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Timing of Payment

How Uncertain Alternatives and Legal Systems affect the Contractual Choice

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Contents

Abstract

Contractual analysis has long been focused on finding the optimal pricing arrangement. However some important aspects of the contractual terms are either assumed away or neglected. This study look at one ubiquitous term - which is the timing of payment. From the view of anthropology or marketing literature, timing of payment is mainly determined by social customs. Although there is an indubitable element of truth in it, all normal social behavior is to some extent customary. A positive economist cannot rest easy with such irrefutable argument. Custom itself is often an implicit compromise of fundamental economic conflict, it deserves an economic explanation.

My dissertation has 3 chapters. The first two chapters study directly the issue of timing of payment under different considerations. Chapter 1 study the situation where consumers have uncertain future alternatives, while chapter 2 study the choice of contracts under different legal environments. The focus of chapter 3 is a bit different. It is also asking a question about timing, but instead of the payment time, it is the bidding time. Under the situation where bidding is a costly activity, the timing of bidding bears some resemblance to the timing of payment. The bidding cost can be viewed as the prepayment studied in Chapter 1. This last chapter thus can be viewed as a generalized timing of payment problem, where part of the "payment" does not goes to the seller.

Chapter 1 studies the situation when consumers with unit demand, having uncertain future alternatives, plan to consume at a fixed date in the future. Under this setting, uncertainty of the future event is a source of price discrimination. One main result is, in the expected profit maximizing mechanism, the monopolist seller gets the same expected profit as if she can observe the new information received by the consumers — the realized uncertain alternatives. This can be implemented by the contract (T, p) where T is paid in advance and p is paid on the date. One interesting finding is that the optimal p does not depend on the realized alternative, if the value of the service and alternative are independent. The expected profit is higher when consumers have more volatile alternatives.

Chapter 2 analyzes the timing of payment under different legal environments. In case of dispute, court will get involved to resolve the conflict. I showed the efficiency of the legal system and the fixed cost of litigation are the keys for the choice of payment timing. My model predicts, generally speaking, ex-ante payment contract is more prevalent in region with low fixed cost of using the legal system. But since under this situation, ex-ante payment contract is

inefficient. I found that full efficiency can be attained by using certain mixed payment contract. When the fixed cost is within certain bounds, ex-ante and ex-post payment contract are identical and perfectly efficient. The model also has some implications for the proposer of the contract. When the seller proposes, a simple ex-ante payment contract dominates ex-post payment and mixed payment contract, which may explain why most of offers in everyday life are made by sellers in a simple ex-ante payment form. And it suggests that primarily mixed payment is a contractual device used by the buyer to mitigate the two sided moral hazard problem.

Chapter 3 considers the effect of bidding cost on bidding behavior. At each period, bidders are allowed to submit or revise their bid with the associated bidding cost. This communicates bidders' information rapidly. A bidder makes a bid in early periods to signal a high valuation. In equilibrium under most cases, the auction effectively ends before the mandated one. The model provides new results concerning profit maximizing reserve price and efficiency reserve price in a costly bidding environment. Also a information based interpretation of delays in bidding is provided.

1 Timing of Payment with Uncertain Alternatives

1.1 Introduction

In a recent New York Times Magazine, an article shows that more and more restaurants start to charge customers for the table per se.[1] In contrast to the tradition where reserving a table is free, now restaurants, particularly for those located in cities with high rent, like New York and San Francisco, charge their customers for reservations. This allow restaurants to better mitigate the risk that they will lose money on a diner who promises to show up and cancels. Throughout the industry, no-shows make up 5 to 10 percent of all reservations. According to Nick Kokonas, a former derivatives trader who is an owner of the exclusive Chicago restaurants Next and Alinea, by selling tickets for the table, the no-show rate drops to less than 2 percent. Fundamentally, the problem faced by both diners and restaurants, is the uncertain alternative that diners have at the time of reservation. Similar situations happen when we pay for concerts, airline tickets and, more recently, car-hailing service Uber.

To study this problem, I consider a simple trading environment between a monopolist seller and consumers. The seller provides a service at a fixed date. She can sell the "tickets" at the date of the service provided, or in advance. Each of the consumers, as different from the standard principal agent model (Baron and Myerson, 1982), has an uncertain alternative at the service-providing date. Instead of treating this as a outside option only, I represent it by a random variable in the utility function, which is only effective when the consumer chooses to take the alternative. By this formulation, consumers take alternatives into account when they form expectations. This is a novel way to model uncertain alternative which gives a representation of the utility with optimized choice.[2]

The unexpected event can be good or bad, a more attractive opportunity or something that stops you to proceed with your plan. For example, a call for party or being sick. In response, people may stick to the plan or change it. It is important to note that the possibility of "quitting" at the service-providing date distinguishes this paper from the dynamic mechanism design literature. Most of the papers usually assume no opt out after the initial participation, hence consider

[1] Available at: http://nyti.ms/1CPlzE9

[2] The closest model in this aspect is Rochet and Stole (2002), which studies the case where the outside option is random. For more details, see the literature review in section 1.6.

only the ex-ante participation constraint but not the subsequent participation constraints.[3] Most of the results in this paper depend on the linkage between the alternative and the service. If the value of the service and the alternative are independent to the consumer, for instance when the alternative of going to a restaurant is a party, then the price being paid in the restaurant will be higher than the case when they are positively correlated, like when the alternative is a similar restaurant. The key lies on how the consumer value these two activities, if they are positively correlated, then the alternative activity is actually a good competitor to the seller. By the well known logic, competition lowers the price. The only difference is that the competition comes not only from another firm but also from another individual. Given the well developed and powerful information technology nowadays, alternatives come from everywhere.

To start with the analysis, I consider the case where the seller cannot price discriminate. This is particularly relevant to the regulated industries. In section 1.3 I derive the optimal uniform prices when the seller sells in advance, at the service providing date, or both. The best strategy is to sell both in advance and at the service-providing date. Under our setting, selling at both dates is weakly better than either one of them and yet none of the date involves zero transaction. The choice of the timing of payment creates imperfect information. Consumers can get full information for their alternative if the seller decides to sell at the service-providing date. However, if the seller decides to sell in advance, only consumers with high enough value will purchase the "ticket". There will be a cutoff value, such that all consumers with value lower than this cutoff, will just give up buying the service. Whereas if the "ticket" sells at the service-providing date, consumers with high realized alternative will not buy the service. The trade off hinges on the distribution of alternative and the distribution of the consumers' value. Actually under our setting, if there is one known alternative or no alternative, the optimal solution is to practice uniform pricing. However, once the alternative becomes uncertain, the optimal selling plan is to price discriminate. Hence the uncertainty of the future event is a source of price discrimination. From section 1.4 onwards, the seller is allowed to price discriminate.

There is a pricing strategy which is not considered above — mixed pricing, where the total payment involves advance payment and payment on the consumption date. After opening up the possibility of mixed payment, in proposition 1.2, I show that the optimal price at the service providing date, does not depend on the distribution of the alternative, if the alternative is independent from the value for the service. The intuition for this result is that since at the service providing date, the decision is binary — either consume or not consume, the actual realization of the alternative does not matter. Given the type reported, there can only be two prices at the service providing date, the price for consuming, and the price for dropping out. This makes

[3] See for example: Eső & Szentes (2013) and Pavan, Segal & Toikka (2014). The usual argument for ignoring the subsequent participation constraints is as follows: in quasilinear environment with unlimited transfers, the principal can ask the agent to post a sufficiently large bond upon acceptance, to be repaid later, so as to make it unprofitable to quit and forfeit the bond at any time during the mechanism. What I do in this paper, is to explicitly deal with this problem, and a closed-form solution for the "bond" is obtained when the alternative and the value is independent. Yet, the "bond" I obtain is not for keeping the agents to participate, but instead to maximize the expected profit subject to initial participation.

no sense if there are more than one price for consuming, people will just report the alternative value such that the lowest price is given. Thus the actual realization of the alternative cannot help boosting profit, and hence means virtually nothing to the seller. This is the reason why the ex-post price does not depend on the distribution of the alternative and the rationale for the main result of this paper.

The main result of this paper is stated as theorem 1 — In the expected profit maximizing mechanism, the monopolist seller get the same expected profit as if she can observe the realization of the uncertain alternatives. This implies that knowing the realization of the uncertain alternative will not raise the profit of the seller. The reason is, in order to provide the incentive to ask consumers to report their type truthfully, the ability of extracting surplus from consumers is limited. Under the case of observable alternatives, the optimal price being charged equals to, given the same w, the price for the lowest type who will buy. Because the price is shown to be decreasing in type. The monopolist seller is extracting the most of what they can get. Whereas in the general case of unobservable alternative, I propose a two part pricing mechanism. The price paid at the service-providing date is less than the one when alternative is observable, but the before part make this up. Hence, the optimal mechanism gets the same profit as when the seller can freely observe the realization of the uncertain alternative. The key assumption is the independence between the value for the service and the uncertain alternative. From the truthful report of the consumers about their types, the seller learns nothing for the uncertain alternative. In section 1.5, I relax the independent assumption. With the assumption of positive correlation between the value and the alternative, the seller expects consumers with higher value, on average gets higher realization of the uncertain alternative, hence charging a lower price.

Another interesting result is that the expected profit is higher when consumers face a more volatile alternative. A mean preserving spread of the distribution function is interpreted as increasing volatility of the random variable. The intuition of this result can be understood as if the seller is providing an insurance. If the consumer gets unlucky, a low realization of the alternative happens, she can consume the service. On the other hand, lucky consumer will enjoy the high value alternative. A mean-preserving spread smooths out and increase the density of the two end. But the lower end is covered by the seller, hence it is only the high end has effect on the consumer well being. As the expected utility of the consumer is higher, there is more surplus to extract. Thus the expected profit is higher.

In section 1.4.5, I analyze a behavioral deviation of the consumers. Consumers may overlook, partly or completely, the possibility of having a future alternative. It is common to see people get shocked by the "unexpected" events. One explanation is that people does not give enough consideration to the possible future alternative he may have. I represent this problem by introducing an extra term ψ, which is the probability that one overlooks the possibility of having future alternatives. Surprisingly, I show that consumers will benefit from this irrationality.

In the next section, I will introduce the model formally. In section 1.3, I study the optimal pricing, where the seller is confined to uniform pricing of pure ex-ante or ex-post pricing. In section 1.4, the possibility of price discrimination and mixed payment is opened up. Most of the results are obtained. Section 1.5 deals with the correlated case of the value for the service and the alternative. In section 1.6, I compare my model with the related literature. Section 1.7 concludes and applies the results to the motivating example. Possible extensions is also discussed.

1.2 Model Setup

Consider a risk neutral monopolist who is providing a service at date 1 with constant marginal cost c, for example a concert or opera. She can sell “tickets” at different points of time, date 0 or date 1.[4] There is a continuum of consumers with mass 1 having valuation θ for the service, where $\theta \sim F[\underline{\theta}, \bar{\theta}]$. Consumers are risk neutral and have unit demand. Thus consumer’s type is denoted by θ. Assume $\underline{\theta} \geq c$, so that trading is efficient for all consumers. Each consumer has a list of uncertain alternatives, I denote the highest valued alternative as $w \sim G[0, \bar{w}]$. The term w denotes the value of the alternative, net of any associated cost.[5] Assume $\bar{w} \geq \bar{\theta}$, so it is possible that all consumers take the alternative. Consumers learn the realization of w at date 1, while θ is known at date 0. Both θ and w are private information, and are assumed to be independent. This means that all consumers have same expected value for the alternative, regardless of his type. The independence assumption will be relaxed in section 1.5.

The utility function of a consumer of type θ is

$$u = \begin{cases} \theta - p & \text{if he consumes the service,} \\ w & \text{if he consumes the alternative.} \end{cases}$$

The price p can either be the ex-ante price p_a, the ex-post price p_p or the sum of both as in section 1.4. If the consumer chooses to buy in advance at date 0, the price will be p_a. However there will not be any refund even if the consumer decides go for the alternative at date 1.[6] Thus the expected utility of a type θ consumer who pays at date 0 is

$$\begin{aligned} U(\theta) &= \theta G(\theta) + \int_{\theta}^{\bar{w}} w dG(w) - p_a \\ &= \bar{w} - \int_{\theta}^{\bar{w}} G(w) dw - p_a. \end{aligned}$$

In the first line, the first term is the expected benefit of the purchase, which happens with

[4] There may be some confusion from the wording “tickets” here, which may imply there is a capacity constraint. However by assumption, the seller can provide unlimited amount of service at a marginal cost of c.

[5] A higher w may also reflect the situation, where no alternative is available but you get sick or the weather is bad on the date of the show, and hence the net value of staying home (the default option) is higher.

[6] Our formulation is actually more general than this, but for the sake of exposition, the refunding interpretation is deferred to section 1.4.

probability $G(\theta)$. Because whatever has been paid is sunk at date 1, and the consumer will not go to enjoy the service, in the case of better alternative arises, when $w \geq \theta$. The second term is the expected benefit from the alternative. The last term is the price the consumer pays for the "ticket". The second line is obtained from the first by integration by parts.

Consumers who buy at date 1 pay p_p. The corresponding expected utility of a type θ consumer is

$$\begin{aligned} U(\theta) &= (\theta - p_p)G(\theta - p_p) + \int_{\theta - p_p}^{\bar{w}} w dG(w) \\ &= \bar{w} - \int_{\theta - p_p}^{\bar{w}} G(w) dw. \end{aligned}$$

In the first line, the first term is again the expected benefit of the purchase, which happens with probability $G(\theta - p_p)$. As the consumer will only consume the service if there is no better alternative, i.e. when $w \leq \theta - p_p$. The second term is the expected benefit from the alternative. The second line is again derived from the first by integration by parts.

1.3 When to sell? Now or later?

To answer the question of the optimal time to sell, I study different cases of market opening. In subsection 1.3.1, only date 0 market is opened. In subsection 1.3.2, only date 1 market is opened. In subsection 1.3.3, both markets are open. Noted that, in all above cases, the monopolist charges an uniform price to all of his customers. This is relevant to any regulated industry where price discrimination is prohibited. However prices can be different at different points of time. The timing is as follows. At date 0 the monopolist announces and commits to a pair of prices (p_a, p_p), where $p_a \in \mathbb{R}_+$ stands for the ex-ante price, and $p_p \in \mathbb{R}_+$ stands for ex-post price. Consumers choose whether to buy at date 0 or not. At date 1, after uncertainties are resolved, those who have already paid choose whether to consume the service. Those have not paid decide whether to buy at date 1. There are three possible cases, i) only the ex-ante market is open and that can be represented as (p_a, ∞); ii) only the ex-post market is open and that can be represented as (p_a, ∞); iii) both ex-ante and ex-post market are open and that can be represented as (p_a, p_p). These representations provide an uniform way to model different market openings.

The main result of this section is stated in the following proposition.

Proposition 1.1. *The optimal uniform pricing scheme is the pair of (p_a, p_p) defined by lemma 1.3. It induces positive demand in both ex-ante and ex-post market, if $F'(\bar{\theta}) = 0$.*

It should be noted that, mixed payment is not studied in this section. Mixed payment means that the total payment involves both advance payment and deferred payment, which will be analyzed in section 1.4.

1.3.1 Ex-ante payment

Let us first consider the case when only the ex-ante market is open. If the consumer chooses not to consume, then her expected utility is the expected value of the alternative. The marginal consumer who is indifferent between purchasing or not purchasing, is denoted by θ_a. Therefore

$$U(\theta_a) = \theta_a G(\theta_a) + \int_{\theta_a}^{\bar{w}} w dG(w) - p_a = \int_0^{\bar{w}} w dG(w).$$

This implies that

$$p_a = \int_0^{\theta_a} G(w) dw. \tag{1.3.1}$$

All consumers with $\theta > \theta_a$ will buy the service in the ex-ante market. The remaining commits to the alternative. By differentiation,

$$\frac{d\theta_a}{dp_a} = \frac{1}{G(\theta_a)} > 0.$$

Since θ_a increases with p_a, a higher p_a implies that less consumer will buy the service. The profit of a monopolist which opens only the ex-ante market is

$$\pi = p_a (1 - F(\theta_a)) - c \int_{\theta_a}^{\bar{\theta}} G(\theta) dF(\theta) \tag{1.3.2}$$

Although consumers pay for the service, they will not necessarily consume it. This is the reason why the term attached to the cost part is different to one attached to the price. Consumers with $w \geq \theta$, will opt for the alternative. The probability of consuming for a type θ consumer is $G(\theta)$. Because p_a is sunk at date 1, there will be over-consumption. The consumers who have already paid act as if the service is free and give up the other valuable alternative which will be taken if they are ask to p_a at date 1.

Lemma 1.1. *The profit maximizing p_a^* and θ_a^* satisfies*

$$p_a^* = \int_0^{\theta_a^*} G(w) dw = G(\theta_a^*) \left(c + \frac{1 - F(\theta_a^*)}{F'(\theta_a^*)} \right). \tag{1.3.3}$$

If $\frac{\int_0^{\theta_a^*} G(w)dw}{G(\theta_a^*)}$ is increasing in θ_a, together with the standard assumption of decreasing hazard rate, there is an unique θ_a. The first assumption is satisfied by the uniform distribution. [7]

[7]Take $G(\cdot) = w/\bar{w}$, $\frac{\int_0^{\theta_a^*} G(w)dw}{G(\theta_a^*)} = \frac{\theta_a^2/2\bar{w}}{\theta_a/\bar{w}} = \frac{\theta_a}{2}$.

1.3.2 Ex-post payment

Let us then consider the case where only the ex-post market is open. Consumer will only consume if the realized value of $w \leq \theta - p_p$, where consumer with $\theta < p_p$ will never buy the service. The profit is

$$\pi = (p_p - c)\int_{p_p}^{\bar{\theta}} G(\theta - p_p)\, dF(\theta). \tag{1.3.4}$$

Lemma 1.2. *The profit maximizing p_p^* satisfies*

$$p_p^* = c + \frac{\int_{p_p^*}^{\bar{\theta}} G\left(\theta - p_p^*\right) dF(\theta)}{\int_{p_p^*}^{\bar{\theta}} G'\left(\theta - p_p^*\right) dF(\theta)}. \tag{1.3.5}$$

Because $p_p^* \geq c$, there is under-consumption. Because the price p_p is charged at a level higher than cost c, consumption will not be efficient. The optimal solution will be interior because the corner solutions are clearly not maximizing profit. While there does not exist "the" highest price, we just need to consider the $p_p = \bar{\theta}$. Because for any $p_p > \bar{\theta}$, there is definitely no one will buy the service. When $p_p = \bar{\theta}$, $\pi = 0$. Thus the optimal solution should be an interior one, as the π^* is positive.[8]

1.3.3 Ex-ante and ex-post payments

Let us now consider the case where both markets are open. There are three choices for consumers, i) buy at date 0 and pay p_a, ii) buy at date 1 and pay p_p, iii) not buy at all. The marginal consumer, who is indifferent between to buy at date 0 and date 1, is denoted by $\widetilde{\theta}$. It can be shown that[9]

$$p_a = \int_{\widetilde{\theta} - p_p}^{\widetilde{\theta}} G(w)\, dw. \tag{1.3.6}$$

It is clear that $\widetilde{\theta} \geq p_p$, otherwise there is no way to satisfy the equation. This implies that consumers who buy at date 0 must have higher value than those who buy at date 1. Recall equation (1.3.1), $p_a = \int_0^{\theta_a} G(w)\, dw$ gives the type (θ_a) which is indifferent between buying at date 0 and not buying at all. By comparing equation (1.3.1) and equation (1.3.6), we know that $\theta_a \leq \widetilde{\theta}$. Hence $\tilde{\theta}$ is a sufficient statistics to determine whether the consumer will buy at date 0.

[8] The sufficient condition for the interior solution is $(p_p - c)\left(G'(0)(1 - F(p_p)) + \int_{p_p}^{\bar{\theta}} G''(\theta - p_p)\, dF(\theta)\right) \leq 2\int_{p_p}^{\bar{\theta}} G'(\theta - p_p)\, dF(\theta)$.

[9] The derivation is as follows, $U_a(\widetilde{\theta}) = U_p(\widetilde{\theta}) \implies \widetilde{\theta}G(\widetilde{\theta}) + \int_{\widetilde{\theta}}^{\bar{w}} w dG(w) - p_a = G(\widetilde{\theta} - p_p)(\widetilde{\theta} - p_p) + \int_{\widetilde{\theta} - p_p}^{\bar{w}} w dG(w)$. Then $p_a = \widetilde{\theta}G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)(\widetilde{\theta} - p_p) - \int_{\widetilde{\theta} - p_p}^{\widetilde{\theta}} w dG(w)$. As a result, $p_a = \int_{\widetilde{\theta} - p_p}^{\widetilde{\theta}} G(w)\, dw$.

To summarize, that means all consumers with $\theta \geq \widetilde{\theta}$ will choose to buy at date 0. Consumers with $\theta \leq p_p$ will not consume at all. The rest will buy at date 1 if no better alternative arises.

By total differentiation of equation (1.3.6),

$$\frac{d\widetilde{\theta}}{dp_a} = \frac{1}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)} > 0$$
$$\frac{d\widetilde{\theta}}{dp_p} = -\frac{G(\widetilde{\theta} - p_p)}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)} < 0$$

As expected, a higher ex-ante price will shift some consumers from ex-ante to ex-post market. But other than shifting some consumers from ex-post to ex-ante, a higher ex-post price will also reduce the total number of potential consumer, as consumers with $\theta \leq p_p$ will not consume at all.

The profit function of the monopolist which opens both ex-ante and ex-post market is

$$\pi = p_a\left(1 - F(\widetilde{\theta})\right) - c\int_{\widetilde{\theta}}^{\bar{\theta}} G(\theta)\,dF(\theta) + (p_p - c)\int_{p_p}^{\widetilde{\theta}} G(\theta - p_p)\,dF(\theta)$$

where $\widetilde{\theta}$ is given by equation (1.3.6). In the following lemma, I assert that the interior solution is the optimal. Besides, there are four corner solutions, i) $p_a = 0$, ii) $p_p = 0$, iii) p_a being so high that no one buy at date 0, where $\widetilde{\theta} = \bar{\theta}$, iv) p_p being so high that no one buy at date 1, where $\widetilde{\theta} = p_p$. The first two cases are clearly not optimal as it gives zero profit. I will also show that the third and fourth corner solutions are not optimal in lemma 1.4 and lemma 1.5.

Lemma 1.3. *The profit maximizing (p_a, p_p) and the cutoff $\widetilde{\theta}$ satisfy*

$$p_a = \left(c + \frac{1 - F(\widetilde{\theta})}{F'(\widetilde{\theta})}\right)\left(G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)\right) + p_p G(\widetilde{\theta} - p_p), \tag{1.3.7}$$

$$p_a = \int_{\widetilde{\theta} - p_p}^{\widetilde{\theta}} G(w)\,dw,$$

$$p_p = c + \frac{\left(1 - F(\widetilde{\theta})\right)G(\widetilde{\theta} - p_p) + \int_{p_p}^{\widetilde{\theta}} G(\theta - p_p)\,dF(\theta)}{\int_{p_p}^{\widetilde{\theta}} G'(\theta - p_p)\,dF(\theta)}. \tag{1.3.8}$$

Proof. The first order condition for profit maximization with respect to p_a is

$$1 - F(\widetilde{\theta}) + \left(c + \frac{p_p G(\widetilde{\theta} - p_p) - p_a}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)}\right)F'(\widetilde{\theta}) = 0.$$

With some manipulation, we get equation (1.3.7). The first order condition for p_p is

$$\frac{\left(p_a - cG(\widetilde{\theta})\right) G(\widetilde{\theta} - p_p) F'(\widetilde{\theta})}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)} + \int_{p_p}^{\widetilde{\theta}} G(\theta - p_p)\, dF(\theta)$$
$$- (p_p - c)\left(\frac{G(\widetilde{\theta} - p_p)^2 F'(\widetilde{\theta})}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)} + \int_{p_p}^{\widetilde{\theta}} G'(\theta - p_p)\, dF(\theta)\right) = 0. \quad (1.3.9)$$

Substitute equation (1.3.7) into equation (1.3.9) to obtain

$$\left[cF'(\widetilde{\theta}) + 1 - F(\widetilde{\theta})\right] G(\widetilde{\theta} - p_p) + \frac{p_p G(\widetilde{\theta} - p_p)^2 F'(\widetilde{\theta})}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)} - \frac{cG(\widetilde{\theta}) G(\widetilde{\theta} - p_p) F'(\widetilde{\theta})}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)}$$
$$+ \int_{p_p}^{\widetilde{\theta}} G(\theta - p_p)\, dF(\theta) - \frac{p_p G(\widetilde{\theta} - p_p)^2 F'(\widetilde{\theta})}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)} - p_p \int_{p_p}^{\widetilde{\theta}} G'(\theta - p_p)\, dF(\theta)$$
$$+ \frac{cG(\widetilde{\theta} - p_p)^2 F'(\widetilde{\theta})}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)} + c \int_{p_p}^{\widetilde{\theta}} G'(\theta - p_p)\, dF(\theta) = 0.$$

Multiply the first term $cF'(\widetilde{\theta})G(\widetilde{\theta} - p_p)$ by $\frac{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)}{G(\widetilde{\theta}) - G(\widetilde{\theta} - p_p)}$, then it can be canceled with the third and seventh terms. Hence

$$\left(1 - F(\widetilde{\theta})\right) G(\widetilde{\theta} - p_p) + \int_{p_p}^{\widetilde{\theta}} G(\theta - p_p)\, dF(\theta) = (p_p - c) \int_{p_p}^{\widetilde{\theta}} G'(\theta - p_p)\, dF(\theta),$$

which implies equation (1.3.8). □

Lemma 1.4. *There exists a pair of (p_a, p_p) which ex-ante and ex-post market demand is positive and yields the same profit as the optimal ex-ante pricing scheme.*

Proof. Define $\bar{p}_p$ as the limit ex-post price such that ex-post demand is zero ($\widetilde{\theta} = p_p$), given $p_a = p_a^*$ as in equation (1.3.3).

$$\int_0^{\bar{p}_p} G(w_i)\, dw_i = p_a^*.$$

Then I can show

$$\left.\frac{\partial \pi(p_a^*, p_p)}{\partial p_p}\right|_{p_p = \bar{p}_p} = 0.$$

The last equation can be easily obtained by inserting $\widetilde{\theta} = p_p$ into equation (1.3.9). That implies operate in both markets is weakly better than only operate in the ex-ante one. □

Lemma 1.5. *There exists a pair of (p_a, p_p) which both ex-ante and ex-post market demand is positive and yields the same profit as the optimal ex-post pricing scheme, if $F'(\bar{\theta}) = 0$.*

Proof. Define $\bar{p}_a$ as the limit ex-ante price such that ex-ante demand is zero ($\widetilde{\theta} = \bar{\theta}$), given $p_p = p_p^*$ as in equation (1.3.5).

$$\int_{\bar{\theta}-p_p^*}^{\bar{\theta}} G(w)\,dw = \bar{p}_a.$$

Then I can show

$$\begin{aligned}\left.\frac{\partial \pi\left(p_a, p_p^*\right)}{\partial p_a}\right|_{p_a=\bar{p}_a} &= F'(\bar{\theta})\left(c + \frac{p_p^* G(\bar{\theta}-p_p^*) - \int_{\bar{\theta}-p_p^*}^{\bar{\theta}} G(w)\,dw}{G(\bar{\theta}) - G(\bar{\theta}-p_p^*)}\right). \\ &= F'(\bar{\theta})\left(c + \bar{\theta} + \frac{\int_{\bar{\theta}-p_p^*}^{\bar{\theta}} w dG(w)}{G(\bar{\theta}) - G(\bar{\theta}-p_p^*)}\right) \geq 0\end{aligned}$$

If $F'(\bar{\theta}) = 0$, the last equation will be equal to zero. That implies operate in both markets is weakly better than only operate in the ex-post one. □

Corollary 1.1. *If $F'(\bar{\theta}) > 0$, operating only the ex-post (date 1) market is profit maximizing.*

Proof. If $F'(\bar{\theta}) > 0$, $\left.\frac{\partial \pi\left(p_a, p_p^*\right)}{\partial p_a}\right|_{p_a=\bar{p}_a} > 0$. That implies that by increasing the ex-ante price to the limit level, essentially closing down the ex-ante market, the seller earns more. □

By now, we have all the components needed for the proof of proposition 1.1,

Proof. As shown in lemma 4 and 5, the profit of a monopolist who operates in a pure ex-ante or ex-post market can be replicated by a pair of (p_a, p_p) where both markets are open, yet those pairs of (p_a, p_p) are not the optimized one, thus the optimal pricing scheme is the one which yields positive demand on both markets. □

1.4 Mixed Payment and Price Discrimination

In this section, we consider the general case when the seller is allowed to practice mixed payment and price discrimination. As a result, mechanism design approach is adopted to analyze the model. From now on, given a pair of (p_a, p_p), the total price the consumer need to pay if he chooses to consume, is the sum of p_a and p_p. Hence, the expected utility of the consumer of type θ, if he decides to buy, is

$$\begin{aligned}U(\theta) &= (\theta - p_p)\,G(\theta - p_p) + \int_{\theta-p_p}^{\bar{w}} w dG(w) - p_a \\ &= \bar{w} - \int_{\theta-p_p}^{\bar{w}} G(w)\,dw - p_a.\end{aligned}$$

With this formulation, the monopolist sells by mixed payment. Although I assume the discount rate to be zero, consumers still have a preference of the timing of payment. To see it, we can compute the marginal rate of substitution (MRS),

$$\text{MRS} = -\left.\frac{dp_a}{dp_p}\right|_{U=\bar{U}} = G(\theta - p_p).$$

Firstly we see that, p_a and p_p are not perfect substitute, as the MRS is not a constant. A higher p_p implies a lower MRS, while p_a plays no role in the MRS. Secondly, consumer with higher θ has higher MRS. For any increment of p_p, consumers with higher θ require more reduction of p_a to be indifferent. This points out that, it costs less to ask consumers with higher θ to pay by p_a, instead of p_p.

To analyze the problem in the general way, another set of variables for the payments is defined. At date 0, after learning his type, consumer reports his type θ at the ex-ante stage and paying $t_0(\hat{\theta})$ as the entry fee. At date 1, each consumer reports his realized w and make a payment $t_1(\hat{\theta}, \hat{w})$ to the monopolist. Before any announcements is made by the consumer, the monopolist commits to a mechanism $\{t_0(\hat{\theta}), t_1(\hat{\theta}, \hat{w})\}$. Define $t = t_0 + t_1$. A consumer's realized net surplus is $u = \theta - t$, if he consumes. The monopolist's profit is $\pi = t - c$. A consumer's pure strategy is a collection of functions $\hat{\theta}(\theta) \in [0, \bar{\theta}]$ and $\hat{w}(\hat{\theta}, \theta, w) \in [0, \bar{w}]$ describing how previous announcements and information map into announcements.

Since the final choice of the consumer is binary, either buy or not. By taxation principle, there is no loss of generality to assume that [10]

$$t_1(\hat{\theta}, \hat{w}) = \begin{cases} t_b(\hat{\theta}) & \text{if buy,} \\ t_n(\hat{\theta}) & \text{otherwise.} \end{cases}$$

It suggests that the exact amount of w does not mean too much to the seller. The seller can do just as well as he does not ask for the reporting of w, but letting consumers to choose among the schedules. Note that when $t_n(\hat{\theta}) \leq 0$, there is refund to the consumer who chooses not to consume. Define

$$T(\theta) = t_0(\theta) + t_n(\theta)$$

as the total price paid for those choose not to consume and

$$p(\theta) = t_b(\theta) - t_n(\theta)$$

[10] Another way to understand is to consider the followings, assuming truth telling of θ, given any direct mechanism $\{q(\theta, w), t(\theta, w)\}$, since by definition $q(\theta, w)$ is multiplicatively separable in both the utility and profit function, the optimal solution of $q(\theta, w)$ will have a cutoff, where one part of it goes to 1, the rest goes to 0. Hence the resulting $t(\theta, w)$ would boils downs to two values only.

as the extra price for those choose to consume.[11] For a consumer whose type is θ and announced $\hat{\theta}$, he will consume if $w \leq \theta - p(\hat{\theta})$.

1.4.1 Simple Cases

In this subsection, three simple cases will be analyzed, namely one type, one alternative, and no alternative. Apart from paving the path to better understand the basics of the model, each of the individual case is interesting in itself, and explains some phenomena. Readers with interest in the general result can skip this subsection.

1.4.1.1 One Type

Consider there is only one type of consumer where θ is known. Example may be the service being provided is linked to a good with a competitive market. In such a case, the resell value of the good is unique, and hence the value of the service linked to it should also be fixed and known. The maximization problem is stated as:

$$\max_{T,p} \pi = T + (p-c)\,G(\theta - p)$$
$$\text{s.t. } U = (\theta - p)\,G(\theta - p) + \int_{\theta-p}^{\bar{w}} w dG(w) - T \geq \int_0^{\bar{w}} w dG(w)$$

The participation constraint is binding and it can be simplified as

$$T = \int_0^{\theta-p} G(w)\,dw.$$

Thus the maximization problem can be rewritten as

$$\max_{p} \pi = \int_0^{\theta-p} G(w)\,dw + (p-c)\,G(\theta-p)\,.$$

The first order condition gives us

$$p^* = c,$$

where the second order condition boils downs to $G'(\cdot) \geq 0$, which is satisfied by assumption. Then we have

$$T^* = \int_0^{\theta-c} G(w)\,dw = \pi^*.$$

The solution is in the same spirit of classical two part tariff solution, where charging the

[11] Because $T(\theta)$ is the choice variable of the following analysis, which is a lumpsum term for the ex-ante payment and the possible refunds. Hence our model will not give an unique solution of refunding.

marginal cost for usage, and let the fixed fee capture the surplus. The only difference is that part of the surplus being captured is actually less than the consumer surplus $\theta - c$.

Lemma 1.6. $T^* < \theta - c$.

Proof.

$$\theta - c - T^* = \theta - c - \int_0^{\theta - c} G(w)\, dw = \int_0^{\theta - c} (1 - G(w))\, dw > 0$$

□

This result is intuitive, as the consumer has an alternative, the maximum amount can be extracted depends on the expected value of the alternative, which is determined by the $G(\cdot)$ function. Thus the higher is the expected value of the alternative, the lower is the consumer surplus can be extracted.

1.4.1.2 One Alternative

Now another extreme case is considered, where there is a known alternative w available to all consumers. One possibility is that a big event is expected to be popular, like world cup, hence it is known to both seller and consumers. As there is no unique optimal schedule of ex-ante and ex-post prices in this case, another way of modeling is used. Define $q(\cdot)$ as the probability that consumption takes place, and $\tau(\cdot)$ as the total payment. The utility of the consumer is,

$$U(\theta) = q(\theta)\theta + (1 - q(\theta))w - \tau(\theta).$$

By the envelope theorem

$$U'(\theta) = q(\theta).$$

With this equation and $q'(\theta) \geq 0$, which is also the second order condition of the utility maximization problem, we have incentive compatibility for all θ. Since this is a standard result, proof is omitted.

The profit function is

$$\begin{aligned}\pi &= \int_{\underline{\theta}}^{\bar{\theta}} (\tau(\theta) - q(\theta)c)\, dF(\theta) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} q(\theta)\left(\theta - w - c - \frac{1 - F(\theta)}{F'(\theta)}\right) dF(\theta).\end{aligned}$$

Thus the optimal $q^*(\theta)$ is

$$q^*(\theta) = \begin{cases} 1 & \text{if } \theta - w - c - \frac{1-F(\theta)}{F'(\theta)} \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

The interpretation of the condition of $q^*(\theta)$ is similar to Bulow and Roberts (1989): the seller's alternative adjusted marginal revenue from the buyer with type θ and alternative w is $\theta - \frac{1-F(\theta)}{F'(\theta)} - w$. The good is sold whenever this expression is greater the marginal cost c, and the total revenue is the area under the alternative adjusted marginal revenue curve.

Define $\tilde{\theta}$ as the marginal type such that $\tilde{\theta} - w - \frac{1-F(\tilde{\theta})}{F'(\tilde{\theta})} = c$. For all $\theta > \tilde{\theta}$, the total payment is

$$\begin{aligned} \tau^*(\theta) &= q(\theta)(\theta - w) - \int_{\underline{\theta}}^{\theta} q(x)\,dx \\ &= (\theta - w) - \theta + \tilde{\theta} \\ &= \tilde{\theta} - w. \end{aligned}$$

If we want to implement this equilibrium by the ex-ante price (T) and ex-post price (p), we have the indifference result.

Lemma 1.7. *Both seller and consumers are indifferent to any $\{T(\theta), p(\theta)\}$, given (i) $T(\theta) + p(\theta) = \tau^*(\theta)$ and (ii) $p(\theta) \leq \theta - w$.*

Proof. For all $\theta > \tilde{\theta}$, $q^*(\theta) = 1$, under the current specification, the utility function can be rewritten as $U(\theta) = \theta - p(\theta) - T(\theta)$, provided that condition (ii) $p(\theta) \leq \theta - w$ is fulfilled, which ensure that the consumer consume with certainty. With this and condition (i), both consumers and sellers are indifferent to any $\{T(\theta), p(\theta)\}$. □

As a result, we do not have an unique optimal schedule of ex-ante and ex-post prices. This points out that the ex-post price will affect the allocation, while ex-ante price is a lump sum payment. Surprisingly, there are no restriction on $p'(\theta)$ here, where we will see in the general analysis, $p'(\theta) \leq 0$.

1.4.1.3 No Alternative

We can study the standard case with no alternative easily by setting $w = 0$ in the previous case. Accordingly,

$$q^*(\theta) = \begin{cases} 1 & \text{if } \theta - c - \frac{1-F(\theta)}{F'(\theta)} \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Define $\hat{\theta}$ as the marginal type such that $\hat{\theta} - c - \frac{1-F(\hat{\theta})}{F'(\hat{\theta})} = 0$. For all $\theta > \hat{\theta}$, the resulting total payment is

$$\tau^*(\theta) = \hat{\theta} = c + \frac{1 - F(\hat{\theta})}{F'(\hat{\theta})}.$$

The standard theory suggests that, under the current setting, the monopoly seller will not practice price discrimination, where all consumers pay the same price.

1.4.2 Observable alternative

After seeing the three special cases, we are just one step towards the general analysis. In the general problem, there are different types of consumers and each of them has an uncertain alternative. In this subsection, we study a case where w is a public information, all else is the same as the general problem. This will be used as a benchmark case, as it generates the same expected profit for the seller in the general case. Apart from this, this case is also interesting to study in itself, as it represents the situation where the seller observes the realized alternative, example includes small village or under a well connected network of sellers and buyers.

Suppose the seller observes w, then we only need to consider incentive constraints for reporting θ. Consider the seller commits to a mechanism $\{q(\theta,w), \tau(\theta,w)\}$ before she observes w, where $q(\theta,w)$ is the probability of consumption takes place, and $\tau(\theta,w)$ is the transfer. The expected utility of a consumer is

$$\begin{aligned} U\left(\theta,\hat{\theta}\right) &= \int_0^{\bar{w}} \left[\theta q(\hat{\theta},w) + \left(1 - q(\hat{\theta},w)\right) w - \tau(\hat{\theta},w)\right] dG(w) \\ &= \int_0^{\bar{w}} \left[w + (\theta - w)\, q(\hat{\theta},w) - \tau(\hat{\theta},w)\right] dG(w). \end{aligned} \tag{1.4.1}$$

By the envelope theorem,

$$U'(\theta) = \int_0^{\bar{w}} q(\theta,w)\, dG(w).$$

Integrating over θ, we obtain

$$\begin{aligned} U(\theta) &= \int_{\underline{\theta}}^{\theta} \int_0^{\bar{w}} q(x,w)\, dG(w)\, dx + U(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\theta} \int_0^{\bar{w}} q(x,w)\, dG(w)\, dx + \int_0^{\bar{w}} w dG(w). \end{aligned} \tag{1.4.2}$$

The second order condition of the utility maximization problem boils downs to $q(\theta,w)$ being increasing in θ. Using equation (1.4.1) and (1.4.2), we obtain

$$\int_0^{\bar{w}} \tau(\theta, w)\, dG(w) = \int_0^{\bar{w}} \left[(\theta - w)\, q(\theta, w) - \int_{\underline{\theta}}^{\theta} q(x, w)\, dx \right] dG(w).$$

The profit function of the seller is

$$\begin{aligned}
\pi &= \int_{\underline{\theta}}^{\bar{\theta}} \int_0^{\bar{w}} \left(\tau(\theta, w) - c q(\theta, w)\right) dG(w)\, dF(\theta) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \int_0^{\bar{w}} \left((\theta - w - c)\, q(\theta, w) - \int_{\underline{\theta}}^{\theta} q(x, w)\, dx \right) dG(w)\, dF(\theta) \\
&= \int_0^{\bar{w}} \int_{\underline{\theta}}^{\bar{\theta}} \left(\theta - w - c - \frac{1 - F(\theta)}{F'(\theta)} \right) q(\theta, w)\, dF(\theta)\, dG(w).
\end{aligned}$$

The seller maximizes π by choosing $q(\theta, w)$. The constraint to the problem is $\int_0^{\bar{w}} q(\theta, w)\, dG(w)$ is weakly increasing in θ. By pointwise maximization,

$$q^*(\theta, w) = \begin{cases} 1 & \text{if } \theta - w - c - \frac{1 - F(\theta)}{F'(\theta)} \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1.4.3}$$

By the standard assumption of decreasing hazard rate, $q^*(\theta, w)$ is increasing in θ, and hence the constraint is satisfied. Define $\underset{\sim}{\theta}(w)$ to be the lowest θ such that $q^*(\theta, w) = 1$, given w. From equation (1.4.3),

$$\underset{\sim}{\theta} - \frac{1 - F(\underset{\sim}{\theta})}{F'(\underset{\sim}{\theta})} = w + c.$$

Because of the assumption of the decreasing hazard rate, $\underset{\sim}{\theta}$ is increasing in w. Given any (θ, w) such that $q^*(\theta, w) = 0$, the optimal transfer is $\tau^*(\theta, w) = 0$. Given any (θ, w) such that $q^*(\theta, w) = 1$, the optimal transfer is

$$\begin{aligned}
\tau^*(\theta, w) &= (\theta - w) - \int_{\underset{\sim}{\theta}(w)}^{\theta} dx, \\
&= \underset{\sim}{\theta}(w) - w \\
&= c + \frac{1 - F(\underset{\sim}{\theta}(w))}{F'(\underset{\sim}{\theta}(w))}.
\end{aligned}$$

Because of the assumption of decreasing hazard rate and $\underset{\sim}{\theta}'(w) \geq 0$, $\tau^*(\theta, w)$ is decreasing in w. Note that the optimal transfer $\tau^*(\theta, w)$ does not depend on θ, thus the payment is like a lucky draw. The type θ matters only for whether the service will be delivered. As a result, the maximized profit is

$$\begin{aligned}
\pi^* &= \int_{\underline{\theta}}^{\bar{\theta}} \int_0^{\bar{w}} \left(\theta - w - c - \frac{1-F(\theta)}{F'(\theta)}\right) q^*(\theta, w)\, dG(w)\, dF(\theta) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \int_0^{\theta - c - \frac{1-F(\theta)}{F'(\theta)}} \left(\theta - w - c - \frac{1-F(\theta)}{F'(\theta)}\right) dG(w)\, dF(\theta) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right) G\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right) - \int_0^{\theta - c - \frac{1-F(\theta)}{F'(\theta)}} w\, dG(w)\, dF(\theta) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \int_0^{\theta - \frac{1-F(\theta)}{F'(\theta)} - c} G(w)\, dw\, dF(\theta). \qquad (1.4.4)
\end{aligned}$$

This profit should be a upper bound for the general problem where w is private information, as the seller have a free access to the information about the alternatives. In what follows, I will show that this profit can be attained by a simple pricing mechanism even when the seller does not observe w.

1.4.3 General Analysis

In this section, I will study the general problem. I will show that the optimal mechanism can be implemented by a simple two part tariff $\{T(\theta), p(\theta)\}$, which depends only on the report of the type. I will show that the seller can achieve the same expected profit as in the case of alternative is observable to the seller. The only difference from the section 1.4.2 is that w is private information and no longer observed by the seller. In the general problem, there are a continuum of consumers having $\theta \in [\underline{\theta}, \bar{\theta}]$ and each of them has an uncertain alternative $w \in [0, \bar{w}]$. Both θ and w are private information to the consumer. The expected utility of a type θ consumer who reports $\hat{\theta}$ is

$$\begin{aligned}
U(\theta, \hat{\theta}) &= G(\theta - p(\hat{\theta}))(\theta - p(\hat{\theta})) - T(\hat{\theta}) + \int_{\theta - p(\hat{\theta})}^{\bar{w}} w dG(w) \\
&= \bar{w} - T(\hat{\theta}) - \int_{\theta - p(\hat{\theta})}^{\bar{w}} G(w)\, dw. \qquad (1.4.5)
\end{aligned}$$

By the envelope theorem,

$$U'(\theta) = G(\theta - p(\theta)).$$

Integrating over θ, we get:

$$\begin{aligned} U(\theta) &= \int_{\underline{\theta}}^{\theta} G(x - p(x))\,dx + U(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\theta} G(x - p(x))\,dx + \int_0^{\bar{w}} w dG(w)\,. \end{aligned} \tag{1.4.6}$$

The second order condition of the utility maximization problem boils down to $p'(\theta) \leq 0$.[12]

Lemma 1.8. *For all $\theta \in [\theta, \bar{\theta}]$, $U(\cdot)$ and $p(\cdot)$ are incentive compatible if*

$$\begin{aligned} &U'(\theta) = G(\theta - p(\theta)) \\ &p'(\theta) \leq 0 \end{aligned}$$

Proof. Assume to the contrary that $U(\theta, \hat{\theta}) > U(\theta, \theta)$, where $\hat{\theta} \neq \theta$. This is equivalent to

$$0 < \int_{\theta}^{\hat{\theta}} U_2(\theta, x)\,dx = \int_{\theta}^{\hat{\theta}} U_2(\theta, x) - U_2(x, x)\,dx = \int_{\theta}^{\hat{\theta}} \int_{x}^{\theta} U_{12}(y, x)\,dydx,$$

where the first equality is using the fact that $U_2(\theta, \theta) = 0$. I can show that

$$U_{12}(y, x) = \frac{\partial G(y - p(x))}{\partial x} = -G'(y - p(x))\,p'(x) \geq 0.$$

If $\hat{\theta} > \theta$, $x \geq \theta$ for all $x \in [\theta, \hat{\theta}]$, and the inequality cannot hold. Similarly, if $\hat{\theta} < \theta$, $x \leq \theta$ for all $x \in [\theta, \hat{\theta}]$, and again we obtain a contradiction. □

By equation (1.4.5) and (1.4.6), we have

$$T(\theta) = \int_0^{\theta - p(\theta)} G(w)\,dw - \int_{\underline{\theta}}^{\theta} G(x - p(x))\,dx. \tag{1.4.7}$$

Corollary 1.2. $T'(\theta) \geq 0$.

Proof. Differentiate equation (1.4.7) with θ,

$$\begin{aligned} T'(\theta) &= \left(1 - p'(\theta)\right) G(\theta - p(\theta)) - G(\theta - p(\theta)) \\ &= -p'(\theta)\,G(\theta - p(\theta)) \geq 0 \end{aligned}$$

We have the last inequality because $p'(\theta) \leq 0$. □

From lemma 1.8, we have a $p(\theta)$ decreasing in θ, if we look for an incentive compatible mechanism. Corollary 1.2 tells us that $T(\theta)$ is increasing in θ, given that $p(\theta)$ is decreasing in

[12]From the first order condition $U_2(\theta, \theta) = 0$, we have $U_{22}(\theta, \theta) = -U_{12}(\theta, \theta)$.

θ. After seeing the general pattern for the prices, we can find out the optimal prices. But first we need the profit function,

$$\begin{aligned}\pi &= \int_{\underline{\theta}}^{\overline{\theta}} \left(T\left(\theta\right)+\left(p\left(\theta\right)-c\right)G\left(\theta-p\left(\theta\right)\right)\right)dF\left(\theta\right)\\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_0^{\theta-p(\theta)} G\left(w\right)dw - \int_{\underline{\theta}}^{\theta} G\left(x-p\left(x\right)\right)dx + \left(p\left(\theta\right)-c\right)G\left(\theta-p\left(\theta\right)\right)\right]dF\left(\theta\right).\end{aligned}$$

The second term can be rewritten as:

$$\begin{aligned}\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\theta} G\left(x-p\left(x\right)\right)dxdF\left(\theta\right) &= \int_{\underline{\theta}}^{\overline{\theta}} G\left(x-p\left(x\right)\right)dx - \int_{\underline{\theta}}^{\overline{\theta}} F\left(\theta\right)G\left(\theta-p\left(\theta\right)\right)d\theta\\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left(1-F\left(\theta\right)\right)G\left(\theta-p\left(\theta\right)\right)d\theta. \qquad (1.4.8)\end{aligned}$$

With equation (1.4.8), profit can be written as

$$\pi = \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_0^{\theta-p(\theta)} G\left(w\right)dw + \left(p\left(\theta\right)-c-\frac{1-F\left(\theta\right)}{F'\left(\theta\right)}\right)G\left(\theta-p\left(\theta\right)\right)\right]dF\left(\theta\right).$$

As a result, the maximization problem can be stated as,

$$\begin{aligned}\max_{\{p(\cdot)\}} \pi &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_0^{\theta-p(\theta)} G\left(w\right)dw + \left(p\left(\theta\right)-c-\frac{1-F\left(\theta\right)}{F'\left(\theta\right)}\right)G\left(\theta-p\left(\theta\right)\right)\right]dF\left(\theta\right),\\ s.t.\ & p'\left(\theta\right)\le 0,\\ & U\left(\theta\right)\ge\int_0^{\bar{w}} wdG\left(w\right),\ \forall\theta.\end{aligned}$$

Since $U'\left(\theta\right) = G\left(\theta-p\left(\theta\right)\right)\ge 0$, the participation constraint can be replaced by $U(\underline{\theta}) = \int_0^{\bar{w}} wdG\left(w\right)$, which can be shown as $T(\underline{\theta}) = \int_0^{\underline{\theta}-p(\underline{\theta})} G\left(w\right)dw$. Ignoring the two constraints, by point-wise maximization,

$$p^*\left(\theta\right) = c+\frac{1-F\left(\theta\right)}{F'\left(\theta\right)}.$$

The second order condition boils downs to $G'\left(\cdot\right)\ge 0$, which is satisfied by assumption.

In order to satisfy the constraint $p'\left(\theta\right)\le 0$, we need the standard assumption for the hazard rate, which is $\frac{1-F(\theta)}{F'(\theta)}$ is decreasing in θ. All convex $F\left(\cdot\right)$ satisfies this requirement, but convexity is only sufficient but not necessary.[13]

[13] A sufficient and necessary condition is $F\left(\theta\right) < 1+\frac{F'(\theta)}{F''(\theta)},\ \forall\theta$.

Proposition 1.2. *The profit maximizing mechanism is*

$$
\begin{aligned}
p^*(\theta) &= c + \frac{1-F(\theta)}{F'(\theta)}, \\
T^*(\theta) &= \int_0^{\theta - c - \frac{1-F(\theta)}{F'(\theta)}} G(w)\,dw - \int_{\underline{\theta}}^{\theta} G\left(x - c - \frac{1-F(x)}{F'(x)}\right) dx, \\
\pi^* &= \int_{\underline{\theta}}^{\overline{\theta}} \int_0^{\theta - \frac{1-F(\theta)}{F'(\theta)} - c} G(w)\,dw\,dF(\theta).
\end{aligned}
\tag{1.4.9}
$$

We can see that the profit maximizing $p^*(\cdot)$ does not depend on the $G(\cdot)$. It is optimal to charge customer at different times, partly ex-ante and partly ex-post, when they are facing uncertain alternatives. With high valued customer pay mainly by ex-ante price, low valued customer pay mainly by ex-post price. We have the usual no distortion at the top result, as $p^*(\bar{\theta}) = c$. Then the corresponding probability of consumption is $G(\bar{\theta} - c)$, which is efficient. However for all $\theta < \bar{\theta}$, there will be downward distortion of the probability of consumption. From here it shows that, $T(\cdot)$ is a lump sum payment which have no effect on the allocation, while $p(\cdot)$ has a important implication on allocation. The following is a concrete example:

Example 1.1. Uniform Distributions

Assume $F(\cdot) = \frac{\theta}{\bar{\theta}}$, $G(\cdot) = \frac{w}{\bar{w}}$, $c = 0$,

$$
\begin{aligned}
p^*(\theta) &= \bar{\theta} - \theta, \\
T^*(\theta) &= \frac{2\theta\bar{\theta} + \bar{\theta}^2}{2\bar{w}}, \\
\pi^* &= \frac{\bar{\theta}^2}{12\bar{w}}.
\end{aligned}
$$

By proposition 1.1, if we restrict to uniform pricing, the monopolist will sell at both pre-sale and on spot market. If we open up the possibility of non-uniform pricing, we will still have on spot market sales, with the condition explained in lemma 1.10. But the pure pre-sale option will not be provided, as shown in lemma 1.9.

Lemma 1.9. *The optimal selling mechanism do not include pure ex-ante payment options.*

Proof. Since $p(\cdot)$ is decreasing in θ, and $p(\bar{\theta}) = c$. Thus there is no payment option which is purely ex-ante. □

Lemma 1.10. *There are payment options which are purely ex-post, when* $(\underline{\theta} - c)F'(\underline{\theta}) \leq 1$.

Proof. Consider the optimal expected utility of $\underline{\theta}$,

$$
U(\underline{\theta}) = \bar{w} - T(\underline{\theta}) - \int_{\underline{\theta} - p(\underline{\theta})}^{\bar{w}} G(w)\,dw = \int_0^{\bar{w}} w\,dG(w).
$$

Therefore,

$$T(\underline{\theta}) = \int_0^{\underline{\theta}-\frac{1}{F'(\underline{\theta})}-c} G(w)\,dw.$$

When $\underline{\theta} - \frac{1}{F'(\underline{\theta})} \leq c$, because by assumption $G(w) = 0,\ \forall w \leq 0$, we have $T(\underline{\theta}) = 0$. Define $\tilde{\theta}$ such that $\tilde{\theta} - \frac{1-F(\tilde{\theta})}{F'(\tilde{\theta})} = c$, thus $T(\theta) = 0,\ \forall \theta \in [\underline{\theta}, \tilde{\theta}]$. Since $p'(\theta) \leq 0$, no one will choose other payment options with $T(\theta) = 0$, but only with $\left(T(\tilde{\theta}), p(\tilde{\theta})\right) = \left(0, c + \frac{1-F(\tilde{\theta})}{F'(\tilde{\theta})}\right)$. □

With the interpretation of Bulow and Roberts (1989), $\theta - \frac{1-F(\theta)}{F'(\theta)}$ is the marginal revenue from consumer with value θ. The definition of $\tilde{\theta}$ is indeed the standard condition where marginal revenue equals marginal cost. By the decreasing hazard rate assumption, marginal revenue is increasing in θ, thus for all $\theta \in [\underline{\theta},\tilde{\theta}]$, marginal revenue is less than marginal cost. Hence they are the portion of the market will not be served, and facing only with the alternative.

There are two ways to understand the condition $(\underline{\theta} - c)F'(\underline{\theta}) \leq 1$: i) the lowest value is very close to the marginal cost c; ii) there are very few people with the lowest value, and they have different reasoning. The pure ex-post payment option is actually decided for consumer with high uncertainty to consume, namely those with very low θ. The lower $\underline{\theta}$ is, the more likely we have those payment options. When $F'(\underline{\theta})$ becomes lower, $p(\underline{\theta})$ becomes higher, since $U(\underline{\theta}) = \int_0^{\bar{w}} w dG(w)$ and $\frac{dT(\theta)}{dp(\theta)}\Big|_{U=\bar{U}} = -G(\theta - p(\theta)) \leq 0$, we are more likely to have a pure ex-post payment option.

Recall that in section 1.4.1, when there is only one certain alternative or no alternative, monopolist seller will not price discriminates consumers. The following two lemmas show clearly, it will no longer be the case. With uncertain alternatives, monopolist seller price discriminates, no matter in the realized sense or expected sense.

Lemma 1.11. *The total price $\tau(\theta) = p(\theta) + T(\theta)$ is decreasing in θ.*

Proof. Define $d\left(\frac{1-F(\theta)}{F'(\theta)}\right)\Big/ d\theta = \delta(\theta)$. By definition, the total price is

$$\tau(\theta) = c + \frac{1-F(\theta)}{F'(\theta)} + \int_0^{\theta - c - \frac{1-F(\theta)}{F'(\theta)}} G(w)\,dw - \int_{\underline{\theta}}^{\theta} G\left(x - c + \frac{1-F(x)}{F'(x)}\right) dx.$$

Differentiate it with respect to θ,

$$\begin{aligned}\tau'(\theta) &= \delta(\theta) + (1-\delta(\theta))\,G\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right) - G\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right)\\ &= \delta(\theta)\left(1 - G\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right)\right) \leq 0.\end{aligned}$$

□

Since high valued consumer pay mainly by ex-ante payment, I have a similar result as in Eső & Szentes (2007) and Nocke, Peitz & Rosar (2011), where there are discounts to ex-ante payment.

Lemma 1.12. *The expected total price $E\tau(\theta) = G(\theta - p(\theta))\,p(\theta) + T(\theta)$ is increasing in θ.*

Proof. Define $d\left(\frac{1-F(\theta)}{F'(\theta)}\right)\Big/ d\theta = \delta(\theta)$. By definition, the expected total price is

$$E\tau(\theta) = G\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right)\left(c + \frac{1-F(\theta)}{F'(\theta)}\right) + \int_0^{\theta - c - \frac{1-F(\theta)}{F'(\theta)}} G(w)\,dw$$
$$- \int_{\underline{\theta}}^{\theta} G\left(x - c + \frac{1-F(x)}{F'(x)}\right)dx.$$

Differentiate it with respect to θ,

$$\begin{aligned} E\tau'(\theta) &= G'\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right)(1-\delta(\theta))\left(c + \frac{1-F(\theta)}{F'(\theta)}\right) + G\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right)\delta(\theta) \\ &\quad + (1-\delta(\theta))\,G\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right) - G\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right) \\ &= G'\left(\theta - c - \frac{1-F(\theta)}{F'(\theta)}\right)(1-\delta(\theta))\left(c + \frac{1-F(\theta)}{F'(\theta)}\right) \geq 0. \end{aligned}$$

□

Theorem 1.1. *The profit maximizing mechanism gives the same outcome as the benchmark case.*

Proof. For the profit, equation (1.4.9) and (1.4.4) are exactly the same. For the allocation, recall equation (1.4.3), $q^*(\theta, w) = 1$ if $\theta - \frac{1-F(\theta)}{F'(\theta)} - c \geq w$. The probability of consumption, for type θ, is $G\left(\theta - \frac{1-F(\theta)}{F'(\theta)} - c\right)$. Where in the profit maximizing mechanism, the consumer consumes whenever $w \leq \theta - p(\theta)$, which happens with probability $G\left(\theta - \frac{1-F(\theta)}{F'(\theta)} - c\right)$. □

This result has some favor of the Theorem 1 of Eső and Szentes (2007), where the seller's expected revenue is the same as if she could observe the "new" information. The main differences are listed as follows. First, the information structure is very different. In Eső and Szentes (2007), buyers get the second period signal to update the value for the object. However, in my model, the value of the service is determined solely by the type of the buyer in the first period. Hence the "new information" affects, in Eső and Szentes (2007), the incentive compatibility constraint, while in my model, w affects the participation constraint. Secondly, whether the buyers get the addition signal in the second period is decided by the seller in Eső and Szentes (2007). In my model, it is just a matter of time, that w will be revealed to the consumer. Thirdly, there is one indivisible object to sell in their model, while in my model, anyone can get the service provided that she pays the price. Fourthly, in the case of very high realization of w, for example, $w \geq \bar{\theta}$, the consumer will not consume the service no matter what the price is, unless it goes negative. However in Eső and Szentes (2007), the buyer with highest value will always buy the good, the only difference is the price the buyer is willing to pay.

Theorem 1.2. *The expected profit is higher when consumers face a more volatile alternative.*

Proof. Consider a mean-preserving spread function of $G(\cdot)$, defined as $\hat{G}(\cdot)$. The profit function becomes

$$\pi^* = \int_{\underline{\theta}}^{\overline{\theta}} \int_0^{\theta - c - \frac{1-F(\theta)}{F'(\theta)}} \hat{G}(w)\, dw dF(\theta).$$

Since $G(\cdot)$ and $\hat{G}(\cdot)$ cross only once, denote the crossing point as $\hat{w}$. Because of the mean-preserving spread, $\hat{G}(\cdot) > G(\cdot)$, for all $w < \hat{w}$. Hence we get the result. □

The intuition of this result can be understand as the seller is providing an insurance-like service. If the consumer gets unlucky, a low realization of the alternative happens, she can consume the service, on the other hand, lucky consumer will enjoy the high value alternative. A mean-preserving spread smooths out and increase the density of the two ends. But the low end is covered by the seller, hence it is only the high end has effect on the consumer well being. As the expected utility of the consumer is higher, there is more surplus to extract. Thus the expected profit is higher. One interpretation for this result is the different size or intensity of social network. People having weak and limited linkage with other people are more likely to face a more volatile alternative. On one hand, it is very likely nothing happens, given the size of the social network. On the other hand, he will not be informed even if there is any, given his halfhearted social network. As a result, the leftovers are big events that affect everyone, for instance blackout or blizzard.

1.4.4 Standard formulation

In this section, I will formulate the problem with the standard mechanism design approach and show that theorem 1 holds under the standard assumptions. The seller announces and commits to a mechanism $\Gamma = (q, \tau)$ at date 0, where $q(\hat{\theta}, \hat{w})$ is the probability that consumer consumes, $\tau(\hat{\theta}, \hat{w})$ is the transfer to the seller. The consumer reports $\hat{\theta}$ at date 0 and $\hat{w}$ at date 1. By backward induction, at date 1, the payoff of a consumer who observes (θ, w) and reports $(\hat{\theta}, \hat{w})$ is

$$\begin{aligned} u(w, \hat{w}; \theta, \hat{\theta}) &= \theta q(\hat{\theta}, \hat{w}) + w(1 - q(\hat{\theta}, \hat{w})) - \tau(\hat{\theta}, \hat{w}) \\ &= w + (\theta - w)\, q(\hat{\theta}, \hat{w}) - \tau(\hat{\theta}, \hat{w}). \end{aligned}$$

The first order condition is

$$(\theta - w)\, q_2(\hat{\theta}, \hat{w}) - \tau_2(\hat{\theta}, \hat{w}) = 0. \tag{1.4.10}$$

The second order condition for the optimal report of $\hat{w}$ is $q_2(\hat{\theta}, w) \leq 0$. By the envelope theorem,

$$u_w(w, w; \hat{\theta}) = 1 - q(\hat{\theta}, w).$$

Taking integration over w gives

$$u(w, w; \hat{\theta}) = \int_0^w 1 - q(\hat{\theta}, x) dx + u(0, \hat{\theta}). \tag{1.4.11}$$

Lemma 1.13. *The two stage mechanism is incentive compatible in the second round after the truthful revelation in the first round, given $q(\theta, w)$ is weakly decreasing in w and*

$$u(w, w; \theta) - u(\hat{w}, \hat{w}; \theta) = \int_{\hat{w}}^w 1 - q(\theta, x)\, dx. \tag{1.4.12}$$

Proof. Equation (1.4.12) is derived from equation (1.4.11). From the definition of consumer payoff, we have

$$u(w, \hat{w}; \theta) = u(\hat{w}, \hat{w}; \theta) + (w - \hat{w})(1 - q(\theta, \hat{w})).$$

Incentive compatibility in the second stage requires $u(w, \hat{w}; \theta) \leq u(w, w; \theta)$, where we rewrite the difference as

$$u(w, \hat{w}; \theta) - u(w, w; \theta) = [u(w, \hat{w}; \theta) - u(\hat{w}, \hat{w}; \theta)] - [u(w, w; \theta) - u(\hat{w}, \hat{w}; \theta)].$$

By definition, we know the terms in the first square bracket equals to $(w - \hat{w})(1 - q(\theta, \hat{w}))$. From equation (1.4.12), the terms in the second square bracket equals to

$$\int_{\hat{w}}^w 1 - q(\theta, x)\, dx \geq \int_{\hat{w}}^w 1 - q(\theta, \hat{w})\, dx = (w - \hat{w})(1 - q(\theta, \hat{w})).$$

Because $q_2(\hat{\theta}, w) \leq 0$, the inequality holds. Hence deviating to report any $\hat{w} \neq w$ is not profitable. □

To complete the analysis, we need to consider the first stage incentive to report truthfully. From equation (1.4.10), we have an implicit function of the optimal report $\hat{w}$ given $(\theta, \hat{\theta}, w)$, denote this by $\hat{w} = \sigma(\theta, \hat{\theta}, w)$. Hence in the first stage, the expected payoff of a consumer of type θ and reports $\hat{\theta}$ is

$$U(\theta, \hat{\theta}) = \int_0^{\bar{w}} w + (\theta - w)\, q(\hat{\theta}, \sigma(\theta, \hat{\theta}, w)) - \tau(\hat{\theta}, \sigma(\theta, \hat{\theta}, w)) dG(w).$$

By the envelope theorem and implicit differentiation,

$$U'(\theta)=\int_0^{\bar{w}}\left[q(\hat{\theta},\sigma(\theta,\hat{\theta},w))-\frac{(\theta-w)\,q_2(\hat{\theta},\sigma(\theta,\hat{\theta},w))^2-\tau_2(\hat{\theta},\sigma(\theta,\hat{\theta},w))q_2(\hat{\theta},\sigma(\theta,\hat{\theta},w))}{(\theta-w)\,q_{22}(\hat{\theta},\sigma(\theta,\hat{\theta},w))-\tau_{22}(\hat{\theta},\sigma(\theta,\hat{\theta},w))}\right]dG(w)\,.$$

The second term in the square bracket equals zero because of equation (1.4.10). Hence the expect payoff of consumer type θ is

$$\begin{aligned} U(\theta) &= \int_{\underline{\theta}}^{\theta}\int_0^{\bar{w}} q(x,\sigma(x,x,w))\,dG(w)\,dx+U(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\theta}\int_0^{\bar{w}} q(x,w)\,dG(w)\,dx+U(\underline{\theta})\,. \end{aligned} \tag{1.4.13}$$

The second equality follows from lemma 1.13, where it is incentive compatible to report the true w after a truth report of θ. By the standard argument, we know that it is incentive compatible to report the truth in first stage given i) $q(\theta,w)$ is weakly increasing in θ and ii) $U'(\theta)=\int_0^{\bar{w}} q(\theta,w)dG(w)$. Equation (1.4.13) is the expected payoff of type θ consumer, and it is same as the one in the observable w case which is equation (1.4.2). Hence the seller's expect profit is the same as in the observable w case, which we have already shown in theorem 1.1.

1.4.5 Behavioral Deviation

In all sections above, consumers are perfectly rational. Recent studies in behavioral and experimental economics show that this is not always true. The advance payment separates or "decouples" the purchase from the consumption and in so doing seems to reduce the perceived cost of the activity (Gourvile and Soman 1998). Most telephone customers choose a flat rate service even though paying by the call would cost them less (Prelec and Loewenstein 1998). These studies are related to my paper in spirit, yet we look at different problems. A related behavioral deviation is to consider consumer have over-focus problem. Very often, when people are planning for the future, most of the attention is placed on the targeted activity, as a result insufficient consideration is given to the possible alternative events that may happen. In the extreme case, people disregard any other possible events, like sickness or accident, that may happen which prevent them from proceeding with their plan. It is not as unusual as it seems, people often get shocked when these alternative events happen, as if they are not expected to happen at all. I believe you can find examples of this kind easily from your daily life.

To model this, an extra term ψ is introduced, which is the probability that the alternative is neglected. The probability ψ can be understood as the reduced form of the complete model for the over-focus problem, which is abstract from here.[14] As a result, the expected value from the alternative is ψw at date 0. Assume $0<\psi\leq 1$. When $\psi=1$, then consumers are

[14] A complete model should be the case where the decision maker faces costs of acquiring, absorbing, and processing information, like Reis 2006.

perfectly aware of their future opportunity. When it converges to zero, consumers are basically over-focusing on the service, completely ignoring the alternative. It should be noted that this is different from discounting the future value. The only thing being "discounted" is the alternative but not the value of the service.[15] A priori, over-focus problem lower the welfare of consumers, because the consumer ignores possible alternatives, however this also lowering the price the seller can charge on consumers. The overall effect is not clear and it is interesting to see what will this brings. This subsection is devoted to the analysis of this irrationality, and I will show that consumer benefits from this irrationality.

The expected utility perceived by the consumer, who has over-focus problem represented by ψ, now becomes

$$\begin{aligned} U(\theta,\hat{\theta}) &= \int_0^{\bar{w}} \left[\theta q(\hat{\theta},\sigma(\theta,\hat{\theta},w)) + \psi w(1-q(\hat{\theta},\sigma(\theta,\hat{\theta},w))) - \tau(\hat{\theta},\sigma(\theta,\hat{\theta},w))\right] dG(w) \\ &= \int_0^{\bar{w}} \left[(\theta-\psi w)\, q(\hat{\theta},\sigma(\theta,\hat{\theta},w)) + \psi w - \tau(\hat{\theta},\sigma(\theta,\hat{\theta},w))\right] dG(w). \end{aligned}$$

Where $\sigma(\theta,\hat{\theta},w)$ is the optimal report of $\hat{w}$ as we have shown in the previous section, which is an implicit function comes from equation (1.4.10). By the envelope theorem,

$$U'(\theta) = \int_0^{\bar{w}} \left[q(\hat{\theta},\sigma(\theta,\hat{\theta},w)) - \frac{\left((\theta-\psi w)\, q_2(\hat{\theta},\sigma(\theta,\hat{\theta},w)) - \tau_2(\hat{\theta},\sigma(\theta,\hat{\theta},w))\right) q_2(\hat{\theta},\sigma(\theta,\hat{\theta},w))}{(\theta-w)\, q_{22}(\hat{\theta},\sigma(\theta,\hat{\theta},w)) - \tau_{22}(\hat{\theta},\sigma(\theta,\hat{\theta},w))}\right] dG(w)$$

If $\psi = 1$, then we have the same equation as in the perfectly rational case. For all $\psi < 1$, the second term in the square bracket is negative. The denominator is just the second order condition of the optimal $\hat{w}$ report, which is assumed to be negative. For the numerator, $(\theta-\psi w)\, q_2(\hat{\theta},\hat{w}) - \tau_2(\hat{\theta},\hat{w}) \leq 0$ because $q_2(\hat{\theta},\hat{w}) \leq 0$. Since $q_2(\hat{\theta},\hat{w}) \leq 0$, the numerator is positive. By lemma 1.13, $\sigma(\theta,\theta,w) = w$, taking integration over θ, we have

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \int_0^{\bar{w}} \left[q(x,w) - \frac{(x-\psi w)\, q_2(x,w) - \tau_2(x,w) q_2(x,w)}{(\theta-w)\, q_{22}(x,w) - \tau_{22}(x,w)}\right] dG(w)\, dx + U(\theta). \quad (1.4.14)$$

As a result, we have the following result.

Lemma 1.14. *Consumers benefit from over-focusing.*

Proof. In equation (1.4.14), the second term in the square bracket is negative, the expected utility perceived by the consumer is higher in the irrational case than the rational case. The actual utility must be higher than the one perceived by the consumer. □

The intuition that consumers benefit by over-focusing is that the seller will charge a lower price. With a lower ψ, given a $q(\cdot)$ and $\tau(\cdot)$, the expected utility of the consumer is lower.

[15] One can also interpret ψ as the difference in discount factor between the alternative and the service, with the normalization to one regarding the service.

As a result, the participation constraint is tighten, hence the maximum price the seller could be charged is lower.

1.4.6 Increasing Hazard rate

When the standard assumption of the hazard rate does not hold, i.e. $\frac{1-F(\theta)}{F'(\theta)}$ is increasing in θ, the incentive constraint will be binding, $p'(\cdot) = 0$. Thus that implies, by corollary 2, a constant $p(\cdot)$ and $T(\cdot)$. As a result, the maximization problem can be stated as follows,

$$\begin{aligned}\max_{T,p} \pi &= \int_{\underline{\theta}}^{\overline{\theta}} \left[T + (p-c) G(\theta - p)\right] dF(\theta), \\ s.t. U(\underline{\theta}) &= \int_0^{\bar{w}} w dG(w).\end{aligned}$$

Since $U(\underline{\theta}) = \bar{w} - T - \int_{\underline{\theta}-p}^{\bar{w}} G(w)\, dw$, the participation constraint can be rewritten as $T = \int_0^{\underline{\theta}-p} G(w)\, dw$. Hence, the maximization problem can be simplified as,

$$\begin{aligned}\max_{p} \pi &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_0^{\underline{\theta}-p} G(w)\, dw + (p-c) G(\theta - p)\right] dF(\theta) \\ &= \int_0^{\underline{\theta}-p} G(w)\, dw + \int_{\underline{\theta}}^{\overline{\theta}} (p-c) G(\theta - p) dF(\theta).\end{aligned}$$

The first order condition gives

$$p^* = c + \frac{\int_{\underline{\theta}}^{\overline{\theta}} G(\theta - p) dF(\theta) - G(\underline{\theta} - p)}{\int_{\underline{\theta}}^{\overline{\theta}} G'(\theta - p) dF(\theta)},$$

where the second order condition is

$$G'(\underline{\theta} - p) + \int_{\underline{\theta}}^{\bar{\theta}} \left[(p-c) G\text{''}(\theta - p) - 2G'(\theta - p)\right] dF(\theta) \leq 0.$$

Thus the profit maximization mechanism is

$$\begin{aligned}p^* &= c + \frac{\int_{\underline{\theta}}^{\overline{\theta}} G(\theta - p) dF(\theta) - G(\underline{\theta} - p)}{\int_{\underline{\theta}}^{\overline{\theta}} G'(\theta - p) dF(\theta)}, \\ T^* &= \int_0^{\underline{\theta}-p^*} G(w)\, dw.\end{aligned}$$

1.5 Correlated Alternatives

In this section, the independence assumption between θ and w is relaxed and we consider that θ and w is positively correlated. Hence people whom have higher θ will have a higher chance to get a higher valued alternative w. This can be interpret in two ways: i) More interested customers have a higher taste for similar events; ii) Wealthy people have more access to higher valued alternatives. The optimal ex-post price $p^*(\cdot)$ will depend on the $G(\cdot)$, and is shown to be lower than the independent case. The optimal ex-post price $p^*(\cdot)$ will be decreasing in θ if the correlation is not too strong, and the sufficient condition is $G(\cdot)$ being convex in w.

The distribution function of w now becomes $G(w,\theta)$. As before, I assume for all $\theta \in \left[\underline{\theta},\overline{\theta}\right]$, $G(0,\theta)=0$, $G(\bar{w},\theta)=1$. I assume first order stochastic dominance (FSD) for the function $G(w,\theta)$ to represent the positive relationship.

Assumption. *(FSD)* $G(w,\theta) \geq G(w,\theta')$ $,\forall\theta' \geq \theta.$

The expected utility of a type θ consumer who reports $\hat{\theta}$ is

$$U(\theta,\hat{\theta}) = G(\theta-p(\hat{\theta}),\theta)(\theta-p(\hat{\theta}))-T(\hat{\theta})+\int_{\theta-p(\hat{\theta})}^{\bar{w}} wG_w(w,\theta)\,dw.$$

By the envelope theorem,

$$\begin{aligned}
U'(\theta,\hat{\theta}) &= (G_w+G_\theta)(\theta-p(\hat{\theta}))+G(\theta-p(\hat{\theta}),\theta)-(\theta-p(\hat{\theta}))G_w+\int_{\theta-p(\hat{\theta})}^{\bar{w}} wG_{w\theta}dw \\
&= G(\theta-p(\hat{\theta}),\theta)+\bar{w}G_\theta(\bar{w},\theta)-\int_{\theta-p(\hat{\theta})}^{\bar{w}} G_\theta(w,\theta)\,dw \\
&= G(\theta-p(\hat{\theta}),\theta)-\int_{\theta-p(\hat{\theta})}^{\bar{w}} G_\theta(w,\theta)\,dw.
\end{aligned}$$

Set $\hat{\theta}=\theta$, integrating over θ,

$$\begin{aligned}
U(\theta) &= \int_{\underline{\theta}}^{\theta} G(x-p(x),x)\,dx-\int_{\underline{\theta}}^{\theta}\int_{x-p(x)}^{\bar{w}} G_\theta(w,x)\,dwdx+U(\underline{\theta}) \\
&= \int_{\underline{\theta}}^{\theta} G(x-p(x),x)\,dx-\int_{\underline{\theta}}^{\theta}\int_{x-p(x)}^{\bar{w}} G_\theta(w,x)\,dwdx+\int_{0}^{\bar{w}} wG_w(w,\underline{\theta})dw. \quad (1.5.1)
\end{aligned}$$

Lemma 1.15. *For all* $\theta \in [\underline{\theta},\bar{\theta}]$, $U(\cdot)$ *and* $p(\cdot)$ *are incentive compatible if*

i) $U'(\theta) = G(\theta-p(\theta),\theta)-\int_{\theta-p(\theta)}^{\bar{w}} G_\theta(w,\theta)\,dw,$

ii) $0 \geq p'(\theta)\left(G_w(\theta-p(\theta),\theta)+G_\theta(\theta-p(\theta),\theta)\right).$

Proof. Assume to the contrary that $U(\theta,\hat{\theta}) > U(\theta,\theta)$, where $\hat{\theta} \neq \theta$. This is equivalent to

$$0 < \int_{\theta}^{\hat{\theta}} U_2(\theta,x)\,dx = \int_{\theta}^{\hat{\theta}} U_2(\theta,x) - U_2(x,x)\,dx = \int_{\theta}^{\hat{\theta}} \int_{x}^{\theta} U_{12}(y,x)\,dydx,$$

where the first equality is using the fact that $U_2(\theta,\theta) = 0$. We can show that

$$\begin{aligned} U_{12}(y,x) &= \frac{\partial G(y-p(x),y) - \int_{y-p(x)}^{\bar{w}} G_\theta(w,y)\,dw}{\partial x} \\ &= -\left(G_w(y-p(x),x) + G_\theta(y-p(x),x)\right)p'(x) \geq 0. \end{aligned}$$

If $\hat{\theta} > \theta$, $x \geq \theta$ for all $x \in [\theta,\hat{\theta}]$, and the inequality cannot hold. Similarly, if $\hat{\theta} < \theta$, $x \leq \theta$ for all $x \in [\theta,\hat{\theta}]$, and again we obtain a contradiction. □

From above, we have the second order condition of the utility maximization problem: [16]

$$p'(\theta)\left(G_w(\theta-p(\theta),\theta) + G_\theta(\theta-p(\theta),\theta)\right) \leq 0. \tag{1.5.2}$$

Thus we have the following lemma,

Lemma 1.16. *$p'(\theta)$ and $G_w(\theta-p(\theta),\theta) + G_\theta(\theta-p(\theta),\theta)$ are of different sign.*

From the assumption I made for the $G(w,\theta)$, for all w and θ, $G_w(w,\theta) \geq 0$ and $G_\theta(w,\theta) \leq 0$. When interdependence is weak such that $G_w(w,\theta) \geq -G_\theta(w,\theta)$, for all w and θ, we have a price schedule decreasing in θ, as in the independent case. However when the interdependence becomes too strong such that $G_w(w,\theta) \leq -G_\theta(w,\theta)$, we have a price schedule increasing in θ.

To derive the optimal pricing schedule, we need to do some manipulation to the profit function:

$$\pi = \int_{\underline{\theta}}^{\overline{\theta}} \left(T(\theta) + (p(\theta)-c)\,G(\theta-p(\theta),\theta)\right)dF(\theta).$$

Since we know

$$T(\theta) = (\theta-p(\theta))\,G(\theta-p(\theta),\theta) + \int_{\theta-p(\theta)}^{\bar{w}} wG_w(w,\theta)\,dw - U(\theta),$$

hence the profit function can now be written as

[16] From the first order condition $U_2(\theta,\theta) = 0$, we have $U_{22}(\theta,\theta) = -U_{12}(\theta,\theta)$.

$$\pi = \int_{\underline{\theta}}^{\overline{\theta}} \left[(\theta - c) G(\theta - p(\theta), \theta) + \int_{\theta - p(\theta)}^{\bar{w}} w G_w(w, \theta) dw - \int_{\underline{\theta}}^{\theta} G(x - p(x), x) dx \right.$$
$$\left. + \int_{\underline{\theta}}^{\theta} \int_{x - p(x)}^{\bar{w}} G_\theta(w, x) dw dx - \int_0^{\bar{w}} w G_w(w, \underline{\theta}) dw \right] dF(\theta).$$

By integration by parts, the third term can be rewritten as,

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} G(x - p(x), x) dx dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} G(x - p(x), x) dx - \int_{\underline{\theta}}^{\overline{\theta}} F(\theta) G(\theta - p(\theta), \theta) d\theta$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} (1 - F(\theta)) G(\theta - p(\theta), \theta) d\theta.$$

Similarly, the fourth term can be rewritten as

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} \int_{x - p(x)}^{\bar{w}} G_\theta(w, x) dw dx dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \int_{x - p(x)}^{\bar{w}} G_\theta(w, x) dw dx - \int_{\underline{\theta}}^{\overline{\theta}} F(\theta) \int_{\theta - p(\theta)}^{\bar{w}} G_\theta(w, \theta) dw d\theta$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} (1 - F(\theta)) \int_{\theta - p(\theta)}^{\bar{w}} G_\theta(w, \theta) dw d\theta.$$

Thus the profit function can be rewritten as

$$\pi = \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta - c - \frac{1 - F(\theta)}{F'(\theta)} \right) G(\theta - p(\theta), \theta) + \left(\frac{1 - F(\theta)}{F'(\theta)} \right) \int_{\theta - p(\theta)}^{\bar{w}} G_\theta(w, \theta) dw \right.$$
$$\left. + \int_{\theta - p(\theta)}^{\bar{w}} w G_w(w, \theta) dw - \int_0^{\bar{w}} w G_w(w, \underline{\theta}) dw \right] dF(\theta).$$

The first order condition with respect to $p(\theta)$ gives

$$p^*(\theta) = c + \frac{1 - F(\theta)}{F'(\theta)} \left(1 + \frac{G_\theta(\theta - p^*(\theta), \theta)}{G_w(\theta - p^*(\theta), \theta)} \right).$$

The second order condition is $\left(p(\theta) - c - \frac{1 - F(\theta)}{F'(\theta)} \right) G_{ww} - \frac{1 - F(\theta)}{F'(\theta)} G_{\theta w} - G_w < 0$. With the first order condition, it can be rewritten as

$$\frac{1 - F(\theta)}{F'(\theta)} \left(\frac{G_\theta}{G_w} G_{ww} - G_{\theta w} \right) - G_w < 0. \tag{1.5.3}$$

If we consider $G(\cdot, \cdot)$ be a convex function, the sufficient condition for second order condition is $\frac{G_\theta}{G_w} \leq \frac{G_{w\theta}}{G_{ww}}$. When $p'(\theta) \leq 0$, $\frac{G_\theta}{G_w} \geq -1$, so the sufficient condition can be simplified into $-1 \leq \frac{G_{w\theta}}{G_{ww}}$. Which requires the correlation effect on G_w cannot be too weak. The condition boils down to $-G_{ww} \leq G_{w\theta}(\bar{\theta}-p(\bar{\theta}),\bar{\theta})$. As a result,

Proposition 1.3. *When values are correlated, the profit maximizing mechanism is*

$$p^*(\theta) = c + \frac{1-F(\theta)}{F'(\theta)}\left(1+\frac{G_\theta(\theta-p^*(\theta),\theta)}{G_w(\theta-p^*(\theta),\theta)}\right) \tag{1.5.4}$$

$$T^*(\theta) = \bar{w} - \int_{\theta-p*(\theta)}^{\bar{w}} G(w,\theta)\,dw - \int_{\underline{\theta}}^{\theta} G(x-p^*(x),x)\,dx$$

$$+\int_{\underline{\theta}}^{\theta}\int_{x-p^*(x)}^{\bar{w}} G_\theta(w,x)\,dwdx - \int_0^{\bar{w}} wG_w(w,\underline{\theta})dw$$

To compare $p^*(\theta)$ with the independent case, we have the following lemma.

Lemma 1.17. *Compare to the independent case, $p^*(\theta)$ is lower in the correlated case.*

Proof. By the second order condition of the utility maximization problem, equation (1.5.2), if i) $p'^*(\theta) \leq 0$, $G_\theta(\theta-p^*(\theta),\theta)+G_w(\theta-p^*(\theta),\theta) \geq 0$. Combined with $G_\theta \leq 0$, we have $-1 \leq \frac{G_\theta}{G_w} \leq 0$. If ii) $p'^*(\theta) \geq 0$, $G_\theta(\theta-p^*(\theta),\theta)+G_w(\theta-p^*(\theta),\theta) \leq 0$. We have $\frac{G_\theta}{G_w} \leq -1$. Thus $p^*(\theta)$ is lower than the independent case. □

From the proof, we get the following corollary directly.

Corollary 1.3. *When $p^*(\theta)$ is decreasing in θ, $p^*(\theta) \geq c$. When $p^*(\theta)$ is increasing in θ, $p^*(\theta) \leq c$.*

The stronger the effect of G_θ, the lower is the $p^*(\theta)$. As it just reflect that consumers having higher expected value from the the uncertain alternative, thus less can be extracted. If $G_\theta(w,\theta) = 0$ for all θ, w, we get the same $p(\theta)$ as in the independent case. The structure of the solution in this case is more complicated and only implicit form is obtained, which makes the comparison of profit difficult.

In the standard analysis, $p'(\theta) \leq 0$ only requires decreasing hazard rate. In this case, we require more than just a decreasing hazard rate.

Lemma 1.18. *Under the correlated case, with the decreasing hazard rate assumption, the optimal $p^*(\theta)$ is decreasing in θ if i) $G_{ww} \geq -G_{w\theta}$, ii) $G_w \geq -G_\theta$ and iii) $G_{\theta\theta}+G_{ww} \leq -2G_{\theta w}$.*

Proof.

$$\frac{dp}{d\theta} = \left[(G_{ww}+G_{w\theta})(c-p)+(G_w+G_\theta)\delta(\theta)+\frac{1-F(\theta)}{F'(\theta)}(G_{\theta\theta}+2G_{\theta w}+G_{ww})\right] \Big/ -\frac{d^2\pi}{dp^2}$$

Where $\delta(\theta) = d\,\frac{1-F(\theta)}{F'(\theta)}\Big/ d\theta$, by the decreasing hazard rate assumption, $\delta(\theta) \leq 0$. By the second order condition for the profit maximization problem, see equation (1.5.3), $d^2\pi\big/dp^2 < 0$. To have a $p'(\theta) \leq 0$, I need the denominator being negative. The first term is negative, since

$G_{ww} \geq -G_{w\theta}$ and $c \leq p$ which is shown in corollary 1.3. The second term is also negative because of the assumption of decreasing hazard rate and $G_w \geq -G_\theta$. For the last term, since $\frac{1-F(\theta)}{F'(\theta)} \geq 0$, so the whole thing boils down to $G_{\theta\theta} + G_{ww} \leq -2G_{\theta w}$. □

We know already condition ii) $G_w \geq -G_\theta$ when $p'(\theta) \leq 0$, is implied by the second order condition of the utility maximization problem. Condition i) is actually implies by the second order condition of the profit maximization problem if $G_{ww} \geq 0$, by equation (1.5.3), $\frac{G_\theta}{G_w} \leq \frac{G_{\theta w}}{G_{ww}}$. Combine with condition ii) we have condition i). The only condition that is new is condition iii). If we take the assumption of $G_{\theta\theta} = 0$, where the probability decreases at a constant rate when θ increases. Condition iii) becomes $G_{ww} \leq -2G_{\theta w}$. Combine with condition i), we have $-G_{\theta w} \leq -2G_{\theta w} \Longrightarrow G_{\theta w}(\theta - p^*(\theta), \theta) \leq 0$ for all θ . Since $\int_0^{\bar{w}} G_w(w,\theta)\,dw = 1$, $\int_0^{\bar{w}} G_{w\theta}(w,\theta)\,dw = 0$. By continuity, this implies there are at least one w which $G_{w\theta}(w,\theta) = 0$. Define $\hat{w}(\theta)$ such that $G_{w\theta}(\hat{w},\theta) = 0$. For simplicity, we restrict our attention to those $G(w,\theta)$ which has unique $\hat{w}(\theta)$. Since I assume $G_{ww} \geq 0$, condition iii) actually requires $\theta - p^*(\theta) \leq \hat{w}(\theta)$. This essentially means that we need the $G(w,\theta)$ put more mass towards high-end. As $\theta - p^*(\theta)$ is increasing in θ, G_w with higher θ will be more skewed to the left. This is actually ensured by the assumption of first order stochastic dominance of $G(w,\theta)$. As a result, lemma 16 can be simplified as follows.

Lemma 1.19. *Under the correlated case, with the decreasing hazard rate assumption, the optimal $p^*(\theta)$ is decreasing in θ if $G(w,\theta)$ is convex in w and satisfies the first order stochastic dominance in θ.*

1.6 Literature Review

There are two lines of research most closely related to my model, one is about advance purchase discount, another is about ticket sale. One common feature is that all papers I discuss below contain individual demand uncertainty. For example consider any sport event, advance booking triggers some consumer to buy in advance, shifting the competition for seats before the event date (DeGraba, 1995). But in my model, “seats” are not limited. This insight introduces the issue of the optimal timing of ticket sales, including the possibility of intertemporal price discrimination where discounts are given to early purchase (Dana, 1998) or offers a menu of options with varying upfront prices and levels of refund in the event of cancellation (Courty and Li, 2000).

Courty and Li (2000) studied price discrimination where consumers with unit demand, know at the time of contracting only the distribution of their true valuations but subsequently learn their actual valuations. Consumers are sequentially screened, as in a menu of refund contracts. Optimal mechanisms depend on informativeness of consumers’ initial knowledge about their valuations, not on uncertainty that affects all consumers. If different types of consumers have very different conditional distributions of valuations, then sequential mechanisms do not yields much greater expected profits than ex-post monopolist pricing. It can be optimal to "subsidize" consumers with smaller valuation uncertainty through low refund to reduce the rent to those who face greater uncertainty and purchase more flexible contracts.

Escobari & Jindapon (2012) showed how an airline monopoly uses refundable and non-refundable tickets to screen consumers who are uncertain about their travel. Their theoretical model predicts that the difference between these two fares diminishes as individual demand uncertainty is resolved. Using an original data set from U.S. airline markets, They find strong evidence supporting their model. Price discrimination opportunities through refund contracts decline as the departure date nears and individuals learn about their demand.

Courty (2003) studied the case of a monopoly ticket agency who sells tickets to consumers who learn new information about their valuations over time. The monopolist can sell early to uninformed consumers and/or late to informed ones, or it can choose to ration tickets and to strategically allow some ticket holders to resell. He show that the following two strategies does equally well: i) sell close to the event date after consumers have learned their valuations; ii) sell early and allow resale. This shows that the selling date, the ticket supply, the ticket price, and the decision to allow resale are complementary pricing instruments that should be chosen jointly as part of a coherent ticketing strategy. I instead focus on the non-transferable service, and hence reselling is not an issue.

Möller and Watanabe (2009) considers advance selling problems. They explains why some goods (e.g. airline tickets) are sold cheap to early buyers, while others (e.g. theater tickets) offer discounts to those who buy late. They derived the profit maximizing selling strategy for a monopolist when aggregate demand is certain but buyers face uncertainty about their individual demands. They have shown that Clearance Sales are more likely to be observed in markets where (1) temporal capacity limits are difficult to implement, (2) marginal costs of capacity are low but sufficiently increasing, (3) prices can be committed to in advance, (4) resale is feasible and (5) rationing is random rather than efficient.

Nocke, Peitz & Rosar (2011) consider an intertemporal setting in which individual uncertainty is resolved over time, advance purchase discounts can serve to price discriminate between consumers with different expected valuations for the product. They characterize the profit maximizing pricing strategy of the monopolist. Furthermore, adopting a mechanism design perspective, they provide a necessary and sufficient condition under which advance purchase discounts implement the monopolist's optimal mechanism. This paper should be the one very closely related to my model.

Rochet and Stole (2002) weakens the assumption of known reservation values by introducing independent randomness into the agents' outside options. They find that some of the received wisdom from mechanism design and nonlinear pricing is not robust and the richer model which allows for stochastic participation affords a more general empirical specification. There are two main differences from my paper in terms of model, first their formulation does not incorporate the random outside options into the utility function. Second, their model have no sequential nature.

Eső and Szentes (2007) analyze a situation where a monopolist is selling an indivisible good to risk-neutral buyers who only have an estimate of their private valuations. The seller can release, without observing, certain additional signals that affect the buyers' valuations. Their main result is that in the expected revenue-maximizing mechanism, the seller makes available all the information that she can, and her expected revenue is the same as if she could observe the part of the information that is "new" to the buyers. They also show that this mechanism can be implemented by handicap auction. In the case of a single buyer, this simplifies to a menu of European call options, which is similar to what I did in this paper.

Due to the sequential structure of my paper, the allocation problem can be analyzed by the dynamic mechanism design approach. However, there are two major differences between my paper and the literature. First, by posting an ex-post price $p(\cdot)$, consumers can opt out in the second stage, whereas in the literature they are forced to participate after the initial participation. The usual argument for this practice is that, under a quasilinear environment with unlimited transfers, the principal can ask the agent to post a sufficiently large bond upon acceptance, to be repaid later, so as to make it unprofitable to quit and forfeit the bond at any time during the mechanism. This points out exactly the second difference, which I explicitly model the "bond" part, which is $T(\cdot)$ in my paper. I want to emphasizes that the role played by $T(\cdot)$ is also different. Instead of keeping the consumer, $T(\cdot)$ is chosen to maximize profit subject to initial participation. Below is the selected literature of dynamic mechanism design.

Eső and Szentes (2013) is a more general version of the previous paper, where they considers a general dynamic contracting problem with adverse selection and moral hazard, in which the agent's type stochastically evolves over time. The main result is that for any fixed decision-action rule implemented by a mechanism, the maximal expected revenue that the principal can obtain is the same as if the principal could observe the agent's orthogonalized types after the initial period. In this sense, the dynamic nature of the relationship is irrelevant: the agent only receives information rents for her initial private information.

Pavan, Segal & Toikka (2014) study mechanism design in dynamic quasilinear environments where private information arrives over time and decisions are made over multiple periods. They provide a necessary condition for incentive compatibility that takes the form of an envelope formula for the derivative of an agent's equilibrium expected payoff with respect to his current type. They characterize the transfers that satisfy the envelope formula and establish a sense in which they are pinned down by the allocation rule.

1.7 Discussion

In this paper, I study the problem of timing of payment when consumers facing uncertain alternatives. To see the relevancy of the results, let us go back to the motivating example in the beginning of this paper. Typically each restaurant has a fixed number of tables, and that is the fundamental reason why restaurants can charge for the reservation. Although there is no capacity constraint in my model, most of the implications are consistent with observations. The price diners pay at the restaurant is not related to the alternative he might have, as predicted by the model when θ and w are independent. However when the alternative is a similar restaurant, this will no longer holds. Consider when there is a new restaurant opened nearby which provides similar food and comparable level of service, the most likely response of the incumbent is to provide discounts, which is consistent with the prediction that p is lower in the correlated case than the independent case. More importantly, the recent trend of charging reservation confirms that the optimal selling mechanism is of simple two part tariff pattern.

With the explosive growth of smart phones, more and more apps and reservation systems are developed. To name a few: Table 8 in San Francisco sells reservations at popular restaurants just days in advance; Zurvu scours the best available tables on OpenTable; Reserve, aims to be a full-fledged digital concierge; SeatMe, allows restaurants to ping eager diners if tables open up at the last minute. Resy, a service that sells reservations for tables at peak times. These services, apart from being cheaper and more efficient than a human reservationist, can also give restaurants access to more intimate portraits of their diners, like usual dining time and average number of people to dine with. These information are relevant to assess the social network of the diner. As predicted by the model, the restaurant will have a higher expected profit with a diner having a more volatile alternative. Diners with a weak and limited social network are more like to have more extreme realization of alternative. Personal data, like dining time and average number of companion, are helpful to infer the size and intensity of someone's social network. Further empirical research can be conduct on whether restaurants really prefer diners of this type.

There is a section devoted to the behavioral deviation where the consumer may overlook, partly or completely, the possibility of having a future alternative. The analysis is based on a reduced form model which provides the most straightforward application of the developed result. For future work, it is interesting to consider the full model where consumers need to allocate limited amount of time to acquire and process the information between the targeted activity and all possible alternatives. In general, consumers will have different amount of time or acquiring and processing ability, hence results in a different level over-focus problem, denoted by ψ. Furthermore ψ can be positively correlated with the value of the service, to represent the situation where people give too much attention to the activity which they are enthusiast. In addition, when the alternative is another restaurant, instead of treating it as an exogenous variable, the model can be extended to formally consider the pricing strategy of the other restaurant. In that case, diners have three choices, restaurant A or restaurant B or the alternative. More interesting works are to be done.

2 Legal Protection and Timing of Payment

2.1 Introduction

In textbook discussions of voluntary exchange, agents trade until mutually beneficial terms of trade are found. Goods then exchange hands and the agents go their respective ways. Similarly, when money is used as a medium of exchange, the emphasis is on the market clearing price, under which seller passes goods to the buyer and buyer passes money to the seller. The implicit assumption is that seller receives his payment at the same time the buyer gets his goods. This standard treatment is certainly a convenient abstraction, which allow economists focus on development of various concept of marginal values. However it is a conception which ignores the fact that agreement on a price is a different thing from the revenue collection of sellers and goods acquisition of buyers.

In general, revenue collection vary from payment before to payment after the supply and consumption of the goods or service. Most merchandise, for example, is purchased under what I will call *ex-ante payment schemes*, where buyers must pay (or obtain credit) and acquire ownership of the merchandise before the consumption take place. But in the case of many personal services, supply and consumption precede payment. This type of payment I will call *ex-post payment schemes*. Moreover payment scheme may differ within the same industry. At fast food restaurants customers must pay before eating, but for most other restaurant, the reverse is true.[1] Other examples of payment schemes are practically as abundant as the number of transactions in the economy. Labor markets typically involve ex-post payments, while many professional services require that a portion of total payment pay in advance, with the balance due after the service is supplied, which I will call *mixed payment*. The goal of this paper is to derive some consistent economic principles which underlies the choice of ex-ante or ex-post payment. Although there are a lot of works done dealing with optimal payment in the principal

[1] Recently there is a nice discussion on Quora (www.quora.com) about the optimal timing of the payment in restaurant. One restaurant owner shared his experience, saying that upfront payment increased table turnover by over 80%. The argument is that customers who have not paid can justify their occupation of a table, by the mere possibility of further ordering. On the other hand, those who have pay will have no moral justification for staying after their meals are finished. However, a higher table turnover does not necessarily associated with higher profit. Another point being raised, is that tips must be pay afterward in order to grade the service, which is one of the key reason why there should be ex-post payment.

agent models, little work are found on how two parties will devise an efficient institutional arrangements for the timing of payment.

One clear risk of using the ex-post payment scheme is buyer default. For example, in the web design industry, the most common problem is that after presenting the work, clients take it without paying. From the blog of Ben Hunt, who was one of the most influential figures on the subject of effective web design.[2] “While clients not paying has been a rarity in our business experience (thankfully!), we’ve had 3 such cases in the last 6 months, which has caused significant consequences for an agency of our size.”[3]. From his experience, he suggested that it is actually mutually beneficial, to have the payments scheduled over time. On one hand, the risk is managed, on the other hand, this provides you incentive to finish the work at the end. So the problem he tries to solve is actually the two sided moral hazard problem.

If we take the contractual view of what seller offer, what really protect the trading parties is the legal system. If there is any party breaches the contract, the innocent party can seek compensation through litigation. Hence the legal system is significantly important to the trading parties. The difference of the legal environment can affect the attractiveness of a market. A country’s law regulate business practices, defines business policies, rights and obligations involved in business transactions. The government of a country defines the legal framework within which firms do businesses. Therefore laws differ from country to country. For example, China has Communists government where business laws are strictly controlled by government to controlled business sectors. Where as India has democratic government and business laws are made to protect small businesses and consumers. Although different countries have different laws and regulations, knowledge of common law, civil law, contract laws, laws governing property rights, product safety and liability for a country helps business people to make business decisions.

The common law system is commonly found in former Great Britain’s colonies and is based on country’s legal history, past court rulings on cases and ways in which laws are applied in specific situations. Judges in a common law system have power to interpret the law under unique circumstances for an individual case. Countries like United States, Australia, India uses common law systems. In civil law system, laws are based on detailed set of written rules and codes. Judges have less flexibility and have power only to apply the law. France, Germany, Russia operate with a civil law system. Some counties have legal system, which is based on religious teachings. Countries like Pakistan, Saudi Arabia, Iran and Middle Eastern nations follow Islamic laws, which is based on holy principles of Koran. It is very important to interpret law according to country and its impact on their commercial activities. Many business transactions are regulated by contract, and contract law governs contract enforcement. Contracts drafted under common law system are tend to be very detailed, where as contracts are much shorter and less specific in civil law system due to already drafted civil codes. Therefore common law sys-

[2] He has written three books and spoken at multiple conferences internationally.
[3]http://webdesignfromscratch.com/business/payment-timing-structure-tips/

tem has long and expensive jurisdiction process. However it has advantage of greater flexibility and allows judges to interpret a contract dispute in particular situation as compare to civil law system. Considering impact of various aspects of legal system in business, it is very important for business people to have good understanding of the legal system where they do business with.

Alongside the formal legal protection, the timing of payment is also an institutional device being used to protect both parties from reneging on a promise. By having part of the payment paid in advance, this reduces the loss to the seller in the case of buyer's default of payment. Even if the seller at the end chooses not to sue the buyer, the seller receives at least partly of the total payment. On the other hand, by having part of the payment paid ex-post, this protects the buyer from the seller's malpractice of providing low quality of goods and service, because he can refuse to pay the remaining part of payment. From this perspective, a mixed payment actually mitigates the problem of the two sided moral problem. A interesting point is that both parties want the timing of the payment shifts to their side, the buyer want to pay ex-post, while the seller want to get paid ex-ante. Although this paper will not provide a solution for the optimal mix of the payments, which is most likely done by bargaining, I show that mixed payment can improve efficiency.

In this paper, I consider a trading of goods or service which quality is buyer-specific, and communicates to the seller during the transaction. When the quality of the goods is of different level, firms are disciplined by competition. Low quality goods or goods with highly variable quality, sell for lower prices. But this requires that some sort of quality standard be agreed upon, and that the quality be easily discernible. When the service rendered is highly personalized, it will not be homogenous among sellers. In this case, the cost of renegotiating with alternative sellers in order to avoid shirking is high and the discipline of competition is less relevant. Examples include photographers, construction work, professional services such as legal advice and management consultancy, and creative work such as architectural design, the writing of software, preparation of advertisements, and research and development.

Consider the simplest setting, where there are one principal and one agent. The agent is paid to provide good or service, whose quality depends on his effort. So here we have a two sided moral hazard problem. On one side, as long studied, principal worries agent shirks. While on the other side, agent worries that principal not paying the agreed amount. To complete the story, we need a third party, which is a court. In the case of dispute, either the seller complains that amount paid is less than the agreed amount, or the buyer finds the quality is lower than asked for, they can go to the court to seek a judgment. Surely there is no such thing as free lunch, and using the court service is costly. The simplest way to represent this, is to consider both party enter into a contest — completing to provide evidence that the other party commits fraud.[4] Apart from the variable cost, there are fixed costs. The fixed cost plays a key role in the model, not only it is an important part of the legal environment which will be defined in the model setup, but also one of the results shows that there is an optimal range of fixed cost to

[4] See Corchón, L. C. (2007).

allow efficient outcome. This hints that reducing fixed cost of litigation need not be beneficial to trade. Also the court efficiency will determine the feasible trading opportunity. A more efficient legal system, as will be defined in the model setup section, will enlarge the feasible set of contracting choice.

There are many different way to call the ex-post payment. Like in construction industry, a contract that provides for Milestone Payments is one where payments are not scheduled by time but by a 'milestone'. This means that payments are due when a certain event or section of work is reached. Typically contractors throwing extra staff and resources at the work with the aim of reaching each milestone extra fast and being able to bank the cash fast. But very often there is no payment at the end, because clients could argue about tiny details or that some particular standard is not met. Clients will then simply terminate the contract. Now a part of the work has been done in record time but with no cost, and clients will simply appoint someone else to finish the work. So there is an advice that contractors should try their best to avoid this type of payment scheme at all.[5]

This example seems to suggests that ex-post payment should not be welcomed by sellers. One of the results in this paper seems to confirm this idea, where ex-ante payment dominates ex-post payment. But it comes with an important condition, which is when the contract is proposed by the seller. However, it seems that in the example above, it is the buyer who propose the contract. Surprisingly, when the contract is proposed by the buyer, we have an equivalent result, ex-ante and ex-post payment gives the same outcome, apart from the case where litigation involves very low fixed cost. Nevertheless, the outcome may not be efficient. I showed that there exists two kind of mixed payment scheme provides efficient outcomes when other schemes could not.

There may be a concern for the alternative solution for this problem, like seller offer guarantees. Reputation based approaches are not complete solutions, particularly for trading parties who are new, small or distant from each other. Since payment terms can be selected to mitigate these concerns, I expect systematic relations between terms of payment and variable that capture the potential for opportunism. In the next section, I will present the model. Then followed by the analysis of litigation under different legal system. In section 2.4 and 2.5, ex-ante and ex-post payment contract is examined in detail. A comparison between these two payment contract is provided in section 2.6. Mixed payment contract is then introduced to the analysis in section 2.7. The role of contract proposer will be shifted to the seller in section 2.8. Literature review is provided in section 2.9. After discussion on some alternative explanation for different payment contract is in section 2.10, section 2.11 concludes.

2.2 Model

I consider a simple trading relationship, one buyer trades with one seller. Both parties are risk neutral. Production technology is linear, $q = e$ where q is quality of the good or service

[5]Contractor debt recovery, http://www.contractorsdebtrecovery.com.au/articles/Milestones.pdf

and e is effort. The buyer pays the price P for the good or service which seller incurs effort cost $e^2/2+E$ to produce. E is the fixed cost to finish the basics, and $e^2/2$ is the variable cost due to the specific requirement of the buyer. The preferences are represented by the following utility functions,

$$EU_B = q - P + V,$$
$$EU_S = P - \frac{e^2}{2} - E + V,$$

where V is the expected values from litigation process which will be analyzed in the next section. Consider a benchmark case which both parties cannot cheat, then there is no need to have any litigation, hence $V = 0$. Clearly we have the efficient effort $\tilde{e} = 1$. As in the standard model, the equilibrium quality is always below this first best level, but we will see that Principal would always minimize the deadweight loss by choosing the more efficient scheme.

The timing of the model will be as follows: First, the buyer offers a contract to the seller, which lists out ex-ante price P_{ante}, required quality q, and ex-post price P_{post}. Second, the seller accepts or rejects the offer. Third, if the seller accepts, the seller receives P_{ante} and chooses his effort e; otherwise, the game ends. Fourth, the buyer chooses whether to fulfill the contract, by deciding whether to pay P_{post}. Fifth, if there is a dispute, either party can bring the case to court. There is no private information and everything above is common knowledge. For the contract which contains a zero ex-ante price or zero ex-post price, I will call that as pure payment contract. For contracts contain positive ex-ante price and ex-post price, I refer them as mixed payment contract. The time line of the model can be represented as follows:

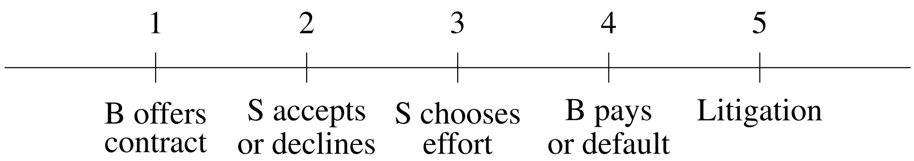

Figure 2.2.1: Time line

In this paper, I adopt the "Perfect Tender Rule" in which the buyer is given the right to reject the goods delivered or cancel the entire contract if the seller's performance is non-conforming.[6] Thus the buyer is given the right to terminate the contract for any deviation by the seller. As a result, the effort choice of the seller will not be interior, the only possible choices are $e = \{0, q\}$, where q is the required quality.

[6]For more details regarding the "Perfect Tender Rule", see appendix.

2.3 Litigation

Now let us make clear the details for the litigation. In the case of dispute, there must be a party which breaks the contract (first), which I call the defaulting party and the other one as the innocent party. Denote the resource spend by i on litigation x_i, where $i, j = B, S$ which is a shorthand notation for the buyer and seller. Party i is the innocent party, while party j is the defaulting party. Let $p\left(x_i, x_j\right)$ to be the probability that agent i wins, so $1 - p(x_i, x_j)$ is the probability that agent j wins, [7]

$$p\left(x_i, x_j\right) = \frac{\alpha x_i}{\alpha x_i + x_j}.$$

Noted that there is a term α attached to x_i. I assume $\alpha > 1$, to represent the advantage enjoyed by the innocent party. Another way to understand α is to consider it represents the efficiency of the legal system. Under this interpretation, a more efficient legal system costs less for the innocent party to achieve the same probability of winning the case. Apart from the variable cost x_i, there are fixed cost of litigation, which is denoted by K. Expert fees, document preparation fees, and investigator fees add to the cost. In arbitration, filing fees can cost thousands and the fee for arbitrators may easily cost thousands for each day of a hearing. In advance of trial, mediation will cost \$500 per half day, not including the attorney's fees.

Certainly there are other indirect costs associated with the litigation. These indirect costs stem from the uncertainty created by litigation, which may deter investment in high-cost jurisdictions. They also may affect companies' borrowing costs and hence their ability to invest, grow, and create jobs. Concerns surrounding litigation can also occupy management time, which may distort or hinder effective business decision making. Apart from that there are hidden costs, like the personal cost and business stress. The business is stressed by occupying key personnel time with the duties of litigation instead of their job responsibilities. Each day of trial will occupy three days of each witness's non-trial time. In addition, employee suffers from personal stress over being a witness or being involved in the litigation or, in the worst case scenario, being the object of a key portion of the litigation – either from their decisions or their actions. All three of these initiators of stress carry direct costs – the loss of the employee's services – and indirect costs – the minimization of these employees. It is a rare employee indeed, who, once accused of "causing" litigation whether by contract decisions, omissions, or direct action is not chastened and therefore hesitant to act. This hesitation leads to indecision, inefficiency, and loss. These costs are difficult to measure but are a significant expense nonetheless. On the personal side, stress manifested as anger and anxiety are common occurrences. Add to that the cloud of uncertainty that hangs over the individual as long as the litigation is unresolved.

[7] The literature has developed from the seminal contributions by Tullock (1967). The contest success function in this paper is similar to the one in Gradstein (1995), with the exception that there is no restriction on α. For more recent development about the contest success function, see Corchón, L., & Dahm, M. (2010).

Combined this equals an emotional, and physical toll that must be added to the financial cost of litigation.

All these factors are too complicated to put into one single model, although they are highly important and relevant, I will not include them as this will certainly make the model too messy to analyze. Certainly one put some of them into analysis, but before that, I believe an understanding of the basic mechanism behind all these other factors is essential for us to know how contracts with different timing of payment will affect the decision makers.

2.3.1 US system

Throughout this paper, we will focus on the US legal system, where each party is responsible for its own spending in court.[8] [9] The expected value from litigation is

$$V_i = C \cdot p\left(x_i, x_j\right) - x_i - K,$$
$$V_j = -C \cdot p\left(x_i, x_j\right) - x_j - K,$$

where C denotes the compensation paid by the losing party to the winning party, K is the fixed cost of litigation. In the case of buyer default, the compensation would be P_{post}.[10] In the case of seller default, the compensation would be P_{ante}, which would leaves buyer not damaged.[11] Simple differentiation give us the following first order conditions,

$$C \frac{\alpha x_j}{\left(\alpha x_i + x_j\right)^2} = 1,$$

$$C \frac{\alpha x_i}{\left(\alpha x_i + x_j\right)^2} = 1.$$

This implies the following lemma, define x_i^*, x_j^* as the equilibrium litigation spending of party i and j respectively:

Lemma 2.1. *In equilibrium, the two parties spend the same amount on litigation.*

[8] For reader interested in seeing different legal system, there is an analysis of the UK system in the appendix.

[9] The contest literature has developed from the seminal contributions by Tullock (1967). Corchón, L (2007) provides a good survey about theory of contest.

[10] This compensation is according to the expectation damage principle. Under the expectation damage measure, the defaulting party pays an amount that puts the other party in the position he would have been in had the contract been performed. For details, see Shavell (1980).

[11] This compensation is according to the restitution damage principle. Under the restitution damage measure the defaulting party returns only the payments made to him. See Shavell (1980).

$$x_i^* = x_j^* = \frac{\alpha C}{(1+\alpha)^2}.$$

Both parties will spend more to compete for a higher compensation (when C is higher), and less when the legal system is more efficient (when α is higher). Thus in equilibrium, the probability of i winning is

$$p^*\left(x_i^*, x_j^*\right) = \frac{\alpha}{1+\alpha}.$$

The equilibrium expected value from litigation is

$$\begin{aligned} V_i^* &= \frac{\alpha}{1+\alpha}C - \frac{\alpha}{(1+\alpha)^2}C - K \\ &= p^{*^2}C - K. \end{aligned}$$

$$\begin{aligned} V_j^* &= -C\frac{\alpha}{1+\alpha} - \frac{\alpha}{(1+\alpha)^2}C - K \\ &= -C\frac{\alpha(2+\alpha)}{(1+\alpha)^2} - K. \end{aligned}$$

Noted that we have a cutoff for litigation decision, the innocent party will sue if and only if $K \leq p^{*^2}C$. I assume that K represents all fixed costs associated with the litigation whether the defendant admits or defenses, so K is not relevant in the decision to admits or defenses. Since $V_j^* > -C$, it is always in the interest of the defaulting party to fight the legal battle.

2.3.1.1 Comparative Statics

We can see how changes in α and C affect the litigation,

$$\frac{\partial p^*}{\partial \alpha} = \frac{1}{(1+\alpha)^2} > 0.$$

The innocent party winning probability increases with the efficiency of the legal system. Notice that $V_i^* = p^{*^2}C$, it is obvious that V_i^* increases in both α and C. With the previous calculation, it is easy to get,

$$\begin{aligned} \frac{\partial V_j^*}{\partial \alpha} &= -C\frac{2}{(1+\alpha)^3} < 0, \\ \frac{\partial V_j^*}{\partial C} &= -\frac{\alpha(\alpha+2)}{(1+\alpha)^2} < 0. \end{aligned}$$

Lemma 2.2. *In terms of absolute value, the innocent party has a stronger marginal effect in changes of α, but weaker in changes of C.*

Proof.

$$\begin{aligned}
\frac{\partial V_i^*}{\partial \alpha} &= \frac{2\alpha C}{(1+\alpha)^3} > \frac{2C}{(1+\alpha)^3} &&= \left|\frac{\partial V_j^*}{\partial \alpha}\right|, \\
\frac{\partial V_i^*}{\partial C} &= \left(\frac{\alpha}{(1+\alpha)}\right)^2 < \frac{\alpha\,(\alpha+2)}{(1+\alpha)^2} &&= \left|\frac{\partial V_j^*}{\partial C}\right|.
\end{aligned}$$

□

2.4 Ex-ante Payment

Let us firstly consider the case where the contract only involves ex-ante payment. After the buyer makes the payment, the seller chooses to shirk or not. If the seller shirks, the buyer chooses to sue or not. The buyer will sue if

$$K \leq p^{*^2} P_{\text{ante}} = P_{\text{ante}} \left(\frac{\alpha}{1+\alpha}\right)^2.$$

As a consequence, P_{ante} need to be sufficiently large to induce buyer to sue. When forming the optimal ex-ante contract, this condition must be met. Otherwise, any P_{ante} lower than K/p^{*^2} is like a gift to the seller, because the buyer will not find it beneficiary to sue. The expected utility of the seller is

$$EU_S = \begin{cases} P_{\text{ante}} - \frac{\tilde{e}^2}{2}, & \text{if the seller works;} \\ P_{\text{ante}} - \frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} - K, & \text{if the seller shirks.} \end{cases}$$

To induce the seller to exert required effort, we need

$$\frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + K \geq \frac{\tilde{e}^2}{2}.$$

The whole problem can be formulated as follows,

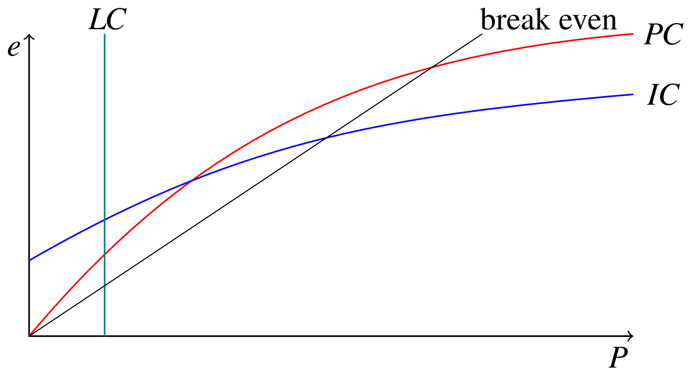

Figure 2.4.1: Constraints for ex-ante payment contract
For the region below the red curve, PC is satisfied. For the region below the blue curve, IC is satisfied. For the region to the right of the teal curve, LC is satisfied. The buyer will not offer any (P,e) below the breakeven line.

$$\max_{\tilde{e},P_{\text{ante}}} \tilde{e} - P_{\text{ante}}$$

$$s.t. \ \frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + K \geq \frac{\tilde{e}^2}{2}, \tag{IC}$$

$$P_{\text{ante}} - \frac{\tilde{e}^2}{2} \geq 0, \tag{PC}$$

$$P_{\text{ante}} \geq K\left(\frac{1+\alpha}{\alpha}\right)^2, \tag{LC}$$

where equation (IC) is the incentive compatibility constraint of the seller, equation (PC) is the participation constraint of the seller, and equation (LC) is the legal constraint for buyer to sue seller, if seller shirks.

There are four types of contract where either IC or PC binds, or both and when the LC and PC binds. It is impossible for none of constraints to be binding, due to the maximization behavior, as shown clearly in figure 2.4.1. The indifference curves of the buyer is the 45 degree line. The optimal solution depends on the intersection point of the three curves IC, PC and LC. Consider the extreme case when $K = 0$, under this case, IC will be underneath PC, LC is on the vertical axis, the only possible tangent point is on the IC. Thus we should expect when K is low, we will only have IC binding, which is case I in the following analysis. When K increases, IC shifts up vertically, and LC shifts right horizontally, leaving PC unchanged. If the tangent point is still on the right of the intersection point of IC and PC, case I follows. If it happens to be at the intersection point of IC and PC, where both IC and PC is binding, then we have case II. When it happens to be the left of the intersection point, where PC is binding, we have case III. The last case, is when K becomes really high, such that the solution is determined by the intersection point of PC and LC.

Before we dive into the analysis of the different cases in details, the result is provided first.

The optimal ex-ante payment contracts is summarized by the following proposition.

Proposition 2.1. *The optimal choice of the ex-ante payment contract depends on the specific legal environment, (α, K), which is summarized in the following table.*

Case	(α, K)	e^*	P^*_{ante}
I	$0 \leq K \leq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$	$\frac{\alpha(2+\alpha)}{(1+\alpha)^2}$	$\frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K\frac{(1+\alpha)^2}{\alpha(2+\alpha)}$
II	$\frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \leq K \leq \frac{1}{2(1+\alpha)^2}$	$\sqrt{2K}\,(1+\alpha)$	$K\,(1+\alpha)^2$
III	$\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$	1	$1/2$
IV	$\frac{\alpha^2}{2(1+\alpha)^2} \leq K \leq \frac{2\alpha^2}{(1+\alpha)^2}$	$\sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right)$	$K\left(\frac{1+\alpha}{\alpha}\right)^2$

To sum up, these four type of contracts correspond with different fixed litigation cost, from low to high, it starts with case I, and finally case IV. This also gives an idea of optimal range K for efficient contracting, with a given α.

Corollary 2.1. *When the fixed cost of litigation is either too high or too low, the efficient contract (case III) will not be offered.*

Proof. See Lemma 2.10. □

In what follows each of the four cases will be analyzed in detail, readers who have no interest in the derivation can skip to the next section.

2.4.1 Case I (Incentive compatibility constraint binds)

Let us consider firstly where only the IC constraint binds. The maximization problem is

$$\max_{\tilde{e}, P_{\text{ante}}} \tilde{e} - P_{ante} \quad s.t.\ \frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + K = \frac{\tilde{e}^2}{2}.$$

Hence,

$$\tilde{e}^* = \frac{\alpha(2+\alpha)}{(1+\alpha)^2},$$
$$P^*_{\text{ante}} = \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K\frac{(1+\alpha)^2}{\alpha(2+\alpha)}.$$

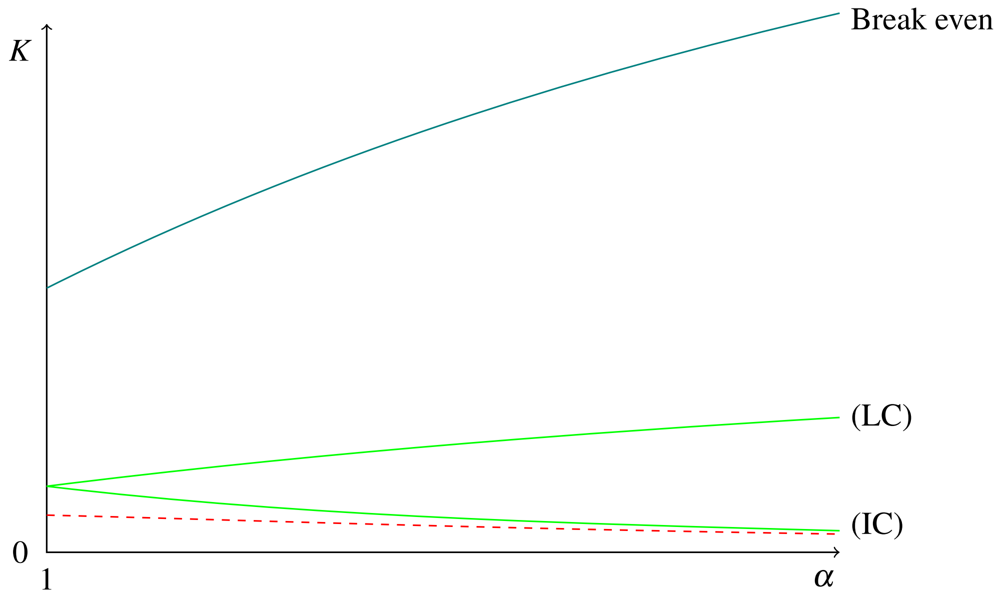

Figure 2.4.2: Optimal choice of ex-ante payment contract
For the region below the red dashed curve, the case I contract will be chosen; For the region between the red dashed curve and the lower green curve (2.4.3), case II contract will be offered. For the region between the two green curves (2.4.3 and 2.4.4), the case III contract will be chosen; For the region between the upper green curve (2.4.4) and the teal curve (2.4.5), the case IV contract will be offered.

In equilibrium,

$$\begin{aligned} EU_B^* &= \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} + K\frac{(1+\alpha)^2}{\alpha(2+\alpha)} > 0, \\ EU_S^* &= \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K\frac{(1+\alpha)^2}{\alpha(2+\alpha)} - \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^4} \\ &= \frac{\alpha(2+\alpha)}{2(1+\alpha)^4} - K\frac{(1+\alpha)^2}{\alpha(2+\alpha)}. \end{aligned}$$

The PC is satisfied if and only if:

$$EU_S \geq 0 \iff \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \geq K. \tag{2.4.1}$$

For the buyer sues the seller when the latter shirks, we need the *LC* being satisfied,

$$\begin{aligned} K &\leq \left(\frac{\alpha}{1+\alpha}\right)^2 P_{\text{ante}} = \frac{\alpha^3(2+\alpha)}{2(1+\alpha)^4} - K\frac{\alpha}{(2+\alpha)} \\ \iff K &\leq \frac{\alpha^3(2+\alpha)^2}{4(1+\alpha)^5}. \end{aligned}$$

It can be shown that equation (2.4.1) is more stringent, as we have

$$\frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} < \frac{\alpha^3(2+\alpha)^2}{4(1+\alpha)^5},$$

where the inequality is a consequence of $\frac{1}{1+\alpha} < \frac{\alpha}{2}$. Since $\alpha > 1$, the inequality is true. Thus the only condition is the one satisfies PC. To summarizes,

Lemma 2.3. *When $K \leq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$, the optimal pure ex-ante payment contract will be,*

$$\tilde{e}^* = \frac{\alpha(2+\alpha)}{(1+\alpha)^2}, \quad P^*_{ante} = \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K\frac{(1+\alpha)^2}{\alpha(2+\alpha)}.$$

The following three lemmas are the comparative statics with respect to K and α.

Lemma 2.4. *The optimal $\tilde{e}^*$ and P^*_{ante} are increasing in α. P^*_{ante} is decreasing in K, while K has no effect on $\tilde{e}^*$.*

Proof.

$$\begin{aligned}
\frac{d\tilde{e}^*}{d\alpha} &= \frac{1}{(1+\alpha)^3} > 0 \\
\frac{d\tilde{e}^*}{dK} &= 0 \\
\frac{dP^*_{\text{ante}}}{d\alpha} &= \frac{1}{(1+\alpha)^3} + \frac{2(1+\alpha)K}{\alpha^2(2+\alpha)^2} > 0 \\
\frac{dP^*_{\text{ante}}}{dK} &= -\frac{(1+\alpha)^2}{\alpha(2+\alpha)} < 0
\end{aligned}$$

□

Lemma 2.5. *The expected payoff of the buyer is increasing with K, while the expected payoff of the seller is decreasing with K.*

Proof.

$$\frac{dEU_B}{dK} = \frac{(1+\alpha)^2}{\alpha(2+\alpha)} > 0, \qquad \frac{dEU_S}{dK} = -\frac{(1+\alpha)^2}{\alpha(2+\alpha)} < 0.$$

□

Lemma 2.6. *The expected payoff of the buyer is increasing with α, if $K \leq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^4}$. While the expected payoff of the seller is increasing with α, if $K \leq \frac{(\alpha^2+2\alpha-1)\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$.*

Proof.

$$\frac{dEU_B}{d\alpha} = \frac{1}{(1+\alpha)^3} - \frac{2(1+\alpha)}{\alpha^2(2+\alpha)^2}K$$
$$\frac{dEU_B}{d\alpha} \geq 0 \iff \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^4} \geq K$$
$$\frac{dEU_S}{d\alpha} = \frac{1-2\alpha-\alpha^2}{(1+\alpha)^5} + \frac{2(1+\alpha)}{\alpha^2(2+\alpha)^2}K$$
$$\frac{dEU_S}{d\alpha} \geq 0 \iff \frac{\left(\alpha^2+2\alpha-1\right)\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \geq K$$

□

2.4.2 Case II (Participation constraint and incentive compatibility constraint bind)

When K becomes larger, the condition in lemma 2.3 may not be satisfied, which ensures both PC and LC to be satisfied. Under that situation, we will have solution at the kink point which both PC and IC are binding:

$$P_{\text{ante}} - \frac{\tilde{e}^2}{2} = \frac{P_{\text{ante}}}{(1+\alpha)^2} - K = 0.$$

Then we get,

$$P^*_{\text{ante}} = K(1+\alpha)^2,$$
$$\tilde{e}^* = \sqrt{2K}(1+\alpha).$$

Under this contract, buyer will always sue seller if he shirks, since $K(1+\alpha)^2 > K\left(\frac{1+\alpha}{\alpha}\right)^2$. The buyer will offer this contract if

$$EU_B^* \geq 0 \Longrightarrow \frac{2}{(1+\alpha)^2} \geq K. \tag{2.4.2}$$

Lemma 2.7. *When $K \leq \frac{2}{(1+\alpha)^2}$, the optimal pure ex-ante payment contract is:*

$$\tilde{e}^* = \sqrt{2K}(1+\alpha), \quad P^*_{ante} = K(1+\alpha)^2.$$

We can show that the case II contract is applicable whenever case I contract is applicable by

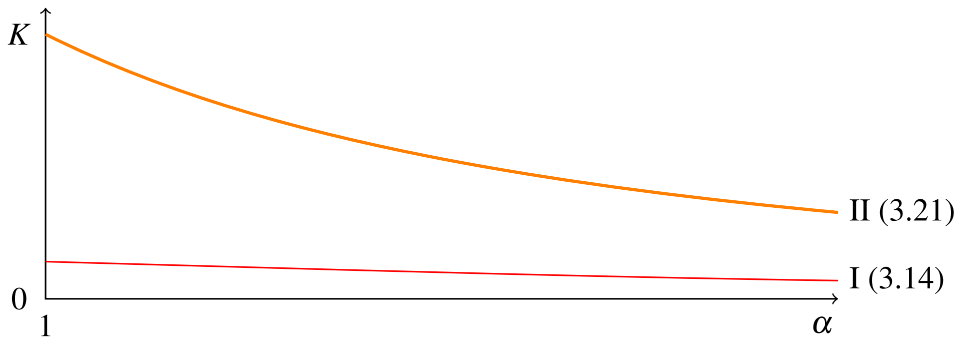

Figure 2.4.3: Legal environment for the two ex-ante payment contracts
The area below the red curve (2.4.1) are the legal environments (α, K) such that the case I ex-ante payment contract is feasible. And the region which below the orange thick curve (2.4.2) is where the case II ex-ante payment contract is feasible. Note that the two curves never intersect.

comparing the conditions.

Lemma 2.8. $\frac{2}{(1+\alpha)^2} \geq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$.

Proof. $\frac{2}{(1+\alpha)^2} \geq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$ is equivalent to

$$4\,(1+\alpha)^4 \geq \alpha^2(2+\alpha)^2,$$

which can be written as $4+16\alpha+20\alpha^2+12\alpha^3+3\alpha^4 \geq 0$. Given $\alpha \geq 1$, the inequality holds. $\square$

To see it in diagram, see figure 2.4.3. The following lemma shows that the case I ex-ante payment contract is superior.

Lemma 2.9. *The case I ex-ante payment contract will be offered whenever it is applicable.*

The intuition of this result is clear, as the kink-point solution is only taken when the tangency is no longer applicable. Another way to see this, by Le Chatelier's principle, case I solution comes with only *IC* constraint, but case II comes with an extra constraint *PC*.

2.4.3 Case III (Participation constraint binds)

When K keep increasing, the solution will go back to the tangency, on the left of the intersection point of *PC* and *IC*, where only PC binds.

$$\max_{\tilde{e}, P_{\text{ante}}} \tilde{e} - P_{\text{ante}} \quad s.t. \; P_{\text{ante}} = \frac{\tilde{e}^2}{2},$$

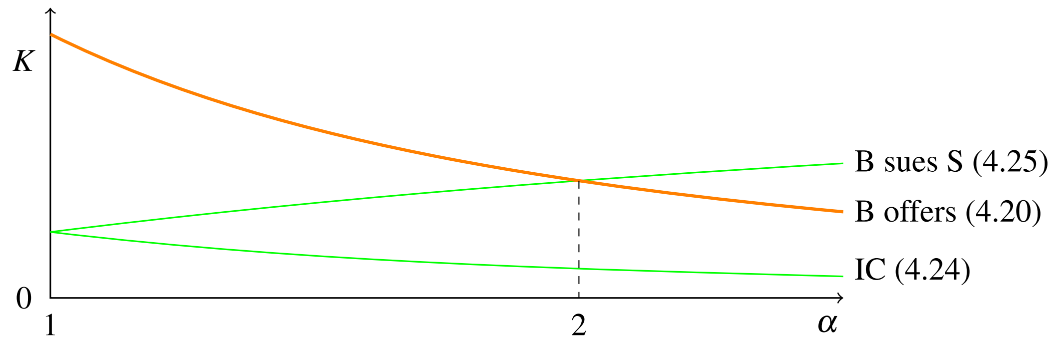

Figure 2.4.4: Legal environment for the second and third ex-ante payment contract
The two green curves (2.4.3) and (2.4.4) are the corresponding constrains for the case III contract to be feasible. This diagram shows when α is low, the feasible region of the case III contract is within the boundary of the case II contract, where under the orange thick curve (2.4.2), but when $\alpha > 2$, this is not true.

$$\tilde{e}^* = 1,\ P_{\text{ante}} = \frac{1}{2},\ EU_P = \frac{1}{2}.$$

There are two constraints need to be satisfied, firstly the *IC* that the seller exert the required effort.

$$K \geq \frac{1}{2\left(1+\alpha\right)^2}. \tag{2.4.3}$$

The other one is that the buyer would sue the seller if the seller shirks,

$$K \leq \frac{\alpha^2}{2\left(1+\alpha\right)^2}. \tag{2.4.4}$$

Lemma 2.10. *When the fixed cost of litigation is medium, $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$, the optimal pure ex-ante payment contract will be,*

$$\tilde{e}^* = 1,\ P_{ante} = \frac{1}{2}.$$

2.4.4 Case IV (Participation constraint & legal constraint bind)

When K becomes really high such that LC lies on the price level associated with the efficient effort level, i.e. when $K = \frac{\alpha^2}{2(1+\alpha)^2}$, LC will have effect on the solution. The solution will be at the intersection point of PC and LC.

$$P_{\text{ante}} = \frac{\tilde{e}^2}{2} = K\left(\frac{1+\alpha}{\alpha}\right)^2$$
$$\tilde{e} = \sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right)$$

To check that the IC is satisfied,

$$\frac{\alpha(2+\alpha)}{(1+\alpha)^2}P_{\text{ante}} + K \geq \frac{\tilde{e}^2}{2} \Longrightarrow K \geq \frac{P_{\text{ante}}}{(1+\alpha)^2}.$$

By substitute $P_{\text{ante}} = K\left(\frac{1+\alpha}{\alpha}\right)^2$ into the inequality, we have $\alpha^2 \geq 1$. Since by assumption $\alpha > 1$, the inequality holds. But the buyer will only offer this contract if this gives him positive payoff.

$$EU_B^* = \tilde{e} - P_{\text{ante}} \geq 0 \Longrightarrow \sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right) \geq K\left(\frac{1+\alpha}{\alpha}\right)^2,$$

which can be simplified as

$$K \leq \frac{2\alpha^2}{(1+\alpha)^2}. \tag{2.4.5}$$

Lemma 2.11. *The optimal pure ex-ante payment contract with binding PC and LC is, if* $\frac{\alpha^2}{2(1+\alpha)^2} \leq K \leq \frac{2\alpha^2}{(1+\alpha)^2}$,

$$\tilde{e}^* = \sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right),\ P_{ante}^* = K\left(\frac{1+\alpha}{\alpha}\right)^2.$$

2.5 Ex-post Payment

Let us make clear the timing of ex-post payment. After receiving the contract from the buyer, the seller chooses to shirk or not. If yes, the game ends, otherwise the buyer chooses to pay or not. If yes, the game ends, otherwise the seller chooses to sue or not. If the seller is not suing buyer, the buyer will not pay for sure. So let us consider the contract that the seller will sue the buyer if he is not paying:

$$P_{\text{post}} \geq \frac{K}{p^{*2}} = \left(\frac{1+\alpha}{\alpha}\right)^2 K. \tag{2.5.1}$$

Equation (2.5.1) is the legal constraint in the ex-post payment case, which defines the minimum feasible payment.

For the buyer,

$$EU_B = \begin{cases} \tilde{e} - P_{\text{post}}, & \text{if pay,} \\ \tilde{e} - P_{\text{post}}\left(\frac{\alpha(2+\alpha)}{(1+\alpha)^2}\right) - K, & \text{otherwise.} \end{cases}$$

So the buyer will pay if $P_{\text{post}} \leq (1+\alpha)^2 K$, which is the condition for the buyer to pay.

The whole problem can be formulated as follows,

$$\max_{\tilde{e}, P_{\text{post}}} \tilde{e} - P_{\text{post}}$$

$$s.t. \; P_{\text{post}} - \frac{\tilde{e}^2}{2} \geq 0, \tag{PC}$$

$$P_{\text{post}} \leq (1+\alpha)^2 K \tag{IC$_B$}$$

$$P_{\text{post}} \geq K\left(\frac{1+\alpha}{\alpha}\right)^2 \tag{LC$_S$}$$

where equation (PC) is the participation constraint of the seller , equation (IC_B) is the incentive constraint of the buyer, and equation (LC_S) is the legal constraint for the seller to sue the buyer, if the buyer do not pay.

When $K = 0$, both IC_B and LC_S lies on the vertical axis, see figure (2.5.1). When K increases, both IC_B and LC_B shifts to the right and there is a gap between them. The higher the K, the larger is the gap. The indifference curves of the buyer are any 45 degree lines. The optimal solution depends on the intersection point of the three curves, PC, IC_B, LC_S. When K is low, the solution is at the intersection point of PC and IC_B, where both PC and IC_B are binding, then that is the case I in the following analysis. When we have medium K, tangent point happens on the PC which lies in the gap between between the IC_B and LC, that is case II in the following subsection. If the solution is at the intersection point of the PC and LC_S, where both PC and LC_S are binding, and that is case III in the following subsection, which happens when K is high.

Before we dive into the analysis of the different cases in details, the result is provided first. The optimal ex-post payment contracts is summarized by the following proposition.

Proposition 2.2. *Optimal ex-post payment contract depends on specific legal environment, (α, K), which is summarized in the following table.*

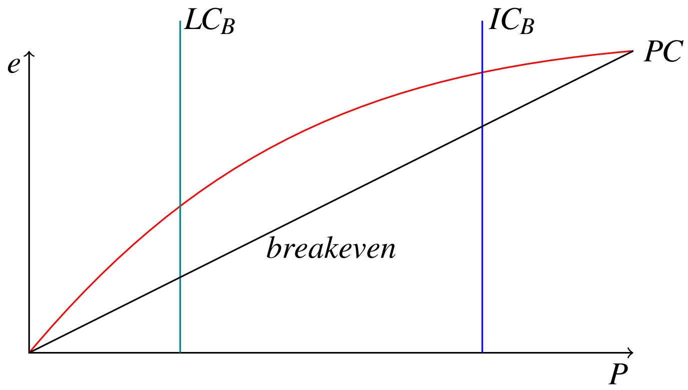

Figure 2.5.1: Constraints for ex-post payment contract

For the region below the red curve, PC satisfies. For the region to the left of the blue curve, IC_B satisfies. For the region to the right of the teal curve, LC_S satisfies. All the region above the black line give the buyer positive payoff, and hence itself is the breakeven line.

Case	(α, K)	e^*	P^*_{ante}
I	$K \leq -\frac{\alpha^2\left(2\sqrt{2(1+\alpha)}-\alpha-3\right)}{2(\alpha-1)\left(\alpha^4+4\alpha^3+3\alpha^2-4\alpha-4\right)}$	$\sqrt{2K}\,(1+\alpha)$	$(1+\alpha)^2 K$
II	$\hat{K}$	$\frac{\alpha}{2+\alpha}$	$\frac{(1+\alpha)^2}{2(2+\alpha)^2}+\left(\frac{1+\alpha}{\alpha}\right)^2 K$
III	$\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$	1	$1/2$
IV	$\frac{\alpha^2}{2(1+\alpha)^2} \leq K \leq \frac{2\alpha^2}{(1+\alpha)^2}$	$\sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right)$	$K\left(\frac{1+\alpha}{\alpha}\right)^2$

where $\hat{K}$ stands for $K \geq -\frac{\alpha^2\left(2\sqrt{2(1+\alpha)}-\alpha-3\right)}{2(\alpha-1)\left(\alpha^4+4\alpha^3+3\alpha^2-4\alpha-4\right)}$,
$K \leq \frac{\alpha^2}{2(2+\alpha)^2\left(\alpha^2-1\right)}$ and $K \leq \frac{\alpha^2}{4(1+\alpha)(2+\alpha)}$

Proof. By lemma 2.12 - 2.16. □

In what follows each of the four cases will be analyzed in detail, readers who have no interest in the derivation can skip to the next section.

2.5.1 Case I (Participation constraint and buyer's incentive compatibility constraint bind)

When K is low, we will have solution where PC and IC

$$P_{\text{post}} = (1+\alpha)^2 K.$$

So seller will always sue buyer if he is not paying, as $P_{\text{post}} = (1+\alpha)^2 K \geq \left(\frac{1+\alpha}{\alpha}\right)^2 K$.

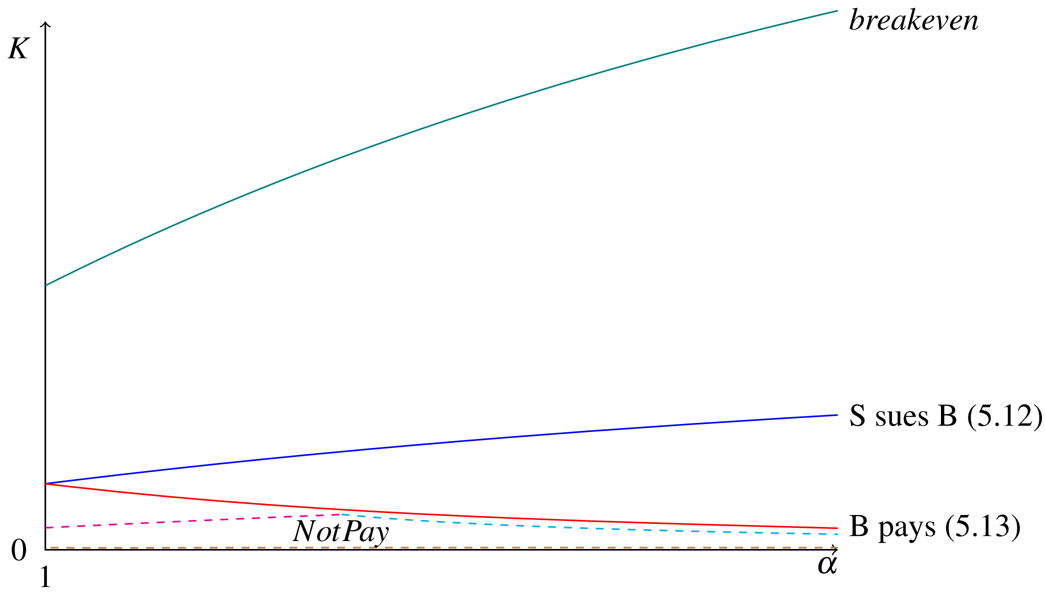

Figure 2.5.2: Optimal ex-post payment contract

For the region below the red curve (2.5.4), excluding the area bounded by the dashed curves, the case I ex-post payment contract will be offered. For the region between blue (2.5.3) and red curve (2.5.4), the case II ex-post payment contract will be offered. For the region bounded by the dashed curves, the buyer-default ex-post payment contract will be offered. For the region between the teal curve (2.5.6) and the blue curve (2.5.3), the case III ex-post payment contract will be offered.

With the binding participation constraint, $P_{\text{post}} - \frac{\tilde{e}^2}{2} = 0$, we have

$$\tilde{e} = \sqrt{2K}\,(1+\alpha)\,.$$

Which is the same as case II of ex-ante payment. Buyer will offer this contract if,

$$EU_B^* \geq 0 \Longrightarrow \tilde{e}^* - P^*_{\text{post}} \geq 0,$$

which is equivalent to

$$K \leq \frac{2}{(1+\alpha)^2}. \tag{2.5.2}$$

Lemma 2.12. *When $K \leq \frac{2}{(1+\alpha)^2}$, the optimal ex-post payment contract is,*

$$\tilde{e}^* = \sqrt{2K}\,(1+\alpha)\,,\; P^*_{post} = (1+\alpha)^2 K.$$

2.5.2 Case II (Participation constraint binds)

Consider first the optimal contract that buyer will pay,

$$\max \tilde{e} - P_{\text{post}} \quad s.t.\, P_{\text{post}} - \frac{\tilde{e}^2}{2} \geq 0,$$

$$\tilde{e}^* = 1, \quad P^*_{\text{post}} = \frac{1}{2}, \quad EU^*_B = \frac{1}{2}.$$

So seller would sue buyer if,

$$K \leq \frac{\alpha^2}{2\,(1+\alpha)^2}. \tag{2.5.3}$$

Buyer would pay if,

$$K \geq \frac{1}{2\,(1+\alpha)^2}. \tag{2.5.4}$$

Notice that, this is exactly the same as the type III contract in the ex-ante payment.

Lemma 2.13. *For medium range of fixed cost, $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$, the optimal ex-post payment contract is efficient, where*

$$\tilde{e}^* = 1,\ P^*_{post} = \frac{1}{2}.$$

2.5.2.1 Comparison: Case II Vs Case I

To compare this contract with the previous one,

$$EU^I_B \leq EU^{II}_B \Longrightarrow (1+\alpha)\left(\sqrt{2K} - K\,(1+\alpha)\right) \leq \frac{1}{2},$$

which is equivalent as

$$K \geq \frac{1}{2\,(1+\alpha)^2}. \tag{2.5.5}$$

But this is exactly the lower bound for the second ex-post payment contract to implement, for any other K, $EU^I_B \leq EU^{II}_B$. So we have the following result.

Lemma 2.14. *The case II ex-post payment contract dominates the case I contract.*

2.5.3 Case III (Participation constraint & seller's legal constraint bind)

When K becomes higher, LC_S will be binding, and the solution is determined by the intersection point of PC and LC_S.

$$p_{post} = \frac{\tilde{e}^2}{2} = K\left(\frac{1+\alpha}{\alpha}\right)^2,$$
$$\tilde{e} = \sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right).$$

IC_B must be satisfied as LC_S is binding, the only condition need to be checked is that the buyer is willing to offer this contract,

$$EU_B = \sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right) - K\left(\frac{1+\alpha}{\alpha}\right)^2 \geq 0,$$

which is equivalent to

$$K \leq \frac{2\alpha^2}{(1+\alpha)^2}. \tag{2.5.6}$$

Lemma 2.15. *The optimal contract when PC and LC are binding, if* $K \leq \frac{2\alpha^2}{(1+\alpha)^2}$, *is*

$$\tilde{e}^* = \sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right),\ p^*_{post} = K\left(\frac{1+\alpha}{\alpha}\right)^2.$$

2.5.3.1 Comparison: Case III Vs Case I

The comparison depends on

$$\sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right) - K\left(\frac{1+\alpha}{\alpha}\right)^2 EU_B^{III} \geq EU_B^{I} \Longrightarrow \geq \sqrt{2K}\,(1+\alpha) - K\,(1+\alpha)^2,$$

which is equivalent to

$$K \geq \frac{2\alpha^2}{(1+\alpha)^4}.$$

Thus we have a lower bound for which Case III ex-post contract is preferred to the Case I contract. A question follows naturally is whether this lower bound goes below the lower bound for the the Case II ex-post payment contract. The following shows that the answer is no.

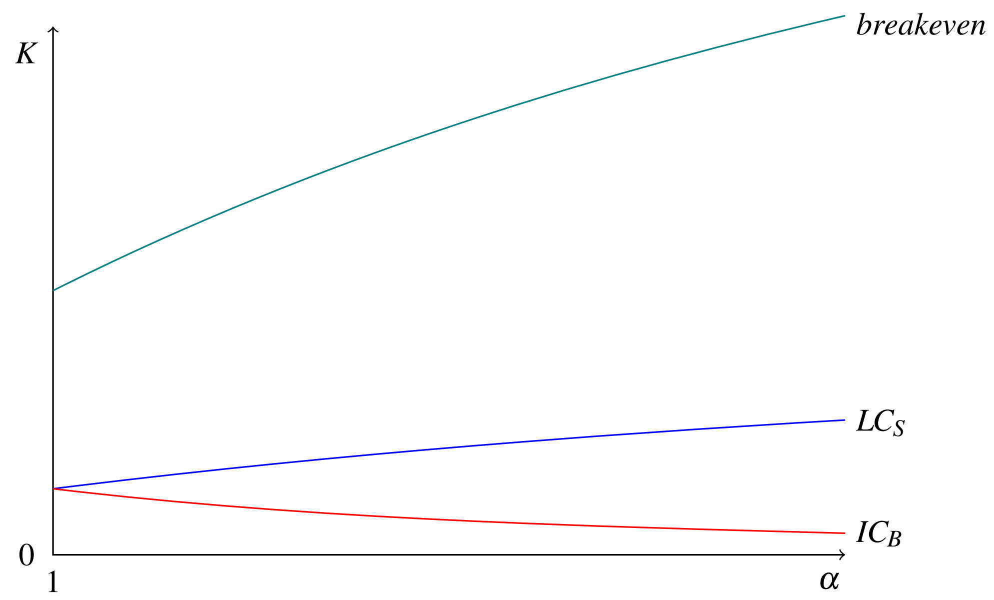

Figure 2.5.3: Legal environment for ex-post payment contract
For the region below the red curve 2.5.4, the case I ex-post payment contract is optimal. For the region between the blue (2.5.3) and red curve (2.5.4), the case II ex-post payment contract is optimal. For the region between the blue (2.5.3) and teal curve 2.5.6, the case III ex-post payment contract is optimal.

$$3\alpha^2 - 2\alpha - 1\frac{2\alpha^2}{(1+\alpha)^4} \geq \frac{1}{2(1+\alpha)^2} \Longrightarrow \geq 0$$

The last inequality holds if $\alpha \geq 1$. So when K increases from zero, the type of ex-post contract will be offered is changed from Case I to Case II and then to Case III. See figure 2.5.3.

By comparing what we have so far, we get an important result.

Proposition 2.3.

For every ex-post payment contract that buyer pays in equilibrium, there is an equivalent ex-ante payment contract, which having the same price and required effort.

Proof.

By lemma 2.10 and 2.13, the case III ex-ante payment contract and the case II ex-post payment contract are essentially the same, except the timing of payment. By lemma 2.7 and 2.12, the case II ex-ante payment contract and the case I ex-post payment contract are essentially the same, except the timing of payment. By lemma 2.11 and 2.15, the case IV ex-ante payment contract and the case III ex-post payment contract are essentially the same, except the timing of payment. So for every ex-post payment contract, there is a corresponding ex-ante payment contract, the only difference is the timing of payment.

□

2.5.4 Case IV (Buyer default)

Proposition 2.3 is true if we only consider the ex-post payment contract that buyer will pay. Now let us consider the optimal ex-post payment contract that buyer will not pay, where $P_{\text{post}} \geq (1+\alpha)^2 K$. Under this case, the seller will sue the buyer for not paying, because $P_{\text{post}} \geq K\left(\frac{1+\alpha}{\alpha}\right)^2$. But still the seller will provide the required effort first. This may seems strange that the seller will do so, if the seller anticipates that the buyer will not pay. Actually the incentive constraint when the seller anticipates buyer default $\left(IC_S^D\right)$ is more stringent than the one he expects no default (IC_S).

$$IC_S \geq IC_S^D \Longrightarrow P_{\text{post}} - \frac{\tilde{e}^2}{2} \geq \left(\frac{\alpha}{1+\alpha}\right)^2 P_{\text{post}} - \frac{\tilde{e}^2}{2} - K,$$

which is equivalent to

$$K \quad \geq \quad -\frac{1+2\alpha}{(1+\alpha)^2} P_{\text{post}}.$$

The last inequality holds by assumption, as all variables are positive. Thus wrong expectation creates no problem. No matter what expectation the seller has, the seller will provide the required effort. The buyer will default, and then the seller sue. So in the following, we are going to maximizes the expected payoff when the buyer defaults, subject to IC_S^D:

$$\max \tilde{e} - P_{\text{post}}\left(\frac{\alpha(2+\alpha)}{(1+\alpha)^2}\right) - K \quad s.t. \ \left(\frac{\alpha}{1+\alpha}\right)^2 P_{\text{post}} - \frac{\tilde{e}^2}{2} - K \geq 0.$$

The solutions are

$$\tilde{e}^* = \frac{\alpha}{2+\alpha},$$
$$P_{\text{post}}^* = \frac{(1+\alpha)^2}{2(2+\alpha)^2} + \left(\frac{1+\alpha}{\alpha}\right)^2 K.$$

The condition of buyer not paying is $P_{\text{post}}^* \geq (1+\alpha)^2 K$, which is equivalent to

$$K \leq \frac{\alpha^2}{2(2+\alpha)^2(\alpha^2-1)}.$$

The buyer will offer this contract if $EU_B^* = \frac{\alpha}{2(2+\alpha)} - \frac{2(1+\alpha)}{\alpha} K \geq 0$, which is equivalent to

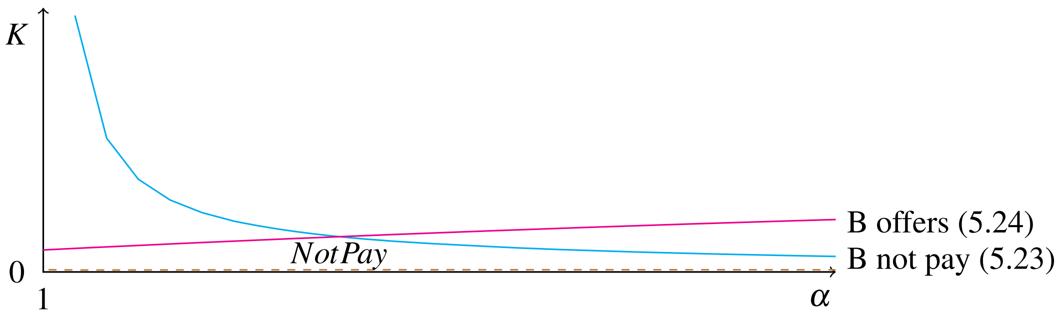

Figure 2.5.4: Legal environment for the buyer default ex-post payment contract
The brown curve is equation 2.5.7. For the region in between the three curves, the buyer default ex-post payment contract is optimal.

$$K \leq \frac{\alpha^2}{4(1+\alpha)(2+\alpha)}.$$

Lemma 2.16. *For low enough litigation fixed cost,* $K \leq \frac{\alpha^2}{2(2+\alpha)^2(\alpha^2-1)}$ *and* $K \leq \frac{\alpha^2}{4(1+\alpha)(2+\alpha)}$, *the optimal buyer default ex-post payment contract is*

$$\tilde{e}^* = \frac{\alpha}{2+\alpha}, \quad P^*_{post} = \frac{(1+\alpha)^2}{2(2+\alpha)^2} + \left(\frac{1+\alpha}{\alpha}\right)^2 K.$$

2.5.4.1 Comparison: Case IV Vs Case I

To compare the buyer default contract with the case I,

$$EU_B^{IV} \geq EU_B^I \Longrightarrow \frac{\alpha}{2(2+\alpha)} - \frac{2(1+\alpha)}{\alpha}K \geq \sqrt{2K}(1+\alpha) - (1+\alpha)^2 K,$$

which is equivalent to

$$K \geq -\frac{\alpha^2\left(2\sqrt{2(1+\alpha)} - \alpha - 3\right)}{2(\alpha-1)(\alpha^4+4\alpha^3+3\alpha^2-4\alpha-4)}. \tag{2.5.7}$$

Note that, for $\alpha > 1$, the numerator is negative while the denominator is positive, thus the right hand side is positive. So the condition says that K cannot be extremely low for the buyer default contract to be chosen.

2.6 Optimal Contract: Ex-ante Vs Ex-post

We have derived various ex-ante and ex-post payment contract in section 2.4 and 2.5, a final step is to ask under a specific legal environment, which contract is going to be offered. The optimal payment scheme depends on the following comparison,

$$
\begin{aligned}
& EU_B^{\text{ante}}(\text{case I}) - EU_B^{\text{post}}(\text{non pay}) \\
=& \frac{\alpha(2+\alpha)}{2(1+\alpha)^2} + K\frac{(1+\alpha)^2}{\alpha(2+\alpha)} - \frac{\alpha}{2(2+\alpha)} + \frac{2(1+\alpha)}{\alpha}K \\
=& \frac{\alpha\left((2+\alpha)^2-(1+\alpha)^2\right)}{2(2+\alpha)(1+\alpha)^2} + K\frac{(1+\alpha)^2+2(1+\alpha)(2+\alpha)}{\alpha(2+\alpha)} \\
>& 0
\end{aligned}
$$

So the case IV buyer default ex-post payment contract is dominated by the case I ex-ante payment contract. In total the optimal payment scheme will be as follows, where P without subscript stands for either P_{ante} or P_{post},

Proposition 2.4. *Optimal pure payment contract depends on specific legal environment, (α, K), which is summarized in the following table.*

Case	(α, K)	e^*	P^*_{ante}
I	$0 \le K \le \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$	$\frac{\alpha(2+\alpha)}{(1+\alpha)^2}$	$\frac{\alpha(2+\alpha)}{2(1+\alpha)^2} - K\frac{(1+\alpha)^2}{\alpha(2+\alpha)}$
II	$\frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \le K \le \frac{1}{2(1+\alpha)^2}$	$\sqrt{2K}(1+\alpha)$	$(1+\alpha)^2 K$
III	$\frac{1}{2(1+\alpha)^2} \le K \le \frac{\alpha^2}{2(1+\alpha)^2}$	1	$1/2$
IV	$\frac{\alpha^2}{2(1+\alpha)^2} \le K \le \frac{2\alpha^2}{(1+\alpha)^2}$	$\sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right)$	$K\left(\frac{1+\alpha}{\alpha}\right)^2$

In diagram, see figure 2.6.1.

Proposition 2.5. *The choice of ex-ante or ex-post payment contract is irrelevant, only when the fixed cost of litigation is relatively low, ex-ante payment contract is certainly chosen.*

Proof. By lemma 2.4. □

To see what level of the fixed cost is low enough to have ex-ante payment contract, consider the legal environment where $\alpha = 1$. Under this legal environment, the ratio $K/e^* = 3/32 \approx 9\%$. This suggests that if the fixed cost of the litigation is less than approximately 9% of the total value of the transaction, ex-ante payment contract is better than ex-post payment contract. If we use the efficient level of quality as a benchmark, the ratio $K/e^* \approx 7\%$, which is not too far from the previous one.

We can understand this result by looking the ex-post payment contract when K is sufficiently low. Consider the extreme case, when $K = 0$. Under this case, no ex-post payment contract can be offered, because IC_B and LC_S requires $P_{\text{post}} = 0$. Thus when K is relatively low, the possible range of P_{post} is severely restricted. If P_{post} goes too high, the buyer has no incentive to pay, but when P_{post} goes too low, the seller will have no incentive to sue even if the buyer defaults. Hence only highly inefficient ex-post payment contract can be offered, whereas ex-ante payment contract do not have this problem. There are no restrictions on the possible range of P_{ante} when $K = 0$, apart from $P_{\text{ante}} \geq 0$. Note that there are still other inefficient contracts offered, with $\tilde{e} < 1$. The following result points out the reason.

Proposition 2.6. *Inefficient contracts are offered due to either too high or too low fixed cost of litigation charged.*

Proof. i) When $\frac{\alpha^2}{2(1+\alpha)^2} \leq K \leq \frac{2\alpha^2}{(1+\alpha)^2}$, the contract with $\tilde{e}^* = \sqrt{2K}\left(\frac{1+\alpha}{\alpha}\right)$, $P^* = K\left(\frac{1+\alpha}{\alpha}\right)^2$ will be offered. If instead the efficient contract is offered, either ex-ante or ex-post payment contract, one party will breach the contract, the other party will be better off not suing the defaulting party, because K is too high. The breaching party will be the seller if the efficient ex-ante payment contract is offered, see equation (2.4.4). While the breaching party will be the buyer if the efficient ex-post payment contract is offered, see equation (2.5.3).
ii) When $\frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6} \leq K \leq \frac{1}{2(1+\alpha)^2}$, $\tilde{e}^* = \sqrt{2K}\,(1+\alpha)$, $\quad P^* = (1+\alpha)^2 K$ is offered. And when $K \leq \frac{\alpha^2(2+\alpha)^2}{2(1+\alpha)^6}$, the case I ex-ante payment contract is offered. These two cases share the same reasoning, the problem is not because K is too high, instead the opposite is true. If instead an efficient contract is offered, either ex-ante or ex-post payment contract, one party will find it better off not to follow the contract and seeking litigation to resolve the conflict, because K is too low. The breaching party will be the seller if the efficient ex-ante payment contract is offered, see equation (2.4.3). While the breaching party will be the buyer if the efficient ex-post payment contract is offered, see equation (2.5.4). □

We understand that, from the proposition above, the cause of inefficient contract is the wrong level of fixed cost of litigation being charged. So naturally that points out a simple solution to the problem.

Proposition 2.7. *Inefficient contracts can be replaced by efficient one when the the fixed cost of litigation is within the range of* $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$.

Proof. By lemma 2.4, for any given α, if the fixed cost of litigation is within the following bounds, $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$, efficient contract will be offered, either ex-ante, ex-post or mixed payment. □

This result sheds some lights on legal reforms. One particular question is whether should the legal sector shares some part of the cost. In the model we presented, a legal reforms can be represented by a higher α, where the funding burden can be represented by a higher K. If one of the purpose of the legal reform is to enhance market efficiency, then there are two general lessons to be taken from our model. If the status quo K is too high, as described in part i) of the proof of proposition 2.6, the best thing to do is to seek external funding. Otherwise, by raising K, it will just make it harder to move into the "efficient region" for a given α. On the other

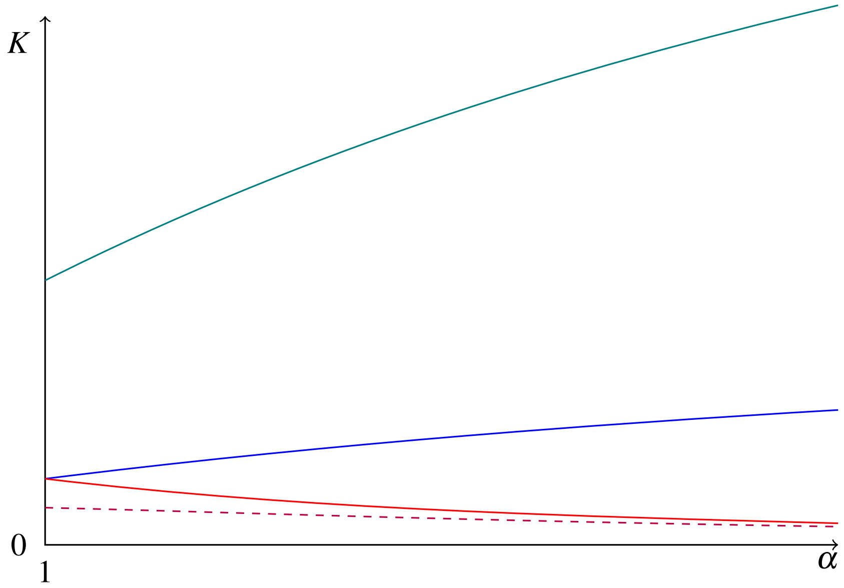

Figure 2.6.1: Optimal contract: ex-ante Vs ex-post
For the region below the purple dashed curve, the case I ex-ante payment contract will be offered. For the region between the purple dashed curve and the red curve (2.4.3) or (2.5.4), the case II ex-ante or case I ex-post payment contract will be offered. For the region between blue (2.4.4) or (2.5.3) and red curve (2.4.3) or (2.5.4), the case III ex-ante or case II ex-post payment contract will be offered. For the region between the teal curve (2.4.5) or (2.5.6) and the blue curve (2.4.4) or (2.5.3), the case IV ex-ante or case III ex-post payment contract will be offered.

hand, when the status quo K is too low, as described in part ii) of the proof of proposition 2.6, the legal reforms better be funded by legal sector itself. The only exception, is that if the status quo is already close to the "efficient region", then partial external funding should be considered.

A question follows naturally is that how to we know the status quo K is too high or too low, given the efficiency of the litigation system α. One defining feature is to look at the output level. There will be overproduction when K is too high, as shown in part iv) of lemma 2.4. And underproduction when K is too low, as shown in part i) and ii) of lemma 2.4. Another way to discern it to look at the price, price will be higher than efficient level when K is too high, and price will be lower than the efficient level when K is too low. By looking at either price or output, and compare to the efficient level, we can get an idea which direction should the legal reform goes.

2.7 Mixed Payment

Up till now, we have only considered the pure form of payment, which is either ex-ante or ex-post, but not both. There is still one type of payment we have not consider yet, which is the mixed payment. There are many possible way to mix the payment, and I will focus on those contract which satisfies the following conditions:

$$P_{\text{ante}} \geq \left(\frac{1+\alpha}{\alpha}\right)^2 K, \tag{LC$_B$}$$

$$P_{\text{post}} \geq \left(\frac{1+\alpha}{\alpha}\right)^2 K, \tag{LC$_S$}$$

$$P_{\text{post}} \leq (1+\alpha)^2 K, \tag{IC$_B$}$$

$$\frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + P_{\text{post}} \geq \frac{\tilde{e}^2}{2} - K, \tag{IC$_S$}$$

$$P_{\text{ante}} + P_{\text{post}} \geq \frac{\tilde{e}^2}{2}. \tag{PC$_S$}$$

Equation (LC_B) is the legal condition for the buyer will sue the seller if the seller shirks. Equation (LC_S) is the legal constraint for the seller to sue the buyer if the buyer defaults on the ex-post price. Equation (IC_B) is the incentive compatibility condition for the buyer to pay the ex-post price. Equation (IC_S) is the familiar incentive constraint to motivate the seller to pay effort. Finally equation (PC_S) is the participation constraint of the seller. Given the number of the constraints we face and the possibility of corner solutions, we are not going to study in details the optimal mixed payment contract, but instead we will show that, when the pure payment contract is inefficient, there are efficient mixed payment contract can be offered and it is in the interest of the buyer to offer that.

2.7.1 Case I

Firstly we consider the solution with binding LC_B: $P_{\text{ante}} = K\left(\frac{1+\alpha}{\alpha}\right)^2$. Then by the binding P$C_S$,

$$P_{\text{post}} = \frac{\tilde{e}^2}{2} - K\left(\frac{1+\alpha}{\alpha}\right)^2.$$

Then we have $EU_S = 0$, $EU_B = \tilde{e} - \frac{\tilde{e}^2}{2}$. Thus $\tilde{e}^* = 1$, and hence $EU_B^* = 1/2$. Since IC will always be satisfied, as long as $\alpha > 1$, the remaining two constraints delineate the effective legal environment for this contract. The first inequality is derived from LC_S, while the second inequality is derived from IC_B.

$$K \leq \frac{\alpha^2}{4(1+\alpha)^2}, \tag{2.7.1}$$

$$K \geq \frac{\alpha^2}{2(1+\alpha)^2(1+\alpha^2)}. \tag{2.7.2}$$

Lemma 2.17. *When the litigation fixed cost is within certain bound, where* $\frac{\alpha^2}{2(1+\alpha)^2(1+\alpha^2)} \leq K \leq \frac{\alpha^2}{4(1+\alpha)^2}$*, the following mixed payment contract is feasible,*

$$P_{ante} = K\left(\frac{1+\alpha}{\alpha}\right)^2, \quad P_{post} = \frac{1}{2} - K\left(\frac{1+\alpha}{\alpha}\right)^2.$$

In order to have non-negative ex-post price, we need

$$K \leq \frac{\alpha^2}{2\,(1+\alpha)^2},$$

where this is satisfied by the LC_S.

An natural question follows is that will IC_S be binding? The answer is if so, the PC_S will not satisfy. By binding IC_S, substitute in $P_{\text{ante}} = K\left(\frac{1+\alpha}{\alpha}\right)^2$ and $e = 1$, we have

$$P_{post} \;=\; \frac{1}{2} - 2K\left(\frac{1+\alpha}{\alpha}\right).$$

This is the P_{post} that satisfies the IC_S, and it can be shown that this P_{post} cannot satisfy the PC_S as follows.

$$\begin{aligned} P_{ante} + P_{post} &= K\left(\frac{1+\alpha}{\alpha}\right)^2 + \frac{1}{2} - 2K\left(\frac{1+\alpha}{\alpha}\right) \\ &= \frac{1}{2} - K\frac{(1+\alpha)(\alpha-1)}{\alpha^2} < \frac{1}{2}. \end{aligned}$$

As a result, it is impossible for both IC_S and PC_S to be binding in this case.

2.7.2 Case II

Another possible mixed payment contract is to set $P_{\text{ante}} = K\,(1+\alpha)^2$. By the binding PC_S, we have

$$P_{\text{post}} = \frac{\tilde{e}^2}{2} - K\,(1+\alpha)^2$$

Again we have $EU_S = 0$, $EU_B = \tilde{e} - \frac{\tilde{e}^2}{2}$. Thus $\tilde{e}^* = 1$, and hence $EU_B^* = 1/2$. Since IC_S will always be binding, the remaining two constraints delineate the effective legal environment for this contract. The first inequality is derived from LC_S, while the second inequality is derived from IC_B.

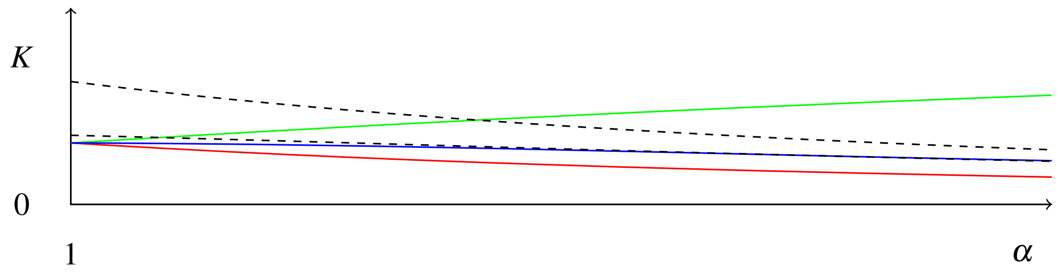

Figure 2.7.1: Feasible legal environment for the mixed payment contracts
The case I mixed payment contract is feasible in the region between the green (2.7.1) and blue curves (2.7.2 or 2.7.3). The case II mixed payment contract is feasible in the region between the red (2.7.4) and blue curves (2.7.2 or 2.7.3). The region between the dashed curve is the region for the inefficient case II ex-ante payment contract or case I ex-post payment contract.

$$K \leq \frac{(\alpha\tilde{e})^2}{2(1+\alpha)^2(1+\alpha^2)}, \tag{2.7.3}$$

$$K \geq \frac{\tilde{e}^2}{4(1+\alpha)^2}. \tag{2.7.4}$$

Lemma 2.18. *When the litigation fixed cost is between,* $\frac{\tilde{e}^2}{4(1+\alpha)^2} \leq K \leq \frac{(\alpha\tilde{e})^2}{2(1+\alpha)^2(1+\alpha^2)}$*, the following mixed payment contract is feasible,*

$$P_{ante} = K(1+\alpha)^2, \quad P_{post} = \frac{\tilde{e}^2}{2} - K(1+\alpha)^2.$$

It can be shown easily that the remaining constraint IC_S is binding. To consider the possible range of this two contracts, see figure 2.7.1.

From equation (2.4.3) and (2.5.5), because $EU_B^* = 1/2$, we know that the mixed payment contracts are preferred to the case II ex-ante payment contract or case I ex-post payment contract. And hence we have the following proposition.

Proposition 2.8. *Mixed payment contracts improve efficiency.*

Proof. Consider the region where $\frac{1}{4(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{4(1+\alpha)^2}$, which is within the feasible region for the inefficient case II ex-ante payment contract and case I ex-post payment contract. Now instead with the mixed payment contract, full efficiency can be attained. See Figure 2.7.1. □

2.8 Who should offer the contract

Up till now, we assume the buyer offers the contract to the seller. In real life, there are a lot of cases where the reverse is true, which sellers decides the price, payment timing and the product quality. Typically it is also a take it or leave it offer. So in the following, the role of contracting party will swap. Similar to the previous analysis, all possible cases of ex-ante and ex-post payment contract will be studied in details. The first result is that, if ex-ante payment contract is used, it is more efficient for have the seller as the contract proposer. The second result is that, when the seller proposes, ex-ante payment contract weakly dominates ex-post payment contract. Finally, when the mixed payment contract is taken into account, in particular the two specific mixed payment contracts we have analyzed previously, ex-ante payment contract still weakly dominates.

2.8.1 Ex-ante Payment

Consider firstly with pure ex-ante payment contract. The problem now can be formulated as follows,

$$\begin{aligned}
&\max_{\tilde{e},P_{\text{ante}}} P_{\text{ante}} - \frac{\tilde{e}^2}{2} \\
&s.t.\ \frac{\alpha(2+\alpha)}{(1+\alpha)^2}P_{\text{ante}} + K \geq \frac{\tilde{e}^2}{2}, && (\text{IC}_S)\\
&\qquad \tilde{e} - P_{\text{ante}} \geq 0, && (\text{PC}_B)\\
&\qquad P_{\text{ante}} \geq K\left(\frac{1+\alpha}{\alpha}\right)^2, && (\text{LC}_B)
\end{aligned}$$

Equation (IC_S) is the incentive constraint for the the seller to discourage shirking, while equation (PC_B) is the participation constraint for the buyer, lastly equation (LC_B) is the legal constraint for the buyer such that buyer will sue the seller in the case of default. Similar to the previous analysis, to envisage the solutions, we can start from $K = 0$, when LC_B becomes $P_{\text{ante}} \geq 0$. In this case, the solution is the tangent point on the PC_B, as the IC_S intersects with the PC_B only at points where $\tilde{e} \geq 1$.[12] When K increases up to some threshold, the LC_B will becomes binding, limiting the offer $\tilde{e} \geq 1$. Under this case, the solution is at the intersection point of LC_B and PC_B. We are now going to analyze these two cases formally.

2.8.1.1 Case I (Buyer's Participation Constraint binds)

When K is small, the only binding constraint is the PC_B. The incentive constraint of the seller must be slack, as shown in footnote 12. The seller's problem is

$$\max P_{\text{ante}} - \frac{\tilde{e}^2}{2}, \qquad s.t.\ \tilde{e} - P_{\text{ante}} \geq 0.$$

[12] When $K = 0$, the intersection point of PC_B and LC_B is $P_{ante} = \tilde{e} = \frac{2\alpha(2+\alpha)}{(1+\alpha)^2}$, it will be greater than 1 if $2\alpha(2+\alpha) \geq (1+\alpha)^2 \Longrightarrow \alpha^2 + 2\alpha - 1 \geq 0$, which is satisfied by the assumption $\alpha \geq 1$.

The solution to the problem is

$$\tilde{e}^* = 1 \quad P^*_{\text{ante}} = 1.$$

There are two corresponding conditions for this contract to be effective. First, it is the IC_S, which requires

$$K \geq \frac{1 - 2\alpha - \alpha^2}{2(1+\alpha)^2}.$$

Since $\alpha \geq 1$ and $K \geq 0$, this condition must be satisfied. In other words, we only need to consider LC_B, which requires

$$K \leq \left(\frac{\alpha}{1+\alpha}\right)^2. \tag{2.8.1}$$

Lemma 2.19. *When the fixed cost of litigation is low enough, $K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$, the optimal seller-proposed ex-ante payment contract is $\tilde{e}^* = P^*_{ante} = 1$.*

2.8.1.2 Case II (Buyer's legal constraint and participation constraint binds)

When K increases to certain threshold, the LC_B will becomes binding. The solution will be at the intersection point of the LC_B and PC_B, where

$$\tilde{e} = P_{\text{ante}} = K\left(\frac{1+\alpha}{\alpha}\right)^2$$

To ensure the IC_S is satisfied, we have condition:

$$K \leq \frac{4\alpha^3}{(1+\alpha)^3}.$$

Because case II is the a corner solution due to the binding LC_B, the EU_S must be lower than case I. Hence we have the optimal ex-ante payment contract when seller is the proposer:

Lemma 2.20. *When $0 \leq K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$, the optimal seller-proposed ex-ante payment contract is $\tilde{e}^* = 1,\ P^*_{ante} = 1$. When $\left(\frac{\alpha}{1+\alpha}\right)^2 \leq K \leq \frac{4\alpha^3}{(1+\alpha)^3}$, the optimal seller-proposed ex-ante payment contract is $\tilde{e} = P_{ante} = K\left(\frac{1+\alpha}{\alpha}\right)^2$.*

From figure 2.8.1, we could notice that the region of efficient contracting is much wider than the previous case when buyer propose the contract. So we have the following proposition.

Proposition 2.9. *For ex-ante payment contract, it is more efficient to have seller as the proposer.*

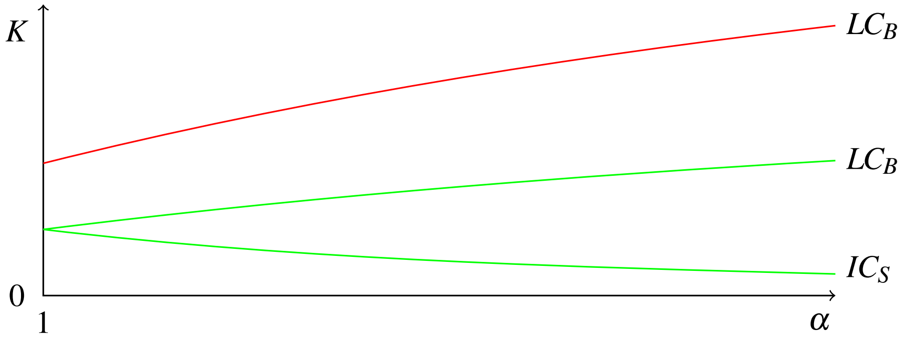

Figure 2.8.1: Legal environment for ex-ante payment contract
For the region below the red curve (2.8.1), the seller-proposed ex-ante payment contract is feasible. For the region between the two green curves (2.4.3 and 2.4.4), the case III buyer-proposed ex-ante payment contract is feasible.

The main intuition lies in that P_{ante} is higher in this case, which can be view as a larger pie to be competed in the litigation, thus allows a wider range of K.

2.8.2 Ex-post Payment

When seller propose ex-post payment contract, the whole problem can be formulated as follows,

$$\max_{\tilde{e},P_{\text{post}}} P_{\text{post}} - \frac{\tilde{e}^2}{2}$$

$$s.t.\ \ \tilde{e} - P_{\text{post}} \geq 0, \tag{PC$_B$}$$

$$P_{\text{post}} \leq K(1+\alpha)^2 \tag{IC$_B$}$$

$$P_{\text{post}} \geq K\left(\frac{1+\alpha}{\alpha}\right)^2 \tag{LC$_S$}$$

where equation (PC_B) is the participation constraint of the buyer, equation (IC_B) is the incentive constraint of the buyer, and equation (LC_S) is the legal constraint for the seller to sue the buyer, if the buyer do not pay. Again we can envisage the solution by considering first an extreme case. When $K = 0$, virtually no ex-post payment contract can be offered, except the one with $\tilde{e} = 0,\ P_{post} = 0$. When K goes positive, we expect the IC_B will be binding, the solution will be the intersection point of IC_B and PC_B. After K moves up to higher level, the IC_B will no longer be binding, and the solution is the tangency point on the PC_B. If K is of a even higher level, the LC_S kicks in, the solution will now be the intersection point of the LC_S and the PC_B. We are going to examine these three cases one by one in the following subsections.

2.8.2.1 Case I (Buyer's incentive compatibility constraint and participation constraint binds)

When K is very low, we will have the following solution,

$$\tilde{e} = P_{post} = (1+\alpha)^2 K$$

The LC_S is satisfied as $K(1+\alpha)^2 \geq K\left(\frac{1+\alpha}{\alpha}\right)^2$. The only condition is that the intersection point is lower than $\tilde{e} \leq 1 \iff K \leq \frac{1}{(1+\alpha)^2}$,

Lemma 2.21. *When $K \leq \frac{1}{(1+\alpha)^2}$, the optimal seller-proposed ex-post payment contract is $\tilde{e}^* = P^*_{post} = (1+\alpha)^2 K$.*

2.8.2.2 Case II (Buyer's participation constraint binds)

When K is higher, the IC_B will be slack, and hence the only binding constraint is the PC_B, and the solution is:

$$\tilde{e}^* = 1, \quad P^*_{\text{post}} = 1.$$

The two corresponding conditions are, firstly from LC_S:

$$K \leq \left(\frac{\alpha}{1+\alpha}\right)^2; \tag{2.8.2}$$

Secondly from IC_B:

$$K \geq \frac{1}{(1+\alpha)^2}. \tag{2.8.3}$$

Lemma 2.22. *When $\frac{1}{(1+\alpha)^2} \leq K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$, the optimal seller-proposed ex-post payment contract is $\tilde{e}^* = 1$, $P^*_{post} = 1$.*

2.8.2.3 Case III (Buyer's legal constraint and participation constraint binds)

When K is even higher, the LC_S kicks in, the solution will be at the intersection of LC_S and PC_B.

$$\tilde{e} = P_{post} = \left(\frac{1+\alpha}{\alpha}\right)^2 K$$

The only condition needs to be consider in this case is the PC_S, which requires

$$K \leq 2\left(\frac{\alpha}{1+\alpha}\right)^2$$

Lemma 2.23. *When* $\left(\frac{\alpha}{1+\alpha}\right)^2 \leq K \leq 2\left(\frac{\alpha}{1+\alpha}\right)^2$*, the optimal seller-proposed ex-post payment contract is* $\tilde{e}^* = P^*_{post} = \left(\frac{1+\alpha}{\alpha}\right)^2 K$*.*

To combine all these three cases, we have the optimal seller-proposed ex-post payment contract.

Lemma 2.24. *The optimal seller-proposed ex-post payment contracts are as follows:*

(α, K)	$\tilde{e}^*$ & P^*_{post}
$0 \leq K \leq \frac{1}{(1+\alpha)^2}$	$(1+\alpha)^2 K$
$\frac{1}{(1+\alpha)^2} \leq K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$	1
$\left(\frac{\alpha}{1+\alpha}\right)^2 \leq K \leq 2\left(\frac{\alpha}{1+\alpha}\right)^2$	$\left(\frac{1+\alpha}{\alpha}\right)^2 K$

From figure 2.8.2, again as we have seen, the range of efficient contracting is wider in ex-post also, when seller proposes. But the result is not as clear cut as the ex-ante case, since some region is not covered. Notice that the the upper constraint (2.8.2) is the same as the one for ex-ante payment (2.8.1), that means, when seller propose, the efficient ex-ante payment contract covers all the efficient ex-post payment contract feasible legal environment, and there is more than that. So we have the following result,

Proposition 2.10. *When the seller offers contract, ex-ante payment contract weakly dominates ex-post payment contract.*

Proof. When $K \geq \left(\frac{\alpha}{1+\alpha}\right)^2$, the case II ex-ante payment contract and case III ex-post payment contract is feasible, and having the same $EU_S = K\left(\frac{1+\alpha}{\alpha}\right)^2\left(1 - \frac{K}{2}\left(\frac{1+\alpha}{\alpha}\right)^2\right)$. However, the coverage of the ex-ante payment contract is wider than the ex-post contract, as $2\left(\frac{\alpha}{1+\alpha}\right)^2 \leq \frac{4\alpha^3}{(1+\alpha)^3}$. The inequality holds as $\alpha \geq 1$. □

From this we know that, when seller proposes, the optimal pure payment contracts is the ex-ante payment contracts shown in lemma 2.20.

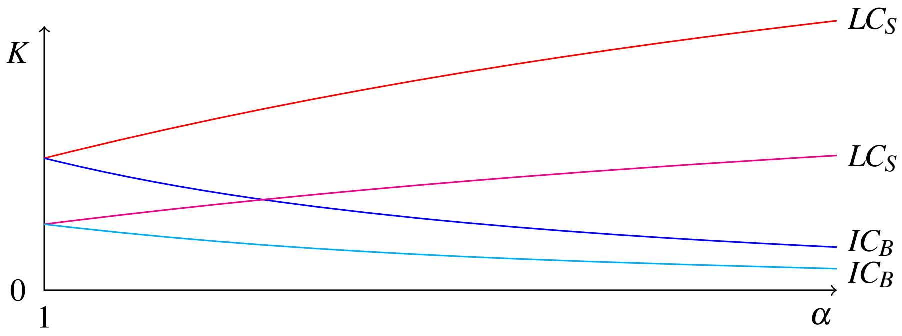

Figure 2.8.2: Legal environment for the ex-post payment contract
For the region between the red (2.8.2) and blue curves (2.8.3), the seller-proposed ex-post payment contract will be offered. For the region between the light red (4.5) and light blue curves (4.6), the case I buyer-proposed ex-post payment contract will be offered.

2.8.3 Mixed Payment

Similar to the buyer-propose mixed payment contract, we have place the same constraints on the seller-propose mixed payment contract,

$$P_{\text{ante}} \geq \left(\frac{1+\alpha}{\alpha}\right)^2 K, \tag{LC$_B$}$$

$$P_{\text{post}} \geq \left(\frac{1+\alpha}{\alpha}\right)^2 K, \tag{LC$_S$}$$

$$P_{\text{post}} \leq (1+\alpha)^2 K, \tag{IC$_B$}$$

$$\frac{\alpha(2+\alpha)}{(1+\alpha)^2} P_{\text{ante}} + P_{\text{post}} \geq \frac{\tilde{e}^2}{2} - K, \tag{IC$_S$}$$

$$P_{\text{ante}} + P_{\text{post}} \leq \tilde{e}. \tag{PC$_B$}$$

Again given the number of the constraints we face and the possibility of corner solutions, we are not going to study in details the optimal mixed payment contract, but instead we will shows that, when the pure payment contract is inefficient, there are efficient mixed payment contract can be offered.

2.8.3.1 Case I

Set $P_{\text{ante}} = \left(\frac{1+\alpha}{\alpha}\right)^2 K$, then by binding PC_B,

$$P_{\text{post}} = \tilde{e} - \left(\frac{1+\alpha}{\alpha}\right)^2 K.$$

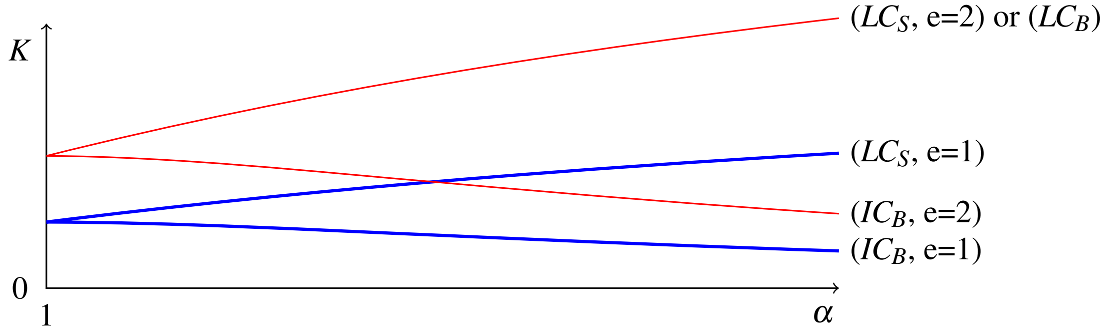

Figure 2.8.3: Legal environment for the case I mixed payment contract
When $\tilde{e} = 1$, for the region between the two thick blue curves (8.15 and 8.16), the case I seller-proposed mixed payment contract is feasible.

Seller will always choose to pay effort if $\tilde{e} \leq 2$, which in order to maximize expected utility, $EU_S = \tilde{e} - \frac{\tilde{e}}{2}$, seller will choose $\tilde{e} = 1$ if feasible. The feasibility depends on the K which is limited by the following two constraints. The first inequality is derived from LC_S while the second inequality is derived from IC_B.

$$K \leq \frac{\alpha^2 \tilde{e}}{2(1+\alpha)^2}, \tag{2.8.4}$$

$$K \geq \frac{\alpha^2 \tilde{e}}{(1+\alpha)^2 (1+\alpha^2)}. \tag{2.8.5}$$

Lemma 2.25. *When the fixed cost of litigation is between $\frac{\alpha^2}{(1+\alpha)^2(1+\alpha^2)} \leq K \leq \frac{\alpha^2}{2(1+\alpha)^2}$, the optimal seller-proposed mixed payment contract is $\tilde{e} = 1$, $P_{ante} = \left(\frac{1+\alpha}{\alpha}\right)^2 K$, $P_{post} = 1 - \left(\frac{1+\alpha}{\alpha}\right)^2 K$.*

In figure 2.8.3, the thick blue curves are the two constraints respectively, assuming $\tilde{e} = 1$. And if $\tilde{e} = 2$, the upper constraint will be the upper red curve (2.8.4, e=2), while the lower constraint is the lower red one (2.8.5, e=2). Notice that the upper constraint (2.8.4) is, when $\tilde{e} = 2$, exactly the same as the constraint for the seller-proposed ex-ante payment contract (2.8.1). So this mixed payment contract can cover as much as the legal environment as the ex-ante one, but there comes a cost. Which is the reduced returns from this mixed payment contract. Only within the blue curves region will the seller earns $\frac{1}{2}$, moving either up or down the $\tilde{e}$ from 1 to escape from the bounds, will reduce marginal returns by $1 - \tilde{e}$.

2.8.3.2 Case II

Just as above, we can set $P_{\text{ante}} = (1+\alpha)^2 K$, by binding PC,

$$P_{\text{post}} = \tilde{e} - (1+\alpha)^2 K.$$

Seller would exert effort so long as $\tilde{e} \leq 2$ and the remaining two constraints are as follows. The first inequality is derived from LC_S while the second inequality is derived from IC_B.

$$K \leq \frac{\alpha^2 \tilde{e}}{(1+\alpha)^2 (1+\alpha^2)}, \tag{2.8.6}$$

$$K \geq \frac{\tilde{e}}{2(1+\alpha)^2}. \tag{2.8.7}$$

Lemma 2.26. *When the fixed cost of litigation is between,* $\frac{1}{2(1+\alpha)^2} \leq K \leq \frac{\alpha^2}{(1+\alpha)^2(1+\alpha^2)}$, *the optimal seller-proposed mixed payment contract is* $\tilde{e} = 1$, $P_{ante} = (1+\alpha)^2 K$, $P_{post} = 1 - (1+\alpha)^2 K$.

In figure 2.8.4, the two constraints are the two thick blue curves respectively, assuming $\tilde{e} = 1$. Which we can see again the feasible region is within the covered area of the ex-ante payment contract. This again shows the inferiority of the mixed payment contract, and we have the following result,

Proposition 2.11. *When the seller proposes, (purely) ex-ante payment contract weakly dominates mixed payment contract.*

Proof. Given the condition which the ex-ante payment contract works is $K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$. Both conditions, (2.8.6) and (2.8.7), are less stringent than $K \leq \left(\frac{\alpha}{1+\alpha}\right)^2$, when $\tilde{e} = 1$. □

The key intuition is that when the seller proposes, he will try to maximize the price, which encourages suing in case of breaching contract, which provides enough incentive for fully efficient contract to be implemented. However when the buyer proposes, he will try to minimize the price, which makes it more restrictive for damaged party willing to sue, thus inefficient contract are proposed, and that opens a room of improvement for mixed contracts, which is not present in the case of seller as the proposer.

2.9 Literature Review

If we consider both trading parties are firms, then the timing of payment problem is studied under the trade credit literature. The main focus of analysis is place on the optimal credit period to be given. If prepayment is selected, the buyer assumes greater product quality risk since buyer cannot inspect the product before payment. Conversely, if trade credit is extended, the seller

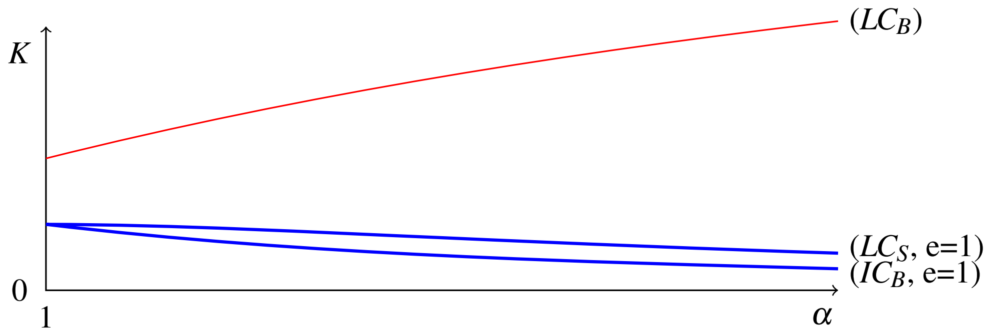

Figure 2.8.4: Legal environment for the case II mixed payment contract
For the region between the two thick blue curves (8.18 and 8.19), the seller-proposed mixed payment contract is feasible.

assumes responsibility for assessing credit risk, financing, and collecting receivables. To put this into a international trade context, recent cases of product adulteration by foreign suppliers have compelled many manufacturers to rethink approaches to deterring suppliers from cutting corners.[13] Recognizing that product liability and product warranty with foreign suppliers are rarely enforceable,[14] some manufacturers turn payment into ex-post contingent on no defects discovery. [15] Which is the case where the K is too high, and outside the bound of any contract with legal protection, and hence the the buyer requests a ex-post payment in order to control the quality of the goods supplied.

An example of ex-post payment in practice is a well-known and widely used financial contract called trade credit.[16] Trade credit is the largest source of external short-term financing for firms both in the US (Petersen and Rajan, 1994) and internationally (Rajan and Zingales, 1995). Ex-post contingent payments (via trade credit) allow the buyers to learn about suppliers' product quality and to withhold contingent payments in case the suppliers produced defective products (Smith, 1987), (Long et al., 1993). Lee and Stowe (1993) argue that ex-post contingent payments (via trade credit) can be thought of as a very strong form of implicit product

[13]Baxter recalled its Heparin in 2008 (Fairclough, 2008), Mattel toys of unapproved lead paint (Story and Barboza, 2007), Pet food due to harmful ingredients such as melamine (Newman, 2007).

[14]Foreign supplier's product liability is rarely enforceable due to different legal systems and inconsistent law enforcement practices in different countries. In some cases, the manufacturer may not even be able to trace the true identity of the fraudulent supplier. And the legal process to claim supplier's product liability can take up to 10 years in foreign countries such as China, the processing cost of a legal case can be very high (Yang, 2007).

Similarly, warranties can be difficult to implement in practice. To get the supplier to pay, the buyer has to prove that a warranty event occurred and that the fault for this event lies with the supplier. And the supplier may be unable to pay, may refuse to pay, or may declare bankruptcy, and courts may need to get involved. For example, as Sherefkin and Armstrong (2003) report, GM demanded \$37.5 million warranty payment from its supplier. But because Oxford Automotive was in bankruptcy, GM and the supplier eventually settled for a smaller amount.

[15]A recent article in the Wall Street Journal (Vandenbosch and Sapp, 2010) supports ex-post payment as a way to "keep the suppliers honest".

[16]For example, trade credit contract "net 30" allows the buyer to delay the payment for 30 days, giving the buyer a 30-day interest-free loan. The trade credit period depends on the industry and on the countries associated with the buyer and the supplier.

warranty, because the buyer can return the product to the supplier and refuse to pay without having to prove that the product is of low quality. More recently, Klapper et al. (2010) articulated that the ex-post contingent payments (via trade credit) reduce the buyers' risk because the buyers have more time to investigate product quality before deciding whether or not to make the contingent payments. For a general review of supply risk problems and solutions, see Tang (2006). Theoretical models that explain trade credit's popularity and the corresponding empirical findings are reviewed in Petersen and Rajan (1997), Biais and Gollier (1997), Ng et al. (1999), and Giannetti et al. (2011).

In the trade credit literature, Long et al. (1993) present an empirical model that builds on the idea articulated by Smith (1987) that trade credit can provide product quality guarantees. By analyzing a sample data that contains all industrial firms from 1984 to 1987, Long et al. (1993) provide empirical evidence to show that the trade credit period increases when defects take more time to discover. Emery and Nayar (1998) present a trade credit model in which the supplier knows the exact time at which the buyer can verify the quality of the product and they show that it is optimal for the supplier to demand payment at the instant before the buyer can verify the product quality. Similarly, Lee and Stowe (1993) propose a signaling model where the quality of the product is known to the supplier but not the buyer and find a separating equilibrium, in which trade credit terms reflect product quality. My model differs from Smith (1987); Lee and Stowe (1993); Emery and Nayar (1998). I do not assume that the quality of the product is exogenous. Instead, I consider the case when the seller can optimally decide on whether or not to shirk, so that the seller's decision is endogenous. Thus I am solving a moral hazard problem rather than a signaling problem.

Babich and Tang (2012) examined simple contingent payment mechanisms to deter suppliers from product adulteration under which the contingent payments are fully controlled by the manufacturer: deferred payment, inspection, and combined mechanisms. They established the conditions under which one mechanism dominates the others. Their study primarily focused on the moral hazard on the seller side, but the moral hazard on the buyer side is not discussed at all. Different from their setup, inspection carries no uncertainty in my paper. Product quality is known immediately once passed to the buyer, which I abstract away from the exact duration of trade credit period.

Dnes, A. W. (1999) consider licensing or franchising agreement in a country with limited rule of law where the franchiser may abscond. The contract needs to be self-supporting, which may be achieved by the careful structuring of the timing of payments. The main idea can be illustrated by the following inequality, $P \leq kL + (1-k)(L-D)$, where P = franchiser's profits from completing contract, L = initial franchise fee, k = probability of franchiser successfully absconding and D = penalty applied to absconding franchiser.[17] So this is essentially an incentive constraint. By singling out L, we have a upper limit of the initial fee (ex-ante payment), $L \leq P + (1-k)D$. So the rest of the payment will pay ex-post to ensure franchiser at least

[17]This is a simplified version by assuming sunk cost to be zero.

break-even. The probability of run-away, k, is exogenous, but in my paper, it is endogenous and both parties have this chance. My paper explicitly model the litigation part which each party decides his spending for the lawsuit.

Faith and Tollison (1980) is the most relevant paper which they directly deal with timing of payment problem. Although there are several interesting claims, it contains no formal model. They suggest timing of payment is an informal institutions that have evolved alongside formal contracts to mitigate agency cost. Their basic claim is that ex-post payment is a rational institutional arrangement to control the significant transaction costs inherent in certain types of exchanges characterized by interpersonal differences in information. There are two cases: Case I - Both traders are equally imperfectly informed ex-ante and equally better informed ex-post. Case II - The seller has an informational advantage ex-ante, but buyer and seller are equally informed ex post. The main problem faced by the seller in the product market, when using ex-post payment is that buyer refuse to pay. But since in general, sellers are competing for buyers, while the reverse is not usually the case, so ex-post payment is a more rational arrangement. Faith and Tollison (1980) paper is full of insight, the main drawback is that the lack of formal modeling of their claims. My paper provides a simple formal model to analyze the timing of payment, while there is no interpersonal differences in information.

Smith and Cox (1985) provides an empirical study of pricing routine legal services. Price may be determined either ex ante or ex post supply. A large cross-sectional survey of law firms does suggest significant variation in whether fixed fee pricing or hourly rate pricing is adopted. Contractual provisions may be understood as a market response to differences in the relative potential for opportunistic cheating by buyers (clients) and by sellers (law firms). But there is a key difference between my paper and theirs. They assume that contracts are perfectly enforceable, so the main focus is on the timing of price determination.

Lee and Png (1990) studies the role of installment payments in relationships characterized by moral hazard and sunk costs. They rule out vertical integration and payments contingent on the product of the contractor. Instead, each payment is negotiated as and when made. In such circumstances, an initial payment serves to redress the weakness of the contractor in ex post renegotiation. If higher effort by the contractor in the first stage increases the marginal product of effort in the second stage, a second installment payment induces the contractor to invest greater effort initially. The contract that they consider is renegotiation-proof, which means there is no need for court to get involved at all, which is the key difference between their study and my paper.

Chen (2004) shows the role of an up-front payment to a contract, with a two-state-two-period model, under the reliance damage measure. He find that in most cases efficiency is not achievable even when an up-front payment is employed. To achieve efficiency, we need three conditions. First, a high enough total payment to make the seller unwilling to breach under the efficient reliance level; second, a high enough up-front payment to make the seller unwilling to sue under the efficient reliance level when the buyer breaches; third, a high enough trading price to make the buyer breach when the low state appears. Lacking any one of these conditions, efficiency fails. The up-front payment therefore plays an indispensable role for efficiency. However my findings suggest that efficiency can be attain even with pure ex-post payment, provided that the legal environment is suitable.

2.10 Some alternative explanations of different payment schemes

There are undoubtedly many other possible explanations for the various payment schemes found in the market. It will be useful to examine some competing explanations. The following discussion does not meant to be a complete list of alternative hypotheses, nor is it a denial of the relevance of alternative explanations. The purpose here is simply to point out the existence of some of the more prominent completing explanations.

One conceivable explanation of the timing of payment is that, depends on the good in question, there is a payment scheme which minimizes the time costs incurred in making payment. This explains multiple cash registers, express line, exact-change requirement. Generally speaking, however, the time cost of paying are liking to be the same with respect to ex-ante or ex-post payment.[18] Ex-ante or ex-post payment would not seems to play a very significant role in reducing the time costs of making payment.

The second potential explanation for the economic distinction between ex-ante and ex-post payment system is that the latter involve an extension of short term credit to customers. But according to this reasoning, there should not be any systematic difference among companies in the timing of collecting revenue. If buyers prefer these short-term loans, competition among sellers will drive all payment go ex-post. What can be said along this line of logic is that firms which choose to extend short term credit by the means of ex-post payment, will try to reducing relevant costs of default. For example, restaurants will have restricted and closely monitored exits if payment is made ex-post.

A firm's size affects its decision of payment scheme. As there is fixed cost associated with managing outstanding credit, it spread over more customer as the firm's customer base expands. Furthermore, the larger the seller's customer base, it is more likely to have more information for some particular type of customers, specifically their credit quality. This led to the prediction that bigger firms would more likely to charge ex-post. It is not very clear whether this holds in general, as counter examples can be found easily.

Another reason why upfront payment will be preferred is because it increases turnover. Consider any restaurant who have fixed number of table, limited operating hours per day, the longer is a customer staying after finished his meal, the less money is making. But why the staying

[18]But not likely to be the same for mixed payment, which requires customer paying twice, in advance and afterward.

time is related to the timing of payment? The argument is mainly about customer psychology, customers who have not paid can justify their occupation of a table, by the mere possibility of further ordering. On the other hand, those who have pay will have no moral justification for staying after their meals are finished. Although this sounds convincing, a higher table turnover does not necessarily associated with higher profit. A related side issue is that, tips must be pay afterward in order to grade the service, which is one of the key reason why there are ex-post payment in restaurant who concerns for their service quality.

The last alternative explanation is that, from an anthropologist or marketing specialist view, timing of payment may be due to certain social customs. Although there is doubtlessly an element of truth in it, all normal social behavior is to some extent customary. A positive economist cannot rest easy with such irrefutable argument. Custom itself is often an implicit compromise of fundamental economic conflict, it deserves an economic explanation.

2.11 Conclusion

This paper provides a simple model to analyze the timing of payment problem. In case of dispute, either the seller fails to perform or the buyer defaults, the court will get involved to resolve the conflict. The efficiency of the legal system and the fixed cost of litigation are the keys for the choice of payment timing. My model predicts, generally speaking, ex-ante payment contract is more prevalent in region with low fixed cost of using the legal system. But since under this situation, ex-ante payment contract is inefficient, by using certain mixed payment contract, full efficiency can be attained. The choice of payment timing becomes unimportant, as the fixed cost goes up to certain range, with respect to the efficiency level of the legal system, both ex-ante and ex-post payment contract are the same and perfectly efficient. Apart from the timing of the payment, the model also have some implications for the proposer of the contract. For ex-ante payment contract, it is more efficient to have the seller as the proposer of the contract, in the sense that it is enforceable in a much wider legal environment. An interesting finding is that, when seller proposes, a simple ex-ante payment contract dominates ex-post payment, which may explain why most of the offers in everyday life are made by the sellers in a simple ex-ante payment form. This suggests that primarily mixed payment is a contractual device used by the buyer to mitigate the two sided moral hazard problem. Besides, this paper also sheds light on the self-funding issue of legal reform.

2.12 Appendix

2.12.1 Perfect Tender Rule

The Uniform Commercial Code (UCC) stipulates in section 2-601 that “if the goods or the tender of delivery fail in any respect to conform to the contract, the buyer may (a) reject the whole; or (b) accept the whole; or (c) accept any commercial unit or units and reject the rest."

Bohlen (1900a; pp. 397f) states that in commercial contracts, courts cannot "force upon [the contract parties] the duty to accept anything differing in the most minute detail from which they have contracted for." In a companion paper, Bohlen (1900b; p. 484) finds clear evidence for the application of strict compliance in state and federal jurisdiction and summarizes that "if either party be guilty of a breach either in delivery or payment, either as to the first or any subsequent portion, then the other party at his option may terminate all of the contract which is still executory."[19]

In Zabriskie Chevrolet, Inc. *v.* Smith,[20] upon driving a new automobile home from the dealer's showroom, the buyer discovered that it was practically inoperable because of a transmission so defective as to require replacement. The court refused to permit the dealer to substitute a new transmission, reasoning that the magnitude of this type of defect upset the peace of mind of the buyer and indeed shattered the buyer's faith in the vehicle. The court reasoned, "Once their faith is shaken, the vehicle loses not only its real value in their eyes, but becomes an instrument whose integrity is substantially impaired and whose operation is fraught with apprehension."

In Joc Oil USA, Inc. *v.* Consolidated Edison Co. of New York,[21] the court agreed that permitting a seller to cure if he had pre-delivery knowledge of the nonconformity but nevertheless reasonably believed the tender would be accepted. "The innocent seller who ships his goods in good faith, reasonably believing that they are conforming and acceptable would be given no relief or redress, but all of the curative power of the statute would be made available to a more culpable seller who delivers his non-conforming wares despite existing knowledge of its [sic] defective, non-conforming qualities. It is difficult to believe that a construction rewarding culpability and penalizing innocence is preferable, or consistent with the remedial intent of the creators of this remedy".[22]

[19] For instance, in a leading common law case, Norrington *v.* Wright (1885), the U.S. Supreme Court rules for contracts governing the sale of goods to be delivered in separate installments and paid for on delivery that if the seller makes a non-conforming, defective delivery with respect to one installment this gives rise to a buyer's right to treat the whole contract as breached and to terminate. In the later case of Fullam *v.* Wright (1907), the court decided accordingly and stipulated that "[w]here there is a contract to sell goods to be delivered in installments and the seller in violation of the contract tenders as a first installment goods inferior to the requirements thereof, the buyer may not only refuse to accept the installment, but he may also rescind the contract *in toto*." Other cases include, Mitsubishi Goshi Kaisha *v.* J. Aron & Co., 16 F.2d 185, 186 (2d Cir. 1926); Buckeye Window Glass Co. *v.* Stewart-Corey Glass Co., 60 Ind. App. 302, 314, 110 N.E. 710, 713 (1915); Goldenberg *v.* Cutler, 189 A.D. 489, 491, 178 N.Y.S. 522, 523 (1919).

[20] 99 N.J. Super. 441, 240 A.2d 195 (1968).

[21] 107 Misc.2d 376, 434 N.Y.S.2d 623 (N.Y. Sup. Ct. 1980).

[22] For more recent cases, see Toshiba Mach. Co. *v.* SPM Flow Control, Inc., 180 S.W.3d 761, 776 (Tex. App.—Fort Worth 2005, pet. granted); D.P. Tech. Corp. *v.* Sherwood Tool, Inc., 751 F. Supp. 1038, 1041-42 & n.6 (D. Conn. 1990); Marlowe *v.* Argentine Naval Comm'n, 808 F.2d 120, 124 (D.C. Cir. 1986); Printing Ctr., Inc. *v.* Supermind Pub. Co., Inc., 669 S.W.2d 779, 784 (Tex. App. —Houston [14th Dist.] 1984, no writ); Tex. Imps. *v.* Allday, 649 S.W.2d 730, 737 (Tex. App.— Tyler 1983, writ ref'd n.r.e.).

2.12.2 UK system

Under the UK system, the losing party is responsible for paying all the litigation expenses. In the litigation stage, if i is the innocent party and j is the faulty one, the expected value of litigation will be:

$$\begin{aligned} V_i &= p\left(x_i, x_j\right)C - \left(1 - p\left(x_i, x_j\right)\right)\left(x_i + x_j + 2K\right) \\ &= \frac{\alpha x_i}{\alpha x_i + x_j}C - \frac{x_j}{\alpha x_i + x_j}\left(x_i + x_j + 2K\right) \qquad (2.12.1) \\ V_j &= -\left(C + x_i + x_j + 2K\right)p\left(x_i, x_j\right) \\ &= -\left(C + x_i + x_j + 2K\right)\frac{\alpha x_i}{\alpha x_i + x_j} \qquad (2.12.2) \end{aligned}$$

where C denotes the compensation paid by the losing party to the winning party, K is the fixed cost of litigation. In the case of buyer default, the compensation would be P_{post}.[23] In the case of seller default, the compensation would be P_{ante}, which would leaves buyer not damaged.[24]

The respective first order conditions of the expected value maximization are:

$$\begin{aligned} \frac{dV_i}{dx_i} &= \frac{\alpha x_j}{\left(\alpha x_i + x_j\right)^2}C + \frac{\alpha x_j}{\left(\alpha x_i + x_j\right)^2}\left(x_i + x_j + 2K\right) - \frac{x_j}{\alpha x_i + x_j} = 0 \\ &= \frac{x_j}{\left(\alpha x_i + x_j\right)^2}\left(C + (\alpha - 1)x_j + 2\alpha K\right) = 0 \\ \frac{dV_j}{dx_j} &= -\frac{\alpha x_i}{\left(\alpha x_i + x_j\right)^2}\left(C + x_i + x_j + 2K\right) - \frac{\alpha x_i}{\alpha x_i + x_j} = 0 \\ &= -\frac{\alpha x_i}{\left(\alpha x_i + x_j\right)^2}\left(C + (1 - \alpha)x_i + 2K\right) = 0 \end{aligned}$$

Define x_i^*, x_j^* as the respective litigation spending of party i and j in equilibrium. Thus we have

$$x_i^* = 0 \text{ or } \frac{C + 2K}{\alpha - 1} \qquad (2.12.3)$$

$$x_j^* = 0 \text{ or } \frac{\alpha\left(C + 2K\right)}{1 - \alpha} \qquad (2.12.4)$$

[23] This compensation is according to the expectation damage principle. Under the expectation damage measure, the defaulting party pays an amount that puts the other party in the position he would have been in had the contract been performed. For details, see Shavell (1980).

[24] This compensation is according to the restitution damage principle. Under the restitution damage measure the defaulting party returns only the payments made to him. See Shavell (1980).

and the second order conditions are:

$$\frac{d^2V_i}{dx_i^2} = \frac{-2\alpha x_j\left(\alpha x_i + x_j\right)\left(c + (\alpha-1)x_j + 2\alpha K\right)}{\left(\alpha x_i + x_j\right)^4}$$
$$\frac{d^2V_j}{dx_j^2} = \frac{-2\alpha x_i\left(\alpha x_i + x_j\right)(c + (1-\alpha)x_i + 2K)}{\left(\alpha x_i + x_j\right)^4}$$

Since by assumption $\alpha > 1$, the solution $x_j = \frac{\alpha(C+2K)}{1-\alpha} < 0$ is rejected. As a result, we have only two Nash equilibria, $\left(x_i^*, x_j^*\right) = \left\{(0,0), \left(\frac{C+2K}{\alpha-1}, 0\right)\right\}$. I will take the second one as the solution as the first one is associated with an undefined winning probability.

Lemma 2.27. *Under the UK system, the optimal litigation spending is*

$$x_i^* = \frac{C+2K}{\alpha-1}$$
$$x_j^* = 0$$

According to this solution, the equilibrium expected value from litigation is

$$\begin{aligned} V_i^* &= \frac{\alpha x_i}{\alpha x_i + x_j}C - \frac{x_j}{\alpha x_i + x_j}\left(x_i + x_j + 2K\right) \\ &= C \qquad (2.12.5) \\ V_j^* &= -\left(C + x_i + x_j + 2K\right)\frac{\alpha x_i}{\alpha x_i + x_j} \\ &= -\left(C + \frac{C+2K}{\alpha-1} + 2K\right) \\ &= -\frac{\alpha}{\alpha-1}(C+2K) \qquad (2.12.6) \end{aligned}$$

This means that, under the UK system, the court is a costless solution to the innocent party. And the innocent party get compensated with certainty. This will completely deter cheating or shirking behavior, and essentially we can obtain the simple first best solution.

3 Costly Sequential Auction

3.1 Introduction

In the standard theory of second price auctions, the highest valuation bidder wins the auction and pay the second highest bid, which is also the second highest realized value, due to the truth telling strategy, see Vickrey (1961). This result is usually taken to be the outcome of sequential English auctions. Bidders submit a bid just over the previous one, up to the level of minimum bid increment, provided that is lower than his valuation. Otherwise he pass. The idea behind this bidding strategy is that, there is nothing to lose, and by doing so, there are potential gains.

This solution is called as ”ratchet solution,” ”straightforward bidding,” or as ”pedestrian bidding”. There are two implications that are very often different from the actual bidding behavior. First, bidding is expected to increase by the minimum amount, and bidders drop out after many bids. In practice, bidding for corporate acquisitions typically involve large jumps, and ends in a few rounds, see Ruback (1983). Cramton (1997) documented frequent large jump bidding for personal communication spectrum rights in US. Borgers and Dustmann (2005), in analyzing the UK sale of licenses for 3G mobile telephone services, found that, there were a significant number of jump bids. Haile and Tamer (2003) also documented jump bidding in a regular English auction and reported that the gap between first-and second-highest bids is usually above the minimum bid increment allowed. Cassady (1967) observes about private auctions that “...[the potential buyer] may offer a high price at an early stage in the proceedings in the hope of scaring off competitors.” The idea is that, in practice, if certain level of competition is anticipated, a high “keep-out” is more likely to be a better strategy than a low initial bid, which hopes no one or not too much competitive bids follows.

Second, the standard analysis does not examine the incentive of waiting to bid. It is very common to see “silence” time in auctions, no matter it is online or holds in real place. Particularly if there is a fixed end, where bidder can adopt sniping strategy, see Roth and Ockenfels (2002).[1] In that case, almost all bidders want to bid at the last second and leaving no time for others to react. Other than the strategic consideration, there is also a more general incentive to wait, which is to learn others value by their action. And this would be one implication of my model.

[1] Some auction website adopt free-end, unlike eBay which is fixed-end. For example, in amazon, if there is bid submitted at the last 10 minutes, it will extend for 10 more minutes.

One of the reason for these discrepancies between the theoretical predictions and empirical observations is that, in some spontaneous auctions, submitting and revising a bid is costly. Hirshleifer and Png (1989) suggest that "In reality, the cost of making and revising takeover bids is far from trivial. It includes fees to counsel, investment bankers, and other outside advisors, the opportunity cost of executive time, and the cost of obtaining financing for the bid." Bidding costs are also affected by federal and state laws governing takeovers. "For instance, federal law specifies that shareholders who tender to one bidder must be allowed to withdraw their shares as long as the offer remains open. This allows shareholders to tender to a competing bidder. The shorter the time available for shareholders to withdraw shares tendered to an earlier bidder, the higher the cost of making a counterbid." And in Daniel and Hirshleifer (1998), " in the US some mandated S.E.C. information filings have to be repeated with each bid revision." Seyhun (2000) notes that unsuccessful takeover bidders experience stock returns of negative 0.7 percent, in contrast with positive 0.7 percent for the successful bidders, which is consistent with the hypothesis here. This paper is interested in generalizing this idea and try to formalize it, and hopefully to have some interesting and meaning results.

I consider a sequential auction model with private valuation and costly bidding process. I show that bidders with low valuations will delay to bid in order to assess the strength of the competition. In which we can consider as a learning process. The standard "pedestrian bidding" describes an outcome that minimizes the rate of learning. The equilibrium in this paper is the one maximizes the rate of learning. Once the bidding process has revealed enough information to persuade a bidder that he is going to lost, he should avoid the bidding cost by quitting immediately. This allows the auction ends rapidly, which economizes the bidding cost — a purely deadweight loss.

3.2 Literature Review

The auction literature is vast and huge. There are a lot of papers discussing jump bidding, since this is not the focus of my paper, so I just list some relevant literature. For theoretical model, see Rothkopf and Harstad (1994), Avery (1998), Isaac and Zillante (2007). For empirical and experimental studies, see Plott and Salmon (2004), Isaac and Zillante (2005), Isaac and Schnier (2005), and Raviv (2008). The most relevant paper found is a working paper by Daniel and Hirshleifer (1998). In which they formalize a sequential auction model, where bidders take turns to bid. And the winner pay what he bid. The key feature of the equilibrium is that bidders would signal their value, not only from how much they bid, but also from the timing that they submit their first bid. By their construction, the signal is perfect, and that economize on bidding cost. The equilibrium they focus on maximizes the rate of learning, and thus the auction ends rapidly. My model share all these features, but I consider a simultaneous setting.

Fishman (1989) examines a setting in which bidding is costless, but the second bidder must pay an initial investigation cost to learn its valuation of the acquisition target. If the first bidder learns of a high valuation it makes a high preemptive bid which deters the second bidder from investigating and entering the contest. Bhattacharyya (1990) provides models based on either an initial investigation cost or a one-time uninformative entry fee. If the second bidder enters then, he assumes that the target firm is sold according to the ratchet solution. He shows that the bid level can be used as a signal of a bidder's valuation. Both Fishman and Bhattacharyya papers assume one-time bidding cost, they find a single jump occurs when the first bidder signals his valuation. Their model exclude delay. But in my paper it is costly to bid at every round and delay is allowed in equilibrium.

Easley and Tenorio (2004) argued that the cost associated with entering online bids and uncertainty about future entry can explain jump bidding in an ascending-price Internet auction. The model is similar to that of Daniel and Hirshleifer, with the difference that even if bidders pass, they will suffer the cost. The motivation for the jumps is bidding costs and the use of the jumps as signals. The model involves two identical risk-neutral bidders with private valuations who will potentially compete over one unit. There is demand uncertainty, and with probability q, the opponent will not find the auction. They find that when costs are zero, the ratchet solution is equilibrium. When costs are positive, the item can either be sold after one or two stages or else remains unsold. They examined their model's assumptions using Yankee-type Internet auctions and found that jump bidding is more likely earlier in the auction and that the incentive for jump bidding increases as competition becomes stronger. They also found that jump bidders place fewer bids and that increased early jump bidding in auctions reduces the total bids placed.

The following section would be the basic setup of my model. Equilibrium is constructed in section 3.4. Various analysis under different cases of bidding cost and reserve price is done at section 3.5. And then efficiency is examined in section 3.6. I conclude in section 3.7.

3.3 Basic Setup

Consider an auction, consists of T periods, there are two bidders and one seller. Bidder i's utility function, $i \in \{1, ..., N\}$, is assumed to be:

$$U^i = \begin{cases} \theta^i - price - cn^i & \text{if he wins} \\ -cn^i & \text{if he bids and loses} \\ 0 & \text{if he does not bid} \end{cases}$$

where θ^i is bidder i's private value, price is the amount he is going to pay to the seller if he wins, c is the per period bidding cost and n^i is number of times that bidder i bids.[2] Bidder's

[2]The quasi-linear functional form is the standard assumption adopt in the literature, and the only difference is the cn^i being added at the end. And actually the bidding cost component can be reduced to c. Since in the following proposed equilibrium, no bidder would bid more than one time, hence, $n^i = 1$.

value is continuously distributed within certain bound, $\theta \sim [0, \overline{\theta}]$. Bidder's value are identical and independently distributed, $\theta \sim F(\cdot)$. All these are common knowledge.

Similarly, seller's profit function take the form:

$$\pi^S = \begin{cases} price - \theta^s & \text{if someone wins} \\ 0 & \text{otherwise} \end{cases}$$

where θ^s denotes the seller's value of the auction item.

The auction run as follows, at each period, every bidder can bid. And they submit their bid directly to auctioneer — no other people observe their bid. At the end of each period, the second highest bid is announced. If there is only one bid, the announced value would be either zero or the reserve price (r), depends on whether the bid submitted is higher than the reserve price (r) or not. The auction ends after period T, and the bidder submitted the highest bid wins and pay the second highest bid being announced at the last period.

So essentially, it is just a multiple period second price sealed bid auction. And by standard argument, bidder's strategy is to tell the truth, they bid θ^i.

The timing of the model is as follows:

1. Auctioneer announces bidding cost (c)
2. Seller announces reserve price (r) and number of period (T)
3. Bidders know their own private value, then decide their bidding strategy

3.4 Equilibrium

We need to construct a sequence of critical values that determines the timing of bidding, where bidders who have value $\theta_t \leq \theta \leq \theta_{t-1}$ will bid in period t. Consider in period $t \in \{1, ..., T-1\}$, bidder with value θ_t, which is constructed as the marginal bidder who is indifferent between bidding in period t or $t+1$, his expected utility would be, if bid in period t:

$$EU_t(\theta_t) = \frac{F(\theta_t)}{F(\theta_{t-1})}(\theta_t - r) - c$$

Since he bid in period t, so he pay bidding cost c. But as he is the lowest possible type to bid in period t, so he only win if the other bidder having value lower than θ_t, that is why we multiply the gain, $\theta_t - r$, with $F(\theta_t)$. And the condition where that bidder θ_t would bid, is that no one bid in any preceding period. That implies the other bidder having value lower than θ_{t-1} with probability $F(\theta_{t-1})$.

And now consider what bidder θ_t will get if he bids in period t+1:

$$EU_{t+1}(\theta_t) = \frac{F(\theta_t)}{F(\theta_{t-1})}\left[\theta_t - c - \frac{F(\theta_{t+1})}{F(\theta_t)}r - \int_{\theta_{t+1}}^{\theta_t} x\frac{f(x)}{F(\theta_t)}dx\right]$$

Bidder θ_t would bid in period t+1, only if no one bid in any preceding periods. And it happens with probability $F(\theta_t)/F(\theta_{t-1})$, which conditional on all other bidders having values lower than θ_{t-1}. The only extra term, compare with the previous equation, is the integral range from θ_t to θ_{t+1}. It represents the expected price to be paid, in the case of the other bidder bids. Which is the expected value of the other bidder, conditional on θ_t being the highest realized value, according to the rules of the auction.

Since by construction bidder θ_t would be indifferent between bidding period t or t+1, so we can equate the the equations above and get,

$$\begin{aligned} EU_t(\theta_t) &= EU_{t+1}(\theta_t) \\ \frac{F(\theta_t)}{F(\theta_{t-1})}(\theta_t - r) - c &= \frac{F(\theta_t)}{F(\theta_{t-1})}\left[\theta_t - c - \frac{F(\theta_{t+1})}{F(\theta_t)}r - \int_{\theta_{t+1}}^{\theta_t} x\frac{f(x)}{F(\theta_t)}dx\right] \\ \int_{\theta_{t+1}}^{\theta_t} x\frac{f(x)}{F(\theta_{t-1})}dx &= c\left[1 - \frac{F(\theta_t)}{F(\theta_{t-1})}\right] + r\left[\frac{F(\theta_t)}{F(\theta_{t-1})} - \frac{F(\theta_{t+1})}{F(\theta_{t-1})}\right] \\ \int_{\theta_{t+1}}^{\theta_t} xf(x)dx &= c\left[F(\theta_{t-1}) - F(\theta_t)\right] + r\left[F(\theta_t) - F(\theta_{t+1})\right] \end{aligned} \tag{3.4.1}$$

Equation (3.4.1) generates a sequence of critical values, $\{\theta_t\}_{t=1}^{T-1}$, which guides bidders when to bid in equilibrium. Since there is an end, the last period critical value would be derives as follows:

$$\begin{aligned} \frac{F(\theta_T)}{F(\theta_{T-1})}(\theta_T - r) - c &= 0 \\ \theta_T &= r + \frac{F(\theta_{T-1})}{F(\theta_T)}c \end{aligned} \tag{3.4.2}$$

We set $\theta_0 = \bar{\theta}$. Equations (3.4.1) and (3.4.2) generates a full sequence of critical values, which guides bidders when to bid. To establish the equilibrium, I need to show that it is optimal to bid as proposed, i.e. bidders having value $\theta_t \leq \theta \leq \theta_{t-1}$ bid in period t.

Proposition 3.1. *No bidders benefit from bidding earlier.*

Proof. For a bidder who has value $\tilde{\theta}$, where $\theta_t \leq \tilde{\theta} \leq \theta_{t-1}$, his equilibrium expected utility is

$$\begin{aligned}
EU(\tilde{\theta}) &= F(\theta_t)(\tilde{\theta}-r-c)+\int_{\theta_t}^{\tilde{\theta}}(\tilde{\theta}-x-c)f(x)dx-\left(F(\theta_{t-1})-F(\tilde{\theta})\right)c \\
&= F(\theta_t)(\tilde{\theta}-r)+\int_{\theta_t}^{\tilde{\theta}}(\tilde{\theta}-x)f(x)dx-F(\theta_{t-1})c \\
&= F(\theta_t)(\tilde{\theta}-r)+\left(F(\tilde{\theta})-F(\theta_t)\right)\tilde{\theta}-\int_{\theta_t}^{\tilde{\theta}}xf(x)dx-F(\theta_{t-1})c \\
&= F(\tilde{\theta})\tilde{\theta}-\int_{\theta_t}^{\tilde{\theta}}xf(x)dx-F(\theta_t)r-F(\theta_{t-1})c. \qquad (3.4.3)
\end{aligned}$$

Take the derivative, we have

$$\frac{dEU(\tilde{\theta})}{d\tilde{\theta}}=F(\tilde{\theta})+f(\tilde{\theta})\tilde{\theta}-f(\tilde{\theta})\tilde{\theta}=F(\tilde{\theta}). \qquad (3.4.4)$$

So $EU(\tilde{\theta})$ is increasing in $\tilde{\theta}$ at an increasing rate.

Consider the expected utility of a bidder who have value $\hat{\theta}\leq\theta_a$, deviating to bid earlier at period a,

$$\begin{aligned}
EU^{de}(\hat{\theta},a) &= F(\theta_a)(\hat{\theta}-r-c)-(F(\theta_{a-1})-F(\theta_a))c \\
&= F(\theta_a)(\hat{\theta}-r)-F(\theta_{a-1})c. \qquad (3.4.5)
\end{aligned}$$

Take the derivative, we have

$$\frac{dEU^{de}(\hat{\theta},a)}{d\hat{\theta}}=F(\theta_a). \qquad (3.4.6)$$

Notice that the derivative is a constant. And it depends only on the choice of bidding deviation — the bidding period a. Now we can rewrite $EU(\tilde{\theta})$ and $EU^{de}(\tilde{\theta},a)$ as

$$EU(\tilde{\theta})=EU(\theta_a)-\int_{\tilde{\theta}}^{\theta_a}F(\theta)d\theta,$$

$$EU^{de}(\tilde{\theta},a)=EU^{de}(\theta_a,a)-\int_{\tilde{\theta}}^{\theta_a}F(\theta_a)d\theta.$$

Since by construction $EU^{de}(\theta_a,a)=EU(\theta_a)$, we have

$$EU(\tilde{\theta})-EU^{de}(\tilde{\theta},a)=\int_{\tilde{\theta}}^{\theta_a}[F(\theta_a)-F(\theta)]\,d\theta\geq 0.$$

The inequality results from $\theta_a\geq\tilde{\theta}$. $\square$

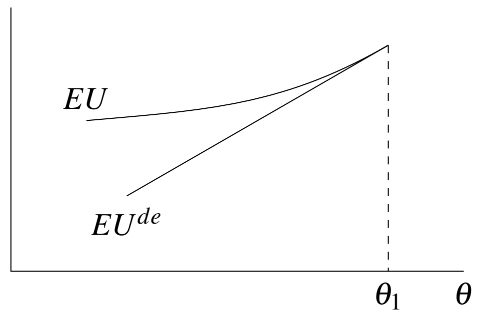

Figure 3.4.1: No benefit for bidding earlier

The same idea can be illustrated graphically. Consider bidder θ_1, he is indifferent to bid between period 1 and 2, so $EU(\theta_1) = EU^{de}(\theta_1, 1)$. And from equation (3.4.4) and (3.4.6), we have $\frac{dEU(\tilde{\theta})}{d\tilde{\theta}} = F(\tilde{\theta})$ and $\frac{dEU^{de}(\hat{\theta})}{d\hat{\theta}} = F(\theta_1)$, hence we have figure (3.4.1):

It clearly shows that bidders who have $\theta < \theta_1$ will not benefit from deviating to bid at period 1. Similar argument can be made for period 2, 3, and the rest. So no bidder will benefit by bidding earlier.

Proposition 3.2. *No bidders benefit from bidding later.*

Proof. Consider the expected utility of a bidder who have value $\hat{\theta} \geq \theta_{z-1}$, deviating to bid later at period z,

$$\begin{aligned} EU^{dl}(\hat{\theta}, z) &= F(\theta_z)\left(\hat{\theta} - r - c\right) + \int_{\theta_z}^{\theta_{z-1}} (\hat{\theta} - x - c) f(x) dx \\ &= F(\theta_{z-1})\hat{\theta} - F(\theta_z) r - F(\theta_{z-1}) c - \int_{\theta_z}^{\theta_{z-1}} x f(x) dx. \end{aligned} \tag{3.4.7}$$

Take the derivative, we have

$$\frac{dEU^{dl}(\hat{\theta}, z)}{d\hat{\theta}} = F(\theta_{z-1}). \tag{3.4.8}$$

Similarly, the derivative is a constant, and it only depends on the choice of bidding period. Now we can rewrite $EU(\tilde{\theta})$ and $EU^{dl}(\tilde{\theta}, z)$ as:

$$EU(\tilde{\theta}) = EU(\theta_{z-1}) + \int_{\theta_{z-1}}^{\tilde{\theta}} F(\theta) d\theta,$$

$$EU^{dl}(\tilde{\theta}, z) = EU^{dl}(\theta_{z-1}, z) + \int_{\theta_{z-1}}^{\tilde{\theta}} F(\theta_{z-1}) d\theta.$$

Since by construction $EU^{dl}(\theta_{z-1}, z) = EU(\theta_{z-1})$, we have

$$EU(\tilde{\theta}) - EU^{dl}(\tilde{\theta}, z) = \int_{\theta_{z-1}}^{\tilde{\theta}} \left[F(\theta) - F(\theta_{z-1})\right] d\theta \geq 0.$$

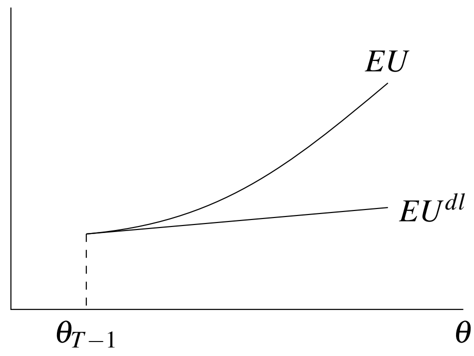

Figure 3.4.2: No benefit for bidding later

The inequality results from $\tilde{\theta} \geq \theta_{z-1}$. □

The same idea can be illustrated graphically. Consider bidder θ_{T-1}, he is indifferent to bid between period T and $T-1$, so $EU(\theta_{T-1}) = EU^{dl}(\theta_{T-1}, T)$. And from equation (3.4.4) and (3.4.8), we have $\frac{dEU(\tilde{\theta})}{d\tilde{\theta}} = F(\tilde{\theta})$ and $\frac{dEU^{dl}(\hat{\theta},z)}{d\hat{\theta}} = F(\theta_{T-1})$. Hence we have figure (3.4.2).

It clearly shows that bidders who have $\theta > \theta_{T-1}$ will not benefit from deviating to bid at period T. Similar argument can be made for period $T-1$, $T-2$, and the rest. So no bidder will benefit by bidding later. And the equilibrium is established.

The proposed equilibrium has following features:

1. The order of bidding follows realized value descending, i.e. Higher value bidders would bid (weakly) earlier, and lower value bidders would bid (weakly) later

2. Bidders bid truthfully.

3. No bidders would submit more than one bid, in equilibrium either bid once or not bidding at all.

4. Whenever there is a bid in any period, no more bids in all subsequent periods.

Firstly, as the sequence of critical value suggests, in equilibrium, a group of highest value bidders bid in the first period, i.e. $\theta > \theta_1$, and then followed by second highest group, $\theta_2 \leq \theta \leq \theta_1$, and it goes on similarly. The second point follows straightly from the rule of the auction. It is a weakly dominant strategy to bid your value in a second price sealed bid auction. And this is linked to point 3, bidders at most will bid once. Since they already tell the truth in the first place, there is no room to bid higher. And since bids are truth-telling, once there is a bid in any preceding period, subsequent bidders will not bid, which is point 4.

3.5 Analysis

Now we turn our attention to the seller side, in general expected revenue is

$$
\begin{aligned}
ER &= \int_{\theta_1}^{\bar{\theta}} x \cdot 2\left[1 - F(x)\right] f(x)dx + 2F(\theta_1)\left[1 - F(\theta_1)\right] r \\
&+ \int_{\theta_2}^{\theta_1} x \cdot 2\left[F(\theta_1) - F(x)\right] f(x)dx + 2F(\theta_2)\left[F(\theta_1) - F(\theta_2)\right] r \\
&\vdots \qquad\qquad\qquad\qquad \vdots \\
&+ \int_{\theta_T}^{\theta_{T-1}} x \cdot 2\left[F(\theta_{T-1}) - F(x)\right] f(x)dx + 2F(\theta_T)\left[F(\theta_{T-1}) - F(\theta_T)\right] r \\
&= \sum_{t=0}^{T-1} \int_{\theta_T}^{\theta_t} x \cdot 2\left[F(\theta_t) - F(x)\right] f(x)dx + \sum_{t=1}^{T} 2F(\theta_t)\left[F(\theta_{t-1}) - F(\theta_t)\right] r.
\end{aligned}
$$

Since seller value the auction item, expected cost is

$$
\begin{aligned}
EC &= \sum_{t=1}^{T} \left\{ \left(\left[F(\theta_{t-1}) - F(\theta_t)\right]^2 + 2F(\theta_t)\left[F(\theta_{t-1}) - F(\theta_t)\right] \right) \theta_s \right\} \\
&= \sum_{t=1}^{T} \left\{ \left(F(\theta_{t-1})^2 - F(\theta_t)^2 \right) \theta_s \right\} \\
&= \left[1 - F(\theta_T)^2\right] \theta_s.
\end{aligned}
$$

Thus the expected profit is

$$
\begin{aligned}
E\pi &= ER - EC \\
&= \sum_{t=0}^{T-1} \int_{\theta_T}^{\theta_t} x \cdot 2\left[F(\theta_t) - F(x)\right] f(x)dx + \sum_{t=1}^{T} 2F(\theta_t)\left[F(\theta_{t-1}) - F(\theta_t)\right] r - \left[1 - F(\theta_T)^2\right] \theta_s.
\end{aligned}
$$

We will go through some cases below, firstly it will be $c = r = 0$.

3.5.1 Zero Bidding Cost and Zero Reserve Price

Let us first consider a very simple case, where $c = r = 0$. From equation (3.4.1), we have $\int_{\theta_{t+1}}^{\theta_t} x f(x)dx = 0$, that implies $\theta_t = \theta_{t+1}$ for all t. So we can apply equation (3.4.2) to get:

$$
\theta_t = 0 \qquad \forall t = 1, \ldots, T
$$

That means all bidders would bid, and they are indifferent to bid in any period. This is actually the classic solution of the Second Price Sealed Bid Auction, where bidders just bid their true value, no matter what others do.

Then expected revenue under this case will be:

$$ER = \int_0^{\bar{\theta}} x \cdot 2\left[1 - F(x)\right] f(x) dx$$

which is the standard result of expected revenue from a single period auction, with 2 independent private value bidders.

3.5.2 Zero Bidding Cost

Now consider $c = 0$, $r > 0$. From equation (3.4.1) and (3.4.2), we get

$$\int_{\theta_{t+1}}^{\theta_t} x f(x) dx = r\left[F(\theta_t) - F(\theta_{t+1})\right] \text{ and } \theta_T = r.$$

One obvious solution is $\theta_t = r$, $\forall t = 1, \ldots, T$. It implies all bidders having value higher than r would bid, and they are indifferent to bid in any period. This again conform the standard result. Expected profit under this case will be

$$E\pi = \int_r^{\bar{\theta}} x \cdot 2\left[1 - F(x)\right] f(x) dx + 2F(r)\left[1 - F(r)\right] r - \left[1 - F(r)^2\right] \theta_s.$$

To find the optimal reserve price, differentiate $E\pi$ with respect to r,

$$\begin{aligned} \frac{\partial E\pi}{\partial r} &= 2F(r)\left[(1 - F(r)) - r f(r) + \theta_s f(r)\right], \\ r^* &= \frac{1 - F(r^*)}{f(r^*)} + \theta_s. \end{aligned} \tag{3.5.1}$$

Take for example, $\theta \sim U[0,1]$, then we have

$$r^* = \frac{(1 + \theta_s)}{2}.$$

3.5.3 Zero Reserve Price

Now consider $r = 0$, $c > 0$. From equation (3.4.1) and (3.4.2), we get

$$\int_{\theta_{t+1}}^{\theta_t} x f(x) dx = c\left[F(\theta_{t-1}) - F(\theta_t)\right], \tag{3.5.2}$$

$$\theta_T = \frac{F(\theta_{T-1})}{F(\theta_T)} c.$$

These two equations will generate a sequence of critical values. For illustration, consider $\theta \sim U[0,1]$, then we have

$$\theta_T \frac{\theta_t^2 - \theta_{t+1}^2}{2} = c(\theta_{t-1} - \theta_t) \text{ and } \theta_T = \frac{\theta_{T-1}}{\theta_T} c.$$

Assume $c = 0.5$, we have

Table 1

	T=1	T=2	T=3	T=4	T=5
θ_1	0.707	0.781	0.811	0.827	0.837
θ_2	/	0.625	0.685	0.715	0.733
θ_3	/	/	0.585	0.632	0.658
θ_4	/	/	/	0.562	0.599
θ_5	/	/	/	/	0.547

The values shown are corrected to the nearest 3 decimal place. For an example, in the column T=3, bidder who have value higher than 0.811 would bid in the first period. Otherwise, provided no one bid in first period, he would bid in second period if his value is between 0.811 and 0.685. And if still there is no bid submitted, bidders with value between 0.685 and 0.585 will bid in the last period . All bidders having value lower than 0.585 would not participate at all.

Several points worth to be highlighted:

- Critical values are monotonically decreasing from the first period to the last period
- θ_1 is monotonically increasing in Total number of periods (T)
- Last period critical value is monotonically decreasing in Total number of periods (T)
- The difference between critical values, $\theta_t - \theta_{t+1}$, is monotonically decreasing in t

The first point illustrates one of the equilibrium feature, where higher value bidders would bid in earlier period. The second point means bidders who bid in the first would be of higher value if the auction is longer. The third point implies that the expected number of actual participation - number of bidders who submit bid, would be higher in longer auction. The last point implies that it is more likely to observe "late bidding" in a longer auction. As the the table above suggests, a bidder with value 0.7 will bid at period 2 in a three period auction, while he bid at period 3 in a four period auction. So when bidders realized values cluster at low level, a lot of bidders will wait to bid at "later" period, or in the extreme case, at last period. This happens when the number of period is too much — it categorize bidders into different group too finely, in the way that no one is in the early period.

3.5.4 Positive Bidding Cost and Reserve Price

Now we move on to the general case, with $c > 0$, $r > 0$. We can modify equation (3.4.1) into,

$$\frac{\int_{\theta_{t+1}}^{\theta_t} xf(x)dx}{F(\theta_t) - F(\theta_{t+1})} = \frac{c\left[F(\theta_{t-1}) - F(\theta_t)\right] + r\left[F(\theta_t) - F(\theta_{t+1})\right]}{F(\theta_t) - F(\theta_{t+1})}$$

$$E\left(x|x \in [\theta_{t+1}, \theta_t]\right) = c\frac{Q_t}{Q_{t+1}} + r \tag{3.5.3}$$

Since we know the conditional expected value is decreasing in t, $\frac{Q_t}{Q_{t+1}}$ is decreasing in t. Then we have,

$$\frac{Q_t}{Q_{t+1}} > \frac{Q_{t+1}}{Q_{t+2}}$$

$$Q_t Q_{t+2} > Q_{t+1}^2 \tag{3.5.4}$$

Proposition 3.3. *Once $Q_t > Q_{t-1}$ in any period t, in all subsequent period j, we have $Q_{t+j} > Q_{t+j-1}$.*

Proof. Since we have $Q_t Q_{t+2} > Q_{t+1}^2$, if $Q_t < Q_{t+1}$, then $Q_{t+1} < Q_{t+2}$. By induction, all subsequent Q_t are higher than the previous one. □

Corollary 3.1. *If $Q_1 < Q_2$, then the last period is the one having the highest probability of bidding.*

Proof. Recall $Q_t = F(\theta_{t-1}) - F(\theta_t)$ is the probability of bidding in period t. By proposition 3.3, we know all Q_t is increasing in t, started from period 1. So $Max\{Q_t\}_{t=1}^{T} = Q_T$.

□

Corollary 3.2. *The probability of bidding can never rise and then fall.*

Proof. By proposition 3, we have this result directly. There are only 3 possible cases. From $Q_t Q_{t+2} > Q_{t+1}^2$, we have $Q_{t+1} < Max\{Q_t, Q_{t+2}\}$. Consider fixing Q_t and Q_{t+2} at some arbitrary value.

1. $Q_t > Q_{t+2} \Longrightarrow Q_{t+1} < Q_t$
2. $Q_t = Q_{t+2} \Longrightarrow Q_{t+1} < Q_t = Q_{t+2}$
3. $Q_t < Q_{t+2} \Longrightarrow Q_{t+1} < Q_{t+2}$

So it is not possible to have $Q_t < Q_{t+1}$ and $Q_{t+1} > Q_{t+2}$. In other words, if we represent it in diagram, we will never have n-shaped curve for Q_t. □

Proposition 3.4. *When Q_t is increasing in t, $Q_{t+j+1} - Q_{t+j}$ increases at increasing rate, and vice versa.*

Proof. From $Q_t Q_{t+2} > Q_{t+1}^2$, take the minimum bound to increase, consider period t is starting point of increase

$$
\begin{aligned}
Q_{t+2} &= \frac{Q_{t+1}^2}{Q_t} \\
Q_{t+3} &= \frac{Q_{t+2}^2}{Q_{t+1}} = \frac{Q_{t+1}^3}{Q_t^2} \\
Q_{t+4} &= \frac{Q_{t+3}^2}{Q_{t+2}} = \frac{Q_{t+1}^4}{Q_t^3} \\
&\vdots \\
Q_{t+i} &= \frac{Q_{t+1}^i}{Q_t^{i-1}}
\end{aligned}
$$

Compute the difference of period j and j+1,

$$
\begin{aligned}
Q_{t+j+1} - Q_{t+j} &= \frac{Q_{t+1}^{j+1}}{Q_t^j} - \frac{Q_{t+1}^j}{Q_t^{j-1}} \\
&= \left(\frac{Q_{t+1}}{Q_t}\right)^j (Q_{t+1} - Q_t)
\end{aligned}
$$

Since we are considering increasing sequence, $Q_{t+1} - Q_t > 0$, that implies the difference is increasing in j. If we consider decreasing sequence, $Q_{t+1} - Q_t < 0$, then it decreases at a decreasing rate. □

We may ask how is the optimal reserve price, with the presence of bidding cost, different from the standard solution. To have a direct comparison, consider $T = 1$. Firstly we look at how bidders behave in this case. From equation (3.4.2), we have

$$
\begin{aligned}
\theta_1 &= r + \frac{F(\theta_0)}{F(\theta_1)} c, \\
F(\theta_1)(\theta_1 - r) &= c. \qquad (3.5.5)
\end{aligned}
$$

This implies, if $c > 0$, then $\theta_1 > r$. Implicitly differentiate θ_1 w.r.t. r,

$$
\frac{\partial \theta_1}{\partial r} = \frac{F(\theta_1)}{F(\theta_1) + f(\theta_1)(\theta_1 - r)}.
$$

Since we know $\theta_1 > r$, and assume $f(\theta) > 0, \quad \forall \theta \in [0, \bar{\theta}]$, we have

$$
0 < \frac{\partial \theta_1}{\partial r} < 1.
$$

Notice that θ_1 is the lowest type that would participate. And in the standard analysis, the lowest type to bid always equals r. However, with bidding cost the difference, $(\theta_1 - r)$, is always positive. And it would at least be equal to c, when $\theta_1 = \bar{\theta}$. If $F(\theta)$ is increasing, the difference would be higher for lower θ_1. And in general, it is

$$(\theta_1 - r) = \frac{c}{F(\theta_1)}.$$

Now we turn our attention to the seller, the expected profit is

$$\pi = \int_{\theta_1}^{\bar{\theta}} x \cdot 2\left[1 - F(x)\right] f(x)dx + 2F(\theta_1)\left[1 - F(\theta_1)\right] r - \left[1 - F(\theta_1)^2\right] \theta_s.$$

To get the optimal reserve price, r, we can compute the FOC,

$$\frac{d\pi}{dr} = \frac{\partial \pi}{\partial \theta_1}\frac{\partial \theta_1}{\partial r} + \frac{\partial \pi}{\partial r}$$

where

$$\begin{aligned}
\frac{\partial \pi}{\partial \theta_1} &= -2\theta_1 \left(1 - F(\theta_1)\right) f(\theta_1) + 2f(\theta_1)\left(1 - F(\theta_1)\right) r - 2F(\theta_1)f(\theta_1)r + 2F(\theta_1)f(\theta_1)\theta_s \\
&= 2f(\theta_1)(r - \theta_1) + 2f(\theta_1)F(\theta_1)\left(\theta_1 - 2r + \theta_s\right), \qquad (3.5.6)
\end{aligned}$$

$$\frac{\partial \theta_1}{\partial r} = \frac{F(\theta_1)}{F(\theta_1) + f(\theta_1)\left(\theta_1 - r\right)},$$

$$\frac{\partial \pi}{\partial r} = 2F(\theta_1)\left(1 - F(\theta_1)\right).$$

Set the FOC equals to 0,

$$\begin{aligned}
\frac{\partial \pi}{\partial \theta_1}\frac{\partial \theta_1}{\partial r} &= -\frac{\partial \pi}{\partial r} \\
\frac{f(\theta_1)(r - \theta_1) + f(\theta_1)F(\theta_1)\left(\theta_1 - 2r + \theta_s\right)}{F(\theta_1) + f(\theta_1)\left(\theta_1 - r\right)} &= F(\theta_1) - 1 \\
\frac{F(\theta_1) + f(\theta_1)F(\theta_1)\left(\theta_1 - 2r + \theta_s\right)}{F(\theta_1) + f(\theta_1)\left(\theta_1 - r\right)} &= F(\theta_1) \\
1 + f(\theta_1)\left(\theta_1 - 2r + \theta_s\right) &= F(\theta_1) + f(\theta_1)\left(\theta_1 - r\right) \\
f(\theta_1)\left(\theta_s - r\right) &= F(\theta_1) - 1 \\
\theta_s + \frac{1 - F(\theta_1)}{f(\theta_1)} &= r^*. \qquad (3.5.7)
\end{aligned}$$

Recalling standard solution, labeled as r^s, from equation (3.5.1),

$$r^s = \theta_s + \frac{1 - F(r^s)}{f(r^s)}.$$

Obviously both r^* and r^s are greater than θ_s. However the comparison between r^* and r^s is not that clear cut, and it depends on the inverse hazard rate.

Proposition 3.5. *If* $\frac{1-F(\theta)}{f(\theta)}$ *is decreasing in* θ*, then* $r^* < r^s$.

Proof. Consider $r^s - r^*$,

$$r^s - r^* = \frac{1 - F(r^s)}{f(r^s)} - \frac{1 - F(\theta_1)}{f(\theta_1)}$$

Proceed by contra-positive, assume otherwise $r^* > r^s$, then we have

$$\frac{1 - F(r^s)}{f(r^s)} < \frac{1 - F(\theta_1)}{f(\theta_1)}$$

and since we know $\theta_1 > r^*$, so $\frac{1-F(\theta)}{f(\theta)}$ is increasing in θ. □

The simplest example of distribution, exhibiting decreasing inverse hazard rate is the uniform distribution. To illustrate, assume $\theta \sim U[0,1]$.

$$1 - \theta' > 1 - \theta'' \iff \theta'' > \theta'$$

$$r^s = \frac{1 + \theta_s}{2}$$

$$r^* = \frac{3(1 + \theta_s) - \sqrt{(1 + \theta_s)^2 + 8c}}{4}$$

It is only when $c = 0$, $r^* = r^s$. For other values of c, $r^* < r^s$.

3.6 Efficiency

In the previous section, we mainly deal with the question of optimal auction. Now we move on to efficiency. To ensure efficiency allocation, we want the object get sold to the bidder i whenever $\theta^i > \theta_s$, and bidder i would be the highest valued bidder, $\theta^i > \theta^j$. Since the auction here we consider is just a multi-period second price auction, given the true-telling bidding strategy, the winner is always the highest valued bidder. There are several possibilities that efficiency is not attained, denote the highest realized value as $\theta^{(1)}$:

1. $\theta_s < \theta^{(1)} < \theta_T$, number of period set "too small" not allowing efficient trading occurs.

2. $\theta_T < \theta^{(1)} < \theta_s$, number of period set "too large" allowing inefficient trading occurs.

So the key point to ensure efficiency is to have an "optimal" number of period where $\theta_T = \theta_s$. And actually this depends on two other factors. Look at equation (3.4.2),

$$\theta_T = r + \frac{F(\theta_{T-1})}{F(\theta_T)}c$$

If there is no bidding cost, $c = 0$, $\theta_T = r$. Then the number of period does not matter any more. And to ensure efficiency, we can simply set $r = \theta_s$.

However, whenever $c > 0$, we want to have $\theta_T = \theta_s + c$, as any lower value of transaction takes place would generate a net loss of value. At first sight, we may need to solve for an infinite pair of (r, T) that satisfy $\theta_T = \theta_s + c$. But recall in the process of bidding, the bidding cost incurred by the bidders is not going to the pocket of seller. It simply dissipates. To minimize the dissipation, we do not want bidders bid in the same period. Thus we would like to have all critical values as close to each other as possible. And to achieve this, we have $T \longrightarrow \infty$. In the limit, we have

$$\lim_{T \longrightarrow \infty} \theta_T = r + c$$

This implies, to ensure efficiency, surprisingly we should set $r = \theta_s$ again. So we have the following result:

Proposition 3.6. *Efficient reserve price does not depend on bidding cost, it always equals seller's value.*

One side issue may be how to compute the critical values when $T \longrightarrow \infty$. We can compute the telescoping sum of the following equations:

$$\begin{aligned}
\int_{\theta_2}^{\theta_1} xf(x)dx &= c\left[F(\theta_0) - F(\theta_1)\right] + r\left[F(\theta_1) - F(\theta_2)\right] \\
\int_{\theta_3}^{\theta_2} xf(x)dx &= c\left[F(\theta_1) - F(\theta_2)\right] + r\left[F(\theta_2) - F(\theta_3)\right] \\
&\vdots \\
\int_{\theta_T}^{\theta_{T-1}} xf(x)dx &= c\left[F(\theta_{T-2}) - F(\theta_{T-1})\right] + r\left[F(\theta_{T-1}) - F(\theta_T)\right]
\end{aligned}$$

which sums to

$$\int_{\theta_T}^{\theta_1} xf(x)dx = c\left[F(\theta_0) - F(\theta_{T-1})\right] + r\left[F(\theta_1) - F(\theta_T)\right].$$

Take limit $T \longrightarrow \infty$, we have $\lim_{T\longrightarrow\infty}\theta_T = r+c$, and in general,

$$\int_{r+c}^{\theta_t} xf(x)dx = c\left[F(\theta_{t-1}) - F(r+c)\right] + r\left[F(\theta_t) - F(r+c)\right]$$

So we can compute the whole chain of $\{\theta_t\}$ recursively, starting from $\theta_0 = \bar{\theta}$. And it can be shown that all propositions and corollaries in section 3.4 are still valid.

3.7 Conclusion

The traditional "pedestrian bidding" solution for sequential ascending auctions predicts that bidding will always proceed by minimal increments. That behavior minimizes the learning rate and maximizes the number of periods needed to find a winner. This paper consider the effect of bidding cost which must be paid whenever a bid is submitted or revised. A bidder makes a bid in early periods to signal a high valuation. Since that reveals bidder's valuation, no matter how small the bidding cost is, other bidders would be deterred to bid in the remaining periods. This equilibrium maximizes the learning rate, so that each bidder needs to bid at most once.

The rapid learning is at the opposite extreme of the traditional "pedestrian bidding" solution. In reality, bidders often bid repeatedly. It could be explained if bidder does not know his valuation for sure, and over the bidding process, he receives more precise signals of his valuation, either exogenously or as a choice of further investigation. He will revise his bid upward if the new information is favorable. This is one possible extension of my model.

When bidders delay before starting to bid, it reveals that a bidder believes he is unlikely to win, and so find it not worthwhile to incur the bidding cost. A bidder who is initially too pessimistic to bid, may gain confidence about his chance of winning, after seeing others pass. The fact that an opponent waits unavoidably "signaled" that his valuation is low, which increases the gains for other bidders to bid.

To sum up, I have provided a model in which, due to bidding cost, at each period a bidder either passes or bid his valuation. This communicates bidders' information rapidly. In equilibrium under most cases, the auction effectively ends before the mandated one. The model provides new results concerning profit maximizing reserve price and efficiency reserve price in a costly sequential auction. Also a information based interpretation of delays in bidding is provided.

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