

# Is there a political support for the double burden on prolonged activity?<sup>1</sup>

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### **Abstract**

In many countries elderly workers are subject to a double distortion when they consider prolonging their activity: the payroll tax and a reduction in their pension rights. It is often argued that such a double burden would not be socially desirable. We consider a setting where it would be rejected by both a utilitarian and a Rawlsian social planner. Furthermore, each individual would also reject it as a citizen candidate. We show that the double burden may nevertheless be (second-best) Pareto efficient and can be supported by a particular structure of social weights biased towards the more productive workers.

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# 1 Introduction

One of the most striking developments of the last fifty years is the trend toward earlier retirement. Clearly one of the factors driving this trend is the long term increase in economic wealth which permits workers to enjoy rising living standards, even as they spend a growing fraction of their lives outside of the workforce. Another important factor of earlier retirement is the expansion of social protection programs which induce workers to stop working long before the standard age and surely the age at which they would retire in the absence of these programs. A number of studies (see, e.g., Gruber and Wise (1999) or Blondal and Scarpetta (1998)) have shown that early retirement is encouraged not only by special provisions of the old age system itself, but also by unemployment related transfer schemes, disability insurance or special early retirement schemes which are available to workers well before the early retirement age of 60 in most countries.

Altogether these programs make prolonging activity a costly decision because it implies paying additional payroll taxes and also declining pension rights.<sup>1</sup> This is what is usually called the double burden of postponing retirement, the first burden being explicit and the second implicit. Interestingly, this second implicit burden varies across countries; see e.g., Gruber and Wise (1999) and Blondal and Scarpetta (1998). It is very important in Belgium and France and very low in Sweden and Japan. Observe that early retirement is very costly in terms of productive resources. As shown by Herbertson and Orszag (2001) its cost amounts to about 7% of GDP in the OECD countries. It also represents a threat to the financial viability of pay-as-you-go systems for it exacerbates the problems raised by the demographic trend (slower population growth and longer life expectancy). Consequently, an increase in effective retirement age via an elimination (or mitigation) of the double burden is often advocated. Yet, such reform often appear to be unpopular and have to face a stiff political resistance.

The question one might raise at this point is: why was early retirement introduced in the first place? Standard answers to that question pertain to labor market considerations

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<sup>1</sup>Total pension rights decrease when continued activity translates into an increase in per-period payments (if any) which is less than actuarially fair.

such as the need to foster employment of jobless youth, the problem raised by old workers with wages much above their productivity or firms in difficulty laying off their aged employees; see e.g. Conde-Ruiz and Galasso (2003).

These are surely reasonable explanations at least for some countries. One would like however to see whether or not early retirement policy could possibly be supported by some type of political majority in a setting where the labor market considerations mentioned above do not matter. Put differently, can there be some political support for the inducement of early retirement which is motivated solely by the inherent features of retirement insurance and specifically its redistributive properties? This is the issue we deal with in this paper by studying the impact of early retirement on the lifetime utility of individuals with different productivities.

We consider a setting where an implicit tax on continued activity would be rejected by both a utilitarian and a Rawlsian social planner. Furthermore, each individual would also reject it as a citizen candidate. We show that the double burden may nevertheless be (second-best) Pareto efficient and can be supported by a particular structure of social weights biased towards the more productive workers. Consequently, it could emerge from the political process as long this process implies that sufficient weight is put on the more productive workers. Similarly, a reform towards a more neutral system may not be supported by the political process.

## 2 The model

Consider a society with workers of productivity  $w_i$ ,  $i = 1, 2, \dots, n$ , where  $w_1 < w_2 < \dots < w_n$ . The number of type  $i$  workers is  $N_i$ , the total number of workers is  $N$  and the proportion of type  $i$  workers is  $\lambda_i = N_i/N$ . The length of life is normalized at 1. The individual labor supply, which represents the length of the working period (number of years spent working), is denoted  $z_i$ . It is interpreted as the *retirement age*. The utility function is:

$$U(d, z) = u(d - h(z)) \tag{1}$$

where  $d$  is total consumption and  $h(z)$  is the disutility of work;  $u(\cdot)$  is increasing and concave:  $u' > 0$ ,  $u'' < 0$  and  $h(\cdot)$  is increasing and convex:  $h' > 0$ ,  $h'' > 0$ . In the remainder of the paper we denote the consumption net of the disutility of labor by  $c = d - h(z)$ .

Total consumption is given by

$$d = w(1 - \tau)z + p - \alpha z,$$

where  $\tau$  is the payroll tax rate of the Pay-As-You-Go (PAYG) pension system. It corresponds to the *explicit tax* on continued activity. The *implicit tax* is represented by the parameter  $\alpha$ . Total pension wealth is given by  $p - \alpha z$ ; when  $\alpha > 0$ , it decreases as the individual decides to retire later. This means that there is implicit taxation on continued activity. The payroll tax rate is assumed to lie between 0 and 1:  $\tau \in [0, 1]$ . Negative values of  $\alpha$  are allowed. In such a case, individuals are induced to work longer. We can write the previous expression as:

$$d = y(1 - \theta) + p,$$

where  $\theta = \tau + \alpha/w$  is the marginal tax rate on continued activity and  $y = wz$  is life cycle income. Observe that  $p$  does not depend on productivity levels.

The optimal retirement decision of an individual with productivity  $w$  is given by the first-order condition

$$w(1 - \theta) - h'(z) = 0. \tag{2}$$

In what follows, we adopt a specific form for the disutility of labor:  $h(z) = \gamma z^2/2$ . This yields the following labor supply function:

$$z = \frac{w(1 - \theta)}{\gamma} = \frac{w(1 - \tau) - \alpha}{\gamma}. \tag{3}$$

Substituting this expression into (1) and taking into account the government budget constraint,

$$p = \alpha \bar{z} + \tau \bar{y},$$

where  $\bar{z}$  and  $\bar{y}$  denote average labor supply and income respectively, we obtain the indirect utility function

$$V(\tau, \alpha; w) = u \left[ \frac{(w(1-\tau) - \alpha)^2}{2\gamma} + \alpha \frac{\bar{w}(1-\tau) - \alpha}{\gamma} + \tau \frac{E(w^2)(1-\tau) - \alpha\bar{w}}{\gamma} \right]. \quad (4)$$

In the remainder of the paper we shall assume that this function is quasi-concave. This means that the upper contour sets of  $V$  in the  $(\tau, \alpha)$  plane are convex. Appendix A shows that  $\bar{w}(\bar{w} - 2w_1) < E(w^2) - \bar{w}^2$ , where  $E(w^2) - \bar{w}^2 > 0$  is the variance of  $w$ , is a *sufficient* condition for this to hold for all levels of  $w$ . It is satisfied in a trivial way when  $w_1 \geq \bar{w}^2/2$  (as in the numerical used in the proof of Proposition 3 below). The condition holds whatever the level of  $w_1$  when  $(E(w^2) - \bar{w}^2)/\bar{w}^2 \geq 1$ , that is when the distribution of productivities involves a sufficient degree of heterogeneity (the coefficient of variation exceeds one).

### 3 Individually optimal policies

In this section, we describe the individuals' most preferred policies  $(\tau, \alpha, p)$ . This individual optimum is defined in the usual way. The policy, *when applied to the entire society*, maximizes the considered individual's welfare. Interestingly, the most preferred policies of *all* individuals (whatever their level of productivity) involve a negative level of  $\alpha$ . Consequently, no individual is in favor of an implicit tax. Everyone prefers that pensions rights  $p - \alpha z$  do not decrease with  $z$ . This point is stated in the following proposition.

**Proposition 1** *The most preferred policy of all the individuals, whatever their level of productivity, involves  $\alpha < 0$ .*

**Proof.** We evaluate  $\partial V/\partial\alpha$  (with  $V$  given by (4)) at the point  $\tau = \tau^*(0)$ ,  $\alpha = 0$ . The optimal value of  $\tau$  is the solution of the first order condition

$$\begin{aligned}\frac{\partial V}{\partial\tau} &= \left[ -y + \bar{y} + \alpha \frac{d\bar{z}}{d\tau} + \tau \frac{d\bar{y}}{d\tau} \right] u'(c) = 0 \\ \Leftrightarrow &\left[ \frac{E(w^2)(1-2\tau) - w^2(1-\tau) + \alpha w - 2\alpha\bar{w}}{\gamma} \right] u'(c) = 0 \\ \Leftrightarrow &\tau^*(\alpha) = \frac{E(w^2) - w^2 + \alpha(w - 2\bar{w})}{2E(w^2) - w^2}.\end{aligned}$$

At  $\alpha = 0$ , this yields

$$\tau^*(0) = \frac{E(w^2) - w^2}{2E(w^2) - w^2}.$$

This is positive if and only if  $w < \sqrt{E(w^2)}$ .

The derivative of the indirect utility function with respect to  $\alpha$  is

$$\begin{aligned}\frac{\partial V}{\partial\alpha} &= \left[ -z + \bar{z} + \alpha \frac{d\bar{z}}{d\alpha} + \tau \frac{d\bar{y}}{d\alpha} \right] u'(c) \\ &= \left[ \frac{\bar{w}(1-2\tau) - w(1-\tau) - \alpha}{\gamma} \right] u'(c).\end{aligned}$$

For individuals with productivity below  $\sqrt{E(w^2)}$ , we obtain

$$\begin{aligned}\frac{\partial V}{\partial\alpha}\Big|_{\alpha=0} &= \left[ \bar{w} \left( 1 - 2 \frac{E(w^2) - w^2}{2E(w^2) - w^2} \right) - w \left( 1 - \frac{E(w^2) - w^2}{2E(w^2) - w^2} \right) \right] u'(c) \\ &= \left[ \frac{w^2}{2E(w^2) - w^2} - w \frac{E(w^2)}{2E(w^2) - w^2} \right] u'(c) \\ &> 0 \Leftrightarrow w > \frac{E(w^2)}{\bar{w}}.\end{aligned}$$

From Jensen's inequality  $\bar{w} < \sqrt{E(w^2)}$ . Therefore  $E(w^2)/\bar{w}$  is larger than  $\sqrt{E(w^2)}$ . This leads to the conclusion that the individuals with productivity below  $\sqrt{E(w^2)}$  want a negative  $\alpha$ .

For individuals with productivity above  $\sqrt{E(w^2)}$ ,

$$\begin{aligned}\frac{\partial V}{\partial\alpha}\Big|_{\alpha=0} &= \left[ \frac{\bar{w} - w}{\gamma} \right] u'(c) \\ &> 0 \Leftrightarrow w < \bar{w}.\end{aligned}$$

Because  $\sqrt{E(w^2)} > \bar{w}$ , high productivity individuals also want a negative  $\alpha$ . ■

To understand the intuition behind this result let us first look at the low productivity workers (those with  $w < \sqrt{E(w^2)}$ ), who prefer a negative  $\alpha$  (an implicit subsidy) in order to induce high productivity individuals to work hard and then tax away their labor income through a relatively high level of  $\tau$ . To be more precise, while both instruments,  $\tau$  and  $\alpha$ , induce some redistribution from the high-skilled to the low-skilled individuals, it turns out that  $\tau$  is the more “powerful” instrument. A positive  $\tau$  generates some redistribution from the individuals with a high labor income ( $wz$ ) to the individuals with a low income. This appears clearly when evaluating  $\partial V/\partial\tau$  at the point  $\tau = 0$ ,  $\alpha = 0$ :

$$\left. \frac{\partial V}{\partial\tau} \right|_{\tau=0, \alpha=0} = (\bar{y} - y) \frac{u'(c)}{\gamma} = (E(w^2) - w^2) \frac{u'(c)}{\gamma}.$$

On the other hand a positive  $\alpha$  redistributes from the aged workers to the early retirees:

$$\left. \frac{\partial V}{\partial\alpha} \right|_{\tau=0, \alpha=0} = (\bar{z} - z) \frac{u'(c)}{\gamma} = (\bar{w} - w) \frac{u'(c)}{\gamma}. \quad (5)$$

Because labor supply is increasing with productivity (see (3)), this results also in some redistribution from high to low productivity individuals. By comparing the two expressions above, one can see that  $\tau$  is a more “powerful” redistributive instrument than  $\alpha$ : for  $w < E(w^2)$ ,  $\partial V/\partial\tau|_{\tau=0, \alpha=0} > \partial V/\partial\alpha|_{\tau=0, \alpha=0}$ . A positive  $\tau$  is then for the poor a more effective instrument than  $\alpha$  to achieve the desired redistribution.

From a poor individual’s perspective, a positive  $\alpha$  may thus be desirable only for efficiency reasons: it could be a means to limit tax distortions (e.g., if the productive would have a higher labor supply elasticity). However with a quadratic disutility of labor the elasticity of labor supply with respect to the net wage is constant: it is the same for the rich and the poor:

$$\frac{\partial z}{\partial w(1-\theta)} \frac{w(1-\theta)}{z} = 1.$$

Therefore the efficiency argument that could lead to a positive  $\alpha$  chosen by the poor is not relevant in our setting.

Let us now turn to the high productivity workers who also prefer a negative  $\alpha$ , but for different reasons, namely because they face a (binding) constraint on  $\tau$ . If they could



set a negative  $\tau$ , they would opt for this solution.<sup>2</sup> Because  $\tau$  is restricted to be non-negative, setting a negative  $\alpha$  is the only way for them to get a favorable redistribution. This is because a negative  $\alpha$  redistributes income from the early retirees (who are the low skill individuals) to the aged workers (the high skills).

We have thus shown that *all* the workers including those with the lowest productivity would vote against a positive  $\alpha$ . In other words, the Rawlsian criterion, or any other welfare criterion or political process which puts all the weight on a single individual implies  $\alpha \leq 0$ . Let us now turn to the utilitarian criterion.

## 4 The utilitarian optimum

As above, we restrict the instruments to be linear with identical rates applying to all individuals. The utilitarian optimum is then obtained by solving.

$$\max_{\tau, \alpha} SW = \sum_{i=1}^n \lambda_i V(\tau, \alpha; w_i).$$

Recall that  $p$  is implicit as a third policy instrument but that the government budget constraint is already incorporated in the indirect utility function.

**Proposition 2** *The utilitarian optimum implies a negative level of  $\alpha$ .*

**Proof.** We first show that the optimal utilitarian tax rate at  $\alpha = 0$ ,  $\tau^u(0)$ , is positive:

$$\begin{aligned} & \left. \frac{\partial SW}{\partial \tau} \right|_{\tau=0, \alpha=0} > 0 \\ \Leftrightarrow & \sum_{i=1}^n \lambda_i [E(w^2) - w_i^2] u'(c_i(0, 0)) > 0. \end{aligned}$$

Denote by  $k$  the first productivity level such that  $E(w^2) - w^2 < 0$ , that is  $E(w^2) - w_i^2 < 0$  iff  $i \geq k$ . Observing that  $c_i(0, 0) = w_i^2/2\gamma$ , we have that  $u'(c_i(0, 0))$  is decreasing with  $i$ . Therefore  $\lambda_i [E(w^2) - w_i^2] u'(c_i(0, 0)) > \lambda_i [E(w^2) - w_i^2] u'(c_k(0, 0))$ ,  $\forall i \neq k$ . This implies

$$\sum_{i=1}^n \lambda_i [E(w^2) - w_i^2] u'(c_i(0, 0)) > \sum_{i=1}^n \lambda_i [E(w^2) - w_i^2] u'(c_k(0, 0)) = 0.$$

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<sup>2</sup>This would be equivalent to introducing a poll tax (because  $p = \tau\bar{y} < 0$ ) and redistributing the proceeds proportionally to income.

We then write the first-order condition that determines the optimal value of  $\tau$  at  $\alpha = 0$ .

$$\begin{aligned} \frac{\partial SW}{\partial \tau} \Big|_{\alpha=0} &= 0 \\ \Leftrightarrow \sum_{i=1}^n \lambda_i [E(w^2) (1 - 2\tau^u(0)) - w_i^2 (1 - \tau^u(0))] u'(c_i(\tau^u(0), 0)) &= 0. \end{aligned} \quad (6)$$

We now evaluate the marginal impact of  $\alpha$  on the utilitarian social welfare at  $\alpha = 0$ :

$$\frac{\partial SW}{\partial \alpha} \Big|_{\alpha=0} = \sum_{i=1}^n \lambda_i [\bar{w} (1 - 2\tau^u(0)) - w_i (1 - \tau^u(0))] u'(c_i).$$

This is positive if and only if

$$\begin{aligned} \frac{E(w^2)}{\bar{w}} \frac{\partial SW}{\partial \alpha} \Big|_{\alpha=0} &> 0 \\ \Leftrightarrow \sum_{i=1}^n \lambda_i [E(w^2) (1 - 2\tau^u(0)) - \frac{w_i E(w^2)}{\bar{w}} (1 - \tau^u(0))] u'(c_i) &> 0 \\ \Leftrightarrow \sum_{i=1}^n \lambda_i [w_i^2 - \frac{w_i E(w^2)}{\bar{w}}] u'(c_i) &> 0, \end{aligned} \quad (7)$$

where we have used (6).

The remainder of the proof follows along the same lines. Denote by  $k$  the lowest productivity level for which  $w_i^2 - w_i E(w^2) / \bar{w} > 0$ ; to be more precise  $k$  is such that  $w_i^2 - w_i E(w^2) / \bar{w} > 0$  iff  $i \geq k$ . Observing that  $\lambda_i [w_i^2 - w_i E(w^2) / \bar{w}] u'(c_i(0, 0)) < \lambda_i [w_i^2 - w_i E(w^2) / \bar{w}] u'(c_k(0, 0))$ ,  $\forall i \neq k$ , and summing, we obtain:

$$\sum_{i=1}^n \lambda_i [w_i^2 - \frac{w_i E(w^2)}{\bar{w}}] u'(c_i(0, 0)) < \sum_{i=1}^n \lambda_i [w_i^2 - \frac{w_i E(w^2)}{\bar{w}}] u'(c_k(0, 0)) = 0.$$

It follows that

$$\frac{E(w^2)}{\bar{w}} \frac{\partial SW}{\partial \alpha} \Big|_{\alpha=0} < 0,$$

so that one cannot have  $\alpha^u > 0$ . ■

This proposition says that a utilitarian planner does not want to introduce an implicit tax. The reasons underlying this result are very similar to those discussed in the Rawlsian case. As the argument in the proof makes clear, this result is indeed driven by the fact that the marginal utility is decreasing with productivity. Consequently,

the utilitarian planner will have a greater concern for the welfare of the poor. From a redistributive point of view, choosing a positive  $\alpha$  is harmful, because it means setting a marginal tax rate higher for low income individuals than for high income ones. Consequently, a positive level of  $\alpha$  could be justified only by efficiency considerations. However, as pointed out before, with constant elasticity of labor supply, there is no reason to differentiate marginal tax rates between individuals (via a positive level of  $\alpha$ ). Summing up, there is no reason to introduce an implicit tax.

## 5 Political outcomes

At this point the case for a positive  $\alpha$  is quite weak. Consider the weighted sum of individual utilities

$$WS = \sum \beta_i V(\tau, \alpha, w_i) \quad (8)$$

where  $\beta_i$  are non negative weights such that  $\sum \beta_i = 1$ . We have seen that

$$\left. \frac{\partial WS}{\partial \alpha} \right|_{\tau^* \geq 0, \alpha=0} < 0$$

for  $\beta_i = \lambda_i$  (utilitarian objective) and  $\beta_i = 1$  for any  $i$  with  $\beta_j = 0$  with  $j \neq i$ . Does that mean that we cannot find any profile of  $\beta_i$  such that the above inequality be reversed?

If this were the case, it would mean that  $\alpha > 0$  can be Pareto efficient. Moreover, interpreting the  $\beta_i$ 's as political weights, it would mean that  $\alpha > 0$  can be the outcome of some political process giving these particular weights to the different productivity groups.<sup>3</sup> If  $V(\cdot)$  had just one argument, such an outcome would not be possible. However we have to remember that low productivity individuals want a positive  $\tau$  and a negative  $\alpha$  whereas the high productivity ones want also a negative  $\alpha$  and they would like a negative  $\tau$  but this is not possible. In other words, given this constraint on  $\tau$ , it is not impossible that with appropriate weights  $\alpha > 0$  and  $\tau > 0$  converge as a solution. The idea underlying this result is that the implicit tax is the result of a compromise: whereas a low tax rate is chosen so as to favor the highly productive workers, the policy maker

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<sup>3</sup>This is a reduced form of the so-called probabilistic voting model, developed by Coughlin and Nitzan (1981) and further elaborated by Lindbeck and Weibull (1987) and Dixit and Londregan (1996). An application of this model to retirement policies can be found in Profeta (2002).

chooses a positive value of  $\alpha$  in order to compensate the low productive workers. The political power of the former must be large, but not too large. Indeed, we have shown in the first section that, should all the political weight be given the most productive individuals, they would choose an implicit subsidy rather than an implicit tax.

Sufficient conditions for  $\alpha$  to be positive are that the optimal tax rate evaluated at  $\alpha = 0$ , denoted  $\tau^p(0)$ , is positive and that the impact on  $WS$  of marginally increasing  $\alpha$  at this point is also positive. Formally:

$$\begin{aligned} & \left. \frac{\partial WS}{\partial \tau} \right|_{\tau=0, \alpha=0} > 0 \\ \Leftrightarrow & \sum_{i=1}^n \beta_i [E(w^2) - w_i^2] u'(c_i(0, 0)) > 0. \end{aligned} \quad (9)$$

and

$$\begin{aligned} & \left. \frac{\partial WS}{\partial \alpha} \right|_{\tau=\tau^p(0), \alpha=0} > 0 \\ \Leftrightarrow & \sum_{i=1}^n \beta_i [\bar{w}(1 - 2\tau^p(0)) - w_i(1 - \tau^p(0))] u'(c_i(\tau^p(0), 0)) > 0. \end{aligned} \quad (10)$$

Using the first-order condition on  $\tau^p(0)$ :

$$\sum_{i=1}^n \beta_i [E(w^2)(1 - 2\tau^p(0)) - w_i^2(1 - \tau^p(0))] u'(c_i(\tau^p(0), 0)) = 0, \quad (11)$$

the second condition (10) becomes

$$\sum_{i=1}^n \beta_i \left[ \frac{w_i}{w} E(w^2) - w_i^2 \right] u'(c_i(\tau^p(0), 0)) < 0. \quad (12)$$

In the appendix we prove that condition (10) cannot be satisfied with only two productivity classes. When there are more than two levels of productivity, we now show that  $\alpha^p > 0$  is possible.

**Proposition 3** *With three productivity levels, there may exist weights  $\beta_1, \beta_2$  and  $\beta_3$  such that the outcome of the political process is an implicit tax on continued activity:  $\alpha^p > 0$ .*

**Proof.** To prove this result, we rely on a numerical example. The utility function is iso-elastic:  $u(x) = x^{1-\varepsilon}/(1-\varepsilon)$ , with  $\varepsilon = 0.95$ . Productivity levels are  $w_1 = 20$ ,  $w_2 = 50$ ,  $w_3 = 80$  and  $\gamma = 90$ ,  $\lambda_1 = \lambda_2 = \lambda_3 = 1/3$ . In such a setting, we obtain that with weights  $\beta_1 = 0.1$ ,  $\beta_2 = 0.2$  and  $\beta_3 = 0.7$ , conditions (9) and (12) are satisfied; the function  $WS$  is maximized at  $\tau = 0.042$  and  $\alpha = 0.249$ .<sup>4</sup> ■

This proposition shows that for some social weights a double burden on postponed retirement can be desirable. In this example the social weights are biased towards workers with high productivity. These workers would like  $\tau$  to be negative. But  $\tau$  is constrained to be non negative and workers with lower productivity would instead ask for a positive  $\tau$ . The solution we obtain is a compromise biased towards the more productive workers. The tax rate is higher than what they want, but much lower than that asked by the less productive workers. In counterpart they have to stand a positive value of  $\alpha$ , which, relatively speaking, favors the less productive workers. As explained in section 3,  $\alpha$  is second best instrument for the poor to redistribute income; when  $\tau$  is restricted to being small, the poor may prefer to set a positive  $\alpha$  (see (5)).

One may wonder why such a compromise is not possible when there are only two types. The formal proof in the Appendix is quite intricate. Roughly speaking the crucial point is that with only two types the policy that maximizes  $WS$  switches directly from  $\tau > 0$ ,  $\alpha < 0$  to  $\tau = 0$ ,  $\alpha < 0$  when the power of the rich is increased, without passing through the region  $\tau \geq 0$ ,  $\alpha > 0$ . In other words there not sufficiently “degrees of freedom” to obtain the result described in Proposition 3.

The practical relevance of our result hinges of course on the plausibility of a political process biased towards the “rich”. While there is admittedly no definitive answer to this issue, there is some evidence that the political process in western democracies may effectively yield such a bias. For instance, Bénabou (2004) argues that “poor and less educated individuals have a lower propensity to register, turn out to vote and give political contributions, than better-off ones. [...] Even for political activities that are time- rather than money-intensive, such as writing to congress, attending meetings,

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<sup>4</sup>The considered distribution of productivities yields  $\bar{w} = 50$  and  $E(w^2) = 3100$ . Consequently we have  $2\bar{w}(\bar{w} - w_1) = 3000 < E(w^2) = 3100$ , so that (13), the sufficient condition for quasi-concavity is satisfied.

trying to convince others, etc., the propensity to participate rises sharply with income and education.” An empirical study of political weights is provided by Bartels (2002) who uses surveys to determine the differential responsiveness of U.S. senators to the preferences of rich and poor constituents. He shows that “In every instance, senators appear to be much more responsive to the opinions of affluent constituents than to the opinions of constituents with modest incomes. On average, [...] the 75th percentile of the income distribution have almost three times as much influence on senators’ general voting patterns as those at the 25th percentile [...]. The preferences of constituents near the top of the income distribution are even more influential, while those in the bottom fifth receive little or no weight, especially from Republican senators.” Summing up, it appears fair to say that the case with a large political power for the rich seems to be more than just a theoretical curiosity.

## 6 Conclusion

The question raised by this paper is whether or not the double burden imposed on elderly workers when they consider postponing retirement can be Pareto efficient and result from some political process. Such a double burden is effectively observed in a number of European countries. The answer is positive which is striking as such a double burden would be consistently rejected by any citizen candidate and by a utilitarian (or Rawlsian) criterion.

We realize that other explanation can be used to justify such a policy which results in a very low rate of labor participation among elderly workers. For instance there are the labor market considerations mentioned in the introduction. To obtain crisper results we have deliberately ignored these aspects. The factors we account for can thus only tell part of the story. However, they may well offer an explanation for the persistence of early retirement at a time when its impact on youth unemployment has been severely questioned; see e.g. Boldrin et al. (1999).

## Appendix

### A Quasi-concavity of the indirect utility: sufficient conditions

The indirect utility function specified by (4) is quasi-concave if

$$D_1 = \begin{vmatrix} 0 & \partial V/\partial\tau \\ \partial V/\partial\tau & \partial^2 V/\partial\tau^2 \end{vmatrix} < 0$$

and

$$D_2 = \begin{vmatrix} 0 & \partial V/\partial\tau & \partial V/\partial\alpha \\ \partial V/\partial\tau & \partial^2 V/\partial\tau^2 & \partial^2 V/\partial\tau\partial\alpha \\ \partial V/\partial\alpha & \partial^2 V/\partial\tau\partial\alpha & \partial^2 V/\partial\alpha^2 \end{vmatrix} > 0.$$

The first condition is always satisfied. Turning to the second condition we successively obtain:

$$\begin{aligned} D_2 &= -\frac{\partial V}{\partial\tau} \begin{vmatrix} \partial V/\partial\tau & \partial V/\partial\alpha \\ \partial^2 V/\partial\tau\partial\alpha & \partial^2 V/\partial\alpha^2 \end{vmatrix} + \frac{\partial V}{\partial\alpha} \begin{vmatrix} \partial V/\partial\tau & \partial V/\partial\alpha \\ \partial^2 V/\partial\tau^2 & \partial^2 V/\partial\tau\partial\alpha \end{vmatrix} \\ &= -\left(\frac{\partial V}{\partial\tau}\right)^2 \frac{\partial^2 V}{\partial\alpha^2} - \left(\frac{\partial V}{\partial\alpha}\right)^2 \frac{\partial^2 V}{\partial\tau^2} + 2\frac{\partial V}{\partial\alpha} \frac{\partial V}{\partial\tau} \frac{\partial^2 V}{\partial\tau\partial\alpha} \\ &= \frac{u'(c)^3}{\gamma^3} [A^2 + B^2(2E(w^2) + w^2) + 2AB(w - 2\bar{w})] \end{aligned}$$

where

$$\begin{aligned} A &= E(w^2)(1 - 2\tau) - w^2(1 - \tau) + \alpha w - 2\alpha\bar{w} \\ B &= \bar{w}(1 - 2\tau) - w(1 - \tau) - \alpha. \end{aligned}$$

This expression is positive if

$$\begin{aligned} &A^2 + B^2(2E(w^2) + w^2) + 2AB(w - 2\bar{w}) > 0 \\ \Leftrightarrow \Phi &\equiv \left(\frac{B}{A}\right)^2 (2E(w^2) + w^2) + \frac{B}{A} 2(w - 2\bar{w}) + 1 > 0. \end{aligned}$$

We solve for the quadratic equation  $\Phi = 0$ :

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= 4(w - 2\bar{w})^2 - 4(2E(w^2) + w^2) \\ < 0 &\Leftrightarrow 2\bar{w}(\bar{w} - w) < E(w^2), \end{aligned} \tag{13}$$

which is the condition stated in Section 2.

### B The case of two productivity levels

We show that with only two productivity levels,  $w_1 < w_2$ , the maximization of (8) always implies  $\alpha^p < 0$  (whatever the weights  $\lambda_i$ ). We proceed in two steps

**Step 1:** With two types, condition (10) is

$$\beta_1 [\bar{w}(1 - 2\tau^p(0)) - w_1(1 - \tau^p(0))] u'(c_1(\tau^p(0), 0)) \\ + \beta_2 [\bar{w}(1 - 2\tau^p(0)) - w_2(1 - \tau^p(0))] u'(c_2(\tau^p(0), 0)) > 0$$

$\Leftrightarrow$

$$\frac{\beta_1}{\beta_2} > - \frac{\bar{w}(1 - 2\tau^p(0)) - w_2(1 - \tau^p(0)) u'(c_2(\tau^p(0), 0))}{\bar{w}(1 - 2\tau^p(0)) - w_1(1 - \tau^p(0)) u'(c_1(\tau^p(0), 0))}.$$

From (11),

$$\frac{\beta_1}{\beta_2} = - \frac{E(w^2)(1 - 2\tau^p(0)) - w_2^2(1 - \tau^p(0)) u'(c_2(\tau^p(0), 0))}{E(w^2)(1 - 2\tau^p(0)) - w_1^2(1 - \tau^p(0)) u'(c_1(\tau^p(0), 0))}.$$

The previous condition then becomes

$$\underbrace{- \frac{E(w^2)(1 - 2\tau^p(0)) - w_2^2(1 - \tau^p(0))}{E(w^2)(1 - 2\tau^p(0)) - w_1^2(1 - \tau^p(0))}}_{\varphi(\tau^p(0))} > \underbrace{- \frac{\bar{w}(1 - 2\tau^p(0)) - w_2(1 - \tau^p(0))}{\bar{w}(1 - 2\tau^p(0)) - w_1(1 - \tau^p(0))}}_{\psi(\tau^p(0))}. \quad (14)$$

We prove now that this inequality can never be satisfied. To do this we make use of the following expression which are easily obtained from (14)

$$\lim_{\tau \rightarrow \left(\frac{\bar{w}-w_1}{2\bar{w}-w_1}\right)^-} \psi(\tau) = +\infty; \quad \lim_{\tau \rightarrow \left(\frac{E(w^2)-w_1^2}{2E(w^2)-w_1^2}\right)^-} \varphi(\tau) = +\infty; \\ \frac{\bar{w} - w_1}{2\bar{w} - w_1} < \frac{E(w^2) - w_1^2}{2E(w^2) - w_1^2}; \\ \tau^p(0) < \frac{E(w^2) - w_1^2}{2E(w^2) - w_1^2}.$$

From these expression we show that  $\varphi(0) = \psi(0) = \lambda_1/\lambda_2$  and  $\psi'(0) > \varphi'(0)$ :

$$\psi'(0) = \frac{\bar{w}(w_2 - w_1)}{(\bar{w} - w_1)^2} > \varphi'(0) = \frac{E(w^2)(w_2^2 - w_1^2)}{(E(w^2) - w_1^2)^2} \\ \Leftrightarrow w_1 + w_2 > \frac{E(w^2)}{\bar{w}}.$$

Observing that  $E(w^2) < w_2\bar{w}$ , this inequality is always satisfied.

**Step 2:** To complete the proof, it remains to show that  $\alpha^p$  evaluated at  $\tau = 0$  is negative. This case needs to be analyzed when  $\tau^p(0) < 0$  (in the case  $\tau^p(0) \geq 0$ , the constraint on  $\tau$  is not binding), that is when

$$\frac{\partial WS}{\partial \tau} \Big|_{\tau=0, \alpha=0} < 0 \\ \Leftrightarrow \frac{\beta_1}{\beta_2} < - \frac{E(w^2) - w_2^2 u'(c_2(0, 0))}{E(w^2) - w_1^2 u'(c_1(0, 0))}.$$



We argue that one cannot have  $\alpha^p$  positive. This would be the case if

$$\begin{aligned} & \left. \frac{\partial WS}{\partial \alpha} \right|_{\tau=0, \alpha=0} > 0 \\ \Leftrightarrow & \sum_{i=1}^2 \beta_i (\bar{w} - w_i) u'(c_i(0, 0)) > 0 \\ \Leftrightarrow & \frac{\beta_1}{\beta_2} > -\frac{\bar{w} - w_2 u'(c_2(0, 0))}{\bar{w} - w_1 u'(c_1(0, 0))}. \end{aligned}$$

Observing that

$$-\frac{E(w^2) - w_2^2}{E(w^2) - w_1^2} = -\frac{\bar{w} - w_2}{\bar{w} - w_1} = \frac{\lambda_1}{\lambda_2},$$

we can conclude that the two conditions on  $\beta_1/\beta_2$  are not compatible and thus that it cannot be the case that  $\alpha^p(0) > 0$ .

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