WORKING PAPERS

N° 17-774

March 2017

“The Horizontally s-shaped laffer curve”

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THE HORIZONTALLY S-SHAPED LAFFER CURVE

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Abstract. In a neoclassical growth model with incomplete markets and heterogeneous, liquidity-constrained agents, the properties of the Laffer curve depend on whether debt or transfers are adjusted to balance the government budget constraint. The Laffer curve conditional on public debt is horizontally S-shaped. Two opposing forces explain this result. First, when government wealth increases, the fiscal burden declines, calling for lower tax rates. Second, because the interest rate decreases when government wealth increases, fiscal revenues may also decline, calling for higher taxes. For sufficiently negative government debt, the second force dominates, leading to the odd shape of the Laffer curve conditional on debt.

Keywords: Laffer Curve, Incomplete Markets, Labor Supply, Public Debt.

JEL Class.: E0, E60

Date: March 2, 2017.

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We would like to thank the editor, C. Michelacci, four anonymous referees, as well as F. Alvarez, C. Cahn, J.-C. Conessa, R. Crino, M. Doepke, C. Hellwig, S. Kankanamge, D. Krueger, E. McGrattan, E. Mendoza, J. Montornes, K. Moran, G. Moscarini, B. Neiman, X. Ragot, T. Sargent, H. Ulhig, and the participants of several conferences for their useful comments and suggestions. The views expressed herein are those of the authors and should under no circumstances be interpreted as reflecting those of the Banque de France.

1. Introduction

Against a backdrop of fiscal consolidation in developed countries, the Laffer curve, i.e., the inverted-U-shaped relation between fiscal revenues and tax rates, has recently received considerable attention; see, among many others, D'Erasmo et al. (2016), Guner et al. (2016), Holter et al. (2014), and Trabandt and Uhlig (2011, 2012). In this context, the Laffer curve has proven a useful tool to quantify the available fiscal space.

In this paper, we study issues related to the shape of the Laffer curve in the context of a neoclassical growth model with incomplete markets and heterogeneous, liquidity-constrained agents (hereafter, IM). We show that in an IM economy, there is no sense in which one can define a Laffer curve abstracting from whether debt or transfers are chosen to balance the government budget constraint. This is because the interest rate itself is not invariant to debt and transfers, contrary to what happens in a representative agent (RA) setup (see Aiyagari and McGrattan, 1998).

To address this issue, we develop the concept of *conditional Laffer curves.* Holding public debt constant, we vary transfers and adjust one tax rate accordingly. This yields a relation linking fiscal revenues to the tax rate conditional on transfers. By holding transfers constant and varying debt, we can similarly define a Laffer curve conditional on public debt. In an RA setup, the two conditional Laffer curves coincide, which is the mere reflection of the irrelevance of public debt and transfers for the equilibrium allocation and price system.[1] In an IM setup, however, the picture changes dramatically.

While the Laffer curve conditional on transfers has the traditional inverted-U shape, its counterpart conditional on debt looks like a horizontal S. In this case, there can be one, two, or three tax rates compatible with a given level of

[1] In other words, given a change in distortionary taxes, the resulting allocation does not depend on transfers and/or public debt, which is just Ricardian equivalence at play.

fiscal revenues. The regular part of this curve (the part that indeed looks like an inverted U) is associated with positive government debt, while the odd part (the part that makes the curve look like a horizontal S) is associated with negative debt levels.

To understand this odd shape, consider a situation such that the debt-output ratio becomes negative, say, because the government is now accumulating assets. There are two effects at work here. Obviously, if government wealth increases, the fiscal burden declines, calling for a lower tax rate to balance the budget constraint. This is the standard force present in an RA framework. However, in an IM context, there is another force at work: The interest rate decreases when government wealth increases. Other things equal, this reduces government revenues, calling for higher taxes. For sufficiently negative government debt, the second force dominates, leading to the oddly shaped Laffer curve conditional on debt.

In practice, the key question is whether the odd portion of the Laffer curve conditional on debt is relevant from an empirical point of view or a mere theoretical curiosity.

Defining debt as government liabilities net of financial assets and using a long data set featuring all the G7 countries, based on Piketty and Zucman (2014), we find occurrences of negative public debt for Japan, Germany, and the United Kingdom. One can alternatively define public debt as government liabilities net of non-financial assets (e.g., administrative buildings, subsoil, intangibles such as artistic originals). This alternative definition is somewhat contentious because the National Accounts assume a zero net return on non-financial assets. However, it provides a rough assessment of government net wealth. Under this definition, negative public debt is pervasive. We conclude from both perspectives, that the odd part of the Laffer curve conditional on debt is not a theoretical curiosity.

To explore these issues, we consider a prototypical neoclassical model along the lines of Aiyagari and McGrattan (1998) and Flodén (2001). In this economy, households are subject to persistent, uninsurable, idiosyncratic productivity shocks and face a borrowing constraint. The model includes distortionary taxes on labor, capital, and consumption. These taxes are used to finance a constant share of government consumption in output, lump-sum transfers, and interest repayments on accumulated debt. While the model is very simple and essentially qualitative, we strive to take it seriously to the data, matching key moments of earning and wealth distributions. We then study the steady-state conditional Laffer curves associated with each of the three taxes considered.

Our main findings are the following. First, when transfers are varied, the Laffer curves in the IM economy look broadly like their RA counterparts. In our benchmark calibration, the revenue-maximizing labor income tax rate hardly differs from its RA counterpart. We reach similar conclusions when considering capital income and consumption taxes. Second, when debt is varied instead of transfers, the regular part of the Laffer curve is similar to its RA counterpart. However, whenever debt is negative, the two curves differ sharply, confirming the insight drawn from the above discussion. A corollary of our results is that the Laffer curves (conditional on transfers) are not invariant to the level of public indebtedness. This is potentially very important in the current context of high public debt-output ratios in the US and other advanced economies. It turns out that the Laffer curves are only mildly affected by the debt-output ratio, provided that the latter is positive. However, for negative levels of public debt, we find that the Laffer curve associated with capital income taxes can be higher than its benchmark counterpart. Our results are robust to a series of model perturbations, such as lower labor supply elasticities, lower shares of government spending, alternative calibration targets for the debt-output ratio, alternative utility functions, and alternative processes for individual productivity.

This paper is related to previous studies investigating taxation and/or public debt in an IM setup. A first strand, exemplified by Aiyagari and McGrattan (1998) and Flodén (2001) , established that a proportional income tax rate changes non-monotonically with debt. However, this literature did not explore how this feature could impact the shape of the Laffer curve. Röhrs and Winter (2015) recently extended this analysis to a carefully calibrated multi-tax environment. However, they also ignored the implications for the Laffer curve. Our paper complements this literature by focusing on how the conditional Laffer curve changes as debt or transfers vary. A second strand has explored the Laffer effect in the context of IM models. For example, Flodén and Lindé (2001) found that the Laffer curve peaks when the labor income tax is approximatively 50% or higher. However, their analysis abstracts from public debt. More recently, using an IM setup, Ljungqvist and Sargent (2008) and Alonso-Ortiz and Rogerson (2010) revisited the effects of labor taxation studied by Prescott (2004). Ljungqvist and Sargent (2008) and Alonso-Ortiz and Rogerson (2010) compared the Laffer curves in IM and RA setups. Focusing on labor income taxes, they found that the prohibitive part of the Laffer curve in the IM case differs only mildly from that in the RA version of their model. However, they too abstract from government debt. Finally, Holter et al. (2014) characterize the impact of the progressivity of the labor tax code on the Laffer curve. They find that progressive labor taxes significantly reduce tax revenues. Guner et al. (2016) conclude that higher progressivity has limited effects on fiscal revenues. Our paper complements these works by further investigating the shape of the Laffer curve conditional on public debt.

The rest of the paper is organized as follows. In Section 2, we review the empirical evidence on negative debt. In Section 3, we expound the IM model, define the steady-state equilibrium under study, and discuss our calibration strategy. We formally introduce the concept of conditional Laffer curves. In Section 4, we discuss our results. We also explore the robustness of our findings. The last section briefly concludes.

2. Historical Evidence on Public Debt

As argued in the introduction, negative public debt plays a central role when analyzing Laffer curves. It is thus important to show that the possibility of negative public debt is empirically relevant. To this end, this section provides a historical review of the public debt dynamics of the G7 countries, covering over a century for some countries.

Here, we consider two definitions of public debt. Let b_g denote the difference between government liabilities and financial assets and k_g denote non-financial assets held by the government. One can simply measure public debt as b_g or alternatively as $b_g - k_g$.

We use the data on the government balance sheet (market value of liabilities, financial assets, and non-financial assets) constructed by Piketty and Zucman (2014) to obtain two measures of public debt.[2] The first indicator corresponds to b_g and the second to $b_g - k_g$. These two measures are expressed as a fraction of national income. The data are available at an annual frequency. The countries are Canada, France, Germany, Italy, Japan, the United Kingdom (UK) and the United States

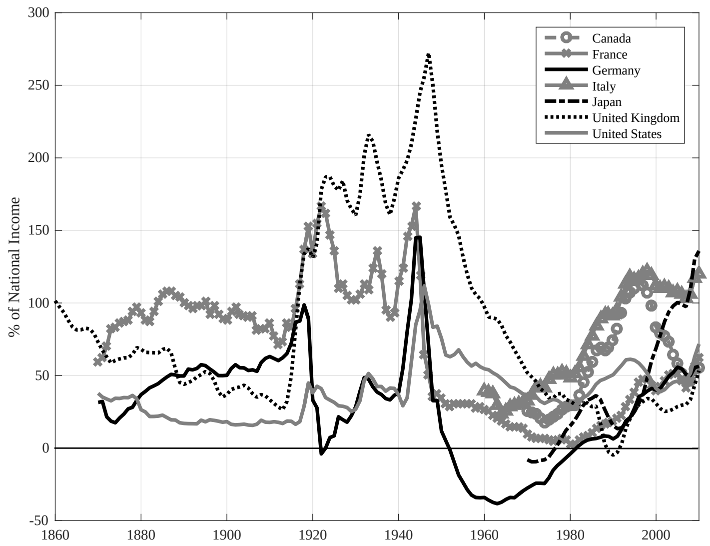
[2] Using the recent international guidelines of the System of National Accounts (SNA) or the European System of Accounts (ESA), Piketty and Zucman (2014) put together a new macro-historical data set on wealth and income, including government wealth and its components. See http://piketty.pse.ens.fr/files/PikettyZucman2013Book.pdf for an exhaustive exposition on the data construction.

(US). For France, Germany, the UK and the US, the sample covers more than one century, whereas the sample starts between 1960 and 1970 for Canada, Italy and Japan. The data for all countries end in 2010.

In the second definition, it is important to clarify the notion of non-financial assets. These include non-produced assets (e.g., land, subsoil, water resources) and produced assets: (i) tangibles, such as dwellings, other non-residential buildings and structures, machinery and equipment, and weapon systems, and (ii) intangibles, such as computer software, entertainment, literacy, and artistic originals. Buildings and structures constitute, by far, the largest component of government non-financial wealth and are mainly owned by regional and local governments. It is important here to emphasize that this alternative definition of public debt is somewhat contentious. This is so because the net return on non-financial assets held by the government is assumed to be zero in the National Accounts. This limits the direct comparison with the public debt concept in our model. However, this alternative definition gives a useful assessment of government net wealth.

Figure 1 reports the first debt definitiong, b_g. The figure shows that in all the countries considered, large fluctuations in public debt are mainly associated with major historical events. After having borrowed 40% of its national income to pay for the Civil War, the US federal government reduced its debt by one-half in the wake of World War I. Subsequently, the debt-to-national-income ratio fluctuated around 40% until World War II. Between 1941 and 1945, the US lent Britain and other countries money to help pay for military costs, and it spent a great deal on its own military expenditures, leading to debt that exceeded one year of national income. Following that war, the US economy grew, and the debt-to-income ratio displayed a downward trend until the mid-1970s when it reached 30%. In the early 1980s, a large increase in defense spending and substantial tax cuts contributed

FIGURE 1. Public Debt - b_g



to ballooning debt. Before the Great Recession, the ratio was below 50%, but the resulting stimulus packages have led to an upward trend.

Similarly, in the 1990s, secular increases in Canadian government services and entitlements pushed debt to 120% of national income. The Canadian government decided to reduce its spending in an attempt to generate surpluses. From 1996 to 2007, the debt-to-income ratio was divided by more than two.

Since the 1970s, starting from a *negative* level, Japan's net debt has increased steadily. Over the the 1990s and 2000s, Japan experienced no increase in nominal income, so the debt-to-income ratio has continuously risen. Japan has been unable to inflate its way out of debt, and it has made tiny interest payments to bondholders.

Following the Napoleonic wars, the UK implemented a long and drastic austerity plan such that the debt represented 26% of national income in 1913. At the end of World War I, the ratio was 180% and remained virtually unchanged until the beginning of World War II. The war caused a sharp increase in debt (reaching 270% in 1947). Unlike France, Germany, Italy and other continental countries, the

UK refused to pursue inflationary default after either World War I or World War II. This explains why the UK displayed only a very gradual decline over the next three decades. From the early 1980s until 2008 (with the notable exception of 1990, where debt was *negative*), the UK's public debt hovered around 30%.

During the 19th century, France experienced rising deficits, and its debt reached 100% of national income by 1890. Most of this increase occurred after 1870 when Germany imposed a costly indemnity on France as a result of the Franco-Prussian War. Consequently, when World War I began, France's debt exceeded 80% of national income. Despite inflation during World War I, the debt-to-income ratio rose to over 170% by the early 1920s. By the beginning of World War II, the ratio had decreased to 100% but shot to over 160% in 1944. France then inflated its way out of debt by imposing heavy losses on bondholders (the rate of inflation exceeded 50% per year between 1945 and 1948). The debt-to-income ratio decreased toward zero until the end of the 1970s and has continually increased since.

Germany inflated its way out of its World War I debts through hyperinflation, wiping out bondholder wealth. The rate of inflation was 17% per year, on average, over the 1913-1950 period. In 1948, Germany used a currency conversion from military marks to Deutsche marks to significantly reduce its debt obligations. Importantly, German public debt was *negative* for roughly 30 years, from the early 1950's to 1980.

Like France, Italy inflated its way out of debt after World War II. However, in stark contrast with France, Italy consistently ran budget deficits after World War II. In the mid-1990s, it reformed its public finances to prevent additional increases in the debt-to-income ratio. During this period, Italy benefited from lower interest rates.

FIGURE 2. Public Debt - $b_g - k_g$

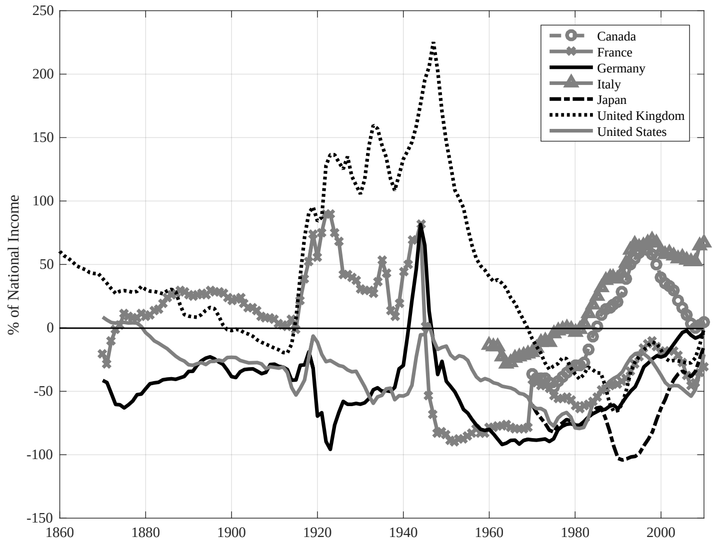


Figure 2 reports $b_g - k_g$, which accounts for government non-financial assets. Although substantial efforts have been undertaken to improve the measurement of these assets, caution is warranted, especially with cross-country comparisons (e.g., statistical methodology, data coverage, sample period). The non-financial assets of the sample countries share three important empirical features.

First, non-financial assets greatly exceed financial assets (by 2 to 5 times). Second, in all countries (except Japan), ratios of non-financial assets to national income exhibit remarkable stability. Consequently, the second indicator is roughly a downward shift of the first indicator. Third, non-financial assets are of the same order of magnitude as government liabilities. It follows that $b_g - k_g$ can be either positive or negative depending on the period. For most countries, regimes of positive debts have been followed by long periods of negative debts (see, for instance, France and the UK).

The period 1950-1980 is especially striking, as it displays large negative debt-to-national-income ratios (see France, Germany and the US) due to large public assets and low debt levels. The ratios were between -50% and -100% in the early

1980s for France, Germany, the UK and the US. However, the ratios increased during the 1990-2000 period and were close to zero in 2010 (due to large increases in government liabilities). Canada displays another interesting pattern: The government debt-to-income ratio was 60% in 1996, subsequently returning to zero in just 15 years when inflation was stable around 2%.

This historical evidence suggests that periods of negative debt (as a fraction of national income) are neither curiosities nor exceptional episodes.

3. Model

3.1. **The Economic Environment.** We consider a discrete time economy without aggregate risk similar to that studied in Aiyagari and McGrattan (1998). Time is indexed by $t \in \mathbf{N}$. The final good Y_t, which is the numeraire, is produced by competitive firms, according to the technology

$$Y_t = K_t^{\theta}(Z_t N_t)^{1-\theta},$$

where $\theta \in (0,1)$ denotes the elasticity of production with respect to capital; K_t and N_t are the inputs of capital and efficient labor, respectively; and Z_t is an exogenous technical progress index evolving according to $Z_{t+1} = (1+\gamma)Z_t$, with $\gamma > 0$ and $Z_0 = 1$. Firms rent capital and efficient labor on competitive markets at rates $r_t + \delta$ and w_t, respectively. Here, $\delta \in (0,1)$ denotes the depreciation rate of physical capital, r_t is the interest rate, and w_t is the wage rate.

The first-order conditions for profit maximization are

$$r_t + \delta = \theta \left(\frac{K_t}{Z_t N_t} \right)^{\theta - 1},$$

$$w_t = (1-\theta) Z_t \left(\frac{K_t}{Z_t N_t} \right)^{\theta}.$$

The economy is populated by a continuum of ex ante identical, infinitely lived households. The typical household utility is given by

$$\mathsf{E}\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{1-\sigma}\left(\frac{c_t}{Z_t}\right)^{1-\sigma}-\frac{\eta}{1+\chi}h_t^{1+\chi}\right]\ |a_0,s_0\right\},$$

where $\mathsf{E}_0\{\cdot|a_0,s_0\}$ is the mathematical expectation conditioned on the individual state at date 0. The individual state consists of initial assets a_0 and the exogenous individual state s_0. Here, $\beta\in(0,1)$ denotes the discount factor, $c_t\geq 0$ is individual consumption, $0\leq h_t\leq 1$ is the individual labor supply, $\sigma>0$ is the relative risk aversion coefficient, $\chi>0$ is the inverse of the Frisch elasticity of labor, and $\eta>0$ is a scaling constant. Consumption appears in deviation from the index of technical progress to ensure a well-behaved, balanced-growth path. This non-standard form is useful whenever $\sigma\neq 1$. In the robustness section, we explore the sensitivity of our results to the limiting case $\sigma=1$, which we interpret as logarithmic utility. In this particular case, our normalization does not play any role and utility is balanced-growth-path consistent.[3]

At the beginning of each period, households receive an individual productivity level $s_t>0$. We assume that s_t is *i.i.d.* across agents and evolves over time according to a Markov process, with bounded support $\mathcal{S}$ and stationary transition function $Q(s,s')$.[4]

An individual agent's efficient labor is s_th_t, with corresponding labor earnings given by $(1-\tau_N)w_ts_th_t$, where τ_N denotes the labor income tax. In addition, agents

[3]This specification yields equivalent results to one in which utility would be a function of the non-normalized level of consumption and $Z_t^{1-\sigma}$ would appear as a scaling factor in front of labor disutility. Benhabib and Farmer (2000) argue that this specification is a reduced-form for technical progress in home production.

[4]The transition Q has the following interpretation: for all $s\in\mathcal{S}$ and for all $\mathcal{S}_0\in\mathscr{S}$, where $\mathscr{S}$ denotes the Borel subsets of $\mathcal{S}$, $Q(s,\mathcal{S}_0)$ is the probability that next period's individual state lies in $\mathcal{S}_0$ when current state is s.

self-insure by accumulating a_t units of assets that pay an after-tax rate of return $(1-\tau_A)r_t$, where τ_A denotes the capital income tax. These assets can consist of units of physical capital and/or government bonds. Once arbitrage opportunities have been ruled out, each asset has the same rate of return. Agents must also pay a consumption tax τ_C. Finally, they perceive transfers T_t. Thus, an agent's budget constraint is

$$(1+\tau_C)c_t + a_{t+1} \leq (1-\tau_N)w_t s_t h_t + [1+(1-\tau_A)r_t]a_t + T_t.$$

Borrowing is exogenously restricted by the constraint

$$a_{t+1} \geq 0.$$

Finally, there is a government in the economy. The government issues debt B_{t+1}, collects tax revenues, provides rebates and transfers, and consumes G_t units of final goods. The associated budget constraint is given by

$$B_{t+1} = (1+r_t)B_t + T_t + G_t - (\tau_A r_t A_t + \tau_N w_t N_t + \tau_C C_t),$$

where C_t and A_t denote aggregate (per capita) consumption and assets held by the agents, respectively.

3.2. **Equilibrium Defined.** In the remainder of this paper, we focus exclusively on the steady state of an appropriately normalized version of the above economy. Growing variables are rendered stationary by dividing them by Z_t. Variables so normalized are indicated with a hat. The ratio of government expenditures to output $g \equiv G/Y$ is assumed constant. It is convenient at this stage to define $b \equiv B/Y$ and $\tau \equiv T/Y$.

We let $\mathcal{A}$ denote the set of possible values for assets $\hat{a}$. We let the joint distribution of agents across assets $\hat{a}$ and individual exogenous states s be denoted $x(\hat{a}, s)$

defined on $\mathscr{A} \times \mathscr{S}$, the Borel subsets of $\mathcal{A} \times \mathcal{S}$. Thus, for all $\mathcal{A}_0 \times \mathcal{S}_0 \in \mathscr{A} \times \mathscr{S}$, $x(\mathcal{A}_0, \mathcal{S}_0)$ is the mass of agents with assets $\hat{a}$ in $\mathcal{A}_0$ and individual state s in $\mathcal{S}_0$.

We can now write an agent's problem in recursive form

$$v(\hat{a}, s) = \max_{\hat{c},h,\hat{a}'} \left\{ \frac{1}{1-\sigma}\hat{c}^{1-\sigma} - \frac{\eta}{1+\chi}h^{1+\chi} + \beta \int_{\mathcal{S}} v(\hat{a}', s')Q(s, \mathrm{d}s') \right\}$$

$$\text{s.t.} \quad (1+\tau_C)\hat{c} + (1+\gamma)\hat{a}' \leq (1-\tau_N)\hat{w}sh + (1+(1-\tau_A)r)\hat{a} + \hat{T}, \tag{1}$$

$$\hat{a}' \geq 0, \quad \hat{c} \geq 0, \quad 0 \leq h \leq 1.$$

For convenience, we restrict $\hat{a}$ to the compact set $\mathcal{A} = [0, \hat{a}_M]$, where $\hat{a}_M$ is a large number.[5] We can thus define a stationary, recursive equilibrium in the following way.

Definition 1. *A steady-state equilibrium is a constant system of prices $\{r, \hat{w}\}$, a vector of constant policy variables $(\tau_C, \tau_A, \tau_N, \hat{T}, \hat{G}, \hat{B})$, a value function $v(\hat{a}, s)$, time-invariant decision rules for an individual's assets holdings, consumption, and labor supply $\{g_a(\hat{a}, s), g_c(\hat{a}, s), g_h(\hat{a}, s\}$, a measure $x(\hat{a}, s)$ of agents over the state space $\mathcal{A} \times \mathcal{S}$, aggregate quantities $\hat{A} \equiv \int \hat{a}\mathrm{d}x$, $\hat{C} \equiv \int g_c(\hat{a}, s)\mathrm{d}x$, $N \equiv \int s g_h(\hat{a}, s)\mathrm{d}x$, and $\hat{K}$ such that:*

(i) The value function $v(\hat{a}, s)$ solves the agent's problem stated in equation (1), with associated decision rules $g_a(\hat{a}, s)$, $g_c(\hat{a}, s)$ and $g_h(\hat{a}, s)$;

(ii) Firms maximize profits and factor markets clear so that

$$\hat{w} = (1-\theta)\left(\frac{\hat{K}}{N}\right)^{\theta},$$

$$r + \delta = \theta\left(\frac{\hat{K}}{N}\right)^{\theta-1};$$

[5] $\hat{a}_M$ is selected so that the decision rule on assets for an individual with the highest productivity and highest discount factor crosses the 45-degree line below $\hat{a}_M$.

(iii) Tax revenues equal government expenses

$$\tau_N \hat{w} N + \tau_A r \hat{A} + \tau_C \hat{C} = \hat{T} + \hat{G} + (r - \gamma)\hat{B};$$

(iv) Aggregate savings equal firm demand for capital plus government debt

$$\hat{A} = \hat{K} + \hat{B};$$

(v) The distribution x is invariant

$$x(\mathcal{A}_0, \mathcal{S}_0) = \int_{\mathcal{A}_0 \times \mathcal{S}_0} \left\{ \int_{\mathcal{A} \times \mathcal{S}} \mathbf{1}_{\{\hat{a}' = g_a(\hat{a}, s)\}} Q(s, s') \mathrm{d}x \right\} \mathrm{d}a' \mathrm{d}s',$$

for all $\mathcal{A}_0 \times \mathcal{S}_0 \in \mathscr{A} \times \mathscr{S}$, where $\mathbf{1}_{\{\cdot\}}$ is an indicator function taking value one if the statement is true and zero otherwise.

For comparison purposes, we also consider a version of the model in which (i) we impose idiosyncratic labor income shocks s_t set to their average value, and (ii) we relax the borrowing constraint. We refer to this environment as the RA environment. Notice that in this RA setup, the distinction between effective labor $H \equiv \int g_h(\hat{a}, s)\mathrm{d}x$ and efficient labor N is no longer useful, since the quantities coincide (up to a multiplicative constant). We thus incorporate a productivity scale factor Ω in front of N_t in the production function to compensate the RA economy for the average labor productivity effect present in the IM economy (i.e., the relative difference between N and H). By doing so, we ensure that in the benchmark calibration described below, all economies share the same interest rate, effective labor H, and stationary production level $\hat{Y}$.

3.3. **The Laffer Curves.** From the government budget constraint, fiscal revenues (in deviation from Z_t) $\hat{R}$ are given by

$$\hat{R} = \tau_N \hat{w} N + \tau_A r \hat{K} + \tau_C \hat{C}.$$

Notice that the level of fiscal revenues R is defined net of fiscal receipts from taxing returns to public bonds.

Traditionally, the steady-state Laffer curve associated with τ_i, $i \in \{N, A, C\}$ is defined as follows. Let τ_i vary over an admissible range, holding the other two taxes constant. The Laffer curve is then the locus $(\tau_i, \hat{R})$, which relates the level of fiscal revenues $\hat{R}$ to the tax rates τ_i. This definition of the Laffer curve correctly takes into account the general equilibrium effects induced by a tax change, as argued by Trabandt and Uhlig (2011). For example, a given change in τ_N will modify x, g_a, g_h, and g_c such that it will also impact all the fiscal bases.

However, notice that in this definition, no reference is made to how the government balances its budget constraint when τ_i varies. Indeed, in equilibrium, we must always have

$$\frac{R}{Y} = g + \tau + [(1 - \tau_A)r - \gamma]b,$$

so that a given change in one of the three tax rates is associated with a corresponding adjustment in either τ or b.[6]

In an RA setup, one can abstract safely from these adjustments, as shown in the following proposition.

Proposition 1. *In a RA setup, the steady-state Laffer curve associated with τ_i, $i \in \{N, A, C\}$ is invariant to which of τ or b is adjusted to balance the government budget constraint.*

Proof. See Appendix A. □

This proposition establishes that in an RA setup, given a change in one of the three distorting taxes, adjusting lump-sum transfers or public debt is of no consequence for the equilibrium allocation and price system, thus implying the

[6] Recall that g is constant in all our experiments.

same Laffer curve. This is just Ricardian equivalence at play, which in the present context, manifests itself notably through the invariance of the after-tax interest rate to changes in τ or b.

In an IM setup, however, the invariance of the after-tax interest rate does not hold. Indeed, the after-tax interest rate is affected by the fact that capital and government bonds provide partial insurance to households. The cost of this insurance is reflected in the lower rate of return on those assets. When the government issues more debt, it effectively decreases the price of capital, thus lowering the insurance cost associated with holding capital. This translates into an increasing interest rate. By the same line of reasoning, since an increase in transfers also provides partial insurance to households, it also translates into an increasing interest rate. Hence, it is a priori unclear how balancing the government budget constraint via either τ or b affects the Laffer curve. As a consequence, in an IM setup, there is no sense in which on can define a Laffer curve independently from the way in which the government budget constraint is balanced.

In order to organize our discussion, it is thus convenient at this stage to define the concept of a steady-state *conditional Laffer curve* as follows.

Definition 2. *Let b be fixed, and let τ vary over an admissible range. Let $\tau_i(\tau)$, $i \in \{N, A, C\}$, denote the tax rate that balances the government budget constraint, holding the other two taxes constant, and let $\hat{R}(\tau)$ denote the associated level of government revenues. The steady-state Laffer curve conditional on transfers is the locus $(\tau_i(\tau), \hat{R}(\tau))$ relating tax rates to fiscal revenues. One can alternatively define the steady-state Laffer curve conditional on debt as the locus $(\tau_i(b), \hat{R}(b))$ by varying b over an admissible range, holding τ constant.*

Definition 2 leads us to the following proposition.

Proposition 2. *In an RA setup, the steady-state conditional Laffer curves* $(\tau_i(\tau), \hat{R}(\tau))$ *and* $(\tau_i(b), \hat{R}(b)$ *coincide, for all* $i \in \{N, A, C\}$.

Proof. See Appendix B. □

This proposition establishes that in an RA setup, the notion of conditional Laffer curves serves no special purpose, since the curves coincide. In the rest of this paper, we focus on analyzing the extent to which they differ in an IM setup.

3.4. **Calibration and Solution Method.** The model is calibrated to the US economy. A period is taken to be a year. Preferences are described by four parameters, σ, χ, η and β. We set $\sigma = 1.5$, as is conventional in the literature, and consider two alternative values for χ. In our benchmark calibration, we set $\chi = 1$, yielding a Frisch elasticity of labor supply equal to 1. Alternatively, we consider $\chi = 2$, yielding a Frisch elasticity of labor supply equal to 0.5. Both values are common in the macroeconomic literature. In each case, we pin down η so that aggregate hours worked $H \equiv \int g_h(a, s)\mathrm{d}x$ equal 0.25. The subjective discount factor β is adjusted so that the after-tax interest rate is equal to 4%, as in Trabandt and Uhlig (2011).

The fiscal parameters b and g are set to match the debt-output ratio and the government consumption-output ratio reported by Trabandt and Uhlig (2011), i.e., $b = 0.63$ and $g = 0.18$, respectively. The tax rates are calibrated to match estimates of effective tax rates computed using the methodology developed by Mendoza et al. (1994). This yields $\tau_N = 0.28$, $\tau_A = 0.38$, and $\tau_C = 0.05$. Using these parameters, the benchmark value of the transfer-output ratio τ is endogenously computed to balance the government budget constraint, yielding $\tau = 7.4\%$. Alternatively, we consider an economy with g set at a much smaller value. See the robustness section for more details.

We assume that $\log(s_t)$ follows an $AR(1)$ process

$$\log(s_t) = \rho_s \log(s_{t-1}) + \sigma_s \epsilon_t, \quad \epsilon_t \sim N(0,1).$$

We interpret $\log(s_t)$ as the residual persistent and idiosyncratic part of the log-wage rate in the specification adopted by Kaplan (2012), once experience and individual fixed effects have been accounted for. In the latter paper, the estimation results based on year effects yield $\rho_s = 0.958$ and $\sigma_s = \sqrt{0.017}$. We approximate this $AR(1)$ process via the Rouwenhorst (1995) method, as advocated by Kopecky and Suen (2010), using $n_s = 7$ points. This yields a transition matrix $\tilde{\pi}$ and a discrete support for individual productivity levels $\{s_1, \dots, s_{n_s}\}$.

In the spirit of Kindermann and Krueger (2014), we then allow for an eighth state corresponding to very high labor productivity. As they argue, such a state is a reduced form for entrepreneurial or artistic opportunities yielding very high labor income.

The final transition matrix is then

$$\pi = \begin{pmatrix} \tilde{\pi}_{11}(1-p_8) & \cdots & \tilde{\pi}_{14}(1-p_8) & \cdots & \tilde{\pi}i_{17}(1-p_8) & p_8 \\ \vdots & & \vdots & & \vdots & \vdots \\ \tilde{\pi}_{71}(1-p_8) & \cdots & \tilde{\pi}_{74}(1-p_8) & \cdots & \tilde{\pi}_{77}(1-p_8) & p_8 \\ 0 & \cdots & 1-p_{88} & \cdots & 0 & p_{88} \end{pmatrix}.$$

Here, p_8 is the probability of reaching the eighth productivity state from any normal productivity level. Additionally, p_{88} is the probability of staying in the high labor income state conditional on being in this state.

This specification of labor income shocks gives us three parameters (p_8, p_{88}, s_8), which we adjust to match, as closely as possible, the Gini coefficient of the wealth distribution ($G_w = 0.82$), the share of wealth held by the richest 20% ($\bar{a}_5 = 0.83$), and the Gini coefficient of the labor earning distribution ($G_e = 0.64$), as reported by Díaz-Gímenez et al. (2011). The calibration is summarized in Table I. In the

Table I. Parameters and Calibration Targets

	Common Parameters					
	$\gamma = 0.02$,	$\theta = 0.38$,	$\delta = 0.07$,	$\tau_N = 0.28$,	$\tau_A = 0.36$,	$\tau_C = 0.05$
	Specific Parameters					
	Benchmark	Low g	High χ	Low b	Log Utility	Alternative s
	$\chi = 1$, $b = 0.63$,	$\chi = 1$, $b = 0.63$,	$\chi = 2$, $b = 0.63$,	$\chi = 1$, $b = -0.50$,	$\chi = 1$, $b = 0.63$,	$\chi = 1$, $b = 0.63$,
	$g = 0.18$, $\sigma = 1.50$	$g = 0.05$, $\sigma = 1.50$	$g = 0.18$, $\sigma = 1.50$	$g = 0.18$, $\sigma = 1.50$	$g = 0.18$, $\sigma = 1.00$	$g = 0.18$, $\sigma = 1.50$
β	0.94	0.95	0.94	0.93	0.95	0.94
η	13.00	8.00	47.00	12.00	9.00	13.00
p_8	1.20%	1.25%	1.20%	1.80%	1.25%	–
p_{88}	85%	85%	85%	74%	85%	–
s_8/s_7	4.40	3.60	5.30	4.40	3.40	–
	Calibration Targets					
$(1-\tau_A)r$	0.04	0.04	0.04	0.04	0.04	0.08
H	0.25	0.25	0.25	0.25	0.25	0.30
G_e	0.63	0.63	0.63	0.63	0.64	0.27
G_w	0.79	0.79	0.79	0.80	0.79	0.63
$\bar{a}_5$	0.83	0.83	0.83	0.83	0.83	0.63

robustness section, we explore the sensitivity of our results to an alternative calibration in which the process for s does not have the extra productivity level and boils down to the $AR(1)$ specification of Kaplan (2012).

The solution method is now briefly described.[7] Given the calibration targets for the debt-output ratio and the tax rates, we postulate candidate values for the interest rate r and aggregate efficient labor N. We then solve the government budget constraint for the transfer-output ratio. To do so, we use the representative firm's first-order conditions, which give us values for $\hat{K}$ and $\hat{w}$, and the aggregate resource constraint, from which we determine $\hat{C}$. Given these values, we solve the agent's problem using the endogenous grid method proposed by Carroll (2006) and adapted to deal with the endogenous labor supply in the spirit of Barillas and Fernandez-Villaverde (2007). Using the implied decision rules, we then solve for the stationary distribution, as in Ríos-Rull (1999), and use it to compute aggregate quantities. We then iterate on r and N and repeat the whole process until the markets for capital and labor clear. For a given N, the interest rate is updated via a hybrid bisection-secant method. The bisection part of the algorithm is activated whenever the secant would update r to a value higher than the RA interest rate (which would result in diverging private savings, as shown in Aiyagari, 1994). Once the market-clearing r is found, N is updated with a standard secant method.[8]

To compute the conditional Laffer curves, we adapt the previous algorithm as follows. We first vary either the transfer-output ratio or the debt-output ratio over pre-specified ranges. At each grid point, given the postulated pair (r, N), the government steady-state budget constraint is balanced by adjusting one of the

[7] Further details are reported in the Technical Appendix.

[8] In a Aiyagari (1994)-like model with an endogenous labor supply, the outer loop on N would not be necessary. In our setup, because we also need to balance the government budget constraint, this extra loop is needed. See the Technical Appendix for further details.

three tax rates considered, holding the other two constant. Given values for the debt-output ratio or the transfer-output ratio, we then solve for the agent's decision rules and for the stationary distribution. We then iterate on r and N as described above.

4. Results

4.1. **Labor Income Taxes.** Figure 3 describes how the conditional Laffer curve associated with labor income taxes τ_N is constructed when the transfer-output ratio $\tau = T/Y$ is varied. Panel A (top left graph) shows the relation between the level of fiscal revenues $\hat{R}(\tau)$ and τ. Panel B (top right graph) shows the corresponding relation between $\tau_N(\tau)$ and τ. Finally, panel C (bottom graph) is a combination of the previous two relations. The black dotted line corresponds to the IM setup, and the dashed gray line is associated with the RA economy.

In both IM and RA economies, the Laffer curve conditional on τ has the classic inverted-U shape, as displayed in panel C. To understand this shape, consider a simplified setup in which $\tau_A = \tau_C = 0$. In this configuration, the government budget constraint simplifies to

$$\frac{R}{Y} = (1-\theta)\tau_N(\tau) = g + \tau + (r-\gamma)b.$$

Since g and b are held constant, assuming differentiability, we obtain from the above equation

$$\frac{\partial\left(\frac{R}{Y}\right)}{\partial\tau} = (1-\theta)\frac{\partial\tau_N}{\partial\tau} = 1 + \frac{\partial r}{\partial\tau}b.$$

In the RA economy, since $\partial r/\partial\tau = 0$ (see the proof of proposition 1), fiscal revenues as a share of GDP R/Y unambiguously increase when τ increases. As the above equation shows, this also implies that labor income taxes increase with τ. Thus, output declines when taxes rise.[9] The level of fiscal revenues $\hat{R}$ is the product of

[9] One can show that this happens whenever consumption and leisure are normal goods.

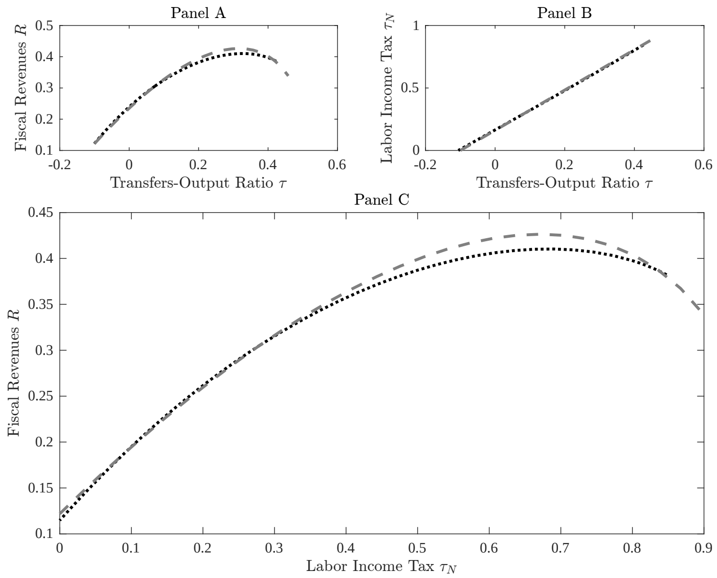


FIGURE 3. Construction of the Laffer Curve Conditional on Transfers - Labor Income Taxes

Note: The black dotted line corresponds to the IM economy, and the dashed gray curve is associated with the RA economy.

a term that declines with τ and another that is an increasing function of τ. This yields the inverted-U shape obtained for $\hat{R}(\tau)$ in the RA setup.

In the IM setup, changes in transfers impact the steady-state interest rate. This is so because higher transfers reduce the self-insurance motive and thus reduce capital accumulation by private agents. We thus expect $\partial r/\partial\tau$ to be positive. Since b is positive in our benchmark calibration, we obtain that R/Y increases with τ. For the same insurance motive, higher transfers also reduce the aggregate labor supply and the capital stock. This is reinforced by the fact that higher transfers come hand in hand with higher labor taxation. Since both N and $\hat{K}$ decline, aggregate output $\hat{Y}$ also declines.

Since in both setups, τ_N is an increasing function of τ (see panel B), the locus $(\tau_N(\tau), \hat{R}(\tau))$ inherits the inverted-U shape obtained for $(\tau, \hat{R}(\tau))$, thus yielding a classic Laffer curve. In the general case, when τ_C and τ_A are non-zero, the above reasoning holds but must also take into account the responses of $\hat{K}$ and $\hat{C}$ to

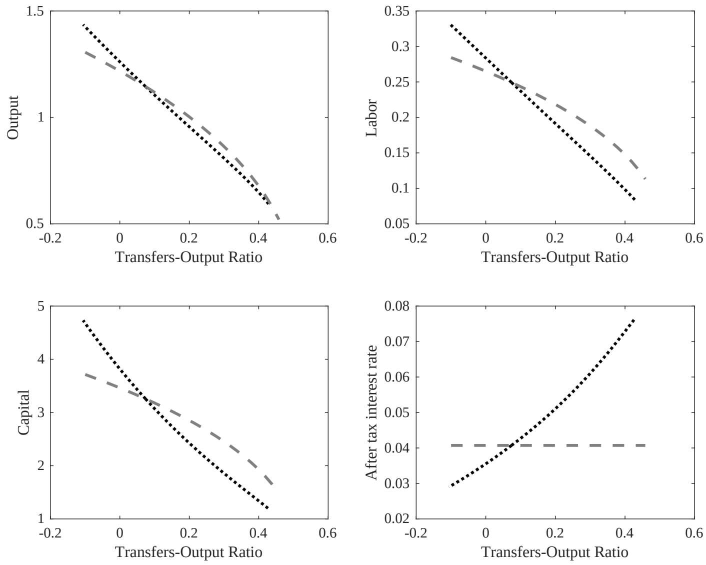


FIGURE 4. Aggregate Variables - Laffer curve on τ_N Conditional on τ

Note: The black dotted line corresponds to the benchmark IM setup for the Laffer curve on τ_N conditional on τ (thus holding b constant). The dashed grey line corresponds to the RA economy.

changes in τ. These endogenous responses combine to define the curves reported in Figure 3.

Notice that the conditional Laffer curve in the IM setup clearly resembles its RA counterpart. If anything, the only difference is that the right-hand side of the Laffer curve in this case declines at a slower pace than its RA counterpart. When transfers are adjusted, resorting to an RA model or to an IM model to characterize the shape and peak of the labor income tax Laffer curve has only mild consequences.

Figure 4 reports key aggregate variables that are useful for understanding the underpinnings of the Laffer curve conditional on transfers. In the RA setup, as previously explained, the after-tax interest rate is invariant to changes in transfers. This, in turn, implies that the capital-labor ratio is fixed. In this setup, an increase in transfers essentially boils down to an increase in the labor income tax, which translates into lower equilibrium hours worked and, thus, output. In contrast, in an IM economy, the interest rate increases as transfer increase. This is the mere reflection of lower precautionary savings. This translates into an even lower capital stock than in the RA setup for sufficiently large transfers. The increase in labor income tax then lowers employment, which also results in lower output.

We turn now to the Laffer curve for labor income taxes τ_N conditional on the debt-output ratio b. Figure 5 describes how this curve is constructed. Panel A (top left graph) shows the relation between $\hat{R}(b)$ and b. Panel B (top right graph) shows the corresponding relation between $\tau_N(b)$ and b. Finally, panel C (bottom graph) is a combination of the previous two relations. The plain black line corresponds to the IM setup and the dashed gray line is associated with the RA economy.

When debt is varied, the conditional Laffer curve now looks like a oriented horizontally S. In the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch, which reaches the usual pattern as labor income taxes decrease. This junction takes place at what appears to be a minimum tax level that is close to 25%. Interestingly, the minimum labor income tax obtains for a debt-output ratio close to -96%. Above this level, there can be one, two, or three tax rates associated with a given level of fiscal revenues. That is, there can be two levels of fiscal revenues associated with the same tax rate on the odd part of the Laffer curve conditional on debt: A high (low) level associated with negative (positive) debt.

What explains the odd shape of the Laffer curve in the left part of Figure 5 when the debt-output ratio is varied? To gain insight into this question, imagine again a simplified setting in which $\tau_C = \tau_A = 0$. Assuming differentiability of fiscal revenues with respect to b, one obtains

$$\frac{\partial \left(\frac{R}{Y}\right)}{\partial b} = (1-\theta)\frac{\partial \tau_N}{\partial b} = (r-\gamma) + b\frac{\partial r}{\partial b}.$$

Now, since public debt crowds out capital in the household's portfolio, we expect $\partial r/\partial b > 0$. Indeed, as shown by Aiyagari and McGrattan (1998), when b is large, $\hat{K}$ decreases, which increases the equilibrium interest rate r. Conversely, when b is

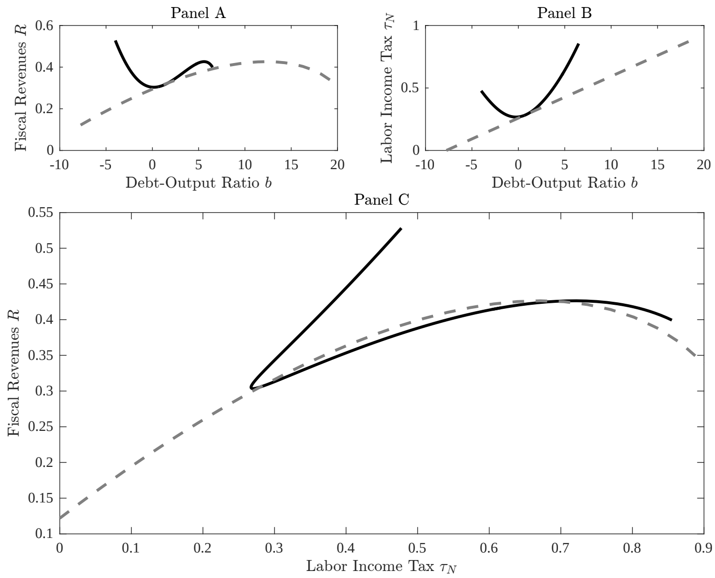


FIGURE 5. Construction of the Laffer Curve Conditional on Debt - Labor Income Taxes

Note: The black line corresponds to the incomplete-market economy, and the dashed gray curve is associated with the RA economy.

negative and large in absolute value, private wealth $\hat{A}$ shrinks, and the aggregate level of capital $\hat{K}$ increases, which decreases the equilibrium interest rate.

Thus, the term $b\partial r/\partial b$ changes sign when b changes sign. For a sufficiently negative debt-output ratio, we can thus observe a change in the sign of $\partial(R/Y)/\partial b$ and, since $R/Y = (1-\theta)\tau_N$, a corresponding change in the sign of $\partial\tau_N/\partial b$.

At the same time, as shown in Figure 6, $\hat{K}$ and N decrease with $\hat{B}$, so $\hat{Y}$ is also decreasing with $\hat{B}$. Thus, the level of fiscal revenues $R(\hat{B})$ is obtained as the product of a relation that changes sign and another that is strictly decreasing, thus yielding a horizontal S shape (see panel A). Now, given the non-monotonic response of $\tau_N(\hat{B})$ (see panel B), the Laffer curve conditional on debt, which is a combination of panels A and B, also exhibits a horizontal S shape (panel C).[10]

[10] It is important at this stage to emphasize that the odd shape of the Laffer curve conditional on debt has nothing to do with pathological behavior of the labor supply. In particular, the Technical Appendix reports that efficient hours are a decreasing function of $\hat{B}$ that closely mimic the behavior of hours worked in an RA setup.

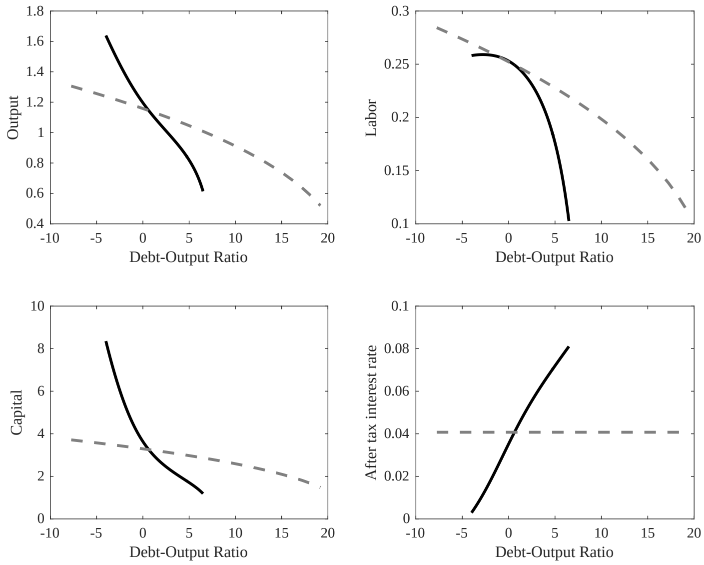


FIGURE 6. Aggregate Variables - Laffer curve on τ_N conditional on b

Note: The black line corresponds to the benchmark IM setup for the Laffer curve on τ_N conditional on b (thus holding τ constant). The dashed grey line corresponds to the RA economy.

Starting from a negative debt-output ratio $\hat{B}$, output, τ_N, and R are large. As the government sells more and more assets, i.e., as $\hat{B}$ increases, output and τ_N decline, so R also declines. This corresponds to the odd part of the Laffer curve. In this region, there are two forces at play. First, as $\hat{B}$ increases, the capital stock decreases, thus implying declining real wages and resulting in declining aggregate labor N. Second, since τ_N also decreases, agents are willing to supply more labor. It turns out that the first force dominates. Once the minimal tax is reached, τ_N and $\hat{R}$ start to increase while $\hat{Y}$ is still declining. This corresponds to the regular part of the Laffer curve, i.e., the part that looks like an inverted U. In this region, increases in τ_N dominate the disincentives of taxation up to the maximal tax rate after which the disincentives start to dominate.

In the general case, when τ_C and τ_A are non-zero, the above reasoning holds but must also take into account the responses of $\hat{K}$ and $\hat{C}$. These endogenous responses combine to define the point at which fiscal revenues exhibit the odd shape identified above. This also defines the minimal labor income tax.

Figure 6 reports key aggregate variables that are useful for understanding the underpinnings of the Laffer curve conditional on debt. The results for the RA model are exactly the same as in Figure 4. In the IM setup, things are radically modified. The interest rate increases steeply as the debt-output ratio increases. This is the crowding-out effect emphasized by Aiyagari and McGrattan (1998). This translates into a steep decline in capital as well. Labor, in turn, is the mirror image of the tax rate. In particular, as public debt becomes increasingly negative, labor starts to decline precisely when taxes start to increase.

4.2. **Capital Income Taxes.** Figure 7 reports three Laffer curves associated with variations in τ_A. The dashed gray curve corresponds to the RA economy. The black dotted line is the Laffer curve conditional on transfers $\hat{T}$ in the IM setup. Finally, the black line is the Laffer curve conditional on debt $\hat{B}$ in the IM economy. To save space, we dispense with a complete description of how the conditional Laffer curves are constructed, as the process closely parallels the previously explained steps.

In the case when transfers are varied, the conditional Laffer curve associated with τ_A has the standard inverted-U shape. It has the overall same shape as the curve that would obtain in the RA economy, as shown in Figure 7.[11]

As was the case for labor income taxes, when the debt-output ratio $\hat{B}$ is varied, we reach very different conclusions (see the black curve in Figure 7). Under this assumption, the Laffer curve also looks like a horizontally oriented S. In the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch that follows the regular pattern as capital income taxes decrease. Once again, this junction takes place at what appears to be a minimum tax level close to 25%. Interestingly, the minimum capital income tax obtains for a debt-output

[11] The figures reporting how key aggregate variables vary with τ and b are reported in Appendix C.

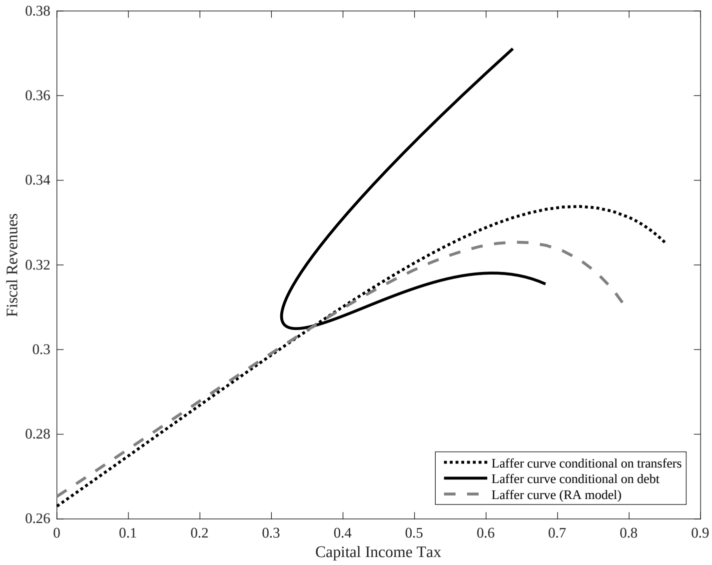


FIGURE 7. Laffer Curves – Capital Income Tax

Note: Level of fiscal revenues as a function of labor income tax τ_A. The black dotted line corresponds to the Laffer curve conditional on transfers, the plain balck line is associated with the Laffer curve conditional on debt, and the dashed grey line corresponds to the RA economy.

ratio close to -129%. Above this level, there can be one, two, or three tax rates associated with a given level of fiscal revenues.

4.3. **Consumption Taxes.** Figure 8 reports three Laffer curves associated with variations in τ_C, defined in the exact way as before. The dashed gray line corresponds to the RA economy as above. The black dotted line is the Laffer curve associated with the IM economy conditional on transfers $\hat{T}$. Finally, the black line is the Laffer curve in the IM economy conditional on the debt-output ratio $\hat{B}$.[12]

As in Trabandt and Uhlig (2011), the Laffer curve associated with τ_C does not exhibit a peak in either the RA setup or the IM setup with adjusted transfers. In the latter, fiscal revenues are slightly higher than in the former. Fundamentally, in both settings, taxing consumption is like taxing labor (both taxes appear similarly in the first-order condition governing labor supply). A difference, though, is that

[12]The figures reporting how key aggregate variables vary with τ and b are reported in Appendix C.

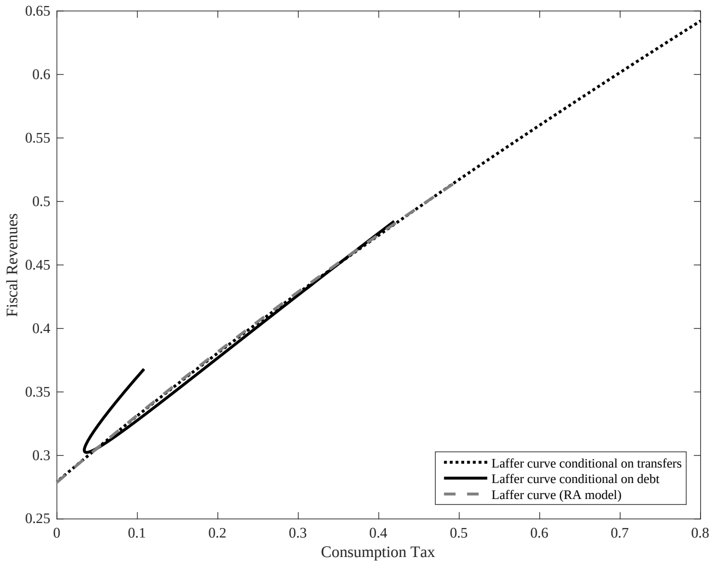


FIGURE 8. Laffer Curves – Consumption Tax

Note: Level of fiscal revenues as a function of labor income tax τ_A. The black dotted line corresponds to the Laffer curve conditional on transfers, the plain black line is associated with the Laffer curve conditional on debt, and the dashed grey line corresponds to the RA economy.

in an IM economy such as ours, agents with low labor productivity choose not to work whenever they hold enough assets. Clearly, those agents would not suffer from labor income taxation but do suffer from consumption taxes. Combined with the relative inelasticity of the labor supply in the IM setup, this explains why the government can raise more revenues in this framework than in the RA setup.

As in the previous sections, when the debt-output ratio $\hat{B}$ is varied, we reach different conclusions (see the black curve in Figure 8). Under this assumption, in the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch that follows the regular pattern as consumption taxes decrease. Again, this junction takes place at what appears to be a minimum tax level that is close to 1.42%, which is associated with a debt-output ratio close to -110.8%. Above this level, there can be two tax rates associated with a given level of fiscal revenues.

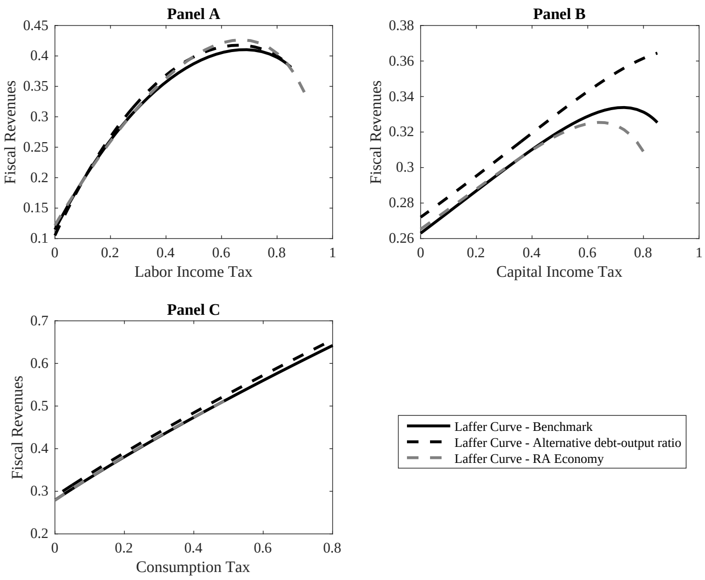


FIGURE 9. Laffer Curves Conditional on τ for Alternative b

Note: Laffer curves conditional on transfers for alternative levels of the steady-state debt-output ratio.

4.4. **Corollary.** Proposition 1 establishes that the Laffer curves associated with τ_i, $i \in \{N, A, C\}$ do not depend on the debt-output ratio in a RA setup. That is, the Laffer curves in an economy with a debt-output ratio of 63% are the same as those in an economy with a debt-output ratio of -50%. However, the previous analyses suggest that we should not expect this property to hold in an IM framework due to the general equilibrium feedback effect of public debt on the after-tax interest rate. This section investigates how variations in the steady-state debt-output ratio impact the Laffer curve conditional on transfers.

The results are reported in Figure 9. Panels A, B, and C report the Laffer curves associated with labor income taxes, capital income taxes, and consumption taxes, respectively. For each tax considered in the analysis, three Laffer curves (conditional on transfers) are drawn. The plain black lines correspond to the benchmark calibration in which $b = 0.63$. The black dashed lines correspond to an alternative economy with $b = -0.5$, holding all the other parameters to their benchmark value. As before, the dashed grey lines correspond to the Laffer curves under the RA model, which we report to facilitate comparison.

Several interesting results emerge. First, panels A and C show that for the range of the debt-output ratios considered here, the Laffer curves associated with labor income taxes τ_N and the consumption taxes τ_C hardly differ. To some extent, this is reassuring given the current fiscal context in the US. However, panels A and C also show that for negative debt levels, the Laffer curves on τ_N and τ_C are somewhat higher than their benchmark counterparts. More striking differences emerge from panel B, which shows the Laffer curves associated with capital income taxation.

4.5. **Robustness.** In our robustness assessment, we explore five alternative calibrations. The first considers a lower elasticity of labor supply ($\chi = 2$), since the Laffer curve has been found to be very sensitive to this parameter (see Trabandt and Uhlig, 2011). The second considers a lower share of government spending. Here, we set g to a smaller number, 5%, adjusting τ to a larger value. As argued by Oh and Reis (2012) and Prescott (2004), a significant share of government spending can be thought as transfers. Also, as argued in Section 2, public debt (net of financial assets) is negative, on average, over the last century in the US, with a value of approximately -50%. In our third robustness check, we recalibrate the government debt-output ratio to match this number. In our fourth robustness check, we explore the sensitivity of the oddly-shaped Laffer curves to the log-utility case ($\sigma = 1$ in the utility function). Finally, in our last robustness check, we drop the exceptional productivity level and adopt the process for idiosyncratic labor productivity estimated by Kaplan (2012).

In each robustness analysis, except for the last one, we recalibrate the model to match the calibration targets as in the benchmark case. The calibration details are reported in Table I. The results are reported in Figures 10 (lower Frisch elasticity),

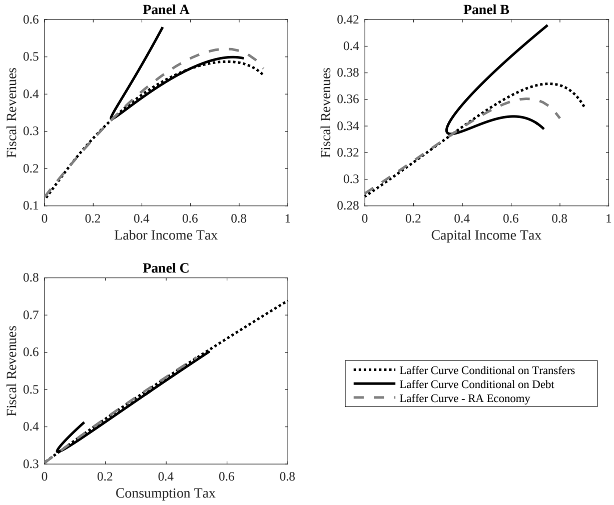


FIGURE 10. Laffer Curves - Lower Elasticity of Labor Supply

Note: The black dotted line is associated with the Laffer curve conditional on transfers, the plain black line is associated with the Laffer curve conditional on debt, and the dashed grey line corresponds to the RA Laffer curve.

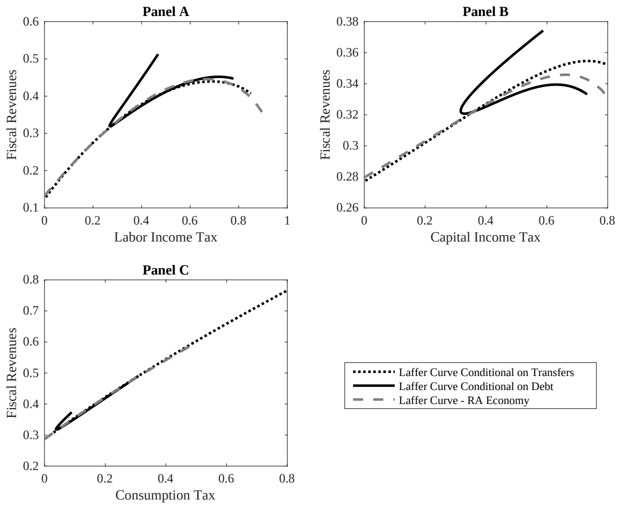


FIGURE 11. Laffer Curves - Lower Share of Government Spending

Note: The black dotted line is associated with the Laffer curve conditional on transfers, the plain black line is associated with the Laffer curve conditional on debt, and the dashed grey line corresponds to the RA Laffer curve.

11 (lower share of government spending), 12 (negative debt-output ratio), 13 (log-utility), and 14 (alternative process for s).

The bottom line is that in all our robustness analyses, our qualitative results hold. In particular, in each alternative calibration, we find that in an IM economy, the Laffer curves conditional on debt look like a horizontal S for labor income

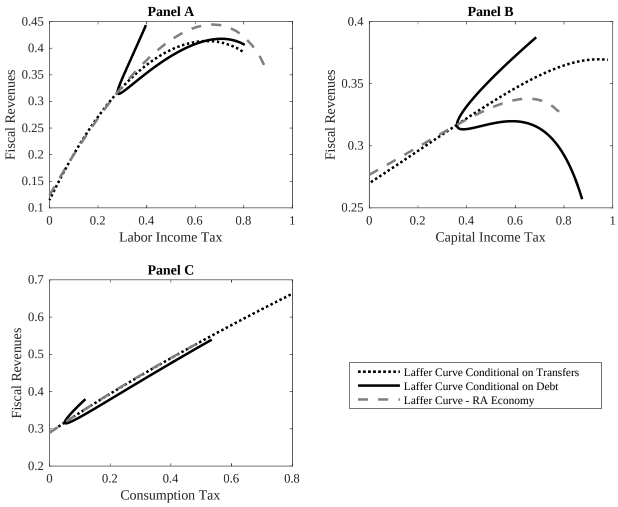


Figure 12. Laffer Curves - Negative Debt-Output Ratio

Note: The black dotted line is associated with the Laffer curve conditional on transfers, the plain black line is associated with the Laffer curve conditional on debt, and the dashed grey line corresponds to the RA Laffer curve.

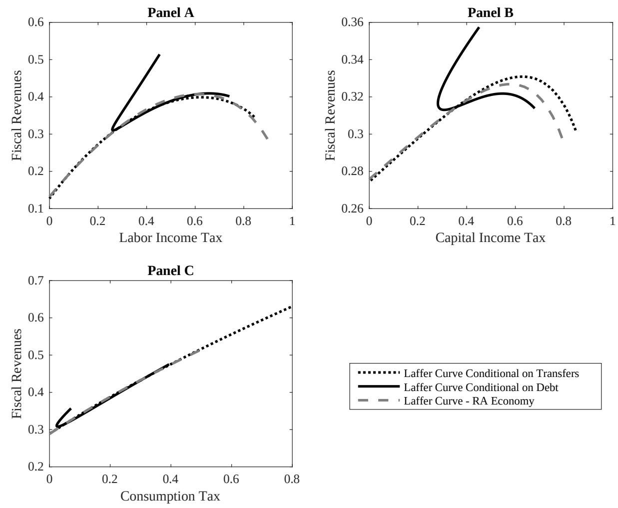


Figure 13. Laffer Curves - Log Utility

Note: The black dotted line is associated with the Laffer curve conditional on transfers, the plain black line is associated with the Laffer curve conditional on debt, and the dashed grey line corresponds to the RA Laffer curve.

and capital income taxes, while the Laffer curves conditional on transfers resemble their RA analogs. Notice that an odd shape appears even for the Laffer curve on consumption taxes conditional on debt.

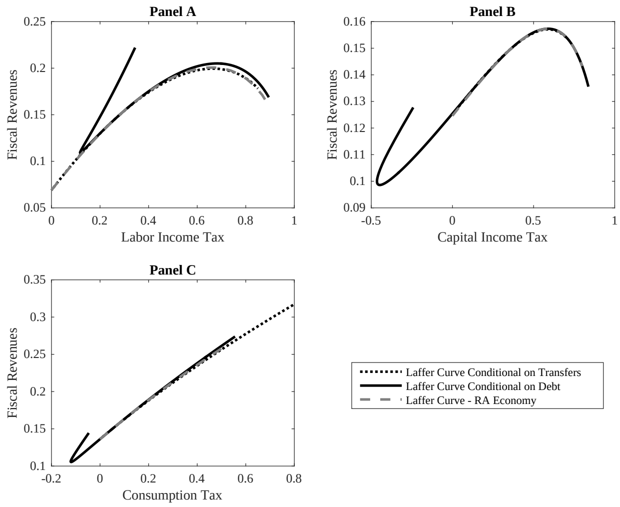


FIGURE 14. Laffer Curves - Alternative Process for s

Note: The black dotted line is associated with the Laffer curve conditional on transfers, the plain black line is associated with the Laffer curve conditional on debt, and the dashed grey line corresponds to the RA Laffer curve.

5. CONCLUSION

In this paper, we have inspected how allowing for liquidity-constrained agents and incomplete financial markets impacts the shape of the Laffer curve. To address this question, we formulated a neoclassical growth model along the lines of Aiyagari and McGrattan (1998). The model was calibrated to the US economy to mimic great ratios as well as moments related to the wealth and earnings distributions. We paid particular attention to which of debt or transfers are adjusted to balance the government budget constraint as taxes are varied. In a RA framework, this does not matter, whereas opting to adjust debt rather than transfers is important in an IM setup.

Our main findings are the following. The properties of the Laffer curve depend on whether it is conditional on debt or conditional on transfers. When we consider Laffer curves conditional on transfers, the results in an IM economy closely resemble their RA analogs. However, when we consider Laffer curves conditional on debt, we obtain a dramatically different picture. Now, the Laffer curves on labor and capital income taxes resemble horizontal Ss, meaning that there can be up to three tax rates associated with the same level of fiscal revenues. These properties appear robust to a series of alternative specifications/calibrations.

Appendix A. Proof of proposition 1

Recall that we defined

$$\tau \equiv \frac{\hat{T}}{\hat{Y}}, \qquad b \equiv \frac{\hat{B}}{\hat{Y}}.$$

In the representative agent version of the model, the steady-state system is

$$\hat{C} + (\gamma + \delta)\hat{K} = (1 - g)\hat{Y}, \tag{A.1}$$

$$(1 + \tau_C)\hat{\Lambda} = \hat{C}^{-\sigma}, \tag{A.2}$$

$$\hat{\Lambda}(1 - \tau_N)\hat{w} = \eta H^{\chi}, \tag{A.3}$$

$$\hat{Y} = \hat{K}^{\theta}(\Omega H)^{1-\theta}, \tag{A.4}$$

$$r + \delta = \theta \frac{\hat{Y}}{\hat{K}}, \tag{A.5}$$

$$\hat{w} = (1 - \theta)\frac{\hat{Y}}{H}, \tag{A.6}$$

$$1 + \gamma = \beta[1 + (1 - \tau_A)r]. \tag{A.7}$$

The system is solved recursively in the usual manner. Combining (A.7) and (A.5), one arrives at

$$\frac{\hat{Y}}{\hat{K}} = \frac{1 + \gamma - \beta[1 - (1 - \tau_A)\delta]}{\beta(1 - \tau_A)\theta}.$$

Using (A.4), this implies

$$\frac{\hat{K}}{H} = \left(\frac{\hat{Y}}{\hat{K}}\right)^{\frac{1}{\theta - 1}} \Omega$$

and

$$\frac{\hat{Y}}{H} = \left(\frac{\hat{Y}}{\hat{K}}\right)^{\frac{\theta}{\theta - 1}} \Omega.$$

Using (A.1), this implies that

$$\frac{\hat{C}}{H} = (1 - g)\frac{\hat{Y}}{H} - (\gamma + \delta)\frac{\hat{K}}{H}.$$

Then, combining (A.2), (A.3), and (A.6), one arrives at

$$\frac{1 - \theta}{\eta}\frac{1 - \tau_N}{1 + \tau_C}\frac{\hat{Y}}{H}\hat{C}^{-\sigma} = H^{\chi},$$

and rearranging yields

$$H = \left(\frac{1-\theta}{\eta} \frac{1-\tau_N}{1+\tau_C} \frac{\hat{Y}}{H} \left(\frac{\hat{C}}{H} \right)^{-\sigma} \right)^{\frac{1}{\chi+\sigma}}.$$

Having solved for H, we can solve for all the other variables. It thus turns out that the steady-state allocation $(\hat{C}, \hat{K}, H, \hat{Y})$ and the steady-state price system $(r, \hat{w})$ do not depend on either τ or b. Tax revenues $\hat{R}$, in turn, depend only on the tax system (τ_N, τ_A, τ_C), the steady-state allocation and the steady-state price system. Thus, for $i \in \{N, A, C\}$, the Laffer curve associated with τ_i is independent of either τ or b.

As an aside, we noted that in practice, we recalibrate η and β so that the RA model and the benchmark IM model have the same H and r given otherwise identical structural parameters (i.e., θ, δ, γ, σ, χ) and identical fiscal parameters (i.e., b, τ, τ_A, τ_N, τ_C). Hence, given a value of r, we back out β via (A.7), yielding

$$\beta = \frac{1+\gamma}{1+(1-\tau_A)r}.$$

Similarly, given r and H, we back out η using

$$\eta = \frac{1-\theta}{H^{\chi+\sigma}} \frac{1-\tau_N}{1+\tau_C} \frac{\hat{Y}}{H} \left(\frac{\hat{C}}{H} \right)^{-\sigma}$$

using the formulas for $\hat{Y}/H$ and $\hat{C}/H$ obtained above.

Appendix B. Proof of proposition 2

Given proposition 1, it is sufficient to establish the existence of a one-to-one relationship between τ and τ_i and between b and τ_i, $i \in \{N, A, C\}$, to prove that the two conditional Laffer curves $(\tau_i(\tau), R(\tau))$ and $(\tau_i(b), R(b))$ coincide in the RA setup.

To this end, let $\tilde{r} \equiv (1-\tau_A)r$ denote the after-tax interest rate. We thus have

$$\tilde{r} = \frac{1+\gamma}{\beta} - 1.$$

Notice that in the RA model, the steady-state $\tilde{r}$ does not depend on any of the three tax rates considered. Fiscal revenues as a share of GDP are then

$$\frac{\hat{R}}{\hat{Y}} = (1-\tau_N)(1-\theta) + \tau_C \frac{\hat{C}}{\hat{Y}} + \frac{\tau_A}{1-\tau_A} \tilde{r} \frac{\hat{K}}{\hat{Y}}.$$

B.1. **Labor income tax.** Let us first consider the labor income tax τ_N. As shown above, $\hat{Y}/\hat{K}$, $\hat{K}/H$, $\hat{Y}/H$ and $\hat{C}/H$ do not depend on τ_N. It follows that

$$\frac{\partial}{\partial \tau_N}\left(\frac{\hat{R}}{\hat{Y}}\right) = 1-\theta > 0.$$

At the same time, we have

$$\frac{\hat{R}}{\hat{Y}} = g + \tau + (\tilde{r}-\gamma)b.$$

Since $\tilde{r}$ is tax invariant, assuming that τ is adjusted and b is fixed, one obtains

$$\frac{\partial \tau}{\partial \tau_N} = \frac{\partial}{\partial \tau_N}\left(\frac{\hat{R}}{\hat{Y}}\right) = 1-\theta > 0.$$

Assuming instead that b is adjusted while τ is fixed, one obtains

$$\frac{\partial b}{\partial \tau_N} = \frac{1}{\tilde{r}-\gamma}\frac{\partial}{\partial \tau_N}\left(\frac{\hat{R}}{\hat{Y}}\right) = \frac{1-\theta}{\tilde{r}-\gamma} > 0,$$

since

$$\tilde{r} - \gamma = (1+\gamma)\left(\frac{1}{\beta} - 1\right) > 0.$$

It follows that the relations between τ_N and τ and between τ_N and b are both strictly increasing and thus one to one. It is equivalent to vary τ_N and adjust τ (b) and to vary τ (b) and adjust τ_N.

B.2. **Consumption tax.** Now, let us consider the consumption tax. Since $\hat{Y}/H$ and $\hat{C}/H$ do not depend on τ_N, it must be the case that

$$\frac{\partial}{\partial \tau_C}\left(\frac{\hat{R}}{\hat{Y}}\right) = \frac{\hat{C}}{\hat{Y}} > 0.$$

Hence, by the same line of reasoning

$$\frac{\partial \tau}{\partial \tau_C} = \frac{\partial}{\partial \tau_C}\left(\frac{\hat{R}}{\hat{Y}}\right) = \frac{\hat{C}}{\hat{Y}} > 0,$$

and

$$\frac{\partial b}{\partial \tau_C} = \frac{1}{\tilde{r}-\gamma}\frac{\partial}{\partial \tau_C}\left(\frac{\hat{R}}{\hat{Y}}\right) = \frac{1}{\tilde{r}-\gamma}\frac{\hat{C}}{\hat{Y}} > 0.$$

It follows that the relations between τ_C and τ and between τ_A and b are both strictly increasing and thus one to one. It is equivalent to vary τ_C and adjust τ (b) and to vary τ (b) and adjust τ_C.

B.3. **Capital income tax.** Finally, consider the capital income tax τ_A. Differentiating the fiscal revenues-output ratio with respect to τ_A yields

$$\frac{\partial}{\partial \tau_A}\left(\frac{\hat{R}}{\hat{Y}}\right) = \tau_C\frac{\partial}{\partial \tau_A}\left(\frac{\hat{C}}{\hat{Y}}\right) + \frac{\tilde{r}}{1-\tau_A}\left[\frac{1}{1-\tau_A}\frac{\hat{K}}{\hat{Y}} + \tau_A\frac{\partial}{\partial \tau_A}\left(\frac{\hat{K}}{\hat{Y}}\right)\right].$$

In turn, using the relation derived in the proof of Proposition 1, we obtain

$$\frac{\partial}{\partial \tau_A}\left(\frac{\hat{K}}{\hat{Y}}\right) = -\frac{\beta\theta(1+\gamma-\beta)}{\{1+\gamma-\beta[1-(1-\tau_A)\delta]\}^2} < 0$$

and

$$\frac{\partial}{\partial \tau_A}\left(\frac{\hat{C}}{\hat{Y}}\right) = \frac{(\gamma+\delta)\beta\theta(1+\gamma-\beta)}{\{1+\gamma-\beta[1-(1-\tau_A)\delta]\}^2} > 0.$$

We thus obtain

$$\frac{\partial}{\partial \tau_A}\left(\frac{\hat{R}}{\hat{Y}}\right) = \frac{\theta(1+\gamma-\beta)[\tau_C\beta(\gamma+\delta)+1+\gamma-\beta(1-\delta)]}{\{1+\gamma-\beta[1-(1-\tau_A)\delta]\}^2} > 0.$$

Thus, by the same line of reasoning, since

$$\frac{\partial}{\partial \tau_A}\left(\frac{\hat{R}}{\hat{Y}}\right) = (\tilde{r}-\gamma)\frac{\partial b}{\partial \tau_A} = \frac{\partial \tau}{\partial \tau_A},$$

it follows that the relations between τ_A and τ and between τ_A and b are both strictly increasing and thus one to one. It is equivalent to vary τ_A and adjust τ (b) and to vary τ (b) and adjust τ_A.

B.4. **Summing up.** For each $i \in \{N, A, C\}$, we found that there exists a one-to-one relationship between τ and τ_i and between b and τ_i. Thus, the Laffer curve obtained by varying τ_i and letting τ (b) adjust coincides with the conditional Laffer curve obtained by varying τ (b) and letting τ_i adjust. By Proposition 1, we thus obtain that in an RA setup, the steady-state conditional Laffer curves coincide exactly.

Appendix C. Additional Results on the Laffer Curves Associated with Capital Income and Consumption Taxes

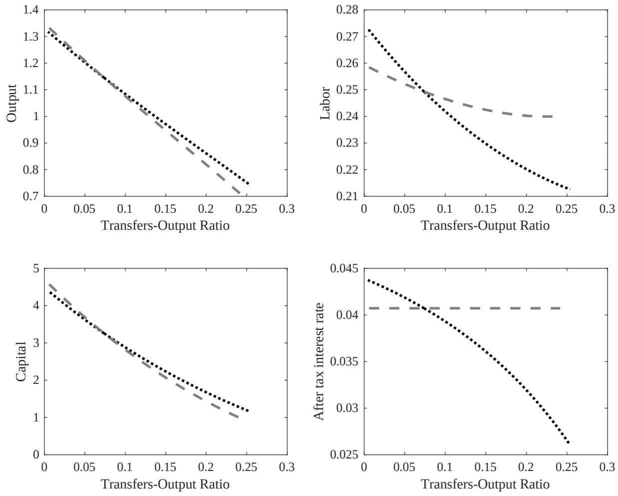


Figure C.1. Aggregate Variables - Laffer Curve on τ_A Conditional on τ

Note: Aggregate variables as τ_A is varied and the transfer-output ratio τ is adjusted. The black dotted line corresponds to the IM economy, and the dashed gray line corresponds to the RA economy.

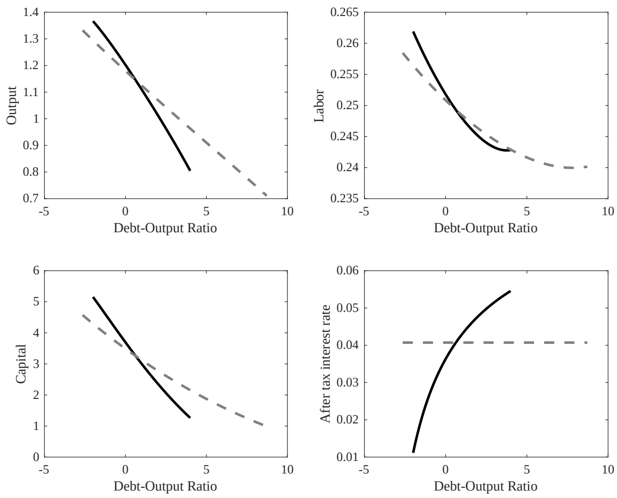


Figure C.2. Aggregate Variables - Laffer Curve on τ_A Conditional on b

Note: Aggregate variables as τ_A is varied and the debt-output ratio b is adjusted. The plain black line corresponds to the IM economy, and the dashed gray line corresponds to the RA economy.

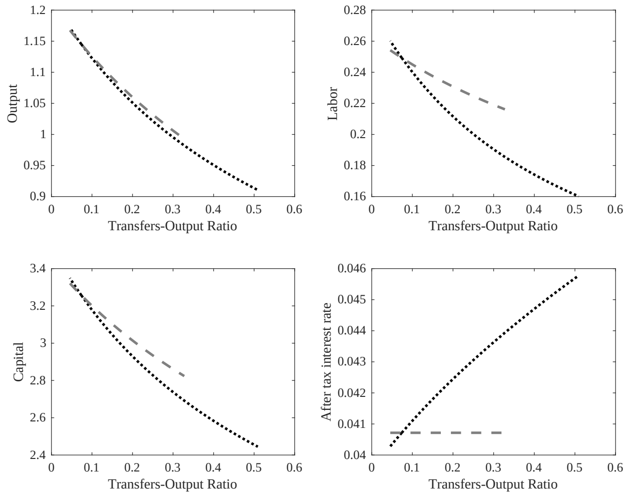


FIGURE C.3. Aggregate Variables - Laffer Curve on τ_C Conditional on τ

Note: Aggregate variables as τ_C is varied and the transfer-output ratio τ is adjusted. The black dotted line corresponds to the IM economy, and the dashed gray line corresponds to the RA economy.

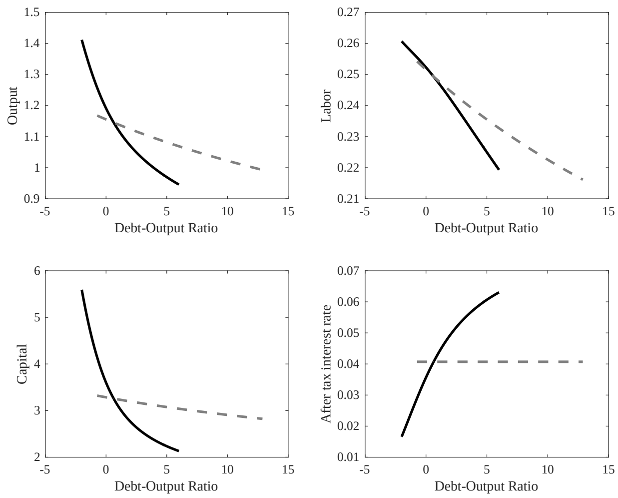


FIGURE C.4. Aggregate Variables - Laffer Curve on τ_C Conditional on b

Note: Aggregate variables as τ_C is varied and the debt-output ratio b is adjusted. The plain black line corresponds to the IM economy, and the dashed gray line corresponds to the RA economy.

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TECHNICAL APPENDIX TO
THE LAFFER CURVE IN AN INCOMPLETE–MARKET ECONOMY
(NOT INTENDED FOR PUBLICATION)

PATRICK FÈVE, JULIEN MATHERON, AND JEAN–GUILLAUME SAHUC

CONTENTS

APPENDIX A. MODEL

A.1. **The Economic Environment.** We consider a discrete time economy without aggregate risk similar to that studied in Aiyagari and McGrattan (1998). Time is indexed by $t \in \mathbf{N}$. The final good Y_t, which is the numeraire, is produced by competitive firms, according to the technology

$$Y_t = K_t^{\theta}(Z_t N_t)^{1-\theta},$$

where $\theta \in (0,1)$ denotes the elasticity of production with respect to capital, K_t and N_t are the inputs of capital and efficient labor, respectively, and Z_t is an exogenous technical progress index evolving according to $Z_{t+1} = (1+\gamma)Z_t$, with $\gamma > 0$ and $Z_0 = 1$. Firms rent capital and efficient labor on competitive markets, at rates $r_t + \delta$ and w_t, respectively. Here, $\delta \in (0,1)$ denotes the depreciation rate of physical capital, r_t is the interest rate, and w_t is the wage rate.

The first order conditions for profit maximization are

$$r_t + \delta = \theta \left(\frac{K_t}{Z_t N_t} \right)^{\theta - 1},$$

$$w_t = (1-\theta) Z_t \left(\frac{K_t}{Z_t N_t} \right)^{\theta}.$$

The economy is populated with a continuum of ex ante identical, infinitely lived households. The typical households has utility given by

$$\mathsf{E} \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} \left(\frac{c_t}{Z_t} \right)^{1-\sigma} - \frac{\eta}{1+\chi} h_t^{1+\chi} \right] |a_0, s_0 \right\},$$

where $\mathsf{E}_0\{\cdot|a_0, s_0\}$ is the mathematical expectation conditioned on the individual state at date 0. The individual state consists of initial assets a_0 and the exogenous individual state s_0. Here, $\beta \in (0,1)$ denotes the discount factor, $c_t \geq 0$ is individual consumption, $0 \leq h_t \leq 1$ is the individual labor supply, $\sigma > 0$ is the relative risk aversion coefficient,

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The views expressed herein are those of the authors and should under no circumstances be interpreted as reflecting those of the Banque de France.

$\chi > 0$ is the inverse of the Frisch elasticity of labor, and $\eta > 0$ is a scaling constant. Consumption appears in deviation from the index of technical progress to ensure a well-behaved balanced-growth path.

At the beginning of each period, households receive an individual productivity level $e(s_t) > 0$. We assume that s_t is *i.i.d*. across agents and evolves over time according to a Markov process, with bounded support $\mathcal{S}$ and stationary transition function $Q(s, s')$.[1]

An individual agent's efficient labor is $e(s_t)h_t$, with corresponding labor earnings given by $(1 - \tau_N)w_t e(s_t)h_t$, where τ_N denotes the labor income tax. In addition, agents self-insure by accumulating a_t units of assets which pay the after-tax rate of return $(1 - \tau_A)r_t$, where τ_A denotes the capital income tax. These assets can consist of units of physical capital and/or government bonds. Once arbitrage opportunities have been ruled out, each asset has the same rate of return. Also, agents must pay a consumption tax τ_C. Finally, they perceive transfers T_t. Thus, an agent's budget constraint is

$$(1 + \tau_C)c_t + a_{t+1} \leq (1 - \tau_N)w_t e(s_t)h_t + [1 + (1 - \tau_A)r_t]a_t + T_t.$$

Borrowing is exogenously restricted by the following constraint

$$a_{t+1} \geq 0.$$

There is finally a government in the economy. The government issues debt B_{t+1}, collects tax revenues, rebates transfers, and consumes G_t units of final good. The associated budget constraint is given by

$$B_{t+1} = (1 + r_t)B_t + T_t + G_t - (\tau_A r_t A_t + \tau_N w_t N_t + \tau_C C_t)$$

where C_t and A_t denote aggregate (per capita) consumption and assets held by the agents, respectively.

A.2. **Equilibrium Defined.** In the remainder of this paper, we focus exclusively on the steady state of an appropriately normalized version of the above economy. Growing variables are rendered stationary by dividing them by Z_t. Variables so normalized are referred to with a hat. In the benchmark specification, the ratio of government expenditures to output $g \equiv \hat{G}/\hat{Y}$ is constant. In the robustness section, we also consider an alternative case in which the level of government expenditures $\hat{G}$ (in deviation from Z_t) is constant.

We let $\mathcal{A}$ denote the set of possible values for assets $\hat{a}$. We let the joint distribution of agents across assets $\hat{a}$ and individual exogenous states s be denoted $x(\hat{a}, s)$, defined on $\mathscr{A} \times \mathscr{S}$, the Borel subsets of $\mathcal{A} \times \mathcal{S}$. Thus, for all $\mathcal{A}_0 \times \mathcal{S}_0 \in \mathscr{A} \times \mathscr{S}$, $x(\mathcal{A}_0, \mathcal{S}_0)$ is the mass of agents with assets $\hat{a}$ in $\mathcal{A}_0$ and individual state s in $\mathcal{S}_0$.

We can now write an agent's problem in recursive form

$$\begin{aligned} v(\hat{a}, s) = \max_{\hat{c}, h, \hat{a}'} & \left\{ \frac{1}{1-\sigma}(\hat{c})^{1-\sigma} - \frac{\eta}{1+\chi}h^{1+\chi} + \beta \int_{\mathcal{S}} v(\hat{a}', s')Q(s, \mathrm{d}s') \right\} \\ \text{s.t.} \quad & (1 + \tau_C)\hat{c} + (1 + \gamma)\hat{a}' \leq (1 - \tau_N)\hat{w}e(s)h + (1 + (1 - \tau_A)r)\hat{a} + \hat{T}, \\ & \hat{a}' \geq 0, \ \ \hat{c} \geq 0, \ \ 0 \leq h \leq 1. \end{aligned} \tag{A.1}$$

For convenience, we restrict $\hat{a}$ to belong to the compact set $\mathcal{A} = [0, \hat{a}_M]$, where $\hat{a}_M$ is a large number.[2] We can thus define a stationary, recursive equilibrium in the following way.

Definition 1. *A steady-state, recursive competitive equilibrium is a constant system of prices $\{r, \hat{w}\}$, a vector of constant policy variables $(\tau_C, \tau_A, \tau_N, \hat{T}, \hat{G}, \hat{B})$, a value function $v(\hat{a}, s)$, time-invariant decision rules for an individual's assets holdings, consumption, and labor supply $\{g_a(\hat{a}, s), g_c(\hat{a}, s), g_h(\hat{a}, s\}$, a measure $x(\hat{a}, s)$ of agents over the state space $\mathcal{A} \times \mathcal{S}$, and aggregate quantities $\hat{A} \equiv \int \hat{a}\mathrm{d}x$, $\hat{C} \equiv \int g_c(\hat{a}, s)\mathrm{d}x$, $N \equiv \int e(s)g_h(\hat{a}, s)\mathrm{d}x$, and $\hat{K}$ such that:*

(i) The value function $v(\hat{a}, s)$ solves the agent's problem stated in equation (A.1), with associated decision rules $g_a(\hat{a}, s)$, $g_c(\hat{a}, s)$ and $g_h(\hat{a}, s)$;

[1] The transition Q has the following interpretation: for all $s \in \mathcal{S}$ and for all $\mathcal{S}_0 \in \mathscr{S}$, where $\mathscr{S}$ denotes the Borel subsets of $\mathcal{S}$, $Q(s, \mathcal{S}_0)$ is the probability that next period's individual state lies in $\mathcal{S}_0$ when current state is s.

[2] $\hat{a}_M$ is selected so that the decision rule on assets for an individual with the highest productivity and highest discount factor crosses the 45-degree line below $\hat{a}_M$.

(ii) Firms maximize profits and factor markets clear, so that

$$\hat{w} = (1-\theta)\left(\frac{\hat{K}}{N}\right)^{\theta},$$

$$r + \delta = \theta\left(\frac{\hat{K}}{N}\right)^{\theta-1},$$

(iii) Tax revenues equal government expenses

$$\tau_N \hat{w} N + \tau_A r \hat{A} + \tau_C \hat{C} = \hat{T} + \hat{G} + (r-\gamma)\hat{B};$$

(iv) Aggregate savings equal firm's demand for capital plus Government's debt

$$\hat{A} = \hat{K} + \hat{B};$$

(v) The distribution x is invariant

$$x(\mathcal{A}_0, \mathcal{S}_0) = \int_{\mathcal{A}_0\times\mathcal{S}_0} \left\{ \int_{\mathcal{A}\times\mathcal{S}} \mathbf{1}_{\{\hat{a}'=g_a(\hat{a},s)\}} Q(s,s')\mathrm{d}x \right\} \mathrm{d}a'\mathrm{d}s',$$

for all $\mathcal{A}_0 \times \mathcal{S}_0 \in \mathscr{A} \times \mathscr{S}$, where $\mathbf{1}_{\{\cdot\}}$ is an indicator function taking value one if the statement is true and zero otherwise.

Appendix B. Solution Method

Here, we describe how the model is solved assuming that the debt-output ratio $b \equiv \hat{B}/\hat{Y}$ and the different taxes are already given and fixed.

We first assume that s takes on values in the discrete set $\mathcal{S} = \{s_1, \dots, s_{n_s}\}$ and has transition matrix π, where n_s denotes the number of states for s.

In general terms, the algorithm involves the following steps.

Algorithm 1. *Solving for the steady-state allocation*

(1) Outer initialization: We first postulate an aggregate demand for effective labor N^d

(2) Generic iteration on N^d

- *(a) Inner initialization: We then postulate an interest rate r*
- *(b) Generic iteration on r*
  - *(i) Given (r, N^d), one can deduce $\hat{w}$, $\hat{K}$, $\hat{Y}$, $\hat{C}$, $\hat{G}$, $\hat{B}$. Using the government budget constraint, one can then back out the level of transfers*

$$\hat{T} = \tau_A r \hat{K} + \tau_N \hat{w} N + \tau_C \hat{C} - [(1-\tau_A)r - \gamma]\hat{B} - \hat{G}.$$

  - *(ii) Given the above, one can then solve for the individual decision rules and for the implied distribution x, from which one can deduce the aggregate supply of assets $\hat{A}$ and an aggregate supply of effective labor N^s*
- *(c) The interest rate r is adjusted until $\hat{A} = \hat{K} + \hat{B}$.*

(3) Then, the aggregate demand for labor N^d is adjusted until $N^d = N^s$.

The procedure used to update the interest rate in step (c) is a hybrid bisection-secant method. After the first iteration, r is updated as follows

$$r_{new} = \theta\left(\frac{\hat{A}-\hat{B}}{N}\right)^{\theta-1} - \delta.$$

Afterwards, the secant method on excess savings is activated. At each iteration, however, if the updated interest rates lies outside the bisection bracket, the algorithm falls back to the bisection method. We use a simple secant method on excess labor supply to update N^d.

Before we proceed, a remark is in order here. One may wonder whether the outer loop (i.e. the iterations on N^d) is necessary in our setup. Imagine a version of the latter without fiscal policy (i.e. a Aiyagari (1994) like model with endogenous labor supply). Postulating an interest rate r would be enough to solve for the individual problem. Using the implied decision rule, one would find the aggregate supply of capital and the aggregate supply of labor and their ratio. This is enough to adjust r until the previous ratio equals the analog ratio backed out from the firms' first order conditions. By Walras law, if ones imposes that the goods market clears, the algorithm ensures automatically that the labor market is cleared. So why is it that we need an extra loop to clear the labor market in our setup? This is so because (i) we also need to balance the government's budget constraint and (ii) solving the individual problem requires that we know the level of transfers. However, knowing $\hat{T}$ requires knowledge of the level of $\hat{Y}$ and hence N (recall that r only gives us K/N). So the fiscal block of the model requires that an extra variable be adjusted. Of course, an alternative strategy could consist in postulating a $\hat{T}$ and then finding the r that clears the capital market. But once equilibrium on the goods and the labor market are imposed, we would still need to adjust $\hat{T}$ to balance the government's budget constraint. One way or another, an extra loop (be it on N^d or $\hat{T}$) is needed.

Turning back to our original problem, each of the above steps entails specific developments which are detailed below.

B.1. **Solving for the Individual Decision Problem.** Given an interest rate r and an aggregate, effective labor demand N^d, we must solve for the agent decision problem. To this end, we resort to the Endogenous Grid Method (EGM), as originally proposed by Carroll (2006). We set a grid of values for next period's assets $\hat{a}'$, denoted by $\mathbf{G}_{1,a}$. In practice, we use $n_{1,a} = 200$ exponentially-spaced points on $\mathbf{G}_{1,a}$. Also, we define a numerical tolerance parameter ϵ_a.

The algorithm for solving for the individual decision rules is initialized by postulating an approximate decision rule for $\hat{a}''$, which we denote by $g_a^{(0)}$. Formally, $g_a^{(0)}$ is a $n_{1,a} \times n_s$ matrix, where n_s denotes the cardinal of the support for s. The typical element $[g_a^{(0)}]_{ij}$ is thus the level of assets that would be chosen at the end of next period if the agent has assets $\hat{a}'_i$ and individual exogenous state s_j at the beginning of next period.

Let us define next period's cash on hand

$$\hat{m}' = (1-\tau_N)\hat{w}e(s')h' + [1+(1-\tau_A)r]\hat{a}' + \hat{T}.$$

The difficulty in applying the EGM in the present setup is that next period's cash on hand $\hat{m}'$ is not known a priori before we solve for next period's labor supply h'. Assuming an interior solution, combining the first-order condition for labor supply and the household's budget constraint, we obtain

$$F(h') \equiv (1+\tau_C)\left(\frac{1}{\eta}\frac{1-\tau_N}{1+\tau_C}\hat{w}e(s')\right)^{\frac{1}{\sigma}}(h')^{-\frac{\chi}{\sigma}} - (1-\tau_N)\hat{w}e(s')h' - R = 0,$$

where we defined

$$R = [1+(1-\tau_A)r]\hat{a}' - (1+\gamma)\hat{a}'' + \hat{T}.$$

Thus, we need to find the root of $F(h)$ at each point of the grids on $\hat{a}'$ and s', conditional on the postulated decision rule $g_a(\hat{a}', s')$. Let the associated roots be denoted $\tilde{h}(\hat{a}', s')$. We then have $g_h(\hat{a}', s') = \tilde{h}(\hat{a}', s')$.[3]

Thus, given r, $\hat{w}$ and $\hat{T}$ (together with the other fiscal instruments, which at this stage are treated as parameters), we resort to the following algorithm.

Algorithm 2. *Solving for the individual decision rules*

(1) *Initialization: We first postulate a decision rule* $g_a^{(0)}$

(2) *At iteration* j*, proceed as follows*

  (a) *Define*

$$R^{(j)}(\hat{a}', s') = [1+(1-\tau_A)r]\hat{a}' - (1+\gamma)g_a^{(j)}(\hat{a}', s') + \hat{T}$$

  and for all $(\hat{a}', s')$ *on* $\mathbf{G}_{1,a} \times \mathcal{S}$*, solve for the roots* $\tilde{h}^{(j)}(\hat{a}', s')$ *of*

$$(1+\tau_C)\left(\frac{1}{\eta}\frac{1-\tau_N}{1+\tau_C}\hat{w}e(s')\right)^{\frac{1}{\sigma}}(h')^{-\frac{\chi}{\sigma}} - (1-\tau_N)\hat{w}e(s')h' - R^{(j)}(\hat{a}', s') = 0$$

  an set

$$g_h^{(j)}(\hat{a}', s') = \tilde{h}^{(j)}(\hat{a}', s').$$

[3]In principle, an interior solution requires that $h \in (0,1)$ given our normalization of the time endowment. In practice, people never want to work more than 1 unit of time and the utility function adopted in the paper ensures that people always devote a positive amount of their time endowment to market activities.

(b) *Define next-period's cash on hand*

$$\hat{m}(\hat{a}',s') = (1-\tau_N)\hat{w}e(s')g_h^{(j)}(\hat{a}',s') + [1+(1-\tau_A)r]\hat{a}' + \hat{T}$$

and solve for today's cash on hand via the Euler equation

$$\hat{m}(\hat{a},s) = \left[(1+(1-\tau_A)r)\frac{\beta}{1+\gamma}\sum_{s'}\pi_{ss'}\left(\hat{m}(\hat{a}',s') - (1+\gamma)g_a^{(j)}(\hat{a}',s')\right)^{-\sigma}\right]^{-1/\sigma} + (1+\gamma)\hat{a}'.$$

(c) *Then, define the piecewise linear interpolant from the abscissas $m(\hat{a},s)$ and associated values $\hat{a}'$, and evaluate this on the abscissas $m(\hat{a}',s')$ to get an updated decision rule $g_a^{(j+1)}(\hat{a}',s')$.*

(d) *If $||g_a^{(j+1)}(\hat{a}',s') - g_a^{(j)}(\hat{a}',s')|| < \epsilon_a$, stop, else return to (a).*

Upon convergence, we define:

(1) Today's labor supply

$$h(\hat{a},s) = \left[\frac{1}{\eta}\left(\frac{\hat{m}(\hat{a},s)-(1+\gamma)\hat{a}'}{1+\tau_C}\right)^{-\sigma}\frac{1-\tau_N}{1+\tau_C}\hat{w}e(s)\right]^{1/\chi};$$

(2) Today's assets

$$\hat{a} = \frac{m(\hat{a},s) - \hat{T} - (1-\tau_N)\hat{w}e(s)h(\hat{a},s)}{1+(1-\tau_A)r}.$$

The levels of today's assets form the endogenous grid of assets, which is denoted by $\mathbf{EG}_{1,a}$.

B.2. **Solving for the Invariant Distribution.** After having solved for the individual decision problem, we must solve for the equilibrium steady-state distribution. To this end, we let $\bar{\pi}$ denote the vector collecting the stationary probability distribution induced by π.

We then set a new grid $\mathbf{G}_{2,a}$ for $\hat{a}'$, much finer than the previous one (in practice, we use 100,000 exponentially spaced points). We use $n_{2,a}$ to denote the number of grid points. Using interpolation techniques, we find the associated grid for $\hat{a}$ corresponding to $\mathbf{G}_{2,a}$, which we label $\mathbf{EG}_{2,a}$. Thus, for each $(\hat{a}',s)$ on $\mathbf{G}_{2,a}\times\mathcal{S}$, the corresponding point on $\mathbf{EG}_{2,a}\times\mathcal{S}$ gives $g_a^{-1}(\hat{a}',s)$. We also define a new grid for h on $\mathbf{G}_{2,a}\times\mathcal{S}$, which we label $\mathbf{G}_{2,h}$. This is done by interpolating the decision rule $g_h(\hat{a}',s')$ for h on $\mathbf{G}_{2,a}$.

We then initialize a CDF $F^{(0)}$ on the joint grid $\mathbf{G}_{2,a}\times\mathcal{S}$ as

$$F^{(0)}(\hat{a}',s') = \frac{\hat{a}'-\hat{a}_{\min}}{\hat{a}_{\max}-\hat{a}_{\min}}\bar{\pi}_{s'}.$$

Notice that this CDF is defined on end-of-period assets $\hat{a}'$. Also, we define a numerical tolerance parameter ϵ_F. Once again, $F^{(0)}$ is an $n_{2,a}\times n_s$ matrix.

Algorithm 3. *Solving for the invariant distribution of agents*

(1) *Initialization: We first postulate an initial CDF on $\mathbf{G}_{2,a}\times\mathcal{S}$, denoted by $F^{(0)}(\hat{a}',s')$.*

(2) *At iteration j, we proceed as follows:*

(a) *For each $s\in\mathcal{S}$, we interpolate $F^{(j)}(\cdot,s)$ on $\mathbf{EG}_{2,a}$. We thus obtain a CDF defined on beginning-of-period assets $\hat{a}$, which we denote by $\hat{F}^{(j)}(\cdot,s)$. We then update the CDF according to*

$$F^{(j+1)}(\hat{a}',s') = \sum_{s=1}^{n_s}\pi_{ss'}\hat{F}^{(i)}(g_a^{-1}(\hat{a}',s),s).$$

(b) *We then relocate the probability masses according to*

$$F^{(i+1)}(\hat{a}',s) = 0 \text{ if } g_a^{-1}(\hat{a}',s) < 0,$$

$$F^{(i+1)}(\hat{a}',s) = \bar{\pi}_s \text{ if } g_a^{-1}(\hat{a}',s) > \hat{a}_M,$$

and then re-normalize $F^{(j+1)}$.

(c) *If $||F^{(i+1)} - F^{(i)}|| < \epsilon_F$, we stop, else we go back to step (a).*

Upon convergence, we use the obtained CDF to compute $\hat{A}$, H, and N^s using the piece-wise linear approximation of F obtained above:

$$\hat{A} = \sum_{s=1}^{n_s} \left(F(\hat{a}_1, s)\hat{a}_1 + \sum_{i=2}^{n_{2,a}} (F(\hat{a}_i, s) - F(\hat{a}_{i-1}, s)) \frac{\hat{a}_i + \hat{a}_{i-1}}{2} \right),$$

$$H = \sum_{s=1}^{n_s} \left(F(\hat{a}_1, s) g_h(\hat{a}_1, s) + \sum_{i=2}^{n_{2,a}} (F(\hat{a}_i, s) - F(\hat{a}_{i-1}, s)) \frac{g_h(\hat{a}_i, s) + g_h(\hat{a}_{i-1}, s)}{2} \right),$$

$$N^s = \sum_{s=1}^{n_s} \left(F(\hat{a}_1, s) e(s) g_h(\hat{a}_1, s) + \sum_{i=2}^{n_{2,a}} (F(\hat{a}_i, s) - F(\hat{a}_{i-1}, s)) e(s) \frac{g_h(\hat{a}_i, s) + g_h(\hat{a}_{i-1}, s)}{2} \right),$$

B.3. **Exogenous Idiosyncratic Shocks and Calibration.** $\mathcal{S}_e$ and $\pi^{(e)}$ are constructed in the following way. We assume that $\log(e_t)$ follows an $AR(1)$ process

$$\log(e_t) = \rho_e \log(e_{t-1}) + \sigma_e \epsilon_t, \quad \epsilon_t \sim N(0,1).$$

We interpret $\log(e_t)$ as the residual persistent and idiosyncratic part of the log-wage rate in the specification adopted by Kaplan (2012), once experience and individual fixed-effects have been accounted for. In the latter paper, the estimation results based on year effects yield $\rho_e = 0.958$ and $\sigma_e = \sqrt{0.017}$. We approximate this $AR(1)$ process via the Kopecky and Suen (2010) method, using $n_e = 7$ points. This yields a transition matrix $\tilde{\pi}$.

In the spirit of Kindermann and Krueger (2014), we then allow for an eighth state corresponding to a very high labor productivity. As they argue, such a states is a reduced form for entrepreneurial or artistic opportunities yielding very high labor income. The final transition matrix is then

$$\pi = \begin{pmatrix} \tilde{\pi}_{11}(1-p_8) & \cdots & \tilde{\pi}_{14}(1-p_8) & \cdots & \tilde{\pi} i_{17}(1-p_8) & p_8 \\ \vdots & & \vdots & & \vdots & \vdots \\ \tilde{\pi}_{71}(1-p_8) & \cdots & \tilde{\pi}_{74}(1-p_8) & \cdots & \tilde{\pi}_{77}(1-p_8) & p_8 \\ 0 & \cdots & 1-p_{88} & \cdots & 0 & p_{88} \end{pmatrix}.$$

Here, p_8 is the probability of reaching the eighth productivity state from any normal productivity level. Also p_{88} is the probability of staying in the high labor income state conditional on being there.

This specification of labor income shocks gives us three parameters (p_8, p_{88}, e_8) which we adjust until we match the Gini coefficient of the wealth distribution, the share of wealth held by the 20% richest, and the Gini coefficient of the labor earning distribution.

APPENDIX C. ADDITIONAL RESULTS ON THE COMPARISON OF LAFFER CURVES FOR ALTERNATIVE DEBT-OUTPUT RATIOS

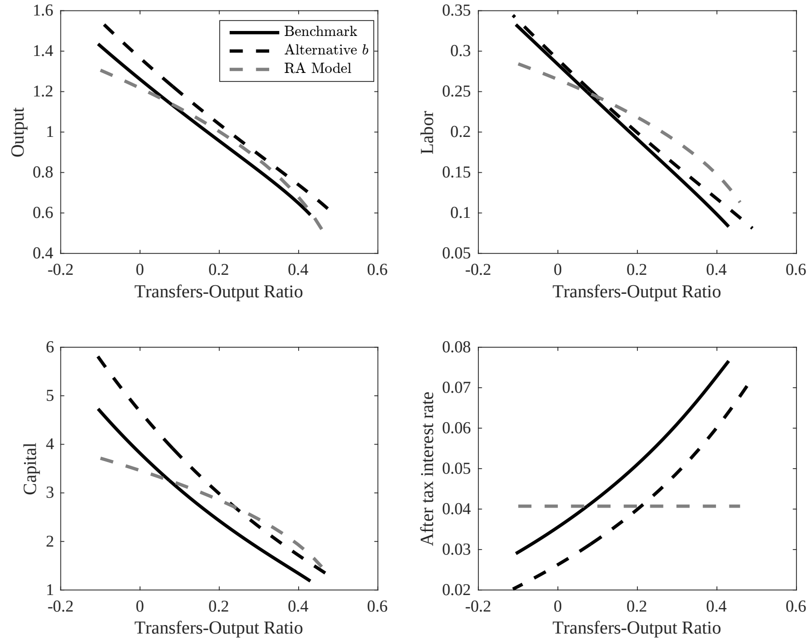

FIGURE C.1. Aggregate Variables - Labor Income Tax τ_N Varied

Note: Aggregate variables as τ_N is varied and the transfer-output ratio τ is adjusted. The plain black line corresponds to the benchmark IM economy. The dashed black line corresponds the IM economy when b is calibrated to -0.5. The dashed grey line corresponds to the RA economy.

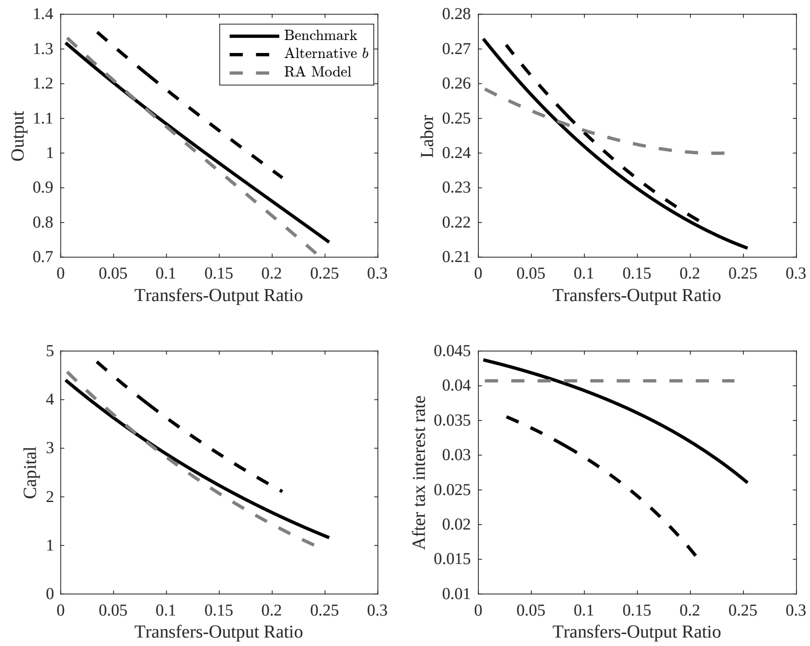

FIGURE C.2. Aggregate Variables - Capital Income Tax τ_A Varied

Note: Aggregate variables as τ_A is varied and the transfer-output ratio τ is adjusted. The plain black line corresponds to the benchmark IM economy. The dashed black line corresponds the IM economy when b is calibrated to -0.5. The dashed grey line corresponds to the RA economy.

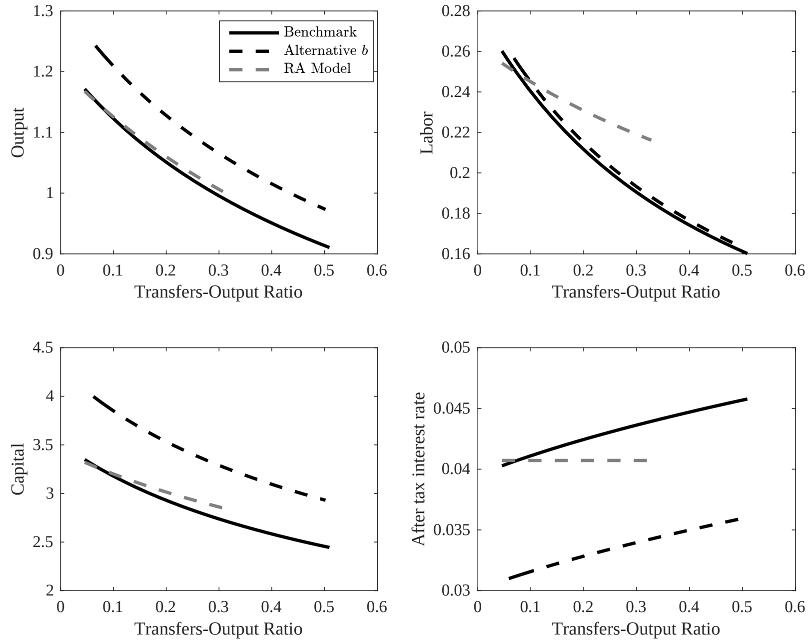

FIGURE C.3. Aggregate Variables - Consumption Tax τ_C Varied

Note: Aggregate variables as τ_C is varied and the transfer-output ratio τ is adjusted. The plain black line corresponds to the benchmark IM economy. The dashed black line corresponds the IM economy when b is calibrated to -0.5. The dashed grey line corresponds to the RA economy.

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