A Voting Model of Federal Standards With Externalities[1]

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April 29, 2003

[1]We are grateful to LEESP, CNRS, and NSF for financial support. The paper has benefited from discussions with and comments from Jenna Bednar, Antoine Loeper, Hervé Moulin, Norman Schofield, and Barry Weingast. We also wish to acknowledge comments from seminar participants at California Institute of Technology, Harvard University, London School of Economics, New York University, Ohio State University, Princeton University, University of Aix-Marseille, University of Malaga, University of Pennsylvania, University of Toulouse, Williams College, and the 2002 meetings of the Public Choice Society. Any remaining shortcomings are our own.

Abstract

This paper proposes a framework for studying policy making in a federal system in the presence of spillover externalities. Local jurisdictions choose local policies by majority rule subject to standards that are set by majority rule at the federal level. We characterize the induced preferences of voters for federal policies, prove existence of local majority rule equilibrium, provide an example of nonexistence of global majority rule equilibrium, and explore the welfare properties of federal standards in the presence of spillovers.

1 Introduction

A common justification for the role of a federal government is to solve problems of externalities between the members of the federation. These externalities can take many forms, indeed it is difficult to imagine public policies that are immune to interjurisdictional externalities. Health and education policies are obvious examples, as are environmental, industrial, and agricultural regulation. Even policies that are nominally local, such as zoning laws and criminal statutes, have significant implications for the welfare of adjoining jurisdictions. If each district makes decisions independently, there will a failure to equate marginal social cost with marginal social benefit, due to the gap between private cost and benefits and social costs and benefits. The potential of a significant welfare-enhancing role of centralized policy making by a federal government is obvious. In fact, if one simply applies the same basic principles from which Coase argued for the merger of two firms when there are production externalities, then the logic is compelling. But are these the right principles to apply? In this paper, we demonstrate that this line of argument can be misleading.

On the one hand, the merger metaphor seems to capture the correct logic, since we are considering public goods, which will not be provided efficiently by markets due to free rider problems. While they can be locally provided, their effects spill over to other districts. Therefore, this puts us in a second best situation, and so traditional economic theory suggests that either mergers or rationally chosen taxes and subsidies should work. In fact, this traditional approach has spawned many papers in the fiscal federalism literature. A typical model of this genre follows roughly the following scenario. Local governmental units apply taxes and subsidies to finance the production of public goods, as a second best method for (partially) correcting for externality-induced inefficiencies. These externalities may be either direct, or indirect, for example due to congestion effects resulting from relocation of residents in response to taxes.[1] These taxes and subsidies are derived by maximizing a utilititarian social welfare function, usually assumed to be the same in all districts, subject to technological constraints, and market conditions of demand and supply. A noncooperative game ensues between the local districts, with each district taking as given the economic and fiscal behavior of the other districts. At a Nash equilibrium, each district chooses its own taxes and subsidies to maximize the same social welfare function, but applied only to their population. This equilibrium is then compared to

[1]See, for example, Gordon (1983) and the references cited there. A number of other papers look at issues related to mobility and "voting with your feet", in the tradition of Tiebout. See for example, Epple and Romer (1991).

a "cooperative" solution in which taxes and subsidies are decided centrally, in a manner that can rationally correct for the externalities between districts (subject, of course, to second-best considerations).[2]

On the other hand, at all levels of government, these decisions – taxes, subsidies, regulations, etc. – are made by political, not economic, institutions. This means that the natural mechanisms for aggregating preferences and deciding policy involve legislatures, elections, and voting, instead of firms, markets, buying, and selling. This key difference – the political dimension – suggests serious limitations of the standard economic approach. These limitations can be viewed as falling into two categories.

The first is normative: the efficiency problem is compounded by a preference aggregation problem. While, with some stretch of the imagination, one can treat firms as unitary actors, such an assumption is quite dubious in the case of voters, politicians, and governments. Voters typically have idiosyncratic preferences over policy choices, and these may differ systematically across jurisdictions. It is this heterogeneity that creates the preference aggregation problem, so federal and local policies are chosen by voting schemes which require a different approach than the standard normative analysis of externalities. In fact, the welfare function to maximize is itself determined endogeously by the political process. Consequently, different districts may implicitly optimize much different welfare functions, and the aggregation of these welfare functions into a "federation" welfare function in some cases may not even be well-defined.

Second, the mechanisms available in the political arena are not as rich as the mechanisms available in an economic setting. In particular, a feature that is virtually universal to political processes is that direct side payments are limited or, in some cases, altogether absent.[3] This changes the nature of equilibrium in the models and the nature of second best solutions. In particular, with voting mechanisms instead of side payments, equilibrium in the resulting game is driven by marginal actors who are pivotal in a voting game. In contrast, equilibrium in market games is determined by marginal utilities and costs, which are the driving forces behind standard efficiency concepts of either the first or second best variety. Unfortunately, there is no guarantee that the preferred policies of the pivotal voters, say the median

[2]Some of the same issues arise in problems of production externalities with multiproduct (or multiplant) production, where cartels or mergers can perform functions similar to those of a central government.

[3]This paper does not provide an explanation for this. There are, in fact, some possibilities for transfers, under the guise of campaign finance, vote trading, cross jurisdictional block grants, and other products of distributive politics. However, to a first approximation transfers can be viewed an very limited.

voters, lead to outcomes closely resembling classical economic efficiency.

In the context of locally provided public goods and multiple jurisdictions, there is even a third difficulty, in that the political decisions at the local and federal levels are dependent on each other. On the one hand, local jurisdictions are constrained in their policy choices by decisions at the federal level. But there are effects in the other direction as well, since federal policies are made by legislative policies that are composed of representatives of the various jurisdictions who anticipate the effect of federal policies on their jurisdictions.

This paper considers a very simple version of the problem of externalities with two levels of government, one level, which we call the local level, and another level, which we call the federal level. A local public good is provided and financed by each jurisdiction at the local level and there are positive spillovers across local jurisdictions.[4] Voters have single peaked preferences along a single issue dimension. Local jurisdictions make decisions independently by majority rule, taking into account the equilibrium policies of other jurisdictions. We assume the political process is open and competitive, so equilibrium outcomes are determined by the preference of the median voter in each district. In the autarchy equilibrium, without any federal policy, the public good is underprovided, relative to the optimal level computed using a utilitarian welfare function constructed from the utility of the median voter in each district. The policies of the federal government are limited to simple constraints on local policies, which we call *federal standards*. They impose lower bounds on the amount of public good that each district must provide. We assume that federal standards are chosen according to majority rule.

The presence of a federal government creates a two-stage game, where the federal standard is decided first by majority rule, followed by a noncooperative game between the median voters of local jurisdictions. Because the federal stage is followed by a local policy making stage, the voter induced preferences over federal standards is complicated. We characterize these preferences and show that they are generally multi-peaked, which can lead to a non-existence problem. However, by extending a result of Kramer and Klevorick (1975) we establish existence of a local majority rule equilibrium, and characterize the range of equilibrium outcomes.

We then illustrate the welfare effects of federal standards by comparing regimes with and without federal standards. When the spillover effects are small, federal standards lead to *worse* outcomes than the autarchy solution: equilibrium federal standards are set *too high* relative to the optimum. That is, while the autarchy solution results in underprovision of the public goods,

[4] The model can be easily translated into a model with negative spillovers.

federal standards overcompensate if the spillover effects are small.

We show that the regime with federal standards always leads to higher production of the public good. Hence, to a first approximation, the relevant consideration is whether or not the spillover effect is sufficiently large to warrant federal intervention. However, in general it is not possible to unambiguously sign the welfare effects, since there are systematic redistributive features to federal standards. In particular, low demanders of the public good from districts who are also relatively low demand districts are made worse off. Not surprisingly, it is the voters from high demand districts who are made better off. The reason for this is that federal standards create a constraint that is only binding on the lowest demand districts, and can actually lead to a reduction in production by the high demand districts.

This paper is not the first to model the political dimension of federalism issues. In Crémer and Palfrey (2000, 2002), we investigate political equilibrium models of federal standards in the absence of externalities. Bednar (2001) models the federation stability problem as a repeated game in which local jurisdictions can "cheat" on public policy agreements. A similar motivation lies behind the analysis of De Figueiredo and Weingast (2001) and the empirical work of Alesina and Spolaore (2002). Crémer and Palfrey (1996, 1999) characterize voter preferences over different rules of representation and degrees of centralization, derived from both individual and jurisdictional characteristics, and study the theoretical implications of these induced preferences for constitutional design. In a different vein, a number of papers are concerned with the issue of interjurisdictional redistribution and the efficiency implications of different federal structures.[5] Finally, there is a large literature in the Tiebout (1956) tradition that investigates mobility across jurisdictions which we do not consider here.[6]

2 The Basic Model

We consider a federation composed of D *districts*, where D is an odd integer greater than or equal to 3. Each district $d = 1, 2, \ldots, D$ has an odd number of voters, and we assume that each district has less than half of the total number of voters in the federation. Each district chooses a level $x_d \in \Re$ of a local public good with positive spillover across districts.

Specifically, we assume that the utility function of voter i in district d

[5]See for example Persson and Tabellini (1996).

[6]See for example Epple and Romer (1991) or Nechyba (1997).

(denoted (i,d)) is given by

$$u_{id}(x) = \begin{cases} t_{id} \ln(x_d + \beta X_{-d}) - x_d & \text{if } x_d + \beta X_{-d} > 0, \\ -\infty & \text{otherwise,} \end{cases}$$

where $X_{-d} \equiv \sum_{d' \neq d} x_{d'}$, $\beta \in (0,1)$ and $t_{id} > 0$.[7] The coefficient β measures the strength of the spillover effect.[8] The voter *type*, t_{id}, is exactly the ideal point of voter (i,d) if $x_{d'} = 0$ for all $d' \neq d$. Whatever X_{-d}, higher types prefer a higher level of the public good in their own districts than do lower types.

Let t_d be the median type in district d. For convenience, we assume that each district has a different median type, and that districts are labeled in order of their median type, so that

$$d < d' \Longleftrightarrow t_d < t_{d'},$$

and within district we assume that the index of voters is ordered by type, so that

$$i < i' \Longleftrightarrow t_{id} < t_{i'd} \ .$$

There is substitutability between production in one's own district and production in the other districts, and the spillover coefficient β measures the degree of substitutability.

2.1 Externality-induced preferences

Due to the spillover effects, a voter's preferences over local public good provision in their own district actually depend on the amount of the public good provided by the other districts. Given any profile of public good production by the other districts, $x_{-d} = (x_1, ...x_{d-1}, x_{d+1}, ...x_D)$, the *conditional ideal point* of voter (i,d), which we denote by $\widehat{x}_{id}(x_{-d})$, is obtained by differentiating u_{id} to get the first order condition

$$\widehat{x}_{id}(x_{-d}) = t_{id} - \beta X_{-d}.$$

The second order conditions for a maximum hold, so this characterizes i's ideal policy, given the policies in the other districts. There are several interesting features of these *externality induced preferences.* First, every voter's ideal point is decreasing in the public good levels of all other districts. This

[7]Implicitly, we are assuming that all districts are the same size, so that their externality effets are symmetric. This is easy to generalize, but at some cost to notation.

[8]The boundary case of $\beta = 0$ that corresponds to no externality was studied in Cremer and Palfrey (2000, 2002).

represents the *substitution effect* of the spillovers. The greater the spillover, the greater the substitution (free riding) effect.

It is also easy to see that all voters are better off as other districts produce more, since the externality is positive. It is this free rider problem that leads to the intuition that federal standards may increase efficiency. However, this free riding problem also leads to complex strategic interactions between the districts, since the (conditional) ideal point of the median voter in a district will depend on the policies adopted by the other districts. A second feature of the induced preferences is that the identity of the median voter in a district is independent of the policies of the other districts, since each ideal point is simply shifted downward by the constant, βX_{-d}. Because they are all shifted down, the order of the conditional ideal points is preserved.

3 Equilibrium without a federal policy

We first consider the case where there is no federal policy. That is, the districts are unconstrained, and each district is free to choose any policy level in their own district. As in Crémer and Palfrey (2000, 2002), the game within a district is modeled as a competitive outcome, driven by the preferences of the median voter. This can be rationalized as the equilibrium of a game between jurisdictions, where the preferences of each district are represented by the preferences of its median voter. A Nash equilibrium of this game will result in a profile of district policies, $x^* = (x_1^*, ..., x_D^*)$ such that, for all d, x_d^* is the conditional ideal point of the median voter in district d, given x_{-d}^*. Since the identity of the median voter in d does not depend on x_{-d}, the first order condition for district d, is

$$x_d^* = t_d - \beta X_{-d}^*,$$

or

$$(1-\beta)x_d^* = t_d - \beta X^*$$

where $X^* = \sum_d x_d^*$ is total public good production. The maximization problem is concave, so this is indeed a maximum. This condition implies a simple but important result. As a consequence of the substitution effect, increased public good levels in one district leads to lower public good provision in all other districts. Combining the first order conditions for all districts and solving gives:

$$X^* = \frac{\sum_d t_d}{1 + (n-1)\beta}$$

$$x_d^* = \frac{t_d - \beta \frac{\sum_d t_d}{1+(n-1)\beta}}{1-\beta} \tag{1}$$

3.1 Socially optimal production

We now show that in equilibrium total production is less than X^{**}, the socially optimal level. To show this, define a socially optimal profile of outputs as a vector $x^{**} = (x_1^*, ..., x_D^*)$ that maximizes the sum of the utilities of the median voters of all districts:

$$\sum_{d=1}^{D} \left(t_d \ln(x_d + \beta X_{-d}) - x_d\right).$$

The first order conditions for a maximum with respect to x_d are

$$\frac{t_d}{x_d^{**} + \beta X_{-d}^{**}} + \sum_{d' \neq d} \beta \frac{t_{d'}}{x_{d'}^{**} + \beta X_{-d'}^{**}} = 1$$

Because t_d and $x_d + \beta X_{-d}$ are strictly positive for all d

$$\frac{t_d}{x_d^{**} + \beta X_{-d}^{**}} < 1$$

which implies $t_d < x_d^{**} + \beta X_{-d}^{**}$. Summing these inequalities yields

$$\sum_{d=1}^{D} t_d < \sum_{d=1}^{D} \left(x_d^{**} + \beta X_{-d}^{**}\right)$$

which implies

$$X^{**} > \frac{\sum_d t_d}{1 + (n-1)\beta} = X^*.$$

4 Equilibrium with a federal policy

As shown above, the free riding problem results in underprovision of the public good when each district decides independently. There is a potential role for federal policy to remedy this problem. In this section, we consider the effect of a simple federal policy called a *standard.* The federal standard, denoted F, imposes a minimum level of the public good that must be produced by each district. There are many examples of such standards. For example, in environmental policy, the US federal government sets water quality standards, air quality standards, and emissions standards for automobiles and industrial plants. States and local jurisdictions in many cases tighten these standards, as does California in the case of emissions standards for new automobiles.

As in Crémer and Palfrey (2000, 2002), we model the federal standard setting process as the first stage of a two stage game. The second stage of the game is analyzed in the same way as in the previous section, but F distorts the induced preferences of the voters, so the equilibrium outputs change with F. This feeds back and changes the voter's induced preferences over F in the first stage.

4.1 Equilibrium in the Second Stage

This section characterizes the equilibrium in the second stage. Because of the nature of the free-riding problem, the imposition of a federal standard has subtle consequences for the inter-district equilibrium choices of x_d. In particular, x_d is not monotonic in F. That is, induced preferences can be either increasing or decreasing in F. However, we will show that $X^*(F)$, the equilibrium total output, is increasing in F.

Recall that the Nash equilibrium of the local standard setting game without federal standards is characterized by D equations of the form

$$x_d^* = t_d - \beta X_{-d}^*.,\ d = 1, 2, ...D.$$

With a federal standard, F, these D conditions become

$$x_d^*(F) = \max\{F, t_d - \beta X_{-d}^*(F)\},\ d = 1, 2, ...D. \tag{2}$$

The solution to this set of equations is unique for each F, and has several properties that are summarized below. First, for any district d, there is some value of F, at which the constraint that $x_d \geq F$ will become binding, given X_{-d}^*. We call this the *critical federal standard for district d.*

Definition 1 *The* ***critical federal standard for district*** *d, F_d, is the minimum value of F for which $x_d^*(F) = F$.*

These critical levels are endogenous, since the critical level of district d will depend on F *and* the public good production of the other districts, which depend on F, and so on. However, it is easy to show that the constraint is first binding on district 1, then district 2, and so forth, so that these levels exist. Furthermore, for any $d > 1$, the output of district d is strictly decreasing for $F \in (F_1, F_d)$. The reason an unconstrained district's output is decreasing in F is that the districts for which F is binding will be producing more, and d's reaction function is downward sloping.

Finally, it is straightforward to show that federal standards do indeed have the expected effect of increasing total output. Obviously total output below F_1 is constant, but total output is *strictly* increasing in F for $F > F_1$. To see this, from equation 2 simple algebra shows that for $d = 1, ..., D-1$, $F \in (F_d, F_{d+1})$,

$$X^*(F) = \frac{(1-\beta)d}{(1-\beta)+(D-d)\beta}F + \frac{\sum_{d'=d+1}^{D} t_{d'}}{(1-\beta)+(D-d)\beta}$$

which is clearly increasing in F. For $F > F_D$, $X^*(F) = DF$ which is also increasing in D. Furthermore, $X^*(F)$ is continuous, hence it is strictly increasing on (F_1, ∞) as claimed.

4.2 An example with three districts

In this subsection, we begin developing an example which illustrates the main points of the paper. There are three districts and three agents in each district, with $\beta = 0.9$. The median voters for the three districts are given respectively by $t_1 = 1$, $t_2 = 2$, and $t_2 = 3$. In each district, there is also a low demand voter of type 0.5 and a high demand voter who has type 5.

Figure 1 shows how the output of each district varies as a function of F.[9] For $F \leq F_1$, none of the districts is constrained and their output is constant. If $F \in (F_1, F_2]$, district 1 is constrained; its output is an increasing function of F, and the two other districts free ride on this increase as their own output is decreasing in F. When district 2 is constrained, the output of district 3 decreases with F at a higher rate. Note that the relationship between output and F only depends on the type of the median voter and not on the preferences of the other voters.

[9] Simple algebra yields $F_1 = -9.3$, $F_2 = -2.5$, and $F_3 = 1.1$. For $F \leq F_1$ we have $x_1 = -9.3$, $x_2 = 0.7$, $x_3 = 10.7$. For $F \in [F_1, F_2]$, we have $x_2 = -3.7 - 0.47F$ and $x_3 = 6.3 - 0.47F$. For $F \in [F_2, F_3]$, we have $x_3 = 3 - 1.8F$. Of course, whenever $F \geq F_i$, $x_i = F$.

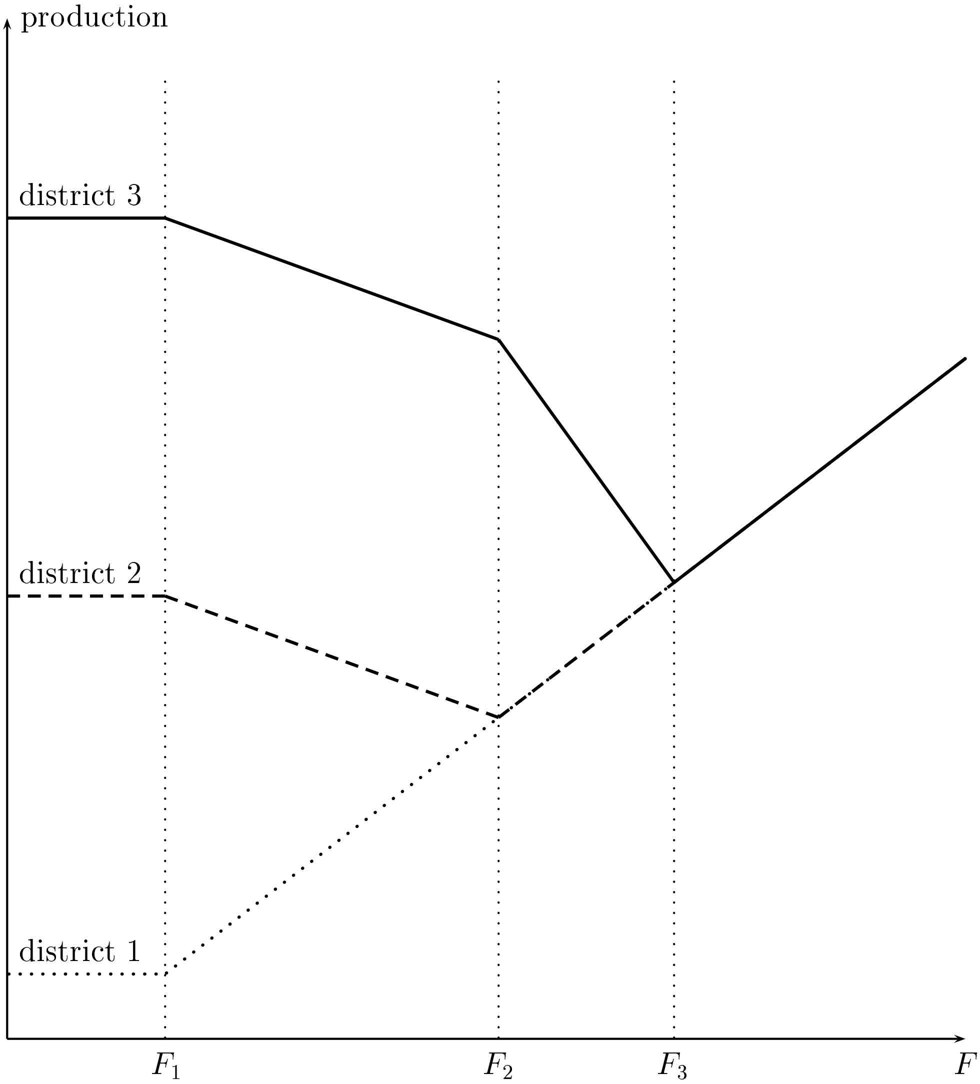

Figure 1: This figure represents the output of the different districts as a function of the federal standard F.

4.3 Induced preferences for F

In the previous section, we analyzed how the final stage inter-district equilibrium responds to variations in F. The majority rule equilibrium in the first stage assumes sophisticated voting, that is, voters vote over levels of the federal standard, F, correctly anticipating the effect of F on (second-stage) equilibrium policies in each district. However, in order to characterize the majority rule equilibrium in the first stage, we need to do more: we must derive the indirect preferences over the federal standard, F, for each voter t_{id}. Note that these indirect preferences are *endogenously determined,* in the sense that the induced preferences of voter (i, d) depends on all the types of all the median voters in the system, through the inter-district equilibrium outcomes. As we will see, this leads to complications similar those resulting from the double-peaked induced preferences of some voters in Crémer and Palfrey (2002).

Figure 2 shows the indirect utility functions of the different voters for the example of the previous section.[10] Notice first that the utility of an agent in district d increases when $F \leq F_d$. Indeed, in that range an increase in F increases total output but decreases the output of district d. We also see that the induced preferences are not single peaked. Indeed, consider the median and the high demand voter in district 2 (the two thickest dashed lines). As we have just discussed, their utilities are increasing on $[F_1, F_2]$. They are decreasing on $[F_2, F_3]$ as district 2 contributes more to total output. However, when F becomes greater than F_3 district 3 cannot free ride any more, and there is a range over which the utility of these agents increase with F.

4.4 Equilibrium in the First Stage

In Crémer and Palfrey (2002), in a different model, we showed existence of a majority rule equilibrium in the first stage despite the fact that preferences were not single peaked; there existed a value for F such that there was no alternative federal standard that was preferred by a majority to F. This result does not hold true for the model of this paper: for instance, as we show later there is no global majority rule equilibrium in our example.

However, a weaker majority rule equilibrium, called *local majority rule equilibrium* ($LMRE$) can be shown to exist. A local majority rule equilibrium is any policy F with the property that there is no policy in the neighborhood of F that a majority of voters prefers to F. This concept was introduced by Kramer and Klevorick, who identified sufficient conditions for

[10] Some of the utility functions have been shifted up or down by a constant amount to improve the lisibility of the figure

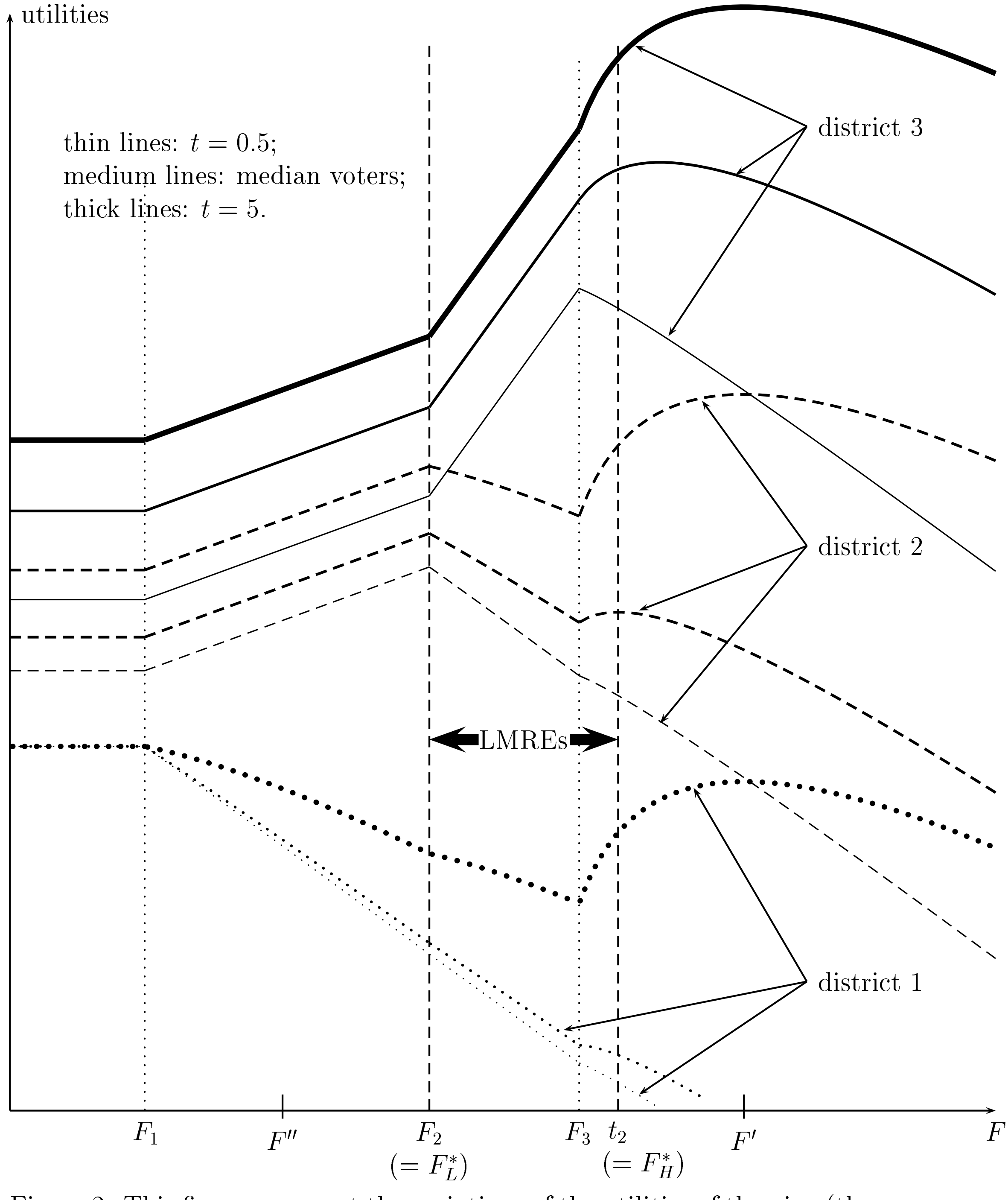

Figure 2: This figure represent the variations of the utilities of the nine (three in each of three districts) voters in our example.

existence (Kramer and Klevorick 1974) and demonstrated a useful application of the concept (Klevorick and Kramer 1973).

In this paper, we use a slightly stronger definition of LMRE and present a different existence argument. We need a stronger definition because all $F < F_1$ are a LMRE, since a small movement away from F changes the utility of no agent. To eliminate these equilibria, we limit attention to *strict* LMRE: those are Fs such that a majority of the voters would be made strictly worse off by a small move away from F. This slight generalization would not warrant another proof, but our proof also has the benefit of being constructive and leading to a full characterization of strict LMRE.

Before proceeding, we use the example to illustrate the difference between LMRE and global majority rule equilibrium. The standard F_2 is a LMRE as a majority of voters would refuse any small move away from F_2. However, it is not a majority rule equilibrium: five agents (all voters in district 3 and the high demand voters of districts 1 and 2) prefer F' to F_2.

We now present the definition of LMRE and our main results.

Definition 2 *A policy $x \in \Re$ is a* (strict) local majority rule equilibrium *if there exist $\epsilon > 0$ such that for (i) for all $x' \in (x, x+\epsilon)$, there exists a voter $i(x')$ such that $U_{i(x')}(x') < U_{i(x')}(x)$ and (ii) for all $x' \in (x-\epsilon, x+\epsilon)$, the number of voters, j, such that $U_j(x') > U_j(x)$ is strictly less than $(N+1)/2$.*

Clearly, an LMRE must be the local maximum of the utility function of at least one voter. We will therefore call a policy a *candidate* if it is the local maximum of the indirect utility function of at least one voter.

For any candidate $F > F_1$, the voters fall into one of four categories, depending on their induced preferences, locally around F. For at least one voter, F is as local maximum. For the remaining voters, the indirect utility function is either increasing in a neighborhood of F, decreasing in a neighborhood of F, or has a local minimum at F. The following definition formally defines these four categories of voters.

Definition 3 *For any candidate $F > F_1$, we will say that*

- i surely votes to the right *of F if there is an open interval (x, y) with $x < F < y$ such that U_i is non-decreasing on that interval. $\mathcal{R}(F)$ is the set of voters that surely vote to the right of F;*

- i surely votes to the left *of F if there is an open interval (x, y) with $x < F < y$ such that U_i is strictly decreasing on that interval. $\mathcal{L}(F)$ is the set of voters that surely vote to the left of F;*

- i votes exactly *for* F *if* F *is a (weak) local maximum of* U_i. $\mathcal{E}(F)$ *is the set of voters that vote exactly for* F*;*

- i votes either to the left or to the right *of* F *if* F *is a strict local minimum of* U_i. $\mathcal{LR}(F)$ *is the set of voters that vote either to the left or to the right of* F.

Our main characterization result is the following.

Theorem 1 *A candidate* F *is a strict LMRE if and only if*

$$|\mathcal{L}(F)| + |\mathcal{LR}(F)| \leq \frac{N-1}{2} \tag{3}$$

and

$$|\mathcal{R}(F)| + |\mathcal{LR}(F)| \leq \frac{N-1}{2}. \tag{4}$$

and

$$F \geq F_1. \tag{5}$$

It is left to show that there exist candidates that satisfy these three inequalities. This is done in the following theorem, which furthermore characterize the lowest and smallest LMRE.

Theorem 2 F_L^**, the smallest candidate such that inequality (4) and (5) hold, and* F_H^**, the greatest candidate such that (3) and (5) hold, are respectively the least and the greatest strict LMRE.*

Corollary 1 (Kramer and Klevorick) *There exists at least one strict LMRE.*

4.4.1 Proofs

For the existence and characterization proof below, we only use the following three properties of the utility functions of the voters. For convenience we will drop the district index and simply write $U_i(F)$ for the indirect utility function of the voters.

Property 1 *For all* i*, the function* U_i *is non-decreasing on* $(-\infty, F_1]$ *and has a finite number of local extrema in the region* $[F_1, \infty)$.

Property 2 *There exist a uniform bound* $M > 0$*, such that for all* i *the function* U_i *is decreasing for* $x \geq M$.

The next assumption ensures that the utility functions are never locally constant.

Property 3 *For any $x \in (F_1, \infty)$ there exists $\eta > 0$ such that, for all i, U_i is strictly monotone on each of the intervals $(x - \eta, x)$ and $(x, x + \eta)$.*

Lemma 1 *If a policy F is a strict LMRE then $F > F_1$.*

Proof. Because $U_i(F)$ is constant on $(-\infty, F_1]$ for all i, all voters are indifferent between F_1 and $F_1 - \varepsilon$, which contradicts the definition of strict LMRE.[11] ■

We next establish that any LMRE is a local maximum of the utility function of at least one agent, and these local maxima are therefore natural "candidate equilibria".

Lemma 2 *If a policy F is a LMRE then there exists an i such that F is a local maximum of U_i.*

Proof. By the previous two lemmas, we know $F > F_1$. Suppose, for all i, that F is not a local maximum. We will show that it is not an LMRE. By property 3, there exists η such that for all i the function U_i is strictly monotone on $(x - \eta, x + \eta)$. Hence, it is either strictly increasing or strictly decreasing for at least $(N + 1)/2$ voters. Assume it is strictly increasing. Then $U_i(x') > U_i(x)$for at least $(N + 1)/2$ voters and for all $x' \in (x, x + \eta)$, which by definition 2 implies it is not an LMRE. ■

Lemma 3 *For any candidate $F > F_1$, $\{\mathcal{L}(F), \mathcal{R}(F), \mathcal{E}(F), \mathcal{LR}(F)\}$ is a partition of the set of voters.*

Proof. Follows immediately from the definition. ■

This enables us to prove theorem 1.

Proof of theorem 1.

i. Necessity.

First, from lemma 1, $F > F_1$ is a necessary condition for a strict LMRE. To establish necessity of the other conditions, we suppose $F > F_1$ is a strict LMRE and show that this implies inequalities (3) and (4). Let $\epsilon(F)$ satisfy

[11] F_1 would not be an equilibrium even if we relaxed the definition of LMREs. Indeed, given that district 1 contains less that half of the voters, there would be a majority in favor of a move to $F + \varepsilon$.

the conditions in the definition of LMRE. Since F is a LMRE, we have $U_i(x) \leq U_i(F)$ for at least $(N+1)/2$ voters for all $x \in (F, F+\epsilon(F))$. Since the functions U_i are strictly monotone on $(F, F+\epsilon(F))$, we get $|\mathcal{E}(F)|+|\mathcal{L}(F)| \geq \frac{N+1}{2}$,which, by lemma 3, implies (4). The proof that inequality (3) holds is similar.

ii. Sufficiency.

Assume now that inequalities (3) and (4) hold, and $F > F_1$. Take any $x' \in (F-\eta, F)$, where η satisfies the condition in property 3. Since U_i is not constant on this open interval for any i, we have $U_i(x') > U_i(F)$ if and only if $i \in \mathcal{L}(F) \cup \mathcal{LR}(F)$. Therefore, by (3) there are fewer than $(N+1)/2$ voters who strictly prefer x' to F. Consider $x' \in (F, F+\eta)$. In this case, we have $U_i(x') > U_i(F)$ if and only if $i \in \mathcal{R}(F) \cup \mathcal{LR}(F)$, and therefore, by inequality (4) there are fewer than $(N+1)/2$ voters who strictly prefer x' to F. Hence F is an LMRE. Strictness follows from local non-constancy and lemma 2. ■

Let $F^{\max}$ be the greatest candidate.[12] For any candidate $F > F_1$, let F^- be the greatest candidate strictly less than F, and for any $F < F^{\max}$, let F^+ be the least candidate strictly greater than F.

Lemma 4 *For any candidate* $F > F_1$:

$$i \in \mathcal{R}(F^-) \cup \mathcal{LR}(F^-) \Rightarrow i \in \mathcal{R}(F) \cup \mathcal{E}(F)$$

Proof. Consider $i \in \mathcal{R}(F^-) \cup \mathcal{LR}(F^-)$. Because U_i is strictly increasing on $(F^-, F^- + \eta)$, and all local extrema of U_i are candidates,[13] the least local maximum of U_i greater than F^- is greater than or equal to F. If it is equal to F, then $i \in \mathcal{E}(F)$; if it is greater than F, then $i \in \mathcal{R}(F)$. Therefore $i \in \mathcal{R}(F) \cup \mathcal{E}(F)$. ■

Proof of theorem 2 and corollary 1. As the final step in showing existence of a strict LMRE, let F_L^* be the least candidate $F > F_1$ for which inequality (4) holds. We will show that F_L^* is a strict LMRE. Strictness follows immediately, so we only need to verify part (ii) of the definition.

Because F_L^* satisfies (4), by lemma 1, if it is not a LMRE it cannot satisfy (3), and we must have

$$|\mathcal{L}(F_L^*)| + |\mathcal{LR}(F_L^*)| \geq \frac{N+1}{2}.$$

[12] We know a greatest candidate exists since U_i is eventually decreasing for all i.

[13] In our model, all local minima of the utility function of a voter are also local maxima for some other voter. This coincidence is not required to prove the results. More generally, one would simply redefine candidates to include all local extrema. In any case, LMREs are always local maxima.

By lemma 3, this implies

$$|\mathcal{E}(F_L^*)| + |\mathcal{R}(F_L^*)| \leq \frac{N-1}{2},$$

and by (4)

$$\left|\mathcal{R}(F_L^{*-})\right| + \left|\mathcal{LR}(F_L^{*-})\right| \leq \frac{N-1}{2},$$

which contradicts the definition of F_L^*. Hence F_L^* is a LMRE. The proof that F_H^* is an LMRE is similar. Existence follows immediately. ■

4.5 Example (continued): Computing LMRE

Theorem 1 enables us to easily identify the set of LMRE for our example; as indicated on figure 2, there are two of them: $F_L^* = F_2$ and $F_R^* = t_2$.

To find global equilibria, we only have to check the two local equilibria, since all global majority rule equilibria must also be local majority rule equilibria. As explained above $F_L^* = F_2$ is not a local equilibrium, as more that half of the voters prefer F' (see figure). Similarly, consider $F_R^* = 2$. It is easy to check that it is also not a global equilibrium, this time because it is "too high". A majority of voters (the three voters of district 1 and the low-demand and median voter in district 2) would prefer the lower federal standard F'' in figure 2.

Therefore, in this example, there are two LMRE and no global LMRE.

5 Welfare effects of federal standards with externalities

We evaluate welfare on the basis of the preferences of the median voters of each district, but the comparison between regimes is complex. First, there can be multiple equilibria (and possible nonexistence of global equilibria), due to the externalities.

Second, standards increase total production which is too low in their absence. A corollary of the result on increased production is that there exists a federal standard (not necessarily an equilibrium), such that the total production of public good equals X^{**}. This follows because $X^*(0) = X^*$, $X^*(F)$ is continuously increasing above F_1, and $X^*(F) = DF$ for $F > F_D$. Hence there exists some point at which $X^*(F) = X^{**}$. Therefore, in principle for any β, there is some federal standard with the property that the resulting total production be efficient.

However, the fact that aggregate production is efficient does not imply that the allocation of the additional output across districts is efficient, and indeed it is inefficient for two reasons. Low demand districts bear the whole burden of the increase, and high demand districts reduce their production in response. In fact, high demand districts may produce even less under the equilibrium federal standard than they do in the autarchy solution, as is true in the example.

In a previous paper (Crémer and Palfrey 2000) we showed that, in the absence of externalities, federal standards can have a negative impact on welfare. It particular, we showed that standards make more voters worse off than better off. The logic is that all voters have an incentive to push for standards up to their ideal point, not taking into account the negative effect this may have on lower demand voters. The resulting equilibrium federal standard is therefore equal to the overall median ideal point. Districts who have medians above this point will be unaffected, while more than half the voters in districts whose median is below the overall median will be worse off.

The example shows that this logic does not extend to the case with externalities. In that example, both the majority of voters and the majority of median voters are better off in either LMRE than without federal standards (see figure 2). Therefore, either of these local equilibria would win if they were voted against a status quo of no standard at all. One can also show that both are more efficient than no standard using various other criteria, such as the utilitarian rule.

6 Separable preferences

The analysis above may seem to rely heavily on the substitution effect, however none of the characterization of local equilibrium depends on this nonseparablity. In this section, we consider the case of separable preferences.

Consider the *separable* utility function

$$u_{id}(x) = v_{id}(x_d) + \beta w_{id}(x_{-d}),$$

where v_{id} is single-peaked, w_{id} is strictly increasing and $\beta > 0$ represents the spillover effect.

In this case, the conditional ideal point of voter (i, d) is the peak of v_{id}, which is independent of the production in the other districts. Hence, in contrast to the nonseparable case, the policy adopted by a district is independent of the policies of the other districts. This implies that $F_d = m_d$ for all d, where m_d is the median peak in district d. Therefore, the equilibrium policies in the second stage, given F, are simply

$$x_d^*(F) = \max[m_d, F], \tag{6}$$

and $F_d = m_d$. As before, the utility of the voters is increasing in F when their district is not constrained, and can have a variety of shapes when it is constrained. As shown in section 4.4.1, theorems 1 and 2 as well as corollary 1 hold in the separable case.

6.1 A bound on LMRE in the separable case

Equation (6) implies that for all d, x_d^* is increasing in F, and strictly increasing if $F \geq m_d$. We also have

$$U_{id}(F) = v_{id}(\max[m_d, F]) + \beta w_{id}(X_{-d}(F)).$$

This implies several interesting results.

Lemma 5 *The function $U_{id}(F)$ is strictly increasing on $(F_1, \max[m_d, t_{id}])$.*

Proof. For $F < m_d$ we have $U_{id}(F) = v_{id}(m_d) + \beta w_{id}(X_{-d}(F))$. Since $F \in (F_1, \max[m_d, t_{id}])$ this implies that X_{-d} is increasing in F, so the second term is increasing since w_{id} is an increasing function. The first term is constant, since m_d is independent of F. Next suppose $t_{id} > m_d$ and $F \in (m_d, t_{id})$. Then $U_{id}(F) = v_{id}(F) + \beta w_{id}(X_{-d}(F))$, and both terms of this sum are increasing in F. ■

Lemma 6 *Let X be such that U_{id} is strictly increasing on (F_1, X) for $(N+1)/2$ voters, and let F^*be a LMRE. Then $F^* \geq X$.*

Proof. Suppose $F \in (F_1, X)$ and U_{id} is increasing on (F_1, X) for $(N+1)/2$ voters. All of these voters are in $\mathcal{R}(F)$ which implies that $|\mathcal{R}(F')| + |\mathcal{LR}(F')| > (N-1)/2$. Therefore F is not a LMRE. ■

Let us define

$$\widetilde{F} = \operatorname*{med}_{(i,d)}(\max[m_d, t_{id}]).$$

Lemma 5 and 6 immediately imply the following corollary:

Corollary 2 *If $F^*$is a LMRE then $F^* \geq \widetilde{F}$.*

Proof. U_{id} is strictly increasing on $[F_1, \max[m_d, t_{id}]]$. Hence, if $F < \widetilde{F}$ the set $\mathcal{R}(F)$ contains at least $(N+1)/2$ voters, and therefore F is not a LMRE. ■

The most relevant of these results for welfare comparisons is corollary 2. That is, the set of LMRE is bounded below by the standard that arises as a global majority rule equilibrium when there are no externalities. This allows some direct comparisons with results in Crémer and Palfrey (2000), where we showed that in the absence of externalities ($\beta = 0$), there was an inefficiently high global equilibrium equal to med$[t_{id}]$.

Corollary 2 shows that a small increase in β from 0 to $\varepsilon > 0$ will lead to an LMRE greater than $\widetilde{F}$, which is greater than med$[t_{id}]$. By continuity, efficiency would require a standard strictly less than med$[t_{id}]$, and hence lower federal standards for values of β close to 0.

Therefore, the negative effects based on utilitarian criteria for welfare (summing utilities) still hold if the spillover effects are small. However, the results that more voters are worse off than are better off may no longer hold, since $\beta > 0$ implies that many of the high valuation voters who are indifferent between regimes when $\beta = 0$, now are strictly better off with federal standards, due to the spillover effects. That is, with $\beta = 0$ a majority would oppose a regime with federal standards, but for small values of $\beta > 0$ a majority would prefer a regime with federal standards to a regime with no federal standards. This implies that even though the externalities are very small, the majority of voters would impose an increase in production on low demand districts.

7 Conclusions

The presence of externalities has complex consequences on equilibrium federal standards. Naive intuition suggests that federal standards may be a valuable way to overcome the free riding problem among districts in a federation. However, this intuition is complicated due to non single peaked preferences and the equilibrium effects of federal standards on the subgame between local districts.

The first result of the paper is that majority rule equilibria may no longer exist. Preferences are not single peaked, since low demand voters are worse off when the standard binds for their district, but then better off when the standard binds for other districts. This can lead to majority rule cycles, as demonstrated in the example.

Second, in spite of the potential cycling problem, local majority rule equilibria are guaranteed to exist. Of particular interest are the strict local majority rule equilibria which create binding constraints on some districts, with these constraints creating secondary effects through the equilibrium in the district subgame. We identified the properties of local majority rule equilibria and characterized the range of these equilibria. The range can be quite large, as demonstrated in the example.

Third, the welfare effects are more complicated than in the original model of Crémer and Palfrey (2000), where there were no spillover effects. The sets of voters who benefit or are made worse off follows an interesting pattern. Federal standards increase the total level of spending on public goods above an inefficiently low level. However, this increase in federal standards is achieved in an inefficient way, because the standards bind first on low demand districts, and last on the highest demand districts, while precisely the opposite pattern would be optimal. This problem can be exacerbated by a substitution effect, whereby high demand districts *reduce* production at the same time that low demand districts are forced to produce more. Thus, low demand voters from low demand districts are made worse off by federal standards, while voters in high demand districts are big winners. Their districts produce less, but they benefit from the spillovers generated by increased production in low demand districts.

Because of the confounding effects of higher total production, but perverse distributive effects across districts, we obtained few unambiguous results about the welfare effects in this model. However, in the case of separable preferences, we are able to obtain stronger conclusions since the subgame between the districts is very simple. In that case, we obtain lower bounds on the LMRE which indicate that if the spillover effects are sufficiently small, federal standards will be set too high, as in Crémer and Palfrey (2000). However, in contrast to that earlier paper, a majority of voters may be made better off even with small spillovers.

While the approach taken here sheds some light on the effectiveness (or ineffectiveness) of federal standards to overcome free riding between districts, it begs the question of what alternatives may be possible, and how well these alternative institutions perform. Hence we see a mechanism design approach as a natural next step in the research agenda. The idea would be to model institutions, in a general way, as game forms that provide the right incentives for more efficient district decisions for local public good production. The use of federal standards, whereby a federation-wide minimum is established is perhaps the simplest class of such mechanisms. More complex mechanisms would allow for the possibility of different standards for different districts, in the form of granting exceptions or exclusions, or possibly employ the use of non-majoritarian methods for voting over mechanisms. Such arrangements could possibly overcome some of the perverse distributive effects of simple federal standards, and would also be consistent with features of some existing federal policies.

REFERENCES

Alesina, A. and E. Spolaore, "On the Number and Size of Nations." *Quarterly Journal of Economics* 112 (1997): 1027-56.

Bednar, J. "Shirking and Stability in Federal Systems." June 2001.

Crémer, J. and T. Palfrey. "In or Out? Centralization by Majority Vote." *European Economic Rev.*. 40 (January 1996):43-60.

Crémer, J. and T. Palfrey. "Political Confederation." *Am. Pol. Sci. Rev.* 93 (March 1999): 69-83.

Crémer, J. and T. Palfrey. "Federal Mandates with Local Agenda-setters," *Review of Economic Design*, forthcoming 2002.

Crémer, J. and T. Palfrey. "Federal Mandates by Popular Demand," *Journal of Political Economy* 108 (October 2000): 905-27.

De Figueiredo, R. and B. Weingast. "Self-enforcing Federalism." 2001.

Epple, D. and Romer, T., "Mobility and Redistribution." *J. P. E.* 99 (1991): 828-58.

Gordon, R. "An Optimal Taxation Approach to Fiscal Federalism," *Quarterly Journal of Economics* 98 (November 1983): 567–87.

Klevorick, A. and G. Kramer. "Social Choice on Pollution Management: The Genossenschaften" *Journal of Public Economics* 2 (1973): 101-46.

Kramer, G. and A. Klevorick. "Existence of a Local Cooperative Equilibrium in a Class of Voting Games" *Review of Economic Studies* 41 (October 1974): 539-47.

Nechyba, T. "Existence of Equilibrium and Stratification in Local and Hierarchical Tiebout Economies with Property Taxes and Voting." *Economic Theory* 10 (August 1997):277-304.

Persson, T. and G. Tabellini, "Fiscal Federal Constitutions: Risk Sharing and Moral Hazard." *Econometrica* 64, no. 3 (1996): 623-46.

Tiebout, C., "A Pure Theory of Local Public Expenditures." *J. P. E.* 64 (1956): 416-24.