Optimal Timing of Carbon Capture Policies Under Alternative CCS Cost Functions*

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Abstract

We determine the optimal exploitation time-paths of three types of perfect substitute energy resources: The first one is depletable and carbon-emitting (dirty coal), the second one is also depletable but carbon-free thanks to a carbon capture and storage (CCS) process (clean coal) and the last one is renewable and clean (solar energy). We assume that the atmospheric carbon stock cannot exceed some given ceiling. These optimal paths are considered along with alternative structures of the CCS cost function depending on whether the marginal sequestration cost depends on the flow of clean coal consumption or on its cumulated stock. In the later case, the marginal cost function can be either increasing in the stock thus revealing a scarcity effect on the storage capacity of carbon emissions, or decreasing in order to take into account some learning process. We show among others the following results: Under a stock-dependent CCS cost function, the clean coal exploitation must begin at the earliest when the carbon cap is reached while it must begin before under a flow-dependent cost function. Under stock-dependent cost function with a dominant learning effect, the energy price path can evolve non-monotonically over time. When the solar cost is low enough, this last case can give rise to an unusual sequence of energy consumption along which the solar energy consumption is interrupted for some time and replaced by the clean coal exploitation. Last, the scarcity effect implies a carbon tax trajectory which is also unusual in this kind of ceiling models, its increasing part been extended for some time during the period at the ceiling.

Keywords: Carbon capture and storage; Energy substitution; Learning effect; Scarcity effect; Carbon stabilization cap.

JEL classifications: Q32, Q42, Q54, Q55, Q58.

Contents

1 Introduction

Carbon dioxide capture and storage (CCS) is a process consisting of the separation of CO_2 from the emissions stream from fossil fuel combustion, transporting it to storage location, and storing it in a manner that ensures its long-run isolation from the atmosphere (IPCC, 2005). Currently, the major CCS effort focus on the removal of CO_2 directly from industrial or utility plants and storing it in secure geological reservoirs. Given that fossil fuels supply over 85% of all primary energy demands, CCS appears as the only technology that can substantially reduce CO_2 emissions while allowing fossil fuels to meet the world's pressing needs (Herzog, 2011). Moreover, CCS technology may have considerable potential to reduce CO_2 at a "reasonable" social cost, given the social costs of carbon emissions predicted for a business-as-usual scenario (Islegen and Reichelstein, 2009). According to Hamilton et al. (2009), the mitigation cost for capture and compression of the emissions from power plants running with gas is about \$52 per metric ton CO_2. Adding the transport and storage costs[1] in a range of \$5-15 per metric ton CO_2, a carbon price of about \$60-65 per metric ton CO_2 is needed to make these plants competitive.

The CCS technology has motivated a large number of empirical studies, mainly through complex integrated assessment models (see for instance McFarland et al. (2003), Kurosawa, 2004, Edenhofer et al., Gerlagh, 2006, Gerlagh and van der Zwaan, 2006, Grimaud et al., 2011). In these models, the only reason to use CCS technologies is to reduce CO_2 emissions[2] and then, climate policies are essential to create a significant market for these technologies. These empirical models generally conclude that an early introduction of sequestration can lead to a substantial decrease in the social cost of climate change. However a high level of complexity for such models, aimed at defining some specific climate policies and energy scenarios, may be required so as to take into account the various interactions at the hand.

The theoretical economic literature on CCS is more succinct. Grimaud and Rouge (2009) study the implications of the CCS technology availability on the optimal use of polluting exhaustible resources and on optimal climate policies within an endogenous growth model. Ayong Le kama et al. (2010) develop a growth model aiming at exhibiting the main driving forces that should determine the optimal CCS policy when the command variable

[1] As explained in Hamilton et al. (2009), the transport and storage costs are very site specific.

[2] As mentioned by Herzog (2009), the idea of separating and capturing CO_2 from the flue gas of power plants did not originate out of concern about climate change. The first commercial CCS plants that have been built in the late 1970s in the United States aimed at achieving enhanced oil recovery (EOR) operations, where CO_2 is injected into oil reservoirs to increase the pressure and thus the output of the reservoir.

of such a policy is the sequestration rate instead of the sequestration flow. Lafforgue et al. (2008-a) characterize the optimal timing of the CCS policy in a model of energy substitution when carbon emissions can be stockpiled into several reservoirs of finite size. However, the outcomes of these models cannot be easily compared since they strongly vary according to a crucial feature: the structure of the CCS cost function.

In the present study, we address the question of the qualitative impacts of such cost function properties on the optimal use of carbon capture and storage. Using a standard Hotelling model for the fossil resource and assuming, as in Chakravorty et al. (2006), that the atmospheric carbon stock should not exceed some critical threshold, we characterize the optimal time paths of energy price, energy consumption, carbon emissions and atmospheric abatement for various types of CCS cost functions. In that sense, we generalize the model of Lafforgue et al. (2008) in which the marginal sequestration cost is assumed to be constant.

The sketch of the model is the following. The energy needs can be supplied by three types of energy resources that are perfect substitutes: The first one is depletable and carbon-emitting (dirty coal), the second one is also depletable but carbon-free thanks to a CCS device (clean coal) and the last one is renewable and clean (solar energy). Hence, we consider two alternative mitigation options allowing to relax the carbon cap constraint: the exploitation of the solar energy and of the clean coal. The design of the optimal energy consumption path thus results from the comparison of the respective marginal costs of these three energy sources. Both the marginal extraction cost of coal and the marginal production cost of the solar energy are assumed to be constant, the former been lower than the later. However, producing clean coal requires an additional CCS cost whose characteristics can vary. We consider alternative structures of the CCS cost function depending on whether the marginal sequestration cost depends on the flow of clean coal consumption or on its cumulated stock. In the later case, the marginal cost function can first be increasing in the stock thus revealing a scarcity effect on the storage capacity of carbon emissions[3]. Second, since as pointed out by Gerlagh (2006) or by Manne and Richels (2004), the cumulated experience in carbon capture generates in most cases some beneficial learning tending to reduce the involved costs, the average cost function can be decreasing in the cumulated clean coal consumption.

We show among others the following results: Under a stock-dependent CCS cost func-

[3]This effect is taken into account in Lafforgue et al. (2008) through the definition of a physical limit of sequestration. In the present study, such a limit in capacity is also tackled in an economical way by assuming that the marginal sequestration cost increases as the carbon reservoir is filled up.

tion, the clean coal exploitation must begin at the earliest when the carbon cap is reached while it must begin before under a flow-dependent cost function. Under stock-dependent cost function with a dominant learning effect, the energy price path can evolve non-monotonically over time. When the solar cost is low enough, this last case can give rise to an unusual sequence of energy consumption along which the solar energy consumption is interrupted for some time and replaced by the clean coal exploitation. Last, the scarcity effect implies a carbon tax trajectory which is unusual in this kind of ceiling models, its increasing part been extended for some time during the period at the ceiling.

The paper is organized as follows. Section 2 presents the model and characterizes the various structures of CCS cost function that are under study. Section 3 describes the optimal path in the case of flow-dependent CCS cost functions by distinguishing different possibilities for the solar energy to be more or less expensive as compared with the clean coal exploitation. Section 4 studies the optimal paths under cost-dependent CCS cost functions according to whether the scarcity effect or the learning effect dominates and according to whether the solar energy cost is high or low. Section 5 investigates the main qualitative dynamical properties of the carbon tax required to enforce the carbon cap constraint that are obtained in the various cases described above, and it compares them. Last Section 6 briefly concludes.

2 The model

Let us consider an economy in which the energy services can be produced from two primary resources, a polluting non-renewable one, say coal, and a clean renewable one, say solar.

2.1 The polluting non-renewable primary resource

Let $X(t)$ be the available stock of coal at time t, X^0 be its initial endowment, $X(0) = X^0 > 0$, and $x(t)$ its instantaneous extraction rate so that:

$$\dot{X}(t) = -x(t),\ X(t) \geq 0,\ t \geq 0 \text{ and } X(0) = X^0 > 0 \tag{1}$$

$$x(t) \geq 0,\ t \geq 0 \tag{2}$$

The average cost of coal exploitation, denoted by c_x, is assumed to be constant, hence equal to its marginal cost. This cost includes all the different costs having to be borne to produce ready-for-use energy services to the final users, that is the extraction cost, the processing cost and the transportation and distribution costs.

Let ζ be the unitary pollutant content of coal so that, absent any abatement policy, the pollution flow which would be released into the atmosphere would amount to $\zeta x(t)$.

2.2 Atmospheric pollution stock

Denote by $Z(t)$ the current level of the atmospheric carbon concentration at time t and by Z^0 the initial concentration inherited from the past: $Z(0) = Z^0 \geq 0$. This atmospheric pollution stock is assumed to be self-regenerating at some constant proportional rate α, $\alpha > 0$.

To get the dynamics of $Z(t)$, we must take into account that its supplying flow can be lower than the potential pollution flow $\zeta x(t)$ generated by coal burning thanks to some carbon capture and sequestration option. Let $s(t)$ be this share of the potential emission flow which is captured and sequestered:

$$s(t) \geq 0 \text{ and } \zeta x(t) - s(t) \geq 0 \tag{3}$$

The dynamics of the atmospheric pollution stock is driven by both the coal consumption policy and the capture and sequestration policy, that is:

$$\dot{Z} = \zeta x(t) - s(t) - \alpha Z(t),\ Z(0) = Z^0 \geq 0 \tag{4}$$

Having adopted this formalization, the next step consists in introducing the CCS average cost as some function of either the current emission captured flow $s(t)$, or of the cumulated captures $S(t)$, $S(t) = S^0 + \int_0^t s(\tau)d\tau$, where $S^0 \equiv S(0)$, in order to take into account the scarcity of accessible sequestering sites and/or the learning effects resulting from the experience in the capture and sequestration activity.

2.3 Clean versus dirty energy services

Instead of expressing the CCS cost as some function of the sequestration flow $s(t)$ and/or of the cumulated sequestration $S(t)$, we proceed formally otherwise by considering two types of fossil energies allowing to produce final energy services together with the clean renewable substitute. We define the clean coal as this part of coal consumption whose emissions are captured and the dirty coal as this part whose emissions are directly released into the atmosphere. Let us denote respectively by $x_c(t)$ and $x_d(t)$ the instantaneous consumption rates of clean and dirty coals. Since $x_c(t) + x_d(t) = x(t)$, then (1) and (2) have to be rewritten as:

$$\dot{X}(t) = -[x_c(t) + x_d(t)],\ X(t) \geq 0\ t \geq 0\ \text{ and } X(0) = X^0 > 0 \tag{5}$$

$$x_c(t) \geq 0 \ \text{ and } \ x_d(t) \geq 0 \tag{6}$$

We denote by $S_c(t)$ be the cumulated clean coal consumption from time 0 up to time t. For the sake of simplicity, we assume that $S_c(0) = 0$, so that:

$$S_c(t) = \int_0^t x_c(\tau)d\tau \Rightarrow \dot{S}_c(t) = x_c(t) \tag{7}$$

equivalently:

$$S_c(t) = \frac{1}{\zeta} S(t) \tag{8}$$

Since only the dirty coal is supplying the atmospheric carbon stock, its dynamics (4) may be simply rewritten as:

$$\dot{Z}(t) = \zeta x_d(t) - \alpha Z(t),\ t \geq 0\ \text{ and } Z(0) = Z^0 \geq 0 \tag{9}$$

2.4 Sequestration costs

Producing energy services from clean coal is more costly than from dirty coal since some additional capture and sequestration costs must be incurred. Let c_s be the additional cost per unit of clean coal. Clearly, the implications of such a way to relax the pollution constraint should depend upon the characteristics of this additional cost.

The CCS average cost c_s may first depend upon the current quantity of clean coal which is consumed, and only upon this flow.

- **CCS.1 Flow-dependent capture cost function:**
  $c_s : \mathbb{R}_+ \to \mathbb{R}_+^*$ is a $\mathcal{C}^2$ function, strictly increasing and strictly convex, $c_s'(x_c) > 0$ and $c_s''(x_c) > 0$ for any $x_c > 0$, with $\lim_{x_c \downarrow 0} c_s(x_c) = \underline{c}_s > 0$.

Under CCS.1, the total additional cost required for consuming clean coal rather than dirty coal thus amounts to $c_s(x_c)x_c$. The associated marginal cost of clean coal, denoted by $c_{ms}(x_c)$, amounts to: $c_{ms}(x_c) = c_s(x_c) + c_s'(x_c)x_c > 0$, and is increasing: $c_{ms}'(x_c) = 2c_s'(x_c) + c_s''(x_c)x_c > 0$.

Second, the CCS cost function may depend upon the cumulated clean coal consumption, which may give rise to two different effects working in quite opposite directions. On the one hand, due to the scarcity of the most accessible sites into which the carbon can be sequestered[4], the average CCS cost may increase with S_c up to some upper bound $\bar{S}_c$ corresponding to the global capacity of such reservoir sites, hence the following constraint:

$$\bar{S}_c - S_c(t) \geq 0 \tag{10}$$

Although not sufficient, a necessary condition for such a condition to be effective is that $\bar{S}_c$ be lower than the maximal cumulated emissions of coal, that is: $\bar{S}_c < X^0$.

On the other hand, the higher S_c, the larger the cumulated experience in carbon capture generating in most cases some beneficial learning tending to reduce the involved costs, in which case the CCS cost function decreases with S_c.

We define stock-dependent capture costs as average capture cost functions depending upon the cumulated clean coal consumption S_c and only the cumulated clean coal consumption, so that at any time t the total additional cost having to be incurred for using the friendly environmental coal instead of the carbon emitting one, amounts to $c_s(S_c(t))x_c(t)$. A stock-dependent capture cost with a dominant effect is a cost function for which the marginal balance sheet between the scarcity and the learning effects does not depend upon the cumulated clean coal consumption. In brief, it is the polar case in which the sign of the derivative of $c_s(S_c)$ does not depend upon S_c and thus, cannot alternate.

In the case of a dominant scarcity effect, c_s must be defined in the range $[0, \bar{S}_c]$.

- **CCS.2 Stock-dependent capture cost with dominant scarcity effect:**
  $c_s : \big[0, \bar{S}_c\big] \to \mathbb{R}_+^*$ is a $\mathcal{C}^2$ function, strictly increasing and strictly convex, $c_s'(S_c) > 0$ and $c_s''(S_c) > 0$ for any $S_c \in \big(0, \bar{S}_c\big)$, with $\lim_{S_c \downarrow 0} c_s(S_c) = \underline{c}_s > 0$.

In the case of a pure dominant learning effect, no restriction has to be put on the global capacity of the reservoirs. Such a constraint would introduce in some sense a scarcity effect blurring the learning effect. The objective of the paper being to isolate the pure learning effect, we neglect an eventual locking of this process that would be involved by a constrained capacity of the reservoirs, even if such a constraint is empirically relevant.

- **CCS.3 Stock-dependent capture cost with dominant learning effect:**
  $c_s : \big[0, X^0\big] \to \mathbb{R}_+^*$ is a $\mathcal{C}^2$ function, strictly decreasing and strictly convex, $c_s'(S_c) < 0$

[4] Lafforgue *et al.* (2008-a) show that the different reservoirs should be completely filled by increasing order of their respective sequestration costs. The present setting assumes that there is no correlation between the extraction and consumption costs and the sequestration costs.

and $c_s''(S_c) > 0$ for any $S_c \in \left(0, X^0\right)$, with $\lim_{S_c \downarrow 0} c_s(S_c) = \bar{c}_s < \infty$ and $c_s(X^0) = \underline{c}_s > 0$.

2.5 The clean renewable primary resource

The other primary resource can be processed at some constant average cost c_y. As for the non-renewable resource this cost includes all the costs having to be supported to supply ready-for-use energy services to the final users. Thus once c_x, possibly c_s, and c_y are supported, the both types of the main primary energy resources are perfect substitutes as far as consuming energy services generates some surplus. Denoting by $y(t)$ the renewable energy consumption, we may define the aggregate energy consumption $q(t)$ as $q(t) = x(t) + y(t) = x_c(t) + x_d(t) + y(t)$, with the usual non-negativity constraint:

$$y(t) \geq 0 \tag{11}$$

The natural flow of solar energy y^n is assumed to be sufficiently large to provide all the energy needs of the society at the marginal cost c_y so that no rent has ever to be charged for an efficient exploitation of the resource. Last, we assume that c_y is larger than c_x to justify the use of coal during some time period. Since relaxing the ceiling constraint can be achieved by using either clean coal or solar energy, the relative competitiveness of these two options may depend upon their respective costs. That is why we will distinguish the cases of a "high" or a "low" solar energy costs in the following analysis. What we mean by "high" or "low" will be made more precise in the next sections.

2.6 Gross surplus generated by energy service consumption

The energy service consumption $q(t)$ is generating an instantaneous gross surplus $u(q(t))$. Function $u(.)$ is assumed to satisfy the following standard assumptions: $u : \mathbb{R}_+ \to \mathbb{R}$ is a $\mathcal{C}^2$ function, strictly increasing and strictly concave verifying the Inada condition: $\lim_{q \downarrow 0} u'(q) = +\infty$.

We denote by $p(q)$ the marginal gross surplus function $u'(q)$, and by $q(p)$ its inverse, i.e. the energy demand function. When the solar energy is the unique energy source, then its optimal consumption would amount to $\tilde{y}$ solution of $u'(q) = c_y$, provided that y^n is not smaller than $\tilde{y}$, what we mean by assuming that y^n is sufficiently large.

2.7 Pollution damages

Turning now to the main focus of the paper, we assume that, as far as the atmospheric pollution stock does not overshoot some critical level $\bar{Z}$, the damages due to the atmospheric carbon accumulation are negligible[5]. However, for pollution stocks that are larger than $\bar{Z}$, the damages would be immeasurably larger than the sum of the discounted gross surplus generated along any path triggering this overshoot. By doing that, we assume a lexicographic structure of the preferences over the set of the time paths of energy consumption and pollution stock. Technically, this lexicographic structure translates into two constraints, the first one on the state variable Z and the second one on the control variable x_d.

Since the overshoot of this critical cap would destroy all that could be gained otherwise, then we must impose:

$$\bar{Z} - Z(t) \geq 0 \; t \geq 0 \tag{12}$$

The other constraint states that, when the ceiling is reached, the maximum quantity of dirty coal which can be consumed is this quantity whose emissions are balanced by the natural regeneration of the atmosphere. Denoting by $\bar{x}_d$ this maximum consumption rate of dirty coal, (9) implies that $\bar{x}_d = \alpha\bar{Z}/\zeta$.

2.8 The social rate of discount and the social planner program

We denote by ρ the instantaneous rate of discount, which is assumed to be constant over time and strictly positive. The social planner program thus consists in determining the paths of x_c, x_d and y that maximize the sum of the discounted net surplus.

3 Flow-dependent CCS cost functions

3.1 Problem formulation and preliminary remarks

Under CCS.1, the social planner program takes the following form:

$$(P) \quad \max_{x_c, x_d, y} \int_0^\infty \{u(x_c(t) + x_d(t) + y(t)) - c_x[x_c(t) + x_d(t)] - c_s(x_c(t))x_c(t) - c_y y(t)\} \, e^{-\rho t} dt$$

subject to constraints (5), (9) and to the inequality constraints (6), (11) and (12).

[5] See Amigues, Moreaux and Schubert (2011) for a model in which the both types of effects are explicitly taken into account.

Let $\mathcal{H}$ be the Hamiltonian in current value of problem (P) (we drop the time argument for notational convenience):

$$\mathcal{H} = u(x_c + x_d + y) - c_x[x_c + x_d] - c_s(x_c)x_c - c_y y - \lambda_X[x_c + x_d] - \lambda_Z[\zeta x_d - \alpha Z]$$

where λ_X and $-\lambda_Z$ are the costate variables of X and Z respectively[6]. Denoting by ν's the Lagrange multipliers associated with the inequality constraints on the state variables and by γ's the multipliers corresponding to the inequality constraints on the control variables, the Lagrangian in current value writes:

$$\mathcal{L} = \mathcal{H} + \nu_X X + \nu_Z[\bar{Z} - Z] + \gamma_{x_c} x_c + \gamma_{x_d} x_d + \gamma_y y$$

The first order optimality conditions are:

$$\frac{\partial \mathcal{L}}{\partial x_c} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_x + \lambda_X + c_{ms}(x_c) - \gamma_{x_c} \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial x_d} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_x + \lambda_X + \zeta\lambda_Z - \gamma_{x_d} \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial y} = 0 \quad \Rightarrow \quad u'(x_c + x_d + y) = c_y - \gamma_y \tag{15}$$

$$\dot{\lambda}_X = \rho\lambda_X - \frac{\partial \mathcal{L}}{\partial X} \quad \Rightarrow \quad \dot{\lambda}_X = \rho\lambda_X - \nu_X \tag{16}$$

$$\dot{\lambda}_Z = \rho\lambda_Z + \frac{\partial \mathcal{L}}{\partial Z} \quad \Rightarrow \quad \dot{\lambda}_Z = (\rho + \alpha)\lambda_Z - \nu_Z \tag{17}$$

together with the usual complementary slackness conditions.

The transversality conditions are:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_X(t)X(t) = 0 \tag{18}$$

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_Z(t)Z(t) = 0 \tag{19}$$

As it is well known, with a constant marginal extraction cost c_x, the mining rent λ_X must grow at the social rate of discount as long as the stock of coal is not exhausted. From (16), we have:

$$X(t) > 0 \Rightarrow \lambda_X(t) = \lambda_{X0}e^{\rho t}, \ \lambda_{X0} = \lambda_X(0) \tag{20}$$

so that $e^{-\rho t}\lambda_X(t)X(t) = \lambda_{X0}X(t)$. Hence from the transversality condition (18), if coal have some positive initial value, i.e. if $\lambda_{X0} > 0$, then its stock must be exhausted in the long run along the optimal path.

[6] Using $-\lambda_Z$ as the costate variable of Z makes it possible to directly interpret $\lambda_Z \geq 0$ as the unitary tax having to be charged for the pollution emissions generated by dirty coal consumption.

Initially, we have $\nu_Z = 0$ as long as the ceiling constraint is not binding. Denoting by $\underline{t}_Z$ the time at which the atmospheric carbon cap $\bar{Z}$ is reached, (17) implies:

$$t \leq \underline{t}_Z \Rightarrow \lambda_Z(t) = \lambda_{Z0} e^{(\rho+\alpha)t}, \quad \text{where } \lambda_{Z0} = \lambda_Z(0) \tag{21}$$

Once the ceiling constraint is no more active and forever, λ_Z must be nil. Denoting by $\bar{t}_Z$ the last time at which the constraint is active, it comes[7]:

$$t \geq \bar{t}_Z \Rightarrow \lambda_Z(t) = 0 \tag{22}$$

3.2 The optimal paths

The dynamics of consumption of the two types of coal is driven by the dynamics of their respective full marginal costs. A common component of these costs is the processing cost c_x augmented by the mining rent $\lambda_X(t)$. We denote by $p^F(t)$ (F for free of tax and free of cleaning cost) this common component:

$$p^F(t) = c_x + \lambda_{X0} e^{\rho t} \Rightarrow \dot{p}^F(t) = \rho \lambda_{X0} e^{\rho t} > 0 \tag{23}$$

In addition to this common component, the full marginal cost of the dirty coal, which is denoted by $c_m^d(x_d)$, must also include the imputed marginal cost of the carbon emissions generated by its consumption:

$$c_m^d(x_d(t)) = p^F(t) + \zeta \lambda_Z(t) \tag{24}$$

The full marginal cost of the clean coal must include the marginal cleaning cost. Thus denoting by $c_m^c(x_c)$ this full marginal cost, we get:

$$c_m^c(x_c(t)) = p^F(t) + c_{ms}(x_c(t)) \tag{25}$$

where $c_{ms}(x_c(t)) = c_s(x_c) + c_s'(x_c)x_c > 0$.

The day-to-day dynamics of exploitation of the two types of coal and solar energy are driven by the dynamics of their instantaneous full marginal costs. Given that we assume a constant marginal cost of the solar energy, free of pollution tax since clean, we may organize the discussion depending on whether this marginal cost of the clean renewable substitute

[7] Solving the ordinary differential equations (9) and (17) respectively results in $Z(t) = \left[Z^0 + \int_0^t \zeta x_d(\tau) e^{\alpha\tau} d\tau\right] e^{-\alpha t}$ and $\lambda_Z(t) = \left[\lambda_{Z0} - \int_0^t \nu_Z(\tau) e^{-(\rho+\alpha)\tau} d\tau\right] e^{(\rho+\alpha)t}$. The transversality condition (19) can thus be written as: $\lim_{t\to\infty} \left[\lambda_{Z0} - \int_0^t \nu_Z(\tau) e^{-(\rho+\alpha)\tau} d\tau\right] \left[Z^0 + \int_0^t \zeta x_d(\tau) e^{\alpha\tau} d\tau\right] = 0$, which implies $\lambda_{Z0} = \int_0^\infty \nu_Z(\tau) e^{-(\rho+\alpha)\tau} d\tau$. Then, $\lambda_Z(t) = \int_t^\infty \nu_Z(\tau) e^{-(\rho+\alpha)(\tau-t)} d\tau$ and, as a consequence, $\lambda_Z(t) = 0$ for any $t \geq \underline{t}_Z$.

is "high" or "low", meaning that either $c_y > u'(\bar{x}_d)$ or $c_y < u'(\bar{x}_d)$ and assuming that the initial coal endowment X^0 is large enough for having the ceiling constraint $\bar{Z} - Z(t) \geq 0$ binding along the optimal path.

3.2.1 The high solar cost case: $c_y > u'(\bar{x}_d)$

Let us assume that solar cost is high. In this case, we show that the optimal path is a five or six phases path when the ceiling constraint is active.

Types of phases

For sufficiently low $\lambda_Z(t)$, that is for $\zeta\lambda_Z(t) < \underline{c}_s$, dirty coal is more competitive than dirty coal and than solar energy, and it thus must be the only source of supplied energy.

Consider now a phase of simultaneous exploitation of the both types of coal and the composition of the resulting energy supply. Denote by $\underline{t}_c$ the time at which clean coal begins to be exploited. If a simultaneous use of both types of coal is possible before the ceiling is attained, $\underline{t}_c < \underline{t}_Z$, then the full marginal costs of the both types of coal must be equal, that is $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = c_{ms}(x_c(t))$. Differentiating this expression with respect to time and solving for $\dot{x}_c$, we get:

$$\dot{x}_c(t) = \frac{\zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)t}}{c'_{ms}(x_c(t))} > 0 \tag{26}$$

where $c'_{ms}(x_c(t)) = 2c'_s(x_c(t)) + c''_s(x_c(t))x_c(t) > 0$. The consumption of clean coal must increase over time during such a phase. Since the energy price $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ is increasing, then the consumption of energy services decreases hence the consumption of the dirty coal must simultaneously decrease.

During a phase along which the ceiling constraint is binding and both types of coal are used, assuming again that it is possible, minimizing the energy production cost implies that the dirty coal must be used as far as possible: $x_d(t) = \bar{x}_d$. The clean coal consumption is thus determined by the condition (13): $u'(x_c(t) + \bar{x}_d) = c_x + \lambda_{X0}e^{\rho t} + c_{ms}(x_c(t))$. Time differentiating this expression and solving for $\dot{x}_c$, we obtain:

$$\dot{x}_c(t) = \frac{\rho\lambda_{X0}e^{\rho t}}{u''(x_c(t) + \bar{x}_d) - c'_{ms}(x_c(t))} < 0 \tag{27}$$

Since the energy consumption $q(t) = x_c(t) + \bar{x}_d$ decreases during such a phase at the ceiling, the energy price must increase.

A crucial problem for characterizing the optimal path is to identify the timing of the different types of phases and their sequencing. The following Proposition 1 states that if the clean coal has to be ever exploited because the ceiling constraint is effective during some phase of the optimal path, then its exploitation must begin before the ceiling constraint is attained. Thus the clean coal use must be seen as some costly device allowing to delay the time at which the ceiling constraint will become effective. Another possibility would be to use the solar energy, but it is assumed to be too costly here, too costly meaning that $c_y > u'(\bar{x}_d)$.

Proposition 1 *Under flow-dependent CCS cost functions CCS.1, assuming that the solar energy cost is high, that clean coal is exploited and that the ceiling constraint is effective along the optimal path, then the clean coal exploitation must begin before the ceiling constraint is active:* $\underline{t}_c < \underline{t}_Z$.

Proof: We first show that $\zeta\lambda_Z(t)$ is always decreasing for $t \in [\underline{t}_Z, \bar{t}_Z)$. During this interval of time, either $x_c(t) = 0$ so that $\zeta\lambda_Z(t) = u'(\bar{x}_d) - p^F(t)$ and $\zeta\dot{\lambda}_Z(t) = -\dot{p}^F(t) < 0$, or $x_c(t) > 0$ so that $\zeta\lambda_Z(t) = c_{ms}(x_c(t))$ and $\zeta\dot{\lambda}_Z(t) = c'_{ms}(x_c(t))\dot{x}_c(t)$, which is also negative from (27). Hence, since we know that $\lambda_Z(t) = \lambda_{Z0}e^{(\rho+\alpha)t}$ for $t \in [0, \underline{t}_Z)$, the maximal value of $\zeta\lambda_Z(t)$ is attained at time $\underline{t}_Z$: $\underline{t}_Z = \text{argmax}\,\{\lambda_Z(t)\}$.

At this point of time, assume that sequestration has not begun yet: $\underline{t}_c > \underline{t}_Z$ so that $x_c(\underline{t}_Z) = 0$. It means that $\zeta\lambda_Z(\underline{t}_Z) < \underline{c}_s$ and then, since $\zeta\lambda_Z(t)$ is decreasing for $t \geq \underline{t}_Z$, we must have $x_c(t) = 0$ for any $t \geq \underline{t}_Z$. If sequestration has not begun yet at time $\underline{t}_Z$, it will never be used thereafter. In order to have any interest, the problem must be such that $\zeta\lambda_Z(\underline{t}_Z) = c_{ms}(x_c(\underline{t}_Z)) > \underline{c}_s$. Consequently, any clean coal consumption phase must begin at some date $\underline{t}_c < \underline{t}_Z$. ■

Proposition 2 below characterizes the behavior the economy during any phase at the ceiling.

Proposition 2 *Under a flow-dependent cleaning cost function, assuming that the cost of solar energy is high, if clean coal has to be used, then there must exist two phases at the ceiling, the first one during which the both types of coal are exploited and the next one during which only dirty coal must be exploited.*

Proof: According to Proposition 1 and (26), the clean coal production is strictly positive when the ceiling is attained. This is possible if and only if $\zeta\lambda_Z(\underline{t}_Z) > \underline{c}_s$. Since the price path must be continuous then there must exist some time interval $(\underline{t}_Z, \underline{t}_Z + \delta)$, $\delta > 0$, during which the clean coal production is still positive and decreasing from (27).

Assume now that clean coal is produced during the entire period at the ceiling. At the end of the period, at time $t = \bar{t}_Z$, we must have $\lambda_Z(\bar{t}_Z) = 0$ as pointed out by (22). Hence, by the price continuity argument, there would exist some time interval $(\bar{t}_Z - \delta, \bar{t}_Z)$ during which $\zeta\lambda_Z(t) < \underline{c}_s$. During such a time interval, the full marginal cost of clean coal would be higher than the energy price, a contradiction. ■

As a consequence, clean coal exploitation allows not only to delay the date at which the ceiling constraint begins to be effective, but also to relax this constraint once it begins to be effective.

The last phase of coal exploitation is the phase of exclusive dirty coal use that follows the phase at the ceiling. Since $\lambda_Z(t) = 0$ from (22), the dirty coal is necessarily less costly than the clean one and the production rate of the later must be nil, implying $u'(x_d(t)) = c_x + \lambda_{X0}e^{\rho t}$. Time differentiating this last expression and solving for $\dot{x}_d$, we get:

$$\dot{x}_d(t) = \frac{\rho\lambda_{X0}e^{\rho t}}{u''(x_d(t))} < 0 \tag{28}$$

Note that, since $c_x + \lambda_{X0}e^{\rho t} > u'(\bar{x}_d)$ along such a phase, then $x_d(t) < \bar{x}_d$ so that $Z(t) < \bar{Z}$.

We denote by $\bar{t}_c$ and t_y, respectively, the time at which the clean coal consumption ends and the time at which the solar energy becomes competitive. A typical optimal path of energy prices and full marginal costs is illustrated in Figure 1 when the coal endowment is sufficiently large to trigger the binding of the ceiling constraint.[8]

Initially, we have $\zeta\lambda_{Z0} < \underline{c}_s$ implying that only dirty coal is used. Since the marginal cost of emissions $\zeta\lambda_Z(t)$ grows at rate $(\rho+\alpha)$, there exists some time $\underline{t}_c$ at which $\zeta\lambda_{Z0}e^{(\rho+\alpha)t} = \underline{c}_s$. Then $\underline{t}_c$ corresponds to the beginning of a phase of simultaneous use of both types of coal although the ceiling is not reached yet. During this phase the consumption of clean coal increases while the consumption of dirty coal decreases. This phase is ending at time $\underline{t}_Z$ when the ceiling is attained and the consumption of dirty coal is precisely equal to $\bar{x}_d$. At this time, a new phase begins, which is still characterized by a simultaneous exploitation of the both types of coal, but now at the ceiling. During this phase, the consumption of

[8]A full analytical characterization of the optimal paths under CCS.1 is given in appendix A.1 for the cases of high and low solar costs.

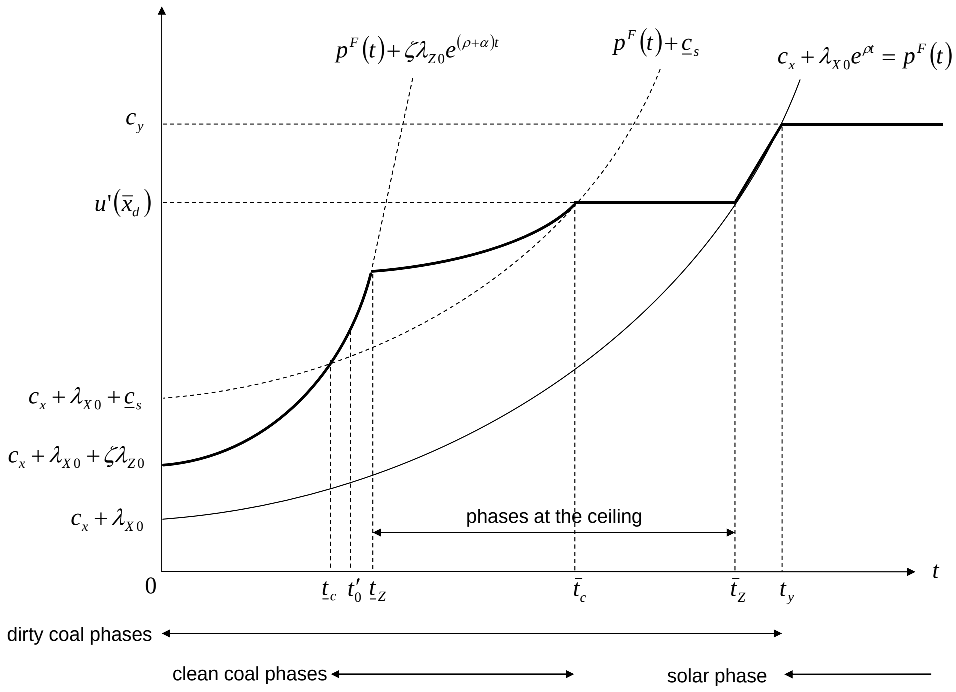

Figure 1: Optimal price path. Flow-dependent CCS average cost and high solar cost: $c_y > u'(\bar{x}_d)$

clean coal decreases while the consumption of dirty coal stays constant and equal to $\bar{x}_d$. The phase stops at time $\bar{t}_c$, when the consumption of clean coal falls to zero.

Note that during the two first phases, the price path is given by the same function $p^F(t)+\zeta\lambda_{Z0}e^{(\rho+\alpha)t}$. The reason is that before the ceiling is attained, the unitary pollution tax must grow at the same proportional rate $\rho+\alpha$. But during the third phase, at the ceiling, $p(t) = u'(x_c(t)+x_d(t)) = p^F(t)+c_{ms}(x_c(t))$. We can write:

$$\lim_{t\uparrow \underline{t}_Z} \dot{p}(t) = \dot{p}^F(\underline{t}_Z) + \zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_Z} > \dot{p}^F(\underline{t}_Z)$$

and, since from (27) $\dot{x}_c(t) < 0$ for any $t \in (\underline{t}_Z, \bar{t}_c)$, we also have:

$$\lim_{t\downarrow \underline{t}_Z} \dot{p}(t) = \dot{p}^F(\underline{t}_Z) + \lim_{t\downarrow \underline{t}_Z}\left[c'_{ms}(x_c(t))\dot{x}_c(t)\right] \leq \dot{p}^F(\underline{t}_Z)$$

Hence, as illustrated in Figure 1, the time derivative of the energy price, while increasing both before and after $\underline{t}_Z$, is discontinuous at $t=\underline{t}_Z$, its speed of growth being abruptly decelerated at this time.

The next phase is still a phase at the ceiling during which only the dirty coal is used at rate $\bar{x}_d$. The energy price is constant and equal to $u'(\bar{x}_d)$ and, from (14), $\lambda_Z(t) =$

$[u'(\bar{x}_d) - (c_x + \lambda_{X0}e^{\rho t})]/\zeta$ goes on to decrease as in the preceding phase. The phase ends at time $\bar{t}_Z$ when λ_Z is nil.

During the following phase, $\lambda_Z = 0$ and the full marginal cost of the dirty coal is $p^F(t)$. The energy price increases up to that time t_y at which the solar energy is becoming competitive: $p^F(t_y) = c_y$. At this time, the stock of coal must be exhausted. Then the solar energy time begins, forever.

The optimal consumption paths of the clean and dirty coals corresponding the price path described above, are illustrated in Figure 2. Although the total coal consumption is always either decreasing or constant, the clean coal consumption first increases, reaches an upper bound and next decreases down to zero. Moreover, clean coal use must begin before attaining the ceiling and must end before leaving it. This result is strongly linked with the increasing CCS marginal cost assumption and, as we shall see in the next section, it is no more valid for stock-dependent structures of marginal costs.

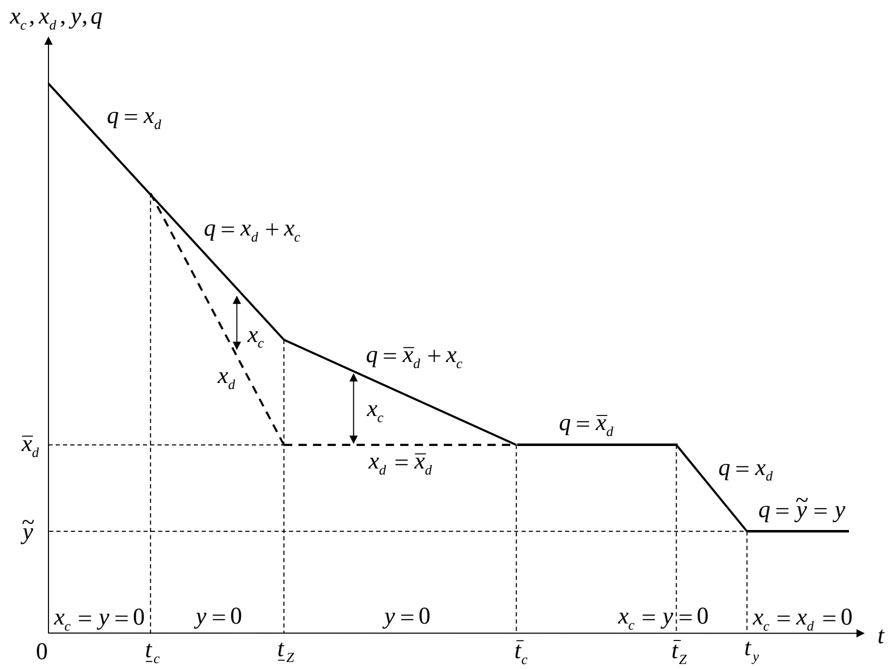

Figure 2: Optimal energy consumption paths. Flow-dependent CCS average cost and high solar cost: $c_y > u'(\bar{x}_d)$

Designing such an optimal path requires some evident necessary conditions. We must impose $c_x < u'(\bar{x}_d) < c_y$, a large enough coal initial endowment and a not too high initial average CCS cost $\underline{c}_s$. This last condition about the $\underline{c}_s$'s value is endogenous but can be more precisely explained by the following test. Assume that the clean coal option is not available and that initial coal endowments are large enough so that the ceiling constraint have to be active. Then the optimal price path is a path as the one illustrated in Figure 3, whose the main characteristics are similar to those underlined in Chakravorty et al. (2006).

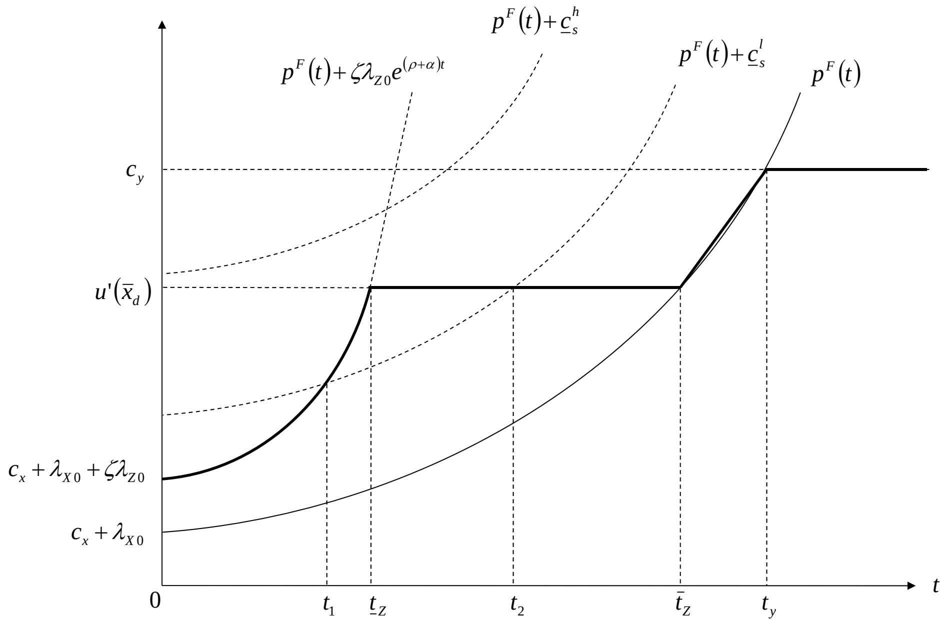

Figure 3: Optimal price path absent the clean coal option

Assume that $\underline{c}_s$ is very high so that the trajectory of $p^F(t) + \underline{c}_s^h$ (superscript h for high) lies above the optimal price path which would be obtained in the absence of the clean coal option, as depicted in Figure 2. It is then clearly never optimal to use the clean coal since its full marginal cost is always higher than the full marginal cost of the dirty coal. On the contrary, if the additional sequestration cost is low enough, $\underline{c}_s^l$ (l for low), then the full marginal cost of the clean coal would be lower than the full marginal cost of the dirty one over the time interval (t_1, t_2) so that the policy consisting in producing energy without clean technology would reveal never optimal.

In the case where the initial atmospheric carbon concentration Z^0 is close to the critical level $\bar{Z}$, CCS appears to be an urgent action in the policy agenda and should be started immediately at time $t = 0$. However, there always exists an initial phase during which the pollution stock increases from its initial level to its critical level since $Z^0 < \bar{Z}$. Thus the optimal scenario is a five phases scenario in which the initial phase $[0, \underline{t}_c)$, as illustrated in Figure 1, disappears. The optimal path looks like the truncated path starting from t'_0, $\underline{t}_c < t'_0 < \underline{t}_Z$, in Figure 1.

The optimal path as illustrated in Figure 1 is entirely characterized once the seven variables λ_{X0}, λ_{Z0}, $\underline{t}_c$, $\underline{t}_Z$, $\bar{t}_c$, $\bar{t}_Z$ and t_y are determined. We detail in Appendix A.1.1 the seven-equation system these variables are solving, resulting in $\zeta\lambda_{Z0} < \underline{c}_s$. When the initial pollution stock is very large, only six parameters have to be determined since $\underline{t}_c$ vanishes, resulting in $\underline{c}_s < \zeta\lambda_{Z0}$.

3.2.2 The low solar cost case: $c_y < u'(\bar{x}_d)$

In the case of a low solar cost, $c_y < u'(\bar{x}_d)$, there may not exist any phase at the ceiling with the energy consumption provided by the dirty coal and the dirty coal only since the solar average cost is undercutting the price $u'(\bar{x}_d)$, which would have to prevail during such a phase. As compared with the high solar cost case, this rises the possibility to have two new types of phases at the ceiling during which solar energy is simultaneously used with either the two types of coal or only the dirty one.

Consider first the possibility of a simultaneous exploitation of the three primary energy sources during a phase at the ceiling. This implies that $p(t) = c_y = p^F(t) + c_{ms}(x_c(t))$, whose time differentiation leads to:

$$\dot{x}_c = -\frac{\dot{p}^F(t)}{c'_{ms}(x_c(t))} < 0 \tag{29}$$

where $\dot{p}^F(t) = \rho\lambda_{X0}e^{\rho t}$.

During such a phase, the clean coal consumption must decrease, the dirty coal consumption is constant and equal to $\bar{x}_d$ since this is a phase at the ceiling, and the total energy consumption is also constant since $p(t) = c_y$. Hence, during such a phase, the solar energy consumption must increase in such a way that it always balances the decrease in clean coal consumption: $\dot{y}(t) = -\dot{x}_c(t)$.

Next, consider a phase at the ceiling during which only dirty coal and solar energy are simultaneously used. Since this is a phase at the ceiling, then $x_d(t) = \bar{x}_d$. Since solar energy is used, then $p(t) = c_y$, hence $q(t) = \tilde{y}$ and $y(t) = \tilde{y} - \bar{x}_d$. The consumption paths of dirty coal and solar energy are both constant during such a phase.

A typical optimal price path is a six phases path as illustrated in Figure 4. The corresponding energy consumption paths are illustrated in Figure 5.

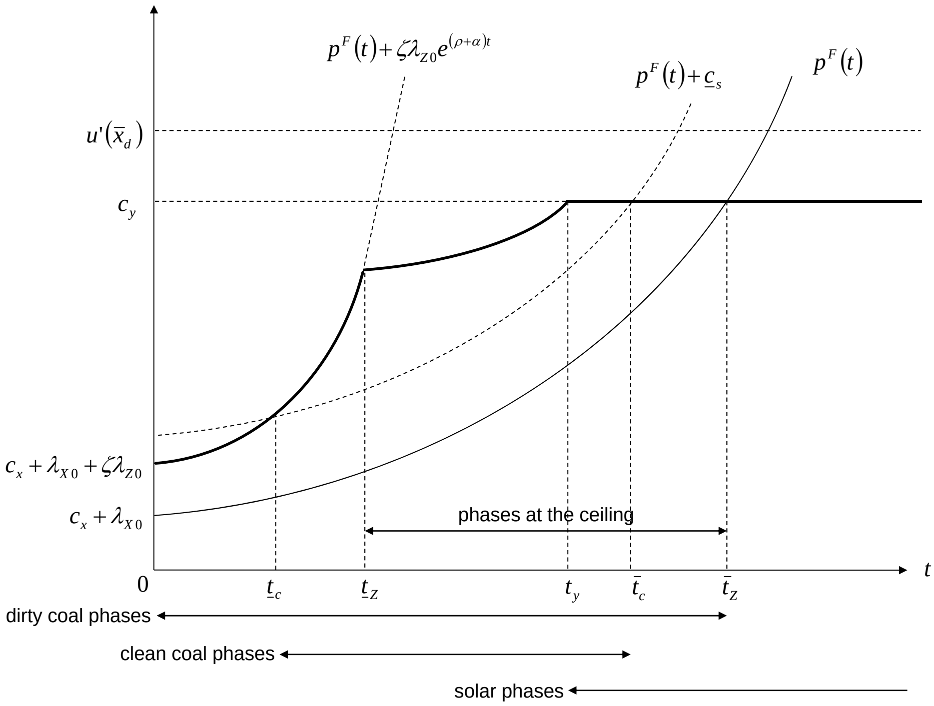

Figure 4: Optimal price path. Flow-dependent CCS average cost and low solar cost: $c_y < u'(\bar{x}_d)$

The three first phases of this optimal path are qualitatively the same as in the high solar cost case: First use dirty coal and only dirty coal, next exploit the both types of coal, that is begin the clean coal exploitation before attaining the ceiling, and third continue with this simultaneous use at the ceiling. From this step, the optimal path differs. Here, the third phase ends when the energy price reaches the marginal cost of solar energy c_y. Then begins phase $(t_y, \bar{t}_c)$ of simultaneous exploitation of the three types of energies – solar, clean and dirty coals – at the ceiling. The phase ends when $p^F(t) + \underline{c}_s = c_y$ so that clean coal is not competitive anymore as compared with solar energy. Since $\underline{c}_s > 0$, dirty coal remains competitive provided that its exploitation rate be maintained at $x_d(t) = \bar{x}_d$ in order to respect the ceiling constraint. Hence the next phase is a phase of simultaneous use of dirty coal and solar energy. This phase must end at $t = \bar{t}_Z$ when $p^F(t) = c_y$ or, equivalently, when $\lambda_Z(t) = 0$. At this time the coal stock must be exhausted. From $\bar{t}_Z$ onwards, solar energy is used alone and forever. Since there is no more pollution flow, the pollution stock $Z(t)$ starts to decrease and the ceiling constraint is no more active and forever.

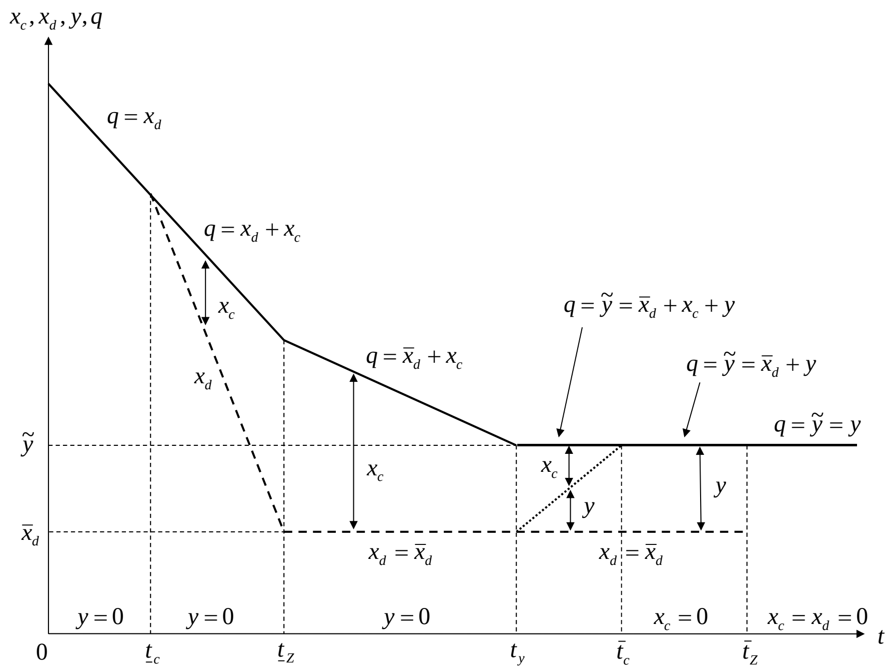

Figure 5: Optimal energy consumption paths. Flow-dependent CCS average cost and low solar cost: $c_y < u'(\bar{x}_d)$

The system of equations allowing to determine the endogenous variables λ_{X0}, λ_{Z0}, $\underline{t}_c$, $\underline{t}_Z$, t_y and $\bar{t}_Z$ in the case of a low solar cost is detailed in Appendix A.1.2.

The main conclusion of this section is that, whatever the marginal cost of the solar clean substitute, either high or low provided that it is constant, assuming that the average abatement cost of the potential pollution flow is an increasing and convex function of the flow of abatement implies that abatement must be activated before the pollution stock constraint begins to bind. Moreover, in the case of low solar costs, the three types of resources – clean coal, dirty coal and solar energy – are simultaneously exploited during the second and the third phases of the period at the ceiling (the third and fourth phases of the scenarios).

As we shall see in the next section, such characteristics of the optimal paths can never be obtained with stock-dependent CCS average cost functions.

4 Stock-dependent CCS cost functions

Although giving rise to contrasted optimal paths according to whether the scarcity effect or the learning one dominates, the optimal paths generated by CCS stock-dependent cost functions have some strongly similar formal features. We first point out these similarities before focusing on the specificities induced by the dominance of each effect.

4.1 Problem formulation and preliminary remarks

Whatever the effect of clean coal cumulative production which is dominant, either the scarcity effect or the learning effect, the social planner problem has the same following general structure:

$$\max_{x_c, x_d, y} \int_0^\infty \{u(x_c(t) + x_d(t) + y(t)) - c_x[x_c(t) + x_d(t)] - c_s(S_c(t))x_c(t) - c_y y(t)\} \, e^{-\rho t} dt$$

subject to constraints (5), (7), (9), to the inequality constraints (6), (11) and (12), all common to the both cases, and to the constraint (10) for the case of a dominant scarcity effect. This last condition is the only one which is differentiating the two dominant effect sub-cases.

Let us denote by λ_S the costate variable of S_c and keep the notations of the previous section for the other costate variables, that is λ_X for X and $-\lambda_Z$ for Z. Then the current valued Hamiltonian of the program reads:

$$\mathcal{H} = u(x_c + x_d + y) - c_x(x_c + x_d) - c_s(S_c)x_c - c_y y - \lambda_X[x_c + x_d] - \lambda_Z[\zeta x_d - \alpha Z] + \lambda_S x_c$$

Also adopting the same notations for the Lagrange multipliers and denoting by ν_S the multiplier associated with constraint (10), the current valued Lagrangian is:

$$\mathcal{L} = \mathcal{H} + \nu_X X + \nu_Z[\bar{Z} - Z] + \nu_S[\bar{S}_c - S_c] + \gamma_{x_c} x_c + \gamma_{x_d} x_d + \gamma_y y$$

with $\nu_S = 0$ for all $S_c \in [0, X^0]$ in the dominant learning effect case, a formal device to include the both CCS.2 and CCS.3 cases in a generic expression of the Lagrangian.

Among the first-order conditions (13)-(17) of the flow-dependent case, the condition (13) relative to the optimal use of x_c must be replaced by:

$$u'(x_c + x_d + y) = c_x + \lambda_X + c_s(S_c) - \lambda_S - \gamma_{x_c} \tag{30}$$

A new condition relative to the dynamics of λ_S must be introduced:

$$\dot{\lambda}_S = \rho\lambda_S + c_s'(S_c)x_c + \nu_S \tag{31}$$

together with the usual complementary slackness condition on ν_S . The associated transversality condition is:

$$\lim_{t\uparrow\infty} e^{-\rho t}\lambda_S(t)S_c(t) = 0 \tag{32}$$

The other first-order conditions (14)-(17) relative to the use of the other primary energies, x_d and y, and to the dynamics of λ_X and λ_Z remain unchanged, as well as the transversality conditions (18) and (19) relative to the long run values of X and Z.

Finally, note that from (31), as long as the clean coal has not yet been exploited, that is during an hypothetic initial phase of exclusive dirty coal consumption, we must have $\dot{\lambda}_S = \rho\lambda_S$, hence:

$$t \leq \underline{t}_c \;\Rightarrow\; \lambda_S(t) = \lambda_{S0}e^{\rho t}, \quad \text{where } \lambda_{S0} \equiv \lambda_S(0) \tag{33}$$

4.2 The case of a dominant scarcity effect

In the case of a dominant scarcity effect, the more the clean coal has been used in the past, the higher its present and future exploitation costs assuming that such exploitation is still possible, that is $S_c(t) < \bar{S}_c$. This suggests that λ_S should be negative.

Proposition 3 *Under a stock-dependent cost function CCS.2 with a dominant scarcity effect, assuming that the clean coal has to be exploited along the optimal path, the costate variable associated with the clean coal cumulated production is negative as long as its exploitation is not yet definitively closed:*

$$\forall t \geq 0: \quad \int_t^\infty x_c(\tau)d\tau > 0 \;\Rightarrow\; \lambda_S(t) < 0 \tag{34}$$

Proof: Solving the non-homogenous differential equation (31) results in:

$$\lambda_S(t) = \left\{\lambda_{S0} + \int_0^t [c_s'(S_c(\tau))x_c(\tau) + \nu_S(\tau)]e^{-\rho\tau}d\tau\right\}e^{\rho t} \tag{35}$$

where $\nu_S(t) \geq 0$. Next, using the transversality condition (32) and the condition $\lim_{t\uparrow\infty} S_c(t) \leq \bar{S}_c$ bounding $S_c(t)$ from above, we obtain the value of λ_{S0}:

$$\lambda_{S0} = -\int_0^\infty [c_s'(S_c(t))x_c(t) + \nu_S(t)]e^{-\rho t}dt$$

Substituting this value for λ_{S0} in the above expression (35) of $\lambda_S(t)$, we finally get:

$$\lambda_S(t) = -\int_t^{\infty} [c_s'(S_c(\tau))x_c(\tau) + \nu_S(\tau)]e^{-\rho(\tau-t)}d\tau \tag{36}$$

which is negative under the qualifying assumption $\int_t^{\infty} x_c(\tau)d\tau > 0$ since $c_s'(S_c) > 0$ under CCS.2. ■

From (36), it should be clear that $\lambda_S(t)$ includes two components. Increasing at time t the cumulated clean coal consumption by $x_c(t)$ units has two effects on the sum of the optimal future discounted[9] net surplus:

- first through the increase in the future sequestration costs by $c_s'(S_c(\tau))x_c(\tau)$, $\tau > t$;

- second through the tightening of the available capacity constraint restricting the size of the stock of carbon which could be stockpiled in the future, this second effect being captured by $\nu_S(\tau)$, $\tau > t$.

It remains to determine the behavior of $\lambda_S(t)$ once the qualifying condition (34) does not hold anymore, that is once the sequestration option is definitively closed, from time $t = \bar{t}_c$ onwards.

Proposition 4 *Under a stock-dependent cleaning cost function with a dominant scarcity effect, once the sequestration is definitively closed:*

- either the carbon reservoir capacity constraint is not binding at the closing time and then $\lambda_S(t) = 0$, more precisely:

$$S_c(\bar{t}_c) < \bar{S}_c \;\Rightarrow\; \lambda_S(t) = 0, \quad t \geq \bar{t}_c \tag{37}$$

- or the carbon stockpiling constraint is effective at the closing time and then:

$$S_c(\bar{t}_c) = \bar{S}_c \;\Rightarrow\; \lambda_S(t) = -\int_t^{\infty} \nu_S(\tau)e^{-\rho(\tau-t)}d\tau, \quad t \geq \bar{t}_c \tag{38}$$

Proof: This result is an immediate implication of (36) which holds at any time. For all $t \geq \bar{t}_c$, $x_c = 0$. If first $S_c(\bar{t}_c) < \bar{S}_c$, then for all $t \geq \bar{t}_c$, $S_c(t) < \bar{S}_c$ hence $\nu_S(t) = 0$ and thus, from (36), $\lambda_S(t) = 0$. Second if $S_c(\bar{t}_c) = \bar{S}_c$ then $S_c(t) = \bar{S}_c$ for all $t \geq \bar{t}_c$ and, from (36) again, we get (38). ■

The important point is that even if sequestration is definitively closed , $\lambda_S(t)$ may be still strictly negative at least for some time. We shall come back soon on the meaning of

[9]Discounted in value at time t.

the analytical expression of λ_S when the reservoir capacity constraint is tight at the closing date of the clean coal exploitation.

Since $\lambda_S(t) < 0$, at least as long as the sequestration is not definitively closed, then the full marginal cost of the clean coal amounts now to:

$$c_m^c(x_c(t)) = p^F(t) + c_s(S_c(t)) - \lambda_S(t) > p^F(t) + c_s(S_c(t)) \qquad (39)$$

This suggests first that, along the optimal path, the clean coal exploitation cannot begin before having attained the pollution cap $\bar{Z}$ (Proposition 5) and, second, that if the clean coal has ever to be used, then its exploitation must be closed before the end of the period at the ceiling (Proposition 6).

Proposition 5 *Under a stock-dependent CCS cost function with a dominant scarcity effect, if clean coal has ever to be used along the optimal path and provided that the ceiling constraint is binding along the path, then its exploitation cannot begin before the ceiling constraint is binding, in brief:* $\underline{t}_c \geq \underline{t}_Z$.

Proof: Assume that the clean coal is exploited while the ceiling is not attained yet: $\underline{t}_c < \underline{t}_Z$. Then, either only the clean coal is used during the time interval $[\underline{t}_c, \underline{t}_Z]$, or there exists a subinterval $[t'_c, t'_Z]$, $\underline{t}_c \leq t'_c < t'_Z \leq \underline{t}_Z$, during which the both types of coal are exploited, or, last, there exists a subinterval $[t''_c, t''_Z]$, $\underline{t}_c \leq t''_c < t''_Z \leq \underline{t}_Z$, during which the clean coal and the solar energy are simultaneously exploited.

First, if only the clean coal is used during $[\underline{t}_c, \underline{t}_Z]$, then from $Z(\underline{t}_c) < \bar{Z}$ and $\dot{Z}(t) = -\alpha Z(t) < 0$ for $t \in [\underline{t}_c, \underline{t}_Z]$, we conclude that $Z(\underline{t}_Z) < \bar{Z}$, a contradiction.

Second, assume that the both types of coal are simultaneously exploited during $[t'_c, t'_Z]$. Then their full marginal costs must be equal. Since the ceiling is not attained yet, the dirty coal full marginal cost amounts to $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ while the clean coal full marginal cost amounts to $p^F(t) + c_s(S_c(t)) - \lambda_S(t)$, $\lambda_S(t) < 0$. Hence:

$$\lambda_S(t) = c_s(S(t)) - \zeta\lambda_{Z0}e^{(\rho+\alpha)t}, \quad t \in [t'_c, t'_Z] \qquad (40)$$

Time differentiating the above equality leads to:

$$\dot{\lambda}_S(t) = c'_S(S(t))x_c(t) - \zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)t}$$

Substituting the left-hand-side of (31) with $\nu_S = 0$ for $\dot{\lambda}_S(t)$, and simplifying, we obtain:

$$\rho\lambda_S(t) = -\zeta(\rho+\alpha)\lambda_{Z0}e^{(\rho+\alpha)t}$$

Last, substitute the right-hand-side of (40) for $\lambda_S(t)$ in the above equality and simplify to get:

$$0 < \rho c_s(S_c(t)) = -\alpha\zeta\lambda_{Z0}e^{(\rho+\alpha)t} < 0, \quad t \in [t'_c, t'_Z]$$

again a contradiction.

Last, we prove in Proposition 8 that clean coal and solar energy may never be simultaneously exploited during any time interval along the optimal path. ■

At this stage, we know that the clean coal exploitation cannot begin before the ceiling is reached. Proposition 6 below shows that it cannot either be introduced after the beginning of the ceiling period.

Proposition 6 *Under a stock-dependent CCS cost function with a dominant scarcity effect, if clean coal has ever to be used along the optimal path, then its exploitation may not start after the beginning of the period at the ceiling:* $\underline{t}_c \leq \underline{t}_Z$.

Proof: Assume that $\underline{t}_Z \leq \underline{t}_c$, then during the time interval $[\underline{t}_Z, \underline{t}_c]$, either $y(t) = 0$ so that $x_d(t) = \bar{x}_d$, or $y(t) > 0$ and $y(t) + x_d(t) = y(t) + \bar{x}_d = \tilde{y}$, depending on wether $c_y \geq u'(\bar{x}_d)$ or $c_y < u'(\bar{x}_d)$, hence $p(t) = \min\{u'(\bar{x}_d), c_y\} \equiv \bar{p}$, $t \in [\underline{t}_Z, \underline{t}_c]$.

Since the clean coal is not competitive at $\underline{t}_Z$, its full marginal cost may not be lower than $\bar{p}$ at this time: $p^F(t)(\underline{t}_Z) + \underline{c}_s - \lambda_{S0}e^{\rho \underline{t}_Z} > \bar{p}$. Moreover, since $p^F(t)$ is increasing and λ_{S0} is negative, we have: $p^F(t)(t) + \underline{c}_s - \lambda_{S0}e^{\rho t} > \bar{p}$, $\forall t \in [\underline{t}_Z, \underline{t}_c]$, so that the clean coal consumption cannot become competitive at $\underline{t}_c$, hence a contradiction. ■

Thus from Propositions 5 and 6 we conclude that the exploitation of the clean coal must begin when the ceiling is attained: $\underline{t}_c = \underline{t}_Z$. The following Proposition 7 shows that its exploitation must be closed before the end of the ceiling period.

Proposition 7 *Under a stock-dependent CCS cost function with a dominant scarcity effect, if clean coal has ever to be used along the optimal path and provided that the ceiling constraint be binding along the path, then its exploitation must be closed before the end of the period at the ceiling.*

Proof: Assume that at the end of the period at the ceiling, the both types of coal are simultaneously used, that is $x_c(\bar{t}_Z) > 0$ and $x_d(\bar{t}_Z) > 0$. At this date, we know from (22) that the shadow marginal cost of the pollution stock must be nil: $\lambda_Z(\bar{t}_Z) = 0$. Then the dirty coal full marginal cost amounts to $p^F(\bar{t}_Z)$ while the clean coal full marginal cost amounts to $p^F(\bar{t}_Z) + c_s(S(\bar{t}_Z)) - \lambda_S(\bar{t}_Z) > p^F(\bar{t}_Z)$. Since the marginal cost of the clean coal is larger than the cost of the dirty one, only the dirty one has to be used, hence a contradiction. ■

Last, Proposition 8 will permit, together with the above propositions, to fully characterize the optimal path provided that the ceiling constraint has to be effective. It shows that the clean coal and the solar energy may never be simultaneously exploited.

Proposition 8 *Under a stock-dependent CCS.2 cost function with a dominant scarcity effect, the clean coal and the solar energy may never be exploited simultaneously along the optimal path.*

Proof: Let us assume that clean coal and solar energy are simultaneously used over some time interval. Their full marginal costs must be equal, that is: $c_y = c_x + \lambda_{X0}e^{\rho t} + c_s(S(t)) - \lambda_S(t)$. Time differentiating, substituting the RHS of (31) (with $\nu_S = 0$ since $S_c(t) < \bar{S}_c$) and simplifying, we get:

$$0 < \lambda_{X0}e^{\rho t} = \lambda_S(t) < 0$$

the RHS of this inequality directly coming from Proposition 3, hence a contradiction. ■

The Propositions 5, 6, 7 and 8 have different implications depending upon wether the cost of the solar energy is high or low.

4.2.1 The high solar cost case: $c_y > u'(\bar{x}_d)$

In this case, we may conclude from the above Propositions 5-8 that, if the ceiling constraint has to be effective and if the clean coal has to be exploited, then the period at the ceiling contains two phases, the first one being a phase during which the both types of coal are used and the second one a phase during which only the dirty coal is exploited. This is due to the fact that, at a price c_y even if only the dirty coal were exploited then x_d would be smaller than $\bar{x}_d$ hence the ceiling constraint could not be active.

A typical optimal path is a five-phases path as illustrated in Figure 6 for the energy price and in Figure 7 for the energy consumptions.[10]

[10] A full analytical characterization of the optimal path under CCS.2 is given in Appendix A.2 for the both cases of high and low solar costs.

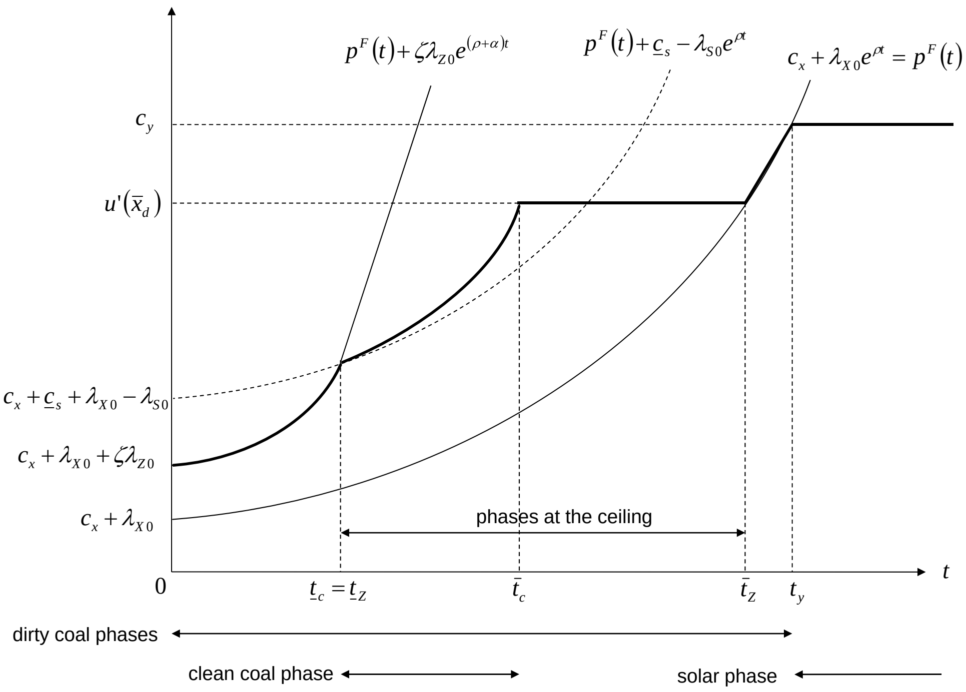

Figure 6: Optimal price path under stock-dependent CCS average costs, with a dominant scarcity effect. The high solar cost case: $c_y > u'(\bar{x}_d)$

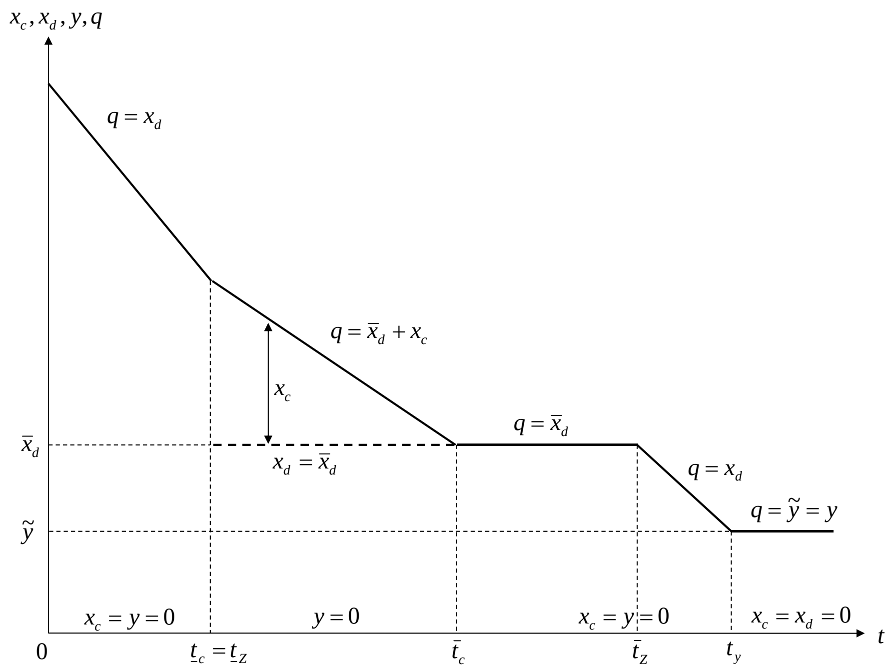

Figure 7: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant scarcity effect. The high solar cost case: $c_y > u'(\bar{x}_d)$

The first phase is a dirty coal phase during which the energy price is equal to $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$. Since only the dirty coal is exploited, its full marginal cost must be lower than the full marginal cost of the clean one, that is:

$$p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t} < p^F(t) + \underline{c}_s - \lambda_{S0}e^{\rho t}$$

Since $\lambda_Z(t)$ is growing at a higher proportional rate than $-\lambda_S(t)$, there exists some time $t = \underline{t}_c$ at which the both prices are equal. From Proposition 5, the ceiling constraint must begin to bind at this time, that is $\underline{t}_c = \underline{t}_Z$.

The second phase is a phase at the ceiling, the both types of coal being simultaneously used. During such a phase, the dirty coal production amounts to $x_d(t) = \bar{x}_d$. From the first-order-condition (30), the clean coal production must be such that $u'(x_c(t) + \bar{x}_d) = p^F(t) + c_s(S(t)) - \lambda_S(t)$. Time differentiating this expression and substituting the RHS of (31) for $\dot{\lambda}_S$ (with $\nu_S = 0$ since $S_c(t) < \bar{S}_c$), results in:

$$\dot{x}_c(t) = \frac{\rho[\lambda_{X0}e^{\rho t} - \lambda_S(t)]}{u''(x_c(t) + \bar{x}_d)} < 0 \tag{41}$$

Clean coal consumption decreases during the phase. Since this consumption is nil during the preceding phase, such a result is possible if and only if the clean coal consumption jumps upwards at the beginning of the second phase, that is at time $t = \underline{t}_Z = \underline{t}_c$. Moreover, this upward jump must be balanced by a downward jump of the same magnitude in the dirty coal consumption trajectory to preserve the continuity of the price path, as illustrated in Figure 6. Such discontinuities can arise thanks to the assumptions of constant full marginal cost of both the clean and the dirty coals at any time, which is the main difference between the stock-dependent CCS cost structure of the present section, and the flow-dependent structure of the previous section.

Another important remark which must be pointed out is that, during this phase of simultaneous exploitation of the both types of coal, we have:

$$\dot{p}(t) = \frac{d}{dt}\left[p^F(t) + c_s(S(t)) - \lambda_S(t)\right] = \dot{p}^F(t) - \rho\lambda_S(t) > \dot{p}^F(t) \tag{42}$$

Moreover, since the energy price $p(t)$ equals $p^F(t) + \zeta\lambda_Z(t)$ from the first-order condition (14) relative to the dirty coal use, then $p^F(t) + \zeta\lambda_Z(t) = p^F(t) + c_s(S(t)) - \lambda_S(t)$, and from (42):

$$\dot{p}^F(t) - \rho\lambda_S(t) = \dot{p}^F(t) + \zeta\dot{\lambda}_Z(t) > \dot{p}^F(t) \;\Rightarrow\; \dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_S(t) > 0 \tag{43}$$

However the instantaneous proportional growth rate of λ_Z is now lower than $\rho+\alpha$ because the ceiling constraint is tight, hence $\nu_Z(t) > 0$ (see (17)). Thus during this phase at the ceiling, the marginal social cost of the atmospheric carbon stock is growing as illustrated in Figure 6. However, the proportional growth rate of λ_Z is lower at the beginning of this phase than at the end of the preceding one, so that $\lim_{t\uparrow \underline{t}_Z} \dot{p}(t) > \lim_{t\downarrow \underline{t}_Z} \dot{p}(t)$, as in the case of flow-dependent cost function when the ceiling is reached.

This second phase ends at time $t = \bar{t}_c$ when the energy price attains the level $u'(\bar{x}_d)$ and, simultaneously, the consumption of clean coal falls down to zero since $x_d(\bar{t}_c) = \bar{x}_d$.

The third phase is a phase at the ceiling during which only the dirty coal is used: $x_d(t) = \bar{x}_d$, $x_c(t) = 0$. During this phase, $\lambda_Z(t) = u'(\bar{x}_d) - p^F(t)$ hence $\dot{\lambda}_Z(t) = -\rho\lambda_{X0}e^{\rho t} < 0$. The marginal social cost of the pollution stock is now decreasing. The phase ends at the time $t = \bar{t}_Z$ when λ_Z is nil.

From $\bar{t}_Z$ onwards, λ_Z is always nil and the next phase is the standard Hotelling phase of exclusive exploitation of the dirty coal up to that time $t = t_y$ at which the increasing energy price attains the level c_y allowing the solar energy to be a competitive substitute of the dirty coal and, simultaneously, the stock of coal is exhausted.

Note that, in this case, $\underline{t}_c = \underline{t}_Z$. Let us denote by $\underline{t}$ this common date: $\underline{t} \equiv \underline{t}_Z = \underline{t}_c$. Thus we have again seven endogenous variables to determine, as in the flow-dependent CCS cost case, but with one date missing and one more initial costate variable: λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}$, $\bar{t}_c$, $\bar{t}_Z$ and t_y. The seven equations system they are solving is detailed in Appendix A.2.1.

The value of λ_S after the end of the sequestration phase:

As pointed out in Proposition 4, when the stockpiling constraint is effective at the end of the sequestration phase, $\lambda_S(t)$ may then be still strictly negative for some time after the closing time of the clean coal exploitation. But how much time? It is clear that any additional stockpiling capacity which would be available only after $\bar{t}_Z$ would be worthless since the pollution ceiling constraint is not binding anymore from $\bar{t}_Z$ onwards. Let us show that the time period during which an additional stockpiling capacity would be exploited if it was available is shorter than $\bar{t}_Z - \bar{t}_c$.

Since we assume that the average CCS cost function is increasing in S_c, the reservoir capacity impacts the optimal scenarios by stopping the availability of stockpiling capacities at an average cost which is at least equal to $c_s(\bar{S}_c)$. The logic of the model would be to assume that any additional capacity $\Delta\bar{S}_c$ could be exploited at an average CCS cost $c_s(S_c)$ which is increasing over the interval $(\bar{S}_c, \bar{S}_c + \Delta\bar{S}_c)$. Over $[0, \bar{S}_c + \Delta\bar{S}_c]$, $c_s(S_c)$ should have the same general properties than over $[0, \bar{S}_c]$. However, in order to show that the time interval during which such an additional capacity has some value is shorter than $\bar{t}_Z - \bar{t}_c$, it is sufficient to show that this is the case even if the average CCS cost is the lowest one, that is equal to $c_s(\bar{S}_c)$.

From (14) and (30), the time $\tilde{t}$ at which the full marginal costs of the both types of coal would be equal while $\lambda_S(t) = 0$, is given as the solution of:

$$c_s(\bar{S}_c) = \zeta\lambda_Z(t)$$

From (14), since $u'(q(t)) = u'(\bar{x}_d)$ over the time interval $[\bar{t}_c, \bar{t}_Z]$, we have:

$$\zeta\lambda_Z(t) = u'(\bar{x}_d) - (c_x + \lambda_{X0}e^{\rho t}), \quad t \in [\bar{t}_c, \bar{t}_Z]$$

together with $\zeta\lambda_Z(\bar{t}_c) = c_s(\bar{S}_c) - \lambda_S(\bar{t}_c) > c_s(\bar{S}_c)$ and $\zeta\lambda_Z(\bar{t}_Z) = 0$. Thus there exists a unique time $\tilde{t}$: $\bar{t}_c < \tilde{t} < \bar{t}_Z$, at which $\zeta\lambda_Z(\tilde{t}) = c_s(\bar{S}_c)$ and from which any additional reservoir capacity is worthless.

4.2.2 The low solar cost case: $c_y < u'(\bar{x}_d)$

As in the case of flow-dependent costs, and for the same reasons, there may not exist a phase at the ceiling during which the dirty coal and only the dirty coal is exploited. Assuming that such a phase could exist, the energy price would have to be equal to $u'(\bar{x}_d)$, a price higher than the solar energy average cost c_y meaning that this alternative energy primary source should have to be exploited, thus a contradiction.

We know from Proposition 5 that if clean coal has to be used, it may not be before the pollution cap $\bar{Z}$ is reached and, from Proposition 7, that clean coal and solar energy may never be exploited simultaneously. Furthermore from Proposition 6, the clean coal exploitation must be closed before the end of the period at the ceiling. Thus if clean coal has to be used and the ceiling constraint has to be active along the optimal path, then the only possible period at the ceiling is a two-phases period. During the first one, the both clean and dirty coals are simultaneously exploited and during the second period, both the dirty coal and the solar energy. Typical paths – four-phases paths in the current case – of energy price and the associated energy consumptions are illustrated in Figures 8 and 9 respectively.

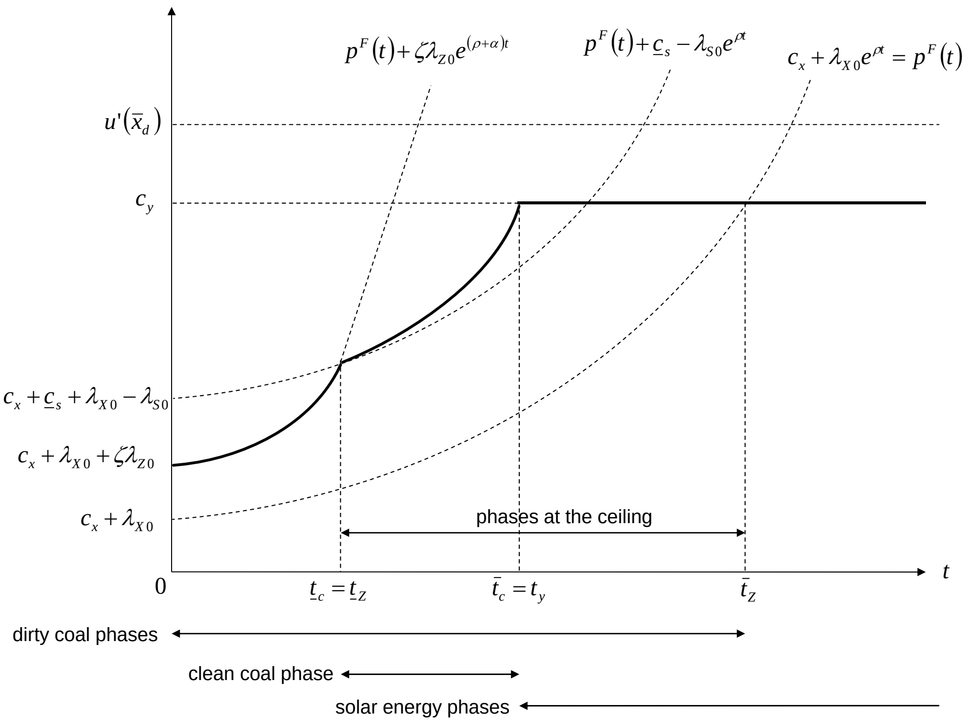

Figure 8: Optimal energy price path under stock-dependent CCS average costs, with a dominant scarcity effect. The low solar cost case: $c_y < u'(\bar{x}_d)$

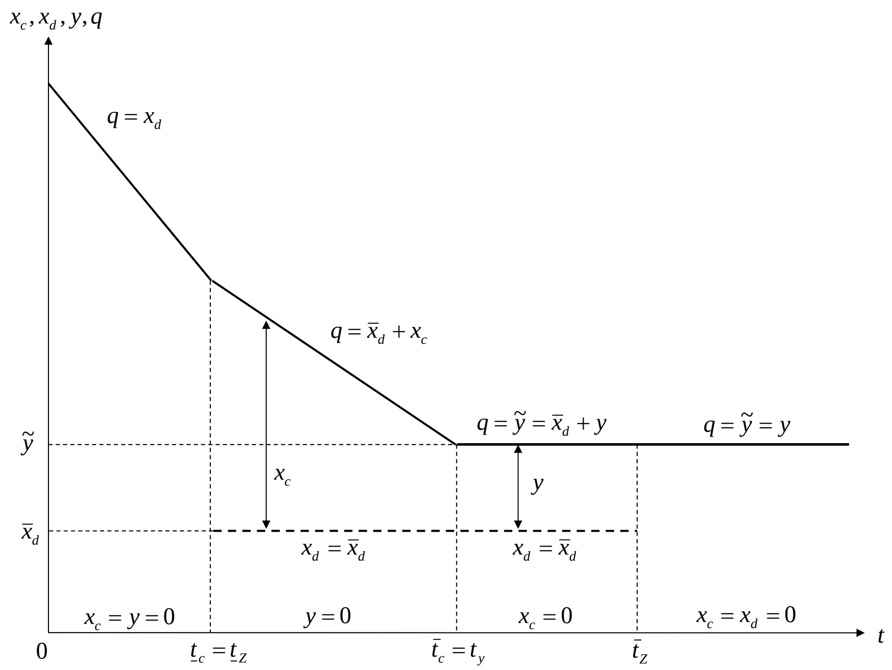

Figure 9: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant scarcity effect. The low solar cost case: $c_y < u'(\bar{x}_d)$

The two first phases are similar to the two first phases of the high solar cost case. The first phase is the usual phase of exclusive use of the dirty coal during which the atmospheric carbon stock grows up to the time $\underline{t}_Z$ at which the carbon cap is attained.

At time $\underline{t}_Z$, the clean coal becomes competitive, $\underline{t}_Z = \underline{t}_c$, and the resulting second phase is a phase of joint exploitation of the two types of coal while at the ceiling: $x_d(t) = \bar{x}_d$ and $x_c(t)$ is decreasing according to (41). Thus at time $t = \underline{t}_Z$, the dirty coal consumption is instantaneously reduced and this downward jump must be balanced by an upward jump of the same magnitude in the clean coal consumption. As in the high solar cost case during this phase:

$$\frac{d}{dt}\left[p^F(t) + c_s(S(t)) - \lambda_S(t)\right] > \dot{p}^F(t) \quad \text{and} \quad \dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_S(t) > 0$$

The argument is the same as the argument leading to expressions (42) and (43). The main difference with the high solar cost case is that now, the phase ends when the energy price is equal to c_y. At this point, the phases of competitiveness of the solar energy begin.

Just before this time t_y, since $c_y < u'(\bar{x}_d)$ and $x_d(t_y) = \bar{x}_d$, then $x_c(t) = \tilde{y} - \bar{x}_d > 0$. However, since the solar energy is competitive just after t_y and, from Proposition 7, both clean coal and solar energy may not be simultaneously used, hence the exploitation of the clean coal must be closed so that $t_y = \bar{t}_c$. Thus the clean coal consumption falls from $\tilde{y} - \bar{x}_d$ down to 0 and the production of the solar energy jumps from 0 up to $\tilde{y} - \bar{x}_d$ to keep the continuity of the energy services consumption path. During this third phase, the production of dirty coal and solar energy are both constant, $x_d(t) = \bar{x}_d$ and $y(t) = \tilde{y} - \bar{x}_d$, while the pollution stock remains at the ceiling level $Z(t) = \bar{Z}$. The associated shadow cost declines: $\lambda_Z(t) = (c_y - c_x - \lambda_{X0}e^{\rho t})/\zeta$. The phase ends at time $t = \bar{t}_Z$ when λ_Z has been reduced to 0, that is when $p^F(t) = c_y$. The exploitation of the dirty coal must be closed and simultaneously, the stock of coal must be exhausted.

The last phase from $\bar{t}_Z$ onwards is a phase of exclusive solar energy consumption, $q(t) = y(t) = \tilde{y}$. Then the pollution stock is gradually eliminated by natural absorption, $Z(t) = Z(\bar{t}_Z)e^{-\alpha(t-\bar{t}_Z)} = \bar{Z}e^{-\alpha(t-\bar{t}_Z)} < \bar{Z}$, $t \geq \bar{t}_Z$.

Note that in this low solar cost case, we have not only $\underline{t}_c = \underline{t}_Z (\equiv \underline{t})$, but also $\bar{t}_c = t_y$. Let us denote by $\hat{t}$ this other common date. Hence, only six variables have to be determined now: λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}$, $\hat{t}$ and $\bar{t}_Z$. The system of six equations that they solve is exposed in Appendix A.2.2.

The value of λ_S after the end of the clean coal exploitation phase:

Here again, λ_S may be strictly negative over some time interval $(\bar{t}_c, \tilde{t})$, $\bar{t}_c = \underline{t}_Z < \tilde{t} < \bar{t}_Z$, occurring at the end of the clean coal exploitation phase when the carbon capture policy is restricted by the reservoir capacity. The argument runs along the same lines than the argument developed in the high solar cost case, but during the phase $[\bar{t}_c, \bar{t}_Z]$, the λ_Z-path is now established from c_y instead of $u'(\bar{x}_d)$ since the energy price path is determined by c_y during this time interval. More precisely, we have:

$$\zeta\lambda_Z(t) = c_y - (c_x + \lambda_{X0}e^{\rho t}), \quad t \in [\bar{t}_c, \bar{t}_Z]$$

together with $\zeta\lambda_Z(\bar{t}_c) = c_s(\bar{S}_c) - \lambda_S(\bar{t}_c) > c_s(\bar{S}_c)$ and $\zeta\lambda_Z(\bar{t}_Z) = 0$. Hence there exists a unique time $t = \tilde{t}$ solving $\zeta\lambda_Z(t) = c_s(\bar{S}_c)$ and defining the date from which λ_S is nil forever.

4.3 The case of a dominant learning effect

Now, the more the clean coal has been used in the past, the lower its marginal additional cost as compared with the dirty coal. This suggests that λ_S should be positive up to the time at which its exploitation is definitively closed.

Proposition 9 *Under a stock-dependent CCS cost function with a dominant learning effect, assuming that the clean coal has to be exploited along the optimal path, the costate variable associated with the clean coal cumulated production is positive as long as its exploitation is not definitively closed:*

$$\forall t \geq 0 : \quad \int_t^\infty x_c(\tau)d\tau > 0 \Rightarrow \lambda_S(t) > 0 \tag{44}$$

Proof: This is a direct implication of (36) with $\nu_S = 0$ and $c'_s < 0$:

$$\lambda_S(t) = -\int_t^\infty c'_s(S_c(\tau))x_c(\tau)e^{-\rho(\tau-t)}d\tau > 0 \quad \blacksquare \tag{45}$$

Integrating by parts (45) we get the following alternative expression of $\lambda_S(t)$ which will turn out to be useful in the proof of Propositions 10, 11 and 12:

$$\lambda_S(t) = c_s(S_c(t)) - \rho\int_t^\infty c_s(S_c(\tau))e^{-\rho(\tau-t)}d\tau \tag{46}$$

Note that in the present case, once the exploitation of the clean coal is definitively closed, then λ_S is nil:

$$\forall t \geq \bar{t}_c : \quad \lambda_S(t) = 0 \tag{47}$$

The following Propositions 10 and 11 show that, as in the case of a dominant scarcity effect, the exploitation of the clean coal cannot begin before the ceiling constraint is binding and must be closed before the end of the ceiling period in the case of a learning effect. However, as we shall see, it may happen that the optimal clean coal exploitation has to begin after the time at which the ceiling is attained. Under a dominant learning effect, the equivalent of Proposition 7 obtained under a dominant scarcity effect does not hold anymore.

Proposition 10 *Under a stock-dependent CCS cost function with a dominant learning effect, if clean coal has ever to be used along the optimal path and provided that the ceiling constraint be active along the path, then its exploitation may not begin before the ceiling constraint is binding:* $\underline{t}_c \geq \underline{t}_Z$.

Proof: The proof runs along the lines of the proof of Proposition 5, but some details of the arguments must be adapted. Assume that $\underline{t}_c < \underline{t}_Z$. First, if during the time interval $[\underline{t}_c, \underline{t}_Z]$ only the clean coal is used, then the argument is the same.

Second, assume that both the dirty and clean coals are exploited during some interval $[t'_c, t'_Z]$. Equating their respective full marginal costs results in:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)t} = c_s(S_c(t)) - \lambda_S(t), \quad t \in (t'_c, t'_Z)$$

Substituting the R.H.S. of (46) for $\lambda_S(t)$, we get:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)t} = \rho \int_t^\infty c_s(S_c(\tau)) e^{-\rho(\tau-t)} d\tau \tag{48}$$

Time differentiate to obtain:

$$\zeta(\rho+\alpha)\lambda_{Z0} e^{(\rho+\alpha)t} = -\rho c_s(S_c(t)) + \rho^2 \int_t^\infty c_s(S_c(\tau)) e^{-\rho(\tau-t)} d\tau$$

that is, taking (48) into account:

$$0 < \zeta \alpha \lambda_{Z0} e^{(\rho+\alpha)t} = -\rho c_s(S_c(t)) < 0, \quad t \in [t'_c, t'_Z]$$

hence a contradiction.

Last we show in Proposition 12 that clean coal and solar energy may never be exploited simultaneously. ■

Proposition 11 *Under a stock-dependent CCS cost function with a dominant learning effect, if clean coal has ever to be used along the optimal path and provided that the ceiling constraint be active along the path, then its exploitation must be closed before the end of the ceiling period.*

Proof: Assume that at $\bar{t}_Z$, the ending time of the ceiling period, the both types of coal are still used, that is $x_c(\bar{t}_Z) > 0$ and $x_d(\bar{t}_Z) = \bar{x}_d$. Equating their full marginal costs and taking into account that $\lambda_Z(\bar{t}_Z) = 0$ from (22), we get:

$$p^F(\bar{t}_Z) = p^F(\bar{t}_Z) + c_s(S_c(\bar{t}_Z)) - \lambda_S(\bar{t}_Z)$$

Substituting the R.H.S. of (46) for $\lambda_S(\bar{t}_Z)$ results in:

$$p^F(\bar{t}_Z) = p^F(\bar{t}_Z) + \rho \int_{\bar{t}_Z}^{\infty} c_s(S_c(\tau))e^{-\rho(\tau-t)}d\tau > p^F(\bar{t}_Z)$$

a contradiction. ■

The last common feature of the optimal paths for the both cases of scarcity and learning dominant effects stands in the impossibility of using simultaneously the clean coal and the solar energy. Here again, the proof has to be adapted from Proposition 8.

Proposition 12 *Under a stock-dependent CCS cost function with a dominant learning effect, the clean coal and the solar energy may never be exploited simultaneously along the optimal path.*

Proof: Assume that the clean coal and the solar energy are simultaneously used during some interval $[t_1, t_2]$. Equating their full marginal costs results in:

$$c_y = c_x + \lambda_{X0}e^{\rho t} + c_s(S_c(t)) - \lambda_S(t), \quad t \in [t_1, t_2]$$

Substituting the R.H.S. of (46) for $\lambda_S(t)$, we get:

$$c_y - c_x = \lambda_{X0}e^{\rho t} + \rho \int_t^{\infty} c_s(S_c(\tau))e^{-\rho(\tau-t)}d\tau \tag{49}$$

Time differentiating, we obtain:

$$0 = \rho[\lambda_{X0}e^{\rho t} - c_s(S_c(t))] + \rho^2 \int_t^{\infty} c_s(S_c(\tau))e^{-\rho(\tau-t)}d\tau$$

and taking (49) into account:

$$0 = \rho[c_y - c_x] - \rho c_s(S_c(t))$$

Time differentiating again, we finally get:

$$0 = -\rho c_s'(S_c(t))x_c(t) > 0, \quad t \in [t_1, t_2]$$

a contradiction ■

Having reviewed the common features of the optimal paths in the cases of scarcity and learning dominant effects, let us turn now to their differences.

From Propositions 10, 11 and 12, the only kind of phases during which the clean coal is used is a phase of joint exploitation of the both types of coal while at the ceiling. Thus if the scarcity and learning dominant effects have different implications, and they should have at least in some cases, this may be because:

- either what happens during this kind of phase is different in the two cases,

- or the position of this phase within the optimal sequence of phases is different in the two cases,

- or the both.

Let us examine first the reasons for which what happens within this kind of phase could be different. During such a phase, $q(t) = x_c(t) + \bar{x}_d$, $t \in [\underline{t}_c, \bar{t}_c]$, and the time derivative of x_c is given formally by (see (41)):

$$\dot{x}_c(t) = \frac{\rho[\lambda_{X0}e^{\rho t} - \lambda_S(t)]}{u''(x_c(t) + \bar{x}_d)} \tag{50}$$

the difference with (41) being that we cannot conclude here about the sign of $\dot{x}_c(t)$ since $\lambda_S(t) > 0$. However, we can show that $x_c(t)$, hence $p(t)$, may follow two types of trajectories and only two during the phase.

First remark that, from (47), $\lambda_S(t)$ is tending to 0 at the end of the phase. Thus, since $\lambda_S(t)$ is necessarily continuous in such a model, there must exist some terminal interval $[\bar{t}_c - \Delta, \bar{t}_c]$, $0 < \Delta \leq \bar{t}_c - \underline{t}_c$, during which $\dot{x}_c(t)$ is negative and the energy price is increasing. We have now to determine what could happen at the beginning of the phase when this terminal interval is strictly shorter than the entire phase, that is when $\Delta < \bar{t}_c - \underline{t}_c$.

The following Proposition 13 states that the sign of $\dot{x}_c(t)$ may change at most only once within the phase.

Proposition 13 *Under a stock-dependent CCS cost function with a dominant learning effect, assuming that there exists a phase during which the both types of coal are exploited while at the ceiling, then during such a phase:*

- either the price of the energy services is monotonically increasing,

- or the price of the energy services is first decreasing and next increasing.

Proof: Assume that $\lim_{t \downarrow \underline{t}_c} \dot{x}_c(t) > 0$. Define t_0 as the first date at which $\dot{x}_c(t)$ alternates in sign, since in this case the sign is changing at least once:

$$t_0 = \inf \{t : \dot{x}_c(t) \leq 0, t \in [\underline{t}_c, \bar{t}_c)\} \Rightarrow \dot{x}_c(t_0) = 0$$

From (30) and (31) respectively, we get:

$$\begin{aligned} u'(x_c(t) + \bar{x}_d) &= c_x + \lambda_{X0} e^{\rho t} + c_s(S_c(t)) - \lambda_S(t) \\ \dot{\lambda}_S(t) &= \rho \lambda_S(t) + c_s'(S_c(t)) x_c(t) = \rho \lambda_S(t) + \dot{c}_s(S_c(t)) \end{aligned}$$

with $\dot{c}_s(S_c(t)) < 0$. Time differentiating the first expression and using the second one, we get:

$$u''(x_c(t) + \bar{x}_d)\dot{x}_c(t) = \rho[\lambda_{X0} e^{\rho t} - \lambda_S(t)]$$

Define $\phi(t) = \lambda_{X0} e^{\rho t} - \lambda_S(t)$. Then $u'' < 0$ implies that:

$$\dot{x}_c(t) > / = / < 0 \Leftrightarrow \phi(t) < / = / > 0$$

Time differentiating $\phi(t)$ and using (31), we get:

$$\dot{\phi}(t) = \rho \lambda_{X0} e^{\rho t} - \rho \lambda_S(t) - \dot{c}_s(S_c(t)) = \rho \phi(t) - \dot{c}_s(S_c(t))$$

Integrating over $[t_0, t]$, $t_0 < t \leq \bar{t}_c$, and taking into account that $\phi(t_0) = 0$, we obtain:

$$\phi(t) = -e^{\rho t} \int_{t_0}^{t} \dot{c}_s(S_c(\tau)) e^{-\rho \tau} d\tau > 0, \quad t \in (t_0, \bar{t}_c]$$

We conclude that, if the sign of $\dot{\phi}(t)$, hence the sign of $\dot{x}_c(t)$ and $\dot{p}(t)$, is changing over $[\underline{t}_c, \bar{t}_c)$, it is only once. ■

The last common characteristics shared by all the paths is about their behavior during the pre-ceiling phase, hence also before the beginning of the clean coal exploitation according to Proposition 10, that is over the time interval $[0, \underline{t}_Z] \subseteq [0, \underline{t}_c]$. During this initial phase, from (35), the shadow full marginal cost of the clean coal amounts to:

$$c_m^c = c_x + \bar{c}_s + (\lambda_{X0} - \lambda_{S0}) e^{\rho t}$$

which may be either increasing or decreasing depending on whether the shadow marginal cost of coal λ_{X0} is larger or smaller than the shadow marginal value of the cumulated experience in cleaning some part of its available stock, λ_{S0}. Such a formulation could prove to be paradoxical since no experience has been yet accumulated. But this is the marginal value of a zero-experience and this marginal value may be very high.

The sign of $\lambda_{X0} - \lambda_{S0}$, which is endogenous, determines the position of the phase of simultaneous exploitation of the both types of coal in the optimal sequence of phases. However, as in the case of a dominant scarcity effect, the types of optimal sequences are depending upon whether the solar energy cost is high or low.

4.3.1 The high solar cost case: $c_y > u'(\bar{x}_d)$

We examine the different possible types of paths according to the sign of $\lambda_{X0} - \lambda_{S0}$.

- Case where $\lambda_{X0} > \lambda_{S0}$

In this case, the shadow marginal value of the experience is relatively low as compared with the coal scarcity rent and the structure of the optimal path is strongly determined by the dominance of this scarcity effect.

Since $\lambda_{X0} > \lambda_{S0}$ and provided that there exists a phase of joint use of the both types of coal while at the ceiling, the clean coal exploitation must precisely begin at the time at which the pollution cap $\bar{Z}$ is reached. The argument is given by Figure 10. At the crossing point of the trajectories $p^F(t) + \bar{c}_s - \lambda_{S0}e^{\rho t}$ and $p^F(t) + \zeta\lambda_{Z0}e^{(\rho+\alpha)t}$ (remind that $p^F(t) = c_x + \lambda_{X0}e^{\rho t}$), either the common full marginal cost is lower than $u'(\bar{x}_d)$ as illustrated in Figure 10, or it is higher (not depicted) so that the clean coal is never competitive. Thus the unique possible optimal sequence of phases is: i) only dirty coal up to the time at which the ceiling is attained and, simultaneously, the clean coal becomes competitive, ii) both the dirty and clean coals while at the ceiling, iii) only dirty coal while at the ceiling, iv) again dirty coal only during a post-ceiling phase, and v) the infinite phase of solar energy use.

The other implication of $\lambda_{X0} > \lambda_{S0}$ is that at time $\underline{t}_c^+$, at the beginning of the phase of joint exploitation of the both types of coal, due to the continuity of $\lambda_S(t)$ in the present case, then:

$$\lambda_{X0}e^{\rho \underline{t}_c^+} - \lambda_S(\underline{t}_c^+) \simeq (\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c^+} > 0 \tag{51}$$

From (50) we conclude that $\dot{x}_c(\underline{t}_c^+) < 0$, hence from Proposition 13, that $\dot{x}_c(t) < 0$ for all t during the phase and the energy price is increasing.

Although the optimal price path depicted by Figure 10 could look quite similar to the optimal price path determined in the case of a dominant scarcity effect with high solar cost as illustrated in Figure 6, these two cases notably differ during the phase of a joint

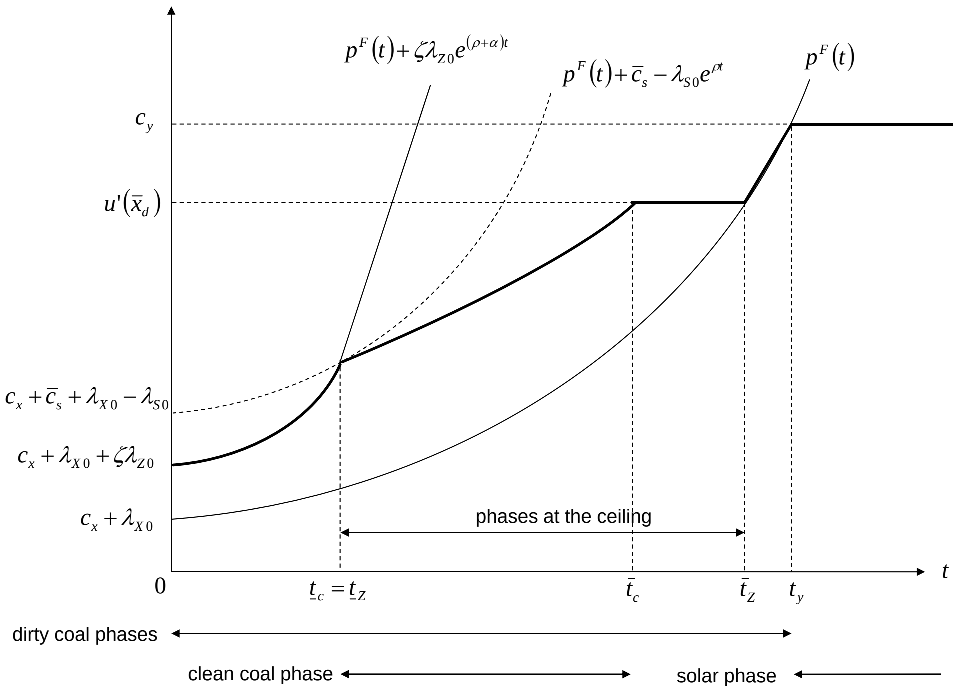

Figure 10: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} > \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$

exploitation of the two types of coal while at the ceiling. In the both cases, we have $\dot{x}_c(t) < 0$ hence $\dot{p}(t) > 0$, but contrary to the case of a dominant scarcity effect, here the shadow marginal cost of the pollution stock $\lambda_Z(t)$ decreases during this phase. From (42) and (43), we obtain now:

$$\dot{p}(t) = \frac{d}{dt}\left[p^F(t) + c_s(S(t)) - \lambda_S(t))\right] = \dot{p}^F(t) - \rho\lambda_S(t) < \dot{p}^F(t) \tag{52}$$

and:

$$\dot{p}^F(t) - \rho\lambda_S(t) = \dot{p}^F(t) + \zeta\lambda_Z(t) < \dot{p}^F(t) \Rightarrow \dot{\lambda}_Z(t) = -\frac{\rho}{\zeta}\lambda_S(t) < 0 \tag{53}$$

However, the qualitative properties of the energy consumption paths (not illustrated) are almost the same as the ones depicted in Figure 7.

- Case where $\lambda_{X0} < \lambda_{S0}$

In this case, the shadow marginal value of the CCS experience is higher than the scarcity rent of coal. This gives rise to some new types of optimal paths, not only because what is happening during the phase of joint exploitation of the both types of coal is different, but also because the position of this phase within the optimal sequence of phases may be different.

Figures 11 and 12 illustrate why the time profile of the energy price and the energy consumption paths are different within this phase of joint exploitation although the optimal sequence of phases is the same as the sequence of the previous subcase $(\lambda_{X0} - \lambda_{S0}) > 0$.

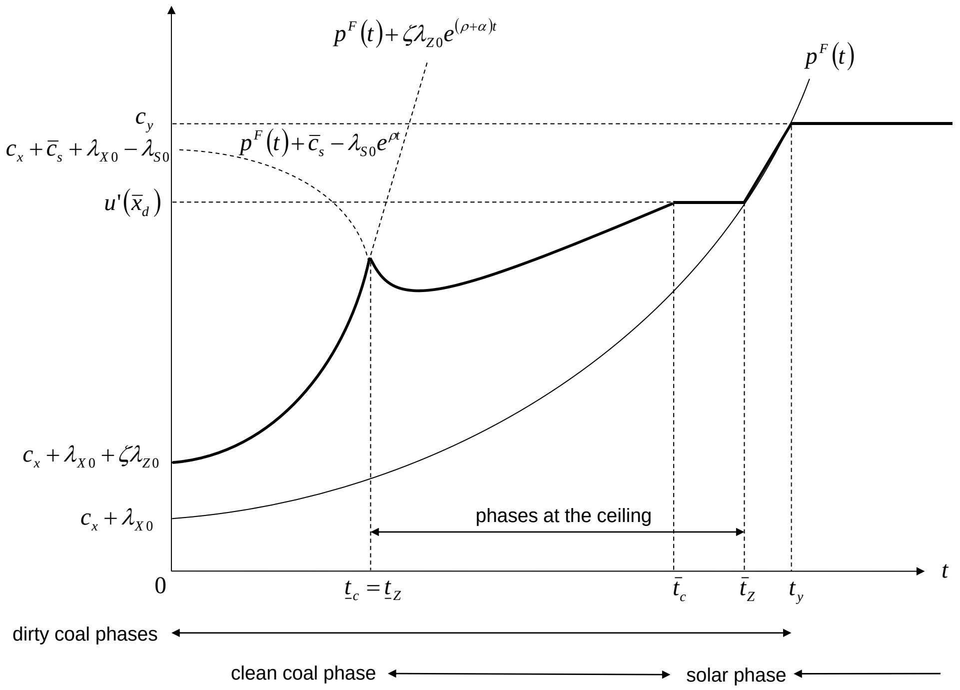

Figure 11: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$ and $\underline{t}_Z = \underline{t}_c$

Since $(\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c} < 0$, then at the beginning of the joint exploitation phase we may have $\lambda_{X0}e^{\rho \underline{t}_c^+} - \lambda_S(\underline{t}_c^+) < 0$ so that $\dot{x}(\underline{t}_c^+) > 0$. From Proposition 13 we know that, in this case, the energy price must be first decreasing and next increasing as illustrated in Figure 11, implying an unusual increase in the total coal consumption once the pollution cap is attained to capitalize on the learning effects. In fact, at the time $\underline{t}_Z = \underline{t}_c$ at which the ceiling is reached, the clean coal becomes also competitive thus triggering a shock – an instantaneous upward jump – in the allocation of its cumulated consumption, contrary to the dominant scarcity effect case.

The other main characteristics of this phase of joint exploitation of the two kinds of coal while at the ceiling is the pattern of the shadow marginal cost of the pollution stock. Clearly, since the price of the energy services is decreasing at the beginning of the phase, then $\lambda_Z(t)$ must be initially decreasing. But an important point is that $\lambda_Z(t)$ also decreases during the second part of the phase when the energy price increases again. The

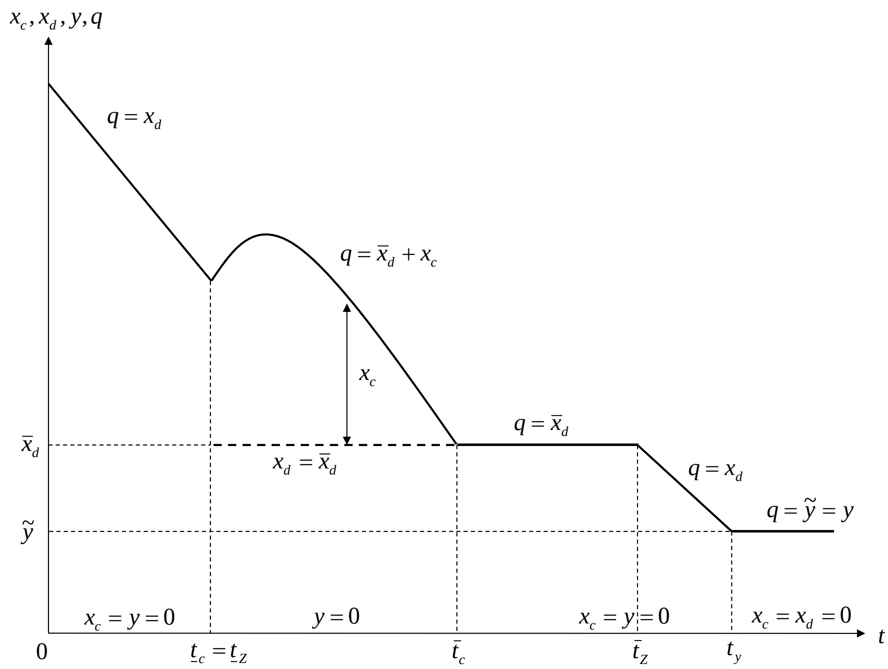

Figure 12: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$ and $\underline{t}_Z = \underline{t}_c$

formal argument is the argument developed to obtain the above relationships (52) and (53), argument which holds whatever is the sign of $\lambda_{X0} - \lambda_{S0}$.

Finally, a last case has to be considered. In Figures 13 and 14, the optimal sequence of phases is modified in the following terms. The clean coal begins to be competitive after the beginning of the period at the ceiling so that $\underline{t}_c$ does not coincide anymore with $\underline{t}_Z$. Consequently, the phase of joint exploitation of the both types of coal takes place within the period at the ceiling and it is flanked by two phases of exclusive dirty coal use: $\underline{t}_Z < \underline{t}_c < \bar{t}_c < \bar{t}_Z$.

Contrary to the above cases of stock-dependent average cost functions, the exploitation of the clean coal begins here smoothly: $\lim_{t \downarrow \underline{t}_c} x_c(t) = 0$. Hence, there is not an abrupt change anymore in the total coal consumption use at time $\underline{t}_c$, contrary to the case where $\underline{t}_c = \underline{t}_Z$.

The system of equations from which the endogenous variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}_Z$, $\underline{t}_c$, $\bar{t}_c$, $\bar{t}_Z$ and t_y can be extracted in the high solar cost case is detailed in Appendix A.3.1 for the both subcases $\lambda_{X0} > \lambda_{S0}$ and $\lambda_{X0} < \lambda_{S0}$. This system contains seven equations when

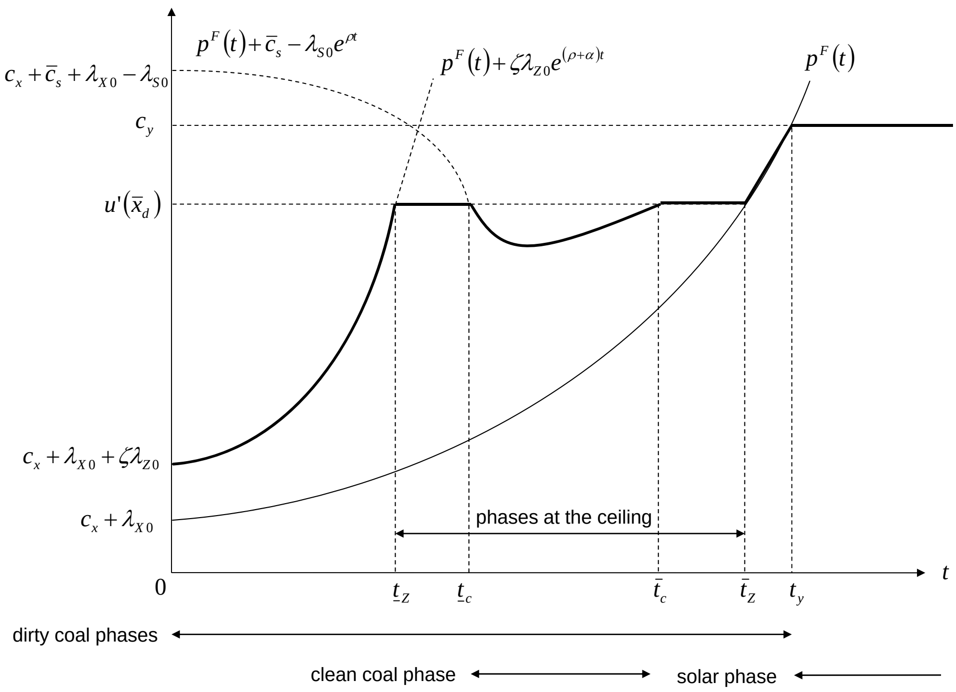

Figure 13: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$ and $\underline{t}_Z < \underline{t}_c$

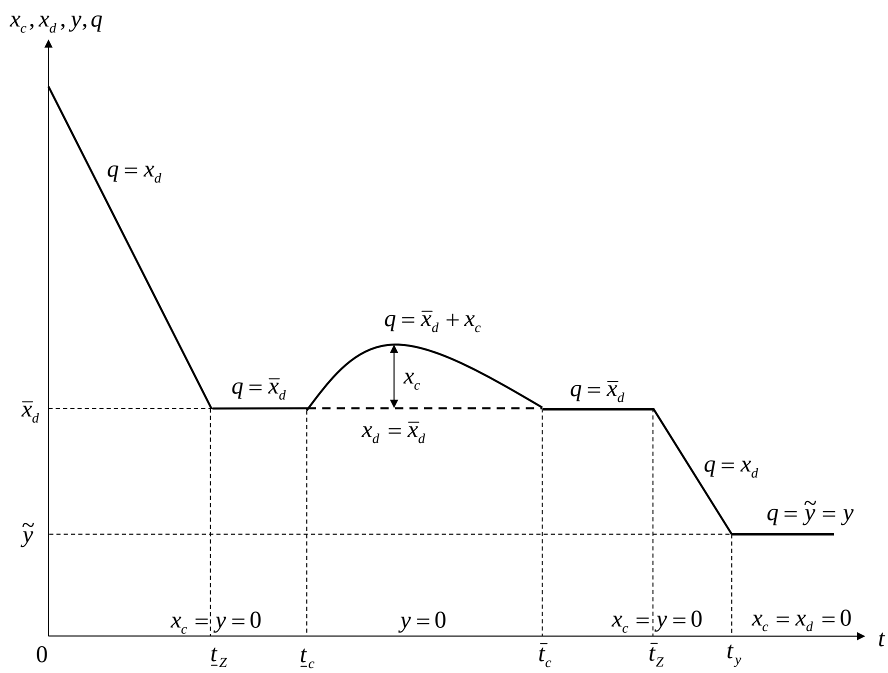

Figure 14: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The high solar cost case: $c_y > u'(\bar{x}_d)$ and $\underline{t}_Z < \underline{t}_c$

$\underline{t}_Z = \underline{t}_c \equiv \underline{t}$, and eight equation when $\underline{t}_Z < \underline{t}_c$.

4.3.2 The low solar cost case: $c_y < u'(\bar{x}_d)$

As in the high solar cost case, various types of optimal paths can appear according to whether $(\lambda_{X0} - \lambda_{S0})$ is positive or negative.

- Case where $\lambda_{X0} > \lambda_{S0}$

Qualitatively, this case is similar to the case in which the scarcity effect dominates and the solar cost is low. According to the arguments developed in the previous paragraph, the phase of joint exploitation of the two types of coal must begin when the ceiling is attained and the energy price must be increasing during this phase although the shadow marginal cost of the pollution stock is decreasing, up to the time at which this price equals c_y instead of $u'(\bar{x}_d) < c_y$, time at which the solar energy becomes competitive. Then, from Proposition 12, the exploitation of the clean coal must cease at this time. The production of solar energy thus substitutes for the production of clean coal while staying at the ceiling up to the time at which $p^F(t) = c_y$. Last the dirty coal exploitation is closed, the coal reserves must be exhausted and the solar energy supplies to totality of the energy needs. Consequently, the price and consumption paths are qualitatively similar to the paths illustrated in Figures 8 and 9 respectively.

- Case where $\lambda_{X0} < \lambda_{S0}$

First, the period of joint exploitation of the two types of coal may precede the period of competitiveness of the solar energy. The associated price and consumption paths are illustrated in Figures 15 and 16 respectively.

However, as illustrated in Figure 17, the phase of competitiveness of the clean coal may also take place once the solar energy is competitive, that is at a date at which the solar energy is already exploited from some time: $t_y = \underline{t}_Z < \underline{t}_c < \bar{t}_c < \bar{t}_Z$. In this case, the exploitation of the solar energy must be interrupted since the energy price falls down the trigger price c_y during the time interval $[\underline{t}_c, \bar{t}_c]$ of joint exploitation of the both kinds of coal. At time $t = \bar{t}_c$, the solar energy becomes competitive again and its production replaces the production of the clean coal. Then, the dirty coal and the solar energy are simultaneously exploited, as in the first phase at the ceiling, up to the time $t = \bar{t}_Z$ at which

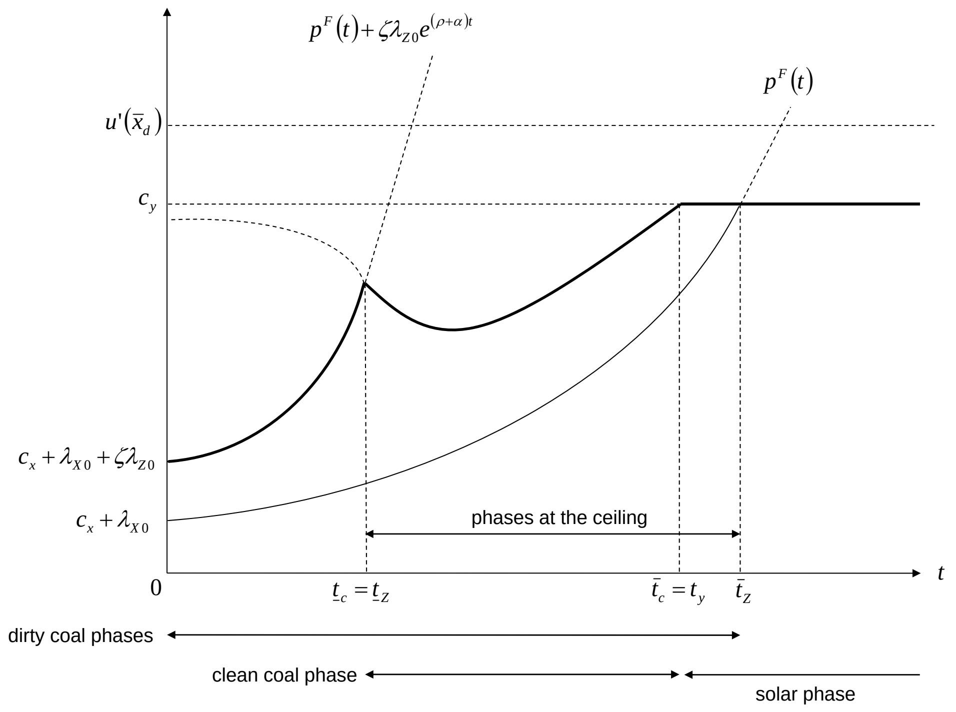

Figure 15: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The low solar cost case: $c_y < u'(\bar{x}_d)$ and $\underline{t}_Z = \underline{t}_c$

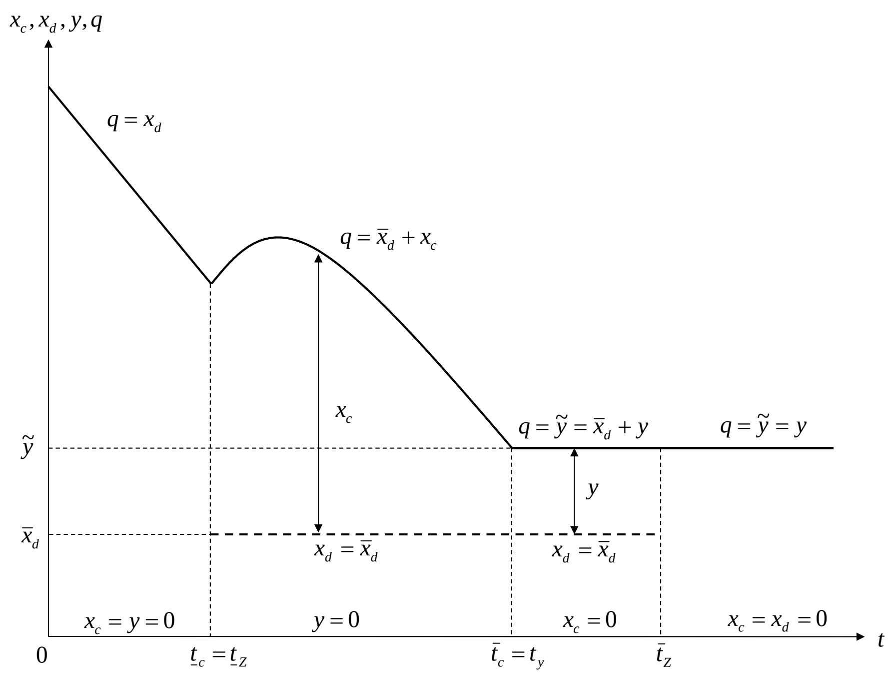

Figure 16: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The low solar cost case: $c_y < u'(\bar{x}_d)$ and $\underline{t}_Z = \underline{t}_c$

$p^F(t) = c_y$ and at which the stock of coal is exhausted. The associated energy consumption paths are illustrated in Figure 18.

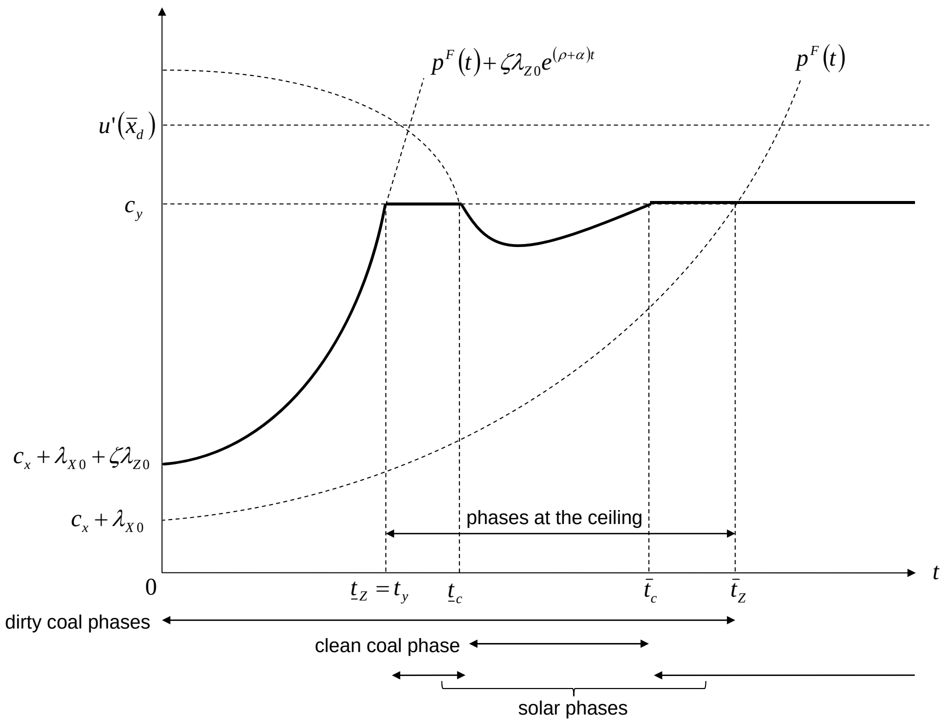

Figure 17: Optimal price path under stock-dependent CCS average costs with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The low solar cost case: $c_y < u'(\bar{x}_d)$ and $\underline{t}_Z < \underline{t}_c$

Last, the full characterization of the optimal path under a CCS.3 cost function in the low solar cost case, that is the determination of the endogenous variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}_Z$, $\underline{t}_c$, $\bar{t}_c$, $\bar{t}_Z$ and t_y, is developed in Appendix A.3.2 for the both subcases $\lambda_{X0} > \lambda_{S0}$ and $\lambda_{X0} < \lambda_{S0}$.

5 Optimal time profile of the carbon tax

The main tax of this model is the carbon tax, the duty having to be charged per unit of carbon emission released into the atmosphere when some part of the energy services are produced from dirty coal.

Whatever the assumptions about the CCS cost functions and about the level of the solar energy cost, the time profile of this tax is, qualitatively, roughly the same: first increasing from some positive level and next declining down to zero at time $\bar{t}_Z$, the end of the period during which the ceiling constraint is binding (see (22)). However, the date at

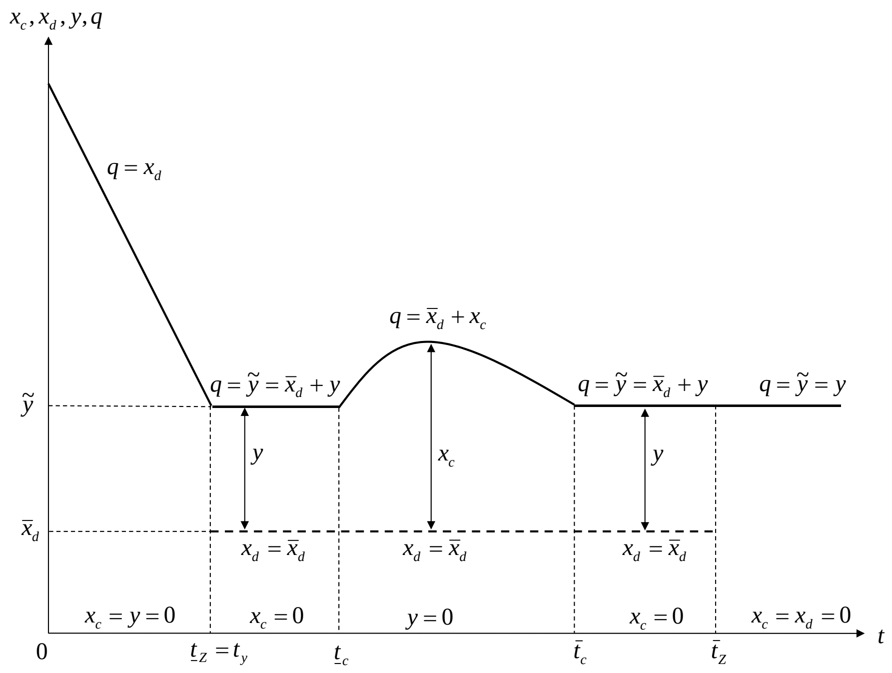

Figure 18: Optimal energy consumption paths under stock-dependent CCS average costs, with a dominant learning effect and $\lambda_{X0} < \lambda_{S0}$. The low solar cost case: $c_y < u'(\bar{x}_d)$ and $\underline{t}_Z < \underline{t}_c$

which the maximum is attained is not necessary the same under all the assumptions. The various possibilities are illustrated in Figure 19 where case a. depicts the flow-dependent CCS cost case, case b. the stock-dependent cost case with a dominant scarcity effect, case c. the stock-dependent cost case with a dominant learning effect when $\underline{t}_Z = \underline{t}_c$ whatever is the sign of $\lambda_{X0} - \lambda_{S0}$ and, last, case d. the stock-dependent cost case with a dominant learning effect when $\lambda_{X0} < \lambda_{S0}$ and $\underline{t}_c > \underline{t}_Z$.

Concerning this date at which the carbon tax reaches its peak, the case of a stock-dependent CCS cost function with a dominant scarcity effect must be contrasted from the other cases. In all the cases, the carbon tax is increasing at the instantaneous proportional rate $(\rho + \alpha)$ up to time $\underline{t}_Z$ at which the ceiling constraint begins to be tight (see (21). But in the case of a stock-dependent CCS cost function with a dominant scarcity effect, the tax is still increasing even after $\underline{t}_Z$, that is during some part of the period at the ceiling although at a lower instantaneous proportional rate (see Figure 19, case b.), contrary to the other cases in which the tax rate begins to decrease once the ceiling is attained (cases a., c. and d.).

The other differences bear on the behavior of the carbon tax rate during the clean coal exploitation period. In the case of a flow-dependent CCS cost function, the tax rate reaches its maximum during this period of clean coal use (case a. in Figure 19), in the case of stock-dependent CCS cost function with a dominant scarcity effect the tax rate is increasing during the phase of clean coal exploitation (case b.) while the rate is declining under stock-dependent cost functions with a dominant learning effect (cases c. and d.).

The last characteristics having to be pointed out is that, as far as the main qualitative properties of the carbon tax trajectory are at stake, the cost of the solar energy, either high or low, does not play an essential role. We conclude that what is really determining this time profile is the nature of the CCS cost function.

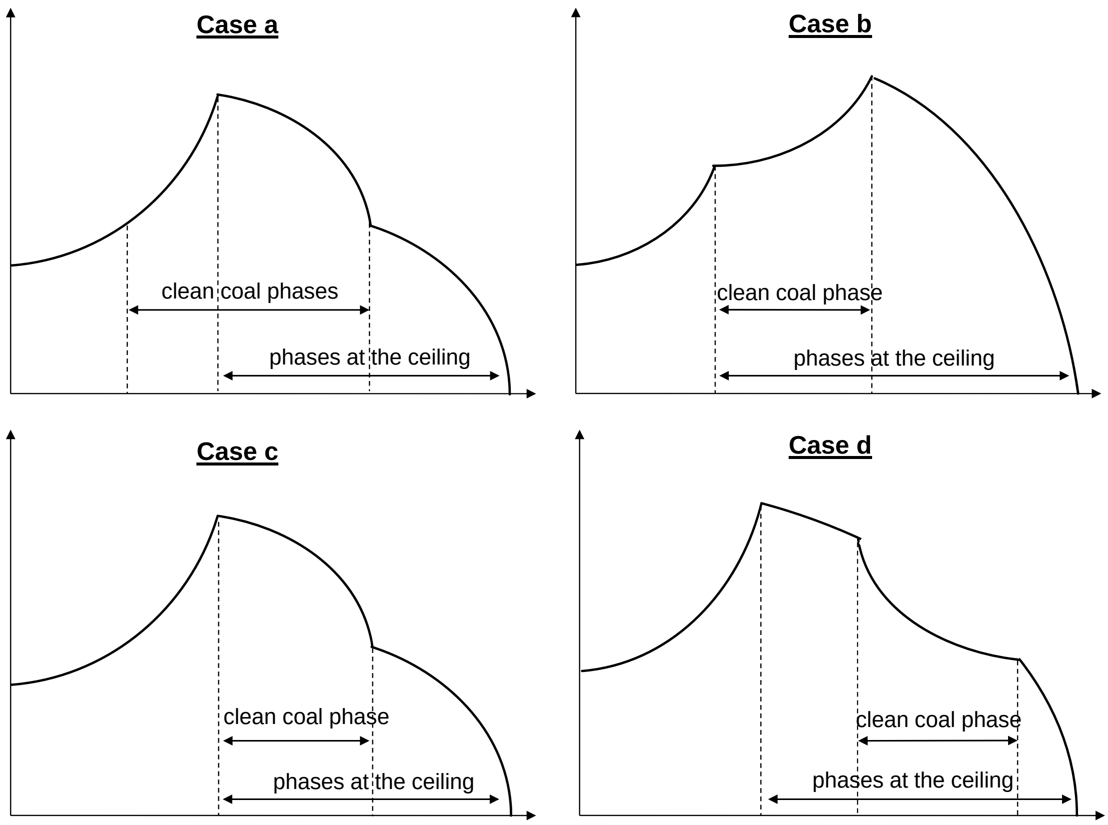

Figure 19: The various optimal time profiles of thee carbon tax.

6 Conclusion

In a Hotelling model, we have characterized the optimal geological carbon sequestration policies for alternative sequestration cost function and thus generalized the study by Lafforgue et al. (2008). The key features of the model were the following. i) The energy needs can be supplied by three types of energy resources that are perfectly substitutable: dirty coal (depletable and carbon-emitting), clean coal (also depletable but carbon-free thanks to a CCS device) and solar energy (renewable and carbon-free). ii) The atmospheric carbon stock cannot exceed some given institutional threshold as in Chakravorty et al. (2006). iii) The CCS cost function depends either on the flow of clean coal consumption or on its cumulated stock. In the later case, the marginal cost function can be either increasing in the stock (dominant scarcity effect) or decreasing (dominant learning effect).

Within this framework, we have shown that, under a stock-dependent CCS cost function, the clean coal exploitation must begin at the earliest when the carbon cap is reached while it must begin before under a flow-dependent cost function. Under stock-dependent cost function with a dominant learning effect, the energy price path can evolve non-monotonically over time. When the solar cost is low enough, this last case can give rise to an unusual sequence of energy consumption along which the solar energy consumption is interrupted for some time and replaced by the clean coal exploitation. Last under stock-dependent cost function, even if the qualitative properties of the price path can be roughly similar in some cases whatever be the dominant effect – scarcity or learning – they can imply some contrasting repercussions on the social marginal cost of the pollution stock. In particular, the scarcity effect can lead to a carbon tax trajectory which is still increasing even after the ceiling has been reached while, in this kind of ceiling models, the tax generally begins to decrease precisely at this date.

References

Amigues J-P., Lafforgue G., Moreaux M. (2011). Optimal CCS and air capture from heterogeneous energy consuming sectors. *LERNA working paper n° 11.16.350*, Toulouse School of Economics.

Amigues J-P., Moreaux M., Schubert K. (2011). Optimal use of a polluting non renewable resource generating both manageable and catastrophic damages. *Annals of Economics and Statistics*, 103, 107-141.

Ayong le Kama A., Lafforgue G., Fodha M. (2009). Optimal carbon capture and storage policies. *LERNA working paper n° 09.24.300*, Toulouse School of Economics.

Chakravorty U., Leach A., Moreaux M. (2011). Would Hotelling Kill the Electric Car? *Journal of Environmental Economics and Management*, 61, 281-296.

Chakravorty U., Magné B., Moreaux M. (2006). A Hotelling model with a ceiling on the stock of pollution. *Journal of Economic Dynamics and Control*, 30, 2875-2904.

Coulomb R., Henriet F. (2010). Carbon price and optimal extraction of a polluting fossil fuel with restricted carbon capture. *Working paper n° 2010-11*, Paris School of Economics.

Edenhofer O., Bauer N., Kriegler E. (2005). The impact of technological change on climate protection and welfare: Insights from the model MIND. *Ecological Economics*, 54, 277-292.

Gerlagh R. (2006). ITC in a global growth-climate model with CCS. The value of nduced technical change for climate stabilization. *Energy Journal*, Special Issue, 55-72.

Gerlagh R., van der Zwaan B.C. (2006). Options and instruments for a deep Cut in CO2 emissions: carbon capture or renewable, taxes or subsidies? *Energy Journal*, 27, 25-48.

Grimaud A., Rougé L. (2009). Séquestration du carbone et politique climatique optimale. *Economie et Prévision*, 190-191, 53-69.

Grimaud A., Lafforgue G., Magné B. (2011). Climate change mitigation options and directed technical change: A decentralized equilibrium analysis. *Resource and Energy Economics*, 33, 938-962.

Hamilton M., Herzog H., Parsons J. (2009). Cost and U.S. public policy for new coal power plants with carbon capture and sequestration. *Energy Procedia, GHGT9 Procedia*, 1, 2511-2518.

Herzog H.J. (2011). Scaling up carbon dioxide capture and storage: From megatons to gigatons. *Energy Economics*, 33, 597-604.

Hoel M., Kverndokk S. (1996). Depletion of fossil fuels and the impacts of global warming. *Resource and Energy Economics*, 18, 115-136.

IPCC (2005). Special report on carbon dioxide capture and storage, Working Group III.

Islegen O., Reichelstein S. (2009). The economics of carbon capture. *The Economist's Voice*, December: The Berkeley Electronic Press.

Kurosawa A. (2004). Carbon concentration target and technological choice. *Energy Economics*, 26, 675-684.

Lafforgue G., Magne B., Moreaux M. (2008-a). Energy substitutions, climate change and carbon sinks. *Ecological Economics*, 67, 589-597.

Lafforgue G., Magne B., Moreaux M. (2008-b). The optimal sequestration policy with a ceiling on the stock of carbon in the atmosphere. In: Guesnerie, R., Tulkens, H. (Eds), *The Design of Climate Policy*. The MIT Press, Boston, pp. 273-304.

Manne A., Richels R. (2004). The impact of learning-by-doing on the timing and costs of CO2 abatement. *Energy Economics*, 26, 603-619.

McFarland J.R., Herzog H.J., Reilly J.M. (2003). Economic modeling of the global adoption of carbon capture and sequestration technologies. In: Gale, J., Kaya, Y. (Eds.), *Proceedings of the Sixth International Conference on Greenhouse Gas Control Technologies*, Vol.2. Elsevier Science, Oxford, pp.1083-1089.

Tahvonen O. (1997). Fossil fuels, stock externalities and backstop technology. *Canadian Journal of Economics*, 22, 367-384.

Toman M.A., Withagen C. (2000). Accumulative pollution, clean technology and policy design. *Resource and Energy Economics*, 22, 367-384.

Appendix

A.1 Full characterization of the optimal price path under CCS.1

A.1.1 The high solar cost case: $u'(\bar{x}_d) < c_y$

Let us denote by $x_c^1(t, \lambda_{Z0})$ and $x_c^2(t, \lambda_{X0})$ the clean coal consumption during the phases $[\underline{t}_c, \underline{t}_Z)$ and $[\underline{t}_Z, \bar{t}_c)$, respectively. During the phase $[\underline{t}_c, \underline{t}_Z)$, $x_c^1(t, \lambda_{Z0})$ reads as the solution of:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)t} = c_s(x_c) + c_s'(x_c) x_c$$

and during the phase $[\underline{t}_Z, \bar{t}_c)$, $x_c^2(t, \lambda_{X0})$ solves:

$$u'(x_c + \bar{x}_d) = c_x + \lambda_{X0} e^{\rho t} + c_s(x_c) + c_s'(x_c) x_c$$

When the atmospheric carbon cap $\bar{Z}$ is sufficiently high and the initial pollution stock Z^0 is sufficiently low so that there exists an initial phase of dirty coal consumption without CCS, then the optimal path is the six-phase path as illustrated in Figure 1. To fully characterize this optimal path, the seven variables λ_{X0}, λ_{Z0}, $\underline{t}_c$, $\underline{t}_Z$, $\bar{t}_c$, $\bar{t}_Z$ and t_y have to be determined. They solve the following system of seven equations:

- The cumulated coal consumption/coal endowment balance equation:

$$\begin{aligned}\int_0^{\underline{t}_Z} q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) dt + \int_{\underline{t}_Z}^{\bar{t}_c} x_c^2(t, \lambda_{X0}) dt & \\ + \bar{x}_d [\bar{t}_Z - \underline{t}_Z] + \int_{\bar{t}_Z}^{t_y} q(c_x + \lambda_{X0} e^{\rho t}) dt &= X^0 \end{aligned} \tag{54}$$

- The atmospheric carbon stock continuity equation at $\underline{t}_Z$:

$$\begin{aligned} Z^0 + \zeta \int_0^{\underline{t}_c} q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) e^{\alpha t} dt & \\ + \zeta \int_{\underline{t}_c}^{\underline{t}_Z} \left[q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) - x_c^1(t, \lambda_{Z0}) \right] e^{\alpha t} dt &= \bar{Z} e^{\alpha \underline{t}_Z} \end{aligned} \tag{55}$$

- The full marginal costs equality equation at the beginning time $\underline{t}_c$ of clean coal exploitation:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)\underline{t}_c} = c_s(0) \tag{56}$$

- The continuity equation of the energy price path at the date $\underline{t}_Z$ at which the ceiling constraint is binding:

$$c_x + \lambda_{X0} e^{\rho \underline{t}_Z} + \zeta \lambda_{Z0} e^{(\rho+\alpha)\underline{t}_Z} = u'\left(x_c^2(\underline{t}_Z, \lambda_{X0}), \bar{x}_d \right) \Leftrightarrow x_c^1(\underline{t}_Z, \lambda_{Z0}) = x_c^2(\underline{t}_Z, \lambda_{X0}) \tag{57}$$

- The continuity equation of the energy price path at the closing time $\bar{t}_c$ of clean coal exploitation:

$$c_x + \lambda_{X0} e^{\rho \bar{t}_c} + c_s(0) = u'(\bar{x}_d) \;\Leftrightarrow\; x_c^2(\bar{t}_c, \lambda_{X0}) = 0 \tag{58}$$

- The continuity equation of the energy price path at the date $\bar{t}_Z$ at which the ceiling constraint ends to be active:

$$c_x + \lambda_{X0} e^{\rho \bar{t}_Z} = u'(\bar{x}_d) \tag{59}$$

- The continuity equation of the energy price path at the time t_y at which solar energy becomes competitive:

$$c_x + \lambda_{X0} e^{\rho t_y} = c_y \tag{60}$$

For any set $\{\lambda_{X0}, \lambda_{Z0}, \underline{t}_c, \underline{t}_Z, \bar{t}_c, \bar{t}_Z, t_y\}$ satisfying the above system of seven equations and such that $\zeta \lambda_{Z0} < c_s(0)$, then the necessary conditions (13)-(17) are satisfied. Since the problem is strictly convex, these conditions are also sufficient.

When the initial pollution stock Z^0 is sufficiently close to $\bar{Z}$ so that the clean coal exploitation must be started immediately, i.e. $\underline{t}_c = 0$, only six variables have to be determined. The equation (55) must be modified as follows:

$$Z^0 + \zeta \int_0^{\underline{t}_Z} \left[q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) - x_c^1(t, \lambda_{Z0}) \right] e^{\alpha t} dt = \bar{Z} e^{\alpha \underline{t}_Z} \tag{61}$$

and the equation (56) must be suppressed.

A.1.2 The low solar cost case $u'(\bar{x}_d) > c_y$

Now $x_c^2(t, \lambda_{X0})$ as defined in the previous paragraph is the clean coal consumption during the phase $[\underline{t}_Z, t_y)$, and we define $x_c^3(t, \lambda_{X0})$, the clean coal consumption during the phase $[t_y, \bar{t}_c)$, as the solution of the following equation:

$$c_y = c_x + \lambda_{X0} e^{\rho t} + c_s(x_c) + c_s'(x_c) x_c$$

First, when Z^0 is large enough and/or c_y is large enough so that the optimal price path is the six-phase path illustrated in Figure 3, the same seven variables λ_{X0}, λ_{Z0}, $\underline{t}_c$, $\underline{t}_Z$, t_y, $\bar{t}_c$ and $\bar{t}_Z$ have to be determined. The system of seven equations they solve now becomes:

- The cumulated coal consumption/coal endowment balance equation:

$$\begin{aligned} \int_0^{\underline{t}_Z} q(c_x + \lambda_{X0} e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t}) dt + \int_{\underline{t}_Z}^{t_y} x_c^2(t, \lambda_{X0}) dt & \\ + \int_{t_y}^{\bar{t}_c} x_c^3(t, \lambda_{X0}) dt + \bar{x}_d [\bar{t}_Z - \underline{t}_Z] \quad &= \quad X^0 \end{aligned} \tag{62}$$

- The equation (55) for the continuity of the atmospheric pollution stock at $\underline{t}_Z$.
- The equations (56) and (57) for the price path continuity at $\underline{t}_c$ and $\underline{t}_Z$, respectively.
- The continuity equation of the energy price path at t_y:

$$u'(x_c^2(t_y, \lambda_{X0}), \bar{x}_d) = c_y \ \Leftrightarrow\ x_c^2(t_y, \lambda_{X0}) = x_c^3(t_y, \lambda_{X0}) \tag{63}$$

- The continuity equation of the energy price path at $\bar{t}_c$:

$$c_x + \lambda_{X0} e^{\rho \bar{t}_c} + c_s(0) = c_y \ \Leftrightarrow\ x_c^3(\bar{t}_c, \lambda_{X0}) = 0 \tag{64}$$

- The continuity equation of the energy price path at $\bar{t}_Z$:

$$c_x + \lambda_{X0} e^{\rho \bar{t}_Z} = c_y \tag{65}$$

Again, when Z^0 is sufficiently close to c_y, it is necessary to immediately begin the CCS activity at $t = 0$, in which case equation (62) has to be substituted for (55) and equation (56) has to be deleted.

A.2 Full characterization of the optimal price path under CCS.2

When the scarcity effect is purely dominant, and whatever the level of the average solar cost c_y as compared with $u'(\bar{x}_d)$, two cases have to be considered depending on whether the reservoir capacity constraint is binding or not at the closing time of the clean coal exploitation (see Proposition 4). This implies that four cases have to be investigated.

A.2.1 The high solar cost case $u'(\bar{x}_d) < c_y$

a. Case where $S_c(\bar{t}_c) < \bar{S}_c$

In this case, the capacity constraint on the cumulated clean coal exploitation is never binding, thus implying that $\nu_S(t) = 0$ for any $t \geq 0$ and that $\lambda_S(t) = 0$ for $t \geq \bar{t}_c$. The expression (36) of the costate variable of the cumulated clean coal production can be simplified into:

$$\lambda_S(t) = -e^{\rho t} \int_t^{\bar{t}_c} c_s'(S_c(\tau)) x_c(\tau) e^{-\rho \tau} d\tau$$

Integrating by parts the above expression results in:

$$\lambda_S(t) = c_s(S_c(t)) - e^{\rho t} \left[c_s(S_c(\bar{t}_c)) e^{-\rho \bar{t}_c} + \rho \int_t^{\bar{t}_c} c_s(S_c(\tau)) e^{-\rho \tau} d\tau \right] \tag{66}$$

The seven endogenous variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}$ (with $\underline{t} = \underline{t}_Z = \underline{t}_c$), $\bar{t}_c$, $\bar{t}_Z$ and t_y solve the following system of seven equations:

- The initial condition on the costate variable $\lambda_S(t)$ which, from (66), results in:

$$\lambda_{S0} = \lambda_S(0) = \underline{c}_s e^{-\rho \underline{t}} - c_s(S_c(\bar{t}_c))e^{-\rho \bar{t}_c} - \rho \int_{\underline{t}}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t} dt \tag{67}$$

- The cumulated coal consumption/coal endowment balance equation:

$$\begin{aligned} \int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t})dt + \int_{\underline{t}}^{\bar{t}_c} q(c_m^c(x_c(t)))dt & \\ + \bar{x}_d[\bar{t}_Z - \bar{t}_c] + \int_{\bar{t}_Z}^{t_y} q(c_x + \lambda_{X0}e^{\rho t})dt &= X^0 \end{aligned} \tag{68}$$

where, from (66), the full marginal cost $c_m^c(x_c(t))$ of the clean coal amounts to:

$$c_m^c(x_c(t)) = c_x + \lambda_{X0}e^{\rho t} + e^{\rho t}\left[c_s(S_c(\bar{t}_c))e^{-\rho \bar{t}_c} + \rho \int_t^{\bar{t}_c} c_s(S_c(\tau))e^{-\rho \tau} d\tau\right], \quad t \in [\underline{t}, \bar{t}_c)$$

- The atmospheric carbon stock continuity equation at time $\underline{t}$:

$$Z^0 + \zeta \int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta \lambda_{Z0} e^{(\rho+\alpha)t})e^{\alpha t} dt = \bar{Z} e^{\alpha \underline{t}} \tag{69}$$

- The continuity equation of the energy price path at the date $\underline{t}$ at which the ceiling constraint is binding and, simultaneously, the clean coal exploitation begins:

$$\zeta \lambda_{Z0} e^{(\rho+\alpha)\underline{t}} = \underline{c}_s - \lambda_{S0} e^{\rho \underline{t}} \tag{70}$$

- The continuity equation of the energy price path at the closing time $\bar{t}_c$ of the clean coal exploitation:

$$c_x + \lambda_{X0}e^{\rho \bar{t}_c} + c_s(S_c(\bar{t}_c)) = u'(\bar{x}_d) \tag{71}$$

- The equations (59) and (60) for the continuity of the energy price path at times $\bar{t}_Z$ and t_y, respectively.

b. Case where $S_c(\bar{t}_c) = \bar{S}_c$

In this case, the reservoir is fulfilled at time $\bar{t}_c$ implying $\lambda_S(\bar{t}_c) < 0$. Here we cannot deduce λ_{S0} from the general expression of $\lambda_S(t)$ as in the previous case. This missing information must be replaced by an additional terminal condition on the cumulated clean coal production: $S_c(\bar{t}_c) = \bar{S}_c$.

Integrating by parts (36), we have now:

$$\lambda_S(t) = c_s(S_c(t)) - e^{\rho t}\left[c_s(\bar{S}_c)e^{-\rho\bar{t}_c} + \rho\int_t^{\bar{t}_c} c_s(S_c(\tau))e^{-\rho\tau}d\tau + \int_t^{\infty}\nu_S(\tau)e^{-\rho\tau}d\tau\right] \quad (72)$$

thus implying:

$$\lambda_{S0} = \underline{c}_s e^{-\rho\underline{t}} - c_s(\bar{S}_c)e^{-\rho\bar{t}_c} - \rho\int_{\underline{t}}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t}dt - \int_{\bar{t}_c}^{\infty}\nu_S(t)e^{-\rho t}dt \quad (73)$$

Replacing into (72) the term $\int_t^\infty \nu_S(t)e^{-\rho t}dt$ by its expression coming from (73), with $\nu_S(t) = 0$ for $t \in [0, \bar{t}_c)$, we obtain after simplifications:

$$\forall t \in [\underline{t}, \bar{t}_c): \qquad \lambda_S(t) = c_s(S_c(t)) - e^{\rho t}\left[\underline{c}_s e^{-\rho\underline{t}} - \rho\int_{\underline{t}}^{t} c_s(S_c(\tau))e^{-\rho\tau}d\tau - \lambda_{S0}\right] \quad (74)$$

$$\text{at time } \bar{t}_c: \qquad \lambda_S(\bar{t}_c) = c_s(\bar{S}_c) - e^{\rho\bar{t}_c}\left[\underline{c}_s e^{-\rho\underline{t}} - \rho\int_{\underline{t}}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t}dt - \lambda_{S0}\right] \quad (75)$$

The seven endogenous variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}$ (with $\underline{t} = \underline{t}_Z = \underline{t}_c$), $\bar{t}_c$, $\bar{t}_Z$ and t_y are determined as the solution of the following seven-equations system:

- The continuity equation of the cumulated clean coal production at $\bar{t}_c$:

$$\int_{\underline{t}}^{\bar{t}_c} x_c(t)dt = \int_{\underline{t}}^{\bar{t}_c} q(c_m^c(x_c(t)))dt - \bar{x}_d[\bar{t}_c - \underline{t}] = \bar{S}_c \quad (76)$$

where, from (74), the full marginal cost $c_m^c(x_c(t))$ of the clean coal is now equal to:

$$c_m^c(x_c(t)) = c_x + \lambda_{X0}e^{\rho t} + e^{\rho t}\left[\underline{c}_s e^{-\rho\underline{t}} - \rho\int_{\underline{t}}^{t} c_s(S_c(\tau))e^{-\rho\tau}d\tau - \lambda_{S0}\right], \quad t \in [\underline{t}, \bar{t}_c)$$

- The cumulated coal consumption/coal endowment balance equation:

$$\int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \bar{x}_d[\bar{t}_Z - \underline{t}] + \bar{S}_c + \int_{\bar{t}_Z}^{t_y} q(c_x + \lambda_{X0}e^{\rho t})dt = X^0 \quad (77)$$

- The equation (69) for the continuity of the atmospheric carbon stock at $\underline{t}$.

- The continuity equation of the energy price path at $\underline{t}$:

$$\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}} = \underline{c}_s - \lambda_{S0}e^{\rho\underline{t}} \quad (78)$$

- The continuity equation of the energy price path at $\bar{t}_c$ which, using (75), implies:

$$\begin{aligned} c_x + \lambda_{X0}e^{\rho\bar{t}_c} + c_s(\bar{S}_c) - \lambda_S(\bar{t}_c) &= u'(\bar{x}_d) \\ \Rightarrow c_x + \lambda_{X0}e^{\rho\bar{t}_c} + e^{\rho\bar{t}_c}\left[\underline{c}_s e^{-\rho\underline{t}} - \rho\int_{\underline{t}}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t}dt - \lambda_{S0}\right] &= u'(\bar{x}_d) \quad (79) \end{aligned}$$

- The equations (59) and (60) for the continuity of the energy price path at times $\bar{t}_Z$ and t_y, respectively.

A.2.2 The low solar cost case $u'(\bar{x}_d) > c_y$

a. Case where $S_c(\bar{t}_c) < \bar{S}_c$

As explained in Section 4.2.2, only the six endogenous variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}$ (with $\underline{t} = \underline{t}_Z = \underline{t}_c$), $\hat{t}$ (with $\hat{t} = \bar{t}_c = t_y$) and $\bar{t}_Z$ have now to be determined. They solve the following system of six equations:

- The equation (67) for the initial condition on $\lambda_S(t)$, with $\bar{t}_c = \hat{t}$.

- The cumulated coal consumption/coal endowment balance equation:

$$\int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \int_{\underline{t}}^{\hat{t}} q(c_m^c(x_c(t)))dt + \bar{x}_d[\bar{t}_Z - \hat{t}] = X^0 \tag{80}$$

where, the full marginal cost $c_m^c(x_c(t))$ has the same expression as in the corresponding high solar cost case for $t \in [\underline{t}, \hat{t})$.

- The equation (69) for the continuity of the atmospheric carbon stock at $\underline{t}$.

- The equation (70) for the continuity of the energy price path at time $\underline{t}$.

- The continuity equation of the energy price path at time $\hat{t}$:

$$c_x + \lambda_{X0}e^{\rho\hat{t}} + c_s(S_c(\hat{t})) = c_y \tag{81}$$

- The equation (65) for the continuity of the energy price path at time $\bar{t}_Z$.

b. Case where $S_c(\bar{t}_c) = \bar{S}_c$

The six endogenous variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}$, $\hat{t}$ and $\bar{t}_Z$ are determined as the solution of the following six-equations system:

- The equation (76) for the continuity of the cumulated clean coal production at $\hat{t}$, with $\hat{t} = \bar{t}_c$.

- The cumulated coal consumption/coal endowment balance equation:

$$\int_0^{\underline{t}} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \bar{x}_d[\bar{t}_Z - \underline{t}] + \bar{S}_c = X^0 \tag{82}$$

- The equation (69) for the continuity of the atmospheric carbon stock at $\underline{t}$.

- The equation (78) for the continuity of the energy price path at $\underline{t}$.

- The equation (79) for the continuity of the energy price path at $\hat{t}$, with $\hat{t} = \bar{t}_c$.

- The equation (65) for the continuity of the energy price path at time $\bar{t}_Z$.

A.3 Full characterization of the optimal price path under CCS.3

Under a stock-dependent CCS cost function with a dominant learning effect, the expression of the costate variable of the cumulated clean coal production is given by (46). Expanding the integral term and simplifying, it comes:

$$\lambda_S(t) = c_s(S_c(t)) - e^{\rho t}\left[c_s(S_c(\bar{t}_c))e^{-\rho\bar{t}_c} + \rho\int_t^{\bar{t}_c} c_s(S_c(\tau))e^{-\rho\tau}d\tau\right] \tag{83}$$

which the same expression as (66) obtained in the dominant scarcity effect case. However, the initial value of λ_S slightly differs since the CCS cost function is now decreasing in S:

$$\lambda_{S0} = \bar{c}_s e^{-\rho\underline{t}_c} - c_s(S_c(\bar{t}_c))e^{-\rho\bar{t}_c} - \rho\int_{\underline{t}_c}^{\bar{t}_c} c_s(S_c(t))e^{-\rho t}dt \tag{84}$$

Finally, since in this case the reservoir that hosts the sequestered carbon emissions is not constrained by any limit in capacity, the associated costate variable must be nil at the closing time of the clean coal exploitation, as specified by (47): $\lambda_S(t) = 0\ \forall t \geq \bar{t}_c$.

A.3.1 The high solar cost case $u'(\bar{x}_d) < c_y$

a. Case where $\lambda_{X0} > \lambda_{S0}$

As mentioned in Section 4.3.1, the energy price and consumption paths are qualitatively very similar to the ones obtained in the dominant scarcity effect case with high solar cost when the capacity constraint on the cumulated clean coal production is never binding. Hence, the seven endogenous variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}$ (with $\underline{t} = \underline{t}_Z = \underline{t}_c$), $\bar{t}_c$, $\bar{t}_Z$ and t_y solve almost the same seven-equations system as in Appendix A.2.1.a:

- The equation (84) for the initial condition on $\lambda_S(t)$.

- The equation (68) for the cumulated coal consumption/coal endowment balance.

- The equation (69) for the continuity of the atmospheric carbon stock at time $\underline{t}$.

- The continuity equation of the energy price path at time $\underline{t}$:

$$\zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}} = \bar{c}_s - \lambda_{S0}e^{\rho\underline{t}} \tag{85}$$

- The equation (71) for the continuity of the energy price path at time $\bar{t}_c$.

- The equations (59) and (60) for the continuity of the energy price path at times $\bar{t}_Z$ and t_y, respectively.

b. Case where $\lambda_{X0} < \lambda_{S0}$

As seen in Section 4.3, when $\lambda_{X0} - \lambda_{S0} < 0$ two subcases have to be considered according to whether the dates at which the carbon cap is reached and at which the clean coal exploitation begins coincide are not.

First, if $\underline{t}_Z = \underline{t}_c \equiv \underline{t}$, then the seven variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}$, $\bar{t}_c$, $\bar{t}_Z$ and t_y exactly solve the same system of equations than the previous one (see Appendix A.3.1 case a.).

Second, if $\underline{t}_Z < \underline{t}_c \equiv \underline{t}$, then we have now to determine eight endogenous variables: λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}_Z$, $\underline{t}_c$, $\bar{t}_c$, $\bar{t}_Z$ and t_y. They solve the following system of seven equations:

- The equation (84) for the initial condition on $\lambda_S(t)$.

- The cumulated coal consumption/coal endowment balance equation:

$$\begin{aligned} \int_0^{\underline{t}_Z} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \int_{\underline{t}_c}^{\bar{t}_c} q(c^c_m(x_c(t)))dt & \\ +\bar{x}_d[(\bar{t}_Z - \underline{t}_Z) - (\bar{t}_c - \underline{t}_c)] + \int_{\bar{t}_Z}^{t_y} q(c_x + \lambda_{X0}e^{\rho t})dt &= X^0 \end{aligned} \tag{86}$$

where, $c^c_m(x_c(t)) = c_x + \lambda_{X0}e^{\rho t} + c_s(S_c(t)) - \lambda_S(t)$, with $\lambda_S(t)$ given by (83).

- The atmospheric carbon stock continuity equation at time $\underline{t}_Z$:

$$Z^0 + \zeta \int_0^{\underline{t}_Z} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})e^{\alpha t}dt = \bar{Z}e^{\alpha \underline{t}_Z} \tag{87}$$

- The continuity equation of the energy price path at time $\underline{t}_Z$:

$$c_x + \lambda_{X0}e^{\rho \underline{t}_Z} + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_Z} = u'(\bar{x}_d) \tag{88}$$

- The continuity equation of the energy price path at time $\underline{t}_c$:

$$c_x + \bar{c}_s + (\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c} = u'(\bar{x}_d) \tag{89}$$

- The equation (71) for the continuity of the energy price path at time $\bar{t}_c$.

- The equations (59) and (60) for the continuity of the energy price path at times $\bar{t}_Z$ and t_y, respectively.

A.3.2 The low solar cost case $u'(\bar{x}_d) > c_y$

a. Cases where $\lambda_{X0} > \lambda_{S0}$ *or where* $\lambda_{X0} < \lambda_{S0}$ *and* $\underline{t}_Z = \underline{t}_c$

The six endogenous variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}$, $\hat{t}$ and $\bar{t}_Z$ are determined as the solution of the following six-equations system:

- The equation (84) for the initial condition on $\lambda_S(t)$.

- The equation (80) for the cumulated coal consumption/coal endowment balance.

- The equation (69) for the continuity of the atmospheric carbon stock at time $\underline{t} = \underline{t}_Z = \underline{t}_c$.

- The equation (85) for the continuity of the energy price path at time $\underline{t}$.

- The equation (81) for the continuity of the price path at time $\hat{t} = \bar{t}_c = t_y$.

- The equation (65) for the continuity of the price path at time $\bar{t}_Z$.

b. Case where $\lambda_{X0} < \lambda_{S0}$ *and* $\underline{t}_Z < \underline{t}_c$

In this last case, the seven endogenous variables λ_{X0}, λ_{Z0}, λ_{S0}, $\underline{t}_Z = t_y$, $\underline{t}_c$, $\bar{t}_c$ and $\bar{t}_Z$ solve the following system:

- The equation (84) for the initial condition on $\lambda_S(t)$.

- The cumulated coal consumption/coal endowment balance equation:

$$\int_0^{\underline{t}_Z} q(c_x + \lambda_{X0}e^{\rho t} + \zeta\lambda_{Z0}e^{(\rho+\alpha)t})dt + \int_{\underline{t}_c}^{\bar{t}_c} q(c_m^c(x_c(t)))dt \\ +\bar{x}_d[(\bar{t}_Z - \underline{t}_Z) - (\bar{t}_c - \underline{t}_c)] = X^0 \tag{90}$$

where, $c_m^c(x_c(t)) = c_x + \lambda_{X0}e^{\rho t} + c_s(S_c(t)) - \lambda_S(t)$, with $\lambda_S(t)$ given by (83).

- The equation (69) for the continuity of the atmospheric carbon stock at time $\underline{t}_Z$.

- The continuity equation of the energy price path at time $\underline{t}_Z = t_y$:

$$c_x + \lambda_{X0}e^{\rho \underline{t}_Z} + \zeta\lambda_{Z0}e^{(\rho+\alpha)\underline{t}_Z} = c_y \tag{91}$$

- The continuity equation of the energy price path at time $\underline{t}_c$:

$$c_x + \bar{c}_s + (\lambda_{X0} - \lambda_{S0})e^{\rho \underline{t}_c} = c_y \tag{92}$$

- The continuity equation of the energy price path at time $\bar{t}_c$:

$$c_x + \lambda_{X0}e^{\rho \bar{t}_c} + c_s(S_c(\bar{t}_c)) = c_y \tag{93}$$

- The equation (65) for the continuity of the price path at time $\bar{t}_Z$.