Downsizing with Labor Sharing and Collusion*

Antonio Estache† Jean-Jacques Laffont‡ Xinzhu Zhang§

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Abstract

In this paper we develop a model with adverse selection on the productive efficiency of workers in the private sector to analyze the downsizing problem in a public enterprise. Moreover, workers are distinguished by an inside productivity factor. Our result shows that reallocation of labor in the optimal downsizing mechanism depends on the comparative advantage of workers in public versus private production and on the size of asymmetric information. In particular, if information asymmetry is small, random downsizing mechanisms may become optimal. We also show that collusion between workers and the manager in charge of downsizing may induce more screening than in the absence of collusion if information asymmetry is large enough. Finally, we study how risk aversion of workers affects the optimal downsizing mechanism.

JEL Classification: D82, L51, J2, O15.

Keywords: Downsizing, Adverse Selection, Collusion, Risk Aversion.

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‡WBI, Washington, DC20433, USA.

§Université des Sciences Sociales (GREMAQ, CNRS and IDEI), Place Anatole France, 31042 Toulouse Cedex, France. Email: laffont@cict.fr.

§Research Center for Regulation and Competition, Chinese Academy of Social Sciences, Beijing 100732, China.
1 Introduction

Because of inflexible labor employment in the public sector, labor redundancy is a phenomenon all over the world wherever the public sector constitutes a large part of the economy. In China, for example, as an overhang of the "low wage and high employment" policy in the pre-reform era, its public sector is now plagued by a serious labor redundancy problem as it is making an arduous effort to build a market economy. According to the newest estimates (Economist, 2000), at least one third to one half of the workers in the state-owned enterprises are working without making any profit. In China Telecom\(^1\), which is the dominant state operator in China's telecommunications market, one of the most profitable markets in China, it is estimated that out of its about half million employees, at least half of them are redundant.

The Chinese government has been well aware of the impact of the labor redundancy problem on the economy and has employed, with the help of international organizations like the World Bank, every method available to solve it. However, until recently it is still an outstanding issue overwhelming the government's reform agenda. One of the difficulties in designing and implementing any downsizing policy is the lack of information on the part of government. Indeed, the government in general knows neither how much nor whom to downsize because it does not have the information about the workers' productivities in the public and private sectors. This situation of asymmetric information poses a great challenge to the government and creates serious adverse selection problems. In practice it is often claimed that, when voluntary downsizing policies are implemented, it is the efficient workers endowed with good outside opportunities who are most likely to leave the public sector. This brain drain problem has caused considerable disturbance to the public sector in the Chinese economy. The objective of downsizing the public sector is to improve efficiency. This requires in most cases a reduction of labor in this sector to achieve a more efficient allocation of labor in the whole economy. However, reallocation of labor does not necessarily mean that all efficient workers should go to the private sector because China as well as other economies still needs an efficient public sector.

In addition to the efficiency concern, the government also needs to worry about the social stability problem which helps to explain the preference for voluntary downsizing mechanisms over mandatory ones. Since 1997, China’s economy has been deeply affected by the East Asia financial crisis and did not fully recover from it yet. Thus, the private sector’s ability to absorb surplus labor in the society is limited. Indeed, China is now in a critical stage of

\(^{1}\text{China Telecom has been restructured into two companies by the Chinese government in 2002.}\)
transition from a planned to a market economy, in which major restructuring of the public sector is underway. The restructuring process is likely to worsen the unemployment problem. Therefore, the downsizing policy is to a large extent closely related to the legitimacy of the Chinese government and it is no surprise that the government has given a high priority to this issue.

In this paper we apply the mechanism design methodology to analyze downsizing of public firms. We focus on labor sharing which is one of the important features of the downsizing programs implemented in China. We are mainly interested in the impact of asymmetric information on optimal downsizing mechanisms. In particular, we want to explore when random mechanisms (which can be interpreted as labor sharing mechanisms so common in China) are employed. In our model workers have different inside productivities or produce goods with different quality levels, which do not affect their own utility levels. Thus, there is a potential conflict between optimality and incentive compatibility. Indeed, when workers with a high opportunity cost of public production are very productive in public production or provide a very high level of quality, optimality requires them to spend as much time as possible in public production. But incentive compatibility requires that they should spend less time in public production than low-productivity workers. If the size of information asymmetry is small, optimality contradicts completely incentive compatibility and pooling occurs. It is an example of the well-known irresponsiveness result (Guesnerie and Laffont, 1984). This result implies that random downsizing mechanisms become optimal.

Then, we introduce the possibility of collusion between the manager of the public firm and workers.\(^2\) We analyze the impact of the threat of collusion on the design of optimal downsizing mechanisms. More specifically, we explore how collusive behavior affects the screening ability of these mechanisms. Modeling the collusion game with hard information as Tirole (1986) we show that a separating equilibrium may occur in the presence of collusion when information asymmetry is large enough, even though a pooling equilibrium would prevail in the absence of collusion. In other words, collusion induces more screening.

The paper that is the closest to the present one is Jeon and Laffont (2000).\(^3\) Both papers analyze efficient downsizing mechanisms with adverse selection. The main difference is that here workers' reservation utility levels in the public sector are independent of their type, which affects only the quality of their production in the public sector and their outside opportunities.

\(^2\)Chong and Lopez-de-Silanes (2002) provide evidence about the importance of the managerial quality of government for the outcome of downsizing mechanisms.

\(^3\)We refer to this paper for a survey of the literature. For recent empirical contributions see the 1999 special issue of the World Bank Economic Review and Estache et alii (2000).
Furthermore, we consider the impact of the threat of collusion and of risk aversion on the characteristics of efficient downsizing mechanisms.4

The rest of the paper is organized as follows. The basic model is presented in Section 3 and the optimal downsizing mechanism is characterized in Section 4. Section 5 extends the analysis to allow for collusion between workers and managers, and for workers’ risk aversion. Section 6 first relates our theoretical results to the downsizing mechanisms observed in China as well as other economies and then some concluding remarks are provided.

## 2 The Model

Let us consider an economy composed of a public firm and a private sector representing the rest of the economy. There is a labor redundancy problem in the public firm and the principal, the benevolent government, wants to implement a downsizing program to solve this problem. Assume there is a continuum of workers of mass 1 employed in the public firm and denote the set of these workers by \( l \).

In this paper we only consider voluntary downsizing5 in the sense that downsizing should be done through a process which induces voluntary participation, that is all workers in the public firm may choose to work full time as they can now with their current status. Assume also that monitoring of output of public production is inefficient and that an improvement of incentive schemes in the public sector is not envisioned. So the principal cannot use the quantity produced as a controllable instrument. Assume in particular that the quantity produced by each worker in the public firm is normalized to 1 both before and after downsizing.

Workers in the public firm are distinguished by their opportunity cost of public production (e.g. the expected competitive wage they would receive in the private sector) which represents an adverse selection parameter in our model. Denote worker \( A_i \)’s type, with \( i \) in \( I \), by \( \theta \) which is his private information. The \( \theta \)'s are independently and identically distributed and take the value of \( \bar{\theta} \) with probability \( \nu \) and \( \theta \) with probability \( 1 - \nu \). Define \( \Delta \theta \equiv \bar{\theta} - \theta > 0 \). We can thus call type \( \bar{\theta} \) and type \( \theta \) agents good type and bad type workers, respectively. Assume workers also have different productivities for public production and a type \( \bar{\theta} \) worker has normalized inside productivity 1 and a type \( \theta \) worker has inside productivity \( \rho \) in \((0, 1)\).

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4Note that our purpose is not to provide a model which represents downsizing in full generality. It is rather to construct a simple model illuminating the issue of labor sharing and the consequences of collusion.

5In their sample of 400 firms, Chong and Lopez-de-Silanes (2002) find that downsizing is voluntary in 41.5% of the cases. See Jeon and Laffont (1999) for an analysis of the theoretical differences between voluntary and compulsory downsizing.
In other words, good type workers for private production are also more efficient for public production. Another way of interpreting the inside productivity factors is to take them as quality measures. Our model thus has two dimensions of private information but the inside productivity or quality characteristic is perfectly correlated with outside productivity. Assume all workers have the same disutility for public production which is normalized to be zero. We then have a model for allocating labor which depends on the distribution of productivities in the private and public sectors. Alternatively, the model can be seen as one in which differently talented workers exert the same effort but provide goods with different levels of quality in the public firm.

In voluntary downsizing, $A_i$ has two choices available: he may either reject the government’s offer of participating in the downsizing mechanism, or accept the offer. $A_i$’s utility is thus given by:

$$U_i = \begin{cases} U^p & \text{if he rejects the offer} \\ U_i(\theta) & \text{if he accepts the offer}, \end{cases}$$

where $U^p(> 0)$ is the same reservation utility level\(^6\) for all workers and $U_i(\theta)$ will be defined below. For our downsizing problem to be interesting we assume $U^p \geq \theta$.

The benevolent government maximizes social welfare:

$$W = S(q) - (1 + \lambda) \sum_{i \in I} t_i + \sum_{i \in I} U_i,$$

where $S(q)$ represents the social surplus generated by public production $q$, $\lambda (> 0)$ is the shadow cost of public funds, and $t_i$ is the monetary transfer from the government to $A_i$. The transfer $t_i$ includes two parts, namely the wage $w_i$ in public production and the severance pay $s_i$ in the private sector. For technical reasons assume $S(\cdot)$ is increasing and strictly concave with $S(0) = \infty$.

According to the revelation principle, we can without loss of generality focus on direct revelation mechanisms. Then, a downsizing mechanism is defined by

$$\{p_i(\hat{\theta}), w_i(\hat{\theta}), s_i(\hat{\theta})\} \text{ with } \hat{\theta} \text{ in } \{\underline{\theta}, \bar{\theta}\},$$

where $\hat{\theta}$ is agent $A_i$’s report to the government about his type and $p_i(\hat{\theta})$ (resp. $1 - p_i(\hat{\theta})$) represents the share of work hours in the private (resp. public) firm when playing the downsizing

\(^6\)For instance, compensations are equal for all workers in public production.
mechanism, \(w_i(\hat{\theta})\) and \(s_i(\hat{\theta})\) are \(A_i\)'s wage and severance pay received from the government with a total expected payment to the agent \(t_i(\hat{\theta}) = (1 - p_i(\hat{\theta}))w_i(\hat{\theta}) + p_i(\hat{\theta})s_i(\hat{\theta})\). Note that to capture the feature of labor sharing in the downsizing mechanism, we assume that downsizing takes the form of a reduction (increase) in the share of work hours in public (private) production. If a worker in the public firm is required to work part time after downsizing, he will try to find a job (part time or full time) in the private sector. Since there is no aggregate uncertainty about types, the share of work hours and the transfer for \(A_i\) depend only upon his own report.

The downsizing mechanism requires that the government offers to each worker the possibility of playing a mechanism composed of a share of work hours spent in private production and the associated transfers. A worker may or may not accept to play the mechanism. If he does not accept the downsizing offer, he keeps the reservation utility \(U^p\); but if he chooses to accept the offer, he must commit to respect the outcome of the mechanism. In particular, when the outcome tells him to work only part time in the public firm, he should abide by the downsizing mechanism by trying to find a job in the private sector and obtaining an expected wage\(^8\) of \(\theta\).

For the sake of expositional simplicity, the following notations are introduced:

\[
\begin{align*}
p_i(\theta) &= p, \quad p_i(\bar{\theta}) = p \\
w_i(\theta) &= w, \quad w_i(\bar{\theta}) = w \\
s_i(\theta) &= s, \quad s_i(\bar{\theta}) = s \\
U_i(\theta) &= \bar{U}, \quad U_i(\bar{\theta}) = \bar{U}
\end{align*}
\]

Denote \(MC^C(\theta)\) as the social marginal cost of public production under complete information. To justify the necessity of downsizing, i.e., to reduce the work hours in public production, we make the following assumption:

\[
S(1) < MC^C(1).
\]

This assumption states that under complete information, if the government keeps all workers spending full time in public production, the social marginal surplus for public production is

\(^7\)Alternatively \(p_i(\hat{\theta})\) can be interpreted as the probability that worker \(i\) is excluded from the public firm. Then, the mechanism has a random component. However, when we refer below to random mechanisms, we mean mechanisms which treat different types similarly and randomly.

\(^8\)However, we will show below that payments can be structured in such a way that workers never regret to have participated.
smaller than the corresponding social marginal cost.

3 The Optimal Downsizing Mechanism

Before characterizing the optimal downsizing mechanism under asymmetric information, let us first consider the benchmark case in which there is complete information about the outside productivity parameters $\theta$s.

For the downsizing mechanism to induce voluntary participation, the following participation constraints must be satisfied for the risk neutral workers: for the bad type,

$$\(1 - p\)w + ps + \theta p \geq U^p;$$

and for the good type,

$$\(1 - p\)w + ps + \theta p \geq U^p.$$  \hspace{1cm} (3)

Expected social welfare is denoted by

$$EW = S(v(1 - p) + (1 - v)\rho(1 - p)) - (1 + \lambda)[v((1 - p)w + ps) + (1 - v)((1 - p)w + ps)] + vU + (1 - v)U,$$

where $v(1 - p) + (1 - v)\rho(1 - p)$ is the public production, and, by abuse of notations, $U \equiv U - U^p$ and $U \equiv U - U^p$ are information rents. From this welfare function one can see immediately that there is a potential conflict between workers and the government because the workers’ inside productivities do not affect their utility levels but affect social welfare. Note also that the allocation of labor depends only on total transfers $t_i = (1 - p)w_i + p_s$, and that the decomposition in wage $w_i$ and severance pay $s_i$ does not matter.

The government’s program under complete information is thus given by:

$$\max_{p, w, \theta} \sum_{i} EW$$

s.t. (2) and (3).

Rewriting expected social welfare in terms of workers’ utilities, we have:

$$EW = S(v(1 - p) + (1 - v)\rho(1 - p)) + (1 + \lambda)(v\theta p + (1 - v)\theta p) - \lambda(vU + (1 - v)U).$$
Proposition 1: Under complete information the optimal shares of work hours in public production can be characterized as follows:

(i) when $\frac{\theta}{\rho} \geq \frac{\theta}{\rho}$, then if $\bar{S}(\cdot)$ is low, $p^* = 1$ and

$$\bar{S}(1 - v)(1 - p^*)\rho = (1 + \lambda)\frac{\theta}{\rho},$$

and if $\bar{S}(\cdot)$ is high, $p^* = 0$ and

$$\bar{S}(\nu(1 - p^*) + (1 - \nu)\rho) = (1 + \lambda)\bar{\theta}.$$  

(ii) if $\theta < \frac{\theta}{\rho}$, then if $\bar{S}(\cdot)$ is low, $p^* = 1$ and

$$\bar{S}(\nu(1 - p^*)) = (1 + \lambda)\bar{\theta},$$

and if $\bar{S}(\cdot)$ is high, $p^* = 0$ and

$$\bar{S}(\nu + (1 - \nu)(1 - p^*)\rho) = (1 + \lambda)\frac{\theta}{\rho}.$$  

Proof: The derivation is trivial, so it is omitted.

(Figure 1 here)

When $\theta \geq \frac{\theta}{\rho}$, the good type workers have a comparative advantage in the private sector or the bad type workers have a comparative advantage in public production (see Figure 1i). We define under symmetric information the following social marginal costs of public production or social opportunity costs of working in the public sector: for the bad type,

$$\bar{MC}^c \equiv (1 + \lambda)\bar{\theta}.$$  

and for the good type,

$$\bar{MC}^c \equiv (1 + \lambda)\bar{\theta},$$

where the superscript $c$ stands for complete information. From the first-order conditions with respect to the shares of work hours in the public firm, the social marginal utility of
public production is equal to the private marginal cost (opportunity cost of public production) multiplied by $1 + \lambda$, because the government must resort to distortive taxation to raise the necessary money, and discounted by inside productivity.

Under complete information the government gives workers the minimal utility levels necessary to induce them to accept the downsizing mechanism, that is the same status quo utility level, $U^p$, to both types of workers. Since the bad type workers have a comparative advantage in public production, if the social value of public production $(S\{\cdot\})$ is low, the optimal production level is determined when the social marginal utility of public production is equal to the social marginal cost of the bad type $MC^c$. In this case the government keeps only the bad type workers spending part time in public production. Note that it is a corner solution for the good type's share of work hours in public production. Since the good type's social marginal cost is higher than the social marginal utility of public production at $q = (1 - \nu)(1 - p^*)\rho$, we have $p^* = 1$. That is the government lays off all the good type workers in the sense of requiring them to spend full time in the private sector.

Similarly, if the social value of public production is high, the optimal production is determined when the social marginal utility of public production is equal to the good type's marginal cost $MC^c$. Then, the government requires the good type workers to spend part time in public production and there is a corner solution for the optimal share of work hours for the bad type workers. Since their social marginal cost is lower than the social marginal utility of public production at $q = \nu + (1 - p^*)(1 - \nu)\rho$, we have $p^* = 0$, or the optimal downsizing mechanism entails that the bad type workers should work full time in public production.

When $\theta < \frac{\theta}{p}$ or $MC^c MC^c$, the good type workers have such a high inside productivity that they have now a lower social opportunity cost of working in the public firm (see Figure 1ii). That is they have a comparative advantage in public production. The optimal solution is therefore reversed: the good type workers should work part time in public production and the bad type workers spend full time in the private sector if the social value of public production is low, and the bad type workers work part time in public production and the good type workers spend full time in the public sector if the social value of public production is high.

Now assume the government has asymmetric information about the $\theta$s. Moreover, let us first focus on the more interesting case in which $\theta < \frac{\theta}{p}$. For the downsizing mechanism to induce truth-telling, the following incentive constraints should be satisfied: for the bad type,

$$ (1 - p)w + ps + \theta p \geq (1 - p)w + ps + \theta p, \quad (8) $$

9
and for the good type,

$$ (1 - p)w + ps + \theta p \geq (1 - p)w + ps + p\theta. $$

(9)

The government’s program under asymmetric information is:

$$ \max_{P^a, w, \lambda, \nu} EW \quad \text{s.t.} \ (2), (3), (8) \text{ and } (9). $$

Under asymmetric information workers may obtain information rents by participating in the downsizing mechanism. Remember that we define a worker’s information rent as the difference between his utility level when he accepts the government’s offer ($U(\theta)$) and his status quo utility level ($U^p$). Since it is costly for the government to leave information rents to workers, the optimal shares of work hours under asymmetric information are determined by the optimal trade-off between efficiency and rent extraction.

To facilitate the exposition of the next proposition let us define

$$ \rho_a = \frac{\theta}{\theta} - \frac{\lambda}{1 + \lambda} \frac{v}{1 - v} \Delta \theta. $$

The next proposition will distinguish two cases according to the value of $\rho$. The first case will correspond to $\rho$ small ($\rho < \rho_a$) when the $\theta$-workers who have lower outside opportunities are also very inefficient in producing the quality of public production. For given $\rho$, this case also occurs when the information asymmetry is low. The opposite case ($\rho \geq \rho_a$) will occur on the contrary for a high information asymmetry.

Solving ($P_a$) we obtain (see Appendix 2 for a proof and Figure 2 for an illustration):

(Figure 2 here)

Proposition 2: i) When $\rho$ is small ($\rho < \rho_a$), optimal regulation under asymmetric information entails a pooling mechanism $p^{**} = \rho^{**} = p^{**}$ characterized by

$$ S((v + (1 - v)p)(1 - p)) = \frac{(1 + \lambda)(v\theta + (1 - v)\theta) - \lambda v \Delta \theta}{v + p(1 - v)}. $$

ii) When $\rho$ is large ($\rho \geq \rho_a$) optimal regulation under asymmetric information entails:
• if $S(\cdot)$ is low, $p^{**} = 1$ and

\[
S((1 - \nu)\rho(1 - p^{**})) = (1 + \lambda)\bar{\theta} - \lambda \frac{\nu}{1 - \nu} \Delta \theta / \rho
\]

• if $S(\cdot)$ is high, $p^{**} = 0$ and

\[
S(\nu(1 - p^{**}) + (1 - \nu)\rho) = (1 + \lambda)\bar{\theta}.
\]

Adding the incentive constraints (8) and (9) yields the monotonicity condition $p \geq \bar{p}$. Therefore, incentive compatibility requires that the good types $\bar{\theta}$ who have higher outside opportunities enjoy in the mechanism a share of work hours in the private sector not lower than the bad types $\theta$. However, this monotonicity condition may conflict with the socially efficient allocation of labor under complete information when the good types are also producing relatively higher quality in public production, i.e., when $p\bar{\theta} < \theta$. Then two cases appear. If the information asymmetry is small enough, even when the virtual cost of the efficient type which includes the cost of information rents is taken into account, the good types are still more efficient with respect to the public sector and screening is not compatible with incentives. The optimal mechanism pools the two types. This case occurs when $\rho$ is not too large, that is $\rho < \rho_a$, or when the information asymmetry $\Delta \theta$ is small enough (see Figure 2i). One can check that asymmetric information leads to less downsizing of the bad type and more downsizing of good type workers (see Appendix 2).

On the other hand, if the information asymmetry $\Delta \theta$ is large enough, screening remains compatible with the monotonicity constraint (see Figure 2ii).

For a low social value of public production, all the good type workers work in the private sector and the bad type workers working part time in the public sector. If the social value of public production is high, the bad type workers spend full time in the public sector and the good type workers work part time in the private sector. One can check that in both cases asymmetric information decreases (resp. increases) the downsizing level of the bad type (resp. good type) workers.

Finally, when $\bar{\theta} \geq \theta / \rho$, the good types are the ones who should work in the private sector under incomplete as well as complete information. Then, there is always a separating allocation with less downsizing of the bad type workers due to asymmetric information.

**Proposition 3**: When $\bar{\theta} \geq \theta / \rho$, optimal regulation under incomplete information entails
If $S(\cdot)$ is low, $p^{**} = 1$ and

$$S((1 - \nu)(1 - p^{**})\rho) = (1 + \lambda)\theta - \lambda \frac{\nu}{1 - \nu}\Delta \theta / \rho. $$

If $S(\cdot)$ is high, $p^{**} = 0$ and

$$S(\nu(1 - p^{**}) + (1 - \nu)\rho) = (1 + \lambda)\theta. $$

If the social value of public production is low, there is less downsizing of the bad type workers while good type workers still work full time in the private sector as under full information. For a high social value of public production, however, the level of downsizing is unchanged for both types of workers. The differential effect of asymmetric information between these two cases can be explained as follows. Asymmetric information obliges the regulator to give up a rent to the efficient type. Indeed, when the $\theta$-type’s participation constraint is binding, the $\theta$-type obtains the information rent $p \Delta \theta$ by mimicking the $\theta$-type. This rent can only be decreased by decreasing $p$. However, in the corner solution such that $p^* = 0$, no downward distortion is possible and asymmetric information does not affect the optimal allocation. It is indeed what happens when the information asymmetry is high and the value of production also high enough.

We observed all along this section that only expected payments $(1 - p)w + ps$ matter. Then, we can choose wages so that workers do not regret to work in the public firm $(w = w = U^p)$. From the participation constraints (2) and (3), we can check that they do not regret working in the private sector either (since $s + \theta \geq U^p$ and $s + \bar{\theta} \geq U^p$).

Propositions 2 and 3 describe the optimal downsizing mechanisms. The implementation of these mechanisms can be thought of proceeding as follows. By solving the equations contained in these propositions, the government defines the targeted downsizing. Then, he offers a menu of contracts $(w, s, p, w, s, p)$ within which workers self-select themselves.

In practice, redundant workers are defined by job category at each key operational function of the enterprise. If we have $k$ such categories, say $S = S(q_1, ..., q_k)$, the targeted downsizing levels for all categories are determined by a system of equations. The marginal productivity of each category $\frac{\partial S}{\partial q_k}$ is equated to its social opportunity cost of working in the public sector. A downsizing mechanism which would not pay attention to the complementarity of the tasks in the firm could seriously disrupt the efficiency of the public firm.

A particular menu is often implemented as follows. A voluntary downsizing is offered for a
transfer \( \bar{t} \). Then, a mandatory random downsizing is imposed on the remaining workers with a proportion \( p \) leaving the firm and with a lower transfer \( t \). This scheme can be interpreted as a mandatory downsizing with a menu

\[
p = 1, \quad \bar{t} = \bar{s}; \\
\bar{p}, \quad t = (1 - p)w + ps \quad \text{for some } w \text{ and } s.
\]

This scheme can be thought of as a large downsizing mechanism focusing on the good type in step 1, and on the bad type in step 2. With rational expectations the good type workers know that if they do not opt for the voluntary mechanism, they will be faced later with the mandatory one.\(^9\)

## 4 Extensions

In this section we extend our basic model to consider the issues of collusion between managers and workers and risk aversion of workers. Our aim is to analyze the impact of these factors on the optimal allocation of labor.

### 4.1 Collusion in Downsizing

Let us now introduce the issue of collusion. We want to illustrate in a very simple setting its impact on the optimal downsizing mechanism. For this purpose we add to the basic principal-agent relationship between the government and workers a hierarchical level representing the manager of the public firm.

Abuse of power by managers is prevalent in practice in the implementation of downsizing programs. Managers often make use of their discretion in the implementation of downsizing programs to unduly favor some workers (e.g., relatives and friends etc.) at the expense of others. Actually, this is one of the main concerns that people have cast on the downsizing programs.

Following Tirole (1986) we assume that the manager’s role is to bridge the government’s

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\(^9\)Recently, the Chinese government, after giving several offers of voluntary downsizing, has strengthened the regulation of voluntary downsizing and imposed mandatory downsizing (Document No. 8, 1999, Ministry of Labor and Social Security). This could be rationalized as a two-step mechanism as above. However, it is likely that the approach was driven pragmatically by the need to save public funds and the desire to limit social unrest.
information gap on $\theta$. He has utility function

$$V = u \geq 0,$$  \hspace{1cm} (10)

where $u$ is the manager’s reward. The manager is risk neutral but is protected by limited liability. To obtain the manager’s participation he should get at least his reservation utility level, which is normalized to be zero.

Assume that the manager is endowed with an information technology under which he learns a private signal ($\sigma = \theta$) about a worker’s outside productivity with probability $\xi$ and learns nothing ($\sigma = \varphi$) with probability $1 - \xi$. For simplicity we assume the manager’s information is known to workers. To make use of the manager’s information the government asks the manager to report the signal he has received, $r \in \{\theta, \bar{\theta}, \varphi\}$. The critical assumption is that the signal the manager’s report is hard information, i.e., when a conclusive signal is reported to the government, it is hard evidence. However, the manager can hide this information and report that the signal is $\varphi$. When an inconclusive signal is obtained, the manager must report $\sigma = \varphi$. Thus, the manager only has discretion when he receives a conclusive signal. The manager’s information technology can be summarized by a table describing the probabilities of the possible cases.

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<tr>
<th>$\sigma$</th>
<th>type $\bar{\theta}$</th>
<th>type $\theta$</th>
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<tbody>
<tr>
<td>$\sigma = \bar{\theta}$</td>
<td>$\nu \xi$</td>
<td>0</td>
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<tr>
<td>$\sigma = \varphi$</td>
<td>$\nu (1 - \xi)$</td>
<td>$(1 - \nu)(1 - \xi)$</td>
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<tr>
<td>$\sigma = \theta$</td>
<td>0</td>
<td>$(1 - \nu) \xi$</td>
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The timing of the downsizing game in the presence of a collusion threat between the manager and workers is as follows: downsizing mechanisms are offered by the government after workers and the manager learn their respective information. After this, side-contracting may take place between them. The government then asks for reports from the manager about workers’ outside productivities. When a conclusive signal is reported for a worker, the downsizing decision is made under full information; but when the manager reports an inconclusive signal, the revelation game under asymmetric information is played as in the previous section.\(^{10}\)

\(^{10}\)Under our assumption about the manager’s information technology, the posterior beliefs of the government
Now let us consider when the threat of collusion matters.\textsuperscript{11} When a good type worker has higher social opportunity cost of working in the public sector ($\rho \geq \rho_a$), in the absence of a collusion threat we know from the last section that a good type worker gets an information rent: $\Pi = p\Delta \theta$. When the manager learns that $\sigma = \bar{\theta}$ and reports this information to the government, the government can extract all the information rent from the good type worker. Thus, the worker has an incentive to bribe the manager to hide this information by reporting that he had an inconclusive signal. To prevent collusion the government must give to the manager a contingent reward which is not less than the stake of collusion, that is the following collusion-proofness constraint must be satisfied:

$$u \geq \frac{1}{1 + \lambda_t} p \Delta \theta,$$

where $u$ is the reward the manager should get if he reports the hard information $\bar{\theta}$ and $\lambda_t$ ($> \lambda$) represents the transaction cost of side-contracting.

When the manager gets a signal $\sigma = \bar{\theta}$, the bad type worker does not obtain any information rent so he has no incentive to bribe the manager. Since it is costly to give up information rents, the manager is offered a zero reward to induce participation and satisfy the limited liability constraint. Similarly, the manager gets a zero reward when he reports an inconclusive signal.

By the collusion-proofness principle we can without loss of generality focus on the downsizing mechanisms in which there is no collusion in equilibrium. Thus, the government’s program under the threat of collusion is:

$$\max_{\rho, p, \xi} \xi \left[ S(v(1 - p^*) + (1 - v)(1 - p^*)\rho) + (1 + \lambda)(vp\bar{\theta} + (1 - v)p\bar{\theta}) \right]$$

$$+ (1 - \xi)\left[ S(v(1 - p) + (1 - v)(1 - p)\rho) \right]$$

$$+ (1 + \lambda)(vp\bar{\theta} + (1 - v)p\bar{\theta}) - \lambda(vU + (1 - v)\bar{U}) - \lambda \xi vU$$

s.t. the same constraints as in $P^a$ and (11).

Since it is costly to leave a rent to the manager, the collusion-proofness constraint is binding.

\textsuperscript{11}In this section we consider only the case $\bar{\theta} < \bar{\theta}'\rho$. However, the results have the same flavor in the opposite case. The threat of collusion induces less downsizing.

\textsuperscript{11}In this section we consider only the case $\bar{\theta} < \bar{\theta}'\rho$. However, the results have the same flavor in the opposite case. The threat of collusion induces less downsizing.
We can thus simplify the program by substituting \( \bar{u} = \frac{1}{1+\lambda t} p\Delta \theta \) into the welfare function. With probability \( \xi \) the downsizing decision is made under complete information; otherwise, the government has asymmetric information about \( \theta \).

When instead \( \rho < \rho_a \), we know from Proposition 2 that in the absence of collusion, screening is not possible and the good type worker obtains an information rent \( p\Delta \theta \). In this case the manager has also discretion when he learns that \( \sigma = \bar{\sigma} \). Collusion is prevented by providing the manager with an incentive reward that is no less than the stake of collusion, i.e. \( \bar{u} \geq \frac{1}{1+\lambda t} p\Delta \theta \). Note that in both cases, by the collusion-proofness principle the presence of collusion is equivalent to the inclusion of a collusion cost in the social welfare function. But the government still needs to satisfy the same incentive constraints as in the absence of collusion.

To simplify the presentation of our next result, we define (here \( ac \) stands for collusion)

\[
\rho_{ac} = \frac{\theta}{\bar{\theta}} - \frac{\lambda}{1+\lambda} \frac{\nu}{1-\nu} \frac{\Delta \theta}{\bar{\theta}} - \frac{1}{1+\lambda t} \frac{\lambda}{1+\lambda} \frac{\nu}{1-\nu} \frac{1-\xi}{\bar{\theta}} \Delta \theta.
\]

Then we have the following proposition:

**Proposition 4.** Optimal regulation entails:

(i) When \( \rho \geq \rho_a \), then if \( S(\cdot) \) is low, \( p^{**} = 1 \) and

\[
S((1-\nu)(1-p^{**})\rho) = (1+\lambda)\theta - \frac{\nu}{1-\nu} \lambda \Delta \theta - \frac{1}{1+\lambda t} \frac{\lambda}{1+\lambda} \frac{\nu}{1-\nu} \frac{1-\xi}{\bar{\theta}} \Delta \theta / \rho,
\]

and if \( S(\cdot) \) is high, then \( p^{**} = p^* \) and \( p^{***} = p^* \).

(ii) When \( \rho_{ac} < \rho < \rho_a \), then if \( S(\cdot) \) is low, \( p^{***} = 1 \) and \( p^{***} \) is the solution of (12). If \( S(\cdot) \) is high, \( p^{***} = 0 \) and

\[
S((1-\nu)\rho + \nu(1-p^{***})) = (1+\lambda)\bar{\theta}.
\]

(iii) When \( \rho < \rho_{ac} \), then \( p^{***} = p^{***} \equiv p^{***} \) and

\[
S((1+\lambda)(\nu \theta + (1-\nu)\bar{\theta}) - \lambda \nu \Delta \theta - \frac{1}{1+\lambda t} \frac{\lambda}{1+\lambda} \frac{\xi}{1-\nu} \nu \Delta \theta / (\nu + (1-\nu)\rho)
\]

Proof: The proof is similar to that in the absence of collusion so it is omitted.

Let us define

\[
\overline{MC}^{ac} = (1+\lambda)\bar{\theta} = \overline{MC}^c
\]

16
and

\[ M \mathcal{C}^{ac} = \frac{1}{\rho} [(1 + \lambda)\theta - \lambda \frac{v}{1 - v}\Delta \theta - \frac{\lambda}{1 + \lambda} \frac{v}{1 - \xi} \Delta \theta] \]

as the good type and the bad type’s virtual social opportunity cost under the threat of collusion. The good type’s social marginal cost is equal to that without collusion. But the bad type’s social marginal cost includes three parts: the first is the social marginal cost under full information, the second is the information cost caused by asymmetric information, and the third is the collusion cost induced by the presence of collusion threat. From these expressions we know that the presence of the threat of collusion decreases further the bad type’s social opportunity cost for public production.

Case (i) describes the situation in which the good type workers have the higher social opportunity cost under the threat of collusion as without collusion. If the social value of public production is low, following the same arguments as in the absence of collusion we obtain that \( 0 < p^{***} < 1 \) and \( p^{**} = 1 \). One can check that \( p^{***} < p^{**} \). In words, the optimal response to the threat of collusion is to implement less downsizing. On the other hand, if the social value of public production is high, since \( p^{***} \) does not affect rent extraction, collusion does not cause any further distortion.

In case (ii) the collusion cost is sufficiently large so that the good type workers have the higher social opportunity cost under the threat of collusion even though they have the lower social marginal cost without collusion. From Proposition 2 we know that the conflict between optimality and incentive compatibility will lead to pooling in the absence of collusion threat. However, efficient downsizing mechanisms in the presence of collusion call for separating equilibrium. Therefore, collusion changes the type of equilibrium from pooling to separating.

More precisely, if the social value of public production is low, we have \( p^{***} < p^{**} \).\(^{12}\) So collusion induces less downsizing of the bad type workers and more downsizing of the good type workers. Similarly, if the social value of public production is high, since \( p^{**} \geq p^{***} = 0 \), there is less downsizing of the bad type workers. But the impact of downsizing on the good type workers is ambiguous.

In case (iii) the collusion cost is small so that, as in the absence of collusion threat, the good type workers have the lower social opportunity cost. In this case optimality contradicts incentive compatibility and pooling occurs. Furthermore, it is easy to verify that \( p^{***} < p^{**} \),

\(^{12}\) It is easy to check that \( \frac{(1 + \lambda)(v\theta - (1 - v)\xi)\Delta \theta}{v + (1 - v)\beta} - \frac{\lambda}{1 + \lambda} \frac{v}{1 - \xi} \Delta \theta < (1 + \lambda)\theta - \frac{v}{1 - v}\lambda \Delta \theta - \frac{\lambda}{1 + \lambda} \frac{v}{1 - \xi} \Delta \theta / \rho \). So \( (v + (1 - v)\beta)(1 - p^{**}) < (1 - v)(1 - p^{***}) \rho \) and one obtains \( p^{***} < p^{**} \).
that is collusion induces less downsizing of workers even though screening is not possible as without collusion.

To summarize, the threat of collusion decreases the optimal level of downsizing of the bad type workers, and reduces the likelihood of the optimality of a pooling downsizing mechanism.

4.2 Risk Aversion

Now assume that workers are risk averse. For simplification we introduce risk-aversion in the following way. Let us keep our basic setting unchanged except that we modify the timing of the game by assuming that workers discover their types after accepting the downsizing mechanism offered by the government.\footnote{This case allows a simple comparison with the risk neutral case and ex post participation constraints. A similar analysis could be carried out with ex post participation constraints and would obviously yield a lower expected social welfare. It is also clear that both cases dominate the case where workers are uninformed about their type. Indeed, the stochastic downsizing which must then be used is always a possible option for the government in the other cases.} We assume that payments have been structured so that workers have no regret after they have chosen their contracts and discovered their types. With this modification of the basic model, we can obtain a new benchmark where workers are risk neutral and the government has asymmetric information about the $\theta$s. The relevant participation constraint is:

$$v((1 - p)\bar{w} + ps + \delta p) + (1 - v)(1 - p)\bar{w} + ps + \delta p) \geq U^p.$$ 

In addition, one obtains the same incentive constraints (8) and (9) as in our basic setting. Then, the first best allocation as characterized in Proposition 1 can be implemented. Indeed, since only \textit{ex ante} participation constraints need to be satisfied, there is enough flexibility in choosing the wage and severance levels to extract all information rents from both types of workers, i.e.

\[ (1 - p)\bar{w} + ps = U^p - \delta p - v_\theta \Delta \theta \]
\[ (1 - p)\bar{w} + ps = U^p - \delta p - v_\theta \Delta \theta. \]

Therefore, asymmetric information imposes no cost on the society if workers are risk neutral and they discover their types after accepting the downsizing mechanism. However, the ex post utility levels are quite different according to the type and this raises the problem of how the downsizing mechanism should be modified if workers are risk averse.

When workers are risk averse, assume that their Von Neumann Morgenstern utility function
is $U(x) = \frac{1}{r}[1 - \exp(-rx)]$ where $r$ is the constant degree of risk aversion. $r = 0$ corresponds to the limiting case where workers are risk neutral as in the previous sections. Note that if workers expect a level of income $y_1$ with probability $\nu$ and $y_2$ with $1 - \nu$, the certainty equivalent of their income is

$$y_1 - \frac{1}{r} \log(\nu + (1 - \nu)e^{-r(y_2 - y_1)}).$$

Then, the relevant participation constraint of workers is

$$EC(p, p) = (1 - p)w + ps + \theta p - \frac{1}{r} \log(1 - \nu + ve^{-r(1 - p)w + ps + \theta p + ((1 - p)w + ps + \theta p)} \geq U_p,$$

where $(1 - p)w + ps + \theta p$ and $(1 - p)w + ps + \theta p$ are their incomes when they have type $\theta$ and type $\theta$, respectively. Only ex post incentive constraints (8) and (9) need to be satisfied as well as (14). Then, the government has the following program:

$$\max_{p, p, w, w, s, s} S(\nu(1 - p) + (1 - \nu)p(1 - p))$$

$$\quad - (1 + \lambda)[\nu((1 - p)w + ps) + (1 - \nu)((1 - p)w + ps)] + EC(p, p),$$

s.t. (8), (9) and (14).

In this case the participation constraint (14) and the good type's incentive constraint are binding. By substituting transfers it is useful to rewrite the objective function as:

$$EW = S(\nu(1 - p) + (1 - \nu)p(1 - p))$$

$$\quad + (1 + \lambda)(\nu \theta p + (1 - \nu)\theta p)$$

$$\quad - (1 + \lambda) \nu \Delta \theta p + \frac{1}{r} \log(1 - \nu + ve^{-r \lambda \theta p}) + U_p.$$

Then, we have the following result:

Proposition 5: If workers are risk averse and discover their types after accepting the downsizing mechanism, the optimal mechanism can be characterized as follows:

(i) When $(1 + \lambda)p\theta \geq (1 + \lambda)\theta - (1 + \lambda)\frac{\nu - \nu \Delta \theta p}{1 - \nu + ve^{-r \lambda \theta p}}$, then if $S(\cdot)$ is low,
\[ p^{****} = 1 \text{ and } p^{****} \text{ satisfies} \]

\[
S((1 - \nu)(1 - p^{****}) \rho) = (1 + \lambda) \Theta - (1 + \lambda) \frac{\nu}{1 - \nu} \Delta \Theta \quad 1 - \frac{e^{-r \Lambda \Theta} p^{****}}{1 - \nu + \nu e^{-r \Lambda \Theta}} / \rho,
\]

and if \( S(\cdot) \) is high, \( p^{****} = 0 \) and \( p^{****} \) is the solution of (5).

(ii) When \((1 + \lambda) \rho \Theta < (1 + \lambda) \Theta - (1 + \lambda) \frac{\nu}{1 - \nu} \Delta \Theta \quad 1 - \frac{e^{-r \Lambda \Theta} p^{****}}{1 - \nu + \nu e^{-r \Lambda \Theta}} \), then \( p^{****} = p^{****} = p^{****} \) and it is the solution of

\[
S((v + (1 - \nu) \rho)(1 - p^{****})) = (1 + \lambda)(v \Theta + (1 - \nu) \Theta) - (1 + \lambda) v \Delta \Theta \quad 1 - \frac{e^{-r \Lambda \Theta} p^{****}}{1 - \nu + \nu e^{-r \Lambda \Theta}} / (v + (1 - \nu) \rho).
\]

The distortions here have a different motivation than in the previous sections. As the contract is accepted ex ante, no expected rent needs to be given up to the workers. (This explains why the distortions are multiplied by the redistribution factor \( 1 + \lambda \) rather than \( \lambda \) here). As the workers are risk averse, they suffer \textbf{ex ante} from the randomness of utility needed for incentive compatibility. Then, the distortion is meant to decrease the risk premium requested by workers to incur the riskiness of the incentive compatible contract. This risk is proportional to the difference of utilities between the two states of nature, which, since it is always the incentive constraint of type \( \Theta \) which is binding, is equal to \( \Delta \Theta \rho \) (which was the information rent before) and this explains the similarity of the distortions. On the top of this distortion, pooling may occur as before if the monotonicity constraint (induced by incentive compatibility, often called second order incentive constraint) is binding. This occurs under the same conditions as in the previous sections with risk neutrality and interim contracting. In comparison with the risk neutral case in which there is always a separating equilibrium, pooling can occur now if the inside productivity factor \( \rho \) is relatively small. So, as can be expected, risk aversion induces less screening.\[15\]

5 Conclusion

In this paper we have characterized the efficient downsizing mechanisms under asymmetric information. More specifically, the optimal downsizing mechanisms have been characterized as a function of the distribution of inside and outside productivities and of the presence of a

\[15\]The case we dealt with before of risk neutrality with ex post participation constraints can be interpreted as infinite risk aversion at the status quo utility level. Similarly we obtained pooling in some cases.
collusion threat. Our results have showed that workers with a high opportunity cost of public production get information rents in the implementation of voluntary downsizing programs. In addition, we have given some foundations to the optimality of random downsizing mechanisms or labor sharing mechanisms. We have showed that pooling may occur when workers who have a high productive efficiency in the private sector turn out to also have a high productivity in the public sector. In this case the optimization of social welfare may be in conflict with incentive compatibility; it changes the type of equilibrium from separating to pooling if the information asymmetry is small. Finally, we have showed that collusion may induce more screening if the information asymmetry is large enough. Similar results obtain with risk averse workers and ex ante contracting.

It is interesting to use our theoretical results to explain some practical downsizing issues in China\textsuperscript{16} and other economies. Our theory helps to explain the claim that there are serious brain drain problems in the sense that many high-productivity people have gone to the private sector in voluntary downsizing mechanisms.

In our adverse selection model, reallocation of labor should be implemented according to relative productivities between the public and private sector and it is affected by information asymmetry. Thus, for the people who have relatively high productivities in the private sector and if the social value of public production is low, efficiency of labor allocation requires them to be laid-off first or spend more time on private production. Moreover, information asymmetry and threat of collusion requires more downsizing of high-productivity workers.

Our results also shed light on the issue of random downsizing mechanisms or lungang that are used in some cases in China as well as other economies. In our model random schemes become optimal when there are two dimensions of information asymmetry (they are perfectly correlated in our model) and the size of information asymmetry is small. Thus, our theory predicts that random downsizing mechanisms will appear in such cases where the managers care not only about workers' opportunity costs, which are closely related to the cost of implementing downsizing mechanisms, but also their inside productivities. However, workers' outside productivity should not differ too much. If the size of information asymmetric is large, random schemes should not arise.

Another issue that our theory can also shed light on is the issue of regret in downsizing mechanisms. In practice, it is very important yet often forgot by the designer of downsizing mechanisms. In China, some cases have been reported that after voluntary downsizing mechanisms were implemented, workers regret having participated in the downsizing mech-

\textsuperscript{16}See Appendix 1 for a description of the downsizing mechanisms in China.
anism and asked for renegotiation. For instance, some staff in the China Mobile Company have participated in the severance pay scheme, in which they were paid a very large amount of compensations by China’s standard. But a couple of years later, these workers were well aware of the fact that the remaining staff became one of the highest paid due to the extraordinary growth of this sector. As a result, they asked for more compensations from the Company. From the perspective of our model, this example illustrates the importance of taking into account the issue of regret in the design of voluntary downsizing mechanisms.

While our model develops an interesting theory about downsizing, we left some important issues unanswered. For instance, one important feature associated with labor sharing is information acquisition. Agents may learn their outside opportunities in the information acquisition process and it is good because workers are risk averse. But it gives also workers an opportunity to take advantage of this additional information and collude against the government. In this case we need to analyze not only the individual incentives of workers but also their group incentives.

Furthermore, labor sharing has a feature that is related to the role that training plays in the downsizing process. Indeed, training is a necessary component of almost all downsizing programs in every country. But its impact on the downsizing mechanisms is not well understood. In China’s case temporary unemployed workers have the option but are not sure to come back because they have to pass some criteria in the training program. Thus, this tournament feature provides incentives for workers to improve human capital, but also the training process can be seen as a screening device.

We plan to pursue these topics in our future research.
References


Notification on Some Issues about the Restriction and Rectification of Early Retirement Programs that Violate the State’s Regulation, Document No. 8, 1999, Ministry of Labor and Social Security, People’s Republic of China.

Appendix 1

Downsizing mechanisms in China:

The Chinese government has implemented both voluntary and mandatory mechanisms to downsize its public sector. As a matter of principle, straight layoffs are rare and most downsizing is implemented in the form of xiagang, under which xiagang workers leave their jobs but are still officially employed and paid for a couple of years, then become unemployed automatically. In practice the government often makes some special arrangements such as retraining and provision of labor market information to help those xiagang workers re-enter the job market.

Many voluntary downsizing schemes have also been implemented so far. According to the government definition, those under various voluntary downsizing schemes are not classified officially as xiagang, but only counted as internal transfers of workers. The earliest voluntary mechanism is called stop paying wage but keep job policy (ting xin liu zhi). Under this policy when someone quits his job in a public firm, the enterprise stops paying his wage including some benefits but the job is reserved so that, at least in principle, the displaced employee can go back to his previous job whenever he wants.

In some cases the separation policy with severance pay is also employed under which anybody who chooses to quit the public sector is offered a lump sum or a flow of payments for a pre-fixed amount of time as a compensation. In this case, incentive compatibility often entails huge costs on the part of the government.

Early retirement is a downsizing policy widely employed in many industries and regions in China. Under this mechanism people who choose to retire before the legal retirement age can be compensated according to a schedule calculated on the basis of how long they have been employed or are ahead of their legal retirement age. As for the severance pay scheme, it may imply a heavy fiscal burden on the government.

Under the previous downsizing policies part of the workers are able to keep their jobs only at the expense of the others who become displaced one way or another. However, lungang policy is designed for the whole labor force, at least as a transitional policy, to share the limited positions with nobody being completely laid off.

Under this mechanism the government sets first a downsizing target for each enterprise and the enterprise in turn allocates to each plant a downsizing target in terms of a total wage after downsizing. In other words, the firm de facto implements the allocated layoff target. Thus, it is the government which determines the scale of downsizing but the decisions on how to implement it are delegated to the managers.
In practice the management at the plant level implements the downsizing target by labor sharing in the sense that the remaining workers are asked to share with those workers who leave their jobs temporarily the total compensations allocated. A worker's compensation includes two parts: a fixed wage and a bonus. When someone becomes temporarily displaced, he or she is still paid the same fixed wage as those who remain on their jobs but there is no bonus which is usually a large part of the worker's total compensation. The temporary unemployed workers may come back to their jobs if they are willing to do so and pass some criteria. Conversely, those workers who are currently employed may become unemployed some day depending on the rule of labor sharing. In this way the whole staff de facto share the required after-downsizing positions.

In most cases, some training programs are designed to select those workers that will keep their jobs in the public sector. Usually all workers are asked to go to these training programs. Then, workers are selected to remain in the enterprise according to their performance in these training programs. In some cases, a random procedure may be employed to determine who will stay in the public sector in the first stage. After some pre-specified period of time, however, the displaced workers can come back to their previous jobs and some others replace them to become temporarily unemployed in the public sector.
Appendix 2

Proof of Proposition 2:

In case (i) in which \( p < p_a \), define a reduced program for the government:

\[
\max_{p \in \mathcal{U}} E \, W \\
\text{s.t. } (8), (9) \text{ and } p \geq p.
\]

It is easy to see that all the constraints in the program are binding. And its solution also satisfies the other constraints in the program \((P^a)\).

Let us now check that \( p^* > p^{**} \) if \( S(\cdot) \) is high, that is there is less downsizing of the bad type workers. Note that since \( p < p_a \), we have

\[
\frac{(1 + \lambda)(v \bar{\theta} + (1 - v)\theta) - \lambda \Delta \theta}{v + (1 - v)p} < \frac{(1 + \lambda)\theta}{\bar{\rho}},
\]

That is \( S((v + (1 - v)p)(1 - p^{**})) < S(v + (1 - v)(1 - p^*)p) \) or \( (v + (1 - v)p)(1 - p^{**}) > v + (1 - v)(1 - p^*)p \), which leads to \( p^* > p^{**} \). Similarly, it is easy to check that

\[
\frac{(1 + \lambda)(v \bar{\theta} + (1 - v)\theta) - \lambda \Delta \theta}{v + (1 - v)p} > \frac{(1 + \lambda)\bar{\theta}}{(1 + \lambda)\bar{\rho}},
\]

so \( (v + (1 - v)p)(1 - p^{**}) < v(1 - p^*) \) and we have \( p^* < p^{**} \).

In case (ii) in which the information asymmetry is large enough, the first order conditions with respect to \( p \) and \( \bar{p} \) are as follows:

\[
S'(v(1 - p^{**}) + (1 - v)p(1 - p^{**})) \leq (1 + \lambda)\bar{\theta} \quad (= \text{if } p^{**} > 0)
\]
\[
pS'(v(1 - p^{**}) + (1 - v)p(1 - p^{**})) \leq (1 + \lambda)\theta - \lambda \frac{v}{1 - v} \Delta \theta \quad (= \text{if } p^{**} > 0)
\]

There are corner solutions for \( p^{**} \) if \( S(\cdot) \) is low and for \( \bar{p}^{**} \) if \( S(\cdot) \) is high. Substituting the corner solutions we obtain the result in this case.
Figure 1i: $\tilde{\theta} \geq \vartheta' \rho$.

Figure 1ii: $\tilde{\theta} < \vartheta' \rho$. 

\[ S(\cdot) \text{ high} \]

\[ S(\cdot) \text{ low} \]
Figure 2i: $\Delta \theta$ small.
First order conditions are incompatible with monotonicity $p \geq p$.

Figure 2ii: $\Delta \theta$ large.