Mixed oligopoly equilibria when firms' objectives are endogenous[1]

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Abstract

We study a vertically differentiated market where two firms simultaneously choose the quality and price of the good they sell and where consumers also care for the average quality of the goods supplied. Firms are composed of two factions whose objectives differ: one is maximizing profit while the other maximizes revenues. The equilibrium concept we model, called Firm Unanimity Nash Equilibrium (FUNE), corresponds to Nash equilibria between firms when there is efficient bargaining between the two factions inside both firms. One conceptual advantage of FUNE is that oligopolistic equilibria exist in pure strategies, even though the strategy space (price, quality) is multi-dimensional.

We first show that such equilibria are inefficient, with both firms underproviding quality. We then assume that the government takes a participation in one firm, which introduces a third faction, bent on welfare maximization, in that firm. We study the characteristics of equilibria as a function of the extent of government's participation. Our main results are twofold. First, government's participation in the firm providing the low quality good decreases efficiency while participation in the firm providing the high quality good increases efficiency. Second, the optimal degree of government's participation in the high-quality firm increases with how much consumers care for average equality.

Key words: mixed oligopoly, vertical differentiation, factions, party-unanimity Nash equilibrium.

JEL Classification: D21, D43, D62, H82.

1 Introduction

In many countries and many sectors of activity, competing firms exhibit different objectives. This is the case for instance where private, profit-maximizing firms compete with public firms, or with private non-profit firms. Examples range from network industries (energy, transportation, telecommunications) to the service sectors (banking, insurance), and from health care provision to education.[1] The objective of the mixed oligopoly literature is precisely to analyze equilibria in industries with competition between a small number of firms whose objectives differ. Until recently, most of the literature has focused on the particular case where a private, profit-maximizing firm competes with a public, welfare-maximizing firm. The papers differ mainly in the timing of the game played by the firms (simultaneous or sequential), and in their control variables (quantity or price).

Recent papers like those of Matsumura (1998) and White (2002) assume that a firm may be semi-public, in which case it maximizes a convex combination of profit and welfare. The typical result obtained is a kind of "paradox", like showing that profit would have been higher if the firm maximized welfare rather than profit. Those results are driven by the strategic interactions **between** firms.

To the best of our knowledge, all papers up to now assume that the objective functions of the firms are exogenous. More precisely, each firm is treated as a unitary actor which is exogenously endowed with some objective. There is no attempt to try to model how firms make decisions, and why they would end up with an objective like a convex combination of profit and surplus. We believe that this depiction of the working of firms is incomplete and may be misleading. For instance, Seabright (2004) contends that "Firms in America or Western Europe are coalitions, products of the eighteenth century political theory of checks and balances that underlies the American Constitution. (p.172)".

Our contribution is to model both the decisions within firms and the competition between firms. We assume that firms are composed of different factions. One faction

[1] Parris, Pestieau and Saynor (1987) contains a quantitative description of the importance of public firms in Western Europe.

represents the interest of the owners, who want to maximize profit. The other faction is composed of managers, whose interests are not perfectly aligned with the owners. We assume that managers maximize revenue, and one can think of several reasons why this may be so, like ego-rents, career concerns (heading the largest firm in an industry means being highly visible), "empire building" temptations, etc. We do not model the principal-agent relationship between owners and managers, but rather assume that decisions within a firm are taken through some form of bargaining between profit-motivated and revenue-motivated agents. As a first step, we are agnostic as to the particular form this bargaining takes, and we only assume that this bargaining is efficient (i.e., it results in an allocation on the Pareto frontier for the two factions inside a firm, given the decision taken by other firms in the industry). A Firm Unanimity Nash Equilibrium (or FUNE) is then a Nash equilibrium between firms coupled with unanimity between factions inside firms — i.e., a vector of firms' actions such that no other action would simultaneously increase both profit and revenue for any firm, given the actions taken by the other firms.

We apply this novel equilibrium concept[2] to the following setting. We assume that two firms offer a vertically differentiated good and that they simultaneously choose the quality and price of the good they offer. We assume that customers care not only about the characteristics (quality and price) of the good they buy, but also about the average quality of the goods consumed. The illustration we have in mind is pollution, with people caring about the average fuel mileage of cars for instance. The individual decision as to which good to buy then exerts a consumption externality on other individuals.

FUNE represents, we hope somewhat more realistically than the traditional approach, what takes place within firms. But there is as well a mathematical payoff to using this equilibrium concept: although firms are competing on a two-dimensional strategy space (price and quality), equilibria exist in pure strategies. Typically, on such a strategy space, if the firms have a single goal (e.g., profit maximization), pure-strategy equilibria do not exist. To achieve existence, authors have resorted to postulating the

[2] FUNE is an adaptation of the concept of party unanimity Nash equilibrium (PUNE), proposed by one of us in the study of political competition (Roemer (1999, 2001)). There, political parties are composed of factions with different goals.

use of mixed strategies, or specifying the game as one with stages. We believe that either resorting to mixed strategies or stage games is a compromise that is difficult to justify; it is conceptually preferable to model oligopolistic competition as a simultaneous move, pure-strategy game. FUNE thus complexifies the conception of what happens inside a firm, in exchange for an intellectually satisfying simplicity in the nature of inter-firm competition.

We start by computing the FUNEs in our setting. There is a two-dimensional manifold of FUNEs, which can be characterized according to the relative bargaining weight of the revenue-maximizing faction in both firms. We analyze the welfare properties of these equilibria and find that they fall short of optimality. This is not surprising since two assumptions of the fundamental welfare theorems are not satisfied: perfect competition and complete markets.

We then add more structure to the intra-firm bargaining by providing an explanation as to how the bargaining weights should be determined at equilibrium. We assume that available managers differ in quality, and that higher-quality managers are attracted by firms generating larger revenues. The quality of the managers affects the bargaining weight of the revenue-maximizing faction, and thus the equilibrium revenues and profits obtained by both firms. We find the unique fixed point of this game, where the quality of the managers attracted in both firms is compatible with the revenues obtained at equilibrium, and where these revenues correspond to the (unique) FUNE with the bargaining weights implied by the managers' quality. We keep this FUNE, characterized by the relative bargaining weights between revenue and profit-maximizing factions in both firms, in the rest of the paper.

The public intervention we study takes the form of the government's taking a participation in one firm. Since it owns part of the firm, the government is entitled to designate a fraction of the directors on the board. We model this as the introduction of a third faction in this firm, with welfare maximization as its objective. To understand the impact of various levels of government intervention, we keep constant the relative bargaining weights between profit- and revenue-maximizing factions in both firms and assume that the bargaining weight of the welfare-maximizing faction increases with (or is a proxy for) the extent of government's participation. A single FUNE corresponds to each vector of bargaining weights, and we study the normative properties of these equilibria as a function of the extent of the government's intervention in the firm. Finally, we study how these normative properties are modified when the intensity of the consumption externality is varied, and when the identity of the firm in which the government invests is modified.

Our main results are as follows. First, in the absence of government intervention, the quality levels provided by both firms are too low, even when there is no consumption externality. Moreover, too many people consume the high quality good. The efficiency of the FUNEs (measured as the total surplus generated by any FUNE allocation compared to the maximum surplus attainable) varies from roughly 50% to 70%. Second, total surplus increases monotonically with the welfare maximizers' bargaining weight in the high quality firm when the externality intensity is large enough. If this externality is very low, total surplus reaches a maximum for an interior value of the welfare maximizers' bargaining weight. Third, we also analyze the situation where the government takes a participation in the low quality firm, and we obtain that total surplus decreases monotonically with the bargaining weight of surplus maximizers! Both qualities decrease with the bargaining weight of the welfare-maximizing faction, moving away from their optimal level. Also, the allocation of consumers across goods worsens as this bargaining weight increases: more and more people buy the high quality good, while optimality would call for fewer buying this good.

2 The model

There is a continuum of consumers, indexed by λ, distributed according to the distribution function F on $[0,\infty[$, with density denoted by f. We denote by $\bar{\lambda}$ the average value of λ and by λ^{med} its median value. Each consumer buys one unit of a good of quality q, and has a utility function

$$V(q, p, \bar{q}; \lambda) = \lambda q + \gamma \lambda \bar{q} - p \tag{1}$$

where $\bar{q}$ is the average quality consumed and p the price. The second term denotes the environmental externality: each individual's utility increases with the average quality of the goods consumed (for instance, the average pollution or fuel mileage of cars). We assume for simplicity that all individuals share the same $\gamma \in [0, 1]$ — i.e., that they have the same relative valuation for average quality (although absolute variation $\gamma\lambda$ varies across individuals).

There are two firms (indexed by subscript $i = 1, 2$), each providing one good. They share the same cost function, which is linear in quantity and convex in quality: $c(q)$ denotes the per unit (of quantity) cost of providing a good of quality q. We assume without loss of generality that $q_1 > q_2$ and we call firm 1 the high quality firm.

When choosing from which firm to buy, individuals do not consider their (infinitesimal) impact on the average quantity of the good consumed. Individual λ then buys from firm 1 if

$$\lambda q_1 - p_1 > \lambda q_2 - p_2$$

i.e., if

$$\lambda > \frac{p_1 - p_2}{q_1 - q_2} = \lambda^*(p_1, q_1, p_2, q_2).$$

We then obtain that

$$\bar{q} = \int_0^{\lambda^*} q_2 f(\lambda) d\lambda + \int_{\lambda^*}^1 q_1 f(\lambda) d\lambda.$$

In each firm, two factions coexist, one maximizing profit while the other maximizes revenue. Profit in firm $i = 1, 2$ is given by

$$\begin{aligned}
\Pi_1(p_1, q_1, p_2, q_2) &= [p_1 - c(q_1)] \left(1 - F(\lambda^*)\right), \\
\Pi_2(p_1, q_1, p_2, q_2) &= [p_2 - c(q_2)] F(\lambda^*),
\end{aligned}$$

while revenue is given by

$$\begin{aligned}
R_1(p_1, q_1, p_2, q_2) &= p_1 \left(1 - F(\lambda^*)\right), \\
R_2(p_1, q_1, p_2, q_2) &= p_2 F(\lambda^*).
\end{aligned}$$

We now introduce our equilibrium concept.

Definition 1 *A Firm Unanimity Nash Equilibrium (FUNE) is a vector* (p_1, q_1, p_2, q_2) *such that*
(i) $\nexists(p_1', q_1')$ *such that* $\Pi_1(p_1', q_1', p_2, q_2) \geq \Pi_1(p_1, q_1, p_2, q_2)$ *and* $R_1(p_1', q_1', p_2, q_2) \geq R_1(p_1, q_1, p_2, q_2)$ *with at least one strict inequality, and*
(ii) $\nexists(p_2', q_2')$ *such that* $\Pi_2(p_1, q_1, p_2', q_2') \geq \Pi_2(p_1, q_1, p_2, q_2)$ *and* $R_2(p_1, q_1, p_2', q_2') \geq R_2(p_1, q_1, p_2, q_2)$ *with at least one strict inequality.*

In words, no firm can find another pair of price and quality that would strictly increase one of its factions' objectives (revenue or profit) without decreasing its other faction's objective.

It will prove easier in the paper to use a slightly different definition of FUNEs. We introduce the following assumption.

Assumption 1 $\log(\Pi_1(p, q, p_2, q_2))$, $\log(R_1(p, q, p_2, q_2))$, $\log(\Pi_2(p_1, q_1, p, q))$ *and* $\log(R_2(p_1, q_1, p, q))$ *are concave in* (p, q).

Roemer (2001, Theorem 8.2.) proves that, if Assumption 1 holds, then any FUNE is also a weighted Nash bargaining solution. We assume from now on that Assumption 1 holds and we make use of the following definition:

Definition 2 *A FUNE is a vector* (p_1, q_1, p_2, q_2) *and a pair* $(a_1, a_2) \in [0, 1]^2$ *such that*
(i) given (p_1, q_1), $(p_2, q_2) = \arg\max (\Pi_2(p_1, q_1, p_2, q_2))^{a_2} (R_2(p_1, q_1, p_2, q_2))^{1-a_2}$, *and*
(ii) given (p_2, q_2), $(p_1, q_1) = \arg\max (\Pi_1(p_1, q_1, p_2, q_2))^{a_1} (R_1(p_1, q_1, p_2, q_2))^{1-a_1}$.

The first order conditions for firm 1 at a FUNE are[3]

$$\frac{a_1}{\Pi_1}\nabla_1\Pi_1 + \frac{1-a_1}{R_1}\nabla_1 R_1 = 0,$$

where

$$\nabla_1\Pi_1 = \left(\frac{\partial\Pi_1}{\partial p_1}, \frac{\partial\Pi_1}{\partial q_1}\right).$$

[3] In order to save on notation, we do not report the arguments for the profit and revenue functions from now on.

These first order conditions can be expanded to obtain

$$\frac{1-a_1}{a_1} = \frac{R_1}{\Pi_1}\frac{1-F(\lambda^*)-[p_1-c(q_1)]\,f(\lambda^*)\frac{\partial\lambda^*}{\partial p_1}}{F(\lambda^*)-1+p_1 f(\lambda^*)\frac{\partial\lambda^*}{\partial p_1}}, \tag{2}$$

$$\frac{1-a_1}{a_1} = \frac{R_1}{\Pi_1}\frac{-c'(q_1)\,(1-F(\lambda^*))-[p_1-c(q_1)]\,f(\lambda^*)\frac{\partial\lambda^*}{\partial q_1}}{p_1 f(\lambda^*)\frac{\partial\lambda^*}{\partial q_1}}, \tag{3}$$

with

$$\frac{\partial\lambda^*}{\partial p_1} = \frac{1}{q_1-q_2} > 0$$
$$\frac{\partial\lambda^*}{\partial q_1} = -\frac{p_1-p_2}{(q_1-q_2)^2} < 0.$$

The first order conditions for firm 2 are obtained similarly.

We prove the following interesting results.

Proposition 1 *Let $c(q) = \delta q^r$ and $r \geq 1$. Let $\hat{\lambda}$ be the solution of the equation*

$$\frac{r-1}{r}\hat{\lambda} = \frac{1-2F(\hat{\lambda})}{f(\hat{\lambda})}.$$

Then, in all FUNEs, $\lambda^*(p_1,q_1,p_2,q_2) = \hat{\lambda}$.

Proof: *See Appendix*

Remark. The proposition immediately implies the following.

1. If $r = 1$, then exactly one-half of the population purchases the high (low) quality good at any FUNE.

2. Let the distribution F be uniform, on any support. Then the fraction of the population who purchase the low quality good, at any FUNE, is a decreasing function of r, approaching one-third of the population in the limit as r becomes large.

We now look at the Pareto efficient allocations in order to compare them with FUNE allocations. Pareto allocations are defined by the triple $(q_1,\, q_2,\, \tilde{\lambda})$. It is easy to see that the high quality good is supplied to all $\lambda > \tilde{\lambda}$ and the low quality good to all $\lambda < \tilde{\lambda}$. This is done with a simple "switching" argument: if $\lambda_1 > \lambda_2$ but λ_1 consumes the low quality good while λ_2 consumes the high quality good, then a trade with side payment can be arranged making both better off.

The usual argument from quasi-linearity shows that Pareto efficiency requires maximization of the sum of consumer surplus and firms' profits

$$W = \int_0^{\tilde{\lambda}} q_2 \lambda dF(\lambda) + \int_{\tilde{\lambda}}^{\infty} q_1 \lambda dF(\lambda) + \gamma\bar{\lambda}\bar{q} - c(q_1)(1 - F(\tilde{\lambda})) - c(q_2)F(\tilde{\lambda}) \tag{4}$$

where $\bar{q} = q_2 F(\tilde{\lambda}) + q_1(1 - F(\tilde{\lambda}))$. We assume from now on in the paper that $c(q) = \delta q^2$. A Pareto optimal allocation $(q_1,\, q_2, \tilde{\lambda})$ that maximizes W solves the following first order conditions:

$$\frac{\partial W}{\partial \tilde{\lambda}} = f(\tilde{\lambda}) \left[\left(\tilde{\lambda} + \gamma\bar{\lambda} \right) (q_2 - q_1) - (c(q_2) - c(q_1)) \right], \tag{5}$$

$$\frac{\partial W}{\partial q_1} = \int_{\tilde{\lambda}}^{\infty} \lambda dF(\lambda) + \gamma\bar{\lambda}(1 - F(\tilde{\lambda})) - (1 - F(\tilde{\lambda}))c'(q_1), \tag{6}$$

$$\frac{\partial W}{\partial q_2} = \int_0^{\tilde{\lambda}} \lambda dF(\lambda) + \gamma\bar{\lambda}F(\tilde{\lambda}) - F(\tilde{\lambda})c'(q_2), \tag{7}$$

We start by examining separately the optimality formulas for the allocation of consumers across goods and for qualities. Equation (5) gives

$$\tilde{\lambda} = \delta(\frac{q_2 + q_1}{2}) - \gamma\bar{\lambda}. \tag{8}$$

This result is very intuitive: for given quality levels, caring for pollution ($\gamma > 0$) induces provision of the high quality good to more people. We will see in a short while how this result is modified when quality levels are set at their optimal level at the same time as the value of λ. As for qualities, putting together (6) and (7), we obtain

$$\bar{\lambda}(1 + \gamma) = \delta\bar{q}, \tag{9}$$

whose intuition is also straightforward: optimal qualities equalize average preferences for quality (including the pollution aspect) and average marginal cost of quality.

We now look at the simultaneous determination of λ and of qualities. Using (9) together with (5) gives

$$\tilde{\lambda} = \bar{\lambda} + \delta(\frac{q_2 + q_1}{2} - \bar{q}).$$

We then obtain that $\tilde{\lambda} = \bar{\lambda}$ if and only if $\bar{\lambda} = \lambda^{med}$. Moreover, with a positively skewed distribution, we have either $\tilde{\lambda} > \bar{\lambda}$ or $\tilde{\lambda} < \lambda^{med}$ — i.e., the only configuration excluded is $\lambda^{med} < \tilde{\lambda} < \bar{\lambda}$.

Finally, observe from (6) and (7) that

$$q_2 + q_1 = \frac{1}{\delta}\left[2\gamma\bar{\lambda} + \frac{\int_{\tilde{\lambda}}^{\infty} \lambda dF(\lambda)}{1 - F(\tilde{\lambda})} + \frac{\int_{0}^{\tilde{\lambda}} \lambda dF(\lambda)}{F(\tilde{\lambda})}\right]$$

so that by (8)

$$\tilde{\lambda} = \frac{1}{2}\left[\frac{\int_{\tilde{\lambda}}^{\infty} \lambda dF(\lambda)}{1 - F(\tilde{\lambda})} + \frac{\int_{0}^{\tilde{\lambda}} \lambda dF(\lambda)}{F(\tilde{\lambda})}\right] \qquad (10)$$

which does not depend either on δ nor on γ! In words, the Pareto efficient allocation of consumers across goods does not depend on the intensity of the externality when qualities are set at their optimal level. Comparing equation (10) and Proposition 1, we see that the goods' allocation in FUNEs is generically not optimal. Moreover, we also have that the optimal difference between quality levels, $q_1 - q_2$, is independent of δ and of γ, since both q_1 and q_2 increase by the same amount with γ and decrease by the same amount with δ.

We now turn to numerical simulations in order to shed more light on the FUNE allocations and on their normative properties.

3 FUNEs without government intervention: general properties

We study numerically[4] FUNEs with a lognormal distribution of λ such that $\bar{\lambda}$=40 and λ^{med}=30 while $c(q) = q^2/3$. We obtain a bidimensional manifold of FUNEs. This manifold is depicted in the (price, quality) space on Figure 1. Figure 1 depicts 3800

[4] Finding FUNEs requires solving 4 equations (first order conditions (2) and (3) for firm 1, and similar equations for firm 2) in 6 unknowns (p_1, q_1, p_2, q_2, a_1 and a_2). We randomly draw a very large number of pairs (a_1, a_2) and attempt to solve for the other 4 unknowns. We also check that the second order conditions are satisfied.

FUNEs which have been sorted by increasing order of the high quality price. Each FUNE vector (p_1, q_1, p_2, q_2) is plotted with the same color, that goes smoothly from blue for the lowest values of p_1 to red for its highest values. The ordered pairs (p_i, q_i) on the diagonal of the price-quality plane are associated with the high quality firm 1, and the ones on the semi-circle are associated with the low quality firm 2. We see there are FUNEs where the two firms play strategies that are close to one another (the blue FUNEs) and ones where they play strategies that are very different (the red end of the spectrum).

[Insert Figure 1 around here]

The main regularities obtained (in all FUNEs) are as follows:

- Profit is lower in firm 1 (the firm producing the high quality good) than in firm 2.
- Revenue is larger in firm 1 than in firm 2.
- The relative bargaining power of the revenue-maximizers, $\alpha_i = (1 - a_i)/a_i$, is higher in firm 1 than in firm 2: $\alpha_1 > \alpha_2$. Moreover, in all FUNEs, $\alpha_1 > 1$ — i.e., the revenue-maximizers have more bargaining power than the profit-maximizers in firm 1 in all FUNEs.

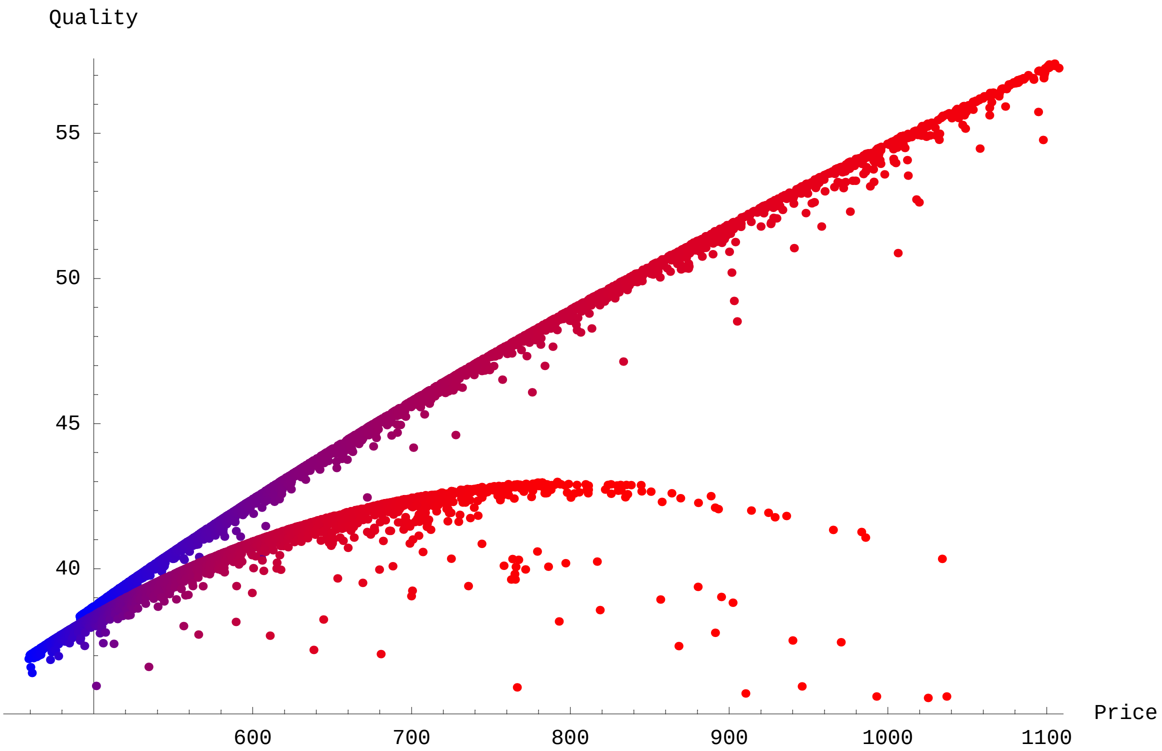
The next Result summarizes the descriptive characteristics of the FUNEs:

Result 1 *In all FUNEs, profit is lower and revenue is larger in the high quality firm, while the relative bargaining weight of the revenue maximizers is larger in that firm than in the low quality firm.*

We now turn to the normative properties of the FUNEs. We obtain the following regularities:

- The qualities offered by both firms at equilibrium are lower than the optimal qualities, whatever the value of γ. The high quality good is especially deficient (its value varies from 38 to 66 in FUNEs while its optimal values goes from 160

Figure 1 : FUNEs in the (Price, Quality) space



for γ=0 to 220 for γ=1; the low quality level varies from 35.7 to 42.5 in FUNEs while its optimal value goes from 43.6 for γ=0 to 103.6 for γ=1).

- Proposition 1 has shown that the equilibrium market share (in volume) $F[\lambda^*]$ is the same in all FUNEs and is not affected by the value of γ. We also know that the optimal market share is not affected by γ either. Comparing both, we obtain that too many people consume the high quality good in all FUNEs, compared to the global optimum: 62.5% vs 14.2%.

- The average quality of goods sold in all FUNEs is lower than the optimal average quality, whatever the intensity of the externality: the first effect mentioned above (both qualities are too low) is larger than the second one (too many people buying the high quality good).

We summarize the normative properties of the FUNEs in the following Result:

Result 2 *In all FUNEs, and whatever the value of the externality intensity λ, the qualities offered by both firms as well as the average quality provided are lower than optimal, while too many people consume the high quality good.*

We measure the efficiency of a FUNE by dividing total surplus in that equilibrium by the maximum surplus attainable (the one corresponding to the optimal qualities and market shares). Table 1 summarizes the results we obtain.

Table 1: Efficiency of FUNEs as a function of γ.

Efficiency:	minimum	maximum	average
$\gamma = 1$	47.9%	70.8%	55.8%
$\gamma = 0.5$	55.5%	76.7%	63.3%
$\gamma = 0.25$	58.7%	76%	65.8%
$\gamma = 0$	59.6%	68.8%	64.8%

The second column in Table 1 gives the minimum efficiency attained over the set of FUNEs, the third column the maximum efficiency and the last column the average efficiency across FUNEs, assuming that they are all equiprobable. We obtain that FUNEs are far from optimal even in absence of externalities ($\gamma = 0$). While the minimum efficiency is monotonically decreasing with the intensity of the externality, it is not the case for the maximum as well as the average efficiency. Clearly, the average efficiency depends on the assumption that all FUNEs are equiprobable, which is a very crude one. For instance, Lee et al. (2005), which uses a solution concept similar to FUNEs but applied to political competition, computes a kernel density function of the bargaining powers (which are the unknown parameters in our problem) to control for the fact that some regions of bargaining-power-space may occur much more frequently than others.

Rather than following this kernel density approach, we proceed here by selecting one FUNE by endogeneizing the factions' bargaining weights inside both firms.

4 Endogenous bargaining weights

Up to now, we have not provided any explanation as to why intra-firms bargaining powers are what they are at equilibrium. In other words, any pair of bargaining powers consistent with Definition 2 constitutes, together with the associated vector of prices and qualities, a FUNE. This is the reason why we obtain a two-dimensional manifold of FUNEs. In this section, we open the bargaining powers' "black box" and provide an explanation as to how their value is determined. Providing such an explanation has an obvious interest by itself. Moreover, it will allow us to narrow down the set of equilibria and indeed to obtain unicity.

We assume that there is a pool of available managers, and that these managers differ in some attribute that we call for the moment "quality". The average quality of this pool is denoted by μ. One half of the managers ends up working in each of the two firms, so if we denote the average quality level of the managers in firm i by μ_i, we obtain that

$$\frac{\mu_1}{2} + \frac{\mu_2}{2} = \mu.$$

Managers are attracted by firms generating a higher revenue level, for the reasons indicated in the introduction (ego rents, etc.). Since managers have to choose between two firms, the average quality of the managers attracted in firm i will be an increasing function of the revenue share (or market share in value) $r_i = R_i/(R_1 + R_2)$ of this firm.

This relationship, which is like a supply function for managerial quality, is summarized by the following increasing function

$$Q(r_i) = \mu_i, \quad i = 1, 2,$$

with $Q(r_1) + Q(r_2) = \mu$. It should be plain that $Q(0.5) = \mu$ and that the steeper the slope of Q, the more mobile the managers.

We need to define what difference the managers' quality makes in each firm. We assume that managers' quality affects the relative bargaining weights of the two factions in each firm. Now, we have that, in all FUNEs, revenue is higher in the firm where the relative bargaining power of the revenue-maximizers is larger than in the other firm. We make the assumption that the relative bargaining weight of the revenue-maximizing faction is increasing in the average quality of the managers in that firm. In that sense, one could talk of manager's cleverness or efficiency, rather than quality, with cleverer/more efficient managers better able to play the boardroom games in order to reinforce their bargaining power as revenue maximizers. Formally, we assume that the relative bargaining weight of the revenue-maximizers is increasing in the managers' quality and represented by the function $a(\mu_i)$.

A FUNE with endogenous bargaining weights (eFUNE) is then a FUNE with the additional requirements that (i) the relative bargaining weight of the revenue-maximizers in each firm is obtained from the average quality of the managers in this firm and (ii) the average quality of the managers in each firm is a function of the equilibrium revenue shares of the firms. Formally,

Definition 3 *A FUNE with endogenous bargaining weight (eFUNE) is a vector* (p_1, q_1, p_2, q_2)*, a pair* $(a_1, a_2) \in [0, 1]^2$ *and an assignment of managers of quality* μ_i *to firm* i *such that*
(i) given (p_1, q_1)*,* $(p_2, q_2) = \arg\max (\Pi_2)^{a_2} (R_2)^{1-a_2}$,
(ii) given (p_2, q_2)*,* $(p_1, q_1) = \arg\max (\Pi_1)^{a_1} (R_1)^{1-a_1}$,
(iii) $Q(\frac{R_i}{R_1+R_2}) = \mu_i, \quad i = 1, 2,$
(iv) $a_i = a(\mu_i), \quad i = 1, 2.$

Generally, there will be locally unique such equilibria, if they exist at all. The unicity argument can be seen from looking at the following mapping: starting from an allocation of managers (μ_1,μ_2), the function $a(\mu_i)$ give the relative bargaining weight in each firm, from which we obtain the corresponding FUNE (which satisfies equations (i) and (ii) in the above definition). We then obtain the revenue share of both firms, which in turn allows us to compute the equilibrium average qualities of managers in both firms via the equation $Q(r_i)$. We are looking for a fixed point of this mapping. The existence problem comes from the fact that there may exist pairs (μ_1,μ_2) for which no FUNE exists.

We use the following functional forms in our simulations: $\mu = 1$, $Q(r_i) = 2r_i$ and $a(\mu_i) = 1/(1 + 5\mu_i^3)$, so that $\alpha(\mu_i) = 5\mu_i^3$. We obtain a unique eFUNE, with p_1=804, q_1=48.4, p_2=617, q_2=40.5, Π_1 = 14.5, R_1 = 503, Π_2 =26.8, R_2=231.5, r_1=68.5%, α_1=12.85 and α_2=1.25. Note that the equilibrium eFUNE qualities are close to the average qualities obtained in all FUNEs assuming that they are all equi-probable (which are 46 for firm 1 and 42.5 for firm 2).

We keep this eFUNE, characterized by the relative bargaining weights between revenue-and profit-maximizing factions in both firms, in the rest of the paper.

5 Government intervention in the high quality firm

From now on, we study the impact of public intervention into this industry. Public intervention is justified by the unappealing normative properties of FUNEs (see section 3 and especially Table 1), and could take several forms. We focus on one public intervention, namely the government's taking a participation in one firm.[5] This section and the next one study participations in the high-quality firm (firm 1) while section 7 studies participation in the low quality firm. Taking a participation in a firm allows the government to appoint a fraction of the managers (or directors) of that firm. In our setting, this means that government intervention introduces a third faction into firm 1. We assume a benevolent government, so that the third faction maximizes total welfare

[5] We assume that there is no government expenditures associated to taking a participation in the firm. We could easily introduce such expenditures by assuming that the government buys shares on the market. The price of these shares would then be related to the profit level attained by that firm before government intervention — i.e., at the eFUNE obtained in the previous section. Note that such expenditures (which constitute a transfer between government and the firm's private owners) would not affect our normative analysis as long as we maximize unweighted surplus.

W in the economy, as given by equation (4). The three factions bargain with each other when deciding about the price and quality of the good offered by firm 1. As previously, each faction is characterized by a bargaining weight inside the firm.

Formally, we have that

Definition 4 *A FUNE with government intervention in firm 1 (FUNE-G1) is a vector* (p_1, q_1, p_2, q_2)*, a triple* $(a_1, b_1, a_2) \in [0,1]^3$ *with* $(a_1, b_1, a_1 + b_1, a_2) \in [0,1]^4$ *such that*
(i) given (p_1, q_1)*,* $(p_2, q_2) = \arg\max (\Pi_2)^{a_2} (R_2)^{1-a_2}$ *,*
(ii) given (p_2, q_2)*,* $(p_1, q_1) = \arg\max (\Pi_1)^{a_1} (R_1)^{1-a_1-b_1} (W)^{b_1}$ *where* W *is given by equation (4).*

Our objective is to study the impact on the industry equilibrium of the government taking a participation in the high quality firm. To do so, we need to relate the extent of the public participation in firm 1 to the bargaining weight of the welfare maximizing faction, b_1. We assume a monotone increasing relationship, so that we can concentrate on the bargaining weight of this faction.

Our benchmark situation is the eFUNE without government intervention obtained in the previous section. To concentrate on the impact of government intervention, we keep the relative bargaining weight of the profit- and revenue-maximizing factions constant in both firms as we increase the bargaining weight of the welfare-maximizers in firm 1. That is, we keep both $(1 - a_2)/a_2$ and $(1 - a_1 - b_1)/a_1$ constant at the level reached in the unique eFUNE (resp. 1.25 and 12.85) and study the FUNE-G1 associated with each value of b_1. To each value of b_1 is associated (at most) one FUNE-G1. Finally, we assume that the intensity of the externality is such that $\gamma = 1$ in equation (1).

We obtain[6] that a FUNE-G1 exists for all values of the bargaining weight of the welfare-maximizing faction $b_1 \in [0, 1]$. As b_1 increases, we have that

- the quality offered by both firms increases monotonically and moves closer to the optimal qualities: q_1 increases from 48.4 to 112.8 when b_1 goes from 0 to 1 (the

[6] Finding FUNE-G1s requires solving 4 equations (first order conditions with respect to price and quality for statements (i) –for firm 2– and (ii) –for firm 1– in Definition 4) in 5 unknowns (p_1, q_1, p_2, q_2 and b_1), given that we keep a_2 and $(1 - a_1 - b_1)/a_1$ constant at the level reached in the unique eFUNE. We choose a value for b_1 and solve for the other 4 unknowns. We also check that the second order conditions are satisfied.

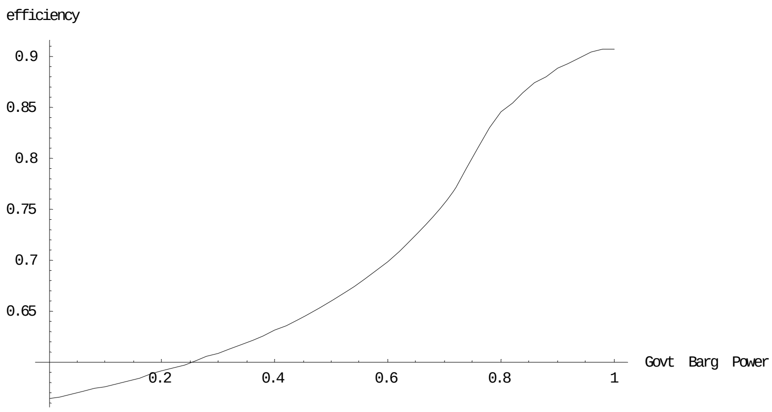


Figure 2: Efficiency of FUNE-G1 as a function of b_1 when $\gamma = 1$.

optimal value of q_1 is 220) while q_2 increases from 40.5 to 81.4 (while the optimal value of q_2 is 103.6);

- the proportion of people buying the low quality good increases (from 37.5% for b_1=0 to 63.8% for $b_1 = 1$) and moves closer to the optimal proportion, 85.8%;
- the efficiency of the FUNE-G1 increases monotonically, from 56% to 91% (see Figure 2).

[Insert Figure 2 around here]

We summarize all this in the next Result:

Result 3 *As the bargaining weight of the welfare maximizing faction in the high quality firm increases, the efficiency of the corresponding FUNE-G1 allocation increases monotonically, with the qualities of both goods increasing and moving closer to their optimal levels, while the proportion of people buying the low quality good increases and also moves closer to its optimal level.*

We thus obtain not only that the government taking a participation (by which we mean introducing a third faction bent on maximizing welfare) in the high quality firm helps increasing the efficiency of the equilibrium allocation, but also that efficiency is monotonically increasing in the bargaining weight of the welfare maximizers. Observe nevertheless that full nationalization falls short of reaching the Pareto optimal allocation, with the low quality firm run by revenue and profit-maximizing factions only.

6 The impact of the externality intensity

In this section, we test the robustness of the results obtained in the previous section to variations in the externality intensity (the value of parameter γ in equation (1)). Note first that the value of γ does not impact the set of FUNEs without government intervention, nor the eFUNE we selected. We thus start from the same eFUNE as in the previous section. On the other hand, γ impacts the efficiency reached in this eFUNE, and also the FUNE-G1 corresponding to any value of b_1.

We have proceeded in the same way as in the previous section, starting from the eFUNE obtained in section 4 and maintaining constant the relative bargaining weights of the profit and revenue-maximizing factions in both firms while we increase b_1. Figure 3 reports by how much total surplus increases (compared to the original eFUNE without welfare maximizers) as a function of the bargaining weight of welfare maximizers (b_1) for several values of the externality intensity γ. We obtain the following results:

- Whatever the value of $b_1 > 0$, the gain in surplus is always positive and is larger when the externality intensity γ is bigger. Moreover, the marginal gain in surplus ($\partial W / \partial b_1$) also increases with the value of γ.

- The surplus gain increases monotonically with b_1 if the externality intensity is large enough ($\gamma \geq 1/4$), so that the maximum gain in surplus is attained when the welfare-maximizing faction has all the bargaining power inside the high quality firm.

- If externality intensity is low enough ($\gamma = 0$ in Figure 3), the gain in surplus reaches a maximum and then decreases slightly. The maximum gain in surplus is attained for $b_1 = 0.78$ when $\gamma = 0$.

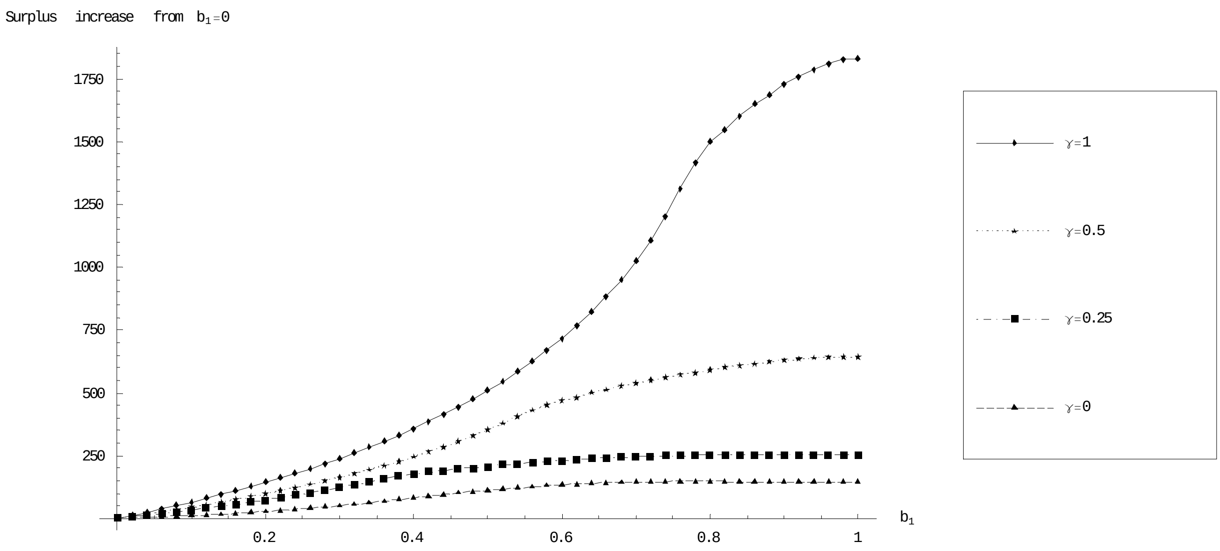


Figure 3: Increases in surplus as a function of b_1 for several values of γ.

- Whatever the value of $\gamma \in [0, 1]$, the quality of both goods and the market share of the low quality good increase with b_1. The high quality level and the fraction of people consuming the low quality good are always lower than their optimal level. As for the low quality level, it is lower than its optimal level, except when γ is very low (zero in Figure 3) and b_1 large ($b_1 > 0.28$). In other words, in the case where γ is low, the lowest quality level is driven above its optimal value as the welfare-maximizers bargaining power increases. This may in turn explain why surplus decreases when b_1 is large enough and $\gamma = 0$.[7]

[Insert Figure 3 around here]

We summarize in the following Result:

Result 4 *Government's participation in the high quality firm is more desirable (i.e., increases more total surplus) when the externality intensity is larger. If this intensity*

[7] Observe that q_2 reaches its optimal level for $b_1 = 0.28$ while total surplus decreases only when b_1 is pushed above 0.78.

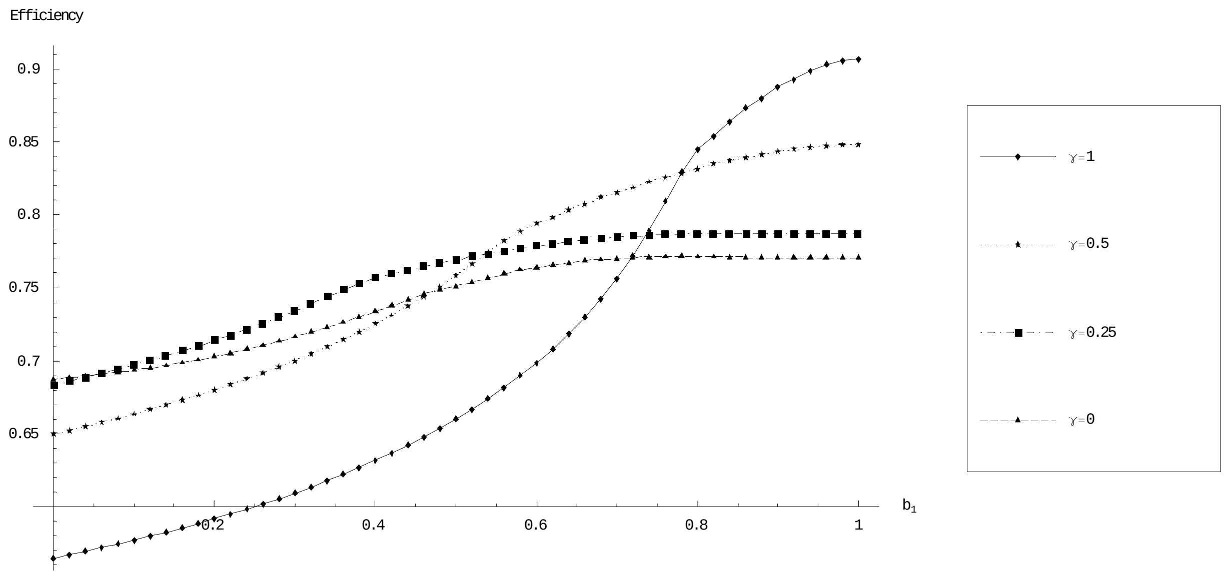


Figure 4: Efficiency as a function of b_1 for several values of γ.

is low enough, maximum surplus is attained for partial nationalization ($b_1 < 1$) while full nationalization ($b_1 = 1$) is called for in the other cases. Increasing b_1 results in increases in both quality levels and in the proportion of people buying the low quality good. The low quality level overshoots its optimal level when individuals do not care for the average quality of good ($\gamma = 0$) and when the bargaining power of the welfare maximizers is large enough.

We now put these surplus gains in perspective by looking at how the efficiency of any FUNE-G1 equilibrium (measured as previously as the ratio of the equilibrium surplus to the maximum, Pareto efficient, surplus), evolves with the welfare maximizers' bargaining weight b_1 for several values of γ (see Figure 4).

[Insert Figure 4 around here]

We obtain the following results:

- When the bargaining weight of the welfare maximizers b_1 is close to zero, efficiency is decreasing with the externality intensity γ: the more people care for the average good quality, the less efficient the "laissez faire" FUNE allocation.

- The ranking is reversed for sufficiently high values of b_1: the efficiency of FUNE-G1 with powerful welfare-maximizers increases when people care more for the average good quality.

- Whatever the value of the externality intensity, the government taking a participation in the high quality firm never allows to reach the Pareto efficient allocation. The gain in surplus from this participation, measured as a proportion of the maximum attainable surplus rather than in absolute terms as in Figure 3, increases with the externality intensity.

We summarize in the following Result:

Result 5 *The gain in efficiency (measured as the ratio of equilibrium surplus to maximum attainable surplus) when the government takes a participation in the high quality firm increases with the externality intensity. If the bargaining weight of the welfare maximizers is low, the efficiency of FUNEs decreases with the externality intensity, while it increases if b_1 is large enough. Neither partial nor total nationalization of the high quality firm allows to attain the Pareto efficient surplus level.*

7 Should the government invest in the low quality firm?

Up to now, we have assumed that the government invests in the high quality firm. One can wonder why this should be so, and whether a better result in terms of efficiency would not be reached if the government invested in the firm offering the low quality good.

Formally, we are looking for FUNE-G2 — i.e., FUNEs with a welfare-maximizing faction in firm 2 but none in firm 1, when $\gamma = 1$.[8] We proceed as in the two previous sections: we start from the eFUNE, keep constant the relative bargaining weights of the revenue- and profit-maximizing factions in both firms and compute the FUNE with

[8] The formal definition is similar to Definition 4 and is left to the reader.

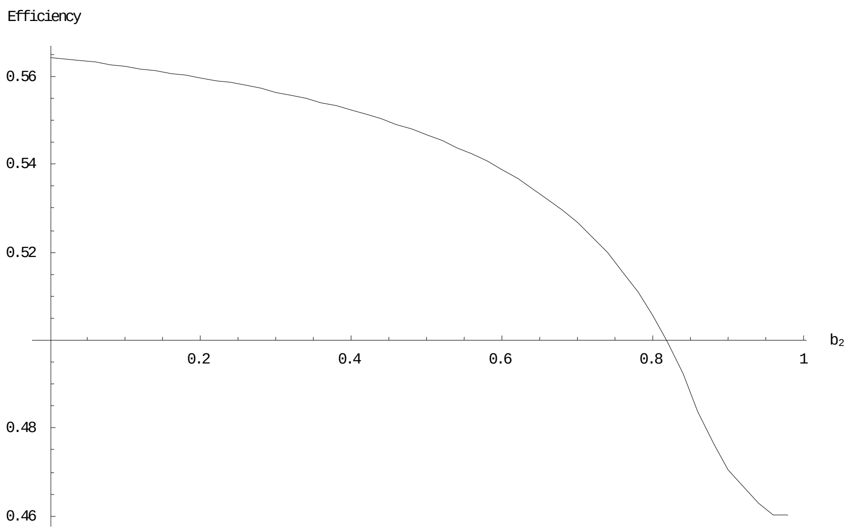


Figure 5: Efficiency of FUNE-G2 as a function of b_2.

government intervention as a function of the bargaining weight of the welfare maximizers (b_2 here). We obtain that a FUNE-G2 exists for all values of the bargaining weight of the welfare-maximizing faction $b_2 \in [0, 1]$. These FUNE-G2 differ strikingly from the case where the government invests in firm 1:

- the quality offered by both firms decreases monotonically and moves away from the optimal qualities: q_1 decreases from 48.4 to 36.7 when b_2 goes from 0 to 1 (the optimal value of q_1 is 220) while q_2 decreases from 40.5 to 33.7 (while the optimal value of q_2 is 103.6);

- the proportion of people buying the low quality good decreases (from 37.5% for b_1=0 to 29.3% for $b_1 = 1$) and moves away from the optimal proportion, 85.8%;

- the efficiency of the FUNE-G2 decreases monotonically, from 56% to 46% (see Figure 5).

[Insert Figure 5 around here]

We summarize all this in the next Result:

Result 6 *As the bargaining weight of the welfare maximizing faction in the low quality firm increases, the efficiency of the corresponding FUNE-G2 allocation decreases monotonically, with the qualities of both goods decreasing and moving away from their optimal levels, while the proportion of people buying the low quality good decreases and also moves away from its optimal level.*

We then obtain a kind of paradox, where increasing the bargaining weight of the welfare maximizers in the low quality firm actually decreases the overall efficiency of the FUNE allocation! Although welfare maximizers should increase the low level quality in order to increase the efficiency of the allocation, the strategic interactions between firms lead to the opposite result.

8 Conclusion

In this paper, we have looked at competition between two firms providing differentiated goods when individuals care for the average quality of the goods supplied and when firms are composed of various factions whose objectives differ. As such, this analysis belongs to the mixed oligopoly literature, which studies competition between firms whose objectives differ. Our main assumptions are that firms are composed of both profit and revenue maximizers, and that these two factions bargain with each other when choosing price and quality of the good their firm is offering. An equilibrium allocation, called Firm Unanimity Nash Equilibrium (FUNE), corresponds to a Nash equilibrium between firms when factions inside each firm bargain efficiently.

Using numerical simulations, we first show that there is a two-dimensional manifold of FUNEs, each characterized by the relative bargaining weight of the revenue maximizers in each firm. The normative properties of FUNEs are bad, with qualities provided being too low and too many consumers buying the high quality good. We then open the bargaining power "black box" and provide an explanation as to how their value is determined. Providing such an explanation has an obvious interest by itself, and also allows us to narrow down the set of equilibria and indeed to obtain unicity. Our next step is then to introduce public intervention in the form of the government taking a participation in one firm. We assume that participation introduces a third faction in this firm, whose objective is to maximize total surplus. We first look at intervention in the high quality firm, and obtain that efficiency is monotonically increasing with the bargaining weight of the welfare maximizing faction when the externality intensity if large enough. This calls for total nationalization of the high quality firm. On the other hand, when the externality intensity is small enough, efficiency is first increasing and then decreasing with the welfare maximizers' bargaining weight. This would then call for partial nationalization of the high quality firm. On the other hand, we obtain strikingly different results when the government takes a participation in the low quality firm, since efficiency is monotonically decreasing with the welfare maximizers' bargaining weights. This last result is reminiscent of White (2002), who shows that one should manipulate the objective of the public firm (in a duopoly where this firm faces a profit-maximizing private firm) in order to reach a better allocation. For instance, instructing the public firm to maximize welfare is not the objective that will actually maximize the welfare level of the equilibrium duopoly allocation.

The approach used in this paper could be extended in several directions. First, we concentrate on one form of government intervention, namely the introduction of welfare-maximizers in one firm's board of directors. It would be interesting to contrast this with other, more light handed, forms of regulations such as price caps for instance. Second, we have assumed that the government faction maximizes welfare. In the light of White (2002) and of our results with public intervention in the low quality firm, it might be interesting to allow for other objectives for the faction appointed by the government, even with a benevolent government. Third, we have abstracted from many shortcomings of the government taking a participation in one firm, like incentive issues or financial expenditures associated with this policy when the cost of public fund is large. We have done this in order to focus on whether there is a case to be made for such a public intervention in the most favorable setting. We have seen that the answer is not always positive, at least when the low quality firm is concerned. Introducing incentive issues would obviously restrict the set of parameters for which taking a participation in the high quality firm is desirable.

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Appendix

Proof. of Proposition 1

Equate the right hand sides of the first order conditions for firm 1 (equations (2) and (3)):

$$\frac{1-F(\lambda^*)-[p_1-c(q_1)]\,f(\lambda^*)\frac{\partial\lambda^*}{\partial p_1}}{F(\lambda^*)-1+p_1 f(\lambda^*)\frac{\partial\lambda^*}{\partial p_1}}=\frac{-c'(q_1)\,(1-F(\lambda^*))-[p_1-c(q_1)]\,f(\lambda^*)\frac{\partial\lambda^*}{\partial q_1}}{p_1 f(\lambda^*)\frac{\partial\lambda^*}{\partial q_1}}.$$

Manipulating this equation reduces it to

$$\frac{c(q_1)}{q_1-q_2}\lambda^*=c'(q_1)\left(\frac{p_1}{q_1-q_2}-\frac{1-F(\lambda^*)}{f(\lambda^*)}\right). \tag{11}$$

Proceeding in like manner for firm 2, we obtain

$$\frac{-c(q_2)}{q_1 - q_2}\lambda^* = c'(q_2)\left(\frac{F(\lambda^*)}{f(\lambda^*)} - \frac{p_2}{q_1 - q_2}\right). \tag{12}$$

Using the fact that

$$\frac{c(q)}{c'(q)} = \frac{q}{r},$$

and dividing equation (11) by $c'(q_1)$ and equation (12) by $c'(q_2)$, and then adding the equations gives

$$\frac{\lambda^*}{r} = \lambda^* + \frac{2F(\lambda^*) - 1}{f(\lambda^*)}.$$

■