WORKING PAPERS

N° TSE-558

February 2015

"Power Distribution in French River Basin Committees"

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Power Distribution in French River Basin Committees.

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Abstract

I study the distribution of voting power between different decision-makers in French river basin committees over the period 1987-2007. To do so, in the first part of the paper, I apply different power measures traditionally used in the literature as well as some other ones lesser known in this context. I compare then the predictions of several indices for the relative power of different decision-makers in different voting situations. In the second part, I describe the methodology to design an optimal decision rule. A simple computational exercise based on this methodology suggests that the residential water users in Adour-Garonne river basin were under represented in the river basin committee during 1989-2006.

Key words: environmental management, water policy, collective decision-making, voting, power indices, optimal decision rule

1 Introduction

Stakeholder participation in the design of environmental policies becomes more and more important for efficient decision-making (Goodhue et al., 2008). However, design and implementation of environmental policies in the presence of several stakeholder groups with conflicting interests as well as the state representatives turns into a difficult and challenging task. In this paper I address this issue in the context of water policy in France.

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Water Agencies (WAs) have been important water policy institutions in France since the middle of 1960s. The general mission of WAs is to protect water against any action which can deteriorate its quality and quantity. The main focus of current policy is on reaching an adequate ecological state of river basin resources while maintaining a balance between available water resources and water needs. In practice, it translates into a set of practical objectives such as reduction of the impact of human activities on aquatic ecosystems, maintaining the natural processes of aquatic ecosystems, promoting the quantitative management of river streams, in particular during the summer, managing ground water resources in a sustainable manner, improving the quality of drinking water, etc. (Adour-Garonne Water Agency, 2012).

The French water policy relies upon the principle of decentralized management of the water resource by river basin. In each of six French Water Agencies there is a River Basin Committee (RBC) acting as a "water parliament". Consisting of elected members of local and parliamentary chambers, water users' representatives and the public administration, the RBCs are in charge of elaborating the environmental objectives of the river basin through voting on different issues.

The main focus of this research is the decision-making at the six French RBCs. Specifically, I analyze how the composition of the RBC and the voting rules relate to voting power of the different (groups of) decision-makers in different types of voting situations. The traditional measures such as Shapley-Shubik index (Shapley and Shubik, 1954) and Banzhaf index (Penrose, 1946 and Banzhaf, 1965) have been widely discussed and applied to analyze many voting institutions such as the EU Council of Ministers[1], the United States Legislative system[2], the Canadian Constitutional Amendment Scheme[3]. The Banzhaf index has been also used to study the design of voting bodies in the EU, US, or IMF[4]. To the best of my knowledge, no one has used the concepts of voting power to examine the distribution of

[1]For example, Laruelle and Widgrén (1998), Napel and Widgrén (2006, 2011), Felsenthal and Machover (2001), Barr and Pasarrelli (2009) among many others.

[2]For example, Mann and Shapley (1962), Straffin (1976), Felsenthal and Macover (1998).

[3]For example, Straffin (1977) among others.

[4]Including Banzhaf (1968), Felsenthal and Machover (2001, 2004), Grofman and Feld (2005), Fidrmuc et al. (2009), Leech and Leech (2009), Miller (2009, 2012), Kirsch and Langner (2011) among others.

power among different decision-makers in the French RBC.

In this paper I apply different power measures traditionally used in the literature as well as some other ones lesser known in this context to analyze the power distribution in two types of voting situations. The first type, "binary issues", in which a decision-maker can either vote "yes" or "no", such as the decision on whether to construct a dam. However, not all the voting situations can be classified as binary, for example, the surplus distribution between the stakeholders. I consider such "distributive issues" as a second type of voting situations. One of the examples of distributive situations is the funding of local projects by the RBC through subsidies. The main difference with the binary setting is that in a distributive setting the set of alternatives is a simplex.

In the context of the RBCs, in the distributive situations mainly three water users (residential, industrial and agricultural) benefit from the surplus distribution, while other decision-makers also vote on the decision. In contrast, in the binary situations, there are may be more beneficiaries, for instance, the ecologists in the example related to the reservoir (non)construction. The later situation is illustrated by the recent protests against the clearing of the Testet wet zone in the Sivens forest (Tarn region, southwestern France) in preparation of the Sivens dam construction. The supporters of the dam construction, including the FDSEA (Departmental Federation of syndicated farmers), claim that the dam is in the public interest as it will ensure irrigation and the development of high-value crops. The opponents of the project, backed by French green party, argue the dam will destroy biodiversity and will only benefit a small number of farmers.

In the binary setting I use the Banzhaf and the Shapley-Shubik indices that are well adapted for this situation. In the context of the distributive situations, the Shapley-Shubik index also seems to be an appropriate power measure (for example, Felsenthal and Machover, 1998). It evaluates a voter's expected relative share in a fixed budget. Apart from the Shapley-Shubik index, I introduce two other measures of power suitable to analyze distributive situations. One of them is the nucleolus, which is not very well known in this context. However, it is becoming more popular as it can be a good alternative to the Shapley-Shubik index[5]. Another power measure applied in my numerical analysis is derived as the vector of expected equilibrium payoffs from a well-known legislative bargaining game due to Baron and Ferejohn (1989). Interestingly, under some conditions it coincides with the nucleolus (Montero, 2006).

In the second part of the paper, I focus on the distributive situations. I emply the power measures to investigate an important issue, the design of an optimal decision rule in RBCs. In fact, the question of finding the optimal voting weights for the representatives of different countries has been already addressed in the literature, but mostly under the binary setting[6]. Under this assumption there are two approaches in the literature. The first one is the utilitarian approach[7] that seek to maximize the total utility of all citizens, and the second one is the egalitarian approach that seek to equalize the power of all citizens measured by the Banzhaf index. Felsenthal and Machover (1998), adopting the egalitarian approach, show that the optimal weights are such that each country's Banzhaf index is proportional to the square root of its population size (Penrose's rule, Penrose, 1946). By comparing the Bz and the square root of the population, they show that larger member states in the EU tend to have too little power and the smaller ones too much. Algaba et al. (2007) apply this theory to analyze the power of the European citizens for 25 and 27 countries. Le Breton, Montero and Zaporozhets (2012) also follow the egalitarian approach, however under the distributive setting with the nucleolus as the power measure. Following this direction I would like to investigate this question for the French RBCs applying three different power measures suitable for the distributive setting.

The paper is organized as follows. In Section 2 the organization and the functioning of WAs in France are briefly described. Section 3 provides a descriptive analysis of power for different groups participating in the decision-making process related to the water policy. In the first part I focus on the Banzhaf and the Shapley-Shubik index to analyze the binary

[5] Recent references include Le Breton et al. (2012), Montero (2005, 2013), Garcia-Valinas and Zaporozhets (2014).

[6] Le Breton, Montero and Zaporozhets (2012) is an exception.

[7] See for example, Barberà and Jackson (2006), Beisbart, Bovens and Hartmann (2005) and Beisbart and Hartmann (2010).

setting. As the analysis demonstrates, in general, the two indices give very close predictions. In the second part I consider distributive situations and compare the performance of the three power measures adapted to the analysis. Additionally, I characterize the conditions under which all three of them give the same result. In Section 4 I address the issue of the optimal design of the RBC. First, I tackle the question whether the three water users in the French RBCs are fairly represented. Then, I explain and apply a methodology of choosing the optimal decision rule. In Section 5 I analyze few cases where the Banzhaf and the Shapley-Shubik indices give significantly different results. Following Straffin (1977), I describe possible modifications of the classical indices which might be more applicable in this situation. Finally, Section 6 provides a summary of the main findings as well as some policy implications and possible extensions.

2 French River Basin Committees

The French WAs have been created in 1966, following the first Water Act of 1964, which institutionalized a decentralized water management system at the hydro-geographical level of the river basin. This system has been reinforced by the subsequent Water Acts of 1992[8] and 2006[9]. The six Water Agencies (Adour-Garonne, Artois-Picardie, Loire-Bretagne, Rhin-

[8]The Water Act of January 2, 1992 instituted the principle and the tools of integrated water management by the RB. The law also translates European directives into French national law. These new tools are the SDAGE (Schémas Directeurs d'Aménagement et de Gestion des Eaux) and the SAGE (Schémas d'Aménagement et de Gestion des Eaux). The SDAGE are designed by the RBCs, while the SAGE are designed at the sub-river basin level, in the framework of the Local Water Commission, which includes 50% elected persons, 25% users and 25% representatives of the State.

[9]The reform of 2006 was devoted to making the system compliant with the Constitution, by reinforcing the role of the RBC, while maintaining the control from the State. The goal of the reform was also to improve operational efficiency and provide enough flexibility in the determination of taxes.

In compliance with the article 34 of the Constitution, the law now sets the rules on tax bases and ceilings for the unit tax rates. The law also provides the main orientations for the multi-year intervention programs, sets the expected level of agencies' budget and leaves to the government the task of supervising the objectives in terms of expenses by major domain of intervention.

Meuse, Rhône-Méditerranée-Corse and Seine-Normandie) are public establishments of administrative nature under the supervision of two ministries: the Ministry of the Environment and the Ministry of Finance.

WAs participate at each river basin level in the national and the European water policies, by developing a strategy originating from an overall view of water issues. WAs contribute to reaching an adequate state of water bodies by reducing the impact of human activities, by preserving water resources and by satisfying user needs through a balance between water resources and rational water use.

Often presented as "Water Parliaments", RBCs participate in the design and adoption of the multi-year intervention programs, they determine the major priorities of the intervention policy of the Agencies, they vote on the tax basis and emission tax rates and the general conditions for attribution of subsidies to the water related projects[10]. The Executive Board first constructs and then submits a proposal to the RBC for approval. The decisions are taken by the majority rule, i.e., to pass a proposal requires approval by more than half of participants. In what follows I assume 100% participation, however some RBCs have explicit quorum requirement.

Each RBC has three colleges: local elected persons, water users (agriculture, industry, residential water users) and representatives of the State (administration). Each college elects among its members the administrators of the WA. The government determines the number of Basin Committee members, including the representation of each category of users (agriculture, tourism, industry, etc.). Representatives of the State from various Ministries as well as from the State prefectures are also included. For example, in the Adour-Garonne RBC in 2012 there were 135 members divided into three colleges: the first college of 54 members representing the local communities, the second college of 54 members representing users and professional bodies and the third college of 27 members representing the State and public

[10]There are several commissions within an RBC, which are delegated by the Executive Board of WA to work on important projects. For example, the Subsidy Commission makes recommendations on major subsidies to be granted to the water-related projects, and the Program Commission deliberates on the multi-year intervention programs.

boards. The first college is composed of representatives from the regions, the large municipalities and and the small municipalities (with a qualification for the municipalities located in either mountain areas or seaside areas). The second college has 9 representatives from agriculture, 27 representatives from the industry and 18 representatives from different associations (consumers, protection of the environment), regional *Social and Economic Councils* and groups of experts.

The composition of the RBCs depends on the geographical range of the basin with the minimum of 58 members in Rhin-Meuse RBC in $1993-1998$ and the maximum of 187 members in Seine-Normandie in $2005-2007$. The distribution of representatives in the six RBCs during the period $1987-2007$ is provided in the Appendix. One may see that the proportion of representatives for local communities, regions and districts is significant, compared to the representatives of water users. Representatives from the agricultural sector are typically more numerous in the RBCs characterized by a higher agricultural activity, as Adour-Garonne and Loire-Bretagne. Representatives of the State have the minority while the number of local elected persons is greater than $1/3$ on average, and representatives of users and socio-professional groups have the majority. In 1999, members of the RBCs and Executive Boards of all Agencies have been renewed with a better representation of urban and rural communities, consumer associations, environmental associations, agriculture and a new representative for small and medium industries. There are now about 40% of elected members for local communities, 40% for water user representatives, and 20% for representatives of the State.

3 Descriptive analysis of power

3.1 Binary Issues: The Banzhaf and the Shapley-Shubik power indices.

In this subsection we consider binary "yes"/"no" decisions by the committee. For simplicity, we assume that the amendments to the proposal are not possible. In this binary setting, *a* *priori* power of a voter is usually measured by the probability of the voter being pivotal. In this context, the two classical voting power measures, namely the Banzhaf and the Shapley-Shubik power indices, are the most used in the literature. I first, recall the formal definitions, and then provide the numerical results for the two indices applied to the six French RBCs in $1987 - 2007$.

In what follows, N denotes a set of the n members of an RBC. We also define a set of *winning coalitions* $\mathcal{W}$: a collection of subsets of N with the following properties:

1) $\emptyset \notin \mathcal{W}$;

2) $N \in \mathcal{W}$;

3) if $S \in \mathcal{W}$ and $T \supset S$ then $T \in \mathcal{W}$ (monotonicity).

The interpretation of the set $\mathcal{W}$ is the following. If S is a set of members voting in favour of a particular decision then the decision is accepted if $S \in \mathcal{W}$ and it fails if $S \notin \mathcal{W}$. Sets that do not belong to $\mathcal{W}$ are called *loosing coalitions.* A pair $(N, \mathcal{W})$ is called a *simple game*, and it fully describes an RBC.

The Shapley-Shubik index (SSI) for a simple game $(N, \mathcal{W})$ is defined as follows. The players vote in a specific order and as the majority is reached the proposal is accepted. The voter whose participation turns the existing coalition from a loosing into a winning one is called *critical* for that ordering. The critical voter is assumed to get the credit for having passed the bill. The SSI is then determined through the assumption of a random voting order:

$$\phi_i = \frac{\text{number of orderings in which } i \text{ is critical}}{\text{total number of orderings}}.$$

One may notice that $\sum_{i=1}^{n} \phi_i = 1$, i.e., the vector $\phi = (\phi_1, \phi_2, ..., \phi_n)$ is normalized.

For example, let us consider the following game with three voters. Player one has two votes, players two and three have one vote each, and the decision is passed if the total number of votes in favour is at least 3. Below we list the six possible orderings in which the players cast their votes:

$$1\underline{2}3,\ 1\underline{3}2,\ 2\underline{1}3,\ 23\underline{1},\ 3\underline{1}2, \text{ and } 32\underline{1}.$$

For each ordering, the critical player (the one who turns the set of his predecessors into a winning coalition) is underlined.

The Shapley-Shubik index ϕ_i of player i is the number of times that player i is underlined divided by six. We obtain:

$$\phi_1 = \frac{4}{6},\ \phi_2 = \phi_3 = \frac{1}{6}.$$

The Banzhaf index of a simple game $(N, \mathcal{W})$ is introduced in a different way. One defines a *swing* for any player i as a winning coalition S containing i and such that i's departure from S would change coalition S from winning to loosing. Let us define:

$$\beta_i' = \frac{\text{number of swings for } i}{2^{n-1}}.$$

Vector $\beta' = (\beta_1', \beta_2', ..., \beta_n')$ is called *absolute Banzhaf power* and, in fact, it is not normalized. The normalized version of this measure, the Banzhaf power index (BZ), is given by:

$$\beta_i = \frac{\beta_i'}{\sum_{i=1}^{n} \beta_i'}.$$

For the above example with three voters, the set of winning coalitions $\mathcal{W}$ consists of $\{1,2\}, \{1,3\}, \{1,2,3\}$. In the first two coalitions both players are critical, however, in the third one only player one is critical. Consequently:

$$\beta_1' = \frac{3}{4},\ \beta_2' = \beta_3' = \frac{1}{4} \text{ and } \beta_1 = \frac{3}{5}, \beta_2 = \beta_3 = \frac{1}{5}.$$

In what follows I provide the numerical results for both the BZ and the SSI for the six French RBC in $1987 - 2007$[11]. The voting situations in RBCs can be represented through *weighted majority* games. The game $(N, \mathcal{W})$ is said to be a *weighted majority game* if there exists an n-tuple $\omega = (\omega_1, ..., \omega_n)$ of non-negative weights with $\omega_1 + \omega_2 + ... + \omega_n = 1$ and a nonnegative quota q such that any $S \in \mathcal{W}$ if and only if the total weight of the players in S exceeds the quota q, i.e., $\sum_{i \in S} \omega_i \geq q$. The pair $[q; \omega]$ is called a *representation* of the game $(N, \mathcal{W})$. In order to run the calculations it is necessary to have information on the number of the representatives in the committees (provided in the Appendix) and the quota. Given that the decisions are taken by the majority rule, the quota is calculated as $q = \left\lfloor \frac{\sum_{i=1}^{n} q_i}{2} \right\rfloor$[12].

[11] I use the computer software for the voting power analysis which is available at http://homepages.warwick.ac.uk/~ecaae/.

[12] For any real number x, $\lfloor x \rfloor$ denotes the smallest integer greater than x.

Remark. It is important to note that operating with the voting weights instead of the set of winning coalitions may be confusing. The same game $(N, \mathcal{W})$ may admit several representations. For example, a majority game with three players has the following set of winning coalitions: $\{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$. At the same time, it can be equivalently represented as $[2;1,1,1]$ or $[51;49,49,2]$. The representation $[2;1,1,1]$ suggests equal voting weights for the three voters. However, in the representation $[51;49,49,2]$ the voting weight of the first voter 24.5 times as much as the voting weight of the third voter. As one may check, the Bz and the SSI are equal to 1/3 for every voter independently of the representation.

Tables 12-17 in the Appendix provide calculations for the distribution of power among different decision-makers in the six RBCs over $1987-2007$. The results indicate that the Bz and the SSI give very close predictions in most cases. Not surprisingly, districts have very high voting power, around 30%. Industrial users have around 15%. There are also many small groups with relatively low values of the Bz and the SSI.

In general, the voting power need not be proportional to voting weights[13]. However, the tables show that the power indices for the RBCs are relatively proportional to the weights. Here we deal with a situation where there is a large number of players and none of them is dominant enough. It seems that for this case Penrose‘s approximation works well (Lindner and Machover, 2004). It states that voting power tends to become proportional to voting weights as the number of voters increases, provided that the distribution of voting weights is not "too unequal". In fact, when the players are put into a smaller number of blocks, the power indices are not proportional to the weights anymore (see discussion in Section 4 and the table for Adour-Garonne RBC in the Appendix).

There are some intuitive properties that the Bz and the SSI satisfy. For example, both indices respect monotonicity: whenever representative i has more votes than representative j, then the voting power of i is higher than the power of j. However, the relation of voting

[13]A striking example is the EU Council of Ministers in 1958 - 1972. During that time it consisted of the representatives of 6 countries: Germany, Italy and France held 4 votes each, Belgium and Netherlands held 2 votes each and Luxembourg held 1 vote. In order to pass a proposal it was necessary to have at least 12 out of 17 votes in favour. Since other member states held an even number of votes, Luxembourg formally was never able to make any difference in the voting process.

weights with the power is not so straightforward. One may think that it is possible to decrease a voter's voting weight within a voting body and at the same time increase his/her power. It is called the paradox of redistribution. The idea is that there is a voter "donating" some weight to another one, so donor looses weight and recipient gains some while the total weight stays the same. The paradox of redistribution states that a donor can gain power or a recipient can loose power. This is the case for industry in Adour-Garonne RBC: in the period $1993 - 1998$ it had 12 votes which is the same as in the period $1999 - 2004$. However, the total number of votes has increased from 81 to 98, implying that the relative number of votes for the industry has dropped. However, the Bz shows an increase in power from 0.136 to 0.159 as well as the SSI indicates an increase from 0.182 to 0.186.

3.2 Distributive Issues

The analysis of distributive issues in RBCs such as taxes and subsidies has some specificities. There are only three groups, namely residential water users, industrial water users and the farmers contributing to the budget by paying different taxes and they may benefit from the redistribution by receiving subsidies to finance different projects. However, everybody in the committee including representatives of the administration participate in the decision-making process. The representatives of water users in the committee are assumed to be selfish, i.e., driven exclusively by their own shares in the proposal. In contrast, the preferences of the other committee members can possibly aggregate the welfare of the 3 categories of users.

Here, I impose a simplifying assumption that each representative of the administration acts on behalf of a single group of users. Formally, the water users are indexed by $j = 1, 2, 3$ and the representatives of the administration - by $k = 4, \ldots, n$. Let us denote by ω_j the voting weight (number of representatives) of sector j for all $j = 1, ..., 3$. All other voters have a weight equal to 1. We denote by M_j (respectively by m_j) the group (respectively the number) of representatives in the set $\{4, ..., n\}$ acting on behalf of user j. We have:

$$\sum_{j=1}^{3} m_j = n - 3.$$

The group of voters voting on behalf of the group $j = 1..3$ has a weight equal to:

$$q_j = \omega_j + m_j. \tag{1}$$

We have obtained a new weighted majority game with three players $[q; q_1, q_2, q_3]$, where the quota q is the same as before. In fact, for three groups there are very few possible games. Following Le Breton, Montero and Zaporozhets (2012), I consider five possible games[14], which can be described as weighted majority games. These are $[1; 1, 0, 0]$ in which player 1 is a dictator, $[3; 2, 1, 1]$ in which player 1 is a veto player[15] but not a dictator, $[2; 1, 1, 0]$ in which players 1 and 2 are veto players, $[3; 1, 1, 1]$ in which all three players are veto players, and $[2; 1, 1, 1]$ which is the simple majority game with no veto players.

Example: Adour-Garonne RBC, 1987-2007.

In this example I would like to group the decision-makers in Adour-Garonne RBC into three groups by attributing the votes of state representatives and other non-users to the three water user groups. To do so in a rigorous way, one would need the data on actual votes in the committee which are not available. Instead, I assume that the representatives of the rural communities, the ministries of agriculture, land development and rural affairs vote in line with the farmers. The representatives of the associations of residential water users, environmental organizations, fishery, water suppliers, tourism, ministry of health, environment and interior cast their votes on behalf of the residential water users. The representatives of the ministry of industry have their votes in line with the industrial users. Other RBC members who are not water users split their votes equally between the three groups. As the result, we get the following distribution of seats between the three user groups.

[14] The game is assumed to be monotonic (adding players cannot turn a winning coalition into a losing one), proper (no two disjoint coalitions can be winning), directed (players can be unambiguously ranked in order of desirability with player 1 being at least as desirable as player 2, who is at least as desirable as player 3), $N \in \mathcal{W}$ and $\emptyset \notin \mathcal{W}$.

[15] A player that belongs to all minimal winning coalitions is called a veto player.

Table 1 Distribution of representatives in Adour - Garonne RBC, 1987 - 2007.

	$1987-1992$	$1993-1998$	$1999-2004$	$2005-2007$
Agriculture	22.67	21.33	25.67	24
Industry&energy	27.67	27.33	28.67	29
Residential	33.67	32.33	43.67	44
Total	84	81	98	97
Quota	43	41	50	49

One may check that for all four periods such distribution of seats and the quota correspond to the simple majority game $[2; 1, 1, 1]$.

The most famous power measure applied in the distributive situations is the SSI[16]. Thus, Felsenthal and Machover (1998) argue that the SSI is a measure of "P-power": it evaluates a voter's expected relative share of a fixed budget, which a winning coalition can obtain. In addition, we apply two alternative power measures, which can be derived as vectors of equilibrium payoffs of positive models of politics.

3.2.1 The Nucleolus

The nucleolus is a solution concept for cooperative games, which was first formulated by Shmeidler (1969). As it is argued in Montero (2006), Le Breton et al. (2012) and Garcia-Valinas and Zaporozhets (2014), the nucleolus is a suitable measure to analyze bargaining over a fixed budget and it can be a good alternative to the SSI. Mashler, Peleg and Shapley (1979) provide the following intuitive meaning of the nucleolus. Suppose there is an arbitrator, who helps the players to decide on the allocation of the common budget. The arbitrator may take the excess of a coalition (the gain/loss that the members of the coalition have if they depart from it) as a measure of dissatisfaction and he may try to decrease it. The coalitions with the negative excess do not want to defect, and the higher the excess of the coalition is, the higher is the coalition dissatisfaction. So, the arbitrator will look

[16] For the definitions and the properties see, for example, Felsenthal and Machover (1998) and Laruelle and Valenciano (2008).

for the payoffs in which the highest excess is as low as possible. If there are several such payoffs, he will proceed in recursive manner: he chooses the outcomes for which the second highest excess is minimal and so on. The formal definition of the nucleolus is provided in the Appendix.

3.2.2 Bargaining and Power

In this section, we describe the power of the players as the expected equilibrium payoffs from a popular legislative bargaining game introduced by Baron and Ferejohn (1989) adapted for our specific setting. As was pointed out before, the main specificity here is that all the decision-makers in RBC participate in the decision on the distribution of the surplus, however, only three water user groups benefit from the final distribution.

The bargaining proceeds as follows. At every round $t = 1, 2, \ldots$ Nature selects a random proposer: player i is selected with probability p_i with $i = 1, .., n$. This player proposes a distribution of the budget (x_1, x_2, x_3) with $x_j \geq 0$ for all $j = 1, 2, 3$ and $\sum_{j=1}^{3} x_i = 1$. Due to our assumption on the behavior of the representatives from the administration, one may say that the probability of player $j = 1, 2, 3$ of being selected as a proposer is equal to $\widehat{p}_j = p_j + \sum_{k \in M_j} p_k$. The proposal is voted upon immediately according to the voting rule represented by a voting game $[q; q_1, q_2, q_3]$ derived above. If the coalition of voters in favor of the proposal is winning, the proposal is implemented and the game ends; otherwise the game proceeds to the next period in which Nature selects a new proposer. Players are risk neutral and discount future payoffs by a factor $\delta_i \in [0, 1)$. A (pure) strategy for player i is a sequence $\sigma_i = (\sigma_i^t)_{t=1}^{\infty}$, where σ_i^t, the tth round strategy of player i, prescribes:

1. A proposal x.

2. A response function assigning "yes" or "no" to all possible proposals by the other players.

The solution concept is *stationary subgame perfect equilibrium* (SSPE). Stationarity requires that players follow the same strategy at every round t regardless of past offers and responses to past offers. Banks and Duggan (2000) have shown that an SSPE always exists[17]

[17] The existence result is provided by Banks and Duggan (2000) in a very general setting in which the space

in this type of bargaining model. In addition, Eraslan (2002) and Eraslan and McLennan (2013) have shown that all SSPE lead to the same expected equilibrium payoffs.

In the case where $\delta_i \to 1$ for all $i = 1, ..., n$, we denote by BF$(N, \mathcal{W})$ the unique vector of equilibrium payoffs attached to the SSPE of the bargaining game[18]. Hereafter, we refer to this vector as the Baron-Ferejohn measure of power attached to the simple game $(N, \mathcal{W})$.

Montero (2006) has analyzed the above bargaining game in the case where $\delta_i = \delta < 1$ for all $i = 1, ..., n$. She shows that if the vector p coincides with the nucleolus, then p is the unique vector of equilibrium payoffs. In her terminology, the nucleolus is a *self-confirming* measure of power.

Table 2 presents the values for the SSI, the nucleolus and the BF for the case of equal recognition probabilities, i.e., $p_j = 1/3$ for $j = 1...3$. Interestingly, the Nucl and the BF give the same predictions for all 5 games. One may notice that in the presence of veto players the two indices, the Nucl and the BF, attribute all power to the veto players and leave the other players with no power. Moreover, the power is equally shared between the veto players if there are several of them. The Nucl and the BF disagree with the SSI only in the case of the veto game $[3; 2, 1, 1]$.

of outcomes can be any convex compact set and the utility functions are concave but otherwise unrestricted.

[18] The BF in the case of $p_i = 1/n$ for all $i = 1...n$ is applied to study the distribution of power among different countries in the EU Council of Ministers in Montero (2007) and Le Breton et al. (2012).

Table 2 Power Values for 5 possible games with 3 groups.

	SSI	**Nucl**	**BF**
$[1;1,0,0]$ dictatorial	$(1,0,0)$	$(1,0,0)$	$(1,0,0)$
$[3;2,1,1]$ veto	$\left(\frac{2}{3},\frac{1}{6},\frac{1}{6}\right)$	$(1,0,0)$	$(1,0,0)$
$[2;1,1,0]$ veto 12	$\left(\frac{1}{2},\frac{1}{2},0\right)$	$\left(\frac{1}{2},\frac{1}{2},0\right)$	$\left(\frac{1}{2},\frac{1}{2},0\right)$
$[3;1,1,1]$ unanimity	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$
$[2;1,1,1]$ majority	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$	$\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

One may notice that the example for Adour-Garonne RBC corresponds to the last row in the table, i.e., the three indices suggest equal distribution of power for the three water user groups.

I would like to characterize the set of possible weights q_1, q_2, q_3 for which the three indices disagree. First, the sets M_i are not known exactly , however, due to the equality (1) the following inequalities should be satisfied:

$$q_i \geq \omega_i \text{ for } i = 1...3.$$

The total number of representatives should stay the same:

$$q_1 + q_2 + q_3 = \sum_{i=1}^{n} q_i.$$

The disagreement appears in the situation with one vetoer. Suppose, it is player 1. Then, we get additional set of inequalities:

$$\begin{aligned} q_1 + q_2 &\geq q \\ q_1 + q_3 &\geq q \\ q_2 + q_3 &< q. \end{aligned}$$

Similar inequalities hold if player 2 or 3 is a vetoer. In the picture below the set of possible weights for which the three indices give different results is drawn.

[Figure is about here]

4 The optimal institutional design

In the previous section we have calculated the power of each representative in the RBC measured by different power measures both for binary and the distributive settings. In this section we would like to address an important issue of the optimal voting rule applying these power measures.

Fairness suggests to allocate the gains equally across all water users. This means that each water user group in the RBC should receive a share proportional to its population size. If there were no intermediate voting bodies, i.e., if the simple game was the majority game with the set of the water users as the set of voters, then all the coordinates of the power index would be equal and proportionality would be fulfilled. Unfortunately, we are in a second best situation: the negotiation takes place across the representatives and the share obtained by the representatives in the RBC is divided among the corresponding water users. We need to evaluate the users' indirect power via their representatives.

Le Breton, Montero and Zaporozhets (2012) have examined this question for the EU citizens under the distributive setting with the Nucl as the power measure. In this section I extend their analysis by applying three power indices appropriate for this situation: the SSI, the Nucl and the BF in the context of RBCs. In what follows "power index" means one of the three power indices if it is not indicated otherwise, and we denote it by $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. The three components of vector α correspond to the shares of the three water users (agricultural, industrial and residential).

The optimization variable here is the simple game $(M, \mathcal{W})$[19], where $M = \{1, 2, 3\}$ is the set of water users' groups, and $\mathcal{W}$ is the set of the winning coalitions. We assume that each group of representatives in the RBC receives a share of the budget equal to the power index

[19] See also Remark 1.

of the simple voting game. Then, each group's payoff is divided equally among corresponding water users.

If we would like to equalize the users' power, we need to choose a voting rule which leads to the power index for the representatives being equal to the water users' population sizes. However, except some very specific cases, it is not possible to find a game for which the vector of users' population sizes coincides with the power index. We will try to find simple games whose power index is as close as possible to the population shares.

We would like to design the simple game $(M, \mathcal{W})$ in such a way that the distance between the induced power index calculated at the water user level and the first best is the smallest possible. Following Le Breton, Montero and Zaporozhets (2012) we consider the quadratic distance. Let us denote by n_i the population size of group $i = 1, 2, 3$ and $n = n_1 + n_2 + n_3$ is the total population in the river basin. Denoting by $\mathcal{S}$ the set of all simple games with 3 players, our problem is defined as follows:

$$\min_{(M,\mathcal{W})\in\mathcal{S}} var\left(\alpha(M, \mathcal{W})\right),$$

and

$$var\left(\alpha(M, \mathcal{W})\right) = \sum_{i=1}^{3} n_i \left[\frac{1}{n} - \frac{\alpha_i}{n_i}\right]^2. \tag{2}$$

The term $\frac{\alpha_i}{n_i}$ indicates how much power (according to any of the three power indices) a water user in group i gets given a specific voting rule. The expression (2) can be simplified into:

$$var\left(\alpha(M, \mathcal{W})\right) = \sum_{i=1}^{3} \frac{(\alpha_i)^2}{n_i} - \frac{1}{n}. \tag{3}$$

The general problem, known as an inverse problem, is to characterize which vectors can be obtained as a power vector for an adequate choice of a simple game, has been formulated recently by Alon and Edelman (2010) for the Bz. They analyze power distributions with most of the power concentrated on the small number of voters. They provide explicit bounds stating that a Bz vector with weights concentrated on $k < n$ players has to be near the Bz vector of a game with $n - k$ dummy players (which is equivalent to the game with k players). Kurz (2014) tightens this bound and obtains similar bounds for several other power indices introduced in the literature. To the best of my knowledge, there are no general results on the inverse problem for the Nucl, the SSI or BF.

Following Le Breton, Montero and Zaporozhets (2012), I illustrate the procedure for solving the problem in the case of 3 voters:

1. Following Subsection 3.2 consider 5 possible games in a given class;
2. Calculate the power index α for each game in the list and find the variance using (3);
3. Choose the game with the minimal variance.

Hereafter we are going to express variance in terms of population shares, $\gamma_i = \frac{n_i}{n}, i = 1, 2, 3$. Without loss of generality, we assume that $\gamma_1 \geq \gamma_2 \geq \gamma_3$. We denote by:

$$\begin{aligned} V_1 &= \frac{1}{\gamma_1} - 1, \\ V_2 &= \frac{1}{4}\left[\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right] - 1, \\ V_3 &= \frac{1}{9}\left[\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3}\right] - 1, \\ V_4 &= \frac{1}{36}\left[\frac{16}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3}\right] - 1. \end{aligned}$$

The results for three power indices are presented in the Table 3 below.

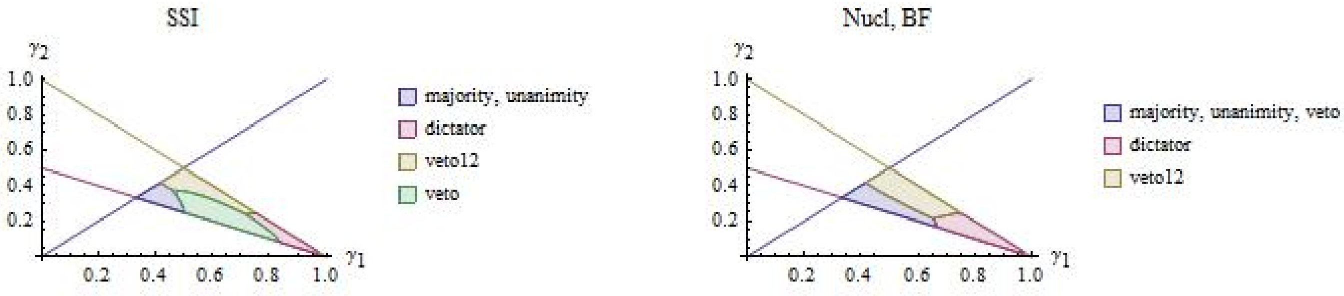
Table 3 Variances for 5 possible games with 3 groups.

	SSI	**Nucl and BF**
$[1; 1, 0, 0]$	$\frac{V_1}{n}$	$\frac{V_1}{n}$
$[3; 2, 1, 1]$	$\frac{V_4}{n}$	$\frac{V_1}{n}$
$[2; 1, 1, 0]$	$\frac{V_2}{n}$	$\frac{V_2}{n}$
$[3; 1, 1, 1]$	$\frac{V_3}{n}$	$\frac{V_3}{n}$
$[2; 1, 1, 1]$	$\frac{V_3}{n}$	$\frac{V_3}{n}$

On the following figure we show the values of the two biggest groups' population shares, γ_1 and γ_2, for which each of the five games is optimal under the SSI, the Nucl and the BF.

One may notice that the majority ($[2; 1, 1, 1]$) and the unanimity ($[3; 1, 1, 1]$) rules can not be distinguished under the three power indices. Not surprisingly, the majority rule is

Figure 1 The optimal rule for three voters under the SSI, the Nucl and the BF.



optimal when the three groups are not too different in terms of their population shares. On the contrary, the dictatorial rule ($[1; 1, 0, 0]$) is optimal in the case where there is a relatively big group. Under the SSI there are four different regions, while under the Nucl and the BF there are only three ones. The Nucl and the BF cannot distinguish between veto ($[3; 2, 1, 1]$), dictatorial and unanimity rules. Under the SSI veto rule is distinguished from other rules.

The following example demonstrates the use of the technique.

Example: Adour-Garonne RBC, 1987-2007 (revisited).

From the calculations in the previous section we conclude that under the assumptions made the decision rule operating in Adour-Garonne RBC is the simple majority rule. It implies that the three groups have equal voting power according to the SSI, the Nucl and the BF. Hereafter, we are going to check whether this rule is optimal, i.e., whether the three water user groups are fairly represented in the committee. In order to do so, we should check whether the sizes of the three water users groups in Adour-Garonne river basins are equal.

In what follows, the data on the employment in industry and agriculture as well as the total employment in the Adour-Garonne river basin in $1989 - 2006$ is used as proxy for the number of different water users. In the first scenario the number of industrial and agricultural water users is approximated by the number of employed people in industry and agriculture respectively. The number of residential water users is approximated by the difference between the total employment and the number of people employed in both industry and agriculture. One difficulty with this assumption is that people working in industry and agriculture are also residential water users. Therefore, in the second scenario, the number of industrial and agricultural water users is approximated by half of the number of employed people in industry and agriculture respectively. The number of residential users is calculated as the difference between the total employment and half of people employed both in industry and agriculture. Of course, one may argue, that a more precise measure should also take into account the number of unemployed and retired people. I will explain below how to proceed in this situation.

Table 18 in the Appendix presents the population shares for the three water user groups in Adour-Garonne river basin in $1989-2006$ under two alternative assumptions. It indicates that the shares for industrial and agricultural users were declining over time whereas the share for residential users was increasing. Under both scenarios the share for residential water users is always the biggest and the share for agriculture is the smallest. Moreover, under the second scenario, the gap in shares between the residential water users and the two other groups increases. Thus, the shares for the three groups are far from being equal, therefore we may conclude that under our specific assumptions, the residential water users are under represented and the two other groups are over represented.

Following the procedure described above we are going to identify the optimal decision rule for both scenarios. To do so, we calculate variances according to the formulas from Table 3. In our case γ_1,γ_2 or γ_3 corresponds to the populational shares for the residential, industrial or agricultural users respectively. As we have seen, the assumption $\gamma_1 \geq \gamma_2 \geq \gamma_3$ is satisfied.

In Table 4 below the four variances are calculated for each scenario.

Table 4 Values of variance calculated for Adour - Garonne river basin, 1989 - 2006.

	first scenario				second scenario			
	V_1	V_2	V_3	V_4	V_1	V_2	V_3	V_4
1989	0.394	0.767	0.829	0.038	0.165	2.128	2.477	0.355
1990	0.385	0.771	0.869	0.044	0.162	2.140	2.559	0.374
1991	0.371	0.803	0.919	0.051	0.156	2.210	2.661	0.397
1992	0.359	0.838	0.959	0.056	0.152	2.286	2.744	0.416
1993	0.343	0.883	1.024	0.065	0.146	2.382	2.877	0.447
1994	0.331	0.911	1.081	0.075	0.142	2.443	2.993	0.474
1995	0.323	0.916	1.136	0.085	0.139	2.456	3.104	0.501
1996	0.318	0.932	1.165	0.090	0.137	2.489	3.164	0.515
1997	0.308	0.976	1.207	0.097	0.133	2.581	3.250	0.535
1998	0.299	0.991	1.274	0.110	0.130	2.615	3.385	0.567
1999	0.288	1.013	1.360	0.127	0.126	2.663	3.559	0.609
2000	0.287	0.988	1.407	0.138	0.125	2.615	3.654	0.632
2001	0.281	0.997	1.470	0.151	0.123	2.634	3.780	0.663
2002	0.272	1.041	1.530	0.162	0.120	2.726	3.902	0.692
2003	0.265	1.078	1.575	0.171	0.117	2.803	3.993	0.714
2004	0.260	1.111	1.600	0.175	0.115	2.872	4.043	0.725
2005	0.246	1.162	1.738	0.204	0.110	2.978	4.323	0.793
2006	0.237	1.199	1.845	0.227	0.106	3.055	4.538	0.845

Under the first scenario the SSI suggests that the veto rule $[3; 2, 1, 1]$ is optimal. The Nucl and the BF cannot distinguish between the two rules, veto rule $[3; 2, 1, 1]$ and the dictatorial rule $[1; 1, 0, 0]$. Under the second scenario the three power indices agree that the dictatorial rule is optimal. In any case, the power indices suggest to reinforce the representation of the residential water users.

If we add the number of unemployed and retired people to the residential water user group, the gap in shares becomes even bigger than in the second scenario as compared to the first one. Therefore, we would expect the dictatorial rule to be optimal in such a case.

In what follows I do the same exercise but take the value added produced by each of the three groups instead of taking groups' population sizes.

Table 5 Values of variance calculated for Adour - Garonne river basin, 1989 - 2006.

	V_1	V_2	V_3	V_4
1990	0.332	0.829	1.164	0.096
1991	0.314	0.845	1.321	0.128
1992	0.309	0.864	1.347	0.132
1993	0.287	0.945	1.478	0.156
1994	0.290	0.990	1.367	0.129
1995	0.307	0.907	1.296	0.119
1996	0.293	0.956	1.383	0.134
1997	0.299	0.983	1.285	0.112
1998	0.296	0.940	1.374	0.133
1999	0.283	0.986	1.465	0.151
2000	0.265	0.990	1.735	0.211
2001	0.278	0.927	1.646	0.194
2002	0.263	0.964	1.838	0.236
2003	0.237	1.068	2.169	0.308
2004	0.230	1.143	2.133	0.296
2005	0.223	1.125	2.425	0.366
2006	0.214	1.184	2.552	0.394
2007	0.212	1.240	2.429	0.362

As before, the Nucl an the BF suggest the two rules, veto rule $[3; 2, 1, 1]$ and the dictatorial rule $[1; 1, 0, 0]$. According to the SSI the veto rule $[3; 2, 1, 1]$ should be optimal before 2003, and the dictatorial rule - between 2003 and 2007.

5 Extensions

In this section I discuss possible modifications of the Bz and the SSI which might be more adapted to particular situations than either classical index. Straffin (1977) has proved that the two indices can be derived from the same basic probabilistic model under different assumptions about voting behavior. The Bz is obtained under the assumption that the voters vote completely independently (independence assumption). In contrast, the SSI is obtained under the assumption that the voters have some common standards or values (homogeneity assumption).

Formally, let us denote by $p = (p_1, ..., p_n)$, the vector with the components $p_i \in [0, 1]$ being the probability that player i votes "yes" on the given proposal. For different proposals, the components p_i are selected from some probability distribution on $[0, 1]$. The *Independence Assumption* states that the p_i's are selected independently from the uniform distribution on $[0, 1]$. This assumption implies that each player will vote in favor of any decision with the probability 1/2. The *Homogeneity Assumption* states that a number p is selected from the uniform distribution on $[0, 1]$, and $p_i = p$ for all i. Straffin (1977) proves that the answer to the question "What is the probability that the bill supported by player i pass?" is given by absolute Banzhaf measure β_i' under the independence assumption, and it is given by the SSI under the homogeneity assumption.

One may argue that the assumption of the uniform distribution is restrictive. Thus, Le Breton, Lepelley and Smaoui (2014) propose to introduce a general probability density function f defined on the interval $[0, 1]$. Then, the modified homogeneity assumption states that a number p is drawn according to the distribution f on $[0, 1]$, and $p_i = p$ for all i. In the case of beta distribution:

$$f(p) = \frac{\Gamma(2\alpha)}{(\Gamma(\alpha))^2} p^\alpha (1-p)^\alpha,$$

where parameter $\alpha > 0$ and Γ is gamma function. The case $\alpha = 1$ represents the uniform distribution. One may calculate:

$$\Pr(x_i = 1 \text{ and } x_j = 1) = \int_0^1 p^2 f(p) dp = \frac{1}{4} + \frac{1}{4(2\alpha + 1)}.$$

Then the covariance is calculated as:

$$cov(x_i, x_j) = \frac{1}{4(2\alpha + 1)},$$

and the correlation coefficient between the votes of two voters is given by:

$$\rho = \frac{1}{2\alpha + 1}.$$

When $\alpha \to 0$ one gets $\rho \to 1$ (perfect positive correlation) and when $\alpha \to \infty$ one gets $\rho \to 0$ (independence).

When $\rho = 1$ we get the block model (Felsenthal and Machover, 1998), in which some representatives always vote in the same way (form blocks). Below, I show an example of gathering different representatives into blocks according to the similarity of their preferences[20]. As a result, there are 11 groups:

1. Farmers (including representatives of the Ministries of Agriculture, Land Development and Rural Affairs);

2. Industry (including energy);

3. Urban communities (residential water users);

4. Rural communities;

5. Environmental associations (including fishery, water suppliers, tourism, Ministries of Health, Environment, the Interior, associations of residential water users);

6. Other communities ;

7. Districts and regions ;

8. Ministry of Industry ;

9. Professional bodies;

10. Other ministries;

11. State prefectures.

The first three categories correspond to water users (those paying emission and water use taxes, and receiving subsidies from the WA), while other categories are special-interest

[20] For a more accurate analysis it is neccessary to have data on the actual votes of the different representatives in the six RBCs, which is not available.

groups, ministries and administration. As one may notice, group 4, the rural communities, is separated from the residential water users, since often these people are also farmers. Their votes can be in line with the votes of farmers or of the residential water users depending on the situation. The numerical results for the Adour-Garonne RBC are provided in the Appendix.

One may think that when assigning some representatives in the same block, their total power should be at least as great as the sum of the power assigned to these representatives in the original setting before formation of the block. It seems also natural to expect that if some voters form a block, this should not increase the relative power of any rival voter. While it may seem intuitive, in general this property does not hold. It is called the paradox of large size or superadditivity property[21].

6 Concluding Remarks

In this paper I apply different power measures to analyze the distribution of power for different decision-makers in the six French RBCs over the period 1987-2007. In order to analyze binary voting situations I apply traditional power indices such as the Bz and the SSI. One of the main messages of this analysis is that the relation between the relative number of votes and the voting power is not straightforward. While the higher number of votes implies the higher power the relation is not linear. In general one should be careful operating with the weights instead of the power measures, as doubling the votes, for example, does not necessarily lead to the doubling the voting power. Additionally, as the analysis of the Section 5 demonstrates, when some decision-makers act as a block one should be careful calculating the voting power of the block as it is not necessarily a sum of the individual powers.

I also show how to proceed with the analysis of the distributive voting situations such as funding local projects through subsidies. I apply the SSI which is well known in this context as well as two other power measures, the nucleolus and the "Baron-Ferejohn" measure, which

[21] See Felthenthal and Machover (1998) for the details.

are less known but suitable in this context. The main conclusion of this exercise is that very often these indices give the same predictions.

There are some insights on the optimal design of an RBC presented in the Section 4. It is important issue since all water users should be equally represented in the RBCs. I provide a descriptive analysis as well as a simple computation exercise for Adour-Garonne river basin which demonstrate the use of the technique. The main conclusion of the exercise is that up to now the residential water users were under represented. Of course, more rigorous analysis is necessary to make assumptions on the voting behavior of the RBC members.

In what follows I would like to explore directions for extending the current research. The power indices applied in this paper are based on a set of assumptions concerning the functioning of the RBCs and the preferences of their members. When considering the distributive voting situations we have assumed that the representatives of the administration act on behalf of a particular water users' group. It is not straightforward to collect direct evidence supporting that assumption. A careful examination of the proceedings reproducing the synthesis of the debates within the committee is a first step in that direction. The same remark is valid for the block models.

In this paper we have considered the conflict between the different decision-makers. While there is a clear evidence supporting such assumption, we could have instead privileged the geographic dimension of the conflict. Indeed, each WA is in charge of the various sub-river basins within the broad hydrographic river basin, with local delegations for each. For example, the Adour-Garonne WA has five such delegations with permanent staff dedicated to local water management issues. Instead of having a dispute among users, we could analyze a dispute among territories. In fact, the balanced composition of the committee in terms of geographic areas may suggest that this characteristic is important. Nevertheless, we would have also to make some assumptions on the preferences of those who are not affiliated to a specific area as the members of the third college. We do not know if there are (as we observe for users) cross subsidies across the territories.

7 Acknowledgements

I am grateful to Alban Thomas for sharing the data for the six river basin committees in France in 1987-2007. I have also benefitted from the discussions with Stefan Ambec, Michel Le Breton, María García-Valinas, François Salanié and Alban Thomas on the earlier drafts.

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8 Appendix

8.1 Distribution of representatives in six RBCs

Table 6 Distribution of Representatives in Adour - Garonne RBC, 1987 - 2007.

	1987 − 1992	1993 − 1998	1999 − 2004	2005 − 2007
Agriculture	5	5	7	7
Industry&energy	12	12	12	13
Urban communities	1	1	2	2
Rural communities	1	1	1	1
Coastal communities	0	0	0	0
Other communities	4	4	8	8
Region	6	6	5	6
District	18	18	18	18
Inter-district	0	0	2	3
Fishery & fish industry	3	3	4	4
Tourism	2	2	2	2
Water supply	3	3	2	2
Residential water users	1	1	3	4
Ecologists	2	2	4	4
Professional bodies	8	8	9	8
Ministry of Environment	1	1	1	1
Ministry of land devt&rural aff	1	1	1	1
Ministry of health	1	1	1	1
Ministry of the Interior	1	0	1	1
Ministry of Industry	1	1	1	1
Ministry of agri	1	0	1	0
Other ministries	6	5	7	4
State prefectures	6	6	6	6
Total	84	81	98	97

Table 7 Distribution of Representatives in Artois - Picardie RBC, 1987 - 2007.

	1987 − 1992	1993 − 1998	1999 − 2004	2005 − 2007
Agriculture	3	3	4	4
Industry&energy	12	12	13	12
Urban communities	1	1	2	2
Rural communities	1	1	1	1
Coastal communities	0	0	0	0
Other communities	3	3	5	5
Region	3	3	3	3
District	17	17	17	17
Inter-district	1	1	1	1
Fishery & fish industry	3	3	2	3
Tourism	1	1	1	1
Water supply	1	2	1	1
Residential water users	2	2	1	2
Ecologists	2	2	3	3
Professional bodies	4	4	5	5
Ministry of Environment	1	1	1	1
Ministry of land devt&rural aff	1	1	1	1
Ministry of health	1	1	1	1
Ministry of the Interior	1	0	1	1
Ministry of Industry	1	1	1	1
Ministry of agri	1	0	1	1
Other ministries	6	5	7	7
State prefectures	2	2	1	2
Total	68	66	73	75

Table 8 Distribution of Representatives in Loire - Bretagne RBC, 1987 - 2007.

	1987 − 1992	1993 − 1998	1999 − 2004	2005 − 2007
Agriculture	7	7	8	7
Industry&energy	17	17	18	18
Urban communities	2	2	2	4
Rural communities	1	1	1	1
Coastal communities	0	0	0	0
Other communities	3	1	7	7
Region	8	8	8	8
District	28	28	28	28
Inter-district	0	2	3	2
Fishery & fish industry	5	5	6	6
Tourism	2	2	3	3
Water supply	2	2	2	1
Residential water users	2	2	4	4
Ecologists	3	3	5	5
Professional bodies	12	11	11	12
Ministry of Environment	1	1	1	1
Ministry of land devt&rural aff	1	1	1	1
Ministry of health	1	1	1	1
Ministry of the Interior	1	0	1	1
Ministry of Industry	1	1	1	1
Ministry of agri	1	0	1	1
Other ministries	6	5	7	5
State prefectures	10	10	10	9
Total	114	110	129	126

Table 9 Distribution of Representatives in Rhin-Meuse RBC, 1987 - 2007.

	1987 − 1992	1993 − 1998	1999 − 2004	2005 − 2007
Agriculture	1	1	2	2
Industry&energy	11	11	12	12
Urban communities	2	1	2	3
Rural communities	1	1	1	1
Coastal communities	0	0	0	0
Other communities	2	2	3	3
Region	3	3	3	3
District	14	14	15	15
Inter-district	0	1	1	4
Fishery & fish industry	3	3	3	3
Tourism	1	1	1	1
Water supply	1	1	1	1
Residential water users	1	1	2	2
Ecologists	2	2	3	3
Professional bodies	5	5	5	5
Ministry of Environment	1	1	1	1
Ministry of land devt&rural aff	1	1	1	1
Ministry of health	1	1	1	1
Ministry of the Interior	1	0	1	1
Ministry of Industry	1	1	1	1
Ministry of agri	1	0	1	1
Other ministries	5	4	6	4
State prefectures	3	3	3	3
Total	61	58	69	71

Table 10 Distribution of Representatives in Rhône - Méditerranée - Corse RBC, 1987 - 2007.

	$1987-1992$	$1993-1998$	$1999-2004$	$2005-2007$
Agriculture	5	5	6	6
Industry	18	18	19	19
Urban communities	2	1	4	4
Rural communities	1	1	1	1
Coastal communities	0	0	0	0
Other communities	3	3	8	11
Region	5	6	6	5
District	28	28	28	26
Inter-district	0	1	1	1
Fishery	5	5	6	6
Tourism	3	3	3	3
Water supply	1	3	3	3
Residential water users	1	1	4	4
Ecologists	3	3	5	5
Professional bodies	8	8	8	8
Ministry of Environment	1	1	1	1
Ministry of land devt	1	1	1	1
Ministry of health	1	1	1	1
Ministry of the Interior	1	0	1	1
Ministry of Industry	1	1	1	1
Ministry of agri	1	0	1	1
Other ministries	7	6	8	6
State prefectures	8	8	8	7
Total	104	104	124	121

Table 11 Distribution of Representatives in Seine-Normandie RBC, 1987 - 2007.

	1987 – 1992	1993 – 1998	1999 – 2004	2005 – 2007
Agriculture	4	4	5	7
Industry&energy	17	17	18	27
Urban communities	1	1	4	12
Rural communities	2	1	1	3
Coastal communities	0	0	0	2
Other communities	3	3	7	21
Region	7	7	7	7
District	25	25	25	25
Inter-district	0	1	1	4
Fishery & fish industry	5	5	5	8
Tourism	2	2	2	3
Water supply	2	2	2	3
Residential water users	2	2	4	6
Ecologists	3	3	6	9
Professional bodies	10	10	10	11
Ministry of Environment	1	1	1	7
Ministry of land devt&rural aff	1	1	1	1
Ministry of health	1	1	1	2
Ministry of the Interior	1	1	1	2
Ministry of Industry	1	1	1	2
Ministry of agri	1	0	1	2
Other ministries	6	5	7	15
State prefectures	8	8	8	8
Total	103	101	118	187

8.2 Power Values for six RBCs

Table 12 Power Values for Adour - Garonne RBC, 1987 - 2007.

	1987 − 1992			1993 − 1998			1999 − 2004			2005 − 2007		
	q_i/n	**Bz**	**SSI**	**q_i/n**	**Bz**	**SSI**	**q_i/n**	**Bz**	**SSI**	**q_i/n**	**Bz**	**SSI**
Agriculture	0.060	0.057	0.056	0.062	0.059	0.058	0.071	0.069	0.069	0.072	0.070	0.070
Industry&energy	0.143	0.135	0.146	0.148	0.139	0.151	0.122	0.121	0.126	0.134	0.133	0.140
Urban communities	0.012	0.011	0.011	0.012	0.012	0.011	0.020	0.019	0.019	0.021	0.020	0.019
Rural communities	0.012	0.011	0.011	0.012	0.012	0.011	0.010	0.010	0.009	0.010	0.010	0.009
Coastal communities	0	0	0	0	0	0	0	0	0	0	0	0
Other communities	0.048	0.045	0.045	0.049	0.047	0.046	0.082	0.079	0.080	0.082	0.080	0.080
Region	0.071	0.068	0.068	0.074	0.071	0.070	0.051	0.049	0.049	0.062	0.060	0.059
District	0.214	0.253	0.248	0.222	0.262	0.258	0.184	0.210	0.210	0.186	0.211	0.212
Inter-district	0	0	0	0	0	0	0.020	0.019	0.019	0.031	0.030	0.029
Fishery & fish industry	0.036	0.034	0.033	0.037	0.035	0.034	0.041	0.039	0.039	0.041	0.040	0.039
Tourism	0.024	0.022	0.022	0.025	0.023	0.022	0.020	0.019	0.019	0.021	0.020	0.019
Water supply industry	0.036	0.034	0.033	0.037	0.035	0.034	0.020	0.019	0.019	0.021	0.020	0.019
Residential water users	0.012	0.011	0.011	0.012	0.012	0.012	0.031	0.029	0.029	0.041	0.040	0.039
Ecologists	0.024	0.022	0.022	0.025	0.023	0.022	0.041	0.039	0.039	0.041	0.040	0.039
Professional bodies	0.095	0.093	0.093	0.099	0.096	0.110	0.092	0.090	0.091	0.082	0.080	0.081
Ministry of Environment	0.012	0.011	0.011	0.012	0.012	0.011	0.010	0.010	0.009	0.010	0.010	0.009
Ministry of land devt&rural aff	0.012	0.011	0.011	0.012	0.012	0.011	0.010	0.010	0.009	0.010	0.010	0.009
Ministry of health	0.012	0.011	0.011	0.012	0.012	0.011	0.010	0.010	0.009	0.010	0.010	0.009
Ministry of the Interior	0.012	0.011	0.011	0	0	0	0.010	0.010	0.009	0.010	0.010	0.009
Ministry of Industry	0.012	0.011	0.011	0.012	0.012	0.011	0.010	0.010	0.009	0.010	0.010	0.009
Ministry of agri	0.012	0.011	0.011	0	0	0	0.010	0.010	0.009	0	0	0
Other ministries	0.071	0.068	0.068	0.062	0.059	0.058	0.071	0.069	0.069	0.041	0.040	0.039
State prefectures	0.071	0.068	0.068	0.074	0.071	0.070	0.061	0.059	0.059	0.062	0.060	0.059

Table 13 Power Values for Artois - Picardie RBC, 1987 - 2007.

	1987 − 1992			1993 − 1998			1999 − 2004			2005 − 2007		
	q_i/n	**Bz**	**SSI**	**q_i/n**	**Bz**	**SSI**	**q_i/n**	**Bz**	**SSI**	**q_i/n**	**Bz**	**SSI**
Agriculture	0.044	0.044	0.040	0.045	0.046	0.041	0.055	0.053	0.051	0.053	0.051	0.050
Industry&energy	0.176	**0.139**	**0.178**	0.182	**0.136**	**0.182**	0.178	0.159	0.186	0.160	0.144	0.165
Urban communities	0.015	0.014	0.013	0.015	0.015	0.013	0.027	0.026	0.025	0.027	0.025	0.024
Rural communities	0.015	0.014	0.013	0.015	0.015	0.013	0.014	0.013	0.012	0.013	0.013	0.012
Coastal communities	0	0	0	0	0	0	0	0	0	0	0	0
Other communities	0.044	0.044	0.040	0.045	0.046	0.041	0.068	0.067	0.064	0.067	0.064	0.063
Region	0.044	0.044	0.040	0.045	0.046	0.041	0.041	0.040	0.038	0.040	0.038	0.037
District	0.250	0.289	0.295	0.258	0.296	0.309	0.233	0.262	0.273	0.227	0.265	0.268
Inter-district	0.015	0.014	0.013	0.015	0.015	0.013	0.014	0.013	0.012	0.013	0.013	0.012
Fishery & fish industry	0.044	0.044	0.040	0.045	0.046	0.041	0.027	0.026	0.025	0.040	0.038	0.037
Tourism	0.015	0.014	0.013	0.015	0.015	0.013	0.014	0.013	0.012	0.013	0.013	0.012
Water supply industry	0.015	0.014	0.013	0.030	0.030	0.027	0.014	0.013	0.012	0.013	0.013	0.012
Residential water users	0.029	0.029	0.027	0.030	0.030	0.027	0.014	0.013	0.012	0.027	0.025	0.024
Ecologists	0.029	0.029	0.027	0.030	0.030	0.027	0.041	0.040	0.038	0.040	0.038	0.037
Professional bodies	0.059	0.059	0.055	0.061	0.062	0.056	0.068	0.067	0.064	0.067	0.064	0.063
Ministry of Environment	0.015	0.014	0.013	0.015	0.015	0.013	0.014	0.013	0.012	0.013	0.013	0.012
Ministry of land devt&rural aff	0.015	0.014	0.013	0.015	0.015	0.013	0.014	0.013	0.012	0.013	0.013	0.012
Ministry of health	0.015	0.014	0.013	0.015	0.015	0.013	0.014	0.013	0.012	0.013	0.013	0.012
Ministry of the Interior	0.015	0.014	0.013	0	0	0	0.014	0.013	0.012	0.013	0.013	0.012
Ministry of Industry	0.015	0.014	0.013	0.015	0.015	0.013	0.014	0.013	0.012	0.013	0.013	0.012
Ministry of agri	0.015	0.014	0.013	0	0	0	0.014	0.013	0.012	0.013	0.013	0.012
Other ministries	0.088	0.094	0.085	0.076	0.080	0.072	0.096	0.101	0.093	0.093	0.094	0.090
State prefectures	0.029	0.029	0.027	0.030	0.030	0.027	0.014	0.013	0.012	0.027	0.025	0.024

Table 14 Power Values for Loire - Bretagne RBC, 1987 - 2007.

	1987 − 1992			1993 − 1998			1999 − 2004			2005 − 2007		
	q_i/n	**Bz**	**SSI**	**q_i/n**	**Bz**	**SSI**	**q_i/n**	**Bz**	**SSI**	**q_i/n**	**Bz**	**SSI**
Agriculture	0.061	0.057	0.057	0.064	0.060	0.059	0.062	0.059	0.059	0.056	0.052	0.052
Industry&energy	0.149	0.133	0.149	0.155	0.134	0.153	0.140	0.129	0.143	0.143	0.132	0.146
Urban communities	0.018	0.016	0.016	0.018	0.017	0.016	0.016	0.015	0.014	0.032	0.030	0.029
Rural communities	0.009	0.008	0.008	0.009	0.008	0.008	0.008	0.007	0.007	0.008	0.007	0.007
Coastal communities	0	0	0	0	0	0	0	0	0	0	0	0
Other communities	0.026	0.024	0.023	0.009	0.008	0.008	0.054	0.051	0.051	0.056	0.052	0.052
Region	0.070	0.065	0.065	0.073	0.068	0.067	0.062	0.059	0.059	0.063	0.060	0.060
District	0.246	0.301	0.296	0.255	0.314	0.309	0.217	0.260	0.257	0.222	0.267	0.263
Inter-district	0	0	0	0.018	0.017	0.016	0.023	0.022	0.021	0.016	0.015	0.014
Fishery & fish industry	0.044	0.041	0.040	0.045	0.042	0.041	0.047	0.044	0.043	0.048	0.045	0.044
Tourism	0.018	0.016	0.016	0.018	0.017	0.016	0.023	0.022	0.021	0.024	0.022	0.022
Water supply industry	0.018	0.016	0.016	0.018	0.017	0.016	0.016	0.015	0.014	0.008	0.007	0.007
Residential water users	0.018	0.016	0.016	0.018	0.017	0.016	0.031	0.029	0.028	0.032	0.030	0.029
Ecologists	0.026	0.024	0.023	0.027	0.025	0.024	0.039	0.037	0.036	0.040	0.037	0.037
Professional bodies	0.105	0.100	0.101	0.100	0.095	0.095	0.085	0.082	0.082	0.095	0.092	0.093
Ministry of Environment	0.009	0.008	0.008	0.009	0.008	0.008	0.008	0.007	0.007	0.008	0.007	0.007
Ministry of land devt&rural aff	0.009	0.008	0.008	0.009	0.008	0.008	0.008	0.007	0.007	0.008	0.007	0.007
Ministry of health	0.009	0.008	0.008	0.009	0.008	0.008	0.008	0.007	0.007	0.008	0.007	0.007
Ministry of the Interior	0.009	0.008	0.008	0	0	0	0.008	0.007	0.007	0.008	0.007	0.007
Ministry of Industry	0.009	0.008	0.008	0.009	0.008	0.008	0.008	0.007	0.007	0.008	0.007	0.007
Ministry of agri	0.009	0.008	0.008	0	0	0	0.008	0.007	0.007	0.008	0.007	0.007
Other ministries	0.053	0.049	0.048	0.045	0.042	0.041	0.054	0.051	0.051	0.040	0.037	0.037
State prefectures	0.088	0.082	0.083	0.091	0.086	0.086	0.078	0.074	0.074	0.071	0.068	0.068

Table 15 Power Values for Rhin-Meuse RBC, 1987 - 2007.

	1987 − 1992			1993 − 1998			1999 − 2004			2005 − 2007		
	q_i/n	**Bz**	**SSI**	q_i/n	**Bz**	**SSI**	q_i/n	**Bz**	**SSI**	q_i/n	**Bz**	**SSI**
Agriculture	0.016	0.016	0.015	0.017	0.017	0.015	0.029	0.028	0.027	0.028	0.028	0.026
Industry&energy	0.180	0.159	0.189	0.190	0.160	0.197	0.174	0.159	0.184	0.169	0.152	0.180
Urban communities	0.033	0.032	0.030	0.017	0.017	0.015	0.029	0.028	0.027	0.042	0.042	0.039
Rural communities	0.016	0.016	0.015	0.017	0.017	0.015	0.014	0.014	0.013	0.014	0.014	0.013
Coastal communities	0	0	0	0	0	0	0	0	0	0	0	0
Other communities	0.033	0.032	0.030	0.034	0.034	0.031	0.043	0.043	0.040	0.042	0.042	0.039
Region	0.049	0.049	0.046	0.052	0.052	0.048	0.043	0.043	0.040	0.042	0.042	0.039
District	0.230	0.253	0.268	0.241	0.263	0.282	0.217	0.240	0.252	0.211	0.232	0.245
Inter-district	0	0	0	0.017	0.017	0.015	0.014	0.014	0.013	0.056	0.056	0.053
Fishery & fish industry	0.049	0.049	0.046	0.052	0.052	0.048	0.043	0.043	0.040	0.042	0.042	0.039
Tourism	0.016	0.016	0.015	0.017	0.017	0.015	0.014	0.014	0.013	0.014	0.014	0.013
Water supply industry	0.016	0.016	0.015	0.017	0.017	0.015	0.014	0.014	0.013	0.014	0.014	0.013
Residential water users	0.016	0.016	0.015	0.017	0.017	0.015	0.029	0.028	0.027	0.028	0.028	0.026
Ecologists	0.033	0.032	0.030	0.034	0.034	0.031	0.043	0.043	0.040	0.042	0.042	0.039
Professional bodies	0.082	0.085	0.078	0.086	0.093	0.083	0.072	0.073	0.069	0.070	0.072	0.068
Ministry of Environment	0.016	0.016	0.015	0.017	0.017	0.015	0.014	0.014	0.013	0.014	0.014	0.013
Ministry of land devt&rural aff	0.016	0.016	0.015	0.017	0.017	0.015	0.014	0.014	0.013	0.014	0.014	0.013
Ministry of health	0.016	0.016	0.015	0.017	0.017	0.015	0.014	0.014	0.013	0.014	0.014	0.013
Ministry of the Interior	0.016	0.016	0.015	0	0	0	0.014	0.014	0.013	0.014	0.014	0.013
Ministry of Industry	0.016	0.016	0.015	0.017	0.017	0.015	0.014	0.014	0.013	0.014	0.014	0.013
Ministry of agri	0.016	0.016	0.015	0	0	0	0.014	0.014	0.013	0.014	0.014	0.013
Other ministries	0.082	0.085	0.078	0.069	0.071	0.065	0.087	0.091	0.084	0.056	0.056	0.053
State prefectures	0.049	0.049	0.046	0.052	0.052	0.048	0.043	0.043	0.040	0.042	0.042	0.039

Table 16 Power Values for Rhône - Méditerranée - Corse RBC, 1987 - 2007.

	1987 − 1992			1993 − 1998			1999 − 2004			2005 − 2007		
	$\mathbf{q}_i/\mathbf{n}$	**Bz**	**SSI**	$\mathbf{q}_i/\mathbf{n}$	**Bz**	**SSI**	$\mathbf{q}_i/\mathbf{n}$	**Bz**	**SSI**	$\mathbf{q}_i/\mathbf{n}$	**Bz**	**SSI**
Agriculture	0.048	0.046	0.044	0.048	0.046	0.043	0.048	0.046	0.045	0.050	0.048	0.046
Industry&energy	0.173	0.131	0.169	0.173	**0.131**	**0.169**	0.153	**0.135**	**0.157**	0.157	0.146	0.164
Urban communities	0.019	0.018	0.017	0.010	0.009	0.008	0.032	0.031	0.029	0.033	0.032	0.031
Rural communities	0.010	0.009	0.008	0.010	0.009	0.008	0.008	0.008	0.007	0.008	0.008	0.007
Coastal communities	0	0	0	0	0	0	0	0	0	0	0	0
Other communities	0.029	0.028	0.025	0.029	0.028	0.025	0.065	0.062	0.061	0.091	0.091	0.089
Region	0.048	0.046	0.044	0.058	0.056	0.053	0.048	0.046	0.045	0.041	0.040	0.038
District	0.269	0.328	0.329	0.269	0.328	0.329	0.226	0.268	0.268	0.215	0.248	0.251
Inter-district	0	0	0	0.010	0.009	0.008	0.008	0.008	0.007	0.008	0.008	0.007
Fishery & fish industry	0.048	0.046	0.044	0.048	0.046	0.043	0.048	0.046	0.045	0.050	0.048	0.046
Tourism	0.029	0.028	0.025	0.029	0.028	0.025	0.024	0.023	0.022	0.025	0.024	0.023
Water supply industry	0.010	0.009	0.008	0.029	0.028	0.025	0.024	0.023	0.022	0.025	0.024	0.023
Residential water users	0.010	0.009	0.008	0.010	0.009	0.008	0.032	0.031	0.029	0.033	0.032	0.031
Ecologists	0.029	0.028	0.025	0.029	0.028	0.025	0.040	0.039	0.037	0.041	0.040	0.038
Professional bodies	0.077	0.076	0.071	0.077	0.076	0.071	0.065	0.062	0.061	0.066	0.064	0.063
Ministry of Environment	0.010	0.009	0.008	0.010	0.009	0.008	0.008	0.008	0.007	0.008	0.008	0.007
Ministry of land devt&rural aff	0.010	0.009	0.008	0.010	0.009	0.008	0.008	0.008	0.007	0.008	0.008	0.007
Ministry of health	0.010	0.009	0.008	0.010	0.009	0.008	0.008	0.008	0.007	0.008	0.008	0.007
Ministry of the Interior	0.010	0.009	0.008	0	0	0	0.008	0.008	0.007	0.008	0.008	0.007
Ministry of Industry	0.010	0.009	0.008	0.010	0.009	0.008	0.008	0.008	0.007	0.008	0.008	0.007
Ministry of agri	0.010	0.009	0.008	0	0	0	0.008	0.008	0.007	0.008	0.008	0.007
Other ministries	0.067	0.066	0.062	0.058	0.056	0.053	0.065	0.062	0.061	0.050	0.048	0.046
State prefectures	0.077	0.076	0.071	0.077	0.076	0.071	0.065	0.062	0.061	0.058	0.056	0.054

Table 17 Power Values for Rhône - Méditerranée - Corse RBC, 1987 - 2007.

	1987 − 1992			1993 − 1998			1999 − 2004			2005 − 2007		
	q_i/n	Bz	SSI	q_i/n	Bz	SSI	q_i/n	Bz	SSI	q_i/n	Bz	SSI
Agriculture	0.039	0.037	0.035	0.040	0.038	0.036	0.042	0.041	0.039	0.037	0.036	0.036
Industry&energy	0.165	0.145	0.168	0.168	0.146	0.170	0.153	0.143	0.159	0.144	0.154	0.156
Urban communities	0.010	0.009	0.009	0.010	0.009	0.009	0.034	0.032	0.031	0.064	0.063	0.063
Rural communities	0.019	0.018	0.017	0.010	0.009	0.009	0.008	0.008	0.008	0.016	0.015	0.015
Coastal communities	0	0	0	0	0	0	0	0	0	0.011	0.010	0.010
Other communities	0.029	0.028	0.026	0.030	0.028	0.027	0.059	0.057	0.056	0.112	0.114	0.116
Region	0.068	0.065	0.063	0.069	0.066	0.064	0.059	0.057	0.056	0.037	0.036	0.036
District	0.243	0.288	0.290	0.248	0.293	0.296	0.212	0.245	0.247	0.134	0.140	0.142
Inter-district	0	0	0	0.010	0.009	0.009	0.008	0.008	0.008	0.021	0.021	0.020
Fishery & fish industry	0.049	0.046	0.044	0.050	0.047	0.045	0.042	0.041	0.039	0.043	0.041	0.041
Tourism	0.019	0.018	0.017	0.020	0.019	0.018	0.017	0.016	0.016	0.016	0.015	0.015
Water supply industry	0.019	0.018	0.017	0.020	0.019	0.018	0.017	0.016	0.016	0.016	0.015	0.015
Residential water users	0.019	0.018	0.017	0.020	0.019	0.018	0.034	0.032	0.031	0.032	0.031	0.030
Ecologists	0.029	0.028	0.026	0.030	0.028	0.027	0.051	0.049	0.048	0.048	0.047	0.046
Professional bodies	0.097	0.097	0.093	0.099	0.099	0.095	0.085	0.083	0.082	0.059	0.057	0.057
Ministry of Environment	0.010	0.009	0.009	0.010	0.009	0.009	0.008	0.008	0.008	0.037	0.036	0.036
Ministry of land devt&rural aff	0.010	0.009	0.009	0.010	0.009	0.009	0.008	0.008	0.008	0.005	0.005	0.005
Ministry of health	0.010	0.009	0.009	0.010	0.009	0.009	0.008	0.008	0.008	0.011	0.010	0.010
Ministry of the Interior	0.010	0.009	0.009	0.010	0.009	0.009	0.008	0.008	0.008	0.011	0.010	0.010
Ministry of Industry	0.010	0.009	0.009	0.010	0.009	0.009	0.008	0.008	0.008	0.011	0.010	0.010
Ministry of agri	0.010	0.009	0.009	0	0	0	0.008	0.008	0.008	0.011	0.010	0.010
Other ministries	0.058	0.056	0.054	0.050	0.047	0.045	0.059	0.057	0.056	0.080	0.079	0.080
State prefectures	0.078	0.075	0.073	0.079	0.076	0.074	0.068	0.066	0.065	0.043	0.041	0.041

8.3 Adour-Garonne river basin, 1989-2006

Table 18 Population Shares for Industrial, Agricultural and Residential Water Users in Adour - Garonne River Basin, 1989 - 2006.

	first scenario			second scenario		
	industry	agriculture	residential	industry	agriculture	residential
1989	0.176	0.106	0.717	0.088	0.053	0.859
1990	0.175	0.103	0.722	0.088	0.051	0.861
1991	0.171	0.099	0.729	0.086	0.050	0.865
1992	0.167	0.097	0.736	0.083	0.049	0.868
1993	0.162	0.094	0.745	0.081	0.047	0.872
1994	0.158	0.090	0.751	0.079	0.045	0.876
1995	0.158	0.087	0.756	0.079	0.043	0.878
1996	0.156	0.085	0.759	0.078	0.043	0.879
1997	0.152	0.084	0.765	0.076	0.042	0.882
1998	0.150	0.080	0.770	0.075	0.040	0.885
1999	0.148	0.076	0.776	0.074	0.038	0.888
2000	0.150	0.073	0.777	0.075	0.036	0.889
2001	0.149	0.070	0.781	0.075	0.035	0.890
2002	0.145	0.068	0.786	0.073	0.034	0.893
2003	0.142	0.067	0.791	0.071	0.034	0.895
2004	0.139	0.067	0.794	0.070	0.033	0.897
2005	0.135	0.063	0.802	0.068	0.031	0.901
2006	0.132	0.059	0.808	0.066	0.030	0.904

8.4 Power Values for block model (11 groups)

Table 19 Power Values for Adour - Garonne RBC (11 groups), 1987 - 2007.

	1987 − 1992			1993 − 1998			1999 − 2004			2005 − 2007		
	q_i/n	Bz	SSI	q_i/n	Bz	SSI	q_i/n	Bz	SSI	q_i/n	Bz	SSI
Agriculture	0.083	0.074	0.074	0.074	0.066	0.065	0.092	0.089	0.086	0.082	0.078	0.076
Industry	0.143	0.135	0.138	0.148	0.139	0.144	0.122	0.107	0.103	0.134	0.138	0.131
Urban communities	0.012	0.009	0.008	0.012	0.008	0.007	0.020	0.018	0.015	0.021	0.021	0.020
Rural communities	0.012	0.009	0.008	0.012	0.008	0.007	0.010	0.009	0.008	0.010	0.011	0.012
Environmental ass.	0.167	0.147	0.156	0.160	0.141	0.150	0.184	0.173	0.188	0.196	0.176	0.187
Other communities	0.048	0.048	0.044	0.049	0.054	0.051	0.082	0.078	0.076	0.082	0.078	0.076
Region and District	0.286	0.356	0.353	0.296	0.373	0.369	0.255	0.297	0.302	0.278	0.323	0.323
Ministry of Industry	0.012	0.009	0.008	0.012	0.008	0.007	0.010	0.009	0.008	0.010	0.011	0.012
Professional bodies	0.095	0.082	0.081	0.099	0.079	0.077	0.092	0.089	0.086	0.082	0.078	0.076
Other ministries	0.071	0.065	0.065	0.062	0.058	0.059	0.071	0.070	0.069	0.041	0.031	0.033
State prefectures	0.071	0.065	0.065	0.074	0.066	0.065	0.061	0.062	0.059	0.062	0.055	0.055

8.5 Nucleolus

The nucleolus is a solution concept for cooperative games, which was first formulated by Shmeidler (1969). In order to formulate it, let us first introduce for any set of winning coalitions $\mathcal{W}$ a characteristic function V:

$$V(S) = \begin{cases} 1 \text{ if } S \in W \\ 0 \text{ otherwise} \end{cases}.$$

Hereafter we will consider the game in the form (N, V).

For convenience, for some vector x we define by

$$x(S) \equiv \sum_{i \in S} x_i \text{ for any } S \subseteq N.$$

A payoff vector $x = (x_1, ..., x_n)$ with $x_i \geq v(i)$ and $x(N) = v(N)$ is called an *imputation*. We denote by $X(N, V)$ the set of all imputations of the game (N, V).

Let x be an imputation, then for any coalition S the *excess* of S is defined as

$$e(S,x) = V(S) - x(S).$$

One may interpret this number as a measure of "dissatisfaction" for coalition S at imputation x. For any imputation x let $S_1, ..., S_{2^n-1}$ be an ordering of the coalitions for which $e(S_l, x) \geq e(S_{l+1}, x)$ for $l = 1, ..., 2^n - 2$. Let $E(x)$ be the vector of excess defined as $E_l(x) = e(S_l, x)$ for all $l = 1, ..., 2^n - 1$. We say that $E(x)$ is *lexicographically* less than $E(y)$ if

$$E_l(x) < E_l(y) \text{ for the smallest } l \text{ for which } E_l(x) \neq E_l(y).$$

We denote this relation by $E(x) \prec_{lex\min} E(y)$.

Definition 1 *The nucleolus is the set of imputations x for which the vector $E(x)$ is lexicographically minimal:*

$$\nu = \nu(N,V) = \{x \in X(N,V) : \nexists y \in X(N,V) : E(y) \prec_{lex\min} E(x)\}.$$

The following recursive procedure is used to characterize the nucleolus. By definition $E_1(x)$ is the largest excess of any coalition relative to x. At the first step of the procedure we find the set X_1 of all imputations x that minimizes $E_1(x)$:

$$\begin{aligned}&\min \epsilon \\ &\text{s.t. } e(S,x) \leq \epsilon \text{ for all } S,\ \emptyset \subset S \subset N\ \cdot \\ &\text{and } x(N) = V(N)\end{aligned}$$

The set X_1 is called the *least core* of c. If it is not a unique point, we find the set X_2 of all x in X_1 that minimizes $E_2(x)$, the second largest excess and so on. This process eventually leads to an X_k consisting of a single imputation, called the *nucleolus* (Shmeidler (1969), Machler, Peleg and Shapley (1979)). The nucleolus minimizes recursively the "dissatisfaction" of the worst treated coalitions.

It appears that the nucleolus of a game in coalitional form exists and it is unique. If the core is not empty, the nucleolus is in the core. Like the Shapley value the nucleolus can be obtained as the unique value satisfying a set of axioms.