediate input of 50% and hence a labor share of output of 50%,[8] implying a value of $\alpha = 0.5$. We calibrate the elasticity of substitution among domestic varieties (σ) to a value of 5, which is standard in the international literature and consistent with the elasticity estimates by Broda and Weinstein (2006), and implies a mark-up of 25%. Note that this number does not represent the profit rate of the economy because firms must pay fixed costs on top of their variable costs. Following equation (2.5), the profit rate of our economy is 17.4%.

Following Bernard et al. (2004) or Fattal-Jaef and Lopez (2014) we choose the Pareto shape parameter γ to be 5.6 which is equal to $\sigma - 0.4$. This choice allows the model to reproduce in the model the standard deviation of log sales of 1.67 in US plants, which is a typical target in models with heterogeneous producers. Finally, regarding trade barriers, we chose to normalize the own country barrier to one ($f_{kk} = 1$ for all k) and set the international fixed cost in the following way: $f_{AB} = f_{BA} = 3$ and $f_{AC} = f_{BC} = f_{CA} = f_{CB} = 4$. Variable trade costs are all set to 20% and except thosebetween A and B which are set to 10%. Recall that the goal of our exercise is to vary τ_{AB} and τ_{BA} and assess the consequences for the reorganization of trade flows across all countries.

Evolution of Exports when τ_{AB} varies

Our main numerical exercise consists in varying the variable trade barriers between A and B and to plot the consequences for exports from A to all countries. First, recall that in models

[8] note that this is not the labor share of value added. In this framework, value added is the sum of labor payment and profit, which represent respectively 50% and 17.4% of gross output (see equation (2.5) for the profit rate net of fixed costs).

Parameter	Value	Note
$L_A = L_B = L_C$	100	Country Size
$w_A = w_B = w_C$	1	One-to-one productivity in the outside good
α	0.5	Intermediate share of output
σ	5	Elasticity of substitution across varieties
γ	4.6	Pareto shape (Fattal-Jaef and Lopez (2014))
$f_{A,A} = f_{B,B} = f_{C,C}$	1	Own country entry cost
$f_{A,B} = f_{B,A}$	3	Fixed entry cost: $A - B$
$f_{A,C} = f_{B,C}$	4	Fixed entry cost: access to C
$f_{C,A} = f_{C,B}$	4	Fixed entry cost: out of C
$\tau_{A,C} = \tau_{B,C}$	1.2	Iceberg cost: access to C
$\tau_{C,A} = \tau_{C,B}$	1.2	Iceberg cost: out of C

of international trade in final goods, a decrease in τ_{AB} and τ_{BA} would not have any impact on firms' marginal cost and hence on pricing. Moreover, models that do not feature an import decisions have a form of separability that enables firms to consider market entry decision one by one, without having to rank different subset of export strategies. In our case, with input-output linkages across countries, this is not the case anymore and a disruption of trade with one trading partner can potentially lead to adverse consequences for trade flows with other partners.

In the graph below (see 2.4), we vary τ_{AB} and τ_{BA} between 1.1 to 1.2, hence modeling the consequences of a doubling of variable trade barriers between those two countries (from 10% to 20% of iceberg cost). We see that while such a change decreases trade flows between the two countries, there is also a negative impact on total exports from A to B. This form of propagation of the trade disruption is specific to models of intermediate input trade.

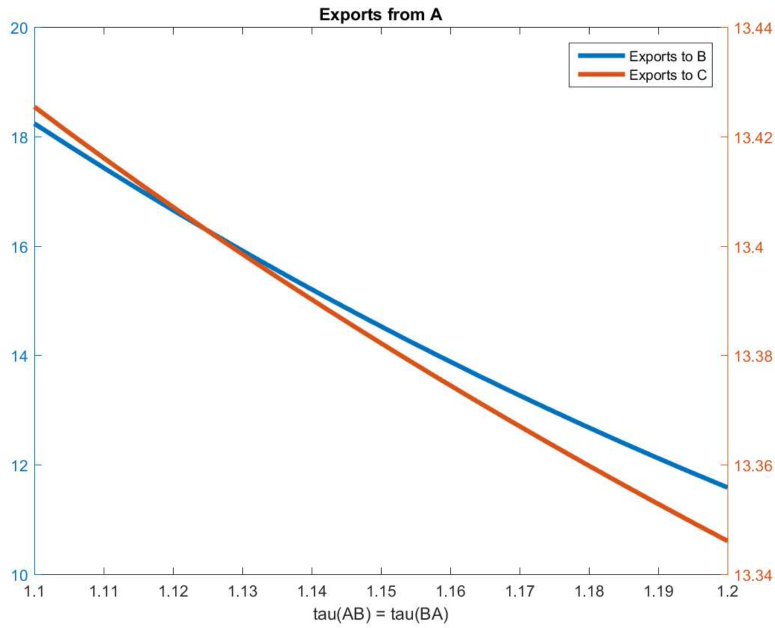

Figure 2.4: Disrupting Trade between A and B
Complementarity effect

Extensive Margin adjustment when τ_{AB} varies

Investigating further the consequences of trade disruption, we compute the percentage of firms from country A that endogenously decide to pay international fixed costs. Keeping track of the percentage of exporters and importers to both destination, we then plot in figure 2.5 the evolution of the fraction of exporters as a function of trade barriers between A and B, keeping everything else constant.

Interestingly, increasing the trade barriers between A and B not only decreases total exports from A to C but also decreases the *extensive margin* of trade, i.e. the number of firms in A that engage in trade with C. The fact that the complementarity observed between total trade flows is also present along the extensive margin provides a new prediction that we can then test in the data. It also provides support for models with an endogenous mass of produced and traded varieties in comparison with competitive ricardian models such as Eaton & Kortum (2002) that feature a *fixed* continuum of goods.

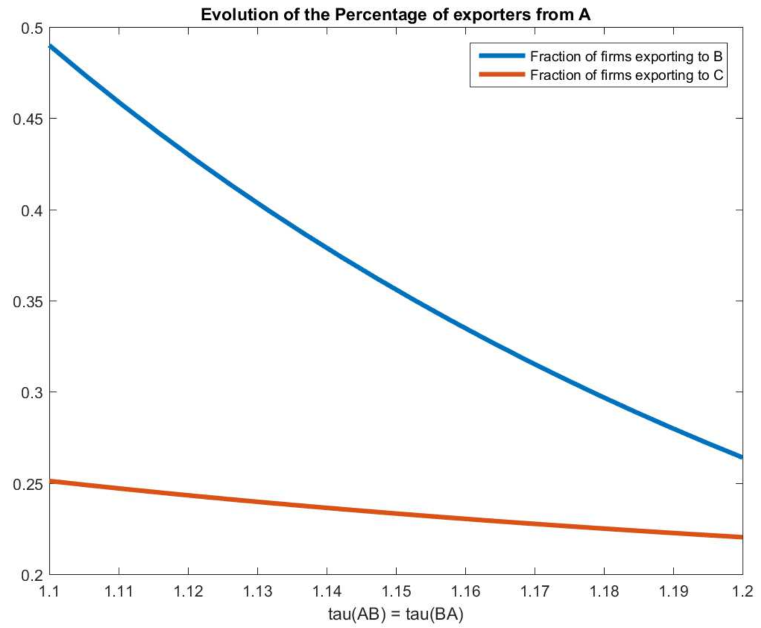

Figure 2.5: Disrupting Trade between A and B
Complementarity effect for the Extensive Margin

Equilibrium switching

As mentioned in section 2.2, the presence of a non-convex decision together with complementarity in firms' payoffs leads to the co-existence of multiple equilibrium. Firms' incentive to pay the fixed cost associated with international trade with another country depends on the number of foreign firms that themselves are engaged in trade (because these firms will represent both the set of new suppliers as well as the set of new customers in the intermediate good market) but it also depends on the efficiency of those firms as measured by their production cost. Hence, unlike in models of trade in final goods in which breaking into a new market always reduces the profit of competing firms, our model features a positive interdependence between firms that gives rise to a coordination issue.

In this section, we showed that for important changes in the fundamentals of the economy, the structure and the pecking order imposed can become unsustainable in equilibrium. In particular, when the trade costs between A and B are very large, then the pecking order specified does not constitute an equilibrium anymore because the incentive compatibility conditions are not satisfied anymore. In such a case, one has to specify a new equilibrium structure with a new pecking order that can be sustained given the trade costs between all country pairs. This is what we do in figure 2.6 where we can see that the pecking order is reversed for large values of iceberg costs between A and B.

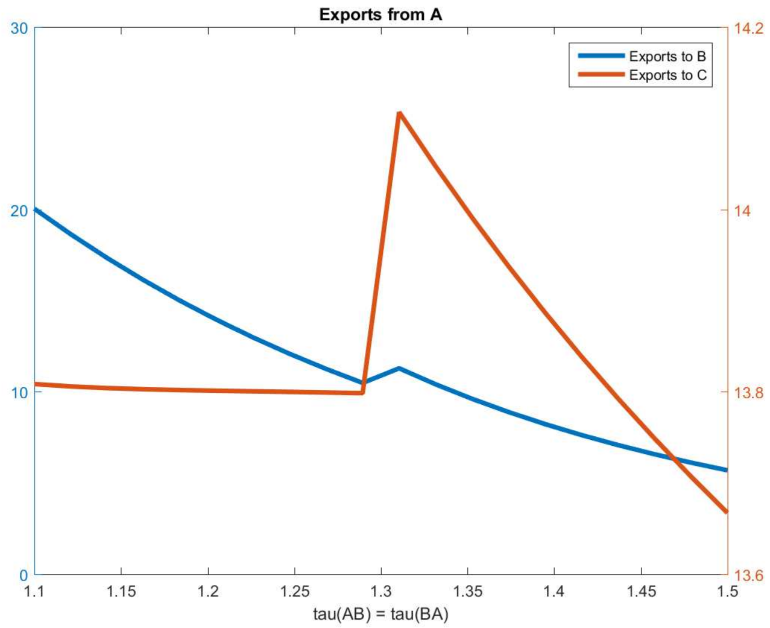

Figure 2.6: Large variations of τ_{AB} and τ_{BA}
Equilibrium Switching

2.4 Empirical support

The model we propose for the study of multi-lateral trade emphasizes trade in intermediates, lower production costs from access to a broader varieties of input, and extensive margins for both exports and imports. The combination of these three ingredients generates complementarities in exporting decisions and the complementarity in multi-lateral aggregate trade flows. In this section, we provide empirical motivation for our modeling choices as well as a reduced form estimate for the existence of trade diversion or trade creation.

First, we provide evidence for our modeling of the dependence of the intensive margins for exports on the intensive margin for imports. Second, we empirically estimate the degree of complementarities amongst multi-lateral trade flows. To that end, we conduct a Difference-in-Differences exercise which measures the association of the change in trade flows between NAFTA and non-NAFTA members with the change in trade barriers among NAFTA members. Recall, however, that our theory features two opposing forces on multi-lateral trade flows: the substitutability force due to trade diversion and increased competition, as is standard in models of trade in final goods, and the complementarity force due to input-output linkages. We interpret the degree of complementarity (or substitutability) from the Difference-in-Differences exercise as the net effect of these two forces. As the numerical exercise illustrates, in theory, either of these two forces can dominate.

Data

The first empirical analysis for the extensive margin below is performed using the dataset from Helpman Melitz and Rubinstein (2008), which contains the following variables: unilateral trade flows from Feenstra's World Trade Flows; distance, which is used as an exogenous, albeit noisy, measure of the variable trade cost; and control variables for geography, institutions, and culture, as listed in the results presented in Table 1. Our second empirical exercise uses Feenstra's World Trade Flow from 1984 to 2014, and on CEPII's Gravity database for data on GDP, population, trade and money agreements.

Empirical evidence on extensive margins

A novel feature of our model is the presence of extensive margins both for exports and for imports, and their interdependence across country-pairs. The benefit of exporting, for a domestic firm, scales with the extensive margin for imports abroad. Hence, the domestic extensive margin for exports, depends positively on the extensive margin for imports. This subsection provides evidence for the salience of this modeling choice.

Due to the lack of availability of firm-level data across many countries, we follow the methodology of Helpman Melitz and Rubinstein (2008) in exploiting zeros and the volume of trade in aggregate data to infer about the probability that a country k exports to a country k'. If country k does not export to k', it must be that even the most productive firm in k does not find it profitable to export to k'. In turn, the probability of country k exporting to country k' is informative about the extensive margin of export in country k.

We use the 1986 cross section with 158 countries; that is $158 * 157 = 24'806$ unidirectional trade flows. Note that each country pair appears twice, once for trade from k to k' and once for trade from k' to k, which allows to identify importing and exporting country fixed effects. Also, more than half of these observations are zero, which we use to estimate the probability that a country exports to another country. To that end, we estimate a latent variable model, where the latent variable measures whether the most productive firm in k could make sufficient profits to recover the fixed cost of exporting to k'.

Trade in intermediates accounts for roughly two-thirds of aggregate trade. In this subsection, we treat all trade as trade in intermediates inputs, which concords with a special version of our model where Ω_k contains only the domestic varieties. We also take the distribution of firms productivities to have a bounded support with upper bound $\bar{\varphi}$. This assumption permits our model to account for zero trade in equilibrium, for some country pairs. Indeed, country k exports to country k' only if the most productive firm in k finds it profitable to export to k'. Using (2.6), this conditions reads:

$$\frac{1}{\sigma}\ r_{k,k'}(\bar{\varphi},(\varphi_{k',k})) > \frac{PB_{k,k'}}{Z_k}\ f_{k,k'}.$$

We make the direct dependence of the revenue for a firm in k from exporting to k' on the intensive margin for imports in k' explicit. It is precisely this dependence that we tease out from the data using the latent variable model below.

Let $T_{k,k'}$ denote a binary variable that takes the value 1 if country k exports to country k'. That is,

$$T_{kk'} = \begin{cases} 1 & \text{if } \ z_{k,k'} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where the latent variable is defined as follows:

$$z_{k,k'} \equiv \ln \frac{\frac{1}{\sigma}\ r_{k,k'}(\bar{\varphi},\varphi_{k',k})}{\frac{PB_k}{Z_k}\ f_{k,k'}}\ .$$

Importantly, the threshold productivity that determines the extensive margins, $\varphi_{k,k'}$, is a function of the latent variable $z_{k,k'}$. Since profits are strictly increasing in a firm's productivity, the larger are the profits of the most productive firm, as measured by $z_{k,k'}$, the lower is the productivity threshold $\varphi_{k,k'}$.

The iceberg cost is assumed to be the distance augmented by a log-normally distributed shock; that is $\tau_{k,k'} \equiv d_{kk'}^{\delta} e^{\epsilon^{\tau}_{k,k'}}$. The fixed cost of exporting is the exponential of the sum of exporting and importing country specific fixed costs, and an error term specific to the country pair; that is $f_{k,k'} \equiv \exp(\phi_{Exp,k} + \phi_{Imp,k'} + \epsilon^{f}_{k,k'})$. Using (R(k)) to substitute for $r_{k,k'}$, we get the following formula[9]:

$$z_{k,k'} = \beta_0 + \beta_k + \beta_{k'} + \delta \ln d_{kk'} + f(\varphi_{k',k}) + \epsilon_{k,k'} \ ,$$

where f is a nonlinear function that accounts for the terms that are direct functions of $\varphi_{k',k}$. This non-linear dependence of the benefit from exporting on the foreign extensive margin would not appear in a model with trade in final goods only.

All else equal, the latent variable and the extensive margins for the country pair k, k' are related one-for-one. We therefore consider the following estimating equation:

$$z_{k,k'} = \alpha + \beta_k + \beta_{k'} + (1-\sigma)d_{kk'} + \tilde{f}(z_{k',k}) + \epsilon_{k,k'} \ ,$$

where $\tilde{f}$ is the composition of f and the mapping from $\varphi_{k',k}$ and $z_{k',k}$.

The latent variable is, however, unobserved, which calls for a two-step estimation method. First, we obtain estimates of z, which we denote $\hat{z}$. Second, we then use $\hat{z}_{k',k}$ as an explanatory variable for $z_{k,k'}$. Our estimating equation is approximated by:

$$z_{k,k'} = \alpha + \beta_k + \beta_{k'} + (1-\sigma)d_{kk'} + \sum_{i=1}^{I} \beta_{zp}\hat{z}^{i}_{k',k} + \epsilon_{k,k'} \ ,$$

where the higher order terms takes into account the nonlinearity of the function $\tilde{f}$.

Results in Table 1 below report the estimation of the probit model for the Linear case where $I = 1$, and a Nonlinear case where $I = 3$. We note that the extensive margin abroad is highly significant as a determinant of the domestic probability of export, and hence of the domestic extensive margin.

Empirical evidence on multi-lateral trade flows

Theoretically, in response to a trade agreement, models of trade in final goods would predict an increase in trade between members, at the expense of a decrease in trade between members and non-members. This substitutability in multilateral trade flows is referred to as *trade creation*

[9]The constant is $\beta_0 = -\ln\sigma + \ln(\alpha^{\alpha}(1-\alpha)^{1-\alpha}) - \ln\bar{\varphi} + \ln\mu$, the exporting country fixed effect is $\beta_k = -\alpha \ln w_k + \phi_{Exp,k}$, the importing country fixed effect is $\beta_{k'} = \ln \mathcal{M}_{k'} + \phi_{Imp,k'}$.

Table 2.1: Determinants of extensive margins kk'

Dependent variable $T_{k,k'}$		
	Linear	Nonlinear
Probability of trade_k'k	1.377***	2.608***
	(9.52)	(3.77)
Log of Bilateral Distance	-0.446***	-0.410***
	(-14.85)	(-10.48)
Common Border Dummy	-0.275**	-0.250**
	(-2.87)	(-2.59)
Islands	-0.240**	-0.220**
	(-3.16)	(-2.85)
Landlock	-0.135	-0.126
	(-1.29)	(-1.20)
Common Legal System Dummy	0.0783**	0.0743*
	(2.62)	(2.47)
Common Language Dummy	0.171***	0.156***
	(4.27)	(3.75)
Colonial Ties Dummy	0.275	0.291
	(0.97)	(1.04)
Currency Union Dummy	0.319*	0.282*
	(2.32)	(2.03)
Free Trade Area Dummy	1.336***	1.271***
	(4.28)	(4.04)
Common Religion (used in HMR)	0.156*	0.139*
	(2.51)	(2.21)
Probability of trade$^2_{k'k}$		-1.929
		(-1.41)
Probability of trade$^3_{k'k}$		0.925
		(1.01)
Importing and exporting country fixed effects not reported		
Observations	24'492	24'492

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

and *trade diversion*. In the presence of input-output linkages, however, the impact of a trade agreement is more subtle. While the forces generating trade creation and trade diversion for trade in final goods are still at play, interdependencies across firms generate complementarities in exporting decisions which aggregate to a force for complementarity in multilateral trade flows. This section presents estimates of the net effect of these two opposing forces for the case of NAFTA.

The sample includes unidirectional bilateral trade between 1984 and 2014 from Feenstra's World Trade Flow. We restrict our sample to countries that were already part of the GATT in 1990[10].

To capture the different effects of NAFTA on trade flows not only within, but also with the trade agreement, we split our sample of unidirectional bilateral trade flows between 150 countries from 1984 to 2014 into the following three groups: country pairs with both members in NAFTA (members), country pairs with only one member in NAFTA for which trade is significant in 1994 (partners), and country pairs that did not trade with a NAFTA member in 1994 (control). Difference-in-differences is the natural identification strategy for a panel setting in which a policy affects groups. In particular, the effect of NAFTA on members (partners respectively) is measured by the difference – between members (partners respectively) and the control group – of the difference in post and pre 1994 trade flows. Our baseline specification controls for all time invariant country-pair heterogeneity as well as aggregate time variation. Standard errors allow for clustering at the country-pair level, which accounts for correlation across either directions in the bilateral relationship as well as for serial correlation across time. We also present results for alternative specifications that include time varying covariates at the country level and different trends across groups.

To account for the 150×149 country pairs, we estimate the within-group transformation[11] of the following model for the log of total imports from j to i at time t:

$$\ln T_{i \to j,t} = \beta_{ij} + \beta_t + \delta_m \ D_t^m + \delta_p \ D_t^p + \epsilon_{ijt} \ . \qquad \text{(Model 1)}$$

[10]Need to better understand the effect of this restriction. More restrictive conditions (conditioning on countries that accessed GATT before 1990) lower the coefficient for trade diversion.

[11]That is

$$\ln T_{i \to j,t} - \bar{m}_{ij} = \beta_t + \delta_m \ D_t^m + \delta_p \ D_t^p + \epsilon_{ijt} \ ,$$

where $\bar{m}_{ij} = \frac{1}{31} \sum_{t=1984}^{2014} m_{ijt}$.

The country-pair fixed effect not only accounts for standard time invariant covariates of gravity models such as geography, institutions and culture, but also for unobserved time invariant heterogeneity. Not included in Model 1, however, is GDP, which varies over time and across countries. We include it in our robustness check and find that it doesn't alter our conclusions. The time dummy not only accounts for the increase in global trade, but also for any global time varying effect. The variables of interests are the two dummy variables that capture the changes attributed to NAFTA. The dummy D_t^m (D_t^p respectively) takes value 1 after 1994 for any country pairs in the group of members (partners respectively) and 0 otherwise.

Before turning to caveats and results, note that under the heroic assumption (which we discuss below) $\mathbb{E}[\epsilon_{ijt}|i,j,t] = 0$, the coefficient associated with the NAFTA dummy measures the aforementioned difference in differences. For the group of partners, we have:

$$\begin{aligned}\delta_p = &(\mathbb{E}[\ln T_{i\to j,t}|i \text{ or } j \text{ partner}, \ t \geq 1994] - \mathbb{E}[\ln T_{i\to j,t}|i \text{ or } j \text{ partner}, \ t < 1994]) - \\ &(\mathbb{E}[\ln T_{i\to j,t}|\text{nor } i \text{ nor } j \text{ partner}, \ t \geq 1994] - \mathbb{E}[\ln T_{i\to j,t}|\text{nor } i \text{ nor } j \text{ partner}, \ t < 1994])\end{aligned}$$

We will interpret e^{δ_p} as the factor by which the proportion of trade between NAFTA members and their partners changed relative to the proportional change in trade for the control group. In other words, a positive δ_m is indicative of trade creation whereas a negative δ_p is indicative of trade diversion. In this reduced form approach, δ_p provides a measure of the net effect of various substitution and complementarity forces on multilateral trade flows. The interpretation of our results is sensitive to endogeneity issues to which we now turn.

Endogeneity of NAFTA: While our baseline specification aims to capture the effect of NAFTA on trade flows, the causality may play in the other direction, which would give rise to endogenous selection into members, partners or controls. In other words, members of the trade agreements may be a selected group for which trade is anticipated to increase. Such reverse causality, and the associated endogenous selection, would make our results overestimate trade creation from NAFTA in the sense that estimates of δ_m would be biased upward. To the contrary, for trade partners, which were *not* selected to be part of NAFTA, estimates of δ_p would be biased downward. Hence our estimates would tend to over-estimate the extent of trade diversion. The selection of countries into members and partners of NAFTA motivates controlling for time varying factors that vary at the group or country level, which we address in the following two ways:

control for group-level trends and for importing and exporting countries' GDP and population.

Controls for group-level trends[12]*:* The difference-in-differences empirical strategy relies on the comparison of the evolution of the groups affected by the policy to the evolution of a control group. Importantly, we account for the group level heterogeneity in trends that may result from self-selection of countries into members, partners or control group, with the inclusion of group trends. The second reduced form model we estimate is:

$$\ln T_{i\to j,t} = \beta_{ij} + \beta_t + \delta_m\ D_t^m + \delta_p\ D_t^p + \gamma_m\ 1_m\ t + \gamma_p\ 1_p\ t + \epsilon_{ijt}. \qquad \text{(Model 2)}$$

where 1_m and 1_p are indicator variables for members and partners respectively. The trends capture deviations, at the group level, from the evolution of world trade (which is accounted for by the time dummy).

Clustering and standard errors: An observation is a unidirectional trade flow for a country-pair in a given year. Importantly, for (Model 1) and (Model 2), standard errors are robust to clustering within country-pairs and across time. That is, we allow for the error term to have a fixed country-pair component common to both (ij) and (ji) and for serially correlation within bilateral country pairs. Under this assumption, there are $150 * 149/2$ clusters. While our current model specifications account for time varying components at the group-level and country-pair heterogeneity, this leaves the possibility for the error term to contain a country level component. This motivates our next reduced form model specification.

Controls for country-level trends, income and population: Shocks at the country level requires to allow for clustering at the country level in the computation of standard errors. Under this assumption, there are still 150 clusters. To improve the precision though, we introduce both importing and exporting country level GDP and GDP per capita. Our third model specification is:

$$\ln T_{i\to j,t} = \beta_{ij}+\beta_t+\delta_m\ D_t^m+\delta_p\ D_t^p+\gamma_m\ 1_m\ t+\gamma_p\ 1_p\ t+\beta^{imp} y_{it}+\beta^{exp} y_{jt}+\beta_c^{imp} y_{it}^{cap}+\beta_c^{exp} y_{jt}^{cap}+\epsilon_{ijt}. \qquad \text{(Model 3)}$$

where y and y^{cap} denote GDP and GDP per capita.

Reduced form estimates do not find evidence of trade diversion.

[12]To do: allow for more flexibility with the inclusion of group level time dummies instead of linear trends.

Table 2.2: Difference-in-Differences

	(1)	(2)	(3)
	ln trade	ln trade	ln trade
1=NAFTA m. post 94	0.274	0.359***	0.151**
	(1.03)	(5.20)	(2.69)
1=NAFTA partn post 94	0.00319	-0.0149	0.0193
	(0.06)	(-0.25)	(0.13)
Trend NAFTA		-0.00555	0.0114*
		(-0.42)	(2.61)
Trend NAFTA partners		0.000199	0.000168
		(1.34)	(0.82)
ln GDPo GDPd p.cap			-0.498***
			(-3.55)
ln GDPo GDPd			1.099***
			(7.95)
Time dummy	Yes	Yes	Yes
Country-pair FE	Yes	Yes	Yes
SE clustered	Country-pair	Country-pair	Country
N	224013	224013	223071

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

2.5 Conclusion

This paper analyzes the consequences of international input-output linkages on the propagation of trade disruption. In our setup, firms participate in global value chain and jointly decide where to import from and to export to. The price and revenue of each firm is then positively linked to those of other firms participating in the supply chain, creating complementarity in firms' sales and profits.

In this context, we show that an increase in trade barriers between two countries has a negative impact on trade flows with other trading partners, giving rise to aggregate trade flow complementarity. This phenomenon implies that disrupting trade between any two countries impacts negatively the whole network of cross-country relations. Further investigation reveals that this complementarity is partly driven by extensive margin adjustments, whereby firms experiencing a negative shock to their input costs revise their import and export decision and contribute to propagate an initially localized disruption to third countries.

2.6 Theoretical Appendix

2.6.1 Price System

The price system in the case of the "natural ranking equilibrium" is defined as

$$
(P)\quad \left\{
\begin{array}{lll}
\mathcal{P}_{M_{A,A}}^{1-\sigma} = & \mathcal{M}_A Z_A^{\sigma-1} \times & \left[\int\limits_{\varphi_{A,A}}^{\varphi_{A,AB}} \left(\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,A}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} \left(\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
\mathcal{P}_{M_{A,AB}}^{1-\sigma} = & \mathcal{M}_A Z_A^{\sigma-1} \times & \left[\int\limits_{\varphi_{A,A}}^{\varphi_{A,AB}} \left(\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,A}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} \left(\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & +\mathcal{M}_B Z_B^{\sigma-1} \times & \left[\int\limits_{\varphi_{B,AB}}^{\varphi_{B,ABC}} \left(\tau_{BA}\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\tau_{BA}\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
\mathcal{P}_{M_{A,ABC}}^{1-\sigma} = & \mathcal{M}_A Z_A^{\sigma-1} \times & \left[\int\limits_{\varphi_{A,A}}^{\varphi_{A,AB}} \left(\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,A}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} \left(\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & +\mathcal{M}_B Z_B^{\sigma-1} \times & \left[\int\limits_{\varphi_{B,AB}}^{\varphi_{B,ABC}} \left(\tau_{BA}\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\tau_{BA}\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & +\mathcal{M}_C Z_C^{\sigma-1} \times & \left[\int\limits_{\varphi_{C,ABC}}^{+\infty} \left(\tau_{CA}\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & & \\
\mathcal{P}_{M_{B,B}}^{1-\sigma} = & \mathcal{M}_B Z_B^{\sigma-1} \times & \left[\int\limits_{\varphi_{B_B}}^{\varphi_{B,AB}} \left(\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,B}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{B,AB}}^{\varphi_{B,ABC}} \left(\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
\mathcal{P}_{M_{B,AB}}^{1-\sigma} = & \mathcal{M}_B Z_B^{\sigma-1} \times & \left[\int\limits_{\varphi_{B_B}}^{\varphi_{B,AB}} \left(\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,B}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{B,AB}}^{\varphi_{B,ABC}} \left(\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & +\mathcal{M}_A Z_A^{\sigma-1} \times & \left[\int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} \left(\tau_{AB}\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\tau_{AB}\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
\mathcal{P}_{M_{B,ABC}}^{1-\sigma} = & \mathcal{M}_B Z_B^{\sigma-1} \times & \left[\int\limits_{\varphi_{B_B}}^{\varphi_{B,AB}} \left(\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,B}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{B,AB}}^{\varphi_{B,ABC}} \left(\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & +\mathcal{M}_A Z_A^{\sigma-1} \times & \left[\int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} \left(\tau_{AB}\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,AB}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\tau_{AB}\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & +\mathcal{M}_C Z_C^{\sigma-1} \times & \left[\int\limits_{\varphi_{C,BC}}^{\varphi_{C,ABC}} \left(\tau_{CB}\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,BC}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{C,ABC}}^{+\infty} \left(\tau_{CB}\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & & \\
\mathcal{P}_{M_{C,C}}^{1-\sigma} = & \mathcal{M}_C Z_C^{\sigma-1} \times & \left[\int\limits_{\varphi_{C_C}}^{\varphi_{C,BC}} \left(\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,C}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{C,BC}}^{\varphi_{C,ABC}} \left(\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,BC}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{C,ABC}}^{+\infty} \left(\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
\mathcal{P}_{M_{C,BC}}^{1-\sigma} = & \mathcal{M}_C Z_C^{\sigma-1} \times & \left[\int\limits_{\varphi_{C_C}}^{\varphi_{C,BC}} \left(\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,C}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{C,BC}}^{\varphi_{C,ABC}} \left(\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,BC}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{C,ABC}}^{+\infty} \left(\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & +\mathcal{M}_B Z_B^{\sigma-1} \times & \left[\int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\tau_{BC}\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
\mathcal{P}_{M_{C,ABC}}^{1-\sigma} = & \mathcal{M}_C Z_C^{\sigma-1} \times & \left[\int\limits_{\varphi_{C_C}}^{\varphi_{C,BC}} \left(\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,C}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{C,BC}}^{\varphi_{C,ABC}} \left(\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,BC}}^{1-\alpha}\right)^{1-\sigma} + \int\limits_{\varphi_{C,ABC}}^{+\infty} \left(\frac{\mu_C}{\varphi}\mathcal{P}_{M_{C,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & +\mathcal{M}_B Z_B^{\sigma-1} \times & \left[\int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\tau_{BC}\frac{\mu_B}{\varphi}\mathcal{P}_{M_{B,ABC}}^{1-\alpha}\right)^{1-\sigma} \right] \\
 & +\mathcal{M}_A Z_A^{\sigma-1} \times & \left[\int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\tau_{AC}\frac{\mu_A}{\varphi}\mathcal{P}_{M_{A,ABC}}^{1-\alpha}\right)^{1-\sigma} \right]
\end{array}
\right.
$$

Dealing with the integrals, noting λ_k constants defined as

$$\lambda_k = \frac{\gamma_k}{\gamma_k - (\sigma - 1)} \mathcal{M}_k \left(\frac{\mu_k}{Z_k}\right)^{1-\sigma}$$

and denoting the LHS variables as $X_{A,A}, X_{A,AB}, ...X_{C,ABC}$, the system of prices for country A collapses to:

$$\begin{cases} X_{A,A} = & \lambda_A \times \Big[(\varphi_{A,A}{}^{\sigma-\gamma_A-1} - \varphi_{A,AB}{}^{\sigma-\gamma_A-1})X_{A,A}^{1-\alpha} \\ & +(\varphi_{A,AB}{}^{\sigma-\gamma_A-1} - \varphi_{A,ABC}{}^{\sigma-\gamma_A-1})X_{A,AB}^{1-\alpha} + \varphi_{A,ABC}{}^{\sigma-\gamma_A-1}X_{A,ABC}^{1-\alpha}\Big] \\ X_{A,AB} = & \lambda_A \times \Big[(\varphi_{A,A}{}^{\sigma-\gamma_A-1} - \varphi_{A,AB}{}^{\sigma-\gamma_A-1})X_{A,A}^{1-\alpha} \\ & +(\varphi_{A,AB}{}^{\sigma-\gamma_A-1} - \varphi_{A,ABC}{}^{\sigma-\gamma_A-1})X_{A,AB}^{1-\alpha} + \varphi_{A,ABC}{}^{\sigma-\gamma_A-1}X_{A,ABC}^{1-\alpha}\Big] \\ & +\lambda_B\tau_{BA}^{1-\sigma} \times \Big[(\varphi_{B,AB}{}^{\sigma-\gamma_B-1} - \varphi_{B,ABC}{}^{\sigma-\gamma_B-1})X_{B,AB}^{1-\alpha} + \varphi_{B,ABC}{}^{\sigma-\gamma_B-1}X_{B,ABC}^{1-\alpha}\Big] \\ X_{A,ABC} = & \lambda_A \times \Big[(\varphi_{A,A}{}^{\sigma-\gamma_A-1} - \varphi_{A,AB}{}^{\sigma-\gamma_A-1})X_{A,A}^{1-\alpha} \\ & +(\varphi_{A,AB}{}^{\sigma-\gamma_A-1} - \varphi_{A,ABC}{}^{\sigma-\gamma_A-1})X_{A,AB}^{1-\alpha} + \varphi_{A,ABC}{}^{\sigma-\gamma_A-1}X_{A,ABC}^{1-\alpha}\Big] \\ & +\lambda_B\tau_{BA}^{1-\sigma} \times \Big[(\varphi_{B,AB}{}^{\sigma-\gamma_B-1} - \varphi_{B,ABC}{}^{\sigma-\gamma_B-1})X_{B,AB}^{1-\alpha} + \varphi_{B,ABC}{}^{\sigma-\gamma_B-1}X_{B,ABC}^{1-\alpha}\Big] \\ & +\lambda_C\tau_{CA}^{1-\sigma} \times \Big[\varphi_{C,ABC}{}^{\sigma-\gamma_C-1}X_{C,ABC}^{1-\alpha}\Big] \end{cases}$$

The same transformation can be done for equations defining price indices in countries B and C. Solving for prices for all strategies s in all countries k comes down to solving a simple system of 9 equations with an equal number of unknowns.

2.6.2 Revenue System

$$
(R_A)\quad\begin{cases}
r_{A,A}(\varphi) = & \left(\frac{p_{A,A}(\varphi)}{\mathcal{P}_{M_{A,ABC}}}\right)^{1-\sigma}\beta X_A \\
& +\mathcal{M}_A \times \int\limits_{\varphi_{A,A}}^{\varphi_{A,AB}} \left(\frac{p_{A,A}(\varphi)}{\mathcal{P}_{M_{A,A}}}\right)^{1-\sigma} \mathcal{P}_{M_{A,A}} M_{A,A}(s)g(s)ds \\
& +\mathcal{M}_A \times \int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} \left(\frac{p_{A,A}(\varphi)}{\mathcal{P}_{M_{A,AB}}}\right)^{1-\sigma} \mathcal{P}_{M_{A,AB}} M_{A,AB}(s)g(s)ds \\
& +\mathcal{M}_A \times \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\frac{p_{A,A}(\varphi)}{\mathcal{P}_{M_{A,ABC}}}\right)^{1-\sigma} \mathcal{P}_{M_{A,ABC}} M_{A,ABC}(s)g(s)ds \\
r_{A,AB}(\varphi) = & \left(\frac{p_{A,AB}(\varphi)}{\mathcal{P}_{M_{A,ABC}}}\right)^{1-\sigma}\beta X_A + \left(\tau_{AB}\frac{p_{A,AB}(\varphi)}{\mathcal{P}_{M_{B,ABC}}}\right)^{1-\sigma}\beta X_B \\
& +\mathcal{M}_A \times \int\limits_{\varphi_{A,A}}^{\varphi_{A,AB}} \left(\frac{p_{A,AB}(\varphi)}{\mathcal{P}_{M_{A,A}}}\right)^{1-\sigma} \mathcal{P}_{M_{A,A}} M_{A,A}(s)g(s)ds \\
& +\mathcal{M}_A \times \int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} \left(\frac{p_{A,AB}(\varphi)}{\mathcal{P}_{M_{A,AB}}}\right)^{1-\sigma} \mathcal{P}_{M_{A,AB}} M_{A,AB}(s)g(s)ds \\
& +\mathcal{M}_A \times \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\frac{p_{A,AB}(\varphi)}{\mathcal{P}_{M_{A,ABC}}}\right)^{1-\sigma} \mathcal{P}_{M_{A,ABC}} M_{A,ABC}(s)g(s)ds \\
& +\mathcal{M}_B \times \int\limits_{\varphi_{B,AB}}^{\varphi_{B,ABC}} \left(\tau_{AB}\frac{p_{A,AB}(\varphi)}{\mathcal{P}_{M_{B,AB}}}\right)^{1-\sigma} \mathcal{P}_{M_{B,AB}} M_{B,AB}(s)g(s)ds \\
& +\mathcal{M}_B \times \int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\tau_{AB}\frac{p_{A,AB}(\varphi)}{\mathcal{P}_{M_{B,ABC}}}\right)^{1-\sigma} \mathcal{P}_{M_{B,ABC}} M_{B,ABC}(s)g(s)ds \\
r_{A,ABC}(\varphi) = & \left(\frac{p_{A,ABC}(\varphi)}{\mathcal{P}_{M_{A,ABC}}}\right)^{1-\sigma}\beta X_A + \left(\tau_{AB}\frac{p_{A,ABC}(\varphi)}{\mathcal{P}_{M_{B,ABC}}}\right)^{1-\sigma}\beta X_B + \left(\tau_{AC}\frac{p_{A,ABC}(\varphi)}{\mathcal{P}_{M_{C,ABC}}}\right)^{1-\sigma}\beta X_C \\
& +\mathcal{M}_A \times \int\limits_{\varphi_{A,A}}^{\varphi_{A,AB}} \left(\frac{p_{A,ABC}(\varphi)}{\mathcal{P}_{M_{A,A}}}\right)^{1-\sigma} \mathcal{P}_{M_{A,A}} M_{A,A}(s)g(s)ds \\
& +\mathcal{M}_A \times \int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} \left(\frac{p_{A,ABC}(\varphi)}{\mathcal{P}_{M_{A,AB}}}\right)^{1-\sigma} \mathcal{P}_{M_{A,AB}} M_{A,AB}(s)g(s)ds \\
& +\mathcal{M}_A \times \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\frac{p_{A,ABC}(\varphi)}{\mathcal{P}_{M_{A,ABC}}}\right)^{1-\sigma} \mathcal{P}_{M_{A,ABC}} M_{A,ABC}(s)g(s)ds \\
& +\mathcal{M}_B \times \int\limits_{\varphi_{B,AB}}^{\varphi_{B,ABC}} \left(\tau_{AB}\frac{p_{A,ABC}(\varphi)}{\mathcal{P}_{M_{B,AB}}}\right)^{1-\sigma} \mathcal{P}_{M_{B,AB}} M_{B,AB}(s)g(s)ds \\
& +\mathcal{M}_B \times \int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\tau_{AB}\frac{p_{A,ABC}(\varphi)}{\mathcal{P}_{M_{B,ABC}}}\right)^{1-\sigma} \mathcal{P}_{M_{B,ABC}} M_{B,ABC}(s)g(s)ds \\
& +\mathcal{M}_B \times \int\limits_{\varphi_{C,ABC}}^{+\infty} \left(\tau_{AC}\frac{p_{A,ABC}(\varphi)}{\mathcal{P}_{M_{C,ABC}}}\right)^{1-\sigma} \mathcal{P}_{M_{C,ABC}} M_{C,ABC}(s)g(s)ds
\end{cases}
$$

Complete expression of the (ΛA) system

$$
(\Lambda A)\quad \begin{cases}
\Lambda_{A,A} = & (1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{\mu_A \mathcal{P}_{M_{A,A}}^{-\alpha}}{Z_A}\right)^{1-\sigma} \times \frac{\gamma_A}{\gamma_A-(\sigma-1)}\left(\varphi_{A,A}^{\sigma-\gamma_A-1} - \varphi_{A,AB}^{\sigma-\gamma_A-1}\right) \times \\
& \left[\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} + \mathcal{M}_A \times [\Lambda_{A,A} + \Lambda_{A,AB} + \Lambda_{A,ABC}]\right] \\
\\
\Lambda_{A,AB} = & (1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{\mu_A \mathcal{P}_{M_{A,AB}}^{-\alpha}}{Z_A}\right)^{1-\sigma} \times \frac{\gamma_A}{\gamma_A-(\sigma-1)}\left(\varphi_{A,AB}^{\sigma-\gamma_A-1} - \varphi_{A,ABC}^{\sigma-\gamma_A-1}\right) \times \Big[\\
& \beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} + \mathcal{M}_A \times [\Lambda_{A,A} + \Lambda_{A,AB} + \Lambda_{A,ABC}] \\
& + \tau_{AB}^{1-\sigma}\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} + \mathcal{M}_B \tau_{AB}^{1-\sigma} \times [\Lambda_{B,AB} + \Lambda_{B,ABC}] \Big] \\
\\
\Lambda_{A,ABC} = & (1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{\mu_A \mathcal{P}_{M_{A,ABC}}^{-\alpha}}{Z_A}\right)^{1-\sigma} \times \frac{\gamma_A}{\gamma_A-(\sigma-1)}\left(\varphi_{A,ABC}^{\sigma-\gamma_A-1}\right) \times \Big[\\
& \beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} + \mathcal{M}_A \times [\Lambda_{A,A} + \Lambda_{A,AB} + \Lambda_{A,ABC}] \\
& + \tau_{AB}^{1-\sigma}\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} + \mathcal{M}_B \tau_{AB}^{1-\sigma} \times [\Lambda_{B,AB} + \Lambda_{B,ABC}] \\
& + \tau_{AC}^{1-\sigma}\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \tau_{AC}^{1-\sigma} \times [\Lambda_{C,ABC}] \Big]
\end{cases}
$$

And we can write the above system in matrix form:

$$\begin{pmatrix}
1-C_{A,A}\mathcal{M}_A & -C_{A,A}\mathcal{M}_A & -C_{A,A}\mathcal{M}_A & 0 & 0 & 0 & 0 & 0 & 0 \\
-C_{A,AB}\mathcal{M}_A & 1-C_{A,AB}\mathcal{M}_A & -C_{A,AB}\mathcal{M}_A & 0 & -C_{A,AB}\tau_{AB}^{1-\sigma}\mathcal{M}_B & -C_{A,AB}\tau_{AB}^{1-\sigma}\mathcal{M}_B & 0 & 0 & 0 \\
-C_{A,ABC}\mathcal{M}_A & -C_{A,ABC}\mathcal{M}_A & 1-C_{A,ABC}\mathcal{M}_A & 0 & -C_{A,ABC}\tau_{AB}^{1-\sigma}\mathcal{M}_B & -C_{A,ABC}\tau_{AB}^{1-\sigma}\mathcal{M}_B & 0 & 0 & -C_{A,ABC}\tau_{AC}\mathcal{M}_C \\
0 & 0 & 0 & 1-C_{B,B}\mathcal{M}_B & -C_{B,B}\mathcal{M}_B & -C_{B,B}\mathcal{M}_B & 0 & 0 & 0 \\
0 & -C_{B,AB}\tau_{BA}^{1-\sigma}\mathcal{M}_A & -C_{B,AB}\tau_{BA}^{1-\sigma}\mathcal{M}_A & -C_{B,AB}\mathcal{M}_B & 1-C_{B,AB}\mathcal{M}_B & -C_{B,AB}\mathcal{M}_B & 0 & 0 & 0 \\
0 & -C_{B,ABC}\tau_{BA}^{1-\sigma}\mathcal{M}_A & -C_{B,ABC}\tau_{BA}^{1-\sigma}\mathcal{M}_A & -C_{B,ABC}\mathcal{M}_B & -C_{B,ABC}\mathcal{M}_B & 1-C_{B,ABC}\mathcal{M}_B & 0 & -C_{B,ABC}\tau_{BC}^{1-\sigma}\mathcal{M}_C & -C_{B,ABC}\tau_{BC}^{1-\sigma}\mathcal{M}_C \\
0 & 0 & 0 & 0 & 0 & 0 & 1-C_{C,C}\mathcal{M}_C & -C_{C,C}\mathcal{M}_C & -C_{C,C}\mathcal{M}_C \\
0 & 0 & 0 & 0 & 0 & -C_{C,BC}\tau_{CB}^{1-\sigma}\mathcal{M}_B & -C_{C,BC}\mathcal{M}_C & 1-C_{C,BC}\mathcal{M}_C & -C_{C,BC}\mathcal{M}_C \\
0 & 0 & -C_{C,ABC}\tau_{CA}^{1-\sigma}\mathcal{M}_A & 0 & 0 & -C_{C,ABC}\tau_{CB}^{1-\sigma}\mathcal{M}_B & -C_{C,ABC}\mathcal{M}_C & -C_{C,ABC}\mathcal{M}_C & 1-C_{C,ABC}\mathcal{M}_C
\end{pmatrix}$$

$$\times \begin{pmatrix} \Lambda_{A,A} \\ \Lambda_{A,AB} \\ \Lambda_{A,ABC} \\ \Lambda_{B,B} \\ \Lambda_{B,AB} \\ \Lambda_{B,ABC} \\ \Lambda_{C,C} \\ \Lambda_{C,BC} \\ \Lambda_{C,ABC} \end{pmatrix} = \begin{pmatrix}
\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} C_{A,A} & + & 0 & + & 0 \\
\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} C_{A,AB} & + & \tau_{AB}^{1-\sigma}\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} C_{A,AB} & + & 0 \\
\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} C_{A,ABC} & + & \tau_{AB}^{1-\sigma}\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} C_{A,ABC} & + & \tau_{AC}^{1-\sigma}\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} C_{A,ABC} \\
0 & + & \beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} C_{B,B} & + & 0 \\
\tau_{BA}^{1-\sigma}\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} C_{B,AB} & + & \beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} C_{B,AB} & + & 0 \\
\tau_{BA}^{1-\sigma}\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} C_{B,ABC} & + & \beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} C_{B,ABC} & + & \tau_{BC}^{1-\sigma}\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} C_{B,ABC} \\
0 & + & 0 & + & \beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} C_{C,C} \\
0 & + & \tau_{CB}^{1-\sigma}\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} C_{C,BC} & + & \beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} C_{C,BC} \\
\tau_{CA}^{1-\sigma}\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} C_{C,ABC} & + & \tau_{CB}^{1-\sigma}\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} C_{C,ABC} & + & \beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} C_{C,ABC}
\end{pmatrix}$$

Solving for the Λs gives us the revenues and finally, profits are just $\pi_i = \frac{1}{\sigma} r_i$.

2.6.3 Verifying Incentive Compatibility

To insure that no firms in neither A nor B nor C has an individual incentive to deviate, we define the deviation price index and profits respectively as follow. In A we have

$$\begin{aligned}
\mathcal{P}_{M_{A,AC}}^{1-\sigma} &= \mathcal{P}_{M_{A,A}}^{1-\sigma} + \lambda_C \tau_{CA}^{1-\sigma} \times \left[{\varphi_{C,ABC}}^{\sigma-\gamma_C-1} \mathcal{P}_{M_{C,ABC}}^{(1-\sigma).(1-\alpha)} \right] \\
\pi_{A,AC}(\varphi_{A,ABC}) &= \frac{1}{\sigma}\left(\frac{\mu_A \mathcal{P}_{M_{A,AC}}^{1-\alpha}}{Z_A}\right)^{1-\sigma} \times \left[\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} + \mathcal{M}_A \times \left[\Lambda_{A,A} + \Lambda_{A,AB} + \Lambda_{A,ABC}\right]\right. \\
&\quad \left. + \tau_{AC}^{1-\sigma} \beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \tau_{AC}^{1-\sigma} \times \Lambda_{C,ABC}\right] \varphi_{A,ABC}^{\sigma-1} - f_{A,C}
\end{aligned}$$

We can then numerically verify if indeed we have $\pi_{A,AB}(\varphi_{A,ABC}) > \pi_{A,AC}(\varphi_{A,ABC})$. In B we have

$$\begin{aligned}
\mathcal{P}_{M_{B,BC}}^{1-\sigma} &= \mathcal{P}_{M_{B,B}}^{1-\sigma} + \lambda_C \tau_{CB}^{1-\sigma} \times \left[{\varphi_{C,ABC}}^{\sigma-\gamma_C-1} \mathcal{P}_{M_{C,ABC}}^{(1-\sigma).(1-\alpha)} \right] \\
\pi_{B,BC}(\varphi_{B,ABC}) &= \frac{1}{\sigma}\left(\frac{\mu_B \mathcal{P}_{M_{B,BC}}^{1-\alpha}}{S_B}\right)^{1-\sigma} \times \left[\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} + \mathcal{M}_B \times \left[\Lambda_{B,B} + \Lambda_{B,AB} + \Lambda_{B,ABC}\right]\right. \\
&\quad \left. + \tau_{BC}^{1-\sigma} \beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \tau_{BC}^{1-\sigma} \times \Lambda_{C,ABC}\right] \varphi_{B,ABC}^{\sigma-1} - f_{B,C}
\end{aligned}$$

We can then numerically verify if indeed we have $\pi_{B,AB}(\varphi_{B,ABC}) > \pi_{B,BC}(\varphi_{B,ABC})$. In C, the deviation toward A writes

$$\begin{aligned}
\mathcal{P}_{M_{C,AC}}^{1-\sigma} &= \mathcal{P}_{M_{C,C}}^{1-\sigma} + \lambda_A \tau_{AC}^{1-\sigma} \times \left[{\varphi_{A,ABC}}^{\sigma-\gamma_C-1} \mathcal{P}_{M_{A,ABC}}^{(1-\sigma).(1-\alpha)} \right] \\
\pi_{C,AC}(\varphi_{C,ABC}) &= \frac{1}{\sigma}\left(\frac{\mu_C \mathcal{P}_{M_{C,AC}}^{1-\alpha}}{Z_C}\right)^{1-\sigma} \times \left[\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_A \times \left[\Lambda_{C,C} + \Lambda_{C,BC} + \Lambda_{C,ABC}\right] + \right. \\
&\quad \left. \tau_{CA}^{1-\sigma} \beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} + \mathcal{M}_A \tau_{CA}^{1-\sigma} \times \Lambda_{A,ABC}\right] \varphi_{C,ABC}^{\sigma-1} - f_{C,A}
\end{aligned}$$

We can then numerically verify if indeed we have $\pi_{C,BC}(\varphi_{C,ABC}) > \pi_{C,AC}(\varphi_{C,ABC})$.

2.6.4 Trade Flows

$$
\begin{aligned}
X_{A\to B} &= \mathcal{M}_A \left(\int\limits_{\varphi_{A,AB}}^{\varphi_{A,ABC}} X_{A\to B}^{AB}(\varphi) g(\varphi) d\varphi + \int\limits_{\varphi_{A,ABC}}^{+\infty} X_{A\to B}^{ABC}(\varphi) g(\varphi) d\varphi \right) \\
&= \frac{\gamma_A \mathcal{M}_A}{\gamma_A - (\sigma - 1)} \left(\frac{\tau_{AB}\mu_A}{Z_A} \right)^{1-\sigma} \times \left[\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} + \mathcal{M}_B \times (\Lambda_{B,AB} + \Lambda_{B,ABC}) \right] \times \\
&\quad \left[\mathcal{P}_{M_{A,AB}}^{(1-\alpha).(1-\sigma)} \left(\varphi_{A,AB}^{\sigma-\gamma_A-1} - \varphi_{A,ABC}^{\sigma-\gamma_A-1} \right) + \mathcal{P}_{M_{A,ABC}}^{(1-\alpha).(1-\sigma)} \varphi_{A,ABC}^{\sigma-\gamma_A-1} \right] \\
&= \lambda_A \times \tau_{AB}^{1-\sigma} \times \left[\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} + \mathcal{M}_B \times (\Lambda_{B,AB} + \Lambda_{B,ABC}) \right] \times \\
&\quad \left[\mathcal{P}_{M_{A,AB}}^{(1-\alpha).(1-\sigma)} \left(\varphi_{A,AB}^{\sigma-\gamma_A-1} - \varphi_{A,ABC}^{\sigma-\gamma_A-1} \right) + \mathcal{P}_{M_{A,ABC}}^{(1-\alpha).(1-\sigma)} \varphi_{A,ABC}^{\sigma-\gamma_A-1} \right]
\end{aligned}
$$

$$
\begin{aligned}
X_{A\to C} &= \mathcal{M}_A \int\limits_{\varphi_{A,ABC}}^{+\infty} \left(\frac{\mu_A \mathcal{P}_{M_{A,ABC}}^{1-\alpha}}{Z_A} \right)^{1-\sigma} \times \tau_{AC}^{1-\sigma} \left[\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \times \Lambda_{C,ABC} \right] \varphi^{\sigma-1} g(\varphi) d\varphi \\
&= \frac{\gamma_A \mathcal{M}_A}{\gamma_A - (\sigma-1)} \times \left(\frac{\mu_A \mathcal{P}_{M_{A,ABC}}^{1-\alpha}}{Z_A} \right)^{1-\sigma} \times \tau_{AC}^{1-\sigma} \times \left[\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \times \Lambda_{C,ABC} \right] \times \varphi_{A,ABC}^{\sigma-\gamma_A-1} \\
X_{A\to C} &= \lambda_A \times \tau_{AC}^{1-\sigma} \times \mathcal{P}_{M_{A,ABC}}^{(1-\alpha).(1-\sigma)} \left[\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \times \Lambda_{C,ABC} \right] \times \varphi_{A,ABC}^{\sigma-\gamma_A-1}
\end{aligned}
$$

$$
\begin{aligned}
X_{B\to C} &= \mathcal{M}_B \int\limits_{\varphi_{B,ABC}}^{+\infty} \left(\frac{\mu_B \mathcal{P}_{M_{B,ABC}}^{1-\alpha}}{Z_B} \right)^{1-\sigma} \times \tau_{BC}^{1-\sigma} \left[\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \times (\Lambda_{C,BC} + \Lambda_{C,ABC}) \right] \varphi^{\sigma-1} g(\varphi) d\varphi \\
&= \frac{\gamma_B \mathcal{M}_B}{\gamma_B - (\sigma-1)} \times \left(\frac{\mu_B \mathcal{P}_{M_{B,ABC}}^{1-\alpha}}{Z_B} \right)^{1-\sigma} \times \tau_{BC}^{1-\sigma} \times \left[\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \times (\Lambda_{C,BC} \Lambda_{C,ABC}) \right] \times \varphi_{B,ABC}^{\sigma-\gamma_B-1} \\
X_{B\to C} &= \lambda_B \times \tau_{BC}^{1-\sigma} \times \mathcal{P}_{M_{B,ABC}}^{(1-\alpha).(1-\sigma)} \left[\beta X_C \mathcal{P}_{M_{C,ABC}}^{\sigma-1} + \mathcal{M}_C \times (\Lambda_{C,BC} \Lambda_{C,ABC}) \right] \times \varphi_{B,ABC}^{\sigma-\gamma_B-1}
\end{aligned}
$$

$$
\begin{aligned}
X_{B\to A} &= \lambda_B \times \tau_{BA}^{1-\sigma} \left[\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} + \mathcal{M}_A \times (\Lambda_{A,AB} + \Lambda_{A,ABC}) \right] \times \\
&\quad \left[\mathcal{P}_{M_{B,AB}}^{(1-\alpha).(1-\sigma)} \left(\varphi_{B,AB}^{\sigma-\gamma_B-1} - \varphi_{B,ABC}^{\sigma-\gamma_B-1} \right) + \mathcal{P}_{M_{B,ABC}}^{(1-\alpha).(1-\sigma)} \varphi_{B,ABC}^{\sigma-\gamma_B-1} \right]
\end{aligned}
$$

$$X_{C\to A} = \lambda_C \times \tau_{CA}^{1-\sigma}\left[\beta X_A \mathcal{P}_{M_{A,ABC}}^{\sigma-1} + \mathcal{M}_A \times \Lambda_{A,ABC}\right] \times \mathcal{P}_{M_{C,ABC}}^{(1-\alpha).(1-\sigma)} \varphi_{C,ABC}^{\sigma-\gamma_C-1}$$

$$\begin{aligned} X_{C\to B} = & \lambda_C \times \tau_{CB}^{1-\sigma}\left[\beta X_B \mathcal{P}_{M_{B,ABC}}^{\sigma-1} + \mathcal{M}_B \times \Lambda_{B,ABC}\right] \times \\ & \left[\mathcal{P}_{M_{C,BC}}^{(1-\alpha).(1-\sigma)}\left(\varphi_{C,BC}^{\sigma-\gamma_C-1} - \varphi_{C,ABC}^{\sigma-\gamma_C-1}\right) + \mathcal{P}_{M_{C,ABC}}^{(1-\alpha).(1-\sigma)} \varphi_{C,ABC}^{\sigma-\gamma_C-1}\right] \end{aligned}$$

2.6.5 Proof of Profits proportional to Revenues

We want to prove that

$$\Pi_k = \frac{\sigma-1}{\gamma\sigma} R_k$$

Proof

Since firms charge a constant markup $\sigma/(\sigma-1)$ over their marginal cost, the total profit made in country k is

$$\Pi_k = \frac{R_k}{\sigma} - \sum_{k'} FC_{k\to k'}$$

where $FC_{k\to k'}$ is the sum of all fixed cost payment of firms from k that access market k'.

We are going to prove the formula for country A and th esame logic applies for the other countries.

First, let me express the fixed cost payment for each destinations:

$$FC_{A\to A} = \mathcal{M}_A \frac{f_{AA}}{Z_A}\left[PB_{A,A}\left(\varphi_{A,A}^{-\gamma} - \varphi_{A,AB}^{-\gamma}\right) + PB_{A,AB}\left(\varphi_{A,AB}^{-\gamma} - \varphi_{A,ABC}^{-\gamma}\right) + PB_{A,ABC}\varphi_{A,ABC}^{-\gamma}\right]$$

$$FC_{A\to B} = \mathcal{M}_A \frac{f_{AB}}{Z_A}\left[PB_{A,AB}\left(\varphi_{A,AB}^{-\gamma} - \varphi_{A,ABC}^{-\gamma}\right) + PB_{A,ABC}\varphi_{A,ABC}^{-\gamma}\right]$$

$$FC_{A\to C} = \mathcal{M}_A \frac{f_{AC}}{Z_A} PB_{A,ABC}\varphi_{A,ABC}^{-\gamma}$$

Now, we express $R_{A,s}$ the total revenues of all firms in A with the international strategy s. Using the slopes $S_{A,k}$, we have

$$\begin{aligned} R_{A,A} &= \mathcal{M}_A \tfrac{\gamma}{\gamma-(\sigma-1)} S_{A,A} \left(\varphi_{A,A}^{\sigma-\gamma-1} - \varphi_{A,AB}^{\sigma-\gamma-1} \right) \\ R_{A,AB} &= \mathcal{M}_A \tfrac{\gamma}{\gamma-(\sigma-1)} S_{A,AB} \left(\varphi_{A,AB}^{\sigma-\gamma-1} - \varphi_{A,ABC}^{\sigma-\gamma-1} \right) \\ R_{A,ABC} &= \mathcal{M}_A \tfrac{\gamma}{\gamma-(\sigma-1)} S_{A,ABC} \varphi_{A,ABC}^{\sigma-\gamma-1} \end{aligned}$$

Total revenues of all firms in A is then the sum og the three terms above. Rearranging the terms, one gets

$$R_A = \mathcal{M}_A \frac{\gamma}{\gamma-(\sigma-1)} \times \left[\varphi_{A,A}^{\sigma-\gamma-1} S_{A,A} + \varphi_{A,AB}^{\sigma-\gamma-1} (S_{A,AB} - S_{A,A}) + \varphi_{A,ABC}^{\sigma-\gamma-1} (S_{A,ABC} - S_{A,AB}) \right]$$

Replacing the thresholds by their expression in 2.8, we get

$$\begin{aligned} R_A &= \mathcal{M}_A \tfrac{\gamma}{\gamma-(\sigma-1)} \Bigg[\varphi_{A,A}^{-\gamma} \sigma \tfrac{PB_{A,A}}{Z_A} f_{AA} + \varphi_{A,AB}^{-\gamma} \sigma \left(\tfrac{PB_{A,AB}}{Z_A} (f_{AA} + f_{AB}) - \tfrac{PB_{A,A}}{Z_A} f_{AA} \right) + \\ &\qquad \varphi_{A,ABC}^{-\gamma} \sigma \left(\tfrac{PB_{A,ABC}}{Z_A} (f_{AA} + f_{AB} + f_{AC}) - \tfrac{PB_{A,AB}}{Z_A} (f_{AA} + f_{AB}) \right) \Bigg] \end{aligned}$$

Again, rearranging terms, we can recognize

$$R_A = \frac{\gamma\sigma}{\gamma-(\sigma-1)} \left(FC_{A\to A} + FC_{A\to B} + FC_{A\to C} \right)$$

Finally, using this expression of $\sum_{k'} FC_{A\to k'}$ into the definition of profits, we get

$$\Pi_A = \frac{\sigma-1}{\gamma\sigma} R_A$$

Chapter 3

Employment in a Network of Input-Output Linkages

By François de Soyres and Shekhar Tomar[1]

Abstract

What is the consequence of a technological improvement in one sector on employment in sectors located downstream in the supply-chain? On the one hand, if material and labor are gross substitute in the production function, the price decrease for the former tends to reduce labor demand for the latter *per unit produced*. On the other hand, the upstream positive technological shock also increases the number of unit produced through a decrease in the marginal cost. The net effect on employment simply depends on the ratio between the elasticity of substitution in the production function and the price elasticity of demand. We estimate those parameters at the sector level using detailed French data and show that employment sensitivity of sectors following a decrease in their material input price are very heterogeneous. Consequences for forecasting the effect of an increase in machine efficiency are discussed.

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3.1 Introduction

Recent developments in the network literature have highlighted the important role of input-output linkages in amplification of sector level shocks. In an interconnected economy, an increase in the efficiency of one sector yields benefits for all other sectors, the magnitude of which depends on the detailed network structure. While those sectoral spillovers are always positive when one looks at gross output production, this is not necessarily the case for usage of one particular input such as labor, or for value added production in general.[2] Following a decrease in the price of a material input, firms and sector re-optimize their input mix in production as well as their production scale.

In this paper, we start by showing in a simple theoretical framework that the consequence of a technological innovation for labor demand in downstream sectors depends on two key elasticities: (i) the elasticity of substitution between labor and material input and (ii) the price elasticity of demand. The first parameter captures the change in the input mix due to a change in relative prices, while the second captures the size of increased sales attracted by the sector following the decrease in marginal cost. Indeed, if total sales is fixed, the employment consequences of a decrease in the price of intermediate input depends solely on the gross substitutability or complementarity between labor and material inputs in the production function. However, when firms increase the share of the cheaper input in their production basket, they also decrease their production costs and hence their price, hereby attracting new customers. Such a change in the scale of production counteracts the reduced labor share per unit produced, so that the overall direction of labor demand is ambiguous. We show that the two elasticities described above are the only parameters one needs to estimate in the data in order to make prediction for employment changes.

We then exploit a very detailed dataset of French firms and estimate those elasticities separately for each sector. Using a panel of French firms matched with employee level data, we can estimate the value of those two parameters using within firms variations and construct a value for the sector level elasticities. Finally, we put together our theoretical and empirical results together and compute the degree of sensitivity of employment in each sector with respect to technological improvements.

[2] Acemoglu and Restrepo (2016) study "the race between machine and man" and the consequences of automation (which could be seen as a technological improvement in the sector producing robots) for labor share.

Our paper is related to several strands of literature on production networks. First, on the theoretical side, after the seminal contribution by Long and Plosser (1983) on business cycles in a network economy, many papers have been interested in the sectoral origin of aggregate volatility in output, diluting the original "diversification" argument, according to which idiosyncratic sectoral or firm-level shocks should wash out in the aggregate due to law of large numbers. Carvalho (2011), Acemoglu et al. (2012) and others have provided necessary conditions under which a networked economy with input-output linkages is able to amplify sectoral shocks to create aggregate fluctuations. While many studies focus on volatility or gross output, we are specifically interested in the employment consequences of technological shocks in the context of input-output linkages.

On the empirical side, Atalay (2015) and Foerster et al. (2011) have tried to test the predictions of these theoretical models and estimate the contribution of sectoral shocks to aggregate volatility. Giovanni et al. (2014) use detailed firm level data from France for this decomposition exercise. More recently Barrot and Sauvagnat (2016) use natural disaster as a proxy for firm level idiosyncratic shocks to study the propagation of these shocks from one firm to another and understand the impact on downstream *output* growth rates and spillovers.

Our paper is also closely related and contributes to the literature on estimating production elasticities. Oberfield and Raval (2016) use cross-sectional firm level balance sheet data to estimate sector level substitution elasticity in the production function. In the absence of precise firm level efficiency wage information, they had to use area level efficiency wages for their estimations. We improve upon their methodology by using detailed firm level data which allows us estimation of firm level efficiency wage, which can be directly used in the estimation of production elasticity. There is another important contribution in the empirical section, which is estimation of substitution elasticity between material inputs and capital-labor. A large part of the previous literature has primarily focused on estimating substitution elasticity between capital and labor but we document the other elasticity as well. Our results highlight substantial heterogeneity in elasticity of substitution between material inputs and capital-labor across sectors.

The rest of the paper is organized as follows. In the next section, we lay down our theoretical framework and derive a prediction for the employment change in a sector following a decrease in the price of its material input. This elasticity depends on two key elasticities for which we derive estimation equations. In section three, we present our dataset and empirical strategy and derive estimated value for the production and the demand elasticities for many sector. We also present consolidated result for sector's sensibility to upstream shocks. Section four offers concluding remarks and avenues for future research.

3.2 Theoretical Framework

This section develops a simple theoretical framework of firms and sectors employment decisions which forms the basis of our empirical work.

3.2.1 Basics

We consider a sector k populated by a large number of identical firms producing a good Y_k using three inputs: labor L_k, capital K_k and material inputs M_k bought from other firms. Since all firms within sector k are symmetric, there is no need for firm specific index and we simply write the generic production function for all firms in sector k as:

$$Y_k = \left[\mu^{\frac{1}{\epsilon_P}} V_k^{\frac{\epsilon_P - 1}{\epsilon_P}} + (1-\mu)^{\frac{1}{\epsilon_P}} M_k^{\frac{\epsilon_P - 1}{\epsilon_P}}\right]^{\frac{\epsilon_P}{\epsilon_P - 1}} \tag{3.1}$$

where $V_k = K_k^{\alpha} L_k^{1-\alpha}$ is a Cobb-Douglas aggregate of labor and capital and $\epsilon_P > 0$ is the elasticity of substitution between basic factors of production and material inputs. μ is a weight that controls for the spending share of basic production factors vis-a-vis material input. As will be clear later, assuming that those weights are constants means that we do not need to estimate them as we use time differences in order to relate changes in relative input prices to changes in relative input usage. Sector k faces an aggregate demand curve characterized by a price elasticity of ϵ_D, such that:

$$D(p_k) = D_0(p_k)^{-\epsilon_D} \tag{3.2}$$

where p_k is the price of sector k's good and ϵ_D is the price elasticity of demand. We do not model further the demand side in the market for goods produced by sector k but rather posit the existence of an aggregate demand function with a (locally) constant price elasticity. Such demand can potentially come from other industries of from final consumers. Sector k is monopolistic and choses its price in order to maximize profit. Since it faces a demand with a constant price elasticity ϵ_D, standard derivations lead to a price at a constant markup over marginal cost:

$$p_k = \frac{\epsilon_D}{\epsilon_D - 1} MC_k = \left(\mu v_k^{1-\epsilon_P} + (1-\mu) q_k^{1-\epsilon_P}\right)^{\frac{1}{1-\epsilon_P}} \tag{3.3}$$

where MC_k is the marginal production cost of sector k which is equal to the price index dual to the CES aggregation in the production function (3.1). We denote by q_k is the price of material inputs and v_k is the price of the $K-L$ bundle which is defined by:

$$v_k = \frac{r_k^\alpha}{\alpha^\alpha} \cdot \frac{w_k^{1-\alpha}}{(1-\alpha)^{1-\alpha}} \tag{3.4}$$

Let us now invert the demand curve (3.2) to get:

$$p_k = D_0^{-\frac{1}{\epsilon_D}} \times Y_k^{-\frac{1}{\epsilon_D}} \tag{3.5}$$

Firms in sector k take input price as given and chose L_k, K_k and M_k in order to maximize their profit, given the demand they are facing.[3] We first model the optimal choice of material input and the capital labor bundle and we will characterize separately the demand for labor and capital below. In this context, total profits in sector k are equal to total revenues minus total costs which can be written:

$$\Pi = p_k Y_k - v_k V_k - q_k M_k = D_0^{-\frac{1}{\epsilon_D}} \times Y_k^{\frac{\epsilon_D - 1}{\epsilon_D}} - v_k V_k - q_k M_k \tag{3.6}$$

where we replace the price using the inverse demand curve (3.5), implying that firms do not take their output price as given. The associated first order conditions are:

$$\{V_k\}: \qquad D_0^{-\frac{1}{\epsilon_D}} Y_k^{-\frac{1}{\epsilon_D}} \frac{\partial Y_k}{\partial V_k} = v \tag{3.7}$$

$$\{M_k\}: \qquad D_0^{-\frac{1}{\epsilon_D}} Y_k^{-\frac{1}{\epsilon_D}} \frac{\partial Y_k}{\partial M_k} = q \tag{3.8}$$

As is usual, firms use production inputs until the marginal revenue product associated with each input equals their price they are paying for those inputs. It is apparent from the above equations that the price elasticity of demand is an important parameter in shaping input demand,

[3] It is equivalent to solve the problem in two step, wherein the first step firms chose inputs to minimize their production price and in a second step they chose their price to maximize profit.

stemming from the fact that the marginal revenue product associated with hiring an additional unit of any input is governed by (i) the impact this additional input has on production cost and (ii) the impact this change in marginal cost and hence on pricing has on final revenue. Using the production function (3.1), we can have an expression for the partial derivative of Y_k with respect to each input. In particular, we have:

$$\frac{\partial Y_k}{\partial V_k} = \left[\mu^{\frac{1}{\epsilon_P}} V_k^{\frac{\epsilon_P - 1}{\epsilon_P}} + (1-\mu)^{\frac{1}{\epsilon_P}} M_k^{\frac{\epsilon_P - 1}{\epsilon_P}}\right]^{\frac{1}{\epsilon_P - 1}} \times \mu^{\frac{1}{\epsilon_P}} \times \frac{\epsilon_P - 1}{\epsilon_P} \times V_k^{\frac{-1}{\epsilon_P}}$$

Using this expression as well as the corresponding equation for $\frac{\partial Y_k}{\partial M_k}$), we can combine the first order conditions and obtain an expression of the ratio of basic input to material demand:

$$\frac{V_k}{M_k} = \frac{\mu}{1-\mu} \times \left(\frac{v_k}{q_k}\right)^{-\epsilon_P}$$

Note that we can also rewrite this equation in terms of factor payment rather than quantity, which will prove useful when we estimate the elasticity of substitution in the next section. Multiplying the above equation on both sides by the ratio of factor prices, yields:

$$\frac{v_k V_k}{q_k M_k} = \frac{\mu}{1-\mu} \times \left(\frac{v_k}{q_k}\right)^{1-\epsilon_P} \tag{3.9}$$

Finally, replacing v_k by its expression in (3.4) and using the fact that total payment to the bundle V_k is simply equal to payment to labor and capital, we obtain:

$$\frac{r_k K_k + w_k L_k}{q_k M_k} = \frac{\mu}{1-\mu} \cdot \left(\alpha^{-\alpha}(1-\alpha)^{\alpha-1}\right)^{1-\epsilon_P} \times \left(\cdot \frac{r_k^{\alpha} w_k^{1-\alpha}}{q_k}\right)^{1-\epsilon_P} \tag{3.10}$$

3.2.2 Hat algebra

We want to get closed form solutions for the percentage changes of variables when there is a positive technological shock in sector located upstream to sector k. In our framework, such a shock would affect sector k's input choice and sales through its impact on the price of material input q_k and ultimately on the marginal production cost. Our goal is to show that the change in employment in sector k depends on two key elasticities, ϵ_P and ϵ_D, which we will then estimate for many sectors using detailed French data.

We denote $\widehat{x}$ the percentage change of any variable x ($\widehat{x} = \frac{dx}{x} = d\log(x)$). Starting with the FOCs in sector k, we first substitute the expression of Y_k using the production function and then log-linearize (3.8) and obtain (assuming that D_0, μ and all elasticities are parameters that do not change):

$$\left(\frac{\epsilon_P(\epsilon_D-1)}{(\epsilon_P-1)\epsilon_D}-1\right)\cdot\frac{\epsilon_P-1}{\epsilon_P}\cdot\left(\frac{\mu^{\frac{1}{\epsilon_P}}V_k^{\frac{\epsilon_P-1}{\epsilon_P}}}{\mu^{\frac{1}{\epsilon_P}}V_k^{\frac{\epsilon_P-1}{\epsilon_P}}+(1-\mu)^{\frac{1}{\epsilon_P}}M_k^{\frac{\epsilon_P-1}{\epsilon_P}}}\widehat{V_k}+\frac{(1-\mu)^{\frac{1}{\epsilon_P}}M_k^{\frac{\epsilon_P-1}{\epsilon_P}}}{\mu^{\frac{1}{\epsilon_P}}V_k^{\frac{\epsilon_P-1}{\epsilon_P}}+(1-\mu)^{\frac{1}{\epsilon_P}}M_k^{\frac{\epsilon_P-1}{\epsilon_P}}}\widehat{M_k}\right)-\frac{1}{\epsilon_P}\widehat{M_k}=\widehat{q_k} \tag{3.11}$$

where we used the usual formula: $\widehat{(x+y)} = s_x\widehat{x} + s_y\widehat{y}$, with $s_x = \frac{x}{x+y}$. The equivalent holds for the first order condition relative to V_k. Let us denote $s_V = \frac{\mu^{\frac{1}{\epsilon_P}}V_k^{\frac{\epsilon_P-1}{\epsilon_P}}}{\mu^{\frac{1}{\epsilon_P}}V_k^{\frac{\epsilon_P-1}{\epsilon_P}}+(1-\mu)^{\frac{1}{\epsilon_P}}M_k^{\frac{\epsilon_P-1}{\epsilon_P}}}$ and equivalently for s_M. Combining the log linear transformations of first order conditions yields the following expressions for $\widehat{V_k}$:

$$\widehat{V_k} = \frac{\epsilon_P^2}{1-(\epsilon_P-1)\left(\frac{\epsilon_P(\epsilon_D-1)}{(\epsilon_P-1)\epsilon_D}-1\right)}\cdot \left[\left(\frac{\epsilon_P-1}{\epsilon_P}\left(\frac{\epsilon_P(\epsilon_D-1)}{(\epsilon_P-1)\epsilon_D}-1\right)s_M-\frac{1}{\epsilon_P}\right)\widehat{v_k}-\frac{\epsilon_P-1}{\epsilon_P}\left(\frac{\epsilon_P(\epsilon_D-1)}{(\epsilon_P-1)\epsilon_D}-1\right)s_M\widehat{q_k}\right] \tag{3.12}$$

We further assume that changes in wage and rental rate of capital are uncorrelated to shocks to the price of material inputs.[4] Taking the expectation over all possible realizations of shocks to upstream sectors, the above expression then simplifies to:

$$\mathbb{E}\widehat{V_k} = \frac{-\epsilon_P^2}{1-(\epsilon_P-1)\left(\frac{\epsilon_P(\epsilon_D-1)}{(\epsilon_P-1)\epsilon_D}-1\right)}\cdot\frac{\epsilon_P-1}{\epsilon_P}\left(\frac{\epsilon_P(\epsilon_D-1)}{(\epsilon_P-1)\epsilon_D}-1\right)\cdot s_M\mathbb{E}\widehat{q_k}$$

Furthermore, using the Cobb-Douglas nature of the $K-L$ bundle, total spendings on labor is simply equal to a share $(1-\alpha)$ of total spendings for the bundle V_k. In turn, this yields a simple

[4] This approach is similar to what is done in Amiti et al (2013). Alternatively, we could assume that shocks to the price of material input are small and the rest of the economy is large enough so that both wages and rental rate of capital are fixed exogenously. In such a case, we would not need to take expectations to get rid of wages and rental rate changes.

relationship between proportional changes in L_k and proportional changes in V_k:

$$w_k L_k = (1-\alpha) v_k V_k \qquad \Longrightarrow \qquad \widehat{L_k} = \widehat{V_k}$$

Finally, after rearranging and considering a positive technological shock upstream, triggering a decrease in the associated price price $\widehat{q_k} = -1$, the change in K and L usage is given by:

$$\widehat{L_k} = \frac{\epsilon_P(\epsilon_P - 1) \cdot \left(\frac{\epsilon_P(\epsilon_D - 1)}{(\epsilon_P - 1)\epsilon_D} - 1 \right)}{1 - (\epsilon_P - 1) \left(\frac{\epsilon_P(\epsilon_D - 1)}{(\epsilon_P - 1)\epsilon_D} - 1 \right)} s_M \tag{3.13}$$

As can be shown numerically, the expression above is strictly positive if and only if

$$\epsilon_D > \epsilon_P$$

Proposition 1

In our partial equilibrium analysis,[5] a technological improvement in a sector k' located upstream to k in the supply chain leads to an increase in employment in k if and only if the elasticity of substitution between the $K - L$ bundle and material inputs ϵ_P is lower than the price elasticity of aggregate demand faced by sector K, ϵ_D.

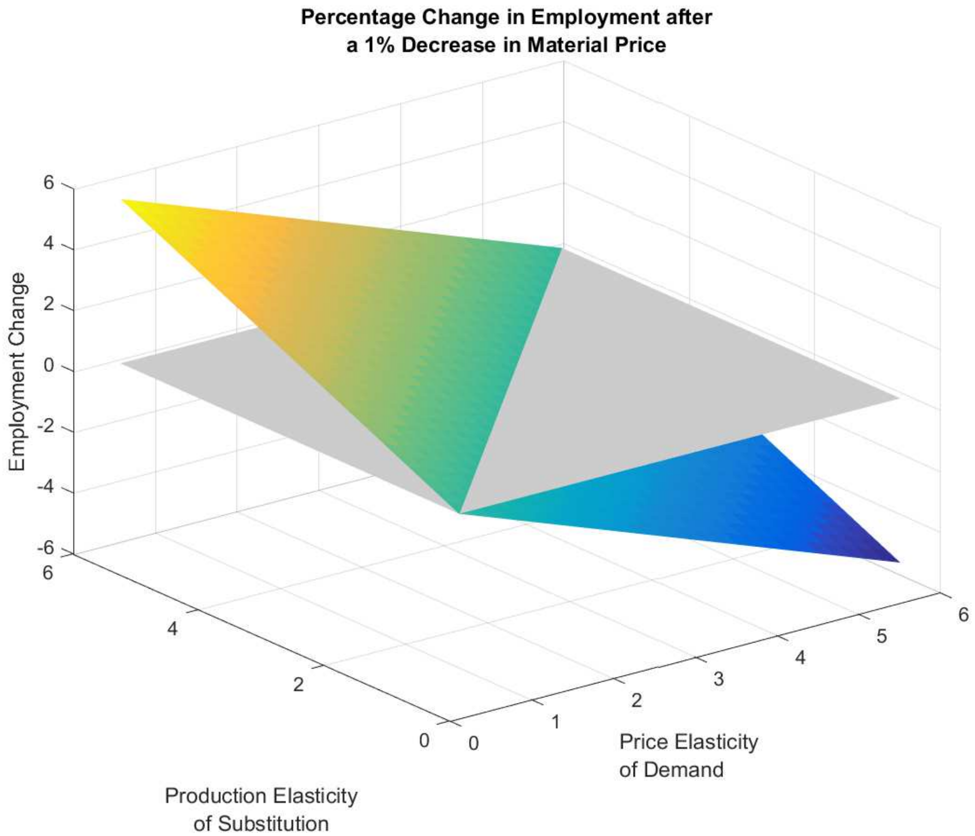
3.2.3 Numerical Explorations

In this section, we quantitatively investigate the consequences of varying the values of the two key parameters: the demand elasticity ϵ_D and the production elasticity ϵ_P. We consider the partial equilibrium model described above with a fixed wages and rental rate of capital, and compute the value of the proportional change in employment after a decrease in material price using equation (3.13). The graph below plots the value of $\widehat{L_k}$ in the vertical axis as a function of both ϵ_P and ϵ_D. We added in grey the reference surface defined by $z = 0$. We see that whenever $\epsilon_D > \epsilon_P$, employment in sector k *increases* when the price of material input *decrease*s, even when labor is gross substitute with materials in the production function.

When $\epsilon_P > 1$, the $K - L$ is gross substitute with materials in the production function. A decrease in material price gives incentives to firms in sector k to demand more material inputs

[5] with fixed wages and rental rate of capital

Figure 3.1: Numerical explorations for the elasticity of employment



relative to labor and capital for each unit produced and the spending *share* of labor decreases which triggers a decrease in marginal cost and hence in the price p_k. Moreover, when $\epsilon_D > \epsilon_P > 1$, the decrease in price leads to an increase in sales that more than overturns the decrease in labor demand. Even though the share of spending devoted to labor decreases, total spendings on all inputs increases more strongly so that the net effect is an increase in total demand for labor. Hence, employment in sector k increases as a result of the positive technological shock upstream. The result is obviously reversed when $\epsilon_D < \epsilon_P$ and the sales increase does not compensate the decrease in labor share, resulting in a net decrease in employment.

When $\epsilon_P < 1$, labor is gross complement with materials in the production function. In such a case, the drop in material price triggers an increase in the labor and capital share in spendings.

3.2.4 Estimation Equations

Before presenting our empirical section, we simply lay down the estimation equations delivered by our theoretical framework. We start by presenting the equations tat enable us to estimate the production elasticity in two different framework and then turn to the demand elasticity.

Looking first at the nested CES production function described above, profit maximization in sector k yields the usual relationships relating relative spending and relative prices:

$$\frac{r_k K_k + w_k L_k}{q_k M_k} = \frac{\mu}{1-\mu} \cdot \left(\alpha^{-\alpha}(1-\alpha)^{\alpha-1}\right)^{1-\epsilon_P} \times \left(\cdot \frac{r_k^{\alpha} w_k^{1-\alpha}}{q_k}\right)^{1-\epsilon_P}$$

Taking the logarithm and considering differences over time within a firm in sector k yields the relationship between *changes* in relative spending and *changes* in relative prices:

$$\Delta \log \left(\frac{r_k K_k + w_k L_k}{q_k M_k}\right) = (1-\epsilon_P) \left(\alpha \Delta \log \left(\frac{r_k}{q_k}\right) + (1-\alpha) \Delta \log \left(\frac{w_k}{q_k}\right)\right) \tag{3.14}$$

Alternatively, positing a production function in which labor and material inputs are directly aggregated in a CES form (without a first step aggregating labor and capital), firms' optimization problem leads to a slightly different realtionship. In particular, without a $K - L$ bundle, the share of labor relative to capital disappears from the relative forst order conditions for labor and material, leading to the following estimation equation:

$$\Delta \log \left(\frac{w_k L_k}{q_k M_k} \right) = (1 - \epsilon_P) \Delta \log \left(\frac{w_k}{q_k} \right) \tag{3.15}$$

Finally, in order to get an indirect measure of the price elasticity of demand, we measure the price cost margin (PCM) in each sector and invert it to get a measure of ϵ_D using

$$\epsilon_D = \frac{PCM}{PCM - 1}$$

3.3 Empirics

The consequence of positive technological shocks in sector i for any sector k located downstream of i in the supply chain is governed by two key elasticities. The goal of this section is to use detailed data on French firms in many industries in order to estimate those elasticities at the sector level. We focus on manufacturing sector corresponding to 1-digit level "C" under NAF nomenclature used by French statistical agency INSEE. There are a total of 24 manufacturing sectors at 2-digit level and our analysis focuses on a subset of 10 sectors for which the number of observation is sufficiently large in each year.

3.3.1 Data

There are two main sources of firm level datasets that we use for this exercise. Our first source of firm level data is the BRN which comes from the French fiscal administration.[6] It contains balance-sheet information collected from the firms' tax fillings as well as detailed information on the firms' balance sheets, including the value of total capital stock, the total wage bill, the average firm level wage rate. Our dataset also contains the sectors and the region in which the firm operates which is an important information given our estimation strategy.

We focus our analysis on the 7 years period stretching from 2003 to 2009 where all variables are labeled in Euro (and not French Francs). Initially, the dataset contains more than 500,000 firms per year. We get rid of firms employing less than 5 employees, which reduces the sample to about 300,000 firms per year.[7] The number of observation per year can be found in table

[6] BRN stands for Benefice Reel Normal, the normal tax regime for French firms.

[7] There are two reasons why we believe small firms should not be used in our estimation. First, firms employing a small number of employees have a "discretionary problem" as adjusting their labor force by one person can

3.1 and the number of observation per sector can be found in appendix, table **??**. Sectors are unevenly represented in our sample and some sectors contain a very low number of observation. We concentrate our analysis on sectors gathering at least 3,000 observations in total.

Table 3.1: Total Observations

	All	More than 5 employees	More than 10 employees
2003	$51,424$	$49,116$	$35,345$
2004	$50,977$	$48,421$	$34,476$
2005	$50,081$	$47,757$	$34,034$
2006	$49,187$	$46,999$	$33,573$
2007	$48,100$	$45,959$	$33,004$
2008	$45,560$	$42,611$	$31,722$

Note: From BRN and DADS merge (Only NAF "C" sectors)

The second dataset is DADS (Déclaration annuelles de données sociales), which collects matched employer-employee information and specifically wage information, which is important for our analysis. The data is collected from mandatory reporting of gross earnings by each firm to the French tax authority. The DADS is a subset of this income tax data, covering all individuals employed in French enterprises and who were born in October of even-numbered years. Each observation in DADS dataset corresponds to a matched employer-employee pair and contains information like number of days worked, total wage, occupation and other employee related information like age, sex etc. Each observation also contains employee id which allows for matching an individual across years as well as firm identifier (SIREN), which allows for matching DADS with BRN data. For the eight years of our analysis, the resulting dataset has roughly 200 million matched employer-employee observations.

In the end we merge the two sources of datasets, BRN and DADS, to get final dataset with approximately $50,000$ observations per year as shown in table 3.1. This is the final merged data we use for running firm level regressions to estimate effect of wages on input choice of firms.

3.3.2 Estimation: Elasticity of substitution

In order to identify elasticity of substitution between various inputs of the firm we use the log-linearized equation (3.14) and (3.15), which come from solving firm's profit maximization

constitute a sizable adjustment in their total wage payment, implying a non-continuous adjustment in their input mix. Second, small firms also experience a higher average growth rate and might expand their input base in a non optimal way based on recruitment opportunities.

problem in the case of a nested CES or full CES production function respectively. Accordingly, we will use two different specifications as baseline estimates of elasticity of substitution. The first comes directly from equation (3.14), which we call nested CES specification, where labor and capital as a bundle are substitutable with intermediates, and it gives us the following regression equation:

$$\log\left(\frac{rK+wL}{qM}\right)_{kjt} = (1-\epsilon_P)(1-\alpha_j)\log{(w)}_{kjt} + \gamma_k + \delta_t + CONTROLS + \epsilon_{kjt} \tag{3.16}$$

where LHS of equation (3.16) gives the ratio of capital and labor spending to intermediates for firm k in sector j at time t. The variable of interest here is the regression coefficient on firm level wage w_{kjt}. The wage used here is the efficiency wage for the firm after controlling for observable measures of skill and worker level characteristics. To estimate the efficiency wage for each firm we use matched employer-employee DADS data to construct residual wage for the the firms. [8] Since, we have firm level panel we can control for firm level heterogeneity by using firm fixed effects γ_k. The time fixed effects δ_t allow us to control for yearly variations across all firms. Lastly, all the regressions that we report controls for firm level variables like age, region, number of employees etc. It is important to note here that we are using firm level data here instead of plant level data because BRN reports capital and intermediate usage at the firm level and not plant level.

The identification in the above equation comes from exploiting the changes in within firm wages as well as across firms over time. Since we have observations for the same firm over multiple periods, we can get rid of bias coming from firm level skill differences or other observable and non-observable factors. Also, this specification allows for firms to have different rental rates of capital which will be captured by firm fixed effects γ_k, under the assumption that this rental rate does not change over time for a given firm.

As a second baseline, we use full (non-nested) CES specification, where capital, labor and intermediates enter the same CES production function with same elasticity of substitution ϵ_P across the three inputs. Log-linearizing profit maximizing condition for non-nested CES production function gives the following regression equation:

[8] The details of residual wage construction are given in Appendix B.

$$\log\left(\frac{qM}{wL}\right)_{kjt} = -(1-\epsilon_P)\log{(w)}_{kjt} + \gamma_k + \delta_t + CONTROLS + \epsilon_{kjt} \tag{3.17}$$

where equation (3.17) is similar to (3.16) with the only difference in LHS coming due to different specification for the production function. The identification strategy in this non-nested case is similar to the one used for nested production function. Since we are interested in difference between ϵ_P across different sectors, we run the above regressions separately for our selection of 10 2-digit sectors belonging to the "C" category, under 1-digit NAF industry classification. The results of our regressions are reported in the next sub section.

Using the French firm level dataset has many advantages and helps us overcome many of the problems persistent with the estimation of production elasticities. Firstly, it gives us information on matched employer-employee data with detailed worker characteristics. This helps us filter out skill and other worker level differences and get precise residuals at the firm level to calculate efficiency wage. Also, since we can match this information with balance sheet data of firms, we can run regressions as stated in equations (3.16) and (3.17) with firm level wages. Otherwise, the lack of individual worker level wage information in balance sheet data does not allow for precise measurement of efficiency wage. So, unlike Oberfield and Raval (2016) who use area level efficiency wage to circumvent this problem, we can directly use efficiency wage at firm level. Our approach thus improves upon their estimation method because we use precise firm level information rather than aggregated area level wages. Secondly, the panel dimension of the dataset allows us to control for individual firm level unobservable heterogeneity. Having firm fixed effects thus allows us to control for differential rental rate of capital across firms and thus we do not have to make strict assumption that capital is completely mobile across firms as in the case of Oberfield and Raval (2016). Thirdly, since we do not use area level wages in our primary regression, we can also control for regional heterogeneity. Oberfield and Raval (2016) instead had to use an IV to control for this regional heterogeneity. In our case, on the other hand we can already control for regional heterogeneity in firm location choice by including region fixed effects.

Endogeneity

The equations (3.16) and (3.17) allow us to estimate firm level elasticity of substitution using both within and across firm variation in wages over time. We make use of the panel dimension of our dataset and control for firm level fixed effects which capture any time invariant firm specific observed and unobserved characteristics. However, one must note that there are various potential sources for endogeniety bias in these regressions. Among others, adjustments costs to capital or labor my distort firm's input choices which would then deviate from the static cost minimization problem as well as impact the wage rates paid by the firm.

To solve potential endogeneity bias, we follow the approach developed in Oberfield and Raval (2016) and use area level efficiency wages instead of firm level wages. Since most firms are small and cannot impact area level wages, using area level wages is a way to correct for the bias arising from firm's deviation from its cost minimization problem as area level wages will be orthogonal to such deviation. Having said that, it is important to highlight that such bias in our case is much smaller than in case of Oberfield and Raval (2016). Indeed, the panel dimension of our dataset allows us to control for any time invariant firm level unobserved heterogeneity through firm fixed effects. Moreover, since Oberfield and Raval (2016) use a single cross-section of area level wages, they could not control for regional differences using region fixed effects. In our case, having access to a panel allows us to correct for this problem by using region fixed problems.

As a robustness check for our OLS results, we thus run two more specifications for each of the nested and non-nested CES case. The first specification is similar to Oberfield and Raval (2016) where we use area level efficiency wage w_{jt} in equations (3.16) and (3.17) instead of firm level wage w_{kjt}. [9] As a second robustness check, we use area level wage w_{jt} as an instrument for firm level wage w_{kjt}. The area level wage w_{jt} satisfies both the conditions necessary for being a good instrument. One, it is highly correlated with firm level efficiency wages once filtering out individual worker level characteristics and other skill level differences. Two, it also satisfies the exclusion restriction since using the panel dimension allows us to capture effects like firm location choice through firm fixed effect and hence error terms are much less likely to be correlated with the instrument.

Estimation Results

Overall, we report three different estimates for each of the two cases corresponding to the nested and non-nested estimation equations (3.16) and (3.17).

1. The first set of estimates is called OLS and correspond to a set of panel regressions for

[9] The area level wages are constructed by taking average across the residuals, across all workers in a given area, from the wage equation.

each sector with firm and year fixed effects for equations (3.16) and (3.17). In this case we use firm level wages as independent variable in the right hand side of the equations.

2. The second set of estimates is called Area and it corresponds to the estimates similar to the ones reported in Oberfield and Raval (2016). Here, instead of using firm level wages, we use area level wages, computed by taking average wage over all firms in a given area. Oberfield and Raval argue that such a specification is attractive because firms may find it costly to adjust capital or labor. Deviations of a firm's capital or labor from static cost minimization due to adjustment costs would then be in the residual, but should be orthogonal to the area level wage rate.

3. The third set of estimates is called IV. In this case, we use area level wages as an instrument for firm level wages.

The point estimates for different sectors in each of these six cases (2 production function assumption $\times$ 3 empirical specification for wages) are reported in table 3.4. Interestingly, there is very large heterogeneity across sectors for these point estimates. Moreover, point estimates and resulting elasticities are quite sensitive to the estimation procedure as well as the assumption about the production function.

Calculation: Elasticity of substitution in production

Equipped with these point estimates PE_j as reported in table 3.4 as well as the value of capital share α_j as reported in table 3.2 for each sector j, we back out the value of the elasticity of substitution in production for each sector. First, assuming a *full CES* aggregation of all factors of production in the production function,[10] the elasticity of substitution is simply given by

$$\epsilon_{P,j} = 1 - PE_j \tag{3.18}$$

with the variance of $\epsilon_{P,j}$ and PE_j being equal. Moreover, positing a nested CES form in the production function yields the following relation between point estimates and $\epsilon_{P,j}$:

$$\epsilon_{P,j} = 1 - \frac{PE_j}{1 - \alpha_j} \tag{3.19}$$

[10] Meaning that there is no $K - L$ bundle that is aggregated as a first step.

We then need to compute standard deviation for this variable. For a ratio $R = X/Y$ where X and Y are independent variables, we use the following formula[11] to compute the variance of R

$$V(R) = \frac{V(X)}{Y^2} + V(Y)\frac{X^2}{Y^4}$$

Hence, noting $\sigma_{\alpha,i}$ and $\sigma_{PE,i}$ the standard errors of α_i and PE_i respectively, the standard error of the production elasticity for sector i is given by

$$\sigma_{\epsilon_P,j} = \sqrt{\frac{\sigma^2_{PE,j}}{(1-\alpha_j)^2} + \sigma^2_{\alpha,j}\frac{PE_j^2}{(1-\alpha_j)^4}} \tag{3.20}$$

As a result, our OLS estimates for the production elasticity are presented in graph 3.3 and 3.2 for the Nested and Non-Nested cases respectively. The first important result to notice here is that sectors are fairly heterogeneous in their elasticity of substitution. In the non nested case (figure 3.2) most of the estimates are not statistically different from one, implying a Cobb-Douglas structure between labour and material inputs as described in Long and Plosser (1983) or Acemoglu et al (2012). More interesting is the case of nested production function presented in figure 3.3. In such a framework with a two step aggregation, some sectors feature an elasticity of production between the $K - L$ bundle and material input larger than 1. This is particularly the case in the chemical industry (sector 20), metallurgy (sector 24) and machine equipment manufacturing (sector 28) which feature an estimated elasticity of substitution with a whole confidence interval lying strictly above one. According to our results, those sectors would then *decrease* the labor content *of each unit produced* in response to a positive shock in the upstream sector.

3.3.3 Estimation: Price elasticity of demand

To estimate the demand elasticity faced by each sector, we assume optimal price setting behavior under monopolistic competition where firms maximize their profits. Under this assumption, the markup of the firm is given in terms of its demand elasticity as $\epsilon_D/(\epsilon_D - 1)$. We estimate the markup across sector by ratio of firm level revenue to cost averaged across all firms in a given sector. To do this we only take firms whose markup lies in $(1, 2)$ and ignore the outliers. The results of this exercise are shown in figure 3.4 and table 3.5. Most of the sectors have a demand

[11]See in "Sampeling Techniques", 3rd Ed. by Cochrane (1977), page 183 for a proof

Figure 3.2: Production Elasticity for 10 2-digits sectors
Non-Nested Production, OLS

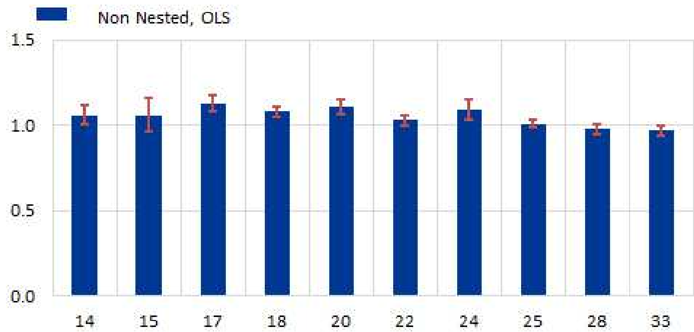


Figure 3.3: Production Elasticity for 10 2-digits sectors
Nested Production, OLS

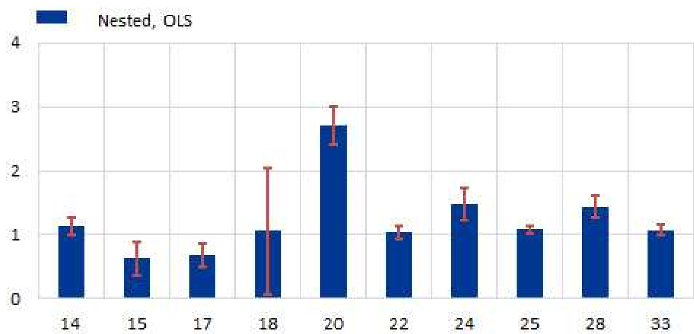
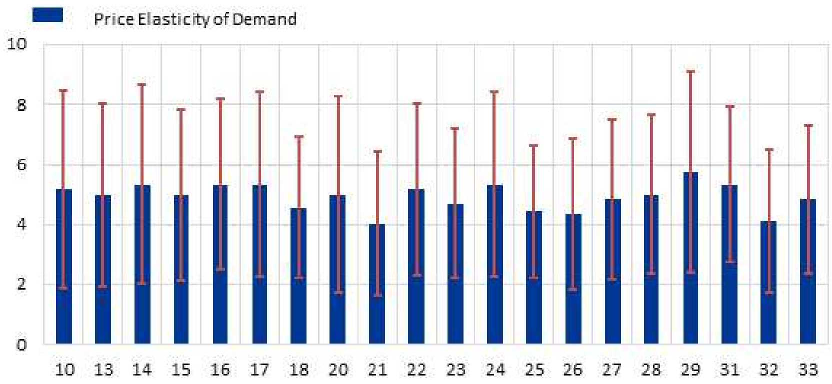


Figure 3.4: Price Elasticity of Demand for 20 2-digits sectors



elasticity between 4 and 5. This way of measuring demand elasticity is in line with other recent work in the literature.

3.4 Sectoral sensitivity to upstream shocks

The sensitivity of employment in sector i with respect to a shock to any sector k located upstream in the supply chain is governed by equation (3.13). As exposed in proposition 1, a positive shock upstream leads to an increase in employment in sector j if and only if:

$$\frac{\epsilon_P}{\epsilon_D} < 1$$

Panel Fixed Effects Regressions

In figures 3.5 and 3.6, we present our estimates of the above ratio for 10 sectors, revealing that the level of this employment elasticity is not constant across sectors under both the baseline estimates of nested and non-nested production function. In terms of *direction*, our OLS estimation yields the prediction that all sectors would increase employment following a positive technological shock upstream.[12] This results stems primarily from the fact that the demand elasticity for most of these sectors lie between 4 and 5 which implies a significatnt increase in the production scale for associated with a decrease in price and marginal cost.

In terms of *magnitude*, however, sectors present significant differences. The chemical industry

[12]Note that this is no longer true in our other specifications where we use area-level wages, see below

Figure 3.5: Ratios of production elasticity to demand elasticity
Non Nested Production, OLS

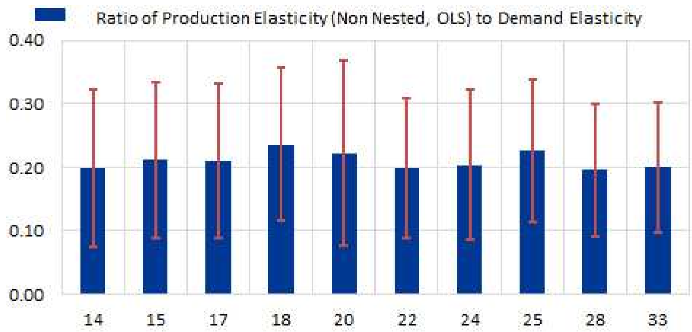
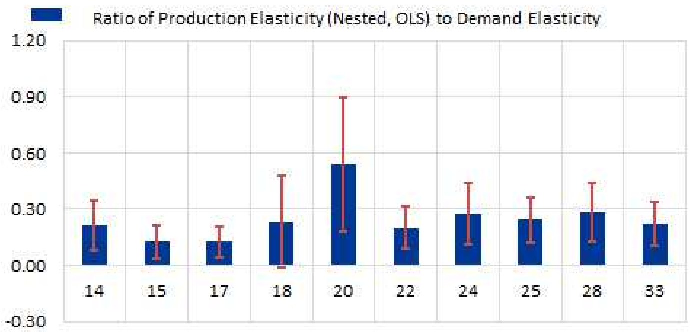


Figure 3.6: Ratios of production elasticity to demand elasticity
Nested Production, OLS



(sector 20) presents a higher ratio of elasticity in the production function to price elasticity of demand than other industries, revealing a lower employment reaction. On the other hand, the leather and shoes industry (sector 15) or the wood industry (sector 16) seems to be particularly sensitive to upstream technological shock with an employment reaction associated with upstream technological shock significantly higher than other industries.

Panel with Area Level wages

Using area level wages in lieu of firm level wages has a significant impact on the results, as presented in figure 3.7 for the non nested case and 3.8 for the case of a nested CES production function. In particular, the nested CES case feature several industries with a ratio of elasticities high enough that the threshold value of one lies in the confidence interval. According to our results, the chemical (sector 20) and metallurgy (sector 24) industries would then *decrease* their

Figure 3.7: Ratios of production elasticity to demand elasticity
Non Nested Production, Area

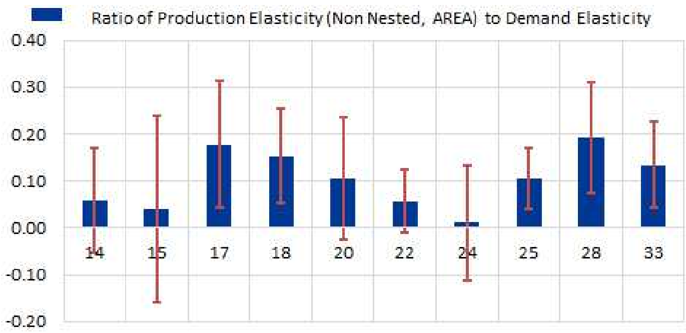
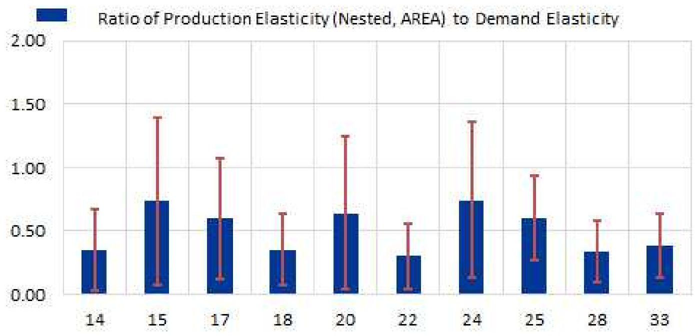


Figure 3.8: Ratios of production elasticity to demand elasticity
Nested Production, Area



employment in response to a positive shock in the upstream sector.

Panel with Area Level wages as Instruments

Finally, figures 3.9 and 3.10 present our estimated ratio of elasticities when the elasticity of substitution in the production function is computed using area level wages as instruments. The result is even more striking in this case as the ratio can take values as low as 0.1 for the clothing industry (sector 14) and as high as 1.2 for the chemical industry (sector 20). Those differences illustrate the fact that assuming identical production and demand elasticities across industries can potentially impose an important bias on the sensitivity of several sectors to upstream technological shocks.

Figure 3.9: Ratios of production elasticity to demand elasticity
Non Nested Production, IV

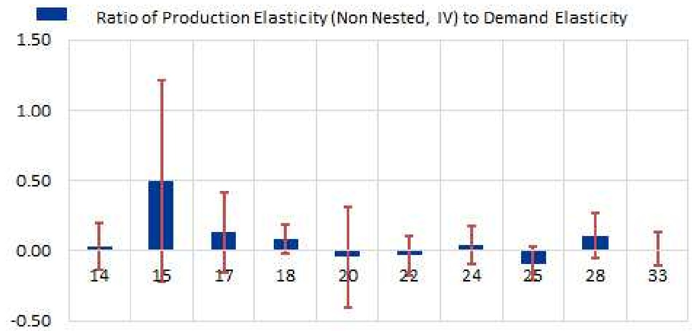
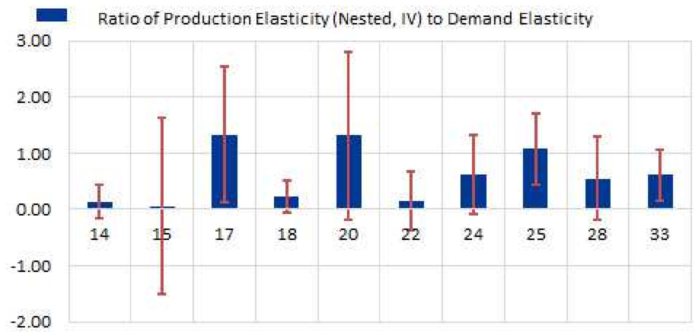


Figure 3.10: Ratios of production elasticity to demand elasticity
Nested Production, IV



3.5 Conclusion

This paper adds to our understanding on propagation of shocks within the network of interconnected sectors. In contrast to other papers in the literature, we highlight how a positive shock in an upstream sector does not always result in hiring more labor in downstream sectors. Using a theoretical model of production with labor, capital and material inputs, we show that the labor response can be summarized by the combination of two crucial elasticities, overall demand elasticity and elasticity of substitution in production, for a given sector.

Using detailed firm level data on French firms, we then estimate these elasticities separately for different industrial sectors. We find a high degree of heterogeneity across sectors for both demand and production elasticities. Interestingly, some sectors feature production elasticities that are high enough that a positive shock in an upstream sector would lead to a decrease in their labor usage. Intuitively, a decrease in the price of material inputs relative to labor triggers an important shift in the optimal input mix chosen by firms in those sectors, depressing labor demand for each unit produced. The associated increase in sales (due to a price decrease) and production is not large enough to compensate this negative force, leading to an overall decrease in employment while total revenues and gross output increase.

Using micro data on French firms in many industrial sectors allows us to take a highly disaggregated view and reveals a high degree of heterogeneity in both production and demand elasticities, leading to different sectoral responses to technological shocks. Such a result sheds a new light on the important debate on the consequences of automation and the digital revolution.[13] Abstracting for changes in the demand side of the economy, an increase in the relative efficiency of machines compare to labor would trigger different consequences across sectors. While textile, chemical or metallurgy might increase employment due to a strong reaction in the production scale, others would experience a contraction of labor usage.

[13]This theme is an important part of the 2017 French presidential campaign. See the excellent book "Le Monde est Clos et le Desir Infini" by Daniel Cohen for an analysis on this phenomenon.

3.6 Theoretical Appendix

3.6.1 Log-Linearization

The log-linear transformation of the first order condition with respect to V_k in firms' profit maximization can be written

$$\left(\frac{\epsilon_P(\epsilon_D-1)}{(\epsilon_P-1)\epsilon_D}-1\right)\cdot\frac{\epsilon_P-1}{\epsilon_P}\cdot\left(\frac{\mu^{\frac{1}{\epsilon_P}}V_k^{\frac{\epsilon_P-1}{\epsilon_P}}}{\mu^{\frac{1}{\epsilon_P}}V_k^{\frac{\epsilon_P-1}{\epsilon_P}}+(1-\mu)^{\frac{1}{\epsilon_P}}M_k^{\frac{\epsilon_P-1}{\epsilon_P}}}\widehat{V_k}+\frac{(1-\mu)^{\frac{1}{\epsilon_P}}M_k^{\frac{\epsilon_P-1}{\epsilon_P}}}{\mu^{\frac{1}{\epsilon_P}}V_k^{\frac{\epsilon_P-1}{\epsilon_P}}+(1-\mu)^{\frac{1}{\epsilon_P}}M_k^{\frac{\epsilon_P-1}{\epsilon_P}}}\widehat{M_k}\right)-\frac{1}{\epsilon_P}\widehat{V_k}=\widehat{v_k} \tag{3.21}$$

3.6.2 Nested-CES derivation

In this appendix, we relax the assumption of Cobb-Douglas aggregation between labor and capital and consider a nested CES production function as:

$$Y_k=\left[\mu^{\frac{1}{\epsilon_P}}\left(\alpha^{\frac{1}{\epsilon_V}}K_k^{\frac{\epsilon_V-1}{\epsilon_V}}+(1-\alpha)^{\frac{1}{\epsilon_V}}K_k^{\frac{\epsilon_V-1}{\epsilon_V}}\right)^{\frac{\epsilon_V}{\epsilon_V-1}\cdot\frac{\epsilon_P-1}{\epsilon_P}}+(1-\mu)^{\frac{1}{\epsilon_P}}M_k^{\frac{\epsilon_P-1}{\epsilon_P}}\right]^{\frac{\epsilon_P}{\epsilon_P-1}} \tag{3.22}$$

3.7 Empirical Appendix

3.7.1 Wage residuals

This section describes the calculation of wage residuals used in the main regressions throughout the paper. The wage residuals are generated by using DADS matched employer-employee dataset as described in detail in the data section. The DADS gives information on wages as well as other characteristics such as age, gender, occupation etc. of the individual in a given year.

The first step is to estimate a wage equation before aggregating the residuals at the firm or area level. We use the following regression to filter out the different effects:

$$\log w_{ikjt}=CONTROLS+O_i+F_k+R_{ikj}+\epsilon_{ikjt} \tag{3.23}$$

where we control for various individual level characteristics. Also, we control for occupation O_i of the individual to filter out the skill bias, as well as firm fixed effects F_k and region dummies R_{ikj}.

After running the above regression, we filter out the residuals to be further used in regressions reported in table 2. We back out the individual level residuals and then calculate average wage residual for each firm by averaging across workers from same firm. Similarly, for area level wage residual we average over all workers in a given geographical area.

Table 3.2: Capital share α_j for different sectors

Sector	Mean	Standard Deviation
10	0.67	0.14
11	0.77	0.14
12	0.79	0.12
13	0.59	0.20
14	0.49	0.19
15	0.49	0.17
16	0.62	0.17
17	0.65	0.17
18	0.56	0.17
19	0.71	0.13
20	0.67	0.18
21	0.71	0.17
22	0.62	0.18
23	0.64	0.18
24	0.66	0.18
25	0.56	0.17
26	0.51	0.19
27	0.52	0.19
28	0.51	0.18
29	0.56	0.17
30	0.56	0.18
31	0.52	0.17
32	0.52	0.17
33	0.44	0.17

Table 3.3: Point estimates from regressions in equations (3.16) and (3.17)

	Non-nested			Nested		
Sector	(OLS)	(Area)	(IV)	(OLS)	(Area)	(IV)
10	-0.16***	-0.04	-0.47	0.05	0.75**	3.71*
	(0.020)	(0.181)	(0.950)	(0.028)	(0.255)	(1.749)
11	-0.263***	-0.28	-0.88	0.26*	-0.63	-0.86
	(0.082)	(0.861)	(1.109)	(0.111)	(1.144)	(1.507)
12	-1.11	19.27	-11.86	0.41	-22.32*	14.12
	(1.191)	(10.26)	(25.16)	(1.030)	(7.571)	(31.12)
13	-0.03	-1.74**	-7.66	-0.18*	1.37*	6.41
	(0.046)	(0.556)	(6.068)	(0.057)	(0.672)	(5.681)
14	-0.06	0.69	0.84	0.00	-1.23*	-1.46
	(0.055)	(0.568)	(0873)	(0.061)	(0.620)	(1.006)
15	-0.06	0.80	-1.48	0.01	-1.43	-0.54
	(0.098)	(0.992)	(3.285)	(0.114)	(1.145)	(3.613)
16	-0.11***	-0.74*	-1.90	0.00	1.07*	3.34
	(0.039)	(0.353)	(1.700)	(0.047)	(0.423)	(2.372)
17	-0.127**	0.05	0.30	0.17**	-.29	-1.08
	(0.048)	(0.476)	(1.481)	(0.063)	(0.617)	(2.019)
18	-0.08**	0.30	0.63	0.03	0.33	0.44
	(0.028)	(0.298)	(0.452)	(0.036)	(0.380)	(0.568)
19	0.98*	4.09	10.29	1.01*	6.81	14.00
	(0.417)	(4.726)	(15.07)	(0.482)	(5.128)	(20.07)
20	-0.11*	0.47	1.22	-0.10	0.45	1.33
	(0.04)	(0.551)	(1.774)	(0.065)	(0.730)	(2.323)
21	-0.24**	-0.71	12.44	0.26	3.42	-35.67
	(0.118)	(1.124)	(32.33)	(0.151)	(1.433)	(87.62)
22	-0.03	0.71*	1.18	-0.04	-0.48	-0.33
	(0.029)	(0.304)	(0.700)	(0.040)	(0.404)	(0.881)
23	-.02	0.51	0.99	-0.04	-0.38	-0.46
	(0.053)	(0.505)	(1.020)	(0.067)	(0.645)	(1.278)
24	-0.09	0.94	0.80	-0.14	-2.09*	-1.92*
	(0.06)	(0.658)	(0.713)	(0.086)	(0.848)	(0.949)
25	-0.01	0.53**	1.40**	-0.06**	-0.40*	-1.12*
	(0.018)	0.177)	(0.505)	(0.020)	(0.200)	(0.548)
26	-0.05	-0.86	-0.72	0.02	0.54	-0.24
	(0.052)	(0.590)	(0.923)	(0.061)	(0.689)	(1.071)
27	0.12*	-0.21	-0.38	-0.29***	0.45	0.87
	(0.050)	(0.553)	(1.067)	(0.060)	(0.641)	(1.305)
28	0.02	0.04	0.46	-0.17***	0.00	-0.50
	(0.03)	(0.300)	(0.775)	(0.035)	(0.351)	(0.905)
29	-.09	0.58	6.68	-0.01	-0.95	-8.75
	(0.059)	(0.592)	(15.42)	(0.070)	(0.690)	(18.15)
30	-0.03	1.89	1.93	-0.21	-0.14	0.23
	(0.111)	(1.147)	(1.691)	(0.134)	(1.391)	(1.870)
31	-0.06	0.57	39.17	-0.04	0.49	14.35
	(0.042)	(0.402)	(227.6)	(0.051)	(0.473)	(86.76)
32	-0.06	-0.20	-2.12	-0.02	0.49	5.21
	(0.036)	(0.394)	(5.034)	(0.044)	(0.466)	(9.048)
33	0.03	0.35	0.95	-0.07*	-0.16	-0.77
	(0.028)	(0.290)	(0.578)	(0.031)	(0.314)	(0.626)

Table 3.4: Number of observations in above regressions

Sector	(1)	(2)	(3)	(4)	(5)	(6)
10	30085	37,326	30,084	30,890	37,331	30,089
11	1841	2,126	1,841	1,842	2,127	1,842
12	22	23	22	22	23	22
13	7,417	8,411	7,417	7,418	8,412	7,418
14	6,771	8,014	6,771	6,772	8,015	6,772
15	2,285	2,617	2,285	2,286	2,618	2,286
16	7,679	9,503	7,679	7,685	9,509	7,685
17	5,157	5,766	5,157	5,159	5,768	5,159
18	13,077	15,935	13,077	13,081	15,939	13,081
19	263	272	263	263	272	263
20	6,941	7,631	6,941	6,941	7,631	6,941
21	1,799	1,907	1,799	1,799	1,907	1,799
22	13,660	15,349	13,660	13,662	15,351	13,662
23	9,410	11,102	9,410	9,413	11,105	9,413
24	3,400	3,667	3,399	3,400	3,667	3,399
25	42,566	50,550	42,565	42,573	50,557	42,572
26	7,155	8,214	7,154	7,158	8,217	7,157
27	5,616	6,322	5,616	5,617	6,323	5,617
28	16,044	18,570	16,044	16,047	18,574	16,047
29	4,880	5,504	4,880	4,882	5,506	4,882
30	2,103	2,415	2,103	2,103	2,415	2,103
31	7,277	8,741	7,277	7,278	8,742	7,278
32	7,907	9,628	7,909	7,907	9,628	7,907
33	20,474	24,870	20,473	20,479	24,876	20,478

Table 3.5: Demand Elasticity ϵ_D

Sector	Markup(Mean)	Markup(SD)	ϵ_D(Mean)	ϵ_D(SD)
10	1.24	0.15	5.16	3.28
13	1.25	0.15	5.00	3.05
14	1.23	0.14	5.34	3.31
15	1.25	0.14	5.00	2.85
16	1.23	0.12	5.34	2.83
17	1.23	0.13	5.34	3.07
18	1.28	0.14	4.57	2.33
20	1.25	0.16	5.00	3.26
21	1.33	0.19	4.03	2.39
22	1.24	0.13	5.16	2.85
23	1.27	0.14	4.70	2.49
24	1.23	0.13	5.34	3.07
25	1.29	0.14	4.44	2.20
26	1.3	0.17	4.33	2.52
27	1.26	0.14	4.84	2.66
28	1.25	0.13	5.00	2.65
29	1.21	0.12	5.76	3.34
31	1.23	0.11	5.34	2.60
32	1.32	0.18	4.12	2.38
33	1.26	0.13	4.84	2.47

Table 3.6: Sector codes under NAF 1-digit "C"

Sector Code	Sector Name
10	Industries alimentaires
11	Fabrication de boissons
12	Fabrication de produits à base de tabac
13	Fabrication de textiles
14	Industrie de l'habillement
15	Industrie du cuir et de la chaussure
16	Travail du bois et fabrication d'articles en bois et en liège, à l'exception des meubles;
17	Industrie du papier et du carton
18	Imprimerie et reproduction d'enregistrements
19	Cokéfaction et raffinage
20	Industrie chimique
21	Industrie pharmaceutique
22	Fabrication de produits en caoutchouc et en plastique
23	Fabrication d'autres produits minéraux non métalliques
24	Métallurgie
25	Fabrication de produits métalliques, à l'exception des machines et des équipements
26	Fabrication de produits informatiques, électroniques et optiques
27	Fabrication d'équipements électriques
28	Fabrication de machines et équipements n.c.a.
29	Industrie automobile
30	Fabrication d'autres matériels de transport
31	Fabrication de meubles
32	Autres industries manufacturières
33	Réparation et installation de machines et d'équipements

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