Selection of boundedly rational firms and the allocation of resources

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October 9, 2006

1 Introduction

The assumption on profit maximization underlies most of economic methodology. When it comes to justifying it as a statement about how firms actually behave, the 'as if' argument is frequently invoked. That argument states that firms which do not maximize profits will be eliminated by competition, because competition drives profits to zero. Hence, non maximizers make losses and are therefore eliminated.

The argument, however, ignores another aspect of competition: that agents who bid more get a large chunk of resources. If there are all sorts of irrational agents participating in the competition process, the highest bidder is unlikely to be a profit maximizer. Rather, it will be an agent which has overestimated its benefits and underestimated its costs.

Consider the following examples:

- Firms compete to sell the same good, they have the same costs, and there are constant returns to scale. A (possibly tiny) fraction of firms "gets it wrong" and charges below cost. The firm which gets it wrong most gets the whole market–and experiences large losses. Thus, in this simple

*I thank Omar Licandro, Ana Ania, and participants at the European Economic Association congress, Vienna, August 2006 for helpful comments and suggestions.

setting, from an allocative perspective, profit maximizers do not matter: the allocation of resources is determined by the firm that has made the largest downward mistake in setting its price.

- Entrepreneurs have different projects and compete for funds. They observe their return with an error. Those who most overestimate their returns are willing to borrow at a higher interest rate, thus overbidding rational entrepreneurs. If making a large mistake is correlated with having a low true return, funds are channel to the wrong projects–and many borrowers will go bankrupt.

- Mobile phone operators are competing in auction for frequencies. Those who most overestimate the value get awarded the frequencies. Again, that results in a misallocation of resources if these operators are at the same time not the most efficient. Recent evidence by Lee and Malmendier (2006) documents such phenomenon on e-bay auctions.

Of course, mistaken agents will eventually be eliminated, because they will run out of funds. Nevertheless, that selection process might have no impact on the allocation of resources. In the first example given above, we only need that there exists one producer selling below cost for rational ones to lose the whole market. Suppose that firms are eliminated if, say, their cumulated losses reach some threshold. Then those producers eventually disappear. But as long as there are *some entrants* that charge below cost, the argument still carries through.

Thus, selection in economics operates very differently from selection in nature. In nature, the less efficient organisms get access to fewer resources, have lower fitness, and eventually disappear from the population. In economics, the less efficient organisms can *temporarily* send market signals that will grant them *more* resources than others. While that does not prevent them from being eventually eliminated, it nevertheless implies that resources may be allocated in a way quite different from what our familiar notion of equilibrium predicts.[1]

An important question, therefore, is whether and how markets can put in place mechanisms that limit the lame ducks' ability to draw resources; and, if such mechanisms emerge, how they affect equilibrium. These mechanisms are bound to be financial institutions that restrict the access to funds of inefficient firms and increase their availability for efficient ones. What sort of institutions should emerge, and how well are they functioning? Does the economy converge to a Walrasian equilibrium, or do inefficient organizations persistently access a large share of the resources?

This paper is a step toward analyzing these issues. I examine the interplay between the allocation of funds by rational savers to imperfectly rational firms, and the bidding process described above. I first set up a model of competition between boundedly rational firms. I assume that firms differ by their cost and set prices in a rigid, naive, fashion. Customers try to get the lowest price. Firms need cash to pay their inputs in advance, and savers-shareholders allocate cash between them so as to maximize their rate of return. Thus, while customers are attracted by firm that tend to charge too low prices, shareholders are attracted by firms that tend to charge too high prices.

I first describe the equilibrium when the rate of return on each firm is observed. In such a case, obviously, firms that charge below cost do not get any funds. Nevertheless, the allocation of funds is not necessarily efficient and the fact that firms follow naive rules has important consequences. It is shown that there are multiple equilibria, and that, relative to the Walrasian equilibrium, some degree of monopoly power is sustained. That is, in some equilibria, firms that charge "too high" prices and could be undercut by profitable firms get all the funds. The reason for multiplicity is a pecuniary externality, by which

[1] A corollary is that limiting the extent of competition may in fact increase the efficiency of the market. In a world with such mistakes, it is easy to think of government interventions that by limiting competition, increase the social surplus. For example, in Saint-Paul (2002), I show how restrictions on transaction prices may increase welfare in a model where people misperceive their opportunity cost of participating in a match.

reallocating funds in favor of a firm affects the customer base of other firms.

In this setting, selection of firms by shareholders and customers therefore tends to allocate resources to firms that "overprice" rather than those that "underprice".

Things can be different, however, if there are new entrants whose price is unobserved. The shareholders' problem is then: which fraction of my savings shall I allocate to new entrants, whose rate of return is unobservable. If entrants get a positive fraction of savings, then entrants that underprice will get a disproportionate fraction of customers, although the expected rate of return of giving funds to entrants must of course remain positive. One can show again that there may be multiple equilibria, for example an equilibrium where all funds go to entrants can coexist with an equilibrium where no funds go to entrants. The former equilibrium will typically involve more bankruptcy, and also more growth if one is in a dynamic setting where entrants are on average more productive than incumbents.

The literature on the evolutionary behavior of firms competing within a single market, is firmly in the "equilibrium selection" tradition. See, for example, Alos-Ferrer et al. (2000), who, following Vega-Redondo (1997), study a model where the evolutionary stable outcome is a subset of the set of Nash equilibria. Here the approach is different in the sense that there is no underlying game, the equilibrium strategies of which agents are supposed to learn. Rather, firms are genuinely making mistakes and we want to know how strong a force 'economic selection' is in correcting these mistakes and leading to the efficient outcome. The novelty of this paper is that selection is made by agents – customers and shareholders – whose fitness depend on different things – profits and utility – and whose behavior tends to select different firms. Nevertheless, this paper is not totally at variance with that literature since some convergence results to the Walrasian equilibrium hold when cash becomes infinitely abundant.

2 A static model of mispricing

In this section, I introduce a simple model that I will use extensively to analyze the issues at hand. The model is extreme in that the only equilibrating mechanism in the economy is the way savers allocate funds between firms. That is, each firm is characterized by a particular behavior and this behavior does not change; all what can happen is that shareholders select their most preferred firms by withdrawing funds from the least successful firms and reallocating them to the most successful ones. In reality – and in a more general setting – we expect both selection and firms' decision to intervene. However, it will still be true that if a firm is intrinsically less good than another at organizing, pricing, buying, delivering, and so on, it should eventually attract less funds.

In the model, firms differ in their cost and only have one decision variable: their price. Unlike the standard model, the price of any given firm is assumed exogenous (while the focus is quite different, that relates the present analysis to the fixed-price equilibrium theory (Bénassy, 1982)). That is, their pricing behavior is an intrinsic characteristic, just like their cost. If a firm, say, charges a price far too high, it cannot increase its profits by lowering it. However, as we shall see, its shareholders can starve it of funds and reallocate funds to a firm with a more profitable pricing behavior. Similarly, a firm which prices below cost makes negative profits, and while this particular firm cannot ever be profitable, shareholders can "close" it by refusing to allocate funds to it.

In order to capture this fund allocation mechanism, I assume that firms need cash in order to buy their inputs in advance. Cash is provided before trade takes place by "savers": in the first period of the model, savers provide cash to firms. After trade has taken place, each firm rebates cash to the savers who have financed it, proportionally to each contribution–thus we can think of these savers as the firm's "shareholders". Each saver's utility is increasing in the amount of cash he ends up with at the end of the game.

More precisely: there are N firms, all producing a single homogenous good. In addition to that good, there are two other goods, "cash" and an input good. Cash is the numéraire, and the price of the input good is normalized to one, which is just a choice of measurement units.

Firm i has a cost c_i and a price p_i. Without loss of generality, I assume that firms are ranked by increasing prices: $p_j \geq p_i, j > i$. Thus the production function for firm i is

$$y_i = x_i / c_i,$$

where y_i is the firm's output and x_i is the amount of input it purchases.

It is important to note that firms are not decision units but automata which mechanically produce and charge a price p_i.

In order to purchase inputs, firms need a good called "cash". It is provided by agents called "savers", who voluntarily provide cash to the firm they want (thus there is no "market" for cash and savers are thought of as shareholders who provide capital to firms). If k_i is the cash of firm i, then it faces the following "cash-in-advance" constraint:

$$x_i \leq k_i.$$

Consumers buy the output good from firms, and pay them with cash. Thus, before trade takes place, firms are endowed with cash by savers and use it to purchase inputs, and after trade has taken place, they hold whatever cash they had left after buying their inputs plus the cash they obtained after selling their product.

While firms are not rational, consumers are, and try to buy from the cheapest producer. The demand curve for the good is $D(p)$. Because of the cash-in-advance constraint, a producer may be unable to serve demand. Those consumers who are not serve then try to buy from the next producer in the price ranking, and so on. Therefore, the allocation mechanism is recursively defined as follows:

- The demand addressed to producer i is $\gamma_i D(p_i)$, with $\gamma_1 = 1$.
- The output of producer i is $y_i = \min(\gamma_i D(p_i), k_i/c_i)$.
- If $\gamma_i D(p_i) \leq k_i/c_i$, then the demand addressed to all producers such that $j > i$ is zero (since $p_j \geq p_i$, there is no additional demand for these producers). That is, $\gamma_j = 0$. In such a case, i is the highest-priced firm with a positive output; we denote the index of this firm by $i_{\max}$.
- If $\gamma_i D(p_i) > k_i/c_i$, then producer i's customers are rationed. Rationing is proportional, so that the demand addressed to producer $i+1$ is $\gamma_{i+1} D(p_{i+1})$, where

$$\begin{aligned}\gamma_{i+1} &= \gamma_i \left(1 - \frac{k_i/c_i}{\gamma_i D(p_i)}\right) \\ &= \gamma_i - \frac{k_i/c_i}{D(p_i)}.\end{aligned}$$

Iterating, it is easy to get the scale factor for demand addressed to any producer $i \geq 2$:

$$\begin{aligned}\gamma_i &= \max\left(1 - \sum_{j=1}^{i-1} \frac{k_j/c_j}{D(p_j)}, 0\right) \\ &= 1 - \sum_{j=1}^{i-1} \frac{y_j}{D(p_j)}, \qquad (1)\end{aligned}$$

where the last inequality comes from the fact that if $\gamma_i > 0$, then $y_j = k_j/c_j$ for all $j < i$; and for the "marginal" producer such that $\gamma_i D(p_i) \leq k_i/c_i$, $y_i = \gamma_i D(p_i)$, so that $0 = \gamma_i - y_i/D(p_i) = 1 - \sum_{j=1}^{i} \frac{y_j}{D(p_j)} = \gamma_{i+1}$, and so on for all producers of higher prices.

The total amount of cash in the economy is K. It is uniformly allocated among a continuum of savers of total mass 1. I shall assume that there is enough cash so that all consumers are served. A sufficient condition for that is[2]

[2] To see this, assume there exists an allocation such that even consumers at the highest price firm are constrained. For this to be the case, it must be that $\gamma_N D(p_N) > k_N/c_N$, or equivalently

$$K \geq \max_i c_i D(p_i) \tag{2}$$

We shall denote by S the set of allocations of output $\{y_i\}$ that derive from an allocation of cash $\{k_i\}$ such that $\sum k_i = K$. Given the previous inequality, this set does not depend on the actual value of K. The preceding arguments can be summarized by Proposition 1, which characterizes how the allocation of output derives from the allocation of cash:

PROPOSITION 1 – Let $i_{\max} = \min\{i, \sum_{j=1}^{i-1} \frac{k_j}{c_j D(p_j)} \geq 1\}$, *which by (2) exists. For any given allocation of cash* $\{k_1, ..., k_N\}$, *the allocation of output is* $y_i = k_i/c_i$ *for all* $i < i_{\max}$,

$$y_{i_{\max}} = \left(1 - \sum_{j=1}^{i_{\max}-1} \frac{k_j}{c_j D(p_j)}\right) D(p_{i_{\max}}),$$

and $y_i = 0$, *for all* $i > i_{\max}$.

I know describe how cash is allocated between firms and then rebated to shareholders at the end of the game.

Assume shareholders are indexed by $\lambda \in [0, 1]$. Then the allocation of cash can be represented by a function $k_i(\lambda)$, which is the density of cash given to firm i by savers λ. One has

$$\int_0^1 k_i(\lambda) d\lambda = k_i, \forall i$$

Furthermore, assuming that all cash must be given to firms, it must also be that–since K is uniformly allocated–

$$\sum_{i=1}^{N} k_i(\lambda) = K, \forall \lambda.$$

$$1 - \sum_{j=1}^{N} \frac{k_j}{c_j D(p_j)} > 0.$$

That cannot be, since, as $\sum k_j = K$, the LHS is smaller than $1 - K/\max_j c_j D(p_j) < 0$.

After trade has taken place, the amount of cash held by firm i, k'_i, is equal to its revenue plus whatever cash it had left after buying its inputs:

$$k'_i = k_i + p_i y_i - c_i y_i. \quad (3)$$

This cash is rebated to shareholders in a proportional way, so that a saver with index λ gets an amount of cash equal to

$$k'_i(\lambda) = \frac{k'_i}{k_i} k_i(\lambda).$$

Note that this is easily interpreted as the firm paying dividends to its shareholders, proportionally to the number of shares they hold.

The utility of savers $V(\lambda)$ is just equal to the total amount of cash they get at the end of the game:

$$V(\lambda) = \sum_{i=1}^{N} k'_i(\lambda).$$

The questions we want to answer in that model are: what is the equilibrium allocation of cash? What is the equilibrium allocation of production between firms? In order to interpret the answers, it is useful to have a benchmark; a natural one is the social optimum, which we discuss next.

3 Optimum

Before describing the equilibrium, we first discuss the allocation of cash which maximizes social welfare.

In the model, there are two kinds of agents: consumers who consume the output good, and savers who consume cash. In principle, one could consider different criteria that differ depending on the relative weight of savers. However, given that cash is the numéraire, the most natural criterion in this partial equilibrium model is to have a weight of 1 on the saver's cash holding and add them to consumer surplus, which is equivalent to the traditional welfare analysis in partial equilibrium models, where producer's surplus is added to consumer's surplus.

The previous section has shown that in equilibrium, firms such that $i < i_{\max}$ produce $y_i = k_i/c_i$, while firm $i_{\max}$ serves all its residual demand: $y_{i_{\max}} = \gamma_{i_{\max}} D(p_{i_{\max}})$. Given proportional rationing, consumer surplus at firm $i \leq i_{\max}$ is given by

$$CS_i = \frac{y_i}{D(p_i)} \int_{p_i}^{+\infty} D(q)dq.$$

Furthermore, the contribution to saver's welfare of that firm is simply k_i' as given by (3). Total social welfare is therefore equal to

$$\begin{aligned} SW &= \sum_{i=1}^{i_{\max}} CS_i + k_i' \\ &= K + \sum_{i=1}^{i_{\max}} y_i \omega_i, \end{aligned}$$

where

$$\omega_i = \frac{1}{D(p_i)} \int_{p_i}^{+\infty} D(q)dq + p_i - c_i.$$

Note that ω_i only depends on firm i's characteristics and does not depend on the allocation of cash between firms.

The preceding steps show that maximizing social welfare is equivalent to maximizing $\sum_{i=1}^{i_{\max}} y_i \omega_i$; furthermore, it is easy to see that picking up an allocation of cash is equivalent to picking up a sequence of y_i which satisfies the proper constraints. That is spelled out in Lemma 1.

LEMMA 1 – The two following properties are equivalent

(A) $\{y_1, ..., y_N\} \in S$

(B) $0 \leq y_i \leq D(p_i) \left[1 - \sum_{j=1}^{i-1} \frac{y_j}{D(p_j)}\right]$, *for all* i, *with* $y_N = D(p_N) \left[1 - \sum_{j=1}^{N-1} \frac{y_j}{D(p_j)}\right]$.

PROOF – That (A) implies (B) comes from the fact that (1) holds and that $y_i \leq \gamma_i D(p_i)$. Conversely, consider an allocation of output which satisfies (B). Let $i_{\max} = \min\{i, y_i = D(p_i) \left[1 - \sum_{j=1}^{N-1} \frac{y_i}{D(p_i)}\right]\}$. Clearly, $y_i = 0$ for $i > i_{\max}$.

Set $k_i = c_i y_i$ for $i < i_{\max}$ and $k_{i_{\max}} = K - \sum_{j=1}^{i_{\max}-1} k_i$. Set $k_i = 0$ for $i > i_{\max}$. Then we can check that this allocation of k_i yields the desired allocation of y_i. To see that, just note that it is associated with the sequence of γ_i's defined by (1), which by construction satisfies $k_i/c_i < \gamma_i D(p_i)$ for $i < i_{\max}$ and $k_{i_{\max}}/c_{i_{\max}} \geq \gamma_{i_{\max}} D(p_{i_{\max}})$. To check the latter, note that it is equivalent to $K \geq \sum_{j=1}^{i_{\max}} c_i y_i = \sum_{j=1}^{i_{\max}} (c_i D(p_i)) \frac{y_i}{D(p_i)}$. As $\sum_{j=1}^{i_{\max}} \frac{y_i}{D(p_i)} = 1$,this expression is always lower than $\max(c_i D(p_i))$, which according to (2) is lower than K. QED.

From Lemma 1 we conclude that the social planner problem is equivalent to maximizing $\sum_{i=1}^{N} y_i \omega_i$, subject to the set of constraints $0 \leq y_i \leq D(p_i) \left[1 - \sum_{j=1}^{i-1} \frac{y_j}{D(p_j)}\right]$. But as these constraints may be rewritten as $\sum_{j=1}^{i} \frac{y_j}{D(p_j)} \leq 1$, for nonnegative values of y_i they can be reduced to a single constraint:

$$\sum_{j=1}^{N} \frac{y_j}{D(p_j)} = 1. \tag{4}$$

From here we can straightforwardly show

LEMMA 2 – Let $s_i = D(p_i)\omega_i = \int_{p_i}^{+\infty} D(q)dq + (p_i - c_i) D(p_i)$ be the total surplus generated by firm i if it had the whole market. The allocation of production that maximizes social welfare is defined by $y_j = h_j D(p_j)$, where h_j is the set of weights which maximize $\sum_{i=1}^{N} h_i s_i$, subject to $h_i \geq 0$ and $\sum_{i=1}^{N} h_i = 1$.

Thus, the social optimum maximizes a weighted average of the social surplus generated by each firm if it had the whole market. That is a convenient property of the proportional rationing scheme: the demand curve faced by any firm is proportional to the demand curve for the whole market, so that its contribution to overall surplus is proportional to s_i.

As a result, the optimal allocation of cash consists in giving all of it to the firm that generates the highest surplus:

PROPOSITION 2 – An allocation of production $\{y_i\}$, $y_i \geq 0$, maximizes *social welfare if and only if:*

$$\{y_1, ..., y_N\} \in S, \text{ i.e. (4) holds,}$$

and

$$y_i > 0 \Longrightarrow i \in \arg\max_i s_i.$$

Note that if the social planner could set prices, the first-best allocation would be simply reached by picking up the lowest cost firm, giving it all the cash and setting its price equal to its cost. Here that strategy may not be optimal, as this firm may misprice its output in such a way that some other firm generates a higher surplus.

An interesting special case is the case where all firms have the same cost c, i.e. all firms have the same technology and the only mistakes they make is when setting their price. As we know that total surplus $\int_p^{+\infty} D(q)dq + (p-c)\,D(p)$ is hump-shaped and maximized at $p = c$, the optimal allocation implies giving all the cash to one of the two firms whose price is closest to c (the one from above and the one from below).

4 Equilibrium

We now characterize the equilibrium of this economy. To do so, we must first define what equilibrium is. Savers want to maximize their ex-post cash-holdings. Thus, we define equilibrium as a situation where an individual saver cannot increase his return by changing his allocation of cash.

DEFINITION – An allocation of cash $\{k_i(\lambda)\}$ *is in equilibrium iff*

$\nexists\ \lambda_0, (\forall\ d\lambda > 0, \exists\ \{\tilde{k}_i(\lambda)\}, \sum \tilde{k}_i(\lambda) = K$ *and* $(\forall i, \forall \lambda \notin [\lambda_0, \lambda_0 + d\lambda], \tilde{k}_i(\lambda) = k_i(\lambda))$, *and* $\sum_i \tilde{k}'_i(\lambda) > \sum_i \tilde{k}_i(\lambda))$.

This formula tells us that an infinitesimal mass of savers around any λ_0 cannot make more money by changing their allocation of savings.

Intuitively, given that dividends are proportional to the amount of cash given to the firm, savers want to allocate cash to the firms with the highest rate of return. Furthermore, given that savers are infinitesimal, the distribution of rates of return is not affected, at the first order, by any reallocation of individual savings.

PROPOSITION 3 – Consider an allocation of capital $\{k_i(\lambda)\}$,*and the associated allocation of output* $\{y_i\}$ *and end-of-period cash* $\{k'_i\}$. *Let* r_i, *the "rate of return" of firm* i, *be defined as*

$r_i = p_i/c_i,\ i < i_{\max}$

$r_{i_{\max}} = 1 + \gamma_{i_{\max}} D(p_{i_{\max}})(p_{i_{\max}} - c_{i_{\max}})/k_{i_{\max}}$

$r_i = 1,\ i > i_{\max}$.

Then the allocation is in equilibrium if and only if

$$k_i > 0 \Longrightarrow i \in \arg\max_i \{r_i\}.$$

PROOF — Consider a small reallocation of cash $\{dk_i\}$.[3] To be feasible it must satisfy $\sum_{i=1}^{N} dk_i = 0$, as well as $dk_i \geq 0$ if $k_i = 0$. Because it is infinitesimal, $i_{\max}$ is unchanged. As dividends are proportional to $k_i(\lambda)$, the net earnings generated by this reallocation are equal to

$$dk' = \sum_{i=1}^{N} \frac{k'_i + dk'_i}{k_i + dk_i} dk_i,$$

where dk'_i is the change in k'_i induced by the change in $\{k_i\}$. Clearly, we have $dk'_i = \frac{p_i}{c_i} dk_i$ for $i < i_{\max}$, $dk'_i = dk_i$ for $i > i_{\max}$, and $dk'_i = dk_i$ for $i = i_{\max}$ (as cash is unused at the margin for that firm). Let us denote by $r_i = \frac{k'_i + dk'_i}{k_i + dk_i}$ the rate of return on equity for firm i. In equilibrium, one must have $dk' \leq 0$ for all feasible reallocations dk_i. That is equivalent to $dk_i < 0$ being infeasible for any firm such that $r_i < r_j$, i.e. to $k_i = 0$. Thus an equilibrium allocation is such that all cash is allocated to firms with the highest rate of return.

[3] More formally, we can assume that an infinitesimal mass $d\lambda$ of savers around some value of λ changes its allocation from $k_i(\lambda)$ to $\tilde{k}_i(\lambda)$, and define $dk_i = d\lambda.(\tilde{k}_i(\lambda) - k_i(\lambda))$.

Next, we can compute r_i. We get $r_i = p_i/c_i$ for all $i < i_{\max}$ (regardless on whether $k_i = 0$ or not), and $r_i = 1$ for all $i > i_{\max}$. The rate of return on firm $i_{\max}$ is finally given by $r_{i_{\max}} = k'_{i_{\max}}/k_{i_{\max}} = 1 + \gamma_{i_{\max}} D(p_{i_{\max}})(p_{i_{\max}} - c_{i_{\max}})/k_{i_{\max}}$. QED

Comparing Proposition 3 to Proposition 2, we see that the market maximizes rates of return, while the social planner maximizes surplus. Furthermore, a firm's rate of return depends on the whole allocation, while a firm's surplus is independent of the allocation.

Proposition 3 only gives us a way of computing the equilibrium, it does not characterize it. We now provide results that characterize the equilibrium. In order to do so, we shall make the following assumption, which holds almost surely:

ASSUMPTION 1 – $i \neq j \Longrightarrow p_i/c_i \neq p_j/c_j$.

This assumption implies that two constrained firms cannot have the same rate of return. Consequently, there cannot be two constrained firms in equilibrium. As all firms but one are constrained, there cannot be more than two firms with a positive output in equilibrium.

Another assumption that we shall make, is that there exists at least one profitable firm:

ASSUMPTION 2 – $\exists i$, $p_i > c_i$.

That assumption guarantees that no non profitable firm gets any cash, since the profitable firms yield a return $r_i \geq 1$ – in the worst possible case, they attract no customer, yielding $r_i = 1$.

In order to rule out some borderline cases, we shall also assume

ASSUMPTION 3 – $\forall i, p_i \neq c_i$.

That assumption also holds almost surely. Note that it does not preclude a firm's price from being arbitrarily close to its cost, but it just prevents it from being right on target.

The following Lemma formalizes the restriction on equilibrium implied by these assumptions:

LEMMA 3 – Assume assumptions 1 and 2 hold; then

(i) in any equilibrium $y_i = 0$ *for at least* $N - 2$ *firms*

(ii) in any equilibrium $k_i = 0$ *if* $y_i = 0$ *or* $p_i < c_i$.

PROOF – Assume there are 3 firms i, j, k, such that $i < j < k$ and $y_i, y_j, y_k > 0$. Then $i < i_{\max}$ and $j < j_{\max}$, implying $r_i = p_i/c_i$ and $r_j = p_j/c_j$. Because of assumption 1, $r_i \neq r_j$. As both firms produce, $k_i, k_j > 0$. Thus, $i, j \in \arg\max_i\{r_i\}$ (Proposition 3), implying $r_i = r_j$, which is a contradiction. This proves (i).

Assume there is a firm such that $p_i < c_i$ and $k_i > 0$. Then $r_i < 1$. Let i' such that $p_{i'} > c_{i'}$. Then $r_i' \geq 1 > r_i$, which violates Proposition 3.

Assume there is a firm such that $k_i > 0$ and $y_i = 0$. Then $r_i = 1$ and there exists $i' < i$ such that $y_i > 0$. Otherwise, firm i would be the cheapest available firm and would attract some consumers, whom it could serve. It must then be that $k_{i'} > 0$, so that $r_{i'} \geq 1$,which in turn implies $p_{i'} \geq c_{i'}$. As $p_{i'} = c_{i'}$ is ruled out, one must actually have $p_{i'} > c_{i'}$. Then, it must be that $r_{i'} > 1 = r_i$, which again violates proposition 3.

Q.E.D.

Putting (i) and (ii) together, we see that at most two firms get cash in any equilibrium. To characterize it, one just has to try and construct equilibria where one firm gets all the cash, and equilibria where output is split between firm i which is constrained and firm $i_{\max}$ which is not. It turns out that the latter are unstable in an economically meaningful sense; that is, if cash is marginally reallocated in favor of one of the two firms, its rate of return moves above that of the other firm, thus reinforcing the initial move. Formally, we can live with the following simple notion of stability:

DEFINITION – An equilibrium is pairwise stable if and only if

(i) $\forall i, j$ *s.t.* $i \neq j, k_i k_j > 0$,

$$\frac{\partial r_j}{\partial k_i} + \frac{\partial r_i}{\partial k_j} > \frac{\partial r_j}{\partial k_j} + \frac{\partial r_i}{\partial k_i}.$$

(ii) $\forall i, j$ *s.t.* $k_i > k_j = 0$, $r_i > r_j$

Condition (i) states that if one reallocates one unit of cash from firm i to j, the change in firm j's rate of return, $\frac{\partial r_j}{\partial k_j} - \frac{\partial r_j}{\partial k_i}$, is lower than that of firm i's rate of return (which is initially equal to r_j), $-(\frac{\partial r_i}{\partial k_i} - \frac{\partial r_i}{\partial k_j})$. Thus, cash would naturally tend to go back to firm i. Condition (ii) states that firms that do get cash get a strictly higher rate of return than firm's that do not. Therefore, moving an infinitesimal unit of cash to these firms would, by continuity, result in a lower rate of return than in the original firm, which would again create an incentive for cash to return to the original firm.

Then:

LEMMA 4 – An equilibrium such that $k_i > 0$ *for more than one firm is not pairwise stable.*

PROOF – We know that in such a case, only two firms, i and $i_{\max}$, get cash. In such an equilibrium, k_i can get any value between 0 and $c_i D(p_i) < K$. One then has $r_i = p_i/c_i$, and $r_{i_{\max}} = 1 + \gamma_{i_{\max}} D(p_{i_{\max}})(p_{i_{\max}} - c_{i_{\max}})/k_{i_{\max}} = 1 + (1 - \frac{k_i}{c_i D(p_i)}) D(p_{i_{\max}})(p_{i_{\max}} - c_{i_{\max}})/(K - k_i)$. Consequently, $\frac{dr_{i_{\max}}}{dk_i} = \frac{\partial r_{i_{\max}}}{\partial k_i} - \frac{\partial r_{i_{\max}}}{\partial k_{i_{\max}}} = \frac{D(p_{i_{\max}})(p_{i_{\max}} - c_{i_{\max}})}{(K-k_i)^2} \left(1 - \frac{K}{c_i D(p_i)}\right) < 0 = \frac{dr_i}{dk_i} = \frac{\partial r_i}{\partial k_i} - \frac{\partial r_i}{\partial k_{i_{\max}}}$. Hence, the stability condition cannot hold.

PROPOSITION 4 – An allocation of cash $\{k_i\}$ *is a pairwise stable equilibrium if and only if there exists* i *such that* *(i)* $k_i = K$; $k_j = 0$, $j \neq i$

(ii) $p_i > c_i$

(iii) $\forall j < i$, $p_j/c_j \leq 1 + \frac{(p_i - c_i)D(p_i)}{K}$.

PROOF – First, we prove that an allocation which satisfies (i)-(iii) is an equilibrium. As firm i gets all the cash, it gets all the demand. Because of (2), it is unconstrained, and we have $r_i = 1 + \frac{(p_i - c_i)D(p_i)}{K}$. As $p_i > c_i$, $r_i > 1$. (iii) then implies that $r_j = p_j/c_j < r_i$ for $j < i$, while as $i = i_{\max}$, we have $r_j = 1 < r_i$ for $j > i$. Thus we have an equilibrium.

To prove that (i)-(iii) is necessary, we rule out other equilibria. Because of Lemmas 3 and 4, we only have to rule out other 1-firm allocations. Consider such an allocation where $k_i = K$, but which violates (ii): then $r_i < 1$. Because of assumption 2, there exists $j \neq i$ such that $r_j \geq 1$. Thus this cannot be an equilibrium. Assume then that (ii) holds but not (iii): that implies that $r_j > r_i$ for some $j < i$. Again, that cannot be an equilibrium. This rules out other 1-firm equilibria.

Q.E.D.

Proposition 4 has a number of implications.

First, there always exists an equilibrium, which we will refer as the "minimal" one. That equilibrium consists in giving all the cash to firm i (the "minimal firm") such that $i = \min\{i, p_i > c_i\}$, which exists since the relevant set is non empty, and satisfies (iii) since the LHS of that condition is lower than 1. Thus, giving all the cash to the lowest price profitable firm is an equilibrium.

Second, the minimal equilibrium need not be unique, as long as a firm's rate of return dominates that of cheaper firms by enough so that condition (iii) holds, we can construct an equilibrium where all cash goes to that firm. Cheaper firms cannot get financed despite being profitable, because the dominant firm has a higher markup and yields a higher rate of return. Therefore, some degree of "monopoly power" is sustainable in equilibrium; there is indeterminacy as savers can coordinate on different levels of monopoly power.

Third, if K is arbitrarily large, (iii) can only hold for the minimal firm. If cash is abundant, the dominant firm's rate of return is close to 1, because it uses only a small fraction of its cash. Deviating by giving a small amount cash to a cheaper, profitable, firm yields a higher return, because it will attract a small number of lucky customers and use all its cash to serve them. Consequently, no cheaper firm may be profitable and one must be at the minimal equilibrium. Therefore, it is the relative scarcity of cash which sustains nonminimal equilibria. If cash is abundant enough, the unique equilibrium is the one which resembles most the Walrasian one, given the constraints imposed on the model, i.e. the minimal one.

Fourth, the equilibria are not optimal from the point of view of social welfare; the firm yielding the highest surplus may not be eligible for equilibrium; for example, it is easy to construct examples where it is not profitable.

Consider now again the case where all firms have the same cost c. We have seen that the highest surplus firm is either the minimal firm or the highest price, unprofitable firm. Therefore, the minimal equilibrium comes as close as possible to the optimum. If there is enough cash, it is the only equilibrium. If the number of firms is large enough, and if prices are drawn from some distribution with full support, its price is arbitrarily close to c, i.e. it is arbitrarily close to maximizing social surplus. In such a situation, the model closely replicates the standard partial equilibrium Walrasian model.

This section has described a situation where savers know the rate of return of each firm prior to financing them. In such a situation, unprofitable firms do not get any cash; relative to the minimal equilibrium, there is a bias in favor of firms with a high markup, not in favor of unprofitable undercutters. The less cash is available, the higher the markup that can be sustained. A priori, an equilibrium is not socially optimal and the social optimum is not an equilibrium. But if costs are the same across firms and cash is abundant, the only equilibrium is the minimal one, which comes close to maximizing surplus. The larger the number of firms, the closer the equilibrium to the social optimum.

5 Equilibrium with new entrants

We now consider a more complex situation where in addition to existing firms whose rate of return is known, there are new entrants whose rate of return is unknown. Savers must now decide how to allocate their cash between incumbents and new entrants.

It is now harder to prove analytical results. Therefore, I confine myself to a simple case, where there is one incumbent, whose rate of return is observed, and one entrant, whose rate of return is not observed. I assume that the incumbent's price and cost, p_I and c_I, are observed. So is the entrant's cost, c_E. However, the entrant's price, p, is drawn from a distribution, with density $f(p)$, and support $[p_{\min}, p_{\max}]$. In order to restrict the number of cases to be analyzed, I will assume that $K > (c_E + c_I)D(p_I)$ and that $K > c_E D(p_{\min})$.

I want to check whether an equilibrium can arise where the entrant gets an amount of capital equal to k_E and the incumbent gets $K - k_E$.

Depending on k_E and on the realization of the shock p, four regimes may arise:

A. The entrant underbids the incumbent, and is constrained by its cash. This configuration arises if the incumbent charges a price $p < p_I$, and if $c_E D(p) > k_E$, or equivalently $p < p^*(k_E) = D^{-1}(k_E/c_E)$. Note that p^* is a decreasing function of k_E.

In such a case, the rate of return to the entrant is

$$r_E(p, k_E) = p/c_E.$$

The incumbent sells $(1 - \frac{k_E}{c_E D(p)})D(p_I)$ units, thus getting a rate of return equal to

$$r_I(p,k_E) = 1 + (p_I - c_I)(1 - \frac{k_E}{c_E D(p)})\frac{D(p_I)}{K - k_E}.$$

B. The entrant underbids the incumbent, and gets the whole market. That happens provided $p < p_I$ and $p > p^*(k_E)$. In this case, we have

$$r_E(p,k_E) = 1 + (p - c_E)\frac{D(p)}{k_E},$$

and

$$r_I(p,k_E) = 1.$$

C. The incumbent underbids the entrant, and is cash-constrained. This happens if $p > p_I$, and $c_I D(p_I) > K - k_E$. We then have

$$r_E(p,k_E) = 1 + (p - c_E)(1 - \frac{K - k_E}{c_I D(p_I)})\frac{D(p)}{k_E},$$

$$r_I(p,k_E) = \frac{p_I}{c_I}.$$

D. Finally, if $c_I D(p_I) < K - k_E$, the incumbent may underbid the entrant and get the whole market. In this case we have

$$r_E(p,k_E) = 1$$

and

$$r_I(p,k_E) = 1 + (p_I - c_I)\frac{D(p_I)}{K - k_E}.$$

These computations allow us to characterize equilibria. A pairwise stable equilibrium is a value of k_E such that

(i) $k_E = 0$ and $E(r_I) = \int_{p_{\min}}^{p_{\max}} r_I(p,k_E) f(p)dp > \int_{p_{\min}}^{p_{\max}} r_E(p,k_E) f(p)dp = E(r_E)$, or

(ii) $k_E = K$ and $E(r_I) < E(r_E)$,or

(iii) $0 < k_E < K$, $E(r_I) = E(r_E)$, and $\frac{d}{dk_E}(E(r_I) - E(r_E)) > 0$.

5.1 Corner equilibria

A straightforward step is to try and transpose the results of Proposition 4 and characterize an equilibrium where one firm gets all the cash. It is easy to compute such corner equilibria:

PROPOSITION 5 –

(i) $k_E = 0$ *is a pairwise stable equilibrium if and only if*

$$\frac{(p_I - c_I)D(p_I)}{K} > \int_{p_{\min}}^{p_I} \frac{p - c_E}{c_E} f(p)dp$$

(ii) $k_E = K$ *is a pairwise stable equilibrium if and only if*

$$\int_{p_{\min}}^{p_{\max}} \frac{p - c_E}{K} D(p)f(p)dp > \frac{p_I - c_I}{c_I} D(p_I)(1 - F(p_I)).$$

PROOF – Integrating cases A,B,C, and D above, we find that

-for $k_E < c_E D(p_I)$,

$E(r_I) - E(r_E) = \int_{p_{\min}}^{p_I} \left[(p_I - c_I)(1 - \frac{k_E}{c_E D(p)})\frac{D(p_I)}{K - k_E} - \frac{p - c_E}{c_E}\right] f(p)dp$
$+(p_I - c_I)\frac{D(p_I)}{K - k_E}(1 - F(p_I))$;

-for $c_E D(p_I) < k_E < K - c_I D(p_I)$,

$E(r_I) - E(r_E) = \int_{p_{\min}}^{\max(p^*(k_E), p_{\min})} \left[(p_I - c_I)(1 - \frac{k_E}{c_E D(p)})\frac{D(p_I)}{K - k_E} - \frac{p - c_E}{c_E}\right] f(p)dp$
$- \int_{\max(p^*(k_E), p_{\min})}^{p_I} (p - c_E)\frac{D(p)}{k_E} f(p)dp + (p_I - c_I)\frac{D(p_I)}{K - k_E}(1 - F(p_I))$;

-for $k_E > K - c_I D(p_I)$,

$E(r_I) - E(r_E) = \int_{p_{\min}}^{\max(p^*(k_E), p_{\min})} \left[(p_I - c_I)(1 - \frac{k_E}{c_E D(p)})\frac{D(p_I)}{K - k_E} - \frac{p - c_E}{c_E}\right] f(p)dp$
$- \int_{\max(p^*(k_E), p_{\min})}^{p_I} (p - c_E)\frac{D(p)}{k_E} f(p)dp + \int_{p_I}^{p_{\max}} \left[\frac{p_I - c_I}{c_I} - (p - c_E)\left(1 - \frac{K - k_E}{c_I D(p_I)}\right)\frac{D(p)}{k_E}\right] f(p)dp.$

Substituting $k_E = K$ and $k_E = 0$ into these formulas yields (i) and (ii). Q.E.D.

5.2 The truncation effect

It is easy to see that Proposition 5 implies that there may be multiple equilibria. Take the simple case where $p_I = c_I$. In such a case, condition (i) is equivalent to

$$E(p - c_E \mid p < p_I) < 0,$$

while condition (ii) is equivalent to

$$E((p - c_E)D(p)) > 0. \tag{5}$$

Clearly, they may both simultaneously hold. For $k_E = 0$ to be an equilibrium, we need that the expectation of the entrant's price, conditional on being lower than the incumbent, is lower than its cost. For $k_E = K$ to be an equilibrium, we just need that the entrant has positive expected profits. What is at work is the usual pecuniary externality, but here it has a richer interpretation. Assume the incumbent gets all the cash, and consider whether it is profitable to give one unit of cash to the entrant, whose price is not observed. Such an entrant will attract customers *only* if it underbids the incumbent. This makes it disproportionately likely that it will charge a price below cost. On the other hand, if the entrant gets all the cash, then it will have all the market even if it charges more than the incumbent. As long as the entrant is on average profitable, savers do not want to give one unit of cash to the incumbent, which – in the case where $p_I = c_I$ – just breaks even. For example, for a constant demand $D(p) = \bar{D}$, condition (5) is equivalent to $E(p - c_E) > 0$. Since $E(p - c_E) > E(p - c_E \mid p < p_I)$, we can construct distributions $f()$ such that multiple equilibria hold.

The equilibrium where all the cash goes to the incumbent is "stagnant", while the equilibrium where it goes to the entrant is "turbulent", in that the entrant will sometimes make losses, and then (in a richer model) go bankrupt. If the entrant has lower costs than the incumbent ($c_E < c_I$), then productivity is higher in the equilibrium where the entrant gets all the cash.

5.3 The demand effect: interior equilibria

Against the truncation effect runs the demand effect, which creates a force for interior equilibria rather than corner equilibria. The demand effect comes from the fact that $D(p)$ is decreasing, so that an unconstrained firm will sell more when it is underpricing than when it is overpricing. This tends to push down the value of $E((p - c_E)D(p))$. On the other hand, a constrained firm does not serve all its demand, and the demand effect disappears for such a firm. In principle, we can thus construct examples where $E(r_E) < E(r_I)$ for $k_E = 0$ and $E(r_I) > E(r_E)$ for $k_E = k$. By continuity, there then exists an interior equilibrium, where both the entrant and the incumbent get a positive fraction of cash. For such a situation to be possible, the demand effect must be strong enough, i.e. the elasticity of demand may be large enough. Let us go back to the case where $p_I = c_I$. To get a configuration such that there is an interior equilibrium and no corner equilibrium, we need that $E((p - c_E)D(p)) < 0 < E(p - c_E \mid p < p_I)$. If $D(p)$ is elastic enough, it will give more weights to low prices, relative to high prices, than truncation at $p = p_I$, and the inequality may hold. For example, take the case where both demand and the distribution of prices are exponential: $D(p) = e^{-\gamma p}$, $f(p) = \lambda e^{-\lambda p}$. We find that $E((p - c_E)D(p)) = -\frac{\lambda}{\lambda+\gamma} c_E + \frac{\lambda}{(\lambda+\gamma)^2}$. Thus, it is negative if and only if

$$c_E > \frac{1}{\lambda + \gamma}.$$

On the other hand, $E(p - c_E \mid p < p_I) = \frac{1}{\lambda} - c_E - \frac{p_I e^{-\lambda p_I}}{1 - e^{-\lambda p_I}}$, which is positive if and only if

$$c_E < \frac{1}{\lambda} - \frac{p_I e^{-\lambda p_I}}{1 - e^{-\lambda p_I}}.$$

A range of c_E exists for which the equilibrium is interior, if $\frac{1}{\lambda+\gamma} < \frac{1}{\lambda} - \frac{p_I e^{-\lambda p_I}}{1 - e^{-\lambda p_I}}$, i.e.

$$\frac{\gamma}{\lambda(\gamma + \lambda)} > \frac{p_I e^{-\lambda p_I}}{1 - e^{-\lambda p_I}}.$$

That is more likely to hold, the greater the slope of the demand curve (the higher γ). It is also more likely if p_I is neither too large nor too small.

6 Conclusion

This paper has considered the broad issue of how markets select between boundedly rational firms. I have narrowed the discussion by focusing on a static model where bounded rationality amounts to charging a fixed, exogenous price, and where the engine of selection is the allocation of cash by rational savers. I have highlighted the existence of a pecuniary externality which is due to the fact that reallocating cash in favor of one firm affects the customer base of another firm.

The key result is that if cash is scarce enough, one can sustain an equilibrium where cash goes to "overpricers" rather than underpricers. Furthermore, the pecuniary externality generates multiple equilibria. When there are entrants with unobserved rates of return, there are equilibria where all cash goes to the entrant and equilibria where all cash goes to the incumbent. Because of the pecuniary externality, these two types of equilibria may co-exist for the same set of parameters. The equilibria where the entrant gets the cash involve "bankruptcy" in the sense that the entrant will sometimes make losses.

From there, where can research go? The present model, in its version with entrants, is a rudimentary theory of venture capital which yields some insights on how the allocation of cash between entrants and incumbents affects the market as a whole. An appealing idea, based on traditional Schumpeterian views of creative destruction, is that entrants are the engine of growth and have lower costs than incumbents. A natural research direction is therefore to extend the model and turn it into a growth model. That would be useful to analyze the role played by bounded rationality and financial institutions in shaping the process of technological change. Such a model would also yield insights on the interactions between growth and 'turbulence', since, as suggested here, faster growth would be associated with a greater fraction of cash going to incumbents, who are more likely to fail.[4]

Another research direction would be relaxing the assumption that savers are rational, and assume that they gradually learn an optimal rule of thumb for allocating their savings between entrants and incumbents. From that process would emerge an endogenous 'attitude' of savers toward risk, that would have a critical impact of the growth process.

Finally, one could clearly allow for a richer behavior of firms and for pricing rules that are not totally rigid (research along these lines in rather different contexts include Anagnostopoulos et al. (2005) and Saint-Paul (2005)). Intuitively, some of this paper's insights should remain to the extent that underpricing and overpricing remain a characteristic of certain firms.

[4] The model would capture the "selection" aspects of capitalism rather than the voluntary rent-seeking aspects, that are emphasized by the Aghion and Howitt (1992) model.

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