Aggregation of Heterogeneous Time Preferences

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We examine an economy whose consumers have different discount factors for utility, possibly not exponential. We characterize the properties of efficient allocations of resources and of the shadow prices that would decentralize such allocations. We show in particular that the representative agent has a decreasing discount rate when, as is usually posited, all of a group's members have a constant discount rate and decreasing absolute risk aversion preferences. We also identify conditions that lead the representative agent to have a rate of impatience that decreases with gross domestic product per capita.

I. Introduction

Time preferences determine individual saving and investment decisions, which are among the most important choices made by economic agents. Following Ramsey (1928) and Samuelson (1937), such decisions are usually represented by assuming that consumers maximize the discounted value of their flow of utility, using a constant rate of impatience. We identify the conditions under which the preferences of the representative agent of a group of consumers also maximize discounted utility.

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We also link the rate of impatience of the representative agent to the distribution of impatience rates in the population.

Frederick, Loewenstein, and O'Donoghue (2002) survey several attempts to estimate individuals' discount rates. Rates differ dramatically across studies, and within studies across individuals. There is no convergence toward an agreed-on or unique rate of impatience. For example, Warner and Pleeter (2001) found that individual discount rates vary between 0 and 70 percent per year. As suggested, for example, by Rader (1981), Jouini and Napp (2003), and Lengwiler (2004), there is no reason to believe that different consumers have identical time preferences for utility streams. This raises the question of the aggregation of heterogeneous time preferences. To examine this question, we consider a simple model in which each agent in a group maximizes a time-additive lifetime utility. The discount rate is heterogeneous across the population, and it may depend on either the time of receipt (hyperbolic discounting) or current consumption. Agents may also have different instantaneous utility functions. We assume that the group is able to allocate consumption within the group in a Pareto-efficient way. That is, we posit an exchange economy in which there is a cake to be shared at each moment, what might be labeled the multiple-cakes problem. We first show that the behavior of the group toward time can be duplicated by a representative agent whose lifetime utility functional is also time-additive. Rubinstein (1974) also examined the question of aggregating heterogeneous rates of impatience, but he derives a solution only for a two-period model and for exponential and logarithmic utility functions.

One of our key findings is that if individuals have heterogeneous constant rates of impatience, the representative agent will not in general use a constant rate to discount the future, as Becker (1992) first observed. More precisely, if individuals have decreasing absolute risk aversion (DARA), as would seem reasonable, then the representative agent will have a declining discount rate. We call this declining discounting. In short, *heterogeneous individual exponential discounting yields a collective discount rate that decreases with the time horizon.* Under some realistic calibrations of the economy, the collective discount factor duplicates either the hyperbolic case discussed by Loewenstein and Prelec (1991) or its simplified beta-delta version (Laibson 1997).

The cornerstone of our result is that individuals will appropriately change the share of resources each gets over time, so as to equalize individual intertemporal marginal rates of substitution. This allows us to define the intertemporal marginal rates of substitution of the representative agent, which will equal the interest rate in a market setting. Obviously, it is Pareto-efficient for the more impatient members to receive a larger share of the period's cake early in life, a share that will decrease with time. However, this desire for an unequal and varying distribution of the cake over time is limited by the limited agents' tolerance for consumption fluctuations. An individual's preference for smoothness is measured by the concavity of her utility function. As shown by Wilson (1968) and Constantinides (1982), it is helpful to use the notion of (absolute) tolerance for consumption fluctuations over time. If $u(\cdot)$ denotes the utility function of an agent, her tolerance for fluctuations is measured by $T(\cdot) = -u'(\cdot)/u''(\cdot)$. This index vitally characterizes the allocation of the aggregate income when it varies over time. Pareto efficiency demands that more tolerant agents bear a larger share of fluctuations of the aggregate income.

Uncertainty plays no role in our analysis; variability in income does. Turning to pure time preferences, we show that the rate of impatience of the representative agent equals a weighted mean of individual rates of impatience. These weights are proportional to the individual tolerances for consumption fluctuations. This is intuitive. The group's rate of impatience determines the group's willingness to transfer aggregate consumption across time. Only those members who are impatient *and* have a large tolerance for consumption fluctuations want such transfers. An impatient agent with a zero tolerance for fluctuations favors a smooth consumption plan. As a result, when aggregating individual consumption plans, the representative agent will have a rate of impatience that is biased in favor of the rates of impatience of the more tolerant members. Except for exponential utility functions, the weights in computing the weighted mean of individual discount rates will evolve over time. Given this, we show how the collective discount rate relates to both the time horizon and the aggregate income in the economy.

II. The Model

We consider a cohort or a group of heterogeneous agents indexed by $i = 1, \ldots, I$. Each member consumes a single commodity in continuous time from date 0 to date N. The natural commodity space is the space L^∞ of functions that are bounded almost everywhere. Agent i's preference order over alternative consumption plans is described by a smooth lifetime utility function $V^i : L^\infty \to R$ defined on the commodity space. We assume that V^i is time-additive:[1]

$$V^i(C) = \int_0^N u^i(C(t),\ t)dt,$$

[1] Following Koopmans (1960), time additivity can be derived from the independence axiom stating that if two intertemporal prospects share a common outcome at a given date, then preference between them is determined solely by the remaining outcomes that differ.

where $C(t)$ is the agent's consumption level at date t, and $u^i(C, t)$ is interpreted as the discounted felicity extracted by agent i consuming C at time t. We assume that u^i is continuous with respect to t and three times differentiable, increasing, and concave with respect to C. We also assume the Inada condition $\lim_{C\to 0} u^i_c(C, t) = +\infty$ for all t. The group is endowed with a flow $z : [0, N] \to R$ of the single consumption good. For simplicity, this collective endowment is risk-free. Finally, $z(t)$ is bounded below by $\underline{z} > 0$ and above by $\bar{z} < \infty$ for all $t \in [0, N]$.

An allocation is characterized by a vector of consumption profiles $C^i : [0, N] \to R$ determining the consumption of agent i at date t. An allocation is feasible if at each instant of time aggregate consumption equals aggregate income. The only restriction that we impose on the cohort's division of the cakes across periods is that it be Pareto-efficient. An allocation is Pareto-efficient if it is feasible and if there is no other feasible allocation that raises the lifetime utility of at least one type without reducing the lifetime utility of the other types. To any such efficient allocation, there exists a weight vector $\boldsymbol{\lambda} = (\lambda^1, \dots, \lambda^I) > 0$ such that it is the solution of the group's following maximization problem:

$$V_{\lambda}(z) = \max_{C^1,\dots,C^I} \sum_{i=1}^{I} \lambda^i V^i(C^i)$$

$$\text{subject to} \sum_{i=1}^{I} C^i(t) = z(t) \quad \forall t \in [0, N]. \tag{1}$$

The reason why this variational problem expresses Pareto efficiency is well known. The locus of individuals' lifetime utilities obtained from feasible allocations is a convex set. It implies that, to every Pareto-efficient allocation, there exists a hyperplane characterized by $(\lambda^1, \dots, \lambda^I)$ that is tangent to this set. Because the set of feasible utility payoffs is closed under our assumptions,[2] a solution to program (1) exists.

The term V_λ can be interpreted as the lifetime utility of the representative agent consuming the flow of aggregate wealth z. In the classic analysis of the static syndicate problem, Wilson (1968) and Constantinides (1982) considered a decision problem that bears parallels to (1). Wilson examined a decision under uncertainty for expected-utility maximizers with heterogeneous utility functions and heterogeneous beliefs. He examined a group whose members have differing utility functions and beliefs, which in turn provide the bases for their individual expected utilities. The group's goal is to choose one among a set of lotteries and

[2] When consumption plans are not restricted to be bounded, the set U of all feasible utility levels of the economy may not be closed, as observed by Araujo (1985), Mas-Colell (1986), and Aliprantis, Brown, and Burkinshaw (1990). A proof of the closedness of U under our assumptions is available on request.

then define a sharing rule for the monetary outcome so as to produce Pareto optimality. Using the additivity property of the expected utility model, Wilson proved that the optimal collective decision policy is isomorphic to the optimal decision policy of a representative agent who also maximizes the expected value of a concave function of consumption per capita in the cohort (see also Constantinides 1982). The existence of a representative agent with such simple aggregative properties has become a cornerstone of theories in finance and macroeconomics. The following proposition presents an equivalent result in our model with time-additive preferences. Its simple proof by contradiction is left to the reader.

PROPOSITION 1. *Representative agent.*—Suppose that the set of all feasible utility levels is closed and that the group allocates wealth efficiently over time according to the vector $\boldsymbol{\lambda} = (\lambda^1, \ldots, \lambda^I)$ of positive Pareto weights. In association with this vector, there exists a representative agent with a time-additive lifetime utility functional $V_\lambda(z) = \int_0^N v(z(t), t)dt$. The representative agent's felicity function $v: [\underline{z}, \bar{z}] \times [0, N] \rightarrow R$ is defined by

$$v(z, t) = \max_{c^1,\ldots,c^I} \sum_{i=1}^{I} \lambda^i u^i(c^i, t)$$

$$\text{subject to } \sum_{i=1}^{I} c^i = z. \tag{2}$$

The associated efficient allocation is characterized by $C^i(t) = c^i(z(t), t)$ for all $t \in [0, N]$ and $i = 1, \ldots, I.$

It is noteworthy that the function v depends on the distribution of Pareto weights $\boldsymbol{\lambda}$. Proposition 1 enables us to decompose the multiperiod maximization program (1) into a sequence of static maximization programs (2). The time additivity of individual preference functionals is, of course, essential to get this result. Notice that the cake-sharing problem (2) has two parameters: the size of the cake z and the time t at which this cake is available. Its solution $(c^1, \ldots, c^N)$ is therefore a function of (z, t). By proposition 1, the optimal solution $(C^1, \ldots, C^N)$ of the intertemporal problem is such that $C^i(t) = c^i(z(t), t)$ for all t and i.

By the concavity of u^i with respect to its first argument, the solution to program (2) is unique. Its first-order condition is written as

$$\lambda^i u_c^i(c^i(z, t), t) = \psi(z, t), \tag{3}$$

for all (z, t) and i, where ψ is the Lagrange multiplier of the feasibility constraint associated with time t and average endowment z. By the envelope theorem, the marginal value of wealth at time t is the Lagrange multiplier associated with time t. Thus we have that

$$v_z(z,\ t) = \psi(z,\ t) \quad \forall(z,\ t). \tag{4}$$

By writing $C^i(t) = c^i(z(t),\ t)$, we have disentangled the two impacts that time has on the efficient sharing of wealth. First, because time affects the marginal utility of every agent in the group, it is likely to affect the efficient sharing of wealth. Second, aggregate wealth changes over time, and individual consumption levels must reflect that. In the following proposition, we summarize the standard results describing the effect of a change in wealth at date t on the efficient allocation and on the marginal value of wealth at that date. Notice that $T^i(c,\ t) = -u^i_c(c,\ t)/u^i_{cc}(c,\ t)$ denotes the absolute tolerance for consumption fluctuations of agent i.

PROPOSITION 2. *Tolerance for consumption fluctuations.*—The marginal propensity to consume out of aggregate wealth of an agent is proportional to this agent's tolerance for consumption fluctuations:

$$c^i_z(z,\ t) = \frac{T^i(c^i(z,\ t),\ t)}{\sum_{j=1}^{I} T^j(c^j(z,\ t),\ t)}. \tag{5}$$

Moreover, the group's absolute tolerance for aggregate consumption fluctuations is the sum of its members' tolerances:

$$T^v(z,\ t) =_{\text{def}} -\frac{v_z(z,\ t)}{v_{zz}(z,\ t)} = \sum_{j=1}^{I} T^j(c^j(z,\ t),\ t). \tag{6}$$

Proof. See, for example, Wilson (1968). QED

This proposition states in (5) that more tolerant agents have larger marginal propensities to consume. It is intuitively appealing that people who are more tolerant of consumption fluctuations should receive a larger share of aggregate fluctuations. All consumption levels are procyclical, but some are more procyclical than others. This result allows us to measure the group's tolerance for aggregate fluctuations, which is just the sum of the members' tolerances.

III. The Group's Rate of Impatience

In the classic case with homogeneous exponential discount factors, individuals' consumption levels vary only with fluctuations in the aggregate endowment $z(\cdot)$. When discount rates are heterogeneous, by contrast, time enters as an additional factor. We examine the partial derivative of individual consumption levels with respect to time. When the average income z remains constant over time, it is intuitive that less patient people will trade later consumption for earlier consumption with those who are more patient. The impatient ones will have a decreasing consumption path, and vice versa.

The instantaneous rate of pure time preference of agent i consuming c at time t equals

$$\delta^i(c,\ t) = -\frac{u^i_{ct}(c,\ t)}{u^i_c(c,\ t)}. \tag{7}$$

It measures the rate at which marginal utility decreases with time when consumption is held constant. The classic discounted utility model assumes that δ^i is independent of $(c,\ t)$. In the case of hyperbolic discounting, δ is independent of c but decreases with t. Given that a nonconstant δ raises a consistency problem, we assume that agents can commit to their future consumption plan at date $t = 0$.

Given the feasibility constraint, it must be that

$$\sum_{i=1}^{I} c^i_t(z,\ t) = 0. \tag{8}$$

When the aggregate wealth remains constant over time, increases in consumption by some members of the group must be compensated by equivalent reductions to others. Fully differentiating the first-order condition (3) yields

$$\lambda^i u^i_{ct}(c^i,\ t) + \lambda^i u^i_{cc}(c^i,\ t)c^i_t = \psi_t,$$

for all $(z,\ t)$. Using (3) to eliminate the Lagrange multiplier λ^i, we can rewrite the above equality as

$$-\delta^i(c^i,\ t) - [T^i(c^i,\ t)]^{-1}c^i_t = \frac{\psi_t}{\psi}$$

or, equivalently,

$$c^i_t = -T^i(c^i,\ t)\left[\frac{\psi_t}{\psi} + \delta^i(c^i,\ t)\right]. \tag{9}$$

Replacing c^i_t in (8) by its expression from (9) yields

$$\frac{\psi_t}{\psi} = -\frac{\sum_{i=1}^{I}\delta^i(c^i(z,\ t),\ t)T^i(c^i(z,\ t),\ t)}{\sum_{i=1}^{I} T^i(c^i(z,\ t),\ t)}. \tag{10}$$

Proposition 3 characterizes the time profile of individual consumption plans when people have heterogeneous discount rates. It flows from properties (9) and (10).

PROPOSITION 3. *Individual consumption path.*—The increase in consumption through time of an agent is decreasing in that agent's discount rate δ^i:

$$c_t^i(z,\ t)\ =\ T^i(c^i(z,\ t),\ t)[\delta^v(z,\ t)\ -\ \delta^i(c^i(z,\ t),\ t)], \tag{11}$$

with

$$\delta^v(z,\ t)\ =\ \frac{\sum_{i=1}^I \delta^i(c^i(z,\ t),\ t)T^i(c^i(z,\ t),\ t)}{\sum_{i=1}^I T^i(c^i(z,\ t),\ t)}. \tag{12}$$

The individual consumption path proposition determines how more patient people should substitute current consumption for future consumption. Notice that the consumption path of agent i increases locally in t if and only if her rate of impatience is smaller than the weighted mean δ^v of individual rates of impatience, where δ^v is the effective prevailing interest rate within the economy. More patient members postpone their consumption to the future in exchange for a positive return on their savings. Because both δ and δ^v are functions of z and t, efficient consumption profiles need not be monotone. In technical terms, proposition 3 requires that the change in individual consumption be increasing in δ^i when agents have the same tolerance for consumption fluctuations. The following corollary, which is a direct consequence of equation (11), exhibits the weaker property of single crossing.

COROLLARY 1. *Single-crossing property.*—Suppose that agents have the same tolerance for consumption fluctuations: $T^i(c,\ t)\ =\ T(c,\ t)$ for all $(c,\ t)$. Suppose that there exist two agents i and j such that $\delta^i(c,\ t) > \delta^j(c,\ t)$ for all $(c,\ t)$. This implies that

$$c^i(z^*,\ t)\ =\ c^j(z^*,\ t) \Rightarrow c_t^i(z^*,\ t) \leq c_t^j(z^*,\ t).$$

We can now turn to the central aim of this paper, which is to characterize the aggregation of individual discount rates. Impatience flows from the fact that, seen from $t = 0$, the marginal value of an increase in consumption decreases with the time at which it takes place. The impatience characterizing the group's preferences can be made more explicit by defining the group's instantaneous rate of impatience as

$$-\frac{v_{zt}(z,\ t)}{v_z(z,\ t)} = -\frac{\psi_t(z,\ t)}{\psi(z,\ t)}. \tag{13}$$

Combining conditions (13) and (10) yields the following result.

Proposition 4. *Collective impatience.*—The instantaneous rate of pure preference for the present of the representative agent defined by (13) is a weighted mean of individual members' instantaneous rates:

$$-\frac{v_{zt}(z,\ t)}{v_z(z,\ t)} = \delta^v(z,\ t) = \frac{\sum_{i=1}^{I}\delta^i(c^i(z,\ t),\ t)T^i(c^i(z,\ t),\ t)}{\sum_{i=1}^{I}T^i(c^i(z,\ t),\ t)}. \tag{14}$$

Not surprisingly, the (implicit) psychological discount rate of the representative agent is a weighted mean of the individual rates of impatience in the cohort. The weights are proportional to the corresponding individual tolerances for consumption fluctuations. This weighting of the mean of δ^i is intuitive. When considering its attitude toward postponing aggregate incomes, the group must take into account the rate of impatience of those members who will have to postpone their consumption. As seen from equation (11), these will be the ones who have a larger tolerance for consumption fluctuations. This is why the collective rate of impatience is biased in favor of the rates of impatience of more tolerant members. To illustrate, consider a cohort with two agents. Agent h has a high discount rate δ^h and is somewhat tolerant of consumption fluctuations. Agent l, by contrast, has a lower discount rate δ^l but has a zero tolerance for consumption fluctuations. Despite his patience, agent 2 will prefer to smooth his consumption completely. Therefore, agent 1 will bear the entire burden of aggregate fluctuations. The cohort's attitude toward time is therefore determined entirely by agent 1's preferences. In particular, the cohort's degree of impatience will be the larger δ^h.

IV. The Term Structure of the Group's Rate of Impatience

As a direct consequence of the fact that δ^v is a weighted mean, it is bounded below and above by the smallest and largest individual rates of impatience:

$$\min_{i=1,\ldots,I}\delta^i(c^i(z,\ t),\ t) \le \delta^v(z,\ t) \le \max_{i=1,\ldots,I}\delta^i(c^i(z,\ t),\ t).$$

It is important to notice that the weights T^i in equation (14) are a function of both z and t. Thus, even if the individual discount rates δ^i are independent of consumption and time, it is generally not true that δ^v is independent of these variables. We now examine the term structure of the collective rate of impatience.

Suppose that all δ^i are independent of c and t; that is, the group's members discount the flow of future felicity exponentially: $u_c^i(c,\ t) = \exp(-\delta^i t)h^i(c)$, where $h^i(c)$ measures the felicity of agent i consuming c. The problem here is to determine whether the cohort as a whole should use exponential discounting when all its members use exponential discounting. When all members have the same discount rate, discounting at that rate is appropriate. With heterogeneous discount rates, fully differentiating condition

$$\delta^v(z,\ t) \ = \ \frac{\sum_{i=1}^I \delta^i T^i(c^i(z,\ t),\ t)}{\sum_{i=1}^I T^i(c^i(z,\ t),\ t)}$$

with respect to t and using condition (11) yields

$$\delta_t^v(z,\ t) \ = \ 2\left(\frac{\sum_{i=1}^I \delta^i T^i T_c^i}{\sum_{i=1}^I T^i}\right)\left(\frac{\sum_{i=1}^I \delta^i T^i}{\sum_{i=1}^I T^i}\right) - \left(\frac{\sum_{i=1}^I \delta^{i^2} T^i T_c^i}{\sum_{i=1}^I T^i}\right)$$
$$- \left(\frac{\sum_{i=1}^I \delta^i T^i}{\sum_{i=1}^I T^i}\right)^2\left(\frac{\sum_{i=1}^I T^i T_c^i}{\sum_{i=1}^I T^i}\right), \tag{15}$$

where T^i and T_c^i are evaluated at $(c^i(z,\ t),\ t)$.

PROPOSITION 5. *Hyperbolic collective impatience.*—Suppose that every agent has a multiplicatively separable utility function with an exponential discount: $u^i(c,\ t) \ = \ \exp{(-\delta^i t)}h^i(c)$. The social rate of impatience δ^v is decreasing (increasing) in t if all felicity functions h^i, $i = 1,\ \ldots,\ I$, have an increasing (decreasing) tolerance for consumption fluctuations.

Proof. Consider a specific $(z,\ t)$ and let x^i and y^i denote, respectively,

$$\frac{T^i(c^i(z,\ t))}{\sum_j T^j(c^j(z,\ t))}$$

and

$$\frac{T^i(c^i(z,\ t))T_c^i(c^i(z,\ t))}{\sum_j T^j(c^j(z,\ t))T_c^j(c^j(z,\ t))}.$$

Under the condition that all T_c^i have a constant sign, we have that y^i is nonnegative for every i and $\sum_i y^i \ = \ 1$. Observe that

$$2\left(\sum_{i=1}^I \delta^i x^i\right)\left(\sum_{i=1}^I \delta^i y^i\right) \le \left(\sum_{i=1}^I \delta^i x^i\right)^2 + \left(\sum_{i=1}^I \delta^i y^i\right)^2. \tag{16}$$

Moreover, we know from Jensen's inequality that

$$\left(\sum_{i=1}^I \delta^i y^i\right)^2 \le \sum_{i=1}^I \delta^{i^2} y^i. \tag{17}$$

Obviously, combining (16) and (17) yields

$$2\left(\sum_{i=1}^I \delta^i x^i\right)\left(\sum_{i=1}^I \delta^i y^i\right) \le \left(\sum_{i=1}^I \delta^i x^i\right)^2 + \sum_{i=1}^I \delta^{i^2} y^i.$$

From equation (15), this is equivalent to

$$\delta_t^v(z,\ t)\frac{\sum_{i=1}^{I} T^i(c^i(z,\ t))}{\sum_{i=1}^{I} T^i(c^i(z,\ t))T_c^i(c^i(z,\ t))} \leq 0.$$

It implies that δ_t^v and T_c^i have opposite signs. QED

To borrow standard terminology from the economics of uncertainty, increasing tolerance for fluctuations means that absolute risk aversion is decreasing (DARA), the traditional assumption. Notice that we do not assume any correlation between rates of impatience and degrees of tolerance for fluctuations. The monotonicity of these degrees of tolerance is the only thing that matters for the slope of the term structure of δ^v. Simple intuition supports this important result. From equation (11), we know that more patient consumers have an increasing consumption profile. Under DARA, their tolerance for consumption fluctuations increases through time. This implies that when time goes forward, consumers with a low δ see their weight growing in the mean $\delta^v(z,\ t)$. This implies that the social rate of impatience decreases with time. In the rest of this section, we present a few illustrations of this result.

Our first illustration involves two agents with respective constant rates of impatience δ^l and $\delta^h > \delta^l$. The two agents have the same felicity function $h(c) = \min\,[b(c-a),\ d(c-a)]$, with $0 < d < b$. This function is piecewise linear with a kink at $c = a$. We consider the case in which b tends to infinity, which means that the left branch of the curve becomes vertical. Parameter a is the minimum level of subsistence. On the relevant domain $[a,\ +\infty[$ of this limit function, agents have a nondecreasing tolerance (DARA), with a zero tolerance at $c = a$ and an infinite tolerance for all $c > a$. We assume that the flow of aggregate incomes is uniformly larger than $2a$ in order to guarantee a bounded value function. In this economy, any Pareto-efficient sharing of the cake produces a consumption pattern that flip-flops from subsistence to surplus, or vice versa. The patient agent enjoys the first path and the impatient agent the second. As a consequence, the social rate of impatience $\delta^v(z,\ t)$ equals δ^h prior to $t = \tau$ and δ^l thereafter. The term structure is a simple downward step function in this case, a special case of hyperbolic discounting that is often referred to as the "beta-delta" model. Phelps and Pollak (1968), then followed by Laibson (1997) and many others afterward, introduced this stepwise functional form to describe observed psychological discount rates.

This discounting functional would emerge as the socially efficient rule for less extreme examples. Let us replace the piecewise-linear felicity function by a power felicity function. The two agents have the same constant relative risk aversion γ. Under the efficient allocation of re-

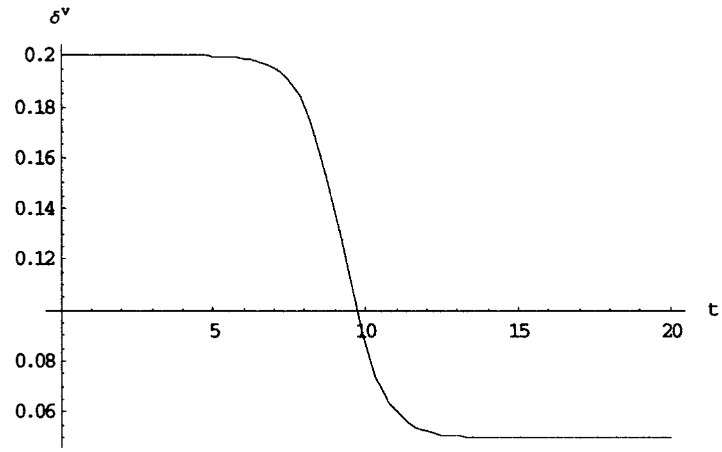

FIG. 1.—The discount rate as a function of time horizon for two agents with $\delta^h = 0.2$ and $\delta^l = 0.05$ when $h^i(c) = c^{0.9}$.

sources with $\lambda^l = \delta^l$ and $\lambda^h = \delta^h$, the group's discount rate as a whole can be written as

$$\delta^v(z, t) = \frac{(\delta^l)^{(1+\gamma)/\gamma} e^{-\delta^l t/\gamma} + (\delta^h)^{(1+\gamma)/\gamma} e^{-\delta^h t/\gamma}}{(\delta^l)^{1/\gamma} e^{-\delta^l t/\gamma} + (\delta^h)^{1/\gamma} e^{-\delta^h t/\gamma}}. \tag{18}$$

When γ tends to zero, this function of t tends to a downward step function with step levels at δ^h and δ^l. The step occurs at time horizon $t^* = [\ln(\delta^h) - \ln(\delta^l)]/(\delta^h - \delta^l)$. In figure 1, we draw this function for $\delta^h = 20$ percent, $\delta^l = 5$ percent, and $\gamma = 0.1$.

These two examples provide additional insights as to why the social rate of impatience should be decreasing. Consider in particular example 2, which is illustrated by figure 1. Consider a marginal investment by the cohort that would move some of the cohort's income from time t to $t - \Delta t$. If t is small, this change in the structure of the cash flows will mostly benefit the impatient agent. It is then intuitive that the social planner uses the (high) rate of impatience of these agents when performing the cost-benefit analysis of this investment project. On the contrary, for an investment project moving some of the cohort's income from a larger time t to $t - \Delta t$, it will be the more patient members who will benefit primarily from this change, because they consume the larger share of the cake at those time horizons. As anticipated by Becker (1992, example 3), the social planner will thus use their smaller rate of impatience to perform the cost-benefit analysis of this alternative investment project. In short, the social planner will use a rate of impatience that is decreasing with the time horizon from δ^h to δ^l.

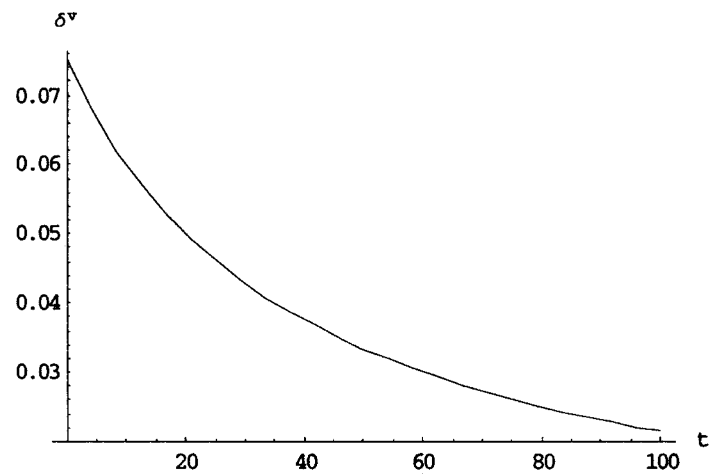

FIG. 2.—The collective rate of impatience δ^v as a function of time when $\gamma = 2$, $\eta = 1$, and $\mu = 5$ percent.

Our last example is interesting because it generates a functional form for the social discount rate that fits some of those that already exist in the literature. Suppose as above that all agents have the same constant relative risk aversion γ. There is a continuum of agents who discount their flow of felicity exponentially. Agents are indexed by their rate of impatience δ. Suppose, moreover, that individual discount rates δ are distributed following a negative exponential law with density $f(\delta) = e^{-\delta/\mu}/\mu$ on support $[0, +\infty]$. The mean rate of impatience in the population equals μ. We consider the Pareto-efficient allocation that corresponds to the weighting function $\lambda(\delta) = \delta/\eta$ for some $\eta > 0$. In this illustration, it can be verified that

$$\delta^v(z, t) = \frac{\eta + \gamma}{t + (\gamma/\mu)}, \tag{19}$$

which is independent of z. When relative risk aversion γ tends to infinity, δ^v tends to μ uniformly for all t. When γ tends to zero, $\delta^v(z, t)$ tends uniformly to η. In figure 2, we draw the discount rate δ^v as a function of time. As seen in (19), the collective discount rate declines with time t as $1/[t + (\gamma/\mu)]$. The discount *factor* can be written as

$$\beta(t) = -\exp\left[\int_0^t \delta^v(z, \tau)d\tau\right] = \left(1 + \frac{\mu t}{\gamma}\right)^{-(\eta+\gamma)}.$$

This is the functional form suggested by Loewenstein and Prelec (1991), who generalized earlier proposals made by Herrnstein (1981) and Ma-

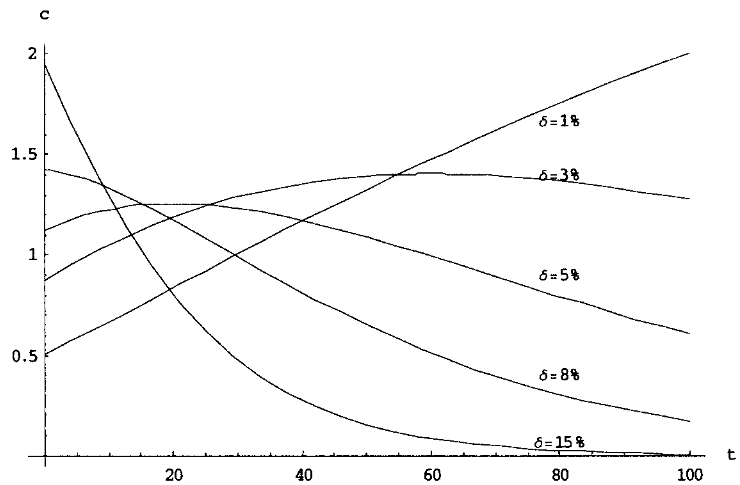

FIG. 3.—Efficient consumption path for agents with different discount rates δ, with $\gamma = 2$, $\eta = 1$, and $\mu = 5$ percent.

zur (1987). It is useful to examine how consumption is allocated in this economy. The set of first-order conditions (3) combined with the feasibility constraints can be solved analytically to yield

$$c^{\delta}(z,\ t) = \frac{\mu z}{\Gamma((\gamma+\eta)/\gamma)}\left(\frac{t}{\gamma}+\frac{1}{\mu}\right)^{(\gamma+\eta)/\gamma}\delta^{\eta/\gamma}e^{-\delta t/\gamma}, \tag{20}$$

where $\Gamma(x) = \int_0 \delta^{x-1}e^{-\delta}d\delta$ is the gamma function. In figure 3, we draw the efficient consumption plan for a few agents when the mean income in the population remains constant over time and is normalized to unity. We see again what drives the declining term structure of the collective discount rate: At $t = 0$, individual consumption levels and individual degrees of tolerance are positively related to the individual rates of impatience. This weighting leads to a social rate of impatience that is greater than μ, the mean rate of impatience in the economy. As time goes forward, most resources go to those with low discount rates, and the social rate of impatience falls below μ. Notice that, following condition (11), the consumption profile of agent δ is locally increasing as long as δ is less than $\delta^v(z, t)$. Because δ^v is decreasing in t, consumption profiles of all agents with a rate of impatience δ less than $\delta^v(z, 0) \simeq 7.5$ percent are hump-shaped; by contrast, those agents with a rate of impatience greater than 7.5 percent have decreasing consumption throughout. In general, efficient consumption profiles are either decreasing or hump-shaped under the assumptions of proposition 5.

In this section, we assumed throughout that all agents discount their flow of felicity at a constant rate, and we obtained that the representative agent will discount at a decreasing rate. As is intuitive, the same result remains true if all individual discount rates are decreasing. The following corollary is a simple extension of proposition 5, and its proof is left to the reader.

Corollary 2. Suppose that agents have multiplicatively separable utility functions with hyperbolic discounting:

$$u^i(c,\ t) = \exp\left[-\delta^i(t)t\right]h^i(c)$$

and $\delta^i_t \leq 0$ for all $i = 1,\ \dots,\ I$. Then the term structure of the collective rate of impatience δ^v is decreasing if all utility functions h^i exhibit DARA.

V. A Wealth Effect on the Group's Impatience

In the standard model of consumption, saving, and growth, rates of impatience are assumed to be independent of consumption levels: $\delta^i_c = 0$. However, it is often observed that wealthier economies are more patient, understanding that causality can flow in either direction. In our notation, this implies that δ^v is decreasing in z. In this section, we examine whether these two assumptions can be compatible.

Observe that we found that δ^v is independent of z in our three examples. These examples illustrate the following proposition.

Proposition 6. Suppose that, for all $i = 1,\ \dots,\ I$, u^i is multiplicatively separable: $u^i(c,\ t) = \beta^i(t)h^i(c)$. The following conditions are equivalent: (1) For any distribution of individual discount factors and of Pareto weights, the collective rate of impatience is independent of the consumption per capita z. (2) All consumers have identical-slope harmonic absolute risk aversion (ISHARA) preferences: $h^i(c) = [(c - a^i)/\gamma]^{1-\gamma}$.

Proof. Consider a specific $(z,\ t)$ and let x^i denote

$$\frac{T^i(c^i(z,\ t))}{\sum_j T^j(c^j(z,\ t))}.$$

Fully differentiating equation (14) with respect to z and using property (5) yields

$$\left[\sum_{i=1}^{I} T^i(c(z,t))\right]\delta^v_z(z,t) = \sum_{i=1}^{I} x^i\delta^i(t)T^i_c(c(z,t)) - \left[\sum_{i=1}^{I} x^i\delta^i(t)\right]\left[\sum_{i=1}^{I} x^iT^i_c(c(z,t))\right]. \tag{21}$$

The right-hand side of this equality can be interpreted as the covariance between δ and T_c. For ISHARA preferences, T^i_c is a constant, which implies that the covariance is zero. When T_c is not a constant, it is always possible to find a counterexample; that is, the consumption level affects discount rates. QED

Rubinstein (1974) obtained the same wealth irrelevancy property in the special case of exponential and logarithmic utility functions. The family of ISHARA preferences is characterized by the property that the derivatives of individual absolute tolerances are a constant across agents. Except in that case, the representative agent need not have a multiplicatively separable utility function. An illustration is given in the following proposition, which assumes that the T_c^i are heterogeneous in the population and that they are correlated with individual rates of impatience. In such a situation, the social rate of impatience is sensitive to the aggregate wealth in the economy.

PROPOSITION 7. Suppose that the members of the group have a rate of impatience that is independent of their consumption: $\delta_c^i = 0$. Suppose also that their tolerance for fluctuations is linear with respect to their consumption: $T_{cc}^i = 0$. The collective rate of impatience at t will be decreasing with the aggregate income if δ^i and T_c^i evaluated at t are anti-comonotone: for all i, j, $[\delta^i(c, t) - \delta^j(c, t)][T_c^i(c, t) - T_c^i(c, t)] \leq 0$.

Proof. It is a direct consequence of (21). QED

Simple intuition supports the result. It flows from the fact that the collective rate of impatience is a weighted mean of individual rates of impatience. When δ^i and T_c^i are anti-comonotone, an increase in wealth differentially increases the weights associated with the lower rates of impatience. An increase in z then pushes δ^v downward.

To illustrate, consider an economy with two agents. Rates of impatience are constant, hence independent of time and consumption levels. The first agent has a low impatience rate, $\delta^l = 5$ percent, and a logarithmic felicity function, which implies that $T_c^l(c, t) \equiv 1$. The second agent has a larger rate of impatience, $\delta^h = 20$ precent, and a constant relative risk aversion $\gamma^h = 10$, which implies that $T_c^h(c, t) \equiv 0.1$. Observe that the conditions of proposition 7 are satisfied in this example. We derived numerically the Pareto-efficient allocation corresponding to equal Pareto weights $\lambda^l = \lambda^h$. Figure 4 shows the term structure of the collective rate of impatience when the aggregate wealth is constant over time and equals either 0.5, 1, or 2. We see that a larger per capita consumption yields a smaller rate of impatience for all time horizons.

VI. Conclusion

It is well known that a group does not or cannot make decisions under certainty in the same manner as individuals. This paper demonstrates that a group will also not treat the same time value of rewards the same way as individual consumers do, even when all consumers are exponential discounters with identical felicity functions. For example, the

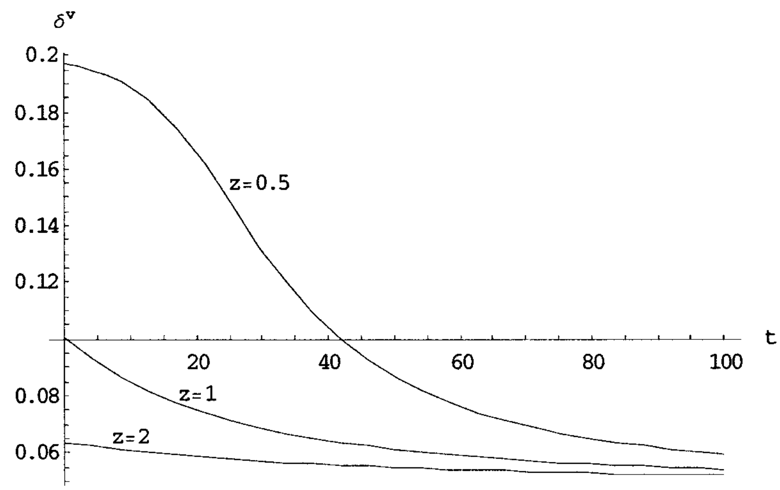

FIG. 4.—Term structure of the collective rate of impatience when $u_c^l(c, t) = e^{-0.05t}c^{-1}$ and $u_c^h(c, t) = e^{-0.2t}c^{-10}$.

group's rate of impatience may well depend on the group's wealth level. Despite this, the basic property of additivity of individual preferences is transmitted to the preferences of the representative agent. This implies that the representative agent of the group has no consumption habits and no anticipatory feelings if its members do not also have such psychological traits.

The main objective of the paper was to identify the appropriate mechanism to aggregate heterogeneous time preferences. That mechanism has a collective rate of impatience at any moment that is a weighted mean of the members' local rates of impatience. Each member's weight is proportional to her degree of absolute tolerance for consumption fluctuations. This aggregation rule implies that the collective rate of impatience is decreasing with respect to the time horizon when wealthier consumers are less averse to consumption fluctuations, a common assumption. This reasoning presupposes, of course, that the group is able to redistribute consumption within the group in response to each agent's degree of impatience. For long horizons, any transfer of the group's wealth across time will mostly affect the more patient agents because they are the ones who have the largest stake on aggregate wealth. Thus, when considering investments affecting cash flows corresponding to these long horizons, the group should use the lower rate of impatience in the group for cost-benefit analysis. On the contrary, for short time horizons, transferring wealth across time affects mainly the consumption flow of the more impatient agents. In the collective cost-benefit analysis for such investments, the larger rate of impatience of these agents should be employed.

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