"A Welfare Assessment of Revenue Management Systems"

Nicolas Dupuis, Marc Ivaldi and Jerome Pouyet

Toulouse
School
of Economics

A Welfare Assessment of Revenue Management Systems[*]

Nicolas Dupuis[†], Marc Ivaldi,[‡] and Jerome Pouyet[§]

January 4, 2015

Abstract

We study the welfare impact of revenue management, i.e. intertemporal price discrimination when the product availability is limited both in time and quantity, and consumers' arrival is random. This practice is particularly relevant, and widely spread, in the transport industry, but little is known about its implications on profits and consumer surplus. We develop a theoretical model of revenue management allowing for heterogeneity in product characteristics, capacity constraints, consumer preferences, and probabilities of arrival. We also introduce dynamic competition between revenue managers. We solve this model computationally and recover the optimal pricing strategies. We find that revenue management is welfare enhancing. Revenue managers face two types of constraints: a limited booking period and fixed capacities. Previous sales affect the relative slackness of these two constraints, explaining price variations. Profits increase as the practice offers more leeway to the seller compared to posting a fixed price throughout the booking period. Total consumer surplus also increases for a wide range of specifications, as revenue management raises the number of sales. In the presence of heterogeneous consumers, consumers with low price sensitivity subsidize ones with high price sensitivity when demand is low but both types benefit from the practice when demand is high. This sheds some light on the impact of revenue management on the surplus of business and leisure passengers.

Keywords: revenue management, transport fares, intertemporal price discrimination, dynamic computational models.

JEL classification: C63, R41.

1 Introduction

We study the welfare impact of revenue management and the dynamics of fares resulting from this pricing strategy. We find that revenue management is generally welfare-enhancing. Revenue management is widely used in transports, especially by airline and railway companies, but also to manage hotel bookings and ticket sales. This form of intertemporal price discrimination allows the seller to manipulate the price of a product whose availability is constrained both in time and in quantity. In a framework in which consumer arrival is uncertain, the seller can react to the demand realization: she increases or decreases her prices given the quantity already sold and the remaining time in which to sell the residual capacity.

Whether revenue management is welfare-enhancing or not is a difficult question which remains, to the extent of our knowledge, unanswered. The practice might seem unfair, and on the ground that it increases firms' profits, raises suspicions that it harms consumers. However, forcing the firm to post a fixed price does not ensure that it will post an affordable one. When restricted to a fixed price, the revenue manager can no longer reward early bookers through rebates or propose last minute deals at the end of the booking period.

We develop a model of revenue management allowing the seller to simultaneously propose different products, heterogeneous in their characteristics but also in their availability, to different types of consumers. For instance, a revenue manager in a railway company could propose several trains on a given day for the same origin-destination leg. These trains could differ in their departure time, number of stops along the way, or number of seats. Each is characterized by a capacity constraint, i.e. its number of remaining seats. As a revenue manager is also likely to react to the prices posted by her competitors, we introduce dynamic competition between revenue managers.

We computationally solve this dynamic program by simulating stochastic arrivals of consumers and their purchasing decisions. We then compute the subsequent optimal reaction

of the revenue manager. Our results are robust to several specifications for the arrival of consumers as we allow for constant and increasing arrival rates.

Our approach allows us to answer the difficult questions of finding the equilibrium pricing strategies and assessing the welfare impact of revenue management despite the absence of a closed-form solution to our model. Another method to understand the strategies played at equilibrium and to carry out this welfare analysis would have been to estimate a structural model of revenue management. It is however difficult to procure sufficiently detailed data on this topic as they are a very sensitive and strategic piece of information.

We first consider a simple model of revenue management with homogeneous products and only one type of consumers. The revenue manager only faces one capacity constraint and one time constraint. This allows us to illustrate the forces at play in revenue management: the intensity of the constraints faced by the revenue manager can be represented by the ratio of remaining units over the remaining time to sell them. For a given number of units, if the remaining time to sell them decreases, the ratio increases, and the revenue manager is pressured into lowering her prices. If, on the other hand, for a given deadline, the revenue manager has less units to sell, the unit-time constraint faced by the revenue manager is relaxed and she will try to sell the last units at a higher price.

We are able to recover a standard feature of the industry pricing: if the arrival rate of consumers is sufficiently high compared to the total capacity, prices start low and increase as time goes by. At the very end of the booking period, prices can dramatically fall as the manager does not want to keep unsold units. When the intensity of demand is low compared to the total capacity, the probability of the revenue manager being constrained by the number of available units is almost zero and intertemporal price discrimination does not occur in practice. We also show that compared to an optimal fixed price, revenue management strongly increases consumer surplus through a higher number of sales without hurting the profits of the company.

We also consider the more complex and realistic case in which heterogeneous consumers have to choose between heterogeneous products, for instance an off-peak and a rush-hour train. The revenue manager now faces one time constraint but several capacity constraints. Solving for the optimal pricing strategy of the revenue manager and for the optimal fixed price, we compare the producer and consumer surpluses between these two pricing strategies. For low intensities of demand compared to the total capacity, revenue management allows price-sensitive consumers to be subsidized by the ones with a lower price elasticity. However both categories of consumers benefit from the practice as the intensity of demand increases for a given total capacity. Applied to the transport industry, this provides some insights about the impact of revenue management on leisure and business passengers.

In the third part of the paper, we introduce both indirect and direct competition in the model, and study their impact in terms of profit and welfare. We find that compared to the monopoly case and holding demand constant, direct competition substantially increases load rates through lower prices. We also study the impact of ex-ante strategic decisions on social welfare, such as allowing revenue managers to choose the total capacity they propose before they compete against each other. In this context, we find that competition between two revenue managers reduces producer surplus only slightly compared to the monopoly case but allows consumer surplus to be twice as large.

We review the literature in section 2 and present the model and our algorithm in sections 3 and 4. Results for the simplest one-product homogeneous-consumers case are presented in section 5. Heterogeneous consumers are introduced in section 6. We analyse dynamic competition between revenue managers and ex-ante competition in capacities in sections 7 and 8.

2 Literature Review

The literature on revenue management is wide both in operations research and economic theory. Our paper fills the gap between the two fields, answering a truly economic problematic in the realistic framework of operations research.

In operations research, Talluri and Van Ryzin (2004, 2005) introduce a choice-based model of revenue management and show the two forces driving the manager's actions: the will to boost demand by selling at a low price, and the incentive to post high prices after a sufficient number of sales.[1] The first force is prevalent at the beginning and at the end of the booking period. They also show that the choice-based approach gives better results than other methods of revenue management. We show that some of their results do not hold when we allow the capacity constraint of the revenue manager to be multidimensional, e.g. when the revenue manager maximizes profits over two different types of products instead of one. In that case, the revenue manager may be willing to post prices which are not efficient according to their definition. McAfee and Velde (2008) study a similar question, restricting to demand functions satisfying constant price elasticity, which allows them to derive a closed-form solution. They also use a reduced-form approach by modelling arrival of consumers and their purchasing decision as a simultaneous process. To the extent of our knowledge, Vulcano et al. (2010) is the first attempt to estimate consumers preferences and to carry out counterfactuals in a revenue management context. Gallego and Van Ryzin (1997) introduce the notion of multiple products and associated limited resources although they adopt a reduced-form approach for the consumers' behaviour. Modarres M. (2009) study the pricing strategy when cancellation is possible and revenue managers do not update their prices every period. McGill and Van Ryzin (1999) provide a comprehensive overview of the issues raised by revenue management. This operations research literature does not

---

[1]A choice-based model is one which explicitly models consumers' choices.

provide any welfare analysis of revenue management, but rather focuses on the best way to implement it from the perspective of the seller. Despite some similarities in the way we model the maximization program of the revenue manager, our aim is radically different. We also extend these models by introducing strategic choices and strategic interactions from and among revenue managers.

The economic literature, starting with Stokey (1979), has dealt with intertemporal price discrimination but not really with revenue management from an operational perspective. For instance, Dana Jr (1999) studies how a firm can use revenue management to smooth demand peaks and reduce capacity costs. He models revenue management as price discrimination between two flights differentiated by their departure time, but does not consider fluctuating prices for a given flight. Related to our topic, although more theoretical, Gershkov and Moldovanu (2009) use a mechanism design approach to solve the revenue management problem. This method has the advantage of providing a model with a closed-form solution. However it stands on a theoretical ground which is far from the practitioners' world. Hörner and Samuelson (2011) study the allocation of a good when consumers are forward-looking. This framework certainly improves the theory of revenue management but is, in our opinion, less useful to study more practical issues such as its welfare impact. Indeed, forward-looking consumers are supposed to be able to observe and react to any price change from the revenue manager from the moment they realize they need a ticket to the end of the booking period. We feel that search costs or aversion towards the risk of not finding a seat might also play a role when booking a ticket. In that case, even forward-looking consumers could behave like impatient ones. In practice, in the transport industry, we feel that consumers looking for a ticket make their purchasing decisions comparing prices of travelling options with close departure dates rather than trying to anticipate future prices. Because of this, we choose to simplify the analysis and model our customers as impatient, which is a standard feature in the operations research literature. This allows us to model a more realistic

framework including several substitutable products to choose from. We also address the possibility of partially forward-looking consumers in one specification of our model by allowing the willingness to purchase the products to increase over time. This represents a situation in which consumers are aware that there might not be any products left if they wait but cannot anticipate the pricing strategy of the revenue manager. Lazarev (2012) empirically studies the welfare impact of intertemporal price discrimination. His approach however does not explicitly take into account revenue management but assumes an exogenous dynamics of fares. Williams (2013) estimates a dynamic model of revenue management and price discrimination using a framework similar to ours. He finds that revenue management benefits consumers on average, which confirms our first result. However, he avoids the problem of substitutability between products by focusing on single flights. We also conduct an analysis of revenue management when the revenue manager can propose several types of products.

Finally, Goodwin (1992) and Wardman and Shires (2003) both conduct reviews of price elasticities' estimates in transports, respectively with a focus on short-term vs. long-term and on the types of consumers. To carry out our analysis, we choose values of parameters giving us price elasticities consistent with the average estimates in this literature.

3 The Model

3.1 Notations

Our revenue management problem features a seller, the revenue manager, proposing several products, constrained in quantity, to a set of potential consumers. The revenue manager can only sell her products during a finite period of time, which corresponds to the booking period in the transport industry. In our model, this period starts at date T and ends at date 1. Consumers who purchased the products consume them at date 0.

We allow products to be heterogeneous in their characteristics and different types of prod-

7

ucts can be simultaneously proposed to consumers by the revenue manager. For instance, a revenue manager can simultaneously propose a first-class ticket and a second-class ticket. All products are constrained in quantity and each type of products has its own constraint. In the previous example, the number of first class tickets that the revenue manager can sell is constrained by the number of first-class seats. Products characteristics are denoted by $i \in I$.[2] X_i denotes the remaining capacity for attribute i. The vector of remaining capacities for all constrained characteristics in I is denoted X.

The range of possible prices for a product with attributes i is denoted P_i and contains a price equal to $+\infty$ for all i. At each period of the booking process, the revenue manager chooses the menu of tariffs she proposes to the consumers. $\tilde{p} \in \prod_i P_i$ is the vector of selected tariff options for all products. We assume that when $\tilde{p}_i = +\infty$ is chosen, the product is sold with probability 0. This tariff option is therefore equivalent to closing the sales of the product. When bought, a product with attributes i is exchanged at price $p_i = \tilde{p}_i$.

We assume that no more than one consumer can buy a product within a period t of the booking process.[3] At each period $t \in \{T, \ldots, 1\}$, a consumer arrives with probability λ_t, observes the available tariffs, and chooses whether or not to purchase. The dummy d_i takes value 1 when a product with attribute i is purchased. When the consumer selects the outside option, $d_0 = 1 - \sum_i d_i = 1$.

In the following, we sometimes use the expressions *market size* or *intensity of demand* when referring to the average rate of arrival $\mathbb{E}(\lambda_t)$.

---

[2]For the sake of simplicity, we chose to model only the products characteristics which are physically constrained. We could easily add another layer of differentiation, such as the flexibility of a ticket. But this would not change the flavour of our results.

[3]Although this might seem a bit restrictive, we can choose T as large as needed to ensure that this assumption is empirically justified.

3.2 The Bellman Equation

At each period t of the booking process, the revenue manager observes the remaining capacity X and chooses a menu of tariffs so as to maximize her overall profit. If she sells a product during the period, she gets the instantaneous profit given by the price of the product plus her continuation value when there is one less unit of product to sell. If she does not sell, she just gets her continuation value for a remaining capacity X. Selling occurs if a consumer arrives and decides to buy. Not selling occurs either because no consumer arrived or the consumer chose not to buy. We denote $Pr(d_i = 1|\tilde{p})$ the probability that a consumer buys a product i conditional on a proposed menu of tariffs $\tilde{p}$. We shall define this probability later when we explicitly model the consumers' behaviour.

Hence, the choices of the revenue manager must satisfy the following Bellman equation:

$$V_t(X) = \max_{\tilde{p} \in \Pi_i P_i} \sum_i \lambda_t Pr(d_i = 1|\tilde{p}) \left[\tilde{p}_i + V_{t-1}(X_i - 1, X_{-i}) \right]$$

$$+ \left[1 - \lambda_t + \lambda_t Pr\left(d_0 = 1|\tilde{p} \right) \right] V_{t-1}(X) \tag{3.1}$$

subject to the final constraints:

$$\begin{cases} V_0(X_i) = 0 & , \forall X_i \\ V_t(X) = 0 & , \text{ whenever } \sum_i X_i = 0 \\ \tilde{p}_i = +\infty & , \text{ if } X_i = 0 \end{cases} \tag{3.2}$$

Denoting $\mathbb{E}(\pi|\tilde{p})$ the instantaneous expected revenue given $\tilde{p}$, we can rewrite Equation

3.1 as:

$$V_t(X) = \max_{\tilde{p} \in \Pi_i P_i} \lambda_t \left[\mathbb{E}(\pi|\tilde{p}) - Pr(d_0 = 0|\tilde{p})V_{t-1}(X) \right.$$

$$\left. + \sum_i Pr(d_i = 1|\tilde{p})V_{t-1}(X_i - 1, X_{-i}) \right] + V_{t-1}(X) \qquad (3.3)$$

subject to the final constraints 3.2.

In the last equation, $Pr(d_0 = 0|\tilde{p})$ represents the total probability of purchase if the menu $\tilde{p}$ of tariffs is chosen.

The three final constraints are respectively given by the fact that sales are closed at the end of the booking process, the limited capacities, and the fact that the revenue manager has to close the sales of a type of products when it is no longer available.

3.3 Demand Modelling

The probability with which a consumer buys a product with characteristics i given $\tilde{p}$ is determined by the way we model demand. We adopt here a multinomial logit approach, in which the utility of a consumer arriving in period t is given by:

$$\begin{cases} u^t(p_i) = v + \alpha u_i - \gamma p_i + \varepsilon_{it} & \text{, when } d_i = 1 \\ u_0^t = \varepsilon_{0t} & \text{, when the outside option is chosen.} \end{cases} \qquad (3.4)$$

In the above equation:

- ε is the random part of the utility and is drawn from a type-I extreme value distribution, with location parameter μ equal to the Euler-Mascheroni constant (≈ 0.5772) and scale parameter $\sigma = 1$. These parameters ensure that ε is a zero-mean random variable with a variance equal to $\frac{\pi^2}{6}$. Those noises are i.i.d across options and consumers.

- $\bar{u}_0 = 0$ in case of no purchase, and $\bar{u}(p_i) = v + \alpha u_i - \gamma p_i$ in case of a purchase of a product

with attributes i at price p_i, are the deterministic parts of the utility. u_i represents the observable characteristics of product i. The parameters α and γ respectively represent the consumers' sensitivity to these characteristics (such as the comfort class) and to the price. v is a scale parameter. Note that we can allow these parameters do depend on consumers' observable characteristics.

Following Ben-Akiva and Lerman (1985), the choice probabilities $Pr(d_i = 1|\tilde{p})$ are given by:

$$
\begin{cases}
Pr(d_i = 1|\tilde{p}) = \dfrac{e^{\bar{u}(p_i)}}{\sum_{j \in I} e^{\bar{u}(\tilde{p}_j)} + 1} \\[4mm]
Pr(d_0 = 1|\tilde{p}) = \dfrac{1}{\sum_{j \in I} e^{\bar{u}(\tilde{p}_j)} + 1}
\end{cases}
\tag{3.5}
$$

Notice that the final constraints of the Bellman equation imply that the revenue manager posts an infinite price p_i when a product i is sold out. Looking at Equation 3.5 above, it means that the probability of purchasing such a good is zero. If all products are sold out, all prices are set to infinity and the probability of choosing the outside option is 1.

Remark 1. *The relationship between γ and the price elasticity of demand $\eta(\tilde{p})$ is easily derived. Denoting $S_i(\tilde{p})$ the market share of a product with attributes i when consumers face a price vector $\tilde{p}$, we get:*

$$
\eta(\tilde{p}) = \frac{\partial S_i}{\partial p_i} \frac{p_i}{S_i} = -\gamma p_i (1 - S_i)
\tag{3.6}
$$

3.4 Comments on the Theoretical Model

Compared to other models of revenue management in the operations research literature, we differentiate ourselves in the following ways:

- We allow for a general product set, described by several capacity constraints.

- We choose a flexible structural approach to model purchasing decisions.

11

- The revenue manager can choose the price of each available product but also which products to propose using the option of an infinite price. By comparison, some revenue management papers assume that if two products are identical in all their characteristics but their prices, they constitute two different products. This leads to unrealistic consumer behaviours: when proposed simultaneously to the low price product, the high price product can be chosen by consumers with some positive probability.

Our model is therefore flexible enough to accommodate realistic pricing and consumption decisions. This flexibility has a cost: our model does not have any closed-form solution and some of the theoretical results that can be found in the literature do not extend here. More details about these issues can be found in Appendix A.

The best approach to solve our model is to use computational techniques.

4   Algorithm

To recover the optimal pricing strategy of the revenue manager and assess the welfare impact of revenue management, we construct an algorithm that solves the revenue manager optimization problem and finds her optimal action at any given period during the booking process and for any value of the state variable X.

First, we set the values of the different parameters driving consumers' choices as well as the choice sets of the revenue manager, $P_i \forall i$. All these values are chosen so as to be as realistic as possible. Values of parameters for the preferences of the consumers were chosen to match price elasticities estimated in the economic literature. (See Goodwin (1992) and Wardman and Shires (2003).) The number of tickets that the revenue manager has to sell matches the average capacity of a plane or a train. In our computations we choose capacities going from 200 seats up to 400 seats. The price grid was constructed based on observations about prices of train tickets in France and intra-European flights.

Given parameters for the preferences of the consumers and product characteristics, we are able to construct the purchase probabilities for each product as well as the expected instantaneous revenue using the demand model described above.

We therefore have all the elements necessary to solve the Bellman equation faced by the revenue manager. We start from the final constraints where the continuation value of the Bellman equation is zero and we compute the value functions at previous dates and for all possible states by iteration. More precisely, we fill a matrix with $T + 1$ rows and $\sum_i X_i$ columns. The first column corresponds to the case in which all products are sold and is therefore filled with zeros. The first row corresponds to the end of the booking period when the revenue manager can no longer sell the products. It is also filled with zeros. We can find the value functions in the second row because we know all values in the preceding row are zero. Using the Bellman equation, the value functions in the second row are the maximum expected revenue conditional on the available products. A maximization program running on the finite set of prices available to the revenue manager gives us this value function as well as the optimal menu of prices. The continuation values in period 1 (second row) are plugged into the Bellman equation, and we can solve it for period 2. We continue this iterative process until period T. This gives us the optimal actions of the revenue manager for each t and X.

Finally, we simulate arrivals of potential consumers during the booking period by random draws from a binomial distribution with parameter λ. We also generate the random shocks ϵ_{it} in the utility of the consumers and recover their purchasing decisions and surplus. Profits and total consumer surplus are computed as the sum of individual purchases and surplus. This being a stochastic process, we repeat the whole operation hundreds of times so as to recover the expectation and variance of the profit and consumer surplus.[4]

---

[4]In the remaining of the paper, we present average profit and consumer surplus across all the simulations. Prices at each period of the booking process are also averaged across all simulations. The number of simulations varies between 500 and 1000 depending on the specification. (See Table 7 of Appendix B.)

5 The Distributive Properties of Revenue Management

Although discrimination often creates an inefficient allocation of goods among consumers, it can sometimes increase the consumer surplus through a higher volume of sales. In this section we show that this is indeed the case with revenue management, using the model developed in section 3, and realistic values of parameters.

We develop here the simplest case possible with homogeneous products and only one type of consumers. Products are only differentiated through their price. Average values of proposed prices, profit and consumer surplus are found using the algorithm described above. The values of parameters we chose are summarized in Table 7 of Appendix B, column (1). We simulate this setup for constant arrival rates:

$$\lambda_t = \lambda, \ \forall t \text{ with } \lambda \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$$

5.1 Price Variations in Revenue Management

In a first setup, we illustrate the revenue management practice using a range of possible prices, P, equal to $\{40, 90\}$.[5] The selected values of parameters and Equation 3.6 allow us to compute the price elasticity of demand in this case: $\eta \approx -0.51$ when $p = 40$ and $\eta \approx -2.1$ when $p = 90$. These are in line with price elasticities' estimates in the rail industry: in Goodwin (1992), the average price elasticity for the demand for train tickets is -0.79 over 92 quoted values. In a more recent review, Wardman and Shires (2003) find an average price elasticity of -0.9 over 456 inter-urban rail demand elasticities in Great Britain, with highest elasticities around -3.2.

Figure 1 represents the change in posted average prices in such a setup for $\lambda = 0.4$ and 0.7. The averaging is performed over all the simulations at a given period of the booking process.

---

[5]These bounds have been chosen after observing the change in ticket prices on the second class Paris-Lyon over the whole booking period.
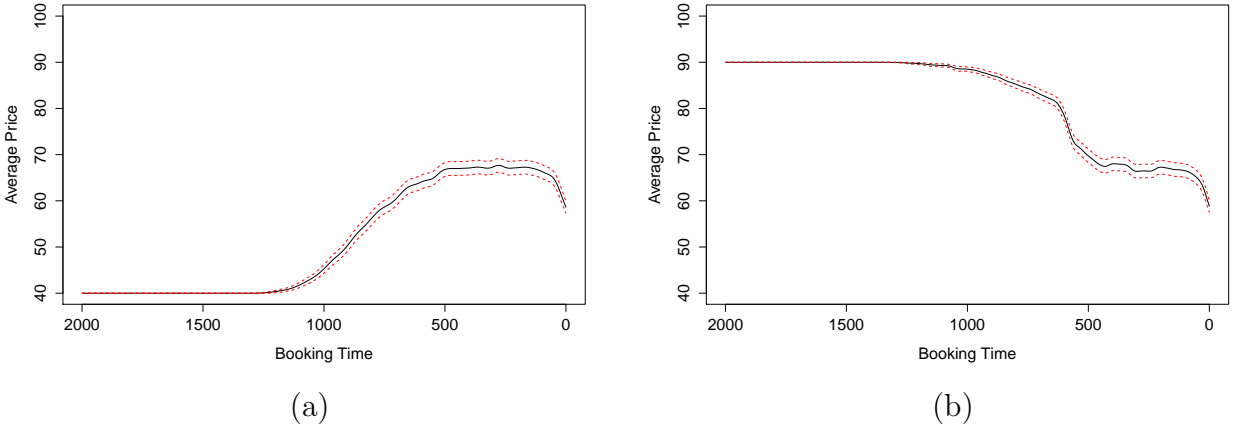
Figure 1: Change in posted average prices as a function of the booking period. Figure (a) deals with a demand of medium intensity, $\lambda = 0.4$; Figure (b) deals with a demand of high intensity, $\lambda = 0.7$. Note: The averaging is performed over all the simulations at a given period of the booking process. To facilitate the reading, we use a non-parametric fit of average prices over the booking period. The confidence interval in red is the smoothed confidence interval of the mean.

We notice that for a medium market size, the revenue manager tends to start with a low price: the time constraint dominates the capacity constraint. In that case, the revenue manager has the incentive to sell fast as many units as possible. When enough sales are made, prices start to increase: the capacity constraint now dominates and the incentive to sell the remaining units at high prices counterbalances the incentive to sell all units. At the end of the booking period, the opportunity cost of having unsold units dominates again and prices go down.

When the market size is larger compared to the total capacity, the frequent arrival of consumers makes it very likely that all units will be sold by the end of the booking period. Therefore, the capacity constraint already dominates at the beginning of the booking period and prices start high. This explains the shape of the price curve when $\lambda = 0.7$.[6]

---

[6]In practice, prices vary through time according to shape (a) rather than shape (b). This would mean that the proposed capacity is quite high compared to the market size. A possible explanation is that given the risk stemming from the random arrival of consumers, revenue managers prefer to underestimate the market size when maximizing their profit. However, such risk aversion is not modelled here. Another explanation

Note that this general shape is not a feature of the particularly simple price structure we adopted in this example. We performed the same exercise with five different prices between 20 and 100 and reported the price variations for $\lambda = 0.5$ in Figure 5 of Appendix H.1. Although variations are less pronounced, the general shape remains the same.

This shape of the price curve is standard in the revenue management literature: it corresponds to the price curve found in McAfee and Velde (2008) and the incentives faced by revenue managers are mentioned in Talluri and Van Ryzin (2004). We can also retrieve this shape looking at transport prices, for instance airline prices, which usually increase as time goes by but can drop at the last minute before the departure of the flight.

5.2   Revenue Management vs. Optimal Fixed Price

Revenue management offers more leeway to the seller than an optimal fixed price strategy. It is therefore expected that a revenue management strategy including the optimal fixed price in the choice set of the seller must (at least weakly) increase her profit compared to the optimal fixed price alone. In this section, we test this prediction and measure to which extent revenue management including the optimal fixed price raises profits and affects consumer surplus.

To do so, we compute the optimal fixed price p^o and the average profit and consumer surplus associated with this price. We then construct the choice set P of the revenue manager to include p^o and some small variations around this price:

$$P = \{p^o - \xi, p^o, p^o + \xi\} \; \xi = 1, 2, \ldots$$

Although the revenue manager's choice set is small, Figures 6 and 7 of Appendix H.2 show that revenue management weakly increases both profits and consumer surplus. The increase

would be that firms choose not to reduce the proposed capacity so as to avoid a shortage of units to sell and possible consumer discontent.

is significant only for market sizes which are high compared to the capacity constraint. When $\lambda \geq 0.7$, revenue management increases profits by 1% compared to the fixed price strategy, and for the same values, the increase in consumer surplus lies between 2% and 3%. These values are significant and indicate that simply adding some leeway in the choice set of the seller can benefit both the company and the consumers.

We find no significant profit increase for $\lambda \leq 0.4$. In these cases, the price posted by the revenue manager is always the optimal fixed price. For these values of the arrival rate, the observation of a high volume of sales during the booking process is very unlikely. The revenue manager does not have any incentive to modify her posted price because she anticipates that the capacity constraint will not be binding. Further discussion of these results can be found in Appendix C.

5.3 Revenue Management and Increasingly Impatient Consumers

Consumers are obliged to choose the outside option if all products are sold out. Although we do not model the consumers' behaviour in a dynamic way, one way to introduce concerns for the future would be to consider consumers with an increasing willingness-to-pay.

Indeed, let us assume a semi-sophisticated consumer who is well aware of the possibility that she might not get a ticket if she waits but cannot fully anticipate the pricing strategy of the revenue manager. If this consumer arrives at the beginning of the booking period, she might think that she has enough time to find another option if she chooses not to buy immediately. Her value for the product is pretty low relative to her value for the outside option. But if she arrives at the end of the booking period, there is a high probability that all products will be sold out soon and she is running out of time to find a decent outside option. Her willingness to buy the product becomes high compared to the outside option.

Taking the example of someone looking for plane tickets, she might feel that she has plenty of other opportunities at the beginning of the booking period and might refuse a

high-price ticket. At the end of the booking period however, her other options may be less enticing or riskier — for instance other means of transportation might not be available anymore. In this case, she might be willing to pay a higher price for her ticket.

We model this situation by allowing the scale parameter in the utility function of the consumer to be a decreasing function of $t \in [T, 1]$. Hence, when purchasing product i, the consumer gets the following utility:

$$u^t(p_i) = v + 1 - \frac{t-1}{T} + \alpha u_i - \gamma p_i + \varepsilon_{it} \tag{5.1}$$

Revenue management might allow the seller to extract even more profit compared to committing to a fixed price strategy. Indeed, the seller can be flexible and adapt her prices to the arrival of consumers who are more willing to pay high prices.

As we want to compare this situation to the case in which consumers have always the same willingness to pay, we choose v in Equation 5.1 to be equal to 1, thus ensuring that the expected utility for purchasing a product at price p_i averaged over the booking period is equal to their expected utility when they have a constant willingness to purchase.

Figures 14 and 15 of Appendix H.2 show our results when consumers have an increasing willingness to purchase the product. For $\lambda \leq 0.4$, revenue management has no effect on profits. When $\lambda \geq 0.5$, the relative impact of revenue management compared to a fixed price on profits is slightly stronger if consumers have an increasing willingness to purchase. Indeed, the increase in profits due to revenue management lies around 1%, which is similar to the case in which consumers have a constant willingness to purchase throughout the booking process. This impact is even stronger in some instances with a peak at 1.5% when $\lambda = 0.5$.

However, the effect of revenue management on consumer surplus in this situation becomes ambiguous as it increases or decreases consumer surplus depending on the value of the arrival rate. More surprisingly, revenue management has some positive effect on the consumer

surplus even for low values of λ: as consumers now have an increasing valuation for the product, the revenue manager has an incentive to deviate from the optimal fixed price when $\lambda \leq 0.4$. When $\lambda \geq 0.6$, the relative impact of revenue management is similar in both cases of constant and increasing willingness to purchase, around $2 - 3\%$. But it appears that for intermediate values of the arrival rate, the impact of revenue management on consumer surplus becomes negative. This demonstrates that in this situation, the fact that revenue management creates surplus through a higher number of sales can be offset by a greater ability to capture consumer surplus. This is confirmed by the observation that the value for the arrival rate yielding the highest profits is also the one yielding the lowest consumer surplus.

5.4   Revenue Management with Noisy Arrival Rates

Another practical issue is the fact that the revenue manager can face some noise in her prediction of the arrival rate of consumers. Even though the prediction is good on average, i.e. over several similar booking periods, for a given booking period unobserved shocks might shift this arrival rate. Formally:

$$\tilde{\lambda} = \lambda + \nu$$

where $\tilde{\lambda}$ is the realized arrival rate and ν is a Gaussian white noise.

Revenue management has the extra advantage over a fixed price to be flexible enough to limit the loss of profits due to these unobserved shifts. Suppose $\tilde{\lambda} > \lambda$. The seller underestimates the size of the market and might post a price which is too low. However, because she sells her products more quickly, she will be able to raise her prices on her remaining inventory over a longer period of time.

To model this situation, we use the same framework as in section 5.2 except that we add

19

a normally distributed noise to λ when we simulate the arrival of consumers.[7] We choose $\nu \sim N(0, 0.02)$. Hence, the optimal pricing strategy of the revenue manager is computed using λ but the actual arrival rate of consumers is simulated using $\tilde{\lambda}$. Other values of parameters remain unchanged.

Figures 16 and 17 of Appendix H.2 show our results when the arrival rate of consumers is noisy. For $\lambda \leq 0.4$, the realized arrival rate of consumers is too low for revenue management to make any difference with an optimal fixed price. In the following, we focus on $\lambda \geq 0.5$. Clearly, the relative impact of revenue management compared to a fixed price on profits is stronger when the arrival rate is noisy. Indeed, revenue management increases profits from 1 to 2% whereas the increase was at most 1% in the absence of noise. The fact that the seller cannot perfectly infer the arrival rate of consumers gives therefore an additional advantage to revenue management compared to a fixed price.

However, the impact on consumer surplus is ambiguous. In some cases, e.g. $\lambda = 0.5$, revenue management lowers consumer surplus by 1 to 2% whereas there is no significant impact in the noiseless case. For higher values of λ, as revenue management leads to more purchases, consumer surplus still increases compared to a fixed price, but to a lesser extent than in the noiseless case. Our interpretation is that when arrival rates are noisy, the sub-optimality of the revenue manager's pricing strategy given $\tilde{\lambda}$ creates inefficiencies. For instance, if the arrival rate is overestimated, prices will be too high for too long at the beginning, and this will preclude consumers to purchase. This phenomenon is not offset by cases in which the arrival rate is underestimated because the increase in surplus due to low prices is bounded by the number of available products.

Remark 2. *The revenue manager could also use variations in proposed prices to try to learn about the real value of the arrival rate during the booking process. She would then be able to update her pricing strategy using her estimated value for $\tilde{\lambda}$. This would increase the impact*

---

[7]The added noise is constant for the whole booking process.

of revenue management on profits. Such a learning model falls beyond the scope of our paper.

6   Heterogeneous Consumers and Products

We now turn to the case in which consumers are heterogeneous and have to choose between two types of products heterogeneous in their quality. Indeed, intertemporal price discrimination can seem unfair to consumers since two buyers can pay largely different prices for the same products. In particular, in the transport industry, business passengers are more likely to pay higher fares since they do not plan their trips in advance and buy their tickets when demand and prices are at their highest. For the same level of service, they might end up paying twice as much as a leisure passenger.

However, some papers in the literature argue that business passengers actually benefit from the presence of leisure passengers since the latter drive peak-demand prices down. (See Dana Jr (1999).)

The situation we model here corresponds to the case in which a revenue manager optimizes profit over a single leg, i.e. same origin-destination, but she can offer two types of products, for instance tickets for a train early in the morning or for a train in the middle of the afternoon. In this case, high quality products are tickets for the more convenient train, which we call the rush-hour train because everyone wants to take it. The less convenient train is called the off-peak train.

In this section, we focus on a transport setting and test whether or not revenue management acts as a redistributive tool (and in which direction) in a general framework in which leisure passengers with a high price elasticity have a constant arrival rate through time and business passengers with a low price elasticity have an increasing arrival rate.[8]

Figure 2 depicts the unconditional probabilities of arrival of each type of consumers when

---

[8]Business passengers can have a low price elasticity because their company covers their travel expenses for instance.

demand is low. Our model for the dynamic arrival of consumers and the extension of the Bellman equation to heterogeneous consumers are thoroughly described in Appendix D.
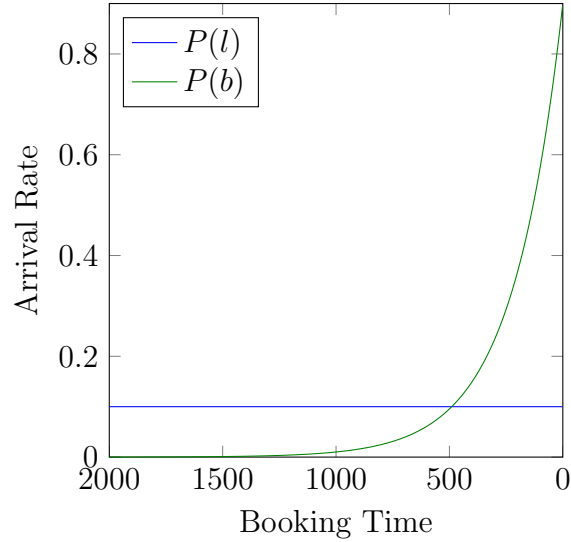

Figure 2: Change in the arrival rates of leisure and business passengers (unconditional probabilities) as a function of the booking period when the intensity of demand is low: $\mathbb{E}(\#arrival) = 400$.

Analysing revenue management as a redistribution tool requires to define different sets of parameters for each group of consumers. We consider here the case of business and leisure passengers. A summary of our simulation setup can be found in Table 7 of Appendix B, column (2). The arrival rates are defined so as to induce an expected number of arrivals over the booking period:

$$\mathbb{E}(arrivals) \in \{400, 600, 800, 1000, 1200, 1400, 1600, 1800\}$$

In each case, the number of consumers' arrivals is in expectation equally divided between business and leisure passengers. Hence, leisure passengers are relatively more frequent at the beginning of the booking period and vice versa.

Both business and leisure passengers have identical preferences except for their price

sensitivity. Using Equation 3.6, the price elasticity of demand is $\eta \approx -0.65$ for a business passenger facing a ticket for the more convenient train at price $p = 100$ and $\eta \approx -1.45$ for a leisure passenger facing a ticket for the less convenient train at price $p = 60$. As a reference, in their meta-analysis of price elasticities in U.K. rail transport industry, Wardman and Shires (2003) find that the price elasticities of demand for inter-urban trains by London business passengers range from -0.54 to -0.63 in the short-run and from -0.79 to -0.92 in the long-run. The same elasticities for the London leisure passengers respectively range from -1 to -1.17 and from -1.47 to -1.72 in the short and long-run.

We compute the optimal fixed price vector $\mathbf{p^o}$ and the associated average profit and consumer surplus. We then compute the same average profit and consumer surplus in the case of a revenue management strategy using the algorithm presented in section 4. The choice set of the revenue manager is defined by:

$$P_i = \{30, 50, 80, 100, 120, 135, 150, 165, 180, 195, 210, 230, 250, 290\}, \; i = 1, 2$$

This choice set covers a wide range of possible prices but is not optimized and has deliberately been chosen to be coarse.

The results are summarized in Tables 1 and 2. The impact of revenue management on profits is even more noticeable than in the previous sections as the effect is positive and significant whether the arrival of consumers is frequent or not.

Table 1 shows that revenue management significantly increases profits compared to an optimal fixed price strategy, even if the set of prices from which the revenue manager can choose has not been optimized. Indeed, revenue management can increase profits from 1.3% up to 13.7%, depending on the market size. The impact on profits is also larger for small market sizes, which greatly contrasts with our results on homogeneous consumers.

The variations in consumer surpluses between business and leisure passengers indicate

Table 1: Profit and consumer surplus under revenue management and a fixed price strategy for different market sizes and heterogeneous consumers.

$\mathbb{E}(arrivals)$	Pricing	$\mathbf{p^o}$	Avg. Profit	Avg. Surplus Business	Avg. Surplus Leisure
400	Fixed	135/135	16202	152	14
	RM		18421	130	86
	Gain from RM		13.7%**	−14.5%**	491%**
600	Fixed	135/135	24333	226	22
	RM		26841	207	99
	Gain from RM		10.3%**	−8.2%**	355%**
800	Fixed	135/135	32502	304	30
	RM		34763	286	95
	Gain from RM		7.0%**	−5.8%**	220%**
1000	Fixed	135/135	40624	378	38
	RM		42581	357	85
	Gain from RM		4.8%**	−5.5%**%	127%**
1200	Fixed	148/136	48186	422	34
	RM		49575	417	70
	Gain from RM		2.9%**	−1.1%**%	107%**
1400	Fixed	164/130	55655	472	35
	RM		56680	461	53
	Gain from RM		1.8%**	−2.3%**	51%**
1600	Fixed	182/140	62463	482	29
	RM		63449	479	31
	Gain from RM		1.6%**	−0.6%	6.5%
1800	Fixed	199/154	68826	479	20
	RM		69741	487	23
	Gain from RM		1.3%**	1.7%**	12%
Significance levels: † : 10%; ∗ : 5%; ∗∗ : 1%					

that revenue management benefits leisure passengers at the expense of business ones for small market sizes. However, as the market size increases, the negative impact of revenue management on business surplus disappears and the effect even becomes significantly positive. The impact on leisure surplus on the other hand is reduced and becomes non-significant. For instance, when the expected number of arrivals is 1800, profits are increased by 1.3% due to revenue management, while the impact on business and leisure passengers is positive with their respective surplus increasing by 1.7% and 12%. The impact on leisure passengers is non-significant. When the expected number of arrival is 400, revenue management increases both profits and leisure consumer surplus by respectively 13.7% and 491%, and lowers business surplus by 14.5%. Revenue management increases social welfare compared to a fixed price strategy for sufficiently high intensities of demand ($\mathbb{E}(arrival) \geq 1600$) but has a more ambiguous impact for lower intensities as the increase in profit and leisure surplus comes at the expense of business passengers.

Table 2 gives us some intuition about why revenue management affects consumer surplus: for low values of demand, revenue management actually decreases the load rates for business passengers to the benefit of leisure passengers. Indeed, the shape of the arrival of the different types of consumers allows the revenue manager to discriminate easily between them, i.e. to fill the train with leisure passengers at the beginning of the booking period and to extract business surplus as much as possible when business demand is high. Higher values of demand call for discrimination between the two trains, as shown by the levels of the optimal fixed prices. The revenue manager wants to allocate leisure passengers to the off-peak train. Revenue management makes this discriminatory process more efficient, which leads to higher load rates, thus higher surplus.

Figures 18 and 19 of Appendix H.3 give us some additional evidence about the discriminatory process. First, Figure 18 shows how often each price of the choice set P is chosen for each train. For each demand intensity, the revenue manager chooses several prices with

Table 2: Comparison of load rates between revenue management and a fixed price strategy for different intensities of demand when consumers are heterogeneous.

$\mathbb{E}(arrivals)$	Pricing	Posted Prices	Avg. Load Rate Rush-Hour Train (B)	Avg. Load Rate Rush-Hour Train (L)	Avg. Load Rate Off-Peak Train (B)	Avg. Load Rate Off-Peak Train (L)
400	Fixed	135/135	33%	4%	20%	3%
	RM		29%	20%	18%	12%
	$\Delta(load)$		$-4\%^{**}$	$16\%^{**}$	$-2\%^{**}$	$10\%^{**}$
600	Fixed	135/135	50%	7%	30%	4%
	RM		46%	24%	28%	15%
	$\Delta(load)$		$-3\%^{**}$	$18\%^{**}$	$-2\%^{**}$	$11\%^{**}$
800	Fixed	135/135	66%	9%	40%	6%
	RM		63%	24%	38%	16%
	$\Delta(load)$		$-3\%^{**}$	$15\%^{**}$	$-2\%^{**}$	$10\%^{**}$
1000	Fixed	135/135	82%	11%	50%	7%
	RM		75%	21%	51%	16%
	$\Delta(load)$		$-7\%^{**}$	$10\%^{**}$	$0.8\%^{**}$	$9\%^{**}$
1200	Fixed	148/136	89%	9%	63%	8%
	RM		84%	14%	65%	18%
	$\Delta(load)$		$-5\%^{**}$	$5\%^{**}$	$2\%^{**}$	$11\%^{**}$
1400	Fixed	164/130	91%	7%	80%	11%
	RM		90%	8%	78%	17%
	$\Delta(load)$		$-0.5\%^{**}$	$1\%^{**}$	$-2\%^{**}$	$5\%^{**}$
1600	Fixed	182/140	93%	4%	87%	10%
	RM		94%	5%	86%	10%
	$\Delta(load)$		$1\%^{**}$	$0.3\%^{**}$	$-0.7\%^{**}$	$0.8\%^{**}$
1800	Fixed	199/154	95%	3%	90%	7%
	RM		96%	3%	91%	7%
	$\Delta(load)$		$1\%^{**}$	$0.2\%^{*}$	$0.5\%^{**}$	0.1%

Significance levels: $\dagger$: 10%; $*$: 5%; $**$: 1%

positive probability through the booking period, which illustrates intertemporal price discrimination. However, for an expected number of arrivals inferior to 800, the distributions of chosen prices for the off-peak and rush-hour trains are almost identical: second-order price discrimination is suboptimal in that case. For an expected arrival greater than 1000, the distribution of prices for the rush-hour train slides to the right of the distribution for the off-peak train: differentiation between the off-peak and the rush-hour train becomes optimal when demand is sufficiently high, i.e. when resources (here sold seats) become limited.

Figure 19 displays the dynamics of fares through the booking period and gives another evidence of this mechanism. For $\mathbb{E}(arrivals) = 600$, i.e. for a small market, the probability that there remains some empty seats in each train at the end of the booking period is almost one and prices evolve in an almost completely deterministic way. Intertemporal price discrimination is only driven by the increasing arrival of high-value consumers and posted prices are *de facto* independent of past sales. The increasing scale shape of the price function comes from the revenue manager trying to extract the business surplus as the arrival rate of business passengers increases.[9] For $\mathbb{E}(arrivals) = 1800$, the price curves are smooth and have the classic revenue management shape, indicating that, as the number of available seats is probably binding, posted prices are now non-monotonic functions of the previous sales.

7   Revenue Management and Intermodal Competition

In this section, we test how competition affects revenue management practices. As transport industry is the most likely economic sector featuring competition between revenue managers, we focus on the two following cases of competition:

- Indirect competition — for instance *Rail Vs. Road*: Larger road infrastructures re-

---

[9]The small irregularities in the shape of the off-peak train price functions are due to the two opposite incentives of the revenue manager: as time goes by, business passengers are more frequent but filling the train also becomes more urgent.

duce driving costs (either time or fuel consumption costs) and thus affect demand for trains. Since the building decisions concerning these infrastructures are mainly political and have long-term impacts on the market, strategic interactions between the revenue manager and the policy maker are non-existent and we model this setup using a reduced-form approach. The level of competition in that case is adjusted via the value of the outside option. We present this part of the analysis of competition in Appendix E.

- Direct competition — for instance *Rail Vs. Air*: As both railway and airline industries use revenue management and are substitutable for distances between 400 and 1000 kilometres, decisions of both revenue managers are highly strategic and must be studied through a game-theoretic approach. We use the concept of subgame-perfect Nash equilibrium to predict probable outcomes of these repeated strategic interactions.

7.1   The Revenue Management Game

Two revenue managers play a stochastic game with finite horizon. Each revenue manager proposes homogeneous products, which can be thought of as tickets for a transport mode, e.g. a plane or train ticket. However, we allow for heterogeneity between revenue managers. In the following, a type of products i is associated with a revenue manager. In that case, $-i$ denotes the products of the other revenue manager.

We assume that consumers are not forward-looking and maximize their utility when making their purchasing decision. We model demand using a multinomial logit approach. The utility of consumers is therefore defined along the lines of section 3.3:

$$\begin{cases} u^t(\theta^i) = v + \alpha^i - \gamma p^i + \varepsilon^{it} & \text{, for product } i \text{ at price } p^i. \\ u_0^t = \varepsilon^{0t} & \text{, when the outside option is chosen.} \end{cases} \tag{7.1}$$

We also assume that at any stage of the game the number of remaining units of each type of products is common knowledge among revenue managers.

The game is defined by:

- 2 revenue managers R and A, which respectively stand for railway and airline.

- The total capacities of both types of products: $\bar{X} = (\bar{X}^R, \bar{X}^A)'$.

- The booking period of length T.

- The state space $X_t \in T \times \{0, \ldots, \bar{X}^R\} \times \{0, \ldots, \bar{X}^A\}$, which represents any possible remaining capacity X at any date t of the booking process.

- At each period, each revenue manager proposes a price in $\tilde{P} = \{\underline{p}^i, \bar{p}^i, +\infty\}$ for $i = R, A$, where $\underline{p}^i$ and $\bar{p}^i$ respectively represent a low and a high price. When a revenue manager chooses $+\infty$, her product is bought with zero probability. It is direct that for any action and state space, action $\tilde{p}^i = +\infty$ for $i = R, A$ is dominated by posting a positive and finite price, unless $X^i = 0$. Indeed, posting the high price yields the maximum possible profit with a positive probability regardless of the competitor's action. Conversely, a revenue manager cannot choose $\tilde{p}^i$ finite when $X^i = 0$. The action $\tilde{p}^i$ chosen at each period by revenue manager i when $X^i \neq 0$ is therefore any element of $\tilde{P}$ or any probability distribution over $\{\underline{p}^i, \bar{p}^i\}$

- The transition probabilities between states $Pr_t : X_t \times \tilde{P} \longmapsto X_{t-1}$, defined by:

$$
\begin{cases}
Pr_t(X_{t-1}^i = X_t^i - 1, X_{t-1}^{-i} = X_t^{-i} | X_t, \tilde{p}) = \dfrac{e^{\bar{u}(\tilde{p}^i)}}{e^{\bar{u}(\tilde{p}^i)} + e^{\bar{u}(\tilde{p}^{-i})} + 1} \\[4mm]
Pr_t(X_{t-1} = X_t | X_t, \tilde{p}) = \dfrac{1}{\sum_{i \in \{R,A\}} e^{\bar{u}(\tilde{p}^i)} + 1}
\end{cases}
, \ i = R, A; \ t = T, \ldots, 1.
$$

$$(7.2)$$

- The gain functions ϕ_t^i at date t for each revenue manager i. These gain functions correspond to the value functions in the monopolistic case. ϕ_t^i represents the profit of revenue manager i from date t onwards if strategies $(\tilde{p}_\tau)_{\tau \leq t}$ are played and remaining capacities are given by X:

$$
\begin{aligned}
\phi_t^i(X, \tilde{p}_t) &= \lambda_t \left\{ Pr(X^i - 1, X^{-i}|X, \tilde{p}) \left[\tilde{p}^i + \phi_{t-1}^i(X^i - 1, X^{-i}, \tilde{p}_{t-1})\right]\right. \\
&\quad \left. + Pr(X^i, X^{-i} - 1|X, \tilde{p})\phi_{t-1}^i(X^i, X^{-i} - 1, \tilde{p}_{t-1})\right\} \\
&\quad + \left[1 - \lambda_t + \lambda_t Pr(X|X, \tilde{p})\phi_{t-1}^i(X, \tilde{p}_{t-1})\right] \\
&= \lambda_t \left\{ Pr(X^i - 1, X^{-i}|X, \tilde{p}) \left[\tilde{p}^i + \phi_{t-1}^i(X^i - 1, X^{-i}, \tilde{p}_{t-1}) - \phi_{t-1}^i(X, \tilde{p}_{t-1})\right]\right. \\
&\quad \left. + Pr(X^i, X^{-i} - 1|X, \tilde{p}) \left[\phi_{t-1}^i(X^i, X^{-i} - 1, \tilde{p}_{t-1}) - \phi_{t-1}^i(X, \tilde{p}_{t-1})\right]\right\} \\
&\quad + \phi_{t-1}^i(X, \tilde{p}_{t-1})
\end{aligned}
\tag{7.3}
$$

Definition 1. *A subgame-perfect equilibrium of this game is defined by:*

- *Consumers maximize their utility when making their purchasing decision.*

- *At each period, each revenue manager plays her best response against the action of the other revenue manager.*

The existence of such an equilibrium is a standard result. This game can be solved by backward induction. Finding an subgame-perfect equilibrium of the game amounts to solving the following Bellman equation for $i = R, A$:

$$
V_t^i(X, \tilde{p}^{-i}) = \max_{p^i} \phi_t^{i*}(X, p^i, \tilde{p}^{-i})
\tag{7.4}
$$

in which ϕ_t^{i*} is the gain function of player i at date t when a subgame-perfect equilibrium is played in the continuation game at $t - 1$.

To deal with the multiplicity of equilibria and properly solve this game, we restrict our set of equilibria *at each continuation game* in the following way:

- If only one equilibrium in pure strategies exists, it is played.

- If several equilibria in pure strategies exist, the one maximizing the joint payoff is played.

- If no equilibrium in pure strategies exists, both revenue managers use mixed strategies.[10]

7.2 Computation of the equilibrium

The only difference between the algorithms used for the duopoly and the monopoly lies in the way we compute the value function, as the action of one player is now influenced by the action of the other. We use backward induction to solve for the equilibrium of this finite sequential game. In the last period, the continuation values are zero and we compute the best response functions of all players.

If there exists a unique equilibrium in pure strategies, we can move up to the previous period using equilibrium profits as continuation values. If there are multiple equilibria, we select one according to the selection rule mentioned above. If only a mixed strategy equilibrium exists, we compute the mixing probabilities using the formula in Appendix F.

The generation of consumers arrival and purchasing decisions is straightforward except that we also draw a realization of the mixed strategies in the case of a mixed equilibrium.

7.3 Mono-Product Duopoly Vs. Multi-Product Monopoly

To investigate the impact of competition between revenue managers, we compare profits and consumer surpluses generated by a mono-product duopoly and a multi-product monopoly.

---

[10]For a derivation of a mixed equilibrium of the game, see Appendix F.

By multi-product monopoly, we mean a situation in which one revenue manager controls the prices of both types of products. This allows us to isolate the impact of competition between revenue managers while preserving the structure of the market. As explained in more details in Appendix G, given our consumer preferences and multinomial logit approach, competition introduces a *de facto* horizontal differentiation between products, even when they have identical characteristics. Compared to a mono-product duopoly, a mono-product monopolist would therefore suppress this differentiation and reduce the total probability of purchasing a product relative to the outside option.

In the following, we assume that a homogeneous population of consumers is indifferent between the two types of products. Values of parameters used in this setup are summarized in Table 7 of Appendix B, column (4). The sets of prices from which the duopoly and monopoly can choose are identical and equal to $\{40, 90\}$. Arrival rates belong to the following set: $\lambda \in \{0.2, 0.3, 0.4, 0.5, 0.7\}$.[11]

Note that the only differentiation here between the products is horizontal and the total capacity for each type is 200 units. Therefore, the profits generated by each revenue managers are perfectly equal. In Table 3, we only report the profit of one firm and not the joint profits of the industry. However, in addition to the consumer surplus generated by each firm, we choose to report the total consumer surplus generated during the booking period. For the multi-product monopoly, we report the monopoly profit and total consumer surplus.

Table 3 shows that, compared to the duopoly, the monopoly yields higher profits and reduces the consumer surplus through lower load rates. As shown in the following section, higher monopoly prices drive these results.

---

[11]For arrival rates greater than 0.7, both means of transportation are full with probability one by the end of the booking period and results are therefore similar to the case in which $\lambda = 0.7$.

Table 3: Profit, consumer surplus, and load rates for various intensities of demand and revenue management pricing with $P = \{40, 90\}$ in the mono-product duopoly and multi-product monopoly.

	Mono-product Duopoly				Multi-product Monopoly		
λ	Avg. Profit	Avg. Surplus	Avg. Tot. Surplus	Avg. Load Rate	Avg. Profit	Avg. Surplus	Avg. Load Rate
0.2	5829	191	522	73%	13516	188	38%
0.3	9021	236	679	99%	20360	281	57%
0.4	12929	166	589	99%	27063	375	75%
0.5	16638	111	523	99%	33649	475	94%
0.7	18000	91	497	100%	36000	497	100%

Note: The average profit, consumer surplus, and load rates for the duopoly correspond to the profit, consumer surplus and load rates generated by one firm.

7.4 Price Dispersion and Competition

We study the impact of competition on price dispersion by comparing the average prices posted during the booking period and their average standard deviation for a multi-product monopoly and a mono-product duopoly. As we assume no vertical differentiation between products, results are perfectly symmetric.

Table 4 shows that competition reduces the average prices posted by revenue managers. In the monopoly, price dispersion is low as the revenue manager always posts the high price. In the duopoly, when the arrival rate of consumers is intermediate, the revenue managers have incentives to switch between the low and high prices, and price dispersion is high.

For extreme values of demand, consumers should expect a low price dispersion as the revenue managers will only post either low prices or high ones.[12]

---

[12]Note that part of these results are due to the coarse choice sets we chose. They might differ if we include the optimal fixed price in the choice sets of the revenue managers.

Table 4: Comparison of average prices and their standard deviations between a multi-product monopoly and a mono-product duopoly for different intensities of demand.

	Mono-Product Duopoly		Multi-Product Monopoly	
λ	Avg. Price	Dispersion	Avg. Price	Dispersion
0.2	40	0	90	0
0.3	45.5	14.9	90	0
0.4	64.8	24.7	90	0
0.5	84.2	13.7	90	0
0.7	90	0	90	0

8 Strategic Decisions and Revenue Management

In this section, we explore to which extent revenue management can affect the strategic decisions of an operations manager. In particular, as revenue managers base their pricing decisions considering the number of unsold units, adjusting the total capacity can have an impact on the seller's profits and the consumer surplus. Hence, in a situation of competition between two revenue managers, such as the one exposed in section 7.1, is there such a thing as an optimal capacity?

8.1 The Capacity Game in the Mono-Product Duopoly

Before competing against each other, firms using revenue management pricing may be able to choose their capacity. In the transport industry, this might translate in choosing the size of planes that an airline company affects to a given origin-destination leg.

As in section 7.1, we suppose that revenue managers maximize their profit for a given total capacity. However, we consider an additional ex-ante stage during which two strate-

gic decision makers choose their capacity constraint among a fixed set of choices $C = \{100, 150, 200, 250\}$. Solving for the equilibrium in the second stage of the game, i.e. the Revenue Management game, we get the payoff matrix and corresponding average consumer surpluses presented in Table 5 for the capacity game.

Table 5: First-stage payoffs and consumer surplus in the static duopolistic capacity game.

	Payoffs ($\times 10^3$)					Consumer surplus			
	100	150	200	250		100	150	200	250
100	9/9	9/13.4	9/14.8	9/15.1	100	250	307	385	483
150	13.4/9	13.1/13.1	12.5/13.9	11.6/14.2	150	309	380	476	579
200	14.8/9	13.9/12.5	12.9/12.8	11.9/13.2	200	382	473	591	698
250	15.1/9	14.2/11.6	13.2/11.8	12.2/12.2	250	481	582	695	826

Note: The figures presented here are averaged across 500 simulations of the revenue management game with a price set $P = \{40, 90\}$ and $\lambda = 0.4$.

Unsurprisingly, the matrix is symmetric. The unique static subgame-perfect Nash equilibrium of the game is given by $C^* = (250, 250)$, i.e. both firms choose the highest possible capacity at equilibrium. This static equilibrium yields a total consumer surplus of 826.

However, this equilibrium is not Pareto optimal since both firms would benefit from a similar increase in their profit by playing $C = (150, 150)$, i.e. by restraining the capacity.[13] If firms were able to commit to limit their capacity to 150 seats, they would be able to secure an increase in profits of 7.3%. This increase in profits is driven by a decrease in the importance of the time constraint relative to the capacity constraint. This leads to a lower opportunity cost of posting a high price and selling units more slowly.

---

[13]In fact, if we consider the repeated game, we can easily implement $(150, 150)$ as an equilibrium, using the static Nash reversion, as long as decision makers are moderately patient:

$$\frac{13.1}{1 - \delta} \geq 14.2 + \delta \frac{12.2}{1 - \delta} \Leftrightarrow \delta \geq 0.55$$

In that case, the consumer surplus strongly decreases and is equal to 380. However, this story of tacit collusion falls beyond the scope of our paper.

8.2   Comparison with the Multi-Product Monopoly

We now compare this result to the case in which the revenue manager is in a situation of monopoly and prior to the booking period chooses the capacity constraints over her two types of products. To make the comparison clear, we choose the case of a multi-product monopoly and no vertical differentiation between the two types. The results are presented in Table 6.

Table 6: First stage payoffs and consumer surplus in the static monopolistic capacity game.

	Payoffs ($\times 10^3$)					Consumer surplus			
	100	150	200	250		100	150	200	250
100	18	22.4	23.8	24.1	100	251	309	386	480
150	22.4	26.3	26.8	26.8	150	310	370	398	403
200	23.8	26.8	27	27.1	200	385	398	378	378
250	24.1	26.8	27.1	27	250	482	405	375	378

Note: The figures presented here are averaged across 1000 simulations with a price set $P = \{40, 90\}$ and $\lambda = 0.4$.

Compared to the duopoly, the situation in which the capacities on both types of products are at their maximum is no longer necessarily played. As long as at least 200 units of each type are available, profits are maximized.[14] Taking the example of transports and assuming that proposing larger trains involves larger fixed costs, the monopoly chooses to restrict the capacity.

We find an increase of 11% in overall profits compared to the duopoly. Consumer surplus on the other hand is less than half the consumer surplus when there is competition between revenue managers. Even in the case of collusion on the proposed capacity, the joint profit and consumer surplus would be similar to the ones under the monopoly, but for a reduced capacity. The welfare effects on competition between revenue managers are therefore positive.

---

[14]The differences in the last two rows and columns of Table 6 are not significant.

9   Conclusion

Revenue management is the main pricing practice in the transport industry and as such, its impact on welfare deserves to be studied. Revenue management in itself has a number of properties which affect profits and consumer surplus compared to other pricing strategies. By offering more leeway to the seller in her choice set, it weakly increases profits compared to an optimal fixed price strategy. We also show in our computations that revenue management as an intertemporal price discrimination practice is only useful when there is a sufficiently high probability that profits are constrained by the number of available units. In this case, revenue management implies a higher consumer surplus compared to a situation in which only the optimal fixed price is posted. Such an increase comes from higher load rates and weakly lower average prices under revenue management. Coming back to transports, revenue management can increase welfare because more potential passengers can afford to travel. Revenue management is also a useful way to optimally respond to demand without actually having to optimize profits over the proposed set of prices. Even using a coarse set of prices to choose from and for different intensities of demand, the revenue manager succeeds in achieving at least as well as an optimal fixed price.

Applying our methodology to heterogeneous consumers, we are able to shed some light on the controversial issue of the *fairness* of revenue management practices. We find that for high values of demand, revenue management benefits all types of consumers. When demand is low, consumers with low price elasticities subsidize the ones with high price elasticities. Applied to transports, when demand is high, revenue management allows leisure passengers to buy cheap tickets without hurting business passengers. But, when demand is low, business passengers no longer benefit from the relatively lower prices they would pay under a fixed price policy.

In the second part of the paper, we study competition between revenue managers and

find classic results: compared to a multi-product monopoly, revenue management in a mono-product duopoly lowers the average price and increases consumer surplus through higher load rates.

We also consider a setup in which revenue managers can choose their total capacity before competing against each other. We find that compared to the monopolistic case, competition between revenue managers does not reduce by much the joint profit of the industry but allows consumer surplus to be twice as high.

References

ANDERSON, S., A. DE PALMA, AND J. THISSE (1989): "Demand for differentiated products, discrete choice models, and the characteristics approach," *The Review of Economic Studies*, 56, 21.

BEN-AKIVA, M. E. AND S. R. LERMAN (1985): *Discrete choice analysis: theory and application to travel demand*, vol. 9, MIT press.

DANA JR, J. (1999): "Using yield management to shift demand when the peak time is unknown," *The Rand Journal of Economics*, 456–474.

FUDENBERG, D. AND D. LEVINE (1988): "Open-loop and closed-loop equilibria in dynamic games with many players," *Journal of Economic Theory*, 44, 1–18.

GALLEGO, G. AND G. VAN RYZIN (1997): "A multiproduct dynamic pricing problem and its applications to network yield management," *Operations Research*, 45, 24–41.

GERSHKOV, A. AND B. MOLDOVANU (2009): "Dynamic revenue maximization with heterogeneous objects: A mechanism design approach," *American Economic Journal: Microeconomics*, 1, 168–198.

GOODWIN, P. B. (1992): "A review of new demand elasticities with special reference to short and long run effects of price changes," *Journal of transport economics and policy*, 155–169.

HÖRNER, J. AND L. SAMUELSON (2011): "Managing Strategic Buyers," *The Journal of Political Economy*, 119, 379–425.

LAZAREV, J. (2012): "The welfare effects of intertemporal price discrimination: an empirical analysis of airline pricing in US monopoly markets," *New York University, working paper*.

McAfee, P. and V. Te Velde (2006): "Dynamic pricing in the airline industry," *forthcoming in Handbook on Economics and Information Systems, Ed: TJ Hendershott, Elsevier.*

McAfee, R. and V. Velde (2008): "Dynamic pricing with constant demand elasticity," *Production and Operations Management*, 17, 432–438.

McGill, J. and G. Van Ryzin (1999): "Revenue management: Research overview and prospects," *Transportation science*, 33, 233.

Modarres M., B. E. (2009): "Dynamic Pricing with Periodic Review and a Finite Set of Prices with Cancellation," *Journal of Industrial and Systems Engineering (JISE)*.

Stokey, N. (1979): "Intertemporal price discrimination," *The Quarterly Journal of Economics*, 355–371.

Talluri, K. and G. Van Ryzin (2004): "Revenue management under a general discrete choice model of consumer behavior," *Management Science*, 15–33.

——— (2005): *The theory and practice of revenue management*, vol. 68, Springer Verlag.

van Ryzin, G. and G. Vulcano (2008): "Computing virtual nesting controls for network revenue management under customer choice behavior," *Manufacturing & Service Operations Management*, 10, 448–467.

Vulcano, G., G. van Ryzin, and W. Chaar (2010): "OM Practice—Choice-Based Revenue Management: An Empirical Study of Estimation and Optimization," *Manufacturing & Service Operations Management*, 12, 371–392.

Wardman, M. and J. Shires (2003): "Review of fares elasticities in Great Britain," .

Williams, K. R. (2013): "Dynamic Airline Pricing and Seat Availability," Tech. rep., mimeo.

A Notes on the Theoretical Model

Our model of revenue management is in the spirit of Talluri and Van Ryzin (2004) who study revenue management in the transport industry. But in their model one product consists of a seat and a price. Therefore the same seat proposed at two different prices actually constitutes two different products. In contrast, we consider that the price is a separate variable.[15] Modelling the price as an additional variable implies that the revenue manager never proposes the exact same product at two different prices. This is useful when we allow for some randomness in the preferences of consumers. Indeed in Talluri and Van Ryzin (2004), the revenue manager can simultaneously propose the same product at two different prices, and when demand is simulated using a multinomial logit approach, consumers choose the highest price with a positive probability, which is unrealistic.

An additional difference is that we allow for different products to have different capacity constraints. Taking the example of two flights departing the same day, once all the tickets for the first one have been sold, the revenue manager can only propose tickets for the second one.

We show that introducing physically constrained characteristics has an impact on the revenue manager's strategy. Talluri and Van Ryzin (2004), show that the revenue manager only proposes menus on the efficient frontier when optimizing her profit function. For them, a menu is efficient if linear combinations of other menus cannot do better in terms of expected revenue $\mathbb{E}(\pi|\tilde{p})$ except by increasing the total probability of purchase. We show that this is not necessarily the case in our model. We take the example of two types of products, each limited to 200 units. Possible prices for each product are $P = \{50, 70, 90, 110\}$. Consumer preferences are defined as in section 5. In Figure 3, we plotted all combinations of

---

[15]We can consider that in Talluri and Van Ryzin (2004), the revenue manager can only control which types of products to propose. Our revenue manager has control over two dimensions: which types of products to propose and at what price.
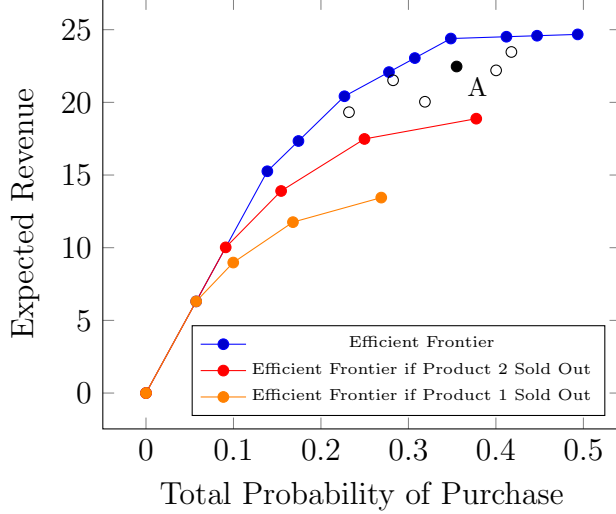
Figure 3: Total probability of purchase (horizontal axis) and the associated expected revenue (vertical axis) of the different possible menus of prices. Efficient frontiers are represented by plain lines. Menu A, which is not on the efficient frontier, is played 12.9% of the time when the arrival rate of the consumer is $\lambda = 0.6$.

products and possible prices according to their total probability of purchase and expected revenue. Product-price combinations that are efficient according to Talluri and Van Ryzin (2004) are represented by coloured dots and linked together. According to the theoretical result of Talluri and Van Ryzin (2004), other combinations should not be proposed by the revenue manager. However, by computationally solving for the optimal pricing of the revenue manager, when $\lambda = 0.6$, the revenue manager proposes an inefficient menu of prices, $(90, 50)$, in 12.9% of the periods of the booking process. This percentage corresponds to an average over 1000 simulations. We attribute this to the fact that we allow for several capacity constraints.

B   Values of Parameters

In Table 7, we summarize the values we use for consumer preferences and train characteristics. Simulations (1) refer to the simulations in the distributive properties of the revenue

42

management. Simulations (2) refer to the case of heterogeneous consumers. Simulations (3) and (4) respectively stand for indirect and direct competition. Finally, simulations (5) are used in robustness checks presented in Appendix **??**.

Table 7: Summary of parameters' values used in the different simulations.

	(1)	(2)	(3)	(4)	(5)
# trains	1	2	1	2	2
X_i	400	200	400	200	200
v	1.5	1.5	1.5	1.5	1.5
α	0	-0.5	0	0	-0.5
γ	0.03	$\gamma^B = 0.01$; $\gamma^L = 0.03$	0.03	0.03	0.03
v_0	1	1	$\{0.1, 0.5, 1, 1.5, 2\}$	1	1
T	2000	2000	2000	2000	2000
# simulations	1000	1000	1000	500	1000

C Discussion of Revenue Management Vs. Optimal Fixed Price

We define the load rate as the ratio between the number of sold units over the total number of available units of the same type. If we take the example of a plane with load rates of 100% for first-class tickets and 80% for economy seats, it means that all seats in first class were booked and 80% of the economy seats in the plane were booked. It is therefore a measure of how full the plane is. Compared to the fixed price strategy, the increase in consumer surplus is driven by higher amounts of sold units or load rates. This is due to the finer choice set of the revenue manager which allows her to post higher or lower prices depending on the past sales. For instance, for $\lambda = 0.9$, when the spread around the optimal price is equal to

5, the revenue manager posts the lowest and highest prices with a relatively high frequency: respectively 21% and 17% of the time. Of course, the larger the spread, the smaller the frequencies at which the lowest and highest prices are posted since they become dominated by the fixed price.

This seems to indicate that revenue management is especially useful for large market sizes (at least when consumers are homogeneous) because in that case it allows the revenue manager to temporarily decrease the posted price when there is no sales and to take some risk in the other case. When demand is low and it is nearly impossible to sell all available units, sales variations never reach the point which induces the revenue manager to change the optimal fixed price. Intertemporal discrimination in this case becomes useless as the changes in the probability of reaching the capacity constraint are negligible compared to the coarseness of the price grid.

Figure 4 displays the average posted price as a function of the booking period for different intensities of demand and a small spread, $\xi = 7$. Obviously, the average level of posted prices increases with the arrival rate of consumers. We also observe that the optimal fixed price is played with probability 1 at the beginning of the booking period. Then, in the middle of the booking period, the average posted price is roughly equal to the optimal fixed price, with some noise or without, respectively when $\lambda = 0.7, 0.9$ and when $\lambda = 0.3$. When $\lambda = 0.5$, it is even slightly higher. This indicates that deviations from the optimal fixed price may occur if the market is sufficiently large, but these deviations might be centred around the optimal fixed price, i.e. the revenue manager might have on average equal incentives to lower or increase the price. At the end of the booking period, the revenue manager tends to post the lowest price more often. This is consistent with the incentives derived from the theoretical model.

Remark 3. *To test whether this result is robust to a more complete choice set, we do the same*
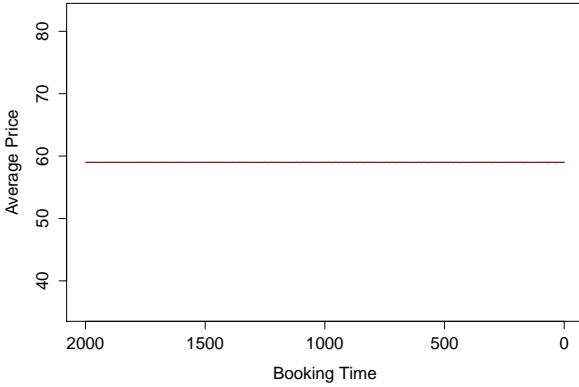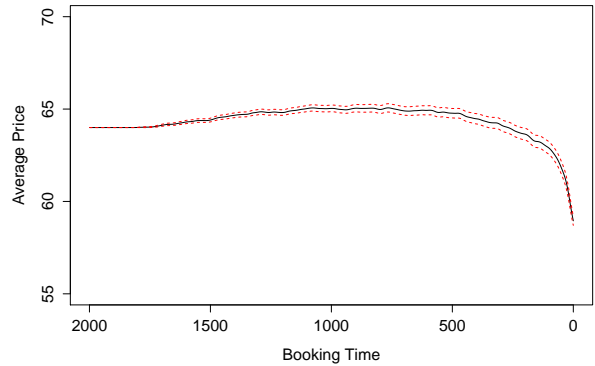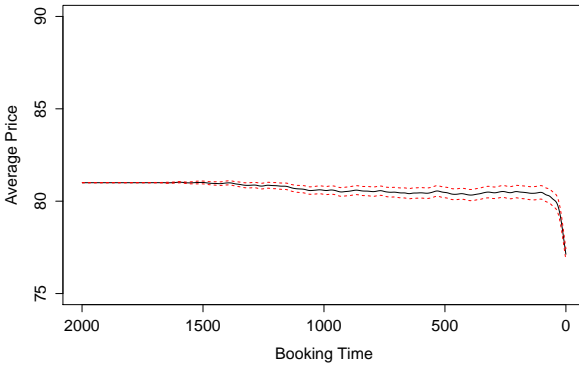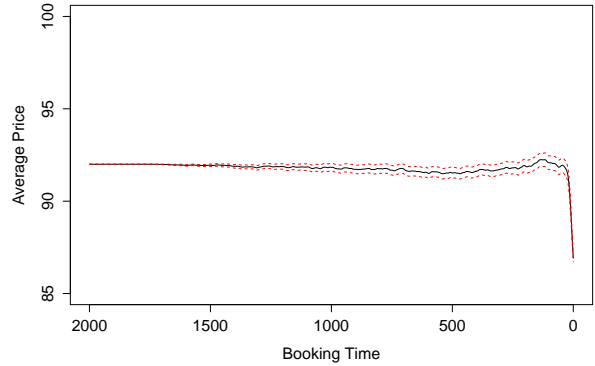
Figure 4: Change in posted average prices as a function of the booking period when the price set is $P = \{p^o - 7, p^o, p^o + 7\}$. Figures (a),(b),(c),(d) deal with increasing intensities of demand, corresponding to an expected arrival of respectively 600, 1000, 1400, and 1800 consumers.

exercise allowing the choice set of the revenue manager to be richer. In these simulations:

$$P = \{p^o, p^o \pm 5i | 1 \leq i \leq \xi\} \;\; for \; \xi = 1, 2, \ldots$$

As shown by Figures 8 and 9 of Appendix H.2, we find similar qualitative results in that case. Consumer surplus does not decrease even if the action set of the revenue manager is richer. The average price throughout the booking period is again very close to the average optimal price (within 1 unit) but the load rates are significantly higher when λ is sufficiently high and the revenue manager actually discriminates. For instance, at $\lambda = 0.9$, the load rate under revenue management is around 99.7% against 98% for optimal pricing.

All these findings are robust to a more general specification including different substitutable types of products.

In a more realistic framework, the revenue manager has to optimize her profit over two types of products from which consumers can choose. We want to test whether or not adding more flexibility in the choices of consumers affects the comparison between revenue management and a fixed price strategy. We suppose that the choice set P of prices available to the revenue manager remains identical between the two types of products. In the case of a fixed price strategy, we also assume that the proposed price is identical for both types. In transports, this could correspond to a situation in which the revenue manager is required to set an identical price for all transports running on a particular origin-destination leg. The parameters used in this section can be found in Table 7 of Appendix B, column (5). $\alpha = -0.5$ means that consumers now prefer product 1 over 2. For instance, if the two types of products refer to two different flights the same day on the same origin-destination leg, this can be interpreted as one flight being more convenient than the other.

We test whether or not revenue management is an improvement over a fixed price strategy and measure to which extent revenue management including the optimal fixed price raises

profits and affects consumer surplus.

To do so, we compute the optimal fixed price p^o for the two types of products and the simulated average profit and consumer surplus associated with this price. We then construct the choice set P of the revenue manager to include the optimal fixed price and some small variations around it:

$$P = \{p^o - \xi, p^o, p^o + \xi\}_{i=1,2} \; \xi = 1, 2, \dots$$

The revenue manager's choice set is again small but Figures 10 and 11 of Appendix H.2 show that revenue management strongly increases both profits and consumer surplus. In fact, all the conclusions we found for one type of products seem to extend to this case. For small market sizes, the effect is not significant. For $\lambda \geq 0.4$, revenue management increases both profits and consumer surplus. For some spread values, profits rise from 4% for $\lambda = 0.5$ up to 12% for $\lambda = 0.9$. This corresponds to large increases in consumer surplus: between 20% and 30% for the same spread values.

The importance of the impact on profit in this case is easily explained by the way we construct the optimal fixed price: we consider a unique fixed price for two types of products, i.e. rule out any possibility to price discriminate between the two types even though one is more attractive than the other. Introducing revenue management offers this possibility despite identical choice sets for the two types. This gives more leeway to the company and considerably raises profits. However, price discrimination between the two types is only profitable when the arrival rate is sufficiently high.

What is more surprising is the huge positive impact on consumer surplus. One might expect that price discrimination between the two types of products allows the firm to extract consumer surplus more easily. We attribute this result to the increase in load rates: more consumers can afford to purchase a product, especially at the beginning of the booking

period. For instance, for $\lambda = 0.5$, 88% of the seats of the less attractive product are sold under revenue management with a spread $\xi = 15$ whereas only 61% of the seats are sold under a fixed price strategy.

Remark 4. *To isolate the sole impact of revenue management as intertemporal price discrimination from the impact of price discrimination between types of products, we also carry out an analysis in which we compute an optimal fixed price for each type. The resulting price vector is denoted* $\mathbf{p^o}$. *We then construct the choice set P of the revenue manager to include for each type, its optimal fixed price and some small variations around this price:*

$$P = \{p_i^o - \xi, p_i^o, p_i^o + \xi\}_{i=1,2} \; \xi = 1, 2, \dots$$

In this case, the results are similar in nature although the importance of the impact of revenue management is much weaker, as seen in Figures 12 and 13 of Appendix H.2. For $\lambda \geq 0.4$, the increase in profit lies between 0.8% and 1.7%, with a more pronounced impact for high intensities of demand. For the same arrival rates, the increase in consumer surplus varies between 1% and 2.7%.

D   Extension to Heterogeneous Consumers

D.1   Increasing Arrival Rates

To simulate two types of consumers randomly arriving at each period, we first have to simulate the probability that one consumer arrives at a given period, then draw the type. Indeed, we cannot have two consumers arriving during the same period. Since in the application to transports, the business type has a probability of arrival increasing as time comes closer to departure and the arrival probability of the leisure type is constant, the overall probability of arrival must increase as well.

Considering this, we define the probability of arrival of one consumer as : $\lambda_t(\rho_l, \rho_{b_t}) = \frac{\rho_l + \rho_b}{\rho_l + 1}$, where ρ_l and ρ_b are the relative rates of arrival of respectively leisure and business passengers. The probabilities of being of type $\tau(= b, l)$ conditional on an arrival are defined as:

$$P(\tau | arrival) = \frac{\rho_\tau}{\rho_\tau + \rho_{-\tau}} \qquad \tau = b, l$$

The unconditional probability of a type-τ arrival is therefore given by:

$$P(\tau) = P(\tau \cap arrival) = \frac{\rho_\tau}{\rho_l + 1} \qquad \tau = b, l$$

We find this way of modelling arrivals convenient since it implies that for all values of ρ_l, $\lambda_t(\rho_l, .)$ goes to 1 as ρ_b goes to 1, which allows us to model a very intense demand towards the end of the booking period.

We want the arrival rate of leisure types to be constant and the one of business types to increase in time. Hence ρ_b has to vary across time. We model it as a discretized version of an exponential distribution:

$$\rho_{b_t} = e^{-\frac{t}{\mu}} \qquad \text{for } t = T, \dots, 1$$

Here, μ is a large number which we use to parametrize the curvature of ρ_{b_t}. As t is large, i.e. far from the departure date, ρ_{b_t} is close to zero and tends to 1 as t goes to 1.

Table 8 gives the correspondence between the expected number of arrivals we want to model and our two parameters, μ and ρ_l.

Table 8: Correspondence between the expected number of arrivals, μ, and ρ_l.

$\mathbb{E}(arrivals)$	400	600	800	1000	1200	1400	1600	1800
μ	223	355	511	710	988	1430	2290	4815
ρ_l	$\frac{1}{9}$	$\frac{3}{17}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{7}$	$\frac{7}{13}$	$\frac{2}{3}$	$\frac{9}{11}$

D.2 The Modified Bellman Equation

The Bellman equation of section 3 naturally extends to this new set-up:

$$V_t(X) = \max_{\tilde{p} \in \Pi_i P_i} \sum_{\tau \in \{b,l\}} \sum_i Pr_t(\tau) Pr(d_i = 1|\tilde{p}, \tau) \left[\tilde{p}_i + V_{t-1}(X_i - 1, X_{-i})\right]$$

$$+ \left[1 - \lambda_t + Pr(l)Pr\left(d_0 = 1|\tilde{p}, \tau = l\right)\right.$$

$$\left. + Pr_t(b)Pr\left(d_0 = 1|\tilde{p}, \tau = b\right)\right] V_{t-1}(X) \tag{D.1}$$

subject to the final constraints:

$$\begin{cases} V_0(X_i) = 0 & , \forall X_i \\ V_t(X) = 0 & , \text{ whenever } \sum_i X_i = 0 \\ \tilde{p}_i = +\infty & , \text{ if } X_i = 0 \end{cases} \tag{D.2}$$

The demand mechanism is modelled as before except that coefficients in the utility functions are type-dependent.

E A Reduced-Form Model of Indirect Competition

To evaluate the impact of indirect competition on revenue management, we study how average profits, consumer surplus and load rates change with the value of the outside option.

E.1 Demand Modelling with Variable Outside Option

The consumer utility when she chooses the outside option is now: $u_0 = \ln(v_0) + \varepsilon_0$, in which $v_0 \in \mathbb{R}^{+*}$ is a parameter capturing the competition intensity and ε_0 is the random part of the utility defined as before. In the benchmark model of section 3.3, $v_0 = 1$. The higher v_0, the higher the intensity of indirect competition.

This new specification yields the following market shares:

$$\begin{cases} Pr(d_i = 1 | \tilde{p}) = \dfrac{e^{\bar{u}(p_i)}}{\sum_{j \in I} e^{\bar{u}(\tilde{p}_j)} + e^{\ln(v_0) + \mathbb{E}(\varepsilon_0)}} = \dfrac{e^{\bar{u}(p_i)}}{\sum_{j \in I} e^{\bar{u}(\tilde{p}_j)} + v_0} \\ \\ Pr(d_0 = 1 | \tilde{p}) = \dfrac{1}{\sum_{j \in I} e^{\bar{u}(\tilde{p}_j)} + e^{\ln(v_0) + \mathbb{E}(\varepsilon_0)}} = \dfrac{v_0}{\sum_{j \in I} e^{\bar{u}(\tilde{p}_j)} + v_0} \end{cases} \tag{E.1}$$

It is obvious that:

$$\lim_{v_0 \to 0} Pr(d_0 = 1 | \tilde{p}) = 0 \text{ and } \lim_{v_0 \to \infty} Pr(d_0 = 1 | \tilde{p}) = 1$$

The other features of the model remain the same.

E.2 Values of Parameters and Results

Values of parameters used in this setup are summarized in Table 7 of Appendix B, column (3). The arrival rates are the same as before, i.e. $\lambda \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$, and the choice set of the revenue manager is given by $P = \{20, 40, 60, 80, 100, 120, 140, 160\}$.

Figures 20 and 21 of Appendix H.4 summarize our results for indirect competition: as competition increases, profits go down while consumer surplus increases. Since we model an increase in competition as an increase in the value of the outside option, these results are quite intuitive: a higher outside option means that the revenue manager's products are less attractive (hence less profits) whereas it also mechanically raises the final consumer surplus

through the utility of consumers who do not purchase.

We also look at the impact of indirect competition on average prices posted during the booking period and on price dispersion. These results are summarized in Figure 22 of Appendix H.4. Unsurprisingly, stronger indirect competition drives prices down. However, its impact on price dispersion is more ambiguous: for low arrival rates, price dispersion is higher when competition is weak; the opposite is true for high arrival rates.

F   Mixed Equilibria

In a mixed equilibrium, players randomize over their action set, here: $\tilde{P} = \{\underline{p}^i, \bar{p}^i, +\infty\}$, $i = R, A$. We have shown that in this special case, $\tilde{p}^i = +\infty$ is always dominated by the other prices unless $X^i = 0$. Therefore, any mixed equilibrium of a continuation game at t is a randomization over $\{\underline{p}^i, \bar{p}^i\}$ for $i = R, A$. In the following, we denote $\sigma^i = P(\tilde{p}^i = \underline{p}^i)$ and by abuse of notation σ^i also denotes the action of player i when she randomizes.[16]

Finding an equilibrium in mixed strategies of the continuation game at t amounts to find σ^i for $i = R, A$:

Suppose i plays σ^i. Then, $-i$ needs to be indifferent between $\underline{p}^{-i}$ and $\bar{p}^{-i}$ to play a mixed strategy, which is summarized by the following condition:

$$\phi^{-i*}_t(X, \sigma^i, \underline{p}^{-i}) = \phi^{-i*}_t(X, \sigma^i, \bar{p}^{-i}) \tag{F.1}$$

where:

$$\phi^{-i*}_t(X, \sigma^i, p) = \sigma^i \phi^{-i*}_t(X, \underline{p}^i, p) + (1 - \sigma^i)\phi^{-i*}_t(X, \bar{p}^i, p)$$

---

[16] Although any pure strategy is a mixed strategy, here we refer to *mixed strategies* if and only if $\sigma^i \in (0, 1)$

Equation F.1 therefore yields:

$$\sigma^i = \frac{\phi_t^{-i*}(X, \vec{p}^i, \bar{p}^{-i}) - \phi_t^{-i*}(X, \vec{p}^i, \underline{p}^{-i})}{\phi_t^{-i*}(X, \vec{p}^i, \bar{p}^{-i}) - \phi_t^{-i*}(X, \underline{p}^i, \bar{p}^{-i}) + \phi_t^{-i*}(X, \underline{p}^i, \underline{p}^{-i}) - \phi_t^{-i*}(X, \vec{p}^i, \underline{p}^{-i})} \qquad \text{(F.2)}$$

G   Multinomial logit approach and horizontal differentiation

Here are more detailed explanations about why overall demand for the proposed products increases if we introduce an additional vertically undifferentiated choice. Assume that given a vector of price p, the utility of buying any product is given by u whether we are in the monopolistic or the duopolistic case.

Then, in the monopolistic case, the probability of buying a product is:

$$p^m(buy|p) = \frac{e^u}{e^u + 1} = 1 - \frac{1}{e^u + 1}$$

In the duopolistic case, the probability of taking of choosing the product of one firm (e.g. firm 1) is given by:

$$p^d(buy\,1|p) = \frac{e^u}{2e^u + 1}$$

which is of course lower than the probability of buying a product in the monopoly. However, the overall probability of buying in the duopoly is given by $1 - \frac{1}{2e^u+1}$, which is higher than in the monopoly.

To give some economic intuition to this result, we say that adding an additional type of products, even vertically undifferentiated, can create horizontal differentiation.

H   Simulation Results

H.1   Dynamics of Prices
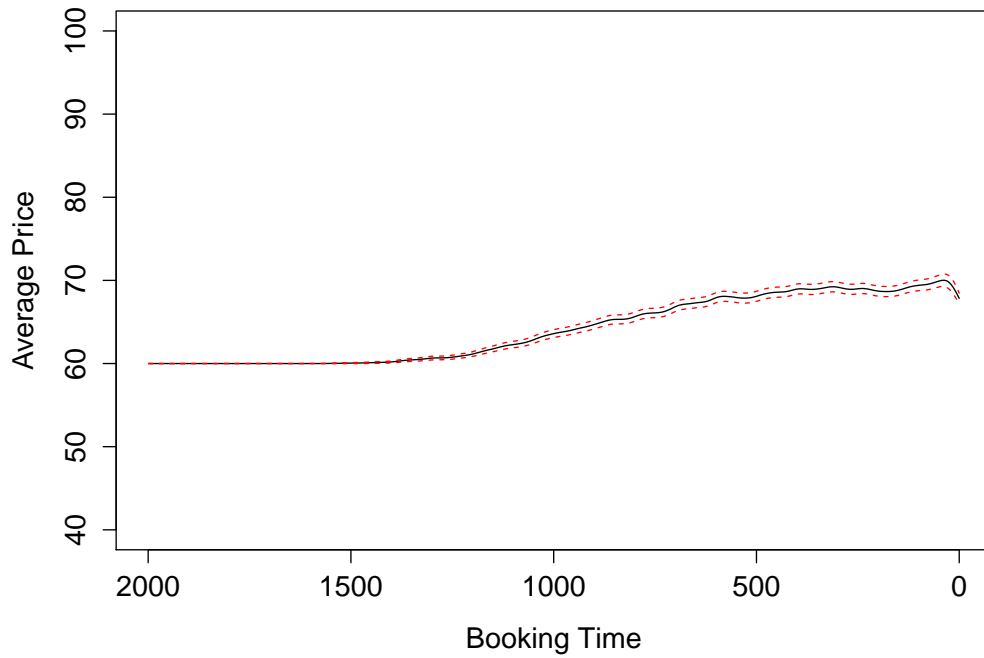
Figure 5: Average change in prices when $\lambda = 0.5$ and the revenue manager chooses the price from $\{20, 40, 60, 80, 100\}$.

H.2 The Distributive Properties of Revenue Management
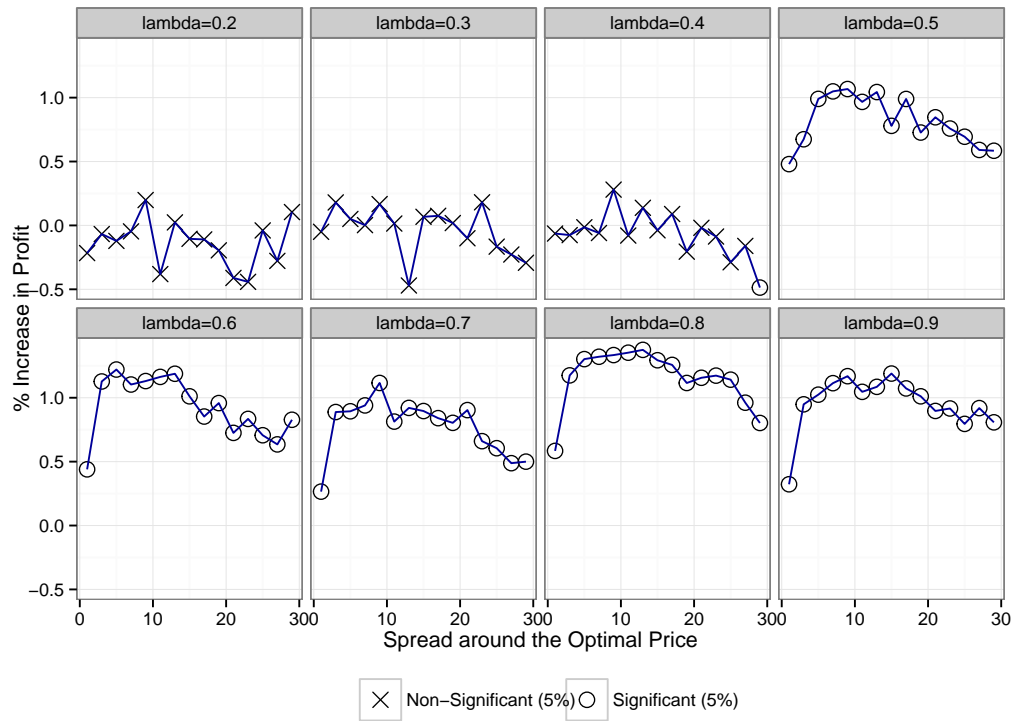
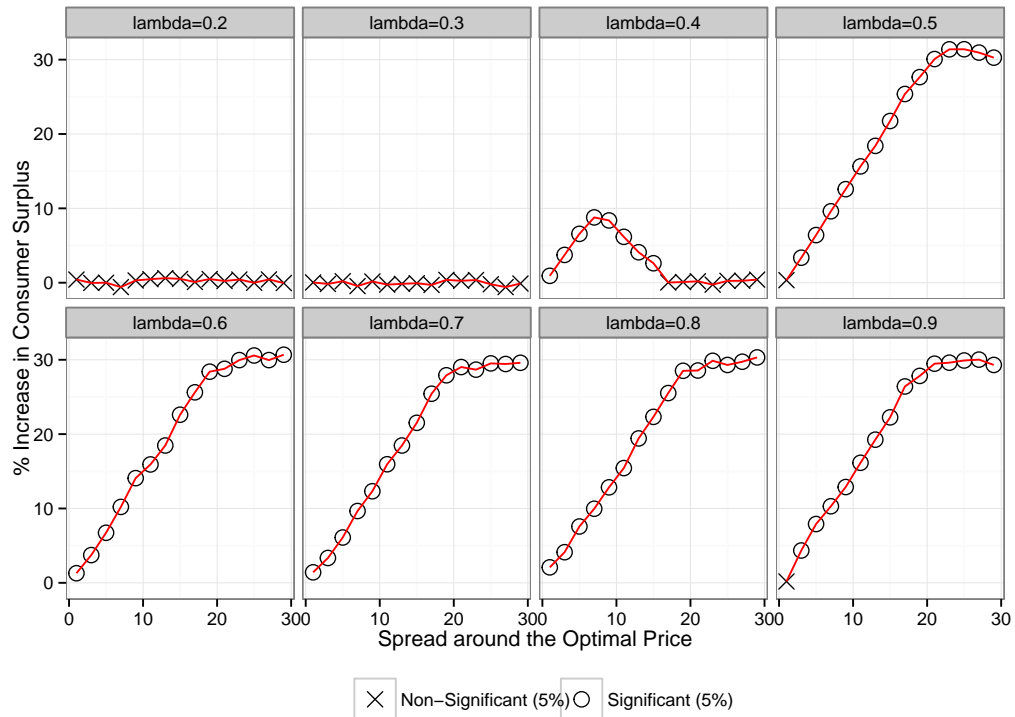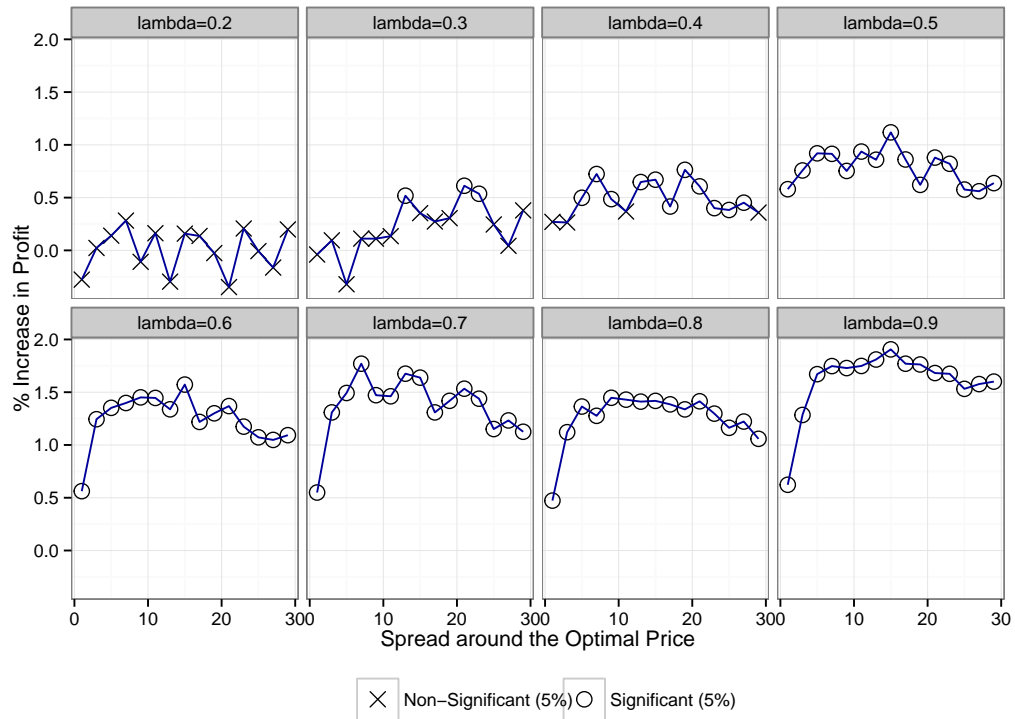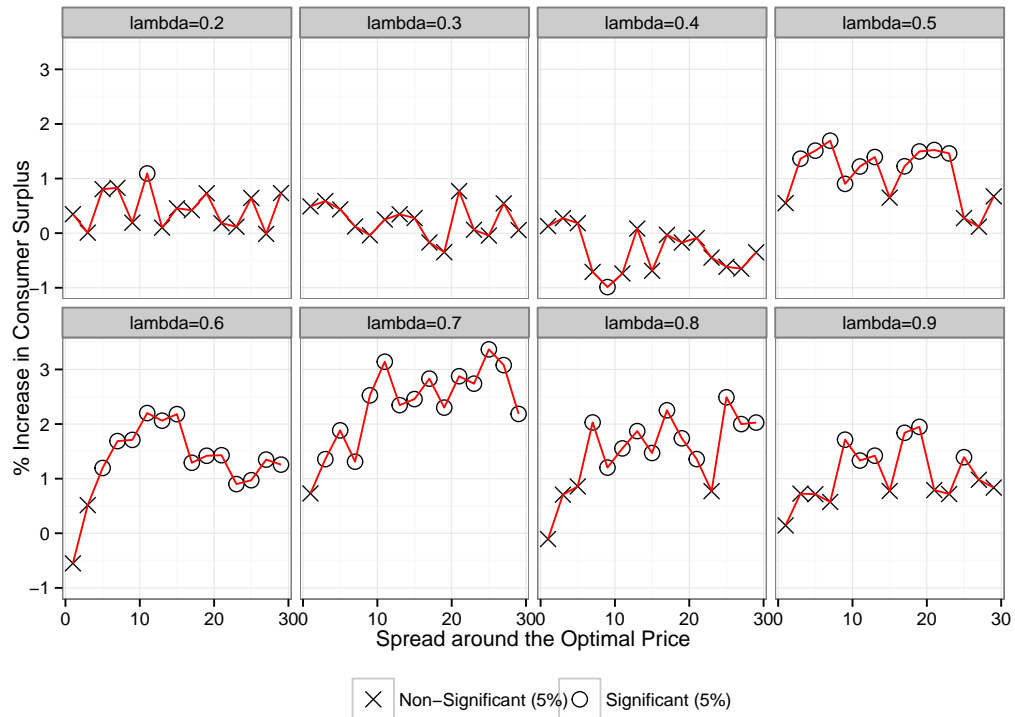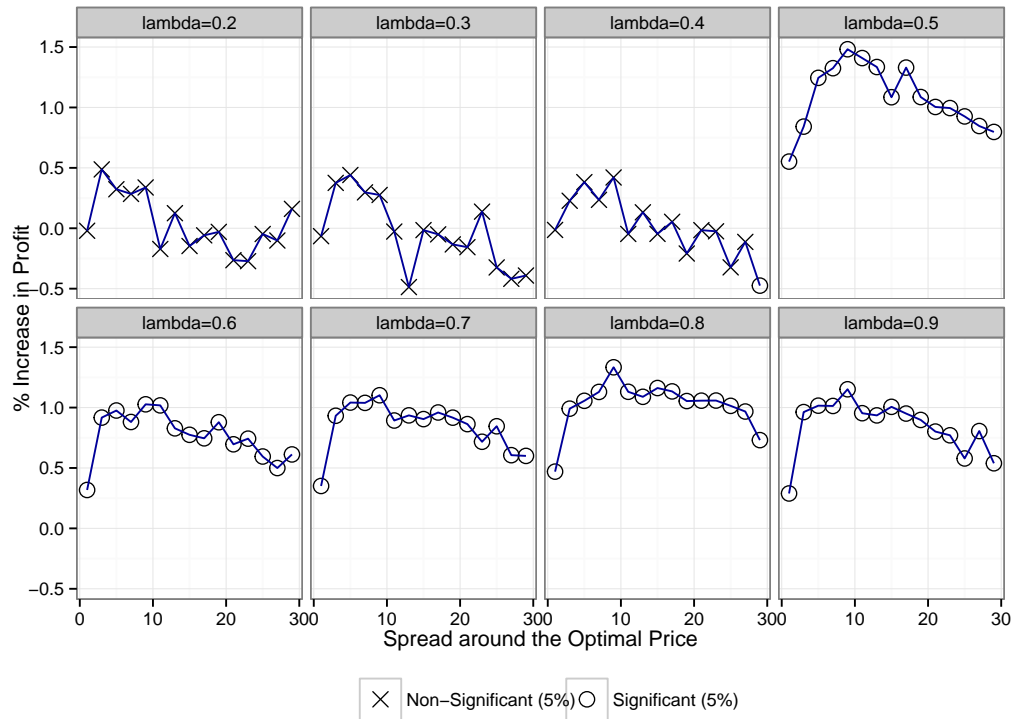
Figure 6: Average change in profit between optimal fixed pricing and revenue management for different intensities of demand for homogeneous products.
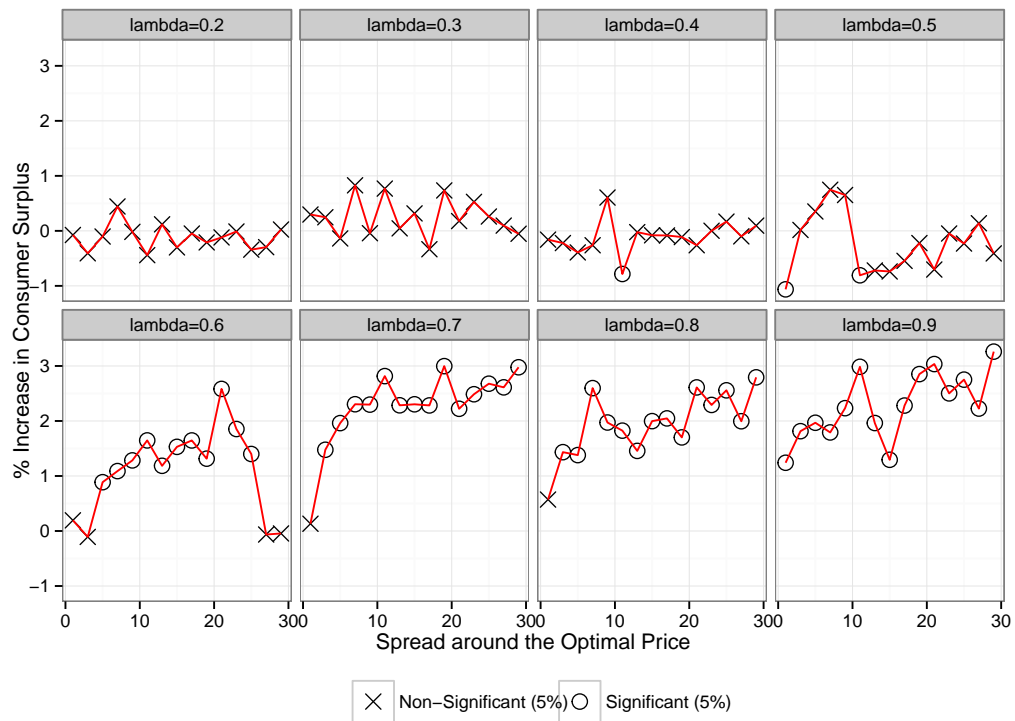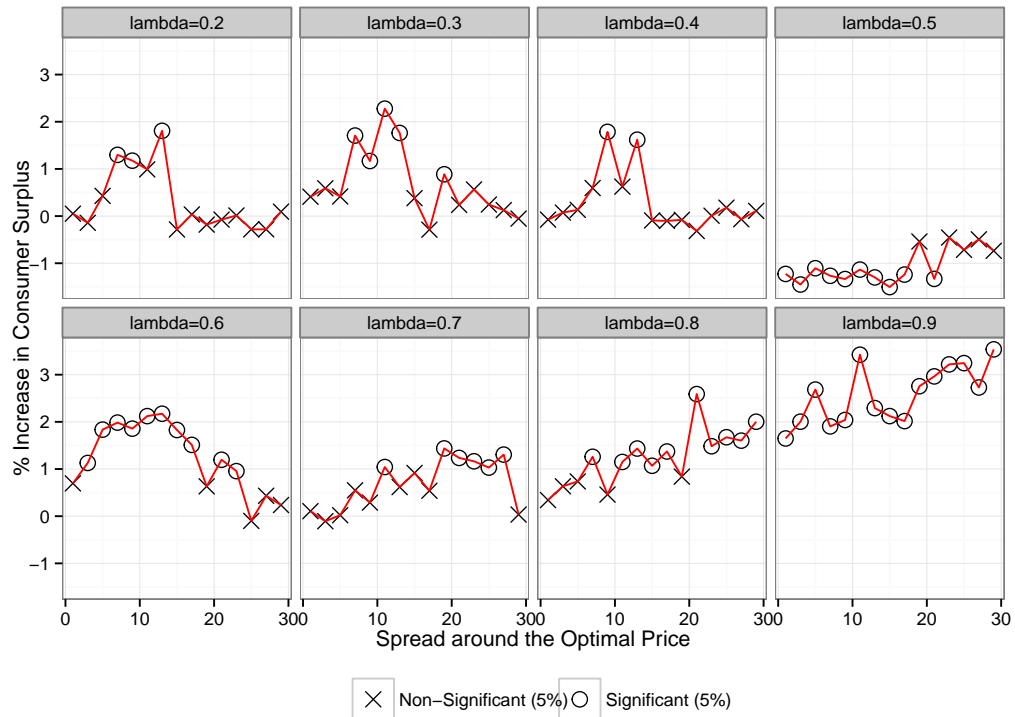
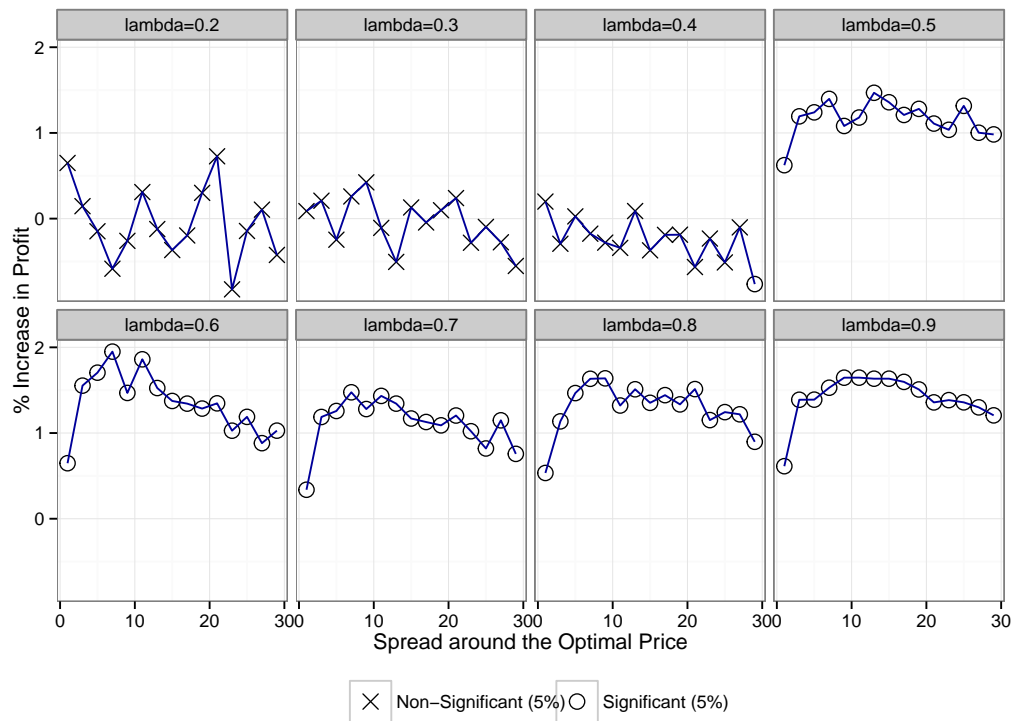Figure 7: Average change in consumer surplus between optimal fixed pricing and revenue management for different intensities of demand for homogeneous products.
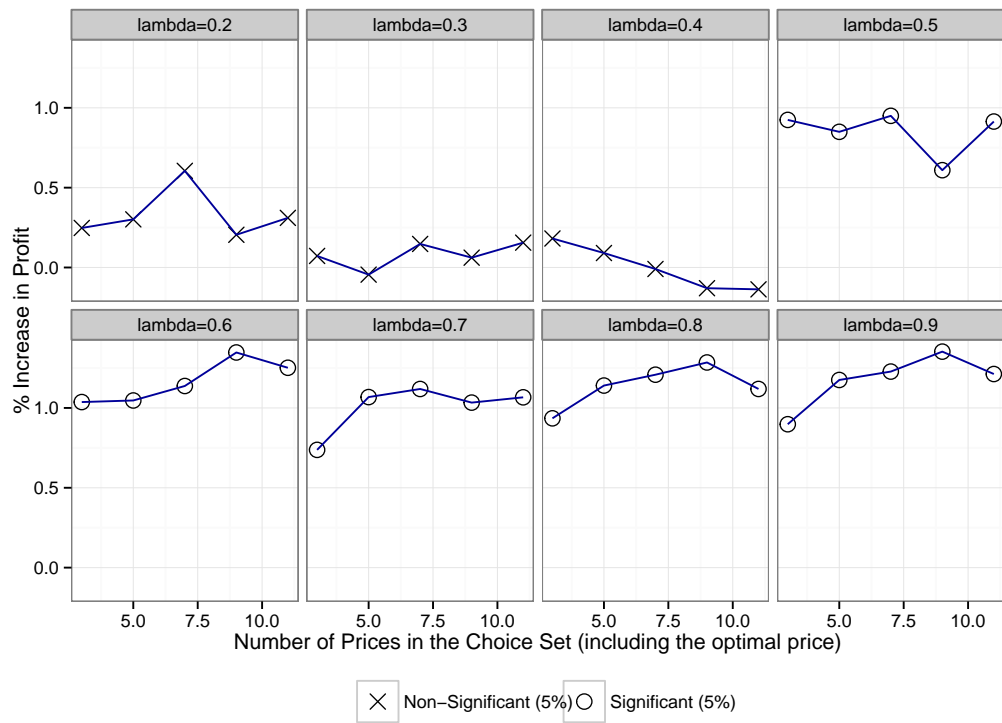
Figure 8: Average change in profit between optimal fixed pricing and revenue management for different intensities of demand for homogeneous products and various sizes of choice sets.
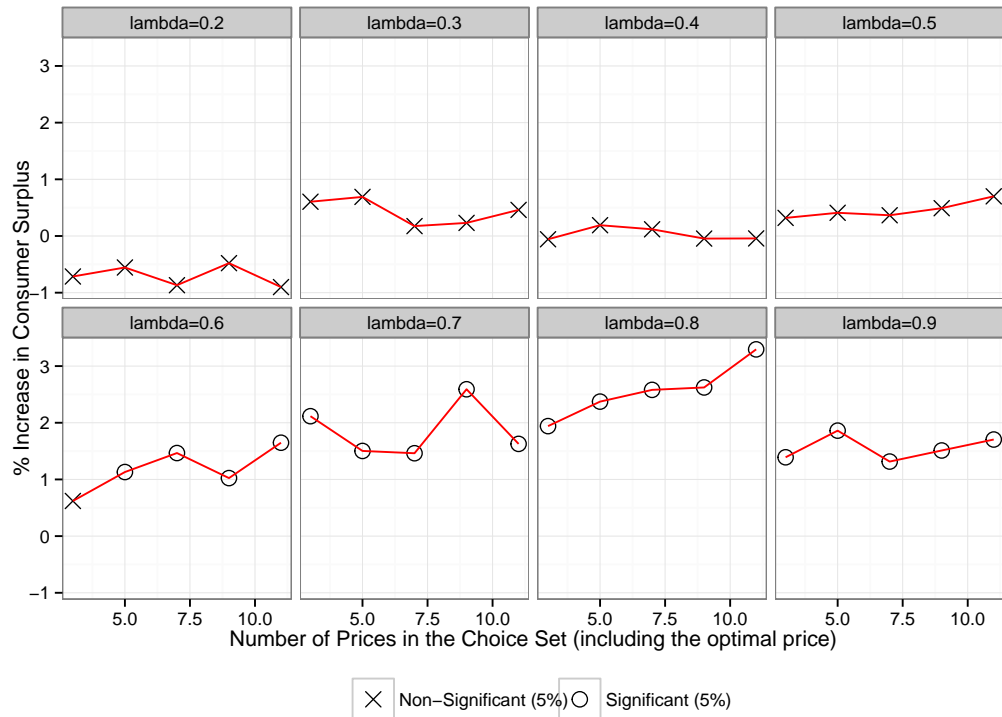
Figure 9: Average change in consumer surplus between optimal fixed pricing and revenue management for different intensities of demand for homogeneous products and various sizes of choice sets.
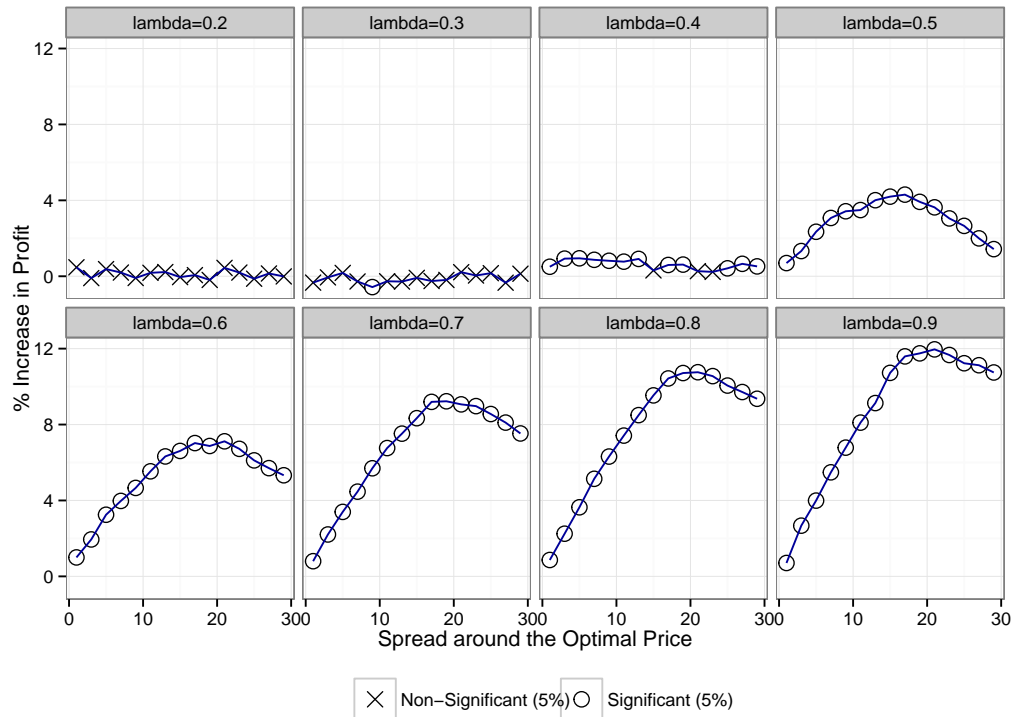
Figure 10: Average change in profit between optimal fixed pricing and revenue management for different intensities of demand and two types of products. The optimal fixed pricing consists here of a unique optimal price for all products.
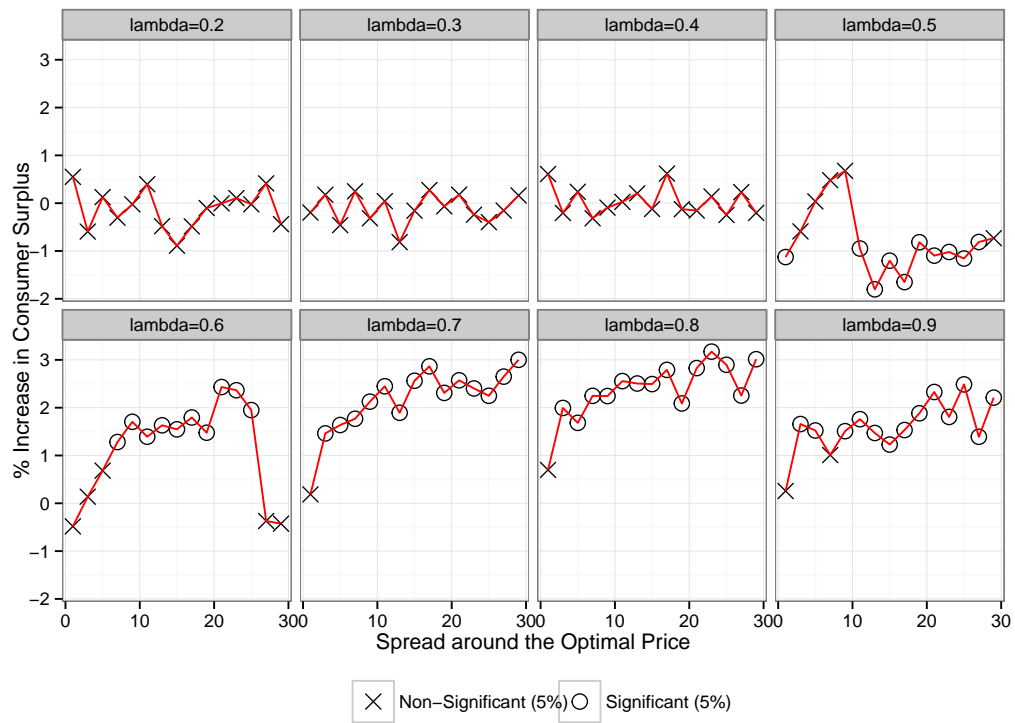
Figure 11: Average change in consumer surplus between optimal fixed pricing and revenue management for different intensities of demand and two types of products. The optimal fixed pricing consists here of a unique optimal price for all products.

Figure 12: Average change in profit between optimal fixed pricing and revenue management for different intensities of demand and two types of products. We compute here a fixed optimal price for each type of products.

Figure 13: Average change in consumer surplus between optimal fixed pricing and revenue management for different intensities of demand and two types of products. We compute here a fixed optimal price for each type of products.

Figure 14: Average change in profit between optimal fixed pricing and revenue management for different intensities of demand when consumers have an increasing willingness to purchase.

Figure 15: Average change in consumer surplus between optimal fixed pricing and revenue management for different intensities of demand when consumers have an increasing willingness to purchase.

Figure 16: Average change in profit between optimal fixed pricing and revenue management for different intensities of demand when the arrival rates of consumers is noisy.

Figure 17: Average change in consumer surplus between optimal fixed pricing and revenue management for different intensities of demand when the arrival rate of consumers is noisy.

H.3 Revenue Management and Heterogeneous Consumers
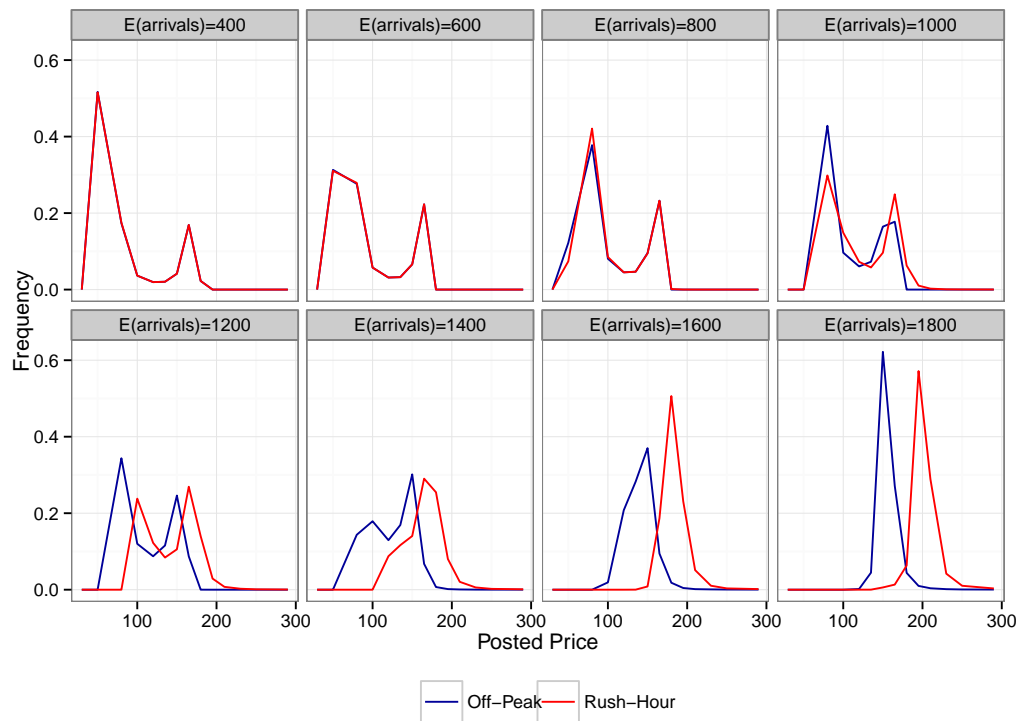
Figure 18: Distribution of the revenue manager's choices for each type of products in the case of heterogeneous consumers.
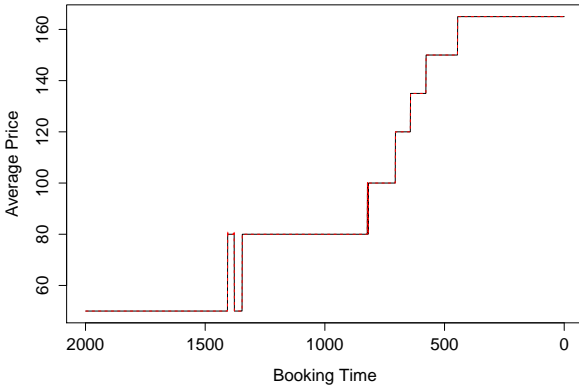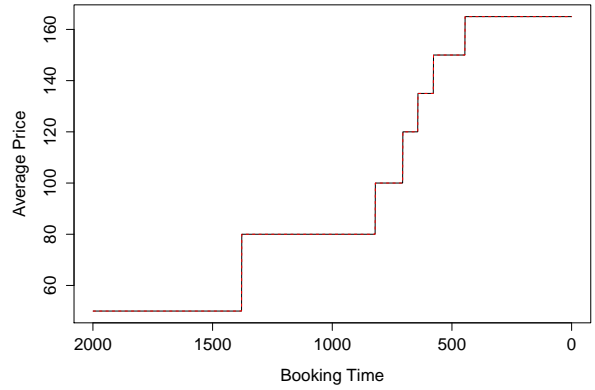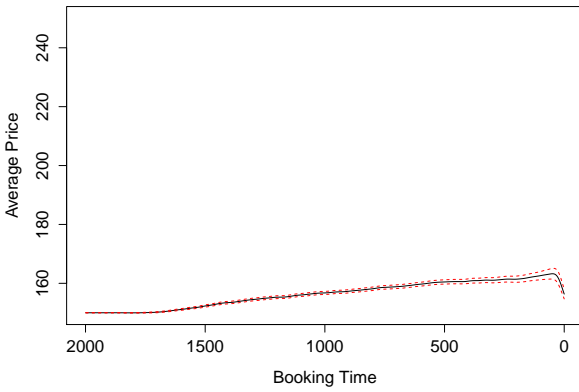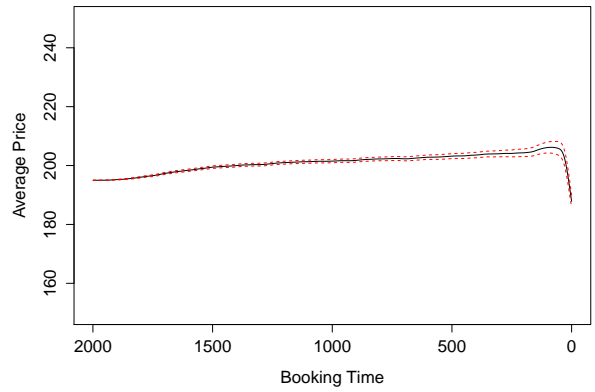
Figure 19: Change in posted average prices as a function of the booking period in the case of heterogeneous consumers. Figures (a) and (b) respectively deal with the off-peak and rush-hour trains for an expected arrival of 600 consumers. Figures (c) and (d) respectively deal with the off-peak and rush-hour trains for an expected arrival of 1800 consumers.

68

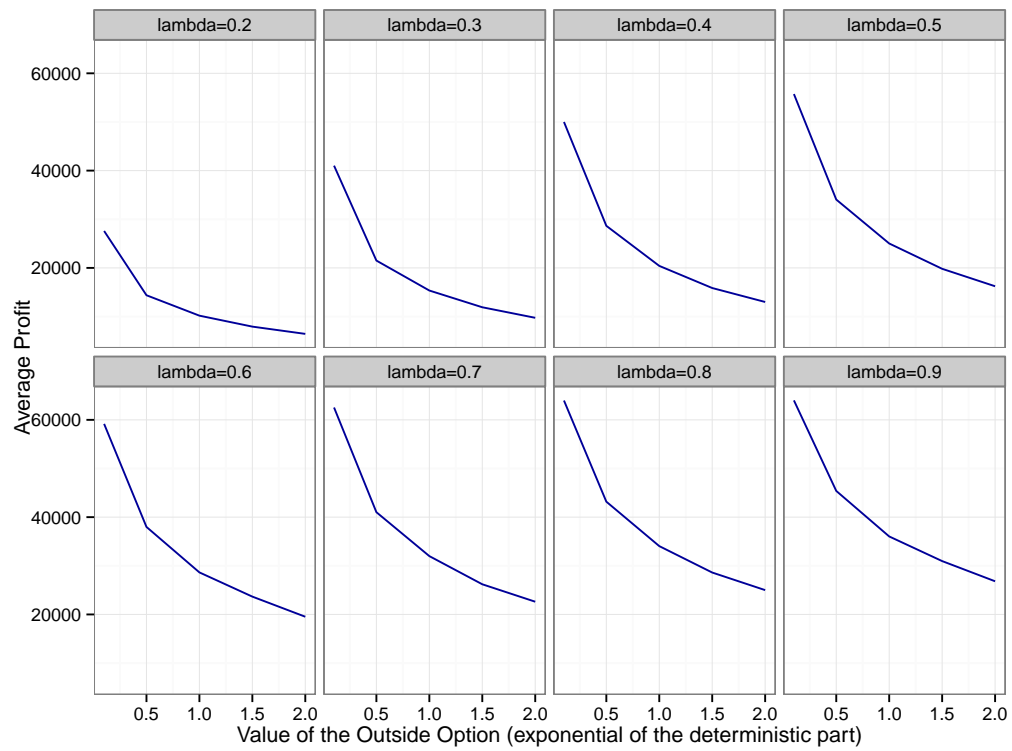# H.4 Revenue Management and Intermodal Competition

Figure 20: Average profit generated by revenue management for different market sizes and different values of the outside option.
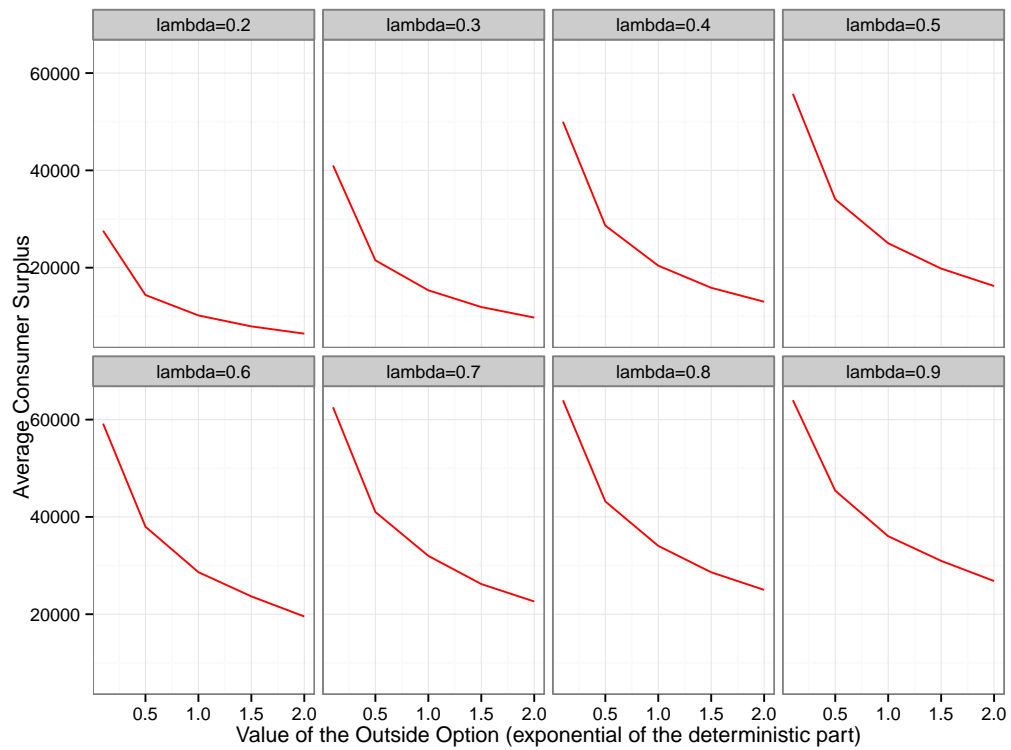
Figure 21: Average consumer surplus generated by revenue management for different market sizes and different values of the outside option.
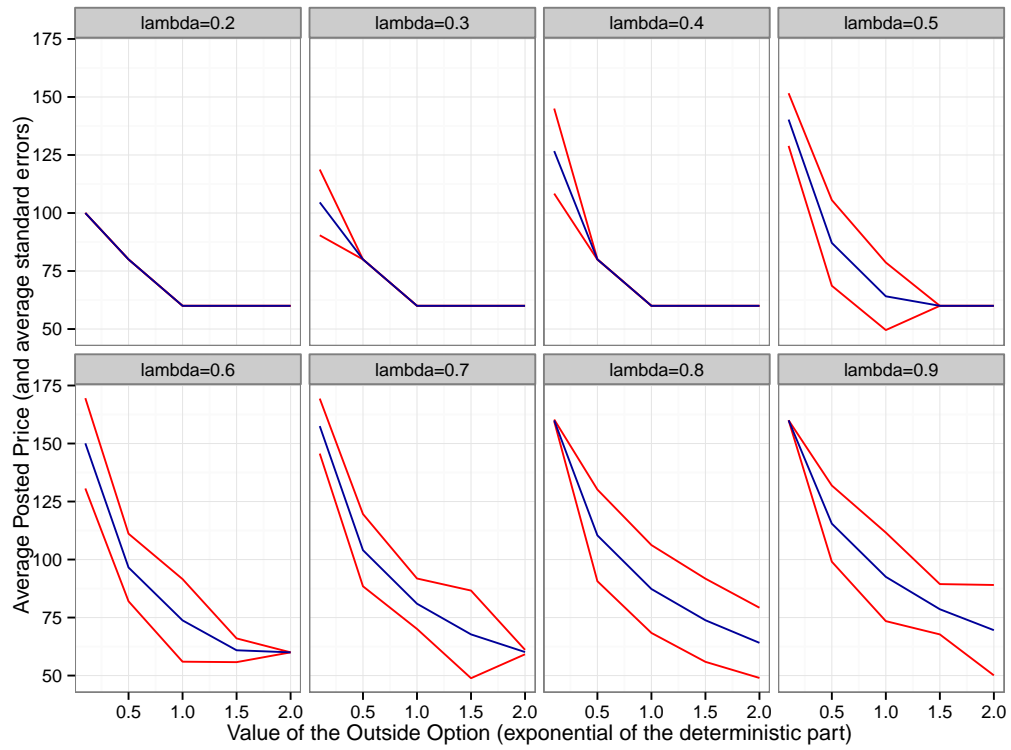
Figure 22: Average price and price dispersion in the case of revenue management for different market sizes and different values of the outside option.