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Oversight and Hierarchies

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Abstract

Political processes in most democracies are characterized by the existence of hierarchies from policy-implementing agency staff to executive appointees at the head of the agency to cabinet members to chiefs of government to voters. Three attributes characterize these hierarchies: (1) uninformed principal; (2) costly information acquisition by the subordinate overseers; and (3) delegated principal-agent relationship to the subordinate overseers. We study how effective such hierarchies are at solving the entailed principal-agent relationships and show that principals can, indeed, improve the agent's performance by hiring intermediate overseers. The effectiveness of a hierarchy significantly depends on its length, however. We show that there always exist conditions under which adding a marginal overseer to a hierarchy of a given length increases the effort level of the agent, and, in fact, can do so in a way that is cost-effective for the principal. Surprisingly, the effectiveness of the hierarchy can decrease with the increase in the information available to the overseers.

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1 Introduction

Political processes at all government levels in most democracies are characterized by the existence of hierarchies from policy-implementing agency staff to executive appointees at the head of the agency to cabinet members or elected administrators to heads of regional governments. Two features of such hierarchies set them apart from most principal-agent relationships studied in political economy. The first is their length, which often goes far beyond the classic binary principal-agent relationship. The second is the fact that voters, as principals of such hierarchies, typically have no special expertise in the particular area of a given agency's policy domain, and neither the ability nor the incentives to acquire it.

How effective are such hierarchies at solving the entailed principal-agent relationships? Is there a satisfactory answer to the question of "who will oversee the overseer?" in the context of such chains of principal-agent relationships? When are principals (voters) better off delegating the oversight, including the creation and enforcement of incentives for their agents, to subordinate overseers, and, in contrast, when are they better off relying on the second-bests (e.g., directly electing heads of state agencies, as they do in California and other states)? While these questions and the corresponding institutional settings are clearly important and related to considerable body of work in political economy (see the discussion below), the conjunction of attributes most relevant to them: (1) uninformed principal; (2) costly information acquisition by the subordinate overseers; and (3) delegated principal-agent relationship to the subordinate overseers appears to have eluded systematic analysis.

A key intuition that underlies our analysis is that a sequence of subordinate overseers can offer the principal a way of implementing effective incentives for oversight that substitutes for the precise information about the agent's choice that the principal does not have. At the core of our main results is the interaction between the credibility of those incentives and the direct and indirect information about the agent's choices. We show that principals can improve the agent performance by hiring intermediate supervisors, but when and how this welfare gain occurs depends on a number of factors. To induce the agent to invest more into effort, the overseers must be willing to invest into oversight — which they are more likely to do when predicting outcome without the knowledge of the agent's effort is harder — and the overseers must be able to commit themselves to retention choices that reward effort by their subordinates. We show that the principal with a single intermediate supervisor has limited abilities to create the incentives that lead to a high effort by the agent even where equilibria with high effort exist in the absence of intermediate supervisors. On the other hand, hiring two or more intermediate supervisors can make the principal strictly better off relative to no intermediate oversight because the addition of the second supervisor makes possible the equilibrium in which the commitment to reward and punish the agent based on his choices is more credible. Adding further overseers can now increase the probability of investment into oversight by their subordinates, which can further increase the agent's choice — but only so long as the highest ranking overseer remains sufficiently uncertain about the outcome to invest into oversight himself. More generally, we provide conditions under which adding marginal overseers to an oversight chain of a given length not only improves the agent performance, but also lowers the cost to the principal of the oversight hierarchy itself. We also show that another factor affecting the value of intermediate oversight is the exogenous informational environment: holding fixed the level of principal's informedness, gains from introducing intermediate oversight decrease in the exogenous information about the action and its consequences available to overseers. Observing the success or failure consequences of agent's actions makes intermediate oversight counterproductive, while the increase in the likelihood of observing those actions and the state on which they are contingent without oversight effort can decrease the agent's action.

In the next section to follow, we briefly discuss the previous work on oversight in administrative hierarchies. Section 3 provides a formal description of our model. Section 4 presents some baseline results. Section 5 studies the equilibria of our main model. Section 6 turns to the question of welfare comparisons, developing results on the welfare effects of adding marginal overseers, optimal distribution of compensation to overseers, and the value of exogenous information. Section 7 shows that the equilibria analyzed in Section 5 are robust to the possibility of stochastic action outcome revelation to the overseers. Section 8 shows that hiring a vertical oversight hierarchy can be attractive for the principal compared to contracting with outside auditors. Section 9, then, concludes with a brief discussion of the relationship between our results and some of the previous findings.

2 Previous Work on Oversight in Hierarchies

An extensive literature in industrial organization has studied output-contingent contracts in the context of the classic binary principal-agent relationship.

A number of papers, starting with Williamson (1967) have examined incentives and oversight in hierarchies. The influential papers that have focused on moral hazard include Calvo and Wellisz (1978) and Qian (1994). These models focus on the environment in which the set of individuals at a given level of hierarchy can be large (it is a singleton in our model) and formalize the notion of the "loss of control" that results from increasing the set of supervised subordinates for a given supervisor. In these models, the loss of control provides a limit on the efficiency of supervision. We depart from this literature in two crucial ways: we focus on the problem of creating the incentives for costly strategic supervision, which these papers do not study, and we abstract away from the problem of the loss of control by focusing on the hierarchies with a single decision-maker/task at each level.[1]

Strausz (1997) studies the problem of creating such incentives when, as in our model, the principal cannot verify the supervisor's monitoring, but, unlike in our model, assumes the environment in which the intermediate supervisor discovers hard information that can be costlessly shared with the principal, whose contract design can condition on what the supervisor reveals to him. Rahman (2013) studies costly private monitoring but outside the framework of the output-contingent contracts, which we assume are faced by the principal; the contracts he analyzes are also unavailable in our environment.

A series of papers (e.g. Tirole, 1986; Laffont and Tirole, 1991; Faure-Grimaud, Laffont and Martimort, 2003) examines the effect of principals' hiring an independent auditor who can inform him about the agent's choices. The auditor performs only auditing functions – i.e., obtaining information and informing the principal – and does not have the executive ability to reward or

[1]Recent empirical work in political economy analyzes the patterns of task bundling and unbundling among the elected officials (Berry and Gersen, 2008), emphasizing, inter alia, considerations of the loss of control. In practice, task bundling leads to the creation of oversight hierarchies under the elected official and tasks unbundling to the shortening or the elimination of such hierarchies. Our analysis in the present paper may be thought of isolating the effects of hierarchy lengths associated with these different concentrations of tasks, while holding fixed the number of tasks or complexity of responsibility.

punish the agent that intermediate overseers in the hierarchies we analyze do. A key question in those models is when the principal benefits from hiring auditors given the possibility of their collusion with the agents. In the present paper, we abstract away from the possibility of collusion, but study a comparison of principal's welfare with vertical oversight hierarchies and with outside auditors.[2]

Apart from the differences discussed above, another key factor that distinguishes our model is the focus on binary contracts: decisions to retain or fire the agents. Such contracts are an important characteristic of many political agency settings, in which principals do not have access to a large and flexible set of contract terms and are often constrained to use blunt instruments, such as retaining or replacing agents, allocating or not allocating a set budget, or reassigning an agent to a less desirable job. Modeling the Principal's problem as the decision about whether to retain or fire the Agent captures this feature of the political environment in a simple way.

The theoretical work on the political economy of accountability originates with the seminal papers by Barro (1973) and Ferejohn (1986). The typical framework analyzed in those models and the large literature that followed (recent examples include Alt, Bueno de Mesquita and Rose (2011); Ashworth, de Mesquita and Friedenberg (2013); others) is that of a binary relationship between a principal and an agent under distinct informational and institutional circumstances. Further, the principal's choice in these models typically turns on the incentives associated with a single action by a subordinate agent.[3] While the present model is motivated by the concerns with accountability that are at the center of this literature, we depart from it in analyzing the effects on the quality of agent's accountability of a sequence of delegated principal-agent relationships among intermediate overseers – thus considerably enlarging the set of decisions that principal's choice seeks to motivate.

Finally, and particularly relevant to our analysis empirically, there is a large literature on delegation and accountability in studies of comparative politics. An influential line of argumentation in that work identifies parliamentary democracies with a "chain of delegation" running from the

[2] In the interests of comparability to the vertical oversight game, we further depart from the previous models of auditing in assuming that auditors must also incur a cost to become informed.

[3] Some exceptions are Gordon and Hafer (2007), who study the interaction between two agents subordinate to the voters, the legislature directly elected by them, for which it then sets policy; Bueno de Mesquita and Landa (2013), who analyze a model with a sequence of choices by the agent between principal's retention choices; and Le Bihan (2014), who analyzes a model with the agent making separate choices with respect to different policy dimensions.

voters at the top of the hierarchy to the elected legislatures to ministerial cabinets to lower-level oversight relationships within ministries and other government agencies so that "at each link a single principal delegates to one and only one agent, or to several non-competing ones, and in which each agent is accountable to one and only one principal" (Strøm (2003); see also Strøm, Müller and Bergman (2006); Strøm, Müller and Smith (2010)). Strøm (2000) argues, further, that in parliamentary democracies, selection problems are resolved within political parties, which are somewhat orthogonal to these chains, whereas moral hazard problems are more central to the relationships between the occupants of the different links in the chains. In focusing on moral hazard problems within such chains of delegation, the games we analyze share key features with this account of parliamentary democracies.

3 Primitives

We consider games between the Agent (A), a sequence of Overseers $O_i \in \{O_1, O_2, ..., O_n\}$ and the Principal (P). Unless indicated otherwise, the sequence of play is as follows:

(i) Nature chooses a state of the world $\omega \in \{\omega_L, \omega_H\}$ with $0 < \omega_L < \omega_H < 1$, and $\Pr(\omega = \omega_H) := \pi$. In what follows, we refer to ω as "the state."

(ii) After observing the state, A takes an action $a \in \{\underline{a}, \overline{a}\}$ with $0 \leq \underline{a} < \overline{a} \leq 1$. With probability $a\omega$, the action results in policy success (s), while with probability $1 - a\omega$, it results in failure (f). In some of the games we consider below the outcome is observed immediately after a is chosen, while in others it is observed after all the Overseers have made their choices.

(iii) Overseer 1's choices:

(a) O_1 chooses whether to invest in a costly search to learn a and ω. We denote the decision of O_1 to investigate A, $I_1 = 1$ and the decision not to investigate $I_1 = 0$.

(b) If O_1 invests, Nature reveals (a, ω) to O_1 with probability e_1 and reveals nothing with probability $(1 - e_1)$. If O_1 does not invest, he does not learn (a, ω).

(c) Based on the information revealed under (b), O_1 chooses an action $R \in \{F, K\}$ with F describing the case where O_1 fires A and K the case where he retains A.

(iv) Each Overseer O_i in sequence $(O_2, ..., O_n)$, starting with Overseer 2, makes her choices:

(a) After observing the retention decisions of Overseers $O_{j<i}$, O_i chooses whether to invest into a costly search to learn a, and ω. The decision to investigate upon observing a sequence of retention decision $(R^{O_{i-2}}, \ldots, R^{O_1}, R)$ is denoted by $I_i(R^{O_{i-2}}, \ldots, R^{O_1}, R) \in \{0, 1\}$.

(b) If O_i invests, Nature reveals (a, ω) to O_i with positive probability. This probability depends on whether O_{i-1} observed (a, ω). If O_{i-1} observed (a, ω), O_i learns (a, ω) with probability e_i. If, however, O_{i-1} did not observe (a, ω), then O_i learns (a, ω) with probability $e_i^{\emptyset} < e_i$.

(c) Based on the information revealed under (b), O_i chooses an action $R^{O_{i-1}} \in \{F^{O_{i-1}}, K^{O_{i-1}}\}$ with $F^{O_{i-1}}$ describing the case where O_i fires O_{i-1} and $K^{O_{i-1}}$ the case where he retains O_{i-1}.

(v) The policy outcome is realized. P observes the outcome and the retention decisions of all Overseers, but not the level of effort exerted by the Agent, the state of the world, and whether the Overseers investigated or not. Based on his observations, P decides whether to retain O_n.

(vi) Utilities are realized.

The players' preferences are given as follows.

Let $W_A > 0$ be the value to A of being retained in office and 0 be the value of A's outside option.

Further, regardless of whether she is retained, A receives $k(1-a)$ with $k \in (0, 1)$.[4] Thus, A's utility is $IW_A + k(1-a)$, where $I = 1$ if A is retained and 0 otherwise.

Let $W_{O_i} > 0$ be the value to O_i of being retained and 0 be the value of her outside option. Further, if O_i chooses to invest in a search, O_i incurs a cost $c_i > 0$. Thus, O_i's utility is $IW_{O_i} - I_i c_i$, where $I = 1$ if O_i is retained and 0 otherwise.

P receives B in case of success (s) and 0 otherwise.

Throughout, our solution concept is pure strategy Perfect Bayesian Equilibrium.

Before proceeding with the analysis, we comment on some noteworthy features of the oversight hierarchy we model. First, we assume a certain asymmetry in the information that is available to

[4]This functional form is used for simplicity of exposition. We would get similar results with any concave function of $(1-a)$.

actors. In particular, while the Principal and the second Overseer observe at no cost the retention decision of the first Overseer, they do not automatically observe the effort level of the Agent, nor the decision of Overseer 1 to investigate or not. This reflects the fact that retention decisions are in the public record. Second, we assume that the Principal cannot himself invest into becoming informed about the actual effort level of the Agent nor about the decisions of Overseers to investigate or not. If we think of the Principal as representing the voters, this restriction may be thought of as reflecting the limited opportunities for acquiring expertise available to the voters and their limited incentives of doing so, given the low likelihood of being pivotal. Finally, the hierarchies we envision as most empirically relevant to our analysis include, but not are limited to, the structures found in many parliamentary democracies: from the voters at the top of the hierarchy to the elected legislatures to ministerial cabinets to lower-level oversight relationships within ministries and other government agencies.

4 Baseline Models

We begin with baseline results, characterizing the equilibria of two models that are variations on the model described above: (1) the environment without intermediate Overseers and (2) the environment where Overseers observe the policy outcome before deciding whether to investigate or not. The primary value of these baseline results is to shed light on features of the main model, though aspects of these results may be of independent interest as well.

4.1 Classical Moral Hazard: No Overseers

Our first result describes the equilibrium in what is a classic principal-agent moral hazard environment.

Lemma 4.1 *Suppose that there are no Overseers. Then, in equilibrium, the Agent chooses*

$$\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } W_A \geq k/\omega \\ \underline{a} \text{ otherwise.} \end{cases}$$

The Principal retains the Agent if and only if the outcome is success.

Proof. Suppose the Principal keeps the Agent with probability $r_P(K|s) \in \{0, 1\}$ when the outcome is success and keeps the Agent with probability $r_P(K|f) \in \{0, 1\}$ when the outcome is failure. Then, the utility of the Agent from exerting effort a is $U_A(a, r_P) = k(1-a) + a\omega r_P(K|s)W_A + (1 - a\omega)r_P(K|f)W_A$. Hence, if $\overline{a}\omega r_P(K|s)W_A + (1-\overline{a}\omega)r_P(K|f)W_A \geq k(\overline{a} - \underline{a}) + \underline{a}\omega r_P(K|s)W_A + (1 - \underline{a}\omega)r_P(K|f)W_A$, the Agent wants to exert level of effort $\overline{a}$, whereas, otherwise, the Agent wants to exert effort level $\underline{a}$. Obviously, the equilibrium level of effort by the Agent is then maximized by letting $r_P(K|s) = 1$ and $r_P(K|f) = 0$. ■

Hence, in equilibrium, 1) if $W_A \geq k/\omega_L$ the Agent will choose to exert high effort in both states of the world, i.e. $\hat{a}(\omega) = \overline{a}$ for all ω, 2) if $k/\omega_L > W_A \geq k/\omega_H$, the Agent will choose to exert high effort in the high state and low effort in the low state, i.e. $\hat{a}(\omega_H) = \overline{a}$, and $\hat{a}(\omega_L) = \underline{a}$, and 3) if $W_A < k/\omega_H$, the Agent will choose to exert low effort in both states of the world, i.e. $\hat{a}(\omega) = \underline{a}$ for all ω. If we think about the Principal as seeking to choose an optimal hierarchical oversight structure, then this result may be seen as establishing the lower bound on the Principals welfare: if a given hierarchy yields welfare below this bound, the Principal is better off not delegating oversight and simply implementing an outcome-based retention rule for the Agent.

4.2 Policy Outcome is Realized Before Overseers Make Their Choices

Our next result concerns the environment in which the Overseers observe the outcome before making their choices. As the Proposition below shows, this is a particularly unattractive world for the Principal: he cannot induce effort on the part of any members of the hierarchy above the Agent.

Proposition 4.1 *Suppose there are n Overseers, $n = 1, 2$, and the Overseers observe the outcome before making their retention decision. Then there exists no equilibrium in which any of the Overseers investigates. The equilibrium level of the Agent's effort is as described in Lemma 4.1. In equilibrium, the Principal and the Overseers each choose to retain if and only if the outcome is success.*

Proof. WLOG suppose that every Overseer up to O_{n-1} has kept the actor who is below him in the hierarchy, i.e. the sequence of retention decision is $(K^{O_n - 1}, K)$. If

$$r_P(K^{O_n}|s, K^{O_{n-1}}, K^{O_{n-2}}, \ldots, K^{O_1}, K) > r_P(K^{O_n}|s, F^{O_{n-1}}, K^{O_{n-2}}, \ldots, K^{O_1}, K),$$

then O_n strictly prefers to keep O_{n-1} upon observing $(s, K^{O_{n-2}}, \ldots, K^{O_1}, K)$ independently of (a, ω) and thus $I_n(s, K^{O_{n-2}}, \ldots, K^{O_1}, K) = 0$. By the same argument, $I_n(s, K^{O_{n-2}}, \ldots, K^{O_1}, K) = 0$ if $r_P(K^{O_n}|s, K^{O_{n-1}}, K^{O_{n-2}}, \ldots, K^{O_1}, K) \leq r_P(K^{O_n}|s, F^{O_{n-1}}, K^{O_{n-2}}, \ldots, K^{O_1}, K)$. In other words, if O_n observes the outcome, he never exerts any effort. But then, as O_n observes the policy outcome before before making his retention decision and never exerts any effort, his retention decision is based on the same information as the retention decision of the Principal. Repeating the argument for O_n we can thus show that O_{n-1} never exerts any effort if O_{n-1} observes the policy outcome before deciding whether to retain O_{n-2}. By induction, it is thus the case that none of the Overseers exerts any effort if the Overseers observe the policy outcome before making their retention decisions.

Now suppose, the Principal keeps O_n if, and only if, the policy outcome is success. Then, O_n is indifferent between keeping and firing O_{n-1} when there is policy success and when there is policy failure and it is thus a best-response for O_n to keep O_{n-1} if, and only if, there is policy success. The same is true in general for any Overseer O_i. In particular, if any Overseer keeps if, and only if, there is success, it is also a best-response for O_1 to keep A if, and only, if there is success. But then, O_1 is using the same retention rule as P is using in Lemma 4.1, which implies that the equilibrium level of the Agent's effort is as described in Lemma 4.1. Finally, it is not possible for the Principal to improve upon this equilibrium, as any improvement would have to come from the fact that Overseers are better informed than the Principal, which we established is never the case in this game. ■

The Overseers cannot do better because by the time of their choices, the outcome is realized, and if the Principal or O_2 chooses to condition her retention rule on a retention action, the immediate subordinate will simply choose a retention action that maximizes her probability of retention – without investing into effort. No Overseer will want to invest into effort because by the time of their choice, the outcome is already determined, and they know that the immediate superior will not condition their retention rule on the subordinate's effort either. O_1's equilibrium rule of *retain if and only if success* will induce the Agent to choose an equilibrium level of effort that is equal to what it would be in the environment with no Overseers. Note that if O_2 chooses to condition O_1's retention on O_1's own retention action with respect to A in a way that is different from *retain if and only if success*, then this will induce O_1 to choose a retention rule for the Agent that cannot increase, but can decrease, the Agent's effort.

If we interpret this result from the standpoint of optimal design of oversight hierarchy, holding the Principal as a residual claimant on the Overseers' rewards of office, then the comparison of this environment and the one without the Overseers is clearly in favor of no Overseers. In both cases, the Principal has to pay the Agent a wage of at least k/ω_L to induce the Agent to choose $\overline{a}$ in both states of the world, and a wage of at least k/ω_H to induce effort in the high state. However, now the Principal also has to pay a Wage to the Overseers. Both Lemma 4.1 and this interpretation are, perhaps, somewhat surprising, since in principal-agent settings, more information about the outcome, and, further, having that information available before making choices, is typically associated with better outcomes. Here, because of the nature of the principal-agent relationship within a hierarchy, that information clearly does not help, and, moreover, as the results below indicate, can hurt relative to what the Principal can do with hierarchical oversight when the outcome is realized after the Overseers' actions.

5 Costly Oversight

5.1 Single Overseer

In contrast to the costless oversight setting, the addition of a single Overseer restricts the ability of the Principal to induce the Agent to exert high effort and is thus generally unattractive when oversight is costly. Indeed, as we explain in more detail below, under costly oversight, there is no equilibrium in which the Agent chooses to exert high effort in both states of the world, and there exists a range of parameter values (a, ω) for which, in any equilibrium, the Agent chooses low effort in both states of the world. To understand why this is the case, we first identify two necessary conditions to sustain an equilibrium in which the Agent chooses high effort in some state of the world ω :

Lemma 5.1 *The Agent chooses to exert high effort $\overline{a}$ in state of the world ω only if*

1. *Overseer O_1 investigates, and*
2. *Overseer O_1 retains the Agent upon observing $(\overline{a}, \omega)$ and fires him upon observing $(\underline{a}, \omega)$.*

The utility to the Agent of choosing high effort in some state of the world ω is

$$U_A(\overline{a}|\omega) = k(1-\overline{a}) + I_1 e_1 r_{O_1}(K|\overline{a},\omega)W_A + (1 - I_1 e_1) r_{O_1}(K|\emptyset)W_A,$$

while the utility of choosing low effort is

$$U_A(\underline{a}|\omega) = k(1-\underline{a}) + I_1 e_1 r_{O_1}(K|\underline{a},\omega)W_A + (1 - I_1 e_1) r_{O_1}(K|\emptyset)W_A.$$

As effort is costly to the Agent, $(k(1-\overline{a}) < k(1-\underline{a}))$, the Agent will choose to exert high effort only if the probability of being retained is higher when exerting high effort than when exerting low effort. This has the straightforward implication that the Overseer must retain the Agent upon observing high effort and dismiss the Agent when observing low effort, i.e. $r_{O_1}(K|\overline{a},\omega) = 1 > 0 = r_{O_1}(K|\underline{a},\omega)$. But it also implies that the Overseer needs to investigate to sustain high effort by the Agent. Indeed, if the Overseer O_1 does not investigate, i.e. $I_1 = 0$, then the retention probability of the Agent will be the same when he exerts high effort as when he exerts low effort, namely $r_{O_1}(K|\emptyset)$. But then there is no upside for the Agent of exerting high effort.

Given a hierarchy with a single Overseer, the ability of the Principal to induce the Agent to exert high effort thus depends on whether the Principal can retain the Overseer in such a way that the Overseer is induced (1) to investigate and (2) to commit to the Agent that he will retain him for high effort and dismiss him for low effort. As the following Lemma reveals, however, any retention rule that the Principal may use to incentivize the Overseer to investigate implies a restriction on the willingness of the Overseer to retain the Agent. With a single Overseer, there is thus a fundamental tension between satisfying the first necessary condition for high effort, namely that the Overseer O_1 investigates, and the second necessary condition, namely that the Overseer rewards high effort and punishes low effort.

Lemma 5.2 *The Overseer only investigates if he strictly prefers to keep the Agent upon observing* $(\hat{a}(\omega),\omega)$ *and strictly prefers to fire the Agent upon observing* $(\hat{a}(\omega'),\omega')$ *with* $\omega \neq \omega'$.

Proof. see Appendix. ■

The intuition behind this Lemma is as follows. As the effort level of the Agent has already been determined when the Overseer is deciding whether to investigate, the Overseer can only be investigating in order to increase his probability of retention. Suppose now the statement in Lemma

5.2 does not hold. Then, the Overseer weakly prefers to keep (or to fire) the Agent in all instances and the Overseer cannot improve his probability of retention by becoming informed about the state of the world and the level of effort exerted by the Agent.

The fundamental tension revealed by Lemma 5.2 severely diminishes the ability of the Principal to incentivize the Agent to exert high effort given a hierarchy with a single Overseer. Indeed, the Principal is never able to induce the Agent to exert high effort in both states of the world.

Proposition 5.1 *Suppose there is a single Overseer who has to pay $c_1 > 0$ to observe a, and ω. Then, there do not exist W_A and W_{O_1} such that in equilibrium the Agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω.*

Proof. Given in the text. ■

The argument behind proposition 5.1 goes as follows. Suppose, by contradiction, that there exists an equilibrium in which the Agent chooses high effort in both states of the world, i.e. $\hat{a}(\omega) = \overline{a}$ for all ω. Then, by Lemma 5.1, it must be the case, in such an equilibrium, that the Overseer O_1 investigates. Moreover, by Lemma 5.2, for the Overseer to investigate it must then be the case that he strictly prefers to keep the Agent upon observing $(\overline{a}, \omega)$ and strictly prefers to fire the Agent upon observing $(\overline{a}, \omega')$ with $\omega \neq \omega'$. However, if the Overseer strictly prefers to fire the Agent upon observing that the Agent exerted high effort in state of the world ω', i.e. upon observing $(\overline{a}, \omega')$, then, by Lemma 5.1, it is not a best-response for the Agent to choose $\hat{a}(\omega') = \overline{a}$.

The implication of this result is clear yet striking. When there is a single Overseer and the outcome is realized after the Overseer makes his retention decision, then no matter how high the wages are that the Principal pays to the Agent and the Overseer, it will never be the case that the Agent chooses to exert high effort in both states of the world.

Lemma 5.2 has two further important implications which distinguish the costless oversight setting from the costly one. First, Lemma 5.2 implies a restriction on the retention rules that can be used by the Principal, in any equilibrium in which the Agent is exerting high effort in some state of the world:

Corollary 5.1 O_1 *does not investigate unless either*

1. $r_P(K^{O_1}|s, K) = 1$, $r_P(K^{O_1}|s, F) = 0$, $r_P(K^{O_1}|f, K) = 0$, *and* $r_P(K^{O_1}|f, F) = 1$, *or*

2. $r_P(K^{O_1}|s,K)=0$*,* $r_P(K^{O_1}|s,F)=1$*,* $r_P(K^{O_1}|f,K)=1$*, and* $r_P(K^{O_1}|f,F)=0$*.*

Proof. see Appendix. ■

The most striking implication of this last result is that the Principal must condition his retention decision of the Overseer on the decision by the Overseer to keep or fire the Agent in order for an equilibrium to exist in which the Agent is willing to exert high effort. Otherwise the Overseer is indifferent between keeping and firing the Agent and thus has no incentive to invest in costly information acquisition. But then the Agent cannot be incentivized to exert effort. This necessary condition for a high effort equilibrium to exist in the costly oversight hierarchy is particularly interesting when compared to the case where the Overseer costlessly observes the state of the world and the level of effort exerted by the Agent. There, we established that the Principal could only reduce the level of effort exerted by the Agent by conditioning on the retention decision used by the Overseers. When there is even the slightest cost of investigating for the Overseers, precisely the opposite is true.

Second, Lemma 5.2, jointly with Corollary 5.1, has important implications for the level of effort of the Agent that can be achieved. Indeed, as our next result shows, there is a large range of values of (a,ω) for which no equilibrium exists in which the Agent chooses to exert high effort in any of the states of the world.

Proposition 5.2 *Suppose there is a single Overseer who has to pay* $c_1>0$ *to observe* a*, and* ω*. If* $\overline{a}\omega_H\leq 1/2$*, or* $a\omega\geq 1/2$ *for some* (a,ω) *such that* $(a,\omega)\neq(\overline{a},\omega_H)$*, then the Agent chooses* $\hat{a}(\omega)=\underline{a}$ *for all* ω *in equilibrium.*

Proof. By Lemmata 5.1 and 5.2, O_1 does not investigate if $a\omega\leq 1/2$ for all (a,ω). By Lemma 5.1, if O_1 does not investigate then the Agent chooses $\hat{a}(\omega)=\underline{a}$ for all ω. It follows that if $\overline{a}\omega_H\leq 1/2$, then $\hat{a}(\omega)=\underline{a}$ for all ω. By a similar argument, if $\underline{a}\omega_L\geq 1/2$, we have $\hat{a}(\omega)=\underline{a}$ for all ω. So suppose $\overline{a}\omega_H>1/2$, and $\underline{a}\omega_L<1/2$. There are three cases to consider: 1) $\overline{a}\omega_L,\underline{a}\omega_H\geq 1/2$, 2) $\overline{a}\omega_L\geq 1/2$, $\underline{a}\omega_H<1/2$, and 3) $\overline{a}\omega_L<1/2$, $\underline{a}\omega_H\geq 1/2$. Let us start with the first case: 1) By Lemma 5.1 O_1 then either prefers to fire A or prefers to keep A both upon observing $(\overline{a},\omega_H)$, and upon observing $(\underline{a},\omega_H)$. It follows that A's best-response is to choose $\underline{a}$ when $\omega=\omega_L$. So assume that O_1 prefers to keep A upon observing $(\overline{a},\omega_L)$, as otherwise A chooses $\underline{a}$. But then, note that by

Lemma 5.1 and by the fact that $\underline{a}\omega_L \geq 1/2$, O_1 also prefers to keep A upon observing $(\underline{a}, \omega_H)$. But then, by Lemma 5.2, O_1 does not investigate which implies, by Lemma 5.1 that A chooses $\underline{a}$ when $\omega = \omega_L$. In the second case, i.e. when $\overline{a}\omega_L \geq 1/2$, $\underline{a}\omega_H < 1/2$, by Lemma 5.1 if O_1 prefers to keep A upon observing $(\overline{a}, \omega_H)$ then O_1 also prefers to keep A upon observing $(\overline{a}, \omega_L)$ and prefers to fire A upon observing $(\underline{a}, \omega_H)$, and upon observing $(\underline{a}, \omega_L)$. By Proposition 5.1 it cannot be the case that A chooses $\overline{a}$ in both states of the world. So, WLOG, suppose by contradiction that the Agent chooses $\overline{a}$ in the high state and $\underline{a}$ in the low state. But then, as by Lemma 5.1 O_1 investigates in such an equilibrium, A wants to deviate to $\overline{a}$ in the low state. Finally, in the third case, i.e. when $\overline{a}\omega_L < 1/2$, $\underline{a}\omega_H \geq 1/2$, Lemma 5.1 implies that O_1 either always prefers to keep in the high state, and always prefers to fire in the low state, or the other way around. In either case, A's best-response is then to choose $\hat{a}(\omega) = \underline{a}$ for all ω. ■

Propositions 5.1 and 5.2 do not imply, however, that the Agent never chooses to exert high effort when there is a single Overseer. Indeed, our next proposition identifies conditions under which the Agent chooses to exert high effort in the high state and low effort in the low state. In a section to be added in the Appendix we show that this is the only equilibrium in which the Agent is exerting high effort in the single Overseer case.

Proposition 5.3 *Suppose there is a single Overseer who has to pay $c_1 > 0$ to observe a, and ω. If $\overline{a}\omega_H > 1/2$, and $a\omega < 1/2$ for all (a, ω) such that $(a, \omega) \neq (\overline{a}, \omega_H)$, then there exist $\overline{W}_A, \overline{W}_{O_1}$ such that if $W_A \geq \overline{W}_A$, and $W_{O_1} \geq \overline{W}_{O_1}$ then, in equilibrium, the Overseer investigates and the Agent chooses*

$$\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ otherwise.} \end{cases}$$

In this equilibrium, the Principal chooses the following retention rule: $r_P(K^{O_1}|s, K) = r_P(K^{O_1}|f, F) = 1$, and $r_P(K^{O_1}|s, F) = r_P(K^{O_1}|f, K) = 0$.

Moreover, we have $\overline{W}_A = k(\overline{a} - \underline{a})/e_1$, and $$\overline{W}_{O_1} = \begin{cases} c_1/(e_1\pi(2\overline{a}\omega_H - 1)) \text{ if } \pi \leq \frac{1-2\underline{a}\omega_L}{2(\overline{a}\omega_H - \underline{a}\omega_L)} \\ c_1/(e_1(1-\pi)(1-2\underline{a}\omega_L)) \text{ otherwise.} \end{cases}$$

Proof. see Appendix. ■

The equilibrium in which the Agent chooses to exert high effort appears to brittle, however. Indeed, if $a\omega > 1/2$ for some (a, ω) such that $(a, \omega) \neq (\overline{a}, \omega_H)$, then in equilibrium the Agent chooses $\hat{a}(\omega) = \underline{a}$ for all ω, independently of the level of wages W_A, W_{O_1}. This suggests that if one widens the set of levels of effort that are available to the Agent, equilibria in which the Agent chooses the highest level of effort available, at least in some state of the world, are likely to cease to exist. Indeed, in a robustness section to be written, we show that widening the set of available levels of effort of the Agent (weakly) decreases the level of effort exerted by the Agent when there is a single Overseer. In the limit, if the Agent is allowed to choose his level of effort from a continuum, i.e. $a \in [0, 1]$, the Agent can never be motivated to exert positive effort in equilibrium.

5.2 Two Overseers

In this section we show that significant qualitative differences exist between the single Overseer case and the two Overseers case, when oversight is costly. Indeed, as we establish in proposition 5.4, not only is the range of values of (a, ω) for which equilibria exist in which the Agent chooses to exert high effort at least in one state of the world significantly wider with two Overseers, there also exist parameter values for which, in equilibrium, the Agent chooses to exert high effort in both states of the world with a hierarchy of two Overseers, something we established is never the case with a single Overseer.

Proposition 5.4 *Suppose there is a hierarchy of two Overseers. Then, in equilibrium:*

(i) If $\overline{a}\omega_H > 1/2$, and $\overline{a}\omega_L < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, each Overseer O_i investigates and the Agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω.

(ii) If $\overline{a}\omega_H > 1/2$, and $\underline{a}\omega_L < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, each Overseer O_i investigates and the Agent chooses $\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ otherwise} \end{cases}$.

(iii) If $\overline{a}\omega_L > 1/2$, and $\underline{a}\omega_H < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, each Overseer O_i investigates and the Agent chooses $\hat{a}(\omega) = \begin{cases} \underline{a} \text{ if } \omega = \omega_H \\ \overline{a} \text{ otherwise.} \end{cases}$

Proof. See Appendix. ■

To understand how the addition of a second Overseer can increase the level of effort exerted by the Agent compared to the single Overseer case, we present below the strategy profile that sustains an equilibrium in which the Agent chooses high effort in both states of the world and highlight how this strategy profile induces the first Overseer O_1 (1) to investigate and (2) to reward the Agent for high effort and punish him for low effort. Before presenting this strategy profile in detail, we preview the most important features of the argument. Remember, that in the single Overseer case the Principal necessarily needs to condition his retention decision of O_1 on O_1's own retention decision of the Agent in order to provide O_1 with incentives to investigate. By doing so, however, the Principal restricts the ability of O_1 to commit to a high effort inducing retention rule for the Agent. When there are two Overseers, the Principal uses a retention rule for O_2 which has three main features. (1) The Principal conditions his retention decision of the second Overseer O_2 on whether O_2's decision to keep or fire the first Overseer O_1 matches the policy outcome. This creates incentives for O_2 to investigate in order to learn whether success of failure is more likely and consequently whether he should keep or fire O_1. (2) The retention rule used by the Principal induces Overseer O_2 to fire O_1 whenever O_2 does not observe (a, ω) and induces O_2 to keep O_1 upon observing $(\hat{a}(\omega), \omega)$ for some ω. As a consequence, the first Overseer is incentivized to investigate. Indeed, if the first Overseer O_1 does not investigate, the second Overseer O_2 is likely not to observe (a, ω) and unlikely to observe $(\hat{a}(\omega), \omega)$. If, on the other hand, O_1 investigates, then O_2 is more likely to observe $(\hat{a}(\omega), \omega)$, yet not to observe (a, ω). In other words, investigating increases the probability that O_1 will be retained by Overseer O_2, which gives O_1 incentives to investigate. (3) The decisive aspect of the retention rule used by the Principal is, however, that he does not condition his decision to keep or fire O_2 on the decision of O_1 to keep or fire the Agent. As a consequence, the second Overseer has no incentive to condition his behavior on the retention decision of the first Overseer either. But, if the second Overseer investigates and then retains (or fires) the first Overseer independently on the decision of the first Overseer to keep or fire the Agent, then the first Overseer is always indifferent between keeping and firing the Agent. The first Overseer is then free to commit to any retention rule, and in particular to a retention rule that incentivizes the Agent to exert high effort. In other words, in the presence of the second Overseer, the Principal can use the second Overseer to incentivize the first Overseer to investigate without having to condition on whether the first Overseer kept or fired the Agent. The fundamental tension between Lemmata 5.1 and 5.2 identified in the single Overseer case thus ceases to exist with two Overseers.

To make this logic fully apparent, consider the following strategy profile. As we will show, provided the wages W_A, W_{O_1}, and W_{O_2} are sufficiently high, this profile sustains an equilibrium in which the Agent chooses high effort in both states of the world provided $\overline{a}\omega_H > 1/2, \overline{a}\omega_L < 1/2$ and π is lower than some threshold $\hat{pi}$ to be derived below. We will later discuss how this profile needs to be amended for other parameter values of (a, ω) and π.

(i) The Agent chooses high effort in both states of the world, i.e. $\hat{a}(\omega) = \overline{a}$ for all ω,

(ii) O_1 investigates,

(iii) O_1 retains the Agent upon observing high effort or upon not observing the Agent's effort level and fires the Agent upon observing low effort, i.e. $r_{O_1}(K|a,\omega) = \begin{cases} 1 \text{ if } a = \overline{a} \\ 0 \text{ if } a = \underline{a} \end{cases}$ for all ω, and $r_{O_1}(K|\emptyset) = 1$,

(iv) O_2 investigates upon observing that O_1 kept the Agent and upon observing that O_1 fired the Agent, i.e. $I_2(K) = I_2(F) = 1$,

(v) Overseer O_2 retains Overseer O_1 upon observing high effort in the high state and fires O_1 otherwise, i.e. $r_{O_2}(K^{O_1}|a,\omega_H,R) = \begin{cases} 1 \text{ if } a = \overline{a} \\ 0 \text{ otherwise} \end{cases}$ for all R, $r_{O_2}(K^{O_1}|a,\omega_L,R) = 0$ for all a and all R, $r_{O_2}(K^{O_1}|\emptyset,R) = 0$ for all R,

(vi) The Principal keeps Overseer O_2 either if (1) O_2 kept O_1 and the policy outcome is success or if (2) O_2 fired O_1 and the policy outcome is failure, i.e. $r_P(K^{O_2}|s,K^{O_1},R) = 1$, $r_P(K^{O_2}|s,F^{O_1},R) = 0$, $r_P(K^{O_2}|f,K^{O_1},R) = 0$, and $r_P(K^{O_2}|f,F^{O_1},R) = 1$ for all R.

Let us first establish that it is indeed a best-response for the Agent to choose high effort in both states of the world, given the strategies used by the other players. As the principal-agent relationship is delegated to the first Overseer, the utility to the Agent of exerting high or low effort only depends on the behavior of the first Overseer. In the strategy profile under consideration, the behavior of the first Overseer conforms to the necessary conditions identified in Lemma 5.1 for the Agent to be incentivized to exert high effort, namely O_1 investigates and O_1 rewards the Agent for high effort, yet punishes him for low effort. As a consequence, the utility to the Agent of exerting high effort in state of the world ω is

$$U_A(\overline{a}|\omega) = k(1-\overline{a}) + e_1 W_A + (1-e_1)W_A$$

while the utility of exerting low effort is

$$U_A(\underline{a}|\omega) = k(1-\underline{a}) + (1-e_1)W_A.$$

Hence, if the value of being retained in office is sufficiently high, namely $W_A \geq k(\overline{a}-\underline{a})/e_1$, it is indeed a best-response for the Agent to exert high effort.

Next consider the incentives for Overseer O_1 to investigate. Again, the delegated nature of the principal-agent relationships in the hierarchy implies that the utility to the first Overseer of investigating only depends on the behavior of the second Overseer. In the strategy profile under consideration the second Overseer investigates and retains the first Overseer if, and only if, O_2 observes $(\overline{a}, \omega_H)$. It follows, that the utility to the first Overseer of not investigating is equal to the probability that O_2 observes $(\overline{a}, \omega_H)$, although O_1 does not investigate, times the value of being retained in office, i.e. $\pi e_2^{\emptyset} W_{O_1}$. On the other hand, if Overseer O_1 chooses to pay the cost c_1 of investigating then the probability that the second Overseer observes $(\overline{a}, \omega_H)$ increases to $\pi e_1 e_2 + (1-e_1)\pi e_2^{\emptyset})$. Overall, the utility to Overseer O_1 of investigating is thus $(\pi e_1 e_2 + (1-e_1)\pi e_2^{\emptyset}))W_{O_1} - c_1$. Hence, if the value of holding office is sufficiently high, i.e. if $W_{O_1} \geq \frac{c_1}{e_1(e_2 - e_2^{\emptyset})\pi}$, then it is optimal for the first Overseer to investigate.

Note that the second Overseer O_2 investigates both when O_1 keeps the Agent and when O_1 fires the Agent. Moreover, O_2 does not condition his retention decision of O_1 on O_1's own retention decision of the Agent. It follows that O_1 is always indifferent between keeping and firing the Agent. Hence, any retention rule used by O_1 is a best-response.

We now show that O_2 investigates both upon observing that O_1 kept the Agent and upon observing that O_1 fired the Agent. On the equilibrium path, the first Overseer never fires the Agent, so the decision of the second Overseer to investigate or not is off-the-equilibrium path as well. It is therefore a best-response for the second Overseer to investigate. If O_2 does not investigate upon observing that O_1 kept the Agent, then O_2 does not observe (a, ω) and chooses to fire O_1. Given that the Principal fires O_2 unless, either there is policy success and O_2 kept O_1, or there is policy failure and O_2 fired O_1, the utility to the second Overseer of not investigating, and thus firing O_1, is given by the probability that there is failure times the value of being retained:

$$U_{O_2}(I_2(K)=0, r_P) = \pi(1-\overline{a}\omega_H)W_{O_2} + (1-\pi)(1-\overline{a}\omega_L)W_{O_2}.$$

If O_2 chooses to pay the cost c_2 of investigating, however, then with probability $\pi(e_1e_2+(1-e_1)e_2^{\emptyset})$, O_2 observes $(\overline{a}, \omega_H)$ and decides to keep O_1. In all other cases, O_2 chooses to fire O_1. It follows that the utility to the second Overseer of investigating is

$$\begin{aligned} U_{O_2}(I_2(K)=1, r_P) = \pi(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})\overline{a}\omega_H W_{O_2} + (1-\pi)(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})(1-\overline{a}\omega_L)W_{O_2} \\ +(1-(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset}))U_{O_2}(I_2(K)=0, r_P) - c_2. \end{aligned}$$

Rearranging, we find that it is a best-response for the second Overseer to investigate upon observing that the first Overseer kept the Agent, i.e. $U_{O_2}(I_2(K)=1, r_P) \geq U_{O_2}(I_2(K)=0, r_P)$, if, and only if, $W_{O_2} \geq \overline{W}_{O_2} := c_2/((e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})\pi(2\overline{a}\omega_H-1))$. When deciding whether to investigate the second Overseer thus evaluates the potential benefit of investigating, given by the probability that he learns he should keep O_1 instead of firing him $\pi(e_1e_2+(1-e_1)e_2^{\emptyset})$ times the additional expected gain of keeping instead of firing in that case $(2\overline{a}\omega_H-1)$, against the cost c_2 of investigating.

Next, we show that the second Overseer has no incentive to deviate from his retention rule r_{O_2}. Remember that the Principal keeps the second Overseer if, and only if, either the outcome is success and O_2 retained O_1 or the outcome is failure and O_2 fired O_1. Hence, the second Overseer keeps the first Overseer when success is more likely than failure and fires the first Overseer otherwise. Now suppose first that the second Overseer observes $(\overline{a}, \omega_H)$. If O_2 keeps O_1 he receives an expected payoff of $\overline{a}\omega_H W_{O_2}$. Firing, on the other hand, yields an expected payoff of $(1-\overline{a}\omega_H)W_{O_2}$. Hence, as long as, $\overline{a}\omega_H > 1/2$ success is more likely than failure and it is a best-response for O_2 to keep O_1 upon observing $(\overline{a}, \omega_H)$. Similarly, it is a best-response for O_2 to fire O_1 upon observing $(\overline{a}, \omega_L)$ as long as $\overline{a}\omega_L < 1/2$.

Suppose now that O_2 does not observe (a, ω). If O_2 keeps O_1, he receives an expected payoff of $\pi\overline{a}\omega_H W_{O_2} + (1-\pi)\overline{a}\omega_L W_{O_2}$. If, on the other hand, he fires O_1 he receives $\pi(1-\overline{a}\omega_H)W_{O_2} + (1-\pi)(1-\overline{a}\omega_L)W_{O_2}$. Hence, if the probability of being in the high state is sufficiently low, i.e. $\pi \leq \hat{\pi} := \frac{1-2\overline{a}\omega_L}{2\overline{a}(\omega_H-\omega_L)}$, it is a best-response for the second Overseer to fire the first Overseer upon not observing (a, ω).

Finally, note that the Principal does not condition his retention decision of O_2 on the decision of O_1 to keep or fire the Agent. Hence, O_2 has no incentive to condition his retention decision of O_1 on R.

The structure of the strategy profile will remain the same for other parameter values of (a, ω) and π. First, if the probability of being in the high state is high, $\pi > \hat{\pi}$, the Principal may need to reward Overseer O_2 for firing O_1 when there is success and for keeping O_1 when there is failure in order to get O_2 to prefer firing O_1, when O_2 does not observe (a, ω). Consequently, O_2 will keep O_1 upon observing $\hat{a}(\omega_L)$ and fire him upon observing $\hat{a}(\omega_H)$.

An implication of proposition 5.4 is that the inability for the Principal of inducing high effort by the Agent does not go away completely with the addition of a second Overseer. Indeed, there exists a segment of the parameter space (a, ω) for which, no matter how high the wages are that the Principal pays to the Agent and the Overseers, no equilibrium exists in which the Agent chooses to exert high effort in any of the states of the world.

Lemma 5.3 *Suppose there is a hierarchy of* 2 *Overseers. If* $\overline{a}\omega_H \leq 1/2$*, or if* $\underline{a}\omega_L \geq 1/2$*, then in any equilibrium the Agent chooses* $\hat{a}(\omega) = \underline{a}$ *for all* ω*.*

Proof. By Lemmata **??** and **??**, O_2 needs to investigate in order to have an equilibrium in which A chooses $\hat{a}(\omega) = \overline{a}$ for some ω. Moreover, by Lemma 5.2, O_2 is only willing to investigate if O_2 strictly prefers to keep the Agent upon observing $(\hat{a}(\omega_H), \omega_H)$ and strictly prefers to fire the Agent upon observing $(\hat{a}(\omega_L), \omega_L)$ or vice versa. Finally, by Lemma 5.1, we have either

$$U_{O_2}(K^{O_1}|\overline{a}, \omega_H, R) = \overline{a}\omega_H W_{O_2},$$

$$U_{O_2}(F^{O_1}|\overline{a}, \omega_H, R) = (1 - \overline{a}\omega_H) W_{O_2},$$

$$U_{O_2}(K^{O_1}|\overline{a}, \omega_L, R) = \overline{a}\omega_L W_{O_2},$$

and

$$U_{O_2}(F^{O_1}|\overline{a}, \omega_H, R) = (1 - \overline{a}\omega_L) W_{O_2},$$

or

$$U_{O_2}(K^{O_1}|\overline{a}, \omega_H, R) = (1 - \overline{a}\omega_H) W_{O_2},$$

$$U_{O_2}(F^{O_1}|\overline{a}, \omega_H, R) = \overline{a}\omega_H W_{O_2},$$

$$U_{O_2}(K^{O_1}|\overline{a}, \omega_L, R) = (1 - \overline{a}\omega_L)W_{O_2},$$

and

$$U_{O_2}(F^{O_1}|\overline{a}, \omega_H, R) = \overline{a}\omega_L W_{O_2}.$$

Some simple algebra then establishes the result. ■

The logic behind this result is somewhat reminiscent of the one Overseer case. By Lemma **??** O_2 needs to investigate in equilibrium in order to sustain an equilibrium in which the Agent chooses $\hat{a}(\omega) = \overline{a}$. O_2, just as O_1 in the one Overseer case, is only willing to investigate, however, if there exist (a', ω') and (a'', ω''), with $\omega \neq \omega'$, both occurring with positive probability in equilibrium such that O_2 strictly prefers to keep O_1 upon observing (a', ω') and strictly prefers to fire O_1 upon observing (a'', ω''). When $\overline{a}\omega_H \leq 1/2$ or when $\underline{a}\omega_L \geq 1/2$, there is no retention rule the Principal could use which satisfies this requirement.

An interpretation of this result is that for the Overseers to be willing to investigate, and thus for an equilibrium in which the Agent chooses to exert high effort, at least in some state of the world, the outcome needs to be sufficiently responsive to the effort choice of the Agent so that it is sufficiently hard to guess the outcome for the last Overseer if he does not know the effort choice of the Agent.

5.3 N overseers

In the previous section, we have shown how the addition of a second Overseer increases the range of values (a, ω) for which the Agent can be incentivized to exert high effort in some state of the world and moreover makes it possible for certain parameter values (a, ω) to sustain an equilibrium in which the Agent chooses to exert high effort in both states of the world. Interestingly, the same logic applies when there are more than two Overseers. The Principal will choose a retention rule of the last Overseer O_n that only conditions on whether the decision of O_n to keep or fire O_{n-1} matches the policy outcome or not. For certain parameter values, this retention rule induces O_n (1) to investigate, (2) to fire O_{n-1} when O_n does not observe (a, ω), or when O_n observes $(\hat{a}(\omega), \omega)$ for some ω and (3) to keep O_{n-1} upon observing $(\hat{a}(\omega'), \omega')$ for some $\omega' \neq \omega$. But then, O_{n-1} will have incentives to investigate in order to increase the probability that O_n observes $(\hat{a}(\omega'), \omega')$ and consequently retains O_{n-1}. Moreover, as the Principal does not condition on whether O_{n-1}

keeps or fires O_{n-2}, O_n will retain or dismiss O_{n-1} independently of his retention decision of O_{n-2}. As a consequence, O_{n-1} will be indifferent between keeping and firing O_{n-2}. Hence, it will be a best-response for O_{n-1} (1) to fire O_{n-2} upon not observing (a, ω) and (2) to keep O_{n-2} upon observing $(\hat{a}(\omega), \omega)$ for all ω. This will create incentives for O_{n-2} to investigate in order to increase the probability that O_{n-1} observes $(\hat{a}(\omega), \omega)$. Moreover, O_{n-1} will have no incentive to condition his retention decision of O_{n-2} on whether O_{n-2} retained or dismissed O_{n-3}. As a consequence, O_{n-2} will be indifferent between keeping and firing O_{n-3}. Repeating the argument shows that it is now possible to incentivize O_1 to investigate and to leave O_1 indifferent between keeping and firing the Agent.

Proposition 5.5 *Suppose there is a hierarchy of n Overseers, $n \geq 2$. Then, in the cheapest equilibrium:*

(i) If $\overline{a}\omega_H > 1/2$, and $\overline{a}\omega_L < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, each Overseer O_i investigates upon observing the sequence of retention decisions $(R^{O_{i-2}}, \ldots, R^{O_1}, R)$ and the Agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω.

(ii) If $\overline{a}\omega_H > 1/2$, and $\underline{a}\omega_L < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, each Overseer O_i investigates upon observing the sequence of retention decisions $(R^{O_{i-2}}, \ldots, R^{O_1}, R)$ and the Agent chooses $\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ otherwise} \end{cases}$.

(iii) If $\overline{a}\omega_L > 1/2$, and $\underline{a}\omega_H < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, each Overseer O_i investigates upon observing the sequence of retention decisions $(R^{O_{i-2}}, \ldots, R^{O_1}, R)$ and the Agent chooses $\hat{a}(\omega) = \begin{cases} \underline{a} \text{ if } \omega = \omega_H \\ \overline{a} \text{ otherwise.} \end{cases}$

Proof. See Appendix. ■

Two remarks are in order with respect to Proposition 5.5 and previous results. First of all, an interesting aspect of the analysis so far is that the ability of the Principal of inducing the Agent to choose high effort requires the conditions to be neither very favorable nor very unfavorable to induce the Agent to exert high effort. Indeed, if $\overline{a}\omega_H \leq 1/2$, and thus $a\omega \leq 1/2$ for all (a, ω), or if

$\underline{a}\omega_L \geq 1/2$, and thus $a\omega \geq 1/2$, for all (a, ω), the Agent chooses to exert low effort in both states of the world, in equilibrium.

Second, in contrast to the comparison between the single Overseer case and the $n \geq 2$ Overseers case, a glance at Proposition 5.5 reveals a certain level of continuity between hierarchies of higher magnitude. To be sure, for any values of $\underline{a}, \overline{a}, \omega_L$, and ω_H if a certain level of effort of the Agent is in principle achievable in equilibrium with n Overseers, $n \geq 2$, then it is also achievable with $m \neq n$ Overseers, $m \geq 2$.

As in the two Overseers case, the inability for the Principal of inducing high effort by the Agent does not generally go away with longer hierarchies of Overseers. Indeed there exists a segment of the parameter space (a, ω) for which, no matter how high the wages are that the Principal pays to the Agent and the Overseers, no equilibrium exists in which the Agent chooses to exert high effort in any of the states of the world.

Lemma 5.4 *Suppose there is a hierarchy of n Overseers, $n \geq 2$. If $\overline{a}\omega_H \leq 1/2$, or if $\underline{a}\omega_L \geq 1/2$, then in any equilibrium the Agent chooses $\hat{a}(\omega) = \underline{a}$ for all ω.*

Proof. ∎

In proposition 5.5 we state that the Agent can be incentivized to exert a certain positive level of effort provided his wage W_A and the wages of the Overseers in the hierarchy W_{O_i} exceed certain respective thresholds $\overline{W}_A, \overline{W}_{O_i}$. In the following proposition, we give the expressions for these wage levels $\overline{W}_A, \overline{W}_{O_i}$.

Proposition 5.6 *Suppose there is a hierarchy of n Overseers, $n \geq 2$.*

(i) The lowest wage that induces the Agent to invest high effort in either or both states of the world is independent of the oversight hierarchy and is equal to $\overline{W}_A = k(\overline{a} - \underline{a})/e_1$.

(ii) The following wage levels $\overline{W}_{O_i}$, $i < n$, are the lowest wage levels for which, in the cheapest equilibrium, the Agent exerts high effort in either or both states of the world:

If $n = 2$, we have $\overline{W}_{O_1} = \frac{c_1}{e_1(e_2 - e_2^{\emptyset})\pi}$, whenever $\pi \leq \hat{\pi}(\hat{a}(\omega))$, and $\overline{W}_{O_1} = \frac{c_1}{e_1(e_2 - e_2^{\emptyset})(1-\pi)}$, whenever $\pi > \hat{\pi}(\hat{a}(\omega))$.[5]

[5]The level of $\hat{\pi}(\hat{a}(\omega))$ depends on the equilibrium level of effort of the Agent. If $\hat{a}(\omega_H) = \overline{a}$, we have $\hat{\pi}(\hat{a}(\omega)) := \frac{1 - 2\hat{a}(\omega_L)\omega_L}{2(\hat{a}(\omega_H)\omega_H - \hat{a}(\omega_L)\omega_L)}$. If $\hat{a}(\omega_H) = \underline{a}$, we have $\hat{\pi}(\hat{a}(\omega)) := \frac{2\overline{a}\omega_L - 1}{2(\overline{a}\omega_L - \underline{a}\omega_H)}$.

For all i*, such that* $2 \neq i \leq n-2$*, we have* $\overline{W}_{O_i} = \frac{c_i}{e_i^{\emptyset}(e_{i+1}-e_{i+1}^{\emptyset})}$.

If $n=3$*, then* $\overline{W}_{O_2} = \frac{c_2}{(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})(e_3-e_3^{\emptyset})\pi}$*, whenever* $\pi \leq \hat{\pi}(\hat{a}(\omega))$*, and* $\overline{W}_{O_2} = \frac{c_2}{(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})(e_3-e_3^{\emptyset})(1-\pi)}$*, whenever* $\pi > \hat{\pi}(\hat{a}(\omega))$.

If $n \geq 4$*, then* $\overline{W}_{O_2} = \frac{c_2}{(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})(e_3-e_3^{\emptyset})}$*. Moreover,* $\overline{W}_{O_{n-1}} = \frac{c_{n-1}}{e_{n-1}^{\emptyset}(e_n-e_n^{\emptyset})\pi}$*, whenever* $\pi \leq \hat{\pi}(\hat{a}(\omega))$*, and* $\overline{W}_{O_{n-1}} = \frac{c_{n-1}}{e_{n-1}^{\emptyset}(e_n-e_n^{\emptyset})(1-\pi)}$*, whenever* $\pi > \hat{\pi}(\hat{a}(\omega))$.

(iii) The lowest wage level $\overline{W}_{O_n}$ *at which* O_n *investigates in the cheapest equilibrium is given by the following:*

If $n=2$*, and* $\hat{a}(\omega_H) = \overline{a}$*, we have* $\overline{W}_{O_2} = \frac{c_2}{(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})\pi(2\hat{a}(\omega_H)\omega_H-1)}$ *whenever* $\pi \leq \hat{\pi}(\hat{a}(\omega))$ *and* $\overline{W}_{O_2} = \frac{c_2}{(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})(1-\pi)(1-2\hat{a}(\omega_L)\omega_L)}$ *whenever* $\pi > \hat{\pi}(\hat{a}(\omega))$.

If $n=2$*, and* $\hat{a}(\omega_H) = \underline{a}$*, we have* $\overline{W}_{O_2} = \frac{c_2}{(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})\pi(1-2\hat{a}(\omega_H)\omega_H)}$ *whenever* $\pi \leq \hat{\pi}(\hat{a}(\omega))$*, and* $\overline{W}_{O_2} = \frac{c_2}{(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})(1-\pi)(2\hat{a}(\omega_L)\omega_L-1)}$ *whenever* $\pi > \hat{\pi}(\hat{a}(\omega))$.

If $n \geq 3$*, and* $\hat{a}(\omega_H) = \overline{a}$*, then* $\overline{W}_{O_n} = \frac{c_n}{e_n^{\emptyset}\pi(2\hat{a}(\omega_H)\omega_H-1)}$ *whenever* $\pi \leq \hat{\pi}(\hat{a}(\omega))$*, and* $\overline{W}_{O_n} = \frac{c_n}{e_n^{\emptyset}\pi(1-2\hat{a}(\omega_L)\omega_L)}$ *whenever* $\pi > \hat{\pi}(\hat{a}(\omega))$.

If $n \geq 3$*, and* $\hat{a}(\omega_H) = \underline{a}$*, then* $\overline{W}_{O_n} = \frac{c_n}{e_n^{\emptyset}\pi(1-2\hat{a}(\omega_H)\omega_H)}$ *whenever* $\pi \leq \hat{\pi}(\hat{a}(\omega))$*, and* $\overline{W}_{O_n} = \frac{c_n}{e_n^{\emptyset}\pi(2\hat{a}(\omega_L)\omega_L-1)}$ *whenever* $\pi > \hat{\pi}(\hat{a}(\omega))$.

Proof. See Appendix. ■

As is apparent from proposition 5.6, the expressions for $\overline{W}_{O_2}$ differ from those of the other Overseers. This stems from an asymmetry in the retention rule used by the first Overseer with respect to the Agent. Indeed, in equilibrium, every Overseer $O_{1<i<n}$ fires the subordinate Overseer O_{i-1} upon not observing (a, ω) and retains O_{i-1} upon observing $(\hat{a}(\omega), \omega)$. As a result, the decision of O_i to keep or fire O_{i-1} is an informative signal to O_{i+1} about whether O_i observed (a, ω) or not. Hence, the probability that O_{i+1} learns (a, ω) depends on e_{i+1} and $e_{i+1}^{\emptyset}$, but not on e_i. In contrast, on the equilibrium path, O_1 retains the Agent upon observing $(\hat{a}(\omega), \omega)$ but also upon not observing (a, ω). It follows that O_2 is uncertain as to whether the first Overseer observed (a, ω). As a result the probability that O_2 observes (a, ω) and hence the incentives to investigate depend on e_1, as well as on e_2, and $e_2^{\emptyset}$.[6]

[6] If $\underline{a}\omega_H < 1/2$, then in the *cheapest equilibrium* O_1 fires A upon not observing (a, ω). For the same reasons discussed with respect to the equilibrium profile of proposition 9.1, this equilibrium is brittle and would cease to exist

The comparative statics with respect to the wage levels are generally as expected. In particular, the lowest wage $\overline{W}_{O_i}$ at which O_i investigates in equilibrium is increasing in the cost of investigating c_i, and decreasing in the probability $e_i^{\emptyset}$ that O_i observes (a, ω) upon investigating, although O_{i-1} did not. More interesting is the fact that for all Overseers O_i, with the exception of Overseer O_n, $\overline{W}_{O_i}$ also depends on the probability that the superior Overseer O_{i+1} observes. Indeed, in equilibrium O_i is only retained if O_{i+1} observes (a, ω). Remember also that the probability that O_{i+1} observes (a, ω) increases from $e_{i+1}^{\emptyset}$ to e_{i+1}, when O_i observes (a, ω) himself. The impetus for O_i to investigate is then to observe (a, ω) in order to increase the probability that O_{i+1} observes (a, ω) as well. It follows that $\overline{W}_{O_i}$ is decreasing in $(e_{i+1} - e_{i+1}^{\emptyset})$, as well as in e_{i+1}, but increasing in $e_{i+1}^{\emptyset}$. Indeed, when $(e_{i+1} - e_{i+1}^{\emptyset})$ increases, so does the difference between the probability that O_i is retained in expectation when investigating compared to when he does not investigate, which increases the incentives for O_i to investigate. Similarly, as $e_{i+1}^{\emptyset}$ increases, O_i is more likely to be retained when he does not investigate which reduces the incentives for O_i to investigate. It follows that there is a potential upside and a potential downside to the Principal of $e_i^{\emptyset}$ increasing. Indeed, when $e_i^{\emptyset}$ increases, the incentives for O_i to investigate increase, but at the same time the incentives for O_{i-1} to investigate decrease. We return to this tension at the end of section 6.

Finally, $\overline{W}_{O_n}$ is decreasing in the difference of the expected value of keeping O_{n-1} versus firing O_{n-1} upon observing the state of the world and the level of effort for which O_n would prefer to keep O_{n-1}. Indeed, O_n prefers to fire O_{n-1} upon not observing (a, ω). Hence, O_n investigates to learn whether he should keep O_{n-1} instead. The more it would be a mistake to fire O_{n-1} upon observing the corresponding (a, ω), the more O_n has incentives to investigate.

6 Which Hierarchy is Best?

Based on the characterization of the cheapest equilibria presented in the previous sections, we are now able to give answers to the question about the optimal hierarchy. We proceed in a series of steps. First, we ask under what conditions it is valuable to the Principal to add a marginal Overseer to a given hierarchy. We then proceed to study under what conditions the Principal would prefer

when the set of the Agent's effort levels is expanded. The qualitative nature of the results is the same across these two equilibria. To avoid additional mathematical clutter we thus focus on the more robust case.

to hire a hierarchy of Overseers rather than to monitor the Agent directly. Finally, we study how the effectiveness of the hierarchy depends on how likely the Overseers are to become informed.

6.1 The Value of Marginal Overseers

We now study how the addition of a marginal Overseer affects the level of effort exerted by the Agent. We start by noticing that proposition 5.6 entails interesting dynamics, as captured in the following corollary, with respect to the lowest wage that the Principal needs to pay certain Overseers to induce them to investigate in equilibrium.

Corollary 6.1 *For any number $n, n+1$ hierarchies of Overseers, $n \geq 2$, holding fixed the level of effort exerted by the Agent in the cheapest equilibrium, and holding fixed $\overline{a}, \underline{a}, \omega_L, \omega_H$, and for all $i \in \{1, \ldots, n\}$, c_i, e_i, and $e_i^{\emptyset}$ we have the following*

(i) the lowest Wage that the Principal needs to pay to the $n-1$th Overseer in order for O_{n-1} to investigate in the cheapest equilibrium is lower in the $n+1$ Overseers case than in the n Overseers case;

(ii) the lowest Wage that the Principal needs to pay to the nth Overseer in order for O_n to investigate in the cheapest equilibrium is lower (higher) in the $n+1$ Overseers case than in the n Overseers case, if the difference in probabilities $(e_{n+1} - e_{n+1}^{\emptyset})$ that O_{n+1} observes (a, ω) upon investigating when O_n observed compared to when O_n did not observe is sufficiently high (low);

(iii) the lowest Wage that the Principal needs to pay to the other Overseers in order for those Overseers to investigate in the cheapest equilibrium is not altered by adding an Overseer.

Proof. Follows directly from Proposition 5.6. ■

The intuition for these results is as follows. Consider the case where in equilibrium the Agent chooses high effort in both states of the world. Remember that by Lemma 5.2, for O_n to investigate it must be the case that O_n strictly prefers to fire O_{n-1} upon observing $(\overline{a}, \omega_H)$ and strictly prefers to fire O_{n-1} upon observing $(\overline{a}, \omega_L)$ (or vice versa). As a consequence, O_{n-1} investigates to increase the probability that O_n will observe $(\overline{a}, \omega_H)$. Suppose we add an additional Overseer O_{n+1} at the top of the hierarchy, then O_n is indifferent between keeping and firing O_{n-1} both upon observing $(\overline{a}, \omega_H)$, and upon observing $(\overline{a}, \omega_L)$. In the cheapest equilibrium, O_n now keeps O_{n-1} upon observing $(\overline{a}, \omega)$ for all ω. This, in turn, increases the probability that O_{n-1} is retained upon investigating, and thus decreases the minimum wage level at which O_{n-1} chooses to investigate in equilibrium.

An important implication of these wage shifting dynamics is that there are conditions under which the Principal can induce the Agent to exert a given level of effort more cheaply when hiring additional overseers. Indeed, we have the following result:

Corollary 6.2 *For any number $n \geq 2$ of Overseers, holding fixed $\overline{a}, \underline{a}, \omega_L, \omega_H$, and for all $i \in \{1, \ldots, n\}$, c_i, e_i, and $e_i^{\emptyset}$, if $\overline{a}\omega_H > 1/2$, and $\underline{a}\omega_L < 1/2$, then there exists c_{n+1}, e_{n+1}, and $e_{n+1}^{\emptyset}$ such that the lowest sum of wages that the Principal needs to pay in order to induce the Agent to exert a given level of effort is lower with $n+1$ Overseers than with n Overseers.*

Proof. Follows from corollary 6.1, and proposition 5.6. ■

The logic is straightforward. Corollary 6.1 states that by adding O_{n+1} to the hierarchy, the wages that need to be paid to O_{n-1}, and O_n go down, provided $(e_{n+1} - e_{n+1}^{\emptyset})$ is sufficiently high. If the costs of investigating for O_{n+1} are sufficiently low such that the wage that the Principal needs to pay to O_{n+1} is not too high, then the benefit of adding O_{n+1}, namely lower wages for O_{n-1}, and O_n, will outweigh the additional cost of hiring O_{n+1}.

Holding fixed the vector of wages $(W_A, W_{O_1}, \ldots, W_{O_n})$ we now ask what effect the addition of a marginal Overseer has on the level of effort exerted by the Agent. We first show that there always exist conditions under which adding an Overseer increases the level of effort of the Agent.

Proposition 6.1 *For any number n of Overseers, there is a vector of Wages $(W_A, W_{O_1}, \ldots, W_{O_n})$, and a range of values for $\underline{a}, \overline{a}, \omega_L, \omega_H, e_{n+1}$, and $e_{n+1}^{\emptyset}$ such that adding an Overseer O_{n+1}, and paying O_{n+1} a sufficiently high wage, increases the level of effort exerted by the Agent.*

Proof. Follows from lemmata **??**, **??**, corollary 6.1, and propositions 5.5, 5.6. ■

In corollary 6.1 we established that adding an Overseer to a hierarchy of n Overseers decreases the wage that the Principal needs to pay to the $n-1$th Overseer to induce him to investigate, and may decrease the wage of the nth Overseer when the difference in probabilities $(e_{n+1} - e_{n+1}^{\emptyset})$ is sufficiently high. The logic behind the previous result should then be clear. Consider a wage profile such that the wage paid to $W_{O_{n-1}}$ is too low for O_{n-1} to investigate in equilibrium when there are n Overseers, yet sufficiently high that O_{n-1} would investigate in equilibrium when there are $n+1$ Overseers. Suppose further that the wage paid to the other Overseers is sufficiently high for them to investigate with n Overseers. In this case, when there are only n Overseers, O_{n-1} never investigates. By Lemma **??** this trickles down in the hierarchy and leads O_1 not to investigate either. But then the Agent chooses to exert low effort in both states of the world. If an Overseer is added at the top of the hierarchy and two requirements are satisfied, namely that the wage paid to O_{n+1} is sufficiently high for him to investigate and the difference in probabilities $(e_{n+1} - e_{n+1}^{\emptyset})$ is sufficiently high that the addition of O_{n+1} does not reduce the willingness of O_n to investigate, then the addition of O_{n+1} leads every Overseer to investigate and thus incentivizes the Agent to exert high effort in some state of the world, whenever $\overline{a}\omega_H > 1/2$, and $\underline{a}\omega_L < 1/2$.

More is true, however. Indeed, suppose $\overline{a}\omega_H > 1/2$ and $\overline{a}\omega_L < 1/2$. Then, there exist wage vectors $(W_A, W_{O_1}, \ldots, W_{O_n})$ such that, in equilibrium, the Agent chooses high effort in both states of the world but there also exist wage vectors such that, in equilibrium, the Agent exerts high effort in the high state and low effort in the low state. Suppose π is sufficiently high and $\frac{c_n}{e_n^{\emptyset}\pi(1-2\overline{a}\omega_L)} > W_{O_n} > \frac{c_n}{e_n^{\emptyset}\pi(1-2\underline{a}\omega_L)}$. Then, given the level of W_{O_n} there is no equilibrium in which the Agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω and the last Overseer O_n investigates, although there is an equilibrium in which the Agent exerts high effort in the high state and low effort in the low state, provided the wages are sufficiently high for the Agent and for the Overseers $O_1, \ldots, O_{n-1}$. As shown in corollary 6.1, adding an additional Overseer O_{n+1} may decrease the minimum wage at which O_n can be incentivized to investigate in equilibrium. It follows that the addition of Overseer O_{n+1} may improve the equilibrium level of effort of the Agent from $\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ if } \omega = \omega_L \end{cases}$ to $\hat{a}(\omega) = \overline{a}$ for all ω. Similar reasoning shows that the addition of a marginal Overseer may also reduce the level of effort exerted by the Agent from $\hat{a}(\omega) = \overline{a}$ for all ω to $\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ if } \omega = \omega_L \end{cases}$. This happens, for example, when $\pi > \hat{\pi}$, $W_{O_n} \geq \frac{c_n}{e_n^{\emptyset}\pi(1-2\overline{a}\omega_L)}$, but $\frac{c_n}{e_{n+1}^{\emptyset}\pi(1-2\overline{a}\omega_L)} > W_{O_{n+1}} > \frac{c_n}{e_{n+1}^{\emptyset}\pi(1-2\underline{a}\omega_L)}$. We thus have the following result:

Proposition 6.2 *For any number n of Overseers, there is a vector of Wages $(W_A, W_{O_1}, \ldots, W_{O_n})$, and a range of values for $\underline{a}, \overline{a}, \omega_L$, and ω_H such that $\hat{a}(\omega) = \overline{a}$ for all ω. Holding fixed $(W_A, W_{O_1}, \ldots, W_{O_n})$,* *there exist* e_{n+1}*, and* $e^{\emptyset}_{n+1}$ *and cutpoints* $W^L_{O_{n+1}}, W^H_{O_{n+1}}$ *such that if* O_{n+1} *is added to the hierarchy the following is true. If* $W_{O_{n+1}} > W^H_{O_{n+1}}$*, then* $\hat{a}(\omega) = \overline{a}$ *for all* ω*. If* $W^H_{O_{n+1}} > W_{O_{n+1}} \geq W^L_{O_{n+1}}$*, then* $\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ if } \omega = \omega_L \end{cases}$*. And if* $W^L_{O_{n+1}} > W_{O_{n+1}}$*, then* $\hat{a}(\omega) = \underline{a}$ *for all* ω*.*[7]

6.2 Optimal Oversight Hierarchies

Our next set of results studies more generally when the Principal is better off hiring a hierarchy as opposed to monitor the Agent directly. We first show that, holding fixed the wage of the Agent, there are conditions under which hiring a hierarchy increases the level of effort exerted by the Agent.

Proposition 6.3 *Suppose there are* n *Overseers,* $n \geq 1$*.*

(i) If $W_A < k(\overline{a} - \underline{a})/e_1$*, then* A *chooses* $\hat{a}(\omega) = \underline{a}$ *for all* ω *in equilibrium regardless of the length of the hierarchy of Overseers.*

(ii) If $W_A \geq \max\{k(\overline{a} - \underline{a})/e_1, k/\omega_L\}$*, then, the addition of Overseers can never increase the level of effort exerted by the Agent and reduces it when either* $\overline{a}\omega_H \leq 1/2$*, or* $\overline{a}\omega_L \geq 1/2$*.*

(iii) If $k/\omega_L > W_A \geq \max\{k(\overline{a} - \underline{a})/e_1, k/\omega_H\}$*, then:*

(a) if $\overline{a}\omega_H > 1/2$*, and* $\overline{a}\omega_L < 1/2$*, there exists a wage vector* $(W_{O_1}, \ldots, W_{O_n})$*,* $n \geq 2$*, that improves the probability of success by increasing the level of effort from* $\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ if } \omega = \omega_L \end{cases}$ *to* $\hat{a}(\omega) = \overline{a}$ *for all* ω*.*

(b) if $\overline{a}\omega_L, \underline{a}\omega_H > 1/2$*, and* $\underline{a}\omega_L < 1/2$*, there is no wage vector* $(W_{O_1}, \ldots, W_{O_n})$ *that improves the probability of success.*

(c) if $\overline{a}\omega_L > 1/2$*, and* $\underline{a}\omega_H < 1/2$*, there exists a wage vector* $(W_{O_1}, \ldots, W_{O_n})$*,* $n \geq 2$*, that may improve the probability of success by changing the Agent's effort level from* $\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ if } \omega = \omega_L \end{cases}$ *to* $\hat{a}(\omega) = \begin{cases} \underline{a} \text{ if } \omega = \omega_H \\ \overline{a} \text{ if } \omega = \omega_L \end{cases}$*.*

[7] Adding an Overseer may alter the equilibrium level of effort of the Agent in yet other ways. The details are yet to be filled in.

(iv) *If* $k/\omega_H > W_A \geq k(\overline{a} - \underline{a})/e_1$, *then there is no vector of wages that will decrease the level of effort exerted by the Agent and there is a wage vector* $(W_{O_1}, \ldots, W_{O_n})$, *that increase the level of effort exerted by the Agent and will never decrease it.*

Proof. Follows from combining Lemma 4.1 and Proposition 5.5. ■

An implication of Proposition 6.3 is the existence of conditions under which, given a fixed wage level W_A for the Agent, the hiring of a hierarchy of Overseers may increase the level of effort exerted by the Agent. One necessary condition in this context is that the probability that Overseer O_1 observes (a, ω) upon investigating needs to be sufficiently high. Indeed, a hierarchy of Overseers can only increase the level of effort of the Agent if there exists some ω such that $k/\omega > W_A \geq k(\overline{a} - \underline{a})/e_1$. Hence, the hierarchy of Overseers can only increase the effort level of the Agent if $e_1 > \omega(\overline{a} - \underline{a})$ for some ω. Provided this necessary condition is satisfied, there always exist wage levels W_A for the Agent and parameter values for $\underline{a}, \overline{a}, \omega_L$, and ω_H, such that the Principal can increase the level of effort by the Agent by hiring a hierarchy of n Overseers, $n \geq 2$, and paying them sufficiently high wages.

Even when the Principal cannot induce the Agent to exert high effort in both states of the world by hiring a hierarchy, it may still be the case that hiring a hierarchy improves the probability of a positive policy outcome relative to the no Overseers case. Indeed, if there is a high probability that the state of the world is low, the probability of policy success will be higher if the Agent chooses high effort in the low state and low effort in the high state than if the Agent chooses high effort in the high state and low effort in the low state. In the absence of a hierarchy, there is no equilibrium in which the Agent chooses low effort in the high state and high effort in the low state. As shown in Proposition 5.5 there are conditions under which in equilibrium the Agent chooses $\hat{a}(\omega) = \begin{cases} \underline{a} \text{ if } \omega = \omega_H \\ \overline{a} \text{ if } \omega = \omega_L \end{cases}$. As a result the hiring of an oversight hierarchy may increase the probability of policy success by inducing the Agent to choose low effort in the high state and high effort in the low state rather than high effort in the high state and low effort in the low state.

More importantly, in terms of the question of whether the Principal should hire a hierarchy or not, it is also the case that the Principal may induce a certain level of effort by the Agent more cheaply by hiring a hierarchy:

Corollary 6.3 *For any number of $n \geq 2$ of Overseers, holding fixed $\overline{a}, \underline{a}, \omega_L$, and ω_H, if $\overline{a}\omega_H > 1/2$, and $\underline{a}\omega_L < 1/2$, then there exists c_i, e_i, and $e_i^{\emptyset}$ with $i = 1, \ldots, n$, such that the lowest sum of wages that the Principal needs to pay to the Agent, and the Overseers to induce a certain level of effort by the Agent is lower than the lowest wage the Principal would have to pay to the Agent alone.*

Proof. Follows from corollary 6.2, proposition 5.5, and lemma 4.1. ■

We explain the nature of the result in the two Overseers case. By corollary 6.2, the logic then extends to any number n of Overseers, $n \geq 2$. Suppose $\overline{a}\omega_H > 1/2$, and $\underline{a}\omega_L < 1/2$. With two Overseers, the lowest sum of wages the Principal has to pay to induce the Agent to choose $\hat{a}(\omega) = \overline{a}$, is $\overline{W}_A + \overline{W}_{O_1} + \overline{W}_{O_2} = \frac{k(\overline{a}-\underline{a})}{e_1} + \frac{c_1}{e_1(e_2-e_2^{\emptyset})\pi} + \frac{c_2}{(e_1(e_2-e_2^{\emptyset})+e_2^{\emptyset})\pi(2\overline{a}\omega_H-1)}$, whereas without Overseers, the Principal needs to pay the Agent at least k/ω_L. It is then obvious that if the probability e_1 that the first Overseer O_1 observes (a, ω) upon investigating is sufficiently high and the costs of investigating c_1, and c_2 are sufficiently low, the Principal can induce the Agent to choose high effort in both states of the world at a lower cost with two Overseers than with none.

6.3 When is Information Beneficial?

We now study how the effectiveness of the hierarchy depends on the probability that Overseers become informed about (a, ω) upon investigating. In section **??** we showed that $\overline{W}_A$ is decreasing in e_1, and $\overline{W}_{O_i}$ is decreasing in e_{i+1} for all i. If we consider a fixed wage vector $(W_A, W_{O_1}, \ldots, W_{O_n})$, this implies that the level of effort of the Agent is weakly increasing in e_i. At first sight, this seems rather intuitive and corresponds to the insights obtained via the baseline models of section 4 which showed that for a hierarchy to be beneficial to the Principal the Overseers need to be better informed than the Principal. Remember, however, that e_i is the probability that Overseer O_i observes (a, ω) upon investigating, conditional on O_{i-1} having observed (a, ω). Perhaps surprisingly, results are very different when we consider $e_i^{\emptyset}$, i.e. the probability that O_i observes (a, ω) although O_{i-1} did not. Indeed, in section **??** we identified a tension in how the minimum wages $\overline{W}_{O_i}$ at which O_i can be incentivized to investigate in equilibrium decrease in $e_i^{\emptyset}$, but increase in $e_{i+1}^{\emptyset}$. This begs the question whether hiring a hierarchy to induce a certain level of effort by the Agent is cheaper when the $e_i^{\emptyset}$'s are lower or when they are higher. As it turns out, hiring a hierarchy is cheaper for the Principal when $e_i^{\emptyset} = 0$ for all i. The structure of the cheapest equilibrium is almost identical when

$e_i^\emptyset = 0$ for all i, then when $e_i^\emptyset > 0$. In both cases, O_i provides incentives for O_{i-1} to investigate by firing O_{i-1} whenever O_i does not observe (a, ω) and keeping O_{i-1} whenever O_i observes $(\hat{a}(\omega), \omega)$ at least for some ω. The difference is that each O_i investigates upon observing any sequence of retention decisions $(R^{O_i-2}, \ldots, R)$ when $e_i^\emptyset > 0$. When $e_i^\emptyset = 0$, O_i has no incentive to investigate upon observing that some subordinate Overseer $O_{j<i}$ fired O_{j-1} as this is an indication that (a, ω) was not observed and hence, as $e_i^\emptyset = 0$, that O_i will not be able to observe (a, ω) either. As a result, O_i only investigates in equilibrium upon observing that every Overseer $O_{j<i}$ kept his subordinate O_{j-1}, i.e. upon learning that Overseer O_{i-1} observed (a, ω). This has two consequences: (1) The minimum wage level $\overline{W}_{O_i}$ at which O_i chooses to investigate does not depend on $e_i^\emptyset$ anymore, but on e_i. As an example, we have $\overline{W}_{O_n} = \frac{c_n}{e_n^\emptyset \pi(2\overline{a}\omega - 1)}$ when $e_n^\emptyset > 0$, whereas $\overline{W}_{O_n} = \frac{c_n}{e_n \pi(2\overline{a}\omega - 1)}$ when $e_n^\emptyset = 0$.[8] As $e_i > e_i^\emptyset$, O_i can be incentivized to investigate in equilibrium upon observing that every Overseer kept his subordinate at a wage that is lower when $e_i^\emptyset = 0$. (2) The incentives that O_i provides to O_{i-1} to investigate are strongest, as $\overline{W}_{O_i}$ is decreasing in $(e_{i+1} - e_{i+1}^\emptyset)$. Indeed, if O_{i-1} does not investigate he will not learn (a, ω). This, in turn, implies that O_i will not learn (a, ω) either and will thus fire O_{i-1}. It follows that O_{i-1} receives a utility of 0 when he chooses not to investigate. Hence, in order to have any chance of being retained O_i must investigate. For example, for any Overseer O_j such that $3 \leq j < n - 1$, we have $\overline{W}_{O_j} = \frac{c_j}{e_j e_{j+1}}$ when $e_i^\emptyset = 0$ for all i, whereas $\overline{W}_{O_j} = \frac{c_j}{e_j^\emptyset (e_{j+1} - e_{j+1}^\emptyset)}$ when $e_i^\emptyset > 0$. We thus have the following:

Proposition 6.4 *(i) Consider any wage vector $(W_A, W_{O_1}, \ldots, W_{O_n})$ such that, if $e_i^\emptyset = 0$ for all i, the Agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω in equilibrium. For any such wage vector, and for any $i \geq 2$, there exist $\underline{e}_i^\emptyset, \overline{e}_i^\emptyset > 0$ such that, if $0 < e_i^\emptyset < \underline{e}_i^\emptyset$ or if $\overline{e}_i^\emptyset < e_i^\emptyset < e_i$, the Agent chooses $\hat{a}(\omega) = \underline{a}$ for all ω in equilibrium.*

(ii) Consider any wage vector $(W_A, W_{O_1}, \ldots, W_{O_n})$ such that, if $e_i^\emptyset = 0$ for all i, the Agent does not choose $\hat{a}(\omega) = \overline{a}$ for all ω in equilibrium. Then there does not exist $(e_1, e_2^\emptyset, \ldots, e_n^\emptyset) \geq \mathbf{0}$ such that the Agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω in equilibrium.

Proof. To be added. ■

The first part of Proposition 6.4 states that compared to an environment in which the Overseers can only observe the effort level of the Agent when subordinate Overseers observed the effort of

[8] A full derivation of $\overline{W}_{O_i}$ for all i when $e_i^\emptyset = 0$ can be found in proposition 9.2 in the Appendix.

the Agent as well, the effectiveness of the hierarchy at providing incentives for the Agent to exert effort may be diminished when Overseers can observe the effort level of the Agent even when subordinate Overseers did not. As discussed above this can essentially happen for two reasons. First, when $e_i^{\emptyset} > 0$ is too small the wage W_{O_i} may not be high enough anymore to induce O_i to investigate. Second, when $e_i^{\emptyset}$ is too high, the difference $(e_i - e_i^{\emptyset})$ may be too small and the wage $W_{O_{i-1}}$ may be too low for O_{i-1} to investigate. In both cases, Lemmata **??** and **??** then imply that the Agent chooses $\hat{a}(\omega) = \underline{a}$ in equilibrium. Similar reasoning then establishes the second part of the proposition. What our analysis then shows is that to provide powerful incentives within the hierarchy itself the conditions under which information is acquired matter a great deal. Being better able to observe the effort level of the Agent, although subordinate Overseers failed to monitor the Agent's effort properly, is creating disincentives for subordinate Overseers to investigate which may hurt the Principal.

7 Comparison to Auditors

So far we have shown that the Principal may be able to provide the Agent with better incentives to exert high effort by delegating the power to retain or fire the Agent to two or more Overseers organized hierarchically. Another possibility for the Principal would be to hire auditor(s) who provide the Principal with reports as to whether the Agent should be retained or not but where the decision to keep or fire the Agent remains with the Principal. We now study the advantages and disadvantages of hiring auditors instead of Overseers under the conditions of costly information acquisition.

We first show that there is no reason for the Principal to hire a single auditor.

Proposition 7.1 *Suppose there is a single auditor. Then, the agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω in equilibrium if, and only if, $W_A \geq k/\omega_L$.*

Proof. To be added. ■

The argument is reminiscent of the single Overseer case. Indeed, in Lemma 4.1 we showed that, in the absence of Overseers, the Principal can incentivize the Agent to choose high effort in both states of the world by retaining the Agent when there is policy success and firing him when there is policy failure, provided that $W_A \geq k/\omega_L$. With a single auditor, the Principal retains the power to keep or fire the Agent. Hence, in order to incentivize the Agent to choose $\hat{a}(\omega) = \overline{a}$ for all ω, although $W_A < k/\omega_L$, it must be the case that the auditor provides the Principal with a report that informs the Principal that the Agent exerted high effort and thus leads the Principal to keep the Agent even when the policy outcome is failure. For the report sent out by the auditor to be informative to the Principal about the effort exerted by the Agent, it must be the case that the auditor sends out one report when the Agent exerted high effort and another report when the Agent exerted low effort. Now consider an equilibrium in which the Agent chooses high effort in both states of the world. By Lemma 5.2, the auditor only investigates if he strictly prefers to advise the Principal to retain the Agent upon observing $(\overline{a}, \omega_H)$ and strictly prefers to advise the Principal to fire upon observing $(\overline{a}, \omega_L)$. By Lemma 5.1 this can only be the case if $\overline{a}\omega_H > 1/2 > \overline{a}\omega_L > \underline{a}\omega_L$. This, in turn, implies that the single Auditor sends the same report to the Principal when the Auditor observes $(\overline{a}, \omega_L)$ than when he observes $(\underline{a}, \omega_L)$. But then the Auditor is essentially unable to provide the Principal with a sensible report in the low state. The Principal is then better off disregarding the auditors report and retaining the Agent if, and only if, the policy outcome is success.

Proposition 7.2 *Suppose $k(\overline{a} - \underline{a}) \leq W_A < k/\omega_H$, $e_1 = e_2 = 1$, and $\overline{a}\omega_L < 1/2$ then there do not exist W_{S_1} and W_{S_2} such that in equilibrium the Agent chooses $\hat{a}(\omega) = \overline{a}$.*[9]

Let $e_1 = e_2 = 1$ and $\overline{a}\omega_L < 1/2$ then in the cheapest equilibrium with two auditors we have $\overline{W}_A = k/\omega_H$, $\overline{W}_{S_i} = \begin{cases} c_i/\pi \text{ if } \pi \leq 1/2 \\ c_i/(1-\pi) \text{ if } \pi > 1/2 \end{cases}$. Hence, if $\overline{a}\omega_H$ and $\underline{a}$ are sufficiently high, two overseers is cheaper than two auditors.

8 Conclusion/ Further Research

TBA

[9]This is likely to be also true for $e_1, e_2 \neq 1$. I need to check the robustness of the argument.

References

Alt, James, Ethan Bueno de Mesquita and Shanna Rose. 2011. "Disentangling accountability and competence in elections: evidence from US term limits." *The Journal of Politics* 73(01):171–186.

Ashworth, Scott, Ethan Bueno de Mesquita and Amanda Friedenberg. 2013. "Accountability Traps.".

Barro, Robert J. 1973. "The control of politicians: an economic model." *Public choice* 14(1):19–42.

Berry, Christopher R and Jacob E Gersen. 2008. "Unbundled Executive, The." *U. Chi. L. Rev.* 75:1385.

Bueno de Mesquita, Ethan and Dimitri Landa. 2013. "Moral Hazard with Sequential Policy-Making.".

Calvo, Guillermo A and Stanislaw Wellisz. 1978. "Supervision, loss of control, and the optimum size of the firm." *Journal of Political Economy* 86(5):943–52.

Faure-Grimaud, Antoine, Jean-Jacques Laffont and David Martimort. 2003. "Collusion, delegation and supervision with soft information." *The Review of Economic Studies* 70(2):253–279.

Ferejohn, John. 1986. "Incumbent Performance and Electoral Control." *Public Choice* 50(1/3):5–25.

Gordon, Sanford C and Catherine Hafer. 2007. "Corporate influence and the regulatory mandate." *Journal of Politics* 69(2):300–319.

Laffont, Jean-Jacques and Jean Tirole. 1991. "The politics of government decision-making: A theory of regulatory capture." *The Quarterly Journal of Economics* 106(4):1089–1127.

Le Bihan, Patrick. 2014. "Popular Referendum and Electoral Accountability.".

Qian, Yingyi. 1994. "Incentives and loss of control in an optimal hierarchy." *The Review of Economic Studies* 61(3):527–544.

Strausz, Roland. 1997. "Delegation of monitoring in a principal-agent relationship." *The Review of Economic Studies* 64(3):337–357.

Strøm, Kaare. 2000. "Delegation and accountability in parliamentary democracies." *European journal of political research* 37(3):261–290.

Strøm, Kaare. 2003. "Parliamentary democracy and delegation." *Delegation and accountability in parliamentary democracies* pp. 55–106.

Strøm, Kaare, Wolfgang C Müller and Daniel Markham Smith. 2010. "Parliamentary control of coalition governments." *Annual Review of Political Science* 13:517–535.

Strøm, Kaare, Wolfgang C Müller and Torbjörn Bergman. 2006. *Delegation and accountability in parliamentary democracies.* Oxford University Press.

Tirole, Jean. 1986. "Hierarchies and bureaucracies: On the role of collusion in organizations." *Journal of Law, Economics, & Organization* 2(2):181–214.

Williamson, Oliver E. 1967. "Hierarchical control and optimum firm size." *The Journal of Political Economy* pp. 123–138.

9 Appendix

Incomplete

Proposition 9.1 *Suppose there is a single Overseer who has to pay $c_1 > 0$ to observe a, and ω. If $\overline{a}\omega_H > 1/2$, and $a\omega < 1/2$ for all (a,ω) such that $(a,\omega) \neq (\overline{a},\omega_H)$, then there exist $\overline{W}_A, \overline{W}_{O_1}$ such that if $W_A \geq \overline{W}_A$, and $W_{O_1} \geq \overline{W}_{O_1}$ then, in equilibrium, the Overseer investigates and the Agent chooses*

$$\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ otherwise.} \end{cases}$$

In this equilibrium, the Principal chooses the following retention rule: $r_P(K^{O_1}|s,K) = r_P(K^{O_1}|f,F) = 1$, and $r_P(K^{O_1}|s,F) = r_P(K^{O_1}|f,K) = 0$.

Moreover, we have $\overline{W}_A = k(\overline{a} - \underline{a})/e_1$, and $$\overline{W}_{O_1} = \begin{cases} c_1/(e_1\pi(2\overline{a}\omega_H - 1)) \text{ if } \pi \leq \frac{1-2\underline{a}\omega_L}{2(\overline{a}\omega_H - \underline{a}\omega_L)} \\ c_1/(e_1(1-\pi)(1-2\underline{a}\omega_L)) \text{ otherwise.} \end{cases}$$

Proof. Consider the following strategy profile:

(i) A chooses $\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \\ \underline{a} \text{ otherwise.} \end{cases}$

(ii) O_1 investigates.

(iii)

$$\hat{r}_{O_1}(K|a,\omega_H) = \begin{cases} 1 \text{ if } a = \overline{a} \\ \\ 0 \text{ otherwise} \end{cases},$$

$$\hat{r}_{O_1}(K|a,\omega_L) = 0, \text{ for all } a,$$

$$\hat{r}_{O_1}(K|\emptyset) = 0 \text{ or } 1.$$

(iv) $r_P(K^{O_1}|s,K) = r_P(K^{O_1}|f,F) = 1$, and $r_P(K^{O_1}|s,F) = r_P(K^{O_1}|f,K) = 0$.

(i) A has no incentive to deviate. Indeed, we have

$$\begin{aligned} U_A(\overline{a}, I_1, r_{O_1}|\omega_H) &= k(1-\overline{a}) + e_1 W_A + (1-e_1) r_{O_1}(K|\emptyset) W_A \\ &\geq k(1-\underline{a}) + (1-e_1) r_{O_1}(K|\emptyset) W_A = U_A(\underline{a}, I_1, r_{O_1}|\omega_H), \end{aligned}$$

if, and only if, $W_A \geq \overline{W}_A := k(\overline{a}-\underline{a})/e_1$. Moreover, as $\overline{a} > \underline{a}$,

$$\begin{aligned} U_A(\overline{a}, I_1, r_{O_1}|\omega_L) &= k(1-\overline{a}) + (1-e_1) r_{O_1}(K|\emptyset) W_A \\ &< k(1-\underline{a}) + (1-e_1) r_{O_1}(K|\emptyset) W_A = U_A(\underline{a}, I_1, r_{O_1}|\omega_L). \end{aligned}$$

Hence, A has no incentive to deviate if $W_A \geq \overline{W}_A$.

(ii) O_1 investigates. Consider first the case in which $\hat{r}_{O_1}(K|\emptyset) = 0$. Then, if O_1 does not investigate, he chooses to fire the Agent. Hence, $U_{O_1}(I_1 = 0) = \pi(1-\overline{a}\omega_H)W_{O_1} + (1-\pi)(1-\underline{a}\omega_L)W_{O_1}$. If O_1 chooses to investigate, he will keep the Agent upon observing $(\overline{a}, \omega_H)$ and fire him otherwise. Hence,

$$U_{O_1}(I_1 = 1) = e_1 \left[\pi \overline{a}\omega_H + (1-\pi)\underline{a}\omega_L\right] W_{O_1} + (1-e_1)\left[\pi(1-\overline{a}\omega_H)W_{O_1} + (1-\pi)(1-\underline{a}\omega_L)W_{O_1}\right] - c_1.$$

Rearranging, we find that $U_{O_1}(I_1 = 1) \geq U_{O_1}(I_1 = 0)$ if, and only if, $W_{O_1} \geq c_1/e_1\pi(2\overline{a}\omega_H - 1)$. Similar derivations show that $U_{O_1}(I_1 = 1) \geq U_{O_1}(I_1 = 0)$ if, and only if, $W_{O_1} \geq c_1/e_1(1-\pi)(1-2\underline{a}\omega_L)$, when $r_{O_1}(K|\emptyset) = 1$.

(iii) O_1 has no incentive to deviate from r_{O_1}. We have

$$U_{O_1}(K|\overline{a},\omega_H) = \overline{a}\omega_H W_{O_1} > (1-\overline{a}\omega_H)W_{O_1} = U_{O_1}(F|\overline{a},\omega_H) \text{ if, and only if } \overline{a}\omega_H > 1/2,$$

$$U_{O_1}(K|\underline{a},\omega_H) = \underline{a}\omega_H W_{O_1} \leq (1-\underline{a}\omega_H)W_{O_1} = U_{O_1}(F|\underline{a},\omega_H) \text{ if, and only if } \underline{a}\omega_H \leq 1/2,$$

$$U_{O_1}(K|\overline{a},\omega_L) = \overline{a}\omega_L W_{O_1} \leq (1-\overline{a}\omega_L)W_{O_1} = U_{O_1}(F|\overline{a},\omega_L) \text{ if, and only if } \overline{a}\omega_L \leq 1/2,$$

and

$$U_{O_1}(K|\underline{a},\omega_L) = \underline{a}\omega_L W_{O_1} < (1-\underline{a}\omega_L)W_{O_1} = U_{O_1}(F|\underline{a},\omega_L) \text{ if, and only if } \underline{a}\omega_L < 1/2.$$

(iv) As this is a moral hazard game, the Principal is indifferent between keeping and firing and has no incentive to deviate.

■

Proof of Lemma ??. Denote $r_{O_1}(K|\emptyset)$ the probability that O_1 keeps A upon observing nothing. As O_1 does not investigate O_1 never observes (a,ω). Hence,

$$\begin{aligned} U_A(\underline{a}, r_{O_1}|\omega) &= k(1-\underline{a}) + r_{O_1}(K|\emptyset)W_A \\ &> k(1-\overline{a}) + r_{O_1}(K|\emptyset)W_A \\ &= U_A(\overline{a}, r_{O_1}|\omega) \end{aligned}$$

■

Proof of Lemma ??. Consider a sequence of retention decisions $(R^{O_{i-3}},\dots,R^{O_1},R)$ and suppose that

$$I_i(K^{O_{i-2}},R^{O_{i-3}},\dots,R^{O_1},R) = I_i(F^{O_{i-2}},R^{O_{i-3}},\dots,R^{O_1},R) = 0.$$

Then,

$$\begin{aligned} &U_{O_{i-1}}(K^{O_{i-2}}, I_i(K^{O_{i-2}},R^{O_{i-3}},\dots,R^{O_1},R), r_{O_i}|a,\omega) = \\ &\quad r_{O_i}(K^{O_{i-1}}|\emptyset, K^{O_{i-2}},R^{O_{i-3}},\dots,R^{O_1},R)W_{O_{i-1}}, \end{aligned}$$

for all (a,ω) while

$$\begin{aligned} &U_{O_{i-1}}(F^{O_{i-2}}, I_i(F^{O_{i-2}},R^{O_{i-3}},\dots,R^{O_1},R), r_{O_i}|a,\omega) = \\ &\quad r_{O_i}(K^{O_{i-1}}|\emptyset, F^{O_{i-2}},R^{O_{i-3}},\dots,R^{O_1},R)W_{O_{i-1}}, \end{aligned}$$

for all (a, ω).

Assume $r_{O_i}(K^{O_{i-1}}|\emptyset, K^{O_{i-2}}, R^{O_{i-3}}, \dots, R^{O_1}, R) > r_{O_i}(K^{O_{i-1}}|\emptyset, F^{O_{i-2}}, R^{O_{i-3}}, \dots, R^{O_1}, R)$. Then, O_{i-1} strictly prefers to keep O_{i-2} for all (a, ω) upon observing $(R^{O_{i-3}}, \dots, R^{O_1}, R)$.

But then,

$$\begin{aligned} U_{O_{i-1}}(I_{i-1}(R^{O_{i-3}}, \dots, R^{O_1}, R) = 0|a(\omega), I_i, r_{O_i}) &= r_{O_i}(K^{O_{i-1}}|\emptyset, K^{O_{i-2}}, R^{O_{i-3}}, \dots, R^{O_1}, R)W_{O_{i-1}}, \\ &> r_{O_i}(K^{O_{i-1}}|\emptyset, K^{O_{i-2}}, R^{O_{i-3}}, \dots, R^{O_1}, R)W_{O_{i-1}} - c_{i-1} \\ &= U_{O_{i-1}}(I_{i-1}(R^{O_{i-3}}, \dots, R^{O_1}, R) = 1|a(\omega), I_i, r_{O_i}). \end{aligned}$$

It follows that O_{i-1} chooses not to investigate upon observing $(R^{O_{i-3}}, \dots, R^{O_1}, R)$, i.e.

$$I_{i-1}(R^{O_{i-3}}, \dots, R^{O_1}, R) = 0.$$

A similar argument shows that

$$I_{i-1}(R^{O_{i-3}}, \dots, R^{O_1}, R) = 0$$

if $r_{O_i}(K^{O_{i-1}}|\emptyset, K^{O_{i-2}}, R^{O_{i-3}}, \dots, R^{O_1}, R) \leq r_{O_i}(K^{O_{i-1}}|\emptyset, F^{O_{i-2}}, R^{O_{i-3}}, \dots, R^{O_1}, R)$ ■

Proof of Lemma ??. TBA ■

Proof of Lemma 5.2. WLOG assume O_n weakly prefers to keep A upon observing $(\hat{a}(\omega), \omega, R^{O_{n-2}}, \dots, R)$ and upon observing $(\hat{a}(\omega'), \omega', R^{O_{n-2}}, \dots, R)$ with $\omega \neq \omega'$, then

$$\begin{aligned} U_{O_n}(I_n(R^{O_{n-2}}, \dots, R) = 0, r_P) = \pi \big[&\hat{a}(\omega_H)\omega_H r_P(K^{O_n}|s, K^{O_{n-1}}, R^{O_{n-2}}, \dots, R) + (1 - \hat{a}(\omega_H)\omega_H) r_P(K^{O_n}|f, K^{O_{n-1}} \\ + (1 - \pi) \big[&\hat{a}(\omega_L)\omega_L r_P(K^{O_n}|s, K^{O_{n-1}}, R^{O_{n-2}}, \dots, R) + (1 - \hat{a}(\omega_L)\omega_L) r_P(K^{O_n}|f, K^{O} \end{aligned}$$

while

$$U_{O_n}(I_n(R^{O_{n-2}}, \dots, R) = 1, r_P) = e_1 U_{O_n}(I_n(R^{O_{n-2}}, \dots, R) = 0, r_P) + (1 - e_1) U_{O_n}(I_n(R^{O_{n-2}}, \dots, R) = 0, r_P) - c_n.$$

But then O_n does not want to investigate. ■

Proof of Lemma 5.1. Assume, by contradiction, that

$$r_P(K^{O_n}|s, K^{O_n-1}, R^{O_{n-2}}, \dots, R) = r_P(K^{O_n}|s, F^{O_n-1}, R^{O_{n-2}}, \dots, R).$$

Then, $U_{O_n}(K^{O_{n-1}}|a, \omega, I_{n-1}, \dots, I_1, R^{O_{n-2}}, \dots, R) = a\omega r_P(K^{O_n}|s, K^{O_n-1}, R^{O_{n-2}}, \dots, R)W_{O_n} + (1 - a\omega) r_P(K^{O_n}|f, K^{O_n-1}, R^{O_{n-2}}, \dots, R)W_{O_n}$, while

$U_{O_n}(F^{O_{n-1}}|a,\omega,I_{n-1},\ldots,I_1,R^{O_{n-2}},\ldots,R) = a\omega r_P(K^{O_n}|s,F^{O_n-1},R^{O_{n-2}},\ldots,R)W_{O_n} + (1 - a\omega)r_P(K^{O_n}|f,F^{O_n-1},R^{O_{n-2}},\ldots,R)W_{O_n}$. As

$$r_P(K^{O_n}|s,K^{O_n-1},R^{O_{n-2}},\ldots,R) = r_P(K^{O_n}|s,F^{O_n-1},R^{O_{n-2}},\ldots,R),$$

we thus have $U_{O_n}(K^{O_{n-1}}|a,\omega,I_{n-1},\ldots,I_1,R^{O_{n-2}},\ldots,R) \geq U_{O_n}(F^{O_{n-1}}|a,\omega,I_{n-1},\ldots,I_1,R^{O_{n-2}},\ldots,R)$ for all (a,ω) if

$$r_P(K^{O_n}|f,K^{O_n-1},R^{O_{n-2}},\ldots,R) \geq r_P(K^{O_n}|s,F^{O_n-1},R^{O_{n-2}},\ldots,R),$$

and $U_{O_n}(K^{O_{n-1}}|a,\omega,I_{n-1},\ldots,I_1,R^{O_{n-2}},\ldots,R) \leq U_{O_n}(F^{O_{n-1}}|a,\omega,I_{n-1},\ldots,I_1,R^{O_{n-2}},\ldots,R)$ for all (a,ω) if

$$r_P(K^{O_n}|f,K^{O_n-1},R^{O_{n-2}},\ldots,R) \leq r_P(K^{O_n}|s,F^{O_n-1},R^{O_{n-2}},\ldots,R).$$

In the first case O_n weakly prefers to keep the Agent for all (a,ω), while in the second O_n weakly prefers to fire the Agent for all (a,ω). By Lemma 5.2 this implies that O_n does not investigate.

So far we have established that for O_n to investigate it must be the case that

$$r_P(K^{O_n}|s,K^{O_n-1},R^{O_{n-2}},\ldots,R) \neq r_P(K^{O_n}|s,F^{O_n-1},R^{O_{n-2}},\ldots,R).$$

By a similar argument we also have

$$r_P(K^{O_n}|f,K^{O_n-1},R^{O_{n-2}},\ldots,R) \neq r_P(K^{O_n}|f,F^{O_n-1},R^{O_{n-2}},\ldots,R).$$

Moreover, for O_n to investigate it must be the case that if

$$r_P(K^{O_n}|s,K^{O_n-1},R^{O_{n-2}},\ldots,R) > r_P(K^{O_n}|s,F^{O_n-1},R^{O_{n-2}},\ldots,R)$$

then

$$r_P(K^{O_n}|f,K^{O_n-1},R^{O_{n-2}},\ldots,R) < r_P(K^{O_n}|f,F^{O_n-1},R^{O_{n-2}},\ldots,R),$$

and vice versa, as otherwise, by Lemma 5.2, O_n does not investigate. Recalling that we restrict attention to pure strategies establishes the result. ■

Proposition 9.2 *Suppose there is a hierarchy of n Overseers, $n \geq 2$, with $e_i^{\emptyset} = 0$ for all i.*

(i) If $\overline{a}\omega_H > 1/2$, and $\overline{a}\omega_L < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, then there exists an equilibrium in which each Overseer O_i investigates only upon observing the sequence of retention decisions $(K^{O_{i-2}}, \ldots, K^{O_1}, K)$ and the Agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω.

(ii) If $\overline{a}\omega_H > 1/2$, and $\underline{a}\omega_L < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, then there exists an equilibrium in which each Overseer O_i investigates only upon observing the sequence of retention decisions $(K^{O_{i-2}}, \ldots, K^{O_1}, K)$ and the Agent chooses $\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } \omega = \omega_H \\ \underline{a} \text{ otherwise .} \end{cases}$

(iii) If $\overline{a}\omega_L > 1/2$, and $\underline{a}\omega_H < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, then there exists an equilibrium in which each Overseer O_i investigates only upon observing the sequence of retention decisions $(K^{O_{i-2}}, \ldots, K^{O_1}, K)$ and the Agent chooses $\hat{a}(\omega) = \begin{cases} \underline{a} \text{ if } \omega = \omega_H \\ \overline{a} \text{ otherwise .} \end{cases}$

The following wage levels $\overline{W}_A, \overline{W}_{O_i}$ are the lowest wages for which, in equilibrium, the Agent exerts high effort in both states of the world, and the Overseers investigate. Similar expressions hold for equilibria of type (ii) and (iii).

For all $n \geq 2$, $\overline{W}_A = k(\overline{a} - \underline{a})/e_1$.

If $n = 2$, we have $\overline{W}_{O_1} = c_1/e_1e_2\pi$, and $\overline{W}_{O_2} = c_2/e_1e_2\pi(2\hat{a}(\omega_H)\omega_H - 1)$ whenever $\pi \leq \hat{\pi} := \frac{1-2\hat{a}(\omega_L)\omega_L}{2(\hat{a}(\omega_H)\omega_H - \hat{a}(\omega_L)\omega_L)}$ and $\overline{W}_{O_1} = c_1/e_1e_2(1-\pi)$, and $\overline{W}_{O_2} = c_2/e_1e_2(1-\pi)(1-2\hat{a}(\omega_L)\omega_L)$ whenever $\pi > \hat{\pi}$.

For all $n \geq 3$, we have $\overline{W}_{O_1} = c_1/e_1e_2$. Moreover, $\overline{W}_{O_n} = c_n/e_n\pi(2\hat{a}(\omega_H)\omega_H - 1)$ whenever $\pi \leq \hat{\pi}$, and $\overline{W}_{O_n} = c_n/e_n\pi(1 - 2\hat{a}(\omega_L)\omega_L)$ whenever $\pi > \hat{\pi}$.

If $n = 3$, then $\overline{W}_{O_2} = c_2/e_1e_2e_3\pi$, whenever $\pi \leq \hat{\pi}$, and $\overline{W}_{O_2} = c_2/e_1e_2e_3(1-\pi)$, whenever $\pi > \hat{\pi}$.

Finally, if $n \geq 4$, then $\overline{W}_{O_2} = c_2/e_1e_2e_3$, and $\overline{W}_{O_i} = c_i/e_ie_{i+1}$ for all $3 \leq i \leq n-2$. Moreover, $\overline{W}_{O_{n-1}} = c_{n-1}/e_{n-1}e_n\pi$, whenever $\pi \leq \hat{\pi}$, and $\overline{W}_{O_{n-1}} = c_{n-1}/e_{n-1}e_n(1-\pi)$, whenever $\pi > \hat{\pi}$.

Proof of Proposition 9.2. In what follows we prove the statement for the case in which the Agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω. The proof of the other cases follows the same steps with straightforward adjustments. Let there be n Overseers and consider the following strategy profile:

(i) A chooses $\hat{a}(\omega) = \overline{a}$ for all ω.

(ii) O_1 investigates.

(iii)

$$\hat{r}_{O_1}(K|a,\omega_H) = \begin{cases} 1 \text{ if } a = \overline{a} \\ 0 \text{ otherwise} \end{cases},$$

$$\hat{r}_{O_1}(K|a,\omega_L) = \begin{cases} 1 \text{ if } a = \overline{a} \\ 0 \text{ otherwise} \end{cases},$$

$$\hat{r}_{O_1}(K|\emptyset) = 1.$$

(iv) O_2 investigates both upon observing that O_1 kept the Agent and upon observing that O_1 fired the Agent, i.e. $I_2(K) = I_2(F) = 1$.

(v) O_i, $i \geq 3$, investigates upon observing that all the Overseers $O_{j<i}$, $j \geq 2$, kept their subordinate Overseer and does not investigate otherwise. In particular, $I_i(K^{O_{i-2}}, K^{O_{i-3}}, \ldots, K) = I_i(K^{O_{i-2}}, K^{O_{i-3}}, \ldots, F) = 1$.

(vi) For all O_i, $n \geq i \geq 2$, O_i fires O_{i-1} if O_i does not observe (a, ω). Moreover, whenever O_i observes (a, ω), O_i's decision to keep or fire O_{i-1} is not conditioned on the sequence of retention decisions, i.e. for all (a, ω) and for all $(R^{O_{i-2}}, \ldots, R)$, $(R\prime^{O_{i-2}}, \ldots, R\prime)$, we have $r_{O_i}(K^{O_{i-1}}|a, \omega, R^{O_{i-2}}, \ldots, R) = r_{O_i}(K^{O_{i-1}}|a, \omega, R\prime^{O_{i-2}}, \ldots, R\prime)$.

(vii) For all O_i, $n-1 \geq i \geq 2$, and for all ω, O_i keeps O_{i-1} whenever O_i observes $(\overline{a}, \omega)$.

(viii) If $\pi \leq \hat{\pi}$ then O_n keeps O_{n-1} if O_n observes $(\overline{a}, \omega_H)$, and fires O_{n-1} if O_n observes $(\overline{a}, \omega_L)$.

If $\pi > \hat{\pi}$ then O_n keeps O_{n-1} if O_n observes $(\overline{a}, \omega_L)$, and fires O_{n-1} if O_n observes $(\overline{a}, \omega_H)$.

(ix) If $\pi \leq \hat{\pi}$, then for all sequences of retention decisions $(R^{O_{n-2}}, \ldots, R)$ we have

$$r_P(K^{O_n}|s, K^{O_{n-1}}, R^{O_{n-2}}, \ldots, R) = 1,$$

$$r_P(K^{O_n}|s, F^{O_{n-1}}, R^{O_{n-2}}, \dots, R) = 0,$$

$$r_P(K^{O_n}|f, K^{O_{n-1}}, R^{O_{n-2}}, \dots, R) = 0,$$

and

$$r_P(K^{O_n}|f, F^{O_{n-1}}, R^{O_{n-2}}, \dots, R) = 1.$$

If $\pi > \hat{\pi}$, then for all sequences of retention decisions $(R^{O_{n-2}}, \dots, R)$ we have

$$r_P(K^{O_n}|s, K^{O_{n-1}}, R^{O_{n-2}}, \dots, R) = 0,$$

$$r_P(K^{O_n}|s, F^{O_{n-1}}, R^{O_{n-2}}, \dots, R) = 1,$$

$$r_P(K^{O_n}|f, K^{O_{n-1}}, R^{O_{n-2}}, \dots, R) = 1,$$

and

$$r_P(K^{O_n}|f, F^{O_{n-1}}, R^{O_{n-2}}, \dots, R) = 0.$$

In what follows we show that the so defined strategy profile is an equilibrium, provided $W_A \geq \overline{W}_A$, and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$. We do so in the case where $\pi \leq \hat{\pi}$. Derivations for $\pi > \hat{\pi}$ follow the same steps with straightforward adjustments.

(i) A has no incentive to deviate. Indeed, we have

$$U_A(\overline{a}, I_1, r_{O_1}|\omega) = k(1-\overline{a}) + e_1 W_A + (1-e_1)W_A \geq k(1-\underline{a}) + (1-e_1)W_A = U_A(\underline{a}, I_1, r_{O_1}|\omega),$$

for all ω given that $W_A \geq \overline{W}_A := k(\overline{a} - \underline{a})/e_1$. Hence, A has no incentive to deviate if $W_A \geq \overline{W}_A$.

(ii) O_1 investigates. As O_2 fires O_1 whenever O_2 does not observe (a, ω), and as O_2 cannot observe (a, ω) unless O_1 does so as well, O_1 receives a payoff of 0, whenever he chooses not to investigate, i.e., we have $U_{O_1}(I_1 = 0, I_2, r_{O_2}) = 0$. If $n = 2$, O_1 gets retained if, and only if, O_2 observes $(\overline{a}, \omega_H)$. As $\hat{a}(\omega_H) = \overline{a}$, and $I_2(K) = I_2(F) = 1$ this occurs with probability $e_1 e_2 \pi$ whenever O_1 investigates. It follows that $U_{O_1}(I_1 = 1, I_2, r_{O_2}) = e_1 e_2 \pi W_{O_1} - c_1$. Hence, when there are two Overseers, O_1 investigates as long as $W_{O_1} \geq \overline{W}_{O_1} := c_1/e_1 e_2 \pi$. If $n > 2$, O_1 gets retained when O_2 observes $(\overline{a}, \omega_H)$ and when O_2 observes $(\overline{a}, \omega_L)$, but not otherwise. Again, $\hat{a}(\omega) = \overline{a}$ for all ω, and $I_2(K) = I_2(F) = 1$ then imply that $U_{O_1}(I_1 = 1, I_2, r_{O_2}) =$

$e_1e_2W_{O_1} - c_1$. Hence, if there are more than two Overseers, O_1 investigates as long as $W_{O_1} \geq \overline{W}_{O_1} := c_1/e_1e_2$.

(iii) Note that O_2 investigates both when O_1 keeps the Agent and when O_1 fires the Agent. Moreover, O_2 does not condition his retention decision of O_1 on O_1's own retention decision of the Agent. It follows that O_1 is always indifferent between keeping and firing the Agent. Hence, any retention rule used by O_1 is a best-response.

(iv) We now show that O_2 investigates both upon observing that O_1 kept the Agent and upon observing that O_1 fired the Agent. Suppose first that there are two Overseers, $n = 2$. If O_2 does not investigate upon observing that O_1 kept the Agent, then O_2 does not observe (a, ω) and chooses to fire O_1. Given that the Principal fires O_2 unless, either there is policy success and O_2 kept O_1, or there is policy failure and O_2 fired O_1, we have

$$U_{O_2}(I_2(K) = 0, r_P) = \pi(1 - \overline{a}\omega_H)W_{O_2} + (1 - \pi)(1 - \overline{a}\omega_L)W_{O_2}.$$

If O_2 chooses to investigate, however, then with probability $e_2e_1\pi$, O_2 observes $(\overline{a}, \omega_H)$ and decides to keep O_1. In all other cases, O_2 chooses to fire O_1. It follows that

$$U_{O_2}(I_2(K) = 1, r_P) = e_2e_1\pi\overline{a}\omega_H W_{O_2} + e_2e_1(1-\pi)(1-\overline{a}\omega_L)W_{O_2} + (1-e_2e_1)U_{O_2}(I_2(K) = 0, r_P) - c_2.$$

Rearranging, we find that $U_{O_2}(I_2(K) = 1, r_P) \geq U_{O_2}(I_2(K) = 0, r_P)$ if, and only if, $W_{O_2} \geq \overline{W}_{O_2} := c_2/e_1e_2\pi(2\overline{a}\omega_H - 1)$.

Suppose next that there are three Overseers, $n = 3$. Note that, on the equilibrium path, O_2 only gets retained if O_3 observes $(\overline{a}, \omega_H)$. As O_3 cannot observe (a, ω), when O_2 does not either, we have

$$U_{O_2}(I_2(K) = 0, I_3, r_3) = 0.$$

Moreover, on the equilibrium path, O_1 always keeps the Agent. Hence, O_2 does not learn anything about whether O_1 observed (a, ω) or not upon observing that O_1 kept the Agent. Hence, when O_2 chooses to investigate the probability that O_2 observes (a, ω) is e_1e_2. As O_3 only keeps O_2 upon observing $(\overline{a}, \omega_H)$ the probability that O_2 is retained when investigating is $e_1e_2e_3\pi$. We thus have

$$U_{O_2}(I_2(K) = 1, I_3, r_3) = e_1e_2e_3\pi W_{O_2} - c_2.$$

It follows that O_2 investigates in the three Overseers case if, and only if, $W_{O_2} \geq \overline{W}_{O_2} := c_2/e_1e_2e_3\pi$.

Suppose finally that there are $n \geq 4$ Overseers. As in the three Overseers case we have $U_{O_2}(I_2(K) = 0, I_3, r_3) = 0$. Unlike in the three Overseers case, however, O_3 now keeps O_2 both upon observing $(\overline{a}, \omega_H)$ and upon observing $(\overline{a}, \omega_L)$. Hence, $U_{O_2}(I_2(K) = 1, I_3, r_3) = e_1e_2e_3W_{O_2} - c_2$, and thus O_2 investigates in the $n \geq 4$ Overseers case if, and only if, $W_{O_2} \geq \overline{W}_{O_2} := c_2/e_1e_2e_3$.

In all the cases, O_1 firing the Agent is off-the-equilibrium path. Hence, we can let $I_2(F) = 1$.

(v) As every Overseer $O_{j \neq i}$, $3 \leq j \leq n$, keeps O_{j-1} upon observing $(\hat{a}(\omega), \omega)$ and fires O_{j-1} upon not observing (a, ω), O_i can infer from $O_{l<i}$ firing O_{l-1}, that O_l did no observe (a, ω). Hence, if O_i chooses to investigate, O_i will not learn (a, ω) either. Now suppose $i \leq n-1$, then O_i also knows that O_{i+1} will not observe (a, ω) and will thus fire O_i. It follows

$$U_{O_i}(I_i(R^{O_{i-2}}, \ldots, F^{O_{j-1}}, \ldots, R) = 1, I_{i+1}, r_{i+1}) = -c_i$$
$$< 0 = U_{O_i}(I_i(R^{O_{i-2}}, \ldots, F^{O_{j-1}}, \ldots, R) = 0, I_{i+1}, r_{i+1})$$

and O_i chooses not to investigate upon observing that some Overseer $O_{l<i}$ did not retain his subordinate Overseer. If $i = n$, then O_n chooses to fire O_{n-1} upon not observing (a, ω) and will thus only be retained by the Principal when there is policy failure. We thus have

$$\begin{aligned} U_{O_n}(I_n(R^{O_{i-2}}, \ldots, F^{O_{j-1}}, \ldots, R) = 1, r_P) &= \pi(1 - \overline{a}\omega_H)W_{O_n} + (1-\pi)(1 - \overline{a}\omega_L)W_{O_n} - c_i \\ &< \pi(1 - \overline{a}\omega_H)W_{O_n} + (1-\pi)(1 - \overline{a}\omega_L)W_{O_n} \\ &= U_{O_n}(I_n(R^{O_{n-2}}, \ldots, F^{O_{j-1}}, \ldots, R) = 0, r_P) \end{aligned}$$

Suppose now that the Overseer O_i, $3 \leq i \leq n-1$, observes the sequence of retention decisions $(K^{O_{i-2}}, K^{O_{i-3}}, \ldots, K)$. As every Overseer O_j keeps O_{j-1} upon observing $(\hat{a}(\omega), \omega)$ and fires O_{j-1} upon not observing (a, ω), O_i can infer from $(K^{O_{i-2}}, K^{O_{i-3}}, \ldots, K)$ that every Overseer below him in the hierarchy has observed $(\hat{a}(\omega), \omega)$. Hence, if O_i chooses to investigate, O_i will learn $(\hat{a}(\omega), \omega)$ with probability e_i. Moreover, if O_i observes $(\hat{a}(\omega), \omega)$, O_i will keep O_{i-1}. In equilibrium O_{i+1} will then choose to investigate himself and keep O_i upon observing $(\hat{a}(\omega), \omega)$. It follows that $U_{O_i}(I_i(K^{O_{i-2}}, K^{O_{i-1}}, \ldots, K) = 1, I_{i+1}, r_{i+1}) = e_ie_{i+1}W_{O_i} - c_i$. If O_i does not investigate, however, O_i will not learn (a, ω). Thus O_{i+1} will not learn (a, ω) either and will fire O_i. Hence, $U_{O_i}(I_i(K^{O_{i-2}}, K^{O_{i-1}}, \dots, K) = 0, I_{i+1}, r_{i+1}) = 0$. It follows that O_i chooses to investigate upon observing $(K^{O_i-2}, K^{O_{i-3}}, \dots, K)$ as long as $W_{O_i} \geq \overline{W}_{O_i} := c_i/e_i e_{i+1}$.

Finally, as before, we have

$$U_{O_n}(I_n(K^{O_{n-2}}, K^{O_{n-3}}, \dots, K) = 0, r_P) = \pi(1 - \overline{a}\omega_H)W_{O_n} + (1 - \pi)(1 - \overline{a}\omega_L)W_{O_n}.$$

Upon observing the sequence of retention decisions $(K^{O_{i-2}}, K^{O_{i-3}}, \dots, K)$, O_n knows that every Overseer below him in the hierarchy has observed $(\hat{a}(\omega), \omega)$. Hence, if O_n chooses to investigate he will observe $(\hat{a}(\omega), \omega)$ with probability e_n. As O_n keeps O_{n-1} upon observing $(\overline{a}, \omega_H)$ and fires O_{n-1} otherwise and given the Principal fires O_2 unless, either there is policy success and O_2 kept O_1, or there is policy failure and O_2 fired O_1, we have

$$\begin{aligned} U_{O_n}(I_n(K^{O_{n-2}}, K^{O_{n-3}}, \dots, K) = 1, r_P) &= e_n \pi \overline{a}\omega_H W_{O_n} + e_n(1 - \pi)(1 - \overline{a}\omega_L)W_{O_n} \\ &\quad + (1 - e_n)U_{O_n}(I_n(K^{O_{n-2}}, K^{O_{n-3}}, \dots, K) = 0, r_P) - c_n. \end{aligned}$$

Rearranging, we find that

$$U_{O_n}(I_n(K^{O_{n-2}}, K^{O_{n-3}}, \dots, K) = 1, r_P) \geq U_{O_2}(I_n(K^{O_{n-2}}, K^{O_{n-3}}, \dots, K) = 0, r_P)$$

if, and only if, $W_{O_n} \geq \overline{W}_{O_n} := c_n/e_n\pi(2\overline{a}\omega_H - 1)$.

(vi) Suppose that O_i, $n > i \geq 2$, does not observe (a, ω). Whether O_i keeps or fires O_{i-1}, O_{i+1} will not observe (a, ω) either and will thus fire O_i. Hence, we have $U_{O_i}(K^{O_{i-1}}|\emptyset, \emptyset, \cdot, \dots, \cdot) = U_{O_i}(F^{O_{i-1}}|\emptyset, \emptyset, \cdot, \dots, \cdot) = 0$. Hence, it is a best-response for O_i to fire O_{i-1} upon not observing (a, ω). Now suppose O_n does not observe (a, ω). If O_n keeps O_{n-1}, O_n gets retained only if there is policy success. If O_n fires O_{n-1}, O_n only gets retained if there is policy failure. Hence, as $\pi \leq \hat{\pi}$, we have

$$\begin{aligned} U_{O_n}(K^{O_{n-1}}|\emptyset, \emptyset, \cdot, \dots, \cdot, R^{O_{n-2}}, \dots, R) &= \pi\overline{a}\omega_H W_{O_n} + (1 - \pi)\overline{a}\omega_L W_{O_n} \\ &\leq \pi(1 - \overline{a}\omega_H)W_{O_n} + (1 - \pi)(1 - \overline{a}\omega_L)W_{O_n} \\ &= U_{O_n}(F^{O_{n-1}}|\emptyset, \emptyset, \cdot, \dots, \cdot, R^{O_{n-2}}, \dots, R). \end{aligned}$$

Finally, note that the Principal does not condition his retention decision on $(R^{O_{n-2}}, \dots, R)$. It follows that for all (a, ω), and for all $(R^{O_{n-2}}, \dots, R)$, $(R'^{O_{n-2}}, \dots, R')$, we have

$$U_{O_n}(R^{O_{n-1}}|a, \omega, R^{O_{n-2}}, \dots, R) = U_{O_n}(R^{O_{n-1}}|a, \omega, R'^{O_{n-2}}, \dots, R').$$

Hence, O_n has no incentive to condition his retention decision of O_{n-1} on $(R^{O_{n-2}}, \ldots, R)$. Repeating the argument we find that there is no O_i, $i \geq 2$, who has an incentive to condition his retention decision on $(R^{O_{i-2}}, \ldots, R)$.

(vii) As O_{i+1} chooses not to investigate upon observing that O_i fired O_{i-1} and thus subsequently fires O_i, we have $U_{O_i}(F^{O_{i-1}}|\bar{a}, \omega, R^{O_{i-2}}, \ldots, R) = 0$, for all $n-1 \geq i \geq 2$, and for all sequences of retention decisions $(R^{O_{i-2}}, \ldots, R)$. Suppose now the sequence of retention decisions is $(K^{O_{i-2}}, \ldots, K)$ and O_i, $n-1 \geq i \geq 2$, observes $(\bar{a}, \omega)$. Then, as O_{i+1} investigates upon observing $(K^{O_{i-1}}, \ldots, K)$ and keeps O_i if O_{i+1} observes $(\bar{a}, \omega)$, we have

$$U_{O_i}(K^{O_{i-1}}|\bar{a}, \omega, K^{O_{i+2}}, \ldots, K) = e_{i+1} W_{O_i}.$$

It follows that O_i keeps O_{i-1} upon observing the sequence of retention decisions $(K^{O_{i-1}}, \ldots, K)$ and $(\bar{a}, \omega)$.

Suppose now the sequence of retention decisions is not $(K^{O_{i-2}}, \ldots, K)$, yet O_i, $n-1 \geq i \geq 2$, observes $(\bar{a}, \omega)$, which can only happen off-the-equilibrium path. Whether O_i fires or keeps O_{i-1}, O_{i+1} will not investigate and will thus subsequently fire O_i. It follows that O_i is then indifferent between keeping and firing O_{i-1}. Keeping O_{i-1} in this case is thus a best-response.

(viii) Given r_P, we have

$$\begin{aligned} U_{O_n}(K^{O_{n-1}}, r_P|\bar{a}, \omega_H, I_{n-1}, \ldots, I_1, K^{O_{n-2}}, \ldots, K) &= \bar{a}\omega_H W_{O_n} \\ &> (1 - \bar{a}\omega_H) W_{O_n} \\ &= U_{O_n}(F^{O_{n-1}}, r_P|\bar{a}, \omega_H, I_{n-1}, \ldots, I_1, K^{O_{n-2}}, \ldots, K), \end{aligned}$$

if, and only if, $\bar{a}\omega_H > 1/2$. Similarly,

$$\begin{aligned} U_{O_n}(K^{O_{n-1}}, r_P|\bar{a}, \omega_L, I_{n-1}, \ldots, I_1, K^{O_{n-2}}, \ldots, K) &= \bar{a}\omega_L W_{O_n} \\ &< (1 - \bar{a}\omega_L) W_{O_n} \\ &= U_{O_n}(F^{O_{n-1}}, r_P|\bar{a}, \omega_L, I_{n-1}, \ldots, I_1, K^{O_{n-2}}, \ldots, K), \end{aligned}$$

if, and only if, $\bar{a}\omega_L < 1/2$.

(ix) As this is a moral hazard game, the Principal is always indifferent between keeping and firing O_n and r_P is indeed a best-response.

■

10 Robustness checks A: Outcome Stochastically Revealed to Overseers

So far, we made the stark assumption that only the Principal observes the policy outcome. We now relax this assumption by considering the case in which the outcome is revealed stochastically to the Overseers. Specifically, we assume that, conditional on the outcome not having been revealed so far, Overseer O_i learns the outcome with probability q_i before deciding whether to investigate. Once the outcome is revealed to some Overseer, it is revealed to all the actors of the game. Maybe surprisingly, we show that the attractiveness of the hierarchy to the Principal is not necessarily improved and may even diminish when Overseers are likely to observe the policy outcome. The main reason is that, as in the model of section 4.2 Overseers do not investigate once the outcome is revealed.

Lemma 10.1 *For all $i = 1, 2$, O_i does not investigate if O_i observes the outcome before deciding whether to investigate.*

Proof. Almost identical to argument given in proof of proposition 4.1 ■

As long as $q_1, q_2 < 1$, it is still possible to have an equilibrium in which the Agent chooses to exert high effort in both states of the world and the Overseers investigate upon not observing the outcome. In such an equilibrium, each Overseer keeps his subordinate when there is success and fires him when there is failure. When the outcome is not revealed, the Overseers essentially adopt the same behavior as in the equilibrium that sustains proposition 5.5. In particular, Overseer O_2 fires Overseer O_1 when O_2 does not observe (a, ω) and keeps O_1 upon observing (a, ω) at least for some ω.

Proposition 10.1 *Suppose there is a hierarchy of two Overseers. Suppose further, that, conditionally on the outcome not having been revealed so far, each Overseer observes the policy outcome with probability q_i before deciding whether to investigate.*

(i) If $\overline{a}\omega_H \leq 1/2$, or if $\underline{a}\omega_L \geq 1/2$, then no Overseer ever investigates in equilibrium, and the Agent chooses $\hat{a}(\omega) = \overline{a}$ if, and only if, $W_A \geq k/q_1\omega_L$.

(ii) If $\overline{a}\omega_H > 1/2$, and $\overline{a}\omega_L < 1/2$, then there exist $\overline{W}_A, \overline{W}_{O_i}$, such that, if $W_A \geq \overline{W}_A$ and, for all i, $W_{O_i} \geq \overline{W}_{O_i}$, then there exists an equilibrium in which each Overseer O_i investigates only upon not observing the outcome and observing the sequence of retention decisions K and the Agent chooses $\hat{a}(\omega) = \overline{a}$ for all ω. In the cheapest such equilibrium, we have $\overline{W}_A = \frac{k(\overline{a}-\underline{a})}{q_1(\overline{a}-\underline{a})\omega_L+(1-q_1)e_1}$. Moreover, $\overline{W}_{O_1}$ is increasing in q_2.

Compared to an environment, where Overseers do not observe the outcome, the main difference is that, as Overseers do not condition on the retention decision of their subordinate when the outcome is revealed, the incentives for Overseer O_1 to investigate arise entirely through the retention rule used by O_2 when the policy outcome is not revealed. It follows that the incentives for O_1 to investigate are reduced the more likely Overseer O_2 is to observe the outcome before deciding whether to investigate or not which explains that $\overline{W}_{O_1}$ is increasing in q_2. Moreover, when the probability that the first Overseer observes (a, ω) upon investigating is sufficiently high that $k(\overline{a} - \underline{a})/e_1 < k/\omega_L$, then $\overline{W}_A$ is increasing in q_1 as well. It follows that there are ranges of values for $\overline{a}, \underline{a}, \omega_L, \omega_H, e_1$, such that, given a vector of wages (W_A, W_{O_1}, W_{O_2}), the Agent would exert high effort in both states of the world when the Overseers are certain not to observe the outcome before deciding whether to investigate, but would revert to exerting low effort when each Overseer O_i observes the outcome with conditional probability $q_i > 0$.

This stands in sharp contrast to results one obtains when Overseers costlessly observe a and ω. To see this, consider a generalization of the model of section 4.2 and suppose that there is a single Overseer who observes (a, ω) with probability e_1, and the policy outcome with probability q_1. Assume further that whether the Overseer observes (a, ω) does not depend on whether he observes the policy outcome.

Proposition 10.2 *Suppose a single Overseer who observes (a, ω) with probability e_1 and observes the policy outcome with probability q_1. Then, in equilibrium, the Agent chooses*

$$\hat{a}(\omega) = \begin{cases} \overline{a} \text{ if } W_A \geq \overline{W}_A := \frac{k(\overline{a}-\underline{a})}{e_1+(1-e_1)q_1(\overline{a}-\underline{a})\omega} \\ \underline{a} \text{ otherwise.} \end{cases}$$

Upon observing (a, ω), the Overseer retains the Agent if and only if $a \geq \overline{a}$. Upon not observing (a, ω), the Overseer retains the Agent when there is policy success and fires the Agent when there *is failure. The Principal can guarantee these choices by retaining the Overseer if and only if the outcome is success.*

Proof. Suppose the Principal keeps the Overseer when the policy outcome is success and fires the Overseer when the policy outcome is failure, independently of the retention decision of the Overseer, i.e. $r_P(K^O|s,K) = r_P(K^O|s,F) = 1$ and $r_P(K^O|f,K) = r_P(K^O|f,F) = 0$. Then, the Overseer is always indifferent between keeping and firing the Agent. Moreover, the utility of the Overseer is strictly increasing in the probability of success and hence in the level of effort exerted by the Agent. It follows that the Overseer wants to choose the retention rule that gives the Agent the strongest incentives to choose $\overline{a}$ over $\underline{a}$. Hence, it is optimal for the Overseer to keep the Agent if he observes that the Agent exerted $\overline{a}$ and to fire the Agent if he observes $\underline{a}$. Moreover, as success is more likely when the Agent exerts high effort, the Overseer should interpret policy success as an indication that the Overseer exerted high effort, and policy failure as an indication that the Agent exerted low effort. In order to provide the strongest incentives to choose $\overline{a}$, the Overseer, upon not observing (a, ω), should thus retain A when he observes policy success and fire when he observes policy failure. It follows that $U_A(\overline{a}|\omega) = k(1-\overline{a}) + e_1 W_A + (1-e_1)q_1\overline{a}\omega W_A + (1-e_1)(1-q_1)r_{O_1}(K|\emptyset,\emptyset)W_A$ and $U_A(\underline{a}|) = k(1-\underline{a}) + (1-e_1)q_1\underline{a}\omega W_A + (1-e_1)(1-q_1)r_{O_1}(K|\emptyset,\emptyset)W_A$. Thus, A chooses $\overline{a}$ if, and only if, $W_A \geq \frac{k(\overline{a}-\underline{a})}{e_1+(1-e_1)q_1(\overline{a}-\underline{a})\omega}$. ■

In such a setting $\overline{W}_A$ is decreasing in the probability e_1 that the first Overseer observes (a, ω) and in the probability q_1 that the Overseer observes the policy outcome. In other words, the more likely the Overseer is to observe the policy outcome the better he is able to incentivize the Agent to exert high effort.