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under Risk and Equity Concerns**

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Preface

This thesis consists of three self-contained papers, each one revolving around the general theme of how to characterize the optimal public decision making in the presence of a risky situation that has heterogeneous consequences across the population.

The first essay, "*Social Willingness to Pay under Equity Concerns*", introduces and characterizes the concept of social premium, which defines the amount that society is ready to pay to eliminate a social risk, i.e. a global risk with heterogeneous impacts. Three approaches to equity are considered: utilitarianism (Harsanyi, 1955); concerns for the distribution of individual risks, or ex-ante equity approach (Diamond, 1967; Epstein and Segal, 1992); concerns for the distribution of realized individual risks, or ex-post equity approach (Hammond, 1983; Broome, 1991; Adler, 2012). In particular, in the class of ex-post equity approaches, I mainly focus on the expected equally distributed equivalent criterion proposed by Fleurbaey (2010). For each approach, I decompose the social premium in a risk premium and an inequality premium, and then study their properties in the small and in the large in a way analogous to the classic Arrow-Pratt result. Interestingly, the approximation separates taste parameters (risk aversion and inequality aversion) and risky measures (variability in either individual expected consumption or realized consumption), which allows to discuss the impact of each component on the social premium. Extensions of the model allow for multiplicative risks and dynamic frameworks. Finally, I present two applications: the social cost derived from the risk of major hurricanes in the US and the mitigation policy to avert the risk of climate change. The first is an example of social risk among contemporaries, the second of a risky situation involving different generations.

The second chapter, "*Fairness, Risk and the Social Cost of Carbon*", considers the computation of the social cost of carbon in the presence of risk and inequality, and studies how different assumptions about social preferences affect its size. In particular, I introduce a model that disentangles risk aversion, resistance to intertemporal substitution and intra-generational inequality aversion, based on an ex-post approach to equity. The model

distinguishes itself from the more standard approach of recursive preferences based on [Kreps and Porteus \(1978\)](#) and [Selden \(1978\)](#). After showing how the social cost of carbon can be derived from the adopted social welfare framework, I analytically determine its main components and provide a second order Taylor approximation to understand their relative importance. Then, I estimate the social cost of carbon based on a simplified version of [Nordhaus \(2010\)](#) RICE model and on the climate damages and carbon cycle adopted by [Golosov et al. \(2014\)](#).

Finally, the last chapter, "*Optimal Climate Policy: Prevention and Risk Sharing*", considers a policy that changes the probability distribution of the climate risk (self-protection choice), and explores whether countries should engage in more mitigation when the risk sharing mechanism is inefficient. The essay focuses exclusively on a utilitarian social planner. By building a parallel between the presence of inefficient risk sharing and the introduction of exogenous climate specific background risks, I determine conditions on risk aversion and prudence that guarantee an increase in mitigation under inefficient risk sharing. Generally speaking, mitigation tends to rise either when the climate event is catastrophic (i.e. low probability, high loss), or when the social planner is not very prudent and the inequality risk small. Contrary to intuition, risk sharing and prevention can be complements. If we consider risk sharing as a type of adaptation, the paper suggests that countries should invest in adaptation if they want to induce a large mitigation effort.

Bibliography

- Adler, M. (2012). *Well-Being and Fair Distribution*. Oxford University Press. 6
- Broome, J. (1991). *Weighing Goods. Equality, Uncertainty and Time*. Oxford: Blackwell. 6
- Diamond, P. (1967). Cardinal welfare, individualistic ethics and interpersonal comparisons of utility: Comment. *Journal of Political Economy* 75, 765–66. 6
- Epstein, L. and U. Segal (1992). Quadratic social welfare function. *Journal of Political Economy* 100, 691–712. 6
- Fleurbaey, M. (2010). Assessing risky social situations. *Journal of Political Economy* 118, 649–80. 6
- Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica* 82(1), 41–88. 7
- Hammond, P. (1983). Ex-post optimality as a dynamically consistent objective for collective choice under uncertainty. In P. Pattanaik and M. Salles (Eds.), *Social Choice and Welfare*, pp. 175–205. Amsterdam: North-Holland. 6
- Harsanyi, J. (1955). Cardinal welfare, individualistic ethics and interpersonal comparison of utilities. *Journal of Political Economy* 63, 309–21. 6
- Kreps, D. and E. Porteus (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica* 46, 185–200. 7
- Nordhaus, W. (2010). Economic aspects of global warming in a post-copenhagen environment. *Proceeding of the National Academy of Sciences* 107(26), 11721–26. 7
- Selden, L. (1978). A new representation of preferences over "certain x uncertain" consumption pairs: the "ordinal certainty equivalent" hypothesis. *Econometrica* 46, 1054–60. 7

Chapter 1

Social Willingness to Pay under Equity Concerns

1.1 Introduction

Nowadays, much attention in the public debate is focused on global risks (e.g. climate change, terrorism, pandemics) whose effects are heterogeneous across the population. For example, it has become evident that climate change entails an unequal distribution of impacts, where some countries are likely to suffer more than others due to their geographical location or the relative importance of the agricultural sector, while other countries might even benefit from it (IPCC, 2014). Moreover, because of the long term nature of the externality, there is a dichotomy between those that bear the costs of the policy and those that will benefit from it. Therefore, policies devoted to mitigating the risk of climate change not only modify the probabilities of different scenarios, but, at the same time, they affect the distribution of consumption across various subpopulations, both today and in the future. Governments, then, are required to find the perfect balance between the risk reduction goals and the respect of the interests of the single parties involved. Equity issues are pivotal also in the debate over mortality and morbidity risks. Consider, for instance, a vaccine for a contagious disease that will reduce the risk for most people, but will produce a fatal reaction on few. When deciding whether to release it or not, authorities will have to weigh the interests of the majority against the well-being of the few. Similar issues arise for anti-terrorist policies, like security measures in airports, where governments are required to set the moral line beyond which the safety of the population is more important than the rights of the passengers. These are

just few examples that highlight how our ethical assumptions about both risk and equity can shape the optimal risk prevention policy.

In the paper, I introduce a new concept, the *social premium*, which provides useful guidance on the comparison and selection of different projects in the presence of a social risky situation. As the risk premium defines the amount that the decision maker is ready to pay to eliminate a private risk, the social premium describes the amount that the decision maker is willing to pay to eliminate a *social risk*, defined as a global risk with differentiated impacts across subpopulations. As the standard risk premium, the social premium (and the corresponding social willingness to pay) helps in building an optimal portfolio of public policies by characterizing some basic investment rules, like 'invest in a risk reducing project only if its cost is lower than the social willingness to pay', or 'choose the project with the best compromise between aggregate returns and variability in terms of both risk and inequality'.

If we assume that the decision maker's moral view satisfies utilitarianism (Harsanyi, 1955), then only the characteristics of the aggregate social risk matters, but not the composition of individual (expected or realized) risks. Since the utilitarian decision maker operates behind the 'veil of ignorance', inequality in the distribution of sure wealth across individuals receives the same treatment as variability in the distribution of risky wealth across states. In other words, inequality looks like a second source of risk. Therefore, if we consider that a realization of the social risk corresponds to the outcome received by a particular agent in a particular state of nature, the utilitarian decision maker cares only about the probability distribution of those realizations, but not about how that particular distribution has been formed. As a consequence, for a given distribution of the social risk, she does not distinguish between individual risks that are equally distributed across the population and individual risks that are concentrated on a specific subpopulation, or between individual risks that are positively correlated and can potentially lead to catastrophic outcomes, and individual risks that are negatively correlated.

In order to disentangle fairness concerns from risk concerns, and to be able to discuss how the composition of the social risk affects the optimal risk prevention policy, the literature has proposed two main approaches. The ex-ante approach to equity, advocated e.g. by Diamond (1967) and Epstein and Segal (1992), postulates that the decision maker should be sensitive to the distribution of individual risks, and describes whether the decision maker prefers to grant the same chances to everyone or to converge the risk on few individuals. On the opposite, the ex-post approach to equity, advocated e.g. by Hammond (1983), Broome (1991), focuses on the realizations of the individual risks, and on how they are correlated. In

particular, it describes whether the decision maker prefers fair situations where every agent bears a loss (misery loves company) or unfair situations where only few individuals sustain a loss. The two approaches represent different ethical theories of the interaction between fairness and risk, and the choice between them responds purely to our normative views.

In the paper, I take the three approaches to equity (utilitarianism, ex-ante equity or 'equality of prospects' approach and ex-post equity), and, for each of them, I characterize the social premium and study its properties in the small and in the large, thereby extending the classic Arrow-Pratt local approximation of the individual risk premium to the case of social risks. For the ex-post approach, several functional forms have been proposed in the literature, see e.g. [Adler \(2012\)](#) for a review. However, I will consider exclusively the *expected equally distributed equivalent* criterion proposed by [Fleurbaey \(2010\)](#), as it allows a direct comparison to utilitarianism. In the literature, the three approaches have been described in terms of sensitiveness to the distribution of (expected or realized) well-being. Under the common assumption of homogeneous preferences, I show that the three approaches can be redesigned in terms of sensitiveness to the distribution of (expected or realized) consumption, which makes the interpretation of the social premium more straightforward. In the equality of prospects approach, the decision maker first computes the certainty equivalent for each individual risk, and then aggregates the individual certainty equivalents with a felicity function expressing aversion to the unequal distribution of risks. In the expected equally distributed equivalent approach, the decision maker first computes the equally distributed equivalent for each state, where a state is a collection of realized risks for each individual; then, she aggregates the equally distributed equivalents with a utility function conveying aversion to the risk of having unequal distributions of realizations. Utilitarianism satisfies either decomposition.

I show that, independently of the approach, the social premium is the sum of a risk premium and an inequality premium. However, the way in which those premia are constructed depends on equity preferences. Indeed, in the equality of prospects approach (concern for ex-ante equity), we care about individual risk premia and the society's inequality premium to eliminate inequality in expected individual consumption. On the contrary, in the expected equally distributed equivalent criterion (concern for ex-post equity), we are interested in the risk premium that the representative agent would pay and the inequality premia that society's would be willing to bear conditional on the realization of a given state. The local approximation of the social premia allows me to identify the components of the risk and

inequality premia according to the approach considered. In particular, I show that the approximation separates taste parameters (i.e. risk aversion and inequality aversion) and risky measures (the variability in either individual expected consumption or in realized individual consumption). Moreover, I consider both the case in which initial income is uniformly distributed across the population and the case in which individuals have both heterogeneous incomes and heterogeneous risks. In the latter, the correlation between initial income and individual risk matters when signing the social willingness to pay for a project. Indeed, if poor individuals are more exposed to the risk, the decision maker is more prone to invest in the project irrespectively of her equity preferences.

Abstracting from the correlation between individual risk and initial income, both the equality of prospects approach and the expected equally distributed one grant a larger social premium than the utilitarian one if and only if the decision maker is more inequality averse than risk averse. Indeed, only in that case the decision maker prefers equally distributed risks to concentrated risks in the ex-ante framework, and equal distribution of realizations to polarized consumption in the ex-post approach. Moreover, in the equality of prospects approach, the decision maker cares about the inequality in individual expected consumption and about the average variance of individual risks. In the expected equally distributed equivalent approach, instead, she cares about the variance in average consumption across states and about the expected contingent inequality. The extension of the comparative results in the large is feasible only for the case of equal distribution of initial income. Furthermore, we can also say something about the comparison between the equality of prospects and expected equally distributed equivalent approaches. Indeed, the difference between inequality aversion and risk aversion signs also the comparison between ex-ante and ex-post equity preferences. In general, the expected equally distributed equivalent social framework assigns a larger willingness to pay than the equality of prospects one, as long as risk aversion is lower than inequality aversion.

In the final part of the paper, I discuss two possible applications of the social premium: the willingness to pay to eliminate the risk of hurricanes in the US, and the willingness to pay to remove the risk of climate change. The two examples differ both for the composition of the population and for the correlation between individual risk and initial income. The risk of hurricanes concerns people living in the same period, and will presumably have a negative impact on the degree of inequality in a society. Indeed, I report some contributions to the literature assessing that the risk of hurricane has a larger incidence on poor households, and may even affect the Gini coefficient, at least in the short run ([Miljkovic and Miljkovic](#),

2014; Shaughnessy et al., 2010; Masozera et al., 2007). The risk of climate change, instead, concerns people living in different generations, and is increasing with income, meaning that the future wealthier people are those more exposed to the risk of climate change. The two applications support the comparative results derived in the first part of the paper, and arise the fundamental issue of how to estimate risk aversion and inequality aversion. In particular, I show that the estimated coefficients of intra-/inter-generational inequality aversion are quite pessimistic about the degree of fairness of our society, suggesting that the adoption of either ex-ante or ex-post equity concerns may decrease the willingness to pay for a risk reducing project with respect to the utilitarian framework.

The choice of the optimal social welfare criterion to take into account the distributional impacts of a proposed intervention has inspired an extensive literature (see e.g. Adler (2012) for a review). For choices under certainty, the debate revolves around the shape of the social welfare function. In particular, two social welfare functions are often used in the literature: an utilitarian one, which sums individual utilities, and a prioritarian one, which sums a strictly concave transformation of individual utilities. For choices under uncertainty, an additional issue arises: the object of the concave transformation, which has generated the debate between the ex-ante and the ex-post approach (see e.g. Adler and Sanchirico (2006) for a review). There exists a small literature that examines the economic implications of the ex-ante and the ex-post approaches. Adler et al. (2012) and Fleurbaey and Bovens (2012) analyze the impact of different social welfare functions on the willingness to pay to reduce mortality risk. Adler and Treich (2014) study the impact of inequality aversion on the optimal allocation of the uncertain aggregate wealth. In the literature on discounting, recent contributions (Emmerling, 2011; Fleurbaey and Zuber, 2014a) examine the importance of equity concerns on the optimal social discount rate.

The paper is organized as follows. Section 2 describes in details the three approaches to risk and equity considered in the paper: utilitarianism, equality of prospects and expected equally distributed equivalent. Section 3 defines the social premium for the three different approaches, which describes the maximum amount the social planner is willing to pay to completely eliminate the risk and its unequal consequences. In the same section, we show how the social premium can be decomposed in the usual risk premium, i.e. the willingness to pay to remove the macro risk, and in an inequality premium, which corresponds to the willingness to pay to replace the unequal distribution of impacts with a more egalitarian one. Section 4 analyzes the components of the social premium in the small, and examines how preferences for risk and inequality affect its size and sign. Section 5 turns to the analysis in

the large. Section 6 includes a number of extensions: proportional risks; the popular case of log-normal distribution and iso-elastic felicity functions; a reinterpretation of the model in terms of intergenerational inequality rather than intra-generational inequality. Section 7 describes two potential applications of the concept of social premium (and social willingness to pay): the risk of major hurricanes in the US and the climate change risk. Finally, Section 8 concludes and suggests future research venues.

1.2 Social welfare function

Let \mathbb{N} denote the set of agents, and $i \in \mathbb{N}$ a single individual; we assume that agents are distributed according to $Q(i)$. The set of states of nature is represented by \mathbb{S} ; let s denote an element of \mathbb{S} , and $P(s)$ the probability distribution of states. Moreover, let $Q(i|s)$ and $P(s|i)$ represent the conditional distributions of individuals and states, respectively. Finally, let $c(i, s)$ be the realization of the random variable \tilde{c} , distributed as $F(s, i)$. All agents have the same von-Neumann Morgenstern utility function u , increasing, concave and twice differentiable.

The paper compares three different approaches to assess risky social situations¹. The first criterion is *Utilitarianism*, popularized by Harsanyi (1955) aggregation theorem², where the social planner maximizes the sum of individual expected utilities, which coincides also with the expected value of the sum of individual utilities:

$$W^U = \int_{\mathbb{N}} \int_{\mathbb{S}} u(c(i, s)) dF(s, i) \quad (1.1)$$

where $c(i, s)$ denotes the quantity consumed by agent i in state s . Although the concavity of the utility function incorporates aversion to mean preserving spreads in the distribution of consumption, utilitarianism is indifferent to the distribution of well-being, thereby failing to express concerns for the relatively worse-off. Indeed, consider the following examples:

¹Note that we will consider only approaches that use the von-Neumann Morgenstern utility function as the well-being measure to study equity issues under uncertainty. For a discussion on non-expected utility criteria, see Fleurbaey and Zuber (2014b). Moreover, we leave aside responsibility issues. In other words, the presence of unequal distributions of utilities is due to bad luck and not to suboptimal behavior. For responsibility-sensitive criteria see, for instance, Fleurbaey and Maniquet (2012).

²Harsanyi (1955) proved that the social welfare function is an affine combination of individuals' von Neumann-Morgenstern utility functions if: (i) society maximizes expected social welfare; (ii) individuals maximize expected utility; (iii) society is indifferent between two probability distributions whenever all individuals are (Pareto principle). The result depends on the fact that, behind the "veil of ignorance", aversion to unequal distribution of wealth is identical to risk aversion.

Example 1. Ex-ante equity.

Assume that there are only two individuals, and a good to be split. Under lottery A , the social planner gives the good to one individual; under lottery B , he flips a coin. The distribution of utilities is represented by the following matrices, where rows stand for individuals, and columns for states with equal probabilities. Each cell describes the utility of an agent in a given state of the world:

$$A: \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad B: \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The utilitarian social planner is indifferent between the two policies, as they yield the same social welfare of 1. [Diamond \(1967\)](#) pointed out that a social planner with equity concerns should prefer lottery B to lottery A , as flipping a coin is fairer than handing the prize to only one agent. Indeed, in lottery B each agent gets 0.5 in expected terms, while in lottery A one agent always gets 1 and the other 0.

Example 2. Ex-post equity.

As before, assume that there are two agents and a good to be split. The social planner must decide whether to equally divide the good between the two agents (policy A') or to toss a coin (policy B). The distribution of utilities is described by the following matrices:

$$A': \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \quad B: \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The utilitarian social planner is indifferent between the two lotteries, as social welfare is always equal to 1. However, in lottery A' each agent gets 0.5 no matter the state of the world, while in lottery B one agent gets 1 and the other 0 in each state of the world. Splitting the good in half seems fairer than tossing a coin.

As the examples illustrate, utilitarianism fails to fully capture society's aversion to inequality. There are two ways in which a concern for equity can be restored, according to whether the social planner is interested in granting everyone equal chances (concern for the distribution of expected utilities, as in example 1) or equal outcomes (concern for the distribution of realized utilities, as in example 2). The social planner averse to ex ante inequality cares about the difference in expected utilities ex ante, before the risk is resolved. In contrast, the decision

maker averse to ex-post inequality cares about the difference in utilities ex-post, once the risk is resolved. The ex-ante and the ex-post approaches express different concepts of equity, and they do not always lead to the same evaluation of risks. For instance, in example 1, the social planner concerned with ex-ante equity prefers lottery B to lottery A ; in contrast, the social planner averse to ex-post inequality is indifferent between the two lotteries, as only one agent gets the good, whatever state of the world occurs. Similarly, in example 2, the social planner with concerns for the realized distribution of utilities prefers lottery A' to lottery B ; instead, the social planner concerned with ex-ante equity is indifferent, as individuals have the same expected utility in the two lotteries. There exists an extensive literature that analyzes the choice of the optimal social welfare criterion, and in particular the ex-ante vs the ex-post approaches (see e.g. [Adler and Sanchirico \(2006\)](#) for a review). In the paper, we take an agnostic view of the most suitable approach, as it requires ethical considerations. Instead, we aim at investigating how different attitudes to equity affect the evaluation of risky social projects.

Concerns for either ex-ante or ex-post equity are mathematically described through the use of a social welfare function, that maps individual (realized or expected) utilities into scalars. In the paper, we will constrain our attention to a prioritarian social welfare function³, which corresponds to the introduction of an increasing, strictly concave function of individual (realized or expected) utilities. The convexity of the transforming function, instead, allows to study anti-prioritarian behaviors, i.e. cases in which the social planner gives priority to the best-off. Note that there exist other types of equity-regarding social welfare functions (e.g. rank-weighted and leximin social welfare functions), that will not be analyzed in the paper (see [Adler \(2012\)](#) for a review).

When the social planner cares about the distribution of expected well-being (ex-ante or "Equality of Prospects" approach, which was advocated, e.g., by [Diamond \(1967\)](#), [Epstein and Segal \(1992\)](#)), social welfare is measured by the certainty equivalent W^{EoP} of a transformation of individual expected utilities:

$$\phi(W^{EoP}) = \int_{\mathbb{N}} \phi \left(\int_{\mathbb{S}} u(c(i, s)) dP(s|i) \right) dQ(i) \quad (1.2)$$

³Prioritarianism is a moral view, developed by contemporary philosophers [Parfit \(2000\)](#) and [Nagel \(1995\)](#), that gives greater social weight to increments in well-being that are realized by the worst off individuals. Mathematically, it corresponds indeed to a concave transformation of individual utilities.

where ϕ is an increasing function⁴, representing society’s attitude towards ex-ante inequality in the distribution of well-being. A linear function ϕ means that the decision maker is neutral to ex-ante well-being inequality, as her preferences simplify to the utilitarian social welfare function. In contrast, a concave ϕ is equivalent to ex-ante well-being inequality aversion. In other words, the social planner dislikes mean-preserving spreads in individual expected utilities. Finally, the convexity of the function ϕ stands for ex-ante well-being equality aversion. The ex-ante approach satisfies individual preferences over lotteries (Pareto principle), but, unlike utilitarianism, it does not have the form of an expected social utility. The consequence is the violation of principles of stochastic dominance (Adler, 2012)⁵, and in particular the sure-thing principle⁶.

In the ex-post approach (endorsed e.g. by Hammond (1983), Broome (1991) and Adler (2012)), society cares about the actual inequalities created by risk taking. Among the many functional forms proposed in the literature⁷, we will analyze an *Expected Equally Distributed Equivalent* criterion, proposed by Fleurbaey (2010), where the social planner first computes a state-dependent social welfare function, and then aggregates over states of the world:

$$W^{EEDE} = \int_{\mathbb{S}} \phi^{-1} \left[\int_{\mathbb{N}} \phi(u(c(i, s))) dQ(i|s) \right] dP(s) \quad (1.3)$$

⁴Grant et al. (2010) provide an axiomatic foundation of (1.2) based on an extension of Harsanyi’s theorem and called ”generalized utilitarianism”.

⁵Consider the following example:

$$A : \begin{pmatrix} 9 & 1 \\ 1 & 9 \end{pmatrix} \quad B : \begin{pmatrix} 5 - \epsilon & 5 - \epsilon \\ 5 - \epsilon & 5 - \epsilon \end{pmatrix}$$

If we fix the state, the distribution of individual utilities in B is fairer than the one in A : any continuous social welfare function will rank the vectors $(9, 1)$ and $(1, 9)$ as worse than $(5 - \epsilon, 5 - \epsilon)$, with ϵ sufficiently small. Thus, all the outcomes of lottery B are ranked higher than all the outcomes of lottery A . Therefore, lottery B stochastically dominates lottery A . However, the Equality of Prospects approach will prefer A , as each individual expected utility is higher.

⁶The sure-thing principle (Savage, 1954) says that outcomes that occur regardless of the actions that are chosen should not affect the agent’s preferences. The violation of the principle is clear in Example 1. If state of nature 1 occurs, the results of lottery A and B are the same, as only the first agent gets the prize. Therefore, state 1 should not affect the final decision. If state of nature 2 occurs, the results of A and B are symmetric, and therefore indifferent. By the sure thing principle, the social planner should be indifferent between lottery A and lottery B . Instead, a social planner with ex-ante equity concerns is found to prefer lottery B .

⁷For instance, a common approach is to use the expectation of the sum of transformed utilities (see e.g. Adler and Sanichirico (2006)): $\int_{\mathbb{S}} \int_{\mathbb{N}} \phi(u(c(i, s))) dF(i, s)$. Unlike EEDE, this criterion does not satisfy a weak version of the Pareto principle. Indeed, individual ex-ante preferences are not respected even in the absence of inequalities, as the decision maker ends up maximizing the expected value of $\phi(u(c(i, s)))$ instead of $u(c(i, s))$, thereby implying a stronger risk aversion than the one justified by individual preferences. See Adler (2012) for a discussion of other functional forms of the social welfare function.

where ϕ is increasing⁸, and represents society's aversion towards ex-post inequality in well-being. When ϕ is linear, we are back to the utilitarian criterion. Instead, a concave ϕ stands for ex-post inequality aversion, meaning that the social planner dislikes state-dependent mean-preserving spreads in individual utilities. The opposite with a convex ϕ . Contrary to the EoP approach, the EEDE criterion retains the form of an expected social utility (thus dominance principles are fulfilled), but satisfies only a weak version of Pareto. Indeed, individual ex-ante preferences are respected only when no inequality arises, as the social planner simply maximizes an individual expected utility. Otherwise, social preferences over lotteries are not necessarily aligned to individual preferences for lotteries⁹. Moreover, the EEDE criterion poses separability issues, as it is impossible to make separate evaluations for subgroups of the entire population. Utilitarianism and the Equality of Prospects approach, being linear in the probability distribution of agents, allow to rank lotteries independently of the well-being of individuals who are not affected as between them. In the EEDE approach, instead, also the utility of the unaffected matters, as the equally distributed equivalent in a state of nature depends on the whole vector of utilities in that state.

Function ϕ describes aversion to unequal distributions of well-being. In the monetary setting of the present paper, where the risk affects individual consumption levels, it is more natural to directly consider aversion to the unequal distribution of consumption. For this purpose, set $v = \phi \circ u$, where the function v represents attitudes toward income inequality, and the function u attitudes toward risk. Thus, when v is more concave than u , the social planner prefers an equal distribution of (expected or realized) well-being; when v is less concave than u , the social planner adopts an anti-prioritarian view. It is possible then to rewrite (1.2) and (1.3), respectively, as:

$$W^{EoP} = \int_{\mathbb{N}} v \left(u^{-1} \left(\int_{\mathbb{S}} u(c(i, s)) dP(s|i) \right) \right) dQ(i) \quad (1.4)$$

$$W^{EEDE} = \int_{\mathbb{S}} u \left(v^{-1} \left(\int_{\mathbb{N}} v(c(i, s)) dQ(i|s) \right) \right) dP(s) \quad (1.5)$$

Therefore, the EoP social criterion represents the average value of individual certainty equivalents. In a first stage each individual computes the certainty equivalent to the individual

⁸Grant et al. (2012) provide an axiomatization of expected equally-distributed equivalent-utility social welfare functions based on Harsanyi's theorem.

⁹Consider again the example described in footnote 5. For any ϵ sufficiently small, ex-ante Pareto superiority ranks lottery A as better than lottery B , while the EEDE formula will rank A as worse than B .

lottery he faces. Next, the social planner computes the expectation of the individual certainty equivalents, where each certainty equivalent is evaluated through the lenses of an inequality averse function v . In the EEDE approach, instead, we first determine the equally distributed equivalent (i.e. certainty equivalent for each state), and then we compute the expected value of the equally distributed equivalents. In the utilitarian case, either formulation works by definition with $v = u$.

Said differently, in the EoP welfare approach, the social planner is interested in equalizing individual certainty equivalents, which reflect the risk each individual faces separately. In the EEDE welfare approach, instead, the social planner is interested in the evaluation of the risk faced by an "average agent", where the "average" consumption levels correspond to the equally distributed equivalents of each state.

1.3 Social WTP for risk elimination

1.3.1 Definitions

We are interested in defining a rule to evaluate risky social choices, i.e. situations where the impacts of the risk are heterogeneous across agents (or, equivalently, agents are subject to heterogeneous individual risks). The most natural exercise will be to determine the willingness to pay for an investment that eliminates the social risk, and to show how it is affected by society's concerns for inequality and individuals' aversion to risk. Then, the investment project is implemented whenever the willingness to pay is larger than its cost, taken as exogenous.

Let us assume that each agent in the economy is endowed with income ω , and let \tilde{y} be an unfair social risk, distributed according to $F(i, s)$. Let $y(i, s)$ be the realization of the random variable \tilde{y} for agent i in state s , where $E\tilde{y} < 0$. Individual i 's consumption in state s is then equal to: $c(i, s) = \omega + y(i, s)$. As before, let $P(s)$ be the probability distribution of states of nature, and $Q(i)$ the distribution of agents. In the following, E will denote the expectations computed with the joint probability distribution function $F(i, s)$, E_i the expectations with respect to individuals (i.e. using $Q(i)$), and E_s the expectations with respect to the probability distribution of states $P(s)$. Moreover, $E[\cdot|i]$ and $E[\cdot|s]$ define the conditional expectations.

Since the social risk has heterogeneous impacts across agents, each individual faces a different risk. Individual i 's consumption is distributed as $P(s|i)$, where $E[\tilde{c}|i] \equiv \int c(i, s)dP(s|i) =$

$\omega + E[\tilde{y}|i] < \omega$. Alternatively, we may be interested in the allocation of consumption contingent to a particular state of nature s . Then, the presence of a social risk with heterogeneous impacts means that each state of nature is characterized by a different distribution of consumption. Consumption in state s is distributed among agents according to $Q(i|s)$, with $E[\tilde{c}|s] \equiv \int c(i, s)dQ(i|s) = \omega + E[\tilde{y}|s]$.

We define society's willingness to pay for the elimination of the unfair risk \tilde{y} as the reduction in consumption that agents are ready to bear in order to replace the initial lottery $\tilde{c} = \omega + \tilde{y}$ with the sure amount ω . As there is no inequality in initial income (each agent is endowed with ω), the implementation of this investment eliminates all sources of risk and inequality. In order to assess the value of this project, consider the following definition:

Definition 1. The *social premium* τ is the sure reduction in wealth such that the social planner would be indifferent between the risky situation yielding $\tilde{c} = \omega + \tilde{y}$, and the sure amount $\omega + E\tilde{y} - \tau$.

Then, given the definition of τ , we can recover the social willingness to pay (SWTP) for an investment that eliminates \tilde{y} . If $E\tilde{y} < 0$, SWTP will have two components: the expected social gains by eliminating the risk \tilde{y} and the social premium τ :

$$SWTP = -E\tilde{y} + \tau \quad (1.6)$$

Individuals' risk aversion and society's inequality aversion (either ex-ante or ex-post) affect the social willingness to pay through the social premium τ . Thus, for each welfare approach described in Section 2, we can define the corresponding social premium. In the utilitarian case, it satisfies the condition:

$$Eu(\omega + \tilde{y}) = u(\omega + E\tilde{y} - \tau^U(u, \omega, \tilde{y})) \quad (1.7)$$

The utilitarian social premium is a function of preferences towards risk (represented by u), initial income ω and social risk \tilde{y} . Similarly, we can define the social premia for the EoP and the EEDE cases:

$$E_i v(u^{-1}(E[u(\omega + \tilde{y})|i])) = v(\omega + E\tilde{y} - \tau^{EoP}(u, v, \omega, \tilde{y})) \quad (1.8)$$

and

$$E_s u(v^{-1}(E[v(\omega + \tilde{y})|s])) = u(\omega + E\tilde{y} - \tau^{EEDE}(u, v, \omega, \tilde{y})) \quad (1.9)$$

In contrast to the utilitarian social premium, social preferences towards inequality (represented by v) matters, as well. Thus, both the EoP and the EEDE social premia are a function of: preferences towards risk; preferences towards inequality; initial income; social risk. However, as the EoP and the EEDE approaches follow two different methods of aggregation of individual realized consumptions, their social premia will clearly differ as well.

Note that the amount $w + E\tilde{y} - \tau$ (which can be labelled social certainty equivalent) in the EoP and EEDE cases is viewed as the composition of two certainty equivalents. Under the EoP welfare approach (1.4), the social planner first determines the certainty equivalent of each individual, and then aggregates them through a social felicity function. Thus, the social certainty equivalent represents the equally distributed equivalent of individual certainty equivalents. Under the EEDE approach (1.5), instead, the social certainty equivalent corresponds to the certainty equivalent of equally distributed equivalents. Indeed, the social planner first determines the equally distributed equivalent consumption for each possible state, and then evaluates the expected utility driven by those equally distributed equivalents.

1.3.2 Decomposition of the social premium

In order to understand the difference between the three welfare approaches, it may be useful to identify the various components of the social premium. Since, by definition, the social premium removes both risk and inequality, I show that, in the presence of additive risks, it can be decomposed exactly into an inequality premium and a risk premium.

The inequality premium can be defined as the willingness to pay to eliminate inequality in the society. Assume that there is no individual risk: individual consumption is certain, although unequally distributed across agents, and let $c(i) = \omega + E[\tilde{y}|i]$ denote the consumption of agent i in the absence of risk, which is distributed according to $Q(i)$, and where

$$E[\tilde{y}|i] = \int_{\mathbf{s}} y(i, s) dP(s|i)$$

Then, we have the following definition:

Definition 2. Assume that individual consumption is given by $c(i) = \omega + E[\tilde{y}|i]$, where $E_i c(i) = \omega + E\tilde{y}$. The *inequality premium* χ is the maximum amount that society is willing to pay to eliminate inequality, and replace the unequal consumption $c(i)$ with the equal amount $\omega + E\tilde{y} - \chi$.

The inequality premium defines how much society cares about inequality in the expected realizations of individual risks. It will depend on social preferences towards equity and on the consumption inequality $E[\tilde{y}|i]$. In the utilitarian case, the concavity of the utility function u defines both aversion to risk and aversion to inequality, since inequality looks like a second source of risk for the social planner behind the veil of ignorance. Mathematically:

$$E_i u(\omega + E[\tilde{y}|i]) = u(\omega + E\tilde{y} - \chi^U(u, \omega, E[\tilde{y}|i])) \quad (1.10)$$

In the EoP and EEDE approaches, preferences towards inequality are represented by the concavity of the function v . In the absence of risk, the inequality premia coincide, and are defined as:

$$E_i v(\omega + E[\tilde{y}|i]) = v(\omega + E\tilde{y} - \chi(v, \omega, E[\tilde{y}|i])) \quad (1.11)$$

It is easy to see that the inequality premium increases with respect to the utilitarian case if and only if v is more concave than u , i.e. the social planner is averse to an unequal distribution of well-being. Note that, if v is a constant relative inequality aversion (CRIA) function¹⁰, the inequality premium corresponds to the Atkinson's inequality index (Atkinson, 1970).

So far, the inequality premium has been defined with respect to the expected consumption of agent i , i.e. $c(i) = \omega + E[\tilde{y}|i]$. However, we can introduce a contingent version of the inequality premium, that does not take into account the inequality in expected individual consumption, but the inequality in a specific state of nature. Let $c(s) = \omega + E[\tilde{y}|s]$ denote the average consumption in state s , where

$$E[\tilde{y}|s] = \int_{\mathbb{N}} y(i, s) dQ(i|s)$$

Definition 3. Let us consider the state of nature $s \in \mathbb{S}$. The *contingent inequality premium* χ_s is the maximum amount that society is willing to pay in state s to eliminate inequality in state s , and replace the unequal consumption $c(i, s)$ with the equal amount $\omega + E[\tilde{y}|s] - \chi_s$.

In order to compute the contingent inequality premium, we need to restrict attention to the social aggregation of individual utilities in a specific state of nature. For instance, in the

¹⁰A CRIA function can be defined as: $v(c) = \frac{c^{1-\varphi}}{1-\varphi}$, where φ represents the coefficient of relative inequality aversion.

utilitarian case the contingent inequality premium is defined as:

$$E[u(\omega + \tilde{y})|s] = u(\omega + E[\tilde{y}|s] - \chi_s^U(u, \omega + E[\tilde{y}|s], \tilde{y})) \quad (1.12)$$

Thus, the utilitarian inequality premium is a function of inequality preferences u , social risk \tilde{y} and average contingent consumption $\omega + E[\tilde{y}|s]$. As the EoP social planner cares about inequality in expected individual consumption, the contingent inequality premium plays no role. In the EEDE case, the contingent inequality premium is defined as:

$$E[v(\omega + \tilde{y})|s] = v(\omega + E[\tilde{y}|s] - \chi_s^{EEDE}(v, \omega + E[\tilde{y}|s], \tilde{y})) \quad (1.13)$$

Therefore, the EEDE contingent inequality premium is a function of inequality preferences v , social risk \tilde{y} and average contingent consumption $\omega + E[\tilde{y}|s]$. As before, if v is more concave than u , for each $s \in \mathbb{S}$, the EEDE contingent inequality premium is larger than the utilitarian one.

The following Proposition describes more precisely the relationship that exists between the inequality premia just defined and the social premium. The proof is relegated into the Appendix. Let $\pi_i(u, \omega, \tilde{y})$ denote the standard individual risk premium, defined in the expected utility framework as the amount of money that satisfies $E[u(\omega + \tilde{y})|i] = u(\omega + E[\tilde{y}|i] - \pi_i)$. Moreover, let $\pi(u, \omega, E[\tilde{y}|s])$ be the risk premium in the absence of inequality, which represents the willingness to pay of the representative agent to eliminate risk: $E_s u(\omega + E[\tilde{y}|s]) = u(\omega + E\tilde{y} - \pi)$, where $E[\tilde{y}|s]$ is distributed as $P(s)$. Then, we can show that:

Proposition 1. *For any functions u and v , and for any distributions \tilde{y} , $E[\tilde{y}|s]$ and $E[\tilde{y}|i]$, we have:*

$$\begin{aligned} \tau^{EoP}(u, v, \omega, \tilde{y}) &= E_i \pi_i(u, \omega, \tilde{y}) + \chi(v, \omega, E[\tilde{y}|i] - E_i \pi_i) \\ \tau^{EEDE}(u, v, \omega, \tilde{y}) &= E_s \chi_s(v, \omega, \tilde{y}) + \pi(u, \omega, E[\tilde{y}|s] - E_s \chi_s) \end{aligned}$$

In the utilitarian case, either decomposition holds.

Hence, the EoP social premium is equal to the sum of the average individual risk premium and the amount the social planner would pay to remove the resulting inequality after that each agent has paid π_i to eliminate the risk. In contrast, in the EEDE case the maximum share of income the social planner is willing to pay to remove the social risk is equal to the average willingness to pay to remove contingent inequality and the society's willingness to pay to eliminate the resulting risk.

Individual	States			
	s_1	s_2	s_3	s_4
Ann	1	1	0	0
Bob	1	0	1	0
Prob of states	q_1	q_2	q_3	q_4

Table 1.1: Example of social risk. Assumptions: $i = \{Ann, Bob\}$, $\omega = 1$ and $l = \omega$. Individual consumption risk: $c_i \sim (0, p_i; 1, 1 - p_i)$.

Example

Suppose that there are only two agents in the economy, labelled *Ann* and *Bob*; each of them faces the risk of suffering an income loss l with probability p_i , with $i = \{Ann, Bob\}$. Assume for simplicity that initial wealth ω is normalized to 1, and that $l = \omega$. If individual risks are independent, there are three possible states: (i) both agents suffer a loss, with probability $q_4 = p_{Ann}p_{Bob}$; (ii) with probability $q_1 = (1 - p_{Ann})(1 - p_{Bob})$, no one bears a loss; (iii) with the complementary probability $q_2 + q_3$, only one agent suffers a loss. If the social planner cares about inequality in the distribution of expected utilities (EoP approach), the social premium can be determined as follows. First of all, each agent expresses her/his willingness to pay to remove her/his own risk, that is the amount (s)he is ready to give up to replace the individual lottery $c_i \sim (0, p_i; 1, 1 - p_i)$ with the expected individual consumption $1 - p_i$. Once paid the individual risk premium, agent i ends up consuming $1 - p_i - \pi_i$. However, as the social planner is averse to an unequal distribution of consumption, she will impose an additional tax to eliminate inequality. As a result, each agent will consume $1 - \frac{1}{2}(p_{Ann} + p_{Bob}) - \frac{1}{2}(\pi_{Ann} + \pi_{Bob}) - \chi^{EoP}$.

In the EEDE case, instead, the decomposition works in the opposite way. As the social planner cares about the inequality existing in each state of nature s , she first determines the contingent tax to eliminate inequality (which allows a redistribution of resources). In the example, three states are possible. If state 1 occurs, each agent has 1; thus, $\chi_1 = 0$; similarly, if state 4 occurs, each agent consumes 0, thereby $\chi_4 = 0$. If either state 2 or state 3 occurs, one agent consumes 1, the other 0. The inequality averse social planner prefers to have both agents consume $1 - \frac{1}{2} - \chi$, rather than having agents with asymmetric wealth. In such a way, we end up with a representative agent problem: with probability q_1 there is no loss; with probability q_4 , she bears the loss $l = 1$; with the complementary probability,

she bears the loss $\frac{1}{2} + \chi$. If the social planner is risk averse, she will be willing to pay a premium π to eliminate the risk in her consumption. Each agent in the economy will end up consuming the same amount $1 - \frac{1}{2}(p_{Ann} + p_{Bob}) - (q_2 + q_3)\chi - \pi$.

1.4 Quadratic approximation

To get more insights, we can consider the local approximations of the social premium. Assume that the social risk \tilde{y} is small, meaning that its variance, denoted as σ_y^2 , tends to zero. If the social planner is utilitarian, she will consider inequality as an additional source of risk. In other words, under the veil of ignorance risk and inequality are treated in the same way, and all what the social planner cares about is the total variability in consumption.

Let $A_u(w) \equiv -\frac{u''(w)}{u'(w)}$, with $w \equiv \omega + E\tilde{y}$, denote the Arrow-Pratt coefficient of absolute risk aversion (which, in the utilitarian case, denotes also aversion to consumption inequality). The utilitarian social premium for a small social risk is defined as follows:

Lemma 1. *Let the social risk \tilde{y} be small. The utilitarian social premium is approximately equal to:*

$$\tau^U(u, w, \tilde{y}) \simeq A_u(w) \frac{\sigma_y^2}{2} \quad (1.14)$$

With utilitarian social preferences, the index of risk aversion indicates also the coefficient of inequality aversion. On the contrary, in the EoP and EEDE approaches, aversion to consumption inequality depends on the concavity of the function $v = \phi(u)$. Then, the function v summarizes social attitudes towards inequality in consumption, and it is affected both by individuals' aversion to consumption inequality (represented by u) and by society's aversion to well-being inequality (represented by ϕ).

Let $A_v(w) \equiv -\frac{v''(w)}{v'(w)}$ represent the coefficient of absolute inequality aversion, and $\sigma_{E[y|i]}^2$ the variance of individual expected consumption $E[y|i] = \int_{\mathbb{S}} y(i, s) dP(s|i)$:

$$\sigma_{E[y|i]}^2 = \int_{\mathbb{N}} \left[\int_{\mathbb{S}} y(i, s) dP(s|i) \right]^2 Q(i) - \left[\int y(i, s) dF(i, s) \right]^2$$

This variance reflects the degree of inequality in the distribution of consumption from an ex-ante point of view. We can now state the second order approximation of the social premium under the Equality of Prospects welfare criterion:

Proposition 2. *Let the social risk \tilde{y} be small. The EoP social premium is approximately equal to:*

$$\tau^{EoP}(u, \phi, w, \tilde{y}) \simeq A_u(w) \frac{\sigma_y^2}{2} + [A_v(w) - A_u(w)] \frac{\sigma_{E[y|i]}^2}{2} \quad (1.15)$$

The first component of the EoP social premium corresponds to the Arrow-Pratt approximation of the utilitarian social premium. Instead, the second component represents the effect of ex-ante well-being inequality aversion. The sign and magnitude of this effect depends on

$$A_v(w) - A_u(w) \equiv A_\phi(u(w))u'(w)$$

In particular, provided that inequality affects the individual expected consumption, that is $\sigma_{E[y|i]}^2 \neq 0$, well-being inequality aversion has no effects when the decision maker is neutral to it at w , i.e. when $A_v(w) = A_u(w)$. Moreover, by definition, if individual expected consumption is equally distributed among agents, i.e. $\sigma_{E[y|i]}^2 = 0$, ex-ante well-being inequality aversion plays no role in the determination of the EoP social premium.

Let $E_i \sigma_{y_i}^2$ denotes the average variance in individual consumption risk:

$$E_i \sigma_{y_i}^2 = \int_{\mathbb{N}} \left(\int_{\mathbb{S}} y(i, s)^2 dP(s|i) - \left(\int_{\mathbb{S}} y(i, s) dP(s|i) \right)^2 \right) dQ(i)$$

Then, the variance of the social risk σ_y^2 can be decomposed along two sources of risk:

$$\sigma_y^2 = E \sigma_{y_i}^2 + \sigma_{E[y|i]}^2$$

These two terms capture, respectively, risk and inequality in the distribution of consumption from an ex-ante point of view. The approximation (1.15) can then be rearranged in the following way:

$$\tau^{EoP}(u, \phi, w, \tilde{y}) \simeq A_u(w) \frac{E_i \sigma_{y_i}^2}{2} + A_v(w) \frac{\sigma_{E[y|i]}^2}{2} \quad (1.16)$$

This formulation exactly reflects the decomposition of the social premium outlined in Proposition 1. The first term in (1.15) corresponds to the average individual risk premium, while the second term represents the inequality premium. Thus, risk aversion determines the social planner's reaction to the average variance in individual consumption $E_i \sigma_{y_i}^2$, and inequality aversion determines her reaction to the variance of expected individual consumption $\sigma_{E[y|i]}^2$.

Example. Let us consider again the example introduced in Table 1. To compute the EoP

social premium, we need information on individual risks and on the frequency of types in the population. Each agent i , with $i = \{Ann, Bob\}$, starts with income $\omega = 1$ and faces the individual consumption risk $y(i) \sim (-1, p_i; 0, 1 - p_i)$, whose expected value is $E[y|i] = -p_i$ and variance $\sigma_{y_i}^2 = p_i(1 - p_i)$. The degree of inequality in expected individual consumption is given by: $\sigma_{E[y|i]}^2 = 0.25(p_{Ann} - p_{Bob})^2$. Given that the social risk \tilde{y} is distributed as $\tilde{y} \sim (-1, 0.5(p_{Ann} + p_{Bob}); 0, 1 - 0.5(p_{Ann} + p_{Bob}))$, with mean $E\tilde{y} = -0.5(p_{Ann} + p_{Bob})$ and variance $\sigma_y^2 = 0.5(p_{Ann} + p_{Bob})(1 - 0.5(p_{Ann} + p_{Bob}))$, the EoP social premium becomes:

$$\tau^{EoP} \simeq \tau^U + [A_v(w) - A_u(w)] 0.125(p_{Ann} - p_{Bob})^2$$

where $w = 1 - 0.5(p_{Ann} + p_{Bob})$ and $\tau^U \simeq A_u(w)0.25(p_{Ann} + p_{Bob})(1 - 0.5(p_{Ann} + p_{Bob}))$.

In the EEDE approach, the social planner is averse to the distribution of outcomes contingent to a given state. Let $\sigma_{y_s}^2 \equiv Var(y|s)$ represent the degree of inequality in state s , and $E_s\sigma_{y_s}^2$ the expected contingent inequality:

$$E_s\sigma_{y_s}^2 = \int_{\mathbb{S}} \left(\int_{\mathbb{N}} y(i, s)^2 dQ(i|s) - \left(\int_{\mathbb{N}} y(i, s) dQ(i|s) \right)^2 \right) dP(s)$$

The quadratic approximation of the EEDE social premium yields the following result:

Proposition 3. *Let \tilde{y} be a small risk. The EEDE social premium is approximately equivalent to:*

$$\tau^{EEDE}(u, \phi, w, \tilde{y}) \simeq A_u(w) \frac{\sigma_y^2}{2} + (A_u(w) - A_v(w)) \frac{E_s\sigma_{y_s}^2}{2} \quad (1.17)$$

where the first term coincides with the utilitarian social premium, and the second term determines the effect of ex-post well-being inequality aversion. Provided that the social planner is well-being inequality averse at w , i.e. $A_v(w) > A_u(w)$, the EEDE social premium tends to the utilitarian one when there is no contingent inequality on average, i.e. $E_s\sigma_{y_s}^2 = 0$. Moreover, we can decompose the variance of \tilde{y} along the two sources of variability relevant for the EEDE approach:

$$\sigma_y^2 = E_s\sigma_{y_s}^2 + \sigma_{E[y|s]}^2$$

where $\sigma_{E[y|s]}^2$ denotes the variance of the average contingent individual consumption:

$$\sigma_{E[y|s]}^2 = \int_{\mathbb{S}} \left(\int_{\mathbb{N}} y(i, s) dQ(i|s) \right)^2 dP(s) - \left(\int_{\mathbb{N}} y(i, s) dF(i, s) \right)^2$$

Thus, approximation (1.17) can be rearranged according to the Arrow-Pratt coefficients of u and v :

$$\tau^{EEDE}(u, v, w, \tilde{y}) \simeq A_u(w) \frac{\sigma_{E[y|s]}^2}{2} + A_v(w) \frac{E_s \sigma_{y_s}^2}{2} \quad (1.18)$$

This formulation suggests that risk aversion determines the social planner's reaction to the variance of average contingent individual consumption, while inequality aversion determines her reaction to the average contingent variance. Thus, the first component denotes the risk premium of the representative agent, while the second the expected contingent inequality premium.

Example. Let us calculate the quadratic approximation for the example in Table 1. Unlike in the EoP case, we need information on which states of the world will realize and on the distribution of income in each state of the world. Earlier, we assumed that individual risks were independent. With probability $q_1 + q_2$, the distribution of income between agents is characterized by: $c(s = 2, 3) \sim (0, 0.5; 1, 0.5)$. Average consumption in either state 2 or state 3 is $E[c|s = 2, 3] = 1 - 0.5$, and the variability is given by: $\sigma_{y(s=2,3)}^2 = 0.25$. In states 1 and 4, instead, there is no inequality. As a consequence, the expected contingent inequality is $E\sigma_{y(s)}^2 = 0.25(q_2 + q_3)$ and the EEDE social premium becomes:

$$\tau^{EEDE} \simeq \tau^U + [A_v(w) - A_u(w)] 0.125(q_2 + q_3)$$

1.4.1 Comparison between approaches

From Proposition 2 and 3 we can clearly see that both the EoP and the EEDE approximated social premia are larger than the utilitarian one as long as $A_v(w) \geq A_u(w)$. Therefore, if the social planner is averse to an unequal distribution of (expected or realized) well-being, she will be willing to pay more for the elimination of a social risk. Undoubtedly, it is a practical matter whether society is more inequality averse than individuals are risk averse. As we will see in the application section, the literature that tries to elicit equity preferences is quite pessimistic on the degree of inequality aversion inside our society, thereby suggesting that A_v is indeed lower than A_u .

While both the EoP and the EEDE social premia can be easily contrast to the utilitarian one, the comparison between the first two is not straightforward. However, by rearranging

(1.18) we get the following:

$$\tau^{EEDE}(u, v, w, \tilde{y}) \simeq \tau^{EoP} + \frac{1}{2}(A_v(w) - A_u(w)) (E_i \sigma_{y_i}^2 - \sigma_{E[y|s]}^2) \quad (1.19)$$

Therefore, the EEDE social premium is larger than the EoP one as long as: inequality aversion is larger than risk aversion, and the average variance of individual risks is larger than the variance of average consumption. The second term should be positive, as we are comparing the average individual variance with the variance of the representative agent. Given that in $\sigma_{E[y|s]}^2$ some risk sharing has already taken place, the sum of individual risks should be riskier than the risk faced by a single representative agent. Indeed, all the examples I could think about point in that direction. Let us consider few significant cases.

Case 1. Identical and independent individual risks. Assume that there are N types in the population, whose distribution is discrete and uniform, and that individual risks are identically and independently distributed across agents, with mean μ and variance σ^2 . As there is no heterogeneity in individual risks, $\frac{1}{N} \sum_i \text{Var}(y_i) = \sigma^2$, while $\text{Var}(E[y|i]) = 0$. Thus, $\tau^{EoP} = \tau^U \simeq 0.5A_u\sigma^2$. In each possible state of the world, the average realization of the risk is given by: $E[y|s] = \frac{1}{N} \sum_i y_i$. Given that individual risks are independently distributed, the variance of the sample mean will be equal to: $\text{Var}(\frac{1}{N} \sum_i y_i) = \frac{1}{N^2} \sum_i \text{Var}(y_i)$. As a consequence, $\tau^{EEDE} \simeq \tau^{EoP} + 0.5(A_v - A_u)\frac{N-1}{N}\sigma^2$.

As an example, consider the lottery outlined in Table 1, under the assumption that $p_{Ann} = p_{Bob} = p$. Then, the variance of individual risk is $\sigma^2 = p(1-p)$, which yields $\tau^{EoP} = \tau^U \simeq 0.5A_u p(1-p)$. The per-state mean is distributed as $\frac{1}{N} \sum_i y_i \sim (0, (1-p)^2; -0.5, 2p(1-p); -1, p^2)$, and its variance is equal to $\text{Var}(\frac{1}{N} \sum_i y_i) = 0.5p(1-p)$. Therefore, $\tau^{EEDE} \simeq \tau^{EoP} + 0.5(A_v - A_u)0.5p(1-p)$.

Moreover, note that when the number of individuals grows very large, $N \rightarrow \infty$, the EEDE social premium becomes: $\tau^{EEDE} \simeq 0.5A_v\sigma^2$. Then, the comparison with the EoP social planner depends exclusively on the difference between inequality aversion and risk aversion.

Case 2. Identical and correlated individual risks. As before, assume that individual risks are identically distributed across agents, with mean μ and variance σ^2 . Let ρ represents the average correlation between individual risks. As there is no heterogeneity in individual risks, $\tau^{EoP} = \tau^U \simeq 0.5A_u\sigma^2$. In contrast, if individual risks are correlated, the variance of

the state mean will be equal to $Var\left(\frac{1}{N}\sum_i y_i\right) = \frac{1}{N^2}\sum_i Var(y_i) + \frac{1}{N^2}\sum_i \sum_{j \neq i} Cov(y_i, y_j) = \frac{\sigma^2}{N} + \frac{N-1}{N}\sigma^2\rho$. As a consequence, $\tau^{EEDE} \simeq \tau^{EoP} + 0.5(A_v - A_u)\frac{N-1}{N}(1 - \rho)\sigma^2$.

Consider again the lottery in Table 1, and suppose that individual risks are correlated. The probabilities of states as a function of the coefficient of correlation ρ ¹¹ are given by: $q_1 = (1 - p)[1 - p(1 - \rho)]$; $q_2 + q_3 = 2p(1 - p)(1 - \rho)$ and $q_4 = p[p + r(1 - p)]$ (see Lucas (1995) for details). By computing the variance of the state means, we indeed find that: $Var\left(\frac{1}{N}\sum_i y_i\right) = 0.5p(1 - p)(1 - \rho)$. For instance, when $\rho = 0$, we are back to case 1. If $\rho = 1$, only states 1 and 4 occur, where there is no inequality in the distribution of consumption, which implies that $\tau^{EEDE} = \tau^{EoP} = \tau^U$. Instead, if $\rho = -1$, only states 2 and 3 occur: although agents face the same risk, eventually only one will suffer a loss. In that case, the social planner is ready to pay an extra amount to eliminate also the realized inequality.

Case 3. Heterogeneous individual means. If individual risks are not identically distributed among agents, the EoP social premium will be larger than the utilitarian one, while the positive difference between EEDE and EoP social premia still persists. For instance, let us assume that individual risks have the same variance σ^2 , but different means μ_i . Then, the EoP social premium becomes $\tau^{EoP} \simeq 0.5A_u\sigma^2 + 0.5A_v Var(\mu_i)$. Instead, the difference between τ^{EEDE} and τ^{EoP} is unaffected by the heterogeneity in individual means: $\tau^{EEDE} \simeq \tau^{EoP} + 0.5(A_v - A_u)\frac{N-1}{N}(1 - \rho)\sigma^2$.

Case 4. Heterogeneous individual variances. Now, suppose, instead, that individual risks have the same expected value, but different variances, and let σ_i^2 the variance of agent i . First of all, note that heterogeneity in individual variances does not affect the inequality component of the EoP social premium, but only the risk one: if agents have the same expected consumption but different variances, the EoP social planner behaves as an utilitarian one: $\tau^{EoP} = \tau^U \simeq 0.5A_u E_i \sigma_{y_i}^2$. Heterogeneity in individual variances, instead, matters for the inequality component of the EEDE social premium. In particular, the same formulas for $Var\left(\frac{1}{N}\sum_i y_i\right)$ as in case 1 and 2 apply, by replacing the uniform variance σ^2 with the average one $E_i \sigma_{y_i}^2$.

Case 5. Heterogeneous individual risks. Finally, the last case occur when individual risks differ in both expected value and variance. The positive sign of the difference $E_i \sigma_{y_i}^2 - \sigma_{E[y|s]}^2$ still holds, as we just need to combine the conditions previously found.

¹¹The coefficient of correlation ρ is defined as $\frac{Cov(y_{Ann}, y_{Bob})}{\sqrt{Var(y_{Ann})Var(y_{Bob})}}$

Think for instance at the example outlined in Table 1, and assume that only *Bob* faces a risk, i.e. $p_{Ann} = 0$ and $p_{Bob} = p$. Then, only states 1 and 2 can occur. The average individual variance will be $E_i \sigma_{y_i}^2 = 0.5p(1-p)$, while the variance of expected risk is $Var(E[y|i]) = 0.25p^2$. Thus, the EoP social premium will be equal to: $\tau^{EoP} \simeq 0.25A_u p(1-p) + 0.125A_v p^2$, which is larger than the utilitarian one: $\tau^U \simeq 0.25A_u p(2-p)$ if and only if $A_v \geq A_u$. Moreover, the variance of state means is equal to $\sigma_{E[y|s]}^2 = 0.25p(1-p)$, which implies that $\tau^{EEDE} \simeq \tau^{EoP} + (A_v - A_u)0.125p(1-p)$ ¹².

1.4.2 Income inequality

So far, I have assumed that individuals are endowed with the same initial wealth ω . When we introduce income inequality, the sign and size of the social premium will depend on the correlation between initial wealth and social risk, i.e. whether the social risk hits more heavily the poor or the rich. However, the definition of the social premium and its decomposition into an inequality premium and a risk premium (carefully designed according to the welfare approach considered) carry over in the presence of heterogeneous initial income.

Let individual i 's consumption in state s be equal to $c(i, s) = \omega(i) + y(i, s)$, where the random variable $\tilde{c} = \tilde{\omega} + \tilde{y}$ is distributed as $F(i, s)$. The social premium $\tau(\tilde{c} \rightarrow \tilde{\omega})$ is defined as the amount that society is willing to pay to eliminate the social risk \tilde{y} , and replace the lottery \tilde{c} with the lottery $\tilde{\omega}$. In the utilitarian case, it satisfies the condition:

$$Eu(\tilde{\omega} + \tilde{y}) = E_i u(\tilde{\omega} + E\tilde{y} - \tau^U)$$

The utilitarian social premium is a function of preferences toward risk, inequality in the initial distribution of income $\tilde{\omega}$, and the social risk \tilde{y} . In the EoP and EEDE approaches, instead, preferences towards inequality matter as well. Thus, the social premia for the EoP and EEDE criteria can be defined, respectively, as:

$$E_i v(u^{-1}(E[u(\tilde{\omega} + \tilde{y})|i])) = E_i v(\tilde{\omega} + E\tilde{y} - \tau^{EoP})$$

and

$$E_s u(v^{-1}(E[v(\tilde{\omega} + \tilde{y})|s])) = u(v^{-1}(E_i v(\tilde{\omega} + E\tilde{y} - \tau^{EEDE})))$$

¹²Note that if both individuals are exposed to the risk, the joint probabilities are given by: $q_1 = (1 - p_A)(1 - p_B) + \rho\sqrt{p_A(1 - p_A)p_B(1 - p_B)}$, $q_2 = p_A(1 - p_B) - \rho\sqrt{p_A(1 - p_A)p_B(1 - p_B)}$, $q_3 = p_B(1 - p_A) - \rho\sqrt{p_A(1 - p_A)p_B(1 - p_B)}$, and, finally, $q_4 = p_A p_B + \rho\sqrt{p_A(1 - p_A)p_B(1 - p_B)}$.

It is easy to see that the decomposition illustrated in Section 3.2 still holds.

Let $\sigma_{\tilde{\omega}}^2$ denote the variability in initial income, and $Cov(\omega, y)$ the covariance between initial income ω and the risk \tilde{y} . If the covariance term is null, it means that every agent faces the same risk \tilde{y} , irrespectively of the size of initial resources $\tilde{\omega}$. On the contrary, if the covariance is positive, then rich agents have rosier expectations about the risk \tilde{y} . In other words, if the risk entails an individual loss in expected terms, poor agents will bear larger losses. Finally, if the covariance is negative, rich agents are more negatively hit by the risk. The following proposition states the local approximation of the social premia in the three different welfare approaches when the consumption risk is small (i.e. both the social risk \tilde{y} and inequality in income, represented by $\tilde{\omega}$, are small).

Proposition 4. *Assume that the consumption risk $\tilde{c} = \tilde{\omega} + \tilde{y}$ is small. The quadratic approximation of the social premia is given by:*

- *Utilitarianism:*

$$\tau^U \simeq \frac{1}{2}A_u\sigma_y^2 + A_uCov(\omega, y) \quad (1.20)$$

- *Equality of prospects approach:*

$$\tau^{EoP} \simeq \tau^U + \frac{1}{2}(A_v - A_u) (\sigma_{E[y|i]}^2 + 2Cov(\omega, y)) \quad (1.21)$$

- *Equally Distributed Equivalent approach:*

$$\tau^{EEDE} \simeq \tau^U + \frac{1}{2}(A_v - A_u) (E_s\sigma_{y_s}^2 + 2Cov(\omega, y)) \quad (1.22)$$

If there is no income inequality, i.e. $\tilde{\omega}$ is a degenerate random variable, we recover exactly the approximations described in Lemma 1 and Proposition 2 and 3. When the initial income and the social risk are correlated, the sign of the utilitarian social premium depends on the type of correlation. If the correlation is positive, the social premium is also positive. If the correlation is negative, the social planner may be unwilling to pay for the elimination of the risk. If there is a negative statistical dependence between income ω and social risk y , it means that individuals with low wealth $\omega(i)$ face a smaller risk. In other words, the social risk provides partial insurance against the inequality components, and its elimination has a negative impact on welfare. Moreover, if income and risk are positively correlated, the presence of larger inequality aversion increases the social premium. Indeed, the EoP and the EEDE premia are both larger than the utilitarian one as long as $A_v \geq A_u$. On the contrary,

if income and risk are negatively correlated, the presence of larger inequality aversion may decrease the social premium ¹³.

In addition, the difference between τ^{EoP} and τ^{EEDE} is independent of the correlation between initial income and the social risk. Indeed, the EEDE social premium can be rewritten as:

$$\tau^{EEDE}(u, v, w, \tilde{c}) \simeq \tau^{EoP} + \frac{1}{2}(A_v(w) - A_u(w)) (E_i \sigma_{y_i}^2 - \sigma_{E[y|s]}^2)$$

Note that the term in parenthesis is unaffected by the inequality in initial income. As each agent i has the same wealth $\omega(i)$ in each state of nature, both the variance of individual consumption risk and the variance of the state mean depend exclusively on the variability of the social risk \tilde{y} . As a consequence, the EEDE social premium is always larger than the EoP one, as long as $A_v \geq A_u$, if we take for granted that the term in parenthesis is positive, as argued in the previous section. If the correlation between initial income and social risk is positive, the EEDE social planner is ready to pay more than the EoP one for the elimination of the social risk. In contrast, if the correlation is sufficiently negative, both the EEDE and the EoP decision makers must be subsidized for the elimination of the risk (i.e. the social premium becomes negative), but the EEDE social planner is willing to accept a lower subsidy than the EoP one. The EEDE criterion reacts more strongly than the EoP to a positive correlation, and less strongly than the EoP to a negative correlation.

Finally, note that if individual risks have the same means, i.e. $\sigma_{E[y|i]}^2 = 0$, the presence of inequality in initial income has no effect on the approximated social premium, independently of the social criterion adopted. Indeed, we are back to the quadratic approximations described in Lemma 1 and Proposition 2 and 3. The result does not necessarily hold in the large.

1.5 The social premium in the large

From Pratt (1964), we know that higher risk aversion is associated to a larger premium not only for small risks, but also for ordinary risks. This section explores whether the same results

¹³ Take for instance the EoP case. When the correlation is negative but sufficiently low, i.e. $Cov(y, \omega) \leq B_1 \equiv -0.5\sigma_{E[y|i]}^2$, both premia can still be positive, although the utilitarian social premium will be larger than the EoP one. Similarly for the EEDE case. If $Cov(y, \omega) \leq B_2 \equiv -0.5E\sigma_{c_s}^2$, both the EEDE and the utilitarian premia may be positive, but the utilitarian premium will be larger than the EEDE one. Furthermore, if the covariance term lies in the range $[B_2, B_1]$, with $|B_2| > |B_1|$, the utilitarian social planner pays more than the EoP one for the elimination of the social risk, but less than the EEDE decision maker.

hold also in the presence of equity concerns, expressed by either an Equality of Prospects welfare criterion or by an Expected Equally Distributed Equivalent one.

We can easily prove that, if income ω is homogeneously distributed among agents, an increase in inequality aversion leads to an increase in the social premium.

Proposition 5. *Let $A_u(c)$ and $A_v(c)$ denote the coefficients of, respectively, local risk and inequality aversion, and assume that there is no income inequality: $\tilde{c} = \omega + \tilde{y}$. Then, the following two conditions are equivalent:*

1. $A_v(c) \geq A_u(c)$ for all c
2. the EoP and the EEDE social planners are ready to pay more than the utilitarian one for the removal of the social risk \tilde{y} .

Proposition 5 can be applied not only to compare utilitarian vs EoP/EEDE approach, but also, within the EoP and the EEDE cases, to compare two social planners with different coefficients of absolute inequality aversion.

Corollary 1. *Assume that there is no income inequality, and let $A_v^k(c)$, τ_k^{EoP} , τ_k^{EEDE} be the local inequality aversion and social premia corresponding to the welfare function v_k , with $k = a, b$. Then, the following conditions are equivalent:*

1. $A_v^a(c) \geq A_v^b(c)$ for all c
2. $\tau_a^{EoP} \geq \tau_b^{EoP}$ and $\tau_a^{EEDE} \geq \tau_b^{EEDE}$

Observe that, with an EoP welfare function, only the inequality premium is affected by an increase in inequality aversion. On the contrary, with an EEDE welfare approach both the contingent inequality premia and the risk premium change. An increase in inequality aversion does increase the contingent inequality premium, thereby reducing the average consumption $c_s = \omega + E[\tilde{y}|s] - \chi_s$ in every state. Therefore, an increase in inequality aversion induces a second order stochastically dominated shift in the distribution of average consumption across states of nature. As the utility function u is increasing and concave, an SSD shift increases the risk premium. As a consequence, compared to the utilitarian approach, the EEDE social premium increases due to an increase in both the contingent inequality premium and the risk premium.

In contrast, a comparison in the large between the EEDE and the EoP approaches yields to ambiguous results, since, as we have seen, it depends on both the difference between

risk aversion and inequality aversion, and on the trade-off between realized inequality and expected inequality.

So far, we have been able to study the comparison between utilitarian and EoP/EEDE criteria in the large only for the case of homogeneous initial income. Indeed, the presence of heterogeneous initial income $\omega(i)$ has two effects on the comparison between the EoP/EEDE social premia and the utilitarian one, which make the analysis highly difficult. First of all, the type of correlation between background wealth and social risk matters. In particular, if the correlation is negative (i.e. poor agents face a smaller risk), the social premium may even be negative, and the EoP/EEDE premia lower than the utilitarian one. Second, paraphrasing the classical results in risk theory, the fact that a decision maker is always willing to pay more than another for the complete removal of inequality does not imply that he is ready to pay more for any marginal reduction of inequality. Thus, a priori, an increase in Arrow-Pratt inequality aversion in the EoP and EEDE cases compared to the utilitarian one does not necessarily lead to a larger social premium (see, for instance, the discussion in [Ross \(1981\)](#) on the effects of a change in risk)¹⁴.

¹⁴Take for instance the EoP approach. As the social premium in the EoP approach is equivalent to the sum of the average individual risk premium and an inequality premium, a change in the preferences for equity affects exclusively the inequality premium component. In the presence of heterogeneous income $\omega(i)$, such an inequality premium is defined as the amount χ^{EoP} that society is ready to pay to reduce the inequality in individual certainty equivalents:

$$E_i v(\omega(i) + E[\tilde{y}|i]) - \pi(i) = E_i v(\omega(i) + E\tilde{y} - E\pi(i) - \chi^{EoP})$$

Note that the individual certainty equivalent can be rewritten as: $\omega(i) + E[\tilde{y}|i] - \pi(i) \equiv \omega(i) + \epsilon(i)$. Therefore, the inequality premium χ^{EoP} measures society's willingness to pay to replace lottery $\tilde{\omega} + \tilde{\epsilon}$ with lottery $\tilde{\omega}$. The random variable $\tilde{\epsilon}$ represents the value of the risk for agent i . The larger the realization $\epsilon(i)$, the less agent i suffers from the social risk (because either the expected value or the variance of its impacts on i are small). Thus, in order to see whether an increase in inequality aversion leads to a larger inequality premium we need to: assess how the distribution of individual expected consumption is affected by the removal of inequality; determine how the change in the distribution impacts the willingness to pay. From , we know when a mean-preserving reduction in risk induces a larger risk premium, while there is no general result in the literature for other types of changes in risk. The point is that the change in risk from $\tilde{\omega} + \tilde{\epsilon}$ hardly takes the form of a mean-preserving spread: if $\epsilon(i)$ increases in $\omega(i)$ (i.e. poor agents suffer more from the exposure to the social risk), a reduction in inequality corresponds to a first order stochastic dominant shift in the distribution of individual expected consumption; in contrast, if $\epsilon(i)$ decreases in $\omega(i)$, it is more difficult to determine the effects on the resulting distribution of income.

When it comes to the EEDE approach, things get even more complicated. First, we need to determine whether the contingent inequality premium is increasing in inequality aversion when we remove the inequality component of the social risk. All what we have just written about the inequality premium in the EoP case applies. Assume that the inequality premium is indeed larger in the EEDE case with respect to the utilitarian one. The problem, now, is that the change in inequality aversion will have an impact also on the risk premium, since the equally distributed equivalents on which the risk premium is computed depend on the welfare approach adopted. Thus, compared to the utilitarian case, when using an EEDE criterion we have both a change in inequality preferences and a change in risk.

1.6 Extensions

1.6.1 Proportional social risk

So far we have defined the social premium as a lump-sum uniform tax on income. However, the previous analysis works, with small changes, also for social premia that are proportional to income. Assume that the social risk is multiplicative rather than additive, i.e. final consumption \tilde{c} is the product of initial wealth $\omega(i)$ and a random factor \tilde{y} : $\tilde{c} = \tilde{\omega}\tilde{y}$. Define the proportional social premium $\hat{\tau}$ as the maximum share of income that society is ready to pay to avoid the risk of losing a random share of income. For instance, in the utilitarian case, the proportional social premium is given by:

$$Eu(\tilde{\omega}\tilde{y}) = E_i u((1 - \hat{\tau}^U)\tilde{\omega}E\tilde{y})$$

Then, the social willingness to pay for a proportional social risk will be equal to $(1 - \hat{\tau}^U)E\tilde{y}$. Similar definitions hold for the EoP and the EEDE welfare approaches. As earlier, we can decompose the social premium into an inequality component and a risk component, although it does not correspond any longer to their sum. Assume, for simplicity, that there is no income inequality. With an EoP social welfare function, the proportional social premium is such that:

$$1 - \hat{\tau}^{EoP}(u, v, \omega, \tilde{y}) = (1 - \hat{\chi}^{EoP}) \frac{E_i [(1 - \hat{\pi}_i)E[\tilde{y}|i]]}{E\tilde{y}}$$

where $\hat{\pi}_i(u, \omega, \tilde{y})$ represents the proportional individual risk premium, i.e. the share of individual income $\omega E[\tilde{y}|i]$ that agent i is ready to pay to eliminate the individual risk, while $\hat{\chi}^{EoP}(v, \omega(1 - \hat{\pi}_i)E[\tilde{y}|i], \tilde{y})$ denotes the proportional inequality premium, i.e. the share of certain income $\omega E_i[(1 - \hat{\pi}_i)E[\tilde{y}|i]]$ that the social planner is willing to pay to remove inequality. In the EEDE case, instead:

$$1 - \hat{\tau}^{EEDE}(u, v, \omega, \tilde{y}) = (1 - \hat{\pi}^{EEDE}) \frac{E_s [(1 - \hat{\chi}_s)E[\tilde{y}|s]]}{E\tilde{y}}$$

where $\hat{\chi}_s(v, \omega, \tilde{y})$ is the proportional contingent inequality premium, i.e. the share of contingent income $\omega E[\tilde{y}|s]$ that the social planner pays to remove contingent inequality, and $\hat{\pi}^{EEDE}(u, \omega(1 - \hat{\chi}_s)E[\tilde{y}|s], \tilde{y})$ is the proportional risk premium of the representative agent, i.e. the share of aggregate income $\omega E_s[(1 - \hat{\chi}_s)E[\tilde{y}|s]]$ that the planner offers to eliminate the remaining risk. Similar definitions hold in the presence of inequality in initial income.

Let $R_u(w) \equiv -\frac{u''(w)w}{u'(w)}$ be the coefficient of relative risk aversion, with $w \equiv E\tilde{\omega}\tilde{y}$, and $R_v(w) \equiv -\frac{v''(w)w}{v'(w)}$ be the coefficient of relative inequality aversion. The following Proposition states the quadratic approximations of the social premia in the small. Note that, in the small, the social premium can be exactly decomposed into the sum of an inequality premium and a risk premium, as in the additive risk case.

Proposition 6. *Assume that the consumption risk $\tilde{c} = \tilde{\omega}\tilde{y}$ is small. The quadratic approximation of the proportional social premium is given by:*

- *Utilitarianism:*

$$\hat{\tau}^U \simeq R_u \frac{\sigma_y^2}{2} + (R_u - 1)Cov(\omega, y) \quad (1.23)$$

- *Equality of Prospects approach:*

$$\hat{\tau}^{EoP} \simeq \hat{\tau}^U + \frac{1}{2}(R_v - R_u) (\sigma_{E[y|i]}^2 + 2Cov(\omega, y)) \quad (1.24)$$

- *Expected Equally Distributed Equivalent approach:*

$$\hat{\tau}^{EEDe} \simeq \hat{\tau}^U + \frac{1}{2}(R_v - R_u) (E_s \sigma_{y_s}^2 + 2Cov(\omega, y)) \quad (1.25)$$

If there is no income inequality, i.e. $\omega(i) = \omega$ for all i , the proportional social premium can be directly derived from the additive one:

$$\hat{\tau}(u, v, \omega, \tilde{y}) = \frac{\tau(u, v, \omega, \tilde{y})}{\omega E\tilde{y}}$$

Then, from the approximations for the additive social premium one can infer the approximated proportional social premium, just by replacing the absolute degrees of risk and inequality aversion $A_u(w)$ and $A_v(w)$ with their relative counterpart: $R_u(w)$ and $R_v(w)$. Moreover, note that when there is income inequality, the comparison between the three proportional social premia is equivalent to the additive case. The only difference concerns the role of the correlation between social risk \tilde{y} and initial income. Indeed, if the function expressing equity concerns is logarithmic, i.e. either $R_u = 1$ in the utilitarian case or $R_v = 1$ in the EoP and EEDe ones, the presence of inequality in the distribution of income has no impact on the social premium. Furthermore, when $R_u < 1$ (see $R_v < 1$) a positive correlation between income and risk reduces the social premium, while the opposite occurs for $R_u > 1$ (see $R_v > 1$). The reason lies on the sign of the utility function. Assume that $Cov(\omega, y) > 0$,

and consider the utilitarian case. When $R_u < 1$, the presence of inequality increases the value of the lottery produced by the risk \tilde{y} , simply because a higher variability in the distribution of consumption raises the expected amount to be consumed. As a consequence, for a given risk aversion, the presence of inequality reduces the willingness to pay to eliminate the risk. On the contrary, when $R_u > 1$, inequality reduces the value of the lottery: for a given R_u , the certainty equivalent for the elimination of both risk and inequality decreases as inequality increases. As a result, a larger variability in per capita consumption makes the social planner more willing to eliminate the correlated risk.

Finally, as for additive risks, we can state the properties of the social premia in the large when there is no income inequality:

Proposition 7. *Suppose that risks are multiplicative: $\tilde{c} = \tilde{\omega}\tilde{y}$, and that $\tilde{\omega}$ is a degenerate random variable. Let R_k , with $k = u, v$, denote the coefficient of, respectively, relative risk and relative inequality aversion. Then, both the EoP and the EEDE social planners are willing to pay more than the utilitarian one for the elimination of the social risk \tilde{y} if and only if $R_v \geq R_u$.*

1.6.2 The power-lognormal case

Like the standard risk premium, the quadratic approximations underestimate the true social premium as the size of the risk gets larger. However, Proposition 4 and 6 suggest the conditions under which the quadratic approximations of the social premium are exact. From the traditional theory of risk, we know that the Arrow-Pratt approximation of the additive risk premium is exact under normal distribution of risk and constant absolute risk aversion, while the approximation of the multiplicative risk premium is almost exact under lognormal distribution of the risk and constant relative risk aversion¹⁵ (Gollier 2001). The results can be generalized to the local approximations of the social premia.

For instance, let us consider the case of proportional social risk. Assume that we can identify agents just by their initial income, i.e. all the individuals with income $\omega(i)$ face the same risk. Suppose that consumption per capita is log-normally distributed, both with and without the social risk. Assume that initial income is distributed as a lognormal $\Phi(\cdot; \bar{\omega}, \sigma_\omega^2)$, with mean $\bar{\omega}$ and variance σ_ω^2 , and that the social risk is also log-normally distributed with mean μ and variance σ_y^2 : $y \sim \Phi(\cdot; \mu, \sigma_y^2)$. Finally, let us make the further assumption that

¹⁵Indeed, let $\hat{\tau}(\tilde{c} \rightarrow \tilde{\omega})$ be the approximated proportional social premium, and $\hat{\tau}^*(\tilde{c} \rightarrow \tilde{\omega})$ the actual one. Then, $\hat{\tau}^*(\tilde{c} \rightarrow \tilde{\omega}) = 1 - e^{-\hat{\tau}(\tilde{c} \rightarrow \tilde{\omega})}$ no matter the social welfare approach adopted.

consumption per capita $\tilde{c} = \tilde{\omega}\tilde{y}$ follows a bivariate lognormal distribution, where r denotes the index of correlation between \tilde{y} and $\tilde{\omega}$:

$$\begin{pmatrix} \omega \\ y \end{pmatrix} \sim \Phi \left(\begin{pmatrix} \bar{\omega} \\ \mu \end{pmatrix}, \begin{pmatrix} \sigma_\omega^2 & r\sigma_\omega\sigma_y \\ r\sigma_\omega\sigma_y & \sigma_y^2 \end{pmatrix} \right)$$

Therefore, consumption per capita is log-normally distributed with mean $\bar{\omega} + \mu$ and variance $\sigma_\omega^2 + \sigma_y^2 + 2r\sigma_\omega\sigma_y$. Moreover, the conditional distribution of the risk \tilde{y} for an agent with initial income $\omega(i)$ is also log-normally distributed with mean μ_i and variance σ^2 , which are defined as:

$$y|\omega(i) \sim \Phi \left(\mu + \frac{r\sigma_y}{\sigma_\omega}(\ln \omega(i) - \bar{\omega}), \sigma_y^2(1 - r^2) \right)$$

As a consequence, agents are exposed to different risks according to their initial income. If $r > 0$, rich agents have rosier expectations about the risk they face. Instead, if $r < 0$, rich agents suffer more from the negative impacts of a risk. Note that, with a bivariate log-normal distribution, individual risks differ in terms of expected value, not in terms of variability¹⁶. Finally, assume that u is CRRA (constant relative risk aversion) with elasticity γ , and v is CRIA (constant relative inequality aversion) with elasticity φ ¹⁷. Then, the social premium in the EoP approach is defined as:

$$\begin{aligned} 1 - \tau^{EoP} &= \frac{\left[\int \left[\int (\omega y)^{1-\gamma} d\Phi(y; \mu_i, \sigma^2) \right]^{\frac{1-\varphi}{1-\gamma}} d\Phi(\omega; \bar{\omega}, \sigma_\omega^2) \right]^{\frac{1}{1-\varphi}}}{\int y d\Phi(y; \mu, \sigma_y^2) \left[\int \omega^{1-\varphi} d\Phi(\omega; \bar{\omega}, \sigma_\omega^2) \right]^{\frac{1}{1-\varphi}}} \\ &= \exp \left\{ -\frac{1}{2}\gamma(1 - r^2)\sigma_y^2 - \frac{1}{2}\varphi r^2\sigma_y^2 + (1 - \varphi)r\sigma_y\sigma_\omega \right\} \end{aligned}$$

The first term represents the average individual risk premium, and it depends on the variability of individual risks (i.e. on the variability of $y|\omega(i)$); the last two terms denote the inequality premium, which depends on the inequality in expected individual mean and on the dependence between initial income $\tilde{\omega}$ and social risk \tilde{y} . When $\gamma = \varphi$, we are back to the utilitarian approach. As expected, when r is positive, the EoP social premium is larger than the utilitarian one as long as the social planner is more inequality averse than risk averse,

¹⁶ Alternatively, we can say that individual risk $y|\omega(i)$ is log-normally distributed with mean μ_i and variance σ^2 , where the individual expectations μ_i are normally distributed with mean μ and variance σ_μ^2 . In order to match the notation in the text, $\sigma_\mu^2 \equiv \sigma_y^2 r^2$

¹⁷A constant relative risk aversion function u is of the form: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, while a constant relative inequality aversion function v is such that: $v(c) = \frac{c^{1-\varphi}}{1-\varphi}$

i.e. $\varphi \geq \gamma$. When r is sufficiently negative, the opposite occurs. Finally, when $r = 0$, all agents face the same risk independently of their initial income. As a result, with isoelastic utility functions the social premium is independent of background inequality.

In order to apply the EEDE criterion, instead, we need information on the correlation between individual risks, i.e. we need to know, for example, whether the probability that agent i suffers a large loss is positively/negatively correlated to the fact that agent $j \neq i$ has suffered a loss. Because of this potential correlation, the distribution of realized consumption per capita is not necessarily log-normally distributed, even though initial income $\tilde{\omega}$ and individual risk $\tilde{y}|\omega(i)$ are. However, there is one case in which the EEDE social premium can be easily computed from the information necessary to determine the EoP social premium. If individual risks are independently distributed, by the law of large numbers the distribution of per-capita consumption in each state of the world will resemble the original distribution $\Phi(\cdot; \bar{\omega} + \mu, \sigma_y^2 + \sigma_\omega^2 + 2r\sigma_y\sigma_\omega)$, as previously defined. As a consequence, in that case the EEDE social premium will be equivalent to:

$$\begin{aligned} 1 - \hat{\tau}^{EEDE} &= \frac{[\int (\omega y)^{1-\varphi} d\Phi(\ln \omega + \ln y; \bar{\omega} + \mu, \sigma_y^2 + \sigma_\omega^2 + 2r\sigma_y\sigma_\omega)]^{\frac{1}{1-\varphi}}}{[\int \omega^{1-\varphi} d\Phi(\ln \omega; \bar{\omega}, \sigma_\omega^2)]^{\frac{1}{1-\varphi}} \int y d\Phi(\ln y; \mu, \sigma_y^2)} \\ &= \exp \left\{ -\frac{1}{2}\varphi\sigma_y^2 + (1-\varphi)r\sigma_y\sigma_\omega \right\} \\ &= (1 - \hat{\tau}^{EoP}) \exp \left\{ -\frac{1}{2}(\varphi - \gamma)\sigma_y^2(1 - r^2) \right\} \end{aligned}$$

As long as $\varphi \geq \gamma$, the social premium under ex-post equity concerns is larger than the social premium with ex-ante equity concerns, independently of the relation between risk and initial income. In the extreme cases $r = \{1, -1\}$, EoP and EEDE social premia coincide. Moreover, as, by assumption, there is no difference in the distribution of consumption across states, the EEDE social premium is affected exclusively by equity preferences.

1.6.3 Intergenerational inequality

So far, we have implicitly focused our attention on risks that befall people living in the same period. However, the representations (1.4) and (1.5) have a more general application. Indeed, the set of agents \mathbb{N} can refer either to individuals belonging to the same generation, or to individuals living in different generations, or to a combination of the two.

Assume, for instance, that $\mathbb{N} = [0, N]$ denotes the number of successive generations, and $i \in \mathbb{N}$ a single generation. The distribution function $Q(i)$ can be interpreted as the proportion of individuals living in generation i , with respect to the total number of agents that will ever be alive. Suppose, for simplicity, that individuals in the same generation are identical, i.e. they have the same preferences and consume the same amount (in other words, there is perfect intra-generational risk sharing). In contrast, because of consumption growth, individuals belonging to different generations have unequal consumption possibilities, although the growth process is subject to risk. A priori, the risk can be either identically and independently distributed across generations or time-dependent. The question, then, would be how much society is willing to pay in each period i to fully remove the risk on growth, given the inequality in expected consumption over time and the (possible) heterogeneity in risk.

In the utilitarian case, the social planner living at time 0 evaluates a risk elimination project by summing over time expected individual felicity functions. If N_i is the proportion of people alive in generation i , we would have¹⁸:

$$W^U = \int_0^N N_i Eu(\tilde{c}) di \quad (1.26)$$

In contrast, an EoP social planner cares about inequality in expected utility over time. According to (1.4), she will first determine the certainty equivalent of each generation separately, and then she will sum an increasing transformation of these certainty equivalents over time. Her objective, then, would be to equalize the chances faced by each generation:

$$W^{EoP} = \int_0^\infty N_i v(u^{-1}(Eu(\tilde{c}))) di \quad (1.27)$$

In the EEDE approach, instead, the social planner is interested in equalizing the realized felicity of each generation. Thus, for each possible state of the world (where a state of the world corresponds to a realized consumption stream over time), she identifies the constant consumption over time that is equivalent to the realized growth process. Then, she will consider the expected value of these equally distributed equivalents through the lenses of the

¹⁸The most common approach to the problem of intergenerational equity would be *discounted utilitarianism*, where the social planner maximizes the discounted sum of expected individual utilities. In the discussion, we implicitly set the discount rate to 0. This is because it would not make sense to put a time discount rate in the ex-post approach to equity, given that the social planner maximizes always the same felicity function over time. Therefore, to consistently compare the expected equally distributed equivalent approach to the others I eliminate the exogenous time discounting. It could be reintroduced endogenously by assuming that society faces a risk of extinction, see Dasgupta and Heal (1979); Bommier and Zuber (2008).

utility function u :

$$W^{EEDE} = Eu \left(v^{-1} \left(\int_0^\infty N_i v(\tilde{c}) di \right) \right) \quad (1.28)$$

Note that the EoP approach is reminiscent of Kreps-Porteus/Epstein-Zin preferences (Kreps and Porteus, 1978; Epstein and Zin, 1991), where it is assumed that the decision maker first evaluates the certainty equivalent of future well-being in an atemporal way, and then considers the lifetime evolution of certainty equivalents. However, unlike the representation (1.27), the Kreps-Porteus/Epstein-Zin model disentangles the attitudes toward consumption smoothing over time and across states in a recursive manner, which ensures time consistency. On the contrary, the Equality of Prospects approach described in (1.27) is not linear in probabilities, thereby failing to satisfy time consistency requirements (see also Adler and Sanichirico (2006) for dynamic consistency issues). Similarly, the EEDE approach (1.28) is close to the non-time additive models of inter-temporal decision inspired by Kihlstrom and Mirman (1974), and used, e.g., in Bommier and Zuber (2008) for the study of catastrophe aversion or in Kihlstrom (2009) for the study of asset pricing¹⁹.

This framework can, for instance, contribute to the climate change debate. Because of the inertia in the climate system, harmful emissions produced today will cause economic damages only in the future. As a consequence, any climate policy will reflect the current generation's concerns for intergenerational justice, taking into account that future generations are expected to be wealthier and that the size of the climate risk is uncertain. In that context, the social willingness to pay would correspond to the uniform amount that society is ready to pay, today and in the future, to prevent climate change. It would represent a measure of the demand side for climate policy, which should then be compared with the actual costs of the mitigation project. Most economy-climate models assume a utilitarian social planner²⁰. In section 7, I will indeed present a simple application of the model to the climate change problem.

1.7 Applications

This paper is largely theoretical; its goal was to determine an optimal rule for removing a social risk as a function of the equity preferences. However, it is useful to examine how the framework can be applied in practice. As a consequence, we will consider two potential applications: the social cost to avert the negative impacts of a hurricane, and the climate

¹⁹Although the latter is not time consistent

²⁰Some exceptions: Traeger (2015), Ha Duong and Treich (2004)

policy to avoid any increase in global temperature. The first is a static model, while the second is an intergenerational problem. In either case, the willingness to pay can be interpreted as the macroeconomic cost of a natural catastrophe/climate change, which will serve as a guidance in a cost-benefit analysis exercise. For each case, I provide rough estimates of the probability distribution of the risk, of the potential damages and of the impact on the degree of (either intra- or inter-generational) inequality.

In both examples, we assume that the risk is proportional to income, and that the utility function u has a constant relative risk aversion (CRRA) form with elasticity γ , while the welfare function v has constant relative inequality aversion (CRIA) with elasticity φ . The determination of the coefficient of relative risk aversion γ is an empirical question, which is addressed by examining individuals' choices under uncertainty. From empirical studies on financial decisions, γ is usually taken between 2 and 4 (Gollier, 2001).

In the first example, the coefficient of relative inequality aversion φ stands for intra-generational inequality. Because of its ethical foundations, it is problematic to estimate its value from observed market behavior. What the literature usually does is to elicit inequality preferences either through the use of surveys (Amiel and Cowell, 1999; Carlsson et al., 2002, 2005) or through the study of taxation systems (which should implicitly reveal social preferences towards inequality, Evans (2005)). The estimated coefficient of relative inequality aversion usually goes from 0.2 to 6.24, according to the different studies (Adler, 2012). In the second example, instead, the coefficient φ represents inter-generational inequality aversion. From market data, it is possible to estimate the relative resistance to intertemporal substitution, which, in the climate change literature, is usually set between 1 and 2²¹.

1.7.1 The social cost of hurricanes in the US

A natural disaster (see hurricane) is a good example to study the impact of equity and risk preferences on society's willingness to pay. First of all, the vulnerability to a disaster differs across the population, depending on the geographical location and the capacity to recover from the impacts of a disaster. A natural catastrophe has sizeable effects (at least in the short run), and can potentially lead to a change in the distribution of income. Finally, unlike other rare events (e.g. financial crisis or wars), it substantially represents an exogenous shock, independent of the degree of inequality or the aggregate economic conditions (e.g. Piketty

²¹Few examples: Weitzman (2007) and Nordhaus (2008) propose a value of 2; Stern (2006), Golosov et al. (2014) and Traeger (2015), instead, suggest to use a logarithmic felicity function

and Saez (2013) argue that income inequality may exacerbate financial fragility, thereby imposing additional welfare costs)²².

In order to compute the social willingness to pay for the removal of a natural disaster risk, let us consider the following simplified version of the model presented earlier. Assume that only two aggregate states are possible. With probability p , a natural disaster (see hurricane) occurs, which will reduce individual consumption and, potentially, affect the degree of inequality in the population. Let $c(i, N)$ denote individual consumption with no disaster, and $c(i, L)$ individual consumption after a disaster. Furthermore, suppose that, independently of the occurrence of a disaster, individual consumption has a lognormal distribution, where μ_j and σ_j^2 , $j = N, L$, denote, respectively, the mean and variance of log consumption. The disaster reduces average consumption ($\mu_L < \mu_N$) and, potentially, increases inequality ($\sigma_L^2 \geq \sigma_N^2$). In the example, I assume that $\gamma = 2$, while $\phi = \{0.5, 4\}$.

Because of data availability, we will focus on the US, and in particular on potential damages caused by major hurricanes²³ in the Atlantic coast and Mexico Gulf states in the US. The average annual probability of major hurricane across these states is $p = 3.23\%$ (very high for Florida: 14%, low for Maine: less than 1%; data from US Landfalling Hurricane Probability project²⁴). The average loss is more difficult to determine; averaging Nordhaus

²²There exist several studies that try to assess the immediate and long term economic impacts of a natural disaster (see Deryugina (2011) for a review), both in terms of economic losses and fatalities. Natural catastrophes do not seem to have long-run effects, at least in rich countries (Cavallo et al., 2013). For instance, for the US Deryugina (2011) show that the negative consequences of hurricanes are mitigated through both disaster and non-disaster social safety net programs (as social insurance, unemployment transfers). Moreover, the macroeconomic costs of a natural disaster come mainly from uninsured losses, which characterize mainly developing countries. In the short run, instead, the local effects of a natural catastrophe can be substantial. Hurricane Katrina, which has been the costliest event in the US, caused a 33 % decrease in per-capita income growth, and a 3% decrease in per capita income for the people involved (Masozera et al., 2007). Nordhaus (2006) proposes a lower annual cost for hurricane damages in the US (around 0.071% of GDP), but he estimates it at national level.

Pre-existing socio-economic conditions play an important role in the ability to cope with and recover from the impacts of a disaster. Although rich people (or nations) are not less exposed to natural disasters than poor people, the former do suffer lower long term damages as they have easier access to public and private resources for recovery (Adger, 1996). Moreover, while the largest damages are often born by wealthier people, due to possessions of higher value, the relative impact is usually larger for poor people (see for instance Masozera et al. (2007) for hurricane Katrina). However, few papers focus on the impact of natural disasters on income distribution and inequality (e.g. Miljkovic and Miljkovic (2014) for the US, Shaughnessy et al. (2010) for Katrina, Dell et al. (2012) for the impact of temperature change at the world level), and the results are contradictory. For instance, Miljkovic and Miljkovic (2014), by analyzing the history of hurricanes in the Atlantic Coast and Mexico Gulf states, estimate that the Gini coefficient increases on average by 5.4% for every 100 billion US dollars in hurricane economic damages. Instead, Shaughnessy et al. (2010) show that inequality decreased in New Orleans after Katrina, mainly because of the migration of poorer households.

²³A major hurricane is defined as a hurricane of category 3 or more.

²⁴<http://www.e-transit.org/hurricane/welcome.html>

Welfare approach	Social Willingness to Pay	
	$\phi = 0.5$	$\phi = 4$
No Inequality	0.033%	
Utilitarianism	0.5%	
Equality of Prospects	0.22%	0.75%
Expected Equally Distributed Equivalent	0.16%	1.01%

Table 1.2: Proportional Social Willingness to Pay for removing the risk of damages due to major hurricanes in the US, as a function of the welfare approach. Coefficient of relative risk aversion $\gamma = 2$. Coefficient of relative inequality aversion $\phi = \{0.5, 4\}$.

(2006) and the case of hurricane Katrina, we set it at 1% of per-capita consumption. Moreover, following Miljkovic and Miljkovic (2014), I assume that the occurrence of a disaster will increase the Gini coefficient by about 5%. The distribution of income in the absence of a catastrophe is drawn by the US Census Bureau for the year 2013, and features an average income of 53000 US\$ and a Gini coefficient of 0.45 (data from CIA, 2007). Given the formula for the GINI coefficient $G = 2\Phi(\frac{\sigma}{\sqrt{2}}) - 1$, where $\Phi(\cdot)$ represents the cumulative standard normal distribution, we can easily derive the variance of the distribution of income with and without a disaster. Moreover, I introduce the assumption that also the individual certainty equivalents (essential for the EoP approach) are lognormally distributed²⁵.

Table 1 reports the proportional willingness to pay to avert the risk of catastrophic damages caused by hurricanes as a function of the equity approach. As expected, if the social planner has equity preferences (i.e. either in the EoP or EEDE approaches), the willingness to pay is increasing in inequality aversion, and it is larger than the utilitarian one as long as $\phi \geq \gamma$. Similarly for the comparison between EoP and EEDE: the ex-post approach grants a larger willingness to pay if and only if the social planner is more inequality averse than risk averse. Moreover, as the expected willingness to pay amounts to 0.0323%, the presence of risk has a considerable impact in all cases in which inequality is taken into account.

²⁵The baseline parameters are: $\mu_N = 10.46$, $\sigma_N^2 = 0.83$; $\mu_L = 10.38$, $\sigma_L^2 = 0.96$. To derive the moments of the distribution of individual certainty equivalents, I assume that the distribution can be stretched in the aftermath of a natural disaster, but that the relative position in the social ladder does not change. In other words, the poor stays poor and the rich stays relative richer. Thus, I first compute the certainty equivalent for the agent standing at the 50th percentile ($CE_{0.5}$) and for the agent corresponding to the 20th percentile ($CE_{0.2}$). Thanks to them, I can recover the mean μ_{CE} and the variance σ_{CE}^2 of the distribution of certainty equivalents, given that $CE_{0.5} = e^{\mu_{CE}}$ and $\sigma_{CE} = \frac{\ln CE_{0.2} - \mu_{CE}}{-0.84}$. Thus, I get: $\mu_{CE} = 10.457$ and $\sigma_{CE}^2 = 0.833$.

1.7.2 Climate change policy

As a framework for policy analysis, we consider a very simple climate policy, which corresponds to the fraction of consumption that the social planner is willing to sacrifice, now and throughout the future, to avoid any increase in temperature. Let $\omega_t = c_0 e^{gt}$ be generation t 's income in the absence of climate change, where c_0 represents current consumption and g the constant growth rate. The economic impact of a temperature change ΔT_t at t takes the form of an exponential consumption loss function²⁶ $L(\Delta T_t) = 1 - e^{-\beta \Delta T_t}$. Thus, consumption at date t will be equal to $c_t = \omega_t(1 - L(\Delta T))$. The law of motion of temperature change depends on the concentration of greenhouse gas emissions in the atmosphere, which are a by-product of economic activity. Instead of explicitly modeling the link between GDP-emissions-temperature change, we adopt the approach of Pindyck (2012), who argues that the following exogenous trajectory for ΔT_t is a good fit for a climate-economy model:

$$\Delta T_t = 2s[1 - (1/2)^{t/100}]$$

where s denotes the climate sensitivity, defined as the increase in temperature that would result from a doubling of the atmospheric CO₂e concentration relative to the pre-industrial level. The right value of s is subject to deep uncertainties²⁷. Following the results by IPCC (2007), I build a discrete distribution for the climate sensitivity such that: with probability 0.37, $s = 0$; with probability 0.47, $s = 3$; with probability 0.13, $s = 7$, and with probability 0.03, $s = 10$. In order to calibrate the economic impact β , I take Nordhaus (2008)'s estimate of 1.77% GDP damages caused by an increase in temperature by 2.5°C, which leads to $\beta = 0.007$. Moreover, let us normalize c_0 to 1. Time is discrete, and the horizon lasts till 200 years from now. From historical data, the global growth rate is $g = 2$. Finally, let us assume that risk aversion is larger than resistance to intertemporal substitution: $\gamma = 4$ and $\varphi = 2$. Table 2 summarizes the results of the analysis.

Given the conservative assumptions about the probability distribution of climate sensitivity and the expected economic loss, all the figures are lower than 1% of consumption.

²⁶Most economic studies of climate change relate ΔT to consumption through a damage function $L(\Delta T)$, with $L(0) = 1$ and $L' < 0$; the damage function is typically inverse-quadratic (Nordhaus, 2008) or exponential-quadratic (Weitzman, 2009). Few studies assume that climate change has an impact on capital (Stokey, 1998) or growth (Pindyck, 2012).

²⁷From the survey of 22 scientific studies, the IPCC (2007) estimated an expected value of 2.5°C to 3°C for climate sensitivity. As pointed out by Weitzman (2009), the aggregation of those studies suggests that there is a 17% probability that a doubling of the CO₂e concentration would lead to a mean temperature increase of 4.5°C or more, a 5% probability of a temperature increase of 7°C or more, and a 1% probability of a temperature increase of 10°C or more.

Welfare approach	SWTP %	social cost of climate change %
No risk ($\gamma = \varphi = 4$)	0.36	-1.26
Utilitarianism ($\gamma = \varphi = 4$)	0.37	-1.25
Equality of Prospects	0.89	-0.73
Expected Equally Distributed Equivalent	0.88	-0.74

Table 1.3: Social cost of climate change risk. Coefficient of relative risk aversion $\gamma = 4$. Coefficient of relative resistance to intertemporal substitution $\varphi = 2$.

However, by adopting either an EoP or EEDE approach, the social willingness to pay almost doubles with respect to the utilitarian case. Moreover, note that, as the correlation between income and impacts is negative (rich generations are expected to suffer more from climate change due to the inertia of the climate system), the EoP approach delivers a larger willingness to pay than the EEDE one, as suggested by Proposition 5. Moreover, because of the negative correlation, the average damage over time is equal to 1.6%, and lower than the constant willingness to pay, which explains why the social premium (third column) is negative.

1.8 Conclusion

The goal of the paper was to define a tool for the evaluation of a social risk, which has been defined as a global risk with heterogeneous impacts across the population. As with the classic Arrow-Pratt risk premium for private risks, I characterized a social premium, and studied its properties in the small and in the large as a function of the social planner's preferences for risk and equity. In particular, I compared three different normative approaches to the link between risk and equity: utilitarianism, equality of prospects approach, and expected equally distributed equivalent approach. We have seen that the approaches differ in the way in which individual preferences are aggregated. Utilitarianism cares about the inequality in consumption, but it is neutral to inequality in the distribution of well-being. The equality of prospects approach is sensitive to the distribution of expected individual utilities, i.e. the social planner has preferences over providing each individual with the same chances. Finally, the expected equally distributed equivalent approach focuses on the actual distribution of utilities, and cares about the equalization/polarization of realized well-being. In particular,

we have seen that the social premium is sensitive to the difference between risk aversion and inequality aversion, and to the correlation between individual risk and initial income.

The quadratic approximation of the social premium helps at identifying the key components, in terms of tastes and variability measures, that affect the optimal risk management strategy. Although it applies only in the large, it provides a benchmark for the selection of the risk prevention project. Moreover, as with the standard risk premium, the approximated social premium can be used to define an extended version of the mean-variance preference model, which would provide a simple way to characterize the optimal portfolio of risk management policies.

The paper suggests several extensions. First of all, we have considered only the social premium that the decision maker is willing to pay for a complete removal of the risk and the connected inequality. A natural extension would be the characterization of rules for the partial alleviation of the risk, which could be useful to study self-insurance and self-protection problems. Second, a restrictive assumption I made in the paper was the presence of homogeneous preferences. It would be interesting to extend the paper to the case of heterogeneous individual preferences, in order to understand how the composition of different degrees of risk aversion affect the optimal risk management strategy. Moreover, we have considered only consumption risks, i.e. risks that affect the amount consumed by each individual. However, some social risks, as global warming or pandemics, represent also a population risk, as they can influence the number of people alive in each state. It would be interesting to study the characteristics of the social premium when we introduce population principles in the analysis. Finally, the paper has focused on social frameworks that separates concerns for ex-ante equity and concerns for ex-post equity. It could be useful to consider a more general framework that encompasses both of them.

Bibliography

- Adger, W. (1996). Approaches to vulnerability to climate change. CSERGE Working Paper GEC 96-05. [44](#)
- Adler, M. (2012). *Well-Being and Fair Distribution*. Oxford University Press. [11](#), [13](#), [16](#), [17](#), [43](#)
- Adler, M., Hammit, J., and Treich, N. (2012). The social value of mortality risk reduction: Vsl vs the social welfare function approach. U of Penn Law School, Public Law Research Paper N. 12-29; U of Penn, Inst for Law and Econ Research Paper N. 12-16. [13](#)
- Adler, M. and Sanchirico, C. (2006). Inequality and uncertainty: Theory and legal applications. *University of Pennsylvania Law Review*, 155:279–377. [13](#), [16](#), [17](#), [42](#)
- Adler, M. and Treich, N. (2014). Consumption, risk and prioritarianism. [13](#)
- Amiel, Y. and Cowell, F. (1999). *Thinking about Inequality*. Cambridge University Press. [43](#)
- Atkinson, A. (1970). On the measurement of inequality. *Journal of Economic Theory*, 2(3):244–63. [22](#)
- Bommier, A. and Zuber, S. (2008). Can preferences for catastrophe avoidance reconcile social discounting with intergenerational equity? *Social Choice and Welfare*, 31:415–34. [41](#), [42](#)
- Broome, J. (1991). *Weighing Goods. Equality, Uncertainty and Time*. Oxford: Blackwell. [10](#), [17](#)
- Carlsson, F., Daruvala, D., and Johansson-Stenman, O. (2002). Measuring future grandparents preferences for equality and relative standing. *Economic Journal*, 112:362–83. [43](#)

- Carlsson, F., Daruvala, D., and Johansson-Stenman, O. (2005). Are people inequality averse, or just risk averse? *Economica*, 72:375–96. 43
- Cavallo, E., Galiani, S., Noy, I., and Pantano, J. (2013). Catastrophic natural disasters and economic growth. *Review of Economics and Statistics*, 95(5):1549–61. 44
- Dasgupta, P. and Heal, G. (1979). *Economic Theory and Exhaustible Resources*. Cambridge University Press, Cambridge. 41
- Dell, M., Jones, B., and Olken, B. (2012). Temperature shocks and economic growth: Evidence from the last half century. *American Economic Journal: Macroeconomics*, 4(3):66–95. 44
- Deryugina, T. (2011). The dynamic effects of hurricanes in the us: The role of non-disaster transfer payments. MIT CEEPR Working Paper 2011-007. 44
- Diamond, P. (1967). Cardinal welfare, individualistic ethics and interpersonal comparisons of utility: Comment. *Journal of Political Economy*, 75:765–66. 10, 15, 16
- Emmerling, J. (2011). Discounting and intragenerational equity. 13
- Epstein, L. and Segal, U. (1992). Quadratic social welfare function. *Journal of Political Economy*, 100:691–712. 10, 16
- Epstein, L. and Zin, S. (1991). Substitution, risk aversion and the temporal behavior of consumption and asset returns: an empirical framework. *Journal of Political Economy*, 99:263–86. 42
- Evans, D. (2005). The elasticity of marginal utility of consumption: Estimates for 20 OECD countries. *Fiscal Studies*, 26(2):197–224. 43
- Fleurbaey, M. (2010). Assessing risky social situations. *Journal of Political Economy*, 118:649–80. 11, 17
- Fleurbaey, M. and Bovens, L. (2012). Evaluating life or death prospects. *Economics and Philosophy*, 28:217–49. 13
- Fleurbaey, M. and Maniquet, F. (2012). *Equality of Opportunity: the Economics of Responsibility*. World Scientific Publishing. 14
- Fleurbaey, M. and Zuber, S. (2014a). Discounting, risk and inequality: a general approach. CES Working Paper No 2014.15. 13

- Fleurbaey, M. and Zuber, S. (2014b). Fair management of social risk. Working Paper No 59-2014, Princeton University. 14
- Gollier, C. (2001). *The Economics of Risk and Time*. MIT Press. 43
- Golosov, M., Hassler, J., Krusell, P., and Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88. 43
- Grant, S., Kajii, A., Pollack, B., and Safra, S. (2010). Generalized utilitarianism and harsanyi’s impartial observer theorem. *Econometrica*, 78:1939–71. 17
- Grant, S., Kajii, A., Pollack, B., and Safra, S. (2012). Equally distributed equivalent utility, ex-post egalitarianism and utilitarianism. *Journal of Economic Theory*, 147(4):1545–71. 18
- Ha Duong, M. and Treich, N. (2004). Risk-aversion, intergenerational equity and climate change. *Environmental and Resource Economics*, 28:195–207. 42
- Hammond, P. (1983). Ex-post optimality as a dynamically consistent objective for collective choice under uncertainty. In Pattanaik, P. and Salles, M., editors, *Social Choice and Welfare*, pages 175–205. Amsterdam: North-Holland. 10, 17
- Harsanyi, J. (1955). Cardinal welfare, individualistic ethics and interpersonal comparison of utilities. *Journal of Political Economy*, 63:309–21. 10, 14
- IPCC (2007). Intergovernmental panel on climate change, fourth assessment report. 46
- IPCC (2014). Intergovernmental panel on climate change, fifth report. 9
- Kihlstrom, R. (2009). Risk aversion and the elasticity of substitution in general dynamic portfolio theory: Consistent planning by forward looking, expected utility maximizing investors. *Journal of Mathematical Economics*, 45(9-10):634–63. 42
- Kihlstrom, R. and Mirman, L. (1974). Risk aversion with many commodities. *Journal of Economic Theory*, 8:361–88. 42
- Kreps, D. and Porteus, E. (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica*, 46:185–200. 42
- Lucas, D. (1995). Default correlations and credit analysis. *Journal of Fixed Income*, 4:76–87. 30

- Masozera, M., Bailey, M., and Kerchner, C. (2007). Distribution of impacts of natural disasters across income groups: A case study of new orleans. *Ecological Economics*, 63:299–306. [13](#), [44](#)
- Miljkovic, T. and Miljkovic, D. (2014). Modeling impact of hurricane damages on income distribution in the coastal us. *International Journal of Disaster Risk Science*, 5:265–73. [12](#), [44](#), [45](#)
- Nagel, T. (1995). *Equality and Partiality*. Oxford University Press. [16](#)
- Nordhaus, W. (2006). The economics of hurricanes in the united states. NBER Working Paper 12813. [44](#)
- Nordhaus, W. (2008). *A Question of Balance: Weighing the Options on Global Warming Policies*. Yale University Press, New Haven, CT. [43](#), [46](#)
- Parfit, D. (2000). Equality or priority. In Clayton, M. and Williams, A., editors, *The Ideal of Equality*, pages 81–125. Houndmills: Palgrave. [16](#)
- Piketty, T. and Saez, E. (2013). Top incomes and the great recession: Recent evolutions and policy implications. *IMF Economic Review*, 61(3):456–78. [43](#)
- Pindyck, R. (2012). Uncertain outcomes and climate change policy. *Journal of Environmental Economics and Management*, 63(3):289–303. [46](#)
- Ross, S. (1981). Some stronger measures of risk aversion in the small and in the large. *Econometrica*, 3:621–38. [35](#)
- Savage, L. (1954). *The Foundation of Statistics*. Wiley, and Chapman and Hall. [17](#)
- Shaughnessy, T., White, M., and Brendler, M. (2010). The income distribution effect of natural disasters: An analysis of hurricane katrina. *Journal of Regional Analysis and Policy*, 40(1):84–95. [13](#), [44](#)
- Stern, N. (2006). *The Economics of Climate Change: the Stern Review*. Cambridge University Press. [43](#)
- Stokey, N. (1998). Are there limits to growth? *International Economic Review*, 39:1–31. [46](#)
- Traeger, C. (2015). Analytic integrated assessment and uncertainty. [42](#), [43](#)

Weitzman, M. (2007). The stern review on the economics of climate change. *Journal of Economic Literature*, 45(3):703–24. 43

Weitzman, M. (2009). On modeling and interpreting the economics of catastrophic climate change. *Review of Economics and Statistics*, 91:1–19. 46

Appendix

1.A Proofs

Proof of Proposition 1. In the Equality of Prospects approach, the social premium is defined as:

$$E_i v \left(u^{-1} \left(E[u(\omega + \tilde{y})|i] \right) \right) = v(\omega + E\tilde{y}) - \tau^{EoP}$$

Given the individual risk premium $\pi_i(u, \omega, \tilde{y})$, which satisfies $E[u(\omega + \tilde{y})|i] = u(\omega + E[\tilde{y}|i] - \pi_i)$, we have:

$$E_i v \left(u^{-1} \left(E[u(\omega + \tilde{y})|i] \right) \right) = E_i v(\omega + E[\tilde{y}|i] - \pi_i)$$

By applying the definition of inequality premium χ^{EoP} , the previous condition becomes

$$E_i v(\omega + E[\tilde{y}|i] - \pi_i) = v(\omega + E\tilde{y} - E_i \pi_i - \chi^{EoP})$$

which implies that

$$\tau^{EoP}(u, \phi, \omega, \tilde{y}) = E_i \pi_i(u, \omega, \tilde{y}) + \chi^{EoP}(v, \omega, E[\tilde{y}|i] - \pi_i)$$

In the EEDE case, instead, we first compute the contingent inequality premia and that the risk premium of the representative agent. Thus, given the social premium:

$$E_s u \left(v^{-1} \left(E[v(\omega + \tilde{y})|s] \right) \right) = u(\omega + E\tilde{y} - \tau^{EEDE})$$

From the definition of contingent inequality premium $\chi_s(v, \omega, \tilde{y})$, which satisfies $E[v(\omega + \tilde{y})|s] = v(\omega + E[\tilde{y}|s] - \chi_s)$, we get:

$$E_s u \left(v^{-1} \left(E[v(\omega + \tilde{y})|s] \right) \right) = E_s u(\omega + E[\tilde{y}|s] - \chi_s)$$

The introduction of the risk premium $\pi(u, \omega, E[\tilde{y}|s] - \chi_s)$ yields:

$$E_s u(\omega + E[\tilde{y}|s] - \chi_s) = u(\omega + E\tilde{y} - E_s \chi_s - \pi)$$

which leads to the result:

$$\tau^{EEDDE}(u, \phi, \omega, \tilde{y}) = E_s \chi_s(v, \omega, \tilde{y}) + \pi(u, \omega, E[\tilde{y}|s] - \chi_s)$$

Finally, the utilitarian approach satisfies both decomposition, as it would be characterized by a $v = u$.

Proof of Lemma 1. Assume that the risk $\tilde{y} = E\tilde{y} + k\tilde{\epsilon}$, where $E\tilde{\epsilon} = 0$, and k represents the size of the risk. The aim is to determine the utilitarian social premium $\tau(k)$ as k goes to zero. A second order approximation of the social premium around $k = 0$ yields $\tau(k) \simeq \tau(0) + k\tau'(0) + 0.5k^2\tau''(0)$. Clearly, $\tau(0) = 0$. By differentiating twice the condition defining the social premium (1.7), we can easily see $\tau'(0) = 0$, and that $\tau''(0) \simeq -\frac{u''(\omega+E\tilde{y})}{u'(\omega+E\tilde{y})}Var(y)$. As $Var(c) \equiv \sigma_c^2 = k^2Var(\epsilon)$, the proof is completed.

Proof of Proposition 2. Assume that the risk $\tilde{y} = E\tilde{y} + k\tilde{\epsilon}$, where $E\tilde{\epsilon} = 0$, and that k represents the size of the risk. As in the previous proof, we want to determine the EoP social premium $\tau(k)$ when k tends to zero, where $\tau(0) = 0$. By using the function $v(c) = \phi(u(c))$, condition (1.8) can be rewritten as:

$$E_i v(u^{-1}(E[u(\omega + \tilde{y})|i])) = v(\omega + E\tilde{y} - \tau^{EoP})$$

By differentiating the previous condition with respect to k , we have:

$$E_i v' \frac{1}{u'(h)} E[u'\tilde{\epsilon}|i] = -v'\tau'$$

with $h \equiv u^{-1}(E[u(\omega + \tilde{y})|i])$. When $k = 0$, $\tau' = 0$. Totally differentiating again:

$$E_i v'' \left(\frac{1}{u'(h)} E[u'\tilde{\epsilon}|i] \right)^2 - E_i v' \frac{u''(h)}{u'(h)^3} (E[u'\tilde{\epsilon}|i])^2 + E_i v' \frac{1}{u'(h)} E[u''\tilde{\epsilon}^2|i] = (\tau')^2 v'' - \tau'' v'$$

At $k = 0$, the condition becomes:

$$v'' E_i(E[\tilde{\epsilon}|i])^2 + v' \frac{u''}{u'} [-E_i(E[\tilde{\epsilon}|i])^2 + E[\tilde{\epsilon}^2]] = -\tau'' v'$$

As $E_i(E[\tilde{\epsilon}|i])^2 = Var(E[\tilde{\epsilon}|i])$ and $-E_i(E[\tilde{\epsilon}|i])^2 + E[\tilde{\epsilon}^2] = -Var(E[\tilde{\epsilon}|i]) + Var(\tilde{\epsilon}) = E(Var(\tilde{\epsilon}|i)) = E_i(E[\tilde{\epsilon}^2|i])$, the EoP social premium is approximately equal to:

$$\tau^{EoP} \simeq -\frac{u''}{u'} \frac{E(Var(\tilde{c}|i))}{2} - \frac{v''}{v'} \frac{Var(E[\tilde{c}|i])}{2}$$

Proof of Proposition 3. Assume again that the risk $\tilde{y} = E\tilde{y} + k\tilde{\epsilon}$, with $E\tilde{\epsilon} = 0$. The social premium in the EEDE case is given by:

$$E_s u (v^{-1}(E[v(\omega + \tilde{y})|s])) = u(\omega + E\tilde{y} - \tau(k))$$

Total differentiation by k yields:

$$E_s u' \frac{1}{v'(h)} E[v'\tilde{\epsilon}|s] = -\tau' u'$$

with $h \equiv v^{-1}(E[v(\omega + \tilde{y})|s])$. At $k = 0$, we have that $\tau' = 0$. A second total differentiation leads to:

$$E_s u'' \left(\frac{1}{v'(h)} E[v'\tilde{\epsilon}|s] \right)^2 + E_s u' \frac{-v''(h)}{v'(h)^3} (E[v'\tilde{\epsilon}|s])^2 + E_s u' \frac{1}{v'(h)} E[v''\tilde{\epsilon}^2|s] = \tau'^2 u'' - \tau'' u'$$

When $k = 0$, we obtain:

$$\tau'' = -\frac{u''}{u'} Var(E[\tilde{\epsilon}|s]) - \frac{v''}{v'} [Var(\tilde{\epsilon}) - V(E[\tilde{\epsilon}|s])]$$

which implies that

$$\tau^{EED E} \simeq A_u Var(\tilde{y}) + (A_v - A_u) EVar(\tilde{y}|s)$$

where $Var(\tilde{y}) \equiv \sigma_y^2$ and $EVar(\tilde{y}|s) \equiv E\sigma_{y_s}^2$.

Proof of Proposition 4. We derive the quadratic approximation of the social premium with income inequality under the assumption that both risk and inequality are small. Suppose

that $\tilde{c} = E\omega + E\tilde{y} + k(\tilde{w} + \tilde{\epsilon})$, where $\omega(i) = E\omega + k\omega(i)$ and $y(i, s) = E\tilde{y} + k\epsilon(i, s)$. By assumption, $E_i\omega(i) = 0$, $E\epsilon(i, s) = 0$, and w and ϵ can be correlated.

Under utilitarianism, the social premium for the elimination of the social risk \tilde{y} is defined as:

$$Eu(E\omega + E\tilde{y} + k(\tilde{w} + \tilde{\epsilon})) = E_i u(E\omega + E\tilde{y} + k\tilde{w} - \tau(k))$$

Similar definitions hold for the other two approaches. In order to determine the social premium, we need to fully differentiate twice the condition defining it along the lines of the previous proofs. Then, we recover the following results:

$$\tau^U \simeq \frac{1}{2}A_u (E(w + \epsilon)^2 - Ew^2) = \frac{1}{2}A_u Var(y) + A_u Cov(y, \omega)$$

where $Cov(y, \omega) = E\omega y - E\omega E y = Ew\epsilon$ and $Var(y) = E\epsilon^2$. Moreover:

$$\begin{aligned} \tau^{EoP} &\simeq \frac{1}{2}A_u (E[w + \epsilon]^2 - E[w + E[\epsilon|i]]^2) + \frac{1}{2}A_v (E[w + E[\epsilon|i]]^2 - Ew^2) = \\ &= \frac{1}{2}A_u EVar(y|i) + \frac{1}{2}A_v [Var(E[y|i]) + 2Cov(y, \omega)] \end{aligned}$$

where $Var(E[y|i]) = E[E[\epsilon|i]]^2$, $EVar(y|i) = E\epsilon^2 - E[E[\epsilon|i]]^2$, and $Cov(\omega, y) = E[wE[\epsilon|i]]$. Finally:

$$\begin{aligned} \tau^{EEDE} &\simeq \frac{1}{2}A_u E[E[w + \epsilon|s]]^2 + \frac{1}{2}A_v (E[w + \epsilon]^2 - Ew^2 - E[E[w + \epsilon|s]]^2) = \\ &= \frac{1}{2}A_u Var(E[y|s]) + \frac{1}{2}A_v (EVar(y|s) + 2Cov(\omega, y)) \end{aligned}$$

where $E[E[w + \epsilon|s]]^2 = E(E[\epsilon|s])^2 = Var(E[y|s])$, and $EVar(y|s) = E\epsilon^2 - E(E[\epsilon|s])^2$. By rearranging terms, we get the result.

Proof of Proposition 5. Let us first compare the EoP and the utilitarian approaches. Let us define the utilitarian social premium τ^U : $Eu(\omega + \tilde{y}) = u(\omega + E\tilde{y} - \tau^U)$. If v is an increasing and concave transformation of u , i.e. $v = \phi \circ u$, then $E v(u^{-1}(E[u(\omega + \tilde{y})|i])) = E\phi(E[u(\omega + \tilde{y})|i]) \leq v(\omega + E\tilde{y} - \tau^U) = v(u^{-1}(Eu(\omega + \tilde{y}))) = \phi(Eu(\omega + \tilde{y}))$, which holds by Jensen's inequality as long as ϕ is concave.

Let us now consider the EEDE approach. If v is an increasing and concave transformation of u , the contingent equally distributed equivalent is lower in the EEDE approach

than in the utilitarian one, for each state s : $v^{-1}(Ev(\omega + \tilde{y})) \leq u^{-1}(Eu(\omega + \tilde{y}))$. As a consequence, the social premium is larger in the EEDE rather than in the utilitarian case, or $Eu(v^{-1}(E[v(\omega + \tilde{y})|s])) \leq u(\omega + \tilde{y} - \tau^U)$. Indeed: $Eu(v^{-1}(E[v(\omega + \tilde{y})|s])) = E\phi^{-1}(E[\phi(u(\omega + \tilde{y}))|s]) \leq E\phi^{-1}(\phi(E[u(\omega + \tilde{y})|s])) = Eu(\omega + \tilde{y})$ if and only if ϕ is concave.

Proof of Proposition 6. Assume that both the social risk \tilde{y} and the initial inequality $\tilde{\omega}$ are small, i.e. $\tilde{y} = (1 - k\tilde{\epsilon})E\tilde{y}$ and $\tilde{\omega} = (1 - k\tilde{w})E\tilde{\omega}$, where $E\tilde{\epsilon} = E\tilde{w} = 0$, while $E\tilde{\epsilon}\tilde{w} \neq 0$. For instance, under utilitarianism, the proportional social premium for the elimination of the social risk \tilde{y} is defined as:

$$Eu((1 - k\tilde{w})(1 - k\tilde{\epsilon})\bar{c}) = Eu((1 - k\tilde{w})(1 - \hat{\tau}^U(k))\bar{c})$$

with $\bar{c} \equiv E\tilde{\omega}E\tilde{y}$. Similarly for the other two approaches. By totally differentiating twice the conditions defining the proportional social premium, the second order approximation of the social premium around $k = 0$ can be easily computed as in the previous proofs.

Chapter 2

Fairness, Risk and the Social Cost of Carbon

2.1 Introduction

The social cost of carbon (SCC) plays an increasingly significant role in the economics of climate change. It represents the shadow price of carbon emissions, and it is defined as the net present value of future economic damages caused by an additional ton of carbon emitted today. Estimates of the social cost of carbon are a crucial component in the cost-benefit analysis of climate and energy related policies. The comparison between the social cost of carbon and the cost of controlling emissions determines the optimal amount of greenhouse gas emission reduction. The higher the value for the social cost of carbon, the larger the required mitigation effort. Policy makers can then use the social cost of carbon to set the optimal carbon tax or the emission reduction target in a cap-and-trade system. Moreover, it provides regulators a guidance for the introduction of climate-affecting policies, like energy efficiency standards in buildings and transportation sectors, incentives for the installation of low-carbon technologies and measures for the protection of forests (see, for instance, [Greenstone et al. \(2013\)](#) for the use of the SCC in US federal regulations, and [Pearce \(2003\)](#) for the UK case).

Given the widespread use of the SCC, its "right" value has been the object of a lively debate in the literature. One open issue concerns the use of equity weights in the calculation of the SCC. Given the inequality in the distribution of consumption across space and time, and the inequality in the distribution of damages from climate change, advocates of equity weighting suggest that costs to poor people should be weight more heavily than costs to rich

people. Applied to the SCC, it means that its value should depend both on who bears the costs of the mitigation policy (*ceteris paribus*, the SCC should increase if only rich people pay for the policy) and on the identity of those more exposed to the negative impacts of global warming (*ceteris paribus*, the SCC should be larger if poor people suffers more from climate change).

Moreover, as the costs and benefits of a mitigation policy accrue at different points in time, the problem of how to aggregate damages among contemporaries is intertwined to the problem of how to value consumption of different generations. Being a net present value, the SCC trades off the consumption costs of the current generation due to a decrease in carbon emissions and the benefits for the future generations due to a slow down in climate change. Intergenerational equity concerns, i.e. how much society is willing to give up today to improve the consumption possibilities of the future, are at the heart of the SCC estimates. However, in a world characterized by intra-generational inequality, the problem further complicates. Indeed, the fundamental question underlying the SCC becomes how much an individual living in a relatively poor region today should sacrifice to increase the consumption of an individual living in a relatively rich region tomorrow. Therefore, the use of equity weights among contemporaries interact with the use of utility weights for different generations (aka the discount rate).

In the paper, I reconsider the computation of the social cost of carbon in the presence of risk and inequality, and study how different assumptions about social preferences affect its size. In particular, I introduce a model that disentangles risk aversion, resistance to intertemporal substitution and intra-generational inequality aversion. After showing how the social cost of carbon can be derived from the adopted social welfare framework, I analytically determine its main components and provide a second order Taylor approximation to understand their relative importance. Then, I estimate the social cost of carbon based on a simplified version of [Nordhaus \(2010\)](#) RICE model.

The first part of the paper is devoted to the deterministic case. I assume that the world can be divided into many regions, and that both income and climate change damages are heterogeneously distributed across regions. By rewriting the social cost of carbon in terms of socially efficient discount rates, I show that its size depends mainly on the evolution of inequality over time (whether we expected or not economic convergence in the future) and on the inequality consumption beta of the mitigation project, which represents the correlation between regional income and regional marginal damages. In particular, economic convergence is going to reduce the social cost of carbon, while the impact of the inequality

consumption beta is ambiguous: while a positive beta increases the socially efficient discount rate (as in that case the mitigation policy would increase existing inequality), a positive beta is also accompanying larger average benefits of the policy. By using the predictions about economic growth and emissions paths incorporated in the RICE model (which corresponds to the SRES A2 emission scenario), I find support for both economic convergence and a positive inequality consumption beta. The size of these effects depends on the comparison between inter- and intra-generational inequality aversion. Indeed, varying the coefficient of intra-generational inequality aversion allows to recover different assumptions about equity weighting. If the social planner is not intra-generational inequality averse, we get the representative agent solution. If the coefficients coincide, we are working with a utilitarian social planner, where equity weights are decreasing in regional consumption. When intra- and inter-generational inequality preferences differ, equity weights are not necessarily decreasing in consumption, as the social planner cares about individual consumption levels relatively to the amounts consumed by the other members of the same cohort. As a consequence, an increase in intra-generational inequality aversion magnifies the importance of economic convergence and the size of the inequality consumption beta, thereby reducing the estimated value of the social cost of carbon.

The second part of the paper, instead, introduces consumption risk into the picture, and look at how different assumptions about risk aversion, intragenerational inequality aversion and resistance to intertemporal substitution affect the social cost of carbon figure. In financial literature, the usual approach to disentangle risk aversion and resistance to intertemporal substitution is the one proposed by [Kreps and Porteus \(1978\)](#) and [Selden \(1978\)](#), where the social planner is adverse to an unequal distribution of expected utilities between today and the future. In contrast, I adopt an ex-post approach to equity, based on the expected equally distributed equivalent utility criterion introduced by [Fleurbaey \(2010\)](#), where welfare is not time additive. In that case, the social planner is adverse to an unequal distribution of realized utility across time. In this framework, I show that the size of the social cost of carbon is affected by: the expected economic convergence across regions; an expected inequality consumption beta, which measures the expected correlation between regional income and regional marginal damages; the traditional consumption beta of asset pricing, which, in our case, measures the correlation between average income and average marginal damages inside a generation; the variance of average income inside a generation. A positive consumption beta due to the presence of risk raises the socially efficient discount rate, given that the implementation of the mitigation policy would aggravate the macroeconomic risk. As for

the inequality consumption beta, the effect on the social cost of carbon is ambiguous, as a positive beta also raises the benefits of the policy. The variance of average income, instead, contributes to the standard precautionary effect in discounting, which leads to an increase in the SCC, as an increase in risk raises the willingness to protect future wealth. Moreover, by disentangling intergenerational inequality aversion and risk aversion according to an ex-post approach to equity, also the degree of correlation of regional risks at different points in time matters. However, their weight in the computation of the social cost of carbon is only marginal when we assume a very long time horizon, as the climate change problem would suggest. I show that, as in the deterministic case, an increase in intra-generational inequality aversion reduces the SCC. The impact of risk aversion, instead, goes through the comparison with intergenerational inequality aversion. If the social planner is more risk averse than inequality averse (which corresponds to the concept of correlation aversion, [Richard \(1975\)](#); [Epstein and Tanny \(1980\)](#)), an increase in risk aversion raises the SCC if the consumption beta is negative and risks are positively correlated over time. In contrast, if the planner is more inequality averse than risk averse, she prefers risks that are positively correlated over time, and an increase in risk aversion would reduce the SCC. Then, I apply the results to a calibration exercise that adopts the same climate change-economy model as in deterministic case, but adds risk in the regional economic impacts. Given the type of risk, I find a negative consumption beta, which supports larger values of the SCC with respect to the deterministic case. Moreover, as we expect from the analysis, the SCC is increasing in the coefficient of risk aversion, and decreasing in intra-generational inequality aversion.

The use of equity weighting in climate change has been advocated in the second assessment report of the IPCC ([Pearce et al., 1996](#)). Since then, several papers have analyzed the impact of equity weighting on both the social discount rate for environmental projects (see, e.g., [Azar and Sterner \(1996\)](#), [Gollier \(2010\)](#), [Emmerling \(2011\)](#), [Fleurbaey and Zuber \(2014\)](#)) and on the social cost of carbon (see, e.g., [Tol \(1999\)](#), [Pearce \(2003\)](#), [Hope \(2008\)](#), [Anthoff et al. \(2009\)](#), [Tol \(2010\)](#)). For what concerns the social cost of carbon, two approaches have been followed. The standard approach (described, e.g., in [Tol \(1999\)](#) and adopted by a large part of the literature) applies equity weights to the distribution of consumption at a particular time. Thus, it first computes the net present value of damages accruing to region r at time t by using a regional discount factor, and then aggregates these present values through the use of time specific equity weights. The second approach (defined 'intertemporal approach' by [Nordhaus \(2014\)](#)), instead, applies equity weights to consumption over both time and space ([Anthoff et al., 2009](#); [Emmerling, 2011](#); [Nordhaus, 2014](#)). The equity weight for the

damages suffered by a specific region r at t depend, then, on both future consumption and current consumption. Unlike the former, the weights in the intertemporal approach match the social welfare function adopted; therefore, I will follow this approach below.

With few exceptions, the literature on the use of equity weights in the estimate of the social cost of carbon analyses a utilitarian social welfare function. [Fankhauser et al. \(1997\)](#) study the role of different social welfare functions in the aggregation of individual marginal damages, but they keep the analysis to a static context. [Tol \(2010\)](#) introduces a social welfare function that disentangles resistance to intertemporal substitution and inequality aversion, and estimates the social cost of carbon by adopting the model FUND. However, contrary to the approach taken in this paper, [Tol \(2010\)](#) assumes that the social planner is averse to the unequal distribution of intertemporal regional utilities. Thus, inequality aversion is expressed with respect to the entire consumption path, and there is no room for intra-generational inequality. [Dennig \(2013\)](#) modifies the standard Nordhaus' RICE model by separating inter- and intra-generational inequality aversion, in a way similar to the one suggested in this paper. However, he restricts himself to the deterministic case, and concentrates only to the uniform carbon tax (which does not correspond to the SCC in a multi-regional context). Finally, [Anthoff and Tol \(2010\)](#) study equity weights not through the lenses of social welfare function, but by adopting a national perspective. The weights, then, depend on the degree of altruism of the region under consideration. As I express the social cost of carbon as a function of the socially efficient discount rates, the paper is related to the literature that links risk and fairness concerns in the social discounting. In particular, [Fleurbaey and Zuber \(2014\)](#) analyze the discounting problem in a general framework that incorporates also the ex-post approach to equity adopted in the paper. Contrary to them, I further disentangle inter- and intra-generational equity concerns, and provide a quadratic approximation of the social discount rate to better analyze the relative importance of the different components. In addition, I include an application of the model to the computation of the social cost of carbon.

The paper is structured as follows. Section 2 introduces the welfare framework in the deterministic case, which disentangles inter- and intra-generational inequality aversion. Section 3 shows how to derive the social cost of carbon from the assumed social welfare framework, and analyzes the impacts of equity weights. After rewriting the social cost of carbon in terms of socially efficient discount rates, Section 4 explores the properties of the discount rate in the small, and derives a modified Ramsey equation. Then, Section 5 contains the calibration of a simple climate change-economy model in order to get an estimation of the SCC. Section

6 introduces risk into the picture. I first introduce a social framework that distinguishes between risk aversion and inequality aversion, both inter- and intra-generationally. Then, I characterize properties of the socially efficient discount rate, and apply the results to a calibration exercise. Finally, Section 7 discusses extensions and concludes. In not otherwise stated, proofs, figures and tables are gathered in the Appendix.

2.2 The welfare framework

For the time being, let us focus exclusively on a deterministic environment. Consider a dynamic economy with a discrete set of generations $t \in \{0, \dots, T\}$. Each generation is made of R regions, lives one period and is replaced by the next generation. Let N be the total number of people alive, and N_r the number of individuals living in region $r = 1, \dots, R$ ¹. Individuals derive utility only from a consumption good, and preferences are the same for all individuals and all cohorts. In addition, let us assume that there is heterogeneity in consumption (due to heterogeneity in endowments and technology) among people living in different regions, but not among people living in the same region.

The problem of the social planner is how to evaluate individual consumption streams over time and across regions. The most common approach in the climate change literature is discounted utilitarianism, where the social planner maximizes the discounted sum of regional utilities over time:

$$W^U = \sum_{t=0}^T e^{-\delta t} \sum_{r=1}^R N_r v(c_{r,t}) \quad (2.1)$$

where $c_{r,t}$ denotes the per-capita consumption level in region r and generation t . However, the utilitarian case does not distinguish between inter- and intra-generational equity. Indeed, the function v represents both resistance to intertemporal substitution and aversion to inequality within a given generation. As a consequence, the utilitarian social planner is indifferent between a transfer from a rich to a poor individual belonging to the same generation and a transfer between a rich and a poor belonging to different generations. Since several experiments have proved that preferences along the two dimensions differ (Atkinson et al., 2009; Carlsson et al., 2002), it might be appropriate to disentangle the two. Therefore, we will consider also the following social objective, named *Equally Distributed Equivalent*

¹For ease of exposition, let us assume that there is no population growth and no change in the composition of the regions. The model can be easily extended to a growth population framework.

criterion in the spirit of [Atkinson \(1970\)](#):

$$W^{EDE} = \sum_{t=0}^T e^{-\delta t} N v \left(\phi^{-1} \left(\sum_{r=1}^R \frac{N_r}{N} \phi(c_{r,t}) \right) \right) \quad (2.2)$$

where the function v still represents resistance to intertemporal substitution, while the function ϕ represents aversion to an unequal distribution of consumption within a generation. Thus, disentangling intra- and inter-generational inequality aversion requires a twofold nested welfare function. First, for each generation t , the social planner evaluates individual consumption through the social welfare function ϕ , whose concavity indicates the social preferences for redistribution. Then, given the social welfare function ϕ , the social planner determines the equally distributed equivalent consumption level as proposed by [Atkinson \(1970\)](#):

$$\phi(c_t^{ede}) \equiv \sum_{r=1}^R \frac{N_r}{N} \phi(c_{r,t}) \quad (2.3)$$

The level c_t^{ede} represents the uniform amount of consumption that delivers the same aggregate welfare to the social planner as the original unequal distribution of consumption. It expresses in a nutshell how much the social planner cares about inequality among contemporaries. Finally, the social planner evaluates inter-generational inequality by introducing a second welfare function that trades off consumption across generations. The general formulation would be:

$$W^{EDE} = W(c_0^{ede}, c_1^{ede}, \dots, c_t^{ede}, \dots)$$

In the paper, I assume that the inter-temporal social welfare function is continuous and characterized by exponential discounting², thereby ensuring time separability and stationarity

²The use of time discounting has been the object of a lively debate in the literature. Indeed, time discounting means that generations are not treated equally, as they receive a weight that depends on when they are borne. On the other side, time discounting allows orderings of infinite utility sequences (see [Koopmans \(1960\)](#) for an axiomatic characterization of discounted utilitarianism). [Dasgupta and Heal \(1979\)](#) provide the only sound justification for time discounting, based on the risk of existence of future generations. This is the route taken also by [Stern \(2006\)](#) for justifying a very low rate $\delta = 0.001$, which would stand for a 0.1% yearly probability of life extinction. For a more thorough discussion of time discounting, see, e.g. [Roemer \(2011\)](#). We keep time discounting for a more straightforward comparison with the utilitarian case, but we will eliminate it when we introduce risk.

when T goes to infinity. As a consequence, the social objective becomes³:

$$W^{EDE} = \sum_{t=0}^T e^{-\delta t} N v(c_t^{ede}) \quad (2.4)$$

Note that if ϕ is linear, the social planner is inequality neutral. Then, c_t^{ede} would correspond to the average global consumption at t . As a consequence, it would be equivalent to consider only one aggregate representative agent, with no regard for distributional issues within a generation. In contrast, if $\phi = v$, we are back to the utilitarian case.

2.3 Equity weights and the social cost of carbon

The social cost of carbon represents the economic damages created by an additional ton of carbon released in the atmosphere. Assume that society reduces current carbon emissions by one ton, which will result in some current welfare costs as emissions contribute to production, and, hence, to consumption. Since greenhouse gases are long-lived, the damages caused by one ton of emissions today will lead to a change in consumption at every future time $t \geq 0$, i.e. consumption depends on the entire history of past emissions. Then, the social cost of carbon will be equivalent to the future marginal welfare damages caused by current emissions divided by the current marginal welfare costs of the mitigation policy.

Let $d_{r,t} \equiv \frac{\partial c_{r,t}}{\partial E_0}$ be the marginal increase in individual consumption for region r at time t due to a marginal decrease in global emissions at time $t = 0$. Then, for a given welfare function W , the marginal externality damage in welfare terms is represented by:

$$\Lambda_0 \equiv \frac{\partial W}{\partial E_0} = \sum_{t=0}^T \sum_{r=1}^R n_r \frac{\partial W}{\partial c_{r,t}} d_{r,t}$$

where $n_r \equiv \frac{N_r}{N}$ is the proportion of individuals living in region r , and $W_t^r \equiv \frac{\partial W}{\partial c_{r,t}}$ is the social priority given to the consumption of an individual living in region r at time t .

³Note that there exists a second way to disentangle intra-generational and inter-generational equity preferences, proposed e.g. in Tol (2010). Indeed, I have assumed that the social planner is averse to the unequal distribution of consumption inside each generation. However, she may also consider the over time path of consumption in each region, and be averse to the unequal distribution of the entire path. In that case, the social planner would first evaluate the temporal distribution of consumption inside each region, by computing a sort of time constant equivalent for each region. Then, she would aggregate those indexes of temporal distribution of consumptions across regions. However, as I have assumed that individuals live for only one period, and the time horizon can be infinite, the disentangling approach outlined in the text seems more appropriate.

To compute the social cost of carbon, we need to divide the marginal externality damages Λ_0 by the welfare costs of the policy, which, in this simple framework, is represented by the marginal social utility of current consumption of the regions that are engaging in emission reductions. An important issue is which consumption value should be used for the normalization of the marginal welfare costs of emissions, i.e. which region should bear the costs of the mitigation policy. For instance, [Anthoff et al. \(2009\)](#) use aggregate consumption, while [Nordhaus \(2014\)](#) compares the case where everyone bears the costs to the case where only rich regions sustain the costs of the mitigation policy⁴. Clearly, the normalization matters, as the SCC will be increasing in the consumption level of the regions suffering the costs. There is no valid reason for picking a specific method. Therefore, let us consider a more general setting for the moment, and let us denote by μ_r the share of mitigation costs borne by an individual in region r , with $\sum_{r=1}^R \mu_r = 1$ ⁵. Then, the social cost of carbon due to an additional ton of emissions today is defined as:

$$SCC_0 = \frac{\Lambda_0}{\sum_{r=1}^R \mu_r \frac{\partial W}{\partial c_{r,0}}} = \sum_{t=0}^T \sum_{r=1}^R n_r \frac{W_t^r}{\sum_{r=1}^R \mu_r W_0^r} d_{r,t} \quad (2.5)$$

If the costs are equally divided, then $\mu_r = n_r, \forall r$; if only region k reduces emissions, then $\mu_k = 1$ and $\mu_r = 0, \forall r \neq k$.

The formula (2.5) for the SCC highlights the role of equity weighting. Indeed, it requires to weigh consumption damages in region r at time t by the *intertemporal equity weight* $\omega_{r,t} \equiv \frac{W_t^r}{\sum_{r=1}^R \mu_r W_0^r}$. These weights represent the social priority of an individual living in region r at time t , discounted by the impact of the policy at time 0. As the weights are time and region specific, they will depend on the social preferences for both intra- and inter-generational inequality, and on the evolution of inequality over time. If we use the social

⁴The rationale for using the consumption of a single region in the normalization is that the SCC can then be used in national cost-benefit analysis and directly compared to other domestic regulations. However, it seems a bit odd to assume that a particular region adopts the perspective of a global social planner, thereby putting the same weight to its own damages and the damages suffered by the rest of the world. For a discussion on how the SCC figure could be made compatible to domestic interests, see [Anthoff and Tol \(2010\)](#)

⁵If we denote by $\frac{\partial c_{r,0}}{\partial E_0}$ the marginal costs borne at time $t = 0$ by an individual living in region r , and by $\frac{\partial C_0}{\partial E_0} = \sum_{r=1}^R n_{r,0} \frac{\partial c_{r,0}}{\partial E_0}$ the total marginal cost of the policy, then $\mu_r \equiv \frac{n_{r,0} \frac{\partial c_{r,0}}{\partial E_0}}{\frac{\partial C_0}{\partial E_0}}$

welfare criterion (2.2), the weight for region r at time t becomes:

$$\omega_{r,t} = e^{-\delta t} \frac{\frac{v'(c_t^{ede})}{\phi'(c_t^{ede})} \phi'(c_{r,t})}{\sum_{r=1}^R \mu_r \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})} \phi'(c_{r,0})}$$

By taking the derivative with respect to the consumption of an agent in region r at t , we find that:

$$\frac{\partial \omega_{r,t}}{\partial c_{r,t}} \propto \left\{ A_\phi(c_t^{ede}) - A_v(c_t^{ede}) \right\} \frac{\phi'(c_{r,t})}{\phi'(c_t^{ede})} - A_\phi(c_{r,t}) \quad (2.6)$$

where $A_h = -\frac{h''}{h'}$ denotes the coefficient of absolute inequality aversion of function $h = v, \phi$. Expression (2.6) clarifies the impact of different social preference assumptions on the intertemporal equity weight. If the social planner is utilitarian, preferences for intra-generational equity and preferences for intertemporal substitution coincide: $A_\phi - A_v = 0$. Then, the intertemporal equity weight would become $\omega_{r,t}^U = e^{-\delta t} \frac{v'(c_{r,t})}{\sum_{r=1}^R \mu_r v'(c_{r,0})}$, which is decreasing in individual consumption. Thus, poor agents get a larger weight in the computation for the social cost of carbon. If the social planner does not care about intra-generational inequality, i.e. ϕ is linear, the weights depend exclusively on the generation, and all contemporaneous individuals get the same weight: $\omega_t = e^{-\delta t} \frac{v'(c_t)}{v'(c_0)}$, where $c_t = \sum_{r=1}^R n_r c_{r,t}$. This case would correspond to the representative agent situation, where the social planner cares about discounting (different weights over time), but not about equity. Again, poor generations get a larger weight. When both ϕ and v are linear, we are computing the SCC by using no distributional weight at all, following the traditional cost-benefit analysis rule that asks for no particular treatment of different individuals. Finally, note that in the general EDE case the weight is not necessarily decreasing in individual consumption. Indeed, the social planner cares not only about how much poor an individual is, but also how much poor relatively to his contemporaries. Therefore, if the social planner cares more about intra-generational than inter-generational inequality, $A_\phi - A_v \geq 0$, a poor individual living in a poor generation will get a lower weight than a poor individual living in a rich generation. As a consequence, the weight is not necessarily monotonic in individual consumption. Clearly, if the social planner is more inter-generational than intra-generational inequality averse, the opposite holds: living in a poor generation outweighs living in a very unequal generation. Therefore, by changing the assumptions about the function v and ϕ , it is possible to discuss the various types of equity weighting suggested in the literature.

2.4 The socially efficient discount rate

In order to identify the different components of the social cost of carbon, and the impact of different types of equity weighting (see social preferences), it is more convenient to rewrite expression (2.5) as a discount rate. Let $d_t \equiv \sum_{r=1}^R n_r d_{r,t}$ be the average marginal damage at t . Then:

$$SCC_0 = \sum_{t=0}^T e^{-\rho_t(d_t)t} d_t \quad (2.7)$$

Therefore, to compute the SCC, we need to determine the average damages for all generations t , and then discount them at the rate $\rho_t(d_t)$:

$$\rho_t(d_t) = -\frac{1}{t} \ln \frac{\sum_{r=1}^R n_r W_t^r d_{r,t}}{d_t \sum_{r=1}^R \mu_r W_0^r} \quad (2.8)$$

Let $E^t x \equiv \sum_{r=1}^R n_r x_{r,t}$ represent the average value of a given variable x in period t , and $\hat{E}^0 x \equiv \sum_{r=1}^R \mu_r x_{r,t}$ the average value given the sharing rule μ . By applying the social preferences defined in (2.2), the following result unfolds.

Proposition 8. *Consider the EDE welfare social criterion (2.2). The social discount rate $\rho_t(d_t)$ for a project with heterogeneous benefits $d_{r,t}$ and heterogeneous cost shares μ_r is given by:*

$$\rho_t(d_t) = \delta - \frac{1}{t} \ln \frac{E^t \frac{v'(c_t^{ede})}{\phi'(c_t^{ede})} \phi'(c_{r,t})}{E^0 \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})} \phi'(c_{r,0})} - \frac{1}{t} \ln \frac{E^t \phi'(c_{r,t}) d_{r,t}}{d_t E^t \phi'(c_{r,t})} + \frac{1}{t} \ln \frac{\hat{E}^0 \phi'(c_{r,0})}{E^0 \phi'(c_{r,0})} \quad (2.9)$$

The social discount rate for average damages $\rho_t(d_t)$ has three components. The first two elements in (2.9) represent the discount rate r_t for projects whose benefits and costs are uniform across individuals:

$$r_t \equiv -\frac{1}{t} \ln \frac{\sum_{r=1}^R n_r W_t^r}{\sum_{r=1}^R n_r W_0^r} = \delta - \frac{1}{t} \ln \frac{E^t \frac{v'(c_t^{ede})}{\phi'(c_t^{ede})} \phi'(c_{r,t})}{E^0 \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})} \phi'(c_{r,0})}$$

The last two elements in (2.9) represent the policy-specific inequality premia, due to the fact that marginal damages are heterogeneous (third term) and the costs of the mitigation policy are unevenly distributed (last term). Let $\chi_t(d_t)$ represent the policy specific inequality premium for projects with heterogeneous benefits, and χ_0 the project-specific inequality

premium due to an uneven distribution of costs⁶:

$$\chi_t(d_t) \equiv -\frac{1}{t} \ln \frac{\sum_{r=1}^R n_r W_t^r d_{r,t}}{d_t \sum_{r=1}^R n_r W_t^r} = -\frac{1}{t} \ln \frac{E^t \phi'(c_{r,t}) d_{r,t}}{d_t E^t \phi'(c_{r,t})}$$

$$\chi_0 \equiv \frac{1}{t} \ln \frac{\sum_{r=1}^R \mu_r W_0^r}{\sum_{r=1}^R n_r W_0^r} = \frac{1}{t} \ln \frac{\hat{E}^0 \phi'(c_{r,0})}{E^0 \phi'(c_{r,0})}$$

Therefore, the decomposition (2.9) tells us that, to compute the social cost of carbon, the average damages d_t must be discounted at a rate that takes into account both the existing inequality in the society (expressed by the discount rate r_t) and the inequality inherent in the project. Moreover, the two dimensions are neatly separated. The discount rate r_t describes the social planner's preferences for equity across time and space given that consumption is heterogeneously distributed both within and between generations. The inequality premia, instead, has no intertemporal dimension. They represent the social planner's preferences for the mitigation policy given that its implementation will affect the degree of inequality in society, both today (as costs are not equally distributed) and in the future (as the benefits of the policy are unequally distributed). In the following sections we will separately analyze the three components of the socially efficient discount rate, and study the implications of different assumptions about equity weighting.

2.4.1 Equity adjusted discount rate

From Proposition 1, the discount rate r_t for projects with uniform costs and benefits includes two elements: the rate of time preferences δ , and an economic growth effect that takes into account the change in consumption inequality over time. As usual, the rate of time preference increases the discount rate, as the social planner would be less concerned about the well-being

⁶To see how $\chi_t(d_t)$ and χ_0 are indeed inequality premia, consider a project that delivers $d_{r,t}$ to region r at t for each unit invested, and let ϵ be the size of the investment. Let F_t be the inequality neutral payoff of the project, i.e. the uniform benefit at t that, from the social planner's point of view, is as fair as the heterogeneous benefits $d_{r,t}$:

$$W(\dots, c_{1,t} + \epsilon d_{1,t}, \dots, c_{r,t} + \epsilon d_{r,t}, \dots, c_{R,t} + \epsilon d_{R,t}, \dots) = W(\dots, c_{1,t} + \epsilon F_t, \dots, c_{r,t} + \epsilon F_t, \dots, c_{R,t} + \epsilon F_t, \dots)$$

For a marginal investment, we have:

$$F_t = \frac{\sum_{r=1}^R n_r W_t^r d_{r,t}}{\sum_{r=1}^R n_r W_t^r}$$

Moreover, by definition, the inequality premium is equivalent to: $\chi_t(d_t) = -\frac{1}{t} \ln \frac{F_t}{d_t}$, which delivers exactly the expression above. Similarly, for the inequality premium χ_0 .

of future generations. Similarly, the economic growth effect should increase the discount rate, as the social planner will be less interested in helping the future richer generations. However, if the decision maker is intra-generational inequality averse, she will care not only about the average growth, but also about the effects of growth on the distribution of consumption. Intuitively, we expect the growth effect to increase if there is convergence, i.e. when future inequality is lower than current one, and decrease in the opposite case⁷. However, the size of this effect will depend on the social planner's preferences for intra- and inter-generational inequality, and, presumably, on the difference between the two.

If we do not use intra-generational equity weights in the computation of the social cost of carbon (ϕ is linear), the third effect would just be the usual growth effect of the Ramsey rule: $-\frac{1}{t} \ln \frac{v'(c_t)}{v'(c_0)} \simeq -\frac{v''(c_0)}{v'(c_0)} \frac{c_t - c_0}{t}$. If relative risk aversion is constant and equal to R_v , and the yearly growth rate of the representative agent is constant over time and equal to g , the growth effect amounts to $R_v g$. Instead, if the social planner was utilitarian, the discount rate would be equal to:

$$r_t^U = \delta - \frac{1}{t} \ln \frac{v'(c_t)}{v'(c_0)} - \frac{1}{t} \ln \frac{E^t v'(c_{r,t})}{v'(c_t)} + \frac{1}{t} \ln \frac{E^0 v'(c_{r,0})}{v'(c_0)}$$

Thus, the discount rate is affected by average growth (second term) and by two inequality premia that reflects the degree of inequality at time t and at time 0. Compared to the utilitarian case, EDE social preferences (2.2) entail a change in the individual marginal utility of consumption both at 0 and at t :

$$r_t = \delta - \frac{1}{t} \ln \frac{v'(c_t)}{v'(c_0)} - \frac{1}{t} \ln \frac{E^t \lambda_{r,t} v'(c_{r,t})}{v'(c_t)} + \frac{1}{t} \ln \frac{E^0 \lambda_{r,0} v'(c_{r,0})}{v'(c_0)} \quad (2.10)$$

where, $\forall t$, we have that:

$$\lambda_{r,t} \equiv \frac{v'(c_t^{ede}) \phi'(c_{r,t})}{\phi'(c_t^{ede}) v'(c_{r,t})}$$

The welfare weights $\lambda_{r,t}$ represent the extra weight that the social planner gives to intra-generational equality with respect to the utilitarian decision maker. Therefore, disentangling intra- and inter-generational preferences affect exclusively the inequality premia, which embody how much the planner cares for inequality. In the utilitarian case, $\lambda_{r,t} = 1$ as $v = \phi$. Instead, if $\frac{v'}{\phi}$ is increasing in consumption, $\lambda_{r,t} > 1$ for relatively poor agents, i.e. for those individuals s.t. $c_{r,t} < c_t^{ede}$. On the contrary, when $\frac{v'}{\phi}$ is decreasing, rich people gets a larger

⁷See, e.g., [Gollier \(2010\)](#) and [Emmerling \(2011\)](#) for an analysis of the impact of convergence on the discount rate under risk and utilitarian preferences.

weight with respect to the utilitarian one. The slope of $\frac{v'}{\phi'}$ refers to the comparison between inequality aversion and resistance to intertemporal substitution. The social planner weighs more the marginal utility of relatively poor agents only when she is more interested in intra- than inter-generational equity; in other words, when she is more intra-generationally inequality averse than the social planner.

It would be interesting to compare the discount rate under the three models (no intra-generational equity weights, utilitarianism, EDE) in general terms. However, as already noted in Gollier (2010) and Emmerling (2011), the comparison is feasible only under limited cases. That depends on the fact that we have assumed consumption inequality both today and in the future, besides a (positive) average growth. As a consequence, a change in social preferences does not change the inequality premia proportionally, but the effect will be tuned to the specific distribution of consumption today and in the future. Therefore, we will directly jump to the small economy case. Appendix A gathers some results on the comparison in the large.

Assume that, for all t , $c_{r,t} = c_0(1 + \epsilon_{r,t})$, where $c_0 \equiv E^0 c_{r,0}$, and $\epsilon_{r,t}$ represents the relative difference (growth rate) in region r at time t with respect to the average consumption at time $t = 0$. The variability in the distribution of regional growth rates is denoted $V^t(\epsilon)$ ⁸. Let $R_v = -\frac{v''(c_0)c_0}{v'(c_0)}$ and $R_\phi = -\frac{\phi''(c_0)c_0}{\phi'(c_0)}$ denote, respectively, the coefficient of relative resistance to intertemporal substitution and relative inequality aversion. Moreover, let $P_\phi = -\frac{\phi'''(c_0)c_0}{\phi''(c_0)}$ represent the coefficient of *relative inequality prudence*. As the standard prudence coefficient measures the strength of the precautionary saving motive under future income uncertainty (Leland, 1968; Kimball, 1990), the inequality prudence coefficient indicates the social planner's incentives to transfer wealth because of future income inequality. Then, the following result holds:

Proposition 9. *The quadratic approximation of the equity adjusted discount rate r_t is given by:*

$$r_t \simeq \delta + \frac{1}{t}R_v E^t \epsilon - \frac{1}{2t}R_\phi \left\{ P_\phi - R_\phi + R_v \right\} \left\{ V^t(\epsilon) - V^0(\epsilon) \right\} \quad (2.11)$$

The first two elements constitute the standard Ramsey rule. The equity adjusted discount rate depends on the rate of impatience and on a wealth effect. The wealth effect is positive when average consumption grows over time. It is approximately equal to the product of relative resistance to intertemporal substitution and the annualized growth rate of average consumption.

⁸By definition: $V^t(\epsilon) = E^t(\epsilon_{r,t})^2 - (E^t \epsilon_{r,t})^2$.

The third term is an inequality precautionary effect, which appears when there is inequality in individual consumption levels, either today or in the future. Its intensity is proportional to the change in inequality over time, and to the product of relative inequality aversion and a term representing the difference between intra- and inter-generational equity preferences. If $P_\phi - R_\phi + R_v \geq 0$, this precautionary effect reduces the discount rate, provided that inequality is increasing over time. This suggests that inequality affecting the future tends to raise our willingness to invest for that future. The peculiar shape for the size of the precautionary effect, i.e. $P_\phi - R_\phi + R_v \geq 0$, depends on how the EDE criterion has been constructed. Indeed, as previously discussed, when the social planner cares more about inter- than intra-generational inequality, a poor agent living in an unlucky generation is more rewarded than a poor agent living in a lucky generation. As a consequence, if inequality in a given generation increases, the social planner will assign an extra weight to this generation only if he is more inter- than intra-generational inequality averse. Moreover, note that the precautionary effect due to an increase in inequality is positive if $P_\phi \geq R_\phi$. This condition is equivalent to say that the social planner has decreasing absolute inequality aversion (DAIA) preferences with respect to intra-generationally inequality: $-\frac{\phi''(c_0)}{\phi'(c_0)}$ is decreasing in c_0 . In other words, for a given degree of inequality, the richer the society is, the less the social planner cares about differences in consumption.

The most common functional form used to compute the social cost of carbon is the isoelastic one. Let γ denote the coefficient of relative resistance to intertemporal substitution, and φ the index of relative inequality aversion: $v(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $\phi(c) = \frac{c^{1-\varphi}}{1-\varphi}$. Then, the discount rate r_t becomes:

$$r_t = \delta + \frac{1}{t}\gamma E^t \epsilon - \frac{1}{2t}\varphi [1 + \gamma] \left\{ V^t(\epsilon) - V^0(\epsilon) \right\}$$

As a consequence, the precautionary effect due to an increase in inequality is positive whatever the comparison between intra- and inter-generational inequality aversion.

As already pointed out in the literature, if we expect convergence in the future, the equity adjusted discount rate is lower than the discount rate of the representative agent case (i.e. with no intra-temporal equity weights), independently of whether the social planner is utilitarian or EDE. Compared to the utilitarian welfare framework, the impact of EDE preferences on the discount rate r_t is ambiguous. Assume that $V^t(\epsilon) - V^0(\epsilon) \geq 0$. Disentangling resistance to intertemporal substitution and inequality aversion reduces the discount rate if and only if

$$(P_\phi - R_\phi)(R_\phi - R_v) + R_v(P_\phi - P_v) \geq 0$$

Assuming that $R_\phi \geq R_v$, the discount rate decreases if the function ϕ is DAIA and $P_\phi \geq P_v$. This last condition means that the EDE social planner has a larger precautionary motive than the utilitarian one. In the isoelastic case, the expression reduces to

$$(\varphi - \gamma)(1 + \gamma) \geq 0$$

Therefore, in the isoelastic case, the introduction of EDE preferences reduces the discount rate when inequality increases over time if and only if the social planner cares more about intra-generational than inter-generational inequality.

2.4.2 Inequality premia

Contrary to the equity adjusted discount rate, the inequality premia $\chi_t(d_t)$ and χ_0 reflect the characteristics of the project. If there was only a representative agent in the economy, or if costs and benefits of the project were equally divided, the two premia would disappear. In the presence of heterogeneous costs and benefits, instead, the social planner takes into account the impact of the project on the existing inequality. Intuitively, we expect the two premia to have an opposite effect: the social discount rate $\rho_t(d_t)$ should increase if the costs of the project are borne by the poorest regions and the benefits enjoyed by the richest regions. The following proposition gives credit to this intuition.

Proposition 10. *Assume that ϕ is concave, and $\phi''' > 0$. The socially efficient discount rate $\rho_t(d_t)$ increases if:*

- $d_{r,t}$ and $c_{r,t}$ are positively dependent;
- the weights μ_r induce either a FSD- or a SSD-dominated shift in the distribution of income.

Proof. The first point is proved by using a result by [Tchen \(1980\)](#), which signs changes in statistical dependence among variables. The proof of the second point is based on the fact that introducing an unequal cost sharing rule is equivalent to change the distribution of consumption across regions. See the Appendix. \square

In crude words, when rich individuals suffer more from climate change, an inequality averse social planner attaches a positive inequality premium $\chi_t(d_t)$ to the mitigation policy, as the reduction in emissions will increase inequality in society. In that case, climate change has a fairness component, as it hits mainly rich people; slowing climate change down, then,

brings about more inequality. In contrast, if poor people are likely to suffer more from climate change, the inequality premium $\chi_t(d_t)$ is negative, since implementing the mitigation project reduces the degree of inequality.

A similar argument can be made for the inequality premium at time 0, χ_0 . The implementation of a non-uniform sharing rule is equivalent to a fictitious change in the distribution of individuals across regions: the proportion of individuals living in region r goes from n_r to μ_r . Thus, we can make use of the notions of stochastic dominance to sign the inequality premium. An example of first order stochastic dominance is the monotone likelihood ratio, according to which $\frac{n_r}{\mu_r} \geq \frac{n_{r'}}{\mu_{r'}}$ if $c_{r,0} \geq c_{r',0}$. Therefore, the inequality premium increases if the artificial share of people in the poorest region r' increases more than the share of people in the richest region r ; in other words, if the poorest region bears proportionally a larger share of the mitigation costs. The concept of second order stochastic dominance, instead, is equivalent to a change in the Lorenz curve⁹. Thus, if the Lorenz curve of consumption when distributed according to μ lies always under the Lorenz curve of consumption when distributed according to the shares n_r , the cost sharing rule worsens inequality. As a consequence, if the emission reduction is sustained mainly by rich people, the social planner attaches a negative inequality premium, as the allocation of costs somehow reduces current inequality. The higher is the mitigation effort required to poor agents, the larger χ_0 is.

As with the equity adjusted discount rate, it might be useful to consider a quadratic approximation of the inequality premia. As before, let $c_{r,t} = c_0(1 + \epsilon_{r,t})$ for all $t = 0, \dots, T$, with $c_0 \equiv E^0 c_{r,0}$. Moreover, let $Cov^t(\epsilon, d)$ represent the covariance between individual consumption and individual damages at t ; $\hat{E}^0 \epsilon = \sum_{r=1}^R \mu_r \epsilon_{r,t}$ the average relative difference from initial consumption c_0 with the income distribution induced by the shares μ_r ; $\hat{V}^0(\epsilon)$ the index of variability at time 0 of the relative consumption differences under the income distribution induced by the weights μ_r .

Proposition 11. *The quadratic approximation of the project specific inequality premia is given by:*

$$\chi_t(d_t) + \chi_0 \simeq \frac{1}{t} R_\phi Cov^t(\epsilon, d) - \frac{1}{t} R_\phi \hat{E}^0 \epsilon + \frac{1}{2t} R_\phi P_\phi \left\{ \hat{V}^0(\epsilon) - V^0(\epsilon) \right\} \quad (2.12)$$

The project-specific inequality effect raises the discount rate when rich people are more exposed to the damages (positive covariance term), as the implementation of the project would aggravate existing inequality. Similarly, if the costs of the project befall to poor

⁹See [Atkinson \(1970\)](#) for a comparison of the concepts of change in risk (stochastic dominance) and change in inequality.

individuals, the social planner values less the project as it will increase current inequality; thus, the social discount rate will be larger. Note that, if mitigation costs are equally distributed among regions, then $\hat{V}^0(\epsilon) = V^0(\epsilon)$ and $\hat{E}^0\epsilon = 0$. If the costs are borne mainly by poor people, $\hat{V}^0(\epsilon) > V^0(\epsilon)$ and $\hat{E}^0\epsilon < 0$. The opposite when rich regions sustain the costs of the policy.

Finally, compared to the utilitarian case, both inequality premia have a larger absolute value when $R_\phi \geq R_v$ and $P_\phi \geq P_v$, i.e. the EDE social planner is both more risk averse and more inequality prudent than the utilitarian one. Isoelastic functions satisfy those conditions.

2.4.3 The modified Ramsey equation

As with the standard Ramsey rule, the approximated equity adjusted discount rate (2.11) and the approximated inequality premia (2.12) are exact under isoelastic functions and lognormal distributions. For instance, assume, as a simplification, that costs are equally distributed at time 0, and that the distribution of consumption across regions can be approximated by a lognormal distribution, both at time 0 and at time t . The log of current consumption is characterized by mean μ_0 and variance σ_0^2 . Moreover, assume that $(\ln c_{r,t}, \ln d_{r,t})$ are jointly normal, with means and variances denoted respectively $(\mu_{c,t}, \mu_{d,t})$ and $(\sigma_{c,t}^2, \sigma_{d,t}^2)$. Their index of correlation is denoted R . In addition, suppose that $v'(c) = c^{-\gamma}$ and $\phi'(c) = c^{-\varphi}$. Then, by using the well known property that if $x \sim N(a, b^2)$ then for all $\theta \in \mathbb{R}$, $Ee^{\theta x} = \exp\{\theta a + 0.5\theta^2 b^2\}$, the socially efficient discount rate is exactly equal to:

$$\rho_t(d_t) = \delta + \gamma g - \varphi(1 + \gamma) \frac{\sigma_{c,t}^2 - \sigma_0^2}{2t} + \varphi \beta_t \frac{\sigma_{c,t}^2}{t} \quad (2.13)$$

where $g \equiv \frac{\mu_{c,t} - \mu_0}{t} + \frac{\sigma_{c,t}^2 - \sigma_{c,0}^2}{2t}$ is the constant average growth rate¹⁰, and β_t is the "inequality consumption beta" of the project, which represents the sensitivity of regional damages to regional consumption:

$$\beta_t = \frac{R\sigma_{d,t}}{\sigma_{c,t}} = \frac{Cov(\ln c_{r,t}, \ln d_{r,t})}{\sigma_{c,t}^2} \quad (2.14)$$

Note that our problem looks like a consumption-based capital asset pricing model, except that the presence of risk is replaced by the presence of inequality. Indeed, the inequality premium $\chi_t(d_t)$ plays a role similar to the risk premium in the traditional consumption based capital asset pricing model. A key concept in CCAPM is the so called "consumption β " of

¹⁰Remember that, given the lognormal distribution, the average consumption in period t is given by $E^t e^{\ln c_{r,t}} = \exp\{\mu_{c,t} + \frac{1}{2}\sigma_{c,t}^2\}$.

the project (Lucas, 1978), which represents the correlation between consumption and cash flow. Similarly, we can introduce the concept of *inequality consumption* β , which represents the elasticity of damages to consumption. The inequality consumption β of an investment project can be interpreted as the average percentage increase in damages when aggregate consumption increases by 1%.

Although we assumed that the population is divided into a discrete number of regions, and that individuals living in a given region are homogeneous, the modified Ramsey equation (2.13) can be a useful reference point for the estimation of the social discount rate, and, consequently, for the social cost of carbon. Indeed, the lognormal distribution has proven to be a very good approximation of the world income distribution (Atkinson and Brandolini, 2010). Therefore, to estimate the social discount rate we need information about: the yearly average growth rate, the change in the degree of inequality over time, and the inequality consumption β (which depends on the inequality in marginal damages and their correlation with regional consumption). Finally, we need to calibrate the relative resistance to intertemporal substitution γ and the relative coefficient of inequality aversion φ .

Finally, note that economic convergence always reduces the social cost of carbon, while the impact of a positive inequality consumption beta is ambiguous. Indeed, a large positive β increases the socially efficient discount rate, but at the same time it raises also the average benefits of the mitigation project. Indeed, if we express the regional damages as a function of consumption, $d_{r,t} = c_{r,t}^{\beta_t}$, then the social cost of carbon becomes:

$$\begin{aligned} SCC &= \sum_{t=0}^T e^{-\rho_t(d_t)t} d_t = \sum_{t=0}^T e^{-\rho_t(d_t)t} E^t c_{r,t}^{\beta_t} = \sum_{t=0}^T e^{-\rho_t(d_t)t} e^{\beta_t \mu_{c,t} + 0.5 \beta_t^2 \sigma_{c,t}^2} \\ &= \sum_{t=0}^T \exp \left\{ -r_t + \beta_t (\mu_{c,t} - \varphi \sigma_{c,t}^2) + 0.5 \beta_t^2 \sigma_{c,t}^2 \right\} \end{aligned}$$

Therefore, the social cost of carbon is increasing in β_t as long as $\beta_t \geq \varphi - \frac{\mu_{c,t}}{\sigma_{c,t}^2}$. A positive inequality consumption beta is not necessarily bad news for the social cost of carbon. Note, however, that this positive effect is reduced as intragenerational inequality aversion φ increases. Therefore, the larger the coefficient of inequality aversion is, the more likely it is to have a low social cost of carbon when rich individuals benefit the most from the climate action.

2.5 Calibration exercise

Let us consider a very simple model of climate change, based on Nordhaus (2010) RICE model. The world is divided into 12 regions, as in the RICE model, and we employ projections about population dynamics and GDP growth that match the SRES A2 emission scenario. Let $Y_{r,t}$ denote the total amount of consumption goods in region r , that we take as exogenous; we assume that climate change reduces the actual amount available for consumption, but a mitigation policy $m_{r,t}$ can restore it¹¹. Then, total consumption in region r at time t is given by:

$$C_{r,t} = Y_{r,t}(1 - \Lambda_{r,t})\Omega_{r,t}$$

where $1 - \Lambda_{r,t}$ represents the mitigation costs, such that $\Lambda'_{r,t}(m_{r,t}) > 0$ and $\Lambda_{r,t}(0) = 0$. In the calibration, we assume that the policy maker does not invest in mitigation, i.e. $m_{r,t} = 0$ ¹².

For what concerns the damage function and the carbon cycle, we adopt the model proposed by Golosov et al. (2014). The damage function $1 - \Omega_{r,t}(\Delta S_t)$ depends on the difference between the concentration of emissions at time t , S_t , and the pre-industrial concentration level \bar{S} : $\Delta S_t = S_t - \bar{S}$. The function is exponential, with λ_r indicating the economic impact of an increase in CO₂ concentration on region r :

$$\Omega_{r,t}(\Delta S_t) = e^{-\lambda_r \Delta S_t}$$

The parameters λ_r are estimated by using the damages reported in Nordhaus (2010) RICE model following an increase in temperature by either 2.5°C or 6°C in 2100. A 2.5°C and a 6°C heating should occur, respectively, when S_t equals 1035 and 2342 gigaton of carbon; pre-industrial atmospheric CO₂ concentration is set at $\bar{S} = 581$ ¹³. To be consistent with the stochastic framework that we will introduce later, we will assume that, with a small probability p , temperature will increase by 6°C and damages will be very large (30% of GDP on average). Then, for each region, we can compute a λ_r^{low} and a λ_r^{high} , according to whether the increase in temperature will be small or large. The coefficients used in the

¹¹We follow the most common assumption in the literature that climate change causes a reduction in output level. Some authors have posited that global warming could have an effect also on growth (Pindyck, 2012) or on capital (Stokey, 1998).

¹²As a consequence, the SCC that we will compute does not reflect the optimal level of a carbon policy, but the minimum price that agents have to pay to start the investments in mitigation. Nordhaus (2014) shows that there is not a big difference between the SCC with no controls on carbon emissions and the SCC in the optimal path because the marginal damages in early periods are only slightly affected by the optimization process.

¹³See Golosov et al. (2014) for details.

calibration of the deterministic model corresponds to the expected value of the two extreme cases: $\lambda_r = p\lambda_r^{high} + (1-p)\lambda_r^{low}$. For the probability of larger damages, we take the value reported in [Goloso et al. \(2014\)](#): $p = 6.8\%$. The carbon cycle is characterized by:

$$\Delta S_t = \sum_{\tau=0}^{t+T} (1 - d_\tau) e_{t-\tau}$$

where $e_{t-\tau}$ denotes total emissions in period $t - \tau$. Therefore, the increase in temperature at time t depends on the accumulation of past emissions in the atmosphere. The function d_τ represents a decay rate, and is given by:

$$1 - d_\tau = \alpha_L + (1 - \alpha_L)\phi_0(1 - \alpha)^\tau$$

where α_L indicates the share of carbon emitted that stays forever in the atmosphere; $1 - \alpha_0$ the share of remaining emissions that exit the atmosphere immediately; α the geometric rate of depreciation of the remaining share of emissions. Following [Goloso et al. \(2014\)](#), the parameters are calibrated as: $\alpha_L = 0.2$; $\alpha_0 = 0.393$; $\alpha = 0.0228$. Emissions are a by-product of output:

$$e_{r,t} = \sigma_{r,t}(1 - m_{r,t})Y_{r,t}$$

where $e_{r,t}$ denotes total emissions produced by region r at time t , and $\sigma_{r,t}$ represents the carbon intensity. Investing in mitigation reduces carbon intensity. Estimates of baseline carbon intensity are taken from the RICE model ([Nordhaus, 2010](#)). Even without mitigation, carbon intensity is supposed to decline over time.

First of all, we can compute the per-capita marginal damages. Because of the form of the damage functions, the damages are a linear function of consumption per capita $c_{r,t}$:

$$d_{r,t} = -(\phi_L + (1 - \phi_L)\phi_0(1 - \phi)^t) \sum_{r=1}^R \lambda_r c_{r,t}$$

As a consequence, marginal damages are linear in the amount consumed by each region. As shown in [Goloso et al. \(2014\)](#), if we assume that the proportion of savings is constant over time, as empirical evidence seems to suggest, marginal damages become a linear function of output. Finally, the SCC is normalized as if all regions were bearing an equal share of the policy costs. Time 0 in the calibration exercise corresponds to the year 2010.

The modified Ramsey equation (2.13) can be a useful starting point for determining the socially efficient discount rate. Nordhaus' growth assumptions imply a slow decrease in growth over time, and a decline in inequality among regions. This convergent trend is consistent also with the results by Pinkovskiy and Sala-i-Martin (2009), who found significant convergence for the world as a whole in the period 1970-2006. Figure 4 in the Appendix reports the declining path of the variance of log income based on the SRES A2 projections used in the RICE model. About the correlation between income and impacts, the literature suggests that damages should be slightly negative correlated with income, pointing to the fact that poor countries are more exposed to the negative effects of climate change due to their geographical location and their lack of adaptation resources (see, e.g., Yohe et al. (2006)). However, based on the data adopted by Nordhaus for estimating the damage functions, we find an inequality consumption β positive and quite large (around 0.9), although declining over time (Figure 1). Indeed, although poor countries suffer more in relative terms, rich countries bears more losses because of the larger value of their assets. As a consequence, we should already expect quite a large impact of equity weighting on the discount rate compared to the representative agent case (with no intra-generational equity concerns). Therefore, the social cost of carbon with equity weighting is going to be lower than the social cost of carbon of the representative agent. However, note that the presence of a positive inequality consumption beta is going to mitigate the negative effect of economic convergence on the SCC as β is likely to be larger than $\varphi - \frac{\mu_{c,t}}{\sigma_{c,t}}$ ¹⁴.

The time preference δ has been the subject of a lively debate in the climate change literature. While Weitzman (2007) sets it at 2% and Nordhaus (2008) at 1%, Stern (2006) raises ethical issues in discounting the well-being of future generations, and proposes a value equal to 0.1%. We will run the calibration for three different parameter values: $\delta = \{0, 1, 2\}$.

Finally, we need to calibrate the coefficients of relative resistance to intertemporal substitution γ and relative inequality aversion φ . The first can be estimated from market data. In the climate change literature, intertemporal inequality aversion γ is usually set between 1 and 2¹⁵. For the index of inequality aversion φ , instead, there is much more uncertainty. The determination of its value is an ethical question, as it represents the degree of fairness inside a society, not just individual preferences for equity; as a consequence, it cannot be calibrated

¹⁴For instance, for the year 2015 $\sigma^2 = 1.5$, and $\mu = 8.6$ and $\beta = 0.94$, which implies a positive effect as long as $\varphi < 6.3$.

¹⁵Few examples: Weitzman (2007) and Nordhaus (2008) propose a value of 2; Stern (2006), Golosov et al. (2014) and Traeger (2015), instead, suggest to use a logarithmic felicity function

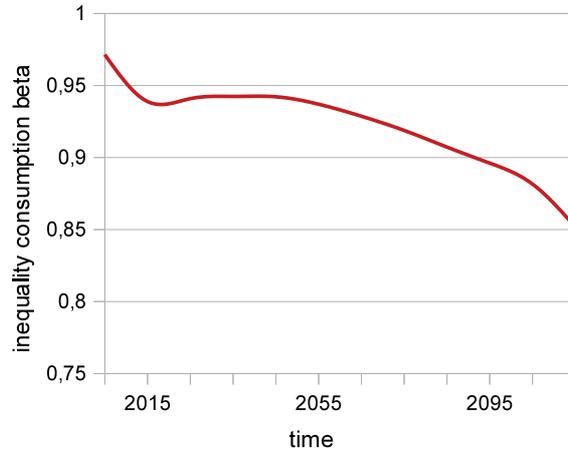


Figure 2.1: Time path of the inequality consumption β based on Nordhaus (2010).

from observed market behavior. What the literature usually does is to elicit inequality preferences either through the use of surveys (Amiel and Cowell, 1999; Carlsson et al., 2002, 2005) or through the study of taxation systems (Evans, 2005). The estimated coefficient of relative inequality aversion usually goes from 0.2 to 6.24, according to the different studies. Note that for the computation of official inequality measures, the US Census Bureau uses a value under unity: $\{0.25, 0.5, 0.75\}$. Moreover, in the climate change literature, Tol (2010) proposes a value equal to 0.7, which is estimated based on international development aid flows made by OECD countries. Atkinson et al. (2009) is a recent study that tries to elicit preferences for inequality and time (and risk) in the climate change case. In contrast to the rest of the literature, they find a coefficient of resistance to intertemporal substitution equal to 9, and a coefficient of inequality aversion in the interval 2-3. From the previous studies, then, it seems appropriate to take a value of φ lower than γ .

Figure 2 shows the socially efficient discount rate for $\gamma = 1$ and $\delta = 0$ and different values of $\varphi = \{0, 0.5, 1, 4\}$. The representative agent case corresponds to the line $\varphi = 0$. The utilitarian social planner is such that $\varphi = \gamma = 1$, while the last two curves correspond to the EDE social planner. The discount rate in the representative case is almost constant, and around 2%, which we could have recovered directly from the Ramsey equation as the growth rate is around 2%, $\delta = 0$ and $\gamma = 1$. The presence of inequality increases the discount rate, because of convergence and a positive correlation between income and impacts. From the modified Ramsey equation (2.13), we can argue that the huge difference between the representative agent case and the cases with equity concerns depends mainly on the size of

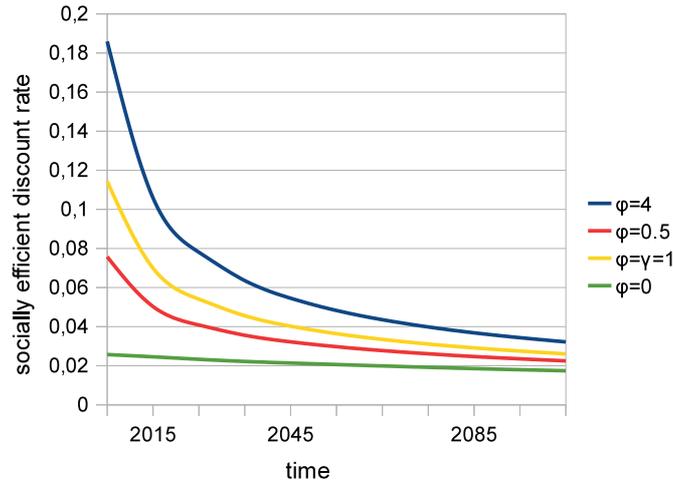


Figure 2.2: Time path of the socially efficient discount rate with $\delta = 0$, $\gamma = 1$ and different assumptions about intra-generational inequality aversion: $\phi = 0$, representative agent; $\phi = 1$, utilitarianism; $\phi = \{0.5, 4\}$, EDE preferences.

the inequality consumption β_t . In the long run, both β_t and the degree of inequality decrease, thereby downsizing the project-specific inequality premium. The socially efficient discount rate is decreasing over time, mainly because the speed of convergence and the inequality consumption β decline over time. When $\varphi \leq \gamma$, the introduction of an equally distributed equivalent welfare approach reduces the discount rate with respect to utilitarianism. The opposite occurs when $\varphi \geq \gamma$.

Given the path of the discount rate over time, we can now determine the social cost of carbon at time 0 implied by those numbers. Let us consider, for instance, the case $\delta = 0$. In the absence of inequality, the social cost of carbon is equal to \$61.83. The introduction of inequality sharply reduces the SCC: it amounts to \$29.87 in the utilitarian case, and \$43.89 in the EDE case with $\varphi = 0.5 < \gamma$. In contrast, when the EDE social planner is more intra-generationally inequality averse than the utilitarian one, $\varphi = 4$, the SCC amounts to just \$16.36. Therefore, the introduction of equity weighting has a negative impact on the SCC, which is partially mitigated when we disentangle intra- and inter-generational inequality aversion, provided that $\varphi \leq \gamma$ (as the existing literature seems to suggest). Table 2 in the Appendix compares the different SCC values when we vary the rate of time preference δ . We should stress how these numbers are just indicative of the optimal social discount rate, as we did not perform any optimization exercise, but made use of both exogenous damages and exogenous consumption paths.

2.6 Equity concerns under risk

2.6.1 The framework

In this section, we analyze how the model can be extended in the presence of uncertainty. As before, we will focus on a framework that is able to distinguish the different preferences of the social planner: resistance to intertemporal substitution, inequality aversion and risk aversion. Assume that uncertainty is described by S states of the world. Let $\mathbb{S} = \{1, \dots, S\}$ represent the set of states of nature, s an element of \mathbb{S} , and π_s the probability of state s , with $\sum_{s \in \mathbb{S}} \pi_s = 1$. Let $c_{r,t,s}$ represent the realized per-capita consumption in state s of individuals living in region r and belonging to generation t . Note that, for simplicity, the risk does not affect either the number of people living in a generation or the generations that will eventually exist or the allocation of individuals across regions. In addition, assume that there is no population growth as in deterministic case.

The most common approach to disentangle attitudes towards consumption smoothing over time and across states is the one proposed by [Kreps and Porteus \(1978\)](#) and [Selden \(1978\)](#), and further extended by [Epstein and Zin \(1989\)](#). Applied to our framework, it would require to consider, for each generation t and state s , the equally distributed equivalent consumption that represents intra-generational inequality. In such a way, we would have a consumption index $c_{t,s}^{ede}$ for each state s and generation t . Then, given a stream of equally distributed equivalents over time and across states, we could apply the Kreps-Porteus-Selden framework to disentangle aversion to the risk faced by future generations and resistance to transfer consumption from the current generation to the future ones (See the Appendix for a more exhaustive description of this social criterion). However, as this model does not satisfy expected utility theory, it introduces preferences for the timing of resolution of uncertainty, even though learning does not have any effect on behavior. While an individual may display preferences for an early/late resolution of uncertainty, it would be more difficult to justify it for a social planner. Moreover, if we want to introduce recursive preferences in a model that further disentangles intra-generational inequality aversion, time consistency will require the decision maker to be ex-ante inter-temporally inequality averse and ex-post intra-generationally inequality averse.

Therefore, we will consider an alternative approach to the distinction of risk aversion and resistance to intertemporal substitution, that is based on the measure of risk aversion suggested by [Khilstrom and Mirman \(1974\)](#), and that was axiomatized by [Fleurbaey \(2010\)](#) and [Grant et al. \(2012\)](#), under the label of expected equally distributed equivalent criterion

(EEDe). Unlike the Kreps-Porteus-Selden framework, the following social objective lies in the realm of expected utility, and aggregates welfare across time in a non-additively manner¹⁶:

$$W^{EEDe} = NT \sum_{s=1}^S \pi_s u \left(v^{-1} \left(\frac{1}{T} \sum_{t=0}^T v(c_{t,s}^{ede}) \right) \right) \quad (2.15)$$

where N represents the number of people alive in each generation and T the number of generations; finally, $c_{t,s}^{ede}$ denotes the equally distributed equivalent of generation t in state s , defined as in (2.3):

$$\phi(c_{t,s}^{ede}) \equiv \sum_{r=1}^R \frac{N_r}{N} \phi(c_{r,t,s}) \quad (2.16)$$

Function u , which is increasing and concave, describes the social planner's aversion to risk. Thus, with respect to the deterministic social preferences (2.2), the social objective (2.15) requires an additional nested function. First, for each generation t and state s , the social planner determines the equally distributed equivalent consumption level $c_{t,s}^{ede}$, which indicates how much she cares about intra-generational inequality if state s occurs. Then, for each state s and generation t , the social planner evaluates inter-generational inequality through the felicity function v , whose concavity expresses resistance to intertemporal substitution. Let \bar{c}_s denote the *constant equivalent consumption* in state s , which is defined as:

$$v(\bar{c}_s) = \frac{1}{T} \sum_{t=0}^T v(c_{t,s}^{ede})$$

Thus, for each state s , the constant equivalent consumption denotes the uniform level of per-capita consumption that is equivalent, from a social point of view, to the unequal stream of equally distributed equivalent consumption across generations.

Now, for each state s we have an index \bar{c}_s that embodies inter-generational equity concerns. The final step will be to aggregate the constant equivalent consumptions across states of nature. Given a probability distribution and a function u expressing preferences for risk, total social welfare coincides with (2.15):

$$W = NT \sum_{s=1}^S \pi_s u(\bar{c}_s)$$

¹⁶Non-time additive preferences to disentangle risk aversion and resistance to intertemporal substitution have been used, for instance, by Ahn (1989) on the equity premium puzzle, Van der Ploeg (1993) to model precautionary saving and Bommier and Rochet (2006) for models of saving and portfolio choices.

The proposed social ordering describes the social planner’s concerns for the distribution of realized consumption levels across time and space. Therefore, it expresses concerns for ex-post fairness, while concerns for ex-ante fairness in lotteries (concerns for the distribution of individual risks) are ignored. Being an expected utility criterion, (2.15) satisfies time consistency and basic principles of stochastic dominance, as statewise dominance or the sure thing principle. In contrast, it does not respect individuals’ ex-ante preferences, unless when there is no inequality, $c_{r,t,s} = c_s$ for all r, t : in that case, the social welfare function simply becomes $NT \sum_{s=1}^S \pi_s u(c_s)$, that is we are just maximizing individuals’ expected utilities. As a consequence, the social planner’s preferences coincide with individual preferences when there is no inequality, but they may deviate from individual preferences when we consider risks with heterogeneous effects across generations. The underlying idea is that the social planner should care for society as a whole, and be free to implement undesirable policies (from the individual point of view) if they serve the needs of the community¹⁷.

Note that the chosen social criterion does not display any explicit rate of time preference. This is because the social objective (2.15) does not change over time: the social planner lives in an a-temporal place. As a consequence, it would not make any sense to treat generations asymmetrically¹⁸. As shown in Dasgupta and Heal (1979) and Bommier and Zuber (2008), an endogenous rate of time preference arises once we add a risk on the existence of future generations, which is independent of the consumption risk.

The main drawback of criterion (2.15) concerns time separability. Specifically, dynamic consistency requires that the evaluation of a given policy depends on the outcome of the entire population (past, present and future), independently of whether they are affected or not by the policy¹⁹. Thus, memory of past choices is a necessary condition for dynamic

¹⁷The trade-off between social rationality (social expected utility) and ex-ante Pareto (individual ex-ante preferences) has been known since Harsanyi (1955), who argued that utilitarianism is the only welfare criterion which ensures the respect of both principles. However, utilitarianism has been criticized for providing insufficient attention to inequality. An alternative approach to the one studied in the paper describes concerns for ex-ante equity, by relaxing social rationality while satisfying ex-ante Pareto. Applied to our problem, it would require the following steps: computing the certainty equivalent for each region r given the generation t ; for each generation t , determining the equally distributed equivalent consumption that is considered as fair as the unequal distribution of certainty equivalents; finally, aggregating the equally distributed equivalents across time. However, besides violating social rationality, this approach is also time inconsistent (Adler and Sanichirico, 2006).

¹⁸As previously explained, the time preference δ was included in the deterministic case only to facilitate the comparison with the standard discounted utilitarian case.

¹⁹Blackorby et al. (2005) suggest that an acceptable social criterion should at least satisfy the "independence of the utilities of the dead" axiom, i.e. the evaluation of a policy should be independent of unconcerned individuals, defined as those individuals who bear no risk and have the same utility in the alternatives that are compared (in other words, the past generations).

consistency²⁰. However, separability is reachable when the function $h = u \circ v^{-1}$ is exponential, i.e. v is logarithmic; in that case, past realized consumption only matters through a positive multiplicative factor, and therefore has no impact on the social planner's preferences. As previously argued, a unitary coefficient for resistance to intertemporal substitution is an acceptable choice. As a consequence, the separability issue poses no harm in our problem.

2.6.2 The social cost of carbon

Let $d_{r,t,s} = \frac{\partial c_{r,t,s}}{\partial E_0}$ be the marginal impact of a decrease in current emissions E_0 on the realized per-capita consumption in state s of an individual living in region r and time t . To compute the social cost of carbon in the presence of risk and inequality, we need to employ a set of *time and state specific regional weights* $\omega_{r,t,s} \equiv \frac{W_{t,s}^r}{\sum_s \pi_s W_{0,s}^r}$, such that $SCC_0 = \sum_{t=0}^T \sum_{s=1}^S \pi_s \sum_{r=1}^R n_r \omega_{r,t,s} d_{r,t,s}$, where $W_{t,s}^r = \frac{\partial W}{\partial c_{r,t,s}}$ represents the social priority given to an individual belonging to region r and time t when the state of nature s occurs. Given the social objective (2.15), these contingent intertemporal equity weights can be expressed as:

$$\omega_{r,t,s} = \frac{\frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_{t,s}^{ede})}{\phi'(c_{t,s}^{ede})} \phi'(c_{r,t,s})}{\frac{v'(c_0^{ede})}{\phi'(c_0^{ede})} \sum_{s=1}^S \pi_s \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \sum_{r=1}^R \mu_r \phi'(c_{r,0})}$$

The sensitivity of the weight to the realized consumption level is proportional to:

$$\frac{\partial \omega_{r,t,s}}{\partial c_{r,t,s}} \propto \left\{ [A_v(\bar{c}_s) - A_u(\bar{c}_s)] \frac{v'(c_{t,s}^{ede})}{v'(\bar{c}_s)} + A_\phi(c_{t,s}^{ede}) - A_v(c_{t,s}^{ede}) \right\} \frac{\phi'(c_{r,t,s})}{\phi'(c_{t,s}^{ede})} - A_\phi(c_{r,t,s}) \quad (2.17)$$

where $A_h = -\frac{h''}{h'}$ denotes the coefficient of absolute risk or inequality aversion of function $h = u, v, \phi$. As in the deterministic case, expression (2.17) helps to understand the relation between equity weights and social preferences. If the social planner is utilitarian, $A_v = A_\phi = A_u$, and the weights are decreasing in consumption. When there is no intra-generational equity weighting (linear ϕ), all contemporaneous individuals get the same weight, although its value is affected by the comparison with the realized consumption of all the other generations if $A_v \neq A_u$. Thus, contingent to the state of nature, a poor generation

²⁰The non time separable model has been discussed also by [Epstein and Zin \(1989\)](#) in comparison to recursive preferences, and discarded exactly because memory of the past is necessary to select the best choice. To be fair, what Epstein and Zin found troubling was the fact that, because of the usual discounting assumptions, memory of the very distant past would become most important. In our case, there is no discounting, so every generation has the same weight in the social welfare function. Still, a separability issue remains, which can be eluded only through a careful selection of the felicity functions.

who happens to be surrounded by rich generations gets a larger weight than a poor generation surrounded by other poor generations. Similarly if we add intra-generational equity weighting (concave/convex ϕ): the weight assigned to an individual depends also on the consumption of the other people living in the same generation and on the goodness of the state, i.e. whether it is a state that is lucky to all generations or only to some of them.

2.6.3 Socially efficient discount rate

As in the deterministic case, the social cost of carbon can be expressed in terms of a discount rate. Let $d_t \equiv \sum_s \pi_s \sum_{r=1}^R n_r d_{r,t,s}$ denote the expected average marginal damage at time t . Then, $SCC_0 = \sum_{t=0}^T e^{-\rho_t(d_t)} d_t$, where the socially efficient discount rate $\rho_t(d_t)$ can be decomposed in three terms:

$$\rho_t(d_t) = r_t + \tau_t(d_t) + \chi_0$$

The first represents the discount rate used for sure projects with uniform costs and benefits:

$$r_t = -\frac{1}{t} \ln \frac{\sum_{s=1}^S \pi_s \sum_{r=1}^R n_r W_{t,s}^r}{\sum_{s=1}^S \pi_s \sum_{r=1}^R n_r W_{0,s}^r}$$

The second term is a project specific risk and inequality premium, which, as the inequality premium in the deterministic case, embodies the correlation between the individual realized damages and the state of the economy:

$$\tau_t(d_t) = -\frac{1}{t} \ln \frac{\sum_{s=1}^S \pi_s \sum_{r=1}^R n_r W_{t,s}^r d_{r,t,s}}{d_t \sum_{s=1}^S \pi_s \sum_{r=1}^R n_r W_{t,s}^r}$$

Finally, we have a project specific inequality premium due to the potential heterogeneity in mitigation costs at time 0.

$$\chi_0 = \frac{1}{t} \ln \frac{\sum_{s=1}^S \pi_s \sum_{r=1}^R \mu_r W_{0,s}^r}{\sum_{s=1}^S \pi_s \sum_{r=1}^R n_r W_{0,s}^r}$$

Note that the risk and inequality premium $\tau_t(d_t)$ is positive as long as individual realized damages are negatively correlated with the individual social priority $W_{t,s}^r$. In other words, the social planner has less incentives to invest in the mitigation project when it benefits mainly those individuals that are assigned a low weight. The weight reflects the social planner's preferences for risk, intra-generational equity and inter-generational consumption

fluctuations. Moreover, as in the deterministic case, the introduction of a heterogeneous cost sharing rule induces an artificial shift in the distribution of individual consumption across regions at time 0. If the shift can be ordered in terms of FSD- or SSD-dominance, we are able to say whether the project specific inequality premium is positive or negative. In particular, if $\sum_{s=1}^S \pi_s W_{0,s}^r$ is decreasing, a FSD-dominant shift increases the socially efficient discount rate; similarly, for a SSD-dominant shift when $\sum_{s=1}^S \pi_s W_{0,s}^r$ is convex. Therefore, the social planner attaches a lower value to the investment if the costs are borne mainly by poor individuals. Note that the characteristics of the socially efficient discount rate under risk and inequality with an EEDE criterion have been analyzed also in [Fleurbaey and Zuber \(2014\)](#).

To simplify the expressions, we need to introduce some further notation. Let $E_s x = \sum_{s=1}^S \pi_s x_{r,t,s}$ the expected value of a variable, $E_r x = \sum_{r=1}^R n_r x_{r,t,s}$ the average value across regions, and $E_t x = \frac{1}{T} \sum_{t=0}^T x_{r,t,s}$ the average value across generations. Moreover, let E_{ij} denote the expectations with respect to two dimensions, and E the expectations with respect to all three dimensions (time, state and space). As before, \hat{E} denotes expectations taken with respect to the distribution μ . Then, we can state the following result:

Lemma 2. *Consider the EEDE social welfare criterion (2.15). The social discount rate $\rho_t(d_t)$ for a project with heterogeneous and risky benefits $d_{r,t,s}$ and heterogeneous cost shares μ_r is given by:*

$$\rho_t(d_t) = -\frac{1}{t} \ln \frac{E_{s,r} \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_{t,s}^{ede})}{\phi'(c_{t,s}^{ede})} \phi'(c_{r,t,s})}{E_{s,r} \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})} \phi'(c_{r,0})} - \frac{1}{t} \ln \frac{E_{s,r} \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_{t,s}^{ede})}{\phi'(c_{t,s}^{ede})} \phi'(c_{r,t,s}) d_{r,t,s}}{d_t E_{s,r} \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_{t,s}^{ede})}{\phi'(c_{t,s}^{ede})} \phi'(c_{r,t,s})} + \frac{1}{t} \ln \frac{\hat{E}_r \phi'(c_{r,0})}{E_r \phi'(c_{r,0})} \quad (2.18)$$

The three terms correspond exactly to the risk free rate for projects with uniform costs and benefits, the project specific risk and inequality premium and the inequality premium due to a non egalitarian allocation of the mitigation policy costs. As there are too many layers, it is hard to analyze the components of the discount rate in general terms, especially in comparison with the standard utilitarian case. Therefore, we jump directly to the small risk and inequality case. Let $c_{r,t,s} = c_0(1 + \epsilon_{r,t,s})$, where $c_0 \equiv E_r c_{r,0}$ and $\epsilon_{r,t,s}$ represents the growth rate contingent to state s in region r at time t with respect to the average consumption at time $t = 0$. Moreover, let $V_r(\epsilon)$ denote the variability in the distribution of regional growth rates at time t contingent to the state of nature s , and V^0 the variability at time 0. In addition, let $V_s(E_r \epsilon)$ be the variance of average growth rate across states of

nature; finally, let $Cov(E_r\epsilon_t, E_r\epsilon_\tau)$ represent the stochastic dependence between the average growth rate at date t , $E_r\epsilon_t$, and the average growth rate at time τ , $E_r\epsilon_\tau$.

Proposition 12. *The quadratic approximation of the discount rate r_t is given by:*

$$r_t \simeq \frac{1}{t}R_v E_{r,s}\epsilon - \frac{1}{2t}R_\phi \left\{ P_\phi - R_\phi + R_v \right\} \left\{ E_s V_r(\epsilon) - V^0(\epsilon) \right\} + \quad (2.19)$$

$$-\frac{1}{t}R_v \left\{ \frac{P_v}{2} + \frac{R_u - R_v}{T} \right\} V_s(E_r\epsilon) - \frac{1}{t}R_v(R_u - R_v) \frac{1}{T} \sum_{\tau \neq t} Cov(E_r\epsilon_t, E_r\epsilon_\tau)$$

We can recognize some familiar components in the quadratic approximation of the discount rate. The first element represents the usual wealth effect, and it depends on the degree of relative resistance to intertemporal substitution and on the expected annualized growth rate of average consumption $E_{r,s}\epsilon \equiv \sum_{s=1}^S \pi_s \sum_{r=1}^R n_r \epsilon_{r,t,s}$.

The second term is an inequality precautionary effect, which appears when there is inequality in individual consumption levels, either today or in the future. This term appeared also in the deterministic discount rate (2.11), although it must be adjusted by the presence of risk. Indeed, unlike the deterministic case, the inequality precautionary effect is proportional to the *expected* change in inequality over time. Therefore, if inequality is on average increasing over time, the social planner will be more willing to invest in a mitigation project, provided that $P_\phi - R_\phi + R_v \geq 0$.

The third term is the standard precautionary effect of the Ramsey rule due to the presence of risk. It measures the social planner's willingness to invest for protecting the future generations from the consumption risk, and it is proportional to the variance of the average growth rate. Its size depends both on preferences for intertemporal substitution and on preferences for risk in a complex way. The reason is the same as the one advanced for the inequality precautionary effect: if the social planner cares more about intergenerational inequality than risk, $R_v - R_u \geq 0$, an unlucky generation living in an unlucky world (i.e. when all generations suffer a loss) gets a lower weight than an unlucky generation living in a lucky world. Therefore, if the risk faced by a given generation increases, the social planner will assign an extra weight to this generation only if she is more risk averse than intertemporal inequality averse. Note that the precautionary effect is more likely to be larger as the number of generations T increases.

Finally, the last term represents an intertemporal correlation effect. It depends on the correlation between average growth in generation t and average growth in the other generations,

and on the difference $R_u - R_v$, which corresponds to the *coefficient of relative intertemporal correlation aversion*²¹. The intertemporal correlation premium describes how much the social planner cares about outcomes that are positively or negatively correlated over time. The preferences for intertemporal correlation are expressed by the function $h = u \circ v^{-1}$; then, the social planner is correlation averse if $R_u - R_v \geq 0$. Alternatively, she is correlation prone. Therefore, the intertemporal correlation premium says that, if the social planner is correlation averse, $R_u - R_v \geq 0$, an increase in intertemporal correlation reduces the discount rate. In contrast, if the social planner prefers to eliminate inequality over time, i.e. $R_u - R_v \leq 0$, an increase in intertemporal correlation increases the discount rate.

The advantage of approximation (2.19) lies on the fact that it allows to disentangle the different components of the risk free discount rate. In particular, it allows to separately identify: the effect of consumption inequality inside a cohort (inequality precautionary effect); the impact of average consumption risk (precautionary effect); the effect of correlation of average consumption across generations. In particular, the correlation effect concerns the evolution of risk over time. If the risks faced by individuals in different generations are independently distributed, the correlation effect disappears. In contrast, if risks have a long run component, a correlation averse social planner is willing to invest more in a mitigation project.

Similarly to the discount rate r_t , we can determine the quadratic approximation of the project specific risk and inequality premium $\tau_t(d_t)$. Let $Cov(d, \epsilon)$ represent the covariance between individual consumption at time t and individual damages at time t , contingent on the state s . Moreover, let $Cov(E_r d_t, E_r \epsilon_t)$ denote the covariance between average consumption at

²¹The concept of correlation aversion has been originally introduced by Richard (1975) under the name of "multivariate risk aversion", and renamed "correlation aversion" by Epstein and Tanny (1980). For instance, assume that the decision maker has preferences over two attributes. Then, he is correlation averse if he prefers a lottery yielding $(H, L), (L, H)$ with equal probabilities, to a lottery yielding $(H, H), (L, L)$ also with equal probabilities, where $H > L$. Applied to our framework, the social planner is inter-temporally correlation averse if she prefers that lotteries concerning different generations are negatively correlated. If a decision maker is correlation averse, then she likes mean preserving spreads in the distribution of well-being across generations. Let $W_s = u(v^{-1}(\frac{1}{T}v(c_{s,t})))$, where $c_{s,t}$ is the average consumption of generation t . The coefficient of correlation aversion is between the average consumption of generation t and average consumption of generation τ can be defined as follows:

$$\rho_{t,\tau} = -\frac{\frac{\partial^2 W_s}{\partial c_{s,t} \partial c_{s,\tau}}}{\frac{\partial W_s}{\partial c_{s,t}}}$$

Then, we can easily prove that, when the variation in consumption is small, $c_{s,t} \simeq c_{s,\tau}$, the coefficient of correlation aversion is exactly equal to $\rho_{t,\tau} = A_u - A_v$. The coefficient of relative intertemporal correlation aversion follows from that definition.

time t and average damages at time t ; finally, let $Cov(E_r d_t, E_r \epsilon_\tau)$ be the covariance between average damages at time t and average consumption in generation τ .

Proposition 13. *The quadratic approximation of the project specific risk and inequality premium is given by:*

$$\tau_t(d_t) \simeq \frac{1}{t} R_\phi E_s Cov(d, \epsilon) + \frac{1}{t} \left\{ R_v + \frac{R_u - R_v}{T} \right\} Cov(E_r d_t, E_r \epsilon_t) + \frac{1}{t} (R_u - R_v) \frac{1}{T} \sum_{\tau \neq t} Cov(E_r d_t, E_r \epsilon_\tau) \quad (2.20)$$

The first term denotes a project specific expected inequality premium. It indicates whether, on average, damages are borne mainly by rich or poor people. If rich individuals are those more exposed to the adverse effects of climate change, the socially efficient discount rate increases. The expected inequality premium is proportional to the coefficient of relative intra-generational inequality aversion.

The second term represents the standard project specific risk premium. If we had a representative agent with utilitarian social preferences, only this term would appear in the formula. It describes the correlation between the risk of the mitigation project for generation t and the macroeconomic risk faced by generation t . When it is positive it means that the mitigation project yields on average larger benefits in states where generation t consumes a lot. Its size depends on both risk aversion and resistance to intertemporal substitution. Moreover, as the risk precautionary effect, it is more likely to be positive as T increases.

Finally, the last term is a project specific correlation premium. It describes the correlation between the risk of the mitigation project for generation t and the macroeconomic risk befalling the other generations. If it is positive, then generation t enjoys large benefits from the project in states where also the other generations consume on average a lot. Then, if the social planner is intertemporally correlation averse, i.e. $A_u - A_v \geq 0$, she will attach a larger weight to projects whose benefits are on average positively correlated with the macroeconomic risk faced by all generations.

From the approximations (2.19) and (2.20) we can easily recover the usual representative agent case, by setting $R_\phi = 0$ and $R_v = R_u$, and the utilitarian case, where $R_\phi = R_v = R_u$. Moreover, as in the deterministic case, the introduction of lognormality of both risk and inequality and iso-elastic utility functions simplifies the approximated socially efficient discount rate²². Assume, for example, that $u'(c) = c^{-\eta}$, $\phi'(c) = c^{-\phi}$ and $v'(c) = c^{-1}$. To

²²The approximation of the socially efficient discount rate under lognormal distributions is exact only when the function v is logarithmic, i.e. the objective function has a multiplicative form. As we have previously seen,

simplify, suppose that, given a state s and a generation t , individual consumption $c_{r,t,s}$ and individual damages $d_{r,t,s}$ are distributed as a bivariate lognormal with, respectively, means $\mu_{s,t}^c$ and $\mu_{s,t}^d$, and variances $\sigma_{c,t}^2$ and $\sigma_{d,t}^2$; let R_t denote their index of correlation. Thus, the risk affects the average consumption for each generation, but not the degree of inequality or the correlation between individual damages and individual consumption. Let us further assume that the contingent means of log consumption and log damages are also normally distributed, with means, respectively, μ_t^c and μ_t^d , and variances $s_{c,t}^2$ and $s_{d,t}^2$; let P_t denote their index of correlation. Finally, let us consider the case where the costs of the policy are uniformly distributed across regions. Then, the socially efficient discount rate is exactly equal to:

$$\begin{aligned} \rho_t(d_t) = & g - \varphi \frac{\sigma_{c,t}^2 - \sigma_{c,0}^2}{t} - \frac{s_{c,t}^2}{t} \left(1 + \frac{\eta - 1}{T}\right) + \varphi \beta_t \frac{\sigma_{c,t}^2}{t} + \left(1 + \frac{\eta - 1}{T}\right) \tilde{\beta}_t \frac{s_{t,c}^2}{t} \\ & + \frac{\eta - 1}{T} \left[\sum_{\tau \neq t} Cov(\mu_{t,s}^d, \mu_{\tau,s}^c) - \sum_{\tau \neq t} Cov(\mu_{t,s}^c, \mu_{\tau,s}^c) \right] \frac{1}{t} \end{aligned} \quad (2.21)$$

where $g \equiv \frac{\mu_t^c - \mu_0^c}{t} + \frac{\sigma_{c,t}^2 - \sigma_{c,0}^2}{2t} + \frac{s_{c,t}^2}{2t}$ is the constant expected growth rate of average consumption, β_t is the (expected) inequality consumption beta as previously defined, and $\tilde{\beta}_t$ is the traditional consumption beta, that measures the elasticity between the benefits of the project for the representative agent and his consumption level:

$$\tilde{\beta}_t = \frac{P_t s_{d,t}}{s_{c,t}} = \frac{Cov(\ln E_r c_{r,t,s}, \ln E_r d_{r,t,s})}{s_{c,t}^2}$$

The last term in (2.21) concerns the correlation between individual risks. It cancels out whenever individual risks are independently distributed across generations. The approximations of the risk free discount rate (2.19) and of the risk and inequality premium (2.20), along with the Ramsey rule for the power-lognormal case, clarify the relative importance of different assumptions. First of all, we can easily deduce that the introduction of intra-generational inequality concerns (positive φ) is going to increase the socially efficient discount rate, and, presumably, reduce the social cost of carbon. That happens both because of an expected economic convergence in the future, and because the inequality consumption β is likely to be positive. Second, risk aversion matters only through intertemporal correlation. As a consequence, its effect is shading as the number of generations T increases. This is

a coefficient of resistance to intertemporal substitution equal to 1 is considered as an acceptable assumption in the literature. Moreover, it implies that social preferences are independent of the past.

in sharp contrast with Kreps-Porteus-Selden preferences, where risk aversion is important to sign the precautionary effect, at least in the short run. The net effect of risk aversion is ambiguous. First of all, it depends on the comparison with resistance to intertemporal substitution (i.e. only the coefficient of relative intertemporal correlation aversion matters). Moreover, an increase in risk aversion reduces the discount rate via the precautionary effect and the correlation effect, while it raises the discount rate via the risk premium and the correlation premium. Finally, the approximations highlight which type of information is necessary for the computation of the discount rate, and, consequently, of the social cost of carbon. In particular, we need to know: the expected evolution of inequality over time, the variance of the aggregate risk for each generation, the expected inequality consumption beta and the aggregate risk consumption beta. In contrast, the correlation between individual risks matters only marginally when T grows very large.

2.6.4 Calibration of the model under uncertainty

Let us consider again the climate change model previously introduced. There are many possible structures for the incorporation of risk, but we will simply assume that until some future date, there is uncertainty regarding the right value of regional economic impacts λ_r : with probability $p = 6.8\%$ the economic impacts are high, λ_r^{high} , and with the complementary probability they are low, λ_r^{low} ; uncertainty will be resolved only in the long run²³. As explained in the deterministic calibration exercise, the lambdas are computed by assuming that the change in temperature at 2100 is either 2.5°C (low) or 6°C (high). Data about the distribution of regional damages and the probability of the high damages case are drawn from Nordhaus (2008) and Nordhaus (2010). Given that the amount consumed depends on the level of damages, this type of risk has effects on both the consumption stream and the benefits of the mitigation project.

The type of risk considered does not have big effects on the distribution of individual risks. In particular, the degree of inequality among regions is almost unaltered across states. That depends mainly on the structure of the carbon cycle and on the way damages have been computed. Figure 5 in the Appendix compares the expected degree of inequality to the inequality found in the deterministic case, and shows how the introduction of risk slightly raises inequality in a cohort. Moreover, in the presence of risk the expected average growth slightly reduces. Given that the risk is small, also its variance is of minor importance (Figure

²³Some examples of works that introduce uncertainty in integrated models of climate change are Kelly and Kolstad (1999), Cai et al. (2012), Traeger (2015).

6 in the Appendix). Finally, we can determine the expected inequality consumption beta and the aggregate risk consumption beta implied by the model. As in the deterministic case, the expected inequality consumption beta is quite large and decreasing over time (Figure 7 in the Appendix). As a consequence, rich regions are expected to suffer large losses from climate change, on average. However, the correlation between individual damages and individual consumption is smaller than in the deterministic case (Figure 8), suggesting that the introduction of risk harms mainly poor regions. In contrast, the type of uncertainty included in the model supports a negative consumption $\tilde{\beta}$ due to the sensitivity of average damages to average consumption (Figure 9 in the Appendix). The sign depends on the fact that, by construction, high benefits of the policy occur when consumption is lower. The consumption beta for climate risks is still an open question, depending on whether we stress the relation between damages and growth (larger damages when the world grows more because there is more to lose), or between climate sensitivity and consumption (a large climate sensitivity yields at the same time large damages and low consumption)²⁴. Our modeling approach belongs to the second type, as we have assumed that the presence of risk in economic impacts altered both the amount available to consume and the benefits of the mitigation project. Finally, note that consumption risks are positively correlated over time, although the importance of the correlation effect is minimal because of the large time horizons.

Figure 3 shows the socially efficient discount rate in the presence of a small risk on the economic impacts of climate change. We maintain the assumptions of the deterministic case concerning inter- and intra-generational inequality aversion; in particular, we set $\gamma = 1$ and we vary φ from 0.5 to 4. From empirical studies on financial choices, the coefficient of risk aversion is usually taken between 2 and 4 (Gollier, 2001). As a consequence, we end up with a social planner that is correlation averse (as $\eta > \gamma$), meaning that he has a preference for mean-preserving spreads in the distribution of well-being across time. We assume $\eta = 4$, and we consider the effects of varying intra-generational inequality (i.e. different assumptions of intra-generational equity weightings) on the discount rate. As in the deterministic case, the discount rate is increasing in φ , as intragenerational inequality aversion quantifies the inequality precautionary effect and the inequality premium, which are both positive due to convergence and positive correlation between individual consumption and individual damages. Moreover, comparing the utilitarian case where $\varphi = \gamma = \eta = 1$, and the case with larger risk aversion η but unitary intra- and inter-generational inequality aversion

²⁴See Gollier (2012) for a discussion of the beta of climate projects.

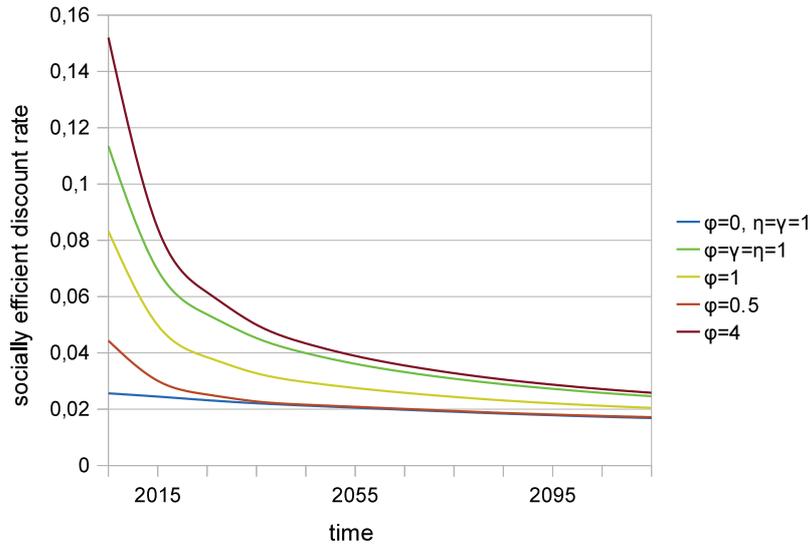


Figure 2.3: Socially efficient discount rate with risk under the assumptions $\gamma = 1$ and $\eta = 4$.

$\varphi = \gamma = 1$, we can evaluate the impact of correlation averse preferences. In particular, a larger risk aversion reduces the discount rate, as it magnifies the risk precautionary effect and the risk premium (that in our example is negative). Finally, compared to the deterministic case, the introduction of risk significantly reduces the discount rate only when we disentangle risk aversion and inter-generational inequality aversion. In other words, the reduction is mainly due to the presence of intertemporal correlation aversion rather than risk.

Table 3 reports the estimated social costs of carbon. Compared to the deterministic case, the utilitarian and the representative agent frameworks support very modest increases in the social cost of carbon, mainly because the risk we considered had minor effects in the short run. For instance, the utilitarian social planner is now ready to spend 29.87\$ compared to the 29.19\$ of the deterministic example. The largest differences occur when we disentangle risk aversion from inequality aversion. For instance, assuming $\varphi = 0.5$, the social cost of carbon is now equal to 52.98\$ compared to the 43.89\$ of the deterministic case. Therefore, the introduction of correlation aversion preferences can have a large impact on the estimated social cost of carbon.

2.7 Conclusion

In the paper, I have analyzed the properties of the social cost of carbon when we adopt a social framework that disentangles risk aversion, intra-generational inequality aversion and resistance to intertemporal substitution. In particular, I have identified which assumptions about the type of risk, the evolution of inequality and the correlation among individual risks are likely to increase the social cost of carbon. Then, I tried to quantify those effects by calibrating a simple model of climate change based on Nordhaus (2010) RICE model. The welfare framework adopted in the paper represents the social planner's preferences for equity in realized risks. Therefore, it disregards equity concerns for the distribution of risks, and, most importantly, it is not separable across time. In particular, this last characteristic has always been considered a drawback by the financial literature (see Epstein and Zin (1989)). The choice of the most suitable social criterion is eventually an ethical issue. The approach proposed in the paper is socially rationale, time consistent and satisfies ex-post fairness. However, it is not time separable (with the exception of the exponential case) and it does not respect ex ante Pareto, unless there is no inequality across space and time.

The range of values found in the literature for the SCC is huge. Tol (2005) surveys 103 estimates from 28 studies and finds a range from $-\$2.5/\text{tC}$ (ton of carbon) to $\$350/\text{tC}$, with a mean value of $\$43/\text{tC}$ and a standard deviation of $\$83/\text{tC}$. Nordhaus (2008) proposes a value of $\$30/\text{tC}$, while Stern (2006) $\$250/\text{tC}$. Nordhaus (2014), in an application of the RICE model, finds a value of $\$44/\text{tC}$. The figures found in the paper are consistently lower, mainly because of the conservative assumptions about damages and the fact that I did not perform any optimization exercise. A better structure for the carbon cycle (Traeger, 2015; Gerlagh and Liski, 2012) coupled with a more exhaustive description of the risks (economic/climate risks) is necessary to put those numbers in perspective.

The paper can be extended in several ways. First of all, the type of risk introduced in the calibration is very simple. A better structure of the stochastic process could help to understand the impact of different sources of risk on the social cost of carbon. For instance, it would be interesting to distinguish between short run and long run risks, given that the latter would have a considerable effect on temporal correlation. Moreover, the ex-post approach to equity is suitable for analyzing population risks, like a risk of extinction or the risk of migrations across regions. Second, we have considered the social cost of carbon today, but we have not studied its time path; that requires assumptions about both the arrival of new information and the type of learning. Third, a more consistent comparison between the model of the paper and the more traditional Kreps-Porteus-Selden framework is required,

to assess the differences in results and the advantages of preferring one of them. Finally, one serious limitation of the model is the implicit assumption of homogeneous preferences. While it could be acceptable for people living in the same cohort, supposing that the future generations will have the same preference orderings as we do is quite restrictive. An extension to a heterogeneous preferences framework would be interesting, but it would require the introduction of assumptions about welfare measurement and interpersonal utility comparison.

Bibliography

- Adler, M. and Sanchirico, C. (2006). Inequality and uncertainty: Theory and legal applications. *University of Pennsylvania Law Review*, 155:279–377. [85](#)
- Ahn, C. (1989). The effect of temporal risk aversion on optimal consumption, the equity premium and the equilibrium interest rate. *Journal of Finance*, 44(5):1411–20. [84](#)
- Amiel, Y. and Cowell, F. (1999). *Thinking about Inequality*. Cambridge University Press. [81](#)
- Anthoff, D., Hepburn, C., and Tol, R. (2009). Equity weighting and the marginal damage costs of climate change. *Ecological Economics*, 68(3):836–49. [62](#), [67](#)
- Anthoff, D. and Tol, R. (2010). On international equity weights and national decision making on climate change. *Journal of Environmental Economics and Management*, 60(1):14–20. [63](#), [67](#)
- Atkinson, A. (1970). On the measurement of inequality. *Journal of Economic Theory*, 2(3):244–63. [65](#), [75](#)
- Atkinson, A. and Brandolini, A. (2010). On analyzing the world distribution of income. *World Bank Economic Review*, 24(1):1–37. [77](#)
- Atkinson, G., Dietz, S., Helgeson, J., Hepburn, C., and Saelen, H. (2009). Siblings, not triplets: Social preferences for risk, inequality and time in discounting climate change. *Economics-The Open Access, Open Assessment E-Journal*, 3(26):1–29. [64](#), [81](#)
- Azar, C. and Sterner, T. (1996). Discounting and distributional considerations in the context of global warming. *Ecological Economics*, 19:169–84. [62](#)
- Blackorby, C., Bossert, W., and Donaldson, D. (2005). *Population Issues in Social-Choice Theory, Welfare Economics and Ethics*. Cambridge University Press, Cambridge. [85](#)

- Bommier, A. and Rochet, J.-C. (2006). Risk aversion and planning horizon. *Journal of the European Economic Association*, 4(4):708–34. [84](#)
- Bommier, A. and Zuber, S. (2008). Can preferences for catastrophe avoidance reconcile social discounting with intergenerational equity? *Social Choice and Welfare*, 31:415–34. [85](#)
- Cai, Y., Judd, K., and Lontzek, T. (2012). Dsice: A dynamic stochastic integrated model of climate change and economy. Working Paper 12-02, RDCEP. [93](#)
- Carlsson, F., Daruvala, D., and Johansson-Stenman, O. (2002). Measuring future grandparents preferences for equality and relative standing. *Economic Journal*, 112:362–83. [64](#), [81](#)
- Carlsson, F., Daruvala, D., and Johansson-Stenman, O. (2005). Are people inequality averse, or just risk averse? *Economica*, 72:375–96. [81](#)
- Dasgupta, P. and Heal, G. (1979). *Economic Theory and Exhaustible Resources*. Cambridge University Press, Cambridge. [65](#), [85](#)
- Davies, J. and Hoy, M. (1995). Making inequality comparisons when lorenz curves intersect. *The American Economic Review*, 85(4):980–86. [104](#)
- Dennig, F. (2013). Inequality in climate change: a modification of RICE. [63](#)
- Emmerling, J. (2011). Discounting and intragenerational equity. [62](#), [71](#), [72](#)
- Epstein, L. and Tanny, S. (1980). Increasing generalized correlation: a definition and some economic consequences. *The Canadian Journal of Economics*, 13(1):16–34. [62](#), [90](#)
- Epstein, L. and Zin, S. (1989). Substitution, risk aversion and the temporal behavior of consumption and asset returns: a theoretical framework. *Econometrica*, 57(4):937–69. [83](#), [86](#), [96](#)
- Evans, D. (2005). The elasticity of marginal utility of consumption: Estimates for 20 OECD countries. *Fiscal Studies*, 26(2):197–224. [81](#)
- Fankhauser, S., Tol, R., and Pearce, D. (1997). The aggregation of climate change damages: a welfare theoretic approach. *Environmental and Resource Economics*, 10:249–66. [63](#)

- Fleurbaey, M. (2010). Assessing risky social situations. *Journal of Political Economy*, 118:649–80. [61](#), [83](#)
- Fleurbaey, M. and Zuber, S. (2014). Discounting, risk and inequality: a general approach. CES Working Paper No 2014.15. [62](#), [63](#), [88](#)
- Gerlagh, R. and Liski, M. (2012). Carbon prices for the next thousand years. Cesifo Working Paper No 3855. [96](#)
- Gollier, C. (2001). *The Economics of Risk and Time*. MIT Press. [94](#), [103](#)
- Gollier, C. (2010). Discounting, inequality and economic convergence. Cesifo Working Paper No 3262. [62](#), [71](#), [72](#)
- Gollier, C. (2012). Evaluation of long-dates investments under uncertain growth trend, volatility and catastrophes. CESifo Working Paper No 4052. [94](#)
- Golosov, M., Hassler, J., Krusell, P., and Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88. [78](#), [79](#), [80](#)
- Grant, S., Kajii, A., Pollack, B., and Safra, S. (2012). Equally distributed equivalent utility, ex-post egalitarianism and utilitarianism. *Journal of Economic Theory*, 147(4):1545–71. [83](#)
- Greenstone, M., Kopits, E., and Wolverton, A. (2013). Developing a social cost of carbon for us regulatory analysis: A methodology and interpretation. *Review of Environmental Economics and Policy*, 7(1):23–46. [59](#)
- Harsanyi, J. (1955). Cardinal welfare, individualistic ethics and interpersonal comparison of utilities. *Journal of Political Economy*, 63:309–21. [85](#)
- Hope, C. (2008). Discount rates, equity weights and the social cost of carbon. *Energy Economics*, 30(3):1011–19. [62](#)
- Kelly, D. and Kolstad, C. (1999). Bayesian learning, growth and pollution. *Journal of Economic Dynamics and Control*, 23:491–518. [93](#)
- Khilstrom, R. and Mirman, L. (1974). Risk aversion with many commodities. *Journal of Economic Theory*, 8:361–88. [83](#)

- Kimball, M. (1990). Precautionary savings in the small and in the large. *Econometrica*, 58:53–73. [72](#)
- Koopmans, T. (1960). Stationary ordinal utility and impatience. *Econometrica*, 28(2):287–309. [65](#)
- Kreps, D. and Porteus, E. (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica*, 46:185–200. [61](#), [83](#), [108](#)
- Leland, H. (1968). Saving and uncertainty: the precautionary demand for saving. *Quarterly Journal of Economics*, 82:465–73. [72](#)
- Lucas, D. (1978). Asset prices in an exchange economy. *Econometrica*, 46:1429–46. [77](#)
- Nordhaus, W. (2008). *A Question of Balance: Weighing the Options on Global Warming Policies*. Yale University Press, New Haven, CT. [80](#), [93](#), [96](#)
- Nordhaus, W. (2010). Economic aspects of global warming in a post-copenhagen environment. *Proceeding of the National Academy of Sciences*, 107(26):11721–26. [60](#), [78](#), [79](#), [81](#), [93](#), [96](#), [109](#)
- Nordhaus, W. (2014). Estimates of the social cost of carbon: Concepts and results from the DICE-2013R model and alternative approaches. *Journal of the Association of Environmental and Resource Economists*, 1(1/2):273–312. [62](#), [67](#), [78](#), [96](#)
- Pearce, D. (2003). The social cost of carbon and its policy implications. *Oxford Review of Economic Policy*, 19(3):362–84. [59](#), [62](#)
- Pearce, D., Cline, W., Achanta, A., Fankhauser, S., Pachauri, R., Tol, R., and Vellinga, P. (1996). The social costs of climate change: Greenhouse damage and the benefits of control. In *IPCC: Economic and Social Dimensions of Climate Change*, pages 183–224. Cambridge University Press. [62](#)
- Pindyck, R. (2012). Uncertain outcomes and climate change policy. *Journal of Environmental Economics and Management*, 63(3):289–303. [78](#)
- Pinkovskiy, M. and Sala-i-Martin, X. (2009). Parametric estimations of the world distribution of income. *NBER Working Paper No. 15433*. [80](#)

- Richard, S. (1975). Multivariate risk aversion, utility independence and separable utility functions. *Management Science*, 22(1):12–21. [62](#), [90](#)
- Roemer, J. (2011). The ethics of intertemporal distribution in a warming planet. *Environmental and Resource Economics*, 48:363–90. [65](#)
- Selden, L. (1978). A new representation of preferences over "certain x uncertain" consumption pairs: the "ordinal certainty equivalent" hypothesis. *Econometrica*, 46:1054–60. [61](#), [83](#), [108](#)
- Stern, N. (2006). *The Economics of Climate Change: the Stern Review*. Cambridge University Press. [65](#), [80](#), [96](#)
- Stokey, N. (1998). Are there limits to growth? *International Economic Review*, 39:1–31. [78](#)
- Tchen, A. (1980). Inequalities for distributions with given marginals. *The Annals of Probability*, 8:814–27. [74](#), [106](#)
- Tol, R. (1999). The marginal costs of greenhouse gas emissions. *The Energy Journal*, 20(1):61–81. [62](#)
- Tol, R. (2005). The marginal damage costs of carbon dioxide emissions: an assessment of the uncertainties. *Energy Policy*, 33(16):2064–74. [96](#)
- Tol, R. (2010). International inequity aversion and the social cost of carbon. *Climate Change Economics*, 1:21–32. [62](#), [63](#), [66](#), [81](#)
- Traeger, C. (2015). Analytic integrated assessment and uncertainty. [80](#), [93](#), [96](#)
- Van der Ploeg, F. (1993). A closed-form solution for a model of precautionary saving. *Review of Economic Studies*, 60(2):385–95. [84](#)
- Weitzman, M. (2007). The stern review on the economics of climate change. *Journal of Economic Literature*, 45(3):703–24. [80](#)
- Yohe, G., Malone, E., Brenkert, A., Schlesinger, M., Meij, H., and Xiaoshi, X. (2006). Global distributions of vulnerability to climate change. *Integrated Assessment Journal*, 6(3):35–44. [80](#)

Appendix

2.A The social discount rate in the deterministic case

Given a social planner with EDE preferences, the social discount rate $\rho(d_t)$ for average damages at t is given by:

$$\rho(d_t) = \delta - \frac{1}{t} \ln \frac{E_t \phi'(c_{r,t}) \frac{v'(c_t^{ede})}{\phi'(c_t^{ede})}}{E_0 \phi'(c_{r,0}) \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})}} - \frac{1}{t} \ln \frac{E_t d_{r,t} \phi'(c_{r,t})}{d_t E_t \phi'(c_{r,t})} + \frac{1}{t} \ln \frac{\hat{E}_0 \phi'(c_{r,0})}{E_0 \phi'(c_{r,0})} \quad (2.22)$$

The first two terms in (2.22) denote the discount rate for projects whose costs and benefits are equally distributed across the population. It is lower under EDE preferences than under utilitarianism if and only if:

$$\frac{E_t \phi'(c_{r,t}) \frac{v'(c_t^{ede})}{\phi'(c_t^{ede})}}{E_0 \phi'(c_{r,0}) \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})}} \geq \frac{E_t v'(c_{r,t})}{E_0 v'(c_{r,0})}$$

The comparison depends on two aspects. First of all, on whether the marginal benefits/marginal costs increase under EDE preferences with respect to the utilitarian case, i.e. on whether $E^t \phi'(c_{r,t}) \frac{v'(c_t^{ede})}{\phi'(c_t^{ede})} \geq E^t v'(c_{r,t})$, $\forall t$. Clearly, the same sign applies both to the numerator and the denominator: either both marginal benefits and marginal costs are larger in the EDE case rather than the utilitarian one, or the opposite holds. Therefore, provided, for instance, that EDE preferences increase both marginal benefits and marginal costs, the second important element is which side increases more. If the increase in marginal benefits is larger than the increase in marginal costs, then the discount rate reduces under EDE preferences.

First of all, we could see if there are some situations when both the numerator and the denominator on the left-hand side are larger than the ones on the right-hand side. By using the diffidence theorem (Gollier, 2001), we can prove that a necessary condition for this result

is: $-\frac{\phi''}{\phi'} \leq -\frac{v''}{v'}$ but $\frac{\phi'''}{\phi'} \leq \frac{v'''}{v'}$, or that the EDE social planner is less inequality averse than the utilitarian one, but more downside inequality averse²⁵. Indeed, when $A_v \geq A_\phi$, we have that:

$$E_t \phi'(c_{r,t}) \frac{v'(c_t^{ede})}{\phi'(c_t^{ede})} \geq E_t \phi'(c_{r,t}) \frac{v'(c_t)}{\phi'(c_t)} \geq E_t v'(c_{r,t})$$

where $c_t = E_t c_{r,t} \geq c_t^{ede}$ by definition. Similarly for the denominator. By applying the diffidence theorem to the last inequality, a necessary condition for it to hold is:

$$v'''(c_t)\phi'(c_t) - \phi'''(c_t)v'(c_t) \geq 0$$

or more downside inequality aversion under utilitarianism. Moreover, in order to have a lower discount rate under EDE we need that the increase in the numerator (marginal benefits) is larger than the increase in the denominator (marginal costs) with respect to the utilitarian case. Under the assumption that $A_v \geq A_\phi$, we need that the index of downside risk aversion increases more in the future than today. This is a rather complicated result to prove, because it depends on average consumption level (c_t vs c_0), on the change of inequality over time, and on how the index of downside inequality aversion responds to variations in both average consumption and inequality.

An easier way to undertake is to simply consider the second order approximation of the discount rate r_t . The last term of this discount rate can be decomposed into:

$$-\frac{1}{t} \ln \frac{E_t \phi'(c_{r,t}) \frac{v'(c_t^{ede})}{\phi'(c_t^{ede})}}{E_0 \phi'(c_{r,0}) \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})}} = -\frac{1}{t} \ln \frac{E_t \phi'(c_{r,t})}{E_0 \phi'(c_{r,0})} - \frac{1}{t} \ln \frac{v'(c_t^{ede})}{\phi'(c_0^{ede})} + \frac{1}{t} \ln \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})}$$

and, then, we can approximate each term separately. Knowing that, $\forall t$, $c_t^{ede} = c_0(1 - \pi_t)$, where the premium π_t is approximatively equal to $\pi_t \simeq -E_t \frac{c_{r,t} - c_0}{c_0} - \frac{1}{2} V \left(\frac{c_{r,t} - c_0}{c_0} \right) \frac{\phi''(c_0)c_0}{\phi'(c_0)}$, and that $E_t \phi'(c_{r,t}) = \phi'(c_0(1 - \psi_t))$, with $\psi_t \simeq -E_t \frac{c_{r,t} - c_0}{c_0} - \frac{1}{2} V \left(\frac{c_{r,t} - c_0}{c_0} \right) \frac{\phi'''(c_0)c_0}{\phi''(c_0)}$, a first-order Taylor approximation around c_0 yields:

$$-\frac{1}{t} \ln \frac{E_t \phi'(c_{r,t})}{E_0 \phi'(c_{r,0})} \simeq -\frac{1}{t} \frac{\phi''(c_0)c_0}{\phi'(c_0)} E_t \frac{c_{r,t} - c_0}{c_0} - \frac{1}{2t} \frac{\phi'''(c_0)c_0^2}{\phi'(c_0)} \left[V \left(\frac{c_{r,t} - c_0}{c_0} \right) - V \left(\frac{c_{r,0} - c_0}{c_0} \right) \right]$$

²⁵The index $\frac{h'''(c_t)}{h'(c_t)}$, with $h = \phi, v$, is usually labelled 'downside inequality aversion' (Davies and Hoy, 1995), and it means that the decision maker prefers spreads in the income distribution that occur at high income levels rather than at the bottom of the distribution. Applied to our framework, marginal benefits/marginal costs increase under EDE preferences when the social planner is less downside inequality averse than the utilitarian one, or more inter- than intra-generational downside inequality averse.

$$\begin{aligned}
-\frac{1}{t} \ln \frac{v'(c_t^{ede})}{\phi'(c_0^{ede})} &\simeq -\frac{1}{t} \ln \frac{v'(c_0)}{\phi'(c_0)} - \frac{1}{t} \left(-\frac{\phi''(c_0)c_0}{\phi'(c_0)} + \frac{v''(c_0)c_0}{v'(c_0)} \right) \left(E_t \frac{c_{r,t} - c_0}{c_0} + \frac{1}{2} \frac{\phi''(c_0)c_0}{\phi'(c_0)} V \left(\frac{c_{r,t} - c_0}{c_0} \right) \right) \\
\frac{1}{t} \ln \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})} &\simeq \frac{1}{t} \ln \frac{v'(c_0)}{\phi'(c_0)} + \frac{1}{2t} \left(-\frac{\phi''(c_0)c_0}{\phi'(c_0)} + \frac{v''(c_0)c_0}{v'(c_0)} \right) \frac{\phi''(c_0)c_0}{\phi'(c_0)} V \left(\frac{c_{r,0} - c_0}{c_0} \right)
\end{aligned}$$

After summing the different components, we get indeed the approximation (2.11). For what concerns the discount rates r_t , as long as $V(\eta_t) \geq V(\eta_0)$, the EDE approach calls for a lower r_t if and only if:

$$R_\phi[P_\phi - R_\phi + R_v] \geq R_v P_v$$

or, after adjusting terms:

$$R_\phi P_\phi - R_v P_v + R_\phi(R_v - R_\phi) \geq 0 \quad (2.23)$$

which corresponds exactly to the necessary condition in the large. Provided that the EDE social planner is less inequality averse than the utilitarian counterpart ($R_v \geq R_\phi$), the discount rate decreases only if the EDE social planner is more downside inequality averse than the utilitarian one ($R_\phi P_\phi \geq R_v P_v$).

However, condition (2.23) can be rearranged as follows:

$$(P_\phi - R_\phi)(R_\phi - R_v) + R_v(P_\phi - P_v) \geq 0 \quad (2.24)$$

Then, a necessary and sufficient condition for a lower discount rate under EDE preferences when inequality is small is: ϕ is DRIA (decreasing inequality aversion: $P_\phi \geq R_\phi$), and ϕ is both more inequality averse and more inequality prudent than v ($R_\phi \geq R_v$ and $P_\phi \geq P_v$). Note that isoelastic utility functions satisfy condition (2.24) as long as $R_\phi \geq R_v$.

The project specific inequality premium $\chi_t(d_t)$ indicates the impact of the project on the degree of inequality inside the society:

$$\chi_t(d_t) = -\frac{1}{t} \ln \frac{E^t d_{r,t} \phi'(c_{r,t})}{d_t E^t \phi'(c_{r,t})}$$

Indeed, if $d_{r,t}$ and $c_{r,t}$ were independent, e.g. each regions were facing the same marginal damages, the inequality premium would be null, as $E^t d_{r,t} \phi'(c_{r,t}) = d_t E^t \phi'(c_{r,t})$. On the contrary, the inequality premium $\chi_t(d_t)$ is positive as long as $E^t d_{r,t} \phi'(c_{r,t}) \leq d_t E^t \phi'(c_{r,t})$.

Lemma 3. *Assume that $c_{r,t}$ and $d_{r,t}$ are positively dependent. Then, the inequality premium $\chi_t(d_t)$ is positive if and only if ϕ is concave.*

Proof. To sign the inequality premium, we can make use of a result by Tchen (1980), according to whom, when $d_{r,t}$ and $c_{r,t}$ are positively dependent, the relation $E^t d_{r,t} \phi'(c_{r,t}) \leq d_t E^t \phi'(c_{r,t})$ holds as long as the function $h(d_{r,t}, c_{r,t}) = d_{r,t} \phi'(c_{r,t})$ is supermodular, i.e. ϕ is concave. \square

A similar argument can be made for the inequality premium χ_0 , which represents the impact of the mitigation costs on the existing inequality at time 0:

$$\chi_0 = \frac{1}{t} \ln \frac{\hat{E}^0 \phi'(c_{r,0})}{E^0 \phi'(c_{r,0})}$$

Note that To ease the notation, let F_n denote the initial distribution of consumption with regional shares $n_{r,0}$, and F_μ the distribution of consumption with shares μ_r .

Lemma 4. *Let $\tilde{c}_\theta \sim F_\theta$, with $\theta = n, \mu$. The inequality premium χ_0 is positive if any of the following conditions holds:*

- \tilde{c}_n FSD-dominates \tilde{c}_μ and ϕ is concave;
- \tilde{c}_n SSD-dominates \tilde{c}_μ and $\phi''' > 0$.

Proof. The inequality premium is positive as long as $\hat{E}^0 \phi'(c_{r,0}) \geq E^0 \phi'(c_{r,0})$. When ϕ is concave, $-\phi'$ is increasing, and $-\hat{E}^0 \phi'(c_{r,0}) \leq -E^0 \phi'(c_{r,0})$ holds when \tilde{c}_μ is first order stochastically dominated by \tilde{c}_n , or $F_\mu \geq F_n$. Similarly, when $-\phi'$ is concave, i.e. $\phi''' > 0$, the fore-mentioned relation holds as long as \tilde{c}_μ is second order stochastically dominated by \tilde{c}_n , or $\int_a^x F_\mu(c)dc \geq \int_a^x F_n(c)dc$, for all $c \in [a, b]$. \square

In conclusion, the discount rate $\rho_t(d_t)$ grows when the project benefits mainly rich people, and the cost of the policy are borne mainly by poor people. In the opposite case, the discount rate lowers.

A second order approximation approximation of the inequality premium $\chi_t(d_t)$ around c_0 and d_t yields:

$$\chi_t(d_t) = -\frac{1}{t} \ln \frac{E_t d_{r,t} \phi'(c_{r,t})}{d_t E_t \phi'(c_{r,t})} \simeq -\frac{1}{t} \frac{\phi''(c_0) c_0}{\phi'(c_0)} E_t \left(\frac{c_{r,t} - c_0}{c_0} \frac{d_{r,t} - d_t}{d_t} \right)$$

where the last term coincides with the covariance between $\frac{c_{r,t} - c_0}{c_0}$ and $\frac{d_{r,t} - d_t}{d_t}$. In order to obtain it, we can take a first order approximation of $E_t \phi'(c_{r,t})$ around c_0 : $E_t \phi'(c_{r,t}) \simeq \phi'(c_0) - c_0 \psi''(c_0)$, where ψ is the inequality premium such that $E_t \phi'(c_{r,t}) = \phi'(c_0(1 - \psi))$. A

second order Taylor approximation of ψ gives: $\psi \simeq -\frac{1}{2} \frac{\phi'''(c_0)c_0}{\phi''(c_0)} V \left(\frac{c_{r,t}-c_0}{c_0} \right)$, where V represents the variance. Similarly for all the other expected values. For the time 0 inequality premium χ_0 , we need to assess an artificial change in the distribution of consumption across regions. A second order approximation around c_0 yields:

$$\chi_0 = \frac{1}{t} \ln \frac{\hat{E}_0 \phi'(c_{r,0})}{E_0 \phi'(c_{r,0})} \simeq \frac{1}{t} \frac{\phi''(c_0)c_0}{\phi'(c_0)} \left[\hat{E}_0 \left(\frac{c_{r,0}-c_0}{c_0} \right) + \frac{1}{2} \frac{\phi'''(c_0)c_0}{\phi''(c_0)} \left(\hat{V} \left(\frac{c_{r,0}-c_0}{c_0} \right) - V \left(\frac{c_{r,0}-c_0}{c_0} \right) \right) \right]$$

where \hat{V} denotes the variance according to the income distribution μ , and V the variance with the original distribution.

2.B The social discount rate in the stochastic case

The following section derives the quadratic approximation of the socially efficient discount rate. The risk and equity adjusted discount rate writes as:

$$r_t = -\frac{1}{t} \ln \frac{N_t}{N_0} - \frac{1}{t} \ln \frac{E_s \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_{s,t}^{ede})}{\phi'(c_{s,t}^{ede})} E_r \phi'(c_{r,s,t})}{E_s \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})} E_r \phi'(c_{r,0})}$$

From the deterministic case, we already know that

$$E_r \phi'(c_{r,s,t}) = \phi'(c_0(1 - \psi_{s,t})) \simeq \phi'(c_0) \left[1 + \frac{1}{2} R_\phi P_\phi V \left(\frac{c_{r,0}-c_0}{c_0} \right) \right]$$

$$\frac{v'(c_0^{ede})}{\phi'(c_0^{ede})} = \frac{v'(c_0(1 - \pi_{s,t}))}{\phi'(c_0(1 - \pi_{s,t}))} \simeq \frac{v'(c_0)}{\phi'(c_0)} \left[1 + \frac{1}{2} R_\phi (R_v - R_\phi) V \left(\frac{c_{r,0}-c_0}{c_0} \right) \right]$$

Moreover

$$E_s \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} = E \frac{u'}{v'}(c_0(1 - E_t \pi_{s,t} - \omega_s)) = \frac{u'}{v'}(c_0(1 - E_{s,t} \pi_{s,t} - E_s \omega_s - \beta)) \simeq h' [1 + R_h (E_{s,t} \pi_{s,t} + E_s \omega_s + \beta)]$$

where $\omega_s \simeq \frac{1}{2} R_v V_t(E_r \epsilon)$ and $\beta \simeq \frac{1}{2} P_h V_s(E_{t,r} \epsilon)$, with $\epsilon \equiv \frac{c_{r,t,s}-c_0}{c_0}$ and $h = \frac{u}{v}$. Similarly for the numerator:

$$E_s \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_{s,t}^{ede})}{\phi'(c_{s,t}^{ede})} E_r \phi'(c_{r,s,t}) = E_s h'(c_0(1 - E_t \pi_{s,t} - \omega_s)) g'(c_0(1 - \pi_{s,t})) \phi'(c_0(1 - \psi_{s,t}))$$

By taking the second order Taylor approximation of the previous expression around $k = 0$, and after performing some tedious computations, we get exactly the result.

For what concerns the social premium

$$\tau_t(d_t) = -\frac{1}{t} \ln \frac{E_s \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_{t,s}^{ede})}{\phi'(c_{t,s}^{ede})} E_r \phi'(c_{r,t,s}) d_{r,t,s}}{d_t E_s \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_{t,s}^{ede})}{\phi'(c_{t,s}^{ede})} E_r \phi'(c_{r,t,s})}$$

In order to obtain the result we have to perform a series of approximations. First of all, let us define the variable $\alpha_{r,t,s} = \frac{d_{r,t,s} - E_r d}{E_r d}$, and consider a risk whose size k goes to zero:

$$E_r \phi'(c_{r,t,s}) d_{r,t,s} = E_r \phi'(c_0(1 + k\epsilon_{r,t,s})) E_r d_{r,t,s} (1 + k\alpha_{r,t,s}) = \phi'(c_0(1 - z_{s,t}(k))) E_r d$$

For small risk, we have $z_{s,t} \simeq -E_r \epsilon + \frac{1}{2} P_\phi V_r(\epsilon) - E_r \alpha \epsilon$. Next, let us write

$$E_s \frac{u'(\bar{c}_s)}{v'(\bar{c}_s)} \frac{v'(c_{t,s}^{ede})}{\phi'(c_{t,s}^{ede})} E_r \phi'(c_{r,t,s}) d_{r,t,s} = E_s h'(c_0(1 - E_t \pi_{s,t} - \omega_s)) g'(c_0(1 - \pi_{s,t})) \phi'(c_0(1 - z_{s,t}(k))) d_t (1 + k\eta_{t,s})$$

with $\eta_{t,s} = \frac{E_r d - d_t}{d_t}$. As before, a second order approximation around $k = 0$ yields the result.

2.C Recursive preferences

Another way to disentangle risk aversion from resistance to intertemporal substitution would be to consider [Kreps and Porteus \(1978\)](#) and [Selden \(1978\)](#) preferences. The analysis is limited to a model with two dates, $t = 0, 1$. Let $c_{s,t}^{ede}$ denote the equally distributed equivalent consumption in state s and generation t :

$$\phi(c_{s,t}^{ede}) = \frac{1}{N} \sum_{r=1}^R N_r \phi(c_{r,t,s})$$

At time 0 there is no risk. In contrast, the certainty equivalent CE_1 of future equally distributed equivalents is evaluated by using an increasing and concave von-Neumann Morgenstern utility function u :

$$u(CE_1) = \sum_s \pi_s u(c_{s,1}^{ede})$$

It represents the value of the risk faced by each individual belonging to the future generation. A time aggregating utility function v is then used to evaluate intertemporal welfare W :

$$W = Nv(c_0^{ede}) + e^{-\delta} Nv(CE_1)$$

where $e^{-\delta}$ can be interpreted as the probability of extinction, which is independent of the consumption risk. The utility function u characterizes attitudes towards risk, while function v characterizes attitudes towards consumption fluctuations over time; finally, ϕ represents attitudes towards intra-generational inequality.

As before, the social cost of carbon can be expressed in terms of social discount rate for a project with uniform and sure benefits $d_t \equiv \sum_{s=1}^S \pi_s \sum_{r=1}^R n_r d_{r,t,s}$, with $n_r \equiv \frac{N_r}{N}$:

$$SCC = \sum_t e^{-\rho t(d_t)} d_t$$

where the social discount rate $\rho_1(d_1)$ between date 0 and date 1 is defined as:

$$\rho_1(d_1) = \delta - \ln \frac{\frac{v'(CE_1)}{u'(CE_1)} \sum_s \pi_s \frac{u'(c_1^{ede})}{\phi'(c_1^{ede})} \sum_r n_r \phi'(c_{r,s}) d_{r,s}}{d_1 \frac{v'(c_0^{ede})}{\phi'(c_0^{ede})} \sum_r n_r \phi'(c_{r,0})}$$

2.D Figures and Tables

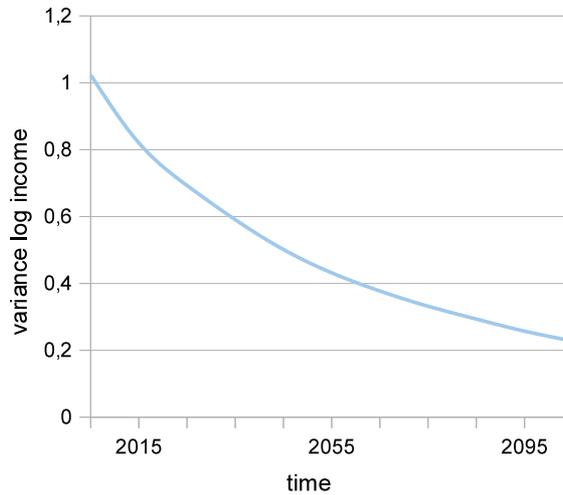


Figure 2.4: Variance of log income based on Nordhaus (2010).

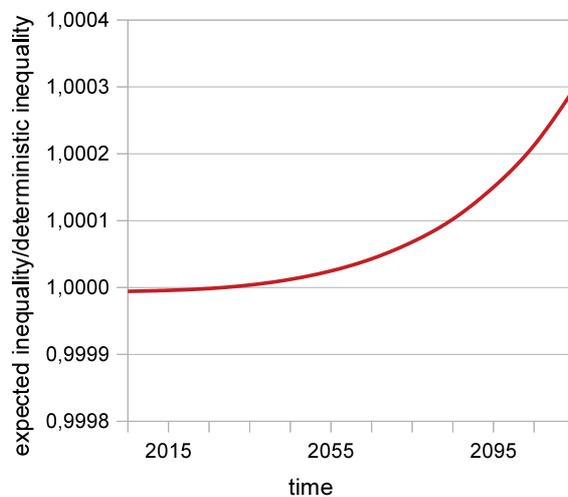


Figure 2.5: Comparison between the degree of inequality in the stochastic model and in the deterministic one. Expected variance of log income over variance of log income in the deterministic case (Figure 3).

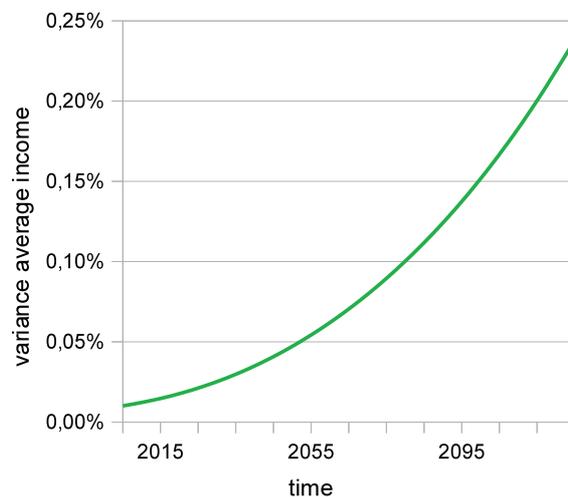


Figure 2.6: Variance of average log income across states.

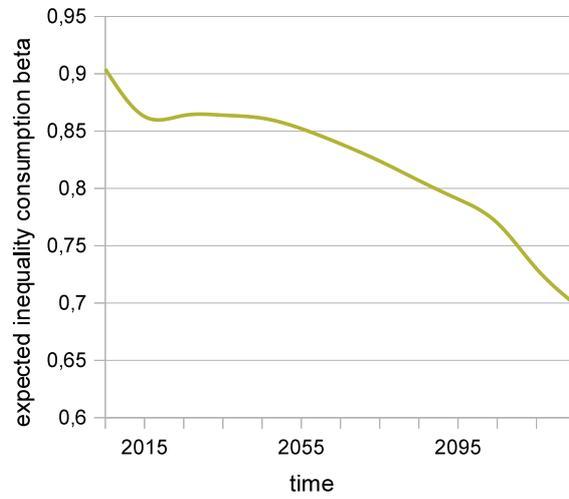


Figure 2.7: Time path of the expected inequality consumption beta.

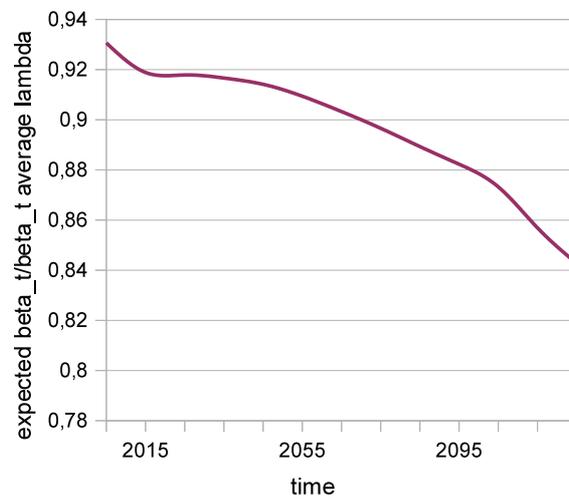


Figure 2.8: Comparison between the expected inequality consumption beta in Figure and the inequality consumption beta derived in the deterministic case (Figure 1).

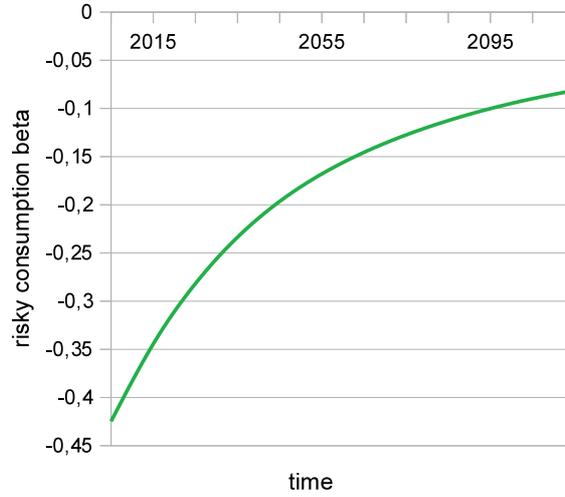


Figure 2.9: Time path of the consumption beta due to the presence of risk.

Region	λ_r low 10^{-5}	λ_r high 10^{-5}	average 10^{-5}
US	2	18.3	3.1
EU	3.2	21.3	4.4
Japan	1.3	21.3	2.7
Russia*	0	20.8	1.4
Eurasia	1.6	17.9	2.7
China	2.9	21.2	4.1
India	3	20.4	4.2
Middle East	6.9	24.7	8.1
Africa	4.6	20.8	5.7
Latin America	21.6	27.9	22.1
Other High Income	5.5	22.4	6.7
Other Asia	5.4	22.1	6.6
Global	3.9	20.7	5.1

Table 2.1: Economic impact of climate change per region based on damages reported in RICE and for an exponential damages function: $1 - \Omega_{r,t}(T_t) = 1 - e^{\lambda_{r,t}T_t}$. Parameters estimated for a temperature change of 2.5°C in 2100 (low) and 6°C (high). As for Russia a low temperature increase would cause benefits and not costs, we set a parameter close to 0.

Social framework	Social cost of carbon in \$		
	$\delta = 0$	$\delta = 1$	$\delta = 2$
Representative agent ($\varphi = 0$)	61.83	14.11	8.41
Utilitarianism ($\varphi = \gamma = 1$)	29.19	6.79	4.03
EDE $\varphi = 0.5$	43.89	10.05	5.97
EDE $\varphi = 4$	16.36	3.41	1.97

Table 2.2: Estimations of the social cost of carbon at time 0 for the deterministic case, with $\gamma = 1$.

Social framework	Social cost of carbon in \$
Representative agent ($\varphi = 0, \eta = \gamma = 1$)	62.32
Utilitarianism ($\varphi = \gamma = \eta = 1$)	29.87
EEDE $\varphi = 0.5$	52.98
EEDE $\varphi = 1$	35.94
EEDE $\varphi = 4$	19.82

Table 2.3: Estimations of the social cost of carbon at time 0 for the stochastic case, with $\gamma = 1$ and $\eta = 4$.

Chapter 3

Optimal climate policy: prevention and risk sharing

3.1 Introduction

One of the main problems in climate policy is the presence of deep uncertainty. The reliability of cost-benefit analysis is questioned both by the limited knowledge of the forces causing the climate to change, and by the uncertainty in evaluating the impacts on society and economy. For instance, for a 2,5°C increase in temperature, the size of the impacts is expected to vary from a slight gain to losses of more than 4% of world GDP. If the temperature increase exceeds the 3°C threshold, uncertainty becomes even more considerable (Tol, 2009; Aldy et al., 2010). In addition, although climate change is very likely to impose net annual costs at the global level, impacts are expected to vary regionally. Some places, like the Sub-Sahara region or the Asian mega-deltas, are highly exposed to the risk of damage, while regions in the North, like Eastern Europe or the former Soviet Union, may benefit from a slight increase in temperature (IPCC, 2007). As a consequence, when designing the optimal climate policy, not only the average increase in temperature matters, but also the distribution of impacts across the world.

As long as risks are shared efficiently and credit markets are perfect, the heterogeneity in impacts has no effect on the optimal policy. Highly exposed countries would be protected by the risk of suffering a severe loss, and, at the same time, poor countries would be compensated for the efforts in reducing emissions. Indeed, the problem of a heterogeneous group of countries can be reduced to the optimal decision of a representative agent (Costantinides, 1982). However, perfect risk sharing is an unrealistic assumption; because of transactions

costs, asymmetric information problems and commitment issues, countries are reluctant to transfer wealth across states of nature.

Notwithstanding the heterogeneity in climate risks faced by different regions, the representative agent assumption still remains the most common tool in the cost benefit analysis of climate change (see e.g. the discounting literature, [Gollier \(2012\)](#)). The aim of the paper is to study the importance of that assumption when countries design the optimal climate mitigation policy. In particular, it investigates whether the presence of uncertain heterogeneous impacts is an argument for stricter mitigation targets. We interpret the climate policy as a self-protection investment, that is as a policy that changes the probability distribution of the climate risk. [Gollier \(2010\)](#) analyses similar issues in a self-insurance context. In particular, we consider a group of countries that invest in a public prevention project in order to reduce the probability of a shift in climate regime. We assume that there are only two possible aggregate states (climate regimes), each of them characterized by a certain set of climate conditions. For example, we can think at them as the regime with no or low increase in global mean temperature ($<2.5^{\circ}\text{C}$) and the regime with a high increase in global mean temperature ($>6^{\circ}\text{C}$). In order to represent the fact that climate change risk has heterogeneous impacts, we assume that each climate regime is characterized by different potential distributions of wealth across countries. As a consequence, the benefits of the mitigation policy are uncertain and heterogeneously distributed. By assumption, the costs of the policy are equally divided among countries (i.e. some Kyoto-type agreement is in place). The group has to decide how much to invest in the policy as a function of the existing risk sharing mechanism. Two cases are compared: in the first one, countries perfectly share the individual risks (representative agent case); in the second one, no risk sharing mechanism is implemented.

We show that the presence of perfect risk sharing has two opposite effects on the optimal prevention policy. On the one hand, risk sharing reduces the risk faced by each country, and, therefore, the need to invest in prevention (income pooling effect). On the other hand, risk sharing relaxes the budget constraint of the most exposed countries, thereby making the investment in prevention relatively less costly (wealth accumulation effect). Indeed, without risk sharing, countries are forced to bear their own impacts. As a consequence, the incentive to protect wealth in case a loss occurs makes countries less willing to invest. The goal of the paper is to characterize under which conditions the income pooling effect is stronger than the wealth accumulation effect. In order to achieve that goal, we build a parallel between the presence of inefficient risk sharing and the introduction of background risks. Indeed, when the social planner is utilitarian, the presence of heterogeneity in realized

incomes across countries looks like a second source of risk. In other words, under the veil of ignorance the social planner does not distinguish between an unequal distribution of sure wealth across countries and an unequal distribution of risky wealth across states. As a consequence, the presence of inefficient risk sharing is equivalent to the introduction of a climate-specific background risk, which represents the degree of inequality characterizing each climate regime. Therefore, we can re-state the problem as determining the effect of uninsurable exogenous risks on the willingness to pay to reduce the probability of loss.

We prove that the impact of state-dependent background risks can be represented in terms of either prudence (when risk is small) or state-dependent increase in risk aversion (when risk is large). Inefficient risk sharing increases prevention if either the probability of loss is sufficiently small or prudence (in the small) and the increase in risk aversion (in the large) are sufficiently low. If countries are sufficiently prudent and the degree of inequality is sufficiently high (which will induce a large increase in risk aversion), the social planner will prefer to accumulate resources for the future rather than to invest in prevention. Therefore, a sufficient condition for increasing prevention under inefficient risk sharing is the presence of a catastrophic event (i.e. an event characterized by low probability and huge loss). The degree of inequality among countries and the assumptions about economic convergence (i.e. whether a large increase in global mean temperature leads to a polarization of wealth or not) only marginally affect this result.

The paper refers to the problem of climate change, but it raises a more general question that finds application in other domains. For instance, the regulation of financial markets is seen as a public self-protection policy to reduce the probability of financial crisis ([Blanchard, 2009](#)). Then, the paper suggests that the degree of government intervention should depend on the efficiency of the risk sharing mechanism. In a world where wealth is not easily transferred across states of nature, the risk of a catastrophic event should induce more regulation.

Literature Review. The paper is related to two strands of literature on decision making under risk. The first one studies self-insurance and self-protection decisions ([Ehrlich and Becker, 1972](#)), where self-insurance is an investment that decreases the size of the loss, while self-protection reduces the probability of a given loss. The second one analyzes the impact of background risks on agent's choices under risk. The main results of the literature on self-insurance and self-protection concern the comparative study of an increase in risk aversion and of prudence. An increase in risk aversion raises self-insurance, but does not necessarily lead to an increase in self-protection ([Dionne and Eeckhoudt, 1984](#); [Jullien et al., 1999](#);

Briys and Schlesinger, 1990), while self-protection and prudence are opponents (Eeckhoudt and Gollier, 2005). In particular, as more risk averse agents care more about their state-contingent consumption possibilities, an increase in risk aversion induces more self-protection only if the probability of loss is sufficiently small, where the threshold is endogenously determined. Otherwise, agents prefer to accumulate wealth in order to have a sufficiently large consumption possibilities set in case a loss occurs. Moreover, as the incentive to accumulate is defined by prudence, a prudent agent has a lower willingness to pay for a risk reduction project than a non-prudent agent. Contrary to those papers, we eliminate the assumption of a representative agent and consider how a group of countries with uncertain unequal wealth invest in self-protection. We still find that prudence and change in risk aversion play an important role. However, in our model, the size of the increase in risk aversion and the size of the degree of absolute prudence matter for the results, not just the presence of prudence or increased risk aversion. Indeed, we find that a prudent social planner increases prevention if he is not too prudent, while the existing literature highlights a negative relationship between prudence and prevention. Moreover, in our case an increase in risk aversion can lead to more prevention even when the probability of loss is high, provided that the increase in risk aversion is sufficiently small.

The second strand of literature related to the paper concerns the impact of background risk. Gollier and Pratt (1996) show that adding an exogenous unfair risk affects the shape of the utility function, in particular risk aversion and prudence. In particular, under risk vulnerability, the presence of a background risk increases risk aversion. Risk vulnerability is satisfied when the coefficient of absolute risk aversion is decreasing and convex. Representing inefficient risk sharing as background risk allows us to state the results in terms of either prudence or risk aversion, and, as a consequence, to draw a parallel with the literature on self-protection. There exists a bunch of papers that analyze the impact of background risk on the willingness to pay to reduce the probability of loss. Eeckhoudt and Hammit (2001) show that the presence of a background financial risk reduces the value of statistical life (which is related to the willingness to pay). The result is mainly driven by the fact that marginal utility in case of survival (i.e. no loss) is larger than marginal utility in case of death (loss). As a consequence, the agent is more interested in protecting the financial wealth in case of survival than in case of death, which decreases the willingness to pay to reduce the mortality risk. On the contrary, in my case marginal utility depends on wealth, and therefore it increases in the loss. As the social planner cares more about the loss state than the no loss state, the impact of background risk is ambiguous. Courbage and Rey (2008) examine

the case of a small risk of loss, and point out that either Decreasing Absolute Risk Aversion or risk vulnerability is required to have an increase in the willingness to pay to reduce the probability of loss in the presence of an independent background risk. In contrast, we do not restrict the analysis to the case of small risk of loss, and introduce aggregate state-dependent background risks. However, we still find that risk vulnerability matters for the results, as under risk vulnerability we can transform the problem in terms of increase in risk aversion. Finally, [Courbage and Rey \(2012\)](#) look at the impact of background risk on self-protection in a two-period model. They find that, according to the configurations of the background risk (either in the first period or in the second, state-dependent or state-independent risk), both prudence and risk aversion are fundamental to sign the effects.

When talking about climate change, there exists a large literature that aims at determining cost benefit rules for the evaluation of prevention projects. The usual approach is to consider a representative agent that marginally modifies his consumption path over time (for a review of the literature on discounting, see [Gollier \(2012\)](#)). Few examples study the case of imperfect financial markets, which prevent risk sharing ([Gollier, 2010](#); [Emmerling, 2011](#); [Fleurbaey and Zuber, 2014](#)). Finally, a recent strand of literature analyzes a willingness to pay approach to climate change ([Pindyck, 2013](#)), where the main goal is to study policies that affect the probability distribution of climate change, similarly to our case.

The paper is developed as follows. Section 2 presents the model and the optimal solution under the two risk sharing assumptions. Section 3 compares the results under perfect risk sharing and no risk sharing. Section 4 interprets the results in the small risk case, while Section 5 reverts to the general case. Section 3.1 discusses the possibility that climate change leads to a convergence of economies. Section 6 considers a framework where climate change has no impact on the degree of inequality. In Section 7 we assume that climate change creates inequalities, while Section 8 studies what happens when there is a change in the degree of inequality across climate regimes. Finally, Section 9 concludes.

3.2 The model

Let us consider a group of countries, uniformly distributed in the set $\Theta = [0, 1]$, where θ denotes the type of country. Countries' preferences are represented by a Von-Neuman Morgenstern utility function $u(c_{\theta,t})$. The variable $c_{\theta,t}$ denotes the quantity consumed in period t by country θ , with u increasing and concave.

The model is static and consists of two periods, $t = \{0, 1\}$. At $t = 0$ countries must allocate their uncertain lifetime wealth between consumption and investment. At $t = 1$ uncertainty is resolved and wealth is observable. We assume that the type of risk to which countries are subject depends on the evolution of the climate between periods. If there is no change in climate, the shock reflects non-weather related constraints. On the contrary, if there is a change in the climate regime (e.g. a substantial increase in average temperature), countries will suffer an economic loss, although the size of potential damages and their distribution across countries are uncertain.

Let $i = b$ define the climate regime with an increase in average temperature, and $i = g$ the regime with no change. For each country θ , regime $i = g$ is characterized by the random variable $\tilde{\omega}_\theta$, while regime $i = b$ by the random variable $\tilde{\omega}_\theta - \tilde{l}_\theta$. Let S_i be the set of possible states of nature contingent on the climate regime i , where each state $s_i \in S_i$ can be represented by a vector of realizations of the random wealth. Finally, ω_{θ, s_g} and $\omega_{\theta, s_b} = \omega_{\theta, s_g} - l_{\theta, s_b}$ denote the realized wealths of agent θ in state $s_g \in S_g$ and $s_b \in S_b$, respectively.

Countries do not know ex-ante which climate regime will prevail in the future. With probability $\pi(e)$ the climate will change, and regime $i = b$ will occur; with probability $1 - \pi(e)$, no change is observed. Countries can slow the pace $\pi(e)$ at which climate is changing by investing in a public prevention project e , such that $\pi'(e) < 0$. Prevention policies might include, for instance, the use of renewable energies or the construction of sinks to capture and store carbon dioxide. The investment is decided by the group as a whole, and the costs are equally divided among countries. Therefore, as the group has mass 1, every country is going to pay $e_\theta = e$. In other words, we assume that there exists some Kyoto-type agreement. Countries has agreed on how to allocate mitigation efforts, but, as climate change will happen only in the future, they might not be able to credibly commit on a system of future transfers.

The expected utility of country θ is represented by:

$$U_\theta = \pi(e)Eu(\tilde{\omega}_\theta - \tilde{l}_\theta - c_\theta) + (1 - \pi(e))Eu(\tilde{\omega}_\theta - c_\theta) \quad (3.1)$$

The problem faced by the group of countries is to determine the optimal amount to invest in prevention to reduce the probability of a change in climate regime. The optimal solution will depend on whether countries agree to redistribute their own resources: if a risk sharing mechanism is in place, countries more exposed to the bad climate shock can be compensated for their intervention; on the contrary, if there exists no risk sharing, each country is forced

to totally bear its own impacts of climate change. Intuitively, two opposite forces determine the impact of risk sharing on optimal prevention effort. A country receiving a transfer is less exposed to the negative consequences of climate change; as a consequence, the need to invest in prevention decreases. At the same time, the transfer relaxes the budget constraint, and makes the investment in prevention relatively less costly. As we will see, the Pareto optimal solution will trade-off the change in risk exposure with the change in marginal costs of effort.

In the following, then, two different economies will be compared: in the first one, no risk sharing occurs; in the second one, an efficient risk sharing mechanism can be implemented.

3.2.1 Inefficient risk sharing

If no risk sharing occurs, the group solves the following problem:

$$\max_e \sum_{\theta} \left\{ \pi(e)Eu(\tilde{\omega}_{\theta} - \tilde{l}_{\theta} - e) + (1 - \pi(e))Eu(\tilde{\omega}_{\theta} - e) \right\} \quad (3.2)$$

From the first order conditions, the optimal levels of investment e is given by:

$$e : \quad -\pi'(e) \sum_{\theta} [Eu(c_{\theta,g}) - Eu(c_{\theta,b})] = \pi(e)Eu'(c_{\theta,b}) + (1 - \pi(e))Eu'(c_{\theta,g}) \quad (3.3)$$

where $c_{\theta,b}$ and $c_{\theta,g}$ indicate the consumption levels of agent θ in states $s_b \in S_b$ and $s_g \in S_g$, respectively. The optimal prevention level trades-off the costs borne in period $t = 0$ and the future expected utility gains from slowing down the changes in climate. The relative size of the costs depends on the consumption smoothing effect. If countries decide to accumulate resources for coping with the future risk, they will lower the amount of resources that can be devoted to prevention. However, thanks to accumulation, they will be more protected in the future in case climate change strikes. Accumulation of resources and prevention can be anti-thetical. The optimal strategy highlights the dilemma that countries (especially developing ones) face: whether to concentrate effort on growth in order to increase future production and consumption possibilities and, thereby, the capacity to adapt to a changing climate, or to devote resources to green investments that should reduce the climate change risk. The problem, thus, is how to define the optimal growth path that conciliates environmental concerns and economic ones.

From the previous conditions, we can define the social willingness to pay for a marginal reduction in the risk of climate change as:

$$SWTP^I = \frac{\sum_{\theta} [Eu(c_{\theta,g}) - Eu(c_{\theta,b})]}{\sum_{\theta} [\pi(e)Eu'(c_{\theta,b}) + (1 - \pi(e))Eu'(c_{\theta,g})]} \quad (3.4)$$

The numerator represents the utility gain from avoiding the loss state, while the denominator the expected marginal cost of spending one more unit in prevention.

3.2.2 Efficient risk sharing

If countries can commit to a perfect risk sharing mechanism, the group has a two-fold decision to take. First of all, how much to invest in the public technology, and then how to reallocate the remaining wealth contingent on the type of climate regime that will prevail. Contrary to the inefficient risk sharing case, countries' wealths are pooled together and then divided among countries. With inefficient risk sharing, instead, each country can only choose how to allocate *its own* wealth between consumption and investment.

Let z_{θ,s_i} be the share of aggregate wealth allocated to country θ in state $s_i \in S_i$. Then, the group's optimal decision can be determined by solving the following problem:

$$\begin{aligned} \max_{e, \{z_{\theta,s_i}\}} \quad & \sum_{\theta} \{ \pi(e)Eu(z_{\theta,s_b}) + (1 - \pi(e))Eu(z_{\theta,s_g}) \} \\ \text{s.t.} \quad & \forall s_i \in S_i : \quad E_{\theta}\omega_{\theta,s_i} = E_{\theta}z_{\theta,s_i} \end{aligned} \quad (3.5)$$

where $E_{\theta}\omega_{\theta,s_i}$ represents the expected resources available in state s_i , $\forall s_i \in S_i$.

If the risk can be shared, the group first chooses the public effort, and then decides how to redistribute the remaining expected resources. Under the veil of ignorance, the distribution of types θ looks like a second source of uncertainty. Therefore, we can transform an interpersonal problem of investment and wealth distribution into a single decision maker problem under two sources of risk: the state s that will prevail in the future, and the type θ , i.e. the particular income that he will receive in state s .

By the mutuality principle, in any risk sharing mechanism the quantity that an agent consumes depends on the state only through the aggregate wealth of the state (Gollier, 2001). Thus, the agent consumes the same amount in all the states characterized by the same aggregate wealth. To ease the comparison between the efficient risk sharing and the

no risk sharing case, we will assume that each climate regime is characterized by only one aggregate wealth level:

Assumption 1. $\forall s_i \in S_i, i = \{b, g\}$, we have that:

$$E_{\theta} \omega_{\theta, s_g} = \omega \quad E_{\theta} \omega_{\theta, s_b} = \omega - l \quad (3.6)$$

Since the economy faces only two aggregate states of nature (climate change vs no climate change), the risk sharing contract will provide two different types of wealth allocations. Moreover, as preferences are homogeneous, each country θ will receive the same state-contingent transfer:

$$z_{\theta, s_g} = \omega \quad \text{and} \quad z_{\theta, s_b} = \omega - l \quad (3.7)$$

Once we have determined the the optimal risk sharing rule, we can found the optimal investment e by maximizing the utility of the representative country:

$$\max_e \quad \pi(e)u(\omega - l - e) + (1 - \pi(e))u(\omega - e) \quad (3.8)$$

Solving for the first order condition:

$$e : \quad -\pi'(e) [u(\omega - e) - u(\omega - l - e)] = \pi(e)u'(\omega - l - e) + (1 - \pi(e))u'(\omega - e) \quad (3.9)$$

Therefore, in the presence of efficient risk sharing the willingness to pay for a reduction in the risk of climate change is given by:

$$SWTPE = \frac{u(\omega - e) - u(\omega - l - e)}{\pi(e)u'(\omega - l - e) + (1 - \pi(e))u'(\omega - e)} \quad (3.10)$$

Thanks to risk sharing, individual losses can be optimally redistributed among countries. Therefore, total welfare is larger in the risk sharing case. Then, the question is how an increase in the efficiency of the risk sharing mechanism affects the collective investment. On the one hand, risk sharing creates income pooling, which allows countries to eliminate part of the uncertainty faced by the group. As a consequence, under standard assumptions, risk sharing should always reduce effort. On the other hand, risk sharing reduces also the marginal costs of investing, as it relaxes the budget constraint of the countries more exposed to the shock. Countries are less worried to end up with a very low wealth in the future, and, therefore, they are more willing to transfer resources to the present in order to invest them in prevention.

3.3 Analysis

Given the definition of social willingness to pay without risk sharing (3.4) and with perfect risks sharing (3.10), the investment in prevention increases under inefficient risk sharing if and only if $SWTP^I \geq SWTP^E$, i.e.:

$$\frac{\sum_{\theta} [Eu(c_{\theta,g}) - Eu(c_{\theta,b})]}{\sum_{\theta} [\pi(e)Eu'(c_{\theta,b}) + (1 - \pi(e))Eu'(c_{\theta,g})]} \geq \frac{u(\omega - e) - u(\omega - l - e)}{\pi(e)u'(\omega - l - e) + (1 - \pi(e))u'(\omega - e)} \quad (3.11)$$

In order to facilitate the comparison between the two conditions, let us express $SWTP^I$ in terms of distance from the optimal quantities consumed in the presence of risk sharing. In other words, we will define the random variables $\tilde{\eta}_{\theta}$ and $\tilde{\epsilon}_{\theta}$, that denote the risk of having a wealth per climate regime different from the average one, in regime g (no climate change) and regime b (climate change), respectively:

$$c_{\theta,s_g} = \omega - e + \eta_{\theta,s_g} \quad \text{and} \quad c_{\theta,s_b} = \omega - l - e + \epsilon_{\theta,s_b} \quad (3.12)$$

where η_{θ,s_g} and ϵ_{θ,s_b} represent the realizations of the shocks. By Assumption 1, $\forall s_i \in S_i$, $E_{\theta}\eta_{\theta,s_g} = E_{\theta}\epsilon_{\theta,s_b} = 0$. Moreover, $E_{s_g}[\omega - e + \eta_{\theta,s_g}] = \omega_{\theta} - e$ and $E_{s_b}[\omega - l - e + \epsilon_{\theta,s_b}] = \omega_{\theta} - l - e$. Therefore, once we fix the state s_i , countries receive on average the expected wealth contingent to the climate regime; η_{θ,s_g} and ϵ_{θ,s_b} represent the distance between individual wealth and average one. In addition, countries are not ex-ante identical in terms of expected wealth: $E_{s_g}\eta_{\theta,s_g} = \eta_{\theta} \neq 0$ and $E_{s_b}\epsilon_{\theta,s_b} = \epsilon_{\theta} \neq 0$. Some countries are expected to suffer more from climate change than others; even if climate change does not occur, countries are heterogeneous in terms of consumption possibilities. Under the veil of ignorance, the ex-ante heterogeneity does not matter, as long as risk can be perfectly shared. From the point of view of the hypothetical social planner, type θ is a source of risk. As the distribution of types is uniform, every country has the same probability of being type θ . As a consequence, the fact that some countries are on average more exposed to the negative shocks is swept away in the optimal risk sharing rule¹.

The variables η and ϵ measure the degree of inequality that characterizes each climate regime. The risk of inequality in state b (with climate change) reflects the uncertain heterogeneous impacts of an increase in average temperature. Because of climate change, countries are expected to suffer a loss, though the size of damages is expected to vary regionally. On

¹The results of the comparison would be different if we assume that the type θ can be ex-ante identified.

the contrary, the shock in state g (no climate change) represents the existing inequalities among countries due to resource constraints and historical patterns.

Two aspects are worth highlighting. First of all, if there is no risk sharing, the group of countries faces both a macro shock (climate change vs no climate change) and a climate-regime-specific idiosyncratic shock (distribution of individual wealth given the macro risk). Thanks to the public investment in prevention, the group can affect the realization of the aggregate shock, but not the distribution of wealth given the climate regime. In other words, countries can "pick" the aggregate state i , but not the specific state of nature s_i . The public project guarantees that the economy as a whole will not suffer a loss from climate change, but it cannot assure that country θ will not experience a loss (not necessarily climate related). Second, the idiosyncratic shock is climate-dependent. In terms of inequality, the no climate change state is not necessarily better. Indeed, we might think that climate change will lead to a polarization of wealth, as poor countries are expected to be more hit by the negative consequences of a rising temperature. In that case, state g has a smaller inequality risk. However, the occurrence of a catastrophic event might also reduce the existing inequalities: the effects of climate change might be so strong and widespread that all differences are swept away. If climate change leads to a convergence of economies, state b has a smaller inequality risk.

The comparison between risk sharing and no risk sharing boils down to determining the impact of inequality shocks on the optimal prevention policy. If there is a risk of inequality, the optimal intervention trades-off the incentive to avoid the change in climate regime versus the incentive to limit inequality. The impact of inequality is twofold. Ex-ante, inequality looks like a second source of risk, which calls for more prevention, although the size of the effort is affected by the differences in inequality between climate regimes. On the other side, an inequality shock means that, conditional on the climate regime, some countries will receive a very low wealth. With no transfer, poor countries will not be able to invest in prevention without harming their consumption possibilities. In other words, the burden of the prevention policies will be relatively larger for poor than for rich countries. Because of the inequality risk, the expected marginal cost of investment is larger.

By using the notion of indirect utility function, and thanks to the definition of inequality shocks $\tilde{\eta}_\theta$ and $\tilde{\epsilon}_\theta$, inefficient risk sharing increases prevention effort (condition 3.11) if and only if:

$$\frac{w_g - v_b}{\pi(e)v'_b + (1 - \pi(e))w'_g} \geq \frac{u_g - u_b}{\pi(e)u'_b + (1 - \pi(e))u'_g} \quad (3.13)$$

with $u_g \equiv u(\omega - e)$, $u_b \equiv u(\omega - l - e)$, $w_g = Eu(\omega - e + \eta_{\theta, s_g})$ and $v_b = Eu(\omega - e - l + \epsilon_{\theta, s_b})$. The inefficiency in the risk sharing mechanism increases both the marginal gain (numerator) and the marginal cost (denominator) of investing in preventive activities. As long as the inequality risk in the regime with no climate change is sufficiently small, no risk sharing increases the value of investment, as countries have a twofold incentive to prevent climate change: to avoid the damages and to reduce inequality. However, if countries are prudent, i.e. $u''' > 0$, also marginal costs increase, since $v'_b > u'_b$ and $w'_g > u'_g$. With risk sharing, the relative cost of mitigation in the climate change regime is uniform across countries, since everyone gets $z_{\theta, s_b} = \omega - l$. On the contrary, if the inequality shock cannot be eliminated, the relative cost of mitigation is larger for the countries that obtain a low wealth. A prudent social planner aims at protecting the wealth of all countries. Therefore, he will be reluctant to increase mitigation in the absence of a risk sharing mechanism.

By rewriting condition (3.13), the inefficiency in the risk sharing mechanism raises effort if and only if:

$$\gamma_1 \pi(e) \leq \gamma_2 (1 - \pi(e)) \quad (3.14)$$

with:

$$\gamma_1 \equiv (u_g - u_b)v'_b - u'_b(w_g - v_b)$$

$$\gamma_2 \equiv u'_g(w_g - v_b) - (u_g - u_b)w'_g$$

The variables γ_1 and γ_2 can be interpreted as the difference in marginal benefits of prevention between risk sharing and no risk sharing, net of the marginal cost borne, respectively, in the climate change and in the no climate change regime. γ_1 is positive if, conditional on the loss state, the presence of an exogenous inequality risk increases the marginal costs of prevention more than the relative marginal benefits:

$$\gamma_1 > 0 \iff \frac{v'_b}{u'_b} > \frac{w_g - v_b}{u_g - u_b} \quad (3.15)$$

while γ_2 is positive if, conditional on the no loss state, the presence of an exogenous risk increases the marginal costs of mitigation less than the relative marginal benefits:

$$\gamma_2 > 0 \iff \frac{w'_g}{u'_g} < \frac{w_g - v_b}{u_g - u_b} \quad (3.16)$$

Therefore, the impact of risk sharing depends on the sign of γ_1 and γ_2 . If both of them are positive, prevention increases if and only if the probability of the climate change regime is low enough, $\pi(e) < \frac{\gamma_2}{\gamma_1 + \gamma_2}$. From the point of view of the no loss state, prevention is beneficial, while, from the point of view of the loss state, exerting some effort makes sense if and only if the probability of avoiding it is sufficiently high. If only γ_1 is positive, the optimal level of prevention reduces, and if only γ_2 is positive, the policy becomes stricter. Indeed, in the former, the increase in the marginal cost of prevention overcomes the benefits from a larger investment, whereas in the latter the opposite effect occurs².

Lemma 5. *Inefficient risk sharing increases prevention if and only if:*

- either $\gamma_1 \geq 0$, $\gamma_2 \geq 0$ and $\pi(e) \leq \frac{\gamma_2}{\gamma_1 + \gamma_2}$
- or $\gamma_1 < 0$ and $\gamma_2 \geq 0$.

By construction, the burden of prevention cost is relatively larger in the loss state than in the no climate change regime. Therefore, inefficient risk sharing increases prevention if either the burden on the loss state is sufficiently small (second case) or the probability of suffering from climate change negligible (first case).

3.3.1 Convergent economies

The goal of the following sections is to determine under which conditions Lemma 5 holds. As we will see, both the shape of the utility function and the characteristics of the risk matter. However, there is one case that violates Lemma 5 for sure. Let us imagine that climate change does not create any inequality at all: if average temperature increases, every country will receive a uniform wealth $\omega - l$. On the contrary, the no climate change regime is characterized by an inequality shock. In this case, the presence of inefficient risk sharing (i.e. an inequality risk) will not induce more prevention with respect to the perfect risk sharing case.

Proposition 14. *Suppose that climate change damages are equally distributed within the population, i.e. $\tilde{\epsilon} = 0$. Under prudence, inefficient risk sharing reduces the optimal prevention level.*

²In Lemma 1, we exclude the case in which both γ_1 and γ_2 are negative since unrealistic. In that case, it would be profitable to invest conditional on being on the loss state, and unprofitable relative to the no loss state.

Proof. If the climate change regime is not risky, i.e. $\tilde{\epsilon} = 0$, $v_b = u_b$. Then, inefficient risk sharing increases prevention if and only if $SWTP^I \geq SWTP^E$, i.e.:

$$\frac{w_g - u_b}{\pi(e)u'_b + (1 - \pi(e))w'_g} \geq \frac{u_g - u_b}{\pi(e)u'_b + (1 - \pi(e))u'_g} \quad (3.17)$$

Compared to the risk sharing case, inequality risk shrinks benefits ($w_g = Eu_g < u_g$), and increases marginal costs because of prudence ($w'_g = Eu'_g > u'_g$ if $u''' > 0$). As a consequence, a prudent agent decreases the prevention investment in the presence of inefficient risk sharing. \square

Prudence measures the willingness to accumulate wealth in the face of a future risk. In terms of shape of the utility function, an agent is prudent if the marginal utility is convex, i.e. $u''' > 0$ (Leland, 1968). If wealth is heterogeneously distributed only in the no climate change case, then the climate policy worsens the inequality problem. Indeed, if no climate change occurs, the less wealthy countries pay a higher fraction of the prevention policy, whereas the relative loss is uniform across countries. With respect to the risk sharing case, we are increasing the cost of the investment. If countries are prudent, they want to protect their future wealth. As a consequence, they are less willing to spend resources in prevention so as not to compromise their future consumption possibilities if no loss occurs.

When only the no climate change regime is unequal, rich countries are expected to gain more from a climate policy. Indeed, climate change sweeps all differences away, which means that it will strike mainly rich countries. However, rich countries, by definition, are more able to cope with the adverse impacts of climate change. Therefore, it would be optimal to reduce the burden on poor countries by setting low prevention targets. Said differently, inefficient risk sharing should increase prevention only when climate change is expected to hit more poor countries. Otherwise, it is sub-optimal to require the global community to invest in a policy that will benefit only those that are already in a advantageous position.

Remark 1. *If wealth is unequally distributed within the population and rich countries are expected to suffer more from climate change, a prudent social planner should set a lax prevention policy.*

Even if the negative externality depends on the activity of all the world, if only rich countries are exposed to the risk, they cannot ask for a tough international policy because of their higher capacity of bearing a loss.

3.4 Small risks

Lemma 1 defines conditions under which inefficient risk sharing increases prevention. However, those conditions are difficult to interpret. The goal of the following sections is to determine when Lemma 5 holds. First of all, let us assume that $\tilde{\eta}$ and $\tilde{\epsilon}$ are small zero-mean risks, where σ_η and σ_ϵ denote the variances of the risks.

To simplify, let us assume first of all that in the absence of climate change there is no inequality at all (i.e. $\tilde{\eta} = 0$): an increasing temperature creates inequality, while if there is no change in climate, countries will be able to eliminate all their differences. If we assume that $\tilde{\eta}$ is a degenerate random variable, γ_2 is positive³, while the sign of γ_1 depends on the degree of absolute prudence. As a direct consequence of Lemma 5, by applying a second order Taylor approximation of v_b and v'_b around $E\tilde{\epsilon} = 0$, we get the following result:

Proposition 15. *Suppose that wealth is equally distributed among countries in the no climate change state, while the regime with climate change is characterized by a small inequality risk $\tilde{\epsilon}$. Inefficient risk sharing increases mitigation if and only if:*

$$P_b \leq \frac{u'_b}{u_g - u_b} \quad (3.18)$$

where $P_b = -\frac{u''_b}{u'_b}$ is the degree of absolute prudence computed at $\omega - l - e$. Otherwise, mitigation increases if and only if $\pi(e) < \bar{\pi}$, where $\bar{\pi}$ is defined as:

$$\frac{\bar{\pi}}{1 - \bar{\pi}} = \frac{u'_g(w_g - v_b) - (u_g - u_b)w'_g}{(u_g - u_b)v'_b - u'_b(w_g - v_b)} \quad (3.19)$$

Proposition 15 says that, when climate change creates inequality, inefficient risk sharing increases prevention if and only if the degree of absolute prudence is lower than the marginal cost of investing in terms of utility loss computed in the climate change regime, which indicates how much countries that bear a loss are willing to pay to reduce the inequality risk. To interpret the result, let us recall that the absolute degree of prudence affects the precautionary premium. [Kimball \(1990\)](#) defines the precautionary premium ψ as the sure reduction in future income that has the same effect on savings as does the introduction of a zero-mean risk on future income. Given a consumption level c and a risk $\tilde{\epsilon}$, such that $E\tilde{\epsilon} = 0$,

³Indeed, as $w_g = u_g$, we have:

$$\gamma_2 = u'_g(u_g - v_b) - u'_g(u_g - u_b) = u_b - v_b > 0$$

the precautionary premium is defined as:

$$Eu'(c + \tilde{\epsilon}) = u'(c - \psi) \quad (3.20)$$

Therefore, at the margin, the behavior of an agent facing risk $\tilde{\epsilon}$ is the same as the behavior of an individual with a lower expected wealth, $c - \psi$. In the small, the precautionary premium is approximatively equal to:

$$\psi \simeq \frac{1}{2}\sigma_\epsilon P \quad (3.21)$$

where σ_ϵ is the variance of the risk, and $P = -\frac{u'''}{u''}$ is the degree of absolute prudence. Therefore, a higher degree of prudence leads to a larger precautionary premium.

Thus, we can reinterpret the conditions of Proposition 15 in terms of precautionary premium. Regardless of the probability of loss, effort increases under inefficient risk sharing if:

$$\psi_b \leq \bar{\psi}_b \equiv \frac{u'_b \sigma_{\epsilon_b}}{2(u_g - u_b)} \quad (3.22)$$

where ψ_b is the precautionary premium correlated to the inequality risk $\tilde{\epsilon}$. Therefore, the behavior of an agent facing a risk in the bad state is equivalent to the behavior of an agent facing the aggregate loss $l + \psi_b$. Condition (3.18) tells us that investment increases if the precautionary motive to protect from risk $\tilde{\epsilon}$ is lower than the marginal cost of investing in terms of utility loss, conditional on the fact that a loss has already been suffered.

Introducing a risk has the same effect on investment as a drop in wealth. A large precautionary premium means that it is more costly for the group of countries to reduce the probability of loss. Therefore, if the reduction in wealth is sufficiently low, countries have an incentive to protect themselves from the loss, and so increase effort. On the contrary, if the drop is high, then they may prefer not to spend money on prevention in order to guarantee themselves a sufficiently high wealth in case an accident occurs.

Let us now turn to the more general case. When both states are characterized by a small inequality risk, by using a second order Taylor approximation around $E\tilde{\eta} = 0$ and $E\tilde{\epsilon} = 0$, the sign of γ_1 and γ_2 depend on:

$$\gamma_1 > 0 \iff P_b > \Gamma_1 \equiv \frac{u'_b}{u_g - u_b} \left(1 - \frac{u''_g \sigma_\eta}{u''_b \sigma_\epsilon} \right) \quad (3.23)$$

$$\gamma_2 > 0 \iff P_g < \Gamma_2 \equiv \frac{u'_g}{u_g - u_b} \left(\frac{u''_b \sigma_\epsilon}{u''_g \sigma_\eta} - 1 \right) \quad (3.24)$$

where $P_b \equiv -\frac{u_b'''}{u_b''}$ and $P_g \equiv -\frac{u_g'''}{u_g''}$. By applying Lemma 1, we determine the following result:

Proposition 16. *Suppose that both inequality risks $\tilde{\epsilon}$ and $\tilde{\eta}$ are small. Inefficient risk sharing increases the optimal level of prevention if and only if:*

- either $P_b < \Gamma_1$ and $P_g < \Gamma_2$
- or $P_b > \Gamma_1$, $P_g < \Gamma_2$ and $\pi(e) < \bar{\pi}$ as defined in (3.19).

where P_b and P_g represent the degree of absolute prudence in the bad and good climate regimes, respectively, and Γ_1 and Γ_2 are defined in (3.23) and (3.24).

A necessary condition for increasing prevention is a small degree of prudence in the good climate regime. Otherwise, conditional on the no loss state, the marginal cost of investing outweighs the marginal gain from avoiding the loss. If P_g is large, the social planner wants to protect income also in the case of no aggregate loss. Therefore, he refuses to use consumption to finance a prevention project.

As long as P_g is small enough, effort increases regardless of the probability of loss if and only if the absolute degree of prudence in the bad climate regime is sufficiently low. As in Proposition 15, the higher the precautionary premium that should be paid in the loss state when increasing prevention, the lower the incentive to invest. Moreover, by confronting Proposition 15 and (3.23), we see that the probability threshold has reduced. As a consequence, the presence of multiple state-dependent risks makes the representative country less willing to undertake prevention so as not to compromise the uncertain state-contingent wealth.

Moreover, it appears that the prudence effect is lower when the bad climate regime is characterized by a large inequality and the good climate regime by a moderate heterogeneity (i.e. high σ_ϵ and low σ_η). The presence of risk in the bad climate regime is equivalent to an increase in the risk of loss in the sense of Rothschild and Stiglitz (1970). As a consequence, if climate change is expected to sharpen the existing inequalities, prudent countries should spend more resources in prevention.

The size of the loss ambiguously affects the optimal prevention policy. Indeed, by differentiating conditions (3.23) and (3.24) with respect to the loss l , and rearranging terms, we can prove that:

$$\frac{\partial \Gamma_1}{\partial l} > 0 \iff P_b > A_b \quad (3.25)$$

$$\frac{\partial \Gamma_2}{\partial l} > 0 \iff P_b > \Gamma_1 \quad (3.26)$$

where $A_b = -\frac{u''_b}{u'_b}$ is the degree of absolute risk aversion conditional on the bad climate regime. Therefore, the probability threshold for investing more on prevention, Γ_1 , is increasing in the size of the loss if the absolute risk aversion is decreasing, i.e. $P_b > A_b$. However, if we assume decreasing absolute prudence, the likelihood of a large loss makes the social planner more prudent, therefore more willing to accumulate resources for the future rather than spending them on mitigation activities. Moreover, conditional on the good state, the marginal benefits of inducing prevention increases only if countries are sufficiently prudent, $P_b > \Gamma_1$, i.e. in the case of a catastrophic event.

3.4.1 Example

To stress the effect of the size of the loss on the optimal climate policy, let us consider a specific form of utility function. Take for instance a CRRA utility function; given consumption c :

$$u(c) = \frac{c^{1-\alpha}}{1-\alpha}$$

When the inequality shock appears only in the bad climate regime, Proposition 15 states that a sufficient condition for increasing investment is:

$$P_b \leq \frac{u'_b}{u'_g - u'_b} \iff l \leq (\omega - z) \frac{\mu(\alpha) - 1}{\mu(\alpha)} \quad (3.27)$$

with $\mu(\alpha) \equiv \left(\frac{2}{1+\alpha}\right)^{\frac{1}{1-\alpha}}$. Regardless of the probability of loss, the representative country exerts more effort under inefficient risk sharing if absolute prudence is lower than a given threshold, i.e. if the precautionary premium is sufficiently low. That happens when the loss is small. Indeed, a prudent country wants to accumulate resources to protect its future wealth. Given that prudence is decreasing in consumption, $P = \frac{1+\alpha}{c}$, a low loss makes countries less prudent, and therefore more willing to spend resources in the public project. Note that the threshold is decreasing in the degree of constant relative risk aversion α since $\mu' < 0$. For a given level of loss, the more the social planner is prudent, i.e. the higher is α , the lower is the probability that he will invest.

Similarly, if both states are unequal, we can unambiguously prove that inefficient risk sharing increases prevention only when the loss is sufficiently small:

Lemma 6. *Suppose that $\tilde{\epsilon}$ and $\tilde{\eta}$ are small, and that the bad climate regime leads to more inequality, i.e. $\sigma_\epsilon \geq \sigma_\eta$. If countries have CRRA preferences, inefficient risk sharing increases prevention irrespectively of the probability distribution only if the aggregate loss is sufficiently small.*

A small loss makes the representative country less prudent, and more prone to spend resources in mitigation. As a consequence, inequalities matter when either the expected loss is small or when the expected event is catastrophic, i.e. characterized by a huge loss and a low probability. Otherwise, the willingness to accumulate wealth for the future prevents from investing in the reduction of harmful emissions.

3.5 Large risks

All the previous results are based on the assumption that risks are small. In that case, prudence plays a fundamental role: as we have seen, if the impacts of climate change are expected to be unequally distributed, very prudent countries will increase prevention only if they face a catastrophic event. Otherwise, the risk of inequality calls for less investment and more accumulation of resources in order to allow every country to cope with the expected loss. Unfortunately, in the large there is no clearcut result in terms of prudence. However, as the presence of inefficient risk sharing is equivalent to the introduction of exogenous inequality shocks, we can refer to the literature on background risk to recover the results.

From the literature on background risk (Gollier and Pratt, 1996), we know that the presence of an exogenous uninsurable risk raises risk aversion, i.e. preferences exhibit risk vulnerability, if either absolute risk aversion is decreasing and convex, or absolute risk aversion and absolute prudence are decreasing. Moreover, one necessary condition for risk vulnerability is DARA. Therefore, under risk vulnerability, inefficient risk sharing can be interpreted as an increase in the degree of risk aversion. Analyzing the impact of risk sharing is equivalent to study the effect of an increase in risk aversion. Moreover, as inequality shocks are climate regime dependent, inefficient risk sharing will cause a state-dependent change in risk aversion.

By manipulating conditions (3.15) and (3.16), we can show how the sign of γ_1 and γ_2 depend on the change in the degree of risk aversion when inefficient risk sharing is introduced. In other words, we need to compare the degree of risk aversion of the indirect utility functions v and w with the one of the original function u . For instance, inequality (3.15) can be

rewritten as:

$$\gamma_1 > 0 \iff \frac{v'_b}{u'_b} - \frac{v_g - v_b}{u_g - u_b} > \frac{w_g - v_g}{u_g - u_b} \quad (3.28)$$

where $v_g = v(\omega - e) = Eu(\omega - e + \tilde{\epsilon})$ is the expected utility that the representative country would enjoy in the good regime if the inequality risk was uniform across states, i.e. if $\tilde{\epsilon} =_d \tilde{\eta}$, where " $=_d$ " means equal in distribution. The left-hand-side of (3.28) depends on the change in the degree of risk aversion when the utility function u is replaced by v . Indeed, by defining the function $v(c) = \varphi[u(c)]$, for all consumption levels c , and repeatedly applying the definition of derivative, we have:

$$\begin{aligned} \frac{v'_b}{u'_b} - \frac{v_g - v_b}{u_g - u_b} &= \varphi'_b - \bar{\varphi}' \\ &= \varphi''(u_b - \bar{u}) \\ &= \varphi'(A_v - A_u)(\bar{u} - u_b) \end{aligned} \quad (3.29)$$

where \bar{u} is an average value such that $\bar{\varphi}'(\bar{u}) = \frac{\bar{v}'}{\bar{u}'} = \frac{v_g - v_b}{u_g - u_b}$, φ'' is an average value such that $\varphi'' = \frac{\varphi'_b - \bar{\varphi}'}{u_b - u_c}$, and, by differentiating it: $\varphi'' = \frac{v'}{u'} \left(\frac{v''}{v'} - \frac{u''}{u'} \right)$. Therefore, we can state the following result:

Lemma 7. γ_1 is positive if and only if:

$$A_v - A_u > \beta \frac{w_g - v_g}{u_g - u_b} \quad (3.30)$$

and γ_2 is positive as long as:

$$A_w - A_u > \gamma \frac{v_b - w_b}{u_g - u_b} \quad (3.31)$$

with $\beta \equiv \frac{u'}{v'(\bar{u} - u_b)}$, $\gamma \equiv \frac{u'}{w'(u_g - u_d)}$, and u_d is such that $\frac{w'_d}{u'_d} = \frac{w_g - w_b}{u_g - u_b}$.

The sign of γ_1 and γ_2 depend on the change in risk aversion and on the comparison between the state-dependent inequality risks. Therefore, by Lemma 5, if the increase in risk aversion is sufficiently high, inefficient risk sharing raises the optimal level of mitigation only in the case of a catastrophic risk, i.e. an event with low probability. Instead, if the increase in risk aversion in the bad climate regime is sufficiently low, prevention rises irrespectively of the probability distribution. The increase in risk aversion depends on the size of the risk: the larger the heterogeneity shock, the more the representative country is averse to the risk of suffering a climate damage. The larger the inequality in the bad climate regime, the higher

the probability of having a positive γ_1 ; similarly, if the inequality in the good state is large, it is more likely to have a positive γ_2 . As a consequence, large inequalities should induce an increase in prevention only if the loss is catastrophic. High inequality shocks generally lead to lower efforts.

To interpret Lemma 5, we will first analyze the impact of inefficient risk sharing when both states are characterized by the same degree of inequality. In that case, the climate policy has no effect on the relative distribution of incomes. Then, we will consider the case of state-specific heterogeneity: either the climate policy reduces inequality, or it leads to a polarization of wealth. In the first case, the loss state is more unequal than the no loss one; in the latter, we have the opposite situation.

3.6 Persistent inequality

The presence of inefficient risk sharing can be represented as the introduction of two state-dependent inequality shocks. If we assume that the risks $\tilde{\epsilon}$ and $\tilde{\eta}$ are state-independent, i.e. $\tilde{\epsilon} =_d \tilde{\eta}$, where " $=_d$ " means equal in distribution, then there is no relationship between the severity of the climate change problem and the degree of inequality. Whatever the global loss, the distribution of impacts within the population is constant: rich countries will be relatively richer also in the presence of climate damages.

If we assume that the risks are state independent, then we find the usual result in the literature on self-protection stating that prevention and risk aversion are not monotonically related (Jullien et al., 1999).

Proposition 17. *Suppose $\tilde{\epsilon} =_d \tilde{\eta}$. Under risk vulnerability, inefficient risk sharing raises the optimal mitigation level if and only if $\pi(e) < \bar{\pi}$, where $\bar{\pi}$ is defined as:*

$$\frac{\bar{\pi}}{1 - \bar{\pi}} = \frac{u'_g(w_g - v_b) - (u_g - u_b)w'_g}{(u_g - u_b)v'_b - u'_b(w_g - v_b)} \quad (3.32)$$

Proof. If $\tilde{\epsilon} =_d \tilde{\eta}$, the RHS of (3.30) and (3.31) are equal to 0, since $w = v$. Inefficient risk sharing is equivalent to the introduction of an uninsurable background risk. When absolute risk aversion is decreasing and convex, the introduction of a background risk increases the aversion to other independent risks (Gollier and Pratt, 1996). As v_b and w_g are more concave than u_b and u_g , γ_1 and γ_2 are both positive. By Lemma 5, inefficient risk sharing raises mitigation if and only if $\frac{\pi(e)}{1 - \pi(e)} \leq \frac{\bar{\pi}}{1 - \bar{\pi}}$. \square

When $\tilde{\epsilon} =_d \tilde{\eta}$, the dispersion of individual wealth around the average one is independent of the aggregate wealth. As a consequence, the two aggregate states have the same degree of inequality. In that case, a deterioration of the risk sharing mechanism looks like the introduction of a new source of risk, which can be interpreted as an increase in the aversion to risk of the representative country. However, higher risk aversion does not necessarily imply a larger investment in prevention. Indeed, as explained in [Briys and Schlesinger \(1990\)](#), a higher investment in self-protection would imply a lower wealth if the accident occurs. Therefore, if the marginal utility of wealth in case of loss is high enough, a more risk averse country would find it optimal to reduce effort.

Moreover, the group should increase the investment in prevention if the probability of the high impact state is low enough or if the loss is sufficiently high.

Corollary 2. *Suppose $\tilde{\epsilon} =_d \tilde{\eta}$. If preferences are risk vulnerable, inefficient risk sharing increases mitigation if the aggregate loss l is sufficiently high.*

Proof. By differentiating the threshold probability $\frac{\bar{\pi}}{1-\bar{\pi}}$ with respect to l , we find that it is increasing if and only if the utility functions v and w are more concave than u , i.e. if preferences are risk vulnerable. \square

Applied to the climate change example, the group of countries becomes less tolerant to the risk of suffering from climate change when the impacts are heterogeneous and cannot be efficiently shared. Therefore, agents are more willing to protect their own wealth. If there is no relationship between climate damages and the degree of inequality, it means that poor agents will stay poor irrespectively of the changing climate. If country θ obtains $\omega - e + x$ in the good state, he will get $\omega - l - e + x$ in the bad climate state. As a consequence, since the climate policy cannot solve the inequality problem, poor agents will pay most of the burden of the mitigation investment. In that case, no risk sharing increases effort only if we expect a catastrophic event, i.e. an event characterized by low probability and high loss. Since agents suffer the same absolute loss, but the relative cost of mitigation is larger for the most poor, the increase in prevention would make sense if and only if the probability of avoiding the high impact state is very high.

Remark 2. *If inequality is persistent, i.e. it is independent of the severity of climate change, the group of countries should spend more on mitigation only if the loss is catastrophic. Otherwise, poor countries have to sacrifice too many resources.*

On the contrary, if $\tilde{\epsilon} \neq_d \tilde{\eta}$, each aggregate state is characterized by a different degree of heterogeneity. The presence of climate change may worsen the existing inequality or it may

lead to a concentration of wealth around a mean value. In the former, countries should be more willing to invest, while in the latter they should prefer to save resources.

3.7 Divergent economies

We have seen that inefficient risk sharing is equivalent to a state-dependent change in preferences. Under risk vulnerability, risk aversion increases with respect to the risk sharing case, but the increase is state specific, and depends on the characteristics of $\tilde{\epsilon}$ and $\tilde{\eta}$, respectively. As shown in Section 3.3.1, if only the good state is characterized by an inequality shock, prudent countries will not increase prevention in the presence of inefficient risk sharing as the utility gain from avoiding the loss state has decreased. On the contrary, if climate change does create heterogeneous impacts, i.e. $\tilde{\eta}$ is a degenerate random variable and $w_g = u_g$, the social willingness to pay under inefficient risk sharing $SWTP^I$ is:

$$\frac{u_g - v_b}{\pi(e)v'_b + (1 - \pi(e))u'_g} \quad (3.33)$$

With respect to the risk sharing case, if the agent is prudent, both the marginal cost of investing and the marginal benefit of avoiding the bad outcome have increased (i.e. $v'_b = Eu'_b > u'_b$ and $u_b > Eu_b = v_b$). Therefore, contrary to Proposition 14, the effect is ambiguous.

Note that adding a risk in the bad climate regime looks like an increase in the risk of loss in the sense of Rothschild and Stiglitz. Indeed, we are adding a risk in the low realization of the risk of loss. On the contrary, when only the good state is unequal, we do not have a reduction in risk by pooling incomes.

If $\tilde{\eta}$ is a degenerate random variable, $\gamma_2 > 0$ since $w = u$. Therefore, the effect of risk sharing depends only on the sign of γ_1 . By applying Lemma 7, we get the following result:

Proposition 18. *Suppose that wealth is equally distributed across countries if there is no climate change, i.e. $\tilde{\eta} = 0$. Under risk vulnerability, inefficient risk sharing increases the optimal mitigation level if and only if one of the following conditions holds:*

1. $A_v - A_u \leq Q(l)(u_g - v_g)$
2. $A_v - A_u > Q(l)(u_g - v_g)$ and $\pi(e) < \bar{\pi}$

where A denotes the absolute degree of risk aversion, $\bar{\pi}$ is defined as in (3.19), and $Q(l) \equiv \frac{\beta}{u_g - u_b}$.

Therefore, the impact of inefficient risk sharing on prevention arises from a trade-off between the increase in risk aversion and the size of the pain, i.e. the utility premium $\lambda_g \equiv u_g - v_g$ (Eeckhoudt and Schlesinger, 2009). Assessing the impact of an uninsurable risk on the bad climate state is equivalent to changing the preferences of the representative country in both states and subtracting a function of the benefits stemming from the presence of only one source of risk, $\tilde{\epsilon}$.

In this way, we can draw an analogy with what happens in the case of independent risks (Proposition 17). In that case, the result depended on the direction of the increase in risk aversion. If the representative country has a decreasing and convex degree of absolute risk aversion, it becomes more risk averse under the presence of inequality risk and will invest more if and only if the probability of loss is low enough.

Now, the sign of the increase in risk aversion is not any longer sufficient. On the contrary, we need to look at the size of the change. If the increase is sufficiently high, then $\gamma_1 > 0$, and countries will increase effort if and only if the probability of loss is low. If the increase in risk aversion is lower than an endogenous threshold, $\gamma_1 < 0$, and countries prefer to increase effort regardless the probability of loss. The threshold is proportional to the utility premium, i.e. to the size of the pain associated to the risk $\tilde{\eta}$ if it was affecting also the low impact state.

The function $Q(l)$ is negatively related to the size of the loss⁴. Thus, the larger the loss, the larger the risk aversion effect:

Corollary 3. *Suppose that only the bad climate regime is characterized by heterogeneous impacts. Under risk vulnerability, inefficient risk sharing increases mitigation regardless of the probability of loss only if the size of the aggregate loss is sufficiently small.*

If the loss is small, its incidence on individual wealth is moderate, and the group seeks to reduce the degree of inequality. Given that the inequality risk in the high impact state is like an increase in the risk of loss, the group would like to reduce the probability of the state. However, if the loss is high, the main concern for the group is to keep a sufficiently high wealth in each state, which means that investment is going to decrease if the probability of the loss state is high, i.e. if the probability of averting a loss is low.

⁴Indeed, $Q(l)$ can be rewritten as:

$$Q(l) = u'_d v' l (\omega_d - \omega + l)$$

Moreover, the increase in the degree of risk aversion depends on the degree of inequality. The larger the heterogeneity shock, the more averse the representative agent becomes to the climate risk. Optimal prevention is therefore negatively related to the degree of inequality. With a large inequality, it is more likely that mitigation increases only in the presence of catastrophic events, i.e. when the probability of an aggregate loss is low.

Applied to the climate change example, we are considering a world where there is no income inequality, but countries are expected to be heterogeneously hit by the rising temperature. Therefore, the changing climate creates inequality. In that case, the group should have a two-fold reason for increasing the mitigation level: avoiding the aggregate loss and reducing the inequality. As a consequence, with respect to the case of persistent inequality treated in Proposition 17, we expect a stricter policy. However, we find that the creation of inequality matters only if the loss is relatively small. Otherwise, the usual trade-off between reducing the probability of loss and preserving a sufficiently high wealth in each state tends to prevail.

Remark 3. *If climate change creates inequality and countries are risk vulnerable, the optimal mitigation level increases if:*

- *the event is catastrophic (i.e. low probability and huge loss)*
- *the degree of inequality is small.*

Even though the climate policy helps at reducing inequalities, countries will efficiently increase the mitigation level only in the presence of a catastrophic event. Otherwise, the sacrifice imposed on poor countries would be too large.

3.8 Change in risk across states

In the previous section, we have assumed that only one state was characterized by heterogeneous impacts. Now, let us turn to the general case of two state-dependent inequality risks, $\tilde{\epsilon}$ and $\tilde{\eta}$. For the comparative statics, it is easier to assume that we have an increase in risk moving from one state to another in the sense of [Rothschild and Stiglitz \(1970\)](#). In other words, one risk is a mean preserving spread of the other.

For instance, if the degree of inequality with climate change is larger than the degree of inequality in the good state, we say that the former risk is dominated by the latter in the sense of second order stochastic dominance. Thus, we assume that $\tilde{\epsilon}$ is obtained from $\tilde{\eta}$ by adding

a white noise \tilde{x} to it, with $E[\tilde{x}|\tilde{\eta} = \eta] = 0$ for all η . As a consequence, $v(c) = Ew(c + \tilde{x})$, for all levels of consumption c . If preferences exhibit risk vulnerability, γ_2 is positive, whereas the sign of γ_1 depends on the comparison of risk aversion, as in Proposition 5.

Proposition 19. *Suppose that the state with climate change is characterized by a larger heterogeneity in the distribution of impacts, i.e. $\tilde{\epsilon} =_d \tilde{\eta} + \tilde{x}$, $E\tilde{x} = 0$. Under risk vulnerability, inefficient risk sharing increases the optimal investment in mitigation if and only if one of the following conditions hold:*

1. $A_v - A_u < \lambda_g Q(l)$
2. $A_v - A_u \geq \lambda_g Q(l)$ and $\pi(e) < \bar{\pi}$.

where A denotes the absolute degree in risk aversion, $\lambda_g = w_g - v_g$, $\bar{\pi}$ is defined as in (3.19) and $Q(l)$ is a positive function, decreasing in l .

Similarly to Proposition 18, the increase in investment is regulated by the size of the change in risk aversion. If the presence of risk does not cause a sharp increase in the degree of risk aversion, then effort raises to protect from a higher inequality. If the increase in utility is high enough, investment increases if and only if the low state occurs with low probability. A large loss increases the risk aversion effect. Therefore, inequality matters only if the inequality shock is small or if we expect a catastrophe.

On the contrary, if the good state is more heterogeneous than the climate change one, i.e. $\tilde{\eta}$ is dominated in a SSD sense by $\tilde{\epsilon}$, the no loss state becomes less attractive. Therefore, the group of agents is less prone to invest than in the case with independent risks (Proposition 16). If preferences exhibit risk vulnerability, γ_1 is positive, whereas the sign of γ_2 depends on the size of the increase in risk aversion, as underlined in Lemma 7.

Proposition 20. *Suppose that the good state yields more inequality than the loss state, i.e. $\tilde{\eta} =_d \tilde{\epsilon} + \tilde{x}$, $E\tilde{x} = 0$. Under risk vulnerability, inefficient risk sharing increases the optimal mitigation effort if and only if:*

$$A_w - A_u \geq \lambda_b \hat{Q}(l) \quad \text{and} \quad \pi(e) < \bar{\pi} \quad (3.34)$$

where $\lambda_b = v_b - w_b$, A denotes the degree of absolute risk aversion, and $\hat{Q}(l) = \frac{\gamma}{u_g - u_b}$ is a positive and decreasing function of l .

In this case, climate policy worsens the existing inequality. Compared to Proposition 14, where climate policy created inequality, countries are required to increase their investment

in mitigation only if the loss is catastrophic. The difference between Proposition 14 and Proposition 20 is the degree of inequality if the loss occurs. In the first case, poor countries benefit from the rising temperature because it will sweep away inequalities. In the second case, heterogeneity persists, which increases the risk that the group of countries faces. If the loss is catastrophic, countries agree on spending resources to avoid it as long as the increase in risk aversion is sufficiently high, i.e. the inequality issue severely affects countries' welfare.

3.9 Conclusion

Under risk sharing, countries pool their incomes, and therefore each of them faces a less risky situation. Given that wealth can be optimally re-allocated across the world, those countries that were expecting high impacts are less worried about their future wealth, and, therefore, more reluctant to spend resources in a mitigation technology. However, without risk sharing, every country is forced to bear its own impacts. As a consequence, mitigation becomes individually more costly. A prevention investment reduces the probability of experiencing a high loss, but also the available wealth in case of loss. Without the possibility to get insured, countries need to accumulate resources to guarantee a sufficiently high wealth in case a loss occurs. The mitigation policy will slow down the climate change, but will not necessarily provide a high income to every country. The incentive to protect future wealth makes countries less willing to invest in mitigation, and that incentive is stronger in the absence of a risk sharing mechanism.

The impact of inefficient risk sharing depends also on the assumptions about economic convergence. If poor countries are expected to be more exposed to damages, climate change will lead to a polarization of wealth. In a Pareto optimal solution, the social planner cares both about the size of climate damages and about their distribution. Therefore, the risk of facing higher inequality should a priori induce more prevention. On the contrary, if climate change affects mainly rich countries, we might expect a convergence of wealths. In that case, the social planner has a lower incentive to increase prevention. Changes in climate that sweep away inequalities might be preferred to no change at all. Living in a world where a few are very rich is not necessarily better than a world where everyone is equally poor.

The paper studied the relationship between the collective prevention of an adverse event and the efficiency of the risk sharing mechanism. The main goal was to determine whether inefficient risk sharing calls for a stricter policy. For instance, applied to climate change, the issue is whether we should further cut emissions when the distribution of impacts are

uncertain and countries are reluctant to transfer wealth across different states. Another example concerns the prevention of a financial crisis, and the degree of market regulation given that the distribution of shocks is asymmetric.

We found that the optimal behavior of a social planner can be expressed either in terms of increase in risk aversion or in terms of prudence. In particular, inefficient risk sharing induces more mitigation if countries are either not very risk averse or not very prudent. Prudence means that countries want to protect their wealth; therefore, they are less willing to spend resources in mitigation. On the contrary, the increase in the degree of risk aversion depends on the size of the inequality risk. Large inequality makes countries more averse to the risk of suffering a climate loss. However, they become also less willing to invest in mitigation so as not to reduce their available wealth in case they have to bear a high loss. A sufficient condition for increasing mitigation is the presence of a catastrophic event, i.e. an event characterized by large losses and small probabilities. In that case, inefficient risk sharing is perceived as an additional cost, and implies a greater effort in reducing harmful emissions.

Applied to the climate change example, we can interpret the relation between prevention and risk sharing as a trade-off between mitigation and adaptation in characterizing the optimal risk management strategy (Kane and Shrogen, 2000; Auerswald et al., 2011). Indeed, mitigation is usually depicted as a prevention strategy, since it affects the probability of experiencing climate-related losses. Adaptation is like an insurance strategy because it reduces the vulnerability of the agents, i.e. the size of the loss, but it has no impact on the way the climate changes. Although the definition of adaptation is quite cloudy, risk sharing can be seen as a form of adaptation because it reduces the size of the loss each agent experiences. Therefore, contrary to the existing literature, the paper suggests that mitigation and adaptation can be complements. In order to induce the implementation of strict climate mitigation policies, countries should first of all invest in adaptation and solve the inefficiencies in the risk sharing mechanism. If the risk transfer system is not reliable, countries can efficiently achieve only a low mitigation level.

Bibliography

- Aldy, J., A. Krupnick, R. Newell, I. Parry, and W. Pizer (2010). Designing climate mitigation policy. *Journal of Economic Literature* 48(4), 903–34. 114
- Auerswald, H., K. Konrad, and M. Thum (2011). Adaptation, mitigation and risk taking in climate policy. Cesifo Working Paper No. 3320. 141
- Blanchard, O. (2009). The crisis: Basic mechanisms and appropriate policies. 116
- Briys, E. and H. Schlesinger (1990). Risk aversion and the propensities for self-insurance and self-protection. *Southern Economic Journal* 57(2), 458–67. 117, 135
- Constantinides, G. (1982). Intertemporal asset pricing with heterogeneous consumers and without demand aggregation. *Journal of Business* 55, 253–67. 114
- Courbage, C. and B. Rey (2008). On the willingness to pay to reduce risks of small losses. *Journal of Economics* 95(1), 75–82. 117
- Courbage, C. and B. Rey (2012). Optimal prevention and other risks in a two-period model. *Mathematical Social Science* 63, 213–17. 118
- Dionne, G. and L. Eeckhoudt (1984). Self-insurance, self-protection and increased risk aversion. *Economics Letters* 17, 39–42. 116
- Eeckhoudt, L. and C. Gollier (2005). The impact of prudence of optimal prevention. *Economic Theory*, 989–94. 117
- Eeckhoudt, L. and J. Hammit (2001). Background risks and the value of a statistical life. *Journal of Risk and Uncertainty* 23, 261–79. 117
- Eeckhoudt, L. and H. Schlesinger (2009). On the utility premium of friedman and savage. *Economics Letters* 105, 46–48. 137

- Ehrlich, I. and G. Becker (1972). Market insurance, self-insurance and self-protection. *Journal of Political Economy*, 623–48. [116](#)
- Emmerling, J. (2011). Discounting and intragenerational equity. [118](#)
- Fleurbaey, M. and S. Zuber (2014). Discounting, risk and inequality: a general approach. CES Working Paper No 2014.15. [118](#)
- Gollier, C. (2001). *The Economics of Risk and Time*. MIT Press. [121](#)
- Gollier, C. (2010). Discounting, inequality and economic convergence. Cesifo Working Paper No 3262. [115](#), [118](#)
- Gollier, C. (2012). *Pricing the Planet's Future: the Economics of Discounting in an Uncertain World*. Princeton University Press. [115](#), [118](#)
- Gollier, C. and J. Pratt (1996). Risk vulnerability and the tempering effect of background risk. *Econometrica* 64(5), 1109–23. [117](#), [132](#)
- IPCC (2007). Intergovernmental panel on climate change, fourth assessment report. [114](#)
- Jullien, B., B. Salanié, and F. Salanié (1999). Should more risk averse agents exert more effort? *The Geneva Papers on Risk and Insurance Theory* 24(1), 19–28. [116](#), [134](#)
- Kane, S. and J. Shrogen (2000). Linking adaptation and mitigation in climate change policy. *Climatic Change* 45(1), 74–102. [141](#)
- Kimball, M. (1990). Precautionary savings in the small and in the large. *Econometrica* 58, 53–73. [128](#)
- Leland, H. (1968). Saving and uncertainty: the precautionary demand for saving. *Quarterly Journal of Economics* 82, 465–73. [127](#)
- Pindyck, R. (2013). The climate policy dilemma. *Review of Environmental Economics and Policy* 7(2), 219–37. [118](#)
- Rothschild, M. and J. Stiglitz (1970). Increasing risk: I. a definition. *Journal of Economic Theory* 3, 225–43. [130](#), [138](#)
- Tol, R. (2009). The economic effects of climate change. *Journal of Economic Perspectives* 23(2), 29–51. [114](#)

Appendix

3.A Example: a CRRA utility function

With a CRRA utility function, the conditions on the sign of γ_1 and γ_2 , (3.23) and (3.24) respectively, become:

$$\gamma_1 < 0 \iff (1 + \alpha)y^{\alpha-1} - 2 + \alpha + y^{1+\alpha}\frac{\sigma_\eta}{\sigma_\epsilon} < 0 \quad (3.35)$$

with $y \equiv \frac{\omega-l-z}{\omega-z}$, and

$$\gamma_2 > 0 \iff 2(1 + \alpha) - (1 + \alpha)y^{1-\alpha} - \frac{(1 - \alpha)\sigma_\epsilon}{\sigma_\eta} < 0 \quad (3.36)$$

By differentiating the two inequalities with respect to l , we find that the first expression is decreasing in l as long as:

$$l \leq \bar{l} \equiv (\omega - z) \left(1 - \sqrt{\frac{\sigma_\epsilon}{\sigma_\eta}} \right) \quad (3.37)$$

while the second expression is decreasing in l if $l > \bar{l}$.

If $\sigma_\epsilon \geq \sigma_\eta$, the threshold \bar{l} is negative. As a consequence, in that case a large loss l decreases the expression for γ_2 and increases the expression for γ_1 . When the loss is sufficiently large, it is more likely to have $\gamma_1 > 0$ and $\gamma_2 > 0$. On the contrary, if the loss is small, we should have $\gamma_1 < 0$ and $\gamma_2 > 0$. Countries increase mitigation irrespectively of the probability of loss only if the loss is sufficiently small. Otherwise, they engage in more mitigation only in the presence of a catastrophe.